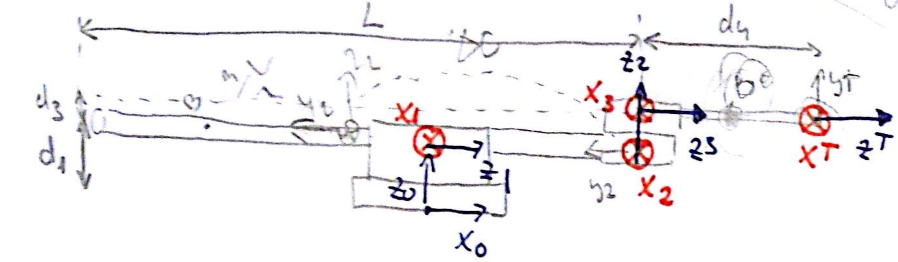
① robot ispituje kakvoću stakla i giba se kontinuirano po liniji

$x = H \quad y = [-w, +w]$



$q(0) = [\frac{\pi}{2}, 0, 0, 0]^T$

$p_0^4 = [d_4, 0, 0]$

	θ	d	α	a	m	Jr	limp
1	$q_1 + \frac{\pi}{2}$	d_1	$\frac{\pi}{2}$	0	$m_1 \sim 0$	$m_2 L^2$	1
2	0	q_2	$-\frac{\pi}{2}$	0	m_2	m_2	1
3	q_3	d_3	$\frac{\pi}{2}$	0	$m_3 \sim 0$	$m_2 \frac{d_4^2}{3}$	1
→4	q_4	d_4	0	0	$m_4 = m_2$	N/A	N/A

$q_1 = [-\frac{\pi}{4}, \frac{\pi}{4}]$

$q_2 = [0, L]$

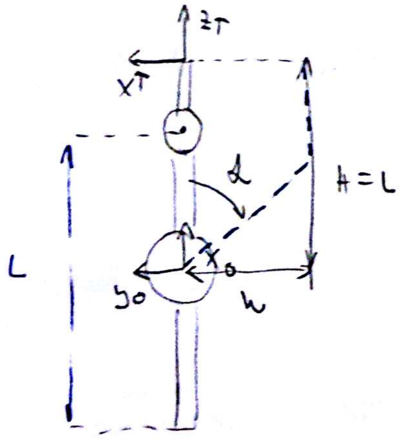
$q_3 = ?$

$$T_0^1 = \begin{bmatrix} -S_1 & 0 & C_1 & 0 \\ C_1 & 0 & S_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_0^2 = \begin{bmatrix} -S_1 & -C_1 & 0 & C_1 q_2 \\ C_1 & -S_1 & 0 & S_1 q_2 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_0^3 = \begin{bmatrix} -S_{13} & 0 & C_{13} & C_1 q_2 \\ C_{13} & 0 & S_{13} & S_1 q_2 \\ 0 & 1 & 0 & d_1 + d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^4 = \begin{bmatrix} -S_{13} & 0 & C_{13} & \boxed{C_1 q_2 + C_{13} d_4} \\ C_{13} & 0 & S_{13} & S_1 q_2 + S_{13} d_4 \\ 0 & 1 & 0 & d_1 + d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad p_0$$

$\frac{\partial p_0}{\partial q_i}$

$$D_4 = m_4 \begin{bmatrix} \frac{1}{3}(d_4^2 + 3q_2(d_4 C_3 + q_2)) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{d_4^2}{3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



①a) NADOPUNITI DH PARAMETRE

ODREDITI q_3 DA JE ALAT KONSTANTNO OKOMIT NA POVRŠINU

$\rightarrow \left.\begin{matrix} x_0 \cdot z_0^4 = 1 \\ y_0 \cdot z_0^4 = 0 \end{matrix}\right\}$ 3. STUPAC IZ T_0^4

$$\begin{bmatrix} z_0^4 \cdot x_0 \\ z_0^4 \cdot y_0 \\ z_0^4 \cdot z_0 \end{bmatrix} = \begin{bmatrix} C_{13} \\ S_{13} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$C_{13} = 1$

$\cos(q_1 + q_3) = 1$

$q_1 + q_3 = 0$

$-\frac{\pi}{4} + q_3 = 0$

$q_3 = \frac{\pi}{4}$

$\frac{\pi}{4} + q_3 = 0$

$q_3 = -\frac{\pi}{4}$

$q_3 = [-\frac{\pi}{4}, -\frac{\pi}{4}]$

1.b) Newton Euler, KUTNA BRZINA

→ ograničenje gibanja zglobova da robot zadrži konstantnu orjentaciju prema stolu.

