Modeliranje i simulacija sustava
Zadaci za vježbu

1. Odredite **diferencijalnu jednadžbu** i nacrtajte **graf toka signala** za sustav opisan prijenosnom funkcijom

$$G(s) = \frac{Y(s)}{U(s)} = \frac{4s^2 + 3s + 1}{1.5s^3 + 2s + 1}$$

2. Odredite **diferencijalnu jednadžbu** i nacrtajte **blokovsku shemu** za sustav opisan prijenosnom funkcijom

$$G(s) = \frac{Y(s)}{U(s)} = \frac{3e^{-2.7t}}{2s^2 + 3s + 1}$$

3. Linearni dinamički sustav s vremenski nepromjenjivim parametrima koji ima dva ulaza i jedan izlaz opisan je prijenosnom funkcijom:

$$Y(s) = \frac{2s^2 + 1.5s + 1}{1.5s^4 + 2s^3 + 1}U_1(s) + \frac{s + 2}{1.5s^4 + 2s^3 + 1}U_2(s)$$

Sustav je potrebno prikazati u prostoru stanja.

4. Linearni dinamički sustav s vremenski nepromjenjivim parametrima koji ima dva ulaza i dva izlaza opisan je matricama u prostoru varijabli stanja:

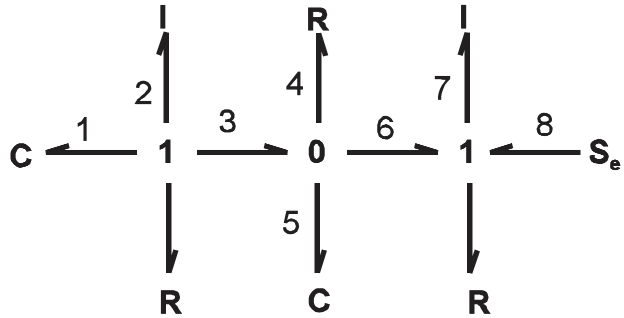
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 0 \\ 0 & 1 & 2 \\ 5 & 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 0 & 0 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 3 & 7 & 0 \\ 8 & 5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Potrebno je odrediti:

a) prijenosnu matricu sustava,
b) blokovsku shemu sustava,
c) diferencijalne jednadžbe sustava.

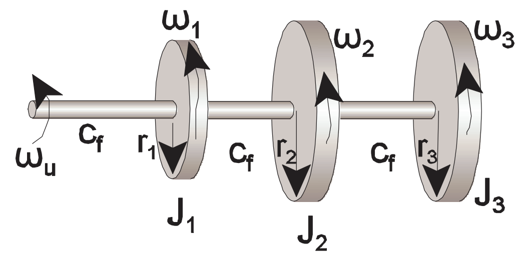
5. Na bond graf sustava prema slici, potrebno je postaviti crtice kauzalnosti.



6. Mehanički rotacijski sustav prikazan je slikom
Parametri sustava su slijedeći:

$c_f=9.8\cdot10^4$ Nm/rad $\quad J_1 = 19.2913$ kgm

$J_2 = 52.9354$ kgm, $\quad J_3 = 30.2913$ kgm



Sustav je potrebno opisati diferencijalnim jednadžbama, te prikazati u prostoru stanja ako je izlazna varijabla ω_3, a ulazna varijabla ω_u.
Sustav je potrebno prikazati blokovskom shemom i bond grafovima.

7. Zadan je rotacijski sustav s dvije mase prema slici, kod kojeg su:

$c_f=9.8\cdot10^4$ Nm/rad

$J_1 = 20.2913$ kgm

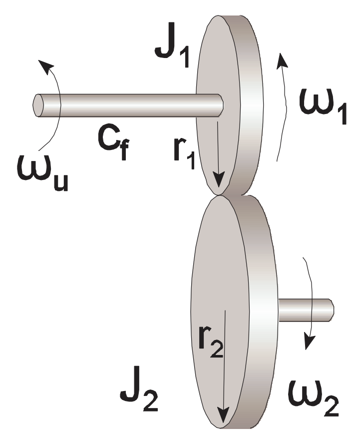
$J_2 = 50.9354$ kgm,

$r_1=0.5$ m,

$r_2=0.75$ m.

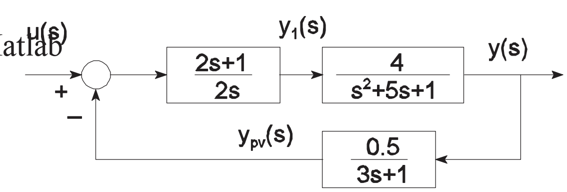
Potrebno je odrediti prijenosnu funkciju sustava

$G(s) = \dfrac{\omega_1(s)}{\omega_u(s)}$, te frekvenciju vlastitih oscilacija.

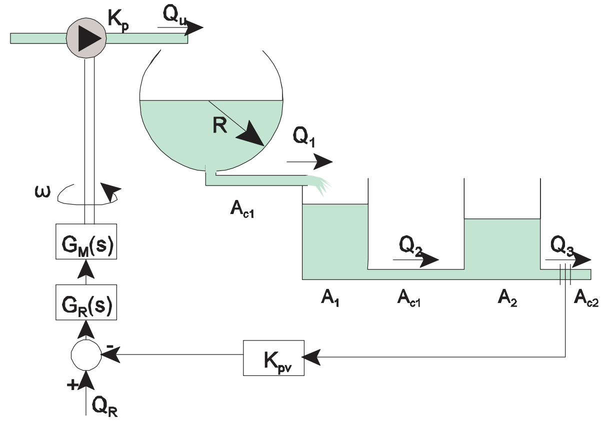


Sustav je osim toga potrebno prikazati bond grafom i blokovskom shemom.

8. Za sustav prema slici potrebno je odrediti simulacijsku shemu sustava za Matlab (Simulink) tako da se koriste samo sumatori, integratori, elementi pojačanja (gain), izvor signala (step), multiplexor i blok za spremanje podataka u varijablu u Workspace-u, ako je u(t)=5S(t) [V].



9. Za sustav prema slici, sa slijedećim parametrima:



$K_p = 1 \left[\frac{m^3}{\min}\cdot\frac{s}{rad}\right]$,

$G_m(s) = \frac{10}{1 + 0.5s} \left[\frac{rad}{s\cdot V}\right]$,

$G_R(s) = \left(1 + \frac{1}{0.5s}\right) \left[\frac{V}{V}\right]$,

$K_{pv} = 1 \left[\frac{V\cdot\min}{m^3}\right]$, $A_1 = 20\ m^2$,

$A_2 = 25\ m^2$, $A_{c1} = A_{c2} = 250\ cm^2$,

R=3m.

Potrebno je odrediti:

a) Odrediti nelinearni matematički model i nacrtati nelinearnu shemu sustava

b) Nacrtati simulacijsku shemu sustava za programski paket matlab, uz upisane vrijednosti parametara, takovu da budu mjerljive veličine Q_1, Q_2, Q_u.

c) Linearizirati proces u radnoj točci određenoj s Q_R=8m^3/min i odrediti prijenosnu funkciju $G(s)=Q_2(s)/Q_R(s)$.

Pretpostavlja se da su strujanja laminarna, da je masa tekućine u cijevima zanemariva i da je brzina tekućine u rezervoarima zanemariva prema brzini u cijevima. Kontrakcija mlaza je 1, a mjerni uređaji ne utječu na protoke.

