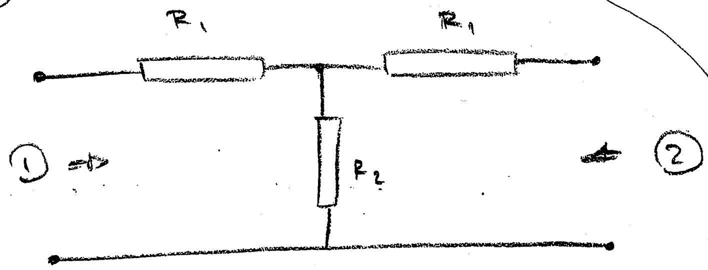
ZADACI 03

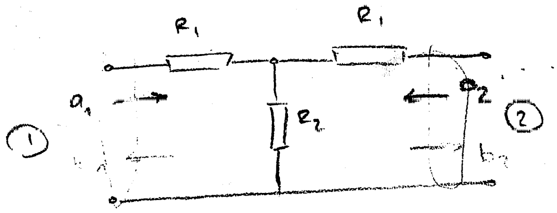
(1) Izračunati raspršnu matricu sklopa prema slici. Karakteristična impedancija sustava je 50 Ω. Čemu služi takav sklop?

$Z_0 = 50$



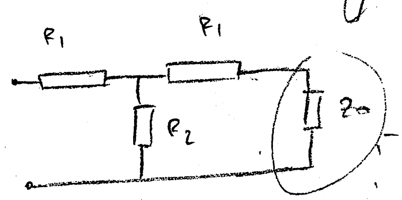
$R_1 = 8.65\ \Omega$

$R_2 = 141.8\ \Omega$



$a_2 = \phi$ ako nema refl., tj. $Z_T = Z_0$

$$\boxed{S_{11} = \Gamma_1 \big|_{a_2=0} = \Gamma_1 \big|_{Z_T = Z_0}}$$



$I_1 = I_2 + I_3$

$I_3 = I_1 - I_2$

$R_2 \cdot I_3 = I_2 (R_1 + Z_0)$

$R_2 (I_1 - I_2) = I_2 (R_1 + Z_0)$

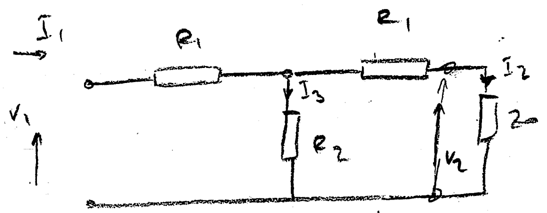
$$I_1 = \frac{I_2 (R_1 + Z_0) + R_2 I_2}{R_2}$$

$$\frac{I_1}{I_2} = \frac{R_1 + R_2 + Z_0}{R_2}$$

$$Z_{ul} = R_1 + \frac{R_2 (R_1 + Z_0)}{R_2 + R_1 + Z_0} = 50.13\ \Omega$$

$$S_{11} = \Gamma_1 \big|_{Z_T = Z_0} = \frac{Z_{ul} - Z_0}{Z_{ul} + Z_0} \approx 0$$

$S_{11} = S_{22} = 0$



$V_1 = I_1 \left[R_1 + R_2 \| (R_1 + Z_0) \right]$

$V_2 = I_2 \cdot Z_0$

$$S_{21} = \frac{b_2}{a_1}\Big|_{a_2=0} = \frac{\frac{V_2^+}{\sqrt{Z_0}}}{\frac{V_1^-}{\sqrt{Z_0}}} = \frac{V_2}{V_1} = \frac{I_2}{I_1} \cdot \frac{Z_0}{R_1 + \frac{R_2 (R_1 + Z_0)}{R_2 + R_1 + Z_0}} = \frac{R_2}{R_1 + \cancel{R_2 + Z_0}} \cdot \frac{Z_0 \cancel{(R_2 + R_1 + Z_0)}}{R_1 (R_2 + R_1 + Z_0) + R_2 (R_1 + Z_0)}$$

$$= 0.705 = S_{12}$$

$$[S] = \begin{bmatrix} 0 & 0.705 \\ 0.705 & 0 \end{bmatrix} \Rightarrow \text{ATENUATOR} \quad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad S \text{ gušenjem}:$$

$$20 \log (0.705) = -3\ \text{dB}$$

? NAPONI

2.) Zadana je raspršna matrica četveroprolaznog sklopa :

$$
\begin{array}{cc}
 & a \\
b & [S] = \begin{bmatrix}
0.1\angle 90^\circ_{11} & 0.707\angle -45^\circ_{12} & 0.707\angle 45^\circ_{13} & 0_{14} \\
0.707\angle -45^\circ_{21} & 0_{22} & 0_{23} & 0.707\angle 45^\circ_{24} \\
0.707\angle -45^\circ_{31} & 0_{32} & 0_{33} & 0.707\angle -45^\circ_{34} \\
0_{41} & 0.707\angle 45^\circ_{42} & 0.707\angle -45^\circ_{43} & 0_{44}
\end{bmatrix}
\end{array}
$$

a) Ima li sklop gubitke ?

b) Da li je sklop recipročan ?

c) Koliki su povratni gubici (return loss) na prolazu 1 ako su svi ostali prolazi prilagođeni ?

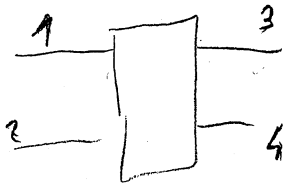
d) Koliki su unešeni gubici (insertion loss) između prolaza 2 i 4 ako su svi ostali prolazi prilagođeni ?

e) Koliki je koeficijent refleksije na prolazu 1, ako je prolaz 3 zaključen kratkim spojem a ~~ostali~~ su prolazi prilagođeni ?

a) $|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 + |S_{41}|^2 = 1$

$1.0096 \neq 1$ $\boxed{\text{sklop nema gubitke već pojačanje}}$!

b) $S_{31} \neq S_{13}$ sklop nije recipročan



c) $\Gamma_1 = S_{11}$ $-20\log(0.1) = \underline{\underline{20\,dB}}$

$RL = -20\log \Gamma_1 = 20\,dB$

d) $IL = -20\log T_{42} = -20\log S_{42} = 3.01\,dB$

e) $\Gamma_1 = ? = \frac{b_1}{a_1}$

$\Gamma_3 = -1 \Rightarrow +1 = \frac{b_3}{a_3} \Rightarrow a_3 = -b_3$ kratki spoj

$\underline{a_2 = a_4 = 0}$ prilagođeni

$b_1 = S_{11}a_1 + S_{12}a_2 + S_{13}a_3 + S_{14}a_4 = S_{11}a_1 - S_{13}b_3 = S_{11}a_1 - S_{13}\cdot S_{31}a_1$

$b_3 = S_{31}a_1 + S_{34}a_4 = S_{31}a_1$

$\Gamma_1 = \frac{b_1}{a_1} = S_{11} - S_{13}\cdot S_{31} = \underline{\underline{0.509\angle 168.7^\circ}}$

(3.) Četveroprolazni sklop zadan je raspršnom matricom:

$$[S] = \begin{bmatrix} 0.6\angle 90^\circ_{11} & 0_{12} & 0_{13} & 0.8\angle 0^\circ_{14} \\ 0_{21} & 0.707\angle 45^\circ_{22} & 0.707\angle -45^\circ_{23} & 0_{24} \\ 0_{31} & 0.707\angle -45^\circ_{32} & 0.707\angle 45^\circ_{33} & 0_{34} \\ 0.8\angle 0^\circ_{41} & 0_{42} & 0_{43} & 0.6\angle 90^\circ_{44} \end{bmatrix}$$

Prolazi 3 i 4 povezani su prilagođenom prijenosnom linijom bez gubitaka električne duljine 100°. Odrediti koeficijent refleksije i povratne gubitke na prolazu 1 te koeficijent prijenosa i unesene gubitke između prolaza 1 i 2. Pretpostaviti da su prolazi 1 i 2 prilagođeni.

$\Gamma_1 = ?$ ✓ $\quad T_{21} = ?$

$RL_1 = ?$ ✓ $\quad IL_{21} = ?$

unesi gubic

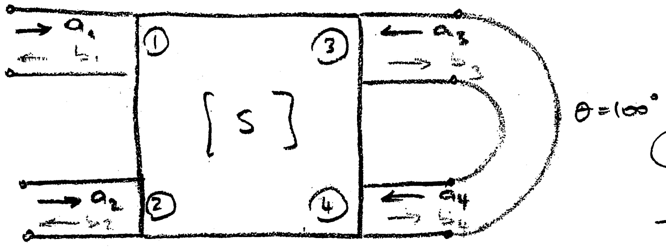
Zadatak riješiti:

a) analitički

b) redukcijom grafa toka

c) primjenom Masonovog pravila.

prolazi 1 i 2 prilagođ.



$a_2 = 0$

$a_4 = b_3 e^{-j\theta}$

$a_3 = b_4 e^{-j\theta}$

a) $\Gamma_1 = \dfrac{b_1}{a_1} = ?$

(1) $b_1 = S_{11} a_1 + S_{12} a_2 + S_{13} a_3 + S_{14} a_4 = S_{11} a_1 + S_{14} a_4 = S_{11} a_1 + S_{14} \cdot b_3 e^{-j\theta}$

(2) $b_2 = S_{22} a_2 + S_{23} a_3 = \cancel{S_{22} a_2}^{0} + S_{23} \cdot b_4 e^{-j\theta}$

(3) $b_3 = S_{32} a_2 + S_{33} a_3 = \cancel{S_{32} a_2}^{0} + S_{33}\, b_4 e^{-j\theta}$