$\omega_0 = 0$

$\omega_1 = \omega_0 + \dot{q}_1 \cdot \hat{z}_0 = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix}$

$\omega_2 = \omega_1 + \dot{q}_2 \cdot \hat{z}_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix}$

$\Downarrow$ 0 - zglob je translacijski

$\omega_3 = \omega_2 + \dot{q}_3 \hat{z}_2 = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix} + \dot{q}_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 + \dot{q}_3 \end{bmatrix}$

$\omega_4 = \omega_3 + \dot{q}_4 \hat{z}_3 = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 + \dot{q}_3 \end{bmatrix}$

$\Downarrow$ 0 - virtualan

$\boxed{\dot{q}_1 + \dot{q}_3 = 0}$ ograničenje gibanja zglobova da se zadrži konstantna okomitost alata na površinu koju se ispituje

1.c) NEWTON - EULER LINIJSKA BRZINA

$v_0 = 0$

$\Delta s_1 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}$

$v_1 = v_0 + \frac{\partial}{\partial t}(\Delta s_1) = v_0 + \vec{\omega_1} \times \vec{\Delta s_1} + |\Delta \dot{s}_1| \, \hat{\Delta s_1} = 0$

0 - u istom su smjeru vektori

$\Downarrow$ 0 rotacijski, konstantne duljine

$v_2 = v_1 + \frac{\partial}{\partial t}(\Delta s_2) = v_1 + \vec{\omega_2} \times \vec{\Delta s_2} + |\Delta \dot{s}_2| \cdot \hat{\Delta s_2}$

$\Delta s_2 = \begin{bmatrix} c_1 q_2 \\ s_1 q_2 \\ 0 \end{bmatrix}$

$\Delta s_2 = q_2 \cdot \hat{z}_1$

$\vec{\omega_2} \times \vec{\Delta s_2} = \begin{vmatrix} x & y & z \\ 0 & 0 & 1 \\ c_1 & s_1 & 0 \end{vmatrix} \dot{q}_1 q_2 = \dot{q}_1 q_2 \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix}$

$$V_2 = \dot{q}_1 q_2 \begin{bmatrix} -S_1 \\ C_1 \\ 0 \end{bmatrix} + \dot{q}_2 \begin{bmatrix} C_1 \\ S_1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\dot{q}_1 q_2 S_1 + \dot{q}_2 C_1 \\ \dot{q}_1 q_2 C_1 + \dot{q}_2 S_1 \\ 0 \end{bmatrix}$$

$$V_3 = V_2 + \frac{\partial}{\partial t}(\vec{\Delta S_3}) = V_2 + \underbrace{\vec{\omega_3} \times \vec{\Delta S_3}}_{\to 0} + \underbrace{\dot{q}_3}_{\to 0 \text{ - rotacijski}} \cdot \hat{z}_2$$

$$\omega_3 \times \Delta S_3 = \begin{vmatrix} x & y & z \\ 0 & 0 & \dot{q}_1 + \dot{q}_3 \\ 0 & 0 & d_3 \end{vmatrix} = 0 \qquad \Delta S_3 = \begin{bmatrix} 0 \\ 0 \\ d_3 \end{bmatrix}$$

$$V_3 = V_2 = \begin{bmatrix} -\dot{q}_1 q_2 S_1 + \dot{q}_2 C_1 \\ \dot{q}_1 q_2 C_1 + \dot{q}_2 S_1 \\ 0 \end{bmatrix}$$

$$V_4 = V_3 + \frac{\partial}{\partial t}\vec{\Delta S_4} = V_3 + \omega_4 \times \Delta S_4 + \underbrace{\dot{\Delta S_4}}_{\to 0 \text{ virtualan}}$$