1.

$$G(s)=\frac{Y(s)}{U(s)}=\frac{4s^2+3s+1}{1,5s^3+2s+1}$$

Diferencijalna jednadžba:

$$1,5s^3Y(s)+2sY(s)+Y(s)=4s^2U(s)+3sU(s)+U(s)$$

$$1,5y'''(t)+2y'(t)+y(t)=4u''(t)+3u'(t)+u(t)$$

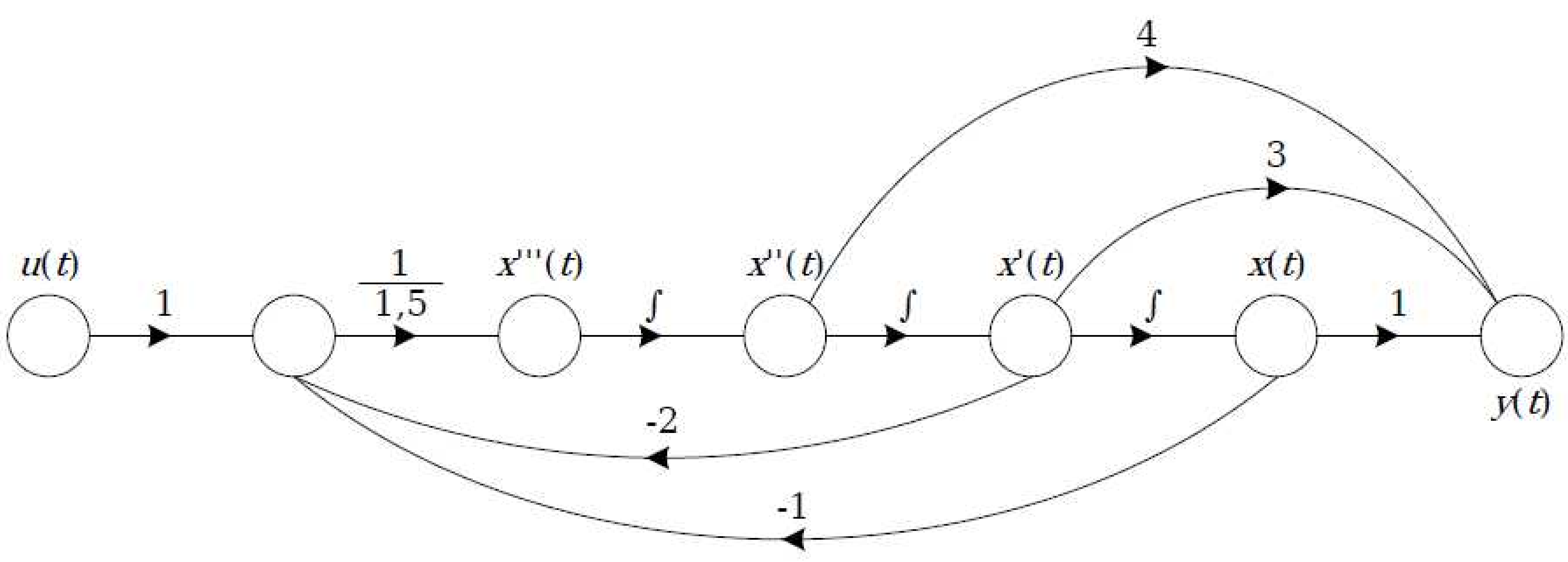
Graf toka signala:

$$\frac{4s^2+3s+1}{1,5s^3+2s+1}=\frac{X(s)}{U(s)}\frac{Y(s)}{X(s)}$$

$$\frac{X(s)}{U(s)}=\frac{1}{1,5s^3+2s+1}\rightarrow u(t)=1,5x'''(t)+2x'(t)+x(t)$$

$$x'''(t)=\frac{1}{1,5}[u(t)-2x'(t)-x(t)]$$

$$\frac{Y(s)}{X(s)}=4s^2+3s+1\rightarrow y(t)=4x''(t)+3x'(t)+x(t)$$



2.

$$G(s) = \frac{Y(s)}{U(s)} = \frac{3e^{-2s}}{2s^2 + 3s + 1}$$

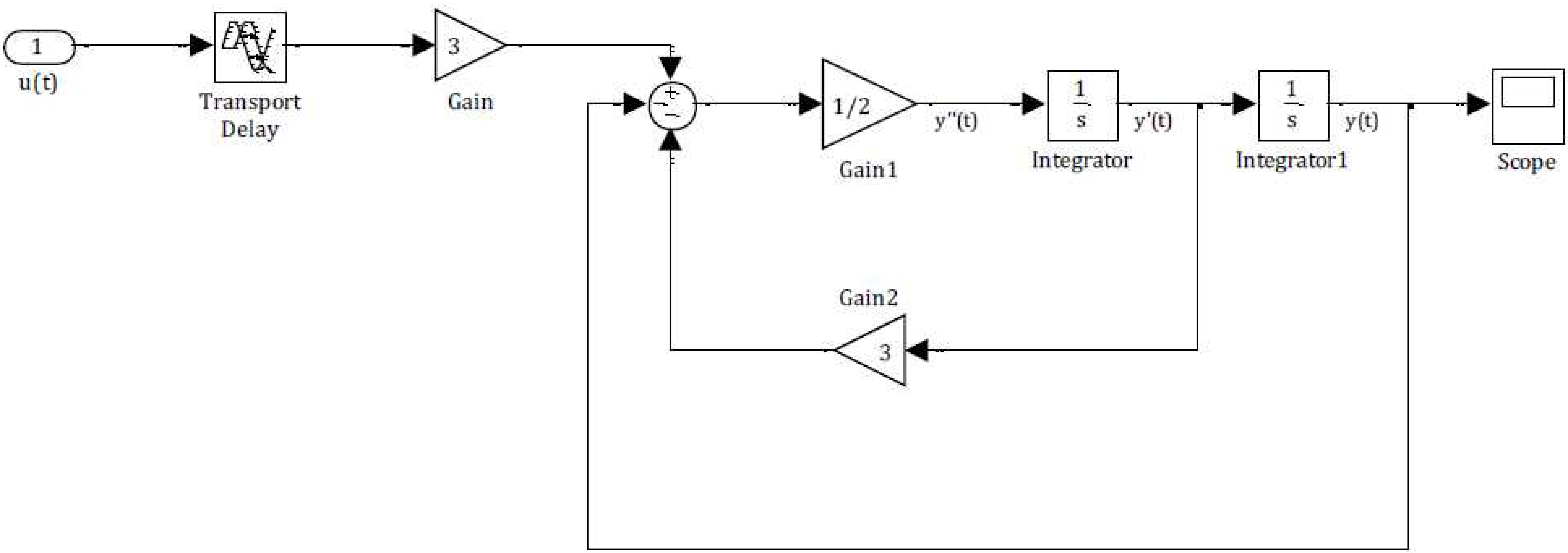
Diferencijalna jednadžba:

$$2s^2Y(s) + 3sY(s) + Y(s) = 3e^{-2s}U(s)$$

$$2y''(t) + 3y'(t) + y(t) = 3u(t-2)S(t-2)$$

Blokovska shema:

$$y''(t) = \frac{1}{2}[3u(t-2)S(t-2) - 3y'(t) - y(t)]$$



4.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}2 & 3 & 0\\ 0 & 2 & 1\\ 5 & 4 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0 & 2\\ 0 & 0\\ 3 & 4\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}3 & 7 & 0\\ 8 & 5 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

Prijenosna matrica sustava:

$$G(s) = C(sI-A)^{-1}B + D = \begin{bmatrix}3 & 7 & 0\\ 8 & 5 & 1\end{bmatrix}\begin{bmatrix}s-2 & -3 & 0\\ 0 & s-2 & -1\\ -5 & -4 & s-1\end{bmatrix}^{-1}\begin{bmatrix}0 & 2\\ 0 & 0\\ 3 & 4\end{bmatrix} =$$

$$= \begin{bmatrix}\dfrac{21s-15}{s^3-5s^2+4s-11} & \dfrac{6s^2+10s+38}{s^3-5s^2+4s-11}\\ \dfrac{3s^2+3s+54}{s^3-5s^2+4s-11} & \dfrac{20s^2-34s+70}{s^3-5s^2+4s-11}\end{bmatrix}$$

Blokovska shema:

