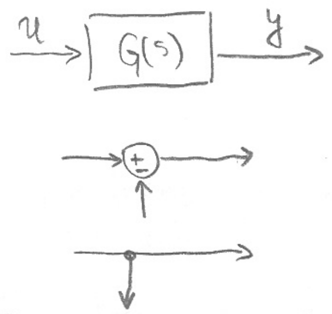
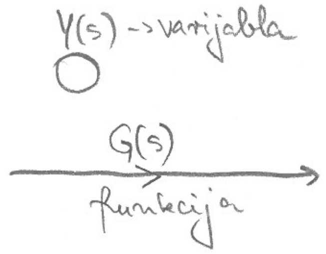
1. OPIS DINAMIČKIH SUSTAVA

GRAFIČKI PRIKAZ:

Blok sheme



Graf toka signala



Vezni dijagrami (Bond)

PRIJENOSNA FUNKCIJA je omjer izlazne i ulazne varijable u domjenu Laplaceovom području uz poč. uvj. jednake nula.

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}$$

$n > m$
realni,
kauzalni

STACIONARNO STANJE: teorem o konačnoj vrijednosti

$$Y(s) = G(s) \cdot U(s)$$

$$y_s = \lim_{t \to \infty} (y(t)) = \lim_{s \to 0} (s \cdot Y(s)) = \lim_{s \to 0} (s \cdot G(s) \cdot U(s))$$

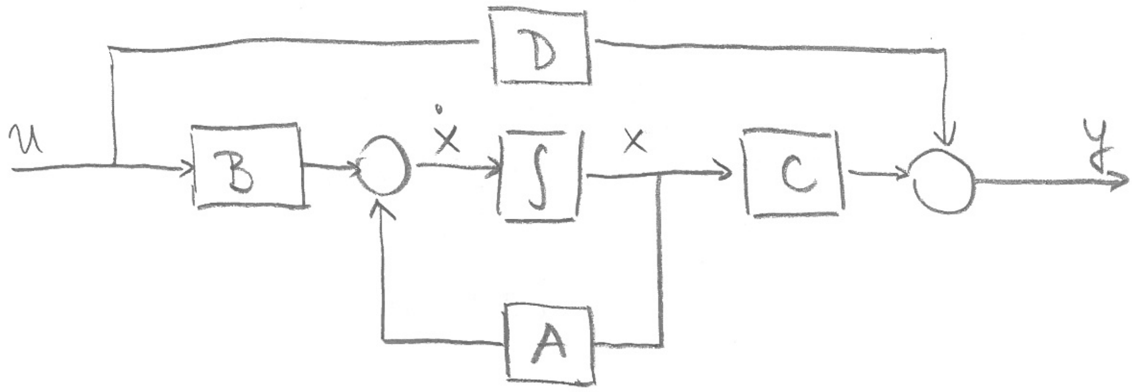
POČETNO STANJE: teorem o početnoj vrijednosti

$$y_s = \lim_{t \to 0} (y(t)) = \lim_{s \to \infty} (s \cdot Y(s)) = \lim_{s \to \infty} (s \cdot G(s) \cdot U(s))$$

ZAPIS U PROSTORU STANJA:

- iz blokovske sheme -> izlazi integratora su varijable stanja
- iz dif. jdb. i/ili prijenosne funkcije

$$\boxed{\begin{aligned} \dot{x} &= A \cdot x + B \cdot u \\ y &= C \cdot x + D \cdot u \end{aligned}}$$



$$G(s) = \left[C\left(sJ - A\right)^{-1} B + D \right]$$

Iterativni (kaskadni) oblik: $G(s) = \frac{Y(s)}{U(s)} = K \frac{s-m_1}{s-p_1} \cdot \frac{s-m_2}{s-p_2} \cdots \frac{s-m_m}{s-p_m}$
(A trokutasta)

Paralelni oblik: $G(s) = \frac{Y(s)}{U(s)} = \frac{k_1}{s-p_1} + \frac{k_2}{s-p_2} + \dots + \frac{k_m}{s-p_m}$
A dijagonalna

1. DZ

$$\frac{d^3 y(t)}{dt^3} + 5.5 \frac{d^2 y(t)}{dt^2} + 2.56 \frac{dy(t)}{dt} + 0.3 y(t) = \frac{du(t)}{dt} + 2u(t)$$

$$\left(s^3 + 5.5 s^2 + 2.56 s + 0.3\right) Y(s) = (s+2)\, U(s)$$

$$G(s) = \frac{s+2}{s^3 + 5.5 s^2 + 2.56 s + 0.3}$$

STAC. STANJE: $y_s = \lim_{t\to\infty} (y(t)) = \lim_{s\to 0} (s \cdot Y(s)) = \lim_{s\to 0} (s \cdot G(s) \cdot U(s))$

$$= \lim_{s\to 0} \left(\cancel{s} \cdot \frac{s+2}{s^3+5.5s^2+2.56s+0.3} \cdot \frac{1}{\cancel{s}} \right) = \frac{2}{0.3} = 6.6667$$

$$G(s) = G_1(s) \cdot G_2(s) = \frac{X(s)}{U(s)} \cdot \frac{Y(s)}{X(s)} = \frac{1}{s^3+5.5s^2+2.56s+0.3} \cdot \frac{s+2}{1}$$

$$G_1(s) = \frac{X(s)}{U(s)} = \frac{1}{s^3 + 5.5s^2 + 2.56s + 0.3}$$

$$\dddot{x} = u - 5.5\ddot{x} - 2.56\dot{x} - 0.3x$$

$$G_2(s) = \frac{Y(s)}{X(s)} = \frac{s+2}{1}$$

$$y = \dot{x} + 2x$$

1. KANONSKI UPRAVLJIV OBLIK $(B = [1\ 0\ 0\ \dots])$

$$x_1 = x \qquad \dot{x}_3 = \dddot{x} = u - 5.5x_3 - 2.56x_2 - 0.3x_1$$

$$x_2 = \dot{x} = \dot{x}_1$$

$$x_3 = \ddot{x} = \dot{x}_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.3 & -2.56 & -5.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot u$$

$$y = \begin{bmatrix} 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

2. KANONSKI OSMOTRIV OBLIK: $(C = [1\ 0\ 0\ \dots])$

$$A_{osm} = A_{upr}^T$$

$$B_{osm} = C_{upr}^T$$

$$C_{osm} = B_{upr}^T$$

3. KASKADNI OBLIK:

$$s^3 + 5.5s^2 + 2.56s + 0.3 = 0$$

$$p_1 = -5 \quad p_2 = -0.2 \quad p_3 = -0.3$$

$$s + 2 = 0$$

$$n_1 = -2$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{s+2}{(s+5)(s+0.2)(s+0.3)} = G_1(s) \cdot G_2(s) \cdot G_3(s)$$

$$= \frac{X_1(s)}{U(s)} \cdot \frac{X_2(s)}{X_1(s)} \cdot \frac{X_3(s)}{X_2(s)}$$

$$y = X_3(s)$$

$$G_1(s) = \frac{X_1(s)}{U(s)} = \frac{1}{s+5} \Rightarrow \dot{x}_1 = u - 5x_1$$

$$G_2(s) = \frac{X_2(s)}{X_1(s)} = \frac{s+2}{s+0.2} \Rightarrow \dot{x}_2 = \dot{x}_1 + 2x_1 - 0.2x_2$$

$$= u - 5x_1 + 2x_1 - 0.2x_2$$

$$= u - 3x_1 - 0.2x_2$$

$$G_3(s) = \frac{X_3(s)}{X_2(s)} = \frac{1}{s+0.3}$$

$$\dot{x}_3 = x_2 - 0.3x_3$$

$$y = x_3$$

A je trokutasta! ✓

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 0 \\ -3 & -0.2 & 0 \\ 0 & 1 & -0.3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \cdot u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

4. PARALELNI OBLIK:

$$\frac{s+2}{(s+5)(s+0.2)(s+0.3)} = \frac{A}{s+5} + \frac{B}{s+0.2} + \frac{C}{s+0.3} \quad / \cdot (s+5)(s+0.2)(s+0.3)$$

$$s+2 = A(s+0.2)(s+0.3) + B(s+5)(s+0.3) + C(s+5)(s+0.2)$$

$$s+2 = A(s^2+0.5s+0.06) + B(s^2+5.3s+1.5) + C(s^2+5.2s+1)$$

$$s+2 = (A+B+C)s^2 + (0.5A+5.3B+5.2C)s + 0.06A + 1.5B + C$$

$$A+B+C=0$$

$$0.5A+5.3B+5.2C=1$$

$$0.06A+1.5B+C=2$$

$$C = 2-0.06A-1.5B$$

$$0.5A+5.3B+5.2(2-0.06A-1.5B)=1$$

$$0.5A-0.312A+5.3B-7.8B+10.4=1$$

$$0.188A-2.5B=-9.4$$

...

$$A=-0.133 \quad B=3.75 \quad C=-3.617$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{-0.133X_1(s)+3.75X_2(s)-3.617X_3(s)}{U(s)}$$

$$\frac{X_1}{U} = G_1 \quad \frac{X_2}{U} = G_2 \quad \frac{X_3}{U} = G_3$$

$$\frac{X_1}{U} = \frac{1}{s+5} \quad \frac{X_2}{U} = \frac{1}{s+0.2} \quad \frac{X_3}{U} = \frac{1}{s+0.3}$$

$$\dot{x}_1 = u - 5x_1$$

$$\dot{x}_2 = u - 0.2x_2$$

$$\dot{x}_3 = u - 0.3x_3$$

$\tilde{A}$ je dijagonalna!

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 0 \\ 0 & -0.2 & 0 \\ 0 & 0 & -0.3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot u$$

$$y = \begin{bmatrix} -0.133 & 3.75 & -3.617 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

3. VEZNI DIJAGRAMI

- vidljivi tok energije

$$\xrightarrow[f]{e} R$$

$\boxed{R\text{-tip}}$	$\boxed{C\text{-tip}}$	$\boxed{J\text{-tip}}$
$\xrightarrow[f]{e} R$	$\xrightarrow[f=q']{e} C$	$\xrightarrow[f]{e=p'} J$
$e = R \cdot f$	$e = \frac{1}{C} \cdot q = \frac{1}{C} \cdot \int f$	$p = J \cdot f$
		$\int e = J \cdot f$

AKTIVNI IZVORI

Izvor napona	Izvor toka
$S_e \xrightarrow{e(t)}$	$S_f \xrightarrow[f(t)]{}$

SPOJEVI:

$\boxed{1\text{-spoj}}$	$\boxed{0\text{-spoj}}$
$\begin{array}{c} e_3 \uparrow f \\ \xrightarrow[f]{e_1} 1 \xrightarrow[f]{e_2} \end{array}$	$\begin{array}{c} e \uparrow f_3 \\ \xrightarrow[f_1]{e} 0 \xrightarrow[f_2]{e} \end{array}$
$\boxed{f = const.}$	$\boxed{e = const.}$
$\boxed{e_1 = e_2 + e_3}$	$\boxed{f_1 = f_2 + f_3}$

Transformator

$$e_1 f_1 = e_2 f_2$$

$$\xrightarrow[f_1]{e_1} \underset{b/a}{TF} \xrightarrow[f_2]{e_2}$$

$$f_2 = \frac{b}{a} f_1$$

$$e_2 = \frac{a}{b} e_1$$

Žirator

$$\xrightarrow[f_1]{e_1} \underset{r}{GY} \xrightarrow[f_2]{e_2}$$

$$e_2 = r f_1$$

$$e_1 = r f_2$$

Upravljivi izvor:

napora

$$\xrightarrow{e_0} S_e \xrightarrow{e}$$

$$e = f(e_0)$$

toka

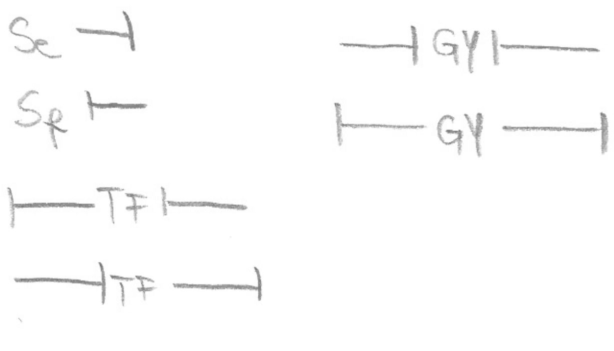
$$\xrightarrow[f_0]{} S_f \xrightarrow[f]{}$$

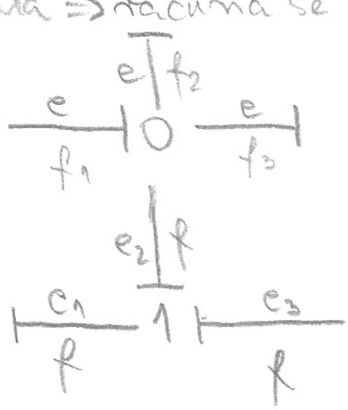
$$f = f(f_0)$$

Kauzalnost:

B djeluje na A naporom e
A odgovara tokom f na napor e

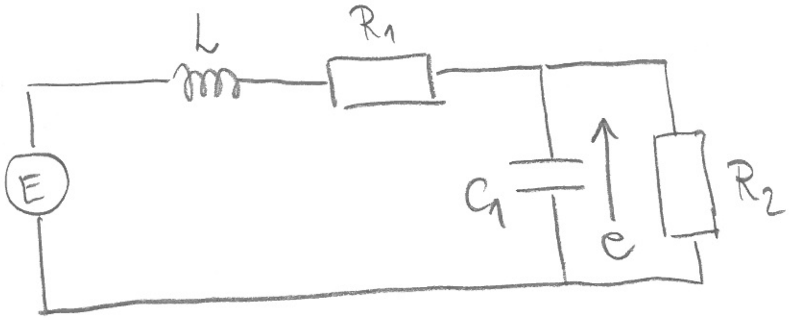
- crtica određuje ulazno-izlazne ovisnosti
  - → crtica uz element ⇒ računa se tok
  - → crtica od elementa ⇒ računa se napor

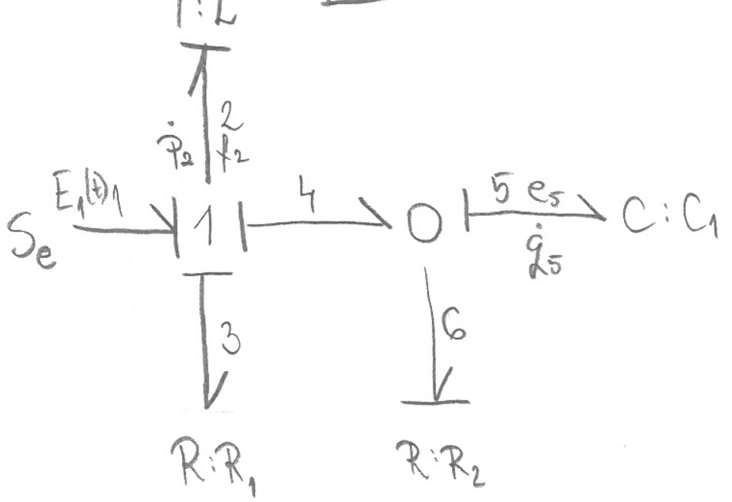




	Integral	Derivacija
I-elem.	$\overset{e=\dot{p}}{\underset{f}{\overline{\qquad\quad}}}\mid I$ $\quad f=\frac{1}{I}\int e$	$\mid\overset{e=\dot{p}}{\underset{f}{\overline{\qquad\quad}}}\ I$ $\quad e=I\cdot\dot{f}$
C-elem.	$\mid\overset{e}{\underset{f=\dot{q}}{\overline{\qquad\quad}}}\ C$ $\quad e=\frac{1}{C}\int f$	$\overset{e}{\underset{f=\dot{q}}{\overline{\qquad\quad}}}\mid C$ $\quad f=C\cdot\dot{e}$

Primjer 1:





Varijable stanja:

p_2, q_5

$$f_2 = \frac{p_2}{J_2} \qquad e_5 = \frac{q_5}{C_5}$$

$$\dot{q}_5 = f_4 - f_6 \qquad \dot{p}_2 = E_1(t) - e_3 - e_4 \qquad e_4 = e_5 = e_6$$

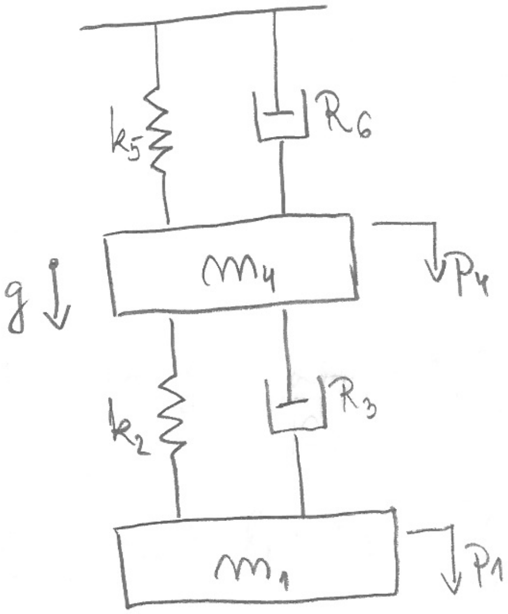
$$\dot{q}_5 = f_2 - \frac{e_6}{R_6} \qquad \dot{p}_2 = E_1(t) - R_3 \cdot f_2 - e_5 \qquad f_4 = f_2$$

$$\dot{p}_2 = E_1(t) - R_3 \cdot \frac{p_2}{J_2} - \frac{q_5}{C_5}$$

$$\dot{q}_5 = \frac{p_2}{J_2} - \frac{q_5}{C_5 \cdot R_6}$$

$$\begin{bmatrix} \dot{p}_2 \\ \dot{q}_5 \end{bmatrix} = \begin{bmatrix} -\frac{R_3}{J_2} & -\frac{1}{C_5} \\ \frac{1}{J_2} & -\frac{1}{C_5 R_6} \end{bmatrix} \begin{bmatrix} p_2 \\ q_5 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} E_1(t)$$

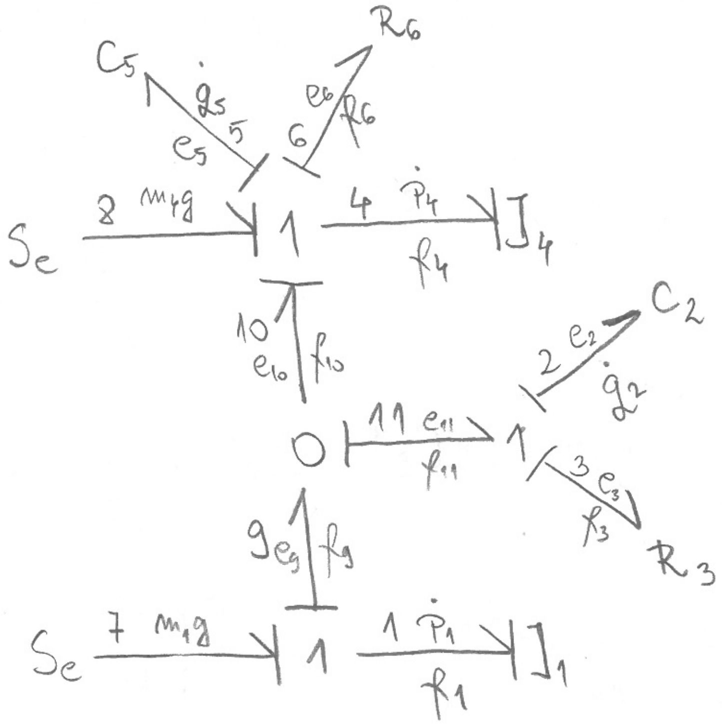
Primjer 2:



Izvori sile:

$e_7 = m_1 g$

$e_8 = m_4 g$



Varijable stanja

- integral sile na J elementu – p
- integral brzine na C elementu – q

$\Downarrow$

$$x = \begin{bmatrix} p_1 \\ q_2 \\ p_4 \\ q_5 \end{bmatrix}$$

Ulazne varijable:
- izvori sile

$$x = \begin{bmatrix} m_1 g \\ m_4 g \end{bmatrix}$$

1-spoj: $\dot{p}_1 = m_1 g - e_9$

$f_1 = f_9 = f_7 = \dfrac{p_1}{J_1}$

$\dot{q}_2 = f_3 = f_{11}$

$e_2 = e_{11} - e_3 = \dfrac{q_2}{C_2}$

$e_3 = R_3 f_3 = R_3 f_{11} = R_3 (f_1 - f_{10}) = R_3 \dfrac{p_1}{J_1} - R_3 \dot{q}_5$

