```
(* 2.3  LABOS: LinearSystems.nb *)

(* 1. Direktno rjeavanje sustava ax+by=e,
cx+dy=f uz uvjet da je determinanta sustava različita od nule *)
Detsustava = Det[{{a, b}, {c, d}}]
```

$$-b\,c+a\,d$$

```
(* ako je Detsustava=0 tada prekidaj postupak, a ako nije nastavi postupak *)

Solve[{a * x + b * y == e, c * x + d * y == f}, {x, y}]
```

$$\left\{\left\{x \to -\frac{-d\,e+b\,f}{-b\,c+a\,d},\ y \to -\frac{-c\,e+a\,f}{b\,c-a\,d}\right\}\right\}$$

```
(* 2. Rjeavanje sustava ax+by=e,
cx+dy=f u matricnom obliku uz uvjet da je determinanta sustava različita od nule *)
Detsustava = Det[{{a, b}, {c, d}}]
```

$$-b\,c+a\,d$$

```
(* ako je Detsustava=0 tada prekidaj postupak, a ako nije nastavi postupak *)
(* kreirajmo matricu sustava M *)
M = {{a, b}, {c, d}}
MatrixForm[M]
```

$$\{\{a, b\}, \{c, d\}\}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

```
(* kreirajmo slobodni stupac *)
Slobodni = {{e}, {f}}
MatrixForm[Slobodni]
```

$$\{\{e\}, \{f\}\}$$

$$\begin{pmatrix} e \\ f \end{pmatrix}$$

```
LinearSolve[M, Slobodni]
```

$$\left\{\left\{\frac{d\,e-b\,f}{-b\,c+a\,d}\right\}, \left\{\frac{c\,e-a\,f}{b\,c-a\,d}\right\}\right\}$$

```
(* ili na jos jedan
 način: rjeenje je produkt inverza od M  sa slobodnim stupcem {e,f} *)
Inverse[M].{e, f}
```

$$\left\{\frac{d\,e}{-b\,c+a\,d}-\frac{b\,f}{-b\,c+a\,d}, -\frac{c\,e}{-b\,c+a\,d}+\frac{a\,f}{-b\,c+a\,d}\right\}$$

```
(* 3. Rjeavanje sustava ax+by=e, cx+dy=
 f Gaussovim eliminacijama uz uvjet da je determinanta sustava različita od nule *)
Detsustava = Det[{{a, b}, {c, d}}]
```

$$-b\,c+a\,d$$

```
(* ako je Detsustava=0 tada prekidaj postupak, a ako nije nastavi postupak *)
(* kreirajmo proirenu matricu sustava MP *)
MP = {{a, b, e}, {c, d, f}}
MatrixForm[MP]
```

```
{{a, b, e}, {c, d, f}}
```

$$\begin{pmatrix} a & b & e \\ c & d & f \end{pmatrix}$$

```
(* izračunajmo reducirani oblik matrice MP *)
Reducirana = RowReduce[MP]
```

$$\left\{\left\{1, 0, \frac{d\,e - b\,f}{-b\,c + a\,d}\right\}, \left\{0, 1, \frac{c\,e - a\,f}{b\,c - a\,d}\right\}\right\}$$

```
MatrixForm[Reducirana]
```

$$\begin{pmatrix} 1 & 0 & \frac{d\,e-b\,f}{-b\,c+a\,d} \\ 0 & 1 & \frac{c\,e-a\,f}{b\,c-a\,d} \end{pmatrix}$$

```
(* rjeenje je zapisano u zadnjem stupcu ove matrice *)

(* 4. Rjeavanje sustava Gaussovim eliminacijama
 uz uvjet da je determinanta sustava jednaka nuli *)
(* rjesavamo sustav   -5y-15z+4u=7,
                      x-2y-4z +3u=6,
                      2x+  4z+ 2u=1,
                      3x+4y+18z+u=4.    *)

(* kreirajmo matricu sustava M *)
M = {{0, -5, -15, 4}, {1, -2, -4, 3}, {2, 0, 4, 2}, {3, 4, 18, 1}}
MatrixForm[M]
```

```
{{0, -5, -15, 4}, {1, -2, -4, 3}, {2, 0, 4, 2}, {3, 4, 18, 1}}
```