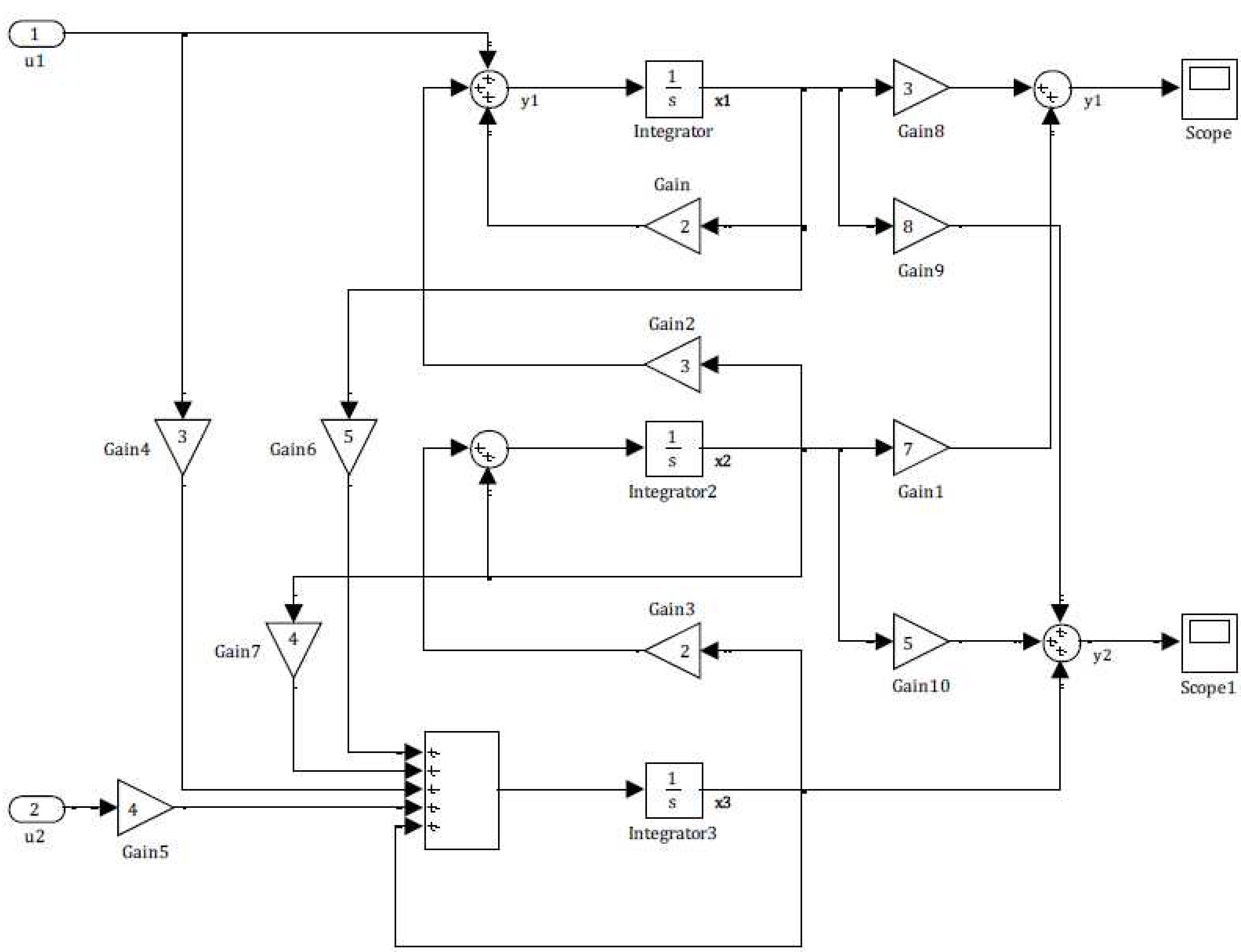
$$\dot{x}_1 = 2x_1 + 3x_2 + 2u_2$$

$$\dot{x}_2 = x_2 + +2x_3$$

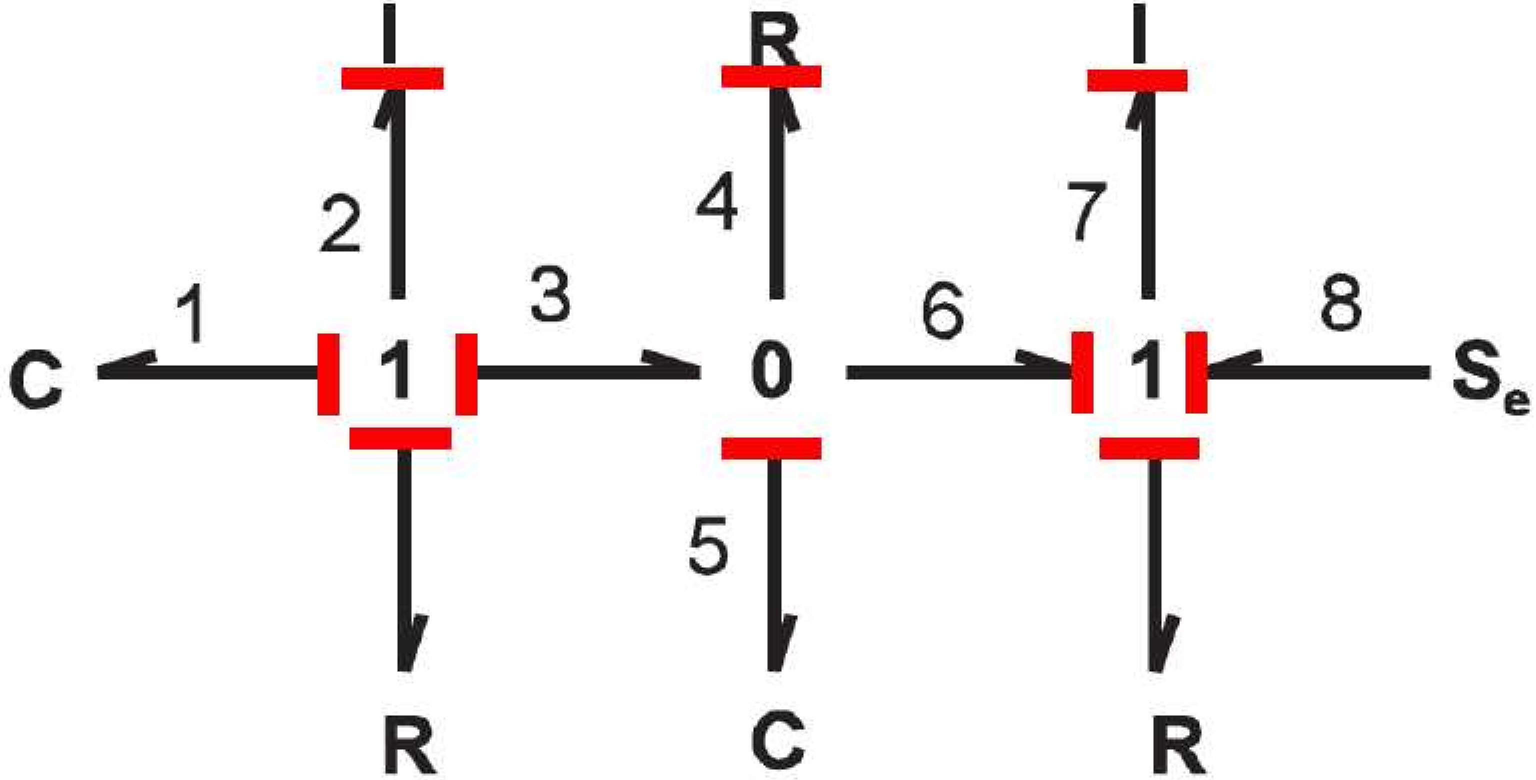
$$\dot{x}_3 = 5x_1 + 4x_2 + x_3 + 3u_1 + 4u_2$$

$$y_1 = 3x_1 + 7x_2$$

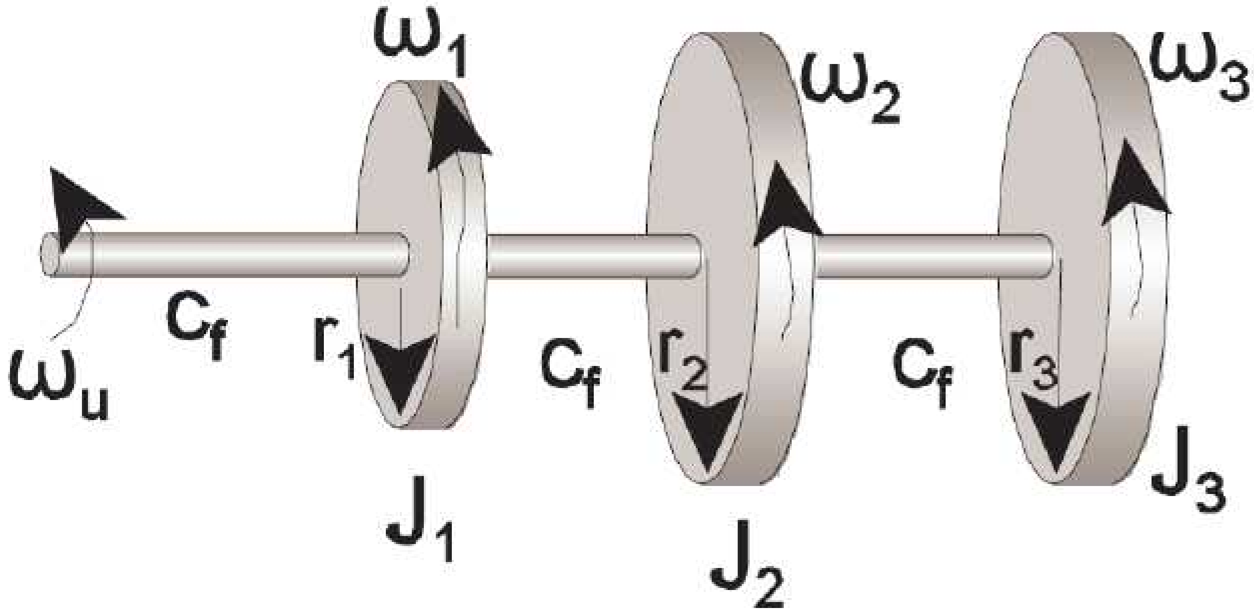
$$y_2 = 8x_1 + 5x_2 + x_3$$



5. Bond graf s crticama kauzalnosti:



6.



