

# BIFURKACIJA (RAČVANJE)

Stabilni linearni sustav  $\rightarrow$  struktorno stabilan sustav  
(nije osjetljiv na male promjene parametara sustava)

Stabilni nelinearni sustav  $\rightarrow$  nije struktorno stabilan

P: Zasto?

0: Ravnotežna stanja nelinearnih sustava ovise o:

- strukturi
- parametrima
- početnim uvjetima
- vanjskim djelovanjima

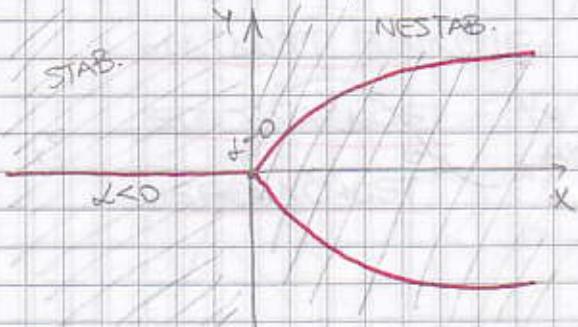
Točke u prostoru parametara sustava u kojima sustav nije struktorno stabilan nazivaju se točke bifurkacije, a parametri u tim točkama BIFURKACIJSKI PARAMETRI (kritični iznosi parametara)

$\rightarrow$  U bifurkacijskim točkama obavezno dolazi do promjene dinamičkih podešenja

pr. Duffingova jednadžba:

$$\ddot{x}(t) + \omega x(t) + x^3(t) = 0 \quad x(t)[\omega + x^2(t)] = 0$$

$\omega$  - jedini parametar sustava



$$\omega < 0 \rightarrow x_e = 0$$

$$\omega > 0 \rightarrow x_e = \sqrt{\omega} \text{ i } x_e = -\sqrt{\omega}$$

$$\omega = 0 \rightarrow \text{točka bifurkacije}$$

Repeleri, atraktori, sedla  $\rightarrow$  ravnotežne točke

- ①  $x_e$  tipa repeler  $\Rightarrow$  u području privlačenja svaša trajektorija koja počinje u tom području privlačenja biti će odbijena od te točke



- ②  $x_e$  tipa atraktor  $\Rightarrow$  privlači

- ③  $x_e$  tipa sedlo  $\Rightarrow$  specijalne ravnotežne točke. Ako se nadete s jedne strane sedla onda privlači a s druge odbija.

### Primjer

$$\dot{x}(t) = f(x, u) = x - x^3 + u$$

$$\text{Nepobudeni sustav} \Rightarrow u=0 \Rightarrow \dot{x}(t) = x - x^3 = 0$$

$$x_{e1} = 0 \rightarrow R$$

$$x_{e2} = 1 \rightarrow A$$

$$x_{e3} = -1 \rightarrow A$$

$$J = \begin{bmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial u} \\ \end{bmatrix} = \begin{bmatrix} 1-3x^2 & 1 \end{bmatrix}$$

$\rightarrow$  lokalna stabilitet  
se određuje sa  
producujućim

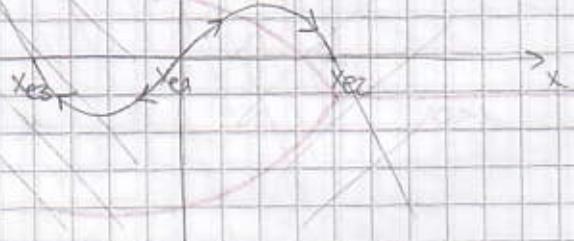
pozitivno privlači  
prirodno  
odnosno

pozitivno privlači  
prirodno  
odnosno

Za  $x_{e1}$ :  $J_1 = 1$  (poz) - nestabil

Za  $x_{e2}$ :  $J_2 = -2$  (neg)  $\downarrow$  stabilni

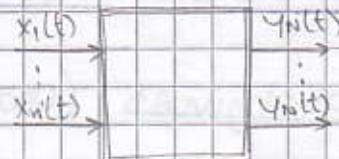
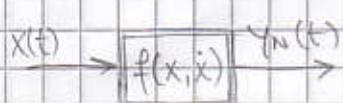
Za  $x_{e3}$ :  $J_3 = -2$  (neg)



## Nelinearnosti

- neželjene - sadržane u procesima (svoštve svakog procesa)
- naučene - ugrađuju se u SAV s ciljem postizanja određenih performansi sustava

### Nelinearni element



→ mogu biti vremenski neprovijenjivi ili vrem. provijenjivi  
 $y_N(t) = f(x, i)$        $y_N(t) = f(x, i, t)$

Tipični nelinearni elementi: Zaslonje, neobičljivost, misterija, težje, izdakost, upor...

Naučene nelinearnosti: reljivi, A/D i D/A pretv., modulatori...

### Slojstva nelinearnosti

SIMETRIČNOST → u odnosu na os ordinatu (parna sim.)

$$\text{pr. } y_N(x) = \sum_{i=1}^{\infty} c_{2i} x^{2i}$$



→ u odnosu na ishodište (neparna sim.)

$$\text{pr. } y_N(x) = \sum_{i=1}^{\infty} c_{2i+1} x^{2i+1}$$



NESIMETRIČNOST - može se simetrirati dvostrukom velikom signalu

### GLATKOST FUNKCIJE

JEDNOZNAČNOST - tamo postoji jedan y

VIŠEZNAČNOST

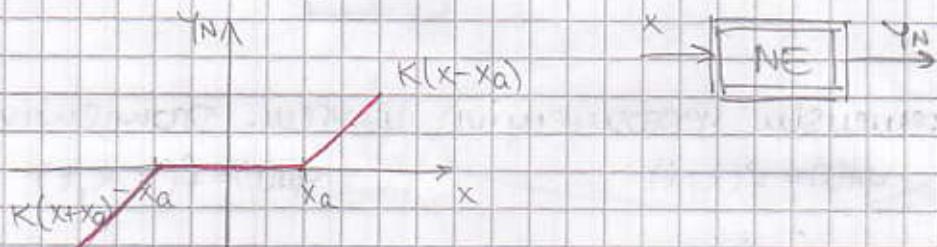
## Tipični NE elementi

članak 10. stranica

Svojstvene i najveće ne linearnosti → staticke karakteristike  
mogu normirati i aproksimativno približiti

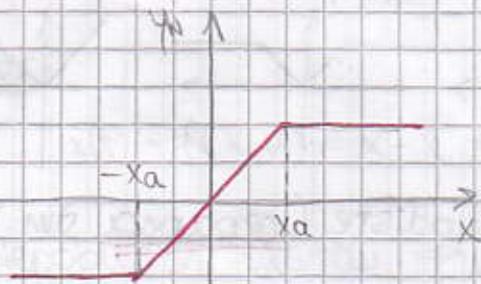
NE s jednoznačnim kont. stat. karakteristikom

### ① Zone neosjetljivosti (Dead zone)



→ ovalnu karakteristiku imaju sve vrste prijenosnika pri malim ulaznim signalima

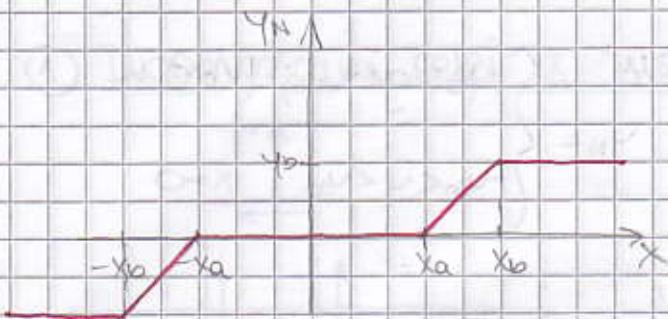
### ② Zasidrenje (ograđivanje) - saturation



$$y_N = \begin{cases} Kx & |x| \leq x_a \\ y_{N1} & |x| > x_a \end{cases}$$

→ sve vrste prijenosnika pri velikim ul. sig.

### ③ Ograničenje (fazidacije) sa zonom neosjetljivosti

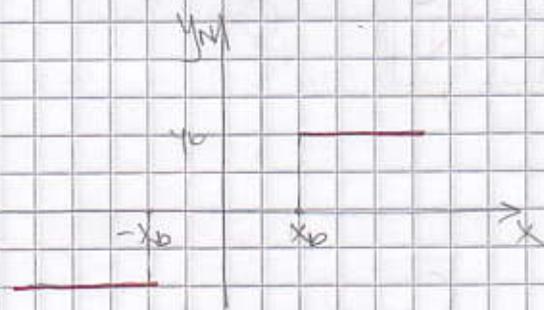


$$Y_N = \begin{cases} 0 & \text{za } |x| \leq x_a \\ y_b \operatorname{sign} x & x_a > x > -x_a \\ K(x-x_a) & x_a > x > x_b \\ K(x+x_a) & -x_b < x < -x_a \\ y_b \operatorname{sign} x & |x| > x_b \end{cases}$$

→ pojačala

NE je jednoznačnim diskontinuiranim stat. karakter.

### ① Dopolozajni relj bez histerese



Polarizirani relj:

- kontakti reljaja su otvoreni

za  $|x| < x_b$

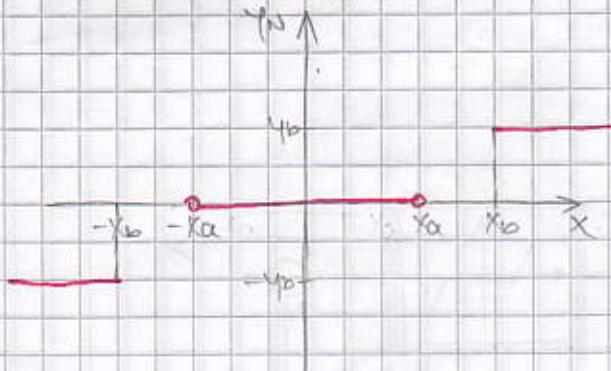
- nije moguće izrediti

čvrđiju ovisnost  $Y_N$

na kontaktu

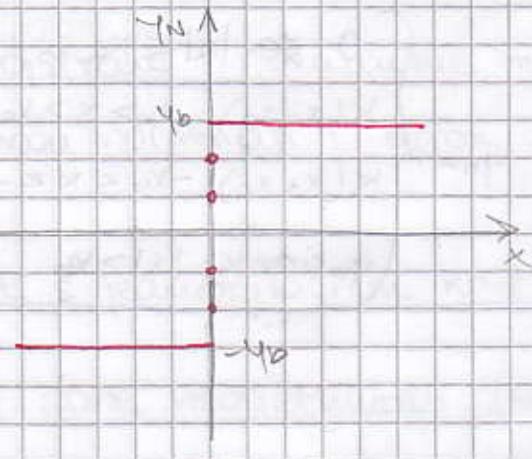
$$Y_N = \begin{cases} y_b \operatorname{sign} x ; & |x| > x_b \\ \text{ne postoji} ; & |x| < x_b \end{cases}$$

### ② Tropoložajni relj bez histerese

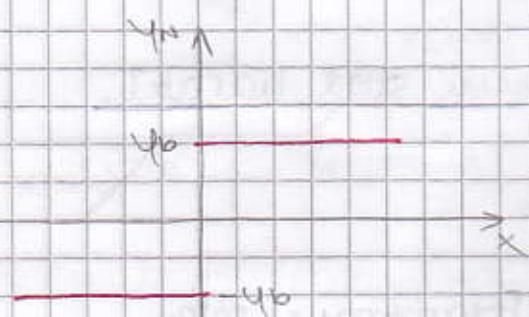


$$Y_N = \begin{cases} y_b \operatorname{sign} x ; & |x| > x_b \\ 0 & ; |x| \leq x_a \\ \text{ne postoji} ; & x_a < |x| < x_b \end{cases}$$

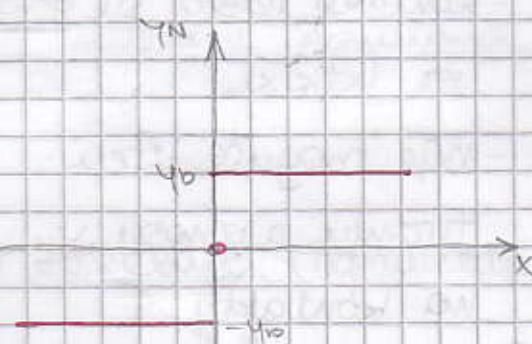
③ Idealni dwopolozajni relej bez histerese



$$y_N = \begin{cases} y_0 \operatorname{sign} x ; |x| > 0 \\ -y_0 < y < y_0 ; x = 0 \end{cases}$$



$$y_N = \begin{cases} y_0 \operatorname{sign} x ; |x| > 0 \\ \text{ne postoji} ; x = 0 \end{cases}$$

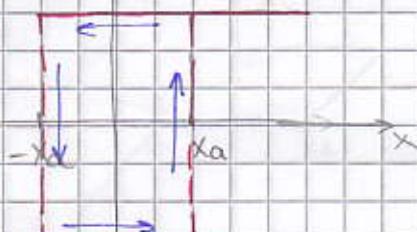


$$y_N = \begin{cases} y_0 \operatorname{sign} x ; |x| > 0 \\ 0 ; x = 0 \end{cases}$$

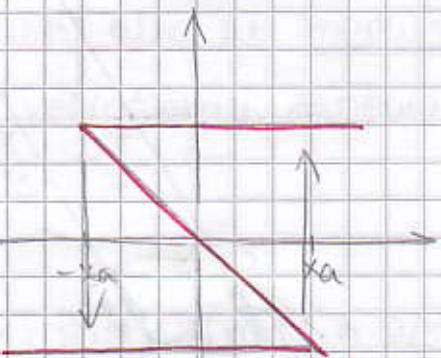
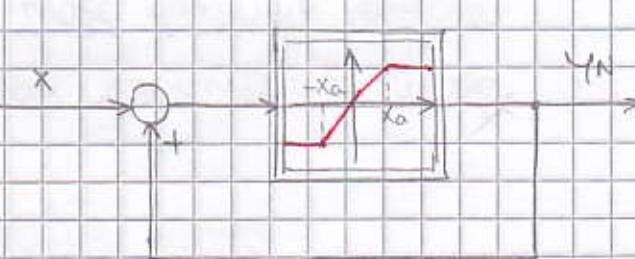
NE s dvoznačním stat. charak.

① Diopoložajni relé s hysterezem

$y_0 \uparrow$

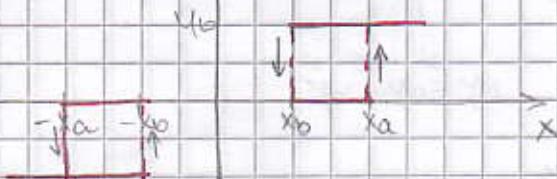


$$y_N = \begin{cases} 1 & ; -x_a < x < \infty \\ -1 & ; -\infty < x < x_a \end{cases}$$



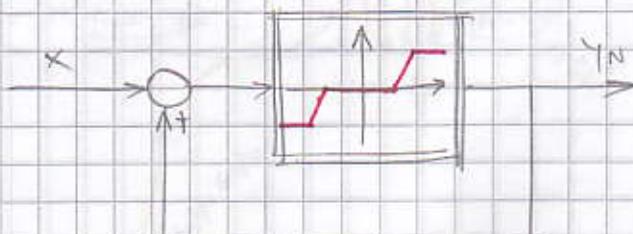
② Trípoloházní relé s hysterezem

$y_N \uparrow$



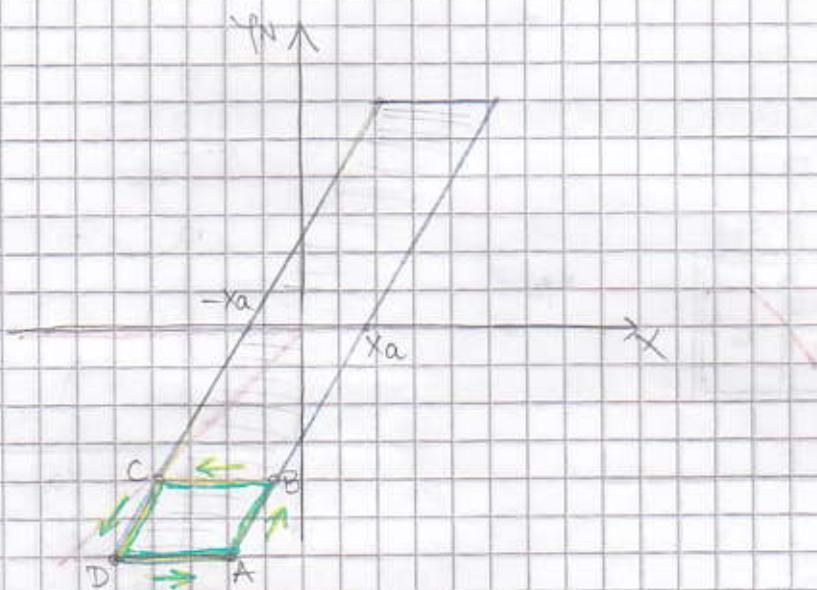
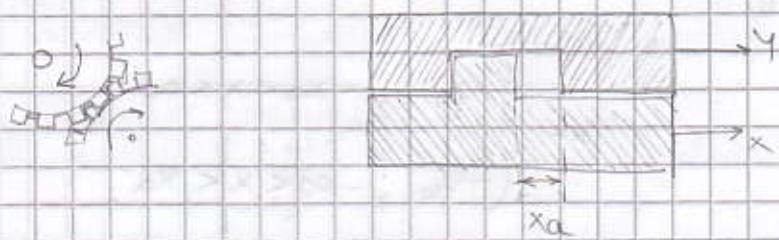
$$y_N = \begin{cases} 1 & ; |x| > x_b \\ 0 & ; -x_b < |x| < x_a \\ -1 & ; |x| > x_a \end{cases}$$

$|x_b| < |x| < |x_a|$ ,  $y_N$  má  
dva vzhledy 0 i  $\pm y_0$



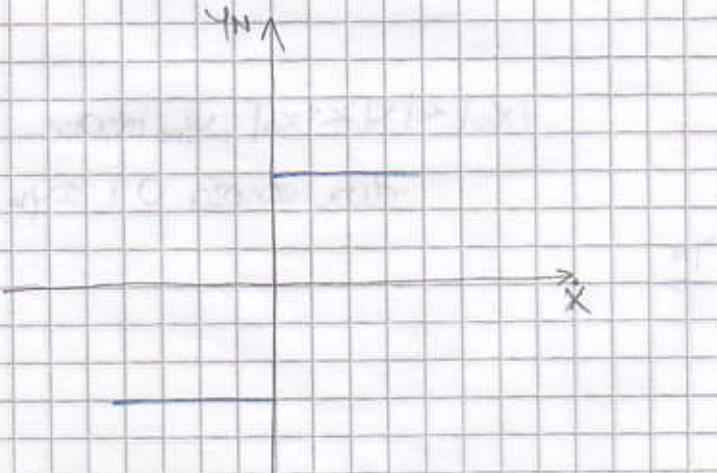
NE s višestručnim stat. karakt.

① Zračnost - javila se kod lupčanika

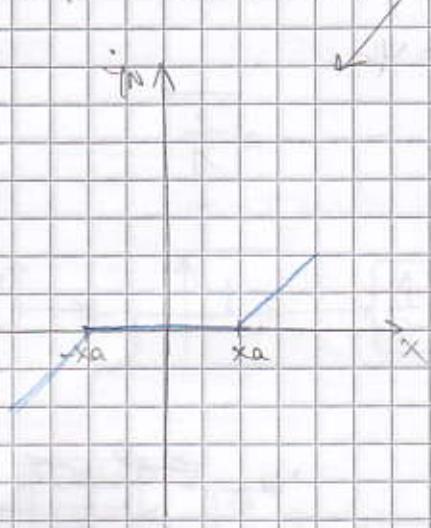


$$F_N = \begin{cases} K(x-x_a) ; & F_N > 0 \\ K(x+x_a) ; & F_N < 0 \\ \text{konec} ; & F_N = 0 ; |x - \frac{x_a}{K}| < x_a \end{cases}$$

② Síra (Coulomb) třeníje (může se approx. dvojpol. reprezentovat bez hist.)  $\rightarrow$  na x-osi je brzda

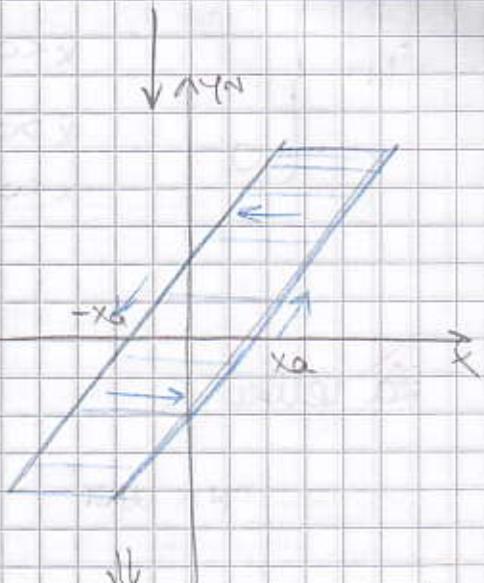


→ approx. možuća sa zavom nosilečtvosti i sa zračnošću



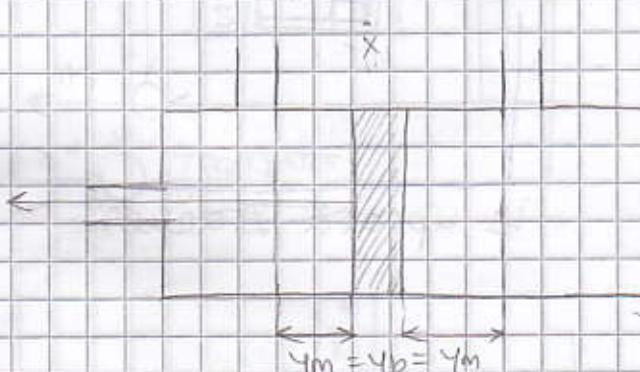
vrijedi akko protusile

i mase gibajućih dijelova  
elem. zanemarivo male

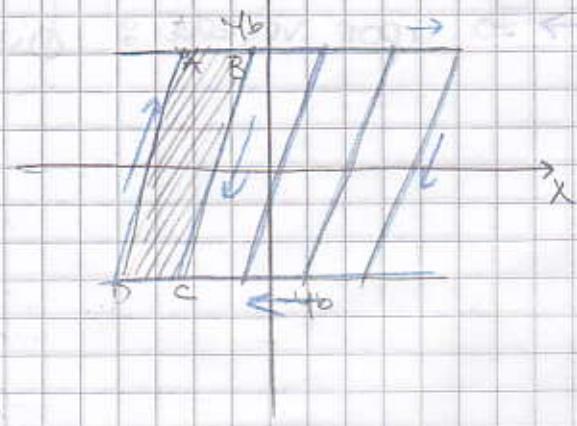
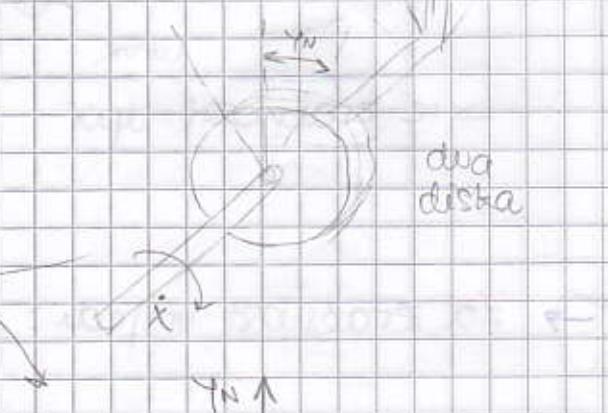
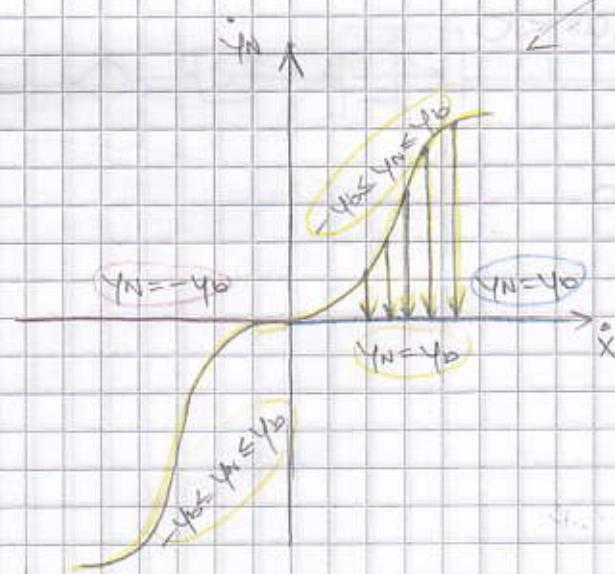


approx. zračnošću se može  
koristiti ali su inercijske  
sile velik. elem. zanemarivo  
male

③ Lijepor - susreće se kod elektromagn., meh. sust., fluidnih sust...



