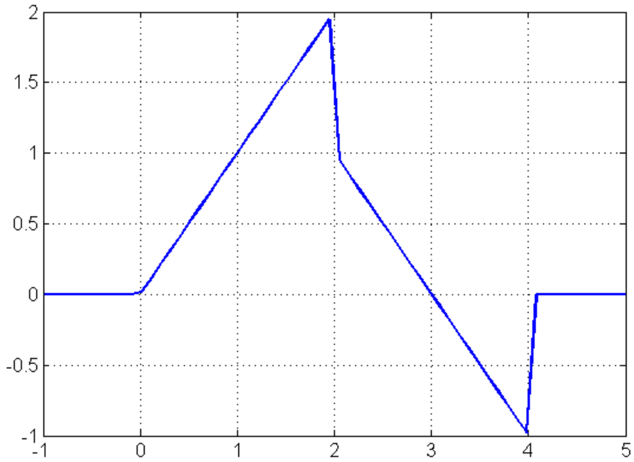
1. **Filtarski slog s potpunom rekonstrukcijom dan je slikom (standardna slika sloga s 2 grane, decimirani). Produkt filtar ima oblik maksimalno glatkog filtra $P_0(z) = (1 + z^{-1})^{2p} Q_{2p-2}(z)$.**

a) Uz p = 1, odredite $Q_{2p-2}(z)$ te filtre ortogonalnog filtarskog sloga tko da $F_0(z)$ i $H_0(z)$ imaju linearnu fazu, te da je zadovoljen uvjet konvergencije wavelet funkcije
b) Odrediti impulsne odzive svih filtara
c) Nacrtati funkciju skale prve razine $\varphi_1(t)$ pridruženu ovom slogu.
d) Nacrtati funkciju skale druge razine $\varphi_2(t)$ pridruženu ovom slogu.

2. **Slično kao 4. zadatak iz primjera 2. MI, samo je zadan drugačiji x(t).**
Nešto, kao: izračunaj i opiši brzu Haarovu oktavnu DWT signala zadanog slikom:

A signal zadan slikom je bio nešto kao:
(Recimo da sam 85% sigurna da je izgledao ovako, samo su ovi odsječci pravci, a moja slika je malo grbava, to ne treba biti tako. Znači signal je tipa x(t)= t na [0, 2], x(t) = -t+3 na [2, 4])



3. **Ulaznom signalu $x[n] = 3n^2 - n + 4$ dodan je bijeli šum. Tako dobiveni signal se razlaže pomoću 'db3' valića (3 nul-momenta).**
   a. Što očekujete, kako će izgledati valićni koeficijenti prve dvije razine razlaganja čistog signala, bijelog šuma, a kako njihovog zbroja?
   b. Navedite i opišite dvije metode potiskivanja šuma. Navedite prednosti svake od metoda.

4. **Filtarski slog realiziran je pomoću koraka podizanja u dvije kaskade. Filtri podizanja prve kaskade su $S_1(z) = 3$ i $T_1(z) = 1 + z^{-1}$. U drugoj kaskadi filtri podizanja su $S_2(z) = -1 + z^{-1}$ i $T_2(z) = 4$. Odredite klasičnu četvorku filtara FS s 2 pojasa i decimacijom: H0(z), H1(z), F0(z) i F1(z).**

5. **Cjelobrojnim 'pohlepnim' algoritmom raspodijelite 4 bita u pojasnom koderu s decimiranim ortogonalnim FS s dvije grane, ako su filtri h0={<u>1</u>, 1}, h1={<u>1</u>, -1}, i x= {<u>2</u>, -2, 5, 1, -1, -1}. Koliki je dobitak pojasnog kodiranja?**