Diferencijalne jednadžbe:

$$m_1 - m_2 = J_1 \frac{d\omega_1}{dt}$$

$$m_2 - m_3 = J_2 \frac{d\omega_2}{dt}$$

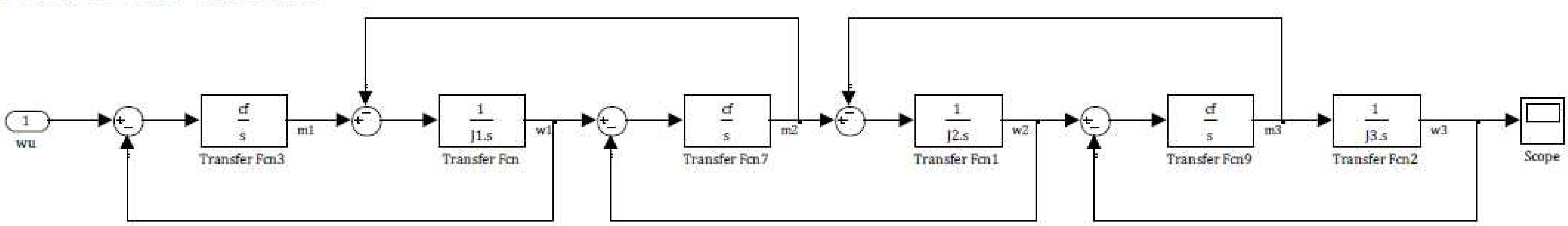
$$m_3 = J_3 \frac{d\omega_3}{dt}$$

$$m_1 = c_f(\varphi_u - \varphi_1) \rightarrow \frac{dm_1}{dt} = c_f(\omega_u - \omega_1)$$

$$m_2 = c_f(\varphi_1 - \varphi_2) \rightarrow \frac{dm_2}{dt} = c_f(\omega_1 - \omega_2)$$

$$m_3 = c_f(\varphi_2 - \varphi_3) \rightarrow \frac{dm_3}{dt} = c_f(\omega_2 - \omega_3)$$

Blokovska shema:



Prostor stanja:

$$x_1 = \omega_3;\ \ x_2 = m_3;\ \ x_3 = \omega_2;\ \ x_4 = m_2;\ \ x_5 = \omega_1;\ \ x_6 = m_1;\ \ y = \omega_3;\ \ u = \omega_u$$

$$\frac{dx_5}{dt} = -\frac{1}{J_1}x_4 + \frac{1}{J_1}x_6$$

$$\frac{dx_3}{dt} = -\frac{1}{J_2}x_2 + \frac{1}{J_2}x_4$$

$$\frac{dx_1}{dt} = \frac{1}{J_3}x_2$$

$$\frac{dx_6}{dt} = c_f(u - x_5)$$

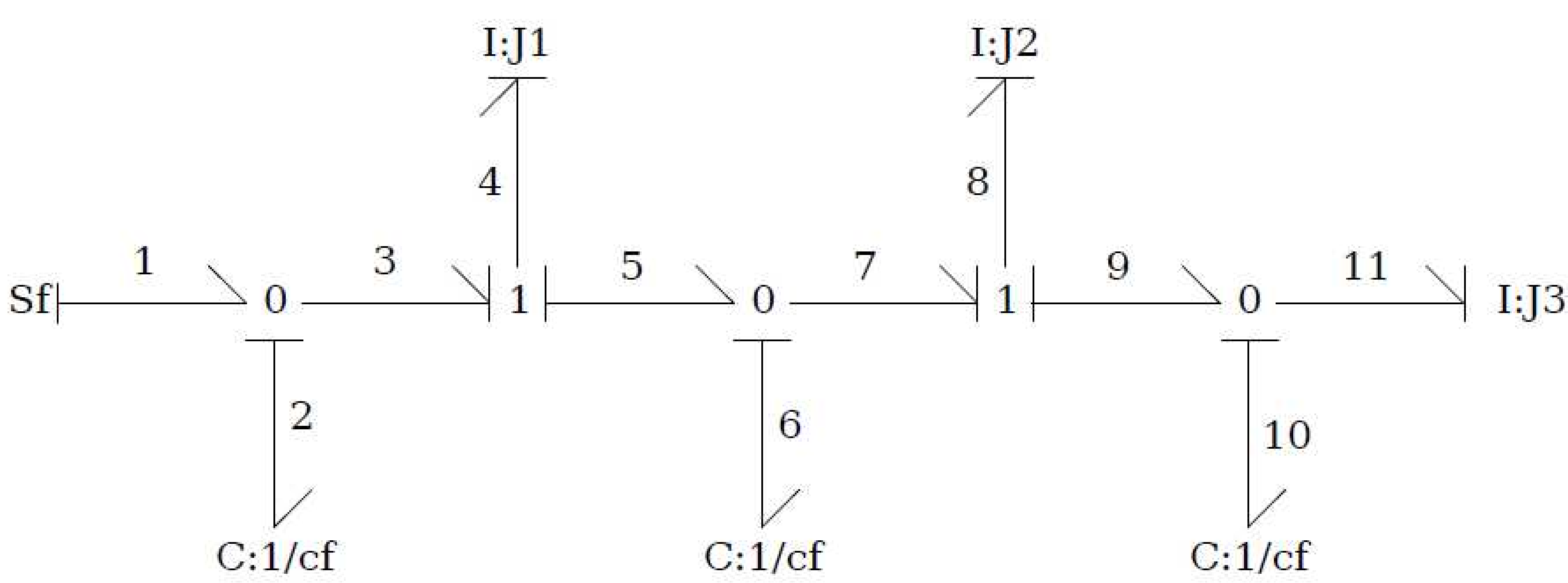
$$\frac{dx_4}{dt} = c_f(x_5 - x_3)$$

$$\frac{dx_2}{dt} = c_f(x_3 - x_1)$$

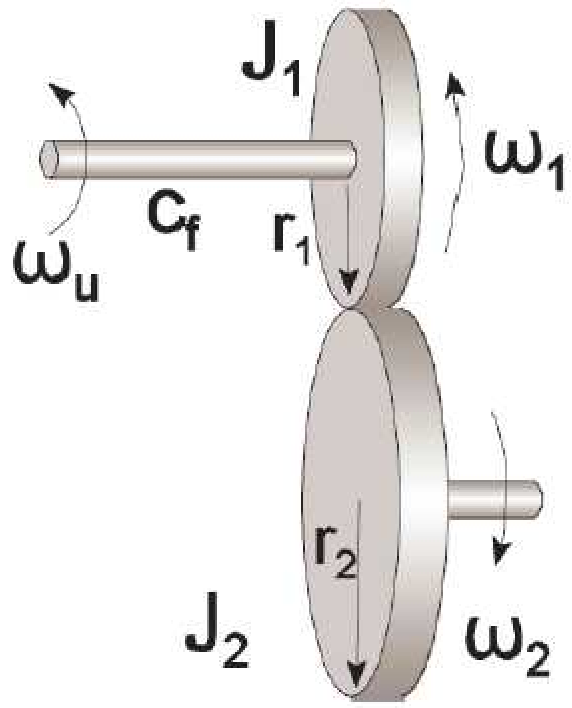
$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dx_3}{dt} \\ \frac{dx_4}{dt} \\ \frac{dx_5}{dt} \\ \frac{dx_6}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{J_3} & 0 & 0 & 0 & 0 \\ -c_f & 0 & c_f & 0 & 0 & 0 \\ 0 & -\frac{1}{J_2} & 0 & \frac{1}{J_2} & 0 & 0 \\ 0 & 0 & -c_f & 0 & c_f & 0 \\ 0 & 0 & 0 & -\frac{1}{J_1} & 0 & \frac{1}{J_1} \\ 0 & 0 & 0 & 0 & -c_f & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ c_f \end{bmatrix} [u]$$

$$[y] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + [0][u]$$

Bond graf:



7.



