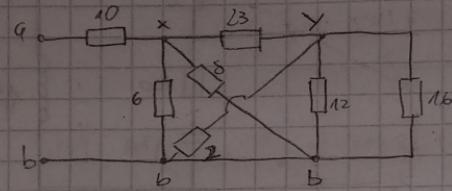


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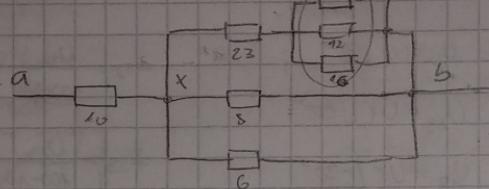
(1)

$$R_{ab} = ?$$



$$R_p' = 1,05 \Omega$$

$$R_n = \frac{261}{21} \Omega$$



$$G_p = \frac{1}{R_n} + \frac{1}{8} + \frac{1}{6}$$

$$R_p = \frac{1}{G_p} = 3 \Omega$$

$$R_{ab} = n \cdot R_p = 13 \Omega$$

(2)

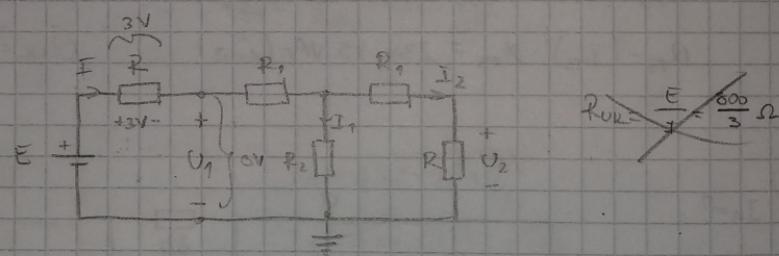
$$E = 8 \text{ V}$$

$$R = 100 \Omega$$

$$U_1 = 5 \text{ V}$$

$$\underline{U_2 = 1 \text{ V}}$$

$$\underline{\underline{R_2 = ?}}$$



$$I_2 = \frac{U_2}{R} = 0,01 \text{ A}$$

$$\underline{\underline{I = \frac{3}{100} = 0,03 \text{ A}}}$$

$$I_1 = I \cdot \frac{R_2 + R}{R_1 + R_2 + R}$$

$$I_2 = I \cdot \frac{R}{R_1 + R_2 + R}$$

$$\underline{\underline{I_1 = I - I_2 = 0,02 \text{ A}}}$$

$$I_1 \cdot R_2 + I \cdot R_1 = U_1$$

$$(I) \left(\frac{R_1 R}{R_1 + R_2 + R} \right) \cdot R_2 + I \cdot R_1 = U_1 / : I$$

$$\cancel{\frac{(R_1 + R) R_2}{R_1 + R_2 + R} + R_1 = \frac{500}{3}}$$

$$\cancel{R_1 R_2 + 100 R_2 + R_1^2}$$

$$I_1 \cdot R_2 = U_1 + I_2 \cdot R_1$$

$$I \cdot \frac{R_1 + R}{R_1 + R_2 + R} R_2 = 1 + I \cdot \frac{R_2}{R_1 + R_2 + R} R_1 / : I$$

$$\cancel{R_1 R_2 + R_1 R_2 = \frac{100}{3} \cdot (R_1 + R_2 + R) + R_1 R_2}$$

$$100 R_2 = \frac{100}{3} R_1 + \frac{100}{3} R_2 + \frac{10000}{3}$$

$$\cancel{\frac{200}{3} R_2 = \frac{100}{3} R_1 + \frac{10000}{3} / 100}$$

$$2 R_2 = R_1 + 100$$

$$R_{\text{tot}} = R + R_1 + \frac{R_2 \cdot (R_1 + R)}{R_1 R_2 + R}$$

$$\frac{800}{3} (R_1 + R_2 + R) = R(R_1 + R_2 + R) + R_1(R_1 + R_2 + R) + R_2(R_1 + R)$$

$$\frac{800}{3} R_1 + \frac{800}{3} R_2 + \frac{80000}{3} = 100 R_1 + 100 R_2 + 10000 + R_1^2 + R_1 R_2 + 100 R_1 + R_1 R_2 + 100 R_2 / \cdot 3$$

$$800(2R_2 - 100) + 800R_2 + 60000 = 300(2R_1 - 100) + 300R_2 + ((2R_2 - 100)^2) + (2R_2 - 100)R_2 + 100(2R_2 - 100) \\ + (2R_2 - 100) + 100R_2$$

