2. CIKLUS

Interakcija težkih nabijenih čestica s tvori:

Specifična ionizacija - gubitek energije po enoti poti:

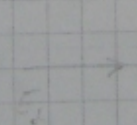
Poluklasičen izvod zakonitosti za spec. ionizacijo:

...

(33.) $^{27}_{13}Al$, α

$1 = b = 1\,fm = 10^{-15}\,m$

$E_\alpha = ?$



$r = R_0 \sqrt[3]{A}$

$R' = r + b =$

$R' = R_0 \sqrt[3]{A} + b$

$R' = 1{,}2 \sqrt[3]{27} + 1$

$R' = 4{,}6\,fm$

$E_{uk,\infty} = E_{uk,R'}$

$E_{kin,\infty} + E_{pot,\infty} = E_{kin,R'} + \frac{Z \cdot z\, e^2}{4\pi\varepsilon_0 \cdot R'}$

$E_{kin,\infty} = E_\alpha$, $E_{pot,\infty} = 0$, $E_{kin,R'} = 0$

$E_\alpha = \frac{Z_{Ni}\, z_\alpha\, e^2}{4\pi\varepsilon_0 \cdot R'} = \frac{13 \cdot 2 \cdot e^2}{4\pi\varepsilon_0 \cdot 4{,}6\,fm}$

$= 1{,}44\,MeV\,fm$

$\boxed{E_\alpha = 8{,}1\,MeV}$

(34.)

$E_p = 10\,MeV$

$b = 10^{-11}\,m = 10^4\,fm$ $\quad p_e = \frac{z \cdot e^2}{2\pi \varepsilon_0 \cdot b \cdot v_p}$

$E_e = ?$

$$E_e = \frac{p_e^2}{2m} = \frac{z_p^2 e^4}{4\pi^2 \cdot \varepsilon_0^2 \cdot b^2 \cdot v_p^2 \cdot 2m} \quad \Big| \cdot 4c^2 \;\; \Big| \cdot m_p \quad z_p = 1$$

$$E_e = \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \cdot \frac{m_p}{b^2 E_p m} \quad \Big| \cdot c^2$$

$(1{,}44^2 (MeV fm)^2)$

$$E_e = 1{,}44^2\, MeV^2 fm^2 \frac{934\,MeV}{10^8 fm^2\; 10\,MeV \cdot 0{,}511\,MeV}$$

$\boxed{E_e = 3{,}8\,eV}$

(40.)

$E_d = 10\,MeV$ $\quad {}^{27}_{13}Al$ $\quad \rho_{Al} = 2{,}7\; g/cm^3$

$\rho\Delta x = 2{,}7\,mg/cm^2$ $\quad \rho\Delta x = 2{,}7 \cdot 10^{-3}\; g/cm^2$

$I_d = 16\,\mu A$ $\quad \Delta x = \frac{2{,}7 \cdot 10^{-3}}{2{,}7} = 10^{-3}\,cm$

$P = ?$

$$-\frac{\Delta E}{\Delta x} = \frac{e^4}{8\pi\varepsilon_0^2 m_e} \cdot \frac{M z^2}{E_d} \cdot \eta \cdot \ln \frac{2E m_e}{M \hbar\nu}$$

$$(-\Delta E) = 2\pi \frac{e^4}{16\pi\varepsilon_0^2 m_e^2 c^4} \cdot \frac{m_e c^2 M c^2}{E_d} \cdot \eta \ln \frac{2E_d m_e c^2 \cdot 2\pi}{M \cdot c^2 (h\nu)} \cdot \Delta x$$