Diferencijalne jednadžbe:

$$m_1 = J_{1uk}\frac{d\omega_1}{dt}$$

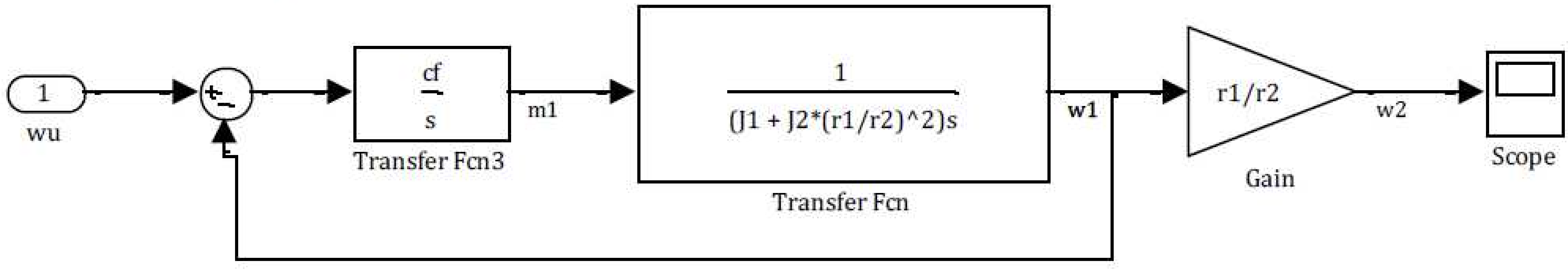
$$J_{1uk} = J_1 + J_2^*$$

$$\omega_1 r_1 = \omega_2 r_2$$

$$J_2\omega_2^2 = J_2^*\omega_1^2 \rightarrow J_2^* = J_2\left(\frac{\omega_2}{\omega_1}\right)^2 = J_2\left(\frac{r_1}{r_2}\right)^2$$

$$m_1 = c_f(\varphi_u - \varphi_1) \rightarrow \frac{dm_1}{dt} = c_f(\omega_u - \omega_1)$$

Blokovska shema:



Prijenosna funkcija:

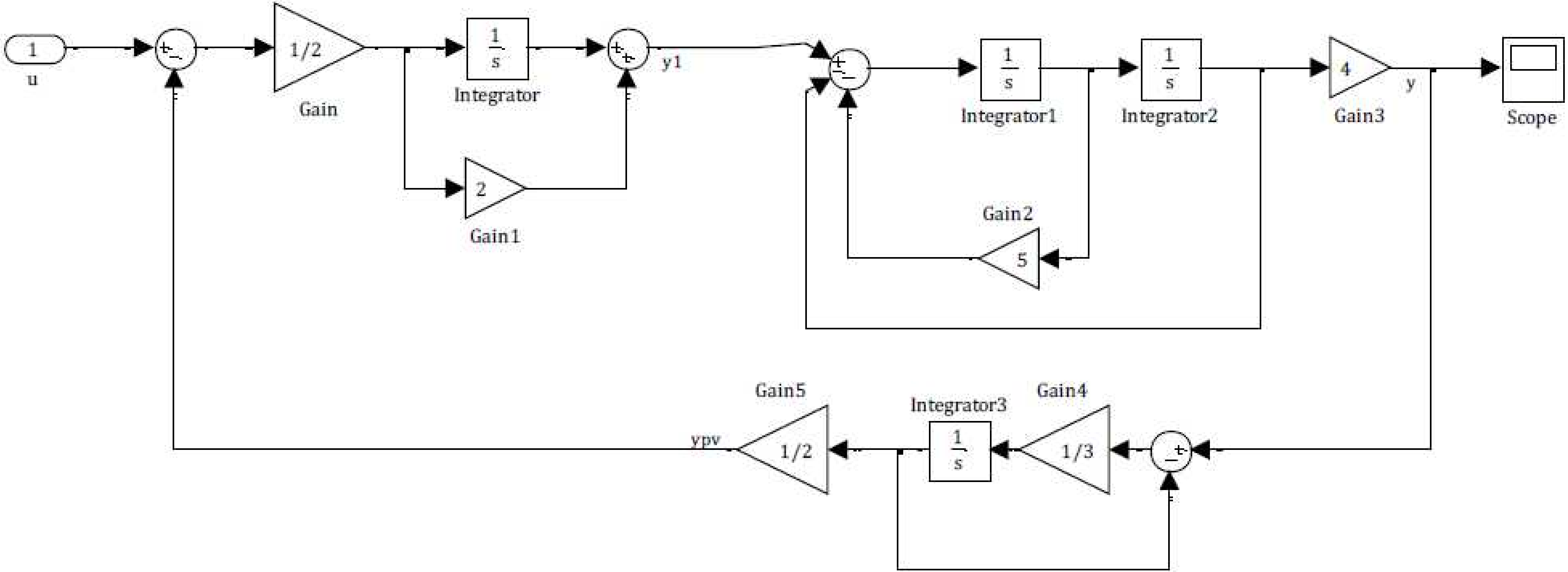
$$M_1 = J_{1uk}s\Omega_1$$

$$sM_1 = c_f(\Omega_u - \Omega_1) \rightarrow M_1 = \frac{c_f}{s}(\Omega_u - \Omega_1)$$

$$\frac{c_f}{s}(\Omega_u - \Omega_1) = J_{1uk}s\Omega_1 \rightarrow \frac{c_f}{s}\Omega_u = \left(J_{1uk}s + \frac{c_f}{s}\right)\Omega_1 = \frac{J_{1uk}s^2 + c_f}{s}\Omega_1$$

$$G(s) = \frac{\Omega_1(s)}{\Omega_u(s)} = \frac{c_f}{J_{1uk}s^2 + c_f} = \frac{9{,}8\cdot 10^4}{42{,}93s^2 + 9{,}8\cdot 10^4}$$

8. Blokovska shema:



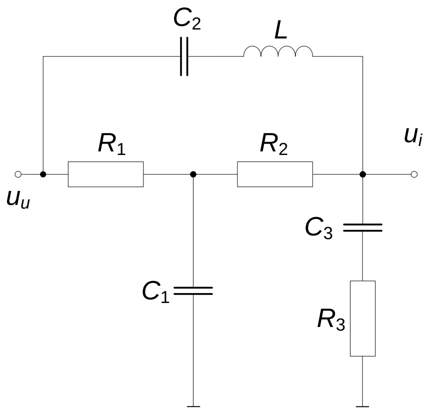
Modeliranje i simuliranje sustava
1. međuispit
2009./2010.

1. Zadan je sustav opisan diferencijalnom jednadžbom:

$$4\frac{d^3y(t)}{dt^3}+3\frac{d^2y(t)}{dt^2}+2\frac{dy(t)}{dt}+y(t)=\frac{d^3u(t)}{dt^3}+2\frac{d^2u(t)}{dt^2}+3\frac{du(t)}{dt}+4u(t).$$

Potrebno je:
(a) nacrtati blokovsku shemu sustava za određivanje odziva na jediničnu skokovitu pobudu uz korištenje osnovnih blokova (integrator, sumator, množenje s konstantom, step funkcija),
(b) izračunati stacionarno stanje sustava pri djelovanju jedinične skokovite pobude;
(c) opisati sustav u prostoru stanja uz izbor izlaza iz integratora na blokovskoj shemi kao varijabli stanja.

2. Pasivnu mrežu prema slici potrebno je prikazati bond grafom, ako je ulazni naponski signal u_u, a na izlazu (u_i) nije priključeno nikakvo trošilo.

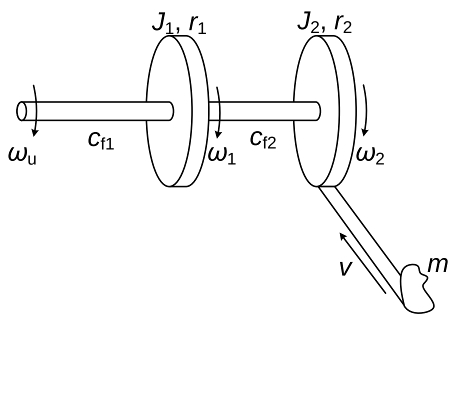


3. Sustav opisan prijenosnom funkcijom

$$G(s)=\frac{Y(s)}{U(s)}=\frac{5}{(1+120s)(1+20s)},$$

potrebno je diskretizirati korištenjem ZOH metode uz vrijeme uzorkovanja $T_d=1$ s te napisati rekurzivnu jednadžbu sustava. Diskretnu prijenosnu funkciju potrebno je zapisati na način da je uz najvišu potenciju u nazivniku koeficijent 1. Ukratko objasnite o čemu treba voditi računa prilikom implementacije dobivene rekurzivne jednadžbe u neko realno hardversko okruženje.

