grupa ...

3. Zadatak Laplaceova razdioba

$$f_x(x) = \frac{1}{2a} e^{-\frac{|x-\mu|}{a}}$$

a) $$\Phi_x(x) = \int_{-\infty}^{\infty} f_x(x)\,dx$$

$$= \frac{1}{2a}\int_{-\infty}^{\infty} e^{-\frac{|x-\mu|}{a}}\,dx = \frac{1}{2a}\left[\int_{-\infty}^{0} e^{-\frac{-(x-\mu)}{a}}\,dx + \int_{0}^{\infty} e^{-\frac{+(x-\mu)}{a}}\,dx\right]$$

$$= \frac{1}{2a}\left[\int_{-\infty}^{0} e^{\frac{x-\mu}{a}}\,dx + \int_{0}^{\infty} e^{\frac{\mu-x}{a}}\,dx\right]$$

$$= \frac{1}{2a}\left[e^{-\frac{\mu}{a}}\int_{-\infty}^{0} e^{\frac{x}{a}}\,dx + e^{\frac{\mu}{a}}\int_{0}^{\infty} e^{\frac{-x}{a}}\,dx\right]$$

$$= \frac{1}{2a}\left[e^{-\frac{\mu}{a}}\cdot a\cdot e^{\frac{x}{a}}\Big|_{-\infty}^{0} - e^{\frac{\mu}{a}}\cdot a\cdot e^{\frac{-x}{a}}\Big|_{0}^{\infty}\right]$$

$$= \frac{1}{2a}\left[a e^{-\frac{\mu}{a}}(1-0) - a e^{\frac{\mu}{a}}(0-1)\right]$$

$$= \frac{1}{2}\left[e^{-\frac{\mu}{a}} + e^{\frac{\mu}{a}}\right]$$

b) $$h(x) = -\int_x f_x(x)\log\left(f_x(x)\right)dx$$

$$= \int_{-\infty}^{\infty} \frac{1}{2a} e^{-\frac{|x-\mu|}{a}} \ln\left(\frac{1}{2a} e^{-\frac{|x-\mu|}{a}}\right)dx$$

$$= \frac{1}{2a}\int_{-\infty}^{\infty} e^{-\frac{|x-\mu|}{a}}\left[\cancel{\ln 1} - \ln 2a - \frac{|x-\mu|}{a}\right]dx$$

$$= -\ln 2 \underbrace{\int_{-\infty}^{\infty} \frac{1}{2a} e^{-\frac{|x-\mu|}{a}}}_{\frac{1}{2}\left[e^{-\frac{\mu}{a}} + e^{\frac{\mu}{a}}\right]} + \frac{1}{2a}\int_{-\infty}^{\infty} -\frac{|x-\mu|}{a} e^{-\frac{|x-\mu|}{a}}\,dx$$

$$= \frac{-\ln 2}{2}\left[e^{-\frac{\mu}{a}} + e^{\frac{\mu}{a}}\right] + \frac{1}{2a}\int_{-\infty} -\frac{|x-\mu|}{a} e^{-\frac{|x-\mu|}{a}}\,dx$$

$$I = \frac{1}{2a}\int_{-\infty}^{\infty} -\frac{|x-\mu|}{a} e^{-\frac{|x-\mu|}{a}} dx$$

$$= \frac{1}{2a}\left[\int_{-\infty}^{0} -\frac{-(x-\mu)}{a} e^{-\frac{-(x-\mu)}{a}} dx + \int_{0}^{\infty} -\frac{+(x-\mu)}{a} e^{-\frac{+(x-\mu)}{a}} dx\right.$$

$$= \frac{1}{2a}\left[\int_{-\infty}^{0} \frac{x-\mu}{a} e^{\frac{x-\mu}{a}} dx + \int_{0}^{\infty} \frac{\mu-x}{a} e^{\frac{\mu-x}{a}} dx\right]$$

