(1.) $x[n] = \{\underline{1}, -2, 5, -2\}$

a) $W_N^{nk} = e^{-j\frac{2\pi nk}{N}}$

$$\begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

b)
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 5 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 10 \\ -4 \end{bmatrix}$$

c) Matrica je unitarna ako vrijedi $U \cdot U^* = U^* \cdot U = I \cdot c$, $c > 0$
stupci unitarne matrice su međusobno ortogonalni

$$U = \begin{bmatrix} 2^{-1/2} & 2^{-1/2} & 0 \\ -2^{-1/2} j & 2^{-1/2} j & 0 \\ 0 & 0 & j \end{bmatrix}$$

$$\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2}\, j & 1/\sqrt{2}\, j & 0 \\ 0 & 0 & j \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2}\, j & -1/\sqrt{2}\, j & 0 \\ 0 & 0 & -j \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(2.) a) STFT transformacijski par

$$X(\tau, \omega) = \int_{-\infty}^{\infty} x(t)\, g^*(t-\tau)\, e^{-j\omega t}\, dt$$

pomak

lokalizacija u vremenu

lokalizacija u frekvenciji

$g(t)$ – lokalni analizirajući otvor

$$x(t) = \frac{1}{2\pi \|g\|^2} \int_\tau \int_\omega X(\tau, \omega)\, g(t-\tau)\, e^{j\omega t}\, dt$$

b) 1) Kao niz Fourierovih transformacija za različite vremenske pomake τ

$$X(\tau,\omega) = \int_t \left[x(t)\, g^*(t-\tau)\right] e^{-j\omega t}\, dt$$

2) Kao slog filtracija za različite frekvencijske pomake ω

$$X(\tau,\omega) = \int_t \left[x(t)\, e^{-j\omega t}\right] g^*(t-\tau)\, dt$$

c) HEISENBERGOV PRINCIP NEODREĐENOSTI

Za svaki transformacijski par $x(t), X(\omega)$ vrijedi

$$\Delta t \cdot \Delta \omega \geq \frac{1}{2} \qquad \left(\Delta_t \cdot \Delta_f \geq \frac{1}{4\pi}\right)$$

$\boxed{\text{STFT}}$

$$\Delta_t^2 = \frac{\int_{-\infty}^{\infty} (t-\tau)^2 \left|g(t-\tau)\, e^{j\omega t}\right|^2 dt}{\int_{-\infty}^{\infty} \left|g(t-\tau)\, e^{j\omega t}\right|^2}$$

$$\Delta_f^2 = \frac{\int_{-\infty}^{\infty} (w-\omega)^2 \left|G(w-\omega)\, e^{j(w-\omega)\tau}\right|^2 dw}{\int_{-\infty}^{\infty} \left|G(w-\omega)\, e^{j(w-\omega)\tau}\right|^2 dw}$$

$\boxed{\text{CWT}}$

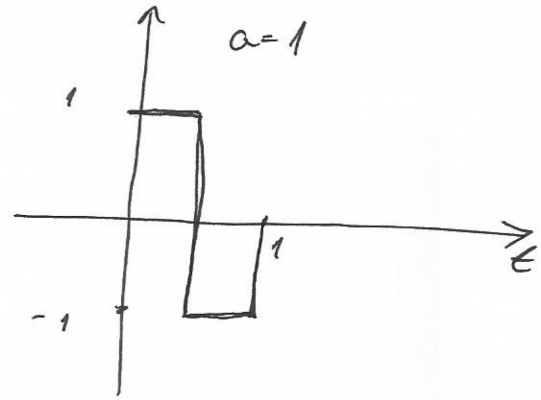
$$\Delta_t^2 = \frac{\int_{-\infty}^{\infty} (t-\tau)^2 \left|\frac{1}{\sqrt{a}}\, \psi\left(\frac{t-\tau}{a}\right)\right|^2 dt}{\int_{-\infty}^{\infty} \left|\frac{1}{\sqrt{a}}\, \psi\left(\frac{t-\tau}{a}\right)\right|^2 dt} = a^2 \Delta_\psi^2$$

$$\Delta_f^2 = \frac{\int_{-\infty}^{\infty} (\omega-\omega_\psi)^2 \left|\sqrt{a}\, \Psi(a\omega)\, e^{-j\omega\tau}\right|^2 d\omega}{\int_{-\infty}^{\infty} \left|\sqrt{a}\, \Psi(a\omega)\, e^{-j\omega\tau}\right|^2 d\omega} = \frac{\Delta_\psi^2}{a^2}$$