(4) $b_4 = S_{41} a_1 + S_{44} a_4 = S_{41} a_1 + S_{44}\, b_3 e^{-j\theta}$

(3):(4) $b_3 = \cancel{S_{32} a_2}^{0} + S_{33} e^{-j\theta} \left[S_{41} a_1 + S_{44} b_3 e^{-j\theta} \right] = S_{33} S_{41} e^{-j\theta} a_1 + S_{33} S_{44} e^{-j2\theta} b_3$

$$b_3 = \frac{S_{33} S_{41} e^{-j\theta}}{1 - S_{33} S_{44} e^{-j2\theta}} a_1$$

=>

$$b_1 = S_{11} a_1 + S_{14} e^{-j\theta} \left[\frac{S_{33} \cdot S_{41} \cdot e^{-j\theta}}{1 - S_{33} S_{44} e^{-j2\theta}} a_1 \right]$$

$$= S_{11} a_1 + \frac{S_{14} S_{33} S_{41} e^{-j2\theta}}{1 - S_{33} S_{44} e^{-j2\theta}} a_1$$

$$\Gamma_1 = \frac{b_1}{a_1} = S_{11} + \frac{S_{14} S_{33} S_{41} e^{-j2\theta}}{1 - S_{33} S_{44} e^{-j2\theta}}$$

$$T_{21} = \frac{b_2}{a_1} = ?$$

$$b_4 = S_{41} a_1 + S_{44} e^{-j\theta} \left[S_{33} b_4 e^{-j\theta} \right]$$

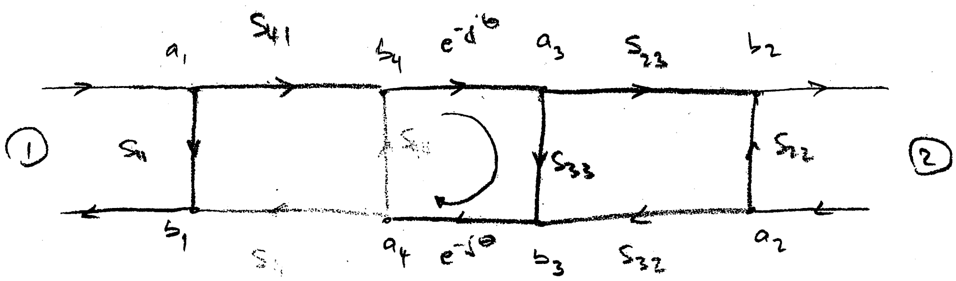
$$b_4 = \frac{S_{41}}{1 - S_{44} S_{33} e^{-j2\theta}} a_1$$

$$b_2 = S_{23} b_4 e^{-j\theta} = S_{23} \cdot \frac{S_{41}}{1 - S_{44} S_{33} e^{-j2\theta}} \cdot e^{-j\theta} a_1$$

$$T_{21} = \frac{b_2}{a_1} = \frac{S_{23} \cdot S_{41} \cdot e^{-j\theta}}{1 - S_{44} S_{33} e^{-j2\theta}}$$

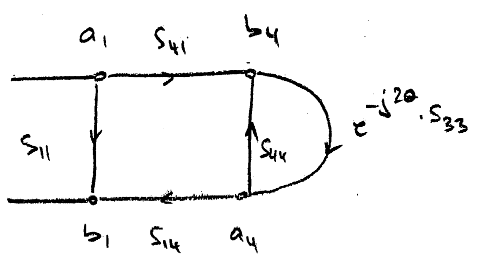
b) $\Rightarrow$

b)



$\Gamma_1 = ?$

→ čvor a_2 nije pobuđen
→ iz čvora b_2 sve izlazi iz mreže
} oba možemo odbaciti !



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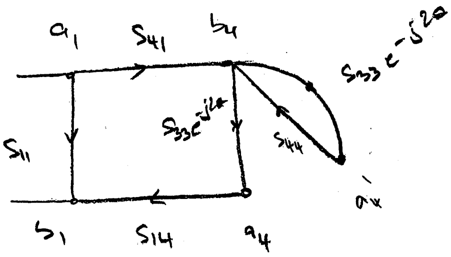

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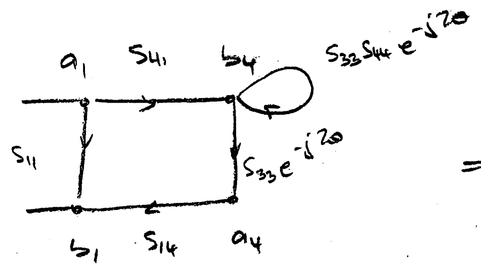

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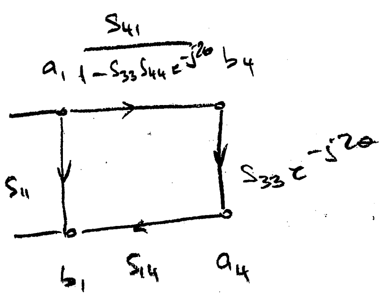

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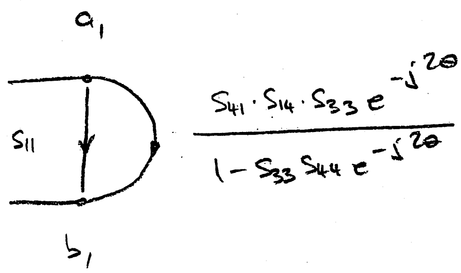

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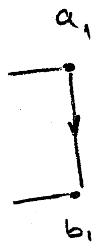
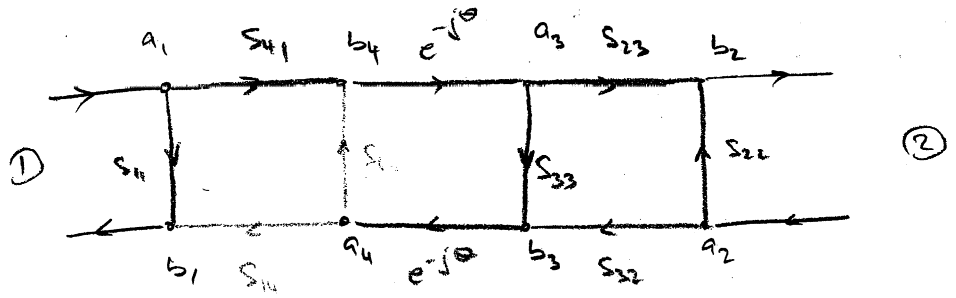


$$S_{11} + \frac{S_{41} \cdot S_{14} \cdot S_{33} e^{-j2\theta}}{1 - S_{33}S_{44}e^{-j2\theta}} = \Gamma_1$$

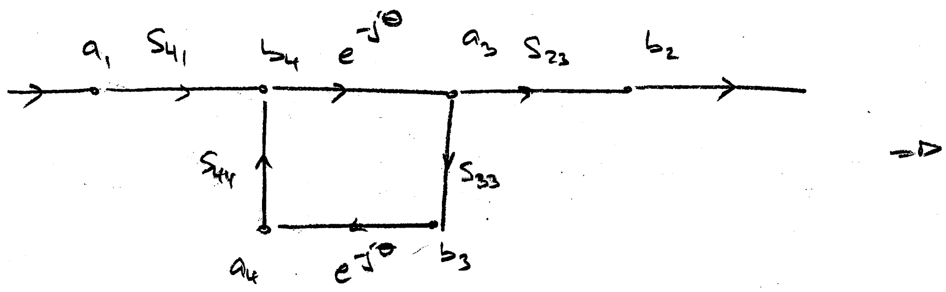
⇒

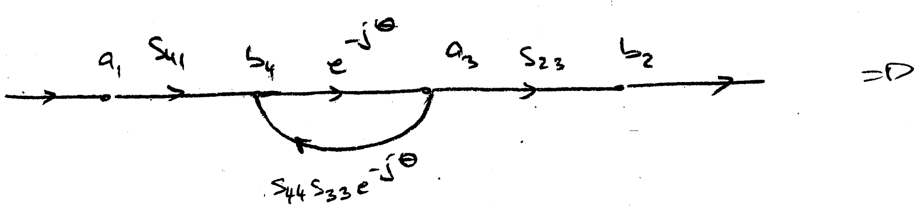
$T_{21} = ? = \frac{b_2}{a_1}$

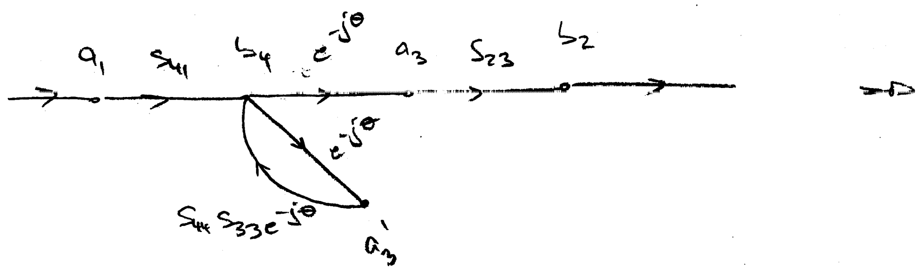


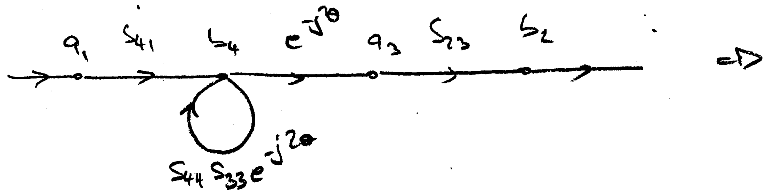
→ čvor a_2 nije pobuđen } oba možemo odbaciti

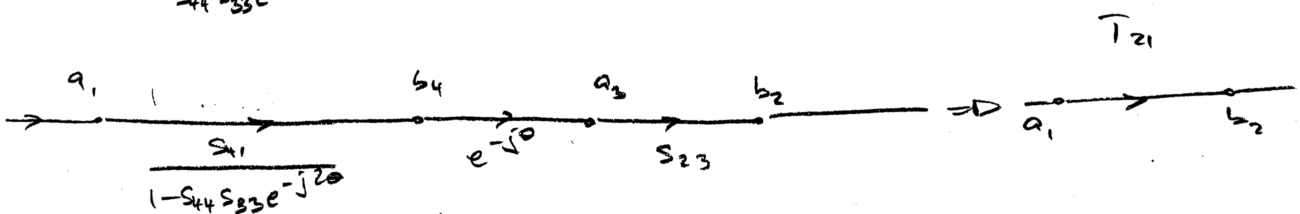
→ iz čvora b_1 sve izlazi iz mreže





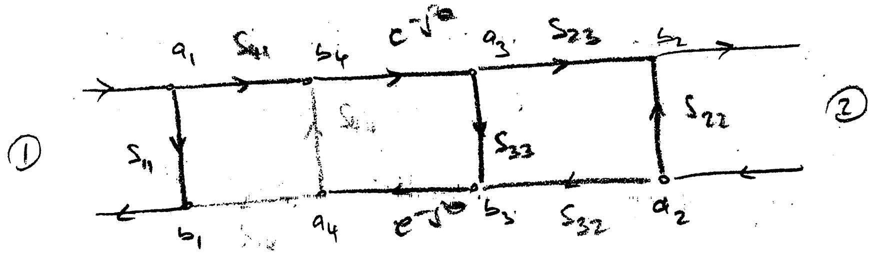






$$T_{21} = \frac{b_2}{a_1} = \frac{S_{41} \cdot S_{23} \cdot e^{-j\theta}}{1 - S_{44} S_{33} e^{-j2\theta}}$$

c)



$$\Gamma_1 = \frac{b_1}{a_1}$$

2 pute $\longrightarrow$ $P_1 = S_{11}$

$P_2 = S_{41}\cdot e^{-j\theta}\cdot S_{33}\cdot e^{-j\theta}\cdot S_{14} = S_{14}\cdot S_{41}\cdot S_{33}\cdot e^{-j2\theta}$