SUPSTITUCIJA: $u = \frac{x-\mu}{a} \Rightarrow au = x-\mu \Rightarrow x = au+\mu$

$$du = \frac{dx}{a} \Rightarrow dx = a\,du$$

$$v = \frac{\mu-x}{a} \Rightarrow av = \mu - x \Rightarrow x = \mu - av$$

$$dv = \frac{-dx}{a} \Rightarrow dx = -a\,dv$$

$$I = \frac{1}{2a}\left[\int_{-\infty}^{0} \mu \cdot e^{u} \cdot a\,du - \int_{0}^{\infty} v e^{v} \cdot a\,dv\right]$$

$$= \frac{1}{2}\left[\int_{-\infty}^{0} u e^{u} du - \int_{0}^{\infty} v e^{v} dv\right] = \Big/ \begin{matrix} p=u & dp=du \\ dQ=e^{u}du & Q=e^{u} \end{matrix} \Big/$$

$$= \frac{1}{2}\left[u e^{u}\Big|_{-\infty}^{0} - \int_{-\infty}^{0} e^{u} du - \int_{0}^{\infty} v e^{v} dv\right] = \Big/ \begin{matrix} p=v & dp=dv \\ dQ=e^{v}dv & Q=e^{v} \end{matrix} \Big/$$

$$= \frac{1}{2}\left[u e^{u}\Big|_{-\infty}^{0} - e^{u}\Big|_{-\infty}^{0} - v e^{v}\Big|_{0}^{\infty} + \int_{0}^{\infty} e^{v} dv\right] =$$

$$= \frac{1}{2}\left[u e^{u}\Big|_{-\infty}^{0} - e^{u}\Big|_{\infty}^{0} - v e^{v}\Big|_{0}^{\infty} + e^{v}\Big|_{0}^{\infty}\right]$$

$$= \frac{1}{2}\left[\frac{x-\mu}{a} e^{\frac{x-\mu}{a}}\Big|_{-\infty}^{0} - e^{\frac{x-\mu}{a}}\Big|_{-\infty}^{0} - \frac{\mu-x}{a} e^{\frac{\mu-x}{a}}\Big|_{0}^{\infty} + e^{\frac{\mu-x}{a}}\Big|_{0}^{\infty}\right]$$

$$= \frac{1}{2}\left[\frac{-\mu}{a} e^{-\frac{\mu}{a}} - e^{-\frac{\mu}{a}} + \frac{\mu}{a} e^{\frac{\mu}{a}} - e^{\frac{\mu}{a}}\right]$$

grupa: 3.RZ1

$$h(x) = \frac{\ln 2}{2}\left[e^{-\frac{\mu}{a}} + e^{\frac{\mu}{a}}\right] + I$$

$$= \frac{\ln 2}{2}\left[e^{-\frac{\mu}{a}} + e^{\frac{\mu}{a}}\right] + \frac{1}{2}\left[-\frac{\mu}{a}e^{-\frac{\mu}{a}} - e^{-\frac{\mu}{a}} + \frac{\mu}{a}e^{\frac{\mu}{a}} - e^{\frac{\mu}{a}}\right]$$

$$= \frac{\ln 2}{2}\left[e^{-\frac{\mu}{a}} + e^{\frac{\mu}{a}}\right] + \frac{1}{2}\frac{\mu}{a}\left[e^{\frac{\mu}{a}} - e^{-\frac{\mu}{a}}\right] - \frac{1}{2}\left[e^{\frac{\mu}{a}} + e^{-\frac{\mu}{a}}\right]$$

$$= \frac{\ln 2 - 1}{2}\left[e^{\frac{\mu}{a}} + e^{-\frac{\mu}{a}}\right] + \frac{\mu}{2a}\left[e^{\frac{\mu}{a}} - e^{-\frac{\mu}{a}}\right]$$

$$= (\ln 2 - 1)\,\mathrm{ch}\left(\frac{\mu}{a}\right) + \frac{\mu}{a}\,\mathrm{sh}\left(\frac{\mu}{a}\right) = /\, z = \frac{\mu}{a} \,/$$

$$= (\ln 2 - 1)\,\mathrm{ch}\, z + z\,\mathrm{sh}\, z$$ ... DIFERENCIJALNA ENTROPIJA IZVORA

$$\sigma_x^2 = E\left[(x - E(x))^2\right] = E\left[x^2 - 2xE(x) + (E(x))^2\right]$$

$$E(x) = \int_{-\infty}^{\infty} x f_x(x)\,dx = \frac{1}{2a}\int_{-\infty}^{\infty} x e^{-\frac{|x-\mu|}{a}}\,dx$$