$\Delta_t = a \cdot \Delta_\psi$ skala · širina matične fje

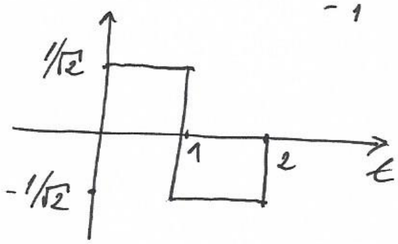
$\Delta_f = \frac{\Delta_\psi}{a}$ širina matične fje / skala

③ CWT $\frac{1}{\sqrt{|a|}}\psi\left(\frac{t-\tau}{a}\right)$



a) $a=2$

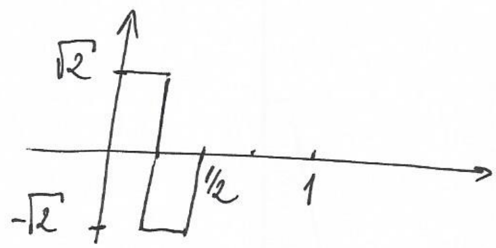
$\frac{1}{\sqrt{2}}\psi\left(\frac{t-\tau}{2}\right)$ → 2x rastezanje



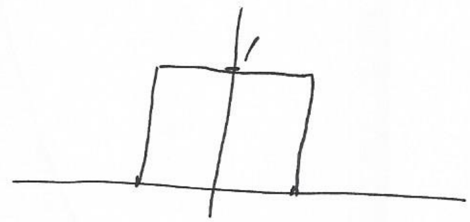
$a=\frac{1}{2}$

stezanje

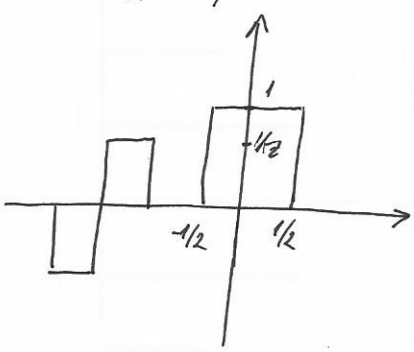
$\sqrt{2}\,\psi\left(2(t-\tau)\right)$



b) $x(t)=\begin{cases}1 & -\frac{1}{2}\le t<\frac{1}{2}\\ 0, & \text{inače}\end{cases}$, $a=2$