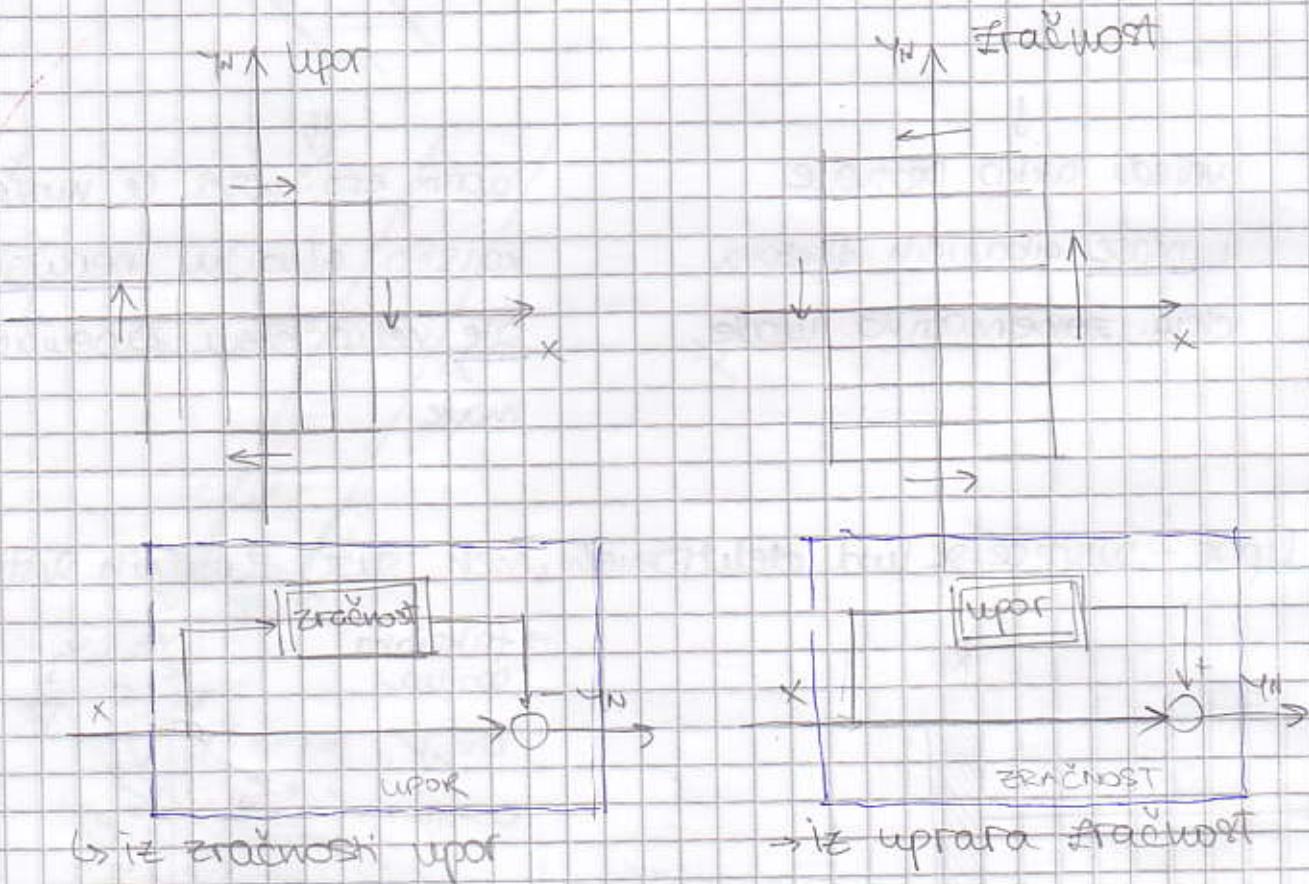
→ fiktivna  
spojka



$$i_N = \begin{cases} k_i & x > 0 \\ 0 & x < 0 \end{cases} \quad -V_0 \leq y_N \leq V_0$$

$$y_N = \begin{cases} V_0 & x > 0 \\ 0 & x < 0 \end{cases}$$

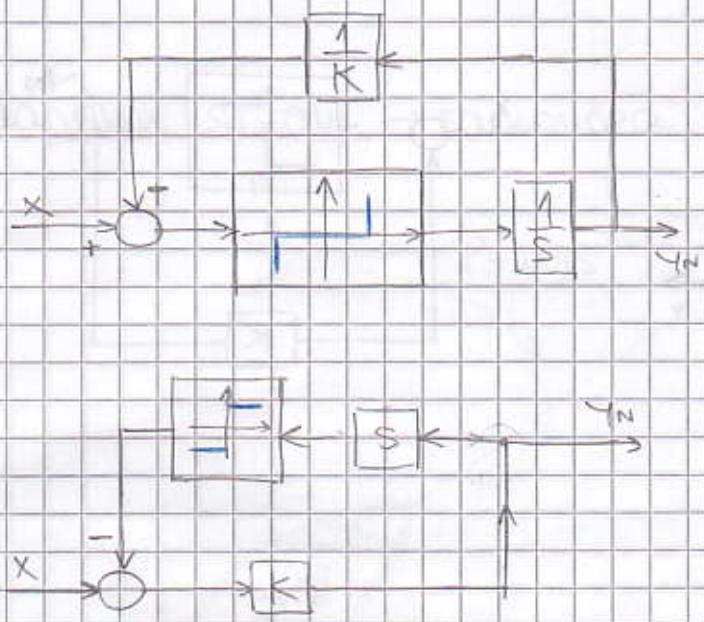
za veliki  $k$



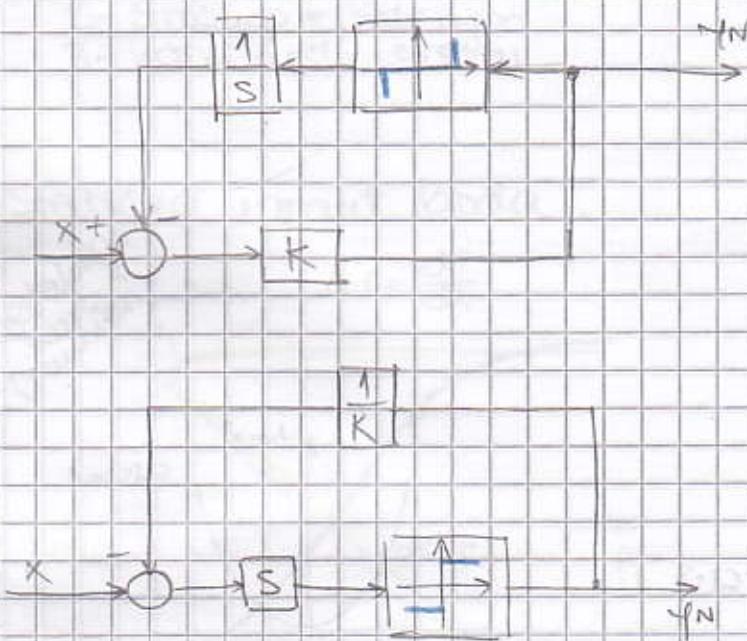
→ Za zračnost vrijedi:  $\int y_N dx < 0$

→ Za upor vrijedi:  $\int y_N dx > 0$

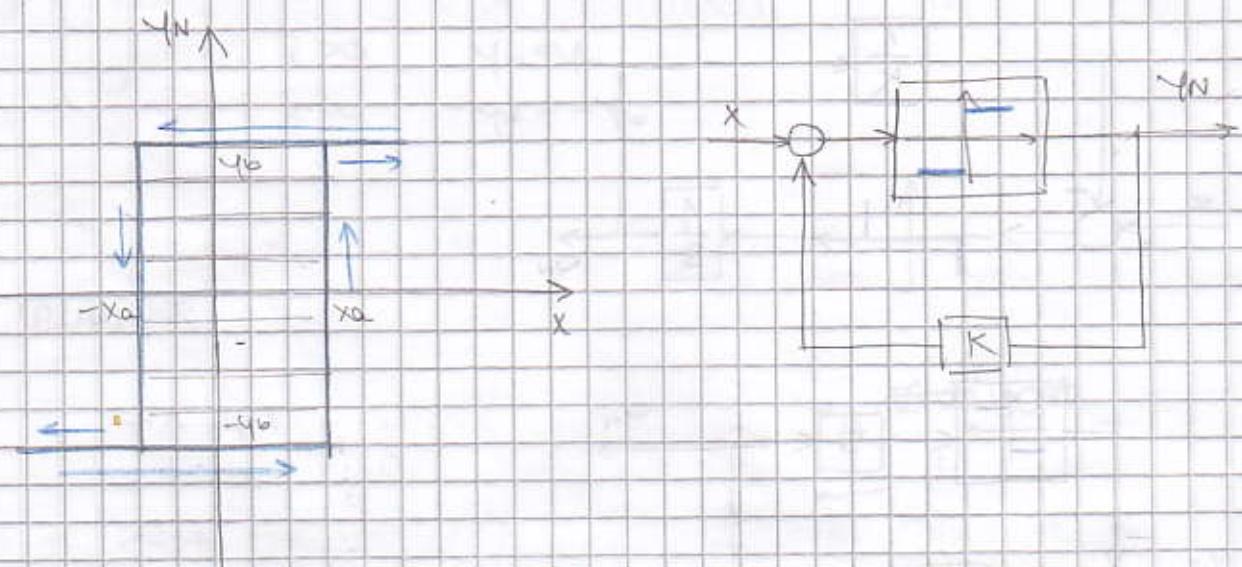
## Simulirajući izračunost



## Simuliranje upora



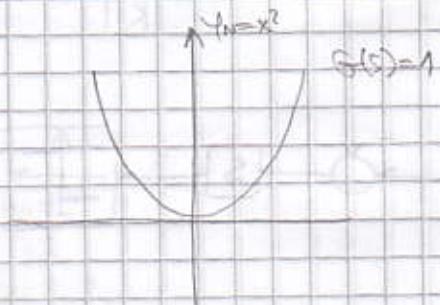
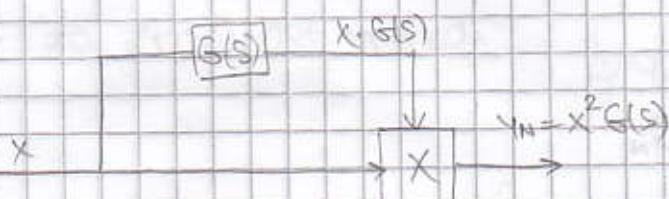
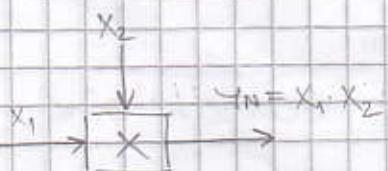
## Magnetska histeresa



## ATIPični NE elementi

(sweze za alg. upravljanje)

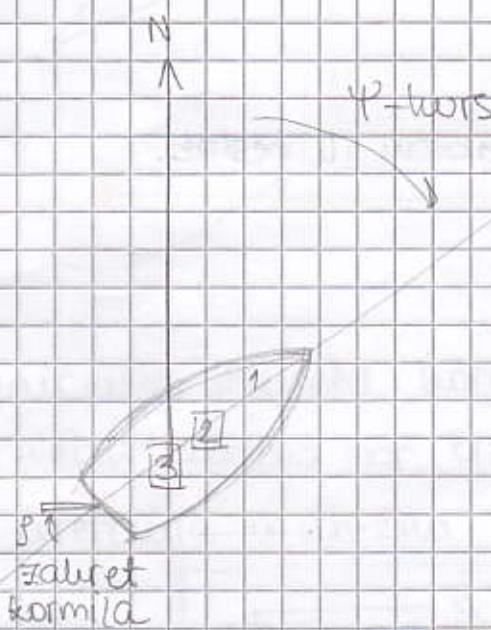
### MODULATOR



Ekstremalni sustavi → rode na osr. extremu

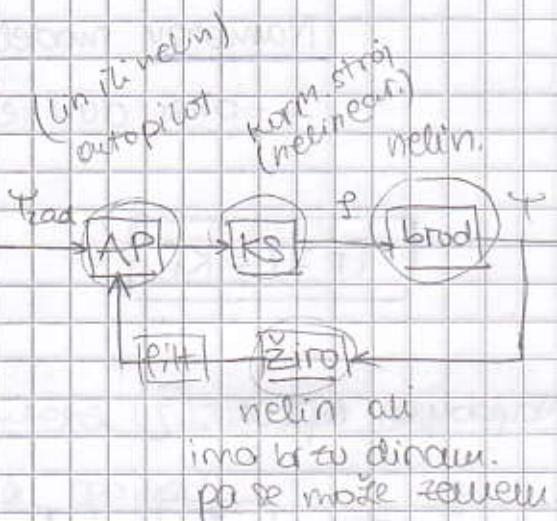
Gofimalni sust. upravljanja

Adaptivni sustavi → samoučedi sustavi

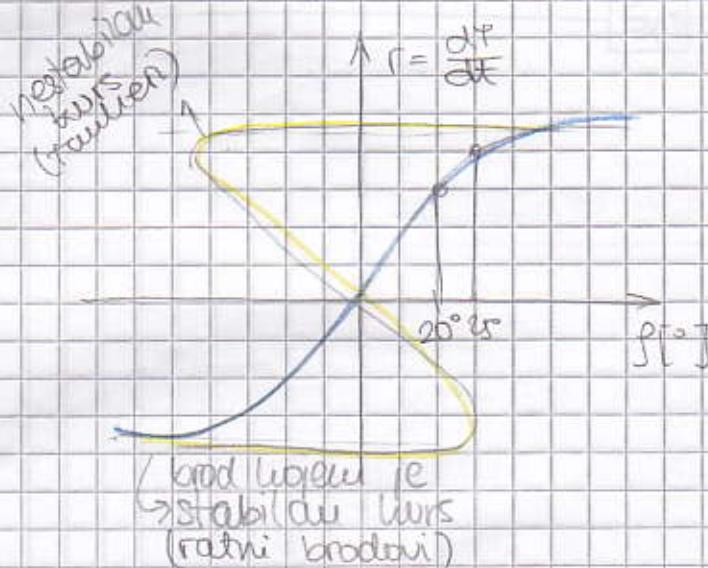


1 - brod

2 - žiroslap + rate gyro  
3 - kompjuterski sustav



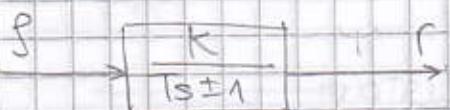
Statička kvalit. broda



→ Uzroko je  $r = \text{konst}$

↳ snima se točka po točku  
u stat. kvalit.

## Aproximacija nelinearnog broda



### Namotov model broda

• ovisi da li je brod stabilan ili nestabil.

$$Tr \pm r = Ks$$

Najbolja aproks:

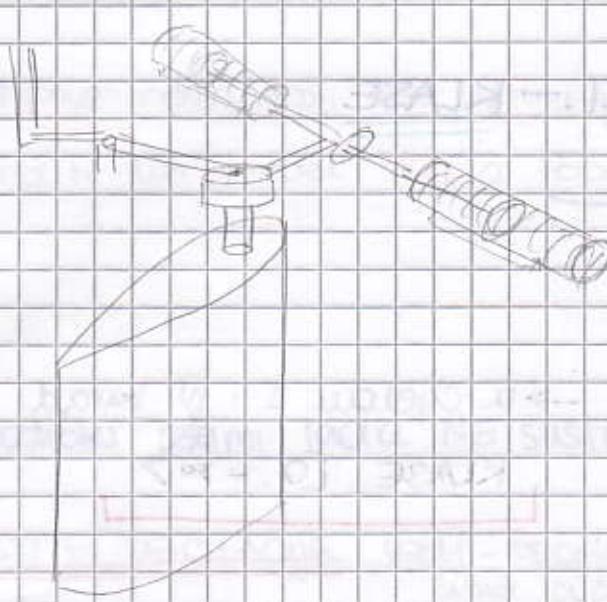


### Norrbinov model

$$Tr \pm r + H(r) = Ks$$

Rusi:  $H(r) = Dr|r|$

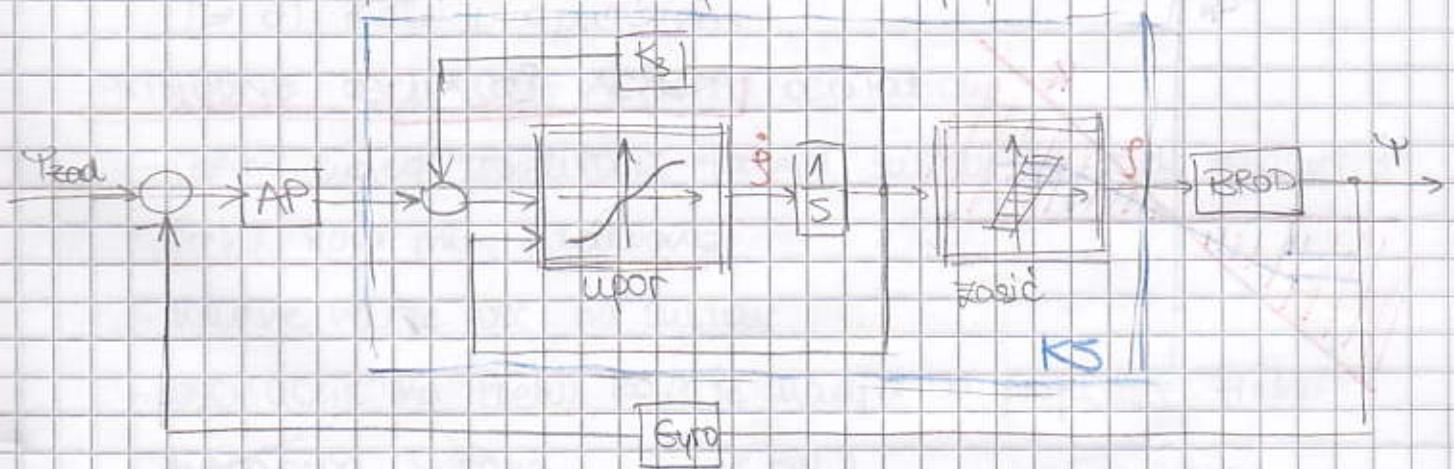
Anuti:  $H(r) = r^3$



→ ograničuje zahreta kormila

→ kod većih kretanja nap. brzine zahreta ( $\dot{\varphi}$ ) max  $7^\circ/\text{sek}$

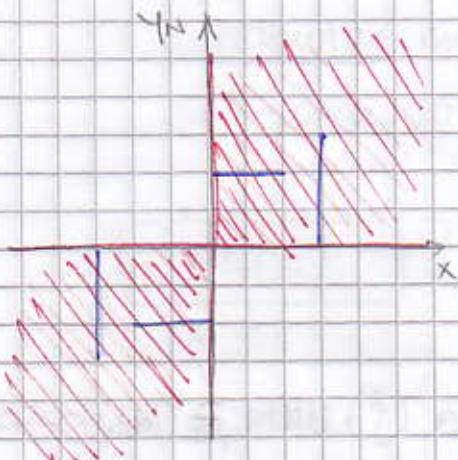
→ lin. kretanje u kružno (filijalna spojka)



## JEDNOZNAČNE NE KARAKT. - KLASE

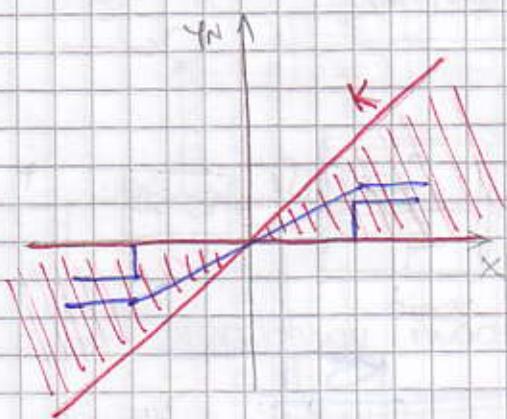
$$Y_N = F(x)$$

$$Y_N = F(t, x)$$



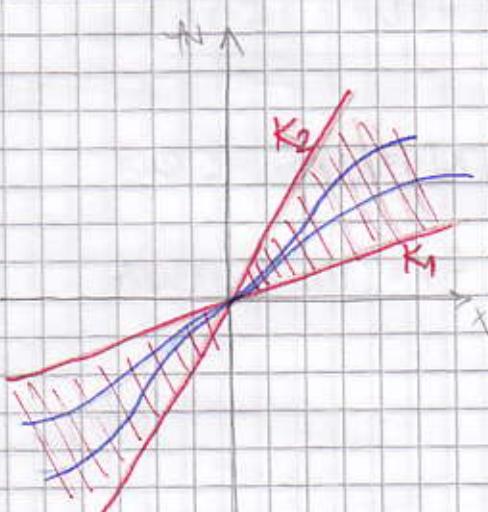
→ u cijelom I i IV kvadr.

KLASE  $[0 - \infty]$



zasićenje u ulazi  $[0 \pi]$

KLASA  $[0 K]$

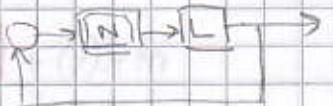


KLASA  $[K, K]$

→ Za tipove nelinearnih koji nisu niti u jednoj klasi ne može se odrediti lin model prema Popovu

## Periodički režimi rada NE sustava

VLASTITE OSCILACIJE (self-oscillations)  
limit cycles

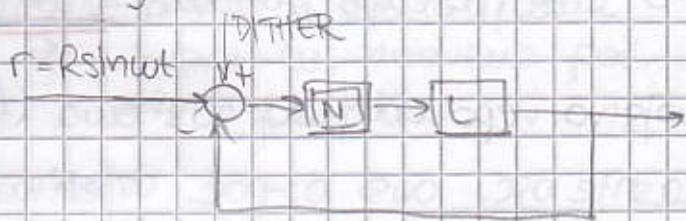


- režim rada nepodauđenog sustava

- sustav radi u režimu stabilizacije (regulator problem)  
↳ režim dječanija radne točke

PRINUĐENE OSCILACIJE (forced oscillations)

- režim u kojem projektant forsira sustav da radi po ampl. i frekv. koja mu odgovara  
- nativne harmon. osc. na sustav  
- oscilacija na frekv. koja je mnošta od propusne frekv.  
linearnog sustava (w > Bw)



- moguće je u sustavu infilirati signal visoke frekv.

(DITHER signal) (w > Bw)

↳ njegova uloga nije ista kao od signala r

↳ on mijenja lin. dio sust. tj. njegovi stat. karakteri

(mijenja nepodigne dijelove stat. karakter.)

↳ može biti harmonički, trokutasti, pravokutni ...

## Opća svojstva periodičkih rješenja velin. dif. jed. II reda

Linearni sustav:

$$m\ddot{x}(t) + \boxed{h}\dot{x}(t) + \boxed{k}x(t) = f(t) \quad (1)$$

Nelinearni sustav:

$$m\ddot{x}(t) + \boxed{\varphi(\dot{x})} + \boxed{\Psi(x)} = f(t) \quad (2)$$

$$m\ddot{x}(t) + \varphi(x, \dot{x}, t) = 0$$

$h$ ,  $\varphi(x)$  - sila otpora koja je prop. brzini kod lin. sust. i u nelin. ovisa o brzini kod nelin. sust.

$k$ ,  $\Psi(x)$  - elastična sila koja je prop. otklonu (poziciji) kod lin. sustava i nelin. f-ja po otklonu kod nelin. sust.

Ako je periodičko rješenje moguće

$$m\ddot{x} + \varphi(\dot{x}) + \Psi(x) = 0 \rightarrow \text{rješenje zove se } \underline{\text{vlastite oscilacije}}$$

Vlastite oscilacije su svojstvo nepokretnog sustava ( $f(t) = 0$ )

Dali će sust. ući u vlastite osc. ovisi o: poč. uvjetima  $\neq 0$

- param. sustava

- svojstvima NE djela sustava

$$(1) \quad \ddot{m}x + \varphi(x) + \psi(x) = 0$$

→ Periodično rješenje od (1) može se dobiti integracijom

Kod lin. sustava

$$\int m\ddot{x} + \boxed{\dot{h}(x)} + kx \Rightarrow \frac{kx^2}{2} - \frac{m\dot{x}^2}{2}$$

$$\int m\ddot{x} + \boxed{\varphi(x)} + \psi(x) \Rightarrow \underbrace{\int \varphi(x)dx}_{\text{poteri učina energetika u sistemu}} - \frac{m\dot{x}^2}{2}$$

→ karakteriziraju energetsku osnovu sustava i okoliša

poteri učina  
energetika u  
sistemu

kinetička  
energija

$$h(x) > 0 \quad \left. \begin{array}{l} \text{sustav poč. energiju islučava} \\ \text{u okolišu (vazdušni sredinu)} \end{array} \right\}$$

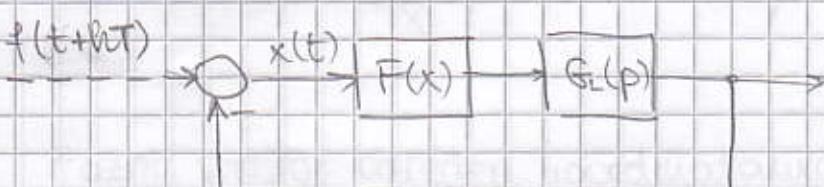
$$\varphi(x) > 0 \quad \left. \begin{array}{l} \text{sust. se popunjava eu.} \\ \text{izvora} \end{array} \right\}$$

DISIPATIVNI  
SUSTAVI

$$h(x) < 0 \quad \left. \begin{array}{l} \text{sust. se popunjava eu.} \\ \text{izvora} \end{array} \right\}$$

$$h(x) = 0 \quad \left. \begin{array}{l} \text{zadržava u oscilacijama} \\ \varphi(x) = 0 \quad \text{ukupna eu. dolazeći poč. uv. ostaje} \\ \text{nepromjenjena} \end{array} \right\}$$

KONZERVATIVNI  
SUSTAVI



$f(t)=0 \rightarrow x(t) = x(t+KT)$  kad je  $x(t)$  periodički u vlastitim  
svojim oscilacijama

$f(t)=f(t+KT) \rightarrow x(t) = x(t+KT)$  primjedbe oscilacije

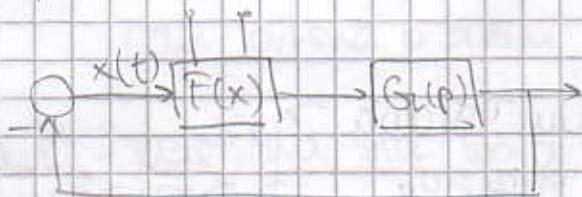
$f(t) \neq f(t + kT)$  →  $x(t) = x(t + kT)$  može je vlastiti  
osc. na ulazu u nel. dio  
nije per. sig.

→ može dovesti do  
per. režima rada

Vlastite oscilacije - moguće kod dissipativnih sustava



↳ promjenjući pot. učinak → dobiva vlastite oscilacije  
potrebno je vanjsko ili unutrašnje napajanje



## Analiza dinamike NE sustava

- prvo treba odrediti matematički model
- treba naći rješenje nelin. dif. jedn.
- postoji samo dva analitička rješenja :
  - Riccati-eva jednadžba
  - Bernoulli-eva jednadžba

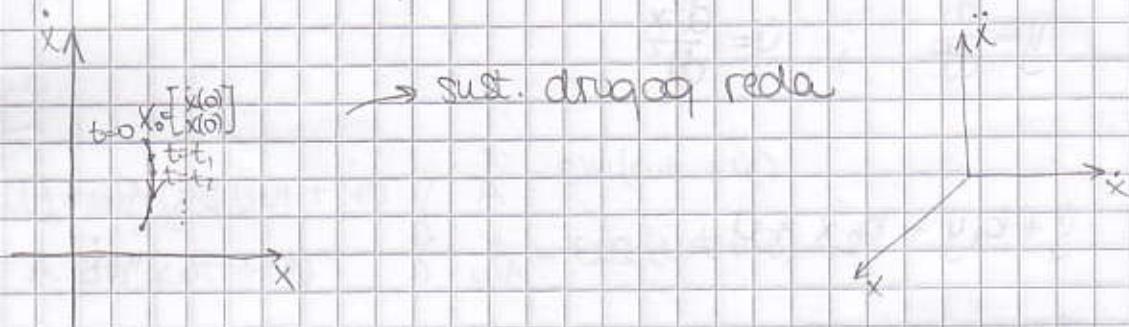
- Numerički postupci (za dobivanje drugih rješenja)
  - simulacija
- Grafoanalitički postupci (koriste se za analizu)
  - Goldfarb

### Metoda faznih trajektorija (trajektorije stanja)

- stanje sustava opisuje se dif. jed.  $n$ -tog reda (<sup>preporučljivo do 3. reda</sup>)

$$a_n \frac{d^n x(t)}{dt^n} + a_{n-1} \frac{d^{n-1} x(t)}{dt^{n-1}} + \dots + a_1 \frac{dx}{dt} + a_0 x(t) = 0 \quad (1)$$

→ prikaz u  $n$  dimenzija



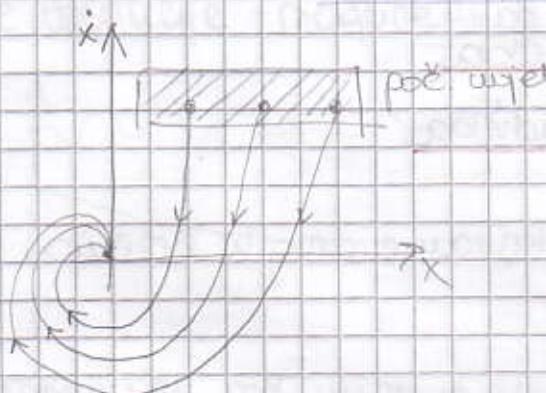
Fazni prostor određen koordinatama  $x, \dot{x}, \dots, \ddot{x}^{(n-1)}$

Fazna točka je točka u tom prostoru koja određuje trenutno stanje sustava.

Srednja f. točka prikazuje stanje sust. u odr. vremenu

Fazna trajektorija je putanja koja se sastoji od faznih točaka.