$\dot{p}_4 = e_{10} + m_4 g - e_5 - e_6$

$f_4 = f_6 = f_{10} = f_8 = \dot{q}_5$

0-spoj: $e_9 = e_{10} = e_{11}$

$f_9 = f_1 = f_{11} + f_{10} = \dot{q}_2 + \dot{q}_5$

$$\dot{p}_1 = m_1 g - e_9 \qquad e_9 = e_{11} \qquad e_{11} = \frac{q_2}{C_2} + e_3$$

$$e_3 = R_3 \frac{p_1}{J_1} - R_3 \dot{q}_5$$

$$e_9 = \frac{q_2}{C_2} + R_3 \frac{p_1}{J_1} - R_3 \dot{q}_5$$

$$\dot{p}_1 = m_1 g - \frac{q_2}{C_2} - R_3 \frac{p_1}{J_1} + R_3 \dot{q}_5$$

$$\dot{q}_2 = f_{11} = f_1 - f_{10} = \frac{p_1}{J_1} - \dot{q}_5$$

$$\dot{p}_4 = e_{10} + m_4 g - e_5 - e_6$$

$$e_{10} = e_9 \qquad e_5 = \frac{q_5}{C_5} \qquad e_6 = R_6 f_6 = R_6 f_{10} = R_6 (f_1 - f_{11}) = R_6 \frac{p_1}{J_1} - R_6 \dot{q}_2$$

$$= R_6 \dot{q}_5$$

$$\dot{q}_5 = \frac{p_4}{J_4}$$

$$\dot{p}_4 = \frac{q_2}{C_2} + R_3 \frac{p_1}{J_1} - R_3 \frac{p_4}{J_4} + m_4 g - \frac{q_5}{C_5} - R_6 \frac{p_4}{J_4}$$

$$= \frac{q_2}{C_2} + R_3 \frac{p_1}{J_1} - (R_3 + R_6) \frac{p_4}{J_4} - \frac{q_5}{C_5} + m_4 g$$

$$\begin{bmatrix} \dot{p}_1 \\ \dot{q}_2 \\ \dot{p}_4 \\ \dot{q}_5 \end{bmatrix} = \begin{bmatrix} \frac{R_3}{J_1} & -\frac{1}{C_2} & +\frac{R_3}{J_4} & 0 \\ \frac{1}{J_1} & 0 & 0 & -\frac{1}{J_4} \\ \frac{R_3}{J_1} & \frac{1}{C_2} & -\frac{(R_3+R_6)}{J_4} & -\frac{1}{C_5} \\ 0 & 0 & \frac{1}{J_4} & 0 \end{bmatrix} \begin{bmatrix} p_1 \\ q_2 \\ p_4 \\ q_5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} m_1 g \\ m_4 g \end{bmatrix}$$

$$J_1 = m_1 \qquad C_2 = \frac{1}{k_2} \qquad J_4 = m_4 \qquad C_5 = \frac{1}{k_5}$$

4. OPIS DISKRETNIH SUSTAVA

$$T_d \leq \frac{T_{min}}{10} \qquad T_{AD} + T_D + T_{DA} < T_d$$

ZOH: $$G_E = \mathcal{L}\left\{S(t) - S(t - T_d)\right\} = \frac{1}{s} - \frac{1}{s}e^{-T_d s} = \frac{1}{s}\left(1 - e^{-T_d s}\right)$$

$$Z\left(1 - e^{-T_d s}\right) = 1 - z^{-1} \qquad \Rightarrow \text{D/A pretvarač}$$

$$G_{kd} = \text{c2d}\left(G_{kk}, T_d, \text{'metoda'}\right)$$

FOH: (dva uzorka se spajaju pravcem)

$$u(t) = u(k) + \frac{t - kT_d}{T_d}\left[u(k+1) - u(k)\right] \qquad kT_d \leq t \leq (k+1)T_d$$

TUSTIN:

$$s = \frac{2}{T_d}\frac{z-1}{z+1}$$

poklapanje nula i polova: $z = e^{T_d s}$

REKURZIVNA FORMULA je način realizacije digitalnih algoritama u digitalnom računalu.

$$y_p(k) = -a_{m-1}y_p(k-1) - \ldots - a_0 y_p(k-n) + b_m u_r(k) + \ldots + b_0 u_r(k-m)$$

DIREKTNA, ITERATIVNA (KASKADNA) I PARALELNA realizacija.

Primjer:

$$G(s) = \frac{s+2}{s^2+4s+3}$$

ZOH $(Td = 0.1s)$

$$G(z) = \frac{z-1}{z} \cdot Z\left\{\frac{1}{s}G(s)\right\}$$

$$\frac{1}{s}G(s) = \frac{1}{s} \cdot \frac{s+2}{s^2+4s+3} = \frac{s+2}{s(s+3)(s+1)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+1}$$

$$s+2 = A(s^2+4s+3) + B(s+1)s + Cs(s+3)$$

$$s+2 = As^2 + A4s + 3A + Bs^2 + Bs + Cs^2 + 3Cs$$

$$s+2 = s^2(A+B+C) + s(4A+B+3C) + 3A$$

$$3A = 2 \qquad A = \frac{2}{3}$$

$$4A + B + 3C = 1$$

$$4 \cdot \frac{2}{3} + B + 3C = 1$$

$$B = 1 - 3C - \frac{8}{3}$$

$$B = -\frac{5}{3} - 3C$$

$$A + B + C = 0$$

$$\frac{2}{3} - \frac{5}{3} - 3C + C = 0$$

$$-1 - 3C + C = 0$$

$$-2C = 1$$

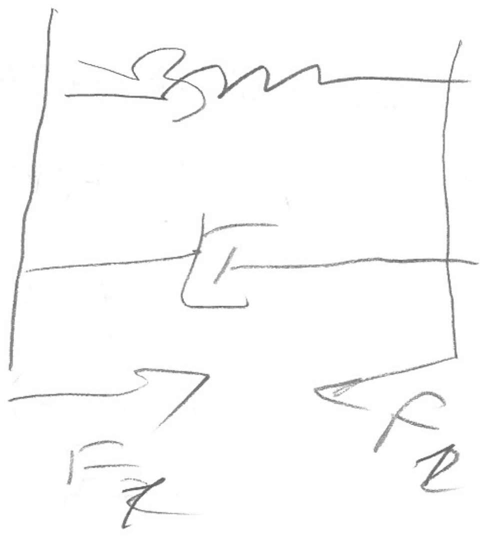
$$C = -\frac{1}{2} \qquad B = -\frac{5}{3} + \frac{3}{2} = -\frac{1}{6}$$

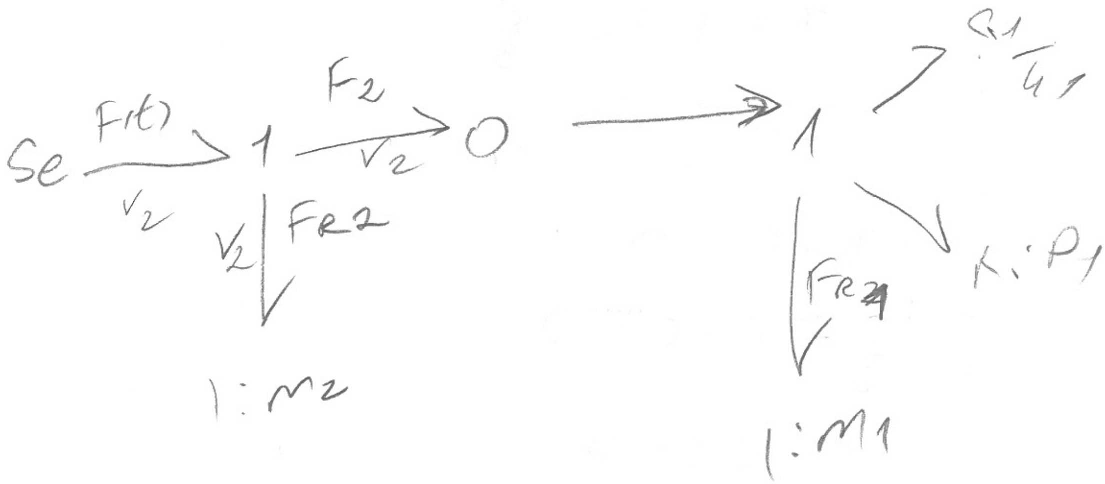
$$\frac{1}{s}G(s) = \frac{2}{3}\cdot\frac{1}{s} - \frac{1}{6}\frac{1}{s+3} - \frac{1}{2}\cdot\frac{1}{s+1}$$

$$\frac{1}{s} = \frac{z}{z-1}$$

$$\frac{1}{s+a} = \frac{z}{z-e^{-aTd}}$$

$$G(z) = \frac{z-1}{z}\left[\frac{2}{3}\cdot\frac{z}{z-1} - \frac{1}{6}\frac{z}{z-e^{-3Td}} - \frac{1}{2}\frac{z}{z-e^{-Td}}\right]$$

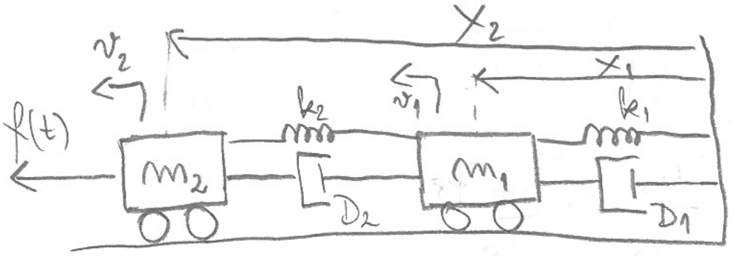




1-spoj

4. MAT. MODELI REALNIH SUSTAVA

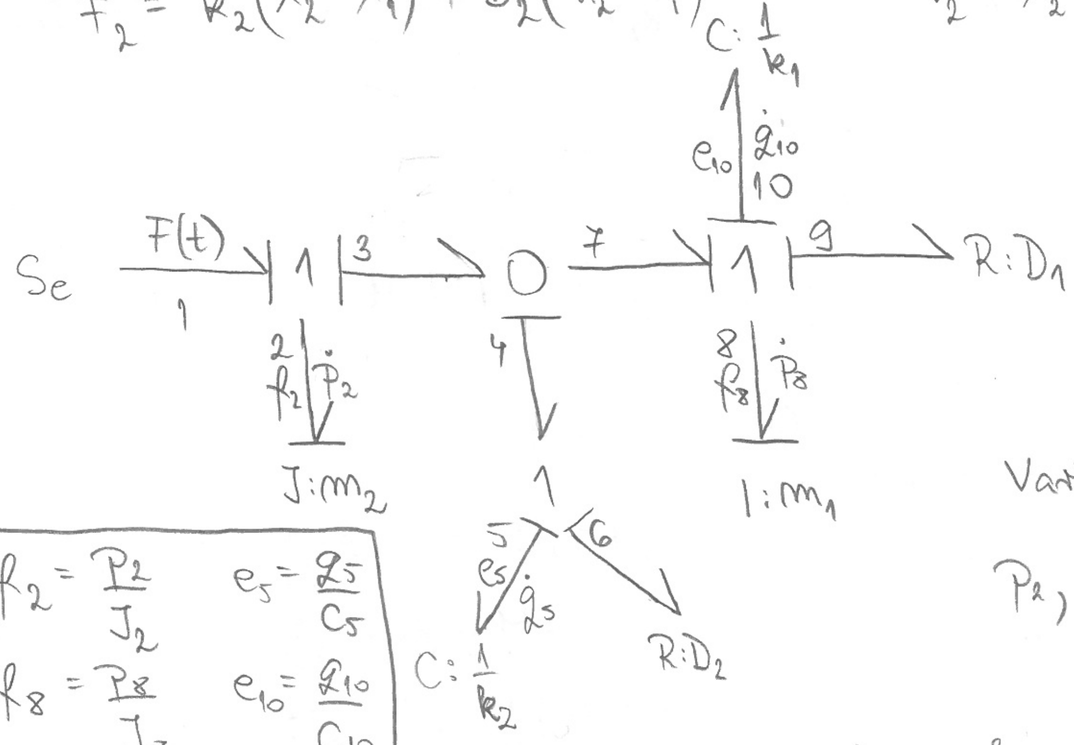
- mehanički sustav s linearnim gibanjem



$m_1 \cdot a_1 = F_2 - F_1$ $\qquad$ $m_2 v_2 = F(t) - F_2$

$F_1 = k_1 \cdot x_1 + D_1 \cdot v_1$ $\qquad$ $v_1 = \dot{x}_1$

$F_2 = k_2(x_2 - x_1) + D_2(v_2 - v_1)$ $\qquad$ $v_2 = \dot{x}_2$



Varijable stanja:

p_2, q_5, p_8, q_{10}

$$\boxed{\begin{array}{ll} f_2 = \frac{p_2}{J_2} & e_5 = \frac{q_5}{C_5} \\ f_8 = \frac{p_8}{J_8} & e_{10} = \frac{q_{10}}{C_{10}} \end{array}}$$

1-spoj $\dot{p}_2 = e_2 = e_1 - e_3$

$f_1 = f_2 = f_3 = \frac{p_2}{J_2}$

0-spoj $e_3 = e_4 = e_7$

$f_3 = f_4 + f_7$

1-spoj: $e_4 = e_5 + e_6$

$e_3 = \frac{q_5}{C_5} + R_6 f_6 = \frac{q_5}{C_5} + R_6 \frac{p_2}{J_2} - R_6 \frac{p_8}{J_8}$

$f_6 = f_4 = \dot{q}_5 = f_4 = f_3 - f_7 = \frac{p_2}{J_2} - \frac{p_8}{J_8}$

1-spoj: $f_7 = f_8 = f_9 = \dot{q}_{10}$

$e_7 = \dot{p}_8 + e_{10} + e_9$

$\dot{p}_8 = e_7 - e_{10} - e_9$

$\dot{p}_8 = e_3 - \frac{q_{10}}{C_{10}} - R_9 f_9$ $\quad f_9 = \frac{p_8}{J_8}$

$$\dot{p}_2 = F(t) - \left(\frac{q_5}{C_5} + \frac{R_6}{J_2} p_2 - \frac{R_6}{J_8} p_8 \right)$$

$$\dot{p}_2 = F(t) - \frac{R_6}{J_2} p_2 - \frac{1}{C_5} q_5 + \frac{R_6}{J_8} p_8$$

$$\dot{q}_5 = \frac{1}{J_2} p_2 - \frac{1}{J_8} p_8$$

$$\dot{p}_8 = \frac{q_5}{C_5} + \frac{R_6}{J_2} p_2 - \frac{R_6}{J_8} p_8 - \frac{q_{10}}{C_{10}} - \frac{R_9}{J_8} p_8$$

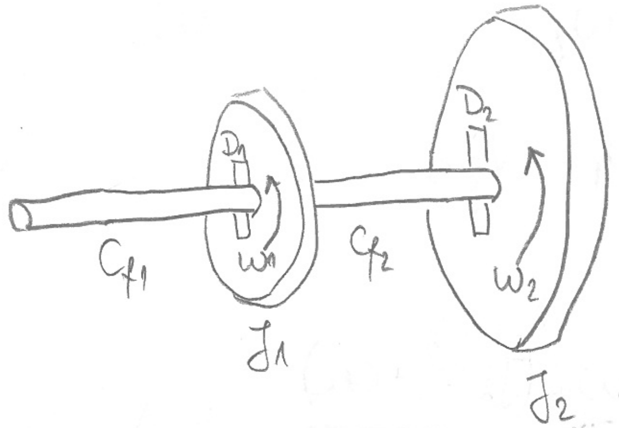
$$\dot{p}_8 = \frac{R_6}{J_2} p_2 + \frac{1}{C_5} q_5 - \frac{R_6 + R_9}{J_8} p_8 - \frac{q_{10}}{C_{10}}$$

$$\dot{q}_{10} = \frac{p_8}{J_8}$$

$J_2 = m_2 \quad J_8 = m_1 \quad C_5 = \frac{1}{k_2} \quad R_6 = D_2 \quad R_9 = D_1 \quad C_{10} = \frac{1}{k_1}$ P

...

6. • SUSTAV S ROTACIJSKIM GIBANJEM



Torzija = opruga

prigušenje:

$$M = D \cdot \omega = D \cdot \dot{\varphi}$$

torzija:

$$M = C_{fi} \cdot \varphi$$

$$\varphi = \int (\omega_1 - \omega_2) dt$$

ravnoteža:

$$\frac{d}{dt}(J\omega) = \sum M$$

primjer (bez D-a):

ravnoteže:

$$J_1 \cdot \dot{\omega}_1 = M_1 - M_2$$

$$J_2 \, \dot{\omega}_2 = M_2$$

$$M_1 = C_{f1} \cdot \int (\omega_u - \omega_1) dt$$

$$M_2 = C_{f2} \cdot \int (\omega_1 - \omega_2) dt$$

$$\Downarrow s \qquad G(s) = \frac{\omega_2(s)}{\omega_u(s)}$$

$$J_1 \cdot s\omega_1 = M_1 - M_2$$

$$J_2 \cdot s\omega_2 = M_2$$

$$M_1 = C_{f1} \cdot \frac{1}{s}(\omega_u - \omega_1)$$

$$M_2 = C_{f2} \cdot \frac{1}{s}(\omega_1 - \omega_2)$$

$$J_1 \cdot s\omega_1 = \frac{1}{s}\left(C_{f1}\omega_u - C_{f1}\omega_1 - C_{f2}\omega_1 + C_{f2}\omega_2\right)$$

$$J_1 \cdot s^2 \omega_1 = C_{f1}\omega_u + C_{f2}\omega_2 - (C_{f1} + C_{f2})\omega_1$$

$$\omega_1 (J_1 s^2 + C_{f1} + C_{f2}) = C_{f1}\omega_u + C_{f2}\omega_2$$

$$J_2 s \omega_2 = C_{f2} \cdot \frac{1}{s} (\omega_1 - \omega_2)$$

$$J_2 s^2 \omega_2 = C_{f2} \omega_1 - C_{f2} \omega_2$$

$$\omega_2 (J_2 s^2 + C_{f2}) = C_{f2} \omega_1$$

$$\omega_1 = \omega_1$$

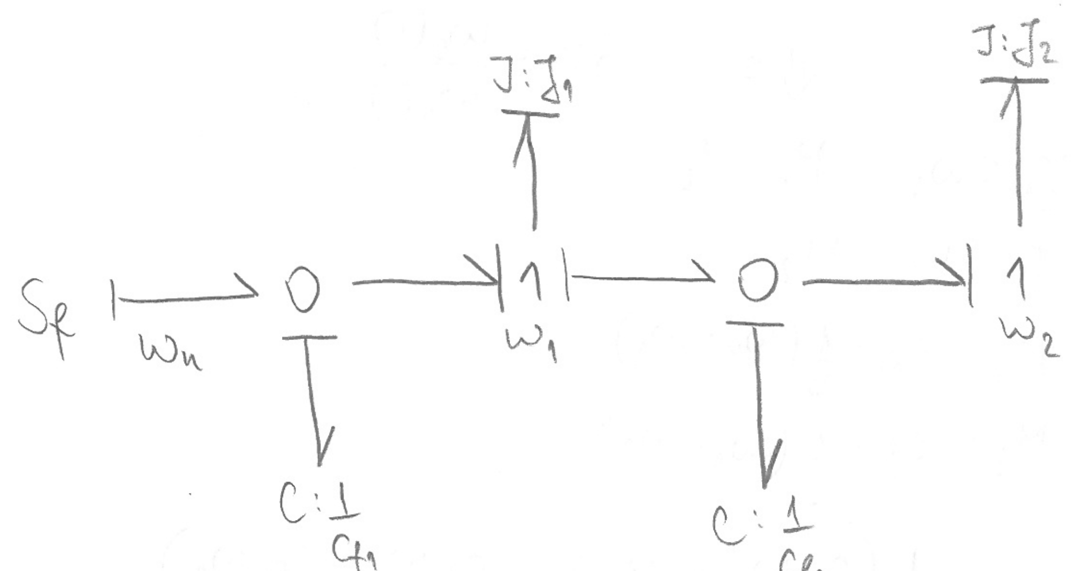
$$\frac{C_{f1} \omega_u + C_{f2} \omega_2}{J_1 s^2 + C_{f1} + C_{f2}} = \frac{\omega_2 \cdot (J_2 s^2 + C_{f2})}{C_{f2}}$$

$$C_{f1} C_{f2} \omega_u + C_{f2}^2 \omega_2 = \omega_2 \cdot (J_2 \cdot s^2 + C_{f2})(J_1 s^2 + C_{f1} + C_{f2})$$

$$C_{f1} C_{f2} \omega_u + C_{f2}^2 \omega_2 = \omega_2 \left(J_1 J_2 s^4 + (C_{f1} + C_{f2}) J_2 s^2 + C_{f2} J_1 s^2 + C_{f2} C_{f1} + C_{f2}^2 \right)$$

$$C_{f1} C_{f2} \omega_u = \omega_2 \left(J_1 J_2 s^4 + (C_{f2} J_1 + C_{f1} J_2 + C_{f2} J_2) s^2 + C_{f1} C_{f2} \right)$$

$$\frac{\omega_2}{\omega_u} = \frac{1}{\frac{J_1 J_2}{C_{f1} C_{f2}} s^4 + \left(\frac{J_1}{C_{f1}} + \frac{J_2}{C_{f2}} + \frac{J_2}{C_{f1}} \right) s^2 + 1}$$



$$T = \frac{2\pi}{\omega}$$

$$\boxed{s_{p1,2} = -\sigma \pm j\omega_p = -\xi \omega_n \pm j \omega_n \sqrt{1 - \xi^2}}$$

prigušene

neprigušene

6. MEHANIKA FLUIDA

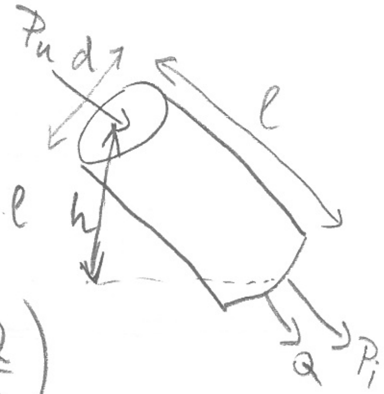
$$\frac{dV}{dt} = \sum_i Q_i \qquad V(h) = \int_0^h A(h)\,dh$$

