MULTIMEDIJSKE TEHNOLOGIJE

AUDITORNE

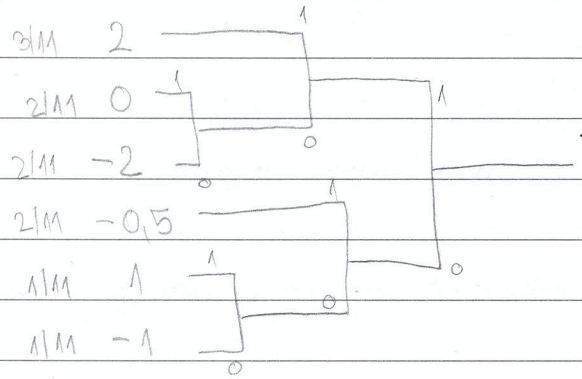
1. −1.1 1.9 0.1 2.1 −0.1 −0.5 −1.9 1.1 −0.6 2.1 −1.9

a) $\Delta = \frac{1}{4}$

$$x_q = \text{round}\left(\frac{x}{\Delta}\right) \cdot \Delta = \text{round}(4 \cdot x)/4$$

−1 2 0 2 0 −0.5 −2 1 −0,5 2 −2

b)



c) $H = -\sum_i p_i \log_2 p_i = 2{,}4817$

$\tilde{d} = \sum p_i d_i = \frac{28}{11}$

$3 \cdot 11 = 33$

$\frac{28}{11} \cdot 11 = 28$

2.

simboli	P	O	T	$
vjerojat.	0.2	0.5	0.2	0.1
inter	$[0, 0.2\rangle$	$[0.2, 0.7\rangle$	$[0.7, 0.9\rangle$	$[0.9, 1\rangle$

POTOTOP

P $[0, 0.2\rangle$ $\quad D_i = D_{i-1} + D_S (G_{i-1} - D_{i-1})$

O $[0.04, 0.14\rangle$ $\quad G_i = D_{i-1} + G_S (G_{i-1} - D_{i-1})$

⋮

P $[0.1214, 0.1216\rangle$

0.002 širina intervala $= p_m$

$p_m = \Pi p_i = 0.002$

2^{-1} 2^{-2} 2^{-3} 2^{-4} 2^{-5} dok ne dodemo do broja koji je u intervalu

0.0 0 0 1 1 1 11 001

$H = -\log_2 p_m = 8.96 \approx 9$ bita

(ako je poruka manja od entropije, nadopisat ϕ)

3.

simbol	0	1	$
p	0.8	0.1	0.1
int.	[0, 0.8⟩	[0.8, 0.9⟩	[0.9, 1⟩

$A_1 = 0.6796875 \rightarrow 0$

$$A_i = (A_{i-1} - D_s)\frac{1}{p_s}$$

$A_2 = (A_1 - D_0)\frac{1}{p_0} = 0.84960937 5 \rightarrow 1$

$A_3 = (A_2 - D_1)\frac{1}{p_1} = 0.4960937 5 \rightarrow 0$

01000$ dok ne bude bor. iz intervala $ ili koliko prije u zad.

4. $\sigma_x^2 = 3$

$H(1) = 4$ bita

$SQNR = 20\log_{10}(48) = 33,62$ dB

a) $SQNR_0$

$$SQNR = 20\log_{10} 2 \cdot H(1) + SQNR_0$$

$$SQNR_0 = 9,54 \text{ dB}$$

b) dif. entropija $h(x) = ?$

$$SQNR_0 = 20\log_{10}\left(\sqrt{12\sigma_x^2} \cdot 2^{-h(x)}\right)$$

$$h(x) = -\log_2\left(\frac{10^{\frac{SQNR_0}{20}}}{\sqrt{12\sigma_x^2}}\right) = 1.0004 \text{ bit}$$

c) $\Delta = ?$

$$\Delta = 2^{h(x) - H(1)}$$

$$\Delta = 0.125$$

5. $X = \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \end{bmatrix} \right\} \quad Y = \{2, -1, 2, 1, 0\}$

a) $\alpha = ?$

↓ srednja vrijednost

$$\Phi_{xx} = E(X \cdot X^T) = \frac{1}{5} \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} \begin{bmatrix} -1 & 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} \begin{bmatrix} 1 & -2 \end{bmatrix} \right\}$$

$$\Phi_{xx} = \frac{1}{5} \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} \right\}$$