$$1600R_2 - 80000 + 800R_2 + 60000 = 600R_2 - 30000 + 2R_2^2 - 100R_2 + 200R_2 - 10000 + 2R_2 - 100 \\ - 100R_2 + 4R_2^2 - 400R_2 + 12000$$

$$2400R_2 = 6R_2^2 + 700R_2 - 30100$$

$$6R_2^2 - 1698R_2 - 30100 = 0$$

$$R_2 = 300 \Omega$$

$$I_1 R_2 = I_2 (R_2 + R_1)$$

$$0,02 R_2 = 0,01 (100 + R_1)$$

$$U_2 + I_2 \cdot R_1 = U_1 - I \cdot R_1$$

$$1 + 0,01 R_1 \stackrel{!}{=} 5 - 0,03 R_1$$

$$R_1 = 100 \Omega$$

$$\frac{R_2}{R_1} = 1$$

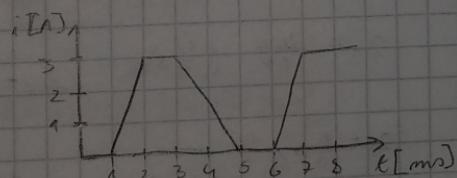
$$U_P = I_2 \cdot (R_2 + R_1)$$

$$R_2 = \frac{U_P}{I_2} = 100 \Omega$$

$$U_P = 2V$$

③

$$Q_U(U_K) = ?$$



$$Q_1 = \frac{3}{1} = 3 \text{ mAs}$$

$$Q_2 = 3 \text{ mAs}$$

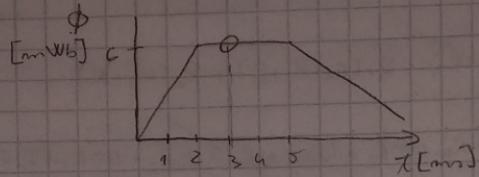
$$Q_{UK} = Q_1 + Q_2 + Q_3$$

$$Q_{UK} = 7,5 \text{ mAs}$$

$$Q_3 = \frac{3}{2} = 1,5 \text{ mAs}$$

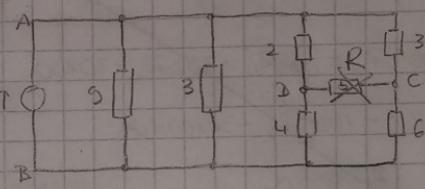
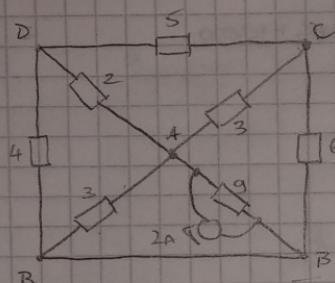


(4) $u_i(t=3 \text{ ms})$



$$\frac{d\phi}{dt} = 0 \quad u_i = 0 \text{ V}$$

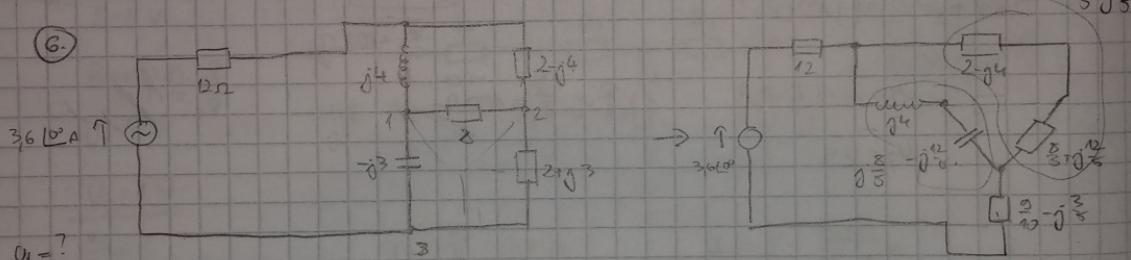
(5)