Bernoulli: $p_u + \rho g h - \frac{\rho v^2}{2} - p_i = 0$

$\rho g h$ ↑ hidrostatski, $\frac{\rho v^2}{2}$ ↑ dinamički

$$Q = A \cdot v = A \cdot \sqrt{\frac{2}{\rho}\left(p_u - p_i + \rho g h\right)}$$

Protoci bez zanemarivanja inercije fluida

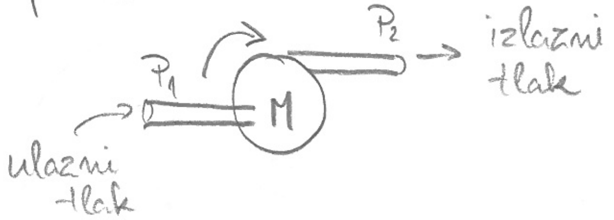


$$A \cdot \sum p_m = \frac{d}{dt}(m v) \qquad v = \frac{Q}{A} \quad V = A l$$

$$A \cdot \left[p_u - p_i + \rho g h - \frac{\rho}{2}\left(\frac{Q}{A}\right)^2\right] = \rho V \frac{d}{dt}\left(\frac{Q}{A}\right)$$

$$\dot{Q} = \frac{A}{\rho \cdot l}\left[p_u - p_i + \rho g h - \frac{\rho}{2}\left(\frac{Q}{A}\right)^2\right] \qquad A = r^2 \pi$$

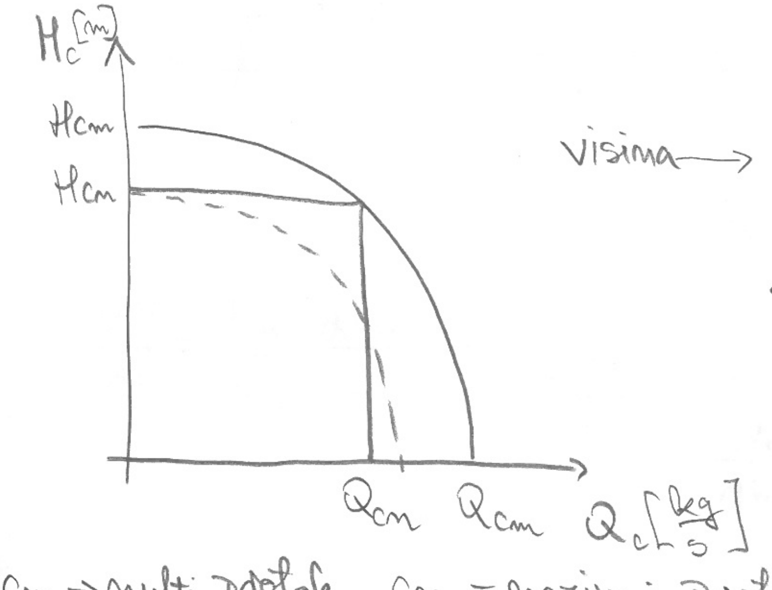
Crpke



$$\Delta p = H = p_2 - p_1$$

↑ energija koju crpka predaje tekućini

$$p_2 > p_1$$



visina ⟶ $H_c = H_{cm} - \left(\frac{Q_c}{Q_{cm}}\right)^2 (H_{cm} - H_{cm})$

$$R_c = \frac{\partial(\Delta p)}{\partial Q} = k \frac{\partial H}{\partial Q}$$

$$Q = k_c \omega$$

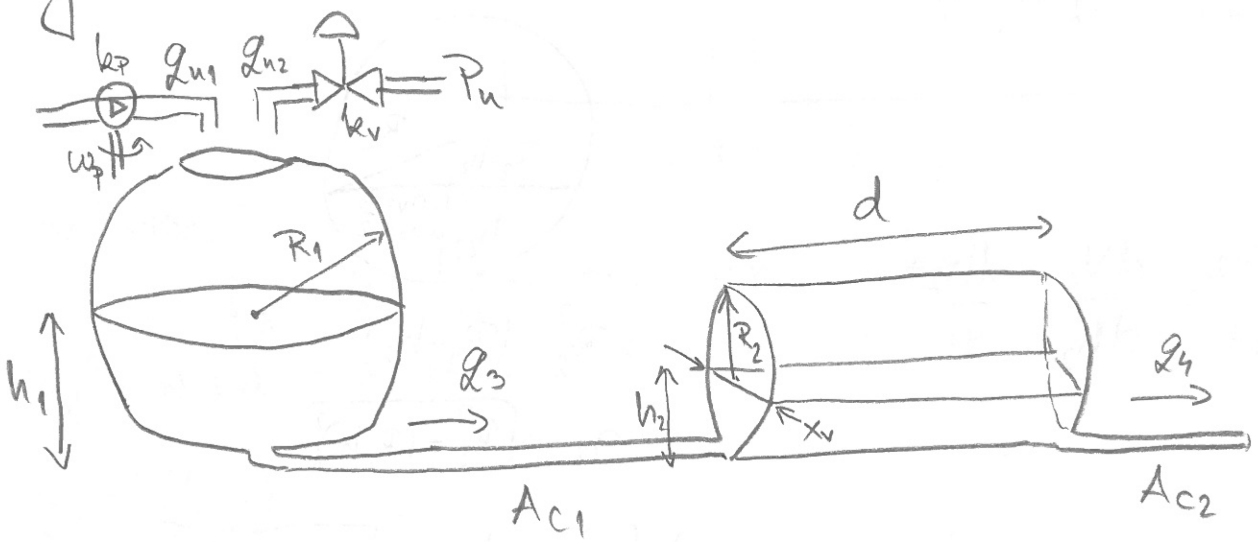
cm → multi protok cn – nazivni protok

Ventili: -> izazivaju pad tlaka

$$Q = k_v \cdot X \cdot \sqrt{\Delta P} \quad \text{-> linearna ovisnost o protoku}$$

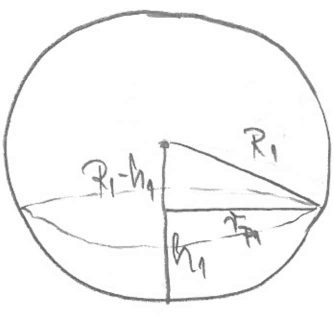
↑ otvor

Primjer:



$$\frac{dV_1}{dt} = q_{u1} + q_{u2} - q_3$$

$$q_{u1} = K_p \cdot \omega \qquad q_{u2} = K_v \cdot X \cdot \sqrt{P_u} \qquad q_3 = A_{c1} \cdot \sqrt{2g(h_1 - h_2)}$$



$$\boxed{V = \int_0^{h_1} A(h_1)\, dh_1 \Rightarrow \frac{dV}{dh_1} = A(h_1)}$$

$$A(h_1) = r_{p1}^2 \pi$$

$$R_1^2 = (R_1 - h_1)^2 + r_{p1}^2 \Rightarrow r_{p1}^2 = \cancel{R_1^2} - \cancel{R_1^2} + 2R_1 h_1 - h_1^2$$

$$r_{p1}^2 = h_1 (2R_1 - h_1)$$

$$A(h_1) = h_1 (2R_1 - h_1) \cdot \pi$$

$$\frac{dV_1}{dt} = \frac{dV_1}{dh_1} \cdot \frac{dh_1}{dt} \Rightarrow \frac{dh_1}{dt} = \frac{\frac{dV_1}{dt}}{\frac{dV_1}{dh_1}} = \frac{q_{u1} + q_{u2} - q_3}{\pi h_1 (2R_1 - h_1)}$$

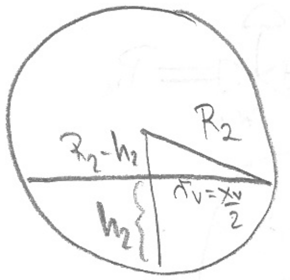
$$\frac{dV_2}{dt} = q_3 - q_4$$

$$V_2 = \int_0^{h_2} A_2(h_2)\,dh_2$$

$$q_3 = A_{c1}\sqrt{2g(h_1 - h_2)}$$

$$A_2 = d \cdot x_v(h_2)$$

$$q_4 = A_{c2}\sqrt{2gh_2}$$



$$\frac{dV_2}{dt} = \frac{dV_2}{dh_2} \cdot \frac{dh_2}{dt}$$

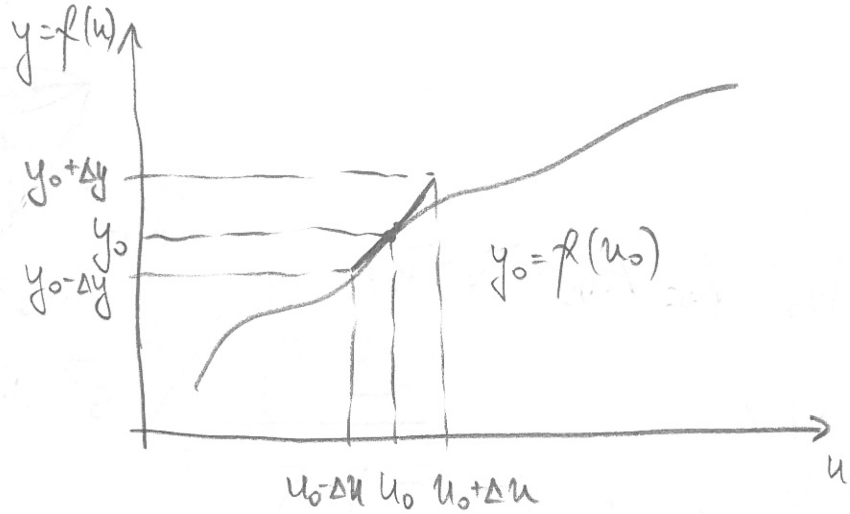
$$R_2^2 = (R_2 - h_2)^2 + r_v^2$$

$$r_v = \sqrt{R_2^2 - (R_2 - h_2)^2}$$

$$A_2(h_2) = d \cdot 2 \cdot \sqrt{R_2^2 - (R_2 - h_2)^2}$$

$$\frac{dh_2}{dt} = \frac{\frac{dV_2}{dt}}{\frac{dV_2}{dh_2}} = \frac{q_3 - q_4}{2d\sqrt{R_2^2 - (R_2 - h_2)^2}}$$

LINEARIZACIJA



- perturbacije -> male promjene oko radne točke
- Taylorov red oko radne točke

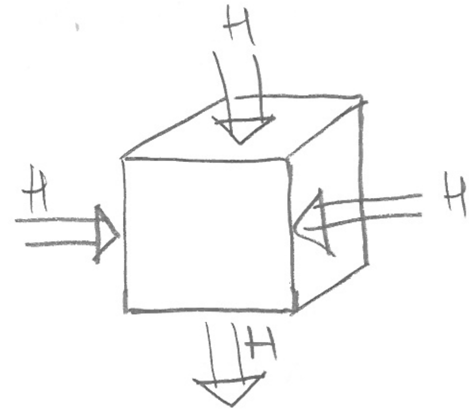
$$y_0 = f(u_0)$$

$$\Delta y = y - y_0$$

$$\Delta u = u - u_0$$

$$\Delta y = \left.\frac{\partial f(u)}{\partial u}\right|_{u=u_0} \Delta u$$

TOPLINSKI PROCESI



$$\frac{dE}{dt} = \sum_i H$$

$$E = m \cdot C \cdot T$$

FLUIDI:

$$\frac{dE}{dt} = \frac{d}{dt}\left(\rho \cdot V \cdot cT\right) = \rho c V \frac{dT}{dt} + \rho c T \frac{dV}{dt}$$

$$= \rho c V \dot{T} + c T \dot{m}$$

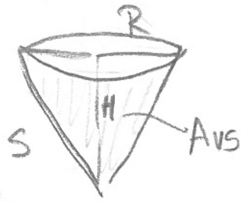
Vođenje:

$$H = \alpha \cdot A \cdot \Delta T$$

$[W]$ $\downarrow$ $\frac{W}{m^2 K}$

Prijenos topline strujanjem:

$$H = \rho \cdot C \cdot T \cdot Q, \quad Q = \frac{dV}{dt}$$



$$H_{vs} = \alpha_{vs} A_{vs} (T_v - T_s)$$

$$A_{vs} = R \pi s$$

$$s = \sqrt{R^2 + H^2} \qquad R = \frac{R_u}{H_u} \cdot H$$

$$H_{vz} = \alpha_{vz} \cdot A_{vz} (T_v - T_z)$$

$$A_{vz} = R^2 \pi$$

$$H_{sz} = \alpha_{sz} \cdot A_{sz} (T_s - T_z)$$

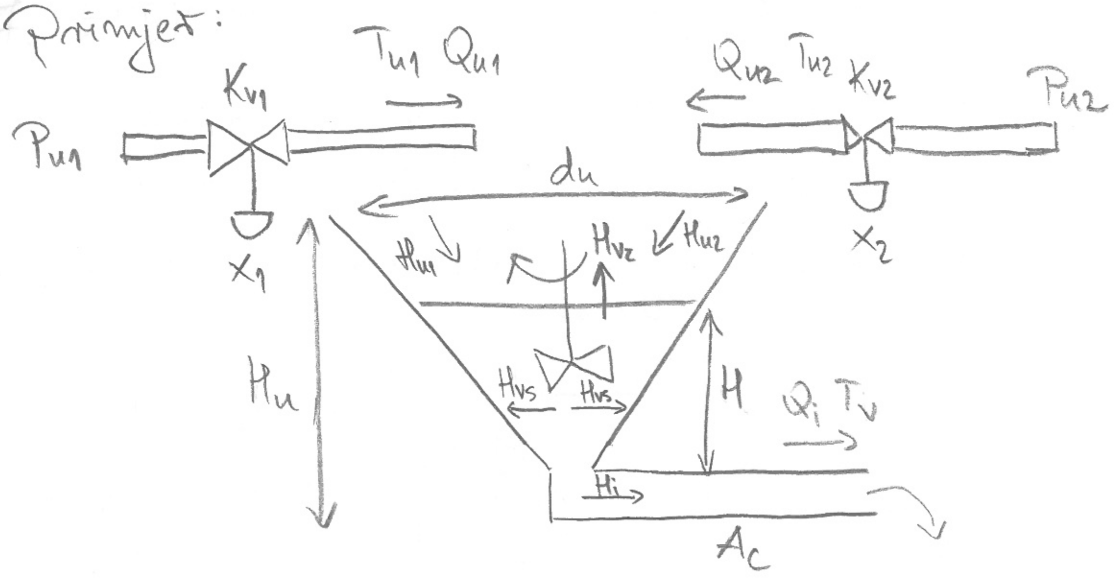
$$A_{sz} = A_{sz1} + A_{sz2}$$

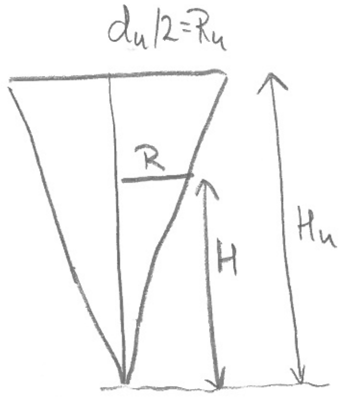
$$A_{sz1} = R_u \pi s_u$$

$$A_{sz2} = A_{sz1} - A_{vs}$$

$$A_{sz} = 2 A_{sz1} - A_{vs}$$

Primjer:





$$\frac{R}{H} = \frac{R_u}{H_u} = \frac{d_u}{2 \cdot H_u}$$

$$R = \frac{H}{H_u} \cdot \frac{d_u}{2}$$

$$Q_{u1} = x_1 \cdot K_{v1} \cdot \sqrt{P_{u1}}$$

$$Q_{u2} = x_2 \cdot K_{v2} \cdot \sqrt{P_{u2}}$$

$$Q_i = A_c \cdot \sqrt{2gH}$$

$$\frac{\partial V}{\partial t} = Q_{u1} + Q_{u2} - Q_i$$

$$V = \int_0^H A(H)\, dH = \int_0^H R^2 \pi\, d(H)$$

$$V = \int_0^H \left(\frac{d_u}{2H_u}\right)^2 \pi \cdot H^2\, dH = \frac{d_u^2 \pi}{4H_u^2} \int_0^H H^2\, dH = \frac{d_u^2 \pi}{4H_u^2} \frac{H^3}{3}$$

$$\frac{dV}{dt} = \frac{dV}{dH} \cdot \frac{dH}{dt} = \frac{d_u^2}{4H_u^2} \cdot \pi \cdot H^2 \frac{dH}{dt}$$

STIJENKA

$$E_s = m_s \cdot C_s \cdot T_s$$

$$\frac{dE_s}{dt} = m_s C_s \frac{dT_s}{dt}$$

$$\frac{dE_s}{dt} = H_{vs} - H_{sz}$$

VODA

$$E_v = m_v \cdot C_v \cdot T_v = V \cdot \rho \cdot c_v \cdot T_v$$

$$\frac{dE_v}{dt} = \rho c_v T_v \frac{dV}{dt} + \rho c_v \cdot V \frac{dT_v}{dt}$$

$$\frac{dE_v}{dt} = H_{u1} + H_{u2} - H_i - H_{vs} - H_{vz}$$

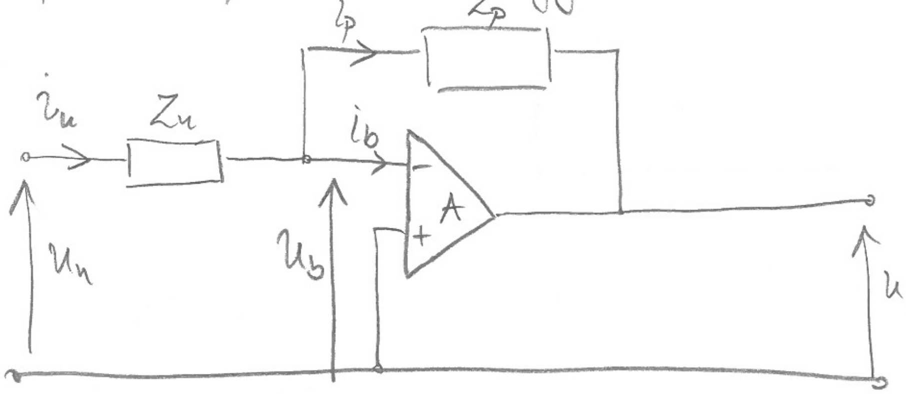
$$H_{u1} = \rho_v \cdot c_v \cdot T_{u1} \cdot Q_{u1}$$

$$H_{u2} = \rho_v \cdot c_v \cdot T_{u2} \cdot Q_{u2}$$

$$H_i = \rho_v \cdot c_v T_v \cdot Q_i$$

7. ELEKTRIČNI I ELEKTROMEHANIČKI SUSTAVI

- elektronički sustavi → pojačala, tiristori ...
- sustavi prijenosa el. energije
- procesi pretvorbe energije → generatori, motori ...



$$u_b = -\frac{1}{A} u_i$$

$$i_u = i_p + i_b$$

$$\frac{u_u - u_b}{Z_u} = i_b + \frac{u_b - u_i}{Z_p}$$

$$u_u - u_b = i_b \cdot Z_u + \frac{Z_u}{Z_p} u_b - \frac{Z_u}{Z_p} u_i$$

$$\frac{1}{A} u_i + \frac{Z_u}{Z_p} \cdot \frac{1}{A} u_i + \frac{Z_u}{Z_p} u_i = i_b Z_u - u_u$$

$$-u_i \left(\frac{1}{A} + \frac{Z_u}{Z_p A} + \frac{Z_u}{Z_p} \right) = i_b Z_u - u_u$$

$$u_i = \frac{\overset{=0}{i_b} Z_u - u_u}{\frac{Z_u}{Z_p} + \underbrace{\frac{1}{A}\left(1 + \frac{Z_u}{Z_p}\right)}_{\approx 0}}$$