$$= \frac{1}{5} \begin{bmatrix} 10 & -2 \\ -2 & 10 \end{bmatrix}$$

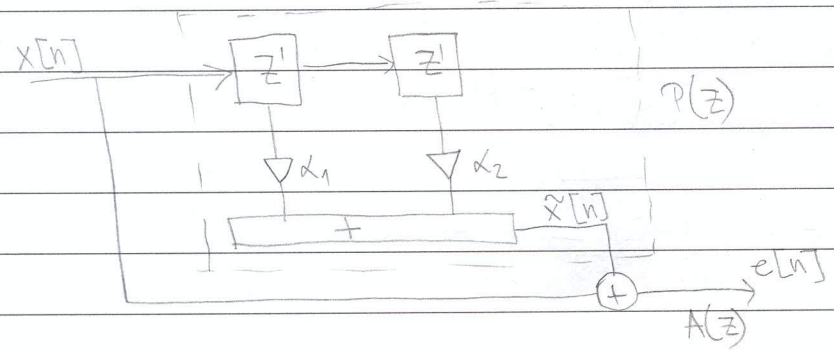
$$\Psi_{xy} = E(X \cdot Y) = \frac{1}{5} \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot 2 + \begin{bmatrix} 2 \\ 1 \end{bmatrix} (-1) + \begin{bmatrix} -1 \\ 2 \end{bmatrix} \cdot 2 + \begin{bmatrix} 2 \\ 0 \end{bmatrix} \cdot 1 \right\}$$

$$= \frac{1}{5} \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 10 & -2 \\ -2 & 10 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

$10\alpha_1 - 2\alpha_2 = -2$ $\quad \alpha_1 = -\frac{5}{48}$

$-2\alpha_1 + 10\alpha_2 = 5$ $\quad \alpha_2 = -\frac{23}{48}$



$$P(z) = \sum_i \alpha_i z^{-i} = \alpha_1 \cdot z^{-1} + \alpha_2 \cdot z^{-2}$$

$$A(z) = 1 - P(z)$$

$$H(z) = \frac{1}{A(z)}$$

$$\tilde{y}(3) = [\alpha^T]\,[x(3)] = \begin{bmatrix} -\frac{5}{48} & \frac{23}{48} \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \frac{51}{48} \qquad e(3) = y(3) - \tilde{y}(3) = \frac{45}{48}$$

6. OTVORENA PETLJA

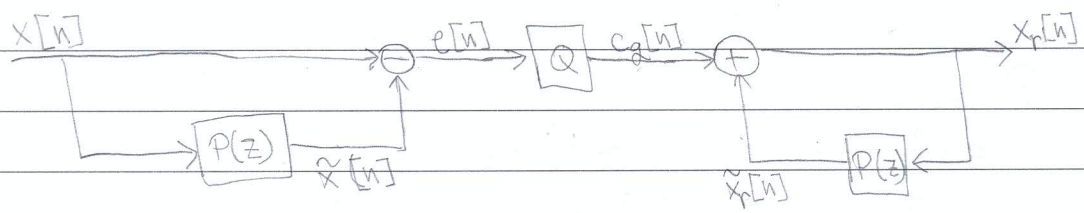
$$P(z) = 2 \cdot z^{-1}$$

$$e_q = \text{round}(2 \cdot e)/2$$

$$x[0] = \frac{1}{2}$$

$$x[1] = \frac{3}{4}$$

$$x[2] = \frac{5}{2}$$



$$\tilde{x}[n] = 2 \cdot x[n-1]$$

$$e[n] = x[n] - \tilde{x}[n]$$

$$\tilde{x}_r[n] = 2 \cdot x_r[n-1]$$

$$x_r[n] = \tilde{x}_r[n] + e_q[n]$$

$n=0$ $\quad \tilde{x}[0] = 2 \cdot x[-1] = 0$

$$e[0] = x[0] - \tilde{x}[0] = \frac{1}{2}$$

$$e_q[0] = \text{round}(2 \cdot e[0])/2 = \frac{1}{2}$$

$$\tilde{x}_r[0] = 0$$

$$x_r[0] = 0 + \frac{1}{2} = \frac{1}{2}$$

$n=1$ $\quad \tilde{x}[1] = 2 \cdot x[0] = 2 \cdot \frac{1}{2} = 1$

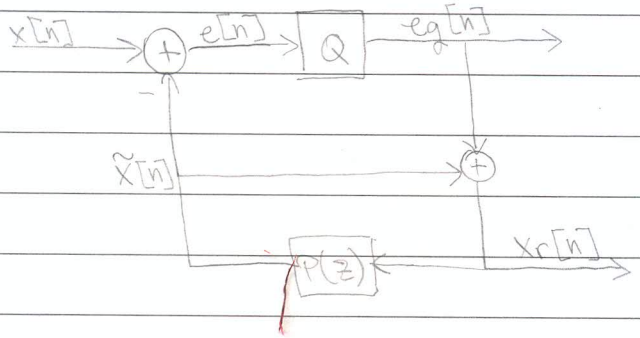
$$e[1] = x[1] - \tilde{x}[1] = \frac{3}{4} - 1 = -\frac{1}{4}$$