$L_1(1) = S_{33}\cdot S_{44}\cdot e^{-j2\theta}$

$L_2(1) = 0$

$L(1) = S_{33}\cdot S_{44}\, e^{-j2\theta}$

$$\Gamma_1 = \frac{b_1}{a_1} = \frac{S_{11}\left[1 - S_{33}\cdot S_{44}\cdot e^{-j2\theta}\right] + S_{14} S_{41} S_{33}\, e^{-j2\theta}}{1 - S_{33} S_{44} e^{-j2\theta}} = S_{11} + \frac{S_{14} S_{41} S_{33}\, e^{-j2\theta}}{1 - S_{33} S_{44}\, e^{-j2\theta}}$$

$$T_{21} = \frac{b_2}{a_1}$$

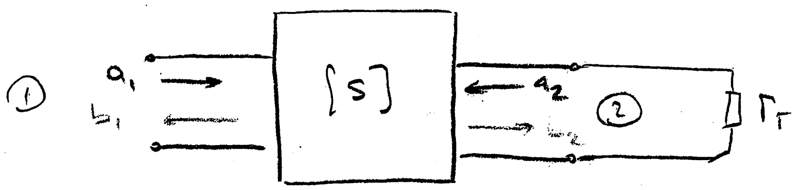
1 put $\longrightarrow$ $P_1 = S_{41}\cdot S_{23}\cdot e^{-j\theta}$

$L_1(1) = 0$

$L(1) = S_{33}\cdot S_{44}\cdot e^{-j2\theta}$

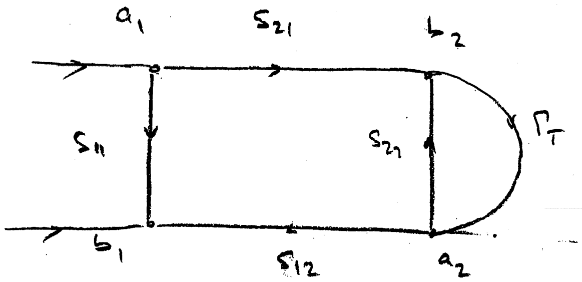
$$T_{21} = \frac{S_{41} S_{23}\, e^{-j\theta}}{1 - S_{33} S_{44}\cdot e^{-j2\theta}}$$

4. Dvoprolazni sklop zadan je raspršnom matricom [S]. Na prolaz 2 spojen je teret koeficijenta refleksije Γ_T. Ako na prolaz 1 ulazi snaga P_{ul}, a teretu se predaje snaga P_T, odrediti omjer P_T / P_{ul}.



$$P_{ul} = \frac{|a_1|^2}{2}\left(1-|\Gamma_{ul}|^2\right) \qquad \frac{P_T}{P_{ul}} = \frac{|b_2|^2\left(1-|\Gamma_T|^2\right)}{|a_1|^2\left(1-|\Gamma_{ul}|^2\right)}$$

$$P_T = \frac{|b_2|^2}{2}\left(1-|\Gamma_T|^2\right)$$



$$\Gamma_{ul} = S_{11} + \frac{S_{21}S_{12}\Gamma_T}{1-S_{22}\Gamma_T}$$

$$T_{21} = \frac{S_{21}}{1-S_{22}\Gamma_T} = \frac{b_2}{a_1}$$

$$\frac{P_T}{P_{ul}} = \left|\frac{S_{21}}{1-S_{22}\Gamma_T}\right|^2 \cdot \frac{1-|\Gamma_T|^2}{1-\left|S_{11}+\frac{S_{21}S_{12}\Gamma_T}{1-S_{22}\Gamma_T}\right|^2} = \frac{|S_{21}|^2\left[1-|\Gamma_T|^2\right]}{\left[1-|S_{22}\Gamma_T|^2\right]\left[1-\left|\frac{S_{11}-S_{11}S_{22}\Gamma_T+S_{21}S_{12}\Gamma_T}{1-S_{22}\Gamma_T}\right|^2\right]}$$

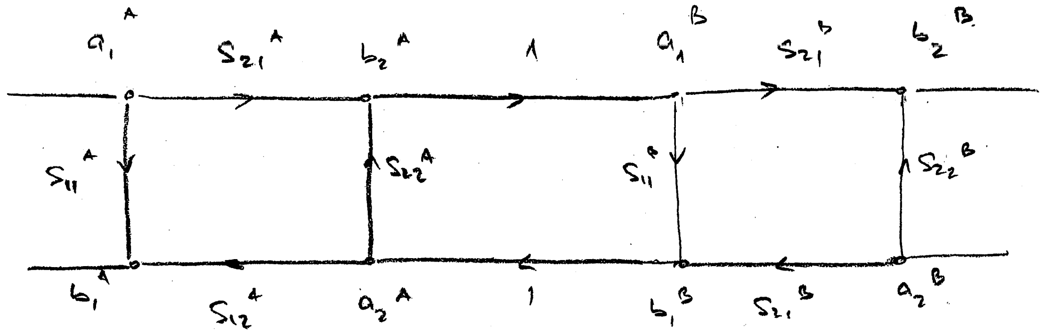
$$= \frac{|S_{21}|^2\left[1-|\Gamma_T|^2\right]}{\left[1-\cancel{|S_{22}\Gamma_T|^2}\right]\dfrac{|1-S_{22}\Gamma_T|^2-|S_{11}-S_{11}S_{22}\Gamma_T+S_{21}S_{12}\Gamma_T|^2}{1-\cancel{|S_{22}\Gamma_T|^2}}} = \frac{|S_{21}|^2\left[1-|\Gamma_T|^2\right]}{|1-S_{22}\Gamma_T|^2-|S_{11}-S_{11}S_{22}\Gamma_T+S_{21}S_{12}\Gamma_T|^2}$$

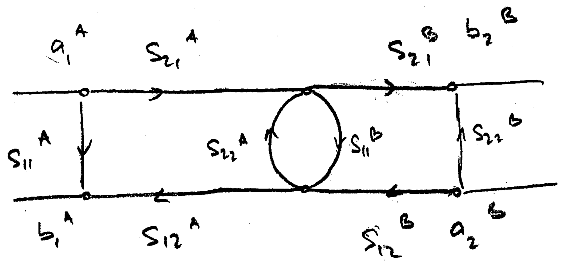
(5.) Dva dvoprolazna sklopa zadana raspršnim matricama $[S^A]$ i $[S^B]$ vezani su u kaskadu prema slici. Odrediti opće izraze za raspršnu matricu kaskade. (5)

Ako su zadane

$$[S^A] = \begin{bmatrix} 0.1\angle 20^\circ_{11} & 0.9\angle 60^\circ_{12} \\ 0.9\angle 60^\circ_{21} & 0.1\angle 20^\circ_{22} \end{bmatrix} \qquad [S^B] = \begin{bmatrix} 0.4\angle -20^\circ_{11} & 0.8\angle -30^\circ_{12} \\ 0.8\angle 30^\circ_{21} & 0.2\angle -50^\circ_{22} \end{bmatrix}$$

izračunati raspršnu matricu kaskade. Električne duljine vodiča koji povezuju dvoprolazne sklopove su zanemarive. Provjeriti da li je raspršna matrica kaskade simetrična, recipročna te da li kaskada ima gubitke i da li je prilagođena.





$$S_{11} = \frac{b_1^A}{a_1^A} = \frac{S_{11}^A(1 - S_{22}^A S_{11}^B) + S_{21}^A S_{11}^B S_{12}^A (1-0)}{1 - S_{22}^A S_{11}^B} = \frac{S_{11}^A - S_{11}^A S_{22}^A S_{11}^B + S_{21}^A S_{11}^A S_{12}^A}{1 - S_{22}^A S_{11}^B}$$

$$= \frac{S_{11}^A(1 - S_{22}^A S_{11}^B) + S_{21}^A S_{11}^A S_{12}^A}{1 - S_{22}^A S_{11}^B} = S_{11}^A + \frac{S_{21}^A S_{11}^B S_{12}^A}{1 - S_{22}^A S_{11}^B}$$

$= 0.36\angle 84.48$

$\Rightarrow$

$$S_{12} = \frac{b_1}{a_2} = \frac{S_{12}^B S_{12}^A}{1 - S_{11}^B S_{22}^A} = 0.75 \angle 30^\circ$$

$$S_{21} = \frac{b_2}{a_1} = \frac{S_{21}^A S_{21}^B}{1 - S_{11}^B S_{22}^A} = 0.75 \angle 30^\circ$$

$$S_{22} = \frac{b_2}{a_2} = \frac{S_{22}^B (1 - S_{11}^B S_{22}^A) + S_{21}^B S_{22}^A S_{21}^B (1-0)}{1 - S_{11}^B S_{22}^A} =$$

$$= S_{22}^B + \frac{S_{21}^B S_{22}^A S_{12}^B}{1 - S_{11}^B S_{22}^A} = 0.132 \angle -38.79$$

$$[S] = \begin{bmatrix} 0.36 \angle 84.48^\circ & 0.75 \angle 30^\circ \\ 0.75 \angle 30^\circ & 0.132 \angle -38.79 \end{bmatrix}$$

SIMETRIČNOST

$S_{11} \neq S_{22}$ $S_{12} \neq S_{12}$ × nije simetrična

RECIPROČNOST

$S_{12} \neq S_{21}$ × nije recipročna

PRILAGODBA

$S_{11} \neq 0$, $S_{22} \neq 0$ × nije prilagođena

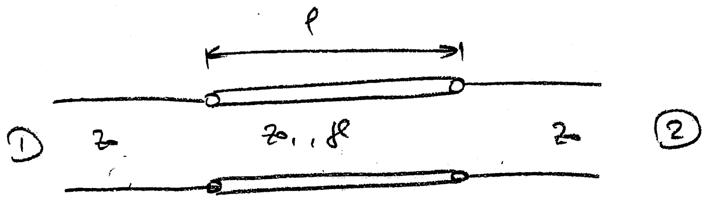
GUBICI

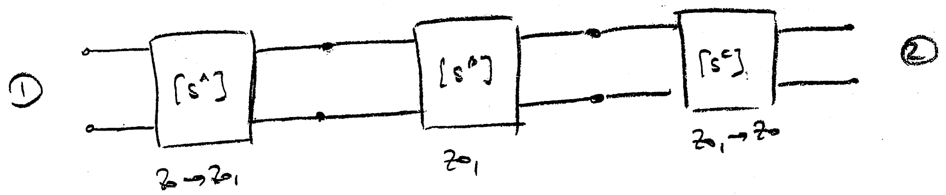
$|S_{11}|^2 + |S_{12}|^2 = 1$

$0.6921 < 1$ mreža ima gubitke

ZADACI 04

(1) Izračunati raspršnu matricu za liniju karakteristične impedancije Z_{01}, koeficijenta širenja γ i duljine l, u sustavu karakteristične impedancije Z_0.

