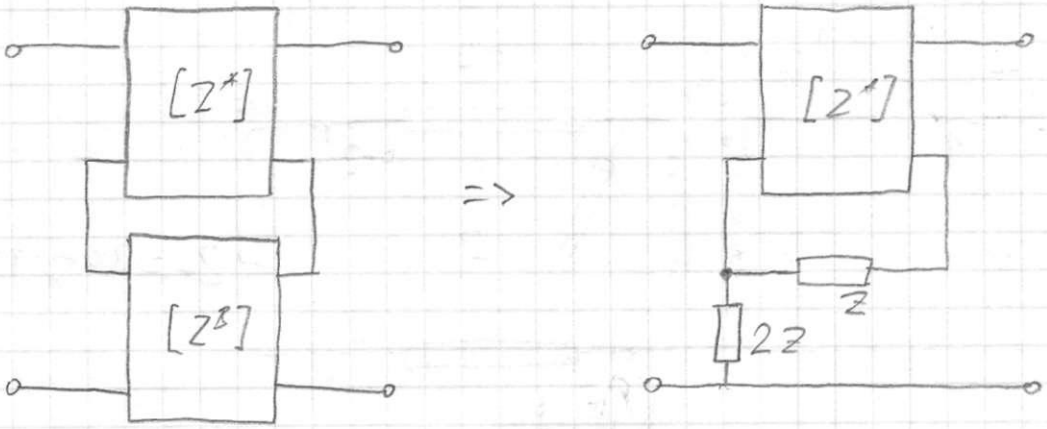
$Z_T = 30 - j40\ \Omega$

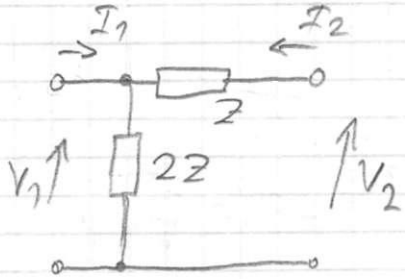
2. MI 2009. god.

Zadatak 1



-> serijski spoj dvopolaznih mreža

za ukupnu matricu Z parametara vrijedi: $[Z] = [Z^A] + [Z^B]$



$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

$Z_{11}^B = \left.\frac{V_1}{I_1}\right|_{I_2=0} \Rightarrow \boxed{Z_{11}^B = Z}$ $\quad Z_{12}^B = \left.\frac{V_1}{I_2}\right|_{I_1=0} \Rightarrow \boxed{Z_{12}^B = 2Z}$

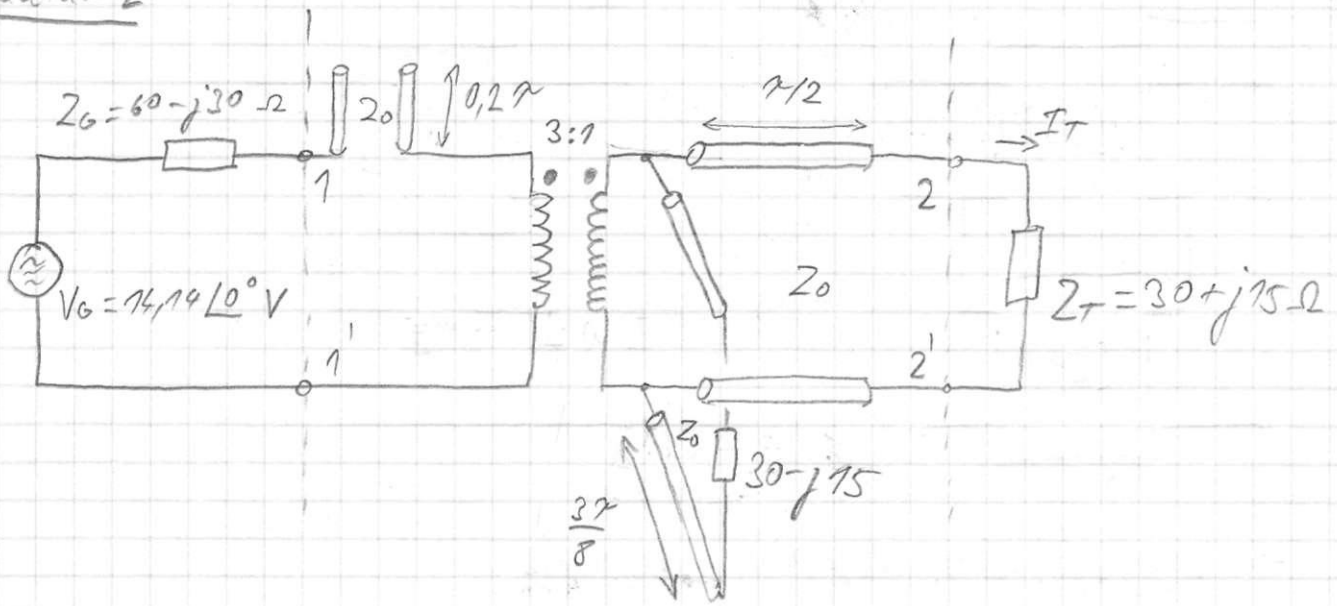
$Z_{22}^B = \left.\frac{V_2}{I_2}\right|_{I_1=0} \Rightarrow \boxed{Z_{22}^B = 3Z}$ $\quad Z_{21}^B = \left.\frac{V_2}{I_1}\right|_{I_2=0} \Rightarrow \boxed{Z_{21}^B = 2Z}$