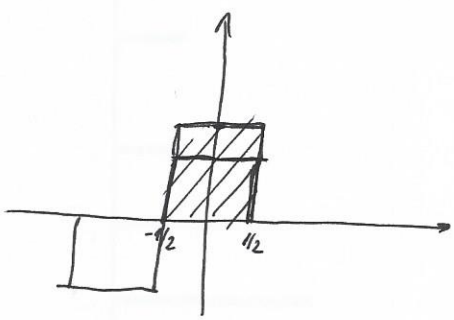


①

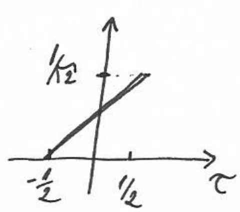


$-\infty<\tau<-\frac{1}{2}$ $\qquad 0$

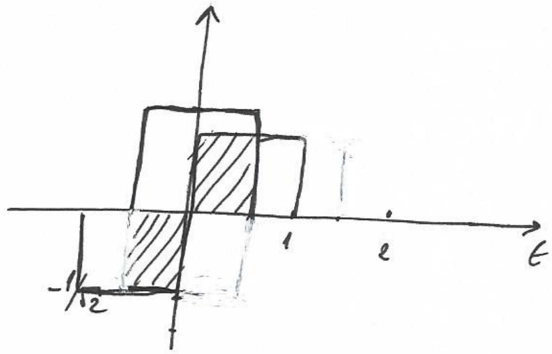
②



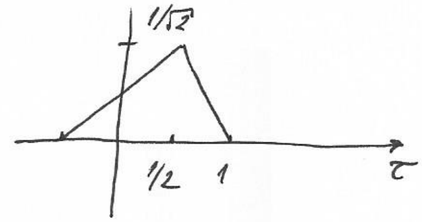
$-\frac{1}{2}<\tau<\frac{1}{2}$



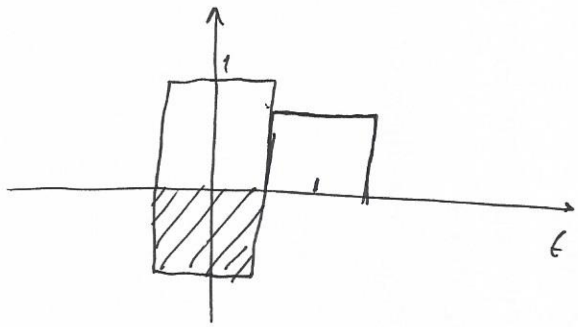
(3)



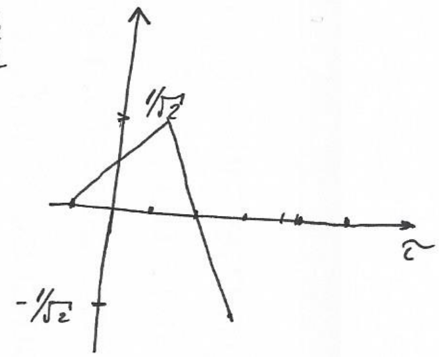
$\frac{1}{2} < \tau < 1$



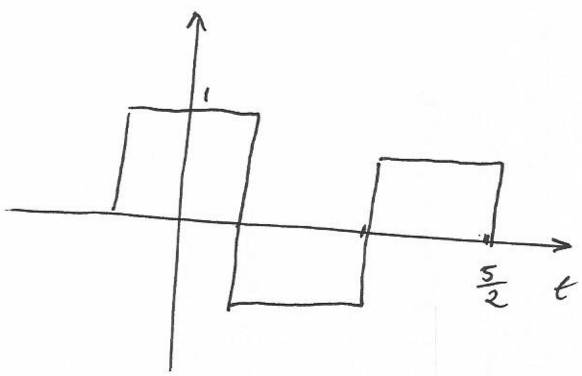
(4)

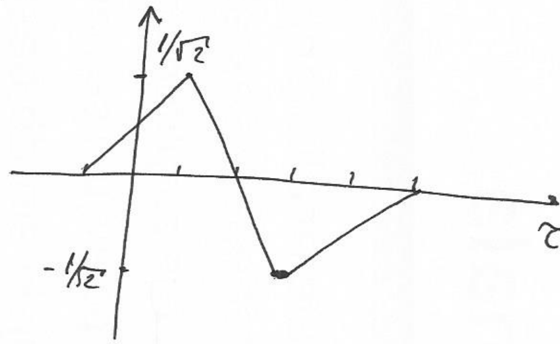


$1 < \tau < \frac{3}{2}$



(5)





4.

$H_0(z) = 2 + 3z^{-1} + 2z^{-2}$ $\quad H_0(e^{j\omega}) = 2 + 3e^{-j\omega} + 2e^{-j2\omega} = e^{-j\omega}(3 + 4\cos(\omega))$

$H_1(z) = 6 - z^{-1} + 6z^{-2}$ $\quad H_1(e^{j\omega}) = 6 - e^{-j\omega} + 6e^{-j2\omega} = e^{-j\omega}(12\cos(\omega) - 1)$

a)

Energetski okvir: $|H_0|^2 + |H_1|^2 = 9 + 24\cancel{\cos(\omega)} + 16\cos^2(\omega) + 144\cos^2 - \cancel{24\cos(\omega)} + 1$

$= 10 + 160\cos^2(\omega)$

min $\Rightarrow \cos(\omega) = 0$

$A = 10$

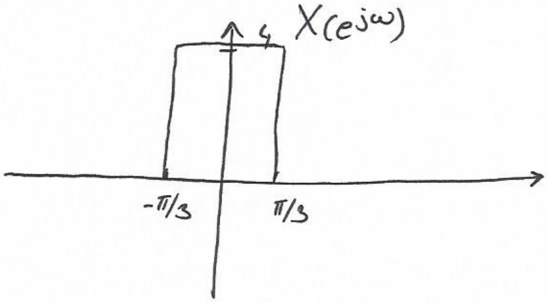
max $\rightarrow \cos(\omega) = 1$

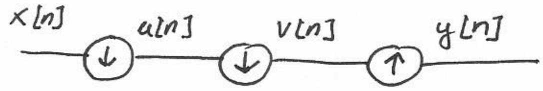
$B = 10 + 160 = 170$

b)

