

# AUDITORNE

- 9) Promatramo prijamnike sa dominirajućim toplotnim šumom, dom. kvantni šum i s opt. pojačalom. Odrediti osjetljivost prijamnika u sva i u slučaju ako promatramo srednju signalu brzinom  $10^9$  / s uz  $BER = 10^{-9}$  na valnoj dužini  $1.55 \mu m$ . Kvantna dpl.  $\eta = 80\%$  Faktor dodatnog šuma  $F_A = 3$ .  $n_{sp} = 1.5$  kod opt. pojačala, a kod PIN diode srednji broj fotona je  $10 \times$  veći nego kod dom. kvantnog šuma.

$$BER = 10^{-9} \Leftrightarrow Q = 6$$

$$\bar{P}_{pr} = \frac{1}{2} \pi h \nu \cdot \frac{1}{T} = \frac{1}{2} \frac{\pi \cdot c}{\eta \cdot \lambda} h \cdot B_0$$

$$Q = \frac{I_1 - I_0}{\sigma_k + \sigma_r}$$

a) APD - kvantni šum

$$\frac{S}{N} = \frac{I^2}{\sigma_k^2 + \sigma_r^2} = \frac{I^2}{\sigma_k^2}$$

$$Q = \frac{I_1 - I_0}{\sigma_k + \sigma_r} \approx \frac{I_1}{\sigma_k} \quad \text{→ zanemariv signal u nuli}$$

$$\left( \frac{S}{N} = Q^2 \right)$$

$$\frac{S}{N} = \frac{\bar{m}}{F_A} \Rightarrow \bar{m} = \left( \frac{S}{N} \right) \cdot F_A = Q^2 F_A$$

$$\begin{aligned} \bar{P}_{pr} &= \frac{1}{2} \frac{ch}{\eta \lambda} \cdot Q^2 \cdot F_A \cdot B_0 = 86.57 \text{ nW} \\ &= -40.6 \text{ dBm} \\ &(-46.8 ??) \end{aligned}$$

b) PIN - termični šum

$$\left. \begin{aligned} \frac{S}{N} &= \frac{I^2}{\sigma_k^2 + \sigma_T^2} = \frac{I^2}{\sigma_T^2} \\ Q &= \frac{I_1 + I_0}{\sigma_k + \sigma_0} = \frac{I_1}{2\sigma_T} \end{aligned} \right\} \frac{S}{N} = 4Q^2$$

$$\left( \frac{S}{N} \right)_{\text{Kvantni}} = Q^2 = \bar{m}_{\text{Kvantni}}$$

$$\bar{m}_{\text{PIN}} = 10 \cdot \bar{m}_{\text{Kvantni}} = 10 \cdot Q^2 = 10 \cdot 6^2 = 360$$

$$\bar{P}_{\text{pr}} = \frac{1}{2} \frac{\pi c}{\eta \lambda} h \cdot B_0 = 289 \text{ mW} = -35.4 \text{ dBm}$$

APD je občutljivi, treba manjša snaga da radujemo s istim BER-om nego kod PIN

c) PIN dioda + prepojačalo

- dominantan kvantni šum

$$\frac{S}{N} = Q^2$$

$$\frac{S}{N} = \frac{G P_{\text{ue}}}{4 \cdot S_{\text{sp}} \Delta f} \cdot \eta$$

$$S_{\text{sp}} = (G - 1) n_{\text{sp}} h \nu$$

↳ spektralna gostota

$$\frac{S}{N} = \frac{G P_{\text{ue}}}{4 (G - 1) n_{\text{sp}} h \nu \Delta f} \cdot \eta = \frac{P_{\text{ue}}}{4 n_{\text{sp}} h \nu \Delta f} \cdot \eta = 2 \cdot \bar{P}_{\text{pr}} \rightarrow$$

$$\left[ \frac{G}{G - 1} \approx 1 \text{ (veliko pojačanje)} \right]$$

$$= \frac{2 \bar{P}_{\text{pr}} \eta}{4 n_{\text{sp}} h \nu \Delta f} = \frac{S}{N} = Q^2$$

$$\bar{P}_{\text{pr}} = \frac{1}{\eta} Q^2 n_{\text{sp}} h \nu B_0 = -40.63 \text{ dBm}$$

