Z-1 gustoća jezgre? – relacija za radijus!

$\rho = ?$

$\rho = \frac{m}{V}$ $\quad m = A \cdot u$ $\quad V_j = \frac{4}{3} R_j^3 \pi$

$V_j = A \cdot V_0$

$\frac{4}{3} R_j^3 \pi = A \frac{4}{3} R_0^3 \pi \Rightarrow R_j = R_0 \sqrt[3]{A}$

$\rho = \frac{m}{V} = \frac{A \cdot u}{\frac{4}{3} R_j^3 \pi} = \frac{Au}{\frac{4}{3} R_0^3 A \pi}$

$\rho = \frac{1.6605 \cdot 10^{-27} kg}{\frac{4}{3} \cdot (1.2 \cdot 10^{-15} m)^3 \pi}$

$\rho = 2.29 \cdot 10^{17} kg/m^3 = 2.29 \cdot 10^{14} g/cm^3$

$R_0 = 1.2 fm = 1.2 \cdot 10^{-15} m$

Z-2 gustoća el. naboja jezgre = ?

br protona = br neutrona

$Z = (A - Z) \Rightarrow A = 2Z$

$\rho_{naboj} = \frac{Q}{V}$ $\quad Q = Z \cdot e$

$\rho_{naboj} = \frac{Z \cdot e}{\frac{4}{3} R_j^3 \pi} = \frac{Z \cdot e}{\frac{4}{3} R_0^3 A \pi} = \frac{Ze}{\frac{4}{3} R_0^3 2Z \pi}$

$\rho_{naboj} = \frac{1.602 \cdot 10^{-19} C}{\frac{4}{3} (1.2 \cdot 10^{-15} m)^3 \cdot 2\pi}$ $\quad 1C = 1As$

$\rho_{naboj} = 1.1066 \cdot 10^{25} As/m^3$

$= 1.1066 \cdot 10^{19} As/cm^3$

Z-3 energija vezanja jezgre deuterija

a) deuterij $\Delta = 0.01410$ AJM

b) tricij $\Delta = 0.01605$ AJM

c) Be^9 $\Delta = 0.01219$ AJM

defekt mase $\Delta = M - A$

a) deuterij 1 proton, 1 neutron : $Z = 1$, $A = 2$, $N = 1$

$E_B = [2 m_p + 2 m_e + (A - Z) m_n - M] c^2$

$E_B = [1 \cdot m_p + 1 \cdot m_e + (2 - 1) m_n - M] c^2$

$\Delta(d) = M - A \Rightarrow M(d) = \Delta(d) + A = (0.0141 + 2)u$

$M(d) = 2.0141u$

$E_B = [1.00727u + \frac{1}{1823} u + 1.00866u - 2.0141u] c^2$

$= [2.3785 \cdot 10^{-3}] uc^2$ $\quad uc^2 = 931.4 MeV$

$E_B = 2.215 MeV$

b) tricij T → 1 proton, 2 neutron $Z = 1$, $A = 3$, $(A - Z) = 2$

$E_B = [Z m_p + Z m_e + (A - Z) m_n - M] c^2$

$\Delta(T) = 0.01605u$ $\quad \Delta(T) = M(T) - A$

$M(T) = \Delta(T) + A = 3.01605u$

$E_B = [1.00727u + \frac{1}{1823} u + 2 \cdot 1.00866u - 3.01605u] c^2$

$= 9.0885 uc^2 = 8.465 MeV$

c) Be^9 $\quad Z = 4$ $\quad A = 9$ $\quad (A - Z) = 5$

$\Delta(Be) = 0.01219u$ $\quad M(Be) = \Delta(Be) + A = 9.01219u$

$E_B = [4 \cdot 1.00727u + 4 \cdot \frac{1}{1823} u + (9 - 4) \cdot 1.00866u - 9.01219u] c^2$

$= 0.06238 uc^2 = 58.104 MeV$

2-4 Min energija za razbijanje jezgre O^{16} na 4 jednaka dijela

$O^{16} \Rightarrow \Delta = M - A = -0.00509$

$He^4 \Rightarrow \Delta = 0.00260$

$$E + M(O^{16})c^2 = 4M(He^4)c^2$$

$M = \Delta + A$

$$E + 16uc^2 + c^2\Delta(O^{16}) = 4(4uc^2 + \Delta(He^4)c^2$$

$$E = 16uc^2 + 4\Delta(He^4)c^2 - 16uc^2 - \Delta(O^{16})c^2$$

$$E = c^2(0.0026 \cdot 4 + 0.00509)u$$

$$E = 0.01549\,uc^2$$

$$E = 14.427\,MeV$$

2-5 Sinteza jezgre Li^6 i $H^2 \rightarrow 2\alpha$

$E_B(Li^6) = 5.33\,MeV$ (po jednom nukleonu)

$E_B(H^2) = 1.11\,MeV$ (-11-)

$E_B(He^4) = 7.08\,MeV$ (-11-) $\leftarrow \alpha$

oslobođena energija $E = ?$

$^6_3Li \rightarrow A=6,\ Z=3$ $\quad \frac{E_B}{A} = 5.33\,MeV \Rightarrow E_B(Li^6) = 6 \cdot 5.33\,MeV$

$E_B(Li^6) = 31.98\,MeV$

$^2_1H \rightarrow A=2,\ Z=1$ $\quad \frac{E_B}{A} = 1.11\,MeV$ $\quad E_B(H^2) = 2.2\,MeV$

$^4_2He \rightarrow A=4,\ Z=2$ $\quad \frac{E_B}{A} = 7.08\,MeV$ $\quad E_B = 4 \cdot 7.08\,MeV$

$E_B = 28.32\,MeV$

$$E_{sinteze} = 2\alpha - [E_B(Li^6) + E_B(H^2)]$$

$E(\alpha) = E_B(He)$

$E = 2 \cdot 28.32\,MeV - (31.98\,MeV + 2.2\,MeV)$

$E = 22.46\,MeV$

2-6 Weizsäckerova formula za energiju vezanja jezgre E_B/A za a) $^{50}_{23}V$ i b) $^{200}_{80}Hg$

$$E_B = aA - bA^{2/3} - c\frac{Z^2}{A^{1/3}} - d\frac{(A-2Z)^2}{A} + \delta\epsilon\frac{1}{A^{3/4}}$$

$a = 14.1\,MeV$
$b = 13.0\,MeV$
$c = 0.58\,MeV$
$d = 19.3\,MeV$
$\epsilon = 33.5\,MeV$

$$\delta = \begin{cases} +1 & \text{parni } A \text{ i } Z \\ 0 & \text{nep. } A \text{ (2 tokaygod)} \\ -1 & \text{nep } Z \text{ i parni } A \end{cases}$$

a) $^{50}_{23}V$ $\quad A=50$ $Z=23$ $\Rightarrow \delta = -1$

$$E_B = 14.1\,MeV\cdot 50 - 13\,MeV\cdot(50)^{2/3} - 0.58\,MeV\frac{23^2}{(50)^{1/3}} - 19.3\,MeV\cdot\frac{(50-2\cdot23)^2}{50} - 1\cdot 33.5\,MeV\frac{1}{50^{3/4}}$$

$$E_B(V) = 437.321\,MeV$$

$$\frac{E_B(V)}{A} = \frac{437.321\,MeV}{50} = 8.746\,MeV/nukl.$$

b) $^{200}_{80}Hg$ $\quad A=200$ $Z=80$ $\Rightarrow \delta = +1$

$$E_B = 14.1\,MeV\cdot 200 - 13\,MeV\cdot 200^{2/3} - 0.58\,MeV\frac{80^2}{200^{1/3}} - 19.3\,MeV\frac{(200-2\cdot80)^2}{200} + 1\cdot 33.5\,MeV\cdot\frac{1}{200^{3/4}}$$

$$E_B(Hg) = 1586.893\,MeV$$

$$\frac{E_B(Hg)}{A} = \frac{1586.893\,MeV}{200} = 7.934\,MeV/nukl$$

2-7 Weizsäckerova formula
E_B je napisana kao f-ja od Z, te zanemarimo spinsku korekciju δ

$$E_B = aA - bA^{2/3} - c\frac{Z^2}{A^{1/3}} - d\frac{(A-2Z)^2}{A} + \delta\epsilon\frac{1}{A^{3/4}}$$

$$E(MeV) = 14.1A - 13A^{2/3} - 0.58\frac{Z^2}{A^{1/3}} - 19\frac{(A-2Z)^2}{A} + 0$$

a) $Z=?$ za max energiju vezanja!
→ tražimo ekstreme f-je, $A=konst$

$Z=?$ $\qquad \left.\frac{\partial E}{\partial Z}\right|_{A=konst} = 0$

$$\left.\frac{\partial E}{\partial Z}\right|_{A=konst} = 0 - 0 - 0.58\frac{2\cdot Z}{A^{1/3}} - 19\frac{2(A-2Z)}{A}\cdot(-2)$$

$$-\frac{1.16Z}{A^{1/3}} + 76\frac{(A-2Z)}{A} = 0 \quad /A$$

$$-1.16A^{2/3}Z + 76A - 152Z = 0 \quad /76$$

$$-0.0152A^{2/3}Z + A - 2Z = 0$$

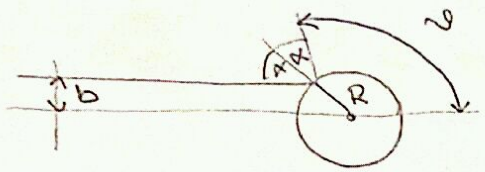
$$2(1 + 0.0152A^{2/3}) = A \Rightarrow Z = \frac{A}{2 + 0.0152A^{2/3}}$$

b) odnos neutrona i protona

$$\frac{A-Z}{Z} = \frac{A}{Z} - 1 = \frac{A}{\frac{A}{2+0.0152A^{2/3}}} - 1 = 1 + 0.0152A^{2/3}$$

2-10

Snop čestica nailjece na kuglu radijusa R i elasticno se odbija udarni presjek za to raspršenje?



broj čestica ΔN, upadnih ukupno N

$$\Delta N = N 2\pi b\, db = N 2\pi b \left(-\frac{db}{d\vartheta} d\vartheta\right)$$

$$\Delta N = -N 2\pi b \frac{db}{d\vartheta} \frac{\sin\vartheta\, d\vartheta}{d\vartheta}$$

$2\pi \sin\vartheta\, d\vartheta = d\Omega$ - prostorni kut

$$\Delta N = -N b \frac{db}{d\vartheta} \cdot \frac{d\Omega}{\sin\vartheta}$$

$$\frac{d\sigma}{d\Omega} = \frac{\Delta N}{N d\Omega}$$

$R \sin\alpha = b$

$2\alpha + \vartheta = \pi \Rightarrow \alpha = \frac{\pi}{2} - \frac{\vartheta}{2}$

$$R \sin\left(\frac{\pi}{2} - \frac{\vartheta}{2}\right) = b \Rightarrow \frac{db}{d\vartheta} = -\frac{1}{2} R \sin\frac{\vartheta}{2}$$

$$\frac{d\sigma}{d\Omega} = R \cos\frac{\vartheta}{2} \left(\frac{1}{2} R \sin\frac{\vartheta}{2}\right) \frac{1}{\sin\vartheta} = \frac{1}{4} R^2$$

totalni udarni presjek

$$\sigma_{tot} = \int_\Omega \frac{d\sigma}{d\Omega} d\Omega = \int_0^\pi \frac{1}{2} R^2 \sin\vartheta\, d\vartheta = R^2 \pi$$

2-11

Rutherfordovo raspršenje α čestica na Al $\vartheta = 90°$ $E(\alpha) = 1\,MeV$ koliko čestica u sekundi, detektira detektor $r = 1cm$, udaljen od mete 5cm, a upadni snop 10^6 č/cm²s

$Z = 13$ (broj mete) $\quad {}^{27}_{13}Al$

$\Theta = \vartheta = 90°$ $\quad E(\alpha) = 1\,MeV$

α čestice → $Z = 2$

detektor
$2r = 1cm$
$l = 5cm$

upadni snop
$\phi_{upadni} = 10^6$ č/cm²s

meta
$r_m = 0.3cm$
$d = 14\mu m = dm = 14 \cdot 10^{-6} m$
$\rho_{Al} = 2.7 g/cm^3$

$dR = ?$

Rutherford

$$\frac{d\sigma}{d\Omega} = \frac{b^2}{16} \cdot \frac{1}{\sin^4\left(\frac{\vartheta}{2}\right)}$$

$$b = \frac{2 \cdot Z \cdot e^2}{4\pi\varepsilon_0 E(\alpha)}$$

(2 = od α čestica, Z = od mete)

$$\frac{e^2}{4\pi\varepsilon_0} = 1.44\,MeV fm$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{2 \cdot Z \cdot 1.44 MeV fm}{1 MeV}\right)^2 \cdot \frac{1}{16 \sin^4\left(\frac{\vartheta}{2}\right)} =$$

$$= (2 \cdot 13 \cdot 1.44 fm)^2 \cdot \frac{1}{16 \sin^4(90/2)} =$$

$$\frac{d\sigma}{d\Omega} = 350.4384 (fm)^2 \qquad 1fm = 10^{-15} m \qquad 1b = 10^{-28} m^2$$

$$= 350.4384 (10^{-15} m)^2 = 350.4384 \cdot 10^{-30} m^2$$

$$\frac{d\sigma}{d\Omega} = 3.5\ b/sr$$

$$\frac{dR}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right) \phi N$$

$$N = \frac{\rho \cdot N_A}{M} \cdot V_m \qquad \text{(br. atoma po jedinici volumena)}$$

$$\frac{dR}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right) \phi \frac{\rho N_A}{M} \cdot V_m \,/ d\Omega$$

$$dR = \left(\frac{d\sigma}{d\Omega}\right) \phi \frac{\rho N_A}{M} \cdot V_m \Omega = 3.5 \cdot 10^{-28} m^2 \cdot 10^6 \frac{1}{cm^2 s} \cdot \frac{2.7 g}{cm} \cdot \frac{6.022 \cdot 10^{26}}{kg} \Rightarrow$$

$dR =$

$V_m = Sdm$

$V_m = r_m^2 dm = (0.3 cm)^2 \cdot 14 \cdot 10^{-4} cm$

$V_m = 1.26 \cdot 10^{-3} cm^3$

... 2-11

$V_m = S\,dm = r_m^2 \cdot dm = (0{,}3\,cm)^2 \cdot 14 \cdot 10^{-4} cm$

$V_m = 1{,}26 \cdot 10^{-4} cm^3$ | $\frac{d\Gamma}{d\Omega} = 35b = 3{,}5 \cdot 10^{-28} m^2$

$\frac{dR}{d\Omega} = \left(\frac{d\Gamma}{d\Omega}\right) \phi N = \left(\frac{d\Gamma}{d\Omega}\right) \phi \cdot \frac{\rho N_A}{M} \cdot V_m / d\Omega$

$dR = \left(\frac{d\Gamma}{d\Omega}\right) \phi \frac{\rho N_A}{M} V_m d\Omega \quad (M = A)$

$dR = 3{,}5 \cdot 10^{-28} m^2 \cdot 10^6 \frac{č}{cm^2 s} \cdot 2{,}7 \frac{g}{cm^3} \cdot 6{,}022 \cdot 10^{26} \frac{\text{atoma}}{kg\,mol} \cdot \frac{1}{27} \cdot 1{,}26 \cdot 10^{-4} cm^3$

$dR = 3{,}5 \cdot 10^{-24} cm^2 \cdot 10^6 \frac{č}{cm^2} \cdot 2{,}7 \cdot 10^{-3} kg \cdot 6{,}022 \cdot 10^{26} \frac{1}{kg\,mol} \cdot \frac{1}{27} \cdot 1{,}26 \cdot 10^{-4}$

~~$dR = 1{,}655 \cdot 10^{-7} č/s$~~

$dR = 2{,}655 č/s$

2-12 broj α čestica koji se rasprši u prostor između 2 kuta na folji zlata

$E(\alpha) = 10 MeV$

$\vartheta_1 = 90°$

$\vartheta_2 = 180°$

$S = 1 cm^2$

$(d) = 4 mg/cm^2 \rightarrow \rho x = 4 mg/cm^2$

$\phi = 10^6 \frac{č}{cm^2 s}$

$^{197}_{79}Au$

α čestica $\rightarrow z = 2$ Au $\rightarrow z = 79$

$R_1 = ?$

$\frac{dR_1}{d\Omega} = \left(\frac{d\Gamma}{d\Omega}\right) \phi N / d\Omega$

$dR_1 = \left(\frac{d\Gamma}{d\Omega}\right) \phi N d\Omega$

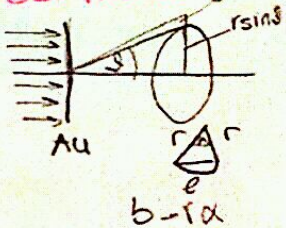
površina detektora

$d\Omega = \frac{\Delta S}{r}$

udaljenost det. od mete

$d\Omega = \frac{\Delta S}{r}$

$d\Omega = \frac{(r\sin\vartheta + r d\vartheta)^2 \pi - (r\sin\vartheta)^2}{r^2} = 2\pi \sin\vartheta d\vartheta$



$dR_1 = \left(\frac{d\Gamma}{d\Omega}\right) \phi N d\Omega \Big/ \int_{\vartheta_1}^{\vartheta_2}$

$R_1 = \int_{\vartheta_1}^{\vartheta_2} N\phi \left(\frac{d\Gamma}{d\Omega}\right) d\Omega = \int_{\vartheta_1}^{\vartheta_2} N\phi \left(\frac{b^2}{16} \cdot \frac{1}{\sin^4(\frac{\vartheta}{2})}\right) d\Omega =$

$= \int_{\vartheta_1}^{\vartheta_2} N\phi \frac{1}{16} \left(\frac{2 \cdot z \cdot 1.44 MeV}{E\alpha}\right)^2 \cdot \frac{1}{\sin^4(\frac{\vartheta}{2})} \cdot 2\pi \sin\vartheta d\vartheta$

$R_1 = \frac{N\phi}{16} \cdot \left(\frac{2 \cdot z \cdot 1.44 MeV}{E\alpha}\right)^2 \cdot 2\pi r \underbrace{\int_{\vartheta_1}^{\vartheta_2} \frac{\sin\vartheta}{\sin^4(\frac{\vartheta}{2})} d\vartheta}_{I}$

$\Rightarrow$

... 2-12 $\vartheta_1 = 90°$ $\vartheta_2 = 180°$

$$I = \int_{\vartheta_1}^{\vartheta_2} \frac{\sin\vartheta}{\sin^4\frac{\vartheta}{2}}\,d\vartheta = \int_{\vartheta_1}^{\vartheta_2} \frac{2\sin\frac{\vartheta}{2}\cos\frac{\vartheta}{2}}{\sin^4\frac{\vartheta}{2}}\,d\vartheta$$

$$= \int_{\vartheta_1}^{\vartheta_2} \frac{2\cos\frac{\vartheta}{2}}{\sin^3\frac{\vartheta}{2}}\,d\vartheta = 2\int_{\vartheta_1}^{\vartheta_2} \frac{\cos\frac{\vartheta}{2}}{\sin^3\frac{\vartheta}{2}}\cdot 2\,d\left(\frac{\vartheta}{2}\right) =$$

$$= 4\int_{\vartheta_1}^{\vartheta_2} \frac{\cos\frac{\vartheta}{2}}{\sin^3\frac{\vartheta}{2}}\,d\left(\frac{\vartheta}{2}\right) = \begin{bmatrix} d\vartheta\cdot 2 = d(2\vartheta) = 2d\vartheta \\ \sin\frac{\vartheta}{2} = t \;/\,d \\ \frac{1}{2}\cos\left(\frac{\vartheta}{2}\right)d\vartheta = dt \\ \frac{d\vartheta}{2} = \frac{dt}{\cos\frac{\vartheta}{2}} \end{bmatrix}$$

promjena granica $\sin\frac{\vartheta_1}{2}$ do $\sin\frac{\vartheta_2}{2}$

$$= 4\int \frac{\cos^{3/2}}{t^3}\frac{dt}{\cos^{3/2}} \;?\;? = 2\left[\frac{1}{\sin^2\left(\frac{\vartheta_1}{2}\right)} - \frac{1}{\sin^2\left(\frac{\vartheta_2}{2}\right)}\right]$$

$$R = \frac{N\Phi}{4}\left(\frac{2\cdot 2\cdot 1.44\,\text{MeV}}{E_\alpha}\right)^2 \pi r^2 \underbrace{\left(\frac{1}{\sin^2\frac{\vartheta_1}{2}} - \frac{1}{\sin^2\frac{\vartheta_2}{2}}\right)}_{1}$$

$$N = \frac{\rho N_A}{M} V_m = \frac{\rho_x N_A}{M}\cdot S$$

$$R = \frac{\Phi}{4}\cdot\left(\frac{2\cdot 2\cdot 1.44\,\text{MeV}}{E_\alpha}\right)^2\pi\cdot\frac{\rho_x N_A}{M}\,S \qquad M = A$$

$$= \frac{10^6\,\check{c}}{4\,cm^2 s}\left(\frac{2\cdot 2\cdot 1.44\,\text{MeV}}{10\,\text{MeV}}\right)^2\pi\cdot\frac{4\,mg/cm^2}{197}\cdot 6.022\cdot 10^{26}\,\frac{\text{mol}}{kg\,\text{mol}}\cdot 1\,cm^2$$

$$= 49.71\cdot 10^6\,\frac{\check{c}}{cm^2 s}\cdot 10^{-3}\,\frac{g}{cm^2}\cdot 10^{26}\cdot\frac{1}{10^3\,g\,\text{mol}}\cdot cm^2 fm^2$$

$$= 49.71\cdot 10^6\,\frac{\check{c}}{cm^2 s}\cdot 10^{-3}\cdot 10^{26}\cdot\frac{1}{10^3\,\text{mol}}\cdot 10^{-30}\,(m^2\cdot 10^{2\cdot 2}\,(cm^2))$$

$$R = 49.71\ \check{c}/s$$

2-13 koliko % deutrona se raspsi između ϑ_1 i ϑ_2?

$E(\text{deut}) = 8\,\text{MeV}$ $\quad$ $Z = 1$ (deuteron)

$\vartheta_1 = 45°$ $\quad$ aluminijska folija

$\vartheta_2 = 90°$

$\rho x = 27\,mg/cm^2$ $\quad$ $^{27}_{13}Al \Rightarrow Z = 13$, $M = A = 27$

$\frac{\Delta N}{N} = ?\ [\%]$

$\frac{d\sigma}{d\Omega} = \frac{\Delta N}{N\cdot n\cdot\Delta\Omega}$ $\qquad$ $n = \rho\frac{N_A}{M}\cdot\Delta x$

$\frac{\Delta N}{N} = n\Delta\Omega\left(\frac{d\sigma}{d\Omega}\right)$ $\qquad$ $b^2 = \left(\frac{2\cdot 2\cdot 1.44\,\text{MeV}}{E_\alpha}\right)^2$

$$\frac{d\sigma}{d\Omega} = \int_{\vartheta_1}^{\vartheta_2}\frac{b^2}{16}\cdot\frac{1}{\sin^4\frac{\vartheta}{2}}\,d\Omega \qquad b = \frac{2\,2e^2}{4\pi\varepsilon_0 E_\alpha} = 1.44\,\text{MeV}\,\frac{2\cdot 2}{E_\alpha}$$

$$= \left(\frac{2\cdot 2\cdot 1.44\,\text{MeV}fm}{E_\alpha}\right)^2\frac{1}{16}\int_{\vartheta_1}^{\vartheta_2}\frac{1}{\sin^4\frac{\vartheta}{2}}\,d\Omega$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{2\cdot 2\cdot 1.44\,\text{MeV}fm}{E_\alpha}\right)^2\cdot\frac{1}{16}\cdot 2\left(\frac{1}{\sin^2\frac{\vartheta_1}{2}} - \frac{1}{\sin^2\frac{\vartheta_2}{2}}\right)$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{1\cdot 13\cdot 1.44\,\text{MeV}fm}{8\,\text{MeV}}\right)^2\frac{1}{16}\cdot 2\underbrace{\left(\frac{1}{\sin^2(45/2)} - \frac{1}{\sin^2(90/2)}\right)}_{4.828}$$

$$\frac{d\sigma}{d\Omega} = 3.3045\ fm^2$$

$$\Delta\Omega = 2\pi\sin\vartheta\,d\vartheta = 2\pi\int_{\vartheta_1}^{\vartheta_2}\sin\vartheta\,d\vartheta = 2\pi\left(-\cos\vartheta\right)\Big|_{\vartheta_1}^{\vartheta_2} = 2\pi\,\frac{\sqrt{2}}{2} = \pi\sqrt{2}$$

$$\frac{\Delta N}{N} = \rho\frac{N_A}{M}\Delta x\,\frac{d\sigma}{d\Omega}\cdot\Delta\Omega =$$

$$= \pi\sqrt{2}\cdot 3.3045\cdot 10^{-30}\,\underbrace{m^2}_{10^{-26}cm^2}\cdot 27\cdot 10^{-3}\,\frac{g}{cm^2}\cdot\frac{6.022\cdot 10^{+26}}{27}\,\frac{\text{mol}}{kg\Rightarrow 10^3 g}$$

$$\frac{\Delta N}{N} = 8.8412\cdot 10^{-5}\,\Delta x$$

Z-14 snop rentgenskih zraka respisi se na kositru

$S = 1cm^2$

$x\rho = 10 mg/cm^2$

$\Phi = 4 \cdot 10^7 fotona/cm^2$ nije pravi I

detektor
$\alpha = 60°$
$\ell = 10cm$
$2r_d = 2cm$
$N_{reg} = 15\% \cdot \Delta R$

Thomsonovo respršenje X zraka (γ-zraka)

~~$Z=?$, $A=?$~~ A = kositrov izotop $\overset{A}{\underset{50}{Sn}}$

$\Phi = \frac{I}{S} \left[\frac{\check{c}}{cm^2 s}\right]$

$d\Omega = \frac{dS}{r^2} = \frac{r_d^2 \pi}{\ell^2} = \frac{1cm^2 \pi}{100 cm^2} = 0.01\pi\, sr$

Thomson

$\sigma_T = \frac{8\pi r_0^2}{3}$ $\quad r_0 = \frac{e^2}{4\pi\varepsilon_0 mc^2} = 2.83fm$

$\frac{d\sigma}{d\Omega} = \frac{r_0^2(1+\cos^2\Theta)}{2}$

$\frac{d\sigma}{d\Omega} = \frac{(2.83fm)^2(1+\cos^2 60)}{2}$

$\frac{d\sigma}{d\Omega} = 5.0055 \cdot 10^{-30} m^2/sr \doteq 5 \cdot 10^{-25} cm^2/sr$

$\frac{dR}{d\Omega} = N\Phi\left(\frac{d\sigma}{d\Omega}\right) / d\Omega$

$dR_1 = N\Phi\left(\frac{d\sigma}{d\Omega}\right) d\Omega$

$N = n_{pm} \cdot S = S \frac{\rho x}{A} N_A \rightarrow$ 22 ATOME,

Thomsonovo respršenje gleda ELEKTRONE

$N(z_{eTh}) = S \frac{\rho x}{A} \cdot N_A \cdot Z$ br. elektrona Thomson

$dR = S \frac{\rho x}{A} \cdot N_A \cdot 2 \cdot \Phi \left(\frac{d\sigma}{d\Omega}\right) d\Omega$

$A = S \rho x N_A 2 \cdot \Phi \left(\frac{d\sigma}{d\Omega}\right) \frac{d\Omega}{dR} \rightarrow \Delta R$; $d\Omega \rightarrow \pi/100$

$A = 1cm^2 \cdot 10 \cdot 10^{-3} \frac{g}{cm^2} \cdot 6.022 \cdot 10^{23} \frac{\check{c}}{g\,mol} \cdot 50 \cdot 4 \cdot 10^7 \frac{1}{cm^2} \cdot 5 \cdot 10^{-26} \frac{cm^2}{sr} \cdot \frac{\pi}{100} \cdot 15\%$

$A = 120.50 \doteq 120$

2-15 % neutrona koji se rasprši kroz olovnu foliju?

$E_n = 14\,MeV$ ^{208}Pb

$\rho x = 0.8\,g/cm^2$

brzi neutroni

$\sigma = 2\pi(R + b + \lambda\!\!\!^{-})^2$

$R = 1.2\,fm\sqrt[3]{M}$

$\lambda\!\!\!^{-} = \frac{hc}{2\pi\sqrt{2EMc^2}}$

$b = 1\,fm$

$M = Au$ $A = 208$ $M = A$

$R = 1.2\,fm\sqrt[3]{M} = 1.2\,fm\sqrt[3]{208} = 7.11\,fm$

$\lambda\!\!\!^{-} = \frac{hc}{2\pi\sqrt{2EMc^2}} = 0.1947\,fm$

$22 \Rightarrow 4.16 \cdot 10^{-26}\,m$

2-16 neutroni $E_n = 15\,MeV$ raspršuju se po ^{58}Ni postotak raspršenja 2.68%. $\rho x = ?$

$E_n = 15\,MeV$ $^{58}Ni \rightarrow A = 58$

$\frac{R}{I}$ - postotak raspršenja

$\frac{R}{I} = 2.68\% = 0.0268$

$Z = 28$

br. neut $= A - Z = 30$

$(\rho x) = ?$

$\sigma = 2\pi(R + b + \lambda\!\!\!^{-})$

broj onih koji dodu do detektora $R = R_0\sqrt[3]{A}$

$b = 1\,fm = 1 \cdot 10^{-15}\,m$ $\lambda\!\!\!^{-} = \frac{\lambda}{2\pi}$ $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_n}}$

$\Phi = \frac{I}{S}$ $R = N\Phi\sigma \rightarrow R = N\frac{I}{S}\sigma$

$\frac{R}{I} = N\sigma = \frac{(\rho x)}{A} \cdot \frac{Sm}{Sm} = N_A\sigma = \frac{(\rho x)}{A} N_A \cdot \sigma$

$(\rho x) = \frac{R}{I} \cdot \frac{A}{N_A\sigma} = 0.0268 \cdot \frac{58}{\quad}$

$R = R_0\sqrt[3]{A} = 1.2\,fm\sqrt[3]{58} = 4.645\,fm$

$b = 10^{-15}\,m$

$\lambda\!\!\!^{-} = \frac{h}{2\pi\sqrt{2Em}} =$

2-17 omjer respršenih X zraka

Thomsonovo respršenje

$\Delta x = 5mm$

$\rho = 2.25 g/cm^3$ } grafit $\quad {}^{12}_{6}C$

$A = 12 \quad Z = 6$

$\sigma = \frac{8\pi}{3} r_0^2$ $\quad r_0 = \frac{e^2}{4\pi\varepsilon_0 mc^2}$ Thomsonovo raspršenje

$$\sigma = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\varepsilon_0 mc^2}\right)^2$$

$\varepsilon_0 = 8.854\ 10^{-12}$

$mc^2 = 931.4 MeV \qquad r_0 \doteq 2.818 \cdot 10^{-15} m$

$e = 1.602 \cdot 10^{-19} C$

$\sigma = 6.6527 \cdot 10^{-29}\ m^2$

$\sigma = \frac{\Delta N}{N \cdot n_{pm}} \qquad n_{pm} = \frac{\rho x}{M_A} N_A$

$$\frac{\Delta N}{N} = \sigma \cdot \rho x \frac{N_A}{M_A}$$

$\rho x = 2.25 \frac{g}{cm^3} \cdot 0.5 \cancel{cm} = 1.125\ g/cm^2$

$\frac{\Delta N}{N} = 6.6527 \cdot 10^{-29} m^2 \cdot 1.125 \frac{g}{cm^2} \cdot \frac{6.022 \cdot 10^{23}/kg}{12}$

$= 6.6527 \cdot 10^{-29} m^2 \cdot 1.125 \frac{g}{cm^2 \cdot 10^{-2 \cdot 2} m^2} \cdot \frac{6.022 \cdot 10^{23}/g}{12}$