$A >>$ Rul $>>$

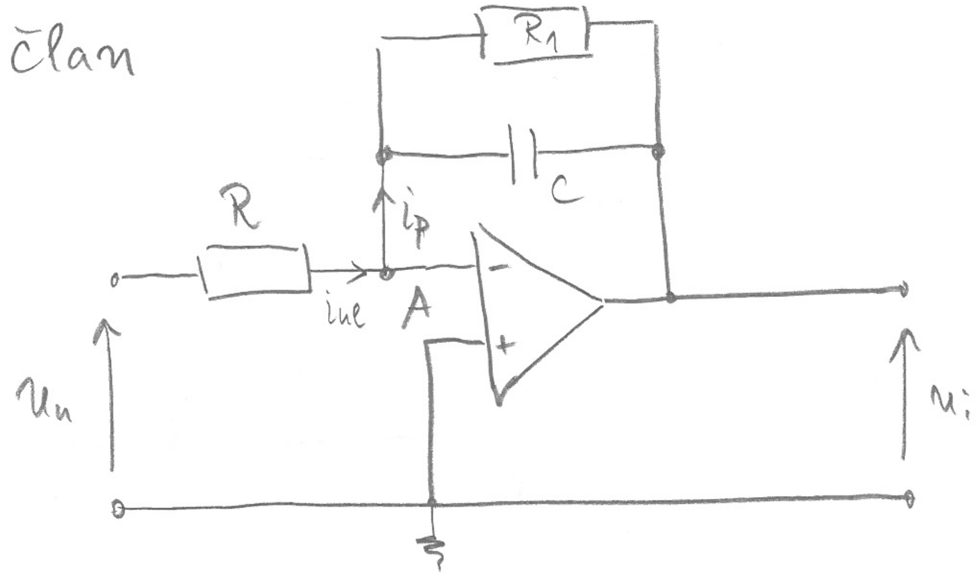
$A >> 1 + \frac{Z_p}{Z_u}$ $\quad i_b = 0$

$$\Downarrow$$

$$u_i = -\frac{Z_p}{Z_u} u_u$$

operacije: množenje, sumiranje, integriranje, funkcije

PT1 član



$$i_{ul} = i_p$$

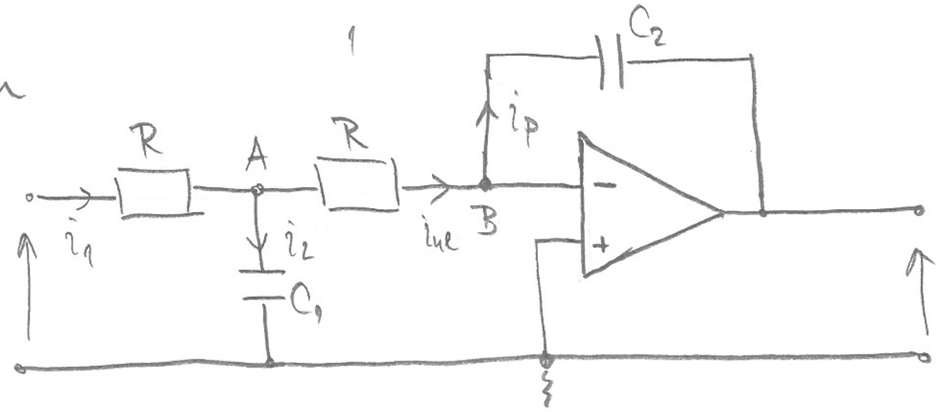
$$u_{ul} = R \cdot i_{ul}$$

$$u_{iz} = -i_p \left(R_1 \parallel \frac{1}{sC}\right)$$

$$u_{iz} = -i_p \frac{R_1 \cdot \frac{1}{sC}}{R_1 + \frac{1}{sC}}$$

$$G(s) = \frac{u_i}{u_u} = -\frac{\cancel{i_p}\,\frac{R_1 \cdot \frac{1}{\cancel{sC}}}{\frac{sR_1C+1}{\cancel{sC}}}}{R \cdot \cancel{i_{ul}}} = -\frac{R_1}{R(1+R_1Cs)} = -\frac{R_1}{R} \cdot \frac{1}{1+R_1Cs}$$

IT1 član



$$B: \; i_{ul} = i_p = \frac{u_A}{R}$$

$$u_B = 0$$

$$i_1 = i_2 + i_{ul}$$

$$\frac{u_u - u_A}{R} = \frac{u_A - 0}{\frac{1}{sC_1}} + \frac{u_A}{R}$$

$$u_u - u_A = RC_1 s\, u_A + u_A$$

$$u_u = u_A(RC_1 s + 2)$$

$$u_A = \frac{u_u}{RC_1 s + 2}$$

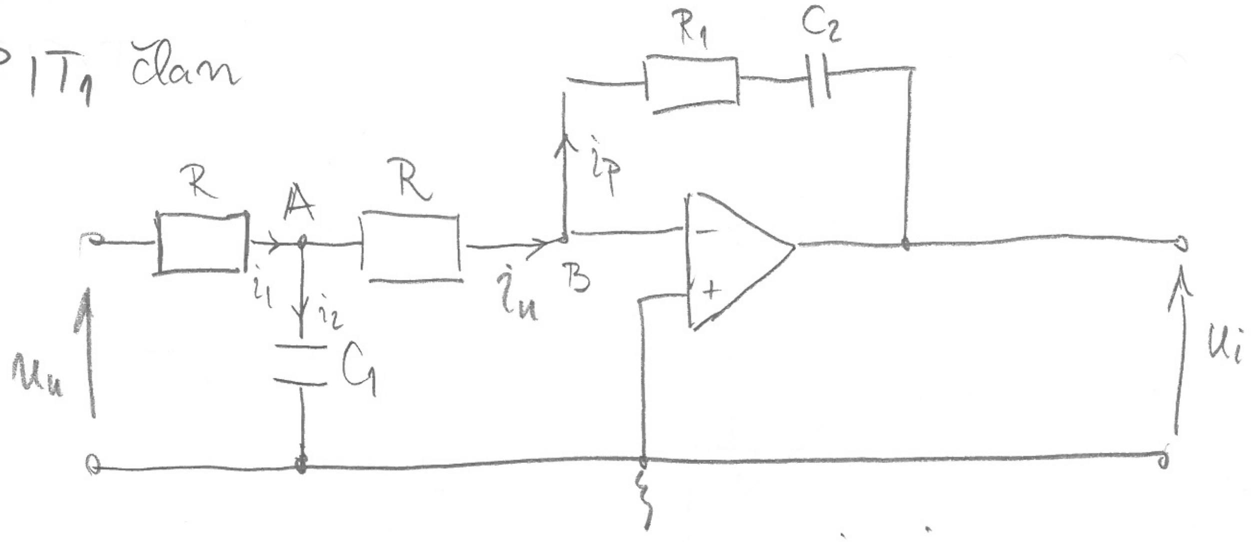
$$u_i = -i_p \cdot \frac{1}{sC_2}$$

$$u_i = -\frac{u_A}{R} \cdot \frac{1}{sC_2}$$

$$u_i = \frac{-u_u}{R(RC_1 s + 2)} \cdot \frac{1}{sC_2}$$

$$\frac{u_i}{u_u} = \frac{1}{RC_2 s(RC_1 s + 2)}$$

PIT$_1$ član



$$i_p = i_u$$

$$i_1 = i_2 + i_u$$

$$\frac{u_u - u_A}{R} = \frac{u_A}{\frac{1}{sC_1}} + \frac{u_A}{R}$$

$$u_u - u_A = RC_1 s\, u_A + u_A$$

$$u_u = u_A (RC_1 s + 2)$$

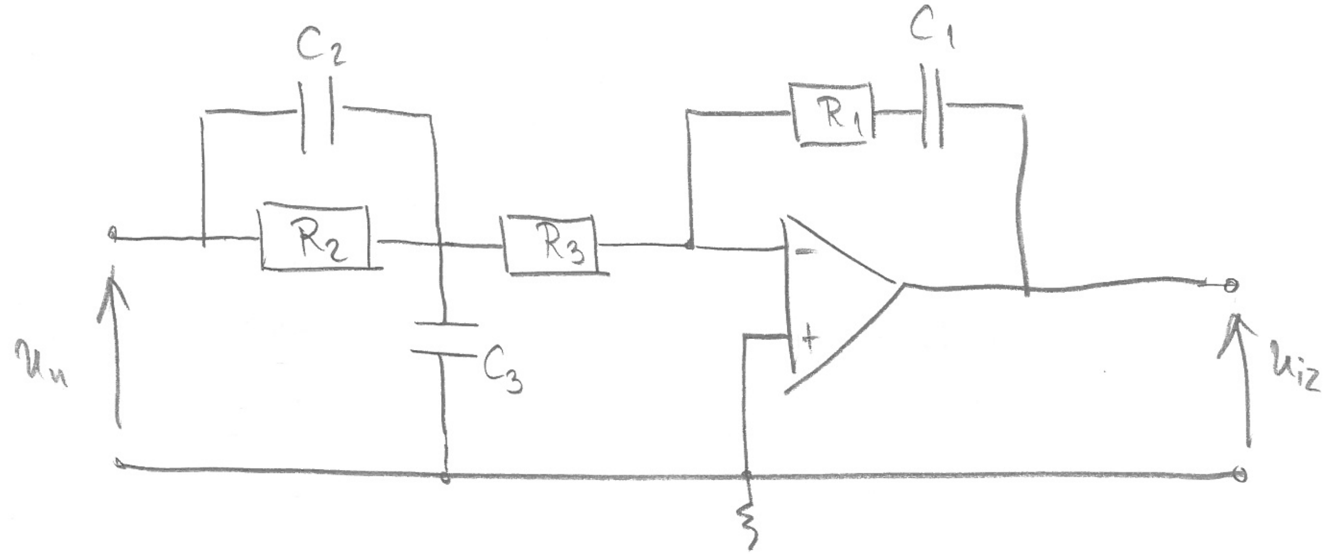
$$u_i = -i_p \cdot \left(R_1 + \frac{1}{sC_2}\right)$$

$$u_i = -\frac{u_A}{R}\left(\frac{R_1 C_2 s + 1}{sC_2}\right)$$

$$u_i = -\frac{1}{R} \cdot \frac{u_u}{RC_1 s + 2} \cdot \frac{R_1 C_2 s + 1}{sC_2}$$

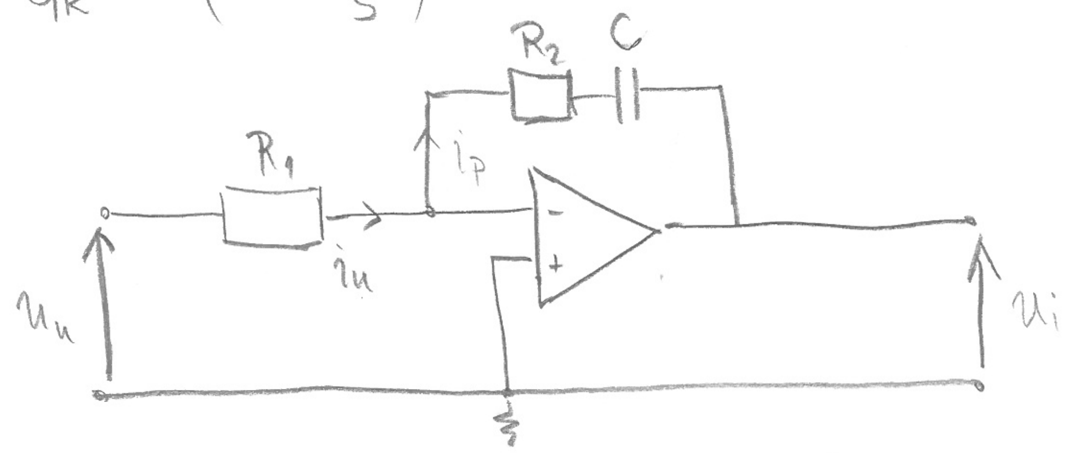
$$\frac{u_i}{u_u} = -\frac{R_1 C_2 s + 1}{2RC_2 s\left(\frac{RC_1}{2} s + 1\right)} = -\frac{\frac{R_1}{2R}\left(1 + \frac{1}{R_1 C_2 s}\right)}{\frac{RC_1}{2} s + 1}$$

PIDT1



$\boxed{P1}$

$G_R = -\left(15 + \frac{10}{s}\right)$ $i_{max} = 2mA$



$i_u = \frac{u_u}{R_1} = i_p$ $u_i = -i_p \cdot \left(R_2 + \frac{1}{sC}\right)$

$$u_i = -\frac{u_u}{R_1}\left(R_2 + \frac{1}{sC}\right)$$

$$\frac{u_i}{u_u} = -\frac{1}{R_1}\left(R_2 + \frac{1}{sC}\right) = -\left(\frac{R_2}{R_1} + \frac{1}{sCR_1}\right)$$

$\frac{R_2}{R_1} = 15 \Rightarrow \boxed{R_1 = \frac{R_2}{15}}$ $\frac{1}{CR_1} = 10 \Rightarrow \boxed{C = \frac{1}{10R_1} = \frac{15}{10R_2}}$

$u_{izm} = 10V$

$i_{izm} = \frac{u_{izm}}{R_2} \leq i_{max}$ $R_2 \geq \frac{u_{izm}}{i_{max}} = \frac{10V}{2mA} = 5k\Omega$ $C = 10\mu F$

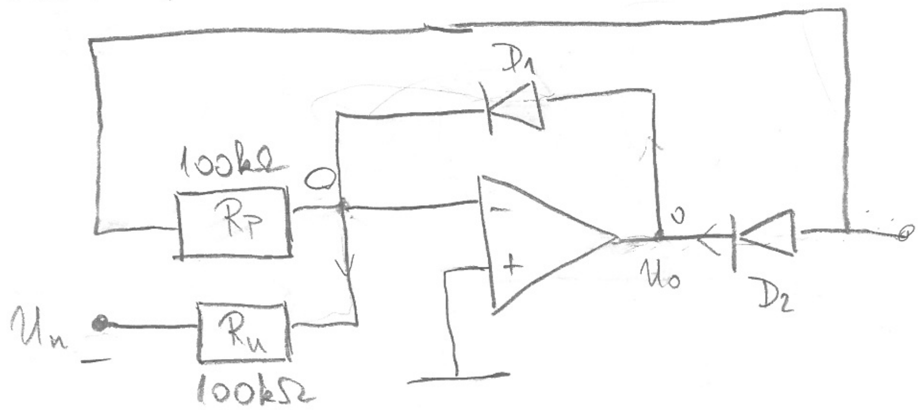
$R_2 = 150\ k\Omega$

$R_1 = 10\ k\Omega$

Nelinearni sklopovi: $U = R \cdot X$

IDEALNO POJAČALO:
1. ∞ pojačanje
2. ∞ ulazni otpor
3. izlazni otpor $= 0$
4. granična frekv. je beskonačna

MRTVA ZONA:



$U_u \leq 0$

$U_o \geq 0$

vodi D_1

$R_P = 0$

$R_u = 100k\Omega$

$A = -\frac{R_P}{R_u} = 0$

$U_a = 0$

$U_u \geq 0$

$U_o \leq 0$

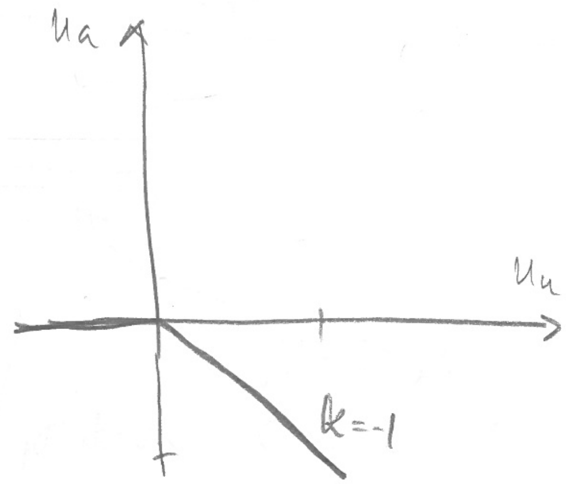
vodi D_2

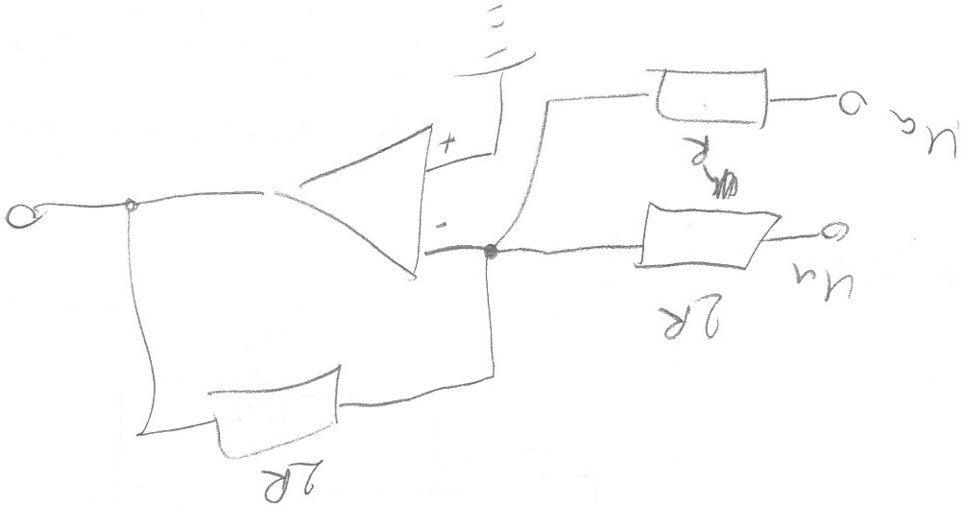
$R_P = 100k\Omega$

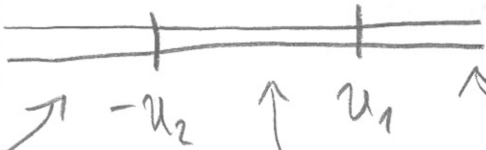
$R_u = 100k\Omega$

$A = -\frac{R_P}{R_u} = -1$

$U_a = -U_u$







$U_u \le -U_2$

$U_u + U_2 \le 0 \Rightarrow$ vodi D_1

$U_1 > -U_2$

$-U_1 < U_2 \Rightarrow U_u - U_1 < 0$

$\Rightarrow D_2$ ne vodi

$U_a = -(U_u + U_2)$

$U_b = 0$

$-U_2 < U_u < U_1$

$-U_2 - U_u < 0$

$-(U_2 + U_u) < 0$

$U_2 + U_u \geq 0$

$U_o \not< 0$ D_1 ne vodi

$U_u < U_1$

$U_u - U_1 < 0$

$U_o > 0$

D_2 ne vodi

$U_b = 0 \quad U_a = 0$

$-U_2 < U_u$

$U_1 < U_u$

$0 < U_u + U_2$

$U_o < 0$ D_1 ne vodi

$(U_u - U_1) > 0$

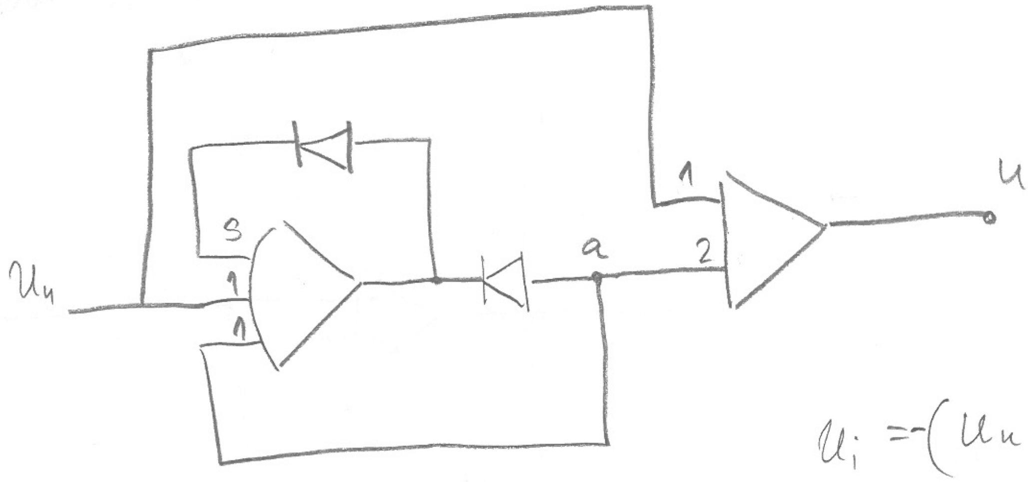
$U_o = < 0$

$D_2 \Rightarrow$ vodi

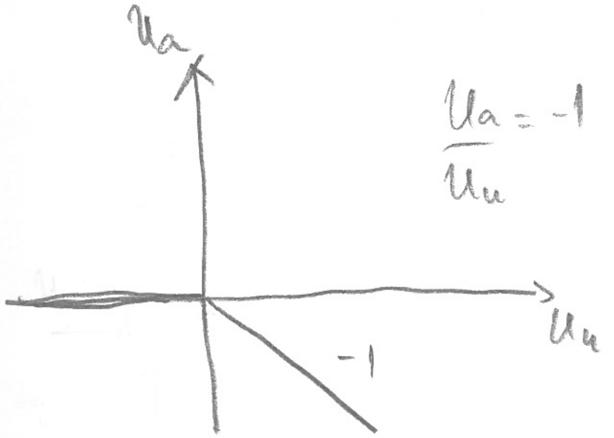
$U_a = 0$

$U_b = -(U_u - U_1)$

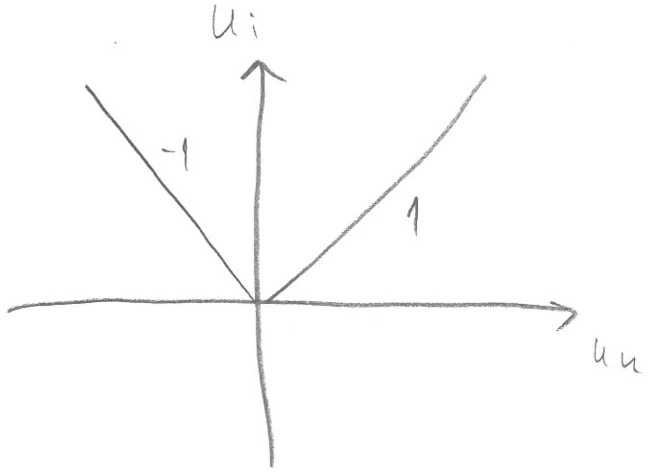
Primjer:

