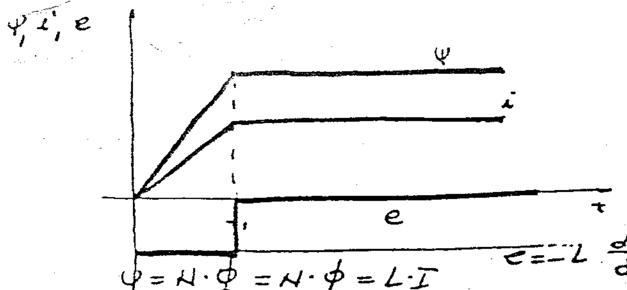
1.5

(3.) Nacrtani su vremenski dijagrami magnetskog toka u svitku

Nacrtajte (kvalitativno) dijagrame vremenske ovisnosti struje i napona



$\psi = N\cdot\Phi = N\cdot\phi = L\cdot I$

$e = -L\,\frac{di}{dt}$

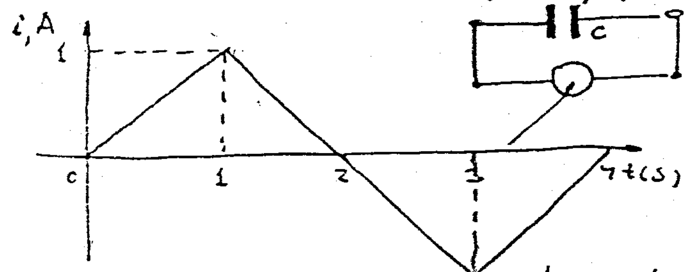
za $(0,t_1)$ $\frac{di}{dt} = const$

$e = -L\cdot const = -k_1$

1.6

(4.) Na idealni strujni izvor priključen je nenabijeni kondenzator $C=1F$. Na slici je prikazan dijagram vremenske ovisnosti struje izvora

Odredite $u_c(t)$ i nacrtajte dijagram njegove vremenske ovisnosti.



Naboj na kondenzatoru je

$Q = C\cdot u \Rightarrow u = \frac{Q}{C} = \frac{1}{C}Q$

$u = \frac{1\cdot Q}{C}$ $\qquad du = \frac{1}{C}\,dQ$

$du_c = \frac{1}{C}\cdot i\cdot dt \qquad i = f(t) \qquad u_c = \frac{1}{C}\int_0^t i\,dt + k$

$u_c = \frac{1}{C}\int_0^t i\,dt + k$ $\qquad$ za $t\in(0,1) \Rightarrow i = t$

za $t\in(1,3) \Rightarrow i = -t+2$

za $t\in(3,4) \Rightarrow i = t-4$

za $t\in(0,1) \Rightarrow u_c = \frac{1}{C}\int_0^t t\,dt + k = \frac{1}{C}\frac{t^2}{2}$ $\qquad C = 1F \Rightarrow u_c = \frac{t^2}{2}$

$\left[\text{za } t\in(1,3) = u_c = \frac{1}{C}\int_0^t (2-t)\,dt = \frac{1}{C}\left[2t - \frac{t^2}{2}\right]\Big|_0^t = \frac{1}{C}\left[2t - \frac{1}{2}t^2\right] = \right]$

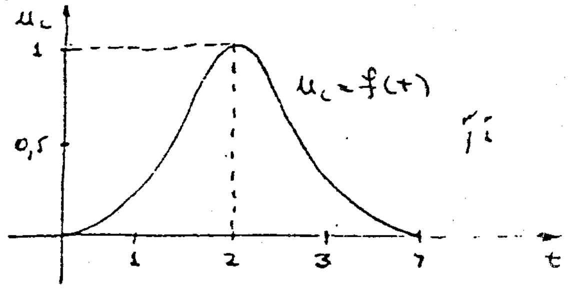
za $t\in(1,3) = u_c = \frac{1}{C}\int_0^1 i\,dt + \frac{1}{C}\int_1^t -(t-2)\,dt = 0,5 + \int_1^t (2-t)\,dt$

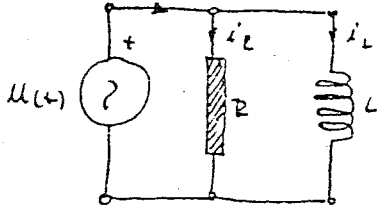
$u_c = 0,5 + \left[2t - \frac{t^2}{2}\right]\Big|_1^t = 0,5 + \left\{\left[2t - \frac{t^2}{2}\right] - \left[2 - 0,5\right]\right\} = 0,5 + \left[2t - \frac{t^2}{2} - \frac{3}{2}\right]$

$u_c = 0,5 - 1,5 + 2t - \frac{t^2}{2} = 2t - \frac{t^2}{2} - 1$

za $t\in(3,4)$ $\qquad u_c = u_3 + \frac{1}{C}\int_3^t i\,dt = (6 - 4,5 - 1) + \int_3^t (t-4)\,dt =$

$= 0,5 + \frac{1}{2}(t-4)^2\Big|_3^t = 0,5 + \frac{1}{2}\left\{(t-4)^2 - 1\right\} = 0,5\,(t-4)^2$





struja kroz otpor i induktivitet teče neovisno jedan o drugom

$i_R = \frac{u}{R} = \frac{u_m \sin t}{R} = u_m \sin t \quad i_R = u_m \sin t$

struja kroz induktivitet = ?

$u = e_s \quad u = L\frac{di}{dt} \quad di_L = \frac{u}{L}dt = \frac{u_m \sin t\, dt}{L} \Rightarrow i = \frac{1}{L}\int u_m \sin t$

$i_L = -\frac{1}{L}\cos t\Big|_0^t + C = -\cos t\Big|_0^t + C \quad$ za $t=0 \quad i_L = 0$

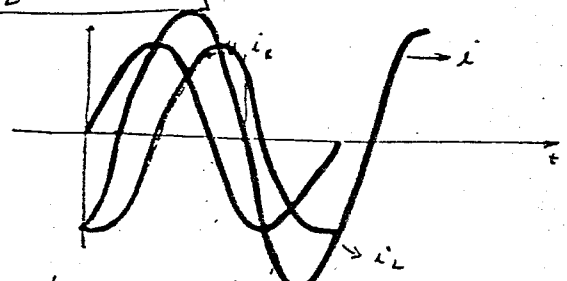
$C = i(t_0) = i(0) = 0$

$i_L = -\cos t\Big|_0^t + 1 = -(\cos t - 1)$ nebitno

$i_L = -\cos t$

$i_R = u_m \sin t$

$i_L = -\cos t \qquad i = i_R + i_L = u_m \sin t - \cos t$

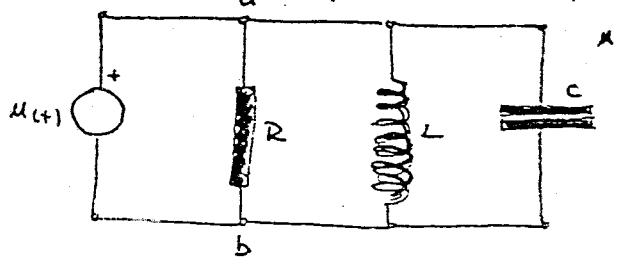


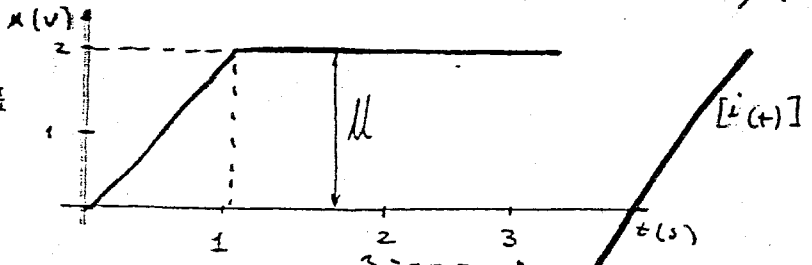
6. Zadan je spoj prema slici. Dan je i vremenska ovisnost napona izvora u vremenu

$R = 2\,\Omega \quad L = 1\,H \quad C = 0{,}5\,F \quad$ za $t \le 0 \quad i_L = 0 \quad u_c = 0$. Odredite 1. $i_L(t)$, $i_C(t)$, i_C

.. 1. x[(1)) c(F)] ; c[

i nacrtajte dijagrame tih struja 2. iznosu otpornika 0,5 Ω, 0,5 Ω 1 Ω, 1,1 Ω i



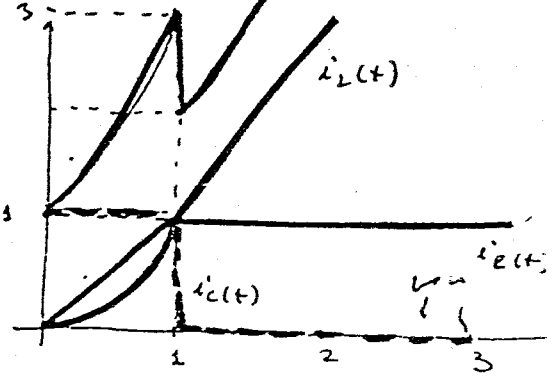


$i_R = \frac{u_{ab}}{R} \Rightarrow$ za $t \in (0,1) \Rightarrow u = 2t\,[V]$

za $t \in (1,\infty) \Rightarrow u = 2\,[V]$

za $t \in (0,1) \Rightarrow i_R = \frac{2t}{2} = t\,[A] \quad i_C = t\,[A]$

za $t \in (1,\infty) \Rightarrow i_R = \frac{2}{2} = 1\,[A] \quad i_C = 1\,[A]$



$i_L = ? \quad u_{ab} = L\frac{di}{dt} \Rightarrow di = \frac{1}{L}u_{ab}\,dt \;/\int$

$i = \frac{1}{L}\int_0^t 2t\,dt = \frac{2}{L}\frac{t^2}{2}\Big|_0^t = \frac{t^2}{L} = t^2$

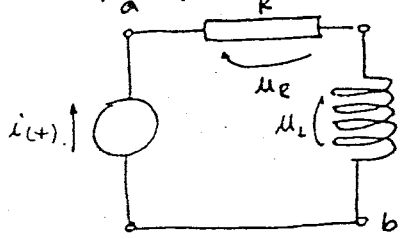
struja kroz induktivitet u nekom trenutku mijenjati će tako da inducirani napon drži ravnotežu narinutom naponu

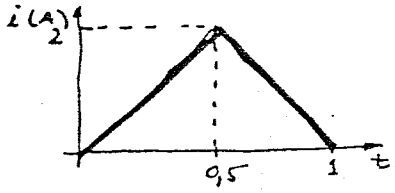
za $t \in (1,\infty) \Rightarrow di = \frac{1}{L}u_{ab}\,dt \Rightarrow i = \frac{1}{L}\int_1^t u_{ab}\,dt + c$

$i = \frac{1}{L}\int_1^t 2\,dt = \frac{1}{L}2(t-1) + i_{(1)} = \frac{1}{L}(2t-2) + 1 = 2t - 2 + 1 = 2t - 1$

Kolika je maksimalna mjednost napona na priključnicama izvora;

Napiši jednadžbu trenutne snage i snage za trenutke 0,25 s i 0,75 s





za $t \in (0, 0,5)$

$\Rightarrow i = 4t$ [A]

$u_R = i \cdot R = 4t \cdot R = 8t$ [V]

$u_L = -L \frac{di}{dt}$ $\Rightarrow$ smjer suprotan poradu struje (Lenzovo pravilo)

$u_L = -L \cdot \frac{d(4t)}{dt} = 4L = 4V$ $\quad \varphi_a = \varphi_b + u_L + i \cdot R \Rightarrow u_{ab} = 4 + 8t$, $t \in (0,0,5)$

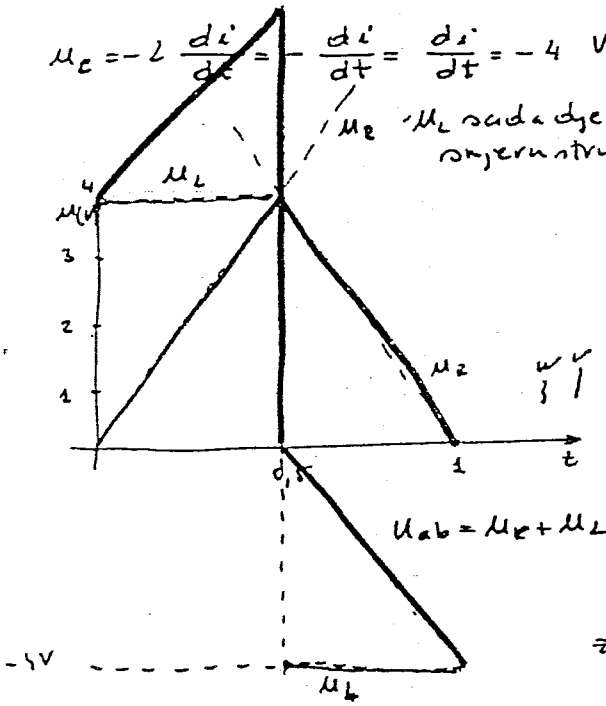
$\boxed{u_{ab} = 8t + 4 \text{ [V]}}$ za $t \in (0,0,5)$

za $t \in (0,5, 1) \Rightarrow i = ?$ $\quad i = kt + l$ $\quad i = 0$ $\quad 0 = kt + l$ $\quad 0 = k + l$ $\quad k = -\frac{2}{\frac{1}{2}} = -4$

$\Rightarrow i = -4t + 4$ $\quad u_R = i \cdot R = (4 - 4t) \cdot 2 = 8 - 8t$ [V]