se sastoji od opt. predpoj. i PIN diode.  
 prenosa  $\lambda = 2.5 \text{ Gb/s}$   $\lambda = 1.55 \mu\text{m}$ ,  $n_{sp} = 2$   
 opt. pojačale a kvantna djel. PIN diode je 85%  
 Ako je BER  $10^{-12}$  odredi osjetljivost prijemnika.  
 Ako je srednja snaga odašiljača  $1 \text{ mW}$  odredi min.  
 maks. duljinu trase, s tim da su gubici  $\alpha = 0.2 \text{ dB/km}$   
 ako je opterećenje prijemnika na  $0.1 \text{ mW}$

$$\left(\frac{S}{N}\right) = Q^2$$

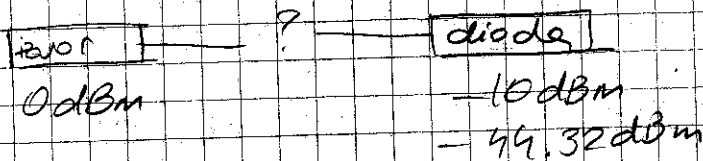
$$\left(\frac{S}{N}\right) = \frac{G P_{ul}}{4 S_{sp} \Delta f} \eta = \frac{G P_{ul} \eta}{4 (G-1) n_{sp} h \nu \Delta f} = \dots \Rightarrow$$

idealni prijemnik

$$P_{01} = -44.32 \text{ dBm}$$

$$P_{prmax} = 0.1 \text{ mW} = -10 \text{ dBm}$$

$$P_{odas} = 1 \text{ mW} = 0 \text{ dBm}$$



$$L_{min} = \frac{(P_{od} - P_{prmax})}{\alpha} = \frac{0 - (-10)}{0.2} = 50 \text{ km}$$

$$L_{max} = \frac{(P_{od} - P_{prmin})}{\alpha} = \frac{0 - (-44.32)}{0.2} = 221.6 \text{ km}$$

# ZADACI

$$\frac{\Delta T}{L} = \left( \frac{n_1}{n_2} \right)^2 \frac{\Delta n}{c} \Rightarrow \frac{\Delta T}{L} = \frac{\Delta n}{c} \Rightarrow \Delta n = \frac{c \Delta T}{L}$$

$n_1 \approx n_2$        $\Delta n$        $\Delta T$

11.  $\frac{\Delta T}{L} = 10 \text{ ns/km}$

$n_1 = 1.46$   
 $L = 10 \text{ km}$

$$\Delta n = 3 \cdot 10^8 \frac{\text{m}}{\text{s}} \cdot 10 \cdot \frac{10^{-9}}{10^3} = 3 \cdot 10^{-3}$$

$$NA = \sqrt{2n \Delta n} = 0.0936$$

$$B_0 L \leq \frac{1}{2} \frac{n_2}{n_1} \frac{c}{\Delta n} \approx \frac{1}{2} \frac{c}{\Delta n} = 5 \cdot 10^{10} \text{ m/s}$$

$$B_0 \leq \frac{5 \cdot 10^{10}}{10^4} = 5 \text{ Mb/s}$$

12.

$$\Delta n = n_1 - n_2$$

$$\alpha_m > 7^\circ$$

$$B_0 L \geq 20 \frac{\text{Mb}}{\text{s}} \text{ km}$$

$$n_1 = 1.46$$

$$NA^{\text{del}} = \sin \alpha_m = 0.12187$$

$$NA = \sqrt{2n \Delta n} > 0.12187$$

$$\Rightarrow \Delta n > \frac{\sin^2 \alpha_m}{2n_1} = 0.005086$$

$$B_0 L = \frac{c}{2\Delta n} > 20 \text{ Mb/s km}$$

$$\Delta n < \frac{c}{2B_0 L} = 0.0075$$

$$\Delta n \in \langle 0.0051, 0.0075 \rangle$$

$$\alpha_m \in \langle 7^\circ, 9.51^\circ \rangle$$

$$B_0 L_{\text{max}} \in \langle 20 \frac{\text{Mb}}{\text{s}} \text{ km}, 29.5 \frac{\text{Mb}}{\text{s}} \text{ km} \rangle$$