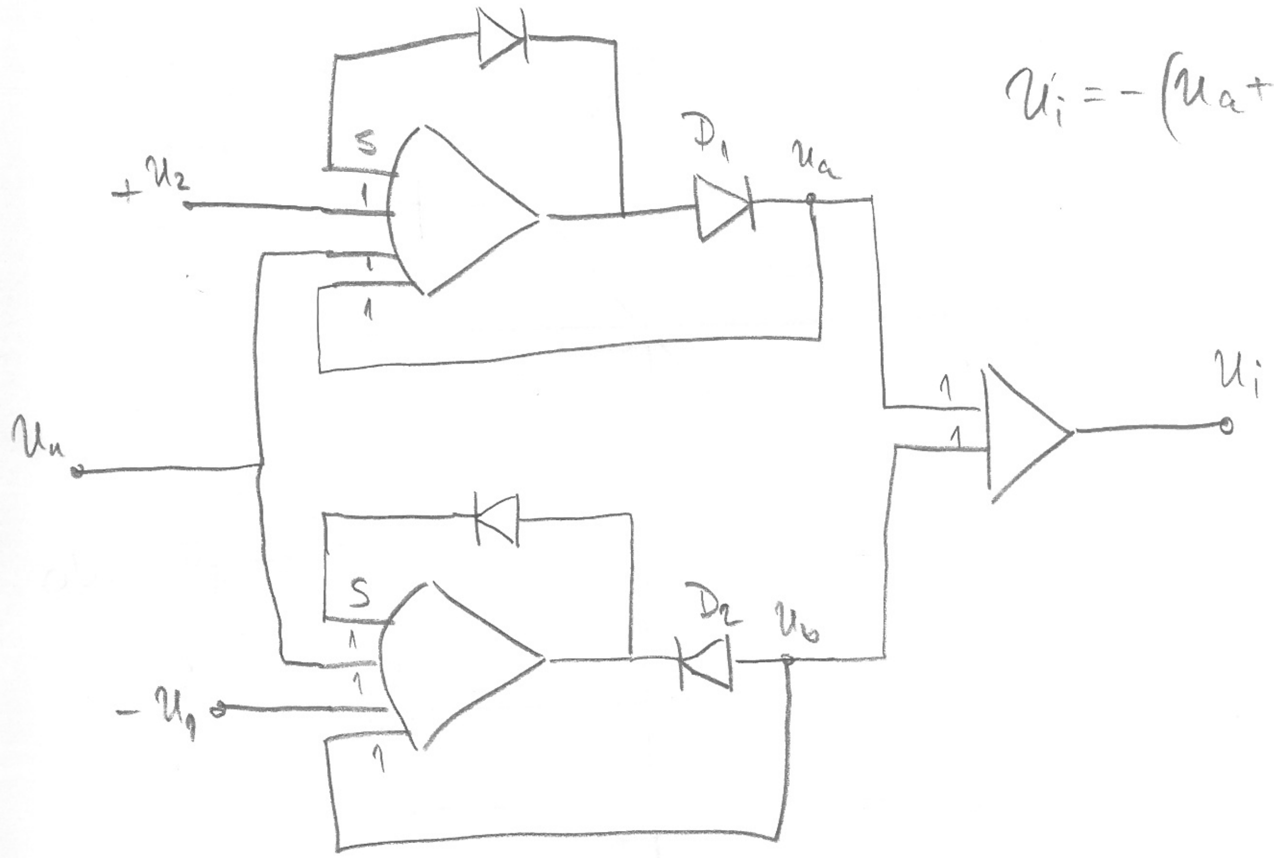


$$u_i = -(u_u + 2u_a)$$



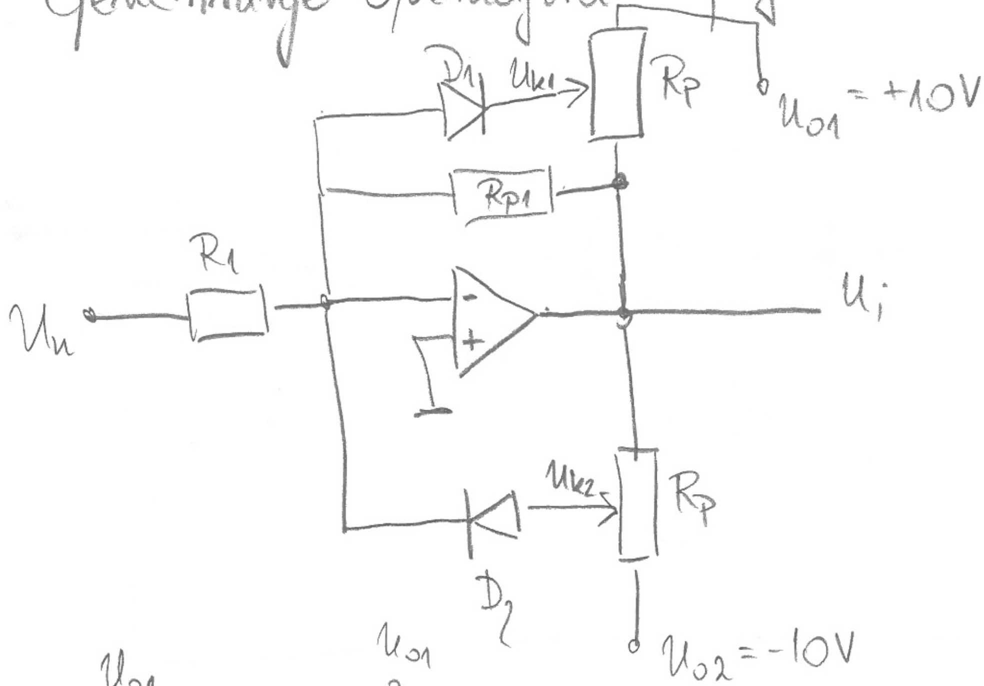
$$\frac{u_a}{u_u} = -1$$





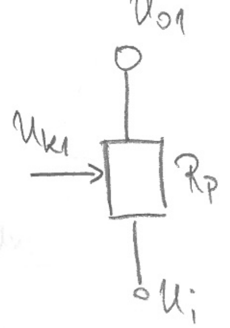
$$u_i = -(u_a + u_b)$$

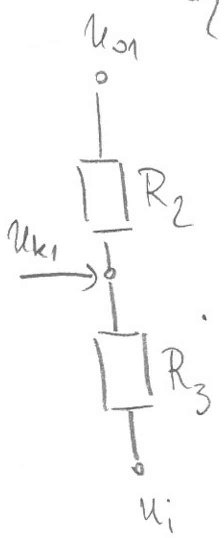
Generiranje operacijskim pojačalima:



$$u_n = 0$$

$$u_i = -\frac{R_{p1}}{R_1} u_n$$





$$R_p = R_2 + R_3$$

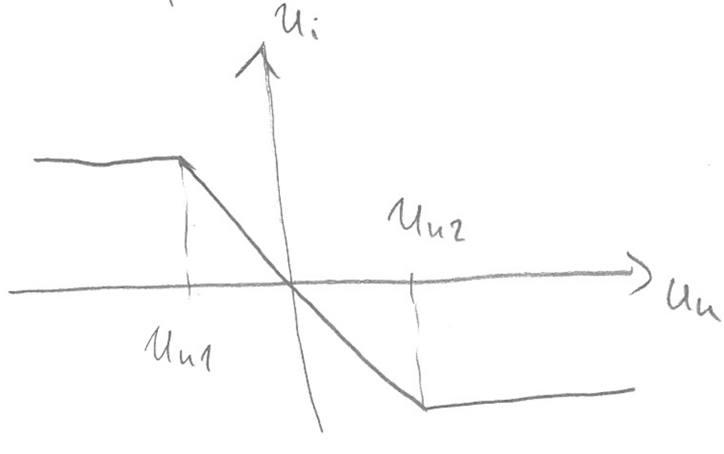
$$K_1 = \frac{R_3}{R_2 + R_3}$$

$$u_{k1} = K_1 (u_{01} - u_i) + u_i$$

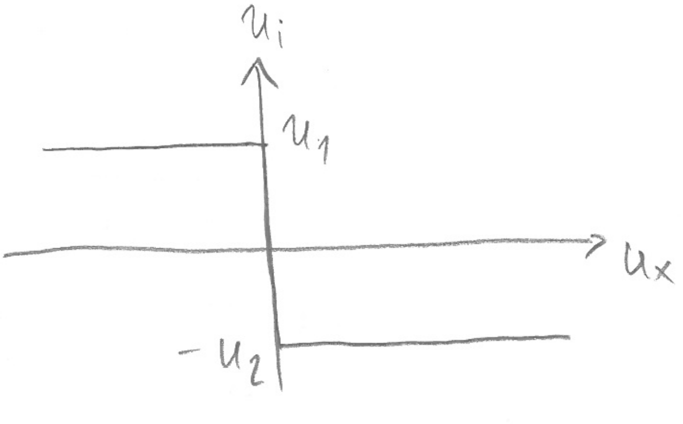
$$\frac{u_{k1} - u_i}{R_3} = \frac{u_{01} - u_i}{R_p}$$

$u_{k1} < 0$

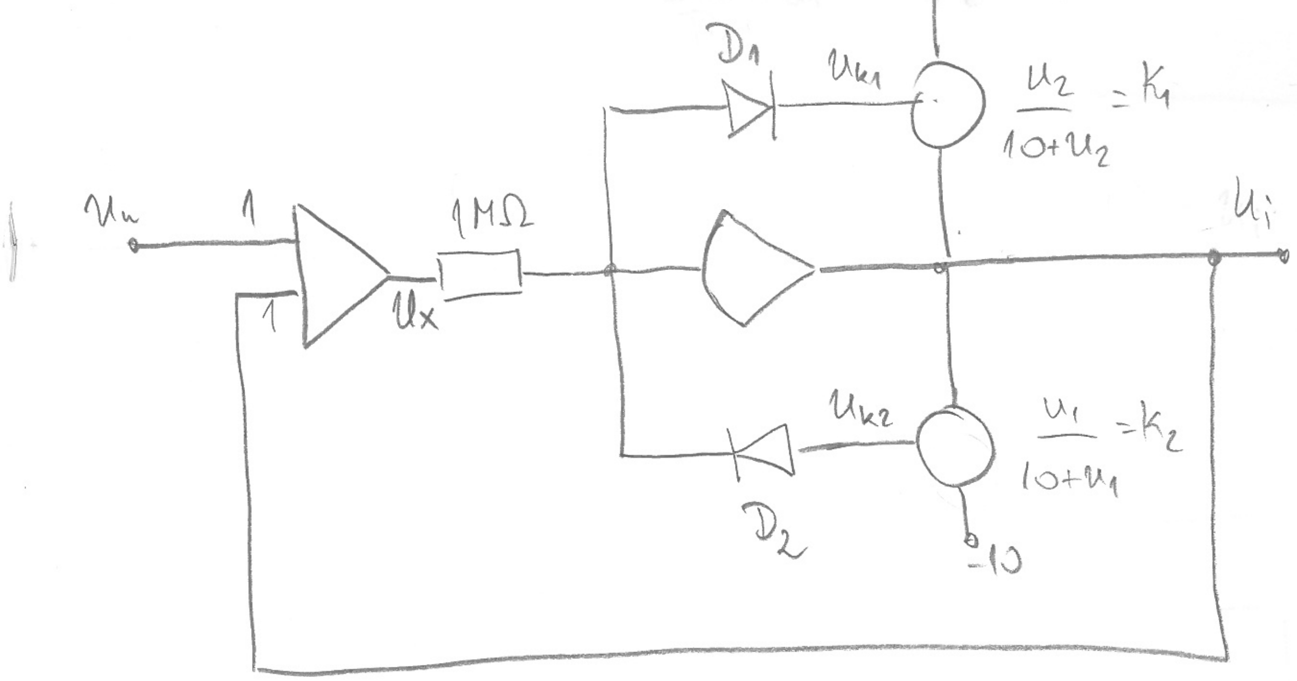
vodi D_1

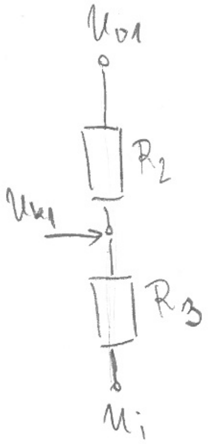


ako $R_{p1} \to \infty$



PRAVOKUTNA HISTEREZA:





$$\frac{u_{01}-u_i}{R_2+R_3} = \frac{u_{k1}-u_i}{R_3}$$

$$\frac{R_3}{R_2+R_3}\left(u_{01}-u_i\right) = u_{k1}-u_i$$

$$u_{k1} = \frac{R_3}{R_2+R_3}\left(u_{01}-u_i\right) + u_i$$

$$u_{k1} = K_1\left(u_{01}-u_i\right) + u_i$$

$u_x = 0^+$ $\quad u_i < 0$

D_1 na rubu

$\Rightarrow u_{k1} = 0$

$$0 = K_1\left(u_{01}-u_i\right) + u_i$$

$$-u_i = K_1 u_{01} - K_1 u_i$$

$$K_1 u_i - u_i = K_1 u_{01}$$

$$u_i\left(K_1-1\right) = K_1 u_{01}$$

$$u_i = \frac{K_1}{K_1-1} u_{01}$$

$$u_i = -\frac{R_3}{R_2} u_{01} = -\frac{u_2}{\cancel{10}} \cdot \cancel{10}$$

$$\frac{K_1}{K_1-1} = \frac{\frac{u_2}{10+u_2}}{\frac{u_2}{10+u_2}-1} = \frac{\frac{u_2}{\cancel{10+u_2}}}{\frac{\cancel{u_2}-10-\cancel{u_2}}{\cancel{10+u_2}}} = \frac{-u_2}{10}$$

$$\frac{K_1}{K_1-1} = \frac{\frac{R_3}{R_2+R_3}}{\frac{R_3}{R_2+R_3}-1} = \frac{\frac{R_3}{\cancel{R_2+R_3}}}{\frac{\cancel{R_3}-R_2-\cancel{R_3}}{\cancel{R_2+R_3}}} = -\frac{R_3}{R_2}$$

$u_x > 0 \implies u_i = -\frac{R_3}{R_2} u_{o1} = -U_2$

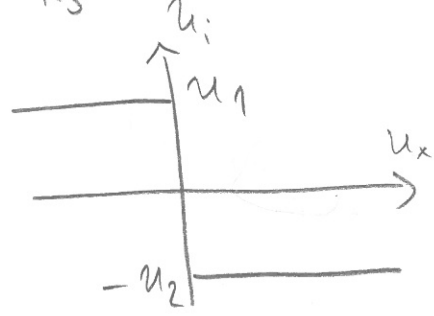
$u_x < 0$

D_2 održava $u_{k_2} = 0$

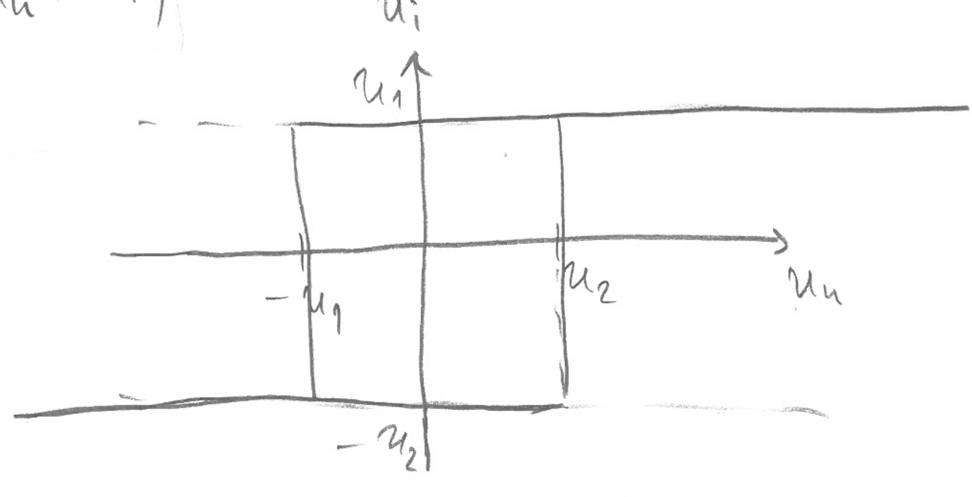
$u_{k_2} = K_2 (u_{o2} - u_i) + u_i$

$$u_i = \frac{K_2}{K_2 - 1} u_{o2} = -\frac{R_4}{R_5} u_{o2} = U_1$$

$$u_i = \begin{cases} -U_2 & \forall \quad u_x > 0 \\ U_1 & \forall \quad u_x < 0 \end{cases}$$