$u_L = -L \frac{di}{dt} = -\frac{di}{dt} = \frac{di}{dt} = -4$ V $\quad \varphi_a = \varphi_b - 4 + 8 - 8t = u_{ab} = 4 - 8t$ [

u_R - u_L sada djeluje u smjeru struje $\quad$ za $t \in (0,5, 1) \Rightarrow \boxed{u_{ab} = 4 - 8t \text{ [V]}}$



max mjednost napona je

$u_{ab_{MAX}} = 8$ V za $t = 0,5$ s

jednadžba trenutne snage

$P = u_{ab} \cdot i$

za $t \in (0, 0,5) \Rightarrow P = (8t + 4) \cdot 4t = 32t^2 + 16t$

za $t \in (0,5, 1) \Rightarrow P = (4 - 8t) \cdot (4 - 4t)$

Sv $t \in (0'2) \Rightarrow 5 = (4 - 8t) \cdot (4 - 4t)$

$\Rightarrow (\quad) \cdot (\quad)$

za $t \in (0,5, 1) \Rightarrow P = (4 - 8t) \cdot (4 - 4t)$

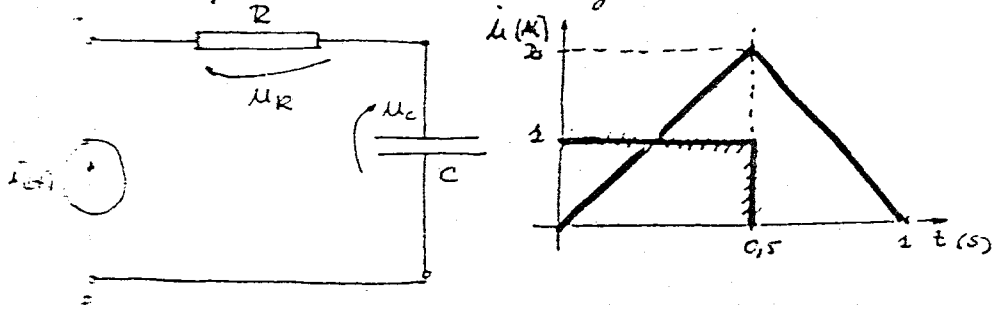
$= 16 - 16t - 32t + 32t^2 = 32t^2 - 48t + 16$

u trenutku $t = 0,25$ s $\Rightarrow P_{(0,25)} = 32 \cdot \frac{1}{16} - 48 \cdot \frac{1}{4} + 16 = 2 - 12 + 16 = 6$ W

u trenutku $t = 0,75$ s $\Rightarrow P_{(0,75)} = 32 \cdot \frac{9}{16} - 48 \cdot \frac{3}{4} + 16 = 18 - 36 + 16 = -2$ W

(5) Na slici nalazi se idealni strujni izvor. Na slici je dijagram promjene struje izvora. $R = 1\,\Omega$ $C = 0{,}5\,F$ i $u_{c(0)} = 0$

Izračunajte 1. napone $u_R(t)$, $u_C(t)$ i $u(t) = u_R(t) + u_C(t)$ i nacrtajte dijagrame promjena navedenih veličina; 2. maksimalnu vrijednost napona na otporniku izvora. 3. jednadžbu trenutne snage



$u_L = i \cdot R \Rightarrow$ za $t \in (0, 0{,}5)$ $u_R = u_R \cdot i \cdot R$ za $t \in (0, 0{,}5)$ $i = 4t$ (A)

$0 < t \leq 0{,}5$ $u_C = 4t \cdot 1 = 4t$ za $t \in (0{,}5, 1)$ $i = -4t + 4 = 4 - 4t$ (A)

$0{,}5 < t < 1$ $u_R = 4 - 4t$

$u_C = ?$ $u_C = \frac{Q}{C}$ $du_C = \frac{dQ}{C} = \frac{1}{C} i \cdot dt \Rightarrow u_C = \frac{1}{C} \int i\,dt$

za $t \in (0, 0{,}5) \Rightarrow i = 4t$ $u_C = \frac{1}{C} \int_0^t 4t\,dt = \frac{1}{0{,}5} \cdot 4 \cdot \frac{t^2}{2} = 4t^2$

za $t \in (0{,}5, 1) \Rightarrow i = 4 - 4t$

$$u_C = \frac{1}{C} \int_{0{,}5}^{t} (4 - 4t)\,dt + u_C(0{,}5)$$

$$u_C = \frac{1}{0{,}5} \left[(4t - 2t^2) \Big|_{0{,}5}^{t} \right] + 1 = 2 \left[(4t - 2t^2) - (2 - \tfrac{1}{2}) \right] + 1 = 2 \left[4t - 2t^2 - \tfrac{3}{2} \right] + 1$$

$$u_C = 8t - 4t^2 - 3 + 1 = 8t - 4t^2 - 2$$

$u_L + u_C = u_{ab} = ?$

za $0 < t \leq 0{,}5$ $u_{ab} = 4t + 4t^2$ za $0{,}5 < t < 1$ $u_{ab} = u_R + u_C$

za $0 < t \leq 0{,}5$

$u_{ab} = 4 - 4t + 8t - 4t^2 - 2$

$p = u_{ab} \cdot i = (4t + 4t^2) \cdot 4t = 16t^3 + 16t^2$

$u_{ab} = -4t^2 + 4t + 2$ [V]

$$\boxed{p = 16t^2(t+1)\ W}$$

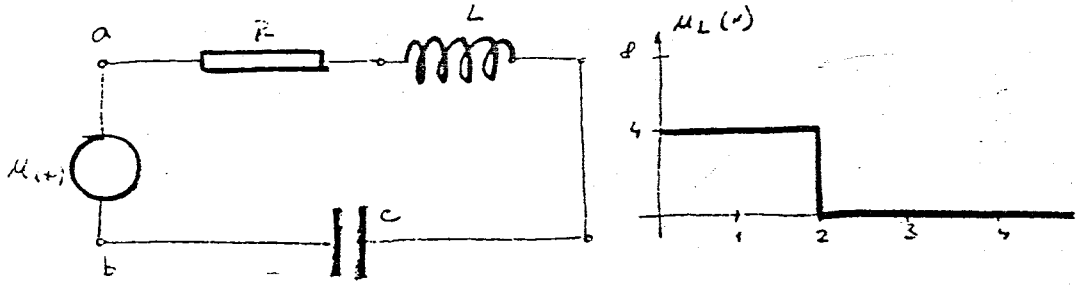
za $0{,}5 < t \leq 1$ $p = u_{ab} \cdot i = \left[-4t^2 + 4t + 2 \right] \cdot (4 - 4t)$ [W]

t	i	$u_R = iR$	k	$u_C = \frac{1}{C} \int_{t_0}^{t_2} i\,dt + k$	$u = u_R + u_C$	$p = u \cdot i$
0 – 0,5	4t	4t	0	$4t^2$	$4t(t+1)$	$16t^2(t+1)$
0,5 – 1	4 – 4t	4 – 4t	1	$-2 + 8t - 4t^2$	$2 + 4t - 4t^2$	$8(2t^3 - 4t^2 + t + 1)$

$p(0{,}25) = 1{,}25$ / $p(0{,}75) = 2{,}75$

(10.) 1.17

U spoju na slici u $t=0$ počinje djelovati izvor napona; $L=1$, $R=0{,}5\,\Omega$, $C=1\,F$. Na slici prikazan je dijagram vremenske ovisnosti napona na stezaljkama sklopke. Napon na kondenzatoru prije $t<0$ jednak je nuli. Treba izračunati $u_R(t)$, $u_C(t)$ i $u(t)$ te nacrtati dijagram vremenske ovisnosti napona.



$$u_L = -L\frac{di}{dt} = L\frac{di}{dt} = 4 \quad \frac{di}{dt} = 4 \quad di = 4\,dt \Rightarrow i = 4t$$

za $0 < t < 2 \Rightarrow$

$$u_R = i \cdot R = 4t \cdot \frac{1}{2} = 2t\,[V] \qquad u_C = \frac{Q}{C} \qquad du_C = \frac{dQ}{C} \qquad u_C = \frac{1}{C}\int_0^t i\,dt =$$

$$u_C = \frac{1}{C}\,2t^2 = 2t^2 \Rightarrow u_C = 2t^2$$

$$u_{ab} = u_R + u_L + u_C = 2t + 4 + 2t^2 = 2t^2 + 2t + 4\,[V]$$

za $t \geq 2$ $\quad u_L = 0 \quad \frac{di}{dt} = 0 \quad di = 0\cdot dt + c \quad i = c = 4$

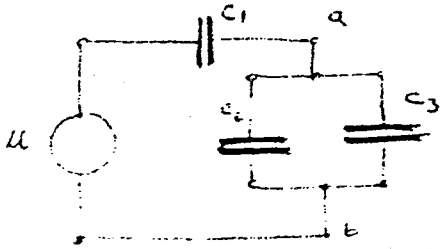
$$u_R = i \cdot R = 8 \cdot \frac{1}{2} = 4V \qquad u_C = \frac{Q}{C} \qquad du = \frac{dQ}{C} = \frac{i\,dt}{C} \qquad u_C = \frac{1}{C}\int_2^t i\,dt + c$$

$$u_C = 8(t-2) + u_{(2)} = 8t - 16 + 8 = 8t - 8 = 8(t-1)\ (V)$$

$$u = u_R + u_L + u_C = 4 + 8t - 8 = 8t - 4 \qquad \boxed{u = 8t - 4\,[V]}$$

(11.) Zadan je spoj prema slici $C_1 = C_2 = C_3 = 1\,F$. Odredite u_{ab}/u

1.28



$C_{23} = C_2 + C_3 = 2F \qquad C_u = ?$

$$\frac{1}{C_u} = \frac{1}{C_1} + \frac{1}{C_1 + C_2} = \frac{1}{1} + \frac{1}{2} = \frac{3}{2}\,F$$

$$C_u = \frac{2}{3}\,F$$

$$Q_1 = Q_{23} = u \cdot C_u = u \cdot \frac{2}{3}$$

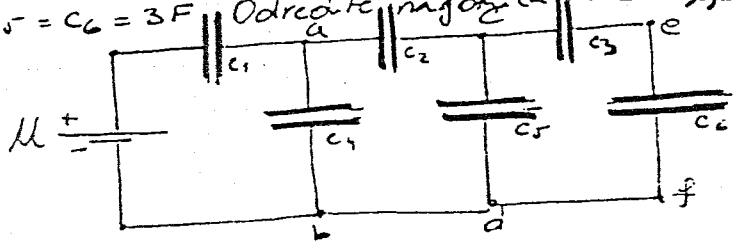
$$u_{ab} = \frac{\frac{2u}{3}}{C_2 + C_3} = \frac{\frac{2u}{3}}{2} = \frac{u}{3} \qquad \frac{u_{ab}}{u} = \frac{\frac{u}{3}}{u} = \frac{1}{3}$$

$$\boxed{\frac{u_{ab}}{u} = \frac{1}{3}}$$

1.2.4

12. U mreži nalazi se izvor istosmernog napona s $U = 10\,V$.

$C_1 = C_2 = C_3 = 1\,F$ $C_4 = C_5 = C_6 = 3\,F$ Odredite energiju na svakom kondenzatoru.



Energija kondenzatora je $W = \frac{1}{2} C U^2$ $C_u = ?$ $C_{56} = \frac{C_5 \cdot C_6}{C_5 + C_6} = \frac{1 \cdot 3}{4} = \frac{3}{4}\,F$

$C_{356} = 3 + \frac{3}{4} = \frac{15}{4}\,F$ $C_{2356} = \frac{1 \cdot \frac{15}{4}}{1 + \frac{15}{4}} = \frac{15}{19}$

$C_{23456} = \frac{15}{19} + 3 = \frac{15}{19} + \frac{57}{19} = \frac{72}{19}\,F$ $C_u = \frac{1 \cdot \frac{42}{19}}{1 + \frac{42}{19}} = \frac{\frac{42}{9}}{\frac{61}{9}} = \frac{42}{61}$ → GREŠ

$C_u = \frac{42}{61}\,F$ $Q = C \cdot U = \frac{42}{61} \cdot 10 = \frac{420}{61} = 8{,}2352\,C$ ($6{,}88\,C$) $Q_1 = 8{,}2352\,C$ ($6{,}88\,C$)

$U_{C_1} = \frac{Q_1}{C_1} = 8{,}2352\,V$ $W_{C_1} = \frac{1}{2} \cdot 1 \cdot (8{,}2352)^2 =$

$C_{23456} = \frac{72}{19}\,F$ $C_u = \frac{C_1 \cdot \frac{72}{19}}{C_1 + \frac{72}{19}} = \frac{\frac{72}{19}}{\frac{19}{19} + \frac{72}{19}} = \frac{\frac{72}{19}}{\frac{91}{19}} = \frac{72}{91}\,F$

$Q = U \cdot C = 10 \cdot \frac{72}{91} = \frac{720}{91} = 7{,}9120$ $U_{C_1} = \frac{Q}{C_1} = 7{,}9120\,V$

$W_1 = \frac{1}{2} C_1 \cdot U_1^2 = \frac{1}{2} \cdot 62{,}6 = 31{,}3\,Ws$

$U_4 = 10 - U_1 = 10 - 7{,}912 = 2{,}088\,V$

$W_4 = \frac{1}{2} C_4 \cdot U_4^2 = \frac{1}{2} \cdot 3 \cdot 4{,}359 = 6{,}539\,Ws$

$C_{235c} = \frac{15}{19}\,F$ $U_{ab} = U_{C_4} = 2{,}088\,V$ $\Rightarrow Q = C \cdot U = \frac{15}{19} \cdot 2{,}088 = 1{,}648\,C$

$Q_2 = 1{,}648\,C$ $U_2 = \frac{Q_2}{C_2} = 1{,}648\,V$ $W_2 = \frac{1}{2} C U^2 = \frac{1}{2} \cdot 1 \cdot 2{,}717 = 1{,}3586\,Ws$

$U_5 = U_{ab} - U_{c_2} = U_{C_4} - U_{C_2} = 2{,}088 - 1{,}648 = 0{,}44\,V$

$W_5 = \frac{1}{2} C_5 \cdot U_5^2 = \frac{1}{2} \cdot 3 \cdot 0{,}1936 = 0{,}2904\,Ws$

$U_{cd} = U_5 = 0{,}44\,V$ $C_{36} = \frac{3}{4}\,F$ $Q = U_5 \cdot C_{36} = 0{,}44 \cdot \frac{3}{4} = 0{,}33\,C$

$U_3 = \frac{0{,}33}{1} = 0{,}33\,V$ $U_6 = 0{,}11\,V$

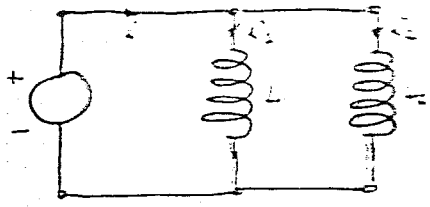
$W_6 = 0{,}258\,Ws$

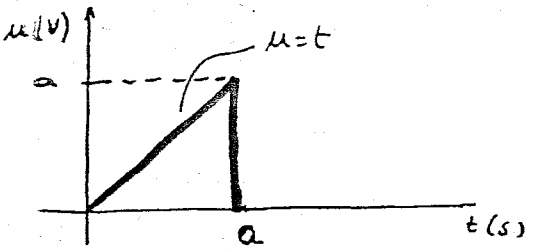
$W_3 = \frac{1}{2} \cdot 1 \cdot 0{,}1089$