ZAVRŠNI 2011/2012

(1) $P_0(z) = (1+z^{-1})^{2p} Q_{2p-2}(z)$

UVJET POTPUNE REKONSTRUKCIJE

$P_0(z) - P_0(-z) = 2z^{-L}$

a) $p=1 \Rightarrow Q_0(z) = g_0 \rightarrow$ konstanta

$P_0(z) = (1+2z^{-1}+z^{-2})\, g_0$

$P_0(z) - P_0(-z) = [\cancel{1}+2z^{-1}\cancel{+z^{-2}}\cancel{-1}+2z^{-1}\cancel{-z^{-2}}]\, g_0 = 2z^{-1} g_0$

$\cancel{2}4z^{-1} g_0 = 2z^{-L} \Rightarrow L = 1$

$g_0 = 1/2$

$P_0(z) = \frac{1}{2}(1+z^{-1})^2$

$H_0(z) = \frac{1}{\sqrt{2}}(1+z^{-1})$

$F_0(z) = \frac{1}{\sqrt{2}}(1+z^{-1})$

SIMETRIČNOST ODZIVA FILTARA

$\Rightarrow$ LINEARNU FAZU POSTIŽEMO:

ANALIZIRAJUĆA!!

1) Oba simetrična, neparne dužine

2) Jedan simetričan, drugi antisimetričan, parne dužine

linearna faza

b) $F_1 = -H(-z)$

$f_1 = h_0(-1)^{n+1}$; $h_1 = f_0(-1)^{n+1}$

$h_0 = \left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}$; $h_1 = \left\{\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right\}$

$f_0 = \left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}$; $f_1 = \left\{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}$

NUŽAN I DOVOLJAN UVJET KONVERGENCIJE

NUŽAN $\Rightarrow$ $H_0(z)\big|_{z=-1} = 0 \quad H_0(z)\big|_{z=1} = \sqrt{2}$

DOVOLJAN $\Rightarrow$ (Mallat 89) NP filtar nema nultočaka na jediničnoj kružnici u rasponu $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$H_1^{(N)}(z) = H_1(z^{2^{N-1}}) \prod_{i=0}^{N-2} H_0(z^{2^i})$

$H_0^{(N)}(z) = \prod_{i=0}^{N-1} H_0(z^{2^i})$

Filtri $H_1^{(N)}(z)$ određuju fje razlaganja waveleta (wavelet funkcije)

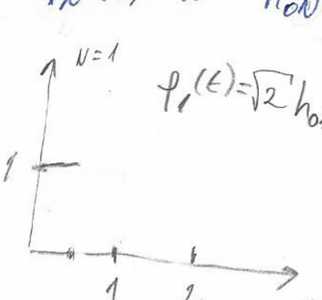
Filtri $H_0^{(N)}(z)$ određuju funkcije skale

stupanj razine

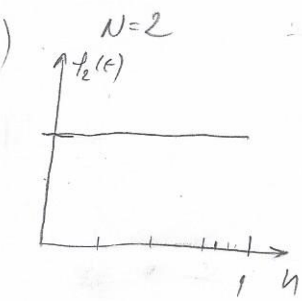
c) $\varphi_N(t) = 2^{N/2} h_{0N}[n]$, $\frac{n}{2^N} \le t \le \frac{n+1}{2^N}$

$N=1$

$\varphi_1(t) = \sqrt{2}\, h_{01}[n]$



d) $N=2$



$H_0^{(1)} = H_0(z) \Rightarrow h_{01} = \frac{1}{\sqrt{2}}\{1, 1\}$

$H_0^{(2)} = H_0(z) \cdot H_0(z^2) = \frac{1}{2}(1+z^{-1})(1+z^{-2}) =$

$= \frac{1}{2}(1+z^{-1}+z^{-2}+z^{-3})$

$h_{02} = \frac{1}{2}\{1, 1, 1, 1\}$

$\varphi_2(t) = 2 \cdot h_{02}[n]$, $\frac{n}{4} \le t \le \frac{n+1}{4}$

$n=0$	$\varphi_2(t) = 1$	$0 \le t \le 1/4$
$n=1$	1	$1/4 \le t \le 1/2$
$n=2$	1	$1/2 \le t \le 3/4$
$n=3$	1	$3/4 \le t \le 1$