Fazni portret - familija f. trajektorija pri svim mogućim poč. uvjetima za dani sustav



ponašanje je isto za svaki poč. uvjet

Jednadžba forme trajektorije za lin. sust. II reda

$$\frac{d^2x}{dt^2} + b_1 \frac{dx}{dt} + b_2 x = 0$$

$$b_1 = 2\zeta\omega_n \quad b_2 = \omega_n^2$$

$$b_1 = \frac{\alpha_1}{\alpha_2} \quad ; \quad b_2 = \frac{\alpha_0}{\alpha_2}$$

$$\frac{dy}{dx} = -b_1 - b_2 \frac{x}{y}$$

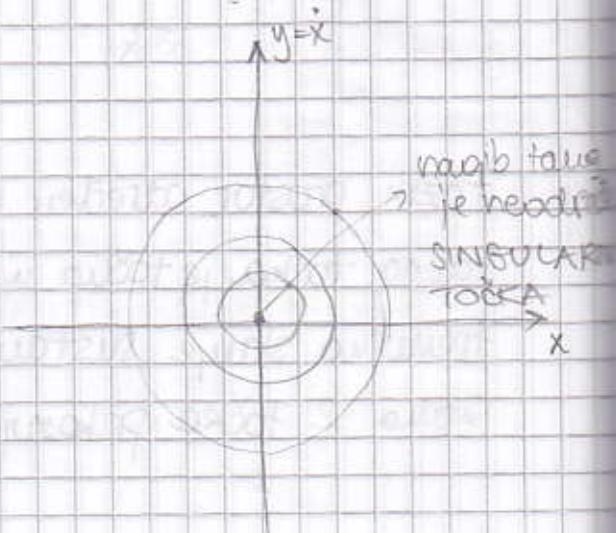
$$y = \frac{dx}{dt} \quad ; \quad \dot{y} = \frac{d^2x}{dt^2}$$

$$\dot{y} + b_1 y + b_2 x = 0$$

$$\frac{dy}{dt} = -b_1 y - b_2 x \quad ; \quad \frac{dx}{dt} \quad (1)$$

$$\frac{dx}{dt} = y \quad (2)$$

$$\frac{dy}{dx} = -b_1 - b_2 \frac{x}{y} \quad (3)$$



Sing. točka ovisi o korišćenima dif. jed.

Postroji 6 tipova singularnih točaka za linearne sustave:

① CENTAR → kod lin.

GRANIČNI KRUGS → kod nelin.

② STABILAN FOKUS

③ NESTABILAN FOKUS

④ STABILAN ČVOR

⑤ NESTABILAN ČVOR

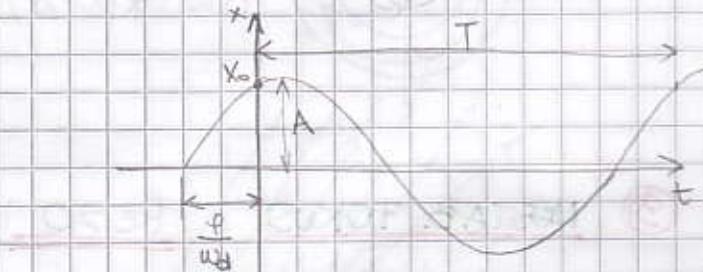
⑥ SEDLO

$$\frac{d^2x}{dt^2} + b_1 \frac{dx}{dt} + b_0 x = 0 \rightarrow \lambda_{1,2} = -\frac{b_1}{2} \pm \sqrt{\frac{b_1^2}{4} - b_0} = -\frac{b_1}{2} \pm i \sqrt{1 - \frac{b_1^2}{4} - b_0} = -\sigma \pm j\omega$$

CENTAR (Im korišteni)

①  $\dot{x}(t) + \omega^2 x(t) = 0$

$\ddot{x}(t) + b_0 x(t) = 0$



$$x(t) = A \sin(\omega t + \varphi) \text{ - općenito rešenje}$$

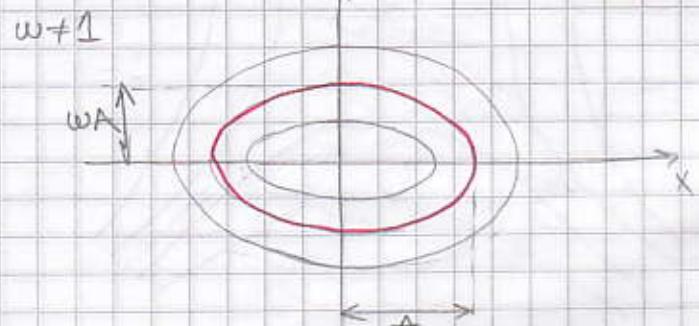
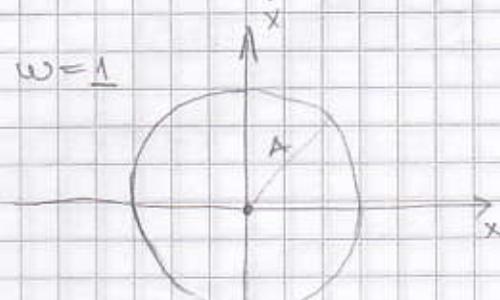
$$\lambda_{1,2} = \pm j\sqrt{b_0} \text{ - smisljene vrijednosti}$$

$$\begin{cases} b_1 = 0 \\ b_0 > 0 \end{cases}$$

$$\left. \begin{array}{l} y = \dot{x}(t) = \omega A \cos(\omega t + \varphi) \\ x(t) = A \sin(\omega t + \varphi) \end{array} \right\} \begin{array}{l} \dot{x} = \sin(\omega t + \varphi) \\ \frac{y}{\omega A} = \cos(\omega t + \varphi) \end{array}$$

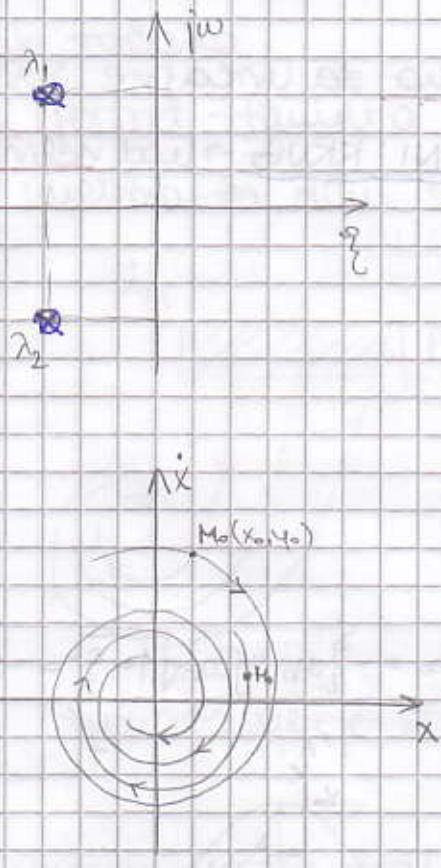
$$\frac{x^2}{A^2} + \frac{y^2}{\omega^2 A^2} = 1 \rightarrow \text{jedn. krugice} \rightarrow \text{fazna trajektorija}$$

Za  $\omega = 1$

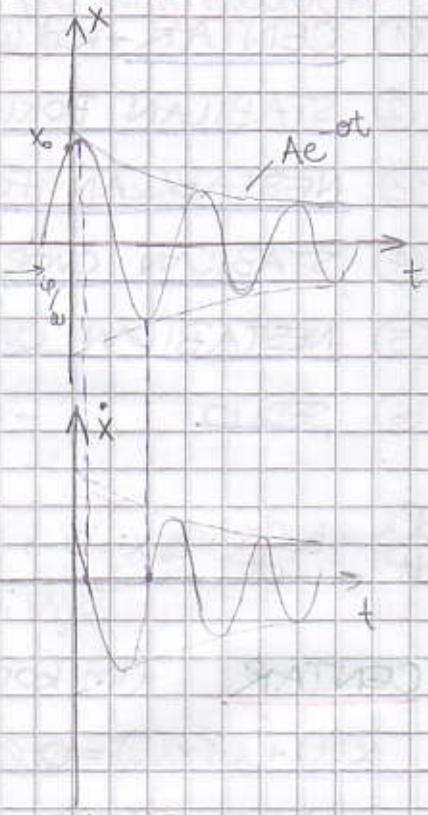


## ② STABILAN FOKUS

$$a_1 > 0, b_1 > 0 \quad x(t) = A \cdot e^{\frac{a_1}{2}t} \sin(\omega_1 t + \varphi)$$



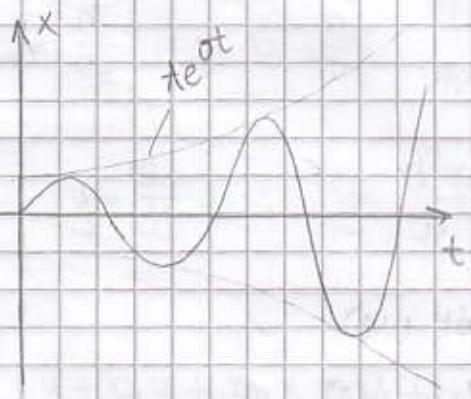
Slojst. vrije vlna -  $\text{Re } < 0$   $\text{Im posto}$



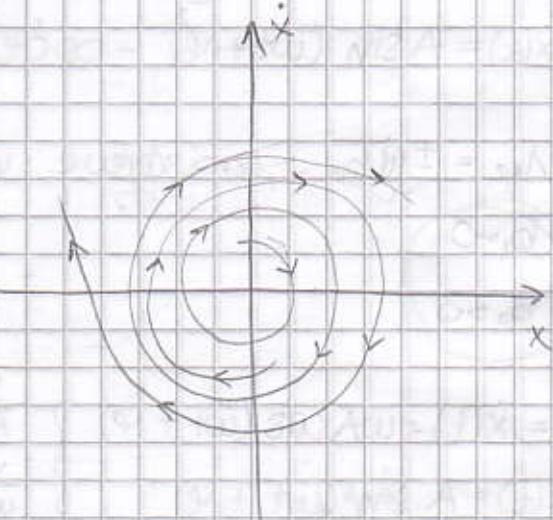
## ③ NESTAB. FOKUS

$$\underline{\text{Re } > 0} \quad \text{Im postoji} \quad b_1 < 0, b_2 > 0$$

- raspirajuće oscilacije



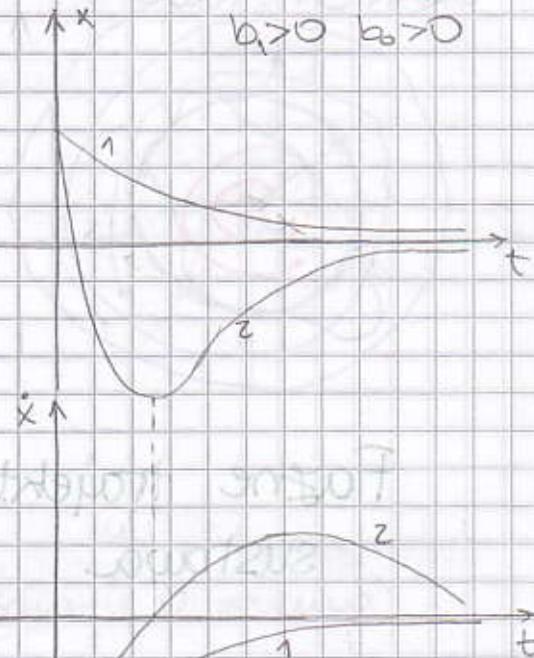
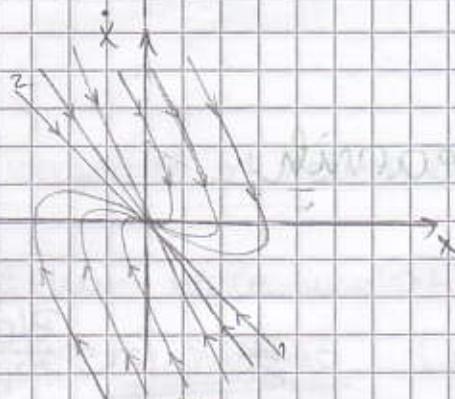
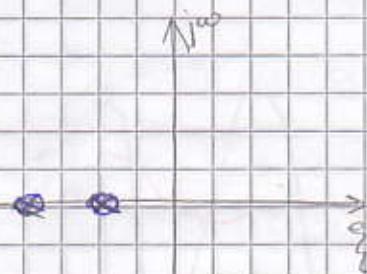
$$x(t) = A \cdot e^{\frac{a_1}{2}t} \sin(\omega_1 t + \varphi)$$



4) STABILAN ČVOR

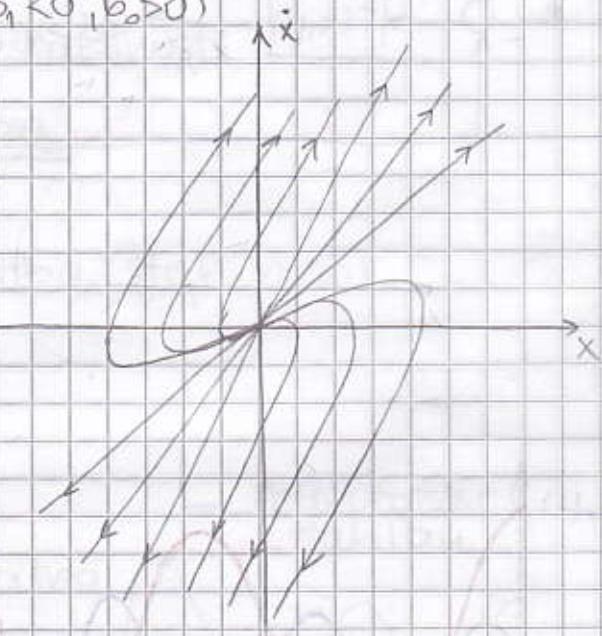
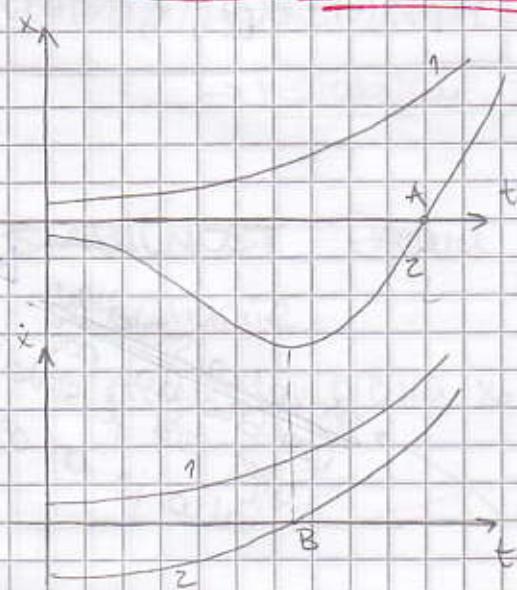
- aperiodično ponašanje

$\text{Re} < 0 \text{ Im} = 0$



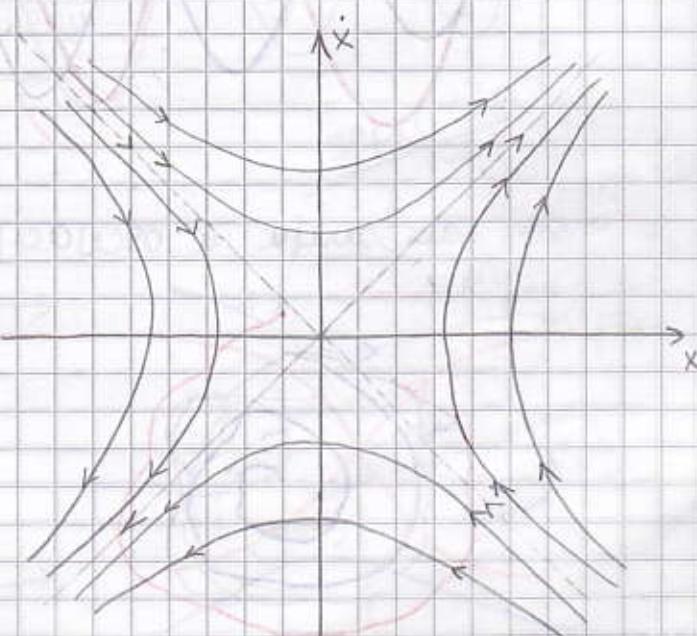
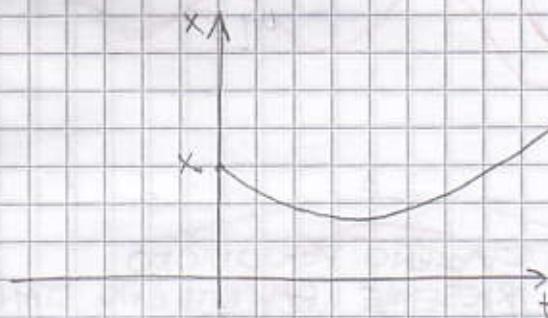
5) NESTAB. ČVOR

$\text{Re} > 0 \text{ Im} = 0 (b_1 < 0, b_0 > 0)$

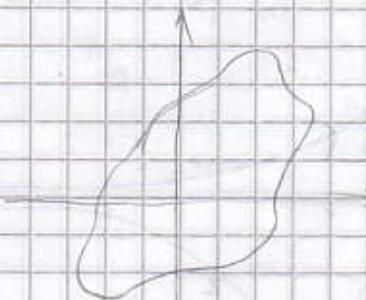


6) SEDLO

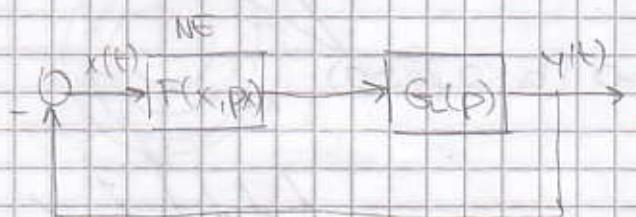
$\text{Re} \pm i\omega_0 < 0, b_1 = 0$



## GRANIČNI KRUG

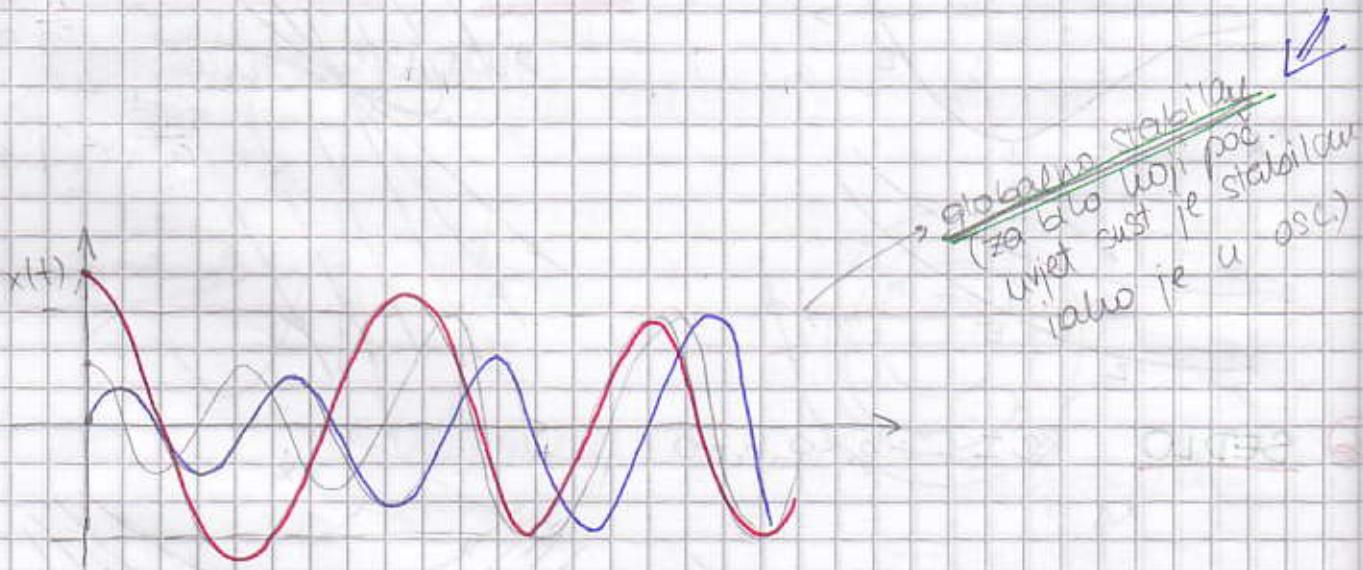


## Fazne trajektorije nelinearnih sustava

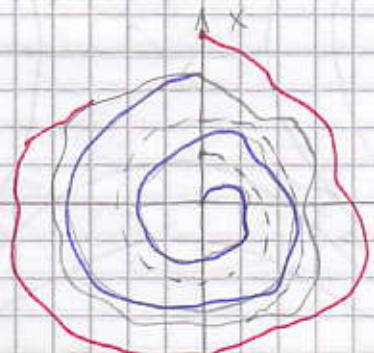


$$p = \frac{d}{dt} \quad G_t = \frac{B(p)}{A(p)}$$

$$A(p)x(t) + B(p)F(x, px) = 0$$



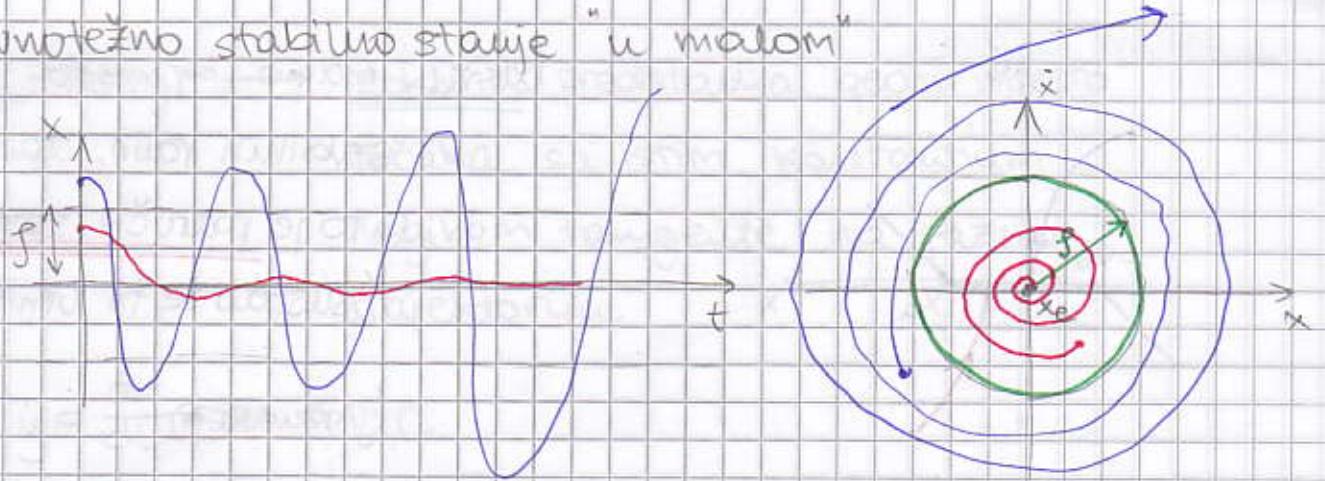
↳ svi završavaju u oscilacijama



STABILNO PERIODIČKO RJEŠENJE (RAVNOSTEŽNO STANJE)

GRANIČNI KRUG (LIMIT CYCLE)

Ravnotežno stabilno stanje "u malom"



$\rho$  - udaljenost od nelinearnog ravn. stanja  $x_0$

- sust. koji kreće unutar delta i završava u  $x_0$  zovemo  
stabilno "u malom" ( $x(0) < \rho$ )

$x(0) > \rho \rightarrow$  sust. završava u nestabilnosti

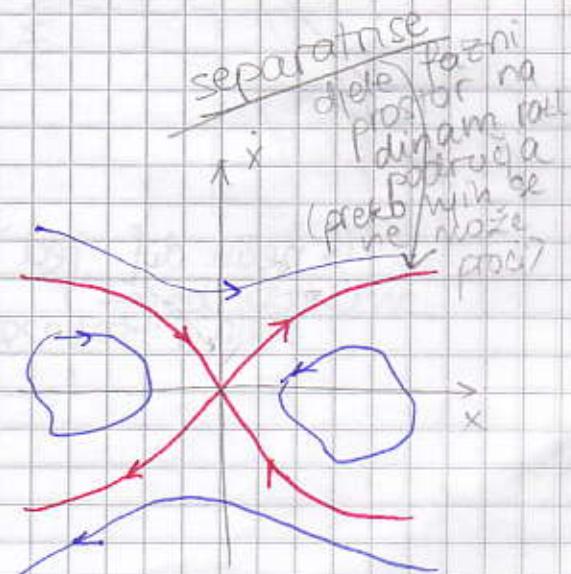
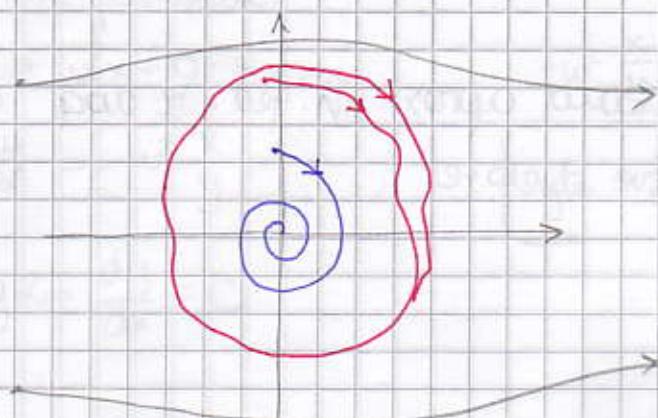
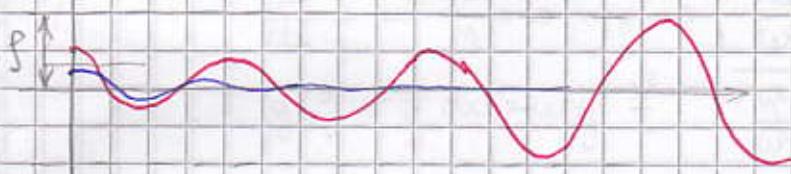
$\rightarrow$  nestabilno "u velikom"

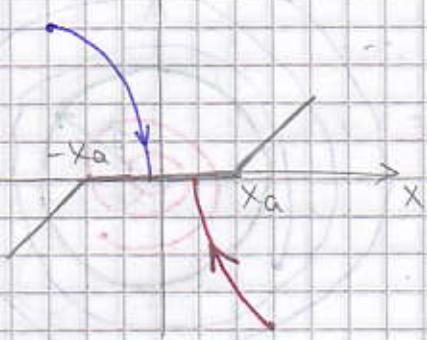
## STABILNOST (velika pojava roba stabilnosti)

- oscilacije

- priqušenje prema  $x_0$

— asimptotski stabilan





$\dot{x} = kx$  imamo  $\approx$  mnogo

stabilnih ravn. stanja  
↳ to je područje zastoja  
(sustav se tu umiri)

(JAM AREA)

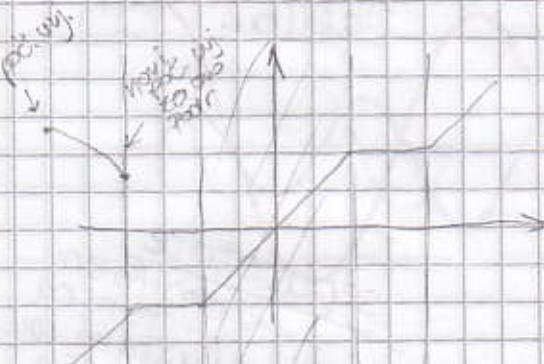
nelin. karakter ima sonu neosjetljivosti

brzina se prestikeva u faznu karakter.

Postupci određivanja faznih trajektorija

- analitički (spajanje rješenja)
- grafički (metoda fokline)
- numerički (simulacijski)

stacionarna karakter.