$W_3 = 0{,}1743$

1.25

(13.) $L_1 = 1H$ $L_2 = 3H$ $i_1(0) = i_2(0) = 0$. U trenutku $t = a/2$ s struja u zajedničkoj grani poprima vrijednost od 1A. Odredite dijagram struje $i(t)$





$i_1(t - \frac{a}{2}) + i_2(t - \frac{a}{2}) = 1\ A$

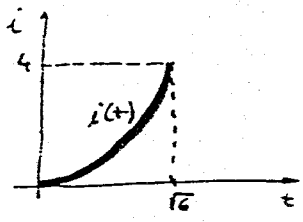
$u = u_L \qquad u = -L\frac{di}{dt} \Rightarrow di = \frac{1}{L}u\,dt \rightarrow i = \frac{1}{L}\int u\,dt$

za $t \in (0, a) \Rightarrow i = \frac{1}{L}\int t\,dt = \frac{1}{2L}t^2 = \frac{1}{2L}t^2 \qquad i_1 = \frac{1}{2L_1}t^2 \quad i_2 = \frac{1}{2L_2}t^2$

$i_1 = \frac{1}{2}t^2 \qquad i_2 = \frac{1}{6}t^2$

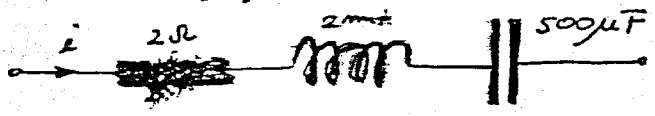
$i = i_1 + i_2 = \frac{1}{2}t^2 + \frac{1}{6}t^2 = \frac{3t^2 + t^2}{6} = \frac{4t^2}{6} = \frac{2t^2}{3}$ $\boxed{i = \frac{2}{3}t^2}$

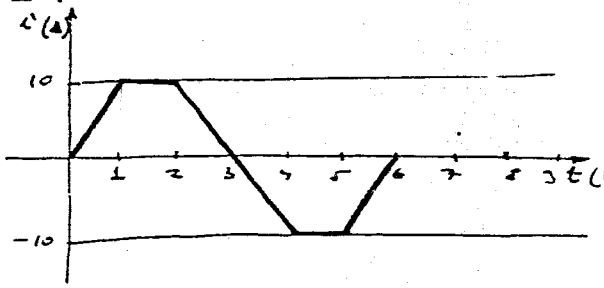
$1 = \frac{2}{3}\frac{a^2}{4} \Rightarrow a^2 = 6 \Rightarrow \underline{a = \sqrt{6}}$ za $t \in (0, \sqrt{6})$ $\boxed{i = \frac{2}{3}t^2}$



1.37

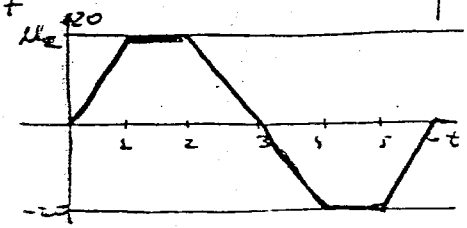
(14.) Kroz uniju zavojnice prema slici teče struja $i(t)$ zadana dijagramom. Izračunaj u_R, u_L, u_C i Q.





$u_2 = i \cdot 2 \qquad u_L = L\frac{di}{dt} \qquad du_C = \frac{dQ}{C} =$

$du = \frac{i\,dt}{C} \Rightarrow u = \frac{1}{C}\int i\,dt$

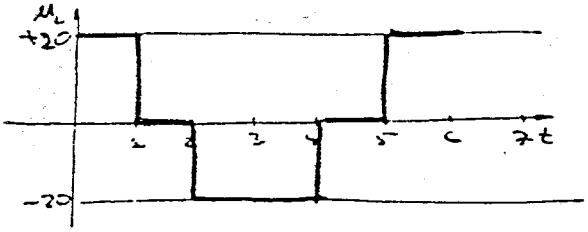


$u_2 = i \cdot 2 = 2i$

$u_L = L\frac{di}{dt}$

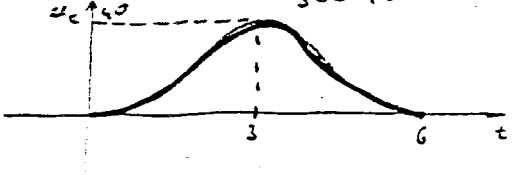
za $0 < t < 1ms \Rightarrow u_L = L \cdot \frac{d}{dt}(10^4 t) = 10^4 L = 10^4 \cdot 2 \cdot 10^{-3} = 2 \cdot 10^1 = 0{,}02 \cdot 10^3 = 20V$

za $2 < t < 4ms \Rightarrow u_L = -20V$



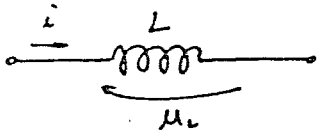
$u_C = \frac{Q}{C} \qquad du_C = \frac{dQ}{C} = \frac{1}{C}i\,dt$

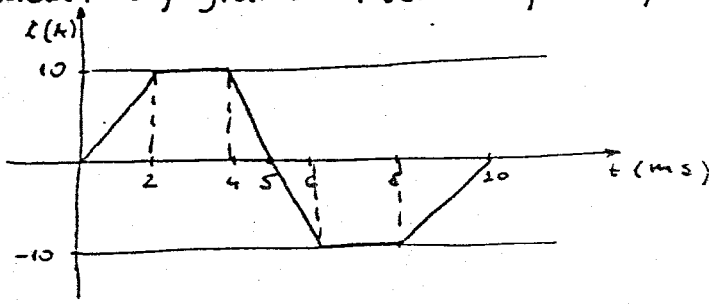
$u_C = \frac{1}{C}\int i\,dt \qquad u_C = \frac{1}{C}\int 10^4 t\,dt = \frac{10^4}{500 \cdot 10^{-6}} \cdot \frac{t^2}{2} = \frac{10^4 t^2}{10 \cdot 10^{-4}} = 10^7 t^2$



1,38

(14) Kroz zavojnicu $L=3\,mH$ teče struja zadana dijagramom. Izračunajte napon u_L snagu $p(t)$ i srednju snagu P.





$u_L = L\frac{di}{dt}$

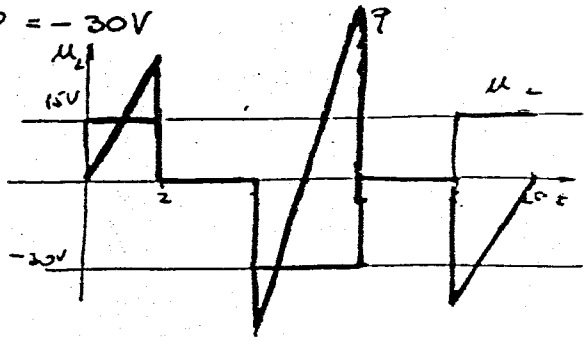
za $0<t<2\,ms \Rightarrow i = 5\cdot 10^3 t$ $\quad u_L = 3\cdot 10^{-3}\cdot 5\cdot 10^3 = 15\,V$

za $2<t<4\,ms \Rightarrow i = 10A \Rightarrow u_L = L\frac{di}{dt} = 0\,V$

za $4<t<6\,ms \Rightarrow i = -10^4 t + k$ $\quad$ za $t=5\,ms$ $i=0$ $\quad 0 = -10^4\cdot 5\cdot 10^{-3} + k$

$0 = k - 5\cdot 10 \Rightarrow k = 50$ $\quad i = -10^4 t + 50$ $\quad u = L\frac{di}{dt} = 3\cdot 10^{-3}\cdot(-10^4)$

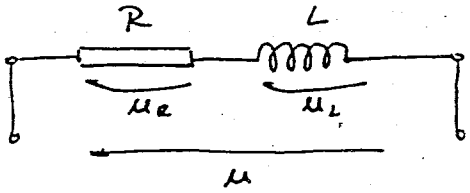
$u = -3\cdot 10 = -30\,V$



Tsrednje = 0

1,39

(15.) Kroz serijski spojene $R=10\,\Omega$ $L=0,5H$ teče struja $i = 0,822\,e^{-20t} + 0,822\sin(377t - 86,9°)$. Izračunajte napone $u_R(t)$ $u_L(t)$ i $u(t)$ prema slici:



$u_R = i\cdot R = 8,22\,e^{-20t} + 8,22\sin(377t - 86,9°)$

$u_L = L\frac{di}{dt} = L\left[0,822\cdot e^{-20t}\cdot(-20) + 0,822\cdot\cos(377t - 86,1°\right.$

$u_L = -8,22\cdot e^{-20t} + 155\cos(377t - 86,16°)$ $\quad u = u_L + u_R = 155\sin 377t$

(16.) Zadana su dva sinusoidalna napona sa ovim podacima

$T=0,001\ s \quad U_{1m}=10V \quad U_{2m}=5V \quad \alpha_{u1}=\frac{\pi}{6} \quad \alpha_{u2}=-\frac{\pi}{4}$

Odredite: f i ω. Jednadžbe trenutnih vrijednosti napona $u_1(t)$, i $u_2(t)=?$

$T=0,001\ s \Rightarrow f=\frac{1}{T}=10^3\ Hz \qquad \omega=2\pi f=6,28\cdot 10^3\ \frac{rad}{s}$

$u_1=10\cdot \sin\left(6280t+\frac{\pi}{6}\right)$

$u_2=5\sin\left(6280t-\frac{\pi}{4}\right)$

(17.) Kroz svitak na N zavoja prolazi vremenski promjenjiv magnetski tok po

2.2 sinusoidalnom zakonu: $\phi_m=0,01Vs \quad T=0,02\ s$. Izračunajte amplitudu induciranog napona uzrokuju $T=0,02\ s \Rightarrow f=50\ Hz \quad \omega=2\pi f=314\ \frac{rad}{s}$

$\phi=\phi_{max}\sin\omega t=0,01\sin\omega t$

$e=-\frac{d\phi}{dt}=-\frac{d}{dt}\left[\phi_{max}\sin\omega t\right]=-\phi_{max}\cdot\omega\cdot\cos\omega t. \qquad E_{max}=\phi_{max}\cdot\omega$

za 1 zavoj $\Longrightarrow \quad e=E_{max}\cos\omega t \qquad E_{max}=0,01\cdot 314$

za 1 zavoj: $E_{max}=3,14\ V$ ili $E_{max}=\pi\ V$

Pošto ima N zavoja

$E_{max}=N\cdot\phi_{max}\cdot\omega=31,4\ V$

$\boxed{E_{max}=31,4\ V}$

(18.) Napon priključen na mehanizam mijenja u po sinusoidalnom zakonu

2.3 $U_{max}=10V \quad f=25\ Hz$. Za koji će najmanji vremenski interval od početka periode napon dostići vrijednost a) $+5V$ b) $+10V$ c) $-5V$ d) proizvoljni predznak (proizvoljni smjer)

$u=U_{max}\sin\omega t \qquad \omega=2\pi f=6,28\cdot 25=157 \qquad \boxed{\omega=157\ \frac{rad}{s}}$

$\boxed{u=10\sin 157t}$

a) $+5=10\sin 157t \Rightarrow \sin 157t=0,5 \qquad 157t=\arcsin 0,5$

a) $\Rightarrow \boxed{t=0,0033\ s}$ $\qquad t=\frac{\arcsin 0,5}{157}$ $\Rightarrow$ preko radijana

b) $b=10V \qquad \sin 157t=1$

$157t=\arcsin 1 \qquad t=\frac{\arcsin 1}{157}=0,01 \Rightarrow \boxed{t=0,01\ s}$

c) $-5=10\sin 157t$

$\sin 157t=-0,5$

$\sin(157t+k\pi)=0,5 \quad | \quad k=1$

$\sin(157t+\pi)=0,5$

$u = 10 \sin 157t$ $\quad \sin 157t = -0{,}5$ $\quad 157t = \frac{7\pi}{6}$ $\quad + \frac{7\pi}{6} = \frac{7}{300}$ s

$\frac{2\pi \cdot 25}{1}$

$t = \frac{7}{300}$ s $\quad$ ww

d) promjenih znak

$u = 10 \sin 157t \quad u = 0$

$\sin 157t = 0 \quad t \cdot 157 = \pi \Rightarrow t = \frac{\pi}{157} = \frac{\pi}{2\pi \cdot 25} = \frac{1}{50} = 0{,}02$ s $\quad t = 0{,}02$ s $\quad$ ww

(19.) Zadana je struja $i_1 = 2\sqrt{2} \sin(\omega t + \frac{\pi}{3})$, a struju i_2 znamo da u mijenja po sinusoidalnom zakonu s frekvenc. ω uz $I_{2m} = 4\sqrt{2}$ A i da iznosti i_1 i i_2

2.4 postoji fazni odnos a) i_2 u podudarnu u fazi sa i_1

b) poredbeni struji i_2 za $\frac{\pi}{4}$

c) zaostaje za i_2 za $\frac{\pi}{6}$

d) i_2 se nalazi u protufazi sa i_1.

