Auditorne vježbe

Nelinearni i optimalni sustavi upravljanja

NiOSU

Sadržaj

- Opisna funkcija višeznačnog nelinearnog elementa
  - Simetrični nelinearni element
  - Nesimetrični nelinearni element
- Vlastite oscilacije
  - Određivanje analitički i grafoanalitički
  - Stabilnost
- Prinudne oscilacije
- Popov kriterij apsolutne stabilnosti

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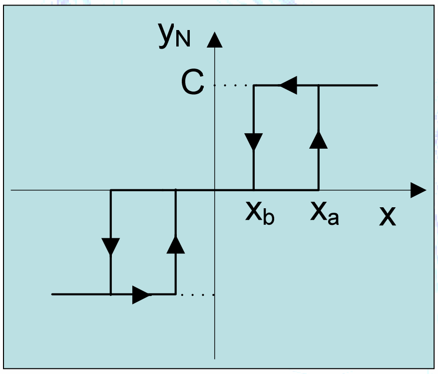
Opisna funkcija višeznačne nelinearnosti

<u>zadatak:</u>

Odredite opisnu funkciju nelinearnog elementa zadanog slikom



$$G_N(X_m) = P(X_m) + jQ(X_m)$$

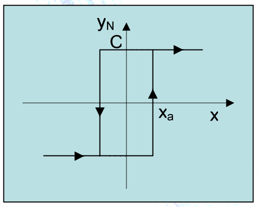
$$P(X_m) = \frac{1}{\pi X_m}\int_0^{2\pi} F(X_m \sin(\omega t))\sin(\omega t)d(\omega t)$$

$$Q(X_m) = \frac{1}{\pi X_m}\int_0^{2\pi} F(X_m \sin(\omega t))\cos(\omega t)d(\omega t)$$

Opisna funkcija višeznačne nelinearnosti

<u>zadatak:</u>

Odredite opisnu funkciju nelinearnog elementa za nesimetrične oscilacije na ulazu.



$$y_N(t) = F_0(A, x_0) + (a(A, x_0) + \frac{b(A, x_0)}{\omega} p)x^*$$

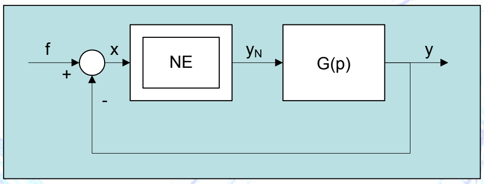
$$F_0(x_0, X_m) = \frac{1}{2\pi}\int_0^{2\pi} F(x_0 + X_m \sin(\omega t))d(\omega t)$$

$$P(X_m) = \frac{1}{\pi X_m}\int_0^{2\pi} F(x_0 + X_m \sin(\omega t))\sin(\omega t)d(\omega t)$$

$$Q(X_m) = \frac{1}{\pi X_m}\int_0^{2\pi} F(x_0 + X_m \sin(\omega t))\cos(\omega t)d(\omega t)$$

Vlastite oscilacije

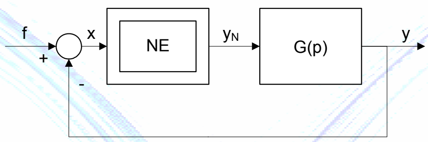
Uvjeti:



1. Harmonička ravnoteža $y(p) = -x(p) \quad \Rightarrow \quad Y_m = X_m$, $\varphi = \pi$

2. Filter hipoteza $\dfrac{Y_{Nk}}{Y_{N1}} = \left|\dfrac{G_L(jk\omega)}{G_L(j\omega)}\right| << 1$

Vlastite oscilacije



$$y(t) = G_N(X_m)\frac{B(p)}{A(p)}x(t)$$

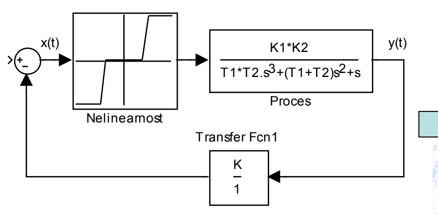
$$x(t) = -y(t)$$

$$\Rightarrow \quad 1 = -G_N(X_m)\frac{B(p)}{A(p)}$$

$$\boxed{D(p) = A(p) + G_N(X_m)B(p) = 0}$$

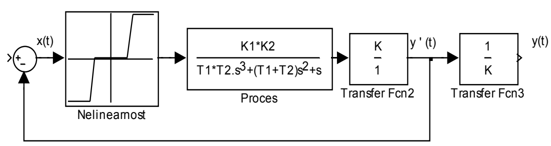
Vlastite oscilacije

A što ako nije jedinična povratna veza?