↳ nelin. dif. jed možemo aproksimirati jed. iz dva dijela ; tako za sve dijelove

$y = f(x, y) \rightarrow$  izoklina predstavlja geom. mjesto točaka sa istim konstantnim nagibom tangente na faznu trajektoriju

$$\boxed{y = \frac{dy}{dt} = \text{kost.} \Rightarrow C}$$

$$C = f(x, y) \rightarrow \text{jed. izokline}$$

↳ ako odabirimo  $C$  po volji dobivamo familiju izoklini

## Priimer FAZNA TRAJEKTORIJA

$$\ddot{x} + w^2 x = 0$$

$$y = \dot{x} \rightarrow \dot{y} + w^2 x = 0 / : y = \frac{dx}{dt}$$

$$\frac{dy}{dt} + w^2 \frac{x}{y} = 0$$

$$\frac{dy}{dx} + w^2 \frac{x}{y} = 0 \rightarrow \left| \begin{array}{l} \frac{y^2}{w^2} + x^2 - A^2 \\ \frac{y^2}{w^2} + \frac{x^2}{A^2} \end{array} \right|$$

$$y = -\int w^2 \frac{x}{y} dx \Leftrightarrow w^2 y \frac{x^2}{2}$$

grafički postupak:

$$y + w^2 x = 0 / : y = \dot{x}$$

$$\frac{dy}{dx} = -w^2 \frac{x}{y}$$

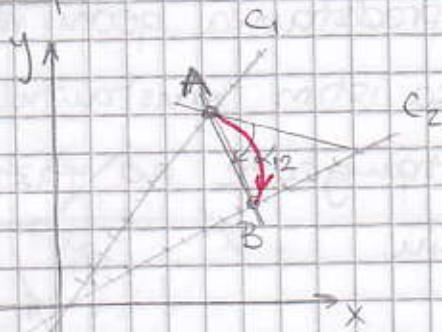
$$\tan \omega = \frac{dy}{dx} = C$$

$$-w^2 \frac{x}{y} = C$$

$$\boxed{y = -\frac{1}{C} w^2 x}$$

→ jed. izokline

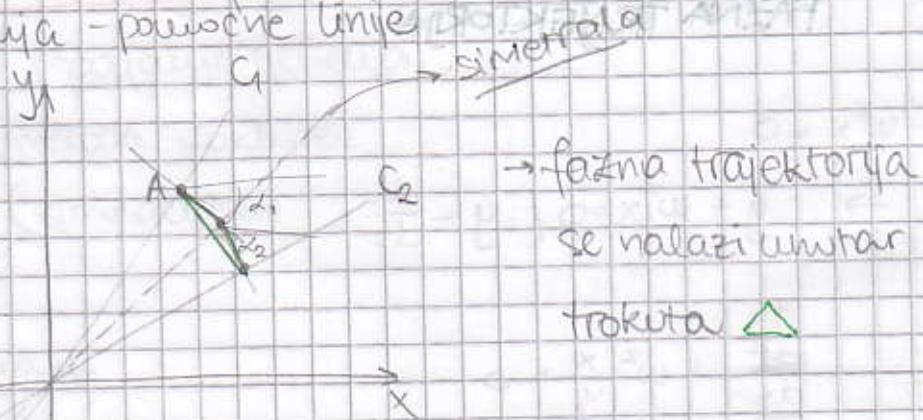
I. način crtanja



$$\alpha_{12} = \arctg C_{12}$$

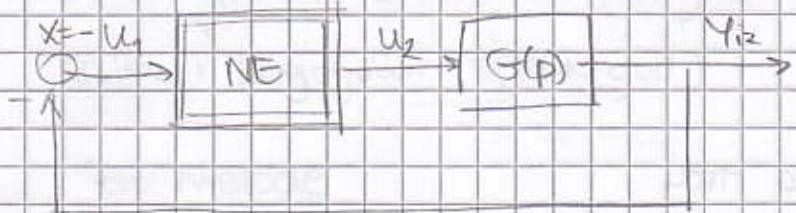
$$C_{12} = \frac{C_1 + C_2}{2}$$

II način crtanja - poluosične linije



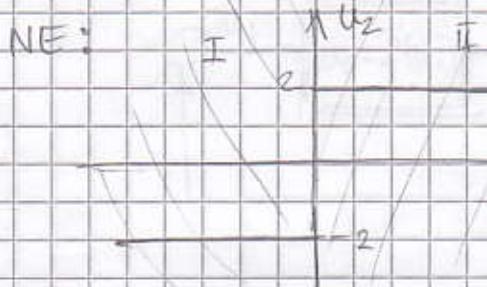
$$\alpha_1 = \arctg C_1$$

$$\alpha_2 = \arctg C_2$$



$$G(p) = \frac{K}{T_1^2 p^2 + T_2 p + 1} + \frac{\gamma}{u_2}$$

$T_1 = 0,5 \text{ s}^2$   
 $T_2 = 1 \text{ s}$



K=1

$$T_1 = 0,5 \text{ s}^2$$

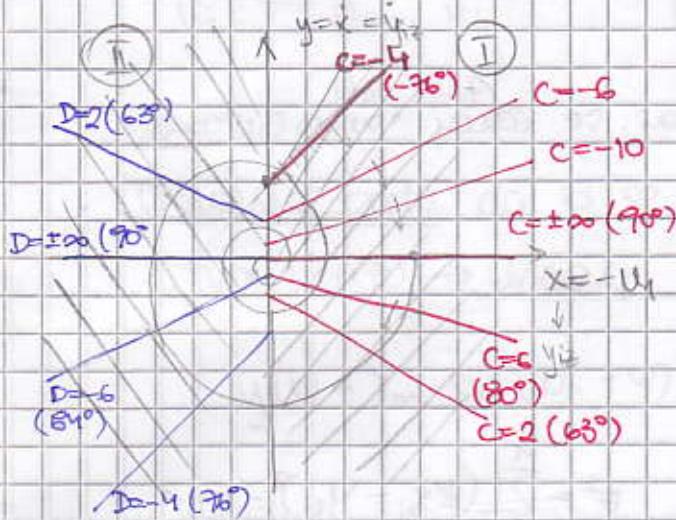
$$T_2 = 1 \text{ s}$$

druk na reakcji bez wsterezy

$$u_1 = -y_{12}$$

$$\text{za } u_1 > 0 : u_2 = 2 \Rightarrow (T_1^2 p^2 + T_2 p + 1) y_{12} = 2K \quad \textcircled{II}$$

$$\text{za } u_1 < 0 : u_2 = -2 \Rightarrow (T_1^2 p^2 + T_2 p + 1) y_{12} = -2K \quad \textcircled{I}$$



$$\textcircled{I} : (0,5 p^2 + p + 1) x = -2$$

$$(p^2 + 2p + 2)x = -4$$

$$\therefore x + 2x + 2x = -4$$

$$\therefore x = -2x - 2x - 4$$

$$\frac{dy}{dt} = -2y - 2x - 4 \quad |: y = \frac{dx}{dt}$$

$$\frac{dy}{dx} = -2 - 2 \frac{x}{y} - \frac{4}{y} - C$$

$$y = \frac{2x+4}{C+2}$$

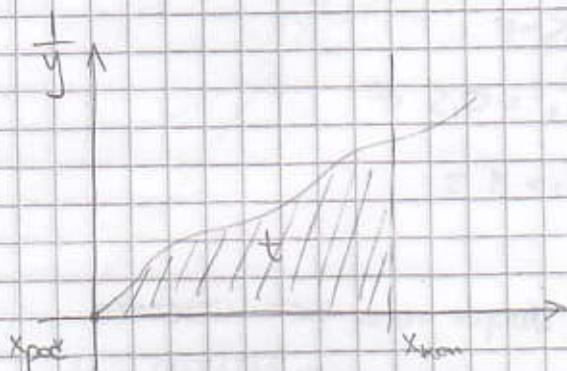
$$\textcircled{II} : \frac{dy}{dx} = \frac{2y + 2x - 4}{y} \Rightarrow$$

$$y = -\frac{2x-4}{D+2}$$

$$\frac{dx}{dt} = y \rightarrow t = \int \frac{1}{y} dx$$

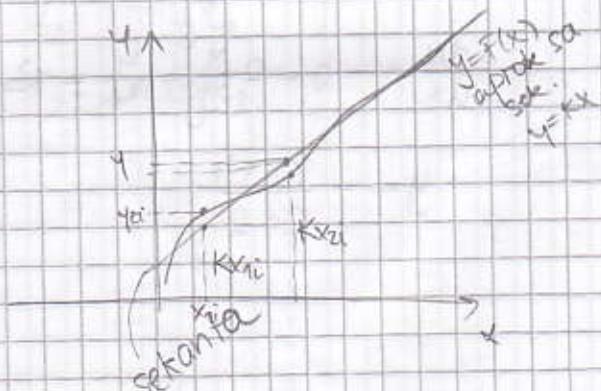
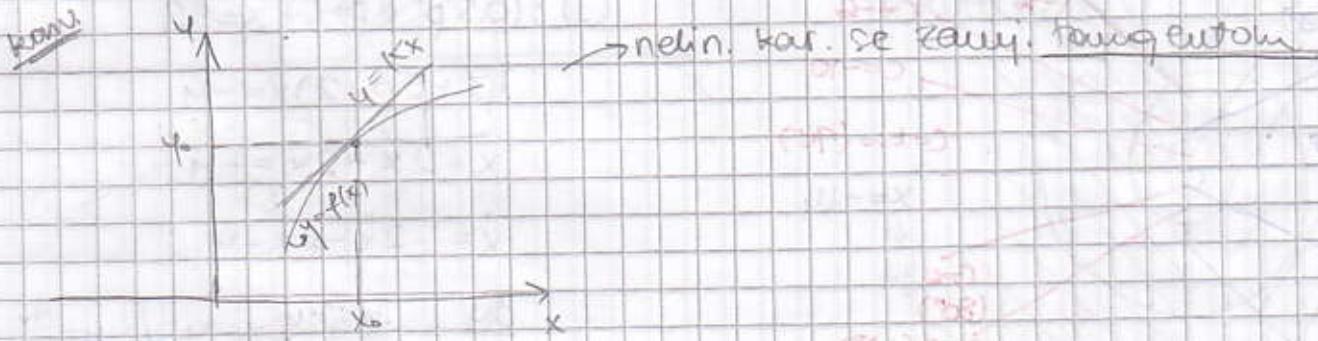
$x_{\text{kon}}$   
 $x_{\text{pot}}$

$y = f(x)$  fazna traj



### Postupci linearizacije

- konvencionalni (tangente, sekante...)  $\rightarrow$  glatka stat. karakter (mole promjene amplituda)
- harmonijska  $\rightarrow$  glatka ali ne mora biti glatka  $\rightarrow$  prednost
- statistička
- kombinirana (harm. i stat.)



$$\epsilon^2 = \sum_{i=1}^n (Kx_{ii} - y_{2i})^2 = \min$$

$$K = \frac{\sum_{i=1}^n y_{2i} x_{ii}}{\sum_{i=1}^n x_{ii}^2}$$

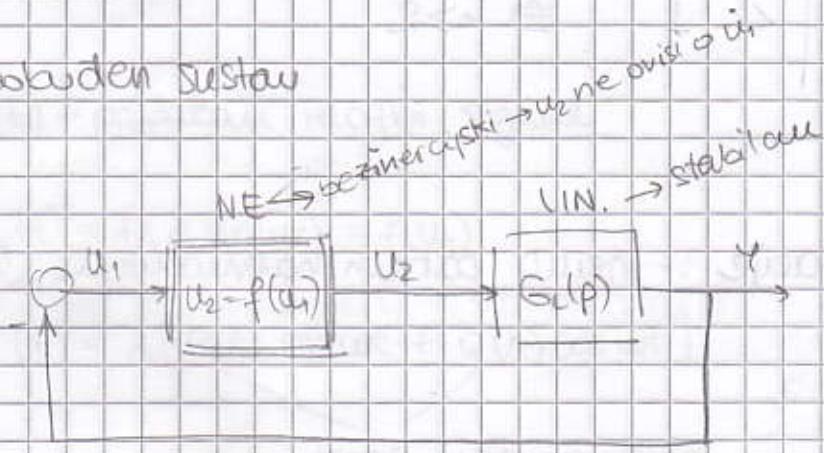
$\rightarrow$  i sek. i tang. ne prebacuju neka bitna svojstva nelin. sust.

# HARMONIJSKA LINEARIZACIJA

Krillov i Bogoliubov (1940 god.)

## Bit metode

- nepodudaren sustav



→ Pretpostavlja se da će se u sust. uspostaviti oscilacije

$$u_1(t) = A \sin \omega t \rightarrow \text{harmon. ulaz}$$

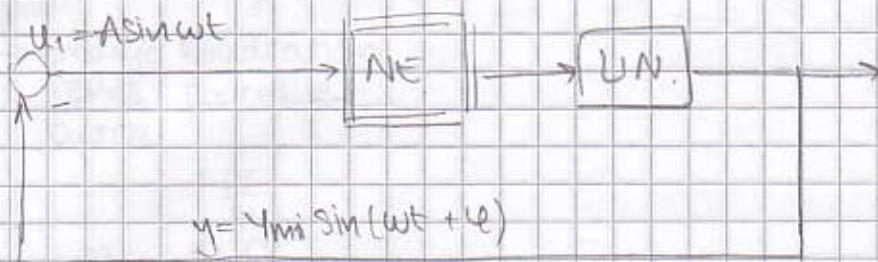
$u_2$  - će imati i osnovnu frekv. i sva više harmonike

$$u_2 : \quad u_{m1}, \omega$$

$$u_{m2}, 2\omega$$

Osnovna pretp. da bi se mogao uspostaviti harm. signal na ulazu → linearni dio quiši sve više harmonike

$$y(t) = Y_m \sin(\omega t + \varphi) \rightarrow \text{FILTER HIPOTEZA}$$



$$\text{jed harmon. ravnoteze: } \underline{|u_1(t) + y(t)| = 0}$$

$$u_1(t) + u_2(t) = 0 \rightarrow \text{dva uvjeta: } A = \gamma_m$$

$$\varphi = \pi$$

$$\left| \frac{U_{m2}}{U_{m1}} \right| \left| \frac{G_L(j\omega_k)}{G_L(j\omega)} \right| \ll 1 \quad \text{za } k \geq 2$$

Simetrične oscilacije: - neuna parnih harmonika (2, 4, 6...)

obično vrijedi:

nagib:

za  $-20 \text{ dB/dek}$  (qušenje)

$$\left| \frac{G_L(j\omega_2)}{G_L(j\omega)} \right| = \frac{1}{2}$$

$$\left| \frac{G_L(j\omega_3)}{G_L(j\omega)} \right| = \frac{1}{3}$$

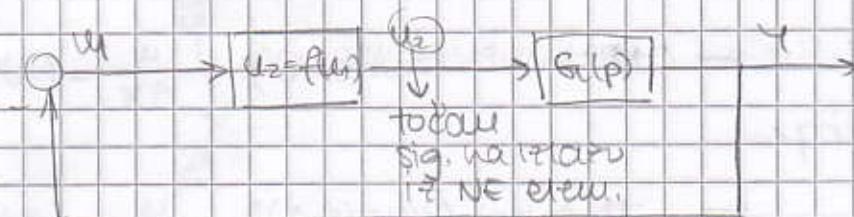
za  $-40 \text{ dB/dek}$  (qušenje)

$$\left| \frac{G_L(j\omega_2)}{G_L(j\omega)} \right| = \frac{1}{4}$$

$$\left| \frac{G_L(j\omega_3)}{G_L(j\omega)} \right| = \frac{1}{9}$$

Govorak postupku wonstimo ako imamo sustav

• koji nam daje u vlastite oscilacije koje nas ne zadovoljavaju



$u_2'(t)$  - približan izrazni signal

$$u_2'(t) = f(A \sin wt) = f(u_1)$$

$$u_2'(t) = A [P(A) \sin wt + Q(A) \cos wt]$$

koef. harmoničke  
linearizacije

→ harm. lin. - ostavlja osnovne harmonike

- aprox. izrazni signal sa pribl. izrazom koji sadrži samo osnovne harm. dok točan sadrži i više harmonike

$P(A)$  i  $Q(A)$  dobivaju se minimiziranjem pogreške

$$\Delta(t) = u_2(t) - u_2'(t)$$

↓ periodička fja ( $T = \frac{2\pi}{\omega}$ )

$$\Delta_m^2 = \frac{1}{T} \int_0^T \Delta^2(t) dt$$

$\Delta_m \rightarrow$  srednje kvadratični  
iznos pogreške  
aprox.

$$\frac{2\pi}{\omega}$$

$$\Delta_m^2 = \frac{\omega}{2\pi} \left( \int_0^{2\pi} [f(A \sin wt) - A[P(A) \sin wt + Q(A) \cos wt]]^2 dt \right)$$

wijs et minimum :

$$\frac{\partial \Delta m^2}{\partial AP(A)} = 0$$

$$\frac{\partial \Delta m^2}{\partial AQ(A)} = 0$$

$$\int_0^{\frac{2\pi}{\omega}} \{ f(A \sin \omega t) - A \} \sin \omega t \, dt = 0$$

$$\int_0^{\frac{2\pi}{\omega}} \{ f(A \sin \omega t) - A \} \cos \omega t \, dt = 0$$

$$\int_0^{\frac{2\pi}{\omega}} f(A \sin \omega t) \sin \omega t \, dt - \int_0^{\frac{2\pi}{\omega}} AP(A) \sin^2 \omega t \, dt = 0$$

$$\int_0^{\frac{2\pi}{\omega}} f(A \sin \omega t) \cdot \cos \omega t \, dt - \int_0^{\frac{2\pi}{\omega}} AP(A) \cos^2 \omega t \, dt = 0$$

$$AP(A) \int_0^{\frac{2\pi}{\omega}} \frac{1 - \cos 2\omega t}{2} \, dt = \int_0^{\frac{2\pi}{\omega}} f(A \sin \omega t) \cdot \sin \omega t \, dt$$

$$AQ(A) \int_0^{\frac{2\pi}{\omega}} \frac{1 + \cos 2\omega t}{2} \, dt = \int_0^{\frac{2\pi}{\omega}} f(A \sin \omega t) \cdot \cos \omega t \, dt$$

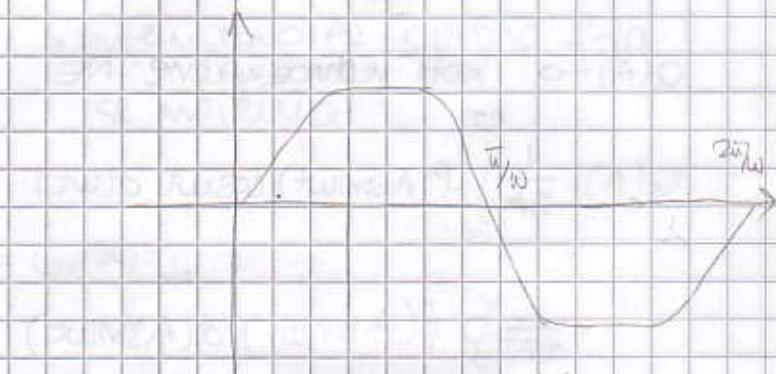
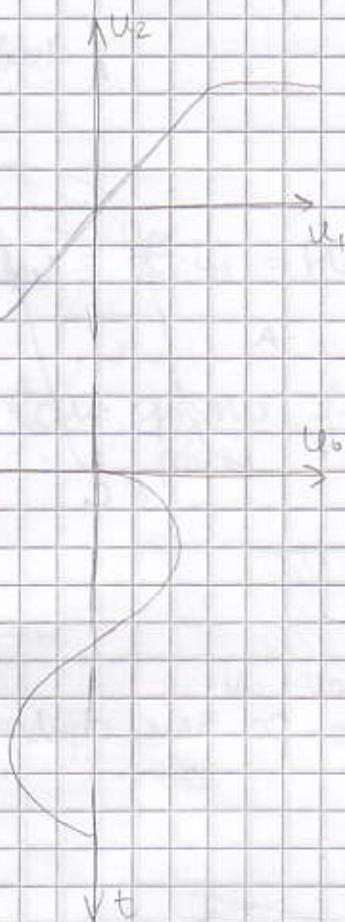
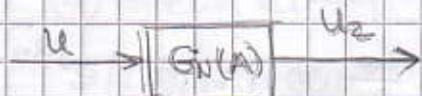
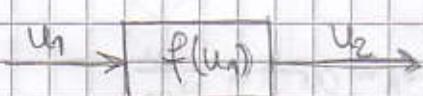
$$\frac{AP(A)}{\omega} \pi = \int_0^{\frac{2\pi}{\omega}} f(A \sin \omega t) \sin \omega t \, dt$$

$$\frac{AQ(A)}{\omega} \pi = \int_0^{\frac{2\pi}{\omega}} f(A \sin \omega t) \cdot \cos \omega t \, dt$$

$$P(A) = \frac{w}{\pi A} \int_0^{\frac{\pi}{w}} f(A \sin wt) \cdot \sin wt dt \rightarrow \text{privi kret. Four. reda}$$

$$Q(A) = \frac{w}{\pi A} \int_0^{\frac{\pi}{w}} f(A \sin wt) \cdot \cos wt dt$$

## OPISNA FUNKCIJA



$$u_1 = A \sin wt = \sum_m A_m e^{j m w t}$$

$$u_2 = u_0 + \sum_{k=1}^{\infty} u_{pk} \sin kwt + \sum_{k=1}^{\infty} u_{ck} \cos kwt$$

$$u_{pk} = \frac{1}{\pi w} \int_0^{\pi} f(A \sin wt) \sin kt dt$$

$$u_{ck} = \frac{1}{\pi w} \int_0^{\pi} f(A \sin wt) \cos kt dt$$

$$u_0 = 0 \text{ za simetrične oscilacije}$$

$$U_2(t) = U_{p1} \sin \omega t + U_{q1} \cos \omega t = \text{Im}(U_{p1} + U_{q1}) e^{j\omega t}$$

Opisna f-a - kompleksni omjer osnovnog ulaznog i izlaznog harmonika

$$|G_N(A)| = \frac{\sqrt{U_{p1}^2 + U_{q1}^2}}{A}$$

$$G_N(A) = P(A) + jQ(A)$$

$$= \frac{U_{p1}}{A} + j \frac{U_{q1}}{A} = |G_N(A)| e^{j\varphi_N}$$

$$\varphi_N = \arctan \frac{U_{q1}}{U_{p1}}$$

$Q(A) = 0$  kod jednozadane NE

$$Q(A) = \frac{1}{\pi A} \int_0^{2\pi} f(A \sin \omega t) \cos \omega t d(\omega t)$$

$$= \frac{1}{\pi A^2} \oint f(A \sin \omega t) d(A \sin \omega t)$$

$$= \frac{1}{\pi A^2} \oint f(u_i) du_i$$

$$\text{od } C_1: u_i = 0 \quad (t=0)$$

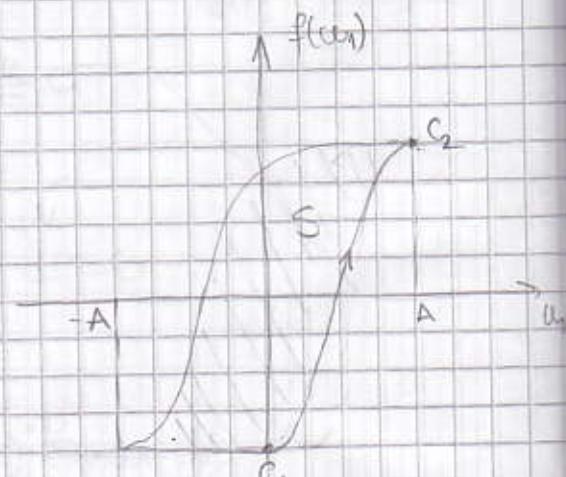
$$\text{do } C_2: u_i = A \quad (t = \frac{\pi}{\omega})$$

$$\text{od } C_2: u_i = -A$$

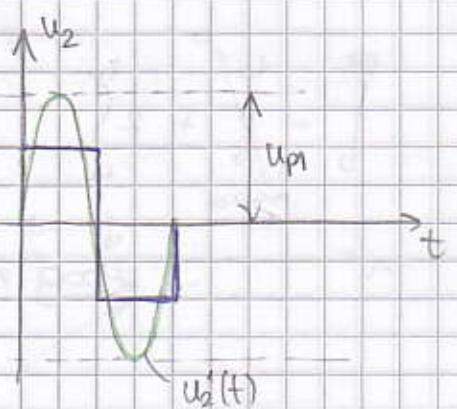
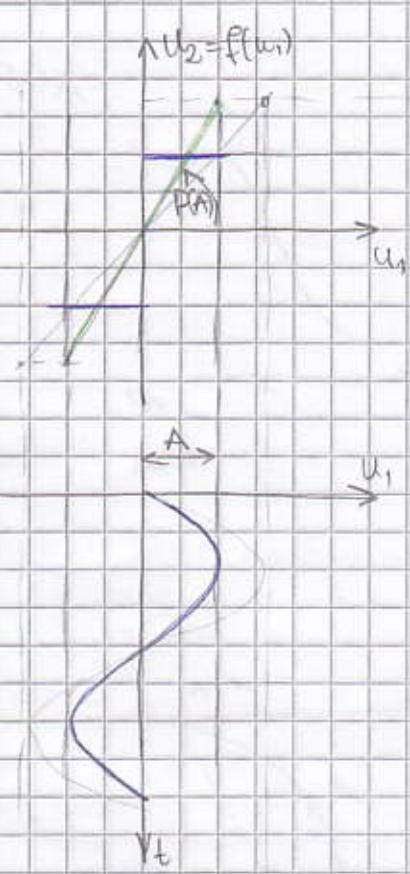
$$\text{do } C_3: u_i = -A \quad (t = \frac{3\pi}{\omega})$$

} integracija po tim dijelovima

$$Q(A) = -\frac{S}{\pi A^2}$$



(4)



$$P(A) = \frac{u_{p1}}{A}$$

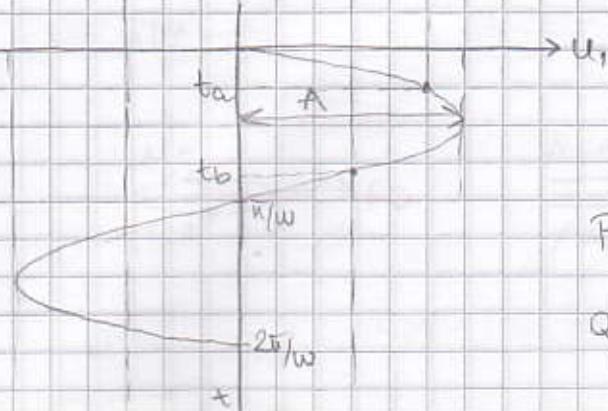
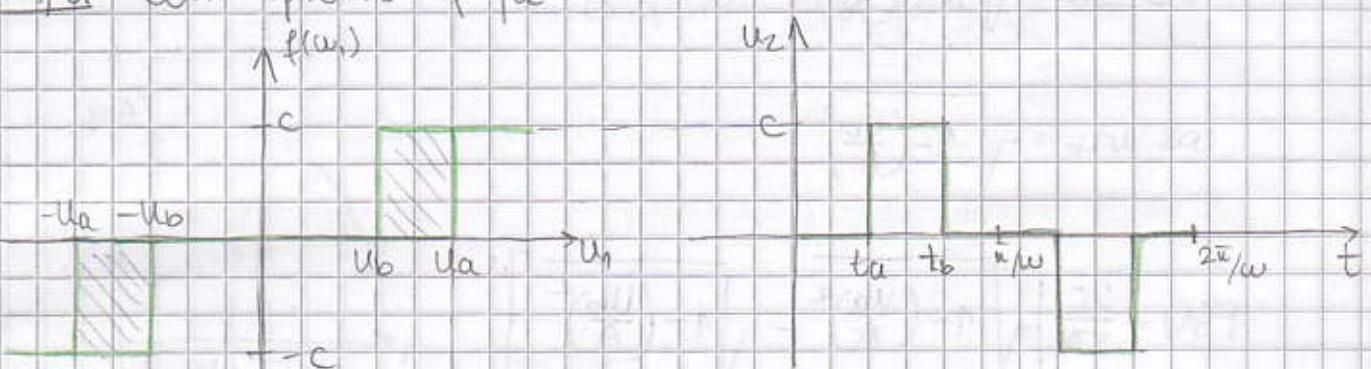
$$Q(A) = 0$$

↗ biełini. karakt. zmiennych  
je prawdziwe

→ mieniącą A, opisną f-ja  
se mienia

$$u_2 = f(u_1) = \frac{u_{p1}}{A} \cdot u_1 = P(A) \cdot u_1 = G_N(A) \cdot u_1$$