$i_1 = 2\sqrt{2} \sin(\omega t + \frac{\pi}{3}) \quad I_{2m+x} = 4\sqrt{2}$ A

a) $i_2 = 4\sqrt{2} \sin(\omega t + \frac{\pi}{3})$

b) $i_2 = 4\sqrt{2} \sin(\omega t + \frac{\pi}{3} + \frac{\pi}{4}) = 4\sqrt{2} \sin(\omega t + \frac{7\pi}{12})$

c) $i_2 = 4\sqrt{2} \sin(\omega t + \frac{\pi}{3} - \frac{\pi}{6}) = 4\sqrt{2} \sin(\omega t + \frac{\pi}{6})$

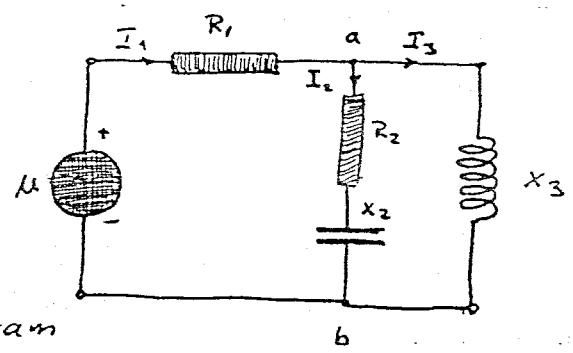
d) $i_2 = 4\sqrt{2} \sin(\omega t + \frac{\pi}{3} + \pi) = 4\sqrt{2} \sin(\omega t + \frac{4\pi}{3})$ $\quad$ ww

(20)

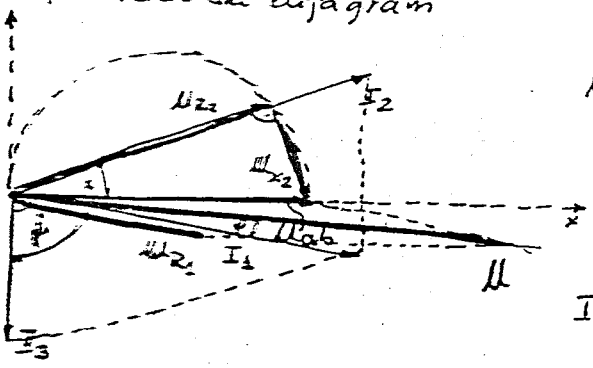
Z. 118

21. Zadana je mreža kojoj je $R_1=5\Omega$ $R_2=6\Omega$ $X_2=8\Omega$ $X_3=10\Omega$ $I_3=10A$

Odredite I_1, I_2, U i P



Nacrtati ću pom vektorski dijagram



$U_{ab} = I_3 \cdot X_3 = 10 \cdot 10 = 100V$

$I_2 = \frac{U_{ab}}{Z_2} = \frac{U_{ab}}{\sqrt{R_2^2 + X_2^2}}$

$I_2 = \frac{100}{\sqrt{36+64}} = \frac{100}{10} = 10A$ $\boxed{I_2=10A}$

$\boxed{I_3=10A}$

$tg\alpha = \frac{X_2}{R_2} \Rightarrow tg\alpha = \frac{8}{6} = \frac{4}{3} \Rightarrow \alpha = arc\,tg\,\frac{4}{3}$ $\alpha = 53,13°$

$\alpha + 90° = 143,13°$ $I_1^2 = I_2^2 + I_3^2 + 2I_2 \cdot I_3 \cdot \cos 143,13°$

$I_1^2 = 100 + 100 + 2 \cdot 100 \cdot (-0,8)$ $I_1^2 = 200 - 160 = 40$ $I_1 = 2\sqrt{10}$

$\boxed{I_1 = 6,324\,A}$

$U_{R_1} = I_1 \cdot R_1 = 6,324 \cdot 5 = \boxed{31,62\,V = U_{R_1}}$ $\boxed{P = U \cdot I_1 \cdot \cos\varphi}$

među kut $\dot{\mu}$

$I_2^2 = I_3^2 + I_1^2 - 2I_1 \cdot I_3 \cdot \cos\dot{\mu} \Rightarrow \cos\dot{\mu} = \frac{I_1^2 + I_3^2 - I_2^2}{2I_1 \cdot I_3} = \frac{40 + 100 - 100}{2 \cdot 2\sqrt{10} \cdot 10} = \frac{40}{40\sqrt{10}}$

$\cos\dot{\mu} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10} \Rightarrow \boxed{\dot{\mu} = 71,56°}$ $90° - \dot{\mu} = 18,435°$ $\boxed{90 - \dot{\mu} = 18,435°}$

$U^2 = U_{ab}^2 + U_{R_1}^2 + 2U_{ab} \cdot U_{R_1} \cdot \cos 18,435$

$U^2 = 10000 + 999,82 + 2 \cdot 100 \cdot 31,62 \cdot 0,9486 = U^2 = 10000 + 1000 + 5998,9464$

$U^2 = 16998,9464$ $\boxed{U = 130,38V}$

$U_{ab}^2 = U_{R_1}^2 + U^2 - 2U_{R_1} \cdot U \cdot \cos\varphi \Rightarrow \cos\varphi = \frac{U_{R_1}^2 + U^2 - U_{ab}^2}{2U_{R_1} \cdot U}$

$P = U \cdot I_1 \cdot \cos\varphi = U \cdot I_1 \cdot \frac{U_{R_1}^2 + U^2 - U_{ab}^2}{2U_{R_1} \cdot U}$

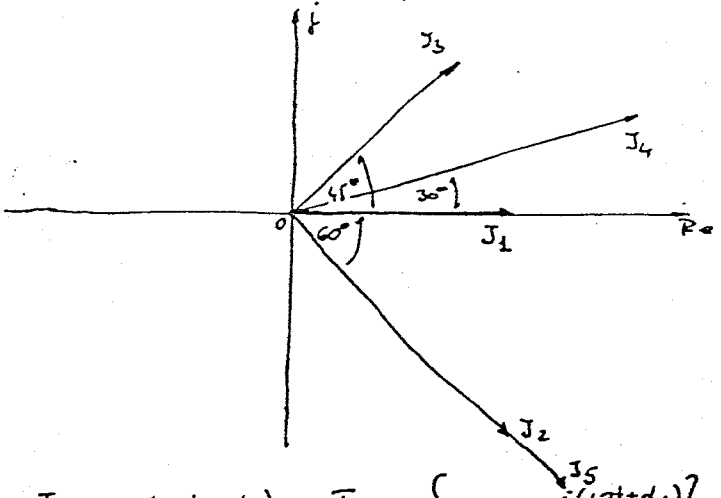
$P = I_1 \cdot \frac{U_{R_1}^2 + U^2 - U_{ab}^2}{2U_{R_1}} = 2\sqrt{10} \cdot \frac{1000 + 16900 - 10000}{2 \cdot 31,62} = \sqrt{10} \cdot 243,84 \approx 790W$

2.5

(22) Izrazi za trenutne vrijednosti struja u:

$i_1 = 2\sin\omega t \quad i_2 = 6\sqrt{2}\sin(\omega t - \frac{\pi}{3}) \quad i_3 = 4\sin(\omega t + \frac{\pi}{4}) \quad i_4 = 8\sqrt{2}\sin(\omega t + \frac{\pi}{6}) \quad i_5 = -10\sin\omega t$

Nacrtati vektorske dijagrame i odrediti izraze za struje u kompl. obliku



$\dot{I}_1 = 2$

$\dot{I}_{2m} = 6\sqrt{2}\, e^{-j\frac{\pi}{3}}$

$\dot{I}_{3m} = 4 \cdot e^{j\frac{\pi}{4}}$

$\dot{I}_{4m} = 8\sqrt{2}\, e^{j\frac{\pi}{6}}$

$\dot{I}_{5m} = 10\, e^{-j\frac{\pi}{3}}$

$I_m \sin(\omega t + \alpha_1) = I_{mag}\left\{ I_m e^{j(\omega t + \alpha_1)} \right\} = I_m\left\{ I_m \cdot e^{j\alpha_1} \cdot e^{j\omega t} \right\} = I_m\left\{ \dot{I}_m e^{j\omega t} \right\}$

$I_m \cos(\omega t + \alpha_1) = Re\left\{ I_m e^{j(\omega t + \alpha_1)} \right\} = Re\left\{ \dot{I}_m e^{j\omega t} \right\}$

$i_1 = I_m\left\{ 2 e^{j\omega t} \right\} \quad i_2 = I_{mag}\left\{ 6\sqrt{2}\, e^{-j\frac{\pi}{3}} e^{j\omega t} \right\} \quad i_3 = I_{mag}\left\{ 4 e^{j\frac{\pi}{4}} e^{j\omega t} \right\}$

$i_5 = I_m\left\{ -10\, e^{j\frac{2\pi}{3}} e^{j\omega t} \right\}$??

2.6

(23) Napon izvora mijenja se po sinusoidalnom zakonu sa ω i zadan je u kompl. obliku

a) $\dot{U}_m = 10\, e^{-j30^\circ}$ b) $\dot{U}_m = 5 + j7$ c) $\dot{U}_m = -5 + j7$. Odrediti trenutnu vrijednost napona $u(t)$

a) $\dot{U}_m = 10\, e^{-j30^\circ} \qquad u(t) = \dot{U}_m \cdot e^{j\omega t} = 10 e^{-j30^\circ} e^{j\omega t} = 10\, e^{-j30^\circ} \cdot e^{j\omega t}$

$u_t = 10\, e^{j(\omega t - \frac{\pi}{6})}$

$u(t) = Im\left\{ 10\, e^{j(\omega t - \frac{\pi}{6})} \right\} = \underline{10\sin(\omega t - 30^\circ)}$??

b) $\dot{U}_m = 5 + j7 \quad |\dot{U}_m| = \sqrt{25 + 49} = \sqrt{74} = |\dot{U}_m| = \sqrt{74} \qquad \cos\varphi = \frac{5}{\sqrt{74}}$

$\boxed{\varphi = 54{,}46}$

$u(t) = Im\left\{ \dot{U}_m \cdot e^{j\omega t} \right\} \qquad \dot{U}_m = \sqrt{74}\, e^{j54{,}46^\circ}$

$u(t) = Im\left\{ \sqrt{74} \cdot e^{j54{,}46^\circ} \cdot e^{j\omega t} \right\} = Im\left\{ \sqrt{74} \cdot e^{j(\omega t + 54{,}46^\circ)} \right\} = \sqrt{74}\sin(\omega t + 54{,}46^\circ)$

c) $\dot{U}_m = -5 + j7 \quad |\dot{U}_m| = \sqrt{74} \quad \boxed{\varphi = 125{,}54^\circ}$

$\dot{U}_m = \sqrt{74}\, e^{j125{,}54^\circ} \qquad u(t) = Im\left\{ \dot{U}_m \cdot e^{j\omega t} \right\} = Im\left\{ \sqrt{74}\, e^{j(125{,}54)^\circ} \cdot e^{j\omega t} \right\}$

$\boxed{u(t) = \sqrt{74}\sin(\omega t + 125{,}54)}$

2.7

(24.) Vektor efektivne vrijednosti napona izražen je kompleksnim brojem $\dot{U} = 6+j8$

Kojim će kompl. brojem biti izražen vektor napona ako ga zakrenemo

a) za 37° suprotno kazaljci na satu

b) za 53° u smjeru kazaljke na satu

a) 37° $\dot{U} = 6+j8$ $\boxed{u(t) = \mathrm{Im}\left\{\dot{U} \cdot e^{j\omega t}\right\}}$

$|\dot{U}| = \sqrt{36+64} = 10$ $\cos\varphi = \frac{6}{10} = \frac{3}{5} \Rightarrow \varphi = 53°$ $\dot{U} = 10\,e^{j53°}$

ako ga zakrenemo u ⊕ smjeru $\dot{U} = 10\,e^{j(53°+37°)} = 10\,e^{j90°} = j10$ $\boxed{\dot{U} = j10}$

b) za 53° u ⊖ smjeru

$|\dot{U}| = 10$ $u(t) = \mathrm{Im}\left\{\dot{U}\,e^{j\omega t}\right\}$ $\boxed{\varphi = 53°}$

$\dot{U} = 10\,e^{j(53°-53°)} = 10$ $\boxed{\dot{U} = 10}$ V

2.8

(25.) Vektor ef. vrijednosti napona izražen je kompleksnim brojem $\dot{U} = \frac{5-j3}{2+jb}$

1) Pri kojoj će vrijednosti b vektor napona biti smješten a) u realnoj osi

b) u imaginarnoj osi

c) pod kutem −45° u odnosu prema pozitivnoj realnoj osi

d) odredite za a) b) c) modul napona

$\dot{U} = \frac{5-j3}{2+jb} \cdot \frac{2-jb}{2-jb}$

$\dot{U} = \frac{10-5jb-6j+j^2 3b}{4+b^2} = \frac{10-j5b-j6-3b}{4+b^2}$

$\dot{U} = \frac{10-3b}{4+b^2} - j\,\frac{6+5b}{4+b^2}$

1) a) u realnoj osi $\Rightarrow \frac{6+5b}{4+b^2} = 0$ $5b = -6$ $\underline{b = -\frac{6}{5}}$

b) u imaginarnoj osi $\Rightarrow \frac{10-3b}{4+b^2} = 0$ $3b = 10$ $\underline{b = \frac{10}{3}}$

c) pod kutem $-\frac{\pi}{4}$ $\dot{U} = \frac{10-3b}{4+b^2} - j\,\frac{6+5b}{4+b^2}$ $\frac{10-3b}{4+b^2} = \frac{6+5b}{4+b^2}$

$10-3b = 6+5b$

$4 = 8b \Rightarrow \boxed{b = \frac{1}{2}}$

2) $|\dot{U}| \Rightarrow$ a) $\frac{10+3\cdot\frac{6}{5}}{4+\frac{36}{25}} = \frac{\frac{50+18}{5}}{\frac{100+36}{25}} = \frac{\frac{68}{5}}{\frac{136}{25}} = \frac{68\cdot 5}{136} = 2{,}5$ V

b) $\dfrac{6+5\cdot\frac{10}{3}}{4+\left(\frac{10}{3}\right)^2} = \dfrac{6+\frac{50}{3}}{4+\frac{100}{9}} = \dfrac{\frac{18+50}{3}}{\frac{36+100}{9}} = \dfrac{\frac{68}{3}}{\frac{136}{9}} = \dfrac{68\cdot 3}{136} = 1{,}5$ ww

c) $|\dot{\mu}| = \sqrt{\left(\frac{10-\frac{3}{2}}{4+\frac{1}{4}}\right)^2+\left(\frac{6+\frac{5}{2}}{4+\frac{1}{4}}\right)^2} = \sqrt{\left[\frac{\frac{20-3}{2}}{\frac{16+1}{4}}\right]^2+\left[\frac{\frac{12+5}{2}}{\frac{16+1}{4}}\right]^2}$

$|\dot{\mu}| = \sqrt{\left(\frac{17\cdot 2}{17}\right)^2+\left(\frac{17\cdot 2}{17}\right)^2} = \sqrt{4+4} = 2\sqrt{2}$ ww

(26.) (2.9) $\dot{\mu} = 2+ja$ i $\dot{I} = 4e^{-j30^\circ}$, a struja je u fazi sa naponom

a) - ponaša se u fazi prema naponu za 45°

b) - ponaša se u fazi prema naponu za -30°

Odredite a za pojedine slučajeve.