$$[Z^B] = \begin{bmatrix} Z & 2Z \\ 2Z & 3Z \end{bmatrix}$$ => mreža je recipročna

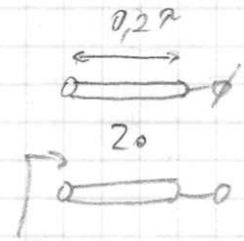
$$[Z] = \begin{bmatrix} Z & -Z \\ 2Z & Z \end{bmatrix} + \begin{bmatrix} Z & 2Z \\ 2Z & 3Z \end{bmatrix} = \begin{bmatrix} Z+Z & -Z+2Z \\ 2Z+2Z & Z+3Z \end{bmatrix}$$

$$[Z] = \begin{bmatrix} 2Z & Z \\ 4Z & 4Z \end{bmatrix} \Rightarrow$$ Z matrica nije recipročna pa ne postoji ekvivalentni T sklop

Zadatak 2



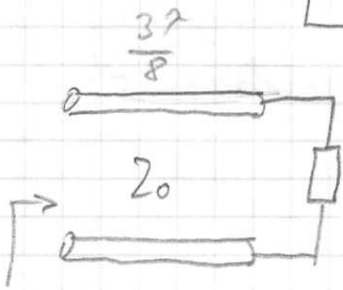
Potrebno je odrediti impedancije odsječaka otvorene linije duljine $0{,}2\lambda$, te linije duljine $3\lambda/8$.



$$Z_{ul_1} = -j\,\frac{Z_0}{\tan\beta l} \qquad \beta l = \frac{2\pi}{\lambda} \cdot 0{,}2\lambda = \frac{2\pi}{5}\,rad$$

$$\beta l = 72°$$

$$\boxed{Z_{ul_1} = -j\,16{,}246\ \Omega = 16{,}246 \angle -90°}$$



$$Z_{ul_2} = Z_0\,\frac{Z_T' + j Z_0 \tan\beta l}{Z_0 + j Z_T' \tan\beta l} \qquad \beta l = \frac{2\pi}{\lambda} \cdot \frac{3\lambda}{8} = \frac{3\pi}{4}$$