$$\begin{pmatrix} 0 & -5 & -15 & 4 \\ 1 & -2 & -4 & 3 \\ 2 & 0 & 4 & 2 \\ 3 & 4 & 18 & 1 \end{pmatrix}$$

```
(* kreirajmo matricu proirenog sustava MP dodavajući slobodni stupac matrici M;
to se najbre radi pomoću funkcije "AppendRows" koja
 se nalazi u paketu <<LinearAlgebra`MatrixManipulation` *)
(* poziv paketa <<LinearAlgebra`MatrixManipulation` *)
<< LinearAlgebra`MatrixManipulation`
(* kreirajmo matricu(vektor) slobodnog stupca *)
Slobstupac = {{7}, {6}, {1}, {4}}
MatrixForm[Slobstupac]
```

```
{{7}, {6}, {1}, {4}}
```

$$\begin{pmatrix} 7 \\ 6 \\ 1 \\ 4 \end{pmatrix}$$

```
(* dodavanje slobodnog stupca *)
MP = AppendRows[M, Slobstupac]
MatrixForm[MP]
Null
```

```
{{0, -5, -15, 4, 7}, {1, -2, -4, 3, 6}, {2, 0, 4, 2, 1}, {3, 4, 18, 1, 4}}
```

$$\begin{pmatrix} 0 & -5 & -15 & 4 & 7 \\ 1 & -2 & -4 & 3 & 6 \\ 2 & 0 & 4 & 2 & 1 \\ 3 & 4 & 18 & 1 & 4 \end{pmatrix}$$

```
(* Gaussove elementarne transformacije nad redcima matrice MP *)
Reducirana = RowReduce[MP]
MatrixForm[Reducirana]
```

$$\left\{\left\{1, 0, 2, 0, -\frac{25}{4}\right\}, \{0, 1, 3, 0, 4\}, \left\{0, 0, 0, 1, \frac{27}{4}\right\}, \{0, 0, 0, 0, 0\}\right\}$$

$$\begin{pmatrix} 1 & 0 & 2 & 0 & -\frac{25}{4} \\ 0 & 1 & 3 & 0 & 4 \\ 0 & 0 & 0 & 1 & \frac{27}{4} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
(* to znači da je varijable "z" suvina,
pa rjeavamo sustav po preostalim varijablama *)
Solve[{x + 2 z == -25/4, y + 3 z == 4, u == 27/4}, {x, y, u}]
```

$$\left\{\left\{x \to \frac{1}{4}(-25 - 8z),\ y \to 4 - 3z,\ u \to \frac{27}{4}\right\}\right\}$$

```
(* ovim je opisana jednoparametarska familija rjeenja *)

(* jedan drugi način za kreiranje matrice sustava M i matrice proirenog sustava MP,
direktnim pretvaranje sustava u matrični
 oblik pomoću funkcije LinearEquationsToMatrices koja se
 nalazi u paketu <<LinearAlgebra`MatrixManipulation` *)
(* poziv paketa <<LinearAlgebra`MatrixManipulation` *)
<< LinearAlgebra`MatrixManipulation`
(* pretvaranje linearnog sustava u listu *)
Pomocna = LinearEquationsToMatrices[{-5 y - 15 z + 4 u == 7,
   x - 2 y - 4 z + 3 u == 6, 2 x + 4 z + 2 y == 1, 3 x + 4 y + 18 z + u == 4 }, {x, y, z, u}]
Null
```

```
{{{0, -5, -15, 4}, {1, -2, -4, 3}, {2, 2, 4, 0}, {3, 4, 18, 1}}, {7, 6, 1, 4}}
```

```
(* ekstrakt matrice sustava iz Pomocna *)
M = Pomocna[[1]]
MatrixForm[M]
```

```
{{0, -5, -15, 4}, {1, -2, -4, 3}, {2, 2, 4, 0}, {3, 4, 18, 1}}
```

$$\begin{pmatrix} 0 & -5 & -15 & 4 \\ 1 & -2 & -4 & 3 \\ 2 & 2 & 4 & 0 \\ 3 & 4 & 18 & 1 \end{pmatrix}$$

```
(* ekstrakt slobodnog stupca iz Pomocna *)
Slobodni = Partition[Pomocna[[2]], 1]
MatrixForm[Slobodni]
```

```
{{7}, {6}, {1}, {4}}
```

$$\begin{pmatrix} 7 \\ 6 \\ 1 \\ 4 \end{pmatrix}$$

```
(* kreiranje proirene matrice sustava *)
MP = AppendRows[M, Slobodni]
MatrixForm[MP]
Null
```

```
{{0, -5, -15, 4, 7}, {1, -2, -4, 3, 6}, {2, 2, 4, 0, 1}, {3, 4, 18, 1, 4}}
```

$$\begin{pmatrix} 0 & -5 & -15 & 4 & 7 \\ 1 & -2 & -4 & 3 & 6 \\ 2 & 2 & 4 & 0 & 1 \\ 3 & 4 & 18 & 1 & 4 \end{pmatrix}$$