Przykład Odr. opisnu f-ju



$$\text{za } A < u_{p1} \Rightarrow G_N(A) \approx 0$$

$$\text{za } A > u_{p1} \Rightarrow G_N(A) = P(A) + Q(A)$$

$$P(A) = \frac{1}{\pi w} \int_{-\pi/w}^{\pi/w} f(u_1) \sin(wt) d(wt)$$

$$Q(A) = \frac{1}{\pi w} \int_{-\pi/w}^{\pi/w} f(u_1) \cos(wt) d(wt)$$

$$\int_0^{2\pi} = \int_0^{t_a} + 2 \int_{t_a}^{t_b}$$

$\downarrow$   
A < u\_a

Zbog neparnosti  $f(u)$

$$t_a: A \sin \omega t_a = u_a \quad \sin \omega t_a = \frac{u_a}{A}$$

$$t_b: A \sin \left[ \omega \left( \frac{\pi}{\omega} - t_b \right) \right] = u_b$$

$$\sin (\pi - \omega t_b) = \frac{u_b}{A}$$

$$t_a < t < t_b \rightarrow f(u) = 0$$

$$P(A) = \frac{2}{\pi A} \int_{t_a}^{t_b} C \sin(\omega t) d(\omega t)$$

$$P(A) = \frac{2C}{\pi A} \left( \cos \omega t_a - \cos \omega t_b \right)$$

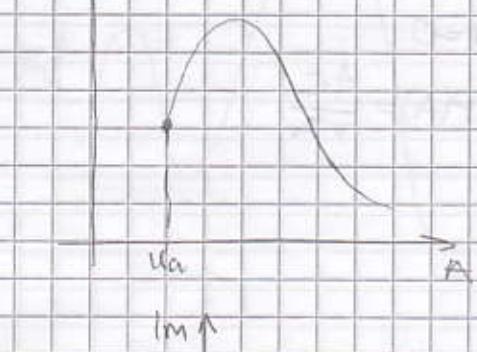
$$\cos \omega t_a = -\sqrt{1 - \left( \frac{u_a}{A} \right)^2}$$

$$\cos \omega t_b = -\sqrt{1 - \left( \frac{u_b}{A} \right)^2}$$

$$P(A) = \frac{2C}{\pi A} \left[ \sqrt{1 - \left( \frac{u_a}{A} \right)^2} - \sqrt{1 - \left( \frac{u_b}{A} \right)^2} \right]$$

$$Q(A) = -\frac{S}{\pi A^2} = -2 \frac{C(u_a - u_b)}{\pi A^2}$$

$P(A)$



$a(A)$



$A = \infty$

Re

A

$A = 1$

$-G_N^{-1}(A)$

Re

A

$A = 1$

$u_2$

c

$-u_a$

$u_a$

$u_1$

$u_{\infty} = u_a$

$Q(A) = 0$

$$G_N(A) = P(A) = \frac{4c}{\pi A} \sqrt{1 - \left(\frac{u_a}{A}\right)^2}$$

( $\Rightarrow$  opisna f-ja je realna

$P(A)$

$u_a$

A

Im

$A = \infty$

$A \leq 1$

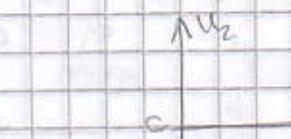
Im

$A = \infty$

$A \leq 1$

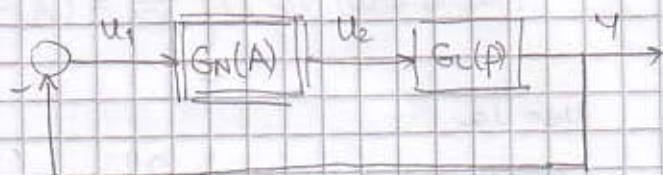
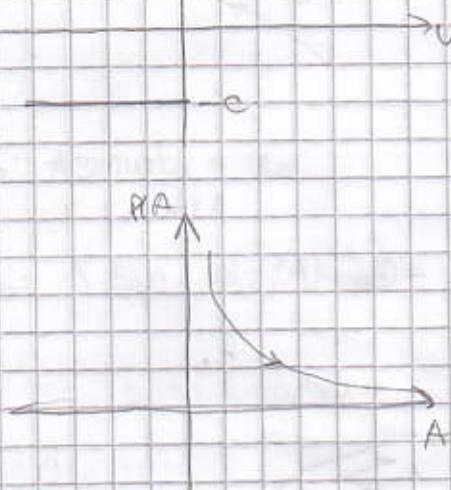
Re

$U_2 = 0$  (dvoopol. reley)



$$Q(A) = 0$$

$$G_N = P(A) = \frac{4C}{\pi A}$$



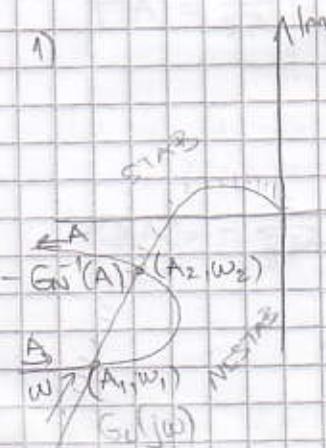
$$1 + G_N(A) \cdot G_L(jw) = 0$$

$$G_N(A) \cdot G_L(jw) = -1$$

RJEŠENJA

1) - grafički (Goldfarb)  $\rightarrow G_L(jw) = -G_N^{-1}(A)$

2) - analitički (Mihailjev)

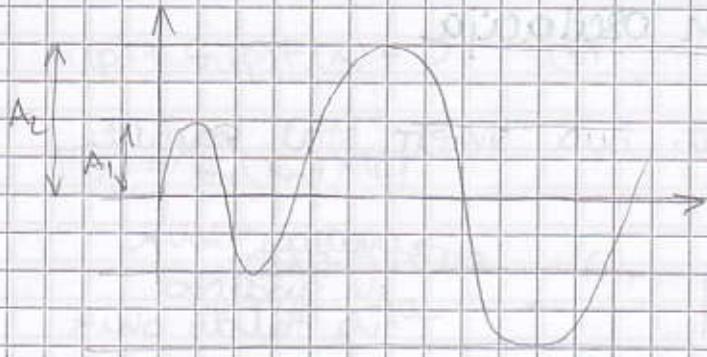


$A_2 > A_1$  } jedno rješenje  
 $w_2 > w_1$  } je stablino

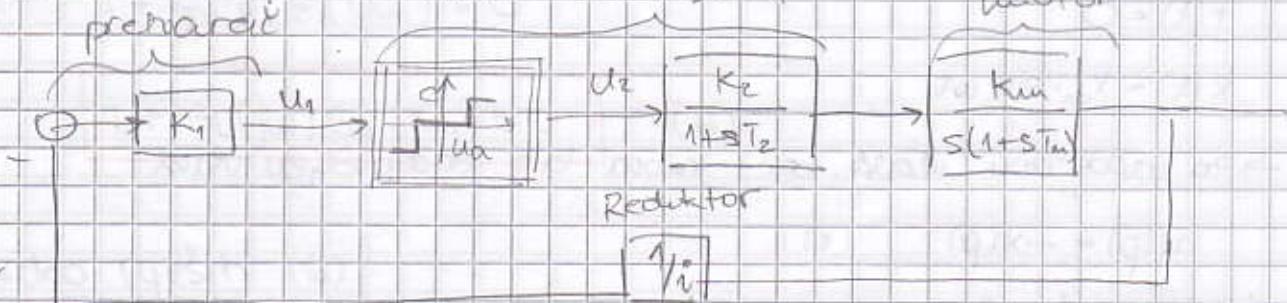
$$U_1 = A_2 \sin \omega t$$

$(A_1, w_1)$  - nestab.

$(A_2, w_1)$  - stab.



Primier Odr fourcun vlast. osc. popadalec srouge



$$K_1 = 1 \quad K_m = 300 \quad u_a = 0,1$$

$$K_2 = 5 \quad T_m = 2 \quad C = 6$$

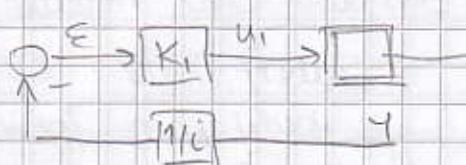
$$T_d = 0,2 \quad i = 7500$$

$$G_L(j\omega) = \frac{K}{j\omega(1+j\omega T_d)(1+j\omega T_m)}$$

$$K = K_1 \cdot K_2 \cdot K_m \cdot \frac{1}{i} = 0,2$$

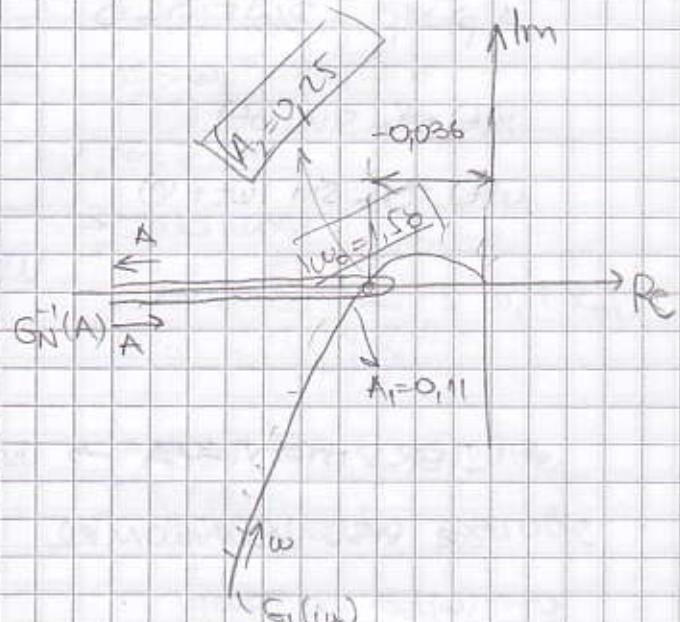
$$G_N(A) = \frac{4C}{\pi A} \sqrt{1 - \left(\frac{\omega_n}{A}\right)^2}$$

$$G_L(j\omega) = -\frac{1}{G_N(A)}$$



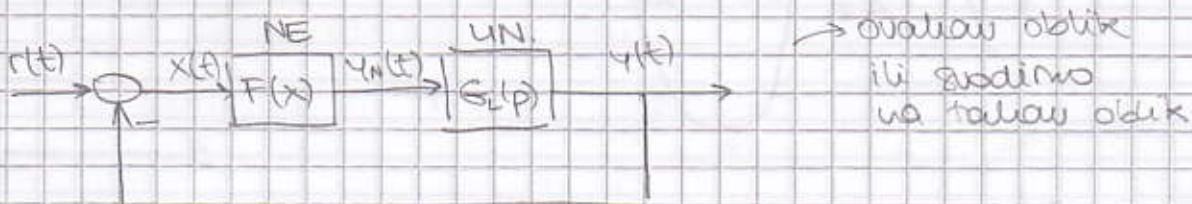
$$[+] \quad \varepsilon(t) = \frac{U_1(t)}{K_1} = 0,25 \sin 1,58t$$

$$y(t) = -i \cdot \varepsilon(t) = -1875 \sin 1,58t$$



## Analiza simetričnih vlastitih oscilacija

↳ dobivaju se kada NE element. ima simetr. stat. karakter.



$$r(t) = 0$$

$$x(t) = X_m \sin \omega t$$

→ za uspostavu vlast. osc. mora biti zadovoljen uvjet

$$y(p) = -x(p) \quad (1)$$

$$p = \frac{d}{dt}$$

$$y(p) = G_L(p)F(x) = \frac{B(p)}{A(p)} F(x) = -X(p)$$

$$A(p)X(p) + B(p)F(x) = 0$$

$$\left. \begin{array}{l} x(t) = X_m \sin(\omega t) \\ y(t) = Y_m \sin(\omega t + \varphi) \end{array} \right\} \left. \begin{array}{l} Y_m = X_m \\ \varphi = \pi \end{array} \right\}$$

WYJET HARMONICKE  
RAVNOSTEŽE

da bi (1) bio zadovoljen

FILTER HIPOTEZA → lin. dio sustava će isfiltrirati

ove više harmonike

$$\left| \frac{G_L(j\omega)}{G_L(j\omega)} \right| \ll 1$$

$$x(p) + G_L(p)F(x) = 0 \quad (3)$$

$$F(x) = G_N(x_m)x(p)$$

↓  
linearizirana  
opisna f-ja

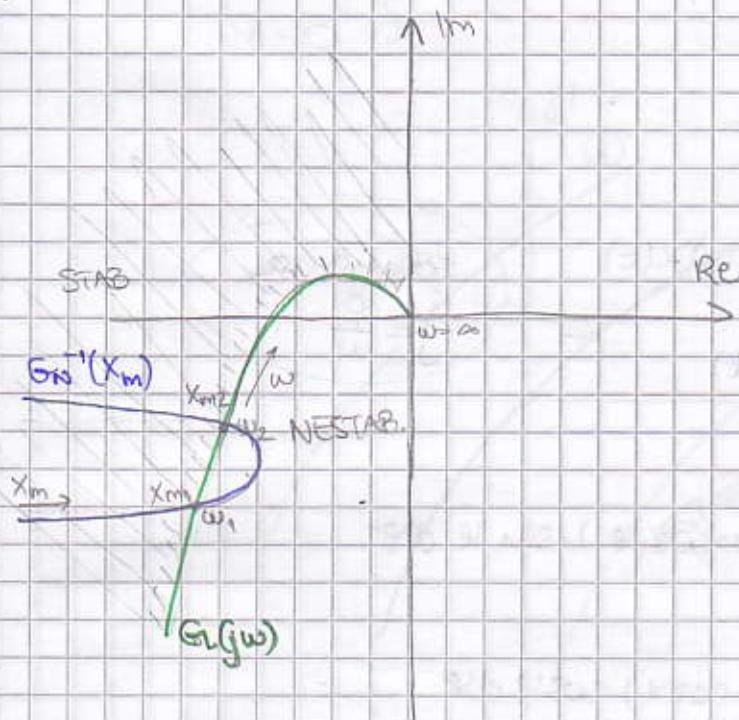
$$x(p) + G_L(p)G_N(x_m)x(p) = 0$$

$$1 + G_L(p)G_N(x_m) = 0$$

$$\boxed{1 + G_L(j\omega)G_N(x_m) = 0} \quad (6)$$

Kako rješiti (6)?

- ① GRAFOANALITIČKI (Goldfarb) → rješenje postoji ako postoji  $G_L(j\omega) = \frac{1}{G_N(x_m)} = -G_N'(x_m)$
- ② ANALITIČKI



→ vlastite osc. stabilne

na višim frekv. i

višoj amplitudi

(poj. u krajini zašto!)

$$\omega_2 > \omega_1$$

$$X_{m2} > X_{m1}$$

→ stabilitet:

moguće osc. s  $\omega_1$  i  $X_{m1}$   
ili sa  $\omega_2$  i  $X_{m2}$

↳ stabilitet se određuje

iscrtovanom područje

lijeko od  $G_L(j\omega)$  → to  
podr. je stabilitet

→ ako se gibaju p

$G_N^{-1}(j\omega)$  i ulazimo

u nestab. podr. onda

je ta točka nestab.

$(\omega_1, X_{m1})$  a ako izlazimo

iz nestab., a ulazimo u stab.  
onda je ta točka stab.  $(\omega_2, X_{m2})$

## (2) ANALITIČKI

$\rightarrow G_p^{-1}(x_m, \omega) \rightarrow$  iwa i inerciju (Goldfrak nije pravý egypt)



$$x(t) + y(t) = 0$$

$$x(t) + G_p(p) F(x, px) \cdot x(t) = 0 \quad (1)$$

$$x(t) = X_m \cdot \sin(\omega t)$$

$$px(t) = X_m \omega \cdot \cos(\omega t)$$

$$y_m(t) = F(X_m \sin(\omega t), X_m \omega \cos(\omega t)) \quad (2)$$

Razvoj (2) u Fourierov red

$$\begin{aligned} y(t) &= P(X_m, \omega) x(t) + j Q(X_m, \omega) x(t) \\ &= P(X_m, \omega) + \frac{j Q(X_m, \omega)}{\omega} p x(t) \end{aligned}$$

Za vrijednost  
 $j = \frac{P}{\omega}$

$$P(X_m, \omega) = \frac{1}{\omega X_m} \int_0^{2\pi} F(X_m \sin \varphi, X_m \omega \cos \varphi) \cdot \sin \varphi d\varphi$$

$$Q(X_m, \omega) = \frac{1}{\omega X_m} \int_0^{2\pi} F(X_m \sin \varphi, X_m \omega \cos \varphi) \cos \varphi d\varphi$$

$$p \rightarrow s \rightarrow j\omega$$

$$y = P(x_m, \omega)x + j \frac{Q(x_m, \omega)}{\omega} x_{\omega}$$

$$\frac{Y}{X} = P(x_m, \omega) + j Q(x_m, \omega) = G_N(x_m, \omega) \quad (3)$$

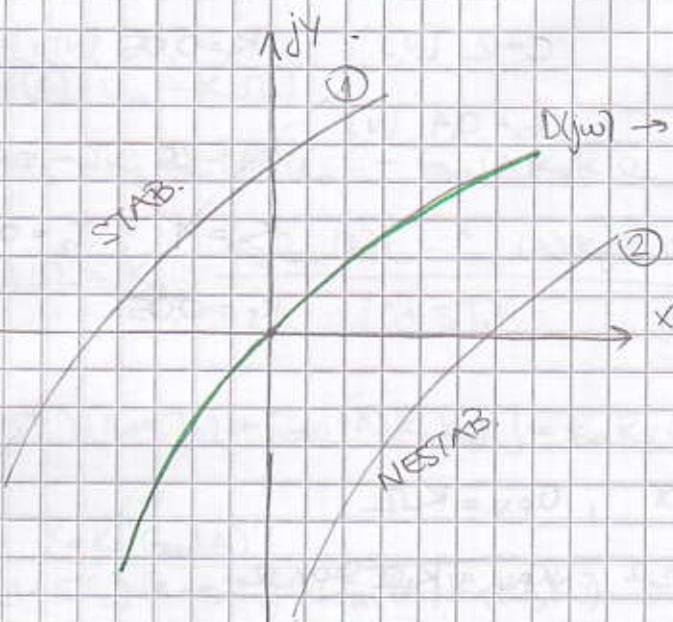
(3)  $\rightarrow$  (1)

$$A(j\omega) + B(j\omega)[P(x_m, \omega) + j Q(x_m, \omega)] = 0$$

$$D(j\omega) = \underbrace{\text{Re}(x_m, \omega)}_X + j \underbrace{\text{Im}(x_m, \omega)}_Y = 0 = 1 + G_N(x_m, \omega) \cdot \underline{G_L(j\omega)}$$

b) treba naći  $x_m$  i  $\omega$

$$\begin{cases} X=0 \\ Y=0 \end{cases} \quad \begin{matrix} \text{z jed.} \\ \text{z nepozn.} \end{matrix}$$



karakt. jed  
zatvoreno  
kruga

(če uje crtač  
krivulju)  
 $D(j\omega)$

$D(j\omega) \rightarrow$  krivulja Mihailova

graf zatvorenog  
kruga

→ ako prolazi  
kroz izvodiste  
postoje vlastite  
oscilacije

→ giba se kroz osolikos kvadrantata kvaliteta je reda

→ ako oblik i izvod. i zavrsna osolika kvad kvalita je potrebuje  
ona je stabilna  $I \rightarrow II \rightarrow III \vee STAB$

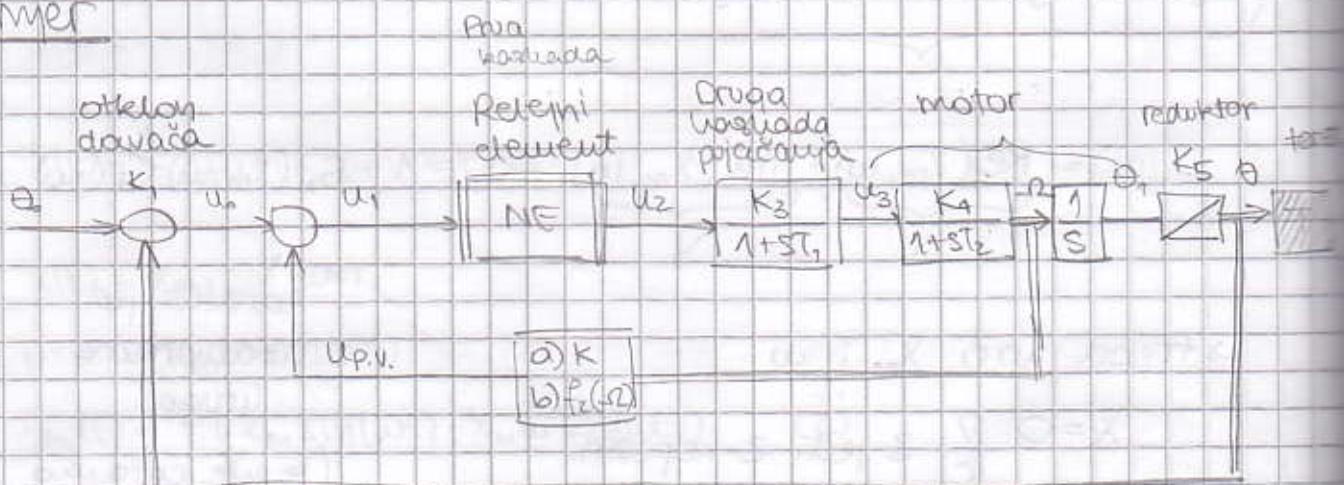
② ide u I pa IV kvad  $\rightarrow$  NESTAB

$$\frac{\partial X}{\partial X_m} \frac{\partial Y}{\partial w} - \frac{\partial Y}{\partial X_m} \frac{\partial X}{\partial w} > 0$$

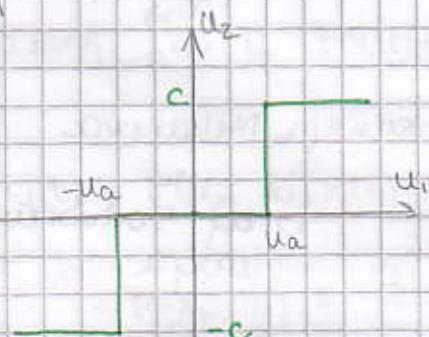
→ uvjet da su vlastite oscitacije stabilne

↳ MITAJLOVYEV KRITERIJ STABILNOSTI

### Primer



Relei



$$c = 2 \text{ [V]}$$

$$u_a = 0,4 \text{ [V]}$$

$$K_1 = 0,05 \text{ [V/lo]} = 2,86 \text{ [V/rod.]}$$

$$0,05 \cdot \frac{120}{4}$$

$$K_3 = 25 ; T_1 = 0,2$$

$$K_4 = 1 ; T_2 = 0,3$$

$$K_5 = 0,05$$

Ad a) Takogenerator , U.p.v. =  $K \Omega$

Ad b)  $f_2(\omega) = K \Omega^2 \operatorname{sign} \omega$  , U.p.v. =  $K \Omega^2 \operatorname{sign} \omega$

$$\omega = \frac{d\theta_1}{dt}$$

Awaliada sestava po param.  $K_1, K$  (rownina,  $K - K_1$ )

$$G_{M1}(A) = P_1(A) = \frac{4C}{KA} \sqrt{1 - \left(\frac{U_0}{A}\right)^2} \rightarrow \text{ima ekstreum, (granica k)}$$

$$U_2 = G_{M1}(A) \cdot U_1$$

[Ad a)]

$$U_{P,V} = K \cdot \Omega = K \cdot \frac{d\Theta_1}{dt}$$

$$U_1 = U_0 - U_{P,V}$$

$$U_2 = G_{M1}(A) \cdot U_1$$

$$(1 + ST_1) U_3 = K_3 U_2$$

$$(1 + ST_2) \Omega = K_4 \cdot U_3$$

$$\Theta_1 = \frac{\Omega}{S}$$

$$\Theta = K_5 \cdot \Theta_1$$

$$U_0 = K_1 (\Theta_0 - \Theta)$$

$\rightarrow$  Unutrašnja petlja  $\rightarrow$  treba poverati  $\Omega$  i  $U_0$

$$U_2 = G_{M1}(A) [U_0 - K \cdot \Omega]$$

$$(1 + ST_1) U_3 = K_3 G_{M1}(A) U_0 - G_{M1}(A) K_3 K \cdot \Omega$$

$$(1 + ST_2) \Omega = K_4 \cdot \frac{K_3 G_{M1}(A) U_0 - G_{M1}(A) K_3 K \cdot \Omega}{(1 + ST_1)}$$

$$\Omega [(1 + ST_2)(1 + ST_1) + G_{M1}(A) K_4 K_3 K] = K_4 K_3 G_{M1}(A) U_0$$

$$\frac{\Omega}{U_0} = \frac{K_4 K_3 G_{M1}(A)}{(1 + ST_2)(1 + ST_1) + G_{M1}(A) K_4 K_3 K} \rightarrow \text{karakter. jed}$$

Zadvoření krug (povzatí  $\Theta_0$  i  $\Theta$ )

$$[(1+sT_1)(1+sT_2) + K_3K_4KG_{N1}(A)]s \cdot \frac{\Theta}{K_3} = K_4K_3G_{N1}(A) \cdot K_1(\Theta_0 - \Theta)$$

$$[s(1+sT_1)(1+sT_2) + K_5K_4KG_{N1}(A)s + K_5K_4K_3K_1G_{N1}(A)]\Theta = K_5K_4K_3K_1G_{N1}(A)\Theta$$

$$\frac{\Theta}{\Theta_0} = \frac{K_5K_4K_3K_1G_{N1}(A)}{s(1+sT_1)(1+sT_2) + K_4K_3KG_{N1}(A)s + K_5K_4K_3K_1G_{N1}(A)}$$

karakt.  
jed.  
zadvoř.  
kruga

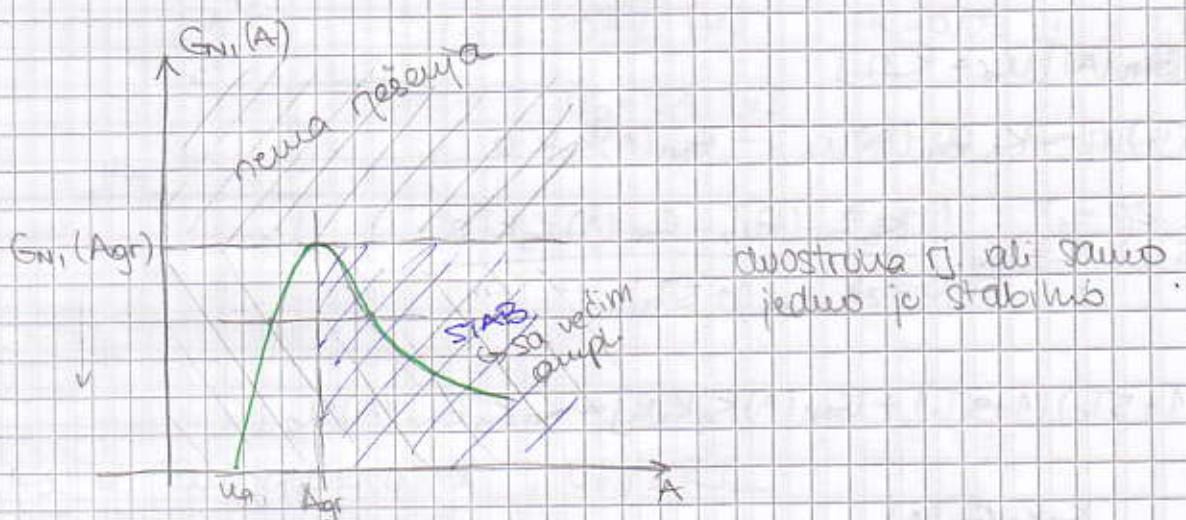
$$T_1 T_2 s^3 + (T_1 + T_2)s^2 + [1 + K_4K_3KG_{N1}(A)]s + K_5K_4K_3K_1G_{N1}(A) = 0 = D(s)$$