a) $\dot{I} = 4e^{-j30^\circ} = 4(\cos 30^\circ - j\sin 30^\circ) = 4\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right) = 2\sqrt{3} - j2$

$\dot{\mu} = 2+ja$

da struja bude u fazi sa naponom moramo imati

$\operatorname{tg}\varphi = \frac{-2}{2\sqrt{3}} = \frac{a}{2} \Rightarrow a = \frac{-4}{2\sqrt{3}} = -\frac{2}{\sqrt{3}} \Rightarrow a = -\frac{2\sqrt{3}}{3}$ ww

b) $\dot{I} = 2\sqrt{3} - j2$ struja zaostaje za naponom za 45° $\dot{\mu} = 2+ja$

$\dot{I} = |\dot{I}|\cdot e^{j\varphi}$

$|\dot{I}| = \sqrt{12+4} = 4$ $\varphi = \arccos\frac{2\sqrt{3}}{4} = -\frac{\pi}{6}$ $\boxed{\varphi = -\frac{\pi}{6}}$

$\dot{I} = 4\cdot e^{-j30^\circ}$

$\dot{\mu} = |\dot{\mu}|\cdot e^{j(-30^\circ+45^\circ)} = |\dot{\mu}|\cdot e^{j15^\circ} = 2-ja$

$\sqrt{4+a^2}\cdot e^{j15} = 2+ja$ $\sqrt{4+a^2}\cdot(0{,}965 + j\,0{,}2588) = 2+ja$

$0{,}965\sqrt{4+a^2} = 2$ $4+a^2 = 4{,}2054 \Rightarrow a^2 = 0{,}2054 \Rightarrow \boxed{a = 0{,}5435}$

$0{,}2588\sqrt{4+a^2} = a\ /^2$ $0{,}0669(4+a^2) = a^2$ $0{,}267 + 0{,}0669a^2 = a^2$

$0{,}267 = 0{,}9331a^2 \Rightarrow a^2 = 0{,}286 \Rightarrow \boxed{a = 0{,}534}$ ww

c)

$a \in -2\sqrt{3}$

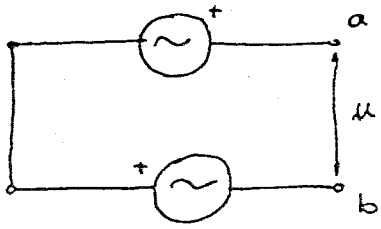
2.10

(27.) U krugu $u_1 = 10\sin(\omega t + 150°)$ i $u_2 = -10\cos\omega t = -10\left\{-\sin\left(\omega t - \frac{\pi}{2}\right)\right\} = 10\sin(\omega t -$

1) prikaži $u_1(t)$ i $u_2(t)$ u kompleksnom obliku

2) odredi izraz za $u(t)$ rezultantni napon

3) ustanovi efektivnu vrijednost napona $u(t)$



1) $u_1 = 10\sin(\omega t + 150°)$ $\quad u_1 = \mathrm{Im}\left\{10\, e^{j(\omega t + 150°)}\right\}$ $\quad u_2 = \mathrm{Re}\left\{-10\, e^{j\omega t}\right\} = \mathrm{Re}\left\{-10\, e^{j\omega t}\right.$

$u_2 = \mathrm{Re}\left\{-10\, e^{j(\omega t + 0)}\right\}$

$u_2 = \mathrm{Im}\left\{10\, e^{j(\omega t - 90°)}\right\}$

2) odredi izraze za $u(t)$ rezultantni napon

$u(t) = u_1(t) + u_2(t) = \mathrm{Im}\left\{10\, e^{j(\omega t + 150°)}\right\} + \mathrm{Im}\left\{10\, e^{j(\omega t - 90°)}\right\}$

$u(t) = \mathrm{Im}\left\{10\, e^{j(\omega t + 150°)} + 10\, e^{j(\omega t - 90°)}\right\}$

$u(t) = \mathrm{Im}\left\{10\left[e^{j\omega t} \cdot e^{j150°} + e^{j(\omega t)} \cdot e^{-j90°}\right]\right\} = \mathrm{Im}\left\{10 \cdot e^{j(\omega t)}\left[e^{-j90°} + e^{j150°}\right]\right\}$

$e^{-j90°} + e^{j150°} = \cos 90° - j\sin 90° + \cos 150° + j\sin 150° = 0 - j - \frac{\sqrt{3}}{2} + j\frac{1}{2}$

$= -\frac{\sqrt{3}}{2} - \frac{1}{2}j \qquad |\ | = \sqrt{\frac{3}{4} + \frac{1}{4}} = 1$

$\varphi = \operatorname{arc\,tg}\frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3} \Rightarrow \varphi = 210°$

$u(t) = \mathrm{Im}\left\{10 \cdot e^{j210°} e^{j\omega t}\right\}$

$u(t) = 10\sin(\omega t + 210°)$

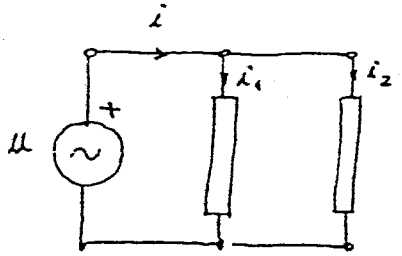
$u(t) = 10\sin(\omega t - 150°)$

3) Efektivna vrijednost napona je $U = \frac{10}{\sqrt{2}} = 7{,}07\,V$ $\quad U = 7{,}07\,V$

2.11

(28) U spoju na slici je $i_1 = \sin(314t - 60°)$ i $i_2 = 5\sin(314t - 15°)$. Odredite

$i_2(t)$



$i_1 = \sin(\omega t - 60°)$

$i_2 = 5\sin(\omega t - 15°)$

$i = i_1 + i_2$

$i_1 = Im\left\{1 \cdot e^{j(\omega t - 60^\circ)}\right\}$ $i_2 = Im\left\{5 \cdot e^{j(\omega t - 15^\circ)}\right\}$

$i = i_1 + i_2 = Im\left\{e^{j\omega t} \cdot e^{-j60^\circ} + 5 \cdot e^{j\omega t} \cdot e^{-j15}\right\} = Im\left\{e^{j\omega t}\left[e^{-j60^\circ} + 5e^{-j15^\circ}\right]\right\}$

$e^{-j60^\circ} + 5 \cdot e^{-j15^\circ} = \cos 60^\circ - j\sin 60^\circ + 5(\cos 15^\circ - j\sin 15^\circ) =$

$= \frac{1}{2} - j\frac{\sqrt{3}}{2} + 5(0{,}965 - j\,0{,}2588) = \frac{1}{2} - j\frac{\sqrt{3}}{2} + 4{,}825 - j\,1{,}294$

$= 5{,}325 - j\,2{,}16 \qquad |I| = \sqrt{4{,}6657 + 28{,}35} = 5{,}7464$

$\varphi = \operatorname{arctg}\frac{-2{,}16}{5{,}325} \Rightarrow \varphi = -22{,}08^\circ \qquad i = Im\left\{6{,}7467 \cdot e^{j\omega t} \cdot e^{-j22{,}08^\circ}\right\}$

$i = Im\left\{5{,}7467 \cdot e^{j(\omega t - 22{,}08^\circ)}\right\}$ $\boxed{i = 5{,}7467 \sin(\omega t - 22{,}08^\circ)}$ ωm

II NAČIN $i_1 = \sin(\omega t - 60^\circ)$ $i_2 = 5\sin(\omega t - 15^\circ)$

$i = i_1 + i_2$ $i = I_m \sin(\omega t + \varphi)$ ⇒ pretpostavljeno rješenje

$i_1 = \sin(\omega t - 60^\circ) = \sin\omega t \cdot \cos 60^\circ - \cos\omega t \cdot \sin 60^\circ = \frac{1}{2}\sin\omega t - \frac{\sqrt{3}}{2}\cos\omega t$

$i_2 = 5\sin(\omega t - 15^\circ) = 5\sin\omega t \cdot \cos 15^\circ - 5\cos\omega t \cdot \sin 15^\circ = 4{,}83\sin\omega t - 1{,}294 \cdot \cos\omega t$

$i = i_1 + i_2 = 5{,}33\sin\omega t - 2{,}16\cos\omega t$

$5{,}33 = C \cdot \cos\varphi \,/^2$ $-2{,}16 = C \cdot \sin\varphi$ $i = i_1 + i_2 = C\sin\omega t \cdot \cos\varphi + C\cos\omega t \cdot \sin\varphi$

$-2{,}16 = C\sin\varphi \,/^2$ $= C[\sin\omega t \cdot \cos\varphi + \cos\omega t \cdot \sin\varphi]$

$28{,}4089 = C^2\cos^2\varphi$ $4{,}665 = C^2\sin^2\varphi$ $+$ $\sin\varphi = \frac{-2{,}16}{5{,}75}$ $= C\sin(\omega t + \varphi)$

$C^2 = 33{,}0745 \Rightarrow \boxed{C = 5{,}75}$ $\boxed{\Rightarrow \varphi = -22{,}06^\circ}$ $\boxed{i = i_1 + i_2 = 5{,}75\sin(\omega t - 22{,}0}$

U ZBIRCI NIJE ZADANA STRUJA i_2 NEGO i

PA JE $i_2 = i - i_1$ .....

III NAČIN ⇒ PRETVORIMO STRUJE (RAZLOŽIMO) NA X i Y KOMPONENTE

TJ. NA REALNI i IMAG. DIO

$\boxed{i_2 = 4{,}8296 - 1{,}2941\,j}$ $\boxed{i_1 = 0{,}5 - 0{,}866\,j}$

92

2.13

(29) Riješite jednadžbu 1° $a\frac{di}{dt} + bi = U_m \sin\omega t$ gdje su a i b konstantni koeficijenti.

$i = I_m \cdot \sin(\omega t + \varphi) \Rightarrow$ rješenje jednadžbe $i = \mathrm{Im}\left\{\dot{I}_{max} e^{j\omega t}\right\}$

$\frac{di}{dt} = \dot{I} \cdot j\omega \cdot e^{j\omega t}$

$a\dot{I}j\omega e^{j\omega t} + b\dot{I}e^{j\omega t} = \dot{U}_m e^{j\omega t} \;/: e^{j\omega t}$ $\quad a\dot{I}j\omega + b\dot{I} = \dot{U}_m$

$\dot{I}[aj\omega + b] = \dot{U}_m$

$\dot{I} = \frac{\dot{U}_m}{b + a\omega j}$ djelovanjem sa $\sqrt{2}$ dobivamo ef. vrijednosti.

$\dot{I} = \frac{\dot{U}}{\underbrace{b + a\omega j}_{\text{KOMPLEKSNA IMPEDANCIJA}}}$ $\quad Z = b + a\omega j$

$|Z| = \sqrt{b^2 + a^2\omega^2}$

$$\boxed{i = \frac{U_m}{\sqrt{b^2 + a^2\omega^2}} \sin\left(\omega t + \operatorname{arctg}\frac{a\omega}{b}\right)}$$

(30) Riješi jednadžbu $\frac{di}{dt} + 10i + 20\int i\,dt = 100\cos\overbrace{20t}^{\omega=20}$ $\quad i = I_{max}\sin\omega t$

2.13

$i = \mathrm{Im}\left\{\dot{I}_{max} e^{j\omega t}\right\}$

$\cos\omega t = \sin(90 - \omega t) = -\sin(\omega t - 90)$

$\dot{I}_{max} \cdot j\omega e^{j\omega t} + 10\dot{I}_{max} e^{j\omega t} + 20 \cdot \frac{1}{j\omega}\dot{I}_{max} e^{j\omega t} = -100\sin(\omega t - \frac{\pi}{2})$

$\dot{I}_{max} j\omega e^{j\omega t} + 10\dot{I}_{max} e^{j\omega t} + 20\frac{1}{j\omega}\dot{I}_{max} e^{j\omega t} = -100 \cdot e^{-j\frac{\pi}{2}} \cdot e^{j\omega t}$

$= -100\, e^{-j\frac{\pi}{2}} \cdot e^{j\omega t} = -$

$\dot{I}_{max} j\omega + 10\dot{I}_{max} - j\frac{20}{\omega}\dot{I}_{max} = -100\, e^{-j\frac{\pi}{2}}$

$\dot{I}j\omega + 10\dot{I} - \dot{I}j\frac{20}{\omega} = -100\, e^{-j\frac{\pi}{2}}$ $\quad \dot{I}\left(10 + j\omega - j\frac{20}{\omega}\right) = -100\, e^{-j\frac{\pi}{2}}$

$\dot{I}\left[10 + j(\omega - \frac{20}{\omega})\right] = -100\, e^{-j\frac{\pi}{2}}$ $\quad \omega = 20$

$\dot{I}\left[10 + j(20 - 1)\right] = -100\, e^{-j\frac{\pi}{2}}$

$\dot{I} = \frac{-100\, e^{-j\frac{\pi}{2}}}{10 + j19} = \frac{-100(0 - j\overbrace{\sin\frac{\pi}{2}}^{1})}{10 + j19} = \frac{+100j}{10 + j19}$

$\dot{I} = \frac{100j \cdot (10 - j19)}{100 + 361} = \frac{1000j + 1900}{461}$

$\dot{I} = 4{,}1214 + 2{,}169j$

$|\dot{I}| = \sqrt{4{,}705 + 16{,}1859} = 4{,}65\ A$ $\quad \operatorname{tg}\varphi = \frac{2{,}169}{4{,}1214} \Rightarrow \varphi = 25{,}88°$