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$$I_B = 0 \text{ A}$$

(6)



$$Z_{12} = 8 \Omega$$

$$Z_1 = -j\frac{4}{10} = -j\frac{12}{5} \Omega$$

$$Z_{UL} = 12 + j\frac{9}{10} - j\frac{3}{5} + Z_P$$

$$Z_{23} = 2 + j3 \Omega$$

$$Z_2 = \frac{8}{5} + j\frac{12}{5}$$

$$Z_P = j\frac{8}{5} \parallel \left(\frac{8}{5} - j\frac{5}{5} \right)$$

$$Z_D = Z_{12} + Z_{23} + Z_{31} = 10 \Omega$$

$$Z_3 = \frac{9}{10} - j\frac{3}{5}$$

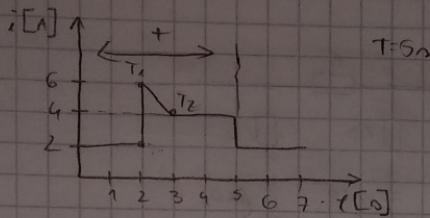
$$Z_P = \frac{32}{45} + j\frac{8}{5}$$

$$Z_{UL} = 13.61 + j5 \Omega$$

$$I_L = I \cdot \frac{\frac{12}{5} - j\frac{8}{5}}{\frac{10}{5}} = 3.94 \angle -23.98^\circ \text{ A}$$

$$U_L = |I_L| \cdot X_L = 15.76 \text{ V}$$

① $I_{ef} = ?$



$$I_{ef} = \sqrt{\frac{1}{T} \int_0^T x(t)^2 dt}$$

$$\int_2^3 y^2 dy = \int_2^3 (4x^2 - 40x + 100) dx = \left(\frac{4x^3}{3} - \frac{40x^2}{2} + 100x \right) \Big|_2^3 =$$

$$y-6 = \frac{(4-6)}{(3-2)} (x-2)$$

$$= 106 - 130,67 = 25,33$$

$$(Q = I \cdot t)$$

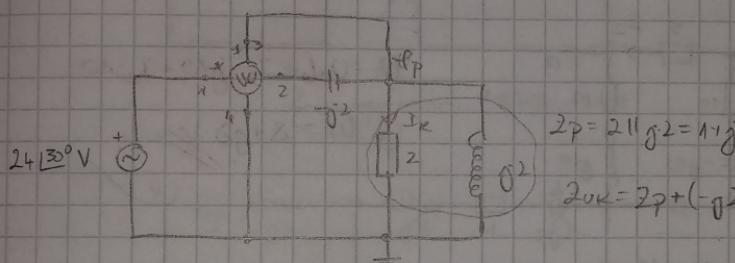
$$y = -2x + 10 / 2$$

$$I_{ef} = \sqrt{\frac{1}{T} (4 \cdot 2 + 25,33 \cdot 1 + 16 \cdot 2)} = 3,615 A$$

$$y^2 = 4x^2 - 40x + 100$$

~~w~~

② $P_W = ?$



$$Z_P = 211 \angle 2 = 11 j$$

$$Z_{LK} = Z_P + (-j2) = 1 - j 2$$

$$I_{LK} = \frac{24 \angle 30^\circ}{Z_{LK}} = 12 \angle 30^\circ = I_W$$

$$\alpha = \angle u - \angle i = 120^\circ - 75^\circ = 45^\circ$$

$$I_R = I_{LK} \cdot \frac{j2}{2+j2} = 12 \angle 120^\circ$$

$$\varphi_P = I_R \cdot R = 12 \angle 120^\circ \cdot 2 = 24 \angle 120^\circ$$

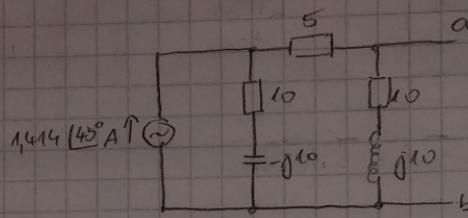
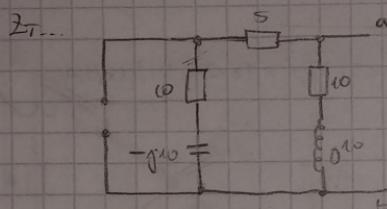
$$P_W = I_W \cdot U_W \cos(\alpha)$$

$$P_W = 288 W$$

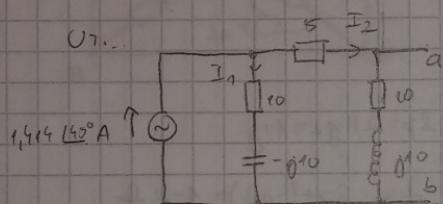
$$U_W = \varphi_P - \alpha = 24 V$$

(9)

$$Z_{Th}, V_{Th} = ? \quad (a, b)$$

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$$Z_T = (15-j10) \parallel (10, j10) = 10+j2 \Omega$$