2.

$$x(t) = \begin{cases} t, & 0 \le t \le 2 \\ -t+3, & 2 \le t \le 4 \end{cases}, \quad \varphi(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & \text{inače} \end{cases}$$

↳ pravokutni puls, ne treba zrcaliti s obzirom a x-os

aproksimacijski koeficijenti ↓ razina ↓ pomak ↓

$$A[m,k] = \int_{-\infty}^{\infty} x(t)\,\varphi(t-m)\,dt$$

$$A[m,1] = \sum_{k=0}^{N_0} h_0[k]\,A[2m+k,0]$$ ← N_0 red imp odziva

HAAR

$$h_0 = \frac{1}{\sqrt{2}}\{1,1\}, \quad h_1 = \{1,-1\}\frac{1}{\sqrt{2}}$$

$$X[m,1] = \sum_{k=0}^{N_0} h_1[k]\,A[2m+k,0]$$

k=0)

$$A[0,0] = \int_0^1 t\,dt = \frac{t^2}{2}\Big|_0^1 = \frac{1}{2}$$

$$A[1,0] = \int_1^2 t\,dt = \frac{t^2}{2} = \frac{3}{2}$$

$$A[2,0] = \int_2^3 (-t+3)\,dt = \left(-\frac{t^2}{2}+3t\right)\Big|_2^3 = -\frac{9}{2}+9+\frac{4}{2}-6 = \frac{1}{2}$$

$$A[3,0] = \int_3^4 (-t+3)\,dt = \left(-\frac{t^2}{2}+3t\right)\Big|_3^4 = -\frac{16}{2}+12+\frac{9}{2}-9 = -\frac{1}{2}$$

k=1)

$$A[0,1] = \frac{1}{\sqrt{2}}A[0+0,0] + \frac{1}{\sqrt{2}}A[2\cdot 0+1,0] = \frac{1}{\sqrt{2}}\left(A[0,0]+A[1,0]\right) = \sqrt{2}$$

$$A[1,1] = \frac{1}{\sqrt{2}}\left(A(2\cdot 1+0,0) + A[2\cdot 1+1,0]\right) = \frac{1}{\sqrt{2}}\left(A[2,0]+A[3,0]\right) = 0$$

$$X[0,1] = \frac{1}{\sqrt{2}}\left(1\cdot A[0+0,0] - 1\cdot A[0+1,0]\right) = \frac{1}{\sqrt{2}}\left(A[0,0]-A[1,0]\right) = -\frac{1}{\sqrt{2}}$$

$$X[1,1] = \frac{1}{\sqrt{2}}\left(1\cdot A[2\cdot 1+0,0] - 1\cdot A[2\cdot 1+1,0]\right) = \frac{1}{\sqrt{2}}\left(A[2,0]-A[3,0]\right) = \frac{1}{\sqrt{2}}$$

k=2)

$$A[0,2] = \frac{1}{\sqrt{2}}\left(1\cdot A[2\cdot 0+0,1] + A[2\cdot 0+1,1]\right) = \frac{1}{\sqrt{2}}\left(A[0,1]+A[1,1]\right) = \frac{1}{2}$$

$A[1,2] = \frac{1}{\sqrt{2}}(A[2,1] \cdots$ nema toga više

$$X[0,2] = \frac{1}{\sqrt{2}}\left(A[2\cdot 0+0,1] - A[2\cdot 0+1,1]\right) = 1$$

3. $x[n] = 3n^2 - n + 4$, 'db3' valić (3 nul-momenta)

a)

b) 1) Hard thresholding - odbacivanje svih wavelet koeficijenata ispod nekog praga

2) Soft thresholding - odbacivanje wavelet koeficijenata ispod praga, a ostale umanjujemo za iznos praga

1) Daje najmanju pogrešku pri rekonstrukciji signala (u smislu najmanjih kvadrata)

2) Redovito bolja u primjenama, osobito kod potiskivanja šuma u slici jer nema naglih skokova u vrijednostima koeficijenata

$\Rightarrow$ Bolje rezultate potiskivanja šuma postiže se wavelet filtarskim slogom bez decimacije

4.