$$\beta l = 135°$$

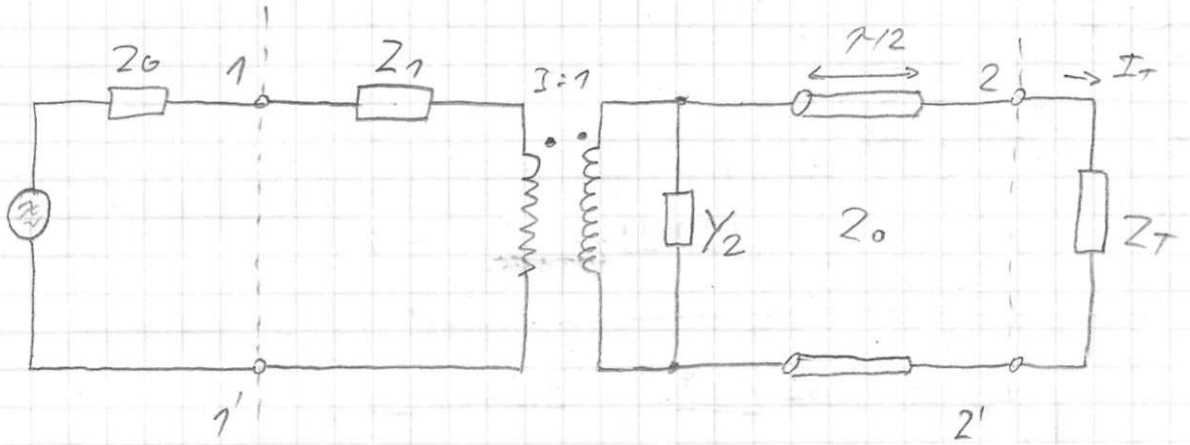
$$Zul_2 = 50 \cdot \frac{30 - j15 - j50}{50 - j(30 - j15)} = 50 \cdot \frac{30 - j65}{35 - j30}$$

$$Zul_2 = 50 \cdot \frac{\sqrt{30^2 + 65^2}\ \angle \arctg \frac{-65}{30}}{\sqrt{35^2 + 30^2}\ \angle \arctg \frac{-30}{35}}$$

▽ OPREZ: Pripazih na kvadrante u kojem se nalaze kutevi

$$Zul_2 = 50 \cdot \frac{73{,}82\ \angle -65{,}2^\circ}{46{,}1\ \angle -40{,}6}$$

$$\boxed{Zul_2 = 80{,}06\ \angle -24{,}6\ \Omega}$$



$$Z_1 = Zul_1 = 16{,}246\ \angle -90^\circ\ \Omega \qquad Y_2 = \frac{1}{Zul_2} = 0{,}0125\ \angle 24{,}6^\circ\ \Omega^{-1}$$

Imamo kaskadu 4 elemenata: impedancije Z_1, transformatora, admitancije Y_2 i odsječka linije duljine $\gamma l/2$. ABCD matrica dvopola mreže između prolaza 1 i 2 jednaka je umnošku ABCD matrica pojedinih elemenata u kaskadi.

$$ABCD(Z_1) = \begin{bmatrix} 1 & 16{,}246\ \angle -90^\circ \\ 0 & 1 \end{bmatrix} \qquad ABCD(trafo) = \begin{bmatrix} 3 & 1 \\ 0 & 1/3 \end{bmatrix}$$

$$ABCD(Y_2) = \begin{bmatrix} 1 & 0 \\ 0{,}0125\ \angle 24{,}6^\circ & 1 \end{bmatrix} \qquad ABCD(linija) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Pogledati u predavanjima točan izraz za ovu matricu

Ukupna ABCD matrica

$$\begin{bmatrix} 1 & 16,245\angle -90^\circ \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 0 & 1/3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0,0125\angle 24,6 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1+5,415\angle -90 \\ 0 & \frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 \\ 0,0125\angle 204,6 & -1 \end{bmatrix}$$

$1+5,145\angle -90 = 1-j5,145 = \sqrt{1^2+5,145^2}\angle \arctan\frac{-5,145}{1}$ (IV. kvadrant)

$= 5,241\angle -79^\circ$

$$\begin{bmatrix} 3 & 5,241\angle -79^\circ \\ 0 & \frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 \\ 0,0125\angle 204,6 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -3+0,065\angle 125,6 & 5,241\angle 101 \\ 0,0041\angle 204,6 & -\frac{1}{3} \end{bmatrix}$$