$u_x = -(u_u + u_i)$



$u_x > 0$

$u_x = -u_u - u_i$

$u_x = -u_u + U_2 > 0$

$u_u < U_2$

$u_u = U_2^+$

$u_x = -U_2^+ + U_2 = 0^-$

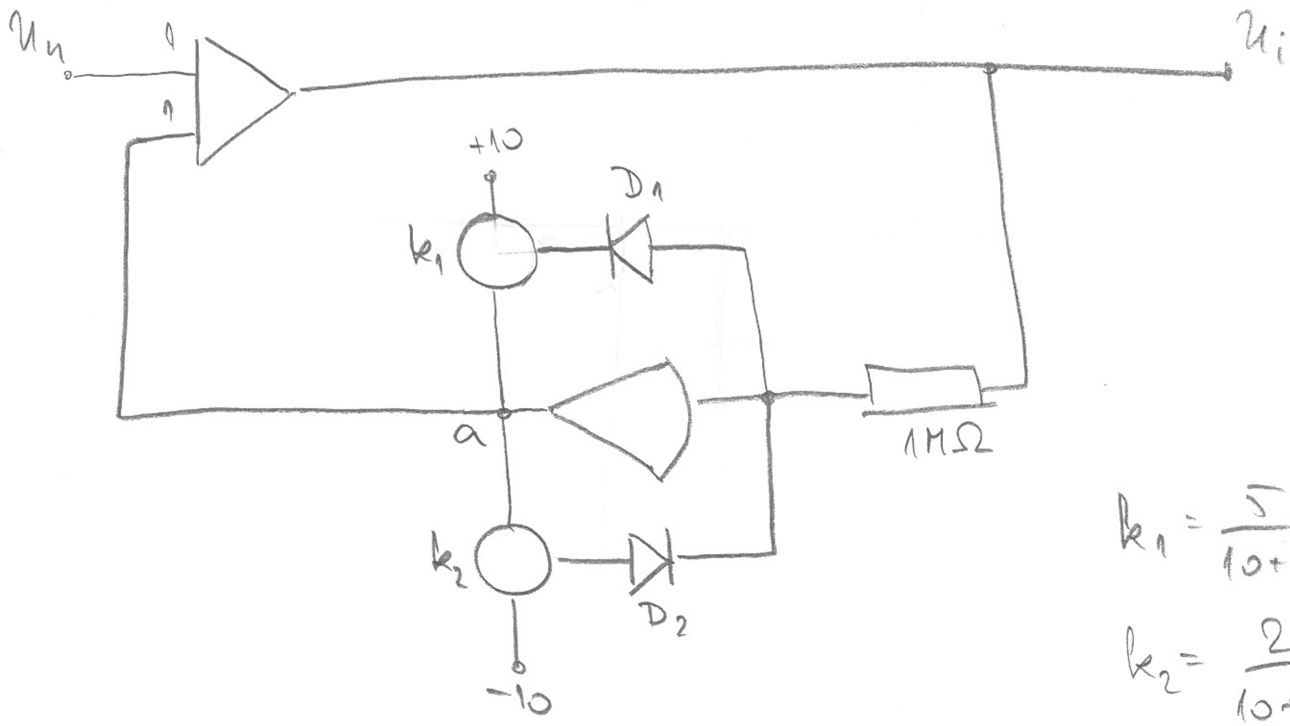
$u_x < 0$

$u_x = -u_u - u_i$

$u_x = -u_u - U_1 < 0$

$-u_u < U_1$

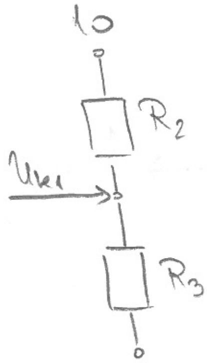
$u_u > -U_1$



$$k_1 = \frac{5}{10+5}$$

$$k_2 = \frac{2}{10+2}$$

$u_i > 0 \Rightarrow$ vodi $D_1 \Rightarrow u_{k1} = 0$



$$\frac{u_{k1} - u_a}{R_3} = \frac{10 - u_a}{R_2 + R_3}$$

$$u_{k1} = \frac{R_3}{R_2 + R_3}(10 - u_a) + u_a$$

$$-u_a = k_1(10 - u_a)$$

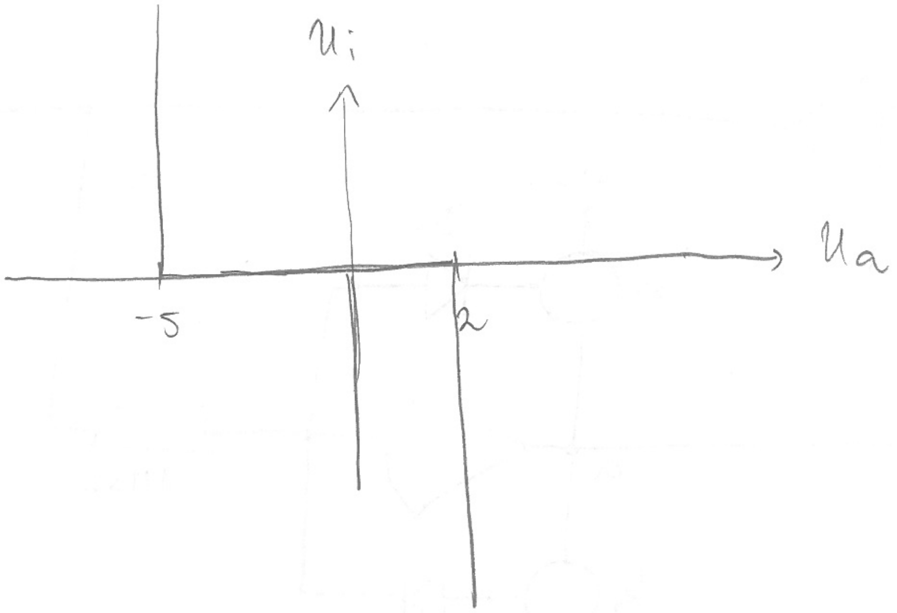
$$-u_a + k_1 u_a = k_1 \cdot 10$$

$$u_a(k_1 - 1) = k_1 \cdot 10$$

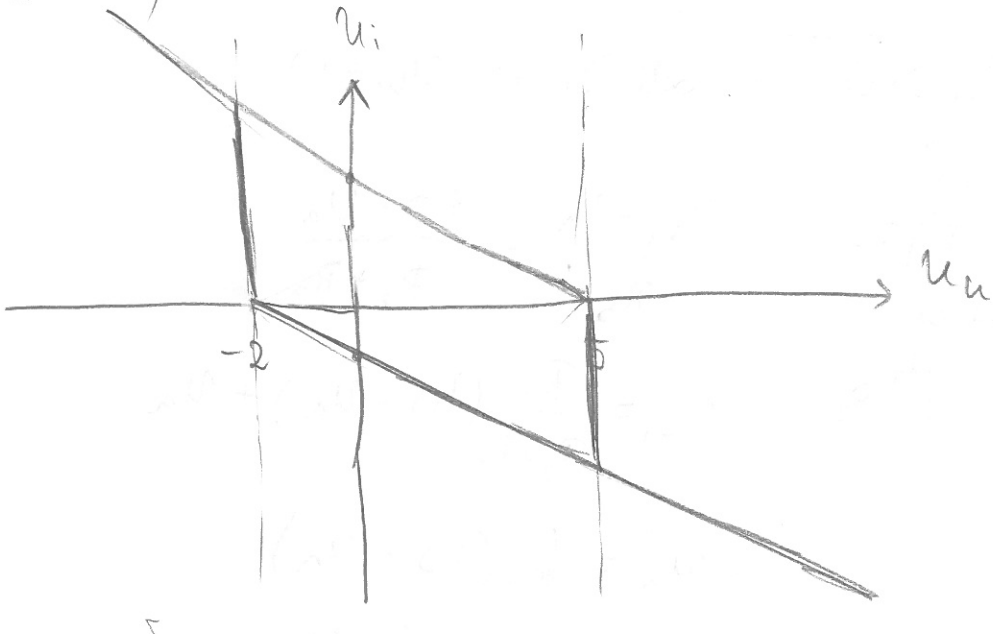
$$u_a = \frac{k_1}{k_1 - 1} \cdot 10 = \frac{\frac{5}{\cancel{10+5}}}{\frac{5-10-5}{\cancel{10+5}}} \cdot 10 = -\frac{5}{\cancel{10}} \cdot \cancel{10} = -5$$

$u_i < 0 \Rightarrow$ vodi $D_2 \Rightarrow u_{k2} = 0$

$$u_a = \frac{k_2}{k_2 - 1} \cdot (-10) = -\frac{2}{\cancel{10}} \cdot (-\cancel{10}) = 2$$



$$u_i = -(u_n + u_a)$$



$u_i > 0$

$-u_n - u_a > 0$

$-u_n + 5 > 0$

$-u_n > -5$

$u_n < 5$

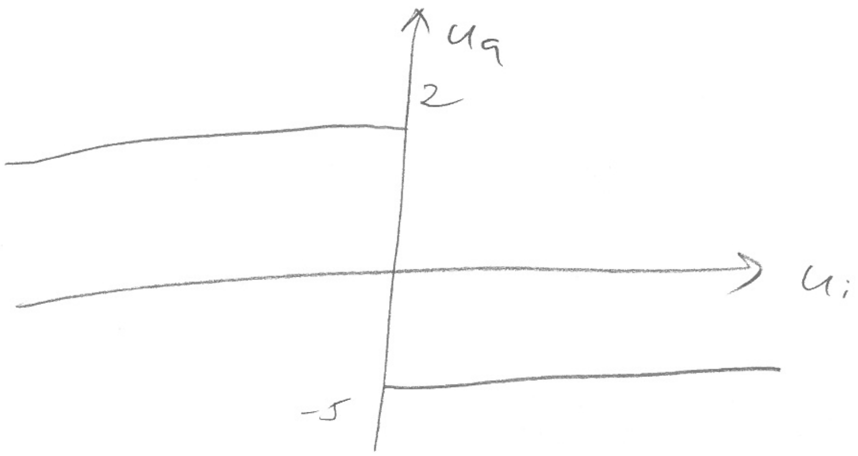
$u_i < 0$

$-u_n - u_a < 0$

$-u_n - 2 < 0$

$-u_n < 2$

$u_n > -2$



$u_i < 0$

$u_a = 2$

$u_i = -(u_u + 2)$

$u_i > 0$

$u_a = -5$

$u_i = -(u_u - 5)$

8. Generiranje funkcije nezavisne varijable

- razvojem u red potencija:

$$y = f(t) \qquad t_0 = 0$$

$$\Rightarrow y(t) = y(t_0) + \frac{1}{1!}\frac{dy}{dt}\Big|_{t=t_0}(t-t_0) + \frac{1}{2!}\frac{d^2y}{dt^2}\Big|_{t=t_0}(t-t_0)^2 + \dots$$

npr

$$y_1 = sh(t) = \frac{e^t - e^{-t}}{2} = \frac{t}{1!} + \frac{t^3}{3!} + \frac{t^5}{5!} + \frac{t^7}{7!} + \dots = \sum_{k=0} \frac{t^{2k+1}}{(2k+1)!}$$

- točnost ovisi o algoritmima numeričke integracije

→ rješavanjem <u>diff. jdbi.</u>

1. tražena f-ja se derivira sve dok nezavisnu varijablu nije moguće eliminirati pomoću tražene f-je i nj. derivacija
2. određuju se početni uvjeti za izlaznu f-ju i nj. derivacije

8.1. Normiranje

- Po amplitudi: $\pm 10V$

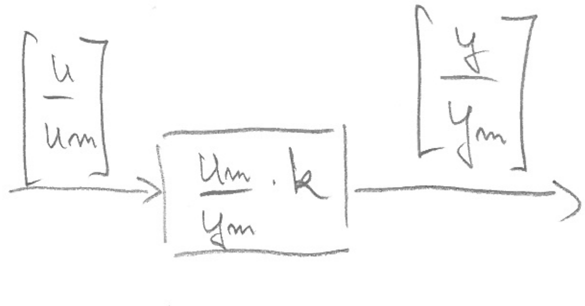
→ dijeljenje jednadžba linearnog elementa s max. vrijednošću izlaza zaokruženom na veću okruglu

→ korigiranje pojačanja

$$y = k \cdot u$$

$$\left[\frac{y}{y_m}\right] = \left(\frac{u_m}{y_m}k\right)\left[\frac{u}{u_m}\right]$$

↑ nove varijable

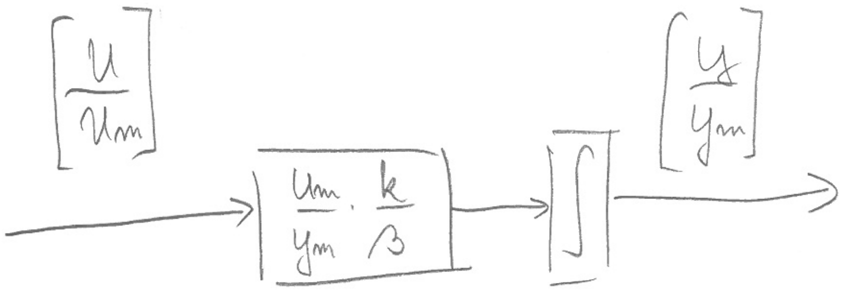


- Po vremenu $\tau = \beta t$

$\beta > 1 \Rightarrow \tau > t$ usporenje vir. vremena

$\beta \in \langle 0, 1]$ -> ubrzanje vir. vremena

$$\boxed{t = \frac{\tau}{\beta}} \qquad \boxed{dt = \frac{1}{\beta} d\tau}$$



9. SUSTAVI S RASPODIJELJENIM PARAMETRIMA

- varijable ovise o vremenu i prostoru

gradijent

$$\vec{\nabla} f = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i} \cdot \vec{e}_i$$

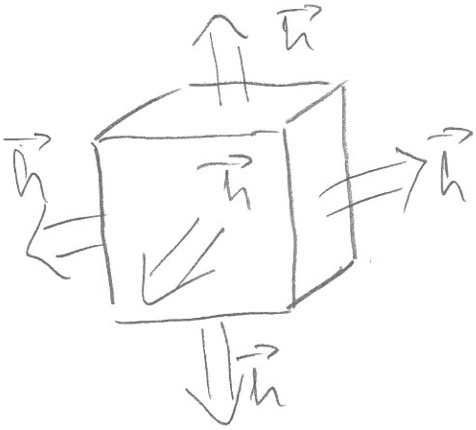
divergencija

$$\mathrm{div}\, \vec{a} = \vec{\nabla} \cdot \vec{a} = \sum_{i=1}^{n} \frac{\partial a_i}{\partial x_i}$$

Laplaceov operator

$$\Delta f = \mathrm{div}\, \vec{\nabla} f = \vec{\nabla}(\vec{\nabla} f) = \sum_{i=1}^{n} \frac{\partial^2 f}{\partial x_i^2}$$

Primjer



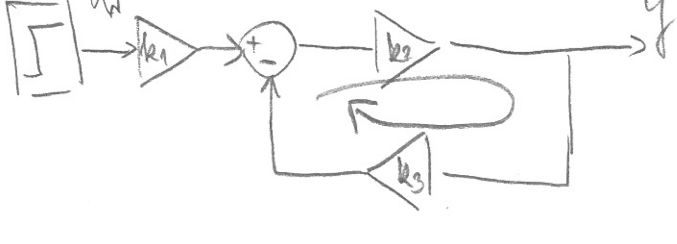
$$E = mcT = \rho\, c \int_V T\, dV$$

$$\frac{dE}{dV} = \rho c T \qquad \frac{dE}{dt} = \int_{\partial\Omega} \cdot \vec{h} \cdot \vec{n} \cdot dA$$

Toplinski tok: $\rho \cdot c \cdot \frac{\partial T}{\partial t} = -k \Delta T$

11. Simuliranje kontinuiranih sustava

Algebarska petlja



$$y = k_2(k_1 u_r - k_3 y)$$

$$y = \frac{k_1 k_2}{1 + k_2 k_3} u_r$$

- ručna promjena
- puštanje Matlabu
- ubacivanje integratora
- ubacivanje niskopropusnog filtra (PT1)

Algoritmi numeričke integracije

a) fiksni/promjenjivi korak integracije

b) jednokoračni / višekoračni postupci

EULER

- fiksni korak
- jednokoračni

$$\frac{dy}{dt} = f(y,t) \qquad y(t=0) = y_0$$

$h = t_{k+1} - t_k \Rightarrow$ korak integracije

$$\hat{y}_{k+1} = \hat{y}_k + y'(t_k)\cdot h$$

$$\hat{y}_{k+1} = \hat{y}_k + f(t_k, \hat{y}_k)\cdot h$$

RUNGE - KUTTA

$$\hat{y}_{k+1} = \hat{y}_k + h\cdot \Phi(t_k, \hat{y}_k, t)$$

2. red:

$$\Phi = ak_1 + bk_2$$

$$k_1 = f(t_k, \hat{y}_k)$$

$$k_2 = f(t_k + ph, \hat{y}_k + qhk_1)$$

$$\boxed{\begin{array}{c} a = 1-b \\ p = q = \frac{1}{2b} \end{array}}$$

za $\boxed{b = \frac{1}{2}} \Rightarrow$ Poboljšana Eulerova ili Heunova

$$\hat{y}_{k+1} = \hat{y}_k + \frac{h}{2}\left[f(t_k, \hat{y}_k) + f(t_{k+1}, \hat{y}_k + hf(t_k, \hat{y}_k))\right]$$

(srednja vrijednost derivacije na početku i kraju intervala integracije)

za $\boxed{b = 1} \Rightarrow$ Modificirana Eulerova

$$\hat{y}_{k+1} = \hat{y}_k + h\cdot f\left(t_k + \frac{h}{2}, \hat{y}_k + \frac{h}{2} f(t_k, \hat{y}_k)\right)$$

(derivacija na pola intervala integracije)

$\boxed{3.\text{ red}}$:

$$y_{k+1} = \hat{y}_k + h \cdot \Phi(t_k, \hat{y}_k, t)$$

$$\Phi = ak_1 + bk_2 + ck_3$$

$\boxed{4.\text{ red}}$:

$$y_{k+1} = y_k + \frac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$k_1 = f(t_k, y_k)$$
$$k_2 = f\left(t_k + \frac{h}{2}, y_k + \frac{h}{2} \cdot k_1\right)$$
$$k_3 = f\left(t_k + \frac{h}{2}, y_k + \frac{h}{2} \cdot k_2\right)$$
$$k_4 = f\left(t_k + h, y_k + hk_3\right)$$

- SVI KONVERGIRAJU

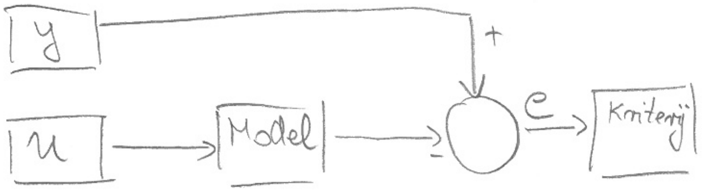
- ODREĐIVANJE KORAKA INTEGRACIJE
  - -> pokušaj/pogreška -> $h = T_{min} \Rightarrow \frac{h}{2} \Rightarrow \frac{h}{2}$
  - -> koeficijent kvalitete -> $\left|\frac{k_3 - k_2}{k_2 - k_1}\right| > 100$ -> smanjiti h
  - -> Bode -> kad najmanja vremenska konstanta ne utječe bitno na prijelaznu pojavu

$$h \leq \begin{cases} T_4 & \text{ako } \omega_4 \geq 10\omega_c \\ \frac{T_4}{k}, \; k = 2 \ldots 5, & \text{ako } \omega_4 < \omega_c \end{cases}$$

  - ako utječe na prijelaznu pojavu $\Rightarrow h \leq \frac{T_3}{k}$, $k = 5 \ldots 10$

- VARIJABILNI KORAK :
  - -> metode za Stiff sustave su brže
  - -> ostale metode su točnije i stabilnije

12. ODREĐIVANJE PARAMETARA MODELA



∘ FUNKCIJA CILJA

• odstupanje pogreške od nule povećava kriterij

svojstva: parna f-ja, ima minimum

• vrijeme trajanja pogreške

integral f-je cilja - do trenutka pada pogreške na 0

ISE: $I = \int_0^\infty e^2(t)\,dt$

ITSE: $I = \int_0^\infty t \cdot e^2(t)\,dt$

ISTSE: $I = \int_0^\infty t^2 e^2(t)\,dt$

IAE: $I = \int_0^\infty |e(t)|\,dt$

ITAE: $I = \int_0^\infty |t \cdot e(t)|\,dt$

Minimizacija funkcije cilja:
-> traži se x za koji f(x) ima minimum
metode: simplex, gradijent

MATLAB f-je za minimizaciju: fgoalattaim (za vektorske)
fminimax
fseminf
linprog
quadprog

$\boxed{\text{fminbnd}}$ -> min jedne varijable u fiksnom intervalu

$$\min_x f(x) \text{ tako da je } x_L < x < x_U$$

x = fminbnd(fun, x1, x2)

[x, fval, exitflag] = fminbnd(...)

fval ↑ vrijednost f-je cilja

exitflag ↑ način završetka optimiranja

npr: $f(x) = (x-3)^2 - 1$ $x \in (0, 5)$

analitički:

$$\frac{df(x)}{dx} = 2(x-3) = 0$$

$$x = 3$$

Matlab:

```
function f = f_cilja(x)
f = (x-3)^2-1;
```

---

x = fminbnd('f_cilja', 0, 5)

rj: x = 3 f = -1

$\boxed{\text{fminsearch}}$ i $\boxed{\text{fminunc}}$

-> min skalarne funkcije više varijabli $\min_x f(x)$

x = fminsearch(fun, x0)

x0 ← poč. vrijednosti parametara

exit_flag: 0 - max_iter dostignut, ali ne i tolerancija

\>0 - uspješno završeno

<0 - funkcija ne konvergira

Opcije :

oldopt = optimset

opt = optimset(oldopt, 'param', vrijednost, ...)

param	vrijednost
Diagnostics	on/off
Display	on/iter/final
GradObj	on/off
MaxFunEvals	broj računanja f-je
MaxIter	broj iteracija
TolFun	skalar
TolX	skalar

$\boxed{\text{fmincon}}$

Primjer:

$$f(x) = \min_{x} \left\{ \int_0^{\rho} t \cdot e(x,t)^2 \, dt \right\}$$

$$G(s) = \frac{1}{\frac{s^2}{\omega_0^2} + \frac{2\xi}{\omega_0} s + 1}$$

9. DZ: Runge Kutta 4. reda

$$\ddot{u}_i + 500\,\dot{u}_i + 1000\,000\,u_i = -200\,\dot{u}_u - 2000\,000\,u_u$$

$$(s^2 + 500s + 1000\,000)\,u_i = (-200s - 2\,000\,000)\,u_u$$

$$\frac{u_i}{u_u} = \frac{-200s - 2\,000\,000}{s^2 + 500s + 1000\,000}$$

$$\frac{u_i}{u_u} = \frac{u_x}{u_u} \cdot \frac{u_i}{u_x} = \frac{1}{s^2 + 500s +}$$

$$\hat{y}_{k+1} = \hat{y}_k + \dot{y}(t_k)\cdot T_s$$

$$\dot{y}(t_k) \downarrow f(t_k, \hat{y}_k)$$

```
function [t,y] = euler(funkcija, y0, Tmax, Ts)

  n = floor(Tmax/Ts);
  y(1) = y0;
  for i = 1:n
      t(i) = n*Ts;
      y(i+1) = y(i) + funkcija(t(i), y(i))·Ts;
  end
end
```

```
f = @(t,y)  (K-y)/T
[t,y] = euler(f, y0, Tmax, Ts)
```

$$\dot{y} = \frac{K-y}{T}$$

$$sTy = K - y$$

$$sTy + y = K$$

$$y(1+sT) = K \qquad \frac{K}{1+sT}$$

$$\ddot{u}_i + A\dot{u}_i + B u_i = C\dot{u}_u + D u_u$$

$y \quad A = -10^6 \quad B = -500 \quad C = -200 \quad D = -2\cdot 10^6$

zad!