4. Zadan je rotacijski sustav s krutom zupčastom letvom prema slici, kod kojeg su $c_{f1}=c_{f2}=100$ Nm/rad, $J_1=J_2=30$ kgm^2, $m=6$ kg, $r_1=r_2=0.6$ m. Ulazna veličina sustava je kutna brzina ω_u, a gubici zbog trenja i zračnosti kod prijenosa energije između diska i letve su zanemarivi.
Potrebno je odrediti prijenosnu funkciju $G(s)=\frac{V(s)}{\Omega_u(s)}$, frekvenciju i period vlastitih oscilacija sustava i prikazati sustav bond grafom.



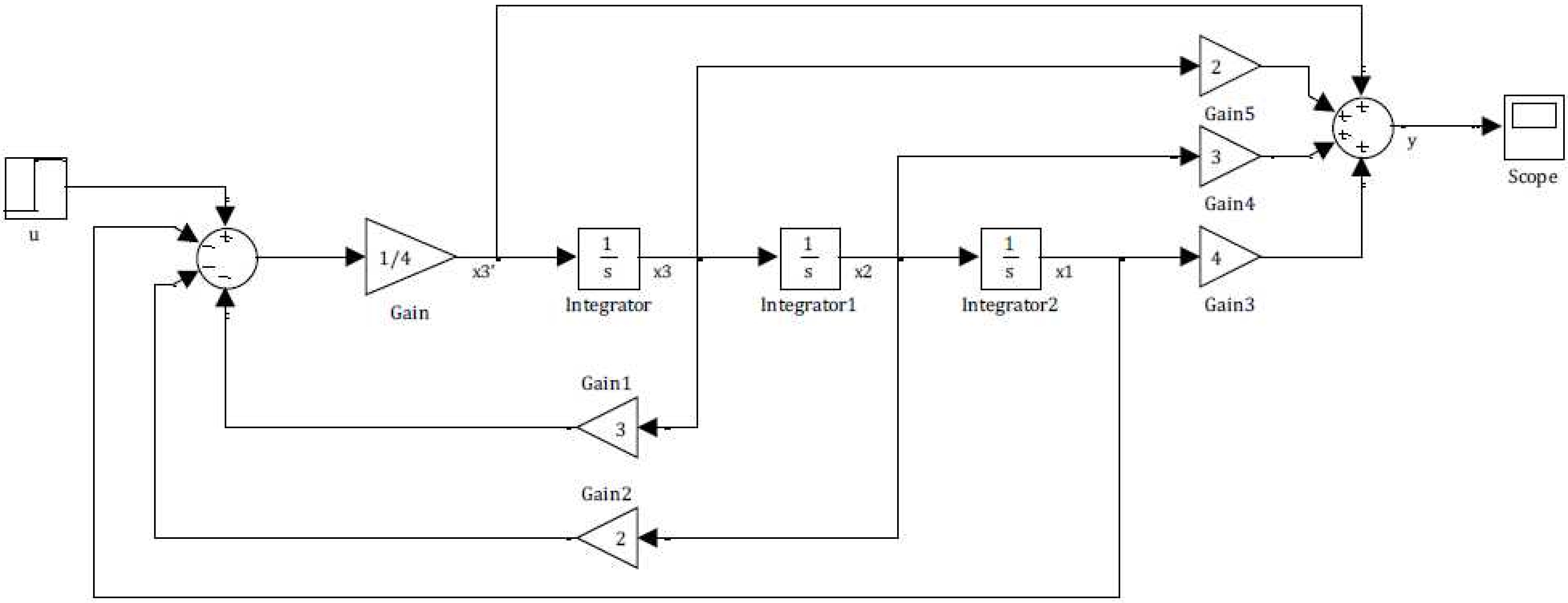
1.

$$G(s)=\frac{Y(s)}{U(s)}=\frac{X(s)}{U(s)}\frac{Y(s)}{X(s)}=\frac{s^3+2s^2+3s+4}{4s^3+3s^2+2s+1}$$

(a)

$$\frac{X(s)}{U(s)}=\frac{1}{4s^3+3s^2+2s+1}\rightarrow x'''(t)=\frac{1}{4}[u(t)-3x''(t)-2x'(t)-x(t)]$$

$$\frac{Y(s)}{X(s)}=s^3+2s^2+3s+4\rightarrow y(t)=x'''(t)+2x''(t)+3x'(t)+4x(t)$$



(b)

$$\lim_{s\to 0}\frac{1}{s}\cdot s\cdot G(s)=\frac{4}{1}=4$$

(c)

$$x_1=x$$

$$\dot{x}_1=x_2$$

$$\dot{x}_2=x_3$$

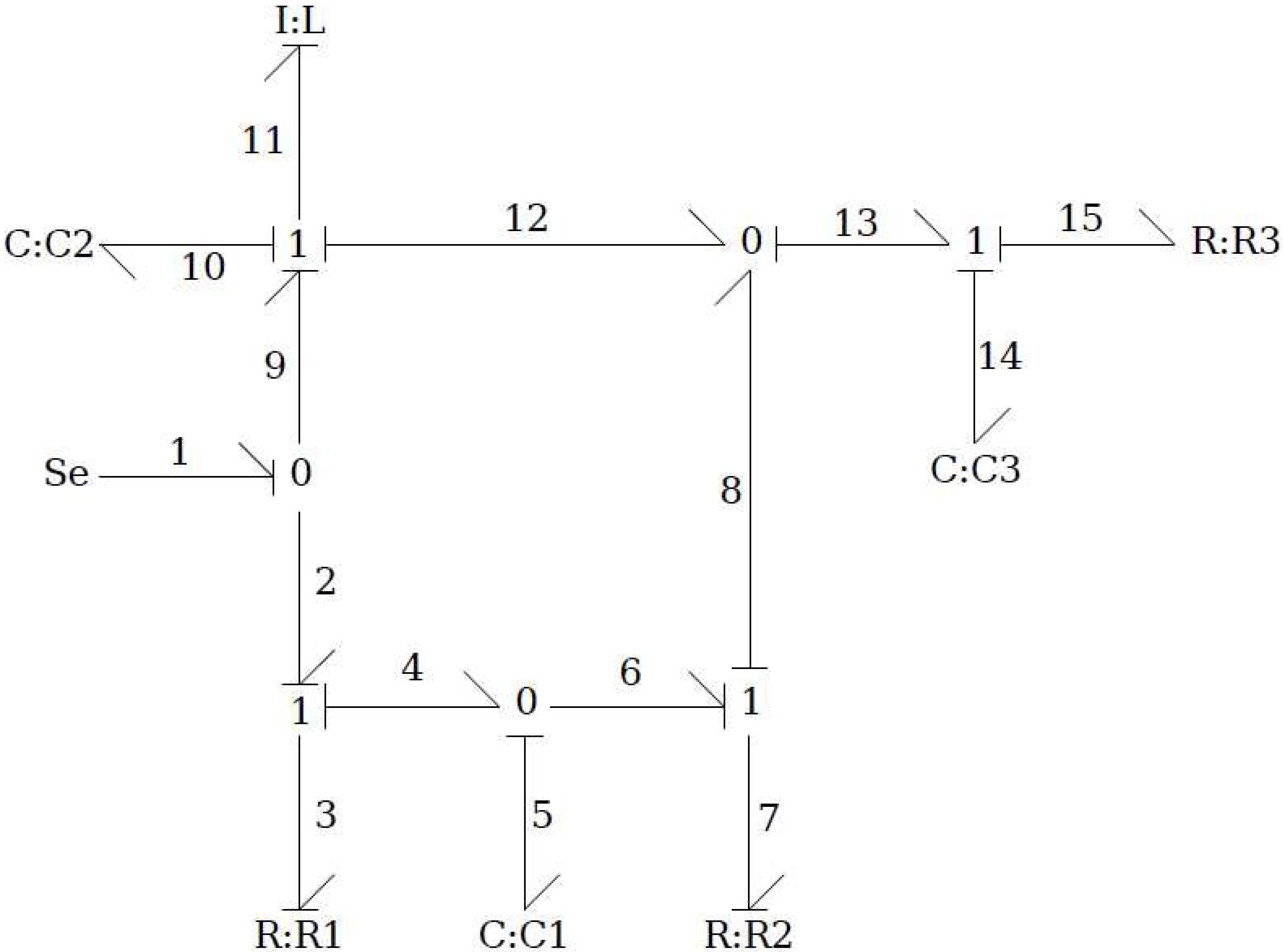
$$\dot{x}_3=\frac{1}{4}(u-3x_3-2x_2-x_1)$$

$$y=\frac{1}{4}(u-3x_3-2x_2-x_1)+2x_3+3x_2+4x_1=\frac{15}{4}x_1+\frac{10}{4}x_2+\frac{5}{4}x_3+\frac{1}{4}u$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0&1&0\\0&0&1\\-\frac{1}{4}&-\frac{2}{4}&-\frac{3}{4}\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}0\\0\\\frac{1}{4}\end{bmatrix}[u]$$

$$[y]=\begin{bmatrix}\frac{15}{4}&\frac{10}{4}&\frac{5}{4}\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}\frac{1}{4}\end{bmatrix}[u]$$

2.