(13)

$$2a = 50 \mu\text{m}$$

$$\Delta = \frac{\Delta n}{n} = 0.01$$

$$n_1 = 1.46$$

$$\lambda_0 = 1.3 \mu\text{m}$$

$$V = \frac{2\pi}{\lambda} a \sqrt{n_1^2 - n_2^2} = \frac{2\pi}{\lambda} a \sqrt{\underbrace{(n_1 - n_2)}_{\Delta n} \underbrace{(n_1 + n_2)}_{2n_1}} = 24.35$$

$$\Delta n = 0.01 n$$

$$M = \frac{\alpha}{\alpha + 2} \cdot \frac{V^2}{2} = (\alpha \approx \infty) \approx \frac{V^2}{2}$$

$$M \approx \frac{V^2}{2} = 31 \text{ modova}$$

$$V < 2.405$$

$$\frac{2\pi}{\lambda} a \sqrt{n_1^2 - n_2^2} < 2.405$$

(14)

$$n_1 = 3.6$$

$$n_2 = 3.55$$

$$\lambda = 1.3 \mu\text{m}$$

$$\theta = 85^\circ$$

$m=0$  (osnovni, dominantni mod)

$$\text{tg} \left( \frac{\pi n_1 d \cos \theta}{\lambda} - \frac{\pi}{2} \right) = \frac{\sqrt{n_1^2 \sin^2 \theta - n_2^2}}{n_1 \cos \theta}$$

$$M = 1 + \text{int} \left\{ 2 \frac{d}{\lambda} \sqrt{n_1^2 - n_2^2} \right\}$$

$$\Rightarrow \frac{d}{\lambda} = \frac{1}{\pi n_1 \cos \theta} \arctg \left( \frac{\sqrt{n_1^2 \sin^2 \theta - n_2^2}}{n_1 \cos \theta} \right) =$$

$$= \frac{1}{\pi \cdot 3.6 \cos 85^\circ} \arctg \left( \frac{\sqrt{3.6^2 \sin^2 85^\circ - 3.55^2}}{3.6 \cos 85^\circ} \right) =$$

$\mu \text{ radijani} / \mu\text{m}$

$$\frac{d}{\lambda} = 1.033$$

$$M = 1 + \text{int} \{ 1.235 \} = 2 \rightarrow \text{šine se 2 moda}$$

увет радиомодности:

$$2 \frac{d}{\lambda} \sqrt{n_1^2 - n_2^2} < 1 \Rightarrow d < 1.087 \mu\text{m}$$

15.

$$n_1 = 3.6$$

$$n_2 = 3.5$$

$$\lambda = 1.55 \mu\text{m} \Rightarrow \text{TE}_0 \text{ (1 moda)}$$

$$\lambda = 1.3 \mu\text{m} \Rightarrow \text{TE}_0, \text{TE}_1 \text{ (2 moda)}$$

a)  $\lambda_1 = 1.55 \mu\text{m}$

$$2 \frac{d}{\lambda_1} \sqrt{n_1^2 - n_2^2} < 1 \Rightarrow d < 1.296 \mu\text{m}$$

b)  $\lambda_2 = 1.3 \mu\text{m}$

$$1 < 2 \frac{d}{\lambda_2} \sqrt{n_1^2 - n_2^2} < 2 \Rightarrow 1.087 \mu\text{m} < d < 2.174 \mu\text{m}$$

$$\Rightarrow 1.087 \mu\text{m} > d > 1.296 \mu\text{m}$$

16.

$$n_1 = 1.48$$

$$n_2 = 1.46$$

$$\lambda = 1.55 \mu\text{m}$$

$$M = 4 \text{ (TE) moda}$$

$$\theta_3 \approx \theta_c$$

$$\theta_c = \arcsin \frac{n_1}{n_2} = 80.57^\circ$$

$$\text{tg} \left( \frac{\pi n_1 d \cos \theta_c}{\lambda} - \frac{\pi}{2} \right) = \frac{\sqrt{n_1^2 \sin^2 \theta_c - n_2^2}}{n_1 \cos \theta_c}$$

$$\Rightarrow d = \left\{ \arctg \left[ \frac{\sqrt{n_1^2 \sin^2 \theta_c - n_2^2}}{n_1 \cos \theta_c} \right] + \frac{3\pi}{2} \right\} \cdot \frac{\lambda}{\pi n_1 \cos \theta_c}$$