$= r_0^2$

$(h\nu)_{Al} = 150\,eV$

$$(-\Delta E) = 2\pi\, r_0^2 \cdot \frac{m_e c^2 \cdot M c^2}{E_d} \cdot \eta \cdot \ln \frac{4\pi E_d m_e c^2}{M c^2 (h\nu)_{Al}}$$

$$\eta = \frac{\rho_{Al}}{M_{Al}} \cdot N_A \cdot Z_{Al} = \frac{2{,}7}{27} \cdot 6{,}02 \cdot 10^{23} \cdot 13 = 7{,}8 \cdot 10^{23}\, cm^{-3}$$

$$(-\Delta E) = 2\pi (2{,}83 \cdot 10^{-13})^2 \cdot \frac{0{,}511\,MeV \cdot 2 \cdot 938\,MeV}{10\,MeV} \cdot 7{,}8 \cdot 10^{23} \cdot$$

$$\cdot \ln \left(\frac{4\pi \cdot 10 \cdot 10^6\,eV \cdot 0{,}511\,MeV}{2 \cdot 938\,MeV \cdot 150\,eV} \right) \cdot 10^{-3}$$

$\boxed{(-\Delta E) = 204\,keV}$

$P = \dot N_d \cdot \Delta E$ $\quad \dot N_d = \frac{I_d}{e}$

$$P = \frac{I_d}{e} \cdot \Delta E = \frac{16 \cdot 10^{-6} \frac{C}{s}}{1{,}6 \cdot 10^{-19}\, C} \cdot 204 \cdot 10^3 \cdot 1{,}6 \cdot 10^{-19}\, J$$

(kulon)

$P = 16 \cdot 10^{-3} \cdot 204\; J/s = \boxed{3{,}3\,W}$

(63.)

$l_{rad} = f(z)$

l_{rad} N pri normalnim uvjetima

$$E = E_0 \cdot e^{-\frac{x}{l_{rad}}}$$

$$\left(-\frac{dE}{dx}\right)_{rad} = \frac{4 r_0^2}{137} \cdot E \cdot N \cdot Z^2 \cdot \ln \frac{183}{Z^{1/3}}$$

$$\frac{dE}{dx} = \frac{-E_0}{l_{rad}} \cdot e^{-\frac{x}{l_{rad}}} \qquad \left(-\frac{dE}{dx}\right) = \frac{E}{l_{rad}}$$

$$l_{rad} = \frac{E}{\left(-\frac{dE}{dx}\right)_{rad}} = \frac{137 \cdot E}{4 r_0^2 E N Z^2 \ln \frac{183}{Z^{1/3}}} = \frac{137}{4 r_0^2 \cdot N \cdot Z^2 \cdot \ln \frac{183}{Z^{1/3}}} =$$

a) delež po masnem številu dušika N_2

$22{,}4\ \ell$ (litre) $\rightarrow 2 \cdot N_A$

$1000\ \ell \rightarrow N$

Z za dušik: $Z = 7$

$N = \frac{2 \cdot 1000 \cdot N_A}{22{,}4} = 2{,}68 \cdot 10^{25}$

$l_{rod} = 355{,}7\ m$

(46.)

$r = 10\ cm = 0{,}1\ m$

$B = 2 \cdot 10^{-2}\ T$

$W_i = 87{,}6\ keV$ (za 1 6vo)

$E_f = ?$

$E_f = E_e + W_i$

$F = e \cdot v \cdot B = \frac{m \cdot v^2}{r} \Rightarrow r = \frac{m \cdot v}{e \cdot B} = \frac{p}{e \cdot B}$

$p = e \cdot B \cdot r$

$\frac{p}{m \cdot c} = \frac{e \cdot B \cdot r}{m \cdot c} = \frac{1{,}6 \cdot 10^{-19} \cdot 2 \cdot 10^{-2} \cdot 0{,}1}{9{,}1 \cdot 10^{-31} \cdot 3 \cdot 10^{8}} = 1{,}17$ $\quad \frac{l \cdot c}{l \cdot c} = \frac{p \cdot c}{mc^2}$

$E_{ke} = \sqrt{p^2c^2 + m^2c^4} - mc^2 = \sqrt{1{,}17^2 \cdot m^2c^4 + m^2c^4} - mc^2 =$

$= mc^2 \left(\sqrt{1{,}17^2 + 1} - 1 \right) = 0{,}511\ MeV \cdot (*) = 0{,}274\ MeV$

$= 274\ keV$

$E_f = E_{k,e} + W_i = 274\ keV + 87{,}6\ keV = 361\ keV$

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(47.)