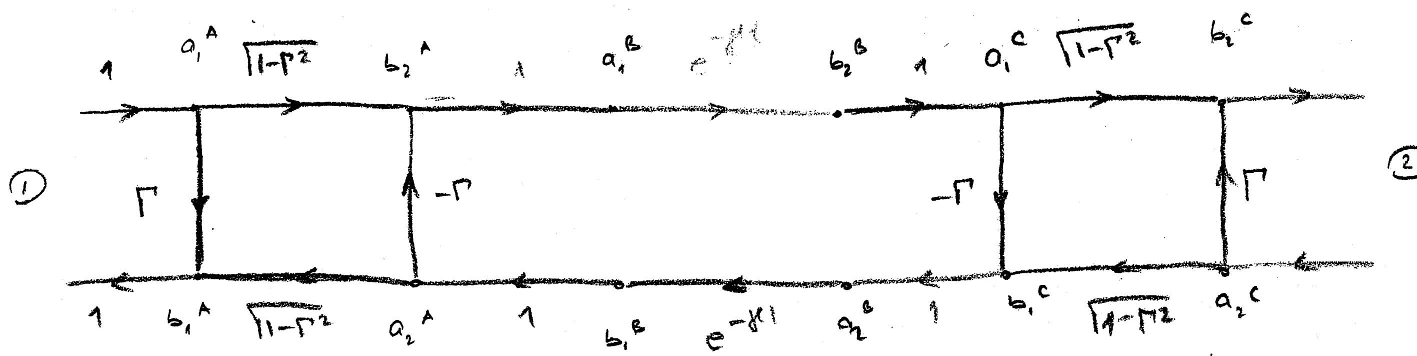




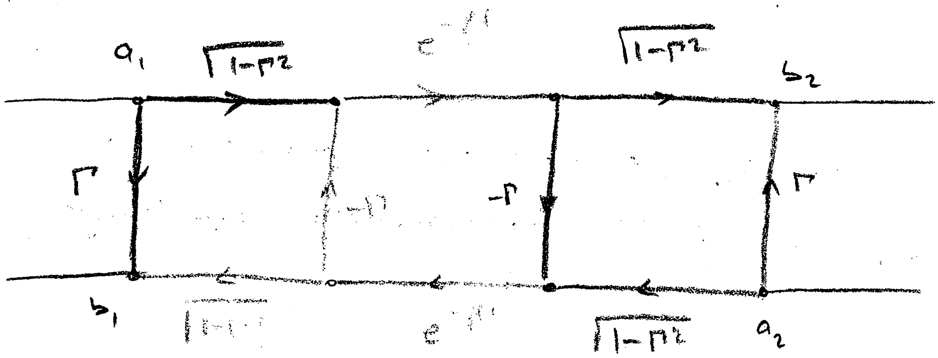
$$[S^A] = \begin{bmatrix} \Gamma & \sqrt{1-\Gamma^2} \\ \sqrt{1-\Gamma^2} & -\Gamma \end{bmatrix} \qquad [S^C] = \begin{bmatrix} -\Gamma & \sqrt{1-\Gamma^2} \\ \sqrt{1-\Gamma^2} & \Gamma \end{bmatrix}$$

$$\Gamma = \frac{Z_{01} - Z_0}{Z_{01} + Z_0}$$

$$[S^B] = \begin{bmatrix} 0 & e^{-\gamma l} \\ e^{-\gamma l} & 0 \end{bmatrix}$$



$\Rightarrow$



$$S_{11}=\frac{b_1}{a_1}\Big|_{a_2=0}=\frac{\Gamma\left(1-\Gamma^2 e^{-2\gamma l}\right)+\overbrace{e^{-2\gamma l}(1-\Gamma^2)(-\Gamma)}^{-\Gamma+\Gamma^3}(1-0)}{1-\Gamma^2 e^{-2\gamma l}}=$$

$$=\frac{\Gamma-\cancel{\Gamma^3 e^{-2\gamma l}}-\Gamma e^{-2\gamma l}+\cancel{\Gamma^3 e^{-2\gamma l}}}{1-\Gamma^2 e^{-2\gamma l}}=\frac{\Gamma\left(1-e^{-2\gamma l}\right)}{1-\Gamma^2 e^{-2\gamma l}}\cdot\frac{e^{\gamma l}}{e^{\gamma l}}=$$

$$=\frac{\frac{Z_{01}-Z_0}{Z_{01}+Z_0}\left(e^{\gamma l}-e^{-\gamma l}\right)}{e^{\gamma l}-\left(\frac{Z_{01}-Z_0}{Z_{01}+Z_0}\right)^2 e^{-\gamma l}}=\frac{\frac{Z_{01}-Z_0}{\cancel{Z_{01}+Z_0}}\left(e^{\gamma l}-e^{-\gamma l}\right)}{\frac{e^{\gamma l}(Z_{01}+Z_0)^2-(Z_{01}-Z_0)^2 e^{-\gamma l}}{(Z_{01}+Z_0)^{\cancel{2}}}}=\frac{(Z_{01}+Z_0)(Z_{01}-Z_0)\left(e^{\gamma l}-e^{-\gamma l}\right)}{\underbrace{e^{\gamma l}(Z_{01}+Z_0)^2-(Z_{01}-Z_0)^2 e^{-\gamma l}}_{(Z_{01}^2+2Z_{01}Z_0+Z_0^2)e^{\gamma l}-(Z_{01}^2-2Z_{01}Z_0+Z_0^2)e^{-\gamma l}}}$$

$$=\frac{(Z_{01}^2-Z_0^2)\left(e^{\gamma l}-e^{-\gamma l}\right)}{(Z_{01}^2+Z_0^2)\left(e^{\gamma l}-e^{-\gamma l}\right)+2Z_{01}Z_0\left(e^{\gamma l}+e^{-\gamma l}\right)}=$$

$$=\frac{(Z_{01}^2-Z_0^2)\sinh\gamma l}{(Z_{01}^2+Z_0^2)\sinh\gamma l+2Z_{01}Z_0\cosh\gamma l}=\underline{\underline{S_{22}}}$$

$$S_{12}=\frac{b_1}{a_2}\Big|_{a_1=0}=\frac{(1-\Gamma^2)\,e^{-\gamma l}}{1-\Gamma^2 e^{-2\gamma l}}=\frac{\left[1-\left(\frac{Z_{01}-Z_0}{Z_{01}+Z_0}\right)^2\right]e^{-\gamma l}}{1-\left(\frac{Z_{01}-Z_0}{Z_{01}+Z_0}\right)^2 e^{-2\gamma l}}=$$

$$=\frac{\left[\frac{(Z_{01}+Z_0)^2-(Z_{01}-Z_0)^2}{\cancel{(Z_{01}+Z_0)^2}}\right]e^{-\gamma l}}{\frac{(Z_{01}+Z_0)^2-(Z_{01}-Z_0)^2 e^{-2\gamma l}}{\cancel{(Z_{01}+Z_0)^2}}}\cdot\frac{e^{\gamma l}}{e^{\gamma l}}=\frac{(Z_{01}+Z_0)^2-(Z_{01}-Z_0)^2}{(Z_{01}+Z_0)^2 e^{\gamma l}-(Z_{01}-Z_0)^2 e^{-\gamma l}}=$$

$$=\frac{4Z_{01}Z_0}{(Z_{01}^2+2Z_{01}Z_0+Z_0^2)e^{\gamma l}-(Z_{01}^2-2Z_{01}Z_0+Z_0^2)e^{-\gamma l}}=\frac{4Z_{01}Z_0}{\left(e^{\gamma l}-e^{-\gamma l}\right)(Z_{01}^2+Z_0^2)+\left(e^{\gamma l}+e^{-\gamma l}\right)(2Z_{01}Z_0)}=$$

$$=\frac{4Z_{01}Z_0}{(Z_{01}^2+Z_0^2)\sinh\gamma l+(2Z_{01}Z_0)\cosh\gamma l}=\underline{\underline{S_{21}}}$$

2. Za cirkulator bez gubitaka koeficijenti refleksije na sva tri prolaza međusobno su jednaki, te ako su preostala dva prolaza prilagođena, iznose 0.2 ∠0°. Kašnjenje signala među susjednim prolazima je 30°. $\Rightarrow -30°$ Gušenje signala u nepropusnom smjeru je 20 dB. Prolaz 2 zaključen je idealnim kratkim spojem koji je udaljen 40° od referentne fazne ravnine prolaza 2. Izračunati raspršnu matricu [s'] na taj način dobivenog dvoprolaznog sklopa. Fazne ravnine 1 i 1', te 3 i 2' se podudaraju.