(→ karakteristická funkce kruga)

$$\text{uz } s=j\omega$$

$$\text{Re: } X(A, \omega) = -(T_1 + T_2)\omega^2 + K_5K_4K_3K_1G_{N1}(A) = 0 \quad \left. \right\} K = f(K_1)$$

$$\text{Im: } Y(A, \omega) = -T_1 T_2 \omega^2 + [1 + K_4K_3KG_{N1}(A)]\omega = 0$$



$$\frac{dG_{N1}(A)}{dA} \rightarrow \text{a a b dobrili Agr}$$

$$\frac{ua^2}{\sqrt{A^2 - ua^2}} - \sqrt{A^2 - ua^2} = 0 \rightarrow \boxed{Agr = \sqrt{2}ua}$$

$$G_{N1}(A_{gr}) = \frac{2C}{T_{kra}}$$

Wziet Minkowska:

$$\frac{\partial X}{\partial A} \frac{\partial Y}{\partial w} - \frac{\partial Y}{\partial A} \frac{\partial X}{\partial w} > 0$$

$$X(A, w) = K^* G_{N1}(A) - (T_1 + T_2) w^2 \rightarrow \frac{\partial X}{\partial A} = K^* \frac{\partial G_{N1}(A)}{\partial A}$$

$$\frac{\partial X}{\partial w} = -2(T_1 + T_2) w \leq 0$$

zdroj  $\rightarrow$  je malející

$$Y(A, w) = [1 + K^{**} G_{N1}(A)]w - T_1 T_2 w^3 \rightarrow \frac{\partial Y}{\partial A} = K^{**} \frac{\partial G_{N1}(A)}{\partial A} w$$

$$\frac{\partial Y}{\partial w} = -2T_1 T_2 w$$

$$K^* \frac{\overset{(2)}{\partial G_{N1}(A)}}{\partial A} \cdot (-2T_1 T_2 w) - (-2(T_1 + T_2) w) K^{**} \frac{\overset{(2)}{\partial G_{N1}(A)}}{\partial A} w > 0$$

$\circlearrowleft$

$$\frac{\partial G_{N1}(A)}{\partial A} < 0$$

$$X(A, w) = 0 \rightarrow w^2 = \frac{K^* G_{N1}(A)}{T_1 + T_2} \Rightarrow w_{gr}^2 = \frac{K^*}{T_1 + T_2} \cdot \frac{2C}{T_{kra}}$$

$$Y(A, w) = 0 \rightarrow K = \frac{T_1 T_2 w^2 - 1}{K_3 K_4 G_{N1}(A)}$$

$$K = \frac{T_1 T_2 K_5 K_1}{T_1 + T_2} - \frac{1}{K_3 K_4 G_{N1}(A)}$$

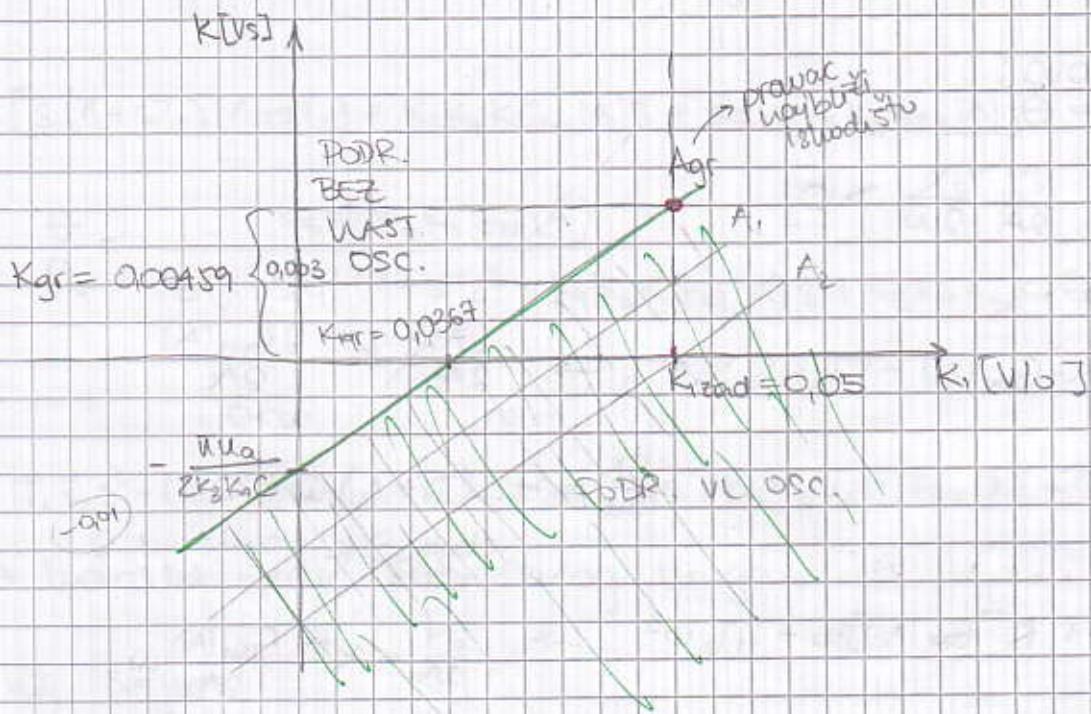
$\circlearrowleft$   
naoživo

$$K = f(K_1)$$

$\hookrightarrow$  pravci

$$K = 0,01 K_1 - \frac{1}{25 G_{N1}(A)}$$

$$K = 0.01 K_1 - \frac{1}{25 Gm(A)}$$



La zadane podatue:  $A_{gr} = 0,57$  [V]

$$G_{M1}(A_{eq}) = 3,18$$

$$K_{\text{gr}} = 0,006 K_{\text{ref}} - 0,0126$$

→ ako odaberemo K veli od 0,00459 sustav neće imati vlastite osc. , ako odaberemo K=K<sub>cr</sub> osc. će sa  $\omega_{ap} = 4,77 \text{ [s}^{-1}]$

$\rightarrow$  also  $\mu \in K < K_{\text{gr}}$

$$K = 0.1006 K_1 - \frac{1}{25 G_{N1}(A)}$$

→ problem weight  $k=0,008$ ;  $k_1=0,005$   
bei doppeltem Gr.

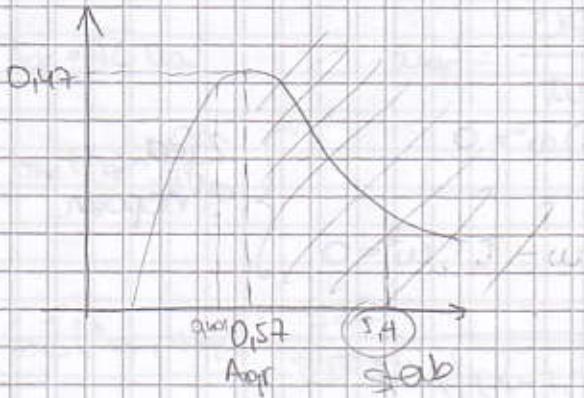
$$G_{N1}(A) = 2.828 = \frac{4.2}{\pi A} \sqrt{1 - \left(\frac{0.4y}{A}\right)^2} / 2$$

$$0,22 = \frac{64(A^2 - 0,4^2)}{\pi^2 A^2} \rightarrow 218A^4 - 64A^2 + 10,24 = 0$$

$$2,18x^2 - 64x + 10,24 = 0$$

$$x_1 = 29,12 \rightarrow A_1 = \sqrt{29,12} = 5,4$$

$$x_2 = 0,16 \rightarrow A_2 = \sqrt{0,16} = 0,401$$



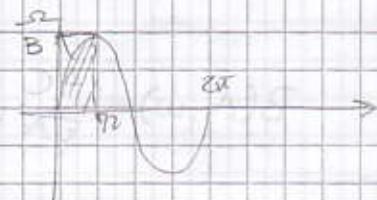
$$\omega_1^2 = \frac{K^* G_{N1}(A)}{T_1 + T_2} = 3,36 [s^{-2}] \quad \omega = 1,83 [s^{-1}]$$

$$u = 5,4 \sin(1,83t)$$

[NEZ] → dvojni nelin. elem. (osim releja)

Adb

$$U_{p.v.} = K \Omega^2 \operatorname{sign} \Omega \approx G_{N2}(B) \Omega$$



Opisna f-ja:

$$\Omega = S\Theta_1 = B \sin(\omega t + \varphi)$$

$$G_{N2}(B) = P_2(B) + jQ_2(B) = -\frac{4}{\pi B} \int_0^\infty K B^2 \sin^2(\omega t) d(\omega t) = \frac{8K}{3\pi} B$$

nelin elem. s inercijom



Karakat. jed. z aktivacijom kružnog

$$T_1 T_2 S^2 + (T_1 + T_2) S^2 + [1 + K_3 K_4 G_{N1}(A) G_{N2}(B)] S + K_1 K_3 K_4 K_5 G_{N1}(A) = 0$$

$s \rightarrow j\omega$

$$\text{Re: } X(A, B, \omega) = K_1 K_3 K_4 K_5 G_{N1}(A) - (T_1 + T_2) \omega^2 = 0$$

$$\text{Im: } Y(A, B, \omega) = [1 + K_2 K_4 G_{N1}(A) G_{N2}(B)] \omega - T_1 T_2 \omega^3 = 0$$

$\left. \begin{array}{l} 2 \text{ jed} \\ 3 \text{ nepozn.} \end{array} \right\}$

$$\frac{\Omega(s)}{U_1(s)} = G_N(A, s) = G_{N1}(A) \cdot \frac{K_3 K_4}{(A + T_1 s)(A + T_2 s)}$$

$$\Omega(s) = \frac{4C}{\pi A} \sqrt{1 - \left(\frac{U_0}{A}\right)^2} \cdot \frac{K_3 K_4}{T_1 T_2 s^2 + (T_1 + T_2)s + 1} U_1(s)$$

$$U_1 = A \sin \omega t$$

Neza ampl.  $B \perp A$  dobiva se iz:

$$\Omega = B \cdot \sin(\omega t + \varphi)$$

$$\boxed{|\Omega| = |G_N(A, s)| \cdot |U_1|}$$

$$B(A, \omega) = \frac{4C}{\pi A} \sqrt{1 - \left(\frac{U_0}{A}\right)^2} \cdot \frac{K_3 K_4}{\sqrt{(1 - T_1 T_2 \omega^2)^2 + (T_1 + T_2)^2 \omega^2}} A$$

$$G_{N2}(B) = \frac{8K}{3\pi} B = K \cdot G_{N2}^+(A, \omega)$$

$$= K \cdot \frac{32C}{3\pi^2 A} \sqrt{1 - \left(\frac{U_0}{A}\right)^2} \cdot \frac{K_3 K_4}{\sqrt{(1 - T_1 T_2 \omega^2)^2 + (T_1 + T_2)^2 \omega^2}} A$$

$\underbrace{\qquad\qquad\qquad}_{G_{N2}^+(A, \omega)}$

$$\text{Re: } X(A, \omega) = K_1 K_3 K_4 K_5 G_{N1}(A) - (T_1 + T_2) \omega^2 = 0$$

$$\text{Im: } Y(A, \omega) = [1 + K_2 K_4 G_{N1}(A) G_{N2}^+(B)] \omega - T_1 T_2 \omega^3 = 0$$

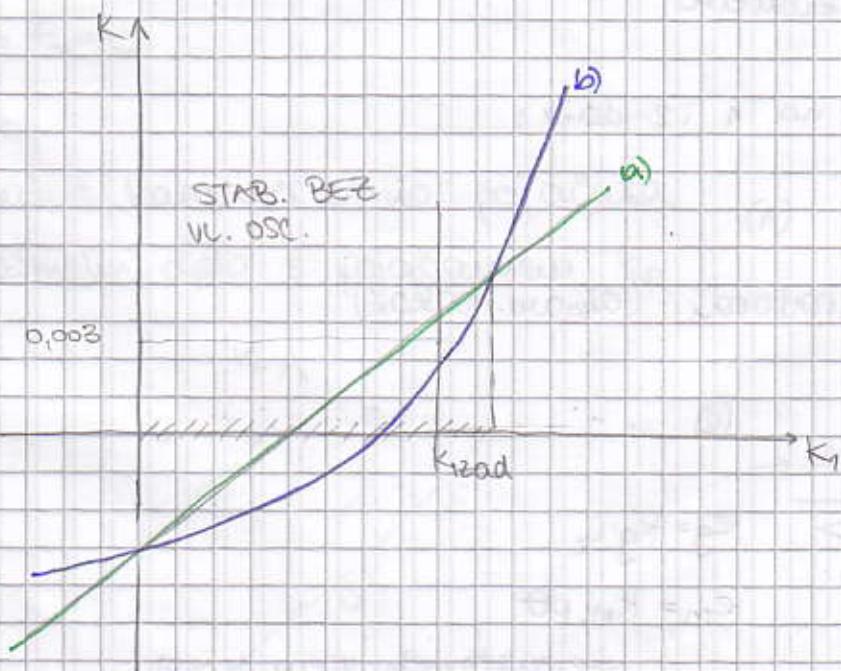
$$Aqr = \sqrt{2} u_a$$

$$\omega_{qr}^2 = \frac{2K_1 K_3 K_u K_s C}{\pi u_a (T_1 + T_2)}$$

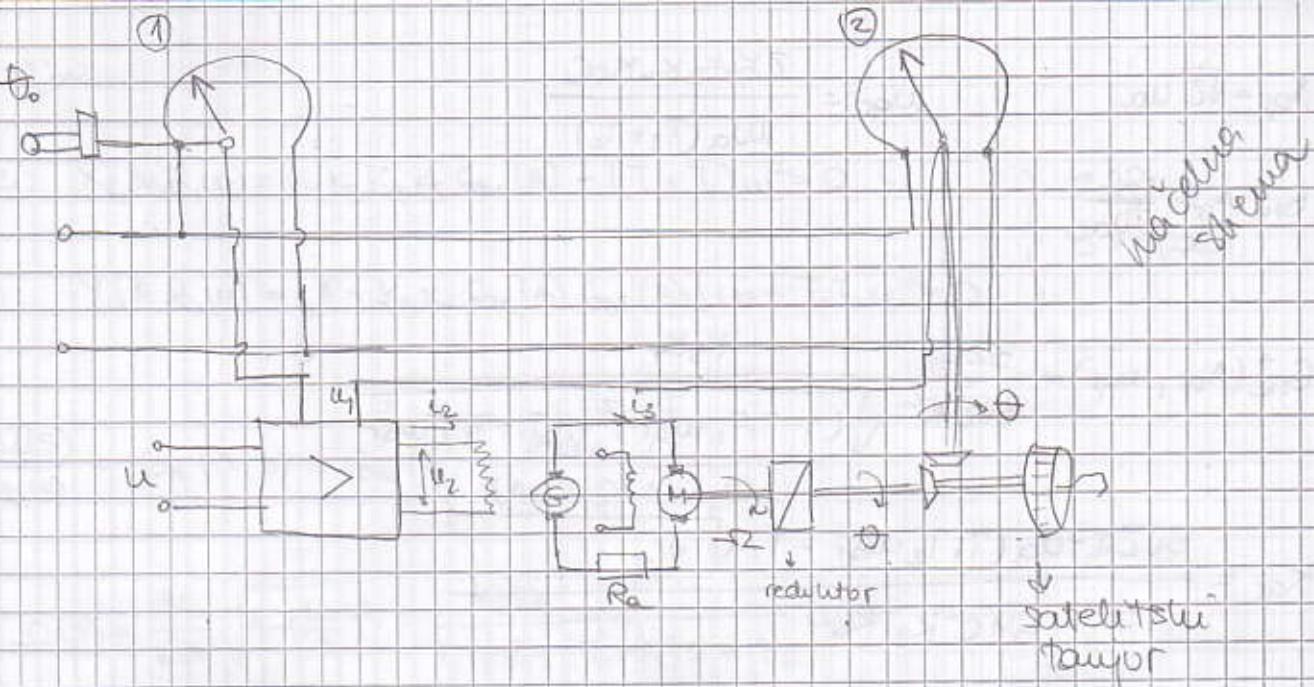
$$G_{NL}(Aqr) = \frac{2C}{\pi u_a}$$

$$G_{NL}(Aqr, \omega_{qr}) = \frac{32C}{3\sqrt{2}\pi^2} \frac{K_3 K_4}{\sqrt{(1 - T_1 T_2 \omega_{qr}^2)^2 + (T_1 + T_2)^2 \omega_{qr}^2}}$$

$$K_{qr} = \frac{3\sqrt{2}\pi^3 u_a (T_1 T_2 \omega_{qr}^2 - 1)}{64 C^2 K_3^2 K_4}$$



$\rightarrow$  da  $K_{izad}$ ,  $K=0,003$  nećemo sada biti u stabilnosti dijelu



→ dalijski prienos zadreta

→ različna izmedu kuta na 1. i 2. fazu:

$$U_1 = K_1(\theta_0 - \theta_1) \quad (1)$$

→ uzbudni krug generatora (diamet. prikaz)

$$(T_1 p + 1) i_2 = K_2 U_2 \quad (2)$$

$$R_g \cdot i_2 = e_g - e_m \Rightarrow e_g = K_g \cdot i_2$$

$$e_m = K_m \cdot p\theta$$

Generatorisch. vrem. konst

$$\textcircled{*} \quad i_3 = \frac{K_3}{R_{L3}} U_2 - \frac{K_m}{R_{L3}} p\theta \quad \rightarrow \text{jed. danoča}$$

$$\left. \begin{aligned} Jp^2\theta + K_{tr}p\theta &= M_{cm} \\ M_m &= K_i \cdot i_3 \end{aligned} \right\} \quad \begin{aligned} \left( \frac{J}{K_{tr}} p^2 + p \right) \theta &= \frac{K_i}{K_{tr}} i_3 \\ T_2' & \downarrow \\ \text{bremeh. vrem. konst} \end{aligned} \quad \rightarrow \text{jed. motora}$$

\textcircled{\*} + jed. motora

$$(T_2' p + 1) p\theta = K_5 (K_3 i_2 - K_4 p\theta)$$

$$(T_2 p + 1) \cdot p \Theta = \frac{K_3 K_5}{1 + K_4 K_5} i_2$$

$$T_2 = \frac{T_2'}{1 + K_5 K_4}$$

$$\frac{\Theta}{i_2} = \frac{K}{p(T_2 p + 1)} \quad (3)$$

$$K = \frac{K_3 K_5}{1 + K_5 K_4}$$

→ jed. liniarnego diela sustawa iz (1) (2) i (3)

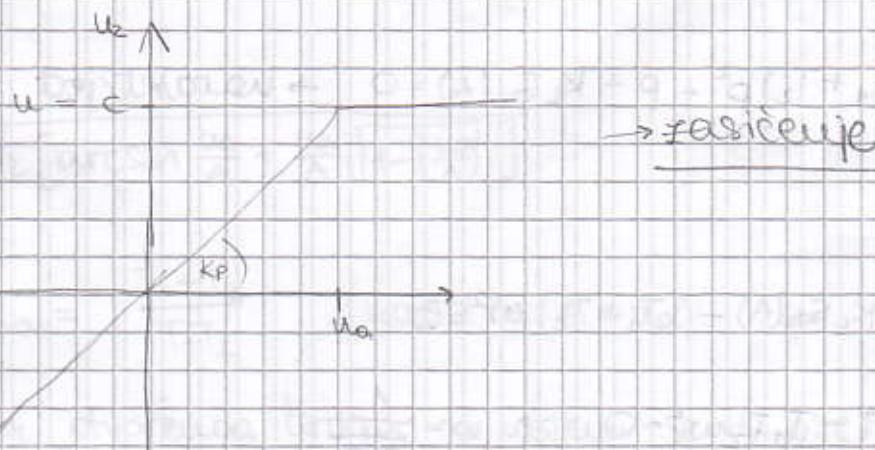
$$(T_1 p + 1)(T_2 p + 1) \cdot p u_1 = -K_L u_2$$

$$\hookrightarrow \Theta_0 = 0$$

$$K_L = \frac{K_1 K_2 K_3 K_5}{1 + K_5 K_4}$$

→ realna karakteristika pojacišala

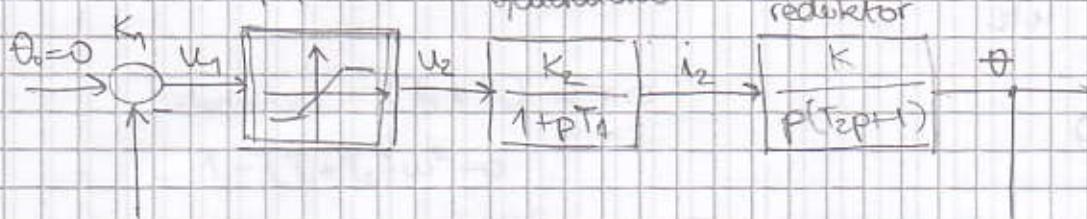
(osim lin. diela s pojacišnjem  $K_p$ )



BUDOVNA SHEMA  
pojacišalo

uređaja  
generatora

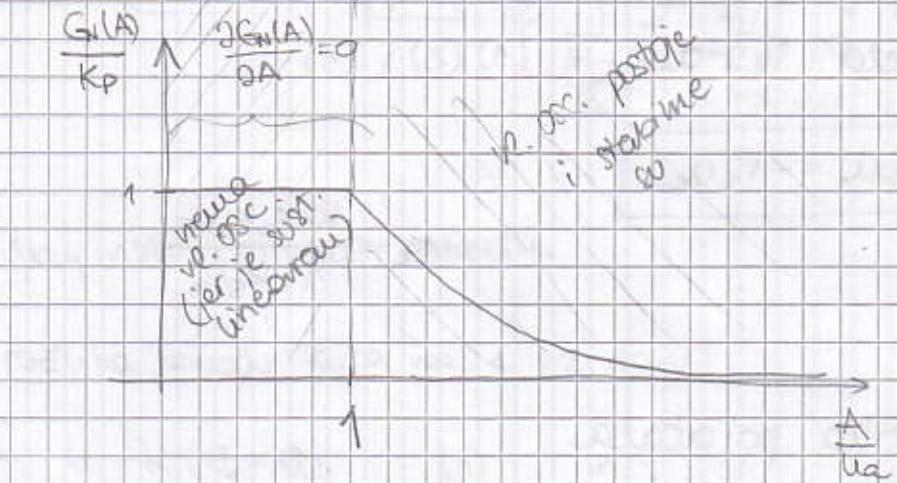
motor +  
redoviktor



$$u_1 = A \cdot \sin(\omega t)$$

$$u_2 = G_N(A) \cdot u_1$$

$$G_N(A) = \begin{cases} K_p, & A < u_a \\ \frac{2K_p}{\pi} \left[ \arcsin \frac{u_a}{A} + \frac{u_a}{A} \sqrt{1 - \left( \frac{u_a}{A} \right)^2} \right], & A > u_a \end{cases} \quad (1)$$



$$[(T_{1p}+1)(T_{2p}+1) + K_L G_N(A)] u_1 = 0$$

$$T_1 T_2 p^3 + (T_1 + T_2) p^2 + p + K_L G_N(A) = 0 \rightarrow \text{charakt. jed.}$$

$$p \rightarrow s \rightarrow j\omega$$

$$X(A, \omega) = K_L G_N(A) - (T_1 + T_2) \omega^2 = 0$$

$$Y(A, \omega) = 1 - T_1 T_2 \omega^2 = 0 \rightarrow \omega = \frac{1}{\sqrt{T_1 T_2}}$$

$$K_L G_N(A) - \frac{T_1 + T_2}{T_1 T_2} = 0$$

$$\text{Zu } G_N(A) = (2)$$

$$\frac{2K_L K_p}{\pi} \left[ \arcsin \frac{u_a}{A} + \frac{u_a}{A} \sqrt{1 - \left( \frac{u_a}{A} \right)^2} \right] = \frac{T_1 + T_2}{T_1 T_2}$$

$$A = f(T_2), \quad \omega = f(T_2)$$

$$A = f(K_2 = K_L K_p)$$

## Mihailov kriterij

$$\frac{\partial X}{\partial A} \frac{\partial Y}{\partial w} - \frac{\partial Y}{\partial A} \frac{\partial X}{\partial w} > 0.$$

$$\frac{\partial X}{\partial A} = K_c \frac{\partial G_N(A)}{\partial A}$$

$$\frac{\partial Y}{\partial A} = 0 \quad \frac{\partial X}{\partial w} = -2(T_1 + T_2)w$$

$$\frac{\partial Y}{\partial w} = -2(T_1 + T_2)w$$

$$-\underbrace{K_c}_{<0} \frac{\partial G_N(A)}{\partial A} \underbrace{2(T_1 + T_2)w}_{>0} > 0$$

$\frac{\partial G_N(A)}{\partial A} < 0 \rightarrow$  vl. osc. postoje i stabilne su samo u sl. kada je  $A > u_a$

$$K_c = \frac{\pi(T_1 + T_2)}{2T_1 T_2 \left[ \arcsin \frac{u_a}{A} + \frac{u_a}{A} \sqrt{1 - \left( \frac{u_a}{A} \right)^2} \right]}$$

$$K_{zgranično} = \frac{T_1 + T_2}{T_1 T_2} \quad (\text{kada je } A = u_a)$$

↳ qf. poj. otvorenenog kružnog na kojem se vl. osc. pojavljuju

$$\rightarrow G_N(A) = K_p \quad (\text{linearan sustav})$$

$$K_{zberitično} = (T_1 + T_2) w^2 = 0$$

$$1 - (T_1 + T_2) w^2 = 0$$

$$K_{zberiti} = \frac{T_1 + T_2}{T_1 T_2} \rightarrow \text{znači daq. pojačanja sust. je nestabilan}$$