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -10\hat{}6 & -500\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}-200\\ -2\cdot 10\hat{}6\end{bmatrix}\cdot u$$

rješenje!

$$\dot{x}_1 = x_2 - 200\,u$$

$$\underset{\ddot{u}_i}{\dot{x}_2} = -10^6 \underset{u_i}{x_1} - 500 \underset{\dot{u}_i}{x_2} - 2\cdot 10^6 u$$

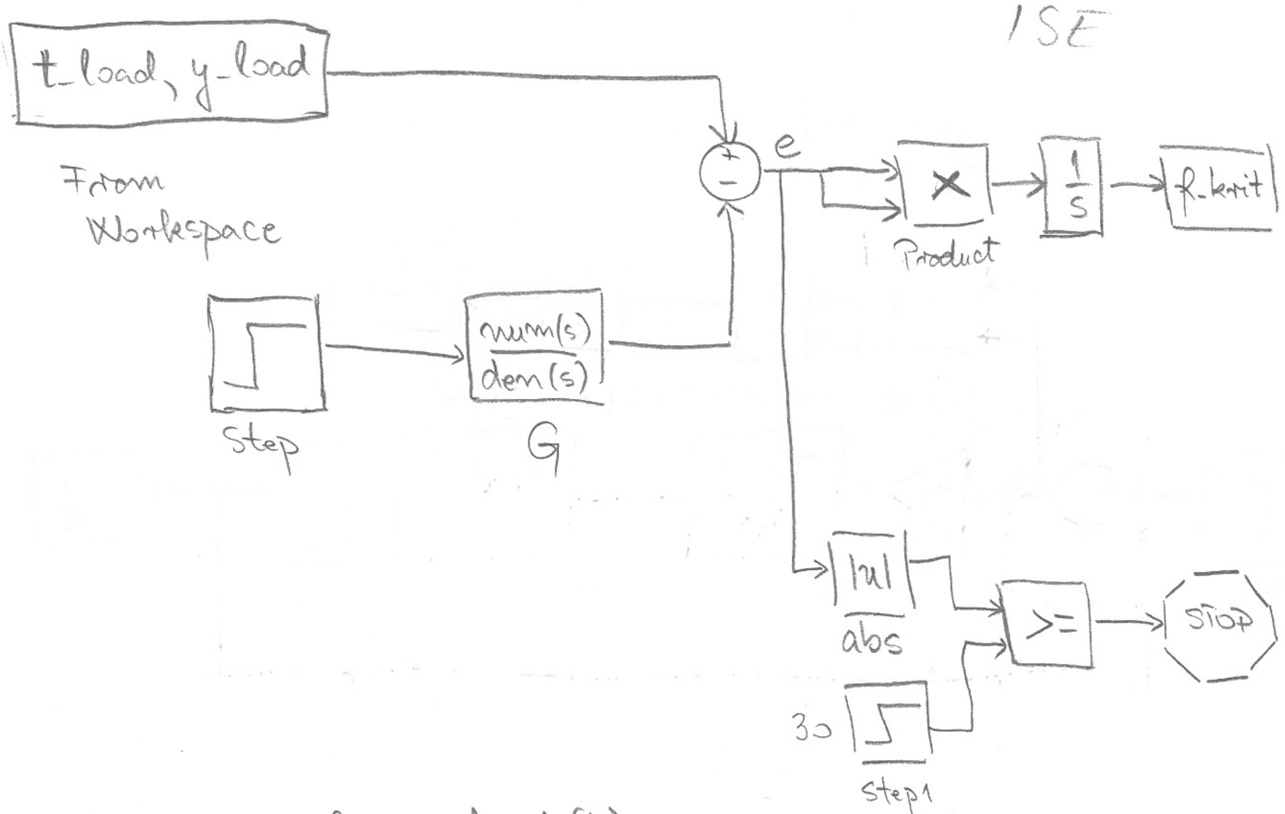
$$\frac{U_i}{U_u} = \frac{s+1}{s^2+s+1}$$

```
function [y,t] = euler(Ts, Tmax)
  brIter = floor(Tmax/Ts);
  t(1)=0
  y(:,1)=0;
  A = [0 1 ; -10^6 -500]
  B = [-200 -2·10^6]
  u = 2;
  for i = 1:brIter
      yD(:,i) = A·y(:,i) + B·u;
      y(:,i+1) = y(:,i) + yD(:,i)·Ts;
      t(i+1) = t(i) + Ts;
  end
end
```

y

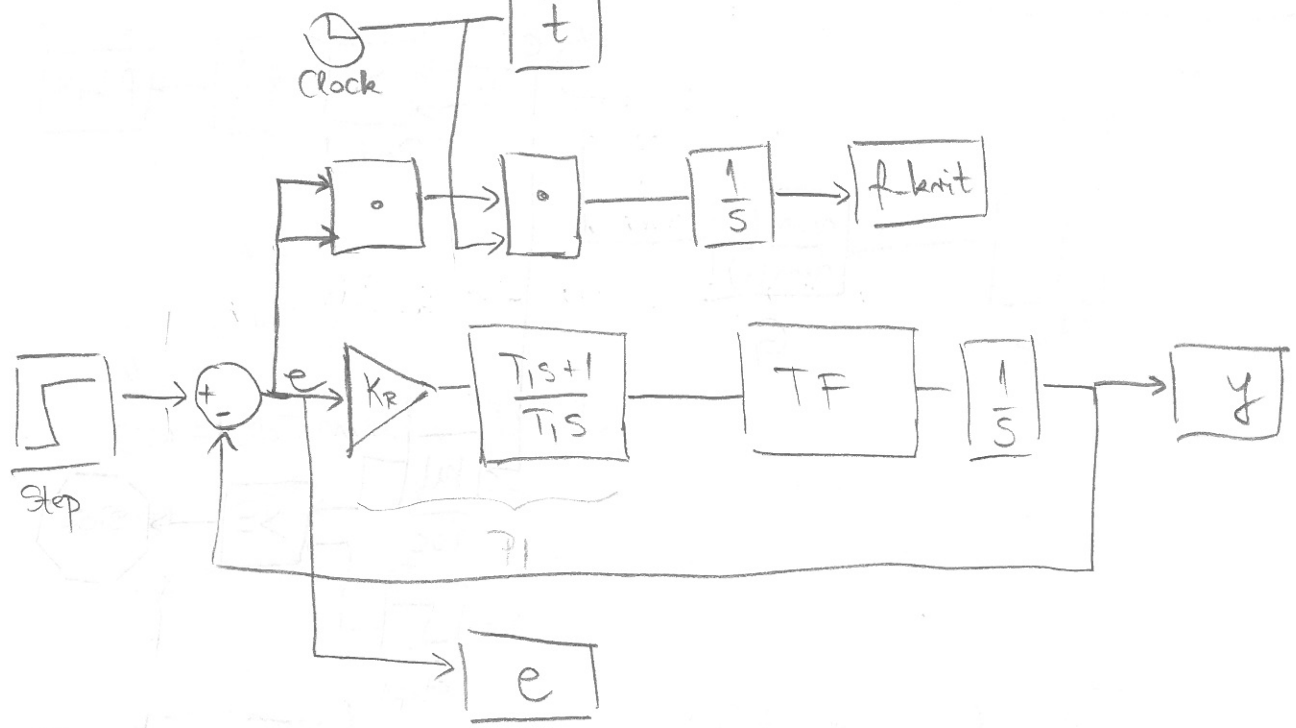
```
clc;

load miss.dat

t_load=t;

y_load=y;

op = optimset('Display','iter');

op = optimset(op,'MaxIter',1000);

op = optimset(op,'TolX',10^-7,'TolFun',10^-7)


x0 = rand([3 1]);

open('id_model.mdl');

[f]= fminsearch('fun_ident',x0,op)

G_i = tf([f(3) 2.f(2)],[1 f(1) f(2)])
```

ISE



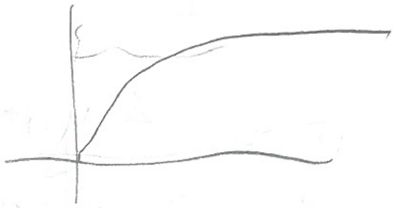

```
function f = fun_ident(k)

t_end = 200;
set_param('id_model/G', 'Numerator', mat2str([k(3) 2*k(2)]), ...
                        'Denominator', mat2str([1 k(1) k(2)]));

sim ('id_model', t_end);

if exist('t') == 1
    if max(t) >= t_end
        f = max(f_krit);
    else
        f = 1e10/max(t);   -> kazmena funkcija za
                              nestabilan sustav
    end
else
    f = 1e13;  -> kazna ako se simulacija ne pokrene
end
end
```

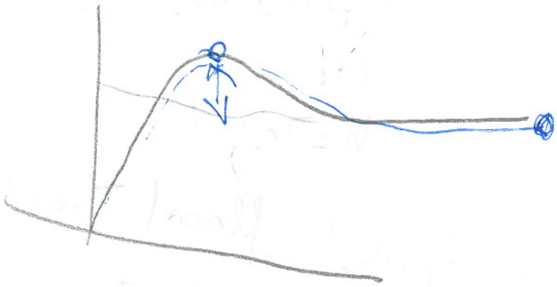
ITSE




```
function [Kr, sigma, tm] = fja(k)

tsim = 100;
for i = 1 : length(k)
    set_param('model/KR', 'Gain', mat2str(k(i)));
    sim('model', tsim);
    if exist('t')
        if max(f_krit) == f_krit(tsim)      (nestabilan odziv)
            sigma(i) = -99;
            tm(i) = -99;
        else
            if max(y) == y(tsim)
                sigma(i) = 0;               (nema nadvisenja)
                tm(i) = -1;
            else
                for ...
```

```
for j = 1:length(t)
    if y(t(j)) == max(y)
        tm(i) = t(j);
        T_max = t(j);
        sigma(i) = (max(y) - y(tsim))/y(tsim)*100;
        break
      end
    end
  end
end
Kr(i) = k(i);
end
plot (Kr(1:end), sigma(1:end));
plot (Kr(1:end), tm(1:end));
end
end
```

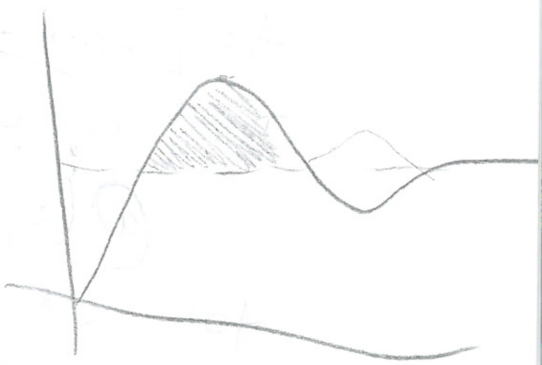




---

proyers

```
for i= 1:length(y)
    if (y(i) > y(end)) && y(i-1)
```



$$G = \frac{\omega^2}{s^2 + 2\xi\omega s + \omega^2}$$

$$\frac{y}{u} = \frac{\omega^2}{s^2 + 2\xi\omega s + \omega^2}$$

$$\ddot{y} + 2\xi\omega\dot{y} + \omega^2 y = \omega^2 u$$

$$x_1 = \dot{y}$$

$$\dot{x}_1 = -2\xi\omega x_1 - \omega^2 y + \omega^2 u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -2\xi\omega & -\omega^2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ y \end{bmatrix} + \begin{bmatrix} \omega^2 \\ 0 \end{bmatrix} \cdot u$$

$$\dot{y} = A y + B u$$

```
function[t,y]=runge_kuttah(Tmax,Ts)
clc;
h=Ts;
n=floor(Tmax/Ts);
x(:,1)=0
t=0:h:Tmax;
A=[-2ξω -ω²; 1 0]
B=[ω² 0];
F=@(t,x,u) A·x +B·u(t);
for i=1:n
    k1 = F(t(i), x(i))·h;
    k2 = F(t(i)+h/2, x(i)+k1/2)·h;
    k3 = F(t(i)+h/2, x(i)+k2/2)·h;
    k4 = F(t(i)+h, x(i)+k3)·h;
    x(i+1) = x(i)+1/6(k1+2k2+2k3+k4);
end
y=x
```

$$G = \frac{1}{s^2+s+1} \cdot \frac{s+1}{1}$$

$$\frac{Y}{U} = \frac{X}{U} \cdot \frac{Y}{X}$$

$$y = \dot{x} + x$$

$$u = \ddot{x} + \dot{x} + x$$

$$x_1 = x$$

$$x_2 = \dot{x}$$

funtion [u]=ulaz(t)

$$y = x_2 + x_1$$

$$u = \dot{x}_2 + x_2 + x_1$$

$$\dot{x}_2 = u - x_2 - x_1$$

$$\dot{x}_1 = x_2$$

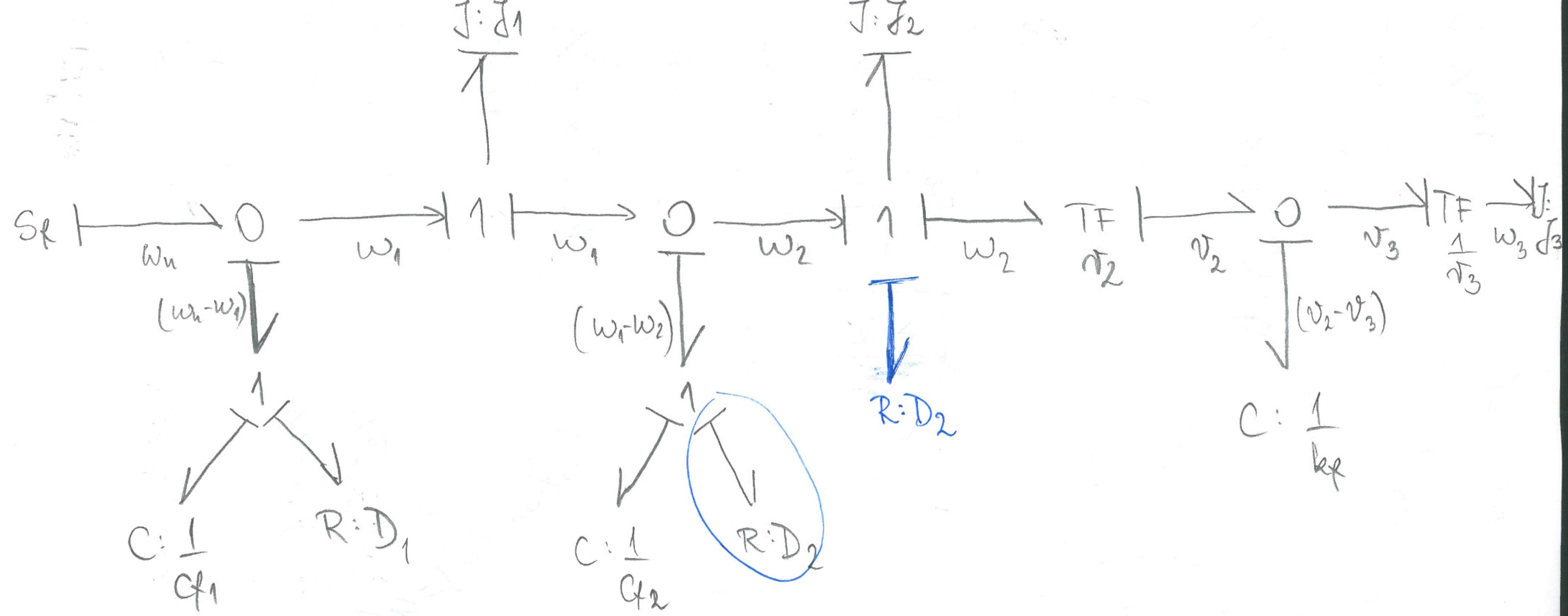
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

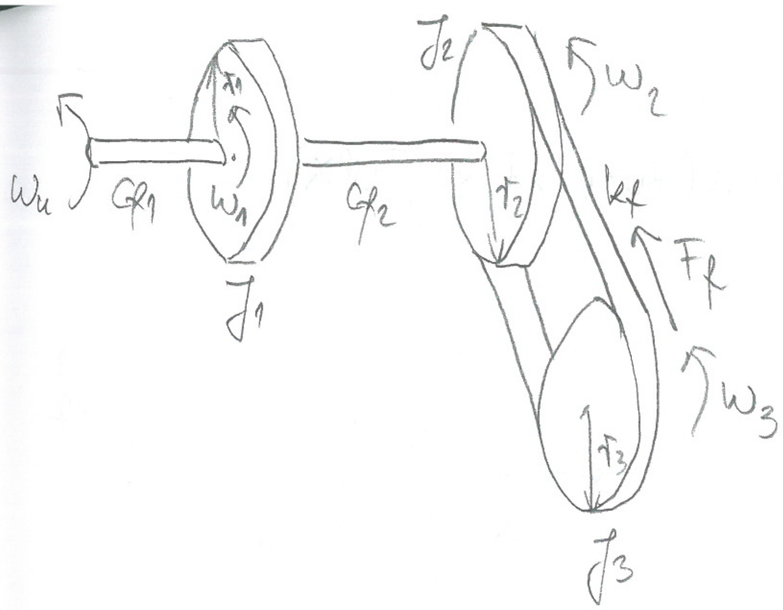
$$x_{k+1} = x_k + f(t, x_k) \cdot h$$

$$f(t, x_k) = A \cdot x_k + B \cdot u$$

$$y_{k+1} = C \cdot x_{k+1}$$



Sf
ω_n
0
$(\omega_n-\omega_1)$
1
C: $\frac{1}{C_{f1}}$
R: D_1
ω_1
1
J: J_1
ω_1
0
$(\omega_1-\omega_2)$
1
C: $\frac{1}{C_{f2}}$
R: D_2
ω_2
1
J: J_2
R: D_2
ω_2
TF r_2
v_2
0
(v_2-v_3)
C: $\frac{1}{k_f}$
v_3
TF $\frac{1}{r_3}$
ω_3
J: J_3



$$\dot{F}_f = k_f \int_{-\infty}^{t} v_2 - v_1 dt$$

$$F_f = k_f \int_{-\infty}^{t} (\omega_2 r_2 - \omega_3 r_3) dt$$

$$M_{Ff2} = F_f r_2$$

$$M_{Ff3} = F_f r_3$$

$$F = k_f \cdot X$$

$$J_1 \dot{\omega}_1 = M_1 - M_2$$

$$J_2 \dot{\omega}_2 = M_2 - M_{Ff2} - M_D$$

$$M_D = D \cdot \omega_2$$

$$J_3 \dot{\omega}_3 = M_{f3}$$

$$M_1 = c_{f1} \int (\omega_u - \omega_1) dt$$

$$M_2 = c_{f2} \int (\omega_1 - \omega_2) dt$$

$$M_{f2} = r_2 \cdot k_f \cdot \int (\omega_2 r_2 - \omega_3 r_3) dt$$

$$M_{f3} = r_3 \cdot k_f \int (\omega_2 r_2 - \omega_3 r_3) dt$$

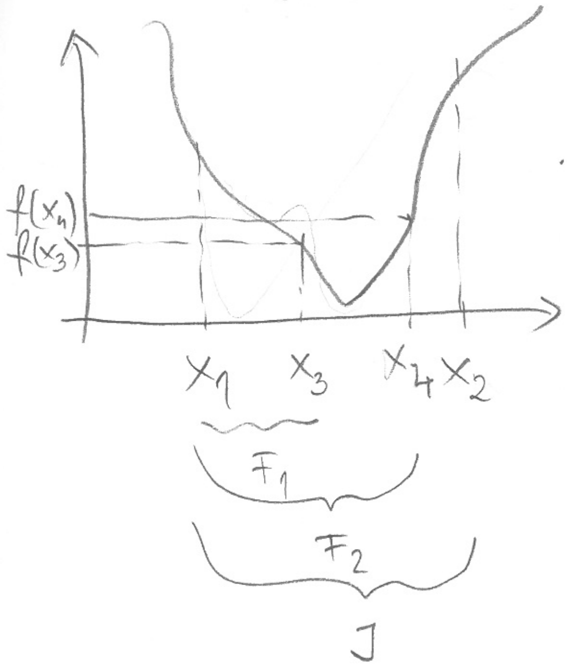
```
function [m]=trazi_min(fun,x1,x2,TolX)

F1=(3-sqrt(5))/2;
F2=(sqrt(5)-1)/2;

J=x2-x1;
Js=J
x3=x1+F1·J;
x4=x1+F2·J;
while J>TolX·Js
  if (fun(x3)>fun(x4))
      x1=x3;          % desno je min
      x3=x4;
      J=x2-x1;
      x4=x1+F2·J
  else
      x2=x4;          % lijevo je min
      x4=x3;
      J=x2-x1;
      x3=x1+F1·J;
  end
end
  m=fun(x1);


function f=fun(x)

f=x.^2+x+3;
end
```

eval([fun '(' num2str(x) ')'])

'fun'

trazi_min(@fun,x1,x2)