$$\vdots$$

$$x_r[1] = \frac{1}{2} \qquad x_r[2] = \frac{5}{2}$$

$e_r[n] = x_r[n] - x[n]$ pogreška rekonstrukcije

7. ZATVORENA PETLJA



$P(z) = 2 \cdot z^{-1}$

$e_q = \text{round}(2 \cdot e)/2$

$x[0] = \frac{1}{2} \quad x[1] = 3/4 \quad x[2] = \frac{5}{2}$

$e[n] = x[n] - \tilde{x}[n]$

$x_r[n] = e_q[n] + \tilde{x}[n]$

$\tilde{x}[n] = 2 \cdot x_r[n-1]$

$n = 0$

$e[0] = x[0] - \tilde{x}[0] = \frac{1}{2} - 0 = \frac{1}{2}$

$\tilde{x}[0] = 2 \cdot x_r[-1] = 0$

$e_q[0] = \frac{1}{2}$

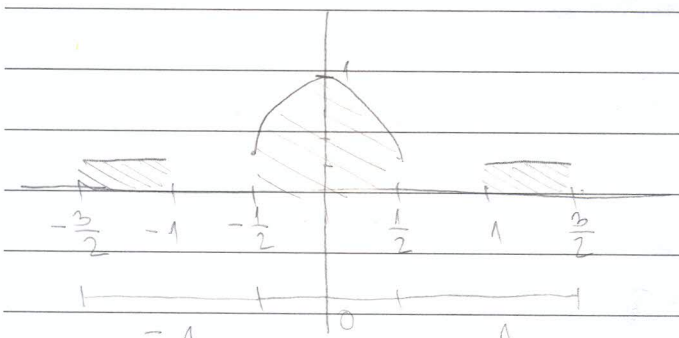
$x_r[0] = e_q[0] + \tilde{x}[0] = \frac{1}{2} + 0 = \frac{1}{2}$

$x_r[1] = \frac{1}{2} \quad x_r[2] = \frac{5}{2}$

8. $E(x) = 0$

$\Delta = 1$

$$f_x(x) = \begin{cases} |3x^2 - 1| & , x \in \left[-\frac{1}{2}, \frac{1}{2}\right] \\ \frac{1}{4} & , x \in \left[-\frac{3}{2}, -1\right] \cup \left[1, \frac{3}{2}\right] \\ 0 & , \text{inače} \end{cases}$$



$p(-1) = p(1) = \frac{1}{8}$

$$p(0) = \int_{-1/2}^{1/2} (-3x^2 + 1)\,dx = \left(-3\frac{x^3}{3} + x\right)\Big|_{-1/2}^{1/2}$$

$= \frac{3}{4}$

$H = -\sum p_i \log p_i = 1.0613 \text{ bit}$

$D = ?$

$D = \frac{\Delta^2}{12}$ vrijedi samo u poseb. uvjetima

$D = \sum D_i$

$D_i = \int_X F_x(x)(x - x_0)^2 dx$

$D_1 = \int_{-\frac{3}{2}}^{-1} \frac{1}{4}(x+1)^2 dx = \frac{1}{96}$

$D_0 = \int_{-1/2}^{1/2} (-3x^2+1)(x-0)^2 dx = \frac{11}{240}$

$D_1 = \int_{1}^{3/2} \frac{1}{4}(x-1)^2 dx = \frac{1}{96}$

$D = \frac{1}{15}$

$\sigma_x^2 = \int_X (x - e(x))^2 F_x(x) dx$

$\sigma_x^2 = \int_{-\frac{3}{2}}^{-1} x^2 \cdot \frac{1}{4} dx + \int_{-1/2}^{1/2} x^2 (-3x^2+1) dx + \int_{1}^{3/2} (x-0)^2 \frac{1}{4} dx$

$\sigma_x^2 = \frac{59}{120}$