↓  
uobičajeno pojačala sust. nije nestab.  
znači  $K_{zberiti}$  ulazi u vl. osc. a ispod je stabilan

$$K_Z = \frac{\pi (T_1 + T_2)}{2T_1 T_2} \left[ \arcsin \frac{u_a}{A} + \frac{u_a}{A} \sqrt{1 - \left( \frac{u_a}{A} \right)^2} \right] \quad (4)$$

$$K_Z = 20 \text{ s}^{-1} \quad u_a = 1 \text{ V}$$

$$T_1 = 0,1 \text{ s} \quad K_h = 100 \text{ V/rad}$$

$$T_2 = 1 \text{ s} \quad u_i = A \cdot \sin(\omega t)$$

$$(u_i) = K_h (\theta) \quad \text{a kuta je isti}$$

$$\theta(t) = A_0 \sin \omega t$$

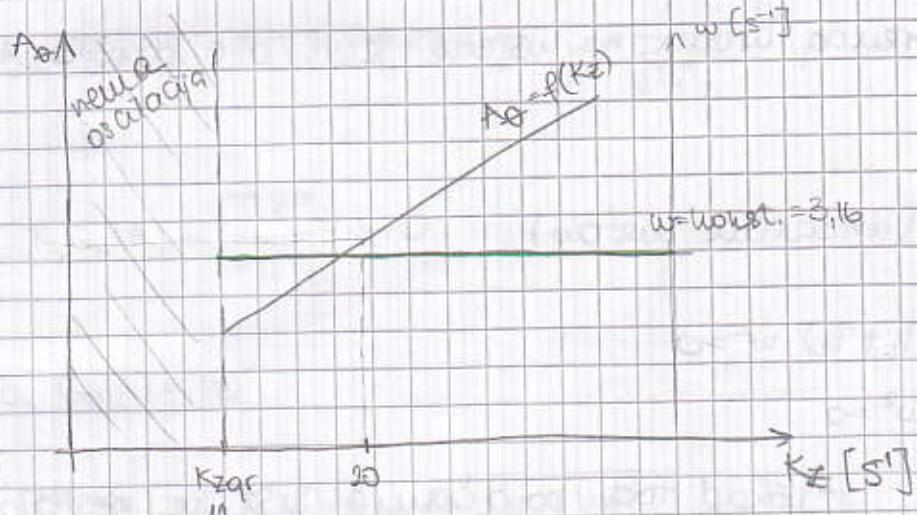
$$K_{Zgr} = \frac{T_1 + T_2}{T_1 T_2} = 1 \text{ s}^{-1}$$

$$K_Z > K_{Zgr} \text{ imamo (4)}$$

$$K_Z = f(A) = \frac{1,1\pi}{0,2 \left[ \arcsin \frac{1}{A} + \frac{1}{A} \sqrt{1 - \frac{1}{A^2}} \right]} \quad , \quad A_0 = \frac{A}{K_h}$$

$$\omega = \frac{1}{\sqrt{T_1 T_2}} = 3,16 \text{ rad/s}$$

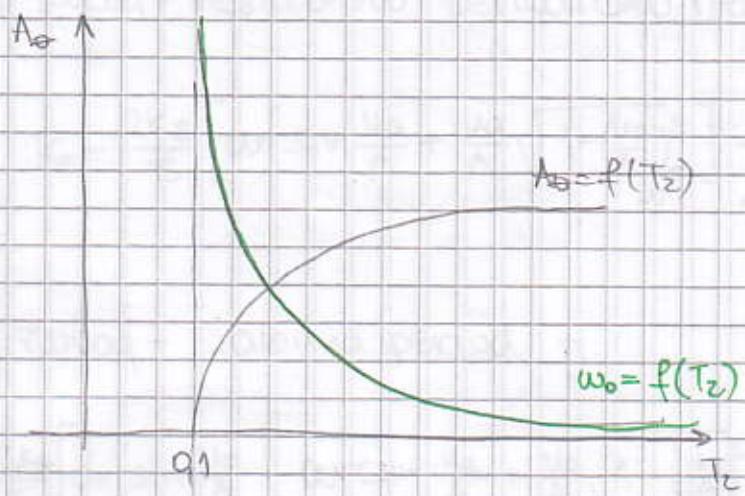
↳ ovisnost amplitude o početku vrijeme



$$\theta(t) = 0,0225 \sin(3,16t)$$

$$u_i(t) = 2,25 \sin(3,16t)$$

→ ovisnost o promjenici vrem. konst.  $T_2$



$$T_{2ar} = \frac{T_1}{(T_1 K_{2ar} - 1)} = 915$$

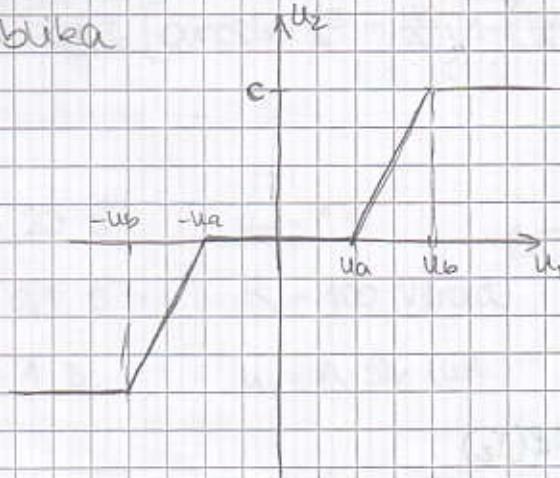
$$T_2 = \frac{0,314}{4 \left( \arcsin \frac{1}{2} + \frac{1}{A} \sqrt{1 - \frac{1}{A^2}} - 3,14 \right)}$$

$$\omega = \sqrt{\frac{10}{T_2}}$$



Z. slučaj

NE obuka



$$X(A, w) = K_c G_n(A) - (T_1 + T_2) w^2 = 0$$

$$Y(A, w) = 1 - T_1 T_2 w^2 = 0$$

$$G_n(A) = \begin{cases} 0, & A < u_a \\ K_p - \frac{2K_p}{\pi} \left[ \arcsin \frac{u_a}{A} + \frac{u_a}{A} \sqrt{1 - \left( \frac{u_a}{A} \right)^2} \right], & u_a \leq A \leq u_b \\ \frac{2K_p}{\pi} \left[ \arcsin \frac{u_b}{A} - \arcsin \frac{u_a}{A} + \frac{u_b}{A} \sqrt{1 - \left( \frac{u_b}{A} \right)^2} - \frac{u_a}{A} \sqrt{1 - \left( \frac{u_a}{A} \right)^2} \right], & A > u_b \end{cases}$$



$$\cdot \frac{\partial X}{\partial A} = K_c \frac{\partial G_n(A)}{\partial A}$$

$$\frac{\partial G_n(A)}{\partial A} < 0 \quad \text{postoje vl. osc. } A > u_b$$

$$\frac{\partial G_n(A)}{\partial A} > 0 \quad \text{vl. osc. nisu stabilne (ne postoje) } u_a < A < u_b$$

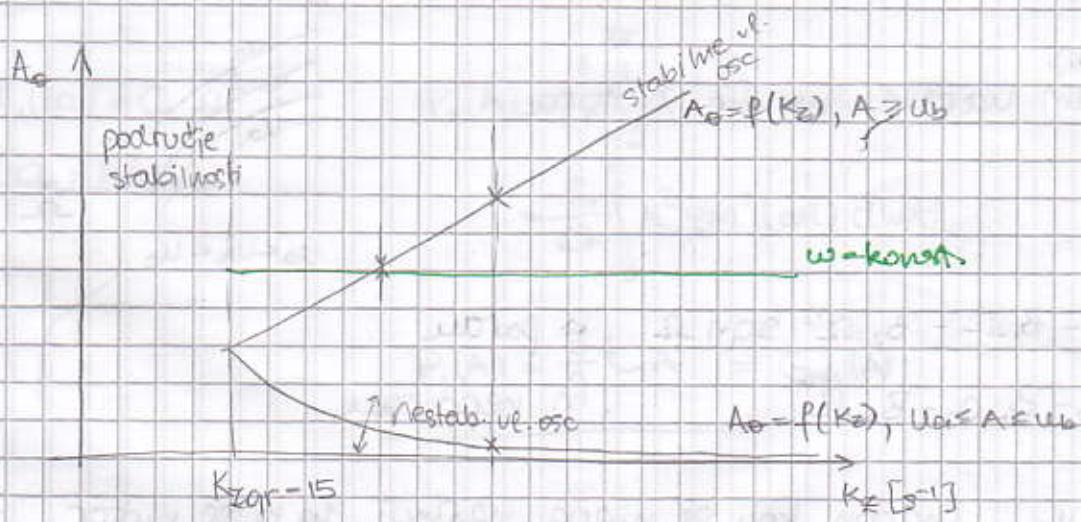
→ veza između amplitude i parametara

① slučaj - nestabilno periodičko rješenje

$$K_2 = \frac{2K_1}{\pi} \left[ \arcsin \frac{U_a}{A} + \frac{U_a}{A} \sqrt{1 - \left( \frac{U_a}{A} \right)^2} \right] = \frac{T_1 + T_2}{T_1 T_2} \quad U_a \leq A \leq U_b$$

② slučaj - stabilna period. rje.

$$\frac{2K_2}{\pi} \left[ \arcsin \frac{U_b}{A} - \arcsin \frac{U_a}{A} + \frac{U_a}{A} \sqrt{1 - \left( \frac{U_b}{A} \right)^2} - \frac{U_a}{A} \sqrt{1 - \left( \frac{U_a}{A} \right)^2} \right] = \frac{T_1 + T_2}{T_1 T_2} \quad A > U_b$$

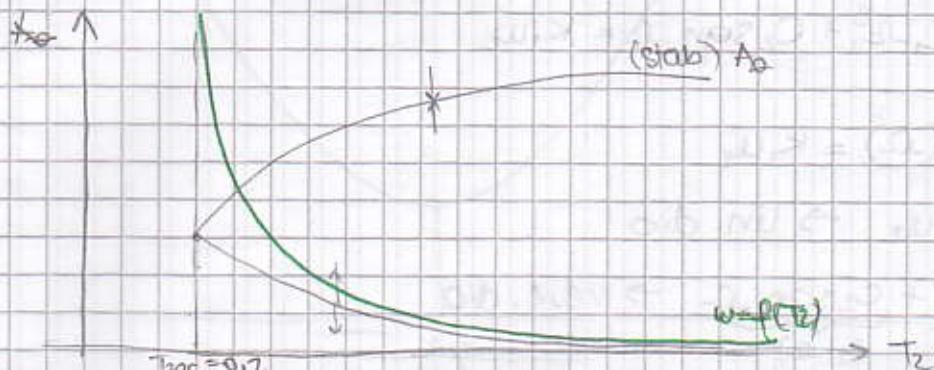


→ uočenjem zone neos. poređalo se podr. stab (K2ap veći)

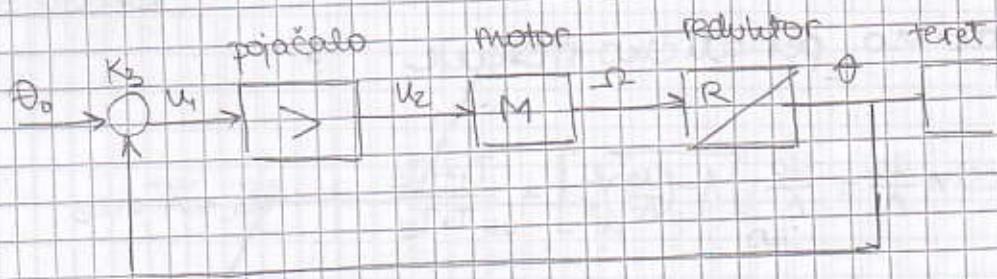
→ isti K2 → daje ampl. A0 (avisi o poč. vrijetu koju učinimo)

→ nestabil. vl. osc. → stabilitet u "malom" → dobivaju se ratnički, doveđe sistem u ravn. stanje, ali bez vl. osc.

→ mali poč. vrijet nemamo vl. osc., ali točnost je manja



## Z. primjer



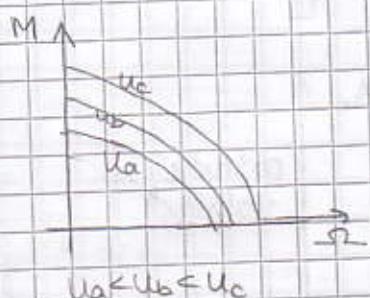
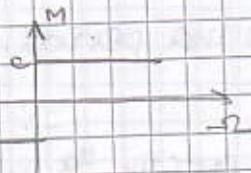
Z. nelinearnosti:

1.) Suhod trenje

bivalazi se u reduktoru

(meh.)

2.) vrem. varant. rezistor. motora



$$M = M_0 - B_1 \Omega - B_2 \Omega^2 \cdot \operatorname{sgn} \Omega, \text{ in paroru}$$

$$M = M_0 - B_1 \Omega - B_2 \Omega^2, \text{ in neparoru}$$

$M_0 = B_0 \cdot U_2$  (napon koji se mora napuniti da bi se motor uspelo pokrenuo)

$$M = M_0 - B_1 \Omega - B_2 \Omega^2$$

$$J \frac{d\Omega}{dt} + B_1 \Omega + B_2 \Omega^2 + C \cdot \operatorname{sgn} \Omega = B_0 U_2 / B_1$$

suhod trenje

$$T_1 = \frac{J}{B_1} \quad K_1 = \frac{B_0}{B_1} \quad C_1 = \frac{B_2}{B_1} \quad C_2 = \frac{C}{B_1}$$

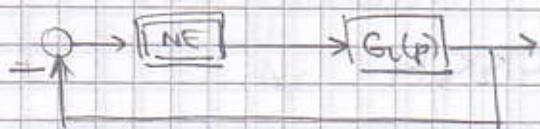
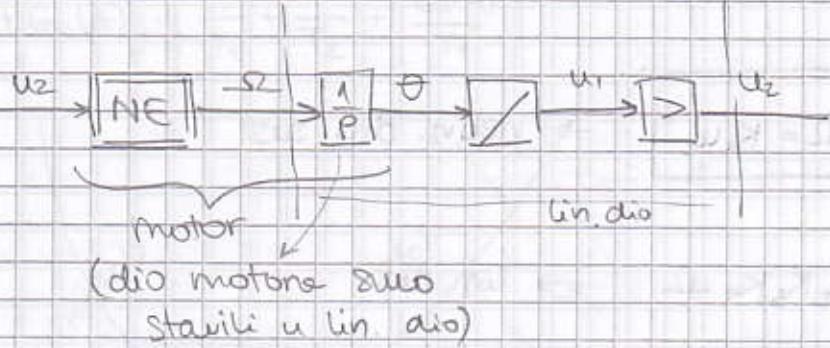
$$T_1 \frac{d\Omega}{dt} + \Omega + C_1 \Omega^2 + C_2 \operatorname{sgn} \Omega = K_1 U_2$$

$$(T_1 s + 1) \Omega + f(\Omega) = K_1 U_2$$

$$(T_1 s + 1) \Omega = K_1 U_2 \rightarrow \text{lin. dio}$$

$$f(\Omega) = C_1 \Omega^2 + C_2 \operatorname{sgn} \Omega \rightarrow \text{vrem. dio}$$

$f_1(\Omega)$        $f_2(\Omega)$



$$f_1(\omega) = C_1 \cdot \omega^3$$

$$\omega = A \sin \omega t$$



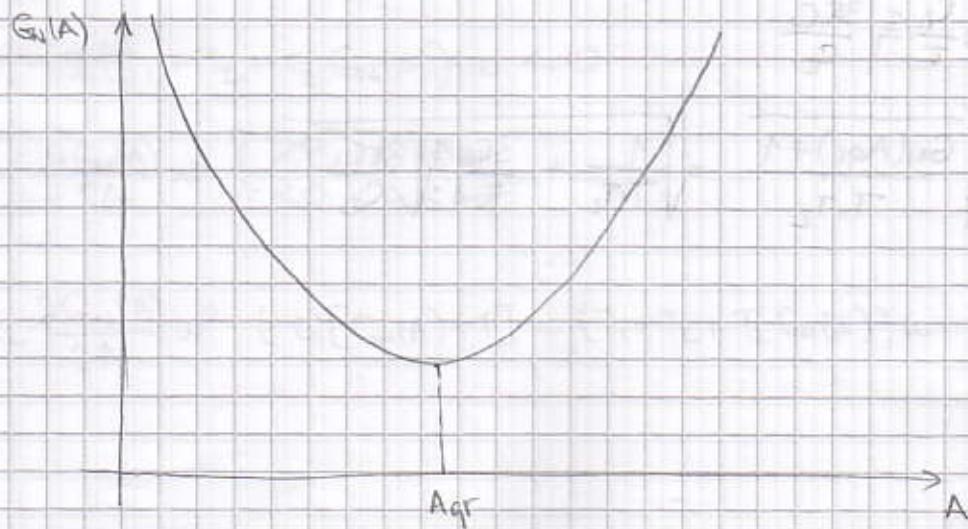
$$\begin{aligned}
 P_1(A) &= \frac{1}{\pi A} \int_0^{2\pi} f_1(\omega) \sin(\omega t) d(\omega t) \\
 &= \frac{C_1}{\pi A} \int_0^{2\pi} A^3 \sin^3(\omega t) d(\omega t)
 \end{aligned}$$

$$P_1(A) = \frac{3}{4} C_1 A^2 = G_{N1}(A)$$

$$f_2 = C_2 \sin \omega t$$

$$G_{N2}(A) = \frac{4C_2}{\pi A}$$

$$G_N(A) = G_{N1}(A) + G_{N2}(A) = \frac{3}{4} C_1 A^2 + \frac{4C_2}{\pi A}$$



$$f(\Omega) = G_N(A) \cdot \Omega$$

$$\boxed{(T_1 s + 1 + G_N(A)) \cdot \Omega = K_4 u_2} \rightarrow \text{nelin. dio sust}$$

$$(1 + sT_2) s u_2 = -K_2 K_3 K_4 \cdot \Omega \rightarrow \text{lin. dio}$$

$$u_1 = K_3 (\theta_0 - \theta)$$

$$s\theta = K_4 \cdot \Omega$$

$$K = K_1 K_2 K_3 K_4$$

$$T_1 T_2 s^3 + [T_1 + T_2 + T_2 G_N(A)] s^2 + [G_N(A) + 1] s + K = 0$$

$$s \rightarrow j\omega$$

$$X(A, \omega) = K - (T_1 + T_2 + T_2 G_N(A)) \omega^2 = 0$$

$$Y(A, \omega) = [G_N(A) + 1] \omega - T_1 T_2 \omega^3 = 0$$

$$G_N(A) = T_1 T_2 \omega^2 - 1$$

$\rightarrow$  auto moderno Pregeušte

$G_N$  s ovim pravcem i mamo  
vel. osc

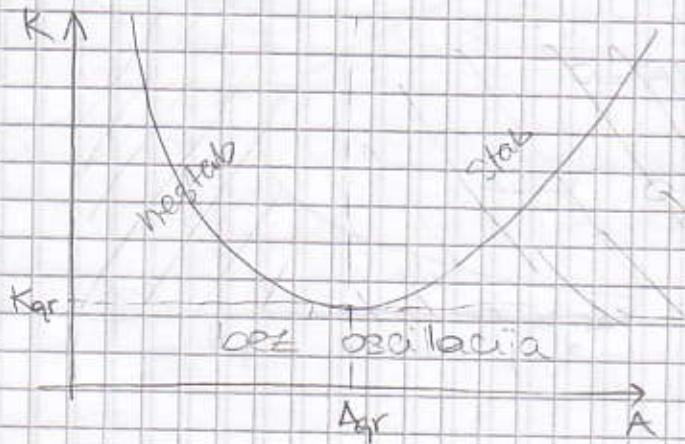
$$\frac{\partial G_N(A)}{\partial A} = \frac{\partial}{\partial A} \left( \frac{3}{4} C_1 A^2 + \frac{4C_2}{\pi A} \right) = 0$$

$$A_{qr} = 2 \sqrt[3]{\frac{C_2}{3C_1 \pi}}$$

$$G_N(A_{qr}) = \frac{3}{\pi} \frac{C_2}{\pi} \sqrt[3]{\frac{3\pi C_1}{C_2}}$$

$$\hookrightarrow \omega_{qr} = \sqrt[3]{\frac{G_N(A_{qr}) + 1}{T_1 T_2}} = \sqrt[3]{\frac{1}{T_1 T_2}} + \frac{3C_2}{\pi} \sqrt[3]{\frac{3\pi C_1}{C_2}}$$

$$K = \left[ 1 + G_N(A) \right] \cdot \left[ \frac{1}{T_1} + \frac{1}{T_2} + \frac{G_N(A)}{T_1} \right]$$



$$K_{pr} = \frac{D}{T_2} + \frac{D}{T_1}$$

$$D = 1 + \frac{3C_2}{\pi} \sqrt{\frac{3K_1}{C_2}}$$

$\exists a$   $K < K_{pr}$  nema vl. osc

$K > K_{pr}$  má vl. osc. (šírau več amplitudu)

Mikhailov kriterij

$$\frac{\partial X}{\partial A} \frac{\partial Y}{\partial w} - \frac{\partial Y}{\partial A} \frac{\partial X}{\partial w} > 0$$

$$\frac{\partial X}{\partial A} = -T_2 \frac{\partial G_N(A)}{\partial A} w^2 \quad \begin{cases} > 0, & A < A_{pr} \\ < 0, & A > A_{pr} \end{cases}$$

$$\frac{\partial Y}{\partial w} = G_N(A) H - 3T_1 T_2 w^2 \quad \left( w^2 = \frac{G_N(A) H}{T_1 T_2} \right) = -2 [G_N(A) H] < 0$$

$$\frac{\partial X}{\partial w} = -2(T_1 + T_2 + T_2 G_N(A)) w < 0$$

$$\frac{\partial Y}{\partial A} = \frac{\partial G_N(A)}{\partial A} w \quad \begin{cases} > 0, & A > A_{pr} \\ < 0, & A < A_{pr} \end{cases}$$

$$-T_2 \frac{\partial G_N(A)}{\partial A} w^2 \cdot (-2)[G_N(A)H] - (T_1 + T_2 + T_2 G_N(A))w \cdot \frac{\partial G_N(A)}{\partial A} w > 0$$

$A > A_{cr} \rightarrow$  stabilne osc.

$A < A_{cr} \rightarrow$  nestabilne osc.

lin. sust.  $C_1=0 \quad C_2=0 \quad G_m(A)=0$

$$T_1 T_2 s^3 + (T_1 + T_2) s^2 + s + k = 0$$

$$K_{cr} = \frac{T_1 + T_2}{T_1 T_2} < \frac{D}{T_2} + \frac{D^2}{T_1} = K_{cr}$$

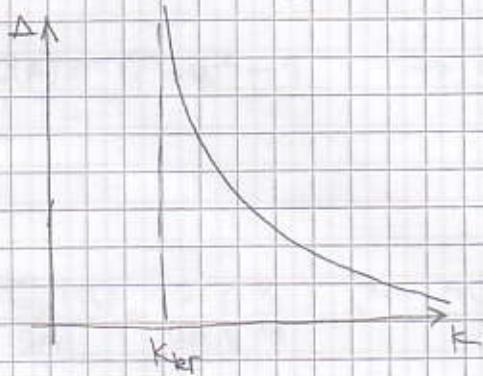
↳ preko Hurwitz

↳ povećanje područja stab

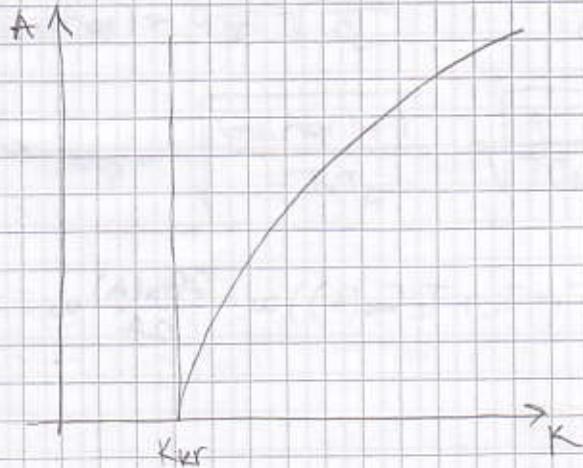
+  
podr. s nestab. vel. osc. koje dovođe

sust. u ravn. stavlja

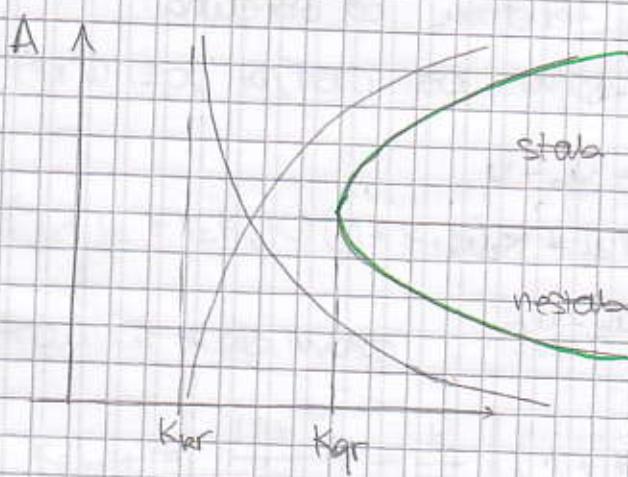
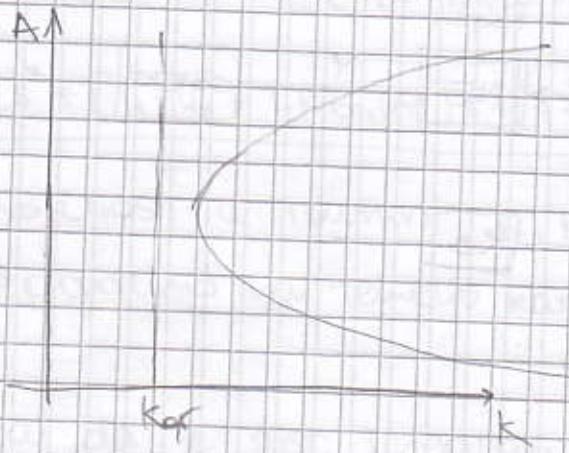
→ ako u sust. postoji samo suho trenje



→ ako postoji samo velik. mlež. karakter.

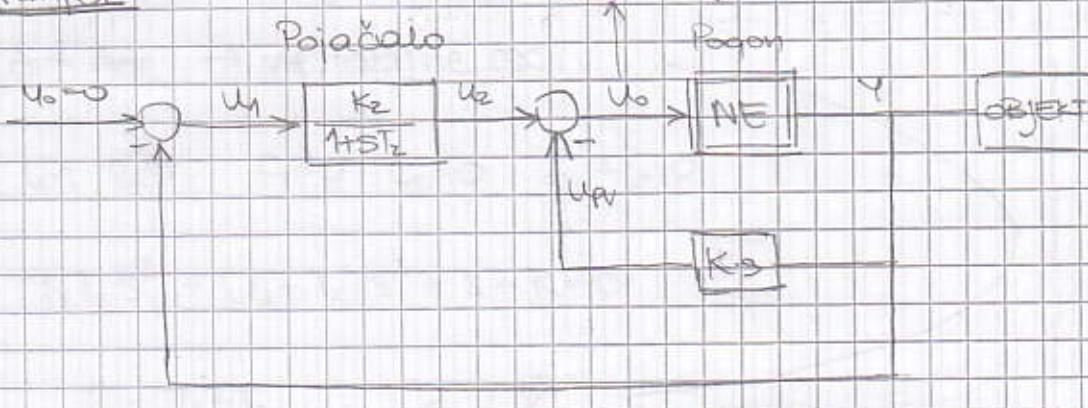


→ imano oboje:



Prijem

komparacija u (1) Asinwt



Komponente:

jedn. pojančalo:  $u_1 = u_0 - u_PV$

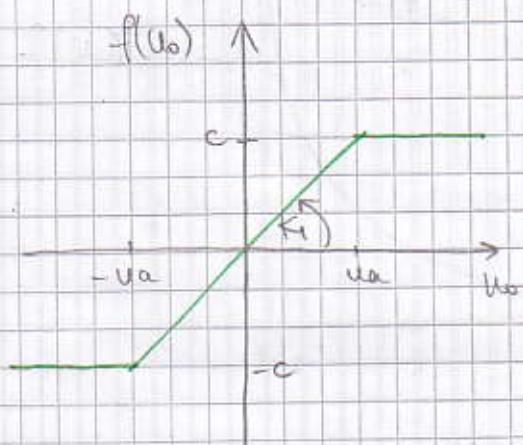
$$(T_2 s + 1) u_2 = K_2 u_1$$

jedn. komparatorda:  $u_3 = u_2 - u_PV$

$$(T_1 s + 1) u_4 = (T_1 s + 1) u_3 = K_3 u_2$$

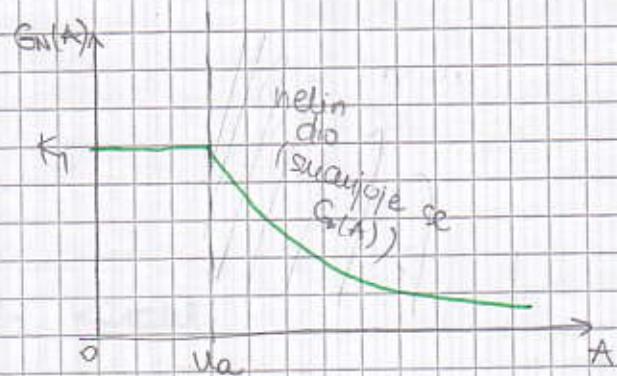
jedn. komparatorda:  $u_4 = u_2 - u_PV$

$$jedn. PV: u_PV = K_3 u_4$$



$$f(u_0) = G_N(A) \cdot u_0$$

$$G_N(A) = \frac{2K_1}{\pi} \left[ \arcsin \frac{u_0}{A} + \frac{u_0}{A} \sqrt{1 - \left( \frac{u_0}{A} \right)^2} \right] A$$



→ računanje karakteristike jedn. sustava

$$(T_1 s + 1) u_4 = G_N(A) [u_2 - K_3 u_4]$$

$$[(T_1 s + 1) s + s K G_N(A)] u_4 = G_N(A) \frac{-K_2}{T_2 s + 1} u_0$$

$$(T_1 s + 1)(T_2 s + 1) S + [(T_2 s + 1) K_S + K_2] G_N(A) = 0$$

$$\boxed{T_1 T_2 s^3 + [T_1 + T_2 + T_2 K G_N(A)] s^2 + [1 + K G_N(A)] s + K_2 G_N(A) = 0}$$

STABILNOST u ravnini  $K_1, K_2$

↳ provodimo na temelju karakteristične jed.