$\vartheta = 30°$

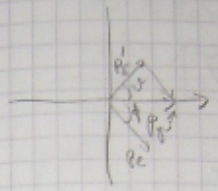
$E_\gamma = 1{,}33\ MeV$

$\phi = ?$

$E_\gamma' = \frac{E_\gamma}{1 + \frac{E_\gamma}{mc^2}(1 - \cos\vartheta)} = \frac{1{,}33\ MeV}{1 + \frac{1{,}33\ MeV}{0{,}511\ MeV}(1 - \cos 30°)} = 0{,}369\ MeV$

$\Downarrow$

formula za energijo raspršene γ svetlobe!!!



$p_\gamma = p_\gamma' \cdot \cos(-\vartheta) + p_e \cdot \cos\phi$

$p_\gamma' \cdot \sin\vartheta = p_e \cdot \sin\phi$

$\sin\phi = \frac{p_\gamma' \sin\vartheta}{p_e} = \frac{E_\gamma' \sin\vartheta}{p_e \cdot c}$

$E^2 = p^2c^2 + m^2c^4$ $\qquad E = mc^2 + E_k$

$\Downarrow$

$p_e \cdot c = \sqrt{E^2 - (mc^2)^2} = \sqrt{(mc^2 + E_k)^2 - (mc^2)^2} = \sqrt{2mc^2 E_k + E_k^2}$

$\sin\phi = \frac{(E_\gamma' \sin\vartheta)\ \text{izračunano}}{\sqrt{2mc^2 E_k + E_k^2}} = \frac{0{,}369\ MeV \cdot 1}{\sqrt{2 \cdot 0{,}511\ MeV \cdot 0{,}96\ MeV + (0{,}96\ MeV)^2}} = 0{,}267$

$\Rightarrow \phi = 15°$

$E_k = E_\gamma - E_\gamma' = 1{,}33 - 0{,}369 = 0{,}96\ MeV$

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(51.) $\lambda_r = 0{,}03\,\text{Å} = 3\cdot10^{-12}\,m$ | $1\,\text{Å} = 10^{-8}\,cm = 10^{-10}\,m$

$\vartheta = 60°$

$E_{k,e} = ?$

$E = h\cdot\nu = h\cdot\frac{c}{\lambda}$ $\quad h = 4{,}14\cdot10^{-15}\,eVs$

$E_\gamma = h\cdot\frac{c}{\lambda} = 4{,}14\cdot10^{-15}\,eVs\cdot\frac{3\cdot10^{8}\,m/s}{3\cdot10^{-12}\,m} = 414\,keV$

$E_{ke} = E_\gamma - E_{\gamma'}$ $\quad E_{\gamma'} = \frac{E_\gamma}{1+\frac{E_\gamma}{mc^2}(1-\cos\vartheta)}$

$E_{\gamma'} = \frac{414\,keV}{1+\frac{414\,keV}{511\,keV}\cdot\frac{1}{2}} = 294{,}6\,keV$

$E_{k,e} = 414 - 294 = \boxed{120\,keV}$

(54.) $E_\gamma = 3\,MeV$

$E_{k,p} = E_{k,e}$

$B = 0{,}1\,T$

$r = ?$

$E_\gamma = 2mc^2 + \overbrace{E_{k,e} + E_{k,p}}^{2E_{k,p}}$

$E_{k,p} = \frac{E_\gamma - 2mc^2}{2} = \frac{(3-1{,}02)}{2}\,MeV$

$E_{k,p} = 0{,}99\,MeV$

$r = \frac{p\,/c}{e\cdot B\,/c} = \frac{\sqrt{E^2-(mc^2)^2}}{e\cdot B\cdot c}$ $\quad E = E_{k,p} + mc^2$

$= 0{,}99 + 0{,}511 = 1{,}5\,MeV$

$r = \frac{\sqrt{(1{,}5)^2-(0{,}511)^2}\cdot(1{,}6\cdot10^{-13})}{1{,}6\cdot10^{-19}\cdot0{,}1\cdot3\cdot10^{8}}\,[m]$