(5.) <u>Decimacija</u> faktorom 2 znači da će novi signal sadržavati svaki drugi uzorak starog signala

<u>Ekspanzija</u> faktorom 2 znači da ćemo stari signal proširiti tako da između svaka dva uzorka umetnemo jednu ∅





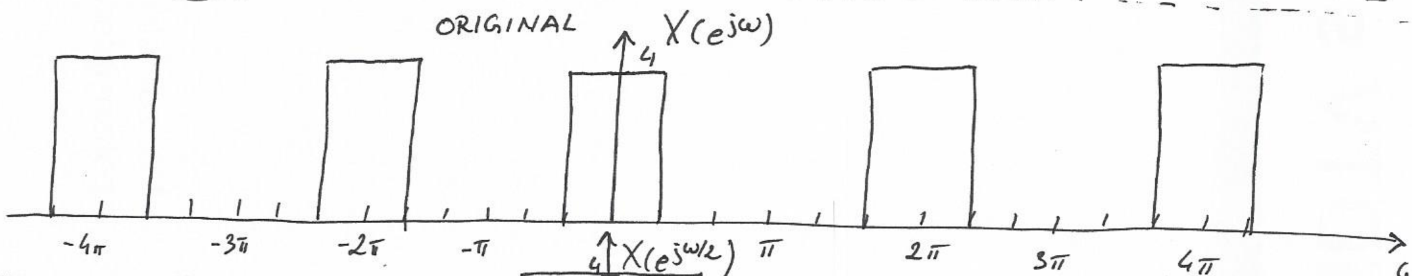
OPĆENITO:

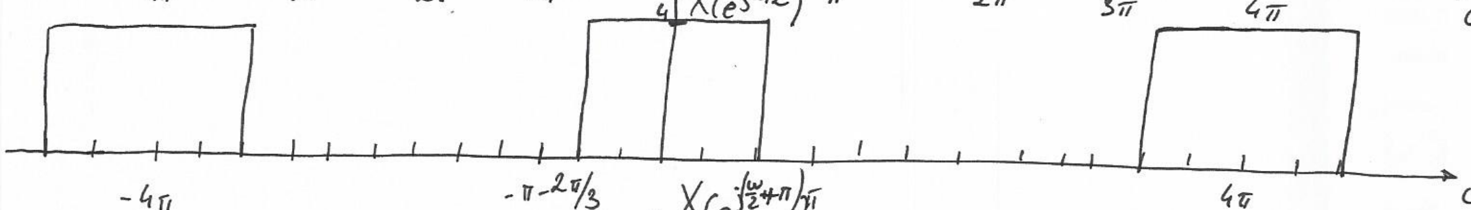
$$U(e^{j\omega}) = \frac{1}{2}\left[X(e^{j\omega}) + X(e^{j(\omega+\pi)})\right]$$

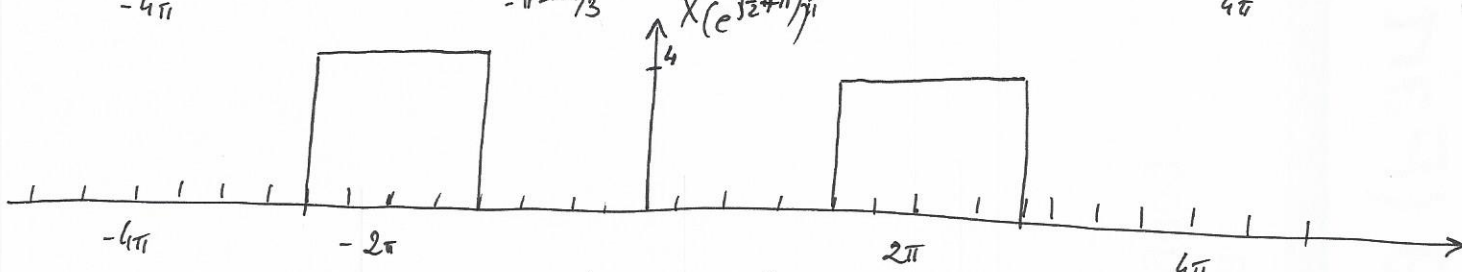
parni, neparni

$$V(e^{j\omega}) = U(e^{j\omega/2}) = \frac{1}{2}\left[X(e^{j\omega/2}) + X(e^{j(\frac{\omega}{2}+\pi)})\right]$$