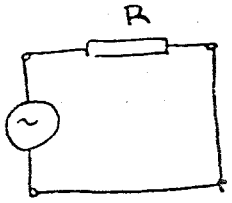
$$\boxed{I = 4{,}65\sin(20t + 26°)}$$

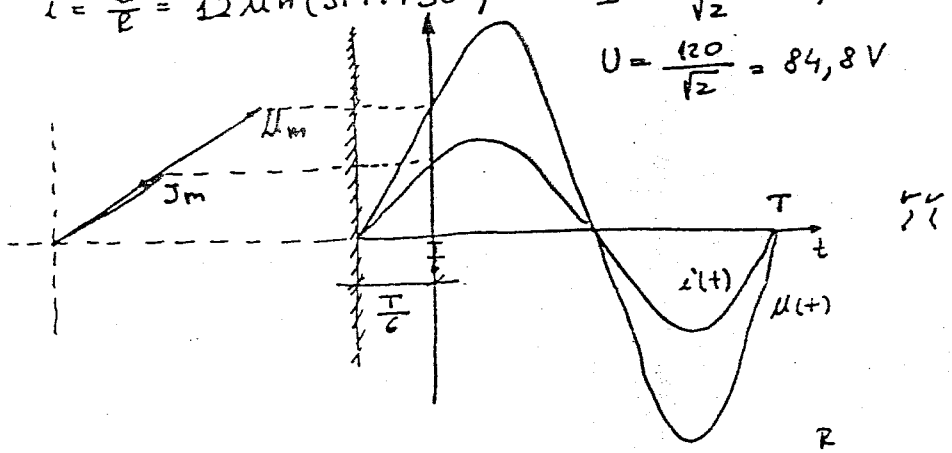
2.14

(31) $u = 120 \sin(314t + 30°)$ i $R = 10\,\Omega$ a) odredi $i(t)$

$i = \frac{u}{R} = 12 \sin(314t + 30°)$ $\quad I = \frac{12}{\sqrt{2}} = 8{,}48\,A$

$U = \frac{120}{\sqrt{2}} = 84{,}8\,V$

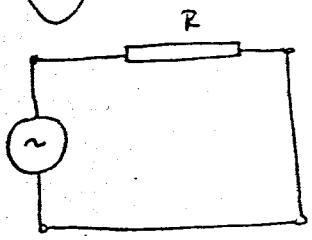




2.15

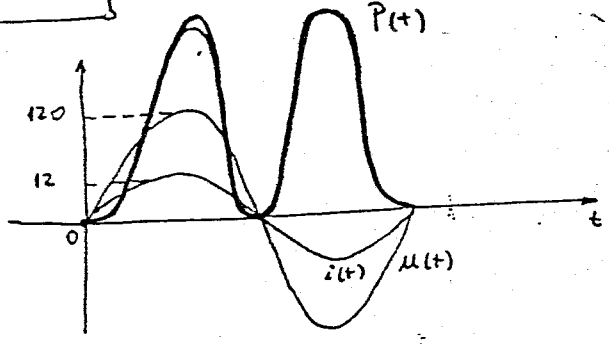
(32.) $u = 120 \sin 314t$ $\quad R = 10\,\Omega$

$i(t) = ?$



Količina potrošene energije

u 1/2 periode $\quad i = \frac{u}{R} = 12 \sin 314t$



$I = \frac{12}{\sqrt{2}} = 8{,}48\,A$

$U = 84{,}8\,V$

$P_{(t)} = u(t) \cdot i(t) = 1440 \sin^2 314t$

$\varphi = 50\,Hz \Rightarrow T = 0{,}02\,s \quad \frac{T}{2} = 0{,}$

$$W = \int_0^{\frac{T}{2}} 1440 \sin^2 314t\, dt = 1440 \int_0^{0{,}01} \sin^2 314t\, dt = 1440 \int_0^{0{,}01} \sin^2 314t\, dt$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x \Rightarrow \sin^2 x = \frac{1 - \cos 2x}{2}$$

$$W = 1440 \int_0^{0{,}01} \frac{1 - \cos 628t}{2}\, dt = 720 \int_0^{0{,}01} (1 - \cos 628t)\, dt = 720 \int_0^{0{,}01} dt - 720 \int_0^{0{,}01} \cos 628t\, dt$$

$$W = 720 \cdot 0{,}01 - \frac{720}{628} \sin 628 \cdot 0{,}01 =$$

$W = 7{,}2 - 1{,}146 \cdot \sin 6{,}28^0 = 7{,}2\,Ws$ $\quad$ ili $P = U \cdot I \cdot \cos\varphi \Rightarrow W = U \cdot I \cdot \cos\varphi \cdot t$

$\boxed{W = 7{,}2\,Ws}$ $\quad$ $\boxed{W = 7{,}2\,Ws}$

2.16

(33.) $i = 2\sqrt{2} \sin \omega t$

$p = 10 - 10\cos 2\omega t = 10(1 - \cos 2\omega t) = 10 \cdot 2 \sin^2 \omega t = 20 \sin^2 \omega t$

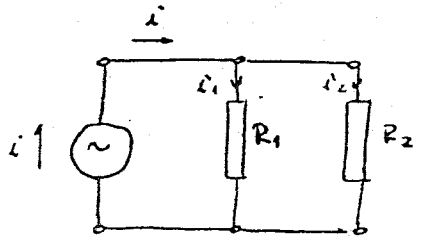
$R = ?$ $\quad R = \frac{P}{i^2} = \frac{20 \sin^2 \omega t}{8 \sin^2 \omega t} = 2{,}5\,\Omega$

$\boxed{R = 2{,}5\,\Omega}$

2.17

(34.) $i = Im\{3\sqrt{2}\, e^{j\omega t}\}$, $R_1 = 1\Omega$ i $R_2 = 2\Omega$

$i_1(t)$, $i_2(t)$ i $P(t) = ?$



$R_u = \frac{1 \cdot 2}{3} = \frac{2}{3}\Omega$

$u(t) = Im\left\{3\sqrt{2} \cdot \frac{2}{3} e^{j\omega t}\right\} = Im\left\{2\sqrt{2}\, e^{j\omega t}\right\}$

$i_1(t) = \frac{u(t)}{R_1} = Im\left\{2\sqrt{2}\, e^{j\omega t}\right\}$ $\quad i_2 = \frac{u(t)}{R_2} = Im\left\{\sqrt{2}\, e^{j\omega t}\right\}$

$u(t) = 2\sqrt{2}\sin(\omega t)$

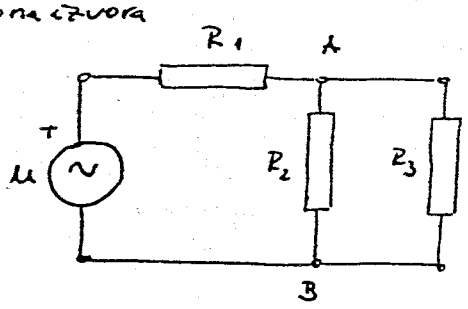
$i_1 = 2\sqrt{2}\sin\omega t$ $\quad i_2 = \sqrt{2}\sin\omega t$ $\quad i = 3\sqrt{2}\sin\omega t$

$p(t) = u(t) \cdot i(t) = \boxed{12\sin^2\omega t = p(t)}$

2.19

(35.) $i_3 = Re\{2(1+j)e^{j\omega t}\}$ $R_1 = 0{,}8\Omega$ $R_2 = 2\Omega$ $R_3 = 3\Omega$

Odredite efektivne vrijednosti svih struja i napona izvora



$i_3 = Re\{2(1+j)e^{j\omega t}\}$

$i_3 = Re\{2\sqrt{2}\, e^{j\frac{\pi}{4}} \cdot e^{j\omega t}\}$

$i_3 = Re\{2\sqrt{2}\, e^{j(\omega t + \frac{\pi}{4})}\}$

$i_3 = 2\sqrt{2}\cos(\omega t + \frac{\pi}{4})$

$\boxed{I_3 = 2A}$ $\quad u_{AB} = i_3 \cdot R_3 = 6\sqrt{2}\cos(\omega t + \frac{\pi}{4})$ $\boxed{U_{AB} = 6V}$

$i_2 = \frac{u_{ab}}{R_2} = \frac{6\sqrt{2}\cos(\omega t + \frac{\pi}{4})}{2} = 3\sqrt{2}\cos(\omega t + \frac{\pi}{4})$ $\boxed{I_2 = 3A}$

$I_1 = I_2 + I_3$

$I_1 = 3 + 2 = 5A$ $\boxed{I_1 = 5A}$ $\quad U = I_1 \cdot Z \Rightarrow R = R_1 + \frac{R_2 \cdot R_3}{R_2 + R_3} =$

$Z = 0{,}8 + \frac{2 \cdot 3}{5}$

$\boxed{U = 10V}$ $\quad R = 0{,}8 + \frac{6}{5} = 0{,}8 + 1{,}2$

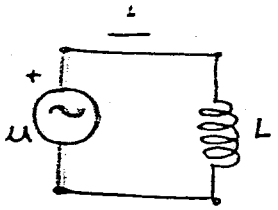
$\boxed{R = 2\Omega}$

2.14

(36.) $i = 5\sin(628t - 60°)$

$L = 0{,}01H$

a) odredi $\Psi(t)$, $u(t)$ i X_L



$u = u_L \quad u = L\frac{di}{dt} \Rightarrow L\,di = u\,dt \Rightarrow di = \frac{1}{L}u\,dt \quad i = \frac{1}{L}\int u\,dt$

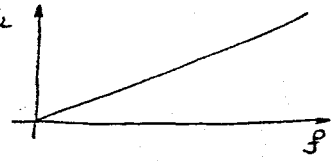
$u = L\frac{di}{dt}. \quad i = 5\sin(628t - 60^\circ)$

$u = 0{,}01 \cdot 5 \cdot 628 \cdot \cos(628t - 60^\circ) = 31{,}4\cos(628t - 60^\circ)$

$u = 31{,}4\sin(628t - 60^\circ + 90^\circ) = \underline{31{,}4\sin(628t + 30^\circ) = u}$

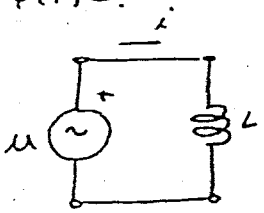
$\psi = L \cdot i = 0{,}05\sin(628t - 60^\circ)$ Vs

$X_L = \omega L = 6{,}28\,\Omega$



(37.) $u = 10\sqrt{2}\sin 5\cdot10^3 t$ i $I = 2$ A a) $L = ?$ b) $\psi(t) = ?$

2.20 $u = \frac{U_{max}}{\sqrt{2}} = 10$V



$X_L = \frac{U}{I} = \frac{10}{2} = 5\,\Omega$

$\omega = 5\cdot10^3 \qquad X_L = \omega L \Rightarrow L = \frac{X_L}{\omega}$

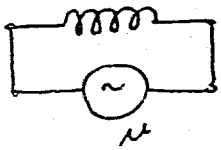
$L = \frac{5}{5\cdot10^3} = 10^{-3}$ $\boxed{L = 0{,}001\,H}$

$\psi(t) = L \cdot i = 0{,}001 \cdot 2\sqrt{2} \cdot \sin(5\cdot10^3 t +$

$\psi(t) = 2\sqrt{2}\cdot10^{-3}\sin(5\cdot10^3 t - 90^\circ)$

(38.) $u = 20\sin 10^3 t \quad L = 0{,}1\,H$

2.21 $i(t) = ? \quad W_L(t) = ? \quad A(t) = ?$ i energiju mag. polja ako je $i = I_m$



$u = L\frac{di}{dt}$

$L\,di = u\,dt \quad di = \frac{1}{L}u\,dt \;/\int$

$i = \frac{1}{L}\int u\,dt = \frac{1}{L}\int 20\sin 10^3 t$

$= \frac{1}{0{,}1}\cdot 20\int \sin 10^3 t\,dt = 200\int \sin 10^3 t\,dt = \frac{200}{10^3}(-\cos 10^3 t)$

$i = -0{,}2\cos 10^3 t = -0{,}2\sin(90 - 10^3 t) = 0{,}2\sin(10^3 t - 90)$

$\underline{i = 0{,}2\sin(10^3 t - 90)} \qquad W = \frac{1}{2}Li^2 = \frac{1}{2}\cdot 0{,}1 \cdot 0{,}04\sin^2(1000t - 90^\circ)$

$W = 0{,}002\sin^2(1000t - 90^\circ)$

$W = 0{,}002\left[\frac{1 - \cos 2(1000t - 90^\circ)}{2}\right] = 0{,}001 - 0{,}001\cos 2(1000t - 90^\circ)$

$p(t) = u \cdot i = 20\sin 10^3 t \cdot (-0{,}2\cos 10^3 t) = -4\sin 10^3 t \cdot \cos 10^3 t =$

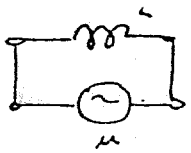
$P(t) = -2\sin 2000t$ W

$W(i = I_{max}) = 0{,}002\sin^2(1000t - 90^\circ)$

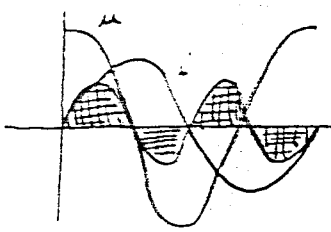
$W(i = I_{max}) = \frac{1}{2}L \cdot 0{,}2^2 = \frac{1}{2}\cdot 0{,}1 \cdot 0{,}04 = \boxed{0{,}002\,Ws = W_{max}}$

2.22

(39) $p = 600 \sin 314t$ $\quad I = 10A$. Odredite L



$p = u \cdot i$



$u = U_m \cdot \cos \omega t \Rightarrow u = U_m \cos \omega t$

$i = I_m \cdot \cos(\omega t - \frac{\pi}{2}) \Rightarrow i = I_m \cos(\omega t - \frac{\pi}{2})$

$p = u \cdot i = U_m \cdot I_m \cos \omega t \cdot \cos(\omega t - \frac{\pi}{2})$

$p = U_m \cdot I_m \cdot \frac{1}{2} \left[\cos(\omega t + \omega t - \frac{\pi}{2}) + \cos(\omega t - \varphi t + \frac{\pi}{2}) \right]$ ($\omega t - \varphi t = 0$)

$p = U_m \cdot I_m \cdot \frac{1}{2} \cos(2\omega t - \frac{\pi}{2}) = U_m \cdot I_m \cdot \frac{1}{2} \sin 2\omega t = p$

$p = P_{max} \sin 2\omega t$ $\quad P_{max} = \frac{1}{2} U_m \cdot I_m \Rightarrow U_m = \sqrt{2} U \Rightarrow P_{max} = U \cdot I$

$I_m = \sqrt{2} I$

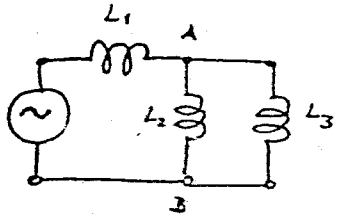
$U = \frac{600}{10} = 60V = U$

$X_L = \frac{U}{I} = 6 \Rightarrow L = \frac{6}{\omega} = \frac{6}{157} = 0{,}0382\,H$ $\quad \boxed{L = 0{,}0382\,H}$