$$\Delta S_4 = \begin{bmatrix} C_{13} d_4 \\ S_{13} d_4 \\ 0 \end{bmatrix} \qquad \omega_4 \times \Delta S_4 = \begin{bmatrix} x & y & z \\ 0 & 0 & \dot{q}_1 + \dot{q}_3 \\ C_{13} d_4 & S_{13} d_4 & 0 \end{bmatrix} = \begin{bmatrix} -(\dot{q}_1 + \dot{q}_3) S_{13} d_4 \\ (\dot{q}_1 + \dot{q}_3) C_{13} d_4 \\ 0 \end{bmatrix}$$

$$V_4 = \begin{bmatrix} -\dot{q}_1 q_2 S_1 + \dot{q}_2 C_1 - (\dot{q}_1 + \dot{q}_3) S_{13} d_4 \\ \dot{q}_1 q_2 C_1 + \dot{q}_2 S_1 + (\dot{q}_1 + \dot{q}_3) C_{13} d_4 \\ 0 \end{bmatrix}$$

2.a) TENZOR INERCIJE $m_1 = m_3 = 0$

$$\Delta c_1 = \begin{bmatrix} 0 & -\frac{d_1}{2} & 0 & 1 \end{bmatrix}^T \quad \Delta c_2 = \begin{bmatrix} 0 & \frac{L}{2} & 0 & 1 \end{bmatrix}^T$$

$$\Delta c_3 = \begin{bmatrix} 0 & -\frac{d_3}{2} & 0 & 1 \end{bmatrix}^T \quad \Delta c_4 = \begin{bmatrix} 0 & 0 & -\frac{d_4}{2} & 1 \end{bmatrix}^T$$

$$D_1' = \frac{\overset{\to 0}{m_1} d_1^2}{12} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$D_1 = 0$$

$$D_2' = \frac{m_2 \cdot L^2}{12} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad D_2 = R_0^2 \cdot D_2' \cdot R_0^{2T}$$

$$D_2 = \begin{bmatrix} -S_1 & -C_1 & 0 \\ C_1 & -S_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -S_1 & C_1 & 0 \\ -C_1 & -S_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \frac{m_2 \cdot L^2}{12}$$

$$D_2 = \begin{bmatrix} -S_1 & 0 & 0 \\ C_1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -S_1 & C_1 & 0 \\ -C_1 & -S_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \frac{m_2 L^2}{12} = \frac{m_2 L^2}{12} \begin{bmatrix} S_1^2 & -S_1 C_1 & 0 \\ -S_1 C_1 & C_1^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C_2 = H \cdot T_0^2 \cdot \Delta C_2 = \begin{bmatrix} -S_1 & -C_1 & 0 & C_1 q_2 \\ C_1 & -S_1 & 0 & S_1 q_2 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ \frac{L}{2} \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -C_1 \frac{L}{2} + C_1 q_2 \\ -S_1 \frac{L}{2} + S_1 q_2 \\ d_1 \\ 0 \end{bmatrix}$$

$$J^2 = \begin{bmatrix} \frac{\partial c_2}{\partial q_1} & \frac{\partial c_2}{\partial q_2} & \frac{\partial c_2}{\partial q_3} \\ \hline z_0 & 0 & z_2 \end{bmatrix} = \begin{bmatrix} S_1 \frac{L}{2} - S_1 q_2 & C_1 & 0 \\ -C_1 \frac{L}{2} + C_1 q_2 & S_1 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$D_2 = A_2^T \cdot m_2 \cdot A_2 + B_2^T \cdot D_2 \cdot B_2$$

$$m_2 \cdot \begin{bmatrix} S_1 (\frac{L}{2} - q_2) & -C_1 (\frac{L}{2} - q_2) & 0 \\ C_1 & S_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} S_1 (\frac{L}{2} - q_2) & C_1 & 0 \\ -C_1 (\frac{L}{2} - q_2) & S_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} =

$$D_2 = m_2 \begin{bmatrix} \left(\frac{L}{2} - g_2\right)^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} s_1^2 & -s_1c_1 & 0 \\ -s_1c_1 & c_1^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \frac{m_2 L^2}{12}$$

$$D_2 = m_2 \begin{bmatrix} \left(\frac{L}{2} - g_2\right)^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \frac{m_2 L^2}{12}$$

$$D_2 = m_2 \left[\begin{bmatrix} \left(\frac{L}{2} - g_2\right)^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right] \frac{m_2 L^2}{12}$$