I:L
11
C:C2
10
1
12
0
13
1
15
R:R3
9
14
Se
1
0
8
C:C3
2
4
6
1
0
1
3
5
7
R:R1
C:C1
R:R2

3.

$$G(z) = (1 - z^{-1})Z\left(\frac{G(s)}{s}\right) = \frac{z-1}{z}Z\left(\frac{5}{s(1+120s)(1+20s)}\right)$$

$$\frac{5}{s(1+120s)(1+20s)} = \frac{5}{2400}\frac{1}{s\left(s+\frac{1}{120}\right)\left(s+\frac{1}{20}\right)} = \frac{5}{2400}\left(\frac{A}{s} + \frac{B}{s+\frac{1}{120}} + \frac{C}{s+\frac{1}{20}}\right)$$

$$\frac{1}{s\left(s+\frac{1}{120}\right)\left(s+\frac{1}{20}\right)} = \frac{A}{s} + \frac{B}{s+\frac{1}{120}} + \frac{C}{s+\frac{1}{20}} \rightarrow A = 2400;\ \ B = -2880;\ \ C = 480$$

$$G(z) = \frac{1}{480}\frac{z-1}{z}\left(2400\frac{z}{z-1} - 2880\frac{z}{z-e^{-\frac{1}{120}}} + 480\frac{z}{z-e^{-\frac{1}{20}}}\right)$$

$$G(z) = \frac{0{,}001022z + 0{,}001002}{z^2 - 1{,}943z + 0{,}9433}$$

4. Diferencijalne jednadžbe:

$$m_1 - m_2 = J_1 \frac{d\omega_1}{dt}$$

$$m_2 = J_{2uk} \frac{d\omega_2}{dt}$$

$$m_1 = c_{f1}(\varphi_u - \varphi_1) \rightarrow \frac{dm_1}{dt} = c_{f1}(\omega_u - \omega_1)$$

$$m_2 = c_{f2}(\varphi_1 - \varphi_2) \rightarrow \frac{dm_2}{dt} = c_{f2}(\omega_1 - \omega_2)$$

$$J_{2uk} = J_2 + mr_2^2$$

$$v = \omega_2 r_2 \rightarrow \omega_2 = \frac{v}{r_2}$$

Prijenosna funkcija:

$$\frac{c_{f1}(\omega_u - \omega_1)}{s} - \frac{c_{f2}(\omega_1 - \omega_2)}{s} = J_1 s \omega_1$$

$$c_{f2}(\omega_1 - \omega_2) = J_{2uk} s^2 \omega_2 \rightarrow c_{f2}\omega_1 = \left(J_{2uk}s^2 + c_{f2}\right)\omega_2 \rightarrow \omega_1 = \frac{J_{2uk}s^2 + c_{f2}}{c_{f2}}\omega_2$$

$$c_{f1}(\omega_u - \omega_1) - c_{f2}(\omega_1 - \omega_2) = J_1 s^2 \omega_1$$

$$c_{f1}\left(\omega_u - \frac{J_{2uk}s^2 + c_{f2}}{c_{f2}}\omega_2\right) - c_{f2}\left(\frac{J_{2uk}s^2 + c_{f2}}{c_{f2}}\omega_2 - \omega_2\right) = \frac{J_1 J_{2uk} s^4 + J_1 c_{f2} s^2}{c_{f2}}\omega_2$$

$$c_{f1}\omega_u - \frac{J_{2uk}c_{f1}s^2 + c_{f1}c_{f2}}{c_{f2}}\omega_2 - J_{2uk}s^2\omega_2 = \frac{J_1 J_{2uk} s^4 + J_1 c_{f2} s^2}{c_{f2}}\omega_2$$

$$c_{f1}\omega_u = \frac{J_1 J_{2uk} s^4 + J_1 c_{f2} s^2 + J_{2uk} c_{f2} s^2 + J_{2uk} c_{f1} s^2 + c_{f1} c_{f2}}{c_{f2}}\omega_2$$

$$\frac{\omega_2}{\omega_u} = \frac{c_{f1}c_{f2}}{J_1 J_{2uk} s^4 + \left(J_1 c_{f2} + J_{2uk} c_{f2} + J_{2uk} c_{f1}\right)s^2 + c_{f1}c_{f2}}$$

$$\frac{\frac{v}{r_2}}{\omega_u} = \frac{c_{f1}c_{f2}}{J_1 J_{2uk} s^4 + \left(J_1 c_{f2} + J_{2uk} c_{f2} + J_{2uk} c_{f1}\right)s^2 + c_{f1}c_{f2}}$$

$$\frac{v}{\omega_u} = \frac{c_{f1}c_{f2}r_2}{J_1 J_{2uk} s^4 + \left(J_1 c_{f2} + J_{2uk} c_{f2} + J_{2uk} c_{f1}\right)s^2 + c_{f1}c_{f2}}$$

$$\frac{v}{\omega_u} = \frac{6000}{964{,}8s^2 + 9432s + 10000}$$

Polovi:

$$964{,}8s^2 + 9432s + 10000 = 0 \rightarrow s_{p1} = -8{,}5661, \qquad s_{p2} = -1{,}21$$

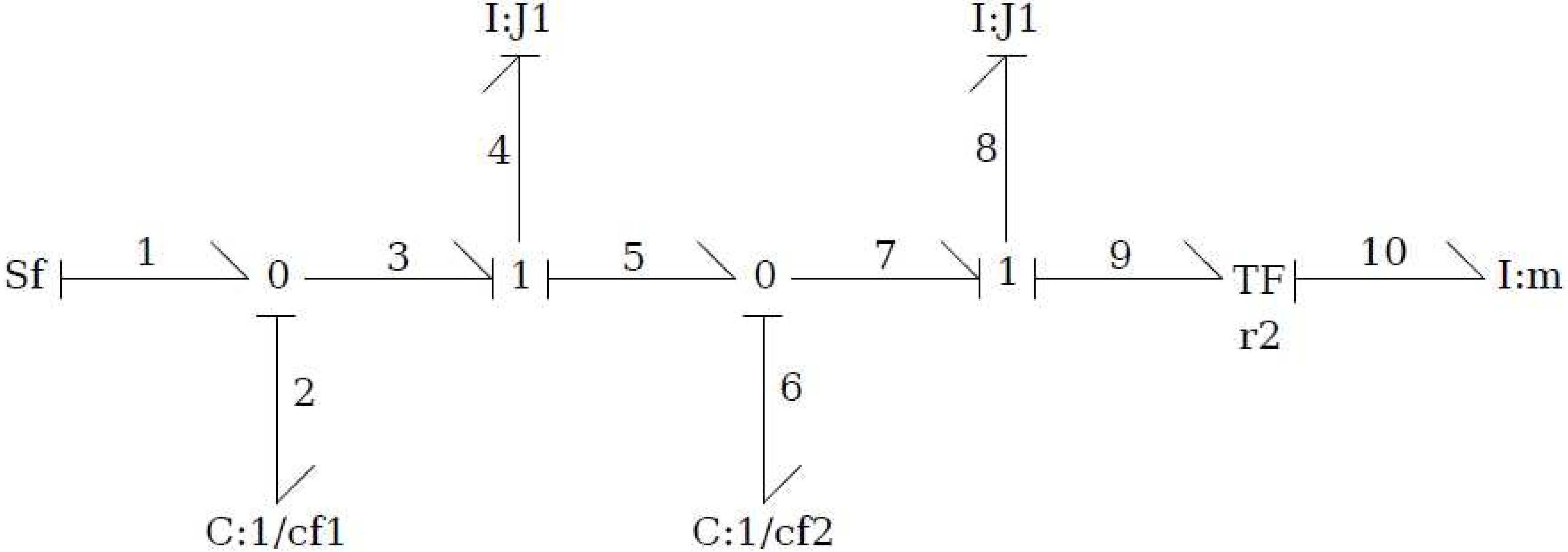
$$s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