$r = 0{,}047\,m = \boxed{4{,}7\,cm}$

(56.) $E_{Eu}^{156} \rightarrow \gamma_1 = 0{,}5\,MeV$ s 60% intenzitás

$\rightarrow \gamma_2 = 2{,}5\,MeV$ s 40% intenzitás

$\rho_{Cu} = 8{,}96\,g/cm^3$

$x_1 = 8\,cm$ $\quad x_1 \rightarrow 20$ áthaladó részecske/minuta

$x_1' = 1\,cm$ $\quad x_1' \rightarrow ?$ áthaladó részecske/minuta

$\left(\frac{\mu_1}{\rho_{Cu}}\right)\Big|_{0{,}5\,MeV} = 0{,}0853\,\frac{cm^2}{g} \Rightarrow \mu_1 = 0{,}764\,cm^{-1}$

$\left(\frac{\mu_2}{\rho_{Cu}}\right)\Big|_{2{,}5\,MeV} = 0{,}0388\,\frac{cm^2}{g} \Rightarrow \mu_2 = 0{,}348\,cm^{-1}$

$N(x) = N_0\,e^{-\mu\cdot x}$

$N(x) = N_0\cdot e^{-\left(\frac{\mu}{\rho}\right)(x\rho)}$

$\gamma_1(0{,}5\,MeV)$: $\quad \frac{N_{10}}{N_1(8cm)} = \frac{N_{10}}{N_{10}\cdot e^{-\mu_1\cdot(8cm)}} = \frac{1}{e^{-0{,}764\cdot8}}$

$= 451{,}24$

$\gamma_2(2{,}5\,MeV)$: $\quad \frac{N_{20}}{N_2(8cm)} = \frac{N_{20}}{N_{20}\,e^{-\mu_2\cdot(8cm)}} = 16{,}2$

$\Rightarrow$ 7

$N_0 = N_{10} + N_{20} = 0{,}6N_0 + 0{,}4N_0$

$$\frac{N_2}{N_1}\Big|_{8cm} = \frac{\left(\frac{0{,}4N_0}{16{,}2}\right)}{\left(\frac{0{,}6N_0}{459{,}24}\right)} \approx 19 = 19:1$$

$x_1' = 1\,cm$

$N_1' = N_{10} e^{-\mu_1 \cdot x_1'}$

$$\frac{N_1'}{N_1} = \frac{N_{10} \cdot e^{-\mu_1 x_1'}}{N_{10} e^{-\mu_1 x_1}} = e^{\mu_1 (x_1 - x_1')} = 210{,}19$$

$\Downarrow$

$210 \frac{otkucaja}{min}$

$$\frac{N_2'}{N_2} = \frac{N_{20} \cdot e^{-\mu_2 x_1'}}{N_{20} \cdot e^{-\mu_2 \cdot x_1}} = e^{\mu_2 (x_1 - x_1')} = 11{,}43$$

$\downarrow$

$\cdot 19 = 217 \frac{otkucaja}{min}$

$210 + 217 = 427 \frac{otkucaja}{min}$

(58.) $1\,Ci = 1\,g$ radija , $\rho_{Ra} = 11{,}4 \frac{g}{cm^3}$

$1\,Ci = 3{,}7 \cdot 10^{10}\,Bq$

$max\ E = 2{,}42\,MeV \Rightarrow \left(\frac{\mu}{\rho}\right) = 0{,}062 \frac{cm^2}{g}$

$N_1 = N_0 \cdot e^{-\left(\frac{\mu}{\rho}\right)\cdot(x\cdot\rho)}$

$$\frac{1\,Ci}{10^6} = 1\,Ci \cdot e^{-0{,}062(x\cdot\rho)} = e^{-0{,}062\cdot(x\cdot\rho)} = 10^{-6}/ln$$