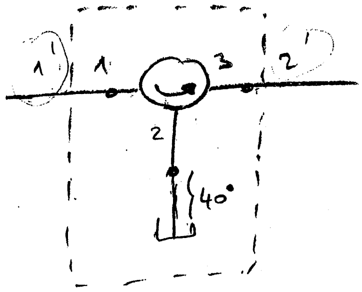
7

$\Gamma_1 = \Gamma_2 = \Gamma_3 = 0.2\angle 0°$

$\varphi = 30°$

$\alpha = 20\text{ dB}$ $\{ 20\log \alpha\, e^{-j\varphi}$

$= 40°$



$$\begin{bmatrix} 11 & 12 & 13 \\ 21 & 22 & 23 \\ 31 & 32 & 33 \end{bmatrix}$$

$S_{11} = S_{22} = S_{33} = 0.2\angle 0°$

$S_{12} = S_{23} = S_{31} = 10^{-\frac{20}{20}} e^{-j30°} = 0.1\angle -30°$

$\Rightarrow |S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 = 1 \Rightarrow$ gubici

$|S_{21}|^2 = 1 - |S_{11}|^2 - |S_{31}|^2 = 1 - (0.2)^2 - (0.1)^2 = 0.95$

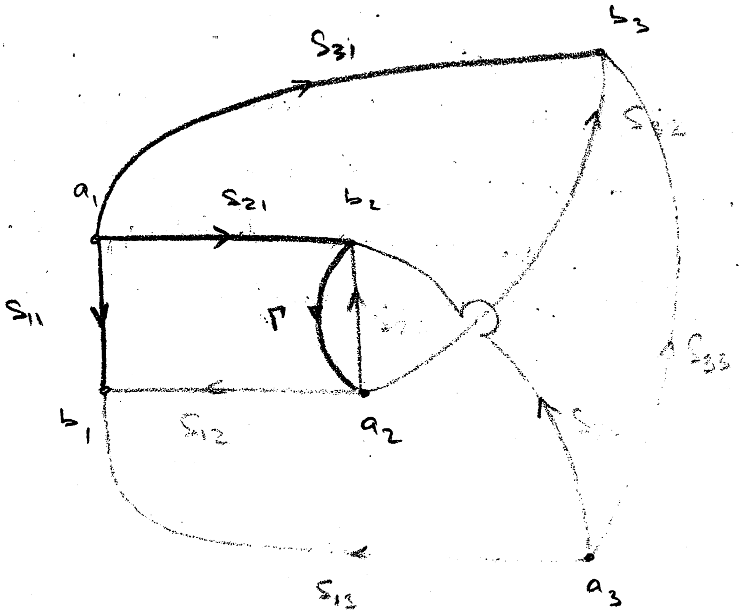
$|S_{21}| = 0.974$

$S_{21} = S_{13} = S_{32} = 0.974\angle -30°$

$$[S] = \begin{bmatrix} 0.2\angle 0°_{11} & 0.1\angle -30°_{12} & 0.974\angle -30°_{13} \\ 0.974\angle -30°_{21} & 0.2\angle 0°_{22} & 0.1\angle -30°_{23} \\ 0.1\angle -30°_{31} & 0.974\angle -30°_{32} & 0.2\angle 0°_{33} \end{bmatrix}$$

$\Rightarrow$

$\Gamma_2 = 1\angle 100^\circ$



$$S_{11}' = \frac{b_1}{a_1} = \frac{S_{11}(1-\Gamma S_{22}) + S_{21}\cdot\Gamma\cdot S_{12}(1-0)}{1-\Gamma\cdot S_{22}} = S_{11} + \frac{S_{21}\cdot S_{12}\cdot\Gamma}{1-\Gamma\cdot S_{22}} = 0.268\angle 15.5^\circ$$

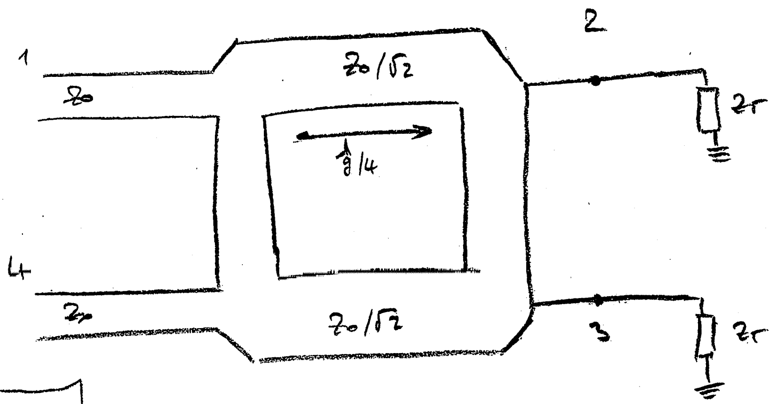
$$S_{22}' = \frac{b_3}{a_3} = \frac{S_{33}(1-\Gamma\cdot S_{22}) + S_{23}\cdot\Gamma\cdot S_{32}(1-0)}{1-\Gamma\cdot S_{22}} = S_{33} + \frac{S_{23}\cdot S_{32}\cdot\Gamma}{1-\Gamma\cdot S_{22}} = 0.268\angle 15.5^\circ$$

$$S_{12}' = \frac{b_1}{a_3} = \frac{S_{13}(1-\Gamma\cdot S_{22}) + S_{23}\cdot\Gamma\cdot S_{12}(1-0)}{1-\Gamma\cdot S_{22}} = S_{13} + \frac{S_{23}\cdot S_{12}\cdot\Gamma}{1-\Gamma\cdot S_{22}} = 0.975\angle -29.44^\circ$$

$$S_{21}' = \frac{b_3}{a_1} = \frac{S_{31}(1-\Gamma\cdot S_{22}) + S_{21}\cdot\Gamma\cdot S_{32}(1-0)}{1-\Gamma\cdot S_{22}} = S_{31} + \frac{S_{21}\cdot S_{32}\cdot\Gamma}{1-\Gamma\cdot S_{22}} = 0.921\angle 44.63^\circ$$

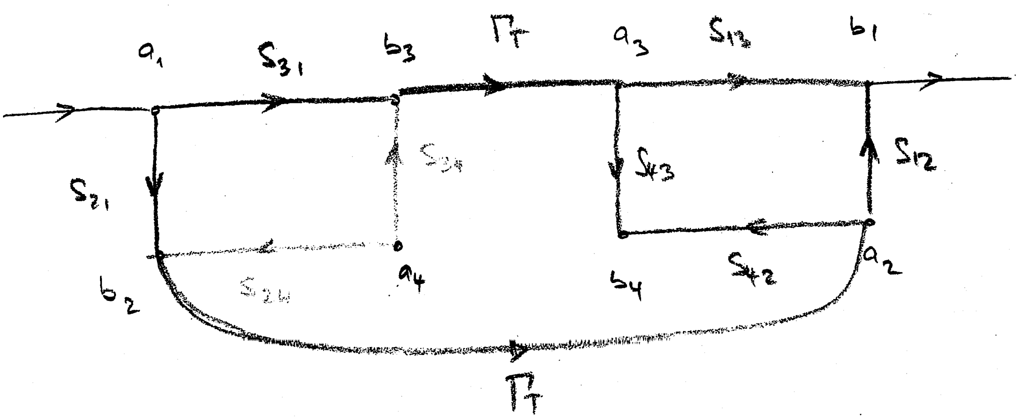
$$[S'] = \begin{bmatrix} 0.268\angle 15.5^\circ & 0.975\angle -29.44^\circ \\ 0.921\angle 44.63^\circ & 0.268\angle 15.5^\circ \end{bmatrix}$$

3. Prolazi 2 i 3 kvadraturnog hibrida zaključeni su teretima impedancije Z_T. Koliko iznosi Z_T ako je novonastali dvoprolazni sklop (prolazi 1 i 4) atenuator s gušenjem od 6 dB i faznim kašnjenjem 0°? Izračunati raspršnu matricu za novi dvoprolazni sklop. Karakteristična impedancija sustava je 50 Ω. 8



$$\boxed{\Gamma_T = \frac{Z_T - Z_0}{Z_T + Z_0}}$$

$$[S] = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & -j & 0 \\ 1 & 0 & 0 & -j \\ -j & 0 & 0 & 1 \\ 0 & -j & 1 & 0 \end{bmatrix}$$



$\Rightarrow$

$$T_{14} = \frac{b_1}{a_4} = \frac{S_{34}\cdot\Gamma_T\cdot S_{13} + S_{24}\cdot\Gamma_T\cdot S_{12}}{1} = \left(\frac{-j}{\sqrt{2}}\right)^2\Gamma(-j) + \left(-\frac{j}{\sqrt{2}}\right)^2\Gamma(-j) = j\cdot\Gamma_T$$

$$T_{41} = \frac{b_4}{a_1} = S_{31}\cdot\Gamma\cdot S_{43} + S_{21}\cdot\Gamma\cdot S_{42} = j\cdot\Gamma_T$$

$$\Gamma_{11} = \frac{b_1}{a_1} = S_{31}\cdot\Gamma\cdot S_{13} + S_{21}\cdot\Gamma\cdot S_{12} = \left(\frac{-j}{\sqrt{2}}\right)^2\Gamma\cdot\underbrace{(-j)(-j)}_{-1} + \left(\frac{-j}{\sqrt{2}}\right)^2\Gamma(1)(1) = 0$$

$$\Gamma_{44} = \frac{b_4}{a_4} = S_{24}\cdot\Gamma\cdot S_{42} + S_{34}\cdot\Gamma\cdot S_{43} = \left(\frac{-j}{\sqrt{2}}\right)^2\Gamma(-j)(-j) + \left(\frac{-j}{\sqrt{2}}\right)^2\Gamma = 0$$

$$[S'] = \begin{bmatrix} \Gamma_{11} & T_{14} \\ T_{41} & \Gamma_{44} \end{bmatrix} = \begin{bmatrix} 0 & j\Gamma_T \\ j\Gamma_T & 0 \end{bmatrix}$$

sklop je simetričan $S_{11} = S_{22}$; $S_{12} = S_{21}$

sklop je recipročan $S_{14} = S_{41}$

sklop je prilagođen $S_{11} = S_{44} = 0$

$$T_{14} = 10^{\frac{-6}{20}} e^{-j0^\circ} = 0.501\angle 0^\circ = T_{41}$$

$$T_{14} = j\Gamma_T = j\cdot\frac{Z_T - Z_0}{Z_T + Z_0}$$

$$Z_T\cdot T_{14} + Z_0 T_{14} = Z_T j - Z_0 j$$

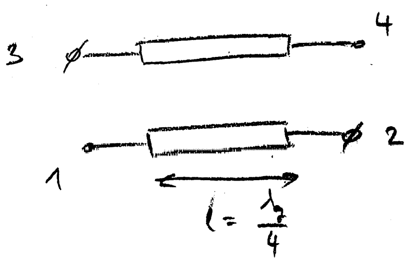
$$Z_T(T_{14} - j) = Z_0(-j - T_{14})$$

$$Z_T = \frac{Z_0(-j - T_{14})}{T_{14} - j} = 29.93 - j\,40.04\ \Omega = 50\angle -53.2^\circ\ \Omega$$

raspršna matrica prilagođenog 6 dB atenuatora:

$$[S'] = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix}$$

4. Linijski sprežnik napona s koeficijentom sprege $C = 10$ dB spojen je na način da su prolazi 2 i 3 otvoreni, dok preostala dva prolaza predstavljaju ulaz (prolaz 1) i izlaz (prolaz 4).
Na središnjoj frekvenciji na kojoj je duljina linija koje čine sprežnik jednaka $\frac{\lambda_g}{4}$ odrediti raspršnu matricu na ovaj način ostvarenog dvoprolaznog sklopa u sustavu karakteristične impedancije $50\,\Omega$.
Koliki su povratni i unešeni gubici te ulazna impedancija ovog sklopa, uz prilagođenje na drugom prolazu?
Odrediti karakteristične impedancije parnog i neparnog moda. Je li ovaj dvoprolazni sklop prilagođen, recipročan i nedisipativan?