ZLATNI REŽ



$$J = x_2 - x_1$$

$$x_3 = x_1 + F_1 \cdot J$$

$$x_4 = x_1 + F_2 \cdot J$$

ako $f(x_3) > f(x_4)$

$$x_1 = x_3$$
$$x_3 = x_4$$
$$J = x_2 - x_1$$
$$x_4 = x_1 + F_2 \cdot J$$

inače

$$x_2 = x_4$$
$$x_4 = x_3$$
$$J = x_2 - x_1$$
$$x_3 = x_1 + F_1 \cdot J$$

$$F_1 + F_2 = 1$$

$$\frac{1}{F_2} = \frac{F_2}{F_1}$$

$$F_2 = \frac{F_1}{F_2} \quad / \cdot F_2$$

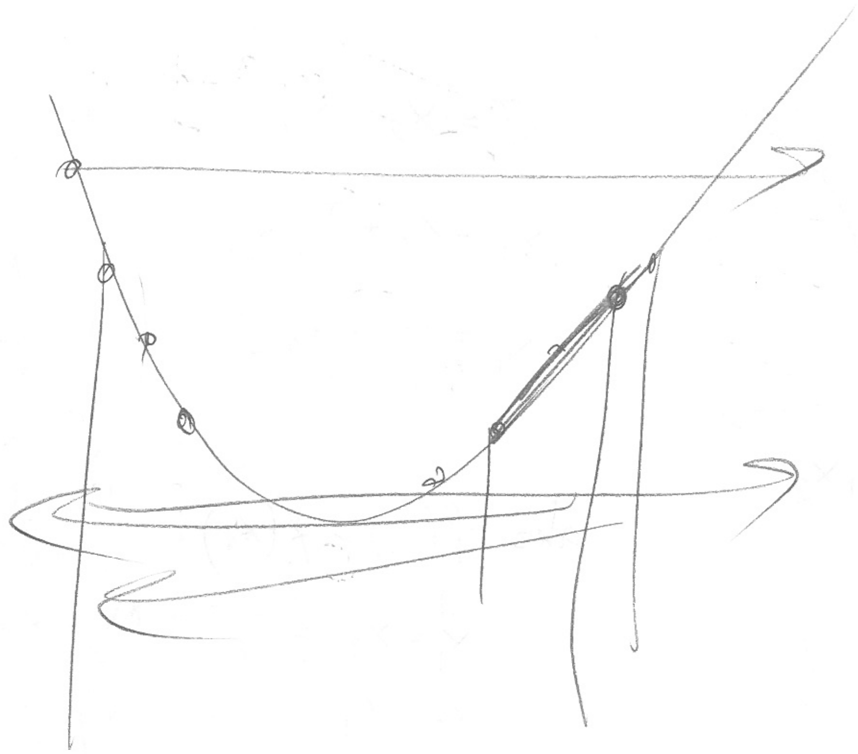
$$F_2^2 = F_1$$

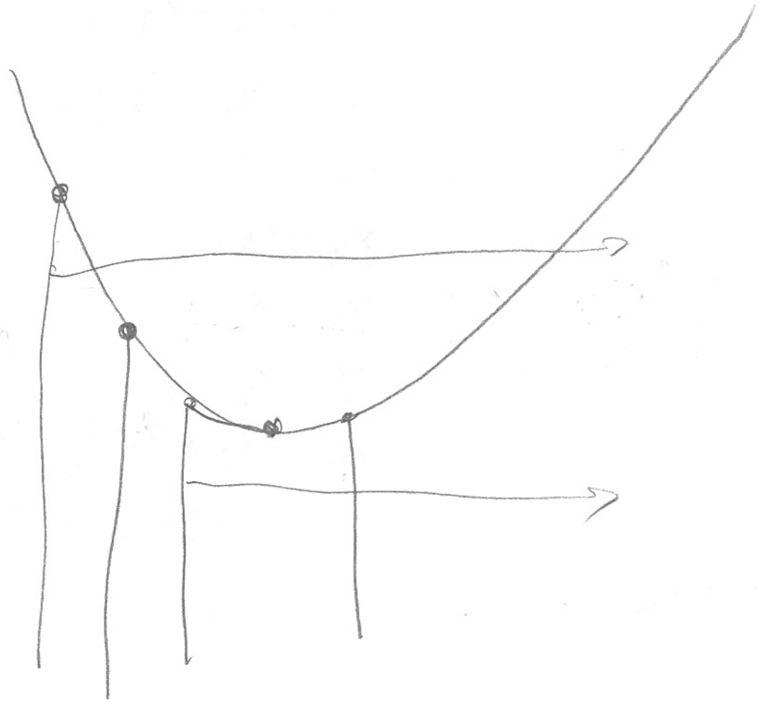
$$F_2 + F_2^2 = 1$$

$$F_2^2 + F_2 - 1 = 0$$

$$F_{2_{1,2}} = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2} = \frac{\sqrt{5}-1}{2}$$

$$F_1 = F_2^2 = \frac{(\sqrt{5}-1)^2}{4} = \frac{5 - 2\sqrt{5} + 1}{4} = \frac{6 - 2\sqrt{5}}{4} = \frac{3 - \sqrt{5}}{2}$$

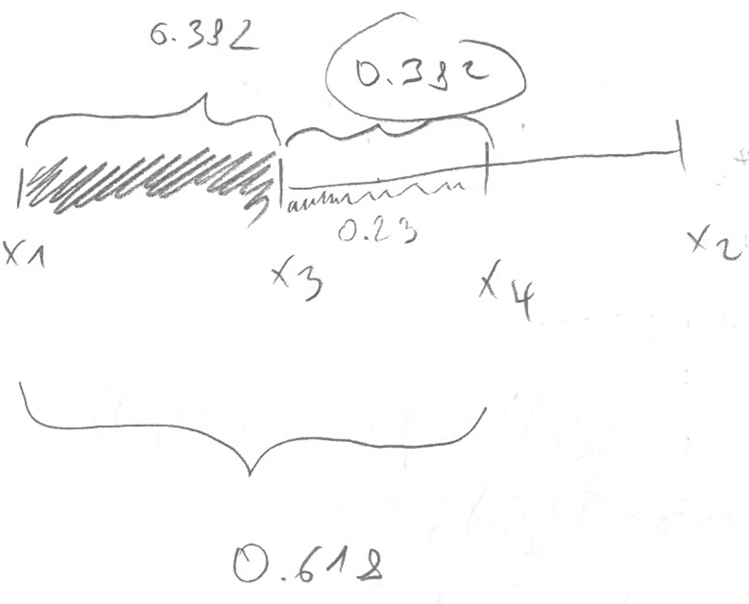




$F_2 - F_1 = 0.6180 - 0.382$

0.236

$1 - 0.382$



$x^2 + x + 3$

$2x + 1$

$2x + 1$

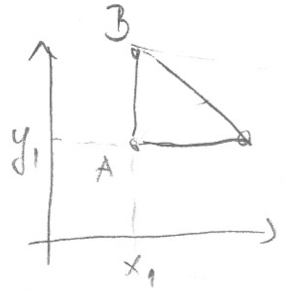
NELDER-MAD
SIMPLEX

```
function f = funkcija(P )
 x=P(1); y=P(2);
 f=2*x.^2 + 3*y.^2 -1;
 end
```

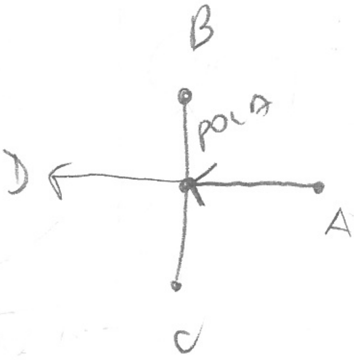
```
function [m, Pm]= najmanji(fun, P0 , TolX)
n=2;
e1=[0 1];
e2=[1 0];
P1 = A + n.e1;
P2= A + n.e2;
P = [P0; P1; P2];
while
 for i=2:1
   for j=1:i
     if fun(P(j,:)) < fun(P(j+1,:))
        temp = P(j,:);
        P(j,:) = P(j+1,:);
        P(j+1,:) = temp;
     end
   end
 end
 A = P(1,:)
 B = P(2,:)
 C = P(3,:)
```

$e^1=[0\ 1]$
$e^2=[1\ 0]$

```
if |fun(A) - fun(C)| < TolX
   break;
```



$$\begin{bmatrix} P_0 \\ P_1 \\ P_2 \end{bmatrix}$$

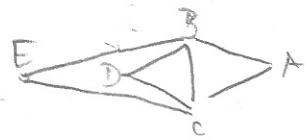


POLA = (B + C)/2 ;
vektor = POLA − A ;
D = POLA + vektor ;

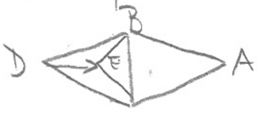
} refleksija

if fun(D) < fun(C)
    E = D + vektor ;
end

} ekspanzija

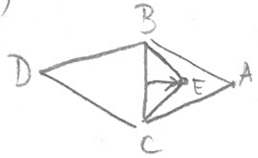


if fun(D) >= fun(C) && fun(D) < fun(A)
    E = D − vektor/2 ;
end



} kontrakcija (vanjska)

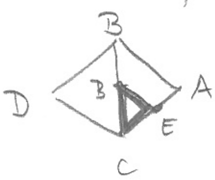
if fun(D) >= fun(A)
    E = D − $\frac{3}{2}$ · vektor ;
end



} kontrakcija (unutarnja)

if fun(D) > fun(C) && fun(D) > fun(B) && fun(D) < fun(A)
    B = C + (B − C)/2 ;
    E = C + (A − C)/2 ;
end

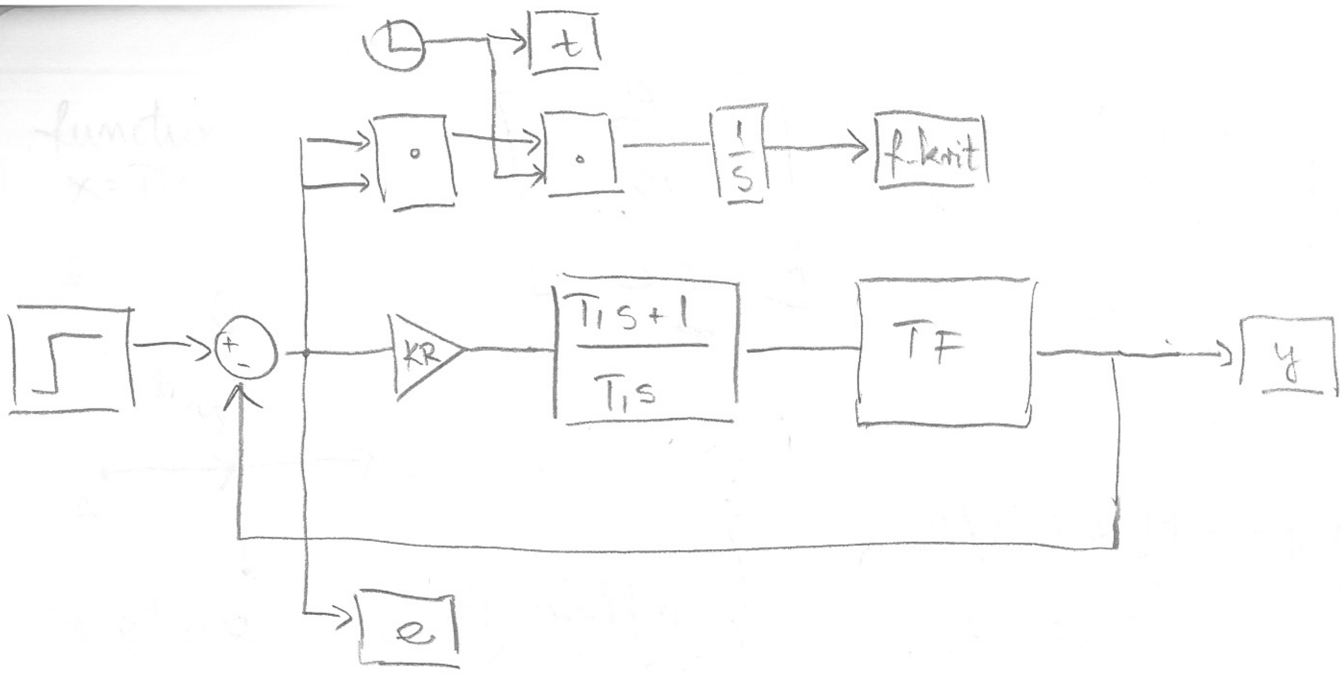
} sažimanje



A = E ;

P = [A ; B ; C] ;

end
m = fun(C) ;
Pm = C ;


t
f_krit
KR
$\frac{T_1 s + 1}{T_1 s}$
TF
y
e
$\frac{1}{s}$

```
function TR = fun(X)
    KR = X(1)
    T1 = X(2)

    set_param(.... KR);
    set_param(.... T1);
    sim('model', Tsim)

    for i = 1:length(t)
        if y(i) >= 0.1*y(end)
            t1 = t(i)
            break

    for i = 1:length
        if y(i) >= 0.9*y(end)
        t2 = t(i)
        break;

    TR = t2-t1;

    return;
end
```

$x^2 + 7 \le 0$

$x^7 + 3 = 0$

$x^2 < 2$

function [c ceq] = nonlcon(x)

$c = x^2 + 7$

$ceq = x^7 + 3$

return [c ceq]

nadvišenje < ~~40%~~ 30%

$t_{nad} = 0.5\,s$

nadvišenje − 30% ≤ 0

tnad − 0.5 = 0

```
function [c ceq] = nonlcon(x)
    T1 = x(2)
    KR = x(1)
    set-param (T1)
    set-param (KR)
    sim ('model')
    [nad idx] = max(y)
     nad = (nad - y(end))/y(end) *100
     tnad = t(idx);
     c = nad - 30;
     ceq = t - 0.5;
end;
```

fmin

function [T, K_R] = moja_funkcija (x_0,

x = fmincon

fmincon (, nonlincon)

fmincon

x = fmincon ('fun', [1 1], 'nonlcon', 'nonlco

```
K = fminbnd('f_cilja', Kmin, Kmax)

function KR = f_cilja(k)
tsim = 100;
set_param('model/KR', 'Gain', mat2str(k));
sim('model', tsim);
if y(tsim) == max(y)
    KR = k;
else
    KR = 0;
end
```

Gradijentna metoda:

odaberi $x_0 \in G$

$i = 1$

$x_i = x0$

$h(x_i) = -\nabla f(x_i)$

sve dok je $h(x_i) \neq 0$

  $h(x_i) = -\nabla f(x_i)$

  $g(t) = f[x_i + t \cdot h(x_i)]$, za sve $t \in [0, +\infty\rangle$

  naći t_0 : $g(t_0) = \min[g(t)]$

  $y = x_i + t_0 \cdot h(x_i)$

  $x_i = y$; $i = i$;

- Gradijent je smjer najvećeg rasta funkcije

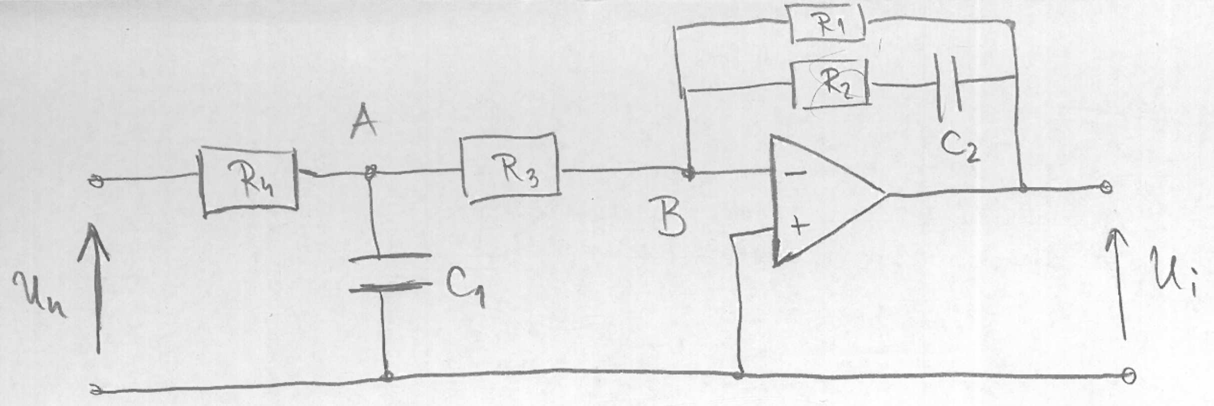
Newton

$$f(x+\Delta x) = f(x) + \frac{1}{1!} \cdot f'(x) \cdot \Delta x + \frac{1}{2!} \cdot f''(x) \cdot \Delta x^2 \Big/ \frac{\partial}{\partial \Delta x}$$

$$0 = 0 + f'(x) + f''(x) \cdot \Delta x$$

$$f''(x) \div / \quad -f'(x) = f''(x) \cdot \Delta x$$

$$-(f''(x))^{-1} \cdot f'(x) = \Delta x$$



PDT 2

$$\frac{u_u - u_a}{R_4} = \frac{u_a - 0}{R_3} + \frac{u_a - 0}{\frac{1}{sC_1}}$$

$$\frac{u_u - u_a}{R_4} = u_a\left(\frac{1}{R_3} + sC_1\right)$$

$$u_u = u_a\left(\frac{R_4}{R_3} + sC_1R_4 + 1\right)$$

$$\frac{u_a - 0}{R_3} = \frac{0 - u_i}{R_1} + \frac{0 - u_i}{R_2 + \frac{1}{sC_2}}$$

$$\frac{u_a}{R_3} = \frac{-\left(R_2 + \frac{1}{sC_2}\right)u_i - R_1u_i}{R_1\left(R_2 + \frac{1}{sC_2}\right)}$$

$$\frac{u_a}{R_3} = -u_i\frac{\left(R_2 + \frac{1}{sC_2} + R_1\right)}{R_1R_2 + \frac{R_1}{sC_2}}$$

$$\frac{1}{R_3}\frac{u_u}{\frac{R_4}{R_3} + sC_1R_4 + 1} = -u_i\frac{\frac{(R_1+R_2)\cdot sC_2 + 1}{\cancel{s}C_2}}{\frac{sR_1R_2C_2 + R_1}{\cancel{s}C_2}} = -u_i\frac{1 + sC_2(R_1+R_2)}{R_1(1 + sC_2R_2)}$$

$$\frac{1}{\cancel{R_3}}\frac{u_u}{\frac{R_4 + sC_1R_3R_4 + R_3}{\cancel{R_3}}}$$

$$\frac{u_i}{u_u} = \frac{1}{R_4 + R_3 + sC_1R_3R_4}\cdot\frac{-R_1(1 + sC_2R_2)}{1 + sC_2(R_1+R_2)} =$$

$$= -R_1\frac{1}{R_4 + R_3\left(1 + \frac{C_1R_3R_4}{R_3+R_4}s\right)}\frac{1 + sC_2R_2}{1 + sC_2(R_1+R_2)}$$

$$K = -\frac{R_1}{R_4 + R_1} \quad T_1 =$$

```
function f=euler(fun,y0,h,Tmax)

k=floor(Tmax/h);

t=0:h:Tmax;
y=zeros(k+1,1);
y(1)=y0;
for i=1:k
      y(i+1)=y(i)+ fun(t(i),y(i)).h;
end
end

fun=@(t,y) 5*sin(t)+y;

euler(@fun,1,2,100)
```

$$y(t) = e^{-t} \sin(\omega t) = y_1(t) \cdot y_2(t)$$

$$y_1(t) = e^{-t}$$

$$y_1'(t) = -e^{-t} = -y_1$$

$$y_1(0) = 1$$

$$y_2(t) = \sin(\omega t)$$

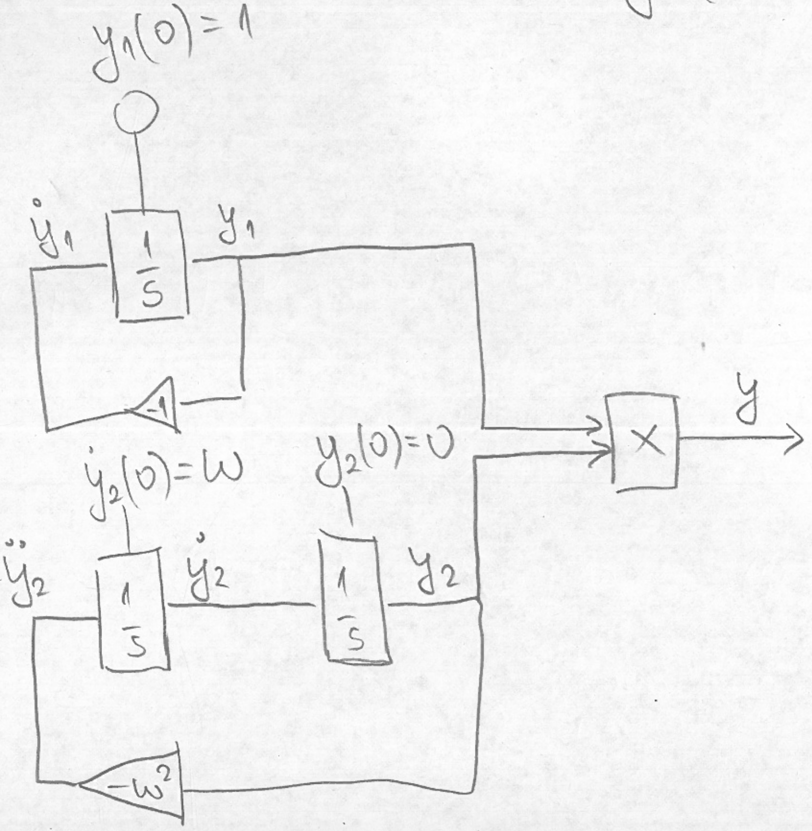
$$y_2'(t) = \omega \cos(\omega t)$$

$$y_2''(t) = -\omega^2 \sin(\omega t)$$

$$= -\omega^2 y_2(t)$$

$$y_2(0) = 0$$

$$y_2'(0) = \omega$$



$y(t) = At^2 \sin(\omega t + \varphi)$ $\quad y(0) = 0$

$\dot{y}(t) = \underbrace{2At \sin(\omega t + \varphi)}_{x_1} + \underbrace{At^2 \omega \cos(\omega t + \varphi)}_{x_2}$ $\quad x1(0) = 0$, $x2(0) = 0$

$\dot{x}_1 = \underbrace{2A \sin(\omega t + \varphi)}_{x_3} + \underbrace{2At\omega \cos(\omega t + \varphi)}_{x_4}$ $\quad x3(0) = 2A\sin\varphi$

$x_3 = 2A \sin(\omega t + \varphi)$ $\quad x_4 = 2At\omega\cos(\omega t + \varphi)$

$\dot{x}_3 = 2A\omega \cos(\omega t + \varphi)$ $\quad \dot{x}3(0) = 2A\omega\cos\varphi$

$\ddot{x}_3 = -2A\omega^2 \sin(\omega t + \varphi)$

$= -\omega^2 x_3$

$\dot{x}_2 = \underbrace{2At\omega\cos(\omega t + \varphi)}_{x_4} - \underbrace{At^2\omega^2 \sin(\omega t + \varphi)}_{-\omega^2 y}$ $\quad x4(0) = 0$

$\dot{x}_4 = \underbrace{2A\omega\cos(\omega t + \varphi)}_{\dot{x}_3} - 2At\omega^2 \sin(\omega t + \varphi)$

$- \omega^2 x_1$