$$V_T = V_{ab} = I_2 \cdot j10 + I_2 \cdot 10$$

$$V_T = 8+j2 \text{ V}$$

$$I_2 = I \cdot \frac{10-j10}{10-j10+15+j10} = 0,8 \text{ A}$$

(10)

$$z_1 = z_2 = z_3 = 120 \angle 75^\circ \Omega$$

$$I_E = \frac{380}{120} = 3,167 \quad V_F = 0,5$$

$$U_F = 380 \text{ V}$$

$$P = ?$$

$$P_{JK} = 3 U_F I_F \cos \varphi$$

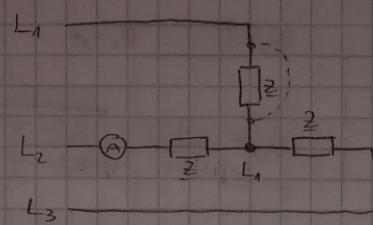
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$$P_{JK} = 2552,9 \text{ W}$$

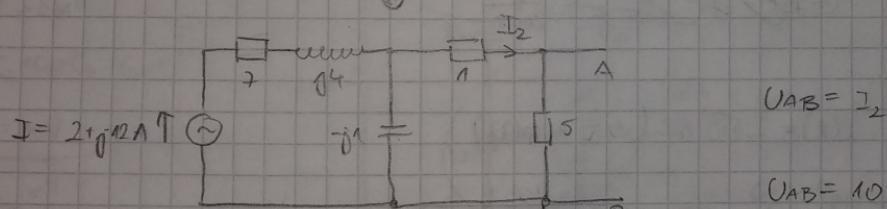
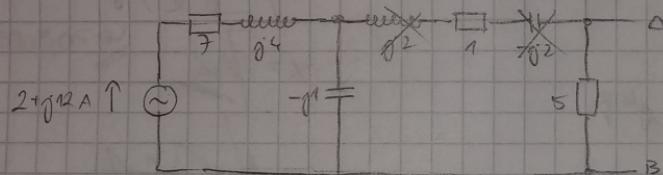
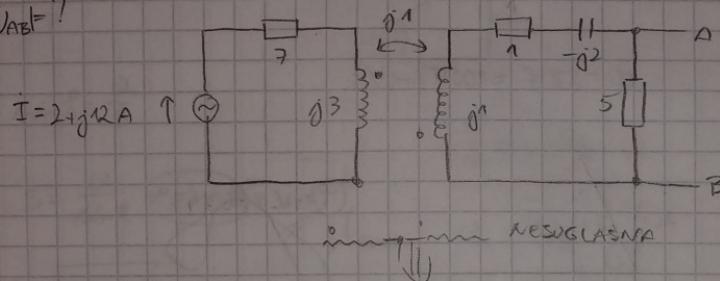
(11) $I_A = 3A$
 $I_{A'} = ?$



1) $U_f = I_A \cdot Z = 3Z$

2) $I_{A'} = \frac{U_f}{Z} = \frac{\sqrt{3}U_f}{Z} = \frac{\sqrt{3} \cdot 3Z}{Z} = 5\sqrt{3}A$

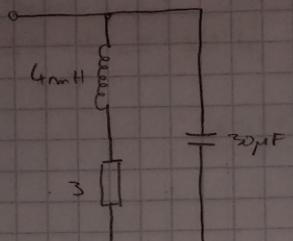
(12) $|U_{AB}| = ?$



$I_2 = I \cdot \frac{-j1}{6+j1} = 2A$

$U_{AB} = 10V$

(13.)



$$\omega_0 = ?$$

$$R_C = 0$$

$$\frac{L}{C} = \frac{400}{3}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \cdot \sqrt{\frac{R_C^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}}$$

$$\omega_0 = 2884,75 \cdot \sqrt{\frac{9 - \frac{400}{3}}{-400}} = 2884,75 \cdot \sqrt{\frac{323}{-400}}$$

$$\omega_0 = 2788 \text{ rad/s}$$

(14.)