→ smatramo da je sust. linearan

$$G_N(A) = K_1$$

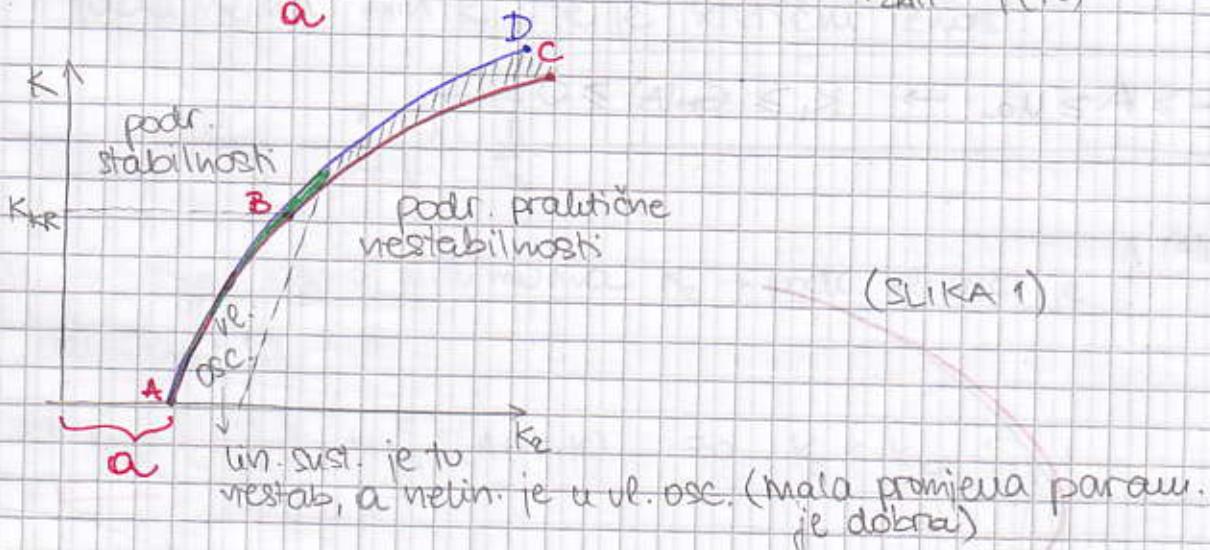
↳ Hurwitzov kriterij za uvjet stabilnosti

$$(T_1 + T_2 + T_2 K_1 K) (1 + K_1 K) - T_1 T_2 K_1 K_2 > 0$$

Granice stabilnosti

$$K_{\text{krit}} = \left[ \frac{T_1 + T_2}{T_1 T_2 K_1} + \frac{K}{T_1} \right] (1 + K_1 K) \rightarrow K_{\text{krit}} \text{ f-ja od } K$$

$K_{\text{krit}} = f(K)$



ispod linijica C i D je podr. preljeđ. nestab

→ BEZOPASNA GRANICA - oduzimajući podr. stab. uđemo u podr. nestab. uči ćemo u ul. osc. (za male promjene  $K_2$ )

→ PRAKT. NESTAB. U VELIKOM - suvijito se podr. stabilnosti  
(za veće promjene  $K_2$  dolazi do rušenja stabilnosti)

## Utjecaj nestabilnosti

↳ opet koristimo Hurwitza

$$(T_1 + T_2 + T_2 K G_N(A)) (1 + K G_N(A)) - T_1 T_2 K_2 G_N(A) > 0$$

$$K_{2qr} = \underbrace{\left[ \frac{T_1 + T_2}{T_1 T_2 G_N(A)} + \frac{K}{T_1} \right] [1 + K G_N(A)]}_a$$

→ zadovoljen  
za drugi dio  
velike pravole  
 $K_2$

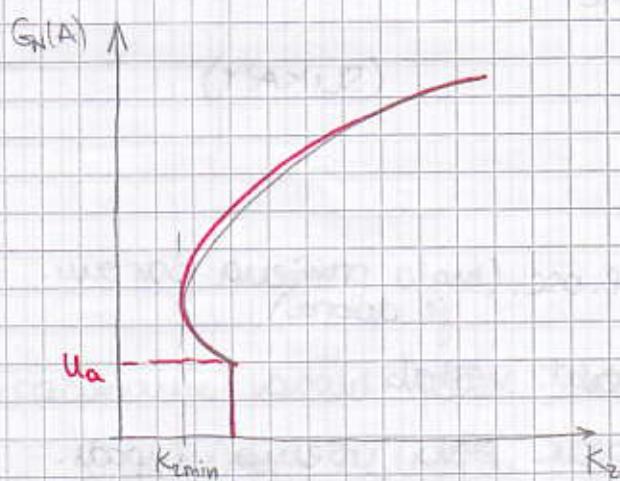
freku. ve. osc. dobivamo iz Re i Im dijela:

$$X(A, \omega) : K_2 G_N(A) + [T_1 + T_2 + T_2 K G_N(A)] \omega^2 = 0$$

$$Y(A, \omega) : 1 + K G_N(A) - T_1 T_2 \omega^2 = 0$$

$$\hookrightarrow \boxed{\omega^2 = \frac{1 + K G_N(A)}{T_1 T_2}}$$

$$\boxed{\infty \geq A \geq u_A \rightarrow K_1 \geq G_N(A) \geq 0}$$



$$K_2 = \infty \rightarrow G_N(A) \rightarrow 0$$

$$G_N(A)_{\max} = K_1$$

→ tražimo min od  $k_2$  pri kojem još postoji vl. osc.

$$\frac{dk_2}{dA} = 0$$

$$\frac{dk_2}{dA} = \underbrace{\frac{dk_2}{dG_N(A)}}_{=0} \cdot \underbrace{\frac{dG_N(A)}{dA}}_{\neq 0} = 0 \rightarrow \boxed{\frac{dk_2}{dG_N(A)} = 0}$$

$$k_{2\min} = k^* = \frac{1}{G_N(A)} \sqrt{1 + \frac{T_1}{T_2}} \quad 0 \leq G_N(A) \leq k_N$$

$$\infty \geq k \geq \frac{1}{k^*} \sqrt{1 + \frac{T_1}{T_2}}$$

Ako su param. sust. tako da uvedi da je

$$K < \frac{1}{k^*} \sqrt{1 + \frac{T_1}{T_2}}$$

tada neva min  $k_2$  te je kritični iznos:

$$k_{KR}^* = \frac{1}{k^*} \sqrt{1 + \frac{T_1}{T_2}}$$

Za  $K < k_{KR}^*$  nema minimuma  $k_2 \rightarrow$  podr. vl. osc. je određeno sa:

$$k_2 \geq \left( \frac{T_1 + T_2}{T_1 T_2 k} + \frac{k}{T_1} \right) (1 + k_1 k) \quad \text{za } K < k_{KR}^*$$

Za  $K > k_{KR}^*$  postoji min.  $k_2$

$$k_{2\min} = \frac{k}{T_1} \left( 1 + \sqrt{1 + \frac{T_1}{T_2}} \right)^2$$

(SLIKA 1)

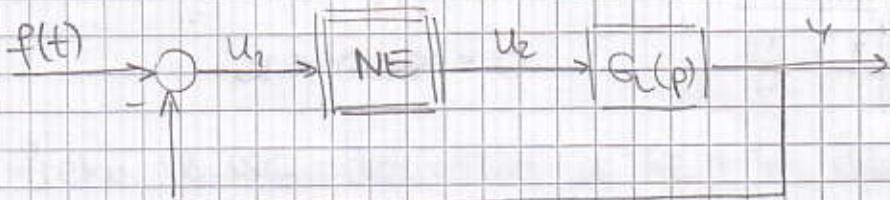
podr. vl. osc. je određeno sa:

$$k_2 \geq \frac{k}{T_1} \left( 1 + \sqrt{1 + \frac{T_1}{T_2}} \right)^2$$

Za  $K > k_{KR}^*$

→ ovaj sustav sa velikim K u pov. vezi ne bi se trebao qualificirati kao linearanu sustav

## Prinudne oscilacije



$$G(p) = \frac{R(p)}{C(p)}$$

↳ rešavamo homogeno diferenc. jed oblika

$$C(p)u_1 + R(p) \cdot f(u_1, pu_1) = 0$$

$$u_1 = A \sin \omega t$$

→ ako A i  $\omega$  nisu zadovoljavajući vrijednosti

signal  $f(t) = B \sin \omega t$  sa ampl. i frekv.

koje nam pao

→ da bi se uspostavile vl. osc.

$$\boxed{\omega_b < BW}$$

ne možemo maximizirati frekv. veću od propusne

→ sada imamo dif. jed. Zatvorimoq sustava

$$C(p)u_1 + R(p)f(u_1, pu_1) = C(p) \cdot f(t) \rightarrow \text{metom. dif. jed.} \quad (2)$$

$$u_1(t) = \boxed{A_b} \sin(\omega_b t + \boxed{\varphi})$$

$A_b$  i  $\varphi$  - nepoznate

Kod prirodnih osc. postoji 2 tipa zadatka

1) Odredi  $A_B$  i  $\varphi$

2) Odredi  $B$  i  $w_B$  s kojim treba posaviti sustav da bi se dobile ul. osc. s  $A_B$

$$f(t) = B \cdot \sin(w_B t) = B \cdot \sin[(w_B t + \varphi) - \varphi] = B \cdot \cos \varphi \sin(w_B t + \varphi) - \\ - B \sin \varphi \cos(w_B t + \varphi)$$

$\underbrace{pu_1(t)}_{A_B w_B}$        $\underbrace{\frac{u(t)}{A_B}}$

↳  $f(t)$  preinatimo tako da dobimo hom. dif. jed. iz vektorom (2)

$$f(t) = \frac{B}{A_B} \left[ \cos \varphi - \frac{\sin \varphi}{w_B} P \right] u(t) \rightarrow \text{writimo u (2)}$$

$$C(p)u_1(t) + R(p)f(u_1, pu_1) = C(p) \cdot \frac{B}{A_B} \left( \cos \varphi - \frac{\sin \varphi}{w_B} P \right) u_1(t)$$

$$\underbrace{\left[ C(p) - C(p) \frac{B}{A_B} \left( \cos \varphi - \frac{\sin \varphi}{w_B} P \right) \right]}_{\text{dodatni član}} u_1(t) + R(p)f(u_1, pu_1) = 0$$

vezovani prirodnim  
osc.

↳ Sada imamo homog. jed. te sve radimo isto kao i pre

FILTER HIPOTEZA :

$$f(u_1, pu_1) = P(A_B, w_B) u_1 + \frac{Q(A_B, w_B)}{w_B} pu_1$$

$$P(A_B, \omega_B) = \frac{1}{\pi A_B} \int_{-\pi}^{\pi} f(A_B \sin \varphi, A_B \omega_B \cos \varphi) \sin \varphi d\varphi$$

$$Q(A_B, \omega_B) = \frac{1}{\pi A_B} \int_0^{2\pi} f(A_B \sin \varphi, A_B \omega_B \cos \varphi) \cos \varphi d\varphi$$

$$\Psi = \omega_B t + \varphi$$

$$C(p) - C(p) \frac{B}{A_B} \left( \cos \varphi - \frac{\sin \varphi}{\omega_B p} \right) + R(p) \left[ P(A_B, \omega_B) + \frac{Q(A_B, \omega_B)}{\omega_B} p \right] = 0$$

↳ uvrštavamo  $p = j\omega_B$  jer pretpostavljamo harmon. rješenje

$$u_i(t) = A_B \sin(\omega_B t + \varphi)$$

$$C(j\omega_B) - C(j\omega_B) \frac{B}{A_B} \underbrace{\left( \cos \varphi - j \sin \varphi \right)}_{e^{-j\varphi}} + R(j\omega_B) \left[ P(A_B, \omega_B) + j Q(A_B, \omega_B) \right] = 0$$

$$A_B \underbrace{C(j\omega_B) + R(j\omega_B) \left[ P(A_B, \omega_B) + j Q(A_B, \omega_B) \right]}_{C(j\omega_B)} = \frac{-j\omega_B}{B e^{-j\varphi}}$$

↓ ali je posao  
kašnije u  
sistemu

Poznato:  $B$  i  $\omega_B$

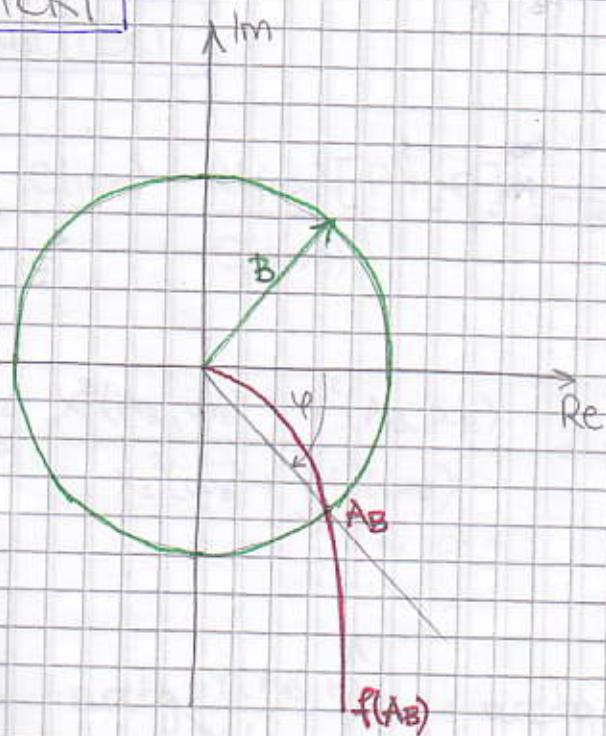
a) grafički

Traži se:  $A_B$  i  $\varphi$

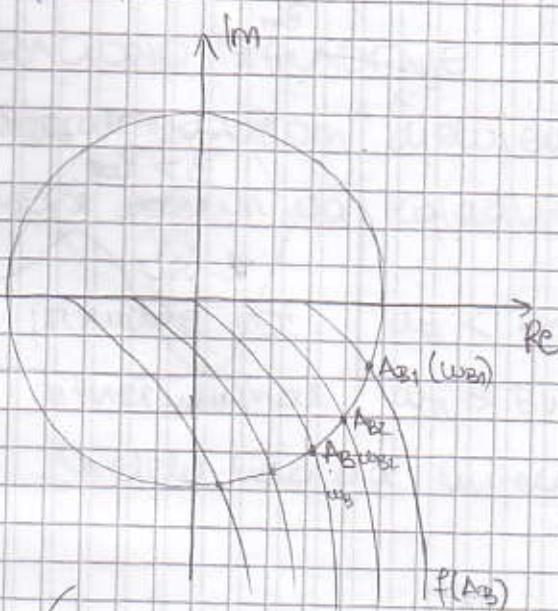
b) analitički

$$f(A_B) = B e^{-j\varphi}$$

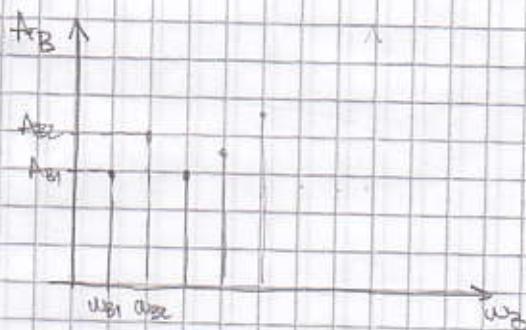
a) GRAFIČKI



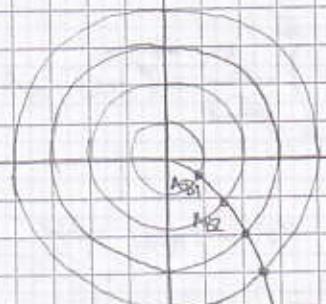
$$A_B = f(w_B)$$



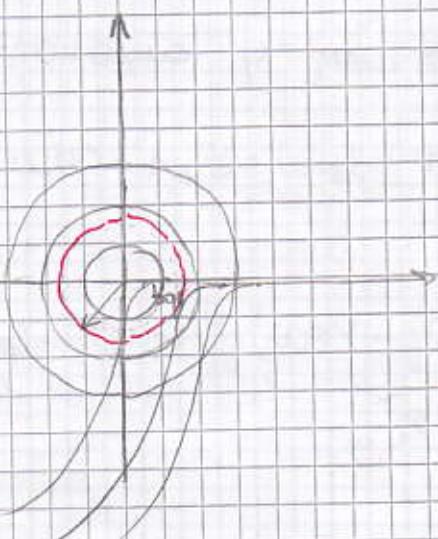
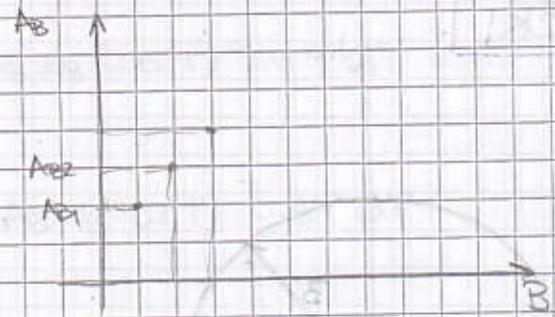
po tomu se čita  $A_B = f(w_B)$



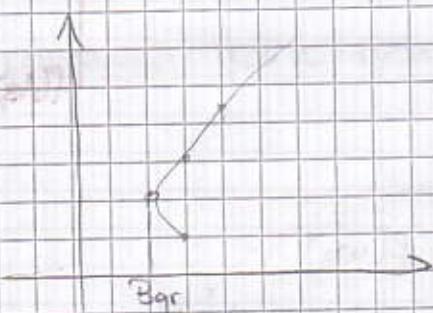
mijenjamo  $B$



$f(A_B)$



$f(A_B)$  uz raze  
 $R_B$



Podr. zaliwacjanya

$B > B_{gr}$

może się

usp.

vl. osc

$w_{B0}$

$w_B$

$B_{gr}$

Podr.  
zaliwacjanya

podr.  
rown.  
stawa

podr.  
vl. osc.

b) ANALITICKI

$$\frac{C(j\omega_B) + R(j\omega_B)[P+jQ]}{C(j\omega_B)} = Be^{-j\varphi}$$

$$A_B^2 \frac{X^2(A_B, \omega_B) + Y^2(A_B, \omega_B)}{X_c^2(\omega_B) + jY_c^2(\omega_B)} = B^2 \rightarrow \text{odredimo } A_B$$

$$\varphi = -\arctg \frac{Y(A_B, \omega_B)}{X(A_B, \omega_B)} + \arctg \frac{Y_c(\omega_B)}{X_c(\omega_B)} \rightarrow \text{odredimo } \varphi$$

→ VIBRACIONO IZGUDENJE

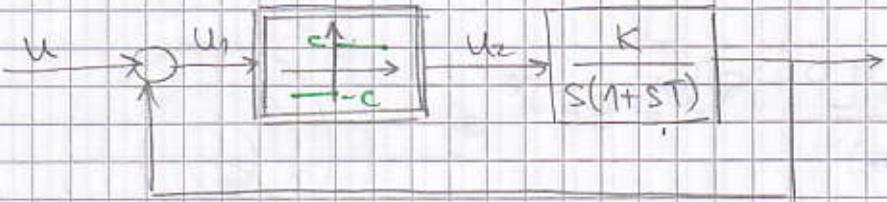
↳ harm. signalom djelujemo na sustav (na neki nacin ga izpladujemo)

a) primorne osc.  $\omega_B < BW$

b) dither signal  $\omega_B > BW$

↳ etelit nao da mijenjanju nelin. karakt. sustava

# Primer



$$u(t) = U_m \sin \omega_n t$$

$$K=10$$

$$U_m = 20 \text{ [V]}$$

$$T=0.1 \text{ [S]}$$

$$\omega_n = 10 \text{ [s}^{-1}]$$

$$\text{NE: } C=10 \text{ [F]}$$

$$u_i(t) = A \sin(\omega_n t + \varphi) ?$$

$$G_N(A) = P(A) = \frac{4C}{\pi A}$$

→ karakter. jed:

$$[s(1+st) + K G_N(A)] u_i(s) = u(s) [s(1+st)]$$

$$u = \frac{U_m}{A} (\cos \varphi - \frac{\sin \varphi}{\omega_n} s) u_i(t)$$

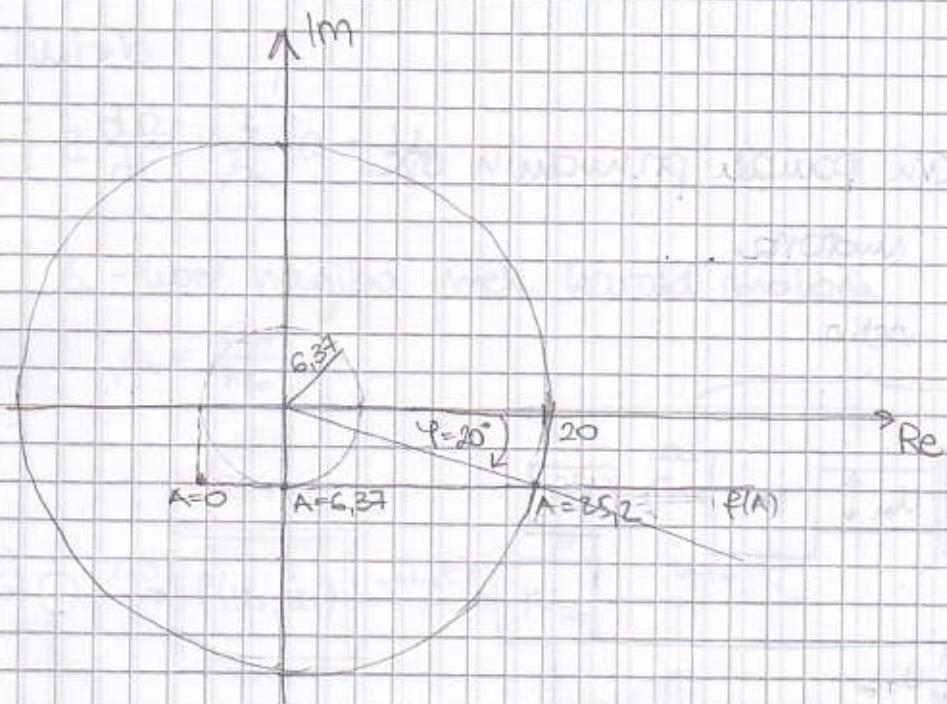
$$s=j\omega_n$$

$$j\omega_n (1+j\omega_n T) + K \frac{4C}{\pi A} = \frac{U_m}{A} (\cos \varphi - j \sin \varphi) (j\omega_n (1+j\omega_n T))$$

$$e^{-j\varphi}$$

$$f(A) = \frac{A \left[ K \frac{4C}{\pi A} - \omega_n^2 T + j\omega_n \right]}{j\omega_n (1+j\omega_n T)} = U_m e^{-j\varphi}$$

$$f(A) = A - 6,37 - j6,37 = 20 \cdot e^{-j\varphi}$$



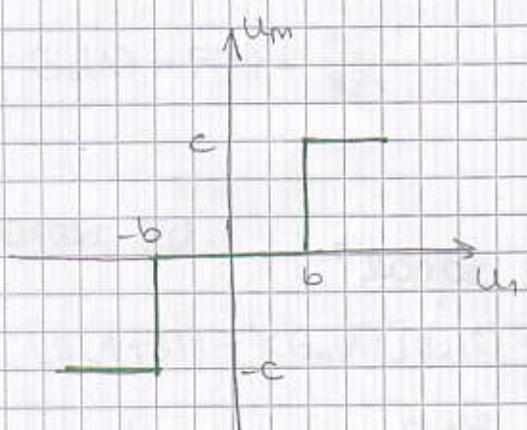
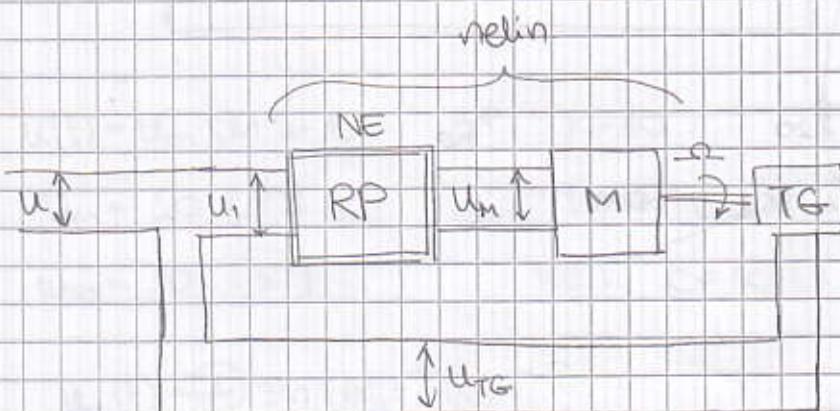
$$u_i(t) = 25,2 \sin(10t + 20^\circ)$$

Kolika je amplit. i frekv. vln. osc?

Što bā dobili da neuvamo primordni signal?

# Priimer

Odr. amp. i fazni pouzak prirodnih osc.  
i brzinu vrtnje motora



$$b = 1 \text{ [V]}$$

$$c = 100 \text{ [V]}$$

$$M_b = 100 \text{ [Nm]}$$

moment  
upuštanja  
motora

$I = 0.002 \text{ [Nm s}^2]$  mom. inercije  
proracunat  
na osnovi  
motora

$M_T = 0$   
teret

$$K_{TG} = 0,01 \text{ Vs / rad}$$

$$\omega_m = 628 \text{ s}^{-1}$$

→ napon na ulazu:

$$u(t) = U_m \sin \omega_n t$$

$$U_m = 8 \text{ V}$$

$$\omega_n = 10 \text{ s}^{-1}$$

kada je  $|u_i| \leq b$

rotor djeluje kao da je  
odspojen (rotor postaje  
zavrsnjak)

$$I \frac{d\alpha}{dt} = 0$$

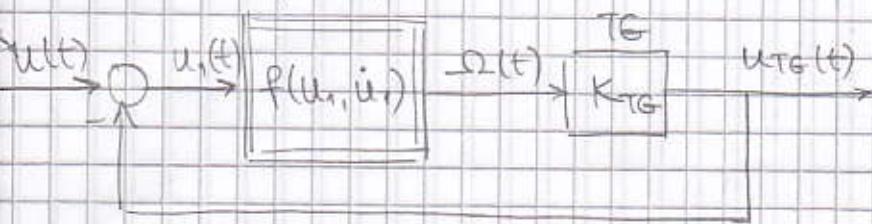
$$|u_n| > b$$

$$I \frac{d\Omega}{dt} + \frac{1}{\beta_0} \Omega = M_0$$

$\beta_0$  - broj nagiba meh. karakter. motora

$$\beta_0 = \frac{\rho_m}{M_0}$$

RPM



$u_r$  - javlja se zbog inercijskih svojstava u motoru, a sam  
NE ih nema

$$u_r(t) = u(t) - u_{TG}(t)$$

$$u_{TG}(t) = K_{TG} \cdot \Omega(t) = K_{TG} f(u_r, u_i)$$