$S_1(z) = 3$

$T_1(z) = 1 + z^{-1}$

$S_2(z) = -1 + z^{-1}$

$T_2(z) = 4$

$$H_p(z) = \left\{ \prod_{i=0}^{K-1} \begin{bmatrix} 1 & 0 \\ T_{K-i}(z) & 1 \end{bmatrix} \begin{bmatrix} 1 & -S_{K-i}(z) \\ 0 & 1 \end{bmatrix} \right\}$$

$$F_p(z) = \prod_{i=1}^{K} \begin{bmatrix} 1 & S_i(z) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -T_i(z) & 1 \end{bmatrix}$$

$$H_p(z) = \left(\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1-z^{-1} \\ 0 & 1 \end{bmatrix} \right) \left(\begin{bmatrix} 1 & 0 \\ 1+z^{-1} & 1 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 1-z^{-1} \\ 4 & 5-4z^{-1} \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 1+z^{-1} & -2-3z^{-1} \end{bmatrix}$$

$$= \begin{bmatrix} 2-z^{-2} & -5-1z^{-1}+3z^{-2} \\ 9+z^{-1}-4z^{-2} & -22-7z^{-1}+12z^{-2} \end{bmatrix} \qquad H_p(z) = \begin{bmatrix} H_{00}(z) & H_{01}(z) \\ H_{10}(z) & H_{11}(z) \end{bmatrix}$$

$$F_p(z) = \left(\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1-z^{-1} & 1 \end{bmatrix} \right) \left(\begin{bmatrix} 1 & -1+z^{-1} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} \right) = \begin{bmatrix} -2-3z^{-1} & 3 \\ -1-z^{-1} & 1 \end{bmatrix} \begin{bmatrix} 5-4z^{-1} & -1+z^{-1} \\ -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -22-7z^{-1}+12z^{-2} & 5+z^{-1}-3z^{-2} \\ -9-z^{-1}+4z^{-2} & 2-z^{-2} \end{bmatrix} \qquad F_p = \begin{bmatrix} F_{01} & F_{11}(z) \\ F_{00}(z) & F_{10}(z) \end{bmatrix}$$

$$\boxed{H_0(z) = H_{00}(z^2) + z^{-1}H_{01}(z^2)} = 2 - z^{-4} - 5z^{-1} - z^{-3} + 3z^{-5} = 2 - 5z^{-1} - z^{-3} - z^{-4} + 3z^{-5}$$

$$\boxed{H_1(z) = H_{10}(z^2) + z^{-1}H_{11}(z^2)} = 9 + z^{-2} - 4z^{-4} - 22z^{-1} - 7z^{-3} + 12z^{-5} = 9 - 22z^{-1} + z^{-2} - 7z^{-3} - 4z^{-4} + 12z^{-5}$$

$$\boxed{F_0(z) = H_p^{-1}(z)}$$

$$\boxed{F_0(z) = F_{00}(z^2) + z^{-1}F_{01}(z^2)} = -9 - z^{-2} + 4z^{-4} - 22z^{-1} - 7z^{-3} + 12z^{-5}$$

$$= -9 - 22z^{-1} - z^{-2} - 7z^{-3} + 4z^{-4} + 12z^{-5}$$

$$\boxed{F_1(z) = F_{10}(z^2) + z^{-1}F_{11}(z^2)} = 2 - z^{-4} + 5z^{-1} + z^{-3} - 3z^{-5} =$$