$\frac{\Delta N}{N} = 0.0375$

2-18 X zrake

intenzitet snopa reducira se raspršenjem na 78% pri prolazu kroz foliju

$\frac{\Delta N}{N} = 78\% = 0.78$

$\Delta x = 0.5 cm$

$\rho = 2.25 g/cm^3$

$Z = ?$

n_e - broj elektrona u meti

n_j - broj jezgara u meti

$\quad {}^{12}_{6}C$

$$\frac{\Delta N}{N} = \sigma_T \cdot n_e$$

$Z = \frac{n_e}{n_j}$ $\quad n_e = \frac{\Delta N}{N}\frac{1}{\sigma_T}$ $\quad n_j = \frac{N_A}{M}\rho x$

broj elektrona u meti

$$Z = \frac{n_e}{n_j} = \frac{\frac{\Delta N}{N}\frac{1}{\sigma_T}}{\frac{N_A}{M}\rho x}$$

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\varepsilon_0 mc^2}\right)^2$$

$e = 1.602 \cdot 10^{-19} C \quad mc^2 = 931 MeV \quad \varepsilon_0 = 8.854 \cdot 10^{-12}$

$1 MeV = 1.602176 \cdot 10^{-13} J \qquad r_0 = 2.83 fm$

$$\sigma_T = \frac{8\pi}{3}\left(\frac{(1.602 \cdot 10^{-19})^2}{4\pi\ 8.854 \cdot 10^{-12} \cdot 931.4 \cdot 1.602176 \cdot 10^{-13} J}\right)^2$$

$\sigma_T = 2.001 \cdot 10^{-35}$

$$Z = \frac{0.78 \cdot \frac{1}{2.001 \cdot 10^{-35}}}{\frac{6.022 \cdot 10^{26}}{12} \cdot 2.25 \frac{g}{cm^3} \cdot 0.5 \cancel{cm}} = 5.86 \cdot 10^{6} \doteq 6$$

7-20 brzina odskoka rezidualne jezgre?

- primjena zakona o očuvanju količine gibanja -

$E_k(\alpha) = 6.9\,MeV$
$^{213}_{z}Ra$

$m_\alpha = 4u$
$m_y = 209u$

$v_y = ?$

$A = 213$
$A - 4 = 209$

$$^{A}_{z}X \rightarrow {}^{A-4}_{z-2}Y + {}^{4}_{2}\alpha$$

$$m_\alpha v_\alpha = m_y v_y$$

$v_x = 0$ → jezgra mirovala na početku

$$v_y = \frac{m_\alpha}{m_y} v_\alpha$$

$$E_\alpha = \frac{m_\alpha v_\alpha^2}{2} \qquad v_\alpha = \sqrt{\frac{2E_\alpha}{m_\alpha}}$$

$$v_y = \frac{m_\alpha}{m_y}\sqrt{\frac{2E_\alpha}{m_\alpha}} = \frac{4u}{209u}\sqrt{\frac{2\cdot 6.9\,MeV}{4u}}$$

$$= \frac{4}{209}\sqrt{\frac{2\cdot 6.9\cdot 1.60217\cdot 10^{-13}\,J}{4\cdot 1.66054\cdot 10^{-27}\,kg}}$$

$$v_y = 3.49\cdot 10^5\,m/s$$

7-21 α raspad jezgre ^{213}Po
oslobođena energija = ?

$E_\alpha = 8.34\,MeV$

$$^{213}_{z}X \rightarrow {}^{209}_{z-2}Y + {}^{4}_{2}\alpha$$

$$E_\alpha = \frac{m_\alpha v_\alpha^2}{2} \qquad E_y = \frac{m_y v_y^2}{2}$$

$$m_\alpha v_\alpha = m_y v_y \qquad v_\alpha = \sqrt{\frac{2E_\alpha}{m_\alpha}}$$

$$m_\alpha \sqrt{\frac{2E_\alpha}{m_\alpha}} = m_y \sqrt{\frac{2E_y}{m_y}} \Big/ \; v_y = \sqrt{\frac{2E_y}{m_y}}$$

$$m_\alpha^2 \frac{2E_\alpha}{m_\alpha} = m_y^2 \frac{2E_y}{m_y}$$

$$E_y = E_\alpha \cdot \frac{m_\alpha}{m_y} = 8.34\,MeV \cdot \frac{4u}{209u}$$

$$E_y = 0.15961\,MeV$$

oslobođena energija

$$E_{uk} = E_\alpha + E_y = 8.34\,MeV + 0.15961\,MeV$$

$$E_{uk} = 8.499\,MeV \doteq 8.5\,MeV$$