$-0{,}062(x\cdot\rho) = -6\,ln\,10$

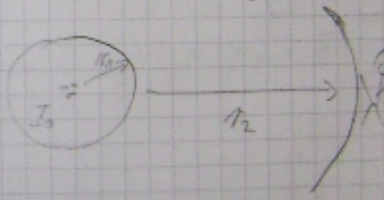
$(x\cdot\rho) = \frac{-6\,ln\,10}{-0{,}062} = 328{,}9407 \frac{g}{cm^2}$

$\Downarrow$

$x = \frac{328{,}9407}{11{,}4} = 29\,cm$

-podpitanje: $r_1 = 29\,cm$

$r_2 = 300\,cm$



$$\frac{\left(\frac{I_0}{4\pi r_2^2}\right)}{\left(\frac{I_0}{4\pi r_1^2}\right)} = \left(\frac{r_1}{r_2}\right)^2 = 9{,}34 \cdot 10^{-3}$$

1 izotopno

Interakcija bla bla: (ZADATAK)

Kolika je duljina folije od kadmija koja reducira fluks termičkih neutrona za faktor... $\rho_{kadm} = 8,65 \cdot 10^{3}\ kg/m^3$, $M = 112,41$.

Do apsorpcije termičkih neutrona dolazi na izotopu kadmija 113 čiji udarni presjek za apsorpciju termičkih neutr. $2\cdot 10^4$ b, a višestrukost 12,26 %

$\sigma_{Cd} = 2\cdot 10^4\ b = 2\cdot 10^{-20}\ cm^2$

$\alpha^{113}_{Cd} = 0,1226$

$\rho_{Cd} = 8,65\ g/cm^3$

$M_{Cd} = 112,41$

$\frac{\phi}{\phi_0} = 1/100$

$x = ?$

$$\phi(x) = \phi_0 \cdot e^{-\Sigma x}$$

$$\frac{\phi}{\phi_0} = e^{-\Sigma x} \quad /\cdot \ln$$

$$\Downarrow$$

$$x = -\frac{\ln(\phi/\phi_0)}{\Sigma}$$

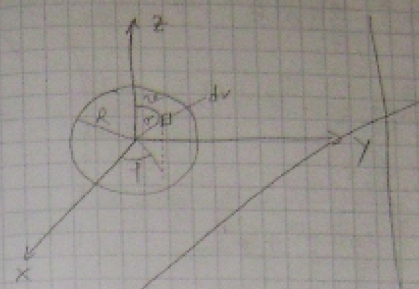
$$\Sigma = n_j \cdot \sigma = \rho \cdot \frac{N_A}{M_{Cd}} \cdot \alpha \cdot \sigma \quad (Cd^{113})$$

$$x = -\frac{M_{Cd} \cdot \ln(\phi/\phi_0)}{\rho \cdot N_A \cdot \alpha \cdot \sigma} = -\frac{112,41 \ln(100^{-1})}{8,65 \cdot 6,022 \cdot 10^{23} \cdot 0,1226 \cdot 2 \cdot 10^{-20}}$$

$$x = 4,05 \cdot 10^{-2}\ cm = 0,4\ mm$$

26.

$$Q_z = \frac{1}{e}\int \rho \cdot r^2 (3\cos^2\vartheta - 1)\, dV =$$



$dV = r^2 \sin\vartheta\, dr\, d\varphi\, d\vartheta$

$$= \frac{\rho}{e}\iiint_V r^2(3\cos^2\vartheta - 1)\, r^2 \sin\vartheta\, dr\, d\varphi\, d\vartheta =$$

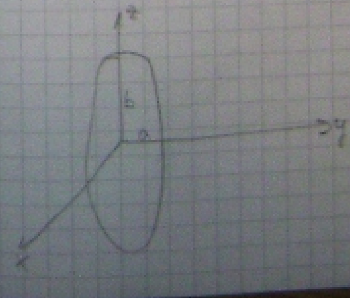
$$= \frac{\rho}{e}\underbrace{\int_0^R r^4 dr}_{\frac{R^5}{5}} \underbrace{\int_0^{2\pi} d\varphi}_{2\pi} \int_0^{\pi} (3\cos^2\vartheta - 1)\sin\vartheta\, d\vartheta =$$