$$= 2 + 5z^{-1} + z^{-3} - z^{-4} - 3z^{-5}$$

(5.) bitovi = 4

$$\sigma_x^2 = \frac{1}{N-1} \sum_{n=0}^{N-1} (x(n) - \bar{x})^2$$

$$x[n] = \{2, -2, 5, 1, -1, -1\}$$

$$G_{SBC} = \frac{2^{-2\bar{r}} \sum_{k=1}^{M} \sigma^2}{\sum_{k=1}^{M} \sigma_k^2 \, 2^{-2r_k}}$$

$$h_0 = \{1, 1\}$$

$$h_1 = \{1, -1\}$$

$$H_0(z) = 1 + z^{-1}$$

$$H_1(z) = 1 - z^{-1}$$

$$X_0(z) = H_0(z)\, X(z) = (1+z^{-1})(2 - 2z^{-1} + 5z^{-2} + z^{-3} - z^{-4} - z^{-5})$$

$$X_0(z) = 2 - \cancel{2z^{-1}} + 5z^{-2} + z^{-3} - \cancel{z^{-4}} - z^{-5} + \cancel{2z^{-1}} - 2z^{-2} + 5z^{-3} + \cancel{z^{-4}} - z^{-5} - z^{-6}$$

$$X_0(z) = 2 + 3z^{-2} + 6z^{-3} - 2z^{-5} - z^{-6}$$

$$X_1(z) = (1 - z^{-1})(2 - 2z^{-1} + 5z^{-2} + z^{-3} - z^{-4} - z^{-5}) = 2 - 2z^{-1} + 5z^{-2} + z^{-3} - z^{-4} - \cancel{z^{-5}} - 2z^{-1} + 2z^{-2} - 5z^{-3} - z^{-4} + \cancel{z^{-5}} + z^{-6}$$

$$X_1(z) = 2 - 2z^{-1} + 7z^{-2} - 4z^{-3} - 2z^{-4} + z^{-6}$$

<u>NAKON DECIMACIJE</u>

$$\left.\begin{aligned} X_0(z) &= 2 + 3z^{-2} + 0z^{-4} - z^{-6} \\ X_1(z) &= 2 + 7z^{-2} - 2z^{-4} + z^{-6} \end{aligned}\right\} \quad \begin{aligned} X_0 &= 2 + 3z^{-1} + 0z^{-2} - z^{-3} \\ X_1 &= 2 + 7z^{-1} - 2z^{-2} + z^{-3} \end{aligned}$$

$$\bar{X_0} = 1 \qquad \bar{X_1} = 2$$

decimirani

$$\left.\begin{aligned} \sigma_1^2 &= \frac{1}{3} \sum_{n=0}^{3} (x(n) - \bar{x})^2 \Rightarrow \sigma_1 = 1{,}8257 \\ \sigma_2^2 &= \frac{1}{3} \sum_{n=0}^{3} (x(n) - \bar{x})^2 \Rightarrow \sigma_2 = 3{,}7417 \end{aligned}\right\} \begin{aligned} r_1 &= 0 \\ r_2 &= 1 \end{aligned}$$

$$\left.\begin{aligned} \sigma_1 &= 1{,}8257 \\ \sigma_2 &= 1{,}871 \end{aligned}\right\} \begin{aligned} r_1 &= 0 \\ r_2 &= 2 \end{aligned}$$

$$G_{SBC} = \frac{2^{-2\bar{r}} \sum_{k=1}^{M} \sigma_k^2}{\sum_{k=1}^{M} \sigma_k^2 \cdot 2^{-2r_k}} =$$

2

$$\left.\begin{aligned} \sigma_1 &= 1{,}8257 \\ \sigma_2 &= 0{,}9354 \end{aligned}\right\} \begin{aligned} r_1 &= 1 \\ r_2 &= 2 \end{aligned}$$

$$= 1{,}0297$$

$$\left.\begin{aligned} \sigma_1 &= 0{,}91285 \\ \sigma_2 &= 0{,}9354 \end{aligned}\right\} \left.\begin{aligned} r_1 &= 1 \\ r_2 &= 3 \end{aligned}\right\} \bar{r} = 2$$