$$[S] = \begin{bmatrix} 0 & -j\alpha & \beta & 0 \\ -j\alpha & 0 & 0 & \beta \\ \beta & 0 & 0 & -j\alpha \\ 0 & \beta & -j\alpha & 0 \end{bmatrix}$$

$$l = \frac{\lambda_g}{4} \Rightarrow \alpha = \sqrt{1 - C^2} = 0.948$$
$$\beta = C = 0.316$$

$$C = 10\,\text{db} = 10^{-\frac{10}{20}} = 0.316$$

2 i 3 otvoreni $\Rightarrow$ $a_2 = b_2$ i $a_3 = b_3$

4 zaključen s Z_0 $\Rightarrow$ $a_4 = 0$

$$[b] = [S] \cdot [a]$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & -j\alpha & \beta & 0 \\ -j\alpha & 0 & 0 & \beta \\ \beta & 0 & 0 & -j\alpha \\ 0 & \beta & -j\alpha & 0 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

$$b_1 = a_2(-j\alpha) + a_3\beta = -j0.948\,b_2 + 0.316\,b_3$$
$$b_2 = a_1(-j\alpha) + a_4\beta = -j\,0.948\,a_1$$
$$b_3 = a_1\beta + a_4(-j\alpha) = 0.316\,a_1$$
$$b_4 = a_2\beta + a_3(-j\alpha) = 0.316\,b_2 - j\,0.948\,b_3$$

$\Rightarrow$

$$S_{11}' = \frac{b_1}{a_1} = -0.798 = S_{22}'$$

$$b_1 = -j0.948(-j0.948)a_1 + 0.316 \cdot 0.316\, a_1 = -0.798\, a_1$$

$$RL = -20 \log |S_{11}'| = 1.95 \text{ dB}$$

$$S_{21}' = \frac{b_4}{a_1} = -j0.59$$

$$b_4 = 0.316(-j0.948 a_1) - j0.948(0.316 a_1) = -j0.59 a_1$$

$$IR = -20 \log |S_{21}'| = 4.58 \text{ dB}$$

prilagođen ✗

$$[S'] = \begin{bmatrix} -0.798 & -j0.59 \\ -j0.59 & -0.798 \end{bmatrix}$$

recipročan ✓

nedisipativan

$|S_{11}|^2 + |S_{21}|^2 = 1$ ✓

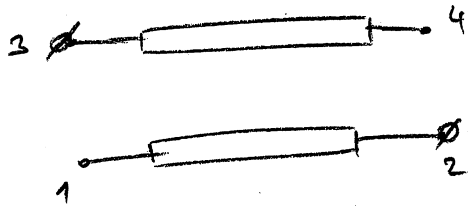
$$Z_{ul} = Z_0 \cdot \frac{1+\Gamma_{ul}}{1-\Gamma_{ul}} = Z_0 \cdot \frac{1+S_{11}'}{1-S_{11}'} = 5.56\ \Omega$$

parni mod: $Z_{0e} = Z_0 \sqrt{\frac{1+C}{1-C}} = 69.35\ \Omega$

neparni mod: $Z_{0o} = \frac{Z_0^2}{Z_{0e}} = 36.04\ \Omega$

(5.) Linijski sprežnik izveden je na supstratu relativne dielektrične konstante 2.5 i debljine 2 mm. Širina pojedine linije je 4.3 mm, a njihova međusobna udaljenost 0.1 mm.
Odrediti karakteristične impedancije parnog i neparnog moda te koeficijent sprege u decibelima.
Ako su prolazi 2 i 3 navedenog sprežnika zaključeni otvorenim krajem, a prolaz 1 i 4 prilagođeni, odrediti koeficijent prijenosa između prolaza 1 i 4.

(10)



$\varepsilon_r = 2.5$

$s = 0.1\ mm$

$w = 4.3\ mm$

$h = 2\ mm$

$Z_{0e}, Z_{0o} = ?$

$C = ?$

$T_{14} = ?$

$$\frac{s}{h} = \frac{0.1}{2} = 0.05$$

$$\frac{w}{h} = \frac{4.3}{2} = 2.15$$

$$\left.\begin{aligned} Z_{0e} &= 75\ \Omega \\ Z_{0o} &= 33\ \Omega \end{aligned}\right\}$$

$$Z_0 = \sqrt{Z_{0e} \cdot Z_{0o}} = 50\ \Omega$$

$$Z_{0e} = Z_0 \sqrt{\frac{1+C}{1-C}}$$

$$\frac{1+C}{1-C} = 2.25$$

$$1 + C = 2.25 - 2.25C$$

$$C = 0.384 = -20\log(x) = 8.3\ dB$$

$$[S] = \begin{bmatrix} 0 & -j\alpha & \beta & 0 \\ -j\alpha & 0 & 0 & \beta \\ \beta & 0 & 0 & -j\alpha \\ 0 & \beta & -j\alpha & 0 \end{bmatrix}$$

$\Rightarrow$

$b_1 = a_2(-j\alpha) + a_3\beta$

$b_2 = a_1(-j\alpha) + a_4\beta$

$b_3 = a_1\beta + a_4(-j\alpha)$

$b_4 = a_2\beta + a_3(-j\alpha)$

$a_2 = b_2$

$a_3 = b_3$

$a_4 = 0$

$\alpha = \sqrt{1-c^2} = 0.923$

$\beta = c = 0.384$

$T_{41} = \frac{b_4}{a_1} = -2j\alpha\beta = -j0.708 = 0.708\angle -90^\circ = T_{14}$

$b_4 = a_1(-j\alpha)\beta + (-j\alpha)\beta a_1 = -2j\alpha\beta a_1$

6. Rezonator duljine $\lambda/2$ ima rezonantnu frekvenciju 5 GHz, a izrađen je od dijela snosne linije čiji su vodiči od bakra vodljivosti $5.81\cdot10^{7}$ S/m. Polumjeri unutarnjeg i vanjskog vodiča su 1 mm, odnosno 4 mm. (11)

Usporediti faktore dobrote za slučaj kada je snosna linija ispunjena zrakom, te za slučaj kada je ispunjena dielektrikom relativne dielektrične konstante 2.08 i tangensa gubitaka $4\cdot10^{-4}$.

? približne vrijednosti (skraćene formule)

$f = 5$ GHz

$\sigma = 5.81\cdot10^{7}$ S/m

$a = 1$ mm

b 4 mm

1° $Q_1 = ?$

$\varepsilon_r = 1$

2° $Q_2 = ?$

$\varepsilon_r = 2.08$

$\operatorname{tg}\delta = 4\cdot10^{-4}$

$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{(j\omega L)(j\omega C)\left(1+\frac{R}{j\omega L}\right)\left(1+\frac{G}{j\omega C}\right)} = j\omega\sqrt{LC}\cdot\sqrt{1-j\left(\frac{R}{\omega L}+\frac{G}{\omega C}\right)-\cancel{\frac{RG}{\omega^2 LC}}}$$

$R \ll \omega L$, $G \ll \omega C$ } $RG \ll \omega^2 LC$

aproksimacija $\sqrt{1-x} \simeq 1+\frac{x}{2}$

$$\gamma \simeq j\omega\sqrt{LC}\left[1+\frac{j}{2}\left(\frac{R}{\omega L}+\frac{G}{\omega C}\right)\right]$$

$$\beta = \omega\sqrt{LC}$$

$$\alpha = \left[\frac{1}{2}\left(R\sqrt{\frac{C}{L}}+G\sqrt{\frac{L}{C}}\right)\right]$$

gubici u vodiču

gubici u dielektriku

$\operatorname{tg}\delta = 0$

$$R_s = \sqrt{\frac{\omega\mu}{2\sigma}} = 1.84\cdot10^{-2}\,\Omega$$

$\varepsilon' = \varepsilon_0\cdot\varepsilon_r = \varepsilon_0$

1°

$$R = \frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right) = 3.58\,\Omega$$

$$C = \frac{2\pi\varepsilon'}{\ln\frac{b}{a}} = 4.01\cdot10^{-11}\text{ F}$$

$$L = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right) = 2.77\cdot10^{-7}\text{ H}$$

$$G = \frac{2\pi\omega\varepsilon''}{\ln\left(\frac{b}{a}\right)} = 0$$

$\beta = 104.703$ rad/m

$\alpha = 0.0215$ $\frac{Np}{m}$

$$Q_0 = \frac{\beta}{2\alpha} = 2434.95$$

$\Rightarrow$

2°

$\varepsilon_r = 2.08$

$tg\,\delta = 4 \cdot 10^{-4}$

$\varepsilon' = \varepsilon_0 \varepsilon_r$

$\varepsilon'' = \varepsilon_0 \varepsilon_r tg\,\delta$

$R_s = 1,84 \cdot 10^{-2}\ \Omega$

$R = 3,58\ \Omega$

$C = 8.34 \cdot 10^{-11}\ F$

$L = 2.77 \cdot 10^{-7}\ H$

$G = 1.04 \cdot 10^{-3}\ \frac{1}{\Omega}$

$\beta = 150.99\ rad/m$

$\alpha = 0,0506\ \frac{Np}{m}$

$Q_0 = \frac{\beta}{2\alpha} = 1491,99$

$\frac{Q_1}{Q_2} = \frac{2434.95}{1491.99} = 1.63$

(7.) Mikrotrakasti rezonator (linijski) duljine $\lambda/2$, zaključen otvorenim krajem, izrađen je od bakra vodljivosti $5.81 \cdot 10^{7}$ S/m na dielektričnom supstratu visine 1.5 mm, relativne dielektrične konstante 2.5 i tangensa gubitaka 10^{-3}. Karakteristična impedancija mikrotrakaste linije je 50 Ω. Kolika je duljina linijskog rezonatora ako mu je rezonantna frekvencija 4 GHz?
Izračunati unutarnji faktor dobrote.
Ukoliko je rezonator paralelno opterećen teretom impedancije 50 Ω, izračunati vanjski faktor dobrote i opterećeni faktor dobrote. Koliki je faktor sprege? Zanemariti rasipna polja na otvorenom kraju i disperziju.

