Primjer: problem za D2

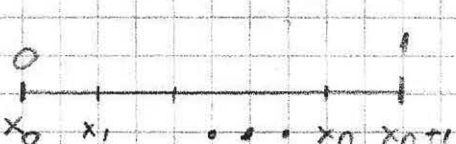
$$-\Delta u = f \quad \text{u} \quad \Omega \subset R^2$$
$$u = 0 \quad \text{na} \quad \partial\Omega$$

U 1-dim:

$$-u''(x) = f(x), \quad x \in (0,1)$$
$$u(0) = 0, \quad u(1) = 0$$

FreeFEM ++ (u 2 dim)

• uniformna mreža n $[0,1]$:



$$0 = x_0 < x_1 < x_2 < x_3 < \dots < x_n < x_{n+1} = 1$$

$$x_1 = h, \; x_2 = 2h, \; \dots, \; x_i = ih, \; \dots, \; x_n = nh, \; x_{n+1} = (n+1)h$$

$$h = \frac{1-0}{n+1} = \frac{1}{n+1}$$

• uvedemo oznake: $u_0 = u(x_0) = u(0)$

$u_i = u(x_i)$

$i = 0, \dots, n+1$

$u_{n+1} = u(x_{n+1}) = u(1)$

• iz rubnih uvjeta znamo: $u_0 = 0$, $u_{n+1} = 0$
• nepoznate vrijednosti: $u_1, u_2, \dots, u_n$
• aproksimirati u'' konačnim diferencijama

$$u'''(x)\Big|_{x=x_i} \approx \frac{u_{i+1} - 2u_i + u_{i-1}}{h^2}$$

$$f(x_i) = f_i$$

$$-\frac{u_{i+1}-2u_i+u_{i-1}}{h^2}=F(x_i) \quad / \cdot h^2$$

$$-u_{i+1}+2u_i-u_{i-1}=h^2F_i, \quad i=1,\dots,n$$

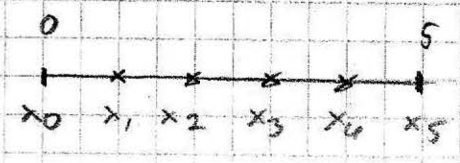
Npr. $n=4$, $h=\frac{1}{5}$

$i=1 \quad -u_2+2u_1-u_0=h^2F_1$

$i=2 \quad -u_3+2u_2-u_1=h^2F_2$

$i=3 \quad -u_4+2u_3-u_2=h^2F_3$

$i=4 \quad -u_5+2u_4-u_3=h^2F_4$



$u_0=0 \quad u_5=0$

Dobili smo linearni sustav:

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = h^2 \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

TRODIJAGONALNA MATRICA SUSTAVA!

Thomasov algoritam $O(n)$

$-u''=F$

+ RUBNI UVJET

rješavanje ODJ

DISKRETIZACIJA

$\rightsquigarrow$ lin. sustav!