$$y(t) = G_N(X_m)\frac{B(p)}{A(p)}x(t)$$

$$x(t) = -Ky(t)$$



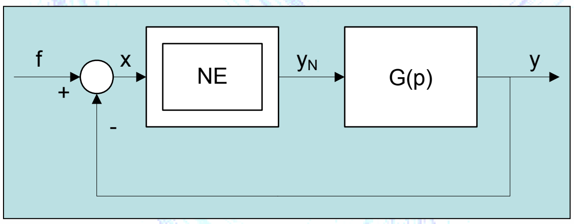
$$1 = G_N(X_m)\frac{B(p)}{A(p)}(-K)$$

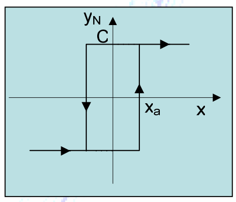
$$D(p) = A(p) + G_N(X_m)B(p)K = 0$$

Vlastite oscilacije

zadatak:

Zadan je nelinearni sustav opisan shemom na slici.





$$G(s) = \frac{K}{(1+T_1 s)(1+T_2 s)}$$

$K = 10$, $T_1 = 1.5$ [s], $T_2 = 1$ [s]

$x_a = 0.1$, $C = 1$

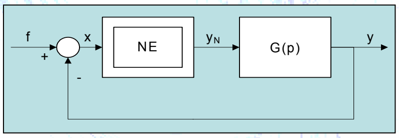
Vlastite oscilacije

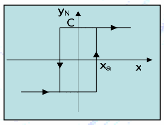
zadatak:

Odredite vlastite oscilacije analitičkim putem.

$$x(t)=X_m\sin(\omega t)$$

$$\boxed{D(j\omega)=A(j\omega)+B(j\omega)G_N(X_m)=0}$$





$$G(s)=\frac{K}{(1+T_1s)(1+T_2s)}$$

K = 10, T_1 = 1.5 [s], T_2 = 1 [s]

x_a = 0.1, C = 1

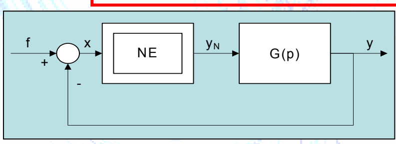
Vlastite oscilacije

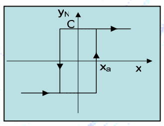
zadatak:

Odredite stabilnost vlastitih oscilacija korištenjem kriterija Petrova.

$$D(j\omega)=R+jI$$

$$\boxed{\frac{\partial R}{\partial X_m}\frac{\partial I}{\partial \omega}-\frac{\partial I}{\partial X_m}\frac{\partial R}{\partial \omega}\rangle 0}$$





$$G(s)=\frac{K}{(1+T_1s)(1+T_2s)}$$

K = 10, T_1 = 1.5 [s], T_2 = 1 [s]

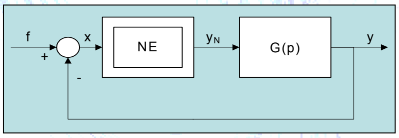
x_a = 0.1, C = 1

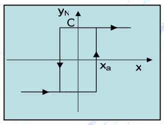
Vlastite oscilacije

zadatak:

Odredite vlastite oscilacije metodom Goldfarba.