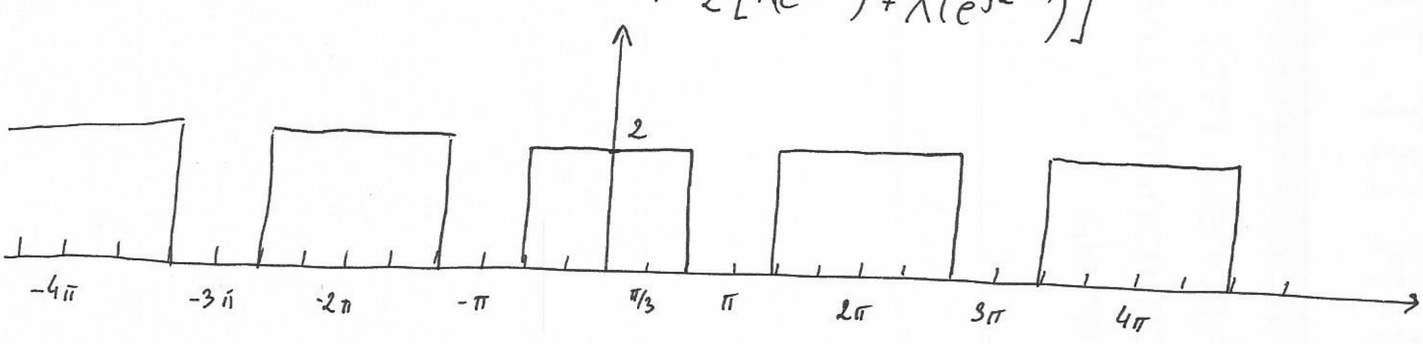
1. Decimacija



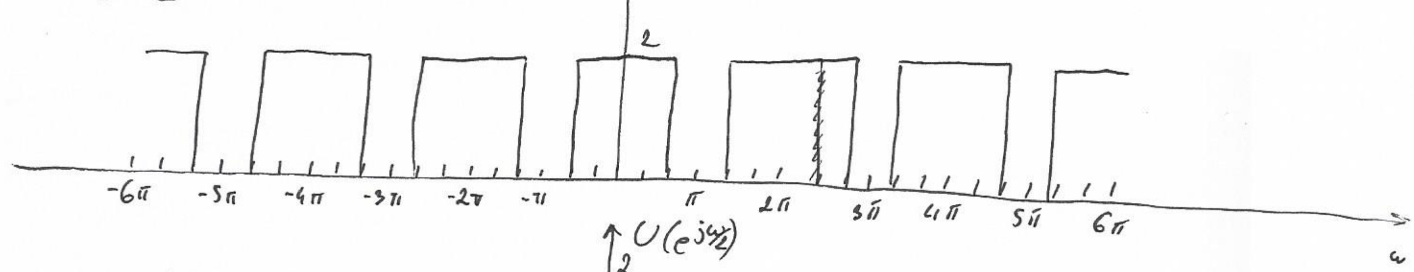


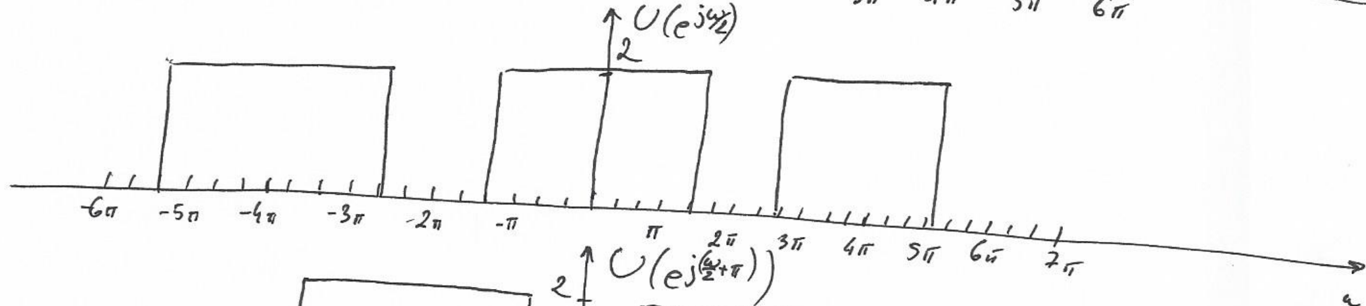


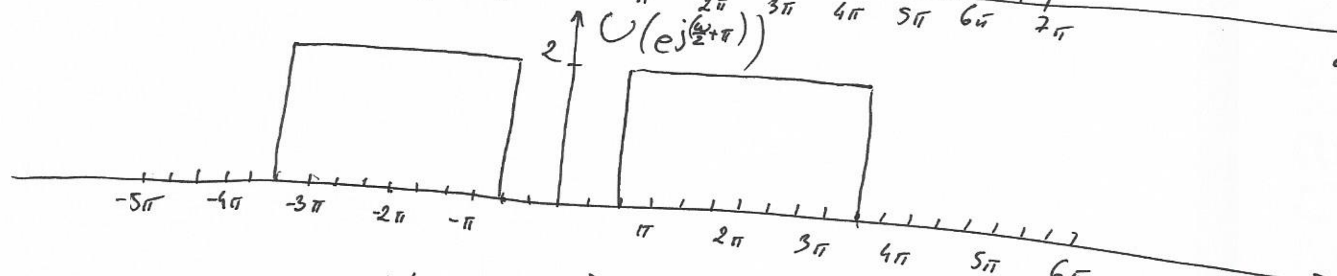
$$U(e^{j\omega}) = \frac{1}{2}\left[X(e^{j\omega/2}) + X(e^{j(\frac{\omega}{2}+\pi)})\right]$$