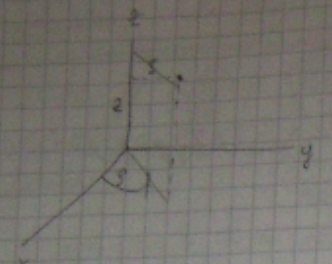
$[-d(\cos\vartheta)]$, $\cos\vartheta = t$

$$= \frac{2\pi\rho}{e}\frac{R^5}{5}\int_{-1}^{1}(3t^2 - 1)\, dt =$$

$$= \frac{2}{5}\frac{\pi \cdot \rho}{e} R^5 \underbrace{(t^3 - t)\Big|_{-1}^{+1}}_{\emptyset} = \emptyset$$

b) elipsoid a, b, Ze





$$\rho = \frac{Z \cdot e}{\frac{4}{3}\pi a^2 b}$$

$A(x, y, z)$

$A(\rho, \varphi, z)$

$dV = \rho \, d\varphi \, d\rho \, dz$

$\eta^2 = \rho^2 + z^2$

$\frac{\rho^2}{a^2} + \frac{z^2}{b^2} = 1$

$Q_2 = \frac{Ze \cdot 3}{e \cdot 4\pi a^2 b} = \frac{3Z}{4\pi a^2 b}$

$$Q_2 = \frac{Z \cdot e \cdot 3}{e \cdot 4\pi a^2 b} \cdot \iiint_V (3z^2 - \eta^2) \, dV = \frac{3Z}{4\pi a^2 b} \iiint_V (2z^2 - \rho^2) \, \rho \, d\rho \, d\varphi \, dz$$

$$Q_2 = \frac{3Z}{4\pi a^2 b} \underbrace{\int_0^{2\pi} d\varphi}_{2\pi} \iint_{\rho z} (2z^2 - \rho^2) \, \rho \, d\rho \, dz$$

$$Q_2 = \frac{3Z}{2a^2 b} \iint_{\rho z} (2z^2 - \rho^2) \, \rho \, d\rho \, dz$$

$\rho_z^2 = a^2 \left(1 - \frac{z^2}{b^2}\right)$

$\iint \rho^3 \, d\rho \, dz = \frac{4}{15} a^4 b$

$\iint z^2 \rho \, d\rho \, dz = \frac{2}{15} a^2 b^3$

12)

$$Q_2 = \frac{3Z}{2a^2 b} \left(2 \iint z^2 \rho \, d\rho \, dz - \iint \rho^3 \, d\rho \, dz \right)$$

$$Q_2 = \frac{3Z}{2a^2 b} \cdot \left(\frac{4}{15} a^2 b^3 - \frac{4}{15} a^4 b \right)$$

$$Q_2 = \frac{4 a^2 b}{15} \cdot \frac{3 \cdot Z}{2a^2 b} \left(b^2 - a^2 \right)$$

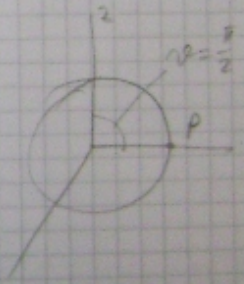
$$Q_2 = \frac{2}{5} Z (b - a) \left(\frac{b + a}{2} \right) \cdot 2$$

$\rightarrow R \qquad \eta = \frac{b - a}{R}$

$$\boxed{Q_2 = \frac{4}{5} \cdot Z \cdot \eta \cdot R^2}$$

27) $r = 4 \text{ fm}$

ρ na ekvatoru $\quad \vartheta = \frac{\pi}{2}$



$$Q_2 = \underbrace{\iiint (\;) \, dV}_{\text{sféra}} + \underbrace{\iiint \rho r^2 (3\cos^2\vartheta - 1) \, dV}_{\text{prstenec}}$$

ϑ

$= -\frac{1}{e} \cdot \eta^2 e$

$= -\eta^2 = -16 \text{ fm}^2$

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