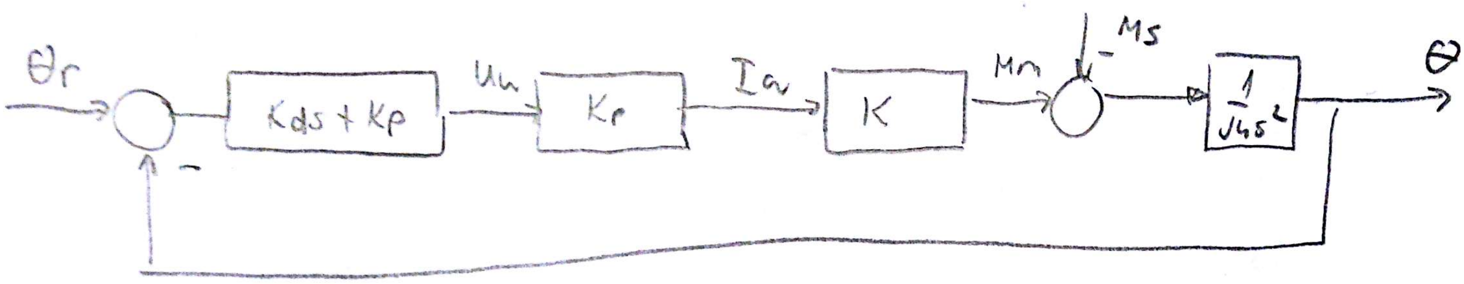
$$D_2 = m_2 \begin{bmatrix} \frac{L^2}{12} + \left(\frac{L}{2} - g_2\right)^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\underline{D_3 = 0 \quad m_3 = 0}$$

D_4 = Mirko izračunao

$$D_{uk} = D_2 + D_4 = \begin{bmatrix} m_2\left(\frac{L^2}{12} + \left(\frac{L}{2} - g_2\right)^2\right) + \frac{m_4}{3}\left(d_4^2 + 3g_2(d_4c_3 + g_2)\right) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{d_4^2}{3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2.b) REGULACIJSKA PETLJA POLOŽAJA ZGLOBA UZ UPRAVLJANJE MOMENTOM, 2c) UZ KOMPENZACIJSKO PROŠIRENJE



MIN MAX METODA

$K_p \cdot K = 1 \qquad m_2 = m_4 \qquad d_4 = \frac{L}{2}$

$$D_{11} = m_2\left(\frac{L^2}{12} + \left(\frac{L}{2} - q_2\right)^2\right) + \frac{m_4}{3}\left(d_4^2 + 3q_2(d_4 C_3 + q_2)\right)$$

$\omega_n = \sqrt{\frac{K_p \cdot J_{ue}}{J_u}} \qquad K_p = \omega_n^2 \cdot \frac{J_{max}}{J_{min}}$

$J_{ue} = J_{min} \qquad K_d = \frac{\varepsilon \cdot 2}{\sqrt{\frac{J_{min}}{K_p \cdot J_{max}}}}$

$J_u = J_{max}$

$q_{2min} = 0 \qquad q_{2max} = L$

$q_{3min} = \frac{\pi}{4} \qquad q_{3max} = 0 \Rightarrow \cos 0 = 1$

$$J_{min}' = J_r + \frac{D_{11}}{i^2} = m_2 L^2 + D_{11min} = m_2L^2 + \frac{5}{12}m_2L^2 = \frac{17}{12}m_2L^2$$

$$D_{11}(q_{2min}, q_{3min}) = m_2\frac{L^2}{12} + m_2\frac{L^2}{4} + \frac{m_4}{3}\left(\frac{L}{2}\right)^2 \qquad m_2 = m_4$$

$$D_{11min} = \frac{m_2L^2}{12} + m_2\frac{L^2}{4} + \frac{m_4L^2}{12} = \frac{5}{12}m_2L^2$$

$$D_{11max} = m_2\left(\frac{L^2}{12} + \left(\frac{L}{2} - L\right)^2\right) + \frac{m_4}{3}\left(\left(\frac{L}{2}\right)^2 + 3L\left(\frac{L}{2} + L\right)\right) \quad \boxed{m_4 = m_2}$$