$$\text{по } \sin \theta_c = \frac{n_2}{n_1}$$

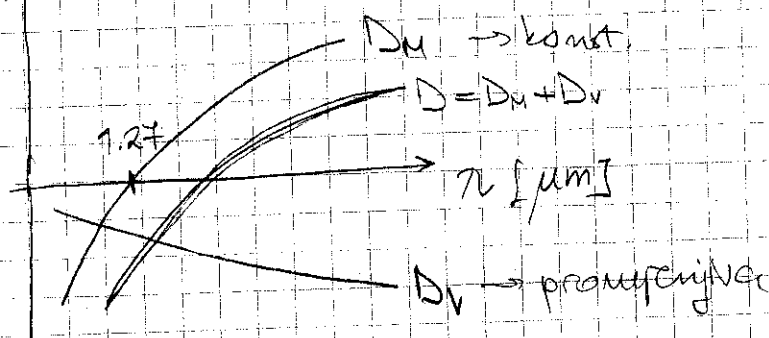
$$d = \frac{3\pi}{2} \cdot \frac{\lambda}{\pi n_1 \cos \theta_c} = 0.59 \mu\text{m}$$

(17)

$$D = D_M + D_V \rightarrow \text{valovodna} \\ \text{materiálna}$$

$$D \left[ \frac{\text{ps}}{\text{nm} \cdot \text{km}} \right]$$

$$D = \frac{\Delta T}{\Delta \lambda L} \left[ \frac{\text{ps}}{\text{nm} \cdot \text{km}} \right]$$



$$D_M \approx 122 \left( 1 - \frac{\lambda_{\text{ND}}}{\lambda} \right)$$

$$D_V = - \frac{\Delta n}{c \lambda} V \frac{d^2(Vb)}{dV^2}$$

$D_V$  (iz grafa na služb. šal.)

$$V=2 \Rightarrow \Rightarrow V \frac{d^2(Vb)}{dV^2} = 0.5$$

$$V=1.5 \Rightarrow \Rightarrow V \frac{d^2(Vb)}{dV^2} = 1.1$$

množa  
↑ disperzia

$$D_M = 122 \left( 1 - \frac{\lambda_{\text{ND}}}{\lambda} \right) = 122 \left( 1 - \frac{1.27}{1.55} \right) = 22 \frac{\text{ps}}{\text{nm} \cdot \text{km}}$$

$$V=1.5 \Rightarrow D=0 \Rightarrow D_V = -22 \frac{\text{ps}}{\text{nm} \cdot \text{km}}$$

$$2a = ?$$

$$D_V = - \frac{\Delta n}{c \lambda} V \frac{d^2(Vb)}{dV^2} \Rightarrow \Delta n = -D_V \frac{c \lambda}{V \frac{d^2(Vb)}{dV^2}} = 0.0033$$

$$V = \frac{2\pi}{\lambda} a \sqrt{n_1^2 - n_2^2} = \frac{2\pi}{\lambda} a \sqrt{(n_1 - n_2)(n_1 + n_2)} \Rightarrow$$

$$V \approx \frac{2\pi}{\lambda} a \sqrt{\Delta n \cdot 2n_1}$$

$$2a = \frac{V \lambda}{\pi \sqrt{\Delta n \cdot 2n_1}} = 4.5 \mu\text{m}$$

$$\begin{aligned}
 2a &= 9 \mu\text{m} \\
 n_1 &= 1.46 \\
 n_2 &= 1.458 \\
 \lambda_{\text{nd}} &= 1.27 \mu\text{m} \\
 L &= 50 \text{ km} \\
 \Delta\lambda &= 5 \text{ nm} \\
 \lambda &= 1.55 \mu\text{m}
 \end{aligned}$$

$$D = \frac{\Delta T}{\Delta\lambda \cdot L} \Rightarrow \Delta T = |D| \cdot \Delta\lambda \cdot L = (D_H + D_V) \cdot \Delta\lambda \cdot L$$

$$D_H = 122 \left(1 - \frac{\lambda_{\text{nd}}}{\lambda}\right) = 122 \left(1 - \frac{1.27}{1.55}\right) = 22.04 \text{ ps/nm.km}$$