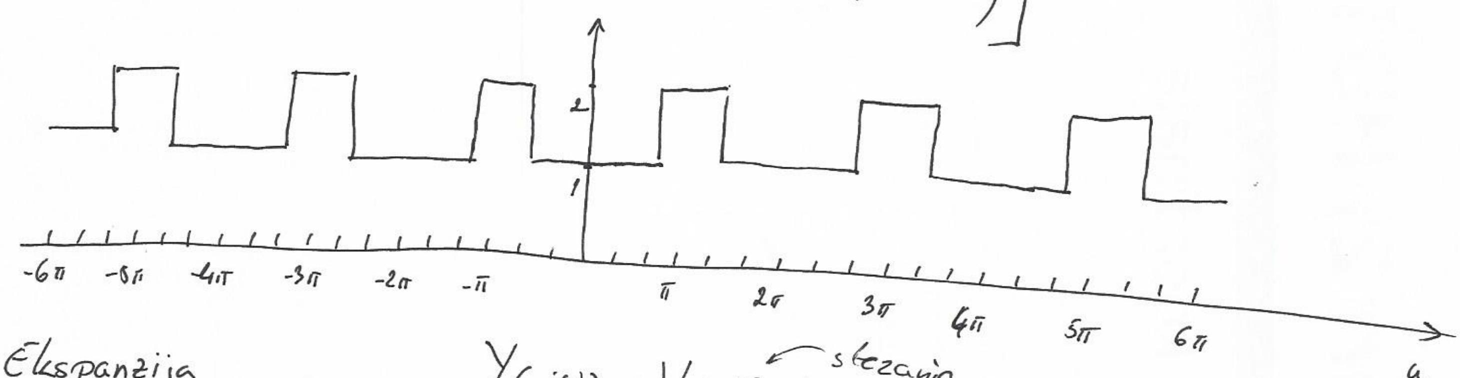
2. Decimacija







$$V(e^{j\omega}) = \frac{1}{2}\left[U(e^{j\omega/2}) + U(e^{j(\frac{\omega}{2}+\pi)})\right]$$



Ekspanzija

$$Y(e^{j\omega}) = V(e^{j2\omega})$$ stezanje