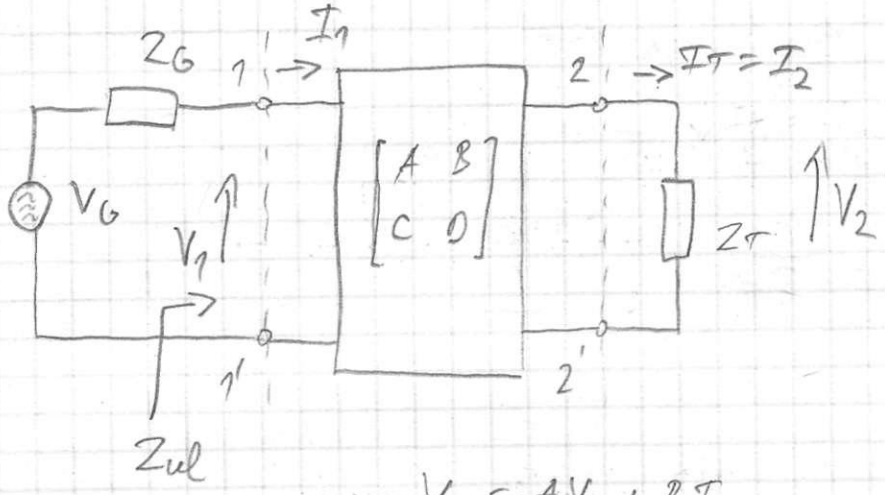
$-3+0,065(\cos 125,6 + j\sin 125,6) = -3-0,038+j0,053$

$= -3,38+j0,053$

$= 3,3787\angle 179,1^\circ$

Konačna ABCD matrica

$$\begin{bmatrix} 3,378\angle 179,1^\circ & 5,241\angle 101 \\ 0,0041\angle 204,6 & -\frac{1}{3} \end{bmatrix}$$



$$V_1 = AV_2 + BI_2$$

$$I_1 = CV_2 + DI_2$$

$$Z_{ul} = \frac{V_1}{I_1}$$

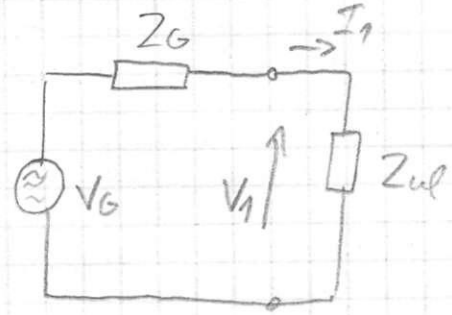
$$Z_{ul} = \frac{V_1}{I_1} = \frac{AV_2 + B\frac{V_2}{Z_T}}{CV_2 + D\frac{V_2}{Z_T}} \Rightarrow Z_{ul} = \frac{AZ_T + B}{CZ_T + D}$$

$$Z_{ul} = \frac{3{,}378 \angle 179{,}1^\circ \cdot 33{,}54 \angle 26{,}56^\circ + 5{,}241 \angle 70^\circ}{0{,}0041 \angle 204{,}6 \; 33{,}54 \angle 26{,}56^\circ - \frac{1}{3}}$$

$$Z_{ul} = \frac{113{,}29 \angle 205{,}66^\circ + 5{,}241 \angle 70^\circ}{0{,}137 \angle 231^\circ{,}16^\circ - \frac{1}{3}} = \frac{-103{,}11 - j\,43{,}91}{-0{,}42 - j\,0{,}106}$$

$$Z_{ul} = \frac{112{,}07 \angle 203^\circ}{0{,}433 \angle 194{,}16^\circ} \Rightarrow \boxed{Z_{ul} = 258{,}82 \angle 8{,}84^\circ}$$

Postavlja se nadomjesna mreža:



$$V_1 = V_G \cdot \frac{Z_{ul}}{Z_{ul} + Z_G} = AV_2 + BI_2 = V_2\left(A + \frac{B}{Z_T}\right)$$

$$\Rightarrow V_2 = V_G \frac{Z_{ul}}{Z_{ul} + Z_G} \cdot \frac{Z_T}{AZ_T + B}$$

$$I_2 = \frac{V_2}{Z_T} \qquad P_T = \frac{1}{2} V_2 I_2^* \qquad P_A = \frac{|V_G|^2}{8R_G}$$

snaga tereta — raspoloživa snaga

Skozja prestava mreži na sklopiščama 1-1':

$$P_{1-1'} = \frac{1}{2} V_1 \cdot I_1^*$$

$$I_1 = \frac{V_1}{Zul}$$