$$D_V = - \frac{\Delta n}{c\lambda} V \frac{d^2(Vb)}{dv^2}$$

$$V = \frac{2\pi}{\lambda} a \sqrt{n_1^2 - n_2^2} = \frac{9\pi}{1.55} \sqrt{1.46^2 - 1.458^2} = 2.03$$

$$\Rightarrow V \frac{d^2(Vb)}{dv^2} = 0.5 \quad (\text{odčitamo iz grafa})$$

$$D_V = - \frac{0.002}{3 \cdot 10^8 \cdot 1.55 \cdot 10^{-6}} \cdot 0.5 = -2.15 \cdot 10^{-6} = -2.15 \frac{\text{ps}}{\text{nm.km}}$$

$$\frac{\text{ps}}{\text{nm.km}} = \frac{10^{-12} \text{ s}}{10^{-9} \text{ m} \cdot 10^3 \text{ m}} = 10^{-6} \frac{\text{s}}{\text{m}^2}$$

$$D = D_H + D_V = 22.04 - 2.15 = 19.89 \text{ ps/nm.km}$$

$$\begin{aligned}
 \Delta T &= |D| \cdot \Delta\lambda \cdot L = 19.89 \left[ \frac{\text{ps}}{\text{nm.km}} \right] \cdot 5 [\text{nm}] \cdot 50 [\text{km}] = \\
 &= 4972.5 \text{ ps} = 4.97 \text{ ns}
 \end{aligned}$$

$$B_0 \leq \frac{1}{2\Delta T} = \frac{1}{2 \cdot 4.97 \cdot 10^{-9}} \approx 100 \text{ Mbit/s}$$

→ zbog  $\Delta\lambda$  je maks. brzina tako mala



19

DCF svetlovod

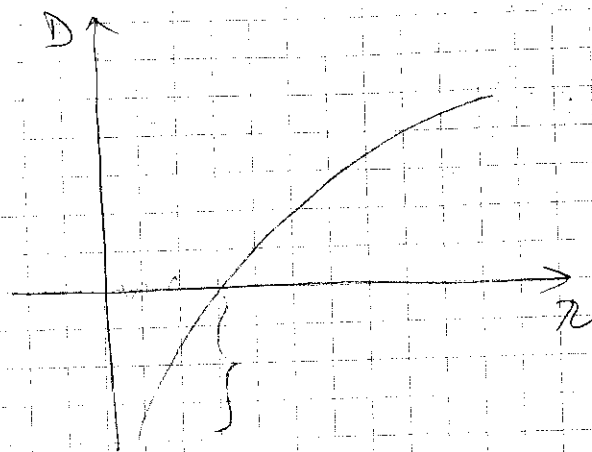
$$L = 30 \text{ km}$$

$$D = 18 \text{ ps/nm.km}$$

$$\alpha = 0.2 \text{ dB/km}$$

$$D_{DCF} = -104 \text{ ps/nm.km}$$

$$\alpha_{DCF} = 0.5 \text{ dB/km}$$



$$DCF \approx -100 \text{ ps/nm.km}$$

$$\Delta T = \Delta \lambda \cdot |D| \cdot L$$

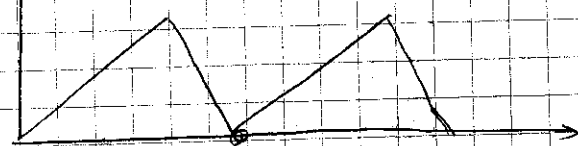
Ako D varira duž trase:

$$\Delta T = \Delta \lambda \int D(z) dz$$

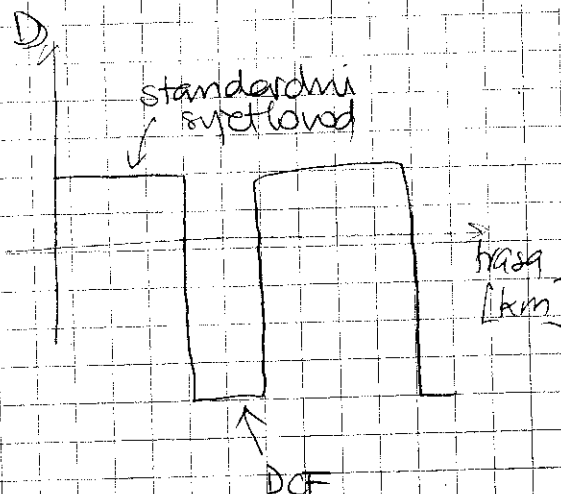
trasa

akumulirana  
dispersija  $\frac{\Delta T}{\Delta \lambda}$

Akumulirana  
D



$D_{akum} = 0$



$$D_{akum} = \int D(z) dz = \int_0^L D dz + \int_L^{L+L_{DCF}} D_{DCF} dz = D \cdot L + D_{DCF} \cdot L_{DCF} = 0$$

$$L_{DCF} = - \frac{D \cdot L}{D_{DCF}} = - \frac{18 \cdot 30}{-104} = 13.85 \text{ km}$$

Gušenje prije DCF sv:

$$g = \alpha \cdot L = 0.2 \cdot 30 = 6 \text{ dB}$$

Gušenje poslije DCF sv:

$$g = \alpha_{DCF} \cdot L_{DCF} + \alpha \cdot L = 0.5 \cdot 13.85 + 0.2 \cdot 30 = 22.9 \text{ dB}$$

$$\lambda = 1.55 \mu\text{m}$$

$$L = 100 \text{ km}$$

$$D = 17 \text{ ps/nm.km}$$

$$\alpha = 0.2 \text{ dB/km}$$

$$\frac{\partial D}{\partial \lambda} = 0.056 \text{ ps/nm}^2 \text{ km}$$

$$D_{DCF} = -104 \text{ ps/nm.km}$$

$$\alpha_{DCF} = 0.5 \text{ dB/km}$$

$$\frac{\partial D_{DCF}}{\partial \lambda} = -0.35 \text{ ps/nm}^2 \text{ km}$$

$$\lambda_1 = 1.5 \mu\text{m}, \lambda_2 = 1.6 \mu\text{m}$$

$$D(\lambda) = D(\lambda_0) + \left. \frac{\partial D}{\partial \lambda} \right|_{\lambda=1.55} (\lambda - \lambda_0)$$

$$1. \text{ korak : } L_{DCF} = - \frac{D \cdot L}{D_{DCF}} = - \frac{17 \cdot 100}{-104} = 16.346 \text{ km}$$

2. korak :

$$\begin{aligned} a) D_{\text{akum}}|_{\lambda_0=1.5 \mu\text{m}} &= D(\lambda) \cdot L + D_{DCF}(\lambda) L_{DCF} = \\ &= \left( D(\lambda_0) + \left. \frac{\partial D}{\partial \lambda} \right|_{\lambda_0} \right) L \cdot \Delta\lambda + \left( D_{DCF}(\lambda_0) + \left. \frac{\partial D_{DCF}}{\partial \lambda} \right|_{\lambda_0} \right) L_{DCF} \cdot \Delta\lambda \end{aligned}$$

$$\oplus = 0$$

$$D_{\text{akum}}|_{\lambda_0} = \left. \frac{\partial D}{\partial \lambda} \right|_{\lambda_0} L \cdot \Delta\lambda + \left. \frac{\partial D_{DCF}}{\partial \lambda} \right|_{\lambda_0} L_{DCF} \cdot \Delta\lambda$$

$$= 0.056 \cdot (1500 - 1550) \cdot 100 - 0.35 \cdot (1500 - 1550) \cdot 16.346$$

$$= -280 + 286 = 6 \frac{\text{ps}}{\text{nm}}$$

$$D_{\text{akum}}(\lambda = 1.5 \mu\text{m}) = 6 \frac{\text{ps}}{\text{nm}}$$

$$b) \lambda_0 = 1.6 \mu\text{m}$$

$$D_{\text{akum}}|_{\lambda_0} = \left. \frac{\partial D}{\partial \lambda} \right|_{\lambda_0} L \cdot \Delta\lambda + \left. \frac{\partial D_{DCF}}{\partial \lambda} \right|_{\lambda_0} L_{DCF} \cdot \Delta\lambda = -6 \frac{\text{ps}}{\text{nm}}$$

Ukupno gubici :

$$\alpha \cdot L + \alpha_{DCF} \cdot L_{DCF} = 20 + 8.173 = 28.173 \text{ dB}$$