$$\zeta = 1$$

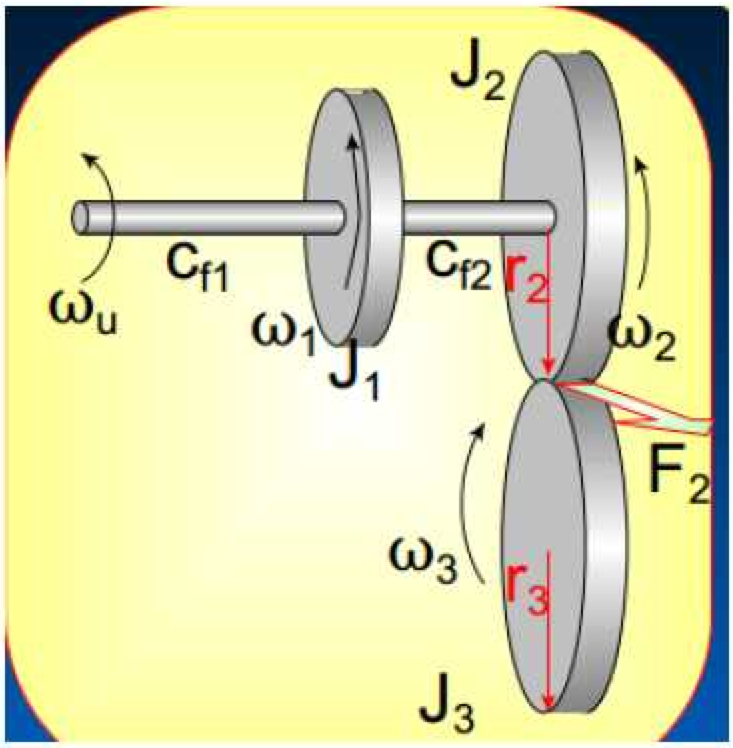
$$\omega_{n1} = 8{,}5661 \rightarrow T_{n1} = \frac{2\pi}{\omega_{n1}}$$

$$\omega_{n2} = 1{,}21 \rightarrow T_{n1} = \frac{2\pi}{\omega_{n2}}$$

Bond graf:



1. Predavanje 5, predzadnji slide.



Diferencijalne jednadžbe:

$$m_1 - m_2 = J_1 \frac{d\omega_1}{dt}$$

$$m_2 - m_{23} = J_2 \frac{d\omega_2}{dt} \text{ ili } m_2 = J_{2uk} \frac{d\omega_2}{dt}$$

$$m_1 = c_{f1}(\varphi_u - \varphi_1) \rightarrow \frac{dm_1}{dt} = c_{f1}(\omega_u - \omega_1)$$

$$m_2 = c_{f2}(\varphi_1 - \varphi_2) \rightarrow \frac{dm_2}{dt} = c_{f2}(\omega_1 - \omega_2)$$

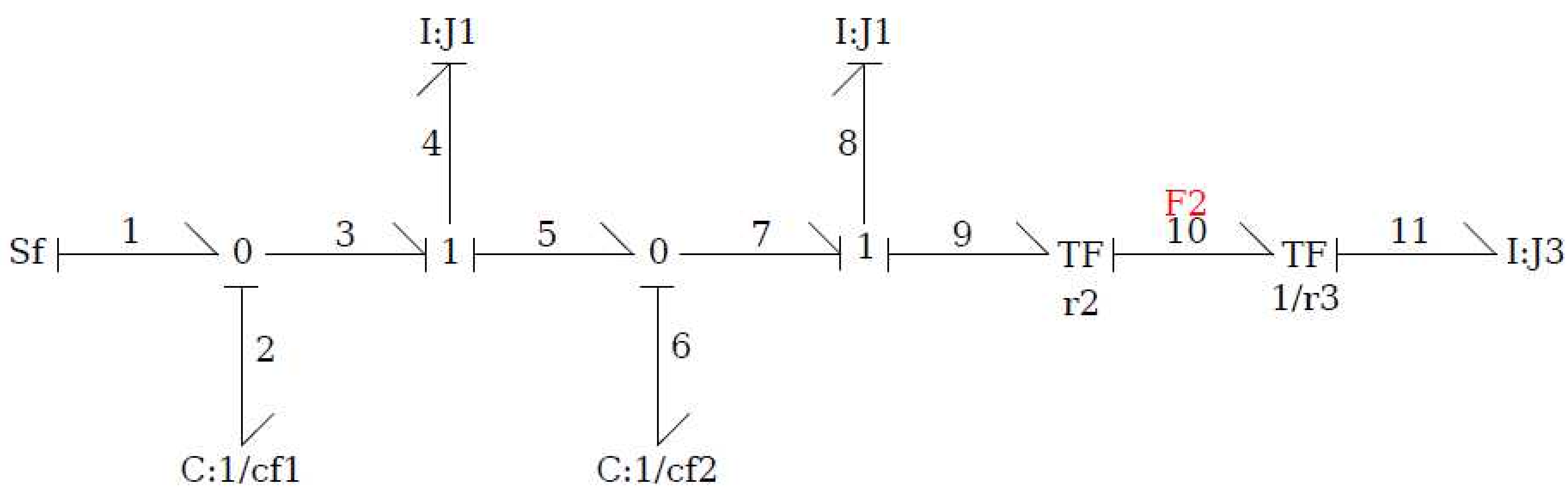
$$m_{23} = F_2 r_2 \text{ i } J_{2uk} = J_2 + J_3 \left(\frac{r_2}{r_3}\right)^2$$

Iz $m_2 - m_{23} = J_2 \frac{d\omega_2}{dt}$ slijedi:

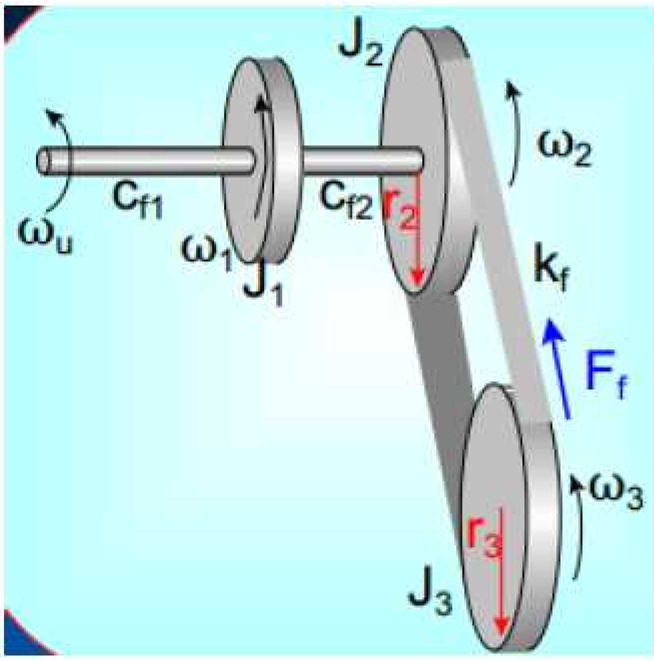
$$m_2 - J_2 \frac{d\omega_2}{dt} = m_{23} \rightarrow m_{23} = J_{2uk} \frac{d\omega_2}{dt} - J_2 \frac{d\omega_2}{dt} = J_3 \left(\frac{r_2}{r_3}\right)^2 \frac{d\omega_2}{dt}$$

$$m_{23} = F_2 r_2 \rightarrow J_3 \left(\frac{r_2}{r_3}\right)^2 \frac{d\omega_2}{dt} = F_2 r_2$$

Bond graf:



2. Predavanje 5, predzadnji slide.



Diferencijalne jednadžbe:

$$m_1 - m_2 = J_1 \frac{d\omega_1}{dt}$$

$$m_2 - m_{23} = J_2 \frac{d\omega_2}{dt}$$

$$m_1 = c_{f1}(\varphi_u - \varphi_1) \rightarrow \frac{dm_1}{dt} = c_{f1}(\omega_u - \omega_1)$$

$$m_2 = c_{f2}(\varphi_1 - \varphi_2) \rightarrow \frac{dm_2}{dt} = c_{f2}(\omega_1 - \omega_2)$$

$$m_{23} = Fr_2$$

$$\frac{dF}{dt} = k_f(v_2 - v_3)$$

$$v_2 = \omega_2 r_2$$

$$v_3 = \omega_3 r_3$$

Bond graf: