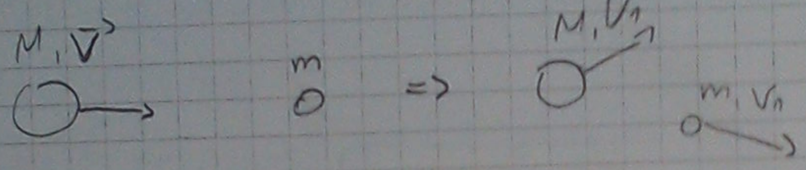
Izvodi

Max. energija predana elektronu u 1 interakciji



$\frac{1}{2}M\cdot V^2 \qquad 0 \qquad \frac{1}{2}M\cdot V_1^2 \qquad \frac{1}{2}m v_1^2$

$$\cancel{\tfrac{1}{2}}M\cdot V^2 = \cancel{\tfrac{1}{2}}M\cdot V_1^2 + \cancel{\tfrac{1}{2}}m\cdot v_1^2 \qquad \text{ZOE}$$

$$M\cdot V = M\cdot V_1 + m v_1 \qquad \text{ZOKG}$$

$$v_1 = \frac{M}{m}(V - V_1)$$

$$MV^2 - MV_1^2 = m v_1^2$$

$$\cancel{M}(V^2 - V_1^2) = \cancel{m}\cdot\frac{M^{\cancel{2}}}{m^{\cancel{2}}}(V - V_1)^2$$

$$V^2 - V_1^2 = \frac{M}{m}(V - V_1)^2 \quad /\cdot m$$

$$mV^2 - mV_1^2 = MV^2 - 2MV\cdot V_1 + MV_1^2$$

$$MV_1^2 + mV_1^2 - 2MVV_1 + MV^2 - mV^2 = 0$$

$$V_{1,2} = \frac{2MV \mp \sqrt{(2MV)^2 - 4(M+m)V^2(M-m)}}{2(M+m)}$$

$$V_{1,2} = \frac{MV \mp \sqrt{(MV)^2 - (M+m)V^2(M-m)}}{M+m}$$

$$v_A = \frac{MV \mp \sqrt{(MV)^2 - (M^2\,m^2)V^2}}{M+m}$$

$$v_A = \frac{MV \mp \sqrt{(\cancel{MV})^2 - (\cancel{MV})^2 + (mV)^2}}{M+m}$$

$$v_A = \frac{MV - mV}{M+m} \qquad \text{zu max } Q$$

$$\boxed{v_A = \frac{(M-m)V}{M+m}}$$

$$\boxed{Q_{max} = \frac{1}{2}MV^2 - \frac{1}{2}MV_A^2} \qquad E = \frac{1}{2}MV^2$$

$$Q_{max} = E - \frac{1}{2}M\left[\frac{(M-m)}{M+m}V\right]^2$$

$$Q_{max} = E\left[1 - \frac{(M-m)^2}{(M+m)^2}\right]$$

$$Q_{max} = E\left[\frac{(M+m)^2 - (M-m)^2}{(M+m)^2}\right]$$

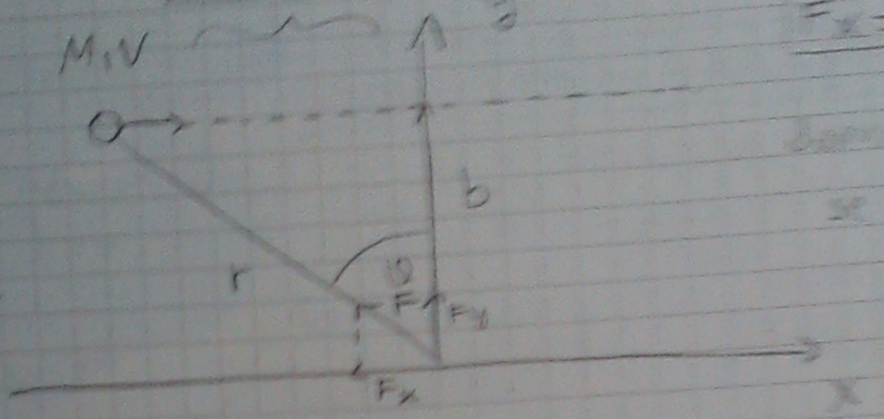
$$Q_{max} = E\left[\frac{\cancel{M^2} + 2Mm + \cancel{m^2} - \cancel{M^2} + 2Mm - \cancel{m^2}}{(M+m)^2}\right]$$

$$\boxed{\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

$$\boxed{Q_{max} = \frac{4mME}{(M+m)^2}} \qquad \boxed{Q_{max}(rel) = \frac{2\gamma^2 mV^2}{1 + 2\gamma\frac{m}{M} + \frac{m^2}{M^2}}}$$

[illegible]

$$\boxed{s = Vt}$$



$$\boxed{F_x = 0}$$

[illegible]

$$\boxed{F = \frac{Z \cdot e^2}{4\pi\varepsilon_0 r^2}}$$

$$F = m a \qquad p = m v$$

$$a = \frac{dV}{dt} \qquad \frac{dp}{dt} = m\frac{dv}{dt}$$

$$F = m\frac{dV}{dt} \qquad \boxed{F = \frac{dp}{dt}}$$

$$dp = F\,dt \;\Big/ \int \quad \Rightarrow \quad p = \int_{-\infty}^{+\infty} F_y\,dt \qquad \boxed{F_y = F \cdot \cos\vartheta}$$

$$p = \int_{-\infty}^{\infty} \frac{Z \cdot e^2}{4\pi\varepsilon_0} \cdot \frac{\cos\vartheta}{r^2}\,dt$$

simetrinost problema

$$p = \frac{Z \cdot e^2}{2 \cancel{4}\pi\varepsilon_0}\,\cancel{2}\int_0^{\infty} \frac{\cos\vartheta}{r^2}\,dt$$

$$p = \frac{Z \cdot e^2}{2\pi\varepsilon_0}\int_0^{\infty} \frac{\cos\vartheta}{r^2}\,dt \qquad \boxed{\cos\vartheta = \frac{b}{r}}$$

$$\boxed{r^2 = (V \cdot t)^2 + b^2}$$

$$p = \frac{Z \cdot e^2 \, b}{2\pi \varepsilon_0} \int_0^{\infty} \frac{dt}{(b^2 + (V \cdot t)^2)^{3/2}}$$

$$\left[\begin{array}{l} Vt = b \, tg\,\vartheta \\ tg\,\vartheta = \frac{Vt}{b} \\ \vartheta = \arctan\left(\frac{Vt}{b}\right) \\ t = 0 \quad \vartheta = 0 \\ t = \infty \quad \vartheta = 90^\circ \end{array}\right]$$

$$tg'\vartheta = \left(\frac{\sin\vartheta}{\cos\vartheta}\right)'$$

$$= \frac{\cos\vartheta \cos\vartheta + \sin\vartheta \sin\vartheta}{\cos^2\vartheta} = \frac{1}{\cos^2\vartheta}$$

$$\boxed{\frac{1}{\cos\vartheta} = \sec\vartheta} \Rightarrow tg'\vartheta = \sec^2\vartheta$$

$$V \cdot dt = b \cdot \sec^2\vartheta \, d\vartheta \Rightarrow dt = \frac{b \cdot \sec^2\vartheta \, d\vartheta}{V}$$

$$I = \int_0^{90} \frac{b \cdot \sec^2\vartheta \, d\vartheta}{V} \cdot \frac{1}{\left[b^2 + (b \, tg\,\vartheta)^2\right]^{3/2}}$$

$$I = \int_0^{90} \frac{b \cdot \sec^2\vartheta \, d\vartheta}{V} \quad \frac{1}{\left[b^2(1 + tg^2\vartheta)\right]^{3/2}}$$

$$I = \int_0^{90} \frac{\cancel{b} \cdot \sec^2\vartheta \, d\vartheta}{V} \quad \frac{1}{b^{\cancel{3}\,2}(1 + tg^2\vartheta)^{3/2}}$$

$$I = \int_0^{90} \frac{\sec^2\vartheta \, d\vartheta}{V \cdot b^2 (1 + tg^2\vartheta)^{3/2}}$$

$$I = \frac{1}{V \cdot b^2} \int_0^{90} \frac{\sec^2\vartheta \, d\vartheta}{(1 + tg^2\vartheta)^{3/2}}$$

$$I_2 = \int_0^{90} \frac{\sec^2\theta}{(1+tg^2(\theta))^{3/2}}$$

$$\boxed{\sin^2 x + \cos^2 x = 1} \quad \Big| : \cos^2 x$$

$$\frac{\sin^2 x}{\cos^2 x} + 1 = \frac{1}{\cos^2 x}$$

$$\boxed{tg^2 x + 1 = \sec^2 x}$$

$$I_2 = \int_0^{90} \frac{\sec^2\theta}{\sec^3\theta}\, d\theta = \int_0^{90} \frac{1}{\sec\theta}\, d\theta$$

$$= \int_0^{90} \cos\theta \, d\theta = \sin\theta \Big|_0^{90} = 1 - 0 = 1$$

$$\Rightarrow I = \frac{1}{v\, b^2} \Rightarrow p = \frac{Z \cdot e^2 \cdot \cancel{b}}{2\pi\varepsilon_0} \frac{1}{v\, b^2}$$

$$\boxed{p = \frac{2Z \cdot e^2}{4\pi\varepsilon_0 v \cdot b}}$$

$$\boxed{-Q = \frac{p^2}{2m}} = \frac{z^2 \cdot e^4}{8\pi \varepsilon_0^2 V^2 b^2 m}$$

$$\left[\begin{array}{l} n \text{ elektronów w } dV \\ b, \; b + db \\ 2\pi n b \cdot db \; dx \\ 2\pi n Q b \cdot db \; dx \\ \quad b_{min}, \; b_{max} \end{array}\right]$$

$$-\frac{dE}{dx} = \frac{z^2 e^4 \cancel{2\pi} n \cancel{b}}{{}_4\cancel{8}\pi^{\cancel{1}} \varepsilon_0^2 V^2 b^{\cancel{2}} m} \int_{b_{min}}^{b_{max}} db$$

$$-\frac{dE}{dx} = \frac{z^2 e^4 \cdot n}{4\pi \varepsilon_0^2 m V^2} \int_{b_{min}}^{b_{max}} \frac{db}{b} = \frac{z^2 e^4 n}{4\pi \cdot \varepsilon_0^2 m V^2} \ln \frac{b_{max}}{b_{min}}$$

$$\frac{b}{V} < \frac{1}{f} \; T \quad \Rightarrow \boxed{b_{max} \approx \frac{V}{f}}$$

$$\lambda_B = \frac{h}{p} = \frac{h}{mV} \quad \Rightarrow \boxed{b_{min} \approx \frac{h}{mV}}$$

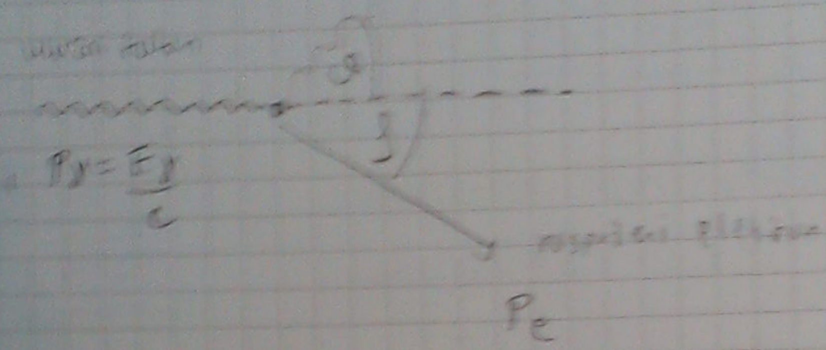
$$\boxed{-\frac{dE}{dx} = \frac{z^2 e^4 n}{4\pi m V^2 \varepsilon_0^2} \ln \frac{mV^2}{h f}} = \frac{e^4}{\frac{8}{2}\pi \varepsilon_0^2 m V^2} z^2 y \ln \frac{mV^2}{h f}$$

$$= \frac{e^4}{8\pi \varepsilon_0^2 m_e} \frac{M z^2}{E} y \ln \frac{2}{2} \frac{M \cdot V^2 \cdot m_e}{M \cdot h f}$$

$$\boxed{-\frac{dE}{dx} = \frac{e^4}{8\pi \varepsilon_0^2 m_e} \frac{M z^2}{E} y \ln \frac{2E m_e}{M \cdot h f}}$$

Comptonův rozptyl [illegible]

[illegible] [illegible] na [illegible] i [illegible] [illegible]



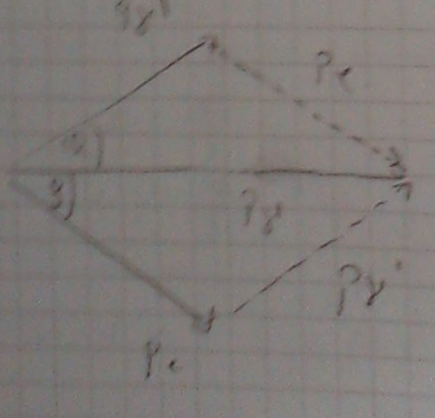
$E_\gamma' \quad p_\gamma' = \frac{E_\gamma'}{c}$

$E_\gamma \quad p_\gamma = \frac{E_\gamma}{c}$

$$\boxed{E_k = E_\gamma - E_\gamma'} \qquad \boxed{E_e = E - m_e c^2} \qquad \boxed{E = m c^2 \gamma}$$

ZoE

$$\boxed{\frac{E_\gamma}{c} = \frac{E_\gamma'}{c}\cos\vartheta + p_e\cos\varphi}, \qquad \boxed{0 = \frac{E_\gamma'}{c}\sin\vartheta - p_e\sin\varphi}$$



$x: \; p_\gamma = p_\gamma'\cos\vartheta + p_e\cos\varphi$

$y: \; p_\gamma'\sin\vartheta = p_e\sin\varphi$

sinusová poučka:

$$\boxed{\frac{\sin\vartheta}{\sin\varphi} = \frac{p_e}{p_\gamma'}}$$

kosinusová poučka:

$$\boxed{p_e^2 = p_\gamma^2 + p_\gamma'^2 - 2p_\gamma p_\gamma'\cos\vartheta} \;\Big|\; c$$

$$(p_e \cdot c)^2 = E_\gamma^2 + E_\gamma'^2 - 2E_\gamma E_\gamma' \cdot \cos\theta = E^2 - m^2c^4$$

$$\left[\begin{array}{l} \boxed{E^2 = p^2c^2 + m^2c^4} \\ \boxed{p^2c^2 = E^2 - m^2c^4} \end{array}\right]$$

$$\left[E_\gamma - E_\gamma' = E - mc^2 \Rightarrow E = E_\gamma - E_\gamma' + mc^2 \right]$$

$$E_\gamma^2 + E_\gamma'^2 - 2E_\gamma E_\gamma' \cos\theta = (E_\gamma - E_\gamma' + mc^2)^2 - m^2c^4$$

$$\left[\begin{array}{l} k = (E_\gamma - E_\gamma')^2 + 2mc^2(E_\gamma - E_\gamma') + m^2c^4 \\ k = E_\gamma^2 - 2E_\gamma E_\gamma' + E_\gamma'^2 + 2mc^2E_\gamma - 2mc^2E_\gamma' + m^2c^4 \end{array}\right]$$

$$\cancel{E_\gamma^2} + \cancel{E_\gamma'^2} - 2E_\gamma E_\gamma' \cdot \cos\theta = \cancel{E_\gamma^2} - 2E_\gamma E_\gamma' + \cancel{E_\gamma'^2} + 2mc^2E_\gamma -$$
$$- 2mc^2E_\gamma' + \cancel{m^2c^4} - \cancel{m^2c^4}$$

$$\cancel{2}mc^2E_\gamma - 2mc^2E_\gamma' - \cancel{2}E_\gamma E_\gamma' = -\cancel{2}E_\gamma E_\gamma' \cos\theta$$

$$E_\gamma'\left[mc^2 + E_\gamma - E_\gamma \cos\theta \right] = mc^2 E_\gamma$$

$$E_\gamma' = \frac{mc^2 E_\gamma}{mc^2 + E_\gamma(1 - \cos\theta)} = \frac{\cancel{mc^2} E_\gamma}{\cancel{mc^2}\left[1 + \frac{E_\gamma}{mc^2}(1 - \cos\theta)\right]}$$

$$\boxed{E_\gamma' = \frac{E_\gamma}{1 + \frac{E_\gamma}{mc^2}(1 - \cos\theta)}}$$

- kut raspršenja elektrona kod Comptonovog efekta

$$\boxed{\frac{E_\gamma}{c} = \frac{E_\gamma'}{c}\cos\vartheta + p_e\cos\varphi} \qquad \boxed{\frac{E_\gamma'}{c}\sin\vartheta = p_e\sin\varphi}$$

$$p_e = \frac{\frac{E_\gamma}{c} - \frac{E_\gamma'}{c}\cos\vartheta}{\cos\varphi}$$

$$p_e = \frac{\frac{E_\gamma'}{\cancel{c}}\sin\vartheta}{\sin\varphi} = \frac{\frac{E_\gamma}{\cancel{c}} - \frac{E_\gamma'}{\cancel{c}}\cos\vartheta}{\cos\varphi}$$

$$\boxed{E_\gamma' = \frac{E_\gamma}{1 + \frac{E_\gamma}{mc^2}(1-\cos\vartheta)}}$$

$$\frac{\frac{\cancel{E_\gamma}}{1 + \frac{E_\gamma}{mc^2}(1-\cos\vartheta)}\sin\vartheta}{\sin\varphi} = \frac{\cancel{E_\gamma} - \frac{\cancel{E_\gamma}}{1 + \frac{E_\gamma}{mc^2}(1-\cos\vartheta)}\cos\vartheta}{\cos\varphi}$$

$$\left[\ \boxed{\sin\vartheta = 2\sin\frac{\vartheta}{2}\cos\frac{\vartheta}{2}} \qquad \boxed{1-\cos\vartheta = 2\sin^2\frac{\vartheta}{2}}\ \right]$$

$$\frac{\sin\vartheta}{\sin\delta} = \frac{1 + \frac{E\gamma}{mc^2}(1-\cos\vartheta) - \cos\vartheta}{\cos\delta} \quad \Big/ \sin\delta$$

$$\sin\vartheta = \mathrm{tg}\,\delta\,(1-\cos\vartheta)\left(\frac{E\gamma}{mc^2}+1\right)$$

$$\frac{\sin\vartheta}{1-\cos\vartheta} = \mathrm{tg}\,\delta\left(\frac{E\gamma}{mc^2}+1\right)$$

$$\frac{\cancel{2}\cancel{\sin}\frac{\vartheta}{2}\cos\frac{\vartheta}{2}}{\cancel{2}\sin^{\cancel{2}}\frac{\vartheta}{2}} = \mathrm{tg}\,\delta\left(\frac{E\gamma}{mc^2}+1\right)$$

$$\boxed{\mathrm{ctg}\,\frac{\vartheta}{2} = \mathrm{tg}\,\delta\left(\frac{E\gamma}{mc^2}+1\right)}$$

Prigušenje neutronskog snopa

$$\boxed{\sigma_t = \sigma_s + \sigma_a}$$

raspršenje — apsorpcija

ukupni

$$\boxed{R = N \cdot \sigma \cdot \phi}$$

$$\boxed{\frac{d\phi}{dx} = N \cdot \sigma \cdot \phi}$$

put od x do $x + dx$

početni uvjet : $\boxed{\phi(0) = \phi_0}$

$$-d\phi = \phi \cdot \sigma_t \cdot N \, dx \quad / : \phi$$

$$-\frac{d\phi}{\phi} = \sigma_t N \cdot dx \quad / \int$$

$$-\int \frac{d\phi}{\phi} = \int \sigma_t \cdot N \, dx$$

$$-\ln \phi = \sigma_t N x + C$$

$$\ln \phi = -\sigma_t N x + C$$

$$\phi = e^{-\sigma_t N x + c}$$

$$\phi = e^{c} e^{-\sigma_t N x}$$

$$\phi = k \cdot e^{-\sigma_t N x}$$

$$\phi = \phi_0 e^{-\sigma_t N x}$$

$\phi(0) = \phi_0$

$\phi(0) = k \rightarrow k = \phi_0$

$$\boxed{\Sigma_t = \sigma_t N} \rightarrow \boxed{\phi = \phi_0 e^{-\Sigma_t x}}$$