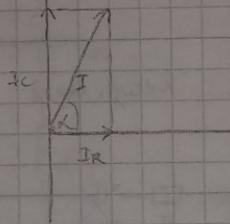
$$\underline{I} = 2 + j4$$

$$\omega = 2\pi f = 100\pi \text{ rad/s}$$

$$i(t) = ?$$

$$\underline{i(t=1)} = ?$$

$$f = 50 \text{ Hz}$$



$$\theta = \arctan \frac{X}{R} = 63,43^\circ$$

$$\rho = \sqrt{R^2 + X^2} / k_0$$

$$X = -2 \Omega$$

$$C = \frac{1}{\omega X} = \frac{1}{200\pi R}$$

~~$$t = R \sqrt{R^2 + X^2} / \omega = 2 \text{ s}$$~~

$$i(t) = 2\sqrt{10} \sin(\omega t + 1,107)$$

$$i(t=1) = 2\sqrt{10} \cdot \sin(100\pi + 1,107)$$

$$i(t=1) = 5,66 \text{ A}$$

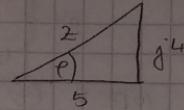
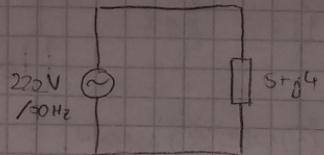
(15)

$$Z = 5 + j4 \Omega$$

$$U_{eff} = 220 \text{ V}$$

$$f = 50 \text{ Hz}$$

$$\cos \varphi = 0,9 (\text{ind.})$$



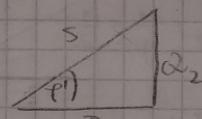
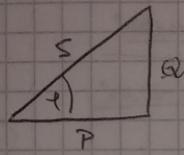
$$C = ?$$

$$\cos \varphi' = 0,9$$

$$\varphi = 38,66^\circ$$

$$\varphi' = 25,84^\circ$$

$$I = \frac{220}{5j4} = 34,36 \text{ A}$$



$$\sin \varphi = \frac{Q}{S}$$

$$S = \frac{Q}{\sin \varphi} = 7358,7 \text{ VA}$$

$$P = \sqrt{S^2 - Q^2}$$

$$P = 5903,05 \text{ W}$$

$$tg \varphi' = \frac{Q_2}{P}$$

$$Q_2 = P \cdot tg \varphi'$$

$$Q_2 = 2853,73 \text{ VAR}$$

$$Q_C = Q_1 - Q_2$$

$$Q_C = 1863,16 \text{ VAR}$$

$$Q_C = \frac{U^2}{\omega C}$$

$$Q_C = \omega C U^2$$

$$C = \frac{Q_C}{\omega U^2} = 122,5 \mu F$$

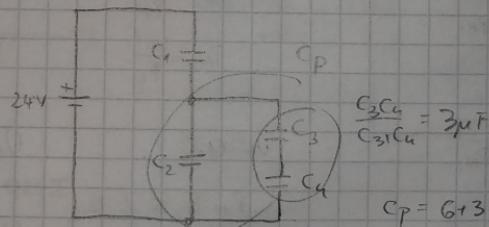
(16)

$$C_1 = 8 \mu F$$

$$C_2 = C_3 = C_4 = 6 \mu F$$

$$U = 24 \text{ V}$$

$$V_{CK} = ?$$



$$C_P = 6 + 3 = 9 \mu F$$

$$U = \frac{Q_C}{C}$$

$$Q_4 = Q_{34}$$

$$C_{UK} = \frac{C_1 \cdot C_P}{C_1 + C_P} = 4,12353 \mu F$$

$$U = ?$$

$$U_4 = \frac{Q_{34}}{C_4}$$

$$C = \frac{Q_{UK}}{U}$$

$$U_4 = 5,65 \text{ V}$$

$$Q_{UK} = C_{UK} \cdot U = 101,65 \mu A$$

$$Q_2 + Q_{34} = Q_{UK}$$

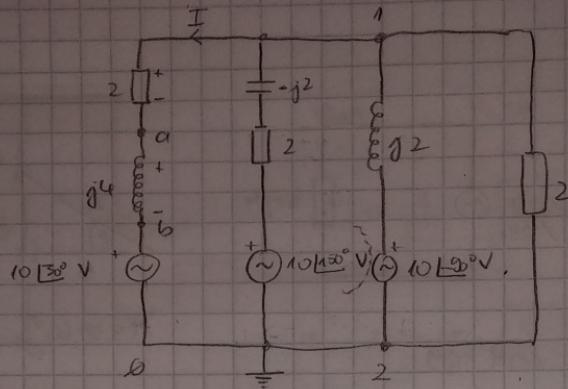
$$\frac{Q_2}{C_2} = \frac{Q_{34}}{C_{34}}$$