(12)

$$l = \frac{\lambda}{2} = \frac{1}{2} \cdot \frac{v_p}{f} = \frac{1}{2} \cdot \frac{c}{f\sqrt{\varepsilon_{ef}}}$$

$\sigma = 5.81 \cdot 10^{7}$ S/m

$h = 1.5$ mm

$\varepsilon_r = 2.5$

$tg\,\delta = 10^{-3}$

$f = 4$ GHz

$l = ?$

$Q_0 = ?$

$Q_v = ?$

$Q_T = ?$

$$\frac{W}{h} > 2 \qquad \frac{W}{h} = \frac{2}{\pi}\left\{B - 1 - \ln(2B-1) + \frac{\varepsilon_r - 1}{2\varepsilon_r}\left[\ln(B-1) + 0.39 - \frac{0.61}{\varepsilon_r}\right]\right\} = 2.83$$

$$B = \frac{377 \cdot \pi}{2 \cdot Z_0\sqrt{\varepsilon_r}} = 7.49$$

$$\varepsilon_{ef} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left(1 + 10\frac{h}{W}\right)^{-0.555} = 2.074$$

$$l = \frac{1}{2} \cdot \frac{c}{f\sqrt{\varepsilon_{ef}}} = 0.026 \text{ m} = 2.6 \text{ cm}$$

$$tgh(A+B) = \frac{tgh A + tgh B}{1 + tgh A \cdot tgh B}$$

$$tgh(jB) = j\,tg\,B$$

ELEMENTI NADOMJESNOG SKLOPA

$Z_T = \infty$ (otvoreni kraj)

$$Z_{ul} = Z_0 \cdot \frac{Z_T + Z_0\, tgh(\gamma l)}{Z_0 + Z_T\, tgh(\gamma l)} = \frac{Z_0}{tgh(\gamma l)} = \frac{Z_0}{tgh[(\alpha + j\beta) l]} = Z_0 \cdot \frac{1 + tgh(\alpha l)\, j\,tg(\beta l)}{tgh(\alpha l) + j\,tg(\beta l)}$$

- promatramo Z_{ul} u okolini rezonantne frekvencije

$$\omega = \omega_0 + \Delta\omega \quad ; \Delta\omega \ll$$

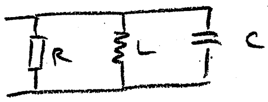
$$\beta l = \frac{\omega}{v_p} \cdot l = \frac{\omega_0 \cdot l}{v_p} + \frac{\Delta\omega \cdot l}{v_p}$$

$$\frac{\omega_0 l}{v_p} = \frac{2\pi f_0}{v_p} \cdot \frac{\lambda}{2} = \frac{\pi f_0}{v_p} \cdot \frac{v_p}{f_0} = \pi \Rightarrow \frac{l}{v_p} = \frac{\pi}{\omega_0}$$

$$\Bigg\} \; \beta l = \pi + \frac{\Delta\omega \cdot \pi}{\omega_0}$$

$$\Rightarrow \operatorname{tg} \beta l = \operatorname{tg}\left(\pi + \frac{\Delta\omega \cdot \pi}{\omega_0}\right) = \operatorname{tg}\left(\frac{\Delta\omega \cdot \pi}{\omega_0}\right) \simeq \frac{\Delta\omega \cdot \pi}{\omega_0}$$

$$R = \frac{Z_0}{\alpha l}$$



$$C = \frac{\pi}{2\omega_0 Z_0} \Rightarrow L = \frac{1}{\omega_0^2 C}$$

$$R_s = \sqrt{\frac{\omega \cdot \mu_0}{2\sigma}} = 1.6 \cdot 10^{-2} \; \Omega$$

$$\frac{W}{h} = 2.83 \Rightarrow W = 2.83 \cdot h = 4.25 \text{ mm}$$

$$l = \frac{\lambda}{2} = 2.6 \text{ cm} \Rightarrow \lambda_g = 5.2 \text{ cm} = 0.052 \text{ m}$$

$$\alpha_c = \frac{R_s}{W \cdot Z_0} = 0.075 \; \frac{Np}{m}$$

$$\alpha_d = \frac{\pi}{\lambda_g} \cdot \frac{\varepsilon_r(\varepsilon_{ef} - 1)}{\varepsilon_{ef}(\varepsilon_r - 1)} \operatorname{tg}\delta = 0.0521 \; \frac{Np}{m}$$

$$\alpha = \alpha_d + \alpha_c = 0.1271 \; \frac{Np}{m}$$

$$R = \frac{Z_0}{\alpha l} = 15\,130.42 \; \Omega$$

$$C = \frac{\pi}{2 \cdot \omega_0 \cdot Z_0} = 1.25 \cdot 10^{-12} \; F$$

$2\pi f$

$$L = \frac{1}{\omega_0^2 C} = 1.26 \text{ nH}$$

$$Q_0 = \omega_0 \cdot R \cdot C = 475.33$$

$$Q_V = \frac{R_T}{\omega_0 L} = 1.578$$

$$\frac{1}{Q_T} = \frac{1}{Q_0} + \frac{1}{Q_V} \Rightarrow Q_T = 1.572$$

$$k = \frac{Q_0}{Q_V} = 301 > 1$$

nadkritična sprega !

(8.) Rezonator je izrađen od dijela snosne linije duljine $\lambda/2$ koja je na jednom kraju zaključena kratkim spojem. Snosna linija je ispunjena zrakom; polumjer unutarnjeg vodiča je 3.5 mm, a vanjskog 7 mm. Vodiči su izrađeni od bakra vodljivosti $5.81 \cdot 10^{7}$ S/m. Rezonantna frekvencija ovog rezonatora je 3 GHz. Nacrtati nadomjesni sklop s koncentriranim elementima za navedeni rezonator u okolini rezonantne frekvencije i odrediti vrijednosti elemenata nadomjesnog sklopa. Odrediti unutarnji faktor dobrote i ulaznu impedanciju rezonatora na rezonantnoj frekvenciji. Ako je rezonator opterećen teretom od 30 Ω, izračunati vanjski i opterećeni faktor dobrote. (13)

KRATKI SPOJ $\Rightarrow$ SERIJSKI TITRAJNI KRUG

$l = \frac{\lambda}{2}$

$\varepsilon_r = 1$

$a = 3.5$ mm

$b = 7$ mm

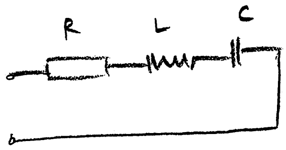
$\sigma = 5.81 \cdot 10^{7}$ S/m

$f = 3$ GHz

$R, L, C = ?$

$Q_0, Z_{ul} = ?$

$R_T = 30\,\Omega \Rightarrow Q_V, Q_T = ?$



$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}} = 40.26\,\Omega$$

$$C = \frac{2\pi\varepsilon'}{\ln\left(\frac{b}{a}\right)} = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{b}{a}\right)} = 8.02 \cdot 10^{-11}\,F$$

$$L = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right) = 1.3 \cdot 10^{-7}\,H$$

$$R = \frac{R_s}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right) = 0.968\,\Omega$$

$$R_s = \sqrt{\frac{\omega\mu}{2\sigma}} = 0.0142\,\Omega$$

$$l = \frac{1}{2} \cdot \frac{c}{f} = 0.05\,m$$

$$\alpha_c = \frac{1}{2}\sqrt{LC} \cdot \frac{R}{L} = 0.012\,\frac{Np}{m}$$

$\Rightarrow$

$R' = Z_0 \cdot \alpha \cdot l = 0.0241\ \Omega$

$L' = \dfrac{Z_0 \cdot \pi}{2 w_0} = 3.35 \cdot 10^{-9}\ H$

$C' = \dfrac{1}{w_0^2 \cdot L'} = 8.401 \cdot 10^{-13}\ F$

$Z_{in,\,RLC} = R' = 0.0241\ \Omega$

$Q_0 = \dfrac{w_0 \cdot L'}{R'} = 2620.16$

$Q_V = \dfrac{w_0 \cdot L'}{R_F} = 2.104$

$Q_T = \left(\dfrac{1}{Q_0} + \dfrac{1}{Q_V} \right)^{-1} = 2.102$

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(9) Prstenasti rezonator izrađen od mikrotrakaste linije širine 2 mm ima prvi rezonantni mod na frekvenciji 1 GHz. Koliki je njegov srednji polumjer ako je izrađen od aluminija vodljivosti $3.82\cdot10^{7}$ S/m, na supstratu debljine 1.6 mm, relativne dielektrične konstante 6.4 i tangensa gubitaka $4\cdot10^{-3}$? Izračunati ulaznu impedanciju u rezonanciji i unutarnji faktor dobrote. Zanemariti disperziju.

$w = 2$ mm

$f_1 = 1$ GHz

$\sigma = 3.82\cdot10^{7}$ S/m

$h = 1.6$ mm

$\varepsilon_r = 6.4$

$\operatorname{tg}\delta = 4\cdot10^{-3}$

$d = ?$

$Z_{ul} = ?$

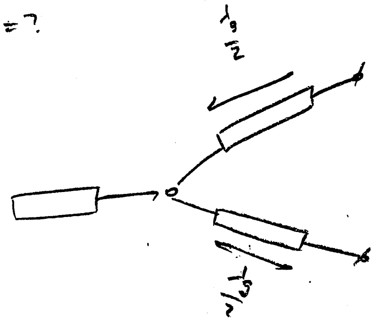
$Q_0 = ?$

$$\varepsilon_{ef} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left(1+10\frac{h}{w}\right)^{-0.555} = 4.49$$

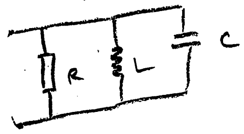
$$\lambda_g = \frac{c}{f\sqrt{\varepsilon_{ef}}} = 0.1415 \text{ m} = 14.15 \text{ cm}$$

$$2\pi d = n\cdot\lambda_g$$

$$d = \frac{n\cdot\lambda_g}{2\pi} = 0.022 \text{ m} = 2.22 \text{ cm}$$



$$l = \frac{1}{2}\cdot\frac{c}{f\sqrt{\varepsilon_{ef}}} = 0.0707 \text{ m} = 7.07 \text{ cm}$$



$$Z_0 = \frac{\eta_0}{\sqrt{\varepsilon_{ef}}}\cdot\frac{1}{\frac{W}{h}+1.98\left(\frac{W}{h}\right)^{0.172}} = 53.79\ \Omega$$

$$\alpha_c = \frac{R_s}{W\cdot Z_0} = 0.0944\ \frac{Np}{m}$$

$$R_s = \sqrt{\frac{\omega\cdot\mu_0}{2\sigma}} = 0{,}0101\ \Omega$$

$$\alpha_d = \frac{\pi}{\lambda_g}\cdot\frac{\varepsilon_r(\varepsilon_{ef}-1)}{\varepsilon_{ef}(\varepsilon_r-1)}\cdot\operatorname{tg}\delta = 0.0818\ \frac{Np}{m}$$

$$\alpha = \alpha_d+\alpha_c = 0.176\ \frac{Np}{m}$$

$$R = \frac{Z_0}{\alpha\cdot l} = 4322.84\ \Omega$$

$$Z_{ul} = R\,||\,R = 2161\ \Omega$$

$$Q_0 = \frac{\beta}{2\alpha} = 126.077$$

$$\beta = \frac{2\pi f}{v_f} = \frac{2\pi f\sqrt{\varepsilon_r}}{c} = 44.37\ \frac{rad}{s}$$

ZADACI 05

(1) Raspršni parametri mikrovalnog tranzistora na frekvenciji 10 GHz izmjereni su u mjernom sustavu karakteristične impedancije 50 Ω:

$S_{11} = 0.45\angle 150^\circ$ $\qquad$ $S_{12} = 0.01\angle -10^\circ$

$S_{21} = 2.05\angle 10^\circ$ $\qquad$ $S_{22} = 0.4\angle -150^\circ$

Na pojačalo su priključeni generator impedancije $Z_G = 20\ \Omega$ i teret impedancije $Z_T = 30\ \Omega$. Ispitati stabilnost tranzistora. Izračunati pogonsko, prijenosno i raspoloživo pojačanje snage.

