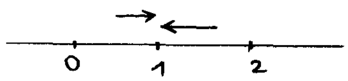
ZADATAK

(i) $\Delta = \{0,1,2\}$ $\qquad s(x) = \begin{cases} x^3+2x+1, & 0 \le x < 1 \\ x^2+2, & 1 \le x \le 2 \end{cases}$

Rj. s nije splajn jer $\lim\limits_{x\to 1^-} s(x) = 4$, $\lim\limits_{x\to 1+} s(x) = 3$



pa s nije neprekidna u $x=1$

(ii) $\Delta = \{0,1,2\}$ $\qquad s(x) = |x-1|^3, \quad x \in [0,2]$

Rj.

$$s(x) = |x-1|^3 = \begin{cases} (1-x)^3, & 0 \le x < 1 \\ (x-1)^3, & 1 \le x \le 2 \end{cases} \qquad s'(x) = \begin{cases} -3(1-x)^2, & 0 \le x < 1 \\ 3(x-1)^2, & 1 \le x \le 2 \end{cases}$$

$$s''(x) = \begin{cases} 6(1-x), & 0 \le x < 1 \\ 6(x-1), & 1 \le x \le 2 \end{cases}$$

- $s\big|_{[0,1\rangle}, \ s\big|_{[1,2]} \in \mathcal{P}_3$
- neprekidnost: $\lim\limits_{x\to 1^-} s(x) = 0$, $\lim\limits_{x\to 1+} s(x) = 0$ $\Rightarrow$ s neprek.
- neprek. 1. der.: $\lim\limits_{x\to 1^-} s'(x) = 0$, $\lim\limits_{x\to 1+} s'(x) = 0$ $\Rightarrow$ s' neprek. u $x=1$
- neprek. 2. der: $\lim\limits_{x\to 1^-} s''(x) = 0$, $\lim\limits_{x\to 1+} s''(x) = 0$ $\Rightarrow$ s'' neprek. u $x=1$

$\Rightarrow$ $s \in C^2[0,2]$ pa je $s \in S_{\Delta,3}$.

Je li $s \in S_{\Delta,4}$?

$s'''(x) = \begin{cases} -6, & 0 \le x < 1 \\ 6, & 1 \le x \le 2 \end{cases}$ nije neprekidna u $x=1$, pa $s \notin S_{\Delta,4}$.

(iii) $\Delta = \{0,1,2\}$ $\quad s(x) = \begin{cases} \frac{1}{2}x^2+x+1, & 0 \le x < 1 \\ -4.5x^2+11x-4, & 1 \le x \le 2 \end{cases}$

Rj. $s|_{[0,1\rangle} \in \mathcal{P}_2, \quad s|_{[1,2]} \in \mathcal{P}_2$

$\lim\limits_{x\to 1^-} s(x) = \frac{5}{2} = \lim\limits_{x\to 1^+} s(x)$ neprekidnost u $x=1$

- $s'(x) = \begin{cases} x+1, & 0 \le x < 1 \\ -9x+11, & 1 \le x \le 2 \end{cases}$

$\lim\limits_{x\to 1^+} s'(x) = 2 = \lim\limits_{x\to 1^-} s'(x) \Rightarrow s \in C^1[0,2]$

- $s''(x) = \begin{cases} 1, & 0 \le x < 1 \\ -9, & 1 \le x \le 2 \end{cases} \Rightarrow s''$ nije neprekidna u $x=1$

$s \in S_{\Delta,2}$ ali $s \notin S_{\Delta,3}$.

ZADATAK.

$$s(x)=\begin{cases}-x^3+3x^2-3x+1, & -1\le x<0\\ p(x) & 0\le x<1\\ x^3-3x^2+3x-1, & 1\le x\le 2\end{cases}$$

$\Delta=\{-1,0,1,2\}$

Postoji li polinom p td $s(x)\in S_{\Delta,3}$? Ako postoji, odredite ga!

Rj. $x_0=-1,\ x_1=0,\ x_2=1,\ x_3=2$

pretp. da traženi polinom p postoji.

$p\in\mathcal{P}_3,\quad p(x)=ax^3+bx^2+cx+d;\quad a,b,c,d\in\mathbb{R}$

s neprek. u $x=0 \iff \lim\limits_{x\to 0^-} s(x)=p(0) \implies d=1$

s neprek. u $x=1 \iff \lim\limits_{x\to 1^-} p(x)=s(1) \implies a+b+c+d=0$

s' neprek. u $x=0 \iff \lim\limits_{x\to 0^-} s'(x)=p'(0) \implies c=-3$

$$s'(x)=\begin{cases}-3x^2+6x-3, & -1\le x<0\\ p'(x) & ,\ 0\le x<1\\ 3x^2-6x+3, & 1\le x\le 2\end{cases}\qquad p'(x)=3ax^2+2bx+c$$

$$s''(x)=\begin{cases}-6x+6, & -1\le x<0\\ p''(x), & 0\le x<1\\ 6x-6, & 1\le x\le 2\end{cases}\qquad p''(x)=6ax+2b$$

s'' neprek. u $x=0 \iff \lim\limits_{x\to 0^-} s''(x) = P''(0) \implies 2b = 6$

$s''(x) = -6x + 6, \quad -1 \le x < 0$

$P''(x) = 6ax + 2b$

$$d = 1$$
$$a + b + c + 1 = 0 \implies a = -1$$
$$c = -3$$
$$b = 3$$

Dakle; s, s', s'' su neprekidne u $x=0$ i s je neprekidna u $x=1$ ako i samo ako vrijedi:

$$P(x) = -x^3 + 3x^2 - 3x + 1, \quad 0 \le x < 1 \qquad (*)$$

Tada je također : $s'(1) = 0, \ P'(1) = 0$

$s'(x) = 3x^2 - 6x + 3, \ 1 \le x \le 2 \qquad s''(x) = 6x - 6, \ 1 \le x \le 2$

$P'(x) = -3x^2 + 6x - 3, \ 0 \le x < 1 \qquad P''(x) = -6x + 6, \ 0 \le x < 1$

$$s''(1) = 0, \quad P''(1) = 0$$

Dakle, polinom $(*)$ čini s kubičnim splajnom.