$$= \frac{1}{2a}\left[\int_{-\infty}^{0} x e^{\frac{x-\mu}{a}}\,dx + \int_{0}^{\infty} x e^{\frac{\mu-x}{a}}\,dx\right] = \frac{1}{2a}\left[e^{-\frac{\mu}{a}}\int_{-\infty}^{0} x e^{\frac{x}{a}}\,dx + e^{\frac{\mu}{a}}\int_{0}^{\infty} x e^{-\frac{x}{a}}\,dx\right]$$

$$= \left/\begin{matrix} u = x & du = dx \\ dv = e^{\frac{x}{a}}dx & v = ae^{\frac{x}{a}} \end{matrix}\right/ = \frac{1}{2a}\left[e^{-\frac{\mu}{a}}\left(axe^{\frac{x}{a}}\Big|_{-\infty}^{0} - \int_{-\infty}^{0} ae^{\frac{x}{a}}\,dx\right) + \right.$$

$$\left. e^{\frac{\mu}{a}}\left(-axe^{-\frac{x}{a}}\Big|_{0}^{\infty} + \int_{0}^{\infty} ae^{-\frac{x}{a}}\,dx\right)\right] = \cdots = \frac{a}{2}\left[e^{\frac{\mu}{a}} - e^{-\frac{\mu}{a}}\right]$$

$$\sigma_x^2 = E\left[x^2 - 2x\frac{a}{2}\left(e^{\frac{\mu}{a}} - e^{-\frac{\mu}{a}}\right) + \frac{a^2}{4}\left(e^{\frac{\mu}{a}} - e^{-\frac{\mu}{a}}\right)^2\right]$$

$$= E\left[x^2 - 2x\frac{a}{2}\left(e^{\frac{\mu}{a}} - e^{-\frac{\mu}{a}}\right) + \frac{a^2}{4}\left(e^{\frac{2\mu}{a}} - 2 + e^{-\frac{2\mu}{a}}\right)\right]$$

-> linearnost očekivanja: $E(ax+b) = aE(x) + b$

$$\sigma_x^2 = E(x^2) + a\left(e^{\frac{k}{a}} - e^{-\frac{k}{a}}\right)E(x) + \frac{a^2}{4}\left(e^{\frac{2k}{a}} - 2 + e^{-\frac{2k}{a}}\right)$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f_x(x)\,dx = \frac{1}{2a}\left[\int_{-\infty}^{0} x^2 e^{\frac{x-k}{a}}\,dx + \int_{0}^{\infty} x^2 e^{\frac{k-x}{a}}\,dx\right]$$

$$= \frac{1}{2a}\left[e^{-\frac{k}{a}}\int_{-\infty}^{0} x^2 e^{\frac{x}{a}}\,dx + e^{\frac{k}{a}}\int_{0}^{\infty} x^2 e^{-\frac{x}{a}}\,dx\right] = \left/ \begin{matrix} u = x^2 & du = 2x\,dx \\ dv = e^{\frac{x}{a}}dx & v = ae^{\frac{x}{a}} \end{matrix} \right/$$

$$= \frac{1}{2a}\left[e^{-\frac{k}{a}}\left(ax^2 e^{\frac{x}{a}}\Big|_{-\infty}^{0} - 2a\int_{-\infty}^{0} x e^{\frac{x}{a}}\,dx\right) + e^{\frac{k}{a}}\left(-ax^2 e^{-\frac{x}{a}}\Big|_{0}^{\infty} + 2a\int_{0}^{\infty} x e^{-\frac{x}{a}}\,dx\right)\right]$$

$$= \dots = \frac{1}{2a}\left[2a^3 e^{-\frac{k}{a}} + 2a^3 e^{\frac{k}{a}}\right] = a^2\left[e^{\frac{k}{a}} + e^{-\frac{k}{a}}\right]$$

$$\sigma_x^2 = a^2\left[e^{\frac{k}{a}} + e^{-\frac{k}{a}}\right] + \left(\frac{a^2}{2} + \frac{a^2}{4}\right)\left[e^{\frac{2k}{a}} - 2 + e^{-\frac{2k}{a}}\right]$$

$$= a^2\left[e^{\frac{k}{a}} + e^{-\frac{k}{a}}\right] + \frac{3a^2}{4}\left[e^{\frac{2k}{a}} - 2 + e^{-\frac{2k}{a}}\right] = \left/ z = \frac{k}{a} \right/$$

$$= 2a^2 \operatorname{ch} z + \frac{3a^2}{2}\operatorname{sh}(2z) - \frac{3a^2}{2}$$ ... VARIJANCA LAPLACEOVOG IZVORA