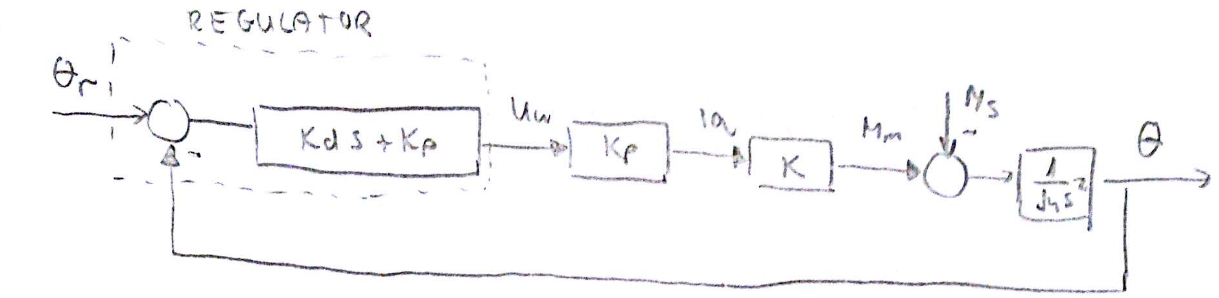
$$= \frac{m_2 L^2}{3} + \frac{m_2}{3}\left[\frac{L^2}{4} + \frac{9}{2}L^2\right] = \frac{23}{12} m_2 L^2$$

$$J_{max}' = \frac{D_{11}^{max}}{1} + m_2 L^2 = \frac{35}{12} L^2 m_2$$

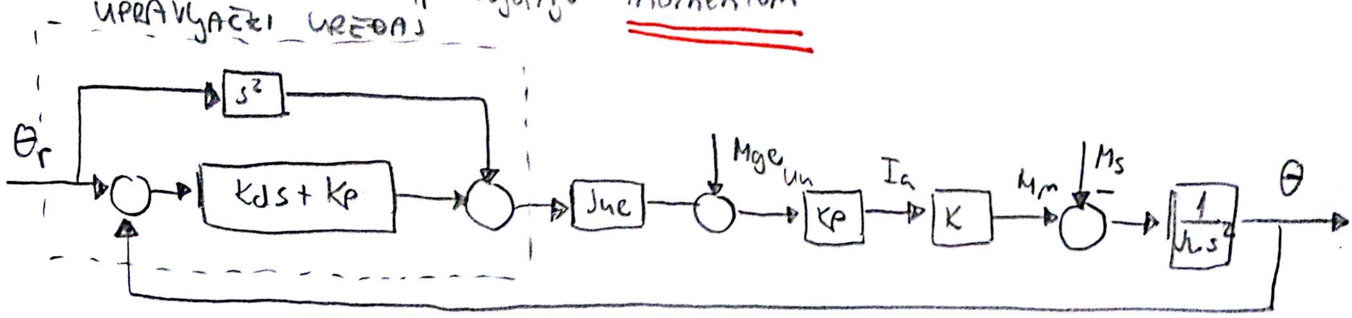
$$K_p = \omega_n^2 \cdot \frac{J_{max}}{J_{min}} = \frac{\frac{35}{12}}{\frac{5}{12}} = 7\omega_n^2$$

$$K_d = \frac{2 \cdot \varepsilon_{min}}{\sqrt{\frac{J_{min}}{K_p \cdot J_{max}}}} = \frac{2 \cdot \varepsilon_{min}}{\sqrt{\frac{\frac{5}{12}}{7 \cdot \frac{35}{12}}}} = 14\,\varepsilon_{min}$$