$P_a = U \cdot I \cdot \sin \varphi^{1} \Rightarrow U = 6\,V \Rightarrow \boxed{L = 0{,}0382\,H}$

(40) $i_2 = Re\{\sqrt{2}(0{,}3 + j0{,}4) e^{j\omega t}\}$ $\quad \omega L_1 = 0{,}8\,\Omega$ $\quad \omega L_2 = 2\,\Omega$ $\quad \omega L_3 = 3\,\Omega$

2.24 Odredite efektivne vrijednosti struja i napona



$i_2 = Re\{\sqrt{2}(0{,}3 + j0{,}4) e^{j\omega t}\}$

$= Re\{(0{,}42426 + j0{,}5656) e^{j\omega t}\}$

$= Re\{0{,}707\, e^{j(\omega t + 53{,}126^\circ)}\}$

$i = 0{,}707 \cos(\omega t + 53{,}126^\circ)$

$\boxed{I_2 = 0{,}5\,A}$

$u = L_1 \frac{di}{dt}$

$u \Rightarrow U_{AB} = 0{,}707 \cdot X_{L_1} \cos[\omega t + 53{,}126 + 80^\circ]$ $\quad U_{AB} = 0{,}5 \cdot 2{,}0 = 1V$ $\quad \boxed{U_{AB} = 1V}$

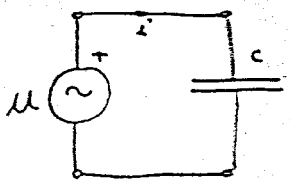
$I_3 = \frac{1}{3} = 0{,}333\,A$ $\quad I_1 = 0{,}5 + 0{,}333 = \frac{1}{2} + \frac{1}{3} = \frac{3+2}{6} = \frac{5}{6}$

$\boxed{I_1 = \frac{5}{6} A}$ $\quad U_L = I_1 \cdot X_{L_1} = \frac{5}{6} \cdot 0{,}8 = \frac{5}{6} \cdot \frac{4}{5} = \frac{2}{3}\,[V]$

$U_{max} = \frac{5}{3}$

2.25

(41.) $u = 120 \sin(314t + 60°)$ i $C = 42\mu F$.



a) $q(t) = ?$ $i(t) = ?$ $X_c(\omega) = ?$

$q = C \cdot u \quad dq = C \cdot du \quad i = \frac{dq}{dt} \quad i\,dt = C \cdot du$ $\boxed{i = C \cdot \frac{du}{dt}}$

$i = C \cdot \frac{du}{dt} = C \cdot 120 \cdot 314 \cdot \cos(314t + 60°) = C \cdot 120 \cdot 314 \cdot \sin[314t + 60° + 90°]$

$i = C \cdot 37680 \sin[314t + 150°]$

$i = 42 \cdot 10^{-6} \cdot 37680 \cdot \sin[314t + 150°] = 1{,}58256 \sin[314t + 150°]$

$\boxed{i = 1{,}58256 \sin[314t + 150°]}$

$i = \frac{dq}{dt} \Rightarrow dq = i\,dt \,/\int \quad q = \int i\,dt$

$q = \int 1{,}58 \sin[314t + 150°]\,dt = \frac{-1{,}58}{314} \cos[314t + 150°]$

$q = -0{,}00504 \sin[90 - (314t + 150)]$

$q = 5 \cdot 10^{-3} \sin[314t + 150 - 90°] = \boxed{5 \cdot 10^{-3} \sin[314t + 60°] = q}$

$X_c = \frac{1}{\omega C}$

$X_c = \frac{10^6}{314 \cdot 42} = 75{,}8\,\Omega$ $\boxed{X_c = 75{,}8\,\Omega}$ $X_c \searrow$ kada $\frac{?}{?}$

(42.) $C = ?$ $I = 1A$ $u = 100\sqrt{2} \sin 10^3 t$ $U = \frac{U_{max}}{\sqrt{2}} = 100V$

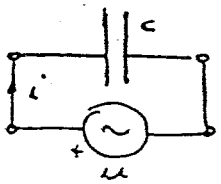
2.26 $X_c = \frac{U}{I} = 100\,\Omega \Rightarrow X_c = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_L} = \frac{1}{1000 \cdot 100} = 10^{-5}\,F$

$C = 10^{-5}\,F$

$\boxed{C = 10\mu F}$

(43.) $u = 10 \sin 10^3 t \quad C = 200\mu F$

2.27 a) $i(t)$, energiju el. polja, $P_c(t)$, energiju el. električnog polja uz $u_c = U_m$



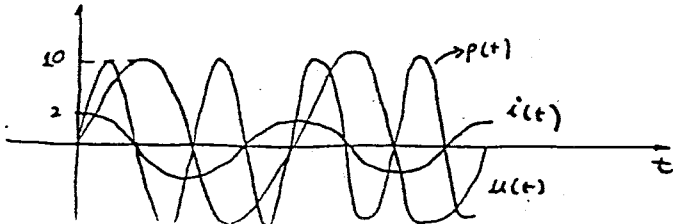
$i = \frac{dq}{dt} = C \frac{du}{dt} = C \cdot 10 \cdot 10^3 \cos 10^3 t = 2 \cdot 10^{-3} \cdot 10^4 \cdot \cos 10^3 t$

$\boxed{i = 2 \cdot \cos 10^3 t\,[A]}$

$W = \frac{1}{2} C u^2 = \frac{1}{2} \cdot 200 \cdot 10^{-6} \cdot 100 \sin^2 10^3 t = 0{,}01 \cdot \sin^2 10^3 t = W(t)$

$P_c(t) = u(t) \cdot i(t) = 10 \sin 10^3 t \cdot 2 \cos 10^3 t = 20 \sin 10^3 t \cdot \cos 10^3 t = 10 \cdot \sin 2 \cdot 10^3 t$

$P(c)(t) = 10 \sin 2 \cdot 10^3 t$ W_c (za $u = U_m$) = opet $= \frac{1}{2} \cdot 200 \cdot 10^{-6} \cdot 10^2 = 10^{-2}$ Ws

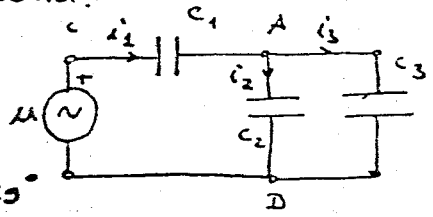


$\boxed{W_{max} = 10^{-2}\,Ws}$

2.30

(44.) Zadan je spoj u kojemu je $\dot{I}_2 = 0{,}4 + j0{,}3$, a $\frac{1}{\omega C_1} = \frac{1}{\omega C_2} = \frac{1}{\omega C_3} = 2\,\Omega$

Odredi efektivne vrijednosti svih struja i napona.



$X_{C_1} = X_{C_2} = X_{C_3} = 2\,\Omega$

$|\dot{I}_2| = \sqrt{0{,}4^2 + 0{,}3^2} \cdot \sqrt{0{,}16 + 0{,}09} = 0{,}5$

$\varphi = \operatorname{arc\,tg} \frac{0{,}3}{0{,}4} = 36{,}869^\circ \qquad \dot{I}_2 = 0{,}5\, e^{j36{,}869^\circ}$

$i_2 = \underset{npr}{Im}\left\{ 0{,}5\, e^{j36{,}87^\circ} \cdot e^{j\omega t} \right\} = Im\left\{ 0{,}5\, e^{j(\omega t + 36{,}87^\circ)} \right\} \qquad I_{MAX} = 0{,}5$

$I_2 = \frac{0{,}5}{\sqrt{2}}$

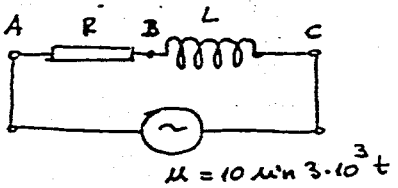
$I_2 = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}\,[A] \qquad U_{AB} = \dot{I}_2 \cdot X_{C_2} = \frac{\sqrt{2}}{4} \cdot 2 = \frac{\sqrt{2}}{2}\,[V]$

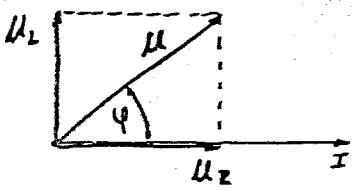
$I_3 = \frac{U_{AB}}{X_{C_3}} = \frac{\frac{\sqrt{2}}{2}}{2} = \frac{\sqrt{2}}{4}\ A \qquad I_1 = I_1 + I_2 = \frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{2}}{2}\ A$

$u_{AC} = I_1 \cdot X_{C_1} = \frac{\sqrt{2}}{2} \cdot 2 = \sqrt{2}\ V \qquad U = U_{AC} + U_{AB} = \sqrt{2} + \frac{\sqrt{2}}{2} = \frac{3\sqrt{2}}{2}\ V$

(45.) Na spitak od radnim otporom $R = 2\,\Omega$ i $L = 1\,mH$ priključen je napon
2.33
$u = 10 \sin 3 \cdot 10^3 t$. Odredi: $i(t)$, $u_L(t)$ i $u_R(t)$

: prikazati frekvencijske karakteristike $z(\omega)$ i $\varphi(\omega)$





$z^2 = R^2 + X_L^2$

$z = \sqrt{R^2 + X_L^2} = \sqrt{4 + 9}$

$z = \sqrt{13}\ \Omega$

$X_L = \omega L = 3 \cdot 10^3 \cdot 10^{-3} = 3\,\Omega$

$I = \frac{U}{z} = \frac{\frac{10}{\sqrt{2}}}{\sqrt{13}} = \frac{10}{\sqrt{13} \cdot \sqrt{2}}\ A \qquad \varphi = \operatorname{arc\,tg} \frac{X_L}{R} = \operatorname{arc\,tg} \frac{3}{2} = \operatorname{arc\,tg} 1{,}5$

$\varphi = 56{,}309^\circ$

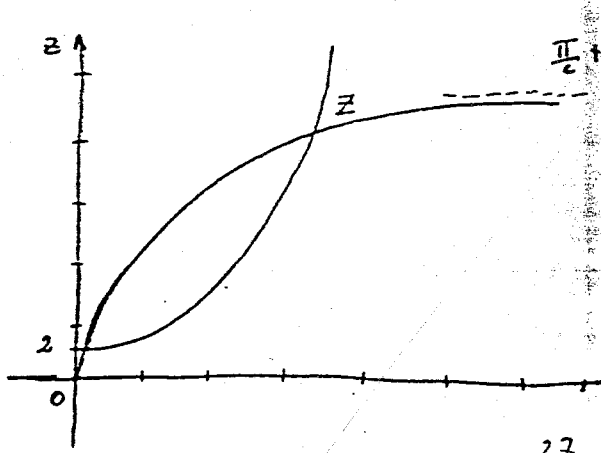
$i = I_{max} \sin(3 \cdot 10^3 t - 56{,}309^\circ) = \frac{10}{\sqrt{13}} \sin(3 \cdot 10^3 t - 56{,}309^\circ)$

$u_R = i \cdot R = \frac{20}{\sqrt{13}} \sin(3 \cdot 10^3 t - 56{,}309^\circ)$

$u_L = \frac{30}{\sqrt{13}} \sin(3 \cdot 10^3 t - 56{,}309^\circ + 90^\circ)$

$z = \sqrt{R^2 + \omega^2 L^2} = \sqrt{4 + 10^{-6}\omega^2}$

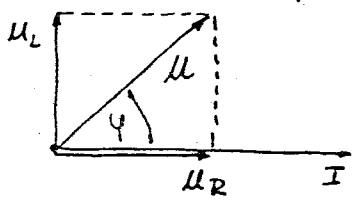
$\varphi = \operatorname{arc\,tg} \frac{\omega L}{R} = \operatorname{arc\,tg} \frac{10^{-3}\omega}{2}$

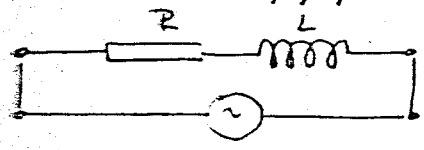


2.34

(46.) Odredi $i(t)$, $p(t)$, $p_R(t)$ i $p_L(t)$ u mreži na kojoj je $u = 100 \sin(314t + 30°)$,

$R = 5\,\Omega$ i $L = 27{,}5\,mH$





$\omega = 314 \frac{rad}{s}$

$\omega L = 8{,}635\,\Omega$

$z^2 = R^2 + X_L^2 \Rightarrow z = \sqrt{R^2 + X_L^2} = \sqrt{25 + 74{,}56} = 9{,}97\,\Omega$

$I = \frac{U}{z} = \frac{100}{\sqrt{2} \cdot 9{,}976} = 7{,}086726\,A$

$I_{MAX} = 10{,}02\,A$

$\varphi = \operatorname{arc\,tg} \frac{\omega L}{R} = \operatorname{arc\,tg} 1{,}72$

$\boxed{\varphi = 60°}$

$i(t) = 10{,}002 \sin(314t + 30° - 60°)$

$i(t) = 10{,}02 \sin(314t - 30°)$ $\quad u_R(t) = i(t) \cdot R = 50 \sin(314t - 30°)$

$p(t) = ?$ $\quad p(t) = u(t) \cdot i(t) = 100 \sin(314t + 30°) \cdot 10 \cdot \sin(314t - 30°)$

$p(t) = 1000 \sin(314t + 30°) \cdot \sin(314t - 30°)$

$p(t) = 1000 \left(-\frac{1}{2}\right) \cdot \left[\cos(314t + 30° + 314t - 30°) - \cos(314t + 30° - 314t + 30°)\right]$

$p(t) = -500 \left[\cos 2 \cdot 314t - \cos 60°\right] = 500 \left[\cos 60° - \cos 2 \cdot 314t\right]$

$p(t) = 500 \left[\frac{1}{2} - \cos 2 \cdot 314t\right] = 250 - 500 \cos 2 \cdot 314t = p(t)$

RADNA SNAGA IZNOSI $p_R = u_R(t) \cdot i(t) = 50 \sin(314t - 30°) \cdot 10 \sin(314t - 30°)$

$p_R(t) = 500 \sin^2(314t - 30°) = 500 \cdot \frac{1}{2} \left[1 - \cos(2 \cdot 314t - 60°)\right]$

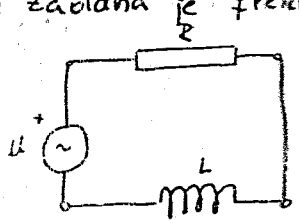
$p_R(t) = 250 \left[1 - \cos(2 \cdot 314 - 60°)\right] = 250 - 250 \cos(2 \cdot 314 - 60°)$ //