$f = 10$ GHz

$$[S] = \begin{bmatrix} 0.45\angle 150^\circ & 0.01\angle -10^\circ \\ 2.05\angle 10^\circ & 0.4\angle -150^\circ \end{bmatrix}$$

$Z_G = 20\ \Omega$

$Z_T = 30\ \Omega$

$$\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.1595 < 1 \checkmark$$

$$k = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = 16.16 > 1 \checkmark \rightarrow \underline{\text{apsolutno stabilan tranzistor}}$$

$$\Gamma_T = \frac{Z_T - Z}{Z_T + Z} = -0.25$$

$$\Gamma_G = \frac{Z_G - Z}{Z_G + Z} = -0.428$$

$$\Gamma_{iz} = S_{22} + \frac{S_{12}S_{21}\Gamma_G}{1 - S_{11}\Gamma_G} = 0.408\angle -150.8^\circ$$

$$\Gamma_{ul} = S_{11} + \frac{S_{12}\cdot S_{21}\Gamma_T}{1 - S_{22}\Gamma_T} = 0.454\angle 150.38^\circ$$

Pogonsko pojačanje: $G_P = \dfrac{1}{1-|\Gamma_{ul}|^2}\cdot|S_{21}|^2\cdot\dfrac{1-|\Gamma_T|^2}{|1-S_{22}\Gamma_T|^2} = 5.93 = \underline{\underline{7.73\ \text{dB}}}$

Prijenosno pojačanje: $G_T = \dfrac{1-|\Gamma_G|^2}{|1-\Gamma_{ul}\Gamma_G|^2}\cdot|S_{21}|^2\cdot\dfrac{1-|\Gamma_T|^2}{|1-S_{22}\Gamma_T|^2} = 5.49 = \underline{\underline{7.39\ \text{dB}}}$

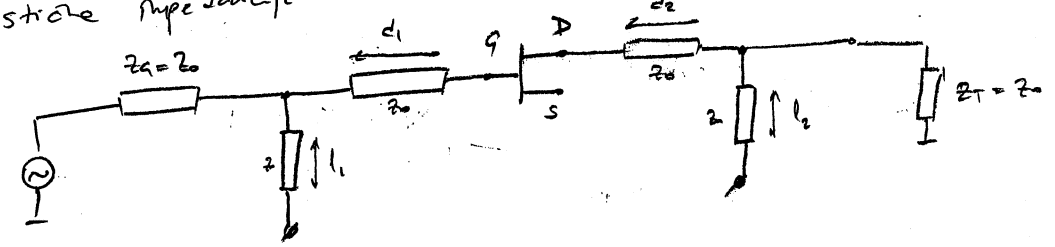
Raspoloživo pojačanje: $G_A = \dfrac{1-|\Gamma_G|^2}{|1-S_{11}\Gamma_G|^2}\cdot|S_{21}|^2\cdot\dfrac{1}{1-|\Gamma_{iz}|^2} = 5.85 = \underline{\underline{7.67\ \text{dB}}}$

(2.) Za GaAs FET zadani su raspršni parametri na frekvenciji 4 GHz za karakterističnu impedanciju sustava 50 Ω:

$S_{11} = 0.72\angle{-116^\circ}$ $S_{12} = 0.03\angle 57^\circ$

$S_{21} = 2.6\angle 76^\circ$ $S_{22} = 0.73\angle{-54^\circ}$

Ispitati stabilnost tranzistora. Projektirati pojačalo za maksimalno pojačanje koristeći na ulazu i izlazu prilagodne mreže s jednim stubom prema slici. Pretpostaviti da su linije bez gubitaka te da je njihova karakteristična impedancija 50 Ω; izračunati prijenosno pojačanje snage.



$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2\,|S_{21}\cdot S_{12}|} = 1.19 > 1$$

$$\Delta = S_{11}\cdot S_{22} - S_{12}\cdot S_{21} = 0.487\angle{-162.2^\circ}$$

$$|\Delta| = 0.487 < 1$$

apsolutno stabilan

maksimalno pojačanje $\Rightarrow$ $\Gamma_{ul} = \Gamma_G^*$ i $\Gamma_{iz} = \Gamma_T^*$

$$\Gamma_{ul} = \Gamma_G^* = S_{11} + \frac{S_{12}\cdot S_{21}\,\Gamma_T}{1 - S_{22}\Gamma_T}$$

$$\Gamma_{iz} = \Gamma_T^* = S_{22} + \frac{S_{12}\cdot S_{21}\,\Gamma_G}{1 - S_{11}\Gamma_G}$$

⋮ ?

$$\underbrace{(S_{11} - \Delta\cdot S_{22}^*)}_{a_1}\Gamma_G^2 + \underbrace{(|\Delta|^2 - |S_{11}|^2 + |S_{22}|^2 - 1)}_{b_1}\Gamma_G + \underbrace{(S_{11}^* - \Delta^* S_{22})}_{c_1} = 0$$

$$\underbrace{(S_{22} - \Delta\cdot S_{11}^*)}_{a_2}\cdot\Gamma_T^2 + \underbrace{(|\Delta|^2 + |S_{11}|^2 - |S_{22}|^2 - 1)}_{b_2}\Gamma_T + \underbrace{(S_{22}^* - \Delta^* S_{11})}_{c_2} = 0$$

$a_1 = 0.37\angle{-123.47^\circ}$ $a_2 = 0.385\angle{-61.08^\circ}$

$b_1 = -0.748$ $b_2 = -0.777$

$c_1 = 0.37\angle 123.4^\circ$ $c_2 = 0.385\angle 61.08^\circ$

$\Rightarrow$

$$(0.37\angle -123.47^\circ)\,\Gamma_g^2 - 0.748\,\Gamma_g + 0.37\angle 123.4 = 0$$

$$\Gamma_{g_{12}} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\Gamma_g \begin{cases} 1.158\angle 123.4^\circ \\ 0.863\angle 123.4^\circ \end{cases}$$

$$(0.385\angle -61.08^\circ)\,\Gamma_T^2 - 0.777\,\Gamma_T + 0.385\angle 61.08^\circ = 0$$

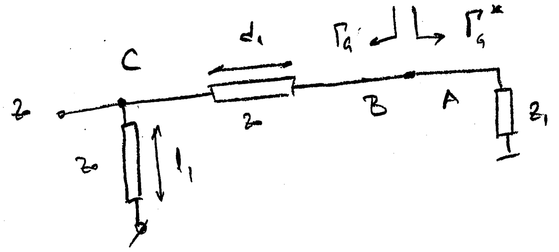
$$\Gamma_T \begin{cases} 1.144\angle 61.08^\circ \\ 0.873\angle 61.08^\circ \end{cases}$$

$\Gamma_g = 0.863\angle 123.4^\circ$

$\Gamma_{ul} = \Gamma_g^* = 0.863\angle -123.4^\circ$

$\Gamma_T = 0.873\angle 61.08$

$\Gamma_{iz} = \Gamma_T^* = 0.873\angle -61.08^\circ$



ulazna mreža:

$\Gamma_g = 0.863\angle 123.4^\circ$

$\Gamma_g^* = 0.863\angle -123.4^\circ$

A: $\dfrac{z_A}{z_0} = \dfrac{1+\Gamma_g^*}{1-\Gamma_g^*} = 0.107 - j0.68 = \dfrac{z_A}{z_0}$

B: $\dfrac{y_B}{y_0} = 0.3 + j1.82$; $w_B = 0.171\lambda$

C: $\dfrac{y_C}{y_0} = 1 - j3.56$; $w_C = 0.291\lambda$

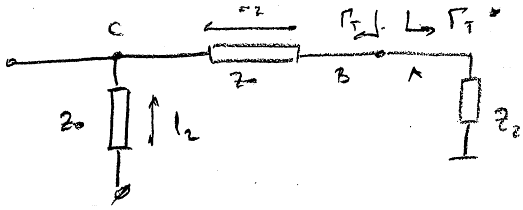
$d_1 = w_C - w_B = 0.12\lambda$

$\dfrac{y_C}{y_0} + \dfrac{y_{st_1}}{y_0} = 1 \Rightarrow \dfrac{y_{st_1}}{y_0} = 1 - \dfrac{y_C}{y_0} = +j3.56$

$w_{st_1} = 0.206\lambda = l_1$

$\Rightarrow$

Izlazna mreža:



$\Gamma_T = 0.873 \angle 61.08$

$\Gamma_T^* = 0.873 \angle -61.08$

A: $\frac{z_2}{z_0} = \frac{1+\Gamma_T^*}{1-\Gamma_T^*} = 0.25 - j1.67 = \frac{z_A}{z_0}$

B: $\frac{y_B}{y_0} = 0.09 + j0.59$; $w_B = 0.085\,\lambda$

C: $\frac{y_C}{y_0} = 1 - j3.64$; $w_C = 0.29\,\lambda$

$d_2 = w_C - w_B = 0.205\,\lambda$

$$\frac{y_C}{y_0} + \frac{y_{st_2}}{y_0} = 1 \Rightarrow \frac{y_{st_2}}{y_0} = 1 - \frac{y_C}{y_0} = +j3.64$$

$w_{st_2} = 0.208\,\lambda = l_2$

Prijenosno pojačanje

$$G_T = \frac{1-|\Gamma_g|^2}{|1-\Gamma_{ul}\Gamma_g|^2} \cdot |S_{21}|^2 \cdot \frac{1-|\Gamma_T|^2}{|1-S_{22}\Gamma_T|^2} = 45.1 \approx \underline{\underline{16.54\ dB}}$$