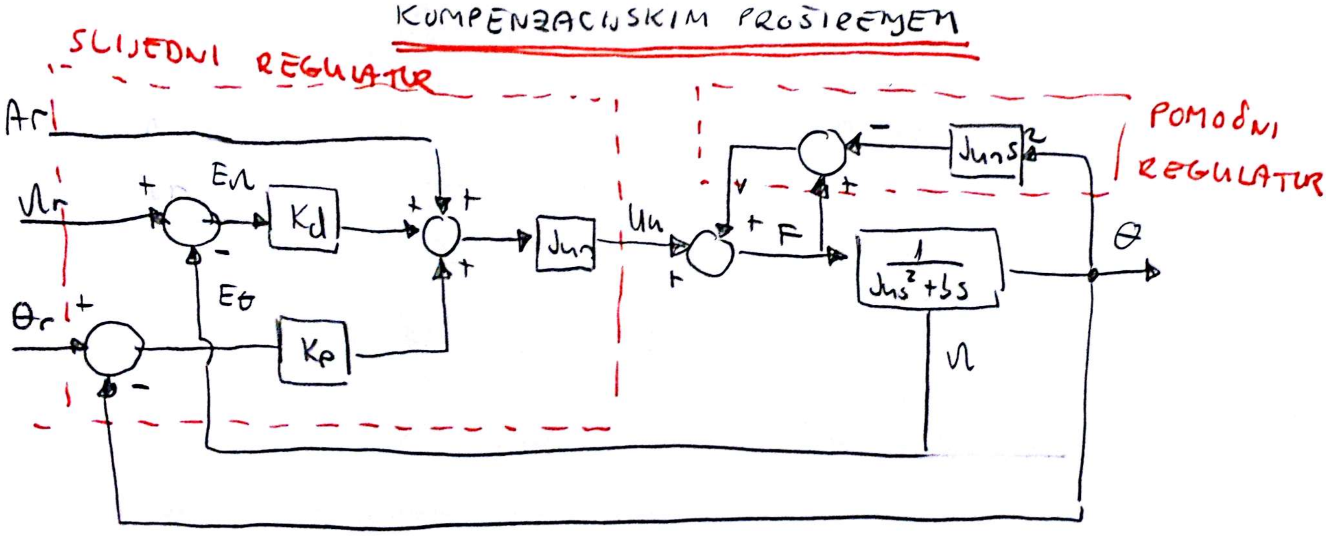
$M_{ge} = ?$



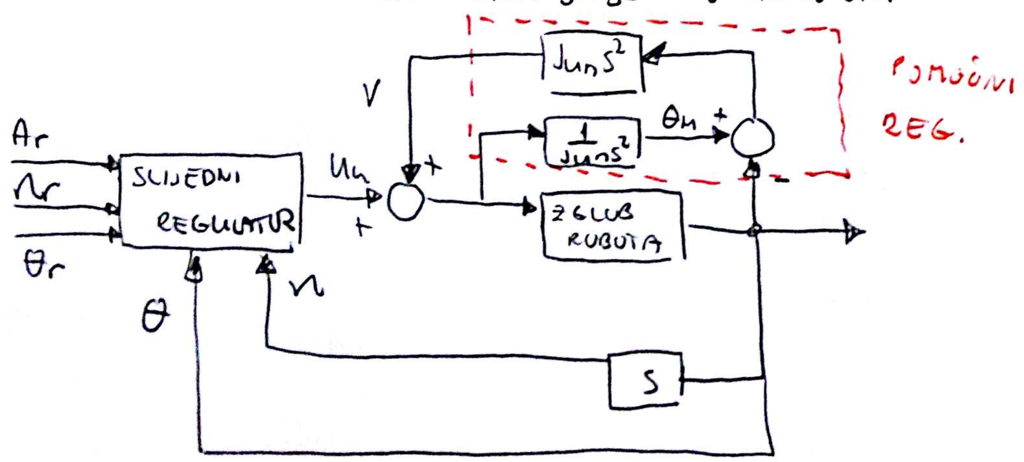
→ Blokovska shema regulacijske petlje položaja uz upravljanje momentom



→ ISTA SHEMA PO UPRAVLJANJU MOMENTOM, ALI S KOMPENZACIJSKIM PROŠIRENJEM



– SHEMA ROBUSNOG UPRAVLJANJA POLOŽAJEM PO HSIAOVOJ METODI uz UPRAVLJANJE PO MOMENTU



SHEMA ADAPTIVNOG UPRAVLJANJA POLOŽAJEM PO HSIAOVOJ METODI uz UPRAVLJANJE PO MOMENTU