$$3Q_{34} = Q_{UK}$$

$$Q_2 = \frac{C_2}{C_{34}} \cdot Q_{34} = 2Q_{34}$$

$$Q_{34} = 33,88 \mu A$$

(17) $U_{ab} = ?$



$$U_{12} = \frac{10\angle 30^\circ + 10\angle 150^\circ + 10\angle -90^\circ}{2j^4 + 2-j^2 + j^2} = \frac{\sqrt{30}}{2} \angle -161,25^\circ$$

$$U_{12} = 7,166 \angle -133,96^\circ \text{ V}$$

$$+ 10\angle 30^\circ + I \cdot j^4 + I \cdot 2 = U_{12}$$

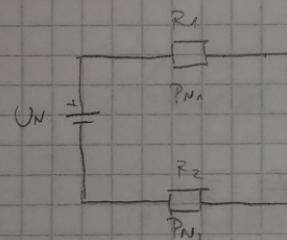
$$(2j^4) I = 17 \angle -143,31$$

$$U_{AB} = I \cdot j^4$$

$$U_{AB} = 13,21 \angle -116,74^\circ \text{ V}$$

$$I = 3,802 \angle 143,25^\circ \text{ A}$$

(18)



$$P_{N1} = \frac{U_N^2}{R_1}$$

$$P_{N2} = \frac{U_N^2}{R_2}$$

$$U_N^2 = U_N^2$$

$$P_{N1} R_1 = P_{N2} R_2 / : P_{N1}$$

$$R_1 = 2R_2$$

$$P_1 = \frac{U_N^2}{R_1} = \frac{\frac{2}{3}U_N^2}{R_2} = \frac{2}{3} \frac{U_N^2}{R_2}$$

$$P_2 = \frac{\frac{1}{3}U_N^2}{R_2} = \frac{1}{3} \frac{U_N^2}{R_2}$$

$$U_1 = U_N \cdot \frac{2R_2}{3R_2} = \frac{2}{3} U_N$$

$$P_{N1} = 50 \text{ W}$$

$$P_{N2} = 100 \text{ W}$$

$$U_{N1} = U_{N2} = U_N$$

$$P_{UK} = ?$$

$$U_2 = U_N \cdot \frac{R_2}{3R_2} = \frac{1}{3} U_N$$

$$P_{UK} = P_1 + P_2$$

$$P_{UK} = \frac{1}{3} \frac{U_N^2}{R_2} = \frac{1}{3} P_{N2}$$

$$P_{UK} = 33,33 \text{ W}$$

$$(19) \quad u(t) = 1 + U_1 \sin(\omega t) [V]$$

$$U_{\text{eff}} = 10 \text{ V}$$

$$\underline{U_1 = ?}$$

$$U_{\text{eff}} = \sqrt{1^2 + \left(\frac{U_1}{R}\right)^2} / 2$$

$$100 = 1 + \frac{U_1^2}{4}$$

$$U_1^2 = 4 \cdot 99$$

$$U_1^2 = 396 \text{ V}^2 \Rightarrow U_1 = 19,9 \text{ V}$$

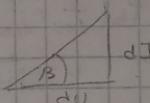
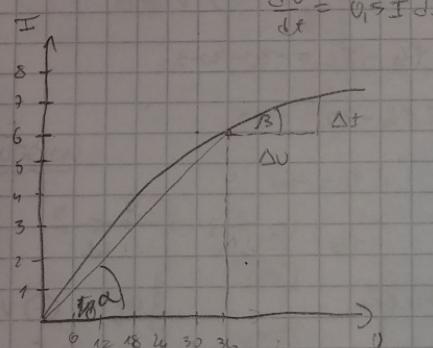
$$U_{1,\text{eff}} = \frac{U_1}{\sqrt{2}} = 19,09 \text{ V}$$

$$(20) \quad U = U_{25} I^2$$

$$I^2 = \frac{U}{U_{25}} = 36 \quad I = 6 \text{ A}$$

$$\frac{U=9 \text{ V}}{\underline{R_s = ?}}$$

$$R_s = \frac{U}{I} = \left(\frac{3}{2} \cancel{R_d}\right) = \frac{3}{2}$$



$$\beta = \arctan \frac{dU}{dI} = \frac{dU}{dI} = \frac{0,5 \frac{I}{dI} dI}{dI} = \frac{1}{2} I$$

$$\frac{R_s}{R_d} = \frac{\frac{1}{2} I}{\frac{1}{2} I} = \frac{1}{2}$$

$$R_s = \frac{U}{I} = \frac{U_{25} I^2}{I} = U_{25} I$$