Isto [illegible] [illegible], za intenzitet :

$$I(x) = I_0 e^{-\Sigma_t x}$$

$$\ln I = -\Sigma_t x \;\Big|\, \frac{\partial}{\partial x}$$

$$-\frac{\partial I}{I} = \Sigma_t \partial x$$

$$-\frac{\partial I}{I} = \Sigma_t \partial x$$

$$\boxed{p(x)\,\partial x = e^{-\Sigma_t \cdot x}\,\Sigma_t \partial x} = \Sigma_t e^{-\Sigma_t x} \partial x$$

$$\boxed{\lambda = \int x\, p(x)\, \partial x}$$

$$\lambda = \Sigma_t \int x e^{-\Sigma_t x} \partial x \qquad \boxed{I = u \cdot v - \int v\, du}$$

$$\left[\begin{array}{ll} u = x & dv = e^{-\Sigma_t} \partial x \\ du = \partial x & v = -\frac{1}{\Sigma_t} e^{-\Sigma_t x} \end{array}\right]$$

$$\lambda = \Sigma_t \left[\frac{-x \cdot e^{-\Sigma_t x}}{\Sigma_t} \Big|_0^{\infty} + \frac{1}{\Sigma_t} \int_0^{\infty} e^{-\Sigma_t x} \partial x \right.$$

$$= \Sigma_t \left[-\frac{1}{\Sigma_t} \underbrace{\lim_{x \to \infty} (x e^{-\Sigma_t x})}_{0} - \frac{1}{\Sigma_t^2} \underbrace{\lim_{x \to \infty} e^{-\Sigma_t}}_{0} + \frac{1}{\Sigma_t^2} \right]$$

$$\lambda = \cancel{\Sigma_t} \frac{1}{\Sigma_t^{\cancel{2}}} \Rightarrow \boxed{\lambda = \frac{1}{\Sigma_t}}$$

Doseg čestica istih brzina

$$\boxed{R = -\int_{E_0}^{0} \frac{dE}{\frac{e^2}{8\pi\varepsilon_0^2 m_e}\frac{MZ^2}{E}\ln\frac{2Em_e}{M\hbar\,\tau}}}$$

$$\boxed{V_1 = V_2}$$

$$\boxed{E = \frac{M\cdot V^2}{2}} \qquad M = \frac{2E}{V^2} \qquad E=0 \quad M=0$$

$$dE = \frac{V^2}{2}\,dM \qquad E = E_0 \quad M = \frac{2E_0}{V^2}$$

$$\Rightarrow E_0 = \frac{MV^2}{2}$$

$$R = -\int_{E_0}^{0} \frac{V^2\,dM}{\frac{\cancel{2}e^2}{{}_4\cancel{8}\pi\varepsilon_0^2 m_e}\frac{\cancel{M}\cdot Z^2\cdot 2}{\cancel{M}\cdot V^2}\ln\frac{\cancel{M}\cdot V^2 m_e}{\cancel{M}\cdot\hbar\,\tau}}$$

$$R = -\int_{E_0}^{0} \frac{V^2\,dM}{\frac{Z^2 e^2}{2\pi\varepsilon_0^2 m_e V^2}\ln\frac{V^2 m_e}{\hbar\cdot\tau}}$$

$$R = -\underbrace{\frac{2\pi\varepsilon_0^2 m_e V^4}{e^2\ln\frac{V^2 m_e}{\hbar\cdot\tau}}}_{K}\frac{1}{Z^2}\int_{\frac{MV^2}{2}}^{0} dM$$

$$R = -\frac{K}{Z^2}M\Big|_{\frac{MV^2}{2}}^{0} = \frac{KMV^2}{2Z^2}$$

$$\frac{R_1}{R_2} = \frac{\cancel{K}\frac{M_1\cancel{V^2}}{\cancel{2}Z_1^2}}{\cancel{K}\cdot\frac{M_2\cancel{V^2}}{\cancel{2}Z_2^2}} \Rightarrow \boxed{\frac{R_1}{R_2} = \frac{M_1 Z_2^2}{M_2 Z_1^2}}$$