$$\boxed{G_L(j\omega) = -\frac{1}{G_N(X_m)}} \qquad \boxed{G_N(X_m) = -\frac{1}{G_L(j\omega)}}$$





$$G(s) = \frac{K}{(1+T_1 s)(1+T_2 s)}$$

K = 10, T_1 = 1.5 [s], T_2 = 1 [s]

x_a = 0.1, C = 1

Vlastite oscilacije

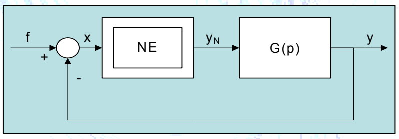
zadatak:

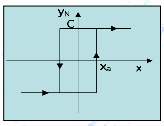
Odredite stabilnost vlastitih oscilacija korištenjem kriterija Mikhailova.

$$\boxed{\frac{\partial Q_L}{\partial \omega}\frac{\partial P_N}{\partial X_m} - \frac{\partial P_L}{\partial \omega}\frac{\partial Q_N}{\partial X_m} < 0}$$

$$P_N = \mathrm{Re}\left\{-G_N^{-1}(X_m)\right\}$$

$$Q_N = \mathrm{Im}\left\{-G_N^{-1}(X_m)\right\}$$





$$G(s) = \frac{K}{(1+T_1 s)(1+T_2 s)}$$

K = 10, T_1 = 1.5 [s], T_2 = 1 [s]

x_a = 0.1, C = 1

Dijagram Popova

POPOV KRITERIJ APSOLUTNE STABILNOSTI

Ravnotežno stanje nepobuđenog nelinearnog sustava upravljanja prikazanog na slici 2.18. u Knjizi str. 103 će biti globalno asimptotski stabilan-apsolutno stabilan ako je sljedeće ispunjeno:

- Linearni dio sustava je vremenski nepromjenjiv, stabilan i u upravljiv
2. Nelinearna funkcija F(x) je klase [0, Kf] uz $0<K_F<$ i zadovoljava uvjet
3. Postoje dva strogo pozitivna realna broja q>0 i proizvoljno malen broj delat>0 tako da za sve omega>=0 vrijedi sljedeća nejednakost

$$F(0)=0$$

$$xF(x)>0;\ x\neq 0$$

$$\int_0^{\pm\infty} F(x)dx=\pm\infty$$

$$0<\frac{F(x)}{x}<k_2;\ \text{za x}\neq 0$$

$$\mathrm{Re}\left\{(1+j\omega)G_L(j\omega)\right\}+\frac{1}{k_2}\geq\delta\rangle 0$$

Dijagram Popova

Postupak:

1. Odredi $\quad \mathrm{Re}\left\{G_L(j\omega)\right\} \quad \mathrm{Im}\left\{G_L(j\omega)\right\}$

2. Odredi

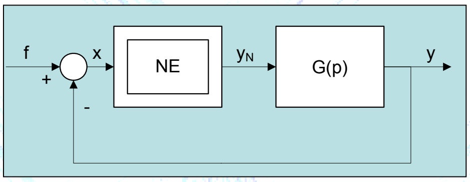
$$\begin{aligned} G_P(j\omega) &= \mathrm{Re}\left\{G_L(j\omega)\right\}+j\omega\,\mathrm{Im}\left\{G_L(j\omega)\right\} \\ &= U(\omega)+jV_P(\omega) \end{aligned}$$

3. Odredi klasu nelinearnosti za koju je sustav apsolutno stabilan

Dijagram Popova

zadatak:

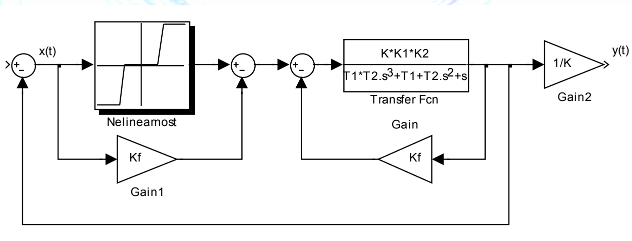
Za koju klasu nelinearnosti će zatvoreni sustav oblika na slici biti apsolutno stabilan?



$$G(s)=\frac{K}{(1+T_1s)(1+T_2s)(1+T_3s)}$$

Dijagram Popova

A što ako je sustav astatički?



Kroz točku $\frac{1}{k+K_f}$ se povlači pravac takav da ne dira dijagram Popova.