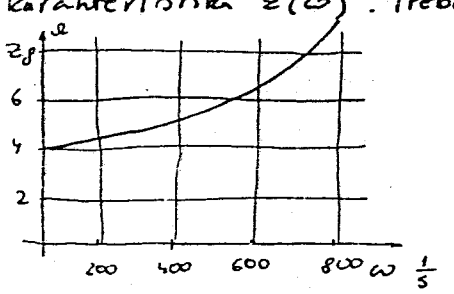
$p_L(t) = u_L(t) \cdot i(t) \Rightarrow u_L(t) = 86{,}35 \sin(314t - 30° + 90°) = 86{,}35 \sin(314t + 90° - 30°)$

$p_L(t) = 86{,}35 \cos(314t - 30°) \cdot 10 \sin(314t - 30°)$ $\quad u_L = 86{,}35 \cos(314t - 30°)$

$p_L(t) = \frac{863{,}5}{2} \sin 2(314t - 30°) = 431{,}75 \sin(2 \cdot 314t - 60°)$ //

① Zadana je frekvencijska karakteristika $z(\omega)$. Treba pronaći R i L





za $\omega = 0\ \frac{rad}{s}$

$z = 4\,\Omega \Rightarrow \boxed{R = 4\,\Omega}$

$z^2 = R^2 + X_L^2$

$X_L^2 = z^2 - R^2 \qquad \omega^2 L^2 = z^2 - R^2 \qquad \omega^2 = \frac{z^2 - R^2}{L^2} \Rightarrow \omega = \frac{1}{L}\sqrt{z^2 - R^2}$

$L = \frac{\sqrt{z^2 - R^2}}{\omega}$ za $\omega = 600\ \frac{rad}{s} \Rightarrow z = 6{,}7\,\Omega$

$L = \frac{\sqrt{44{,}89 - 16}}{600} = 8{,}9 \cdot 10^{-3}\,H \qquad L = 8{,}9 \cdot 10^{-3}\,H$

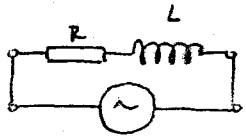
Rješenje

$R = 4\,\Omega$

$L = 9\,mH$

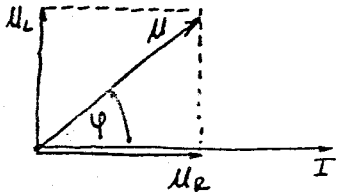
7.36.

② Zadan je spoj na kojem je $u = U_m \sin\omega t$ i $X_L = R$. Nacrtajte dijagram vremenske ovisnosti veličina φ, ω_L i p_L



$u = U_m \sin\omega t$

$X_L = R$



$z^2 = R^2 + X_L^2 = R^2 + R^2 = 2R^2 \qquad z = \sqrt{2}R$

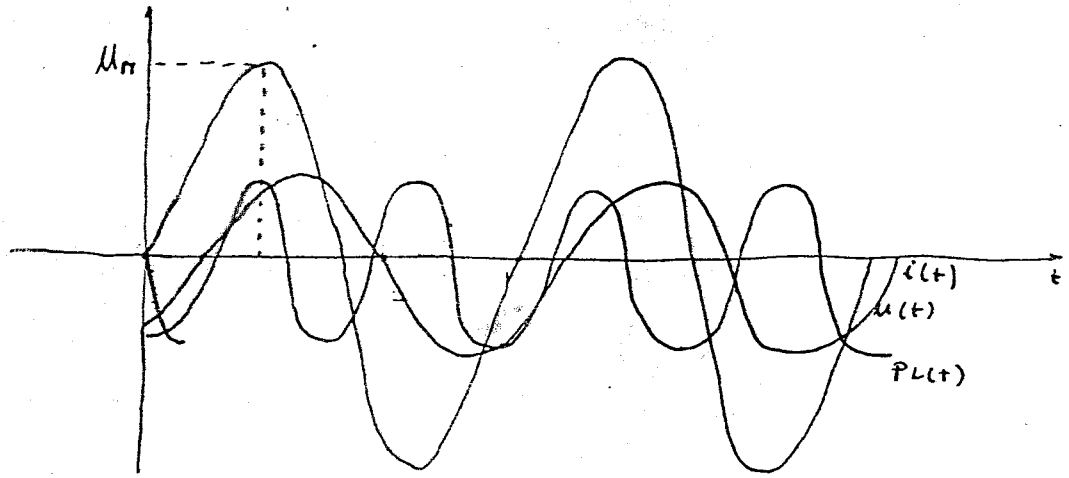
$\varphi = 45° \qquad u = U_m \sin\omega t$

$i = \frac{U_m}{\sqrt{2}R}\sin(\omega t - 45°) \qquad \psi = L \cdot i = \frac{L \cdot U_m}{\sqrt{2}R}\sin(\omega t - 45°)$

$U_L(t) = I_{max} \cdot X_L \cdot \sin(\omega t - 45° + 90°) = \frac{U_m}{\sqrt{2} \cdot X_L} \cdot X_L \sin(\omega t + 45°) = \frac{U_m}{\sqrt{2}}\sin(\omega t + 45°)$

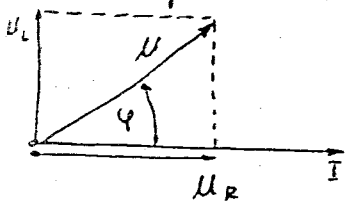
$p_L(t) = U_L(t) \cdot i(t) = \frac{U_m}{\sqrt{2}} \cdot \frac{U_m}{\sqrt{2}R} \cdot \sin(\omega t + 45°) \cdot \sin(\omega t + -45°) = \frac{U_m^2}{2R}\sin(\omega t + 45°) \cdot \cos(\omega$

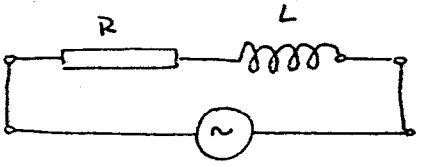
$p_L(t) = \frac{U_m^2}{2R \cdot 2} \cdot 2\sin(\omega t - 45°) \cdot \cos(\omega t - 45°) = \frac{U_m^2}{4R}\sin(2\omega t - \frac{\pi}{2}) = \frac{-U_m^2}{4R} \cdot \cos 2\omega t$



(2.78)

3. Napon izvora je $U = 220V$ $f = 50Hz$, a žarulja je izrađena za napon od 80V pri kojemu teče struja od 0,5A. Koliki mora biti induktivitet da bi ona dobila potrebni napon.





$I \cdot R = 80V \Rightarrow R = \frac{80}{0,5} = 160\Omega$

$U^2 = U_R^2 + U_L^2 \quad (I \cdot Z)^2 = U_R^2 + U_L^2 \quad U_L^2 = 220^2 - 80^2 = 48400 - 6400 = 42000$

$X_L = \frac{U_L}{I} = \frac{204,9}{0,5} = 409,8\Omega$ $\boxed{U_L = 204,9\ V}$

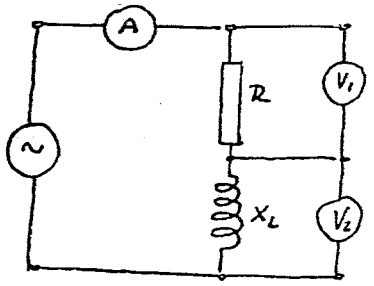
$L = \frac{409,8}{\omega} = \boxed{1,3\ H = L}$

(2.79)

4. Odredite pokazivanje mjernih instrumenata ako je na spoj priključen

1) sinusoidalni napon efektivne vrijednosti $U = 10V$, a pritome je $\dot{z} = \sqrt{2}\, e^{j45°}$

2) istosmjerni napon



$\dot{z} = \sqrt{2}\, e^{j45°}$

$z = \sqrt{2}(\cos 45° + j \sin 45°)$

$z = \sqrt{2}\left(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right)$

$z = 1 + j$

$\Rightarrow R = 1\Omega \quad X_L = 1\Omega$ a) $I = \frac{U}{Z} = \frac{U}{\sqrt{R^2 + X_L^2}} = \frac{10}{\sqrt{2}}$ $\underline{I = 5\sqrt{2}\ A}$

$U_R = I \cdot R = 5\sqrt{2} \cdot 1 = 7,07\ V \quad U_L = I \cdot X_L = 7,07\ V$

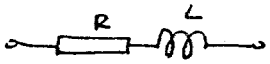
b) Za istosmjerni napon $U_L = 0 \quad U_R = U = 10V$

(2.40)

5. Induktivni svitak je u krugu izvora sinusoidalnog napona $U_1 = 100V$, a kompleksna izvrsna prividna snaga $\dot{S} = 20 + j40$. Odredite snagu koja se troši na toj zavojnici ako je priključena na izvor napona od $U_2 = 100\ V$ (istosmjernog napona)

$\dot{S} = 20 + j40 \Rightarrow P = 20\ W$

$Q = 40\ VAr$



$P = I^2 \cdot R = \frac{U_R^2}{R} \Rightarrow R = \frac{U_R^2}{P} =$

$\varphi = \text{arc tg}\ \frac{40}{20} = \text{arc tg}\ 2$

$\varphi = 63,435°$

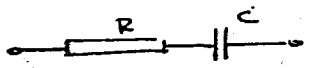
$P = U \cdot I \cdot \cos\varphi \Rightarrow I = \frac{P}{U \cos\varphi} = \frac{20}{100 \cdot 0,447213} = 0,447A$

$P = I^2 \cdot R \Rightarrow R = \frac{20}{0,200019} = 99,9\Omega$

$\boxed{P_2 = 10W}$

2.41

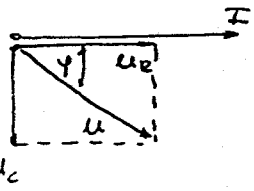
6. $u = 100\sqrt{2}\sin 1000t$ $R = 10\,\Omega$ $C = 100\,\mu F$ Odredi: $i(t)$ $u_R(t)$ $u_L(t)$



$z^2 = R^2 + X_c^2$

$z^2 = 100 + \left[\frac{1}{1000 \cdot 10^{-4}}\right]^2$

$z^2 = 100 + 100$



$z = 10\sqrt{2}\,\Omega$ $I = \frac{U}{z} = \frac{100}{10\sqrt{2}} = \frac{10}{\sqrt{2}}$ A $\underline{I = \frac{10}{\sqrt{2}}\text{ A}}$

$\varphi = \text{arc tg}\,\frac{1}{\omega RC}$

$\Rightarrow \varphi = 45°$

$X_c = 10$

$i = 10\sin(1000t + 45°)$

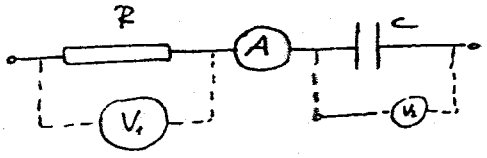
$u_R = i \cdot R = 100\sin(1000t + 45°)$

$u_C = i \cdot X_c = 100\sin(1000t + 45° - 90°) = 100\sin(1000t - 45°)$ ✓✓

2.47

7. Zadan je izraz $\dot{z} = \sqrt{2}\,e^{-j45°}$. Odredite pokazivanje mjernih instrumenata ako je krug priključen na 1. stalni napon $U = 10\,V$

2. izmjenični napon ef. vrijednosti $U = 10\,V$



$z = \sqrt{2}\,e^{-j45°}$

$z = \sqrt{2}(\cos 45° - j\sin 45°)$

$= \sqrt{2}\left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right)$

$= 1 - j$ $z = 1 - j \Rightarrow \underline{R = 1\,\Omega}$ $\underline{X_c = 1\,\Omega}$

1. za stalni napon od 10 V poslije stacionarnog stanja $U_1 = 0\,V$ $U_2 = 10\,V$ $I = 0\,A$

2. $U_{ef} = 10\,V$

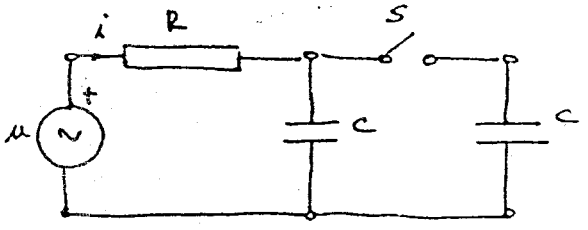
$z^2 = R^2 + X_c^2 = 1 + 1 = 2 \Rightarrow z = \sqrt{2}$ $I = \frac{U}{z} = \frac{10}{\sqrt{2}} = \underline{7{,}07\text{ A}}$

$U_R = I \cdot R = \underline{7{,}07\,V}$

$U_C = I \cdot X_c = \underline{7{,}07\,V}$ ✓✓

2.48

8. Pri otvorenom prekidaču struja u krugu prethodi naponu izvora za kut $\varphi_1 = 45°$. Koliki kut je (φ) pri zatvorenom prekidaču



pri otvorenom prekidaču $\varphi = 45°$

$\varphi = \text{arc tg}\,\frac{X_c}{R} = 45° \Rightarrow \frac{X_c}{R} = 1$

$X_c = R$

Poslije zatvaranja sklopke S $\Rightarrow$ biti će $C_u = C + C = 2C$

$X_{C_u} = \frac{1}{\omega C_u} = \frac{1}{\omega \cdot 2C}$

$X_{C_u} = \frac{1}{2}X_C$

$\varphi_2 = \text{arc tg}\,\frac{X_{C_u}}{R} = \text{arc tg}\,\frac{\frac{1}{2}X_C}{R} = \text{arc tg}\,\frac{\frac{1}{2}R}{R}$

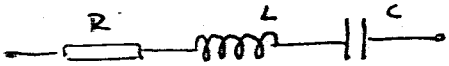
$\varphi = \text{arc tg}\,\frac{1}{2} \Rightarrow \boxed{\varphi = -26{,}56°}$ ✓✓

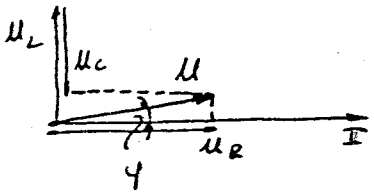
2.49

⑨ U spoju u kojemu je $u = Re\{20 e^{j(\omega t + \dots)}\}$, $R = 4\Omega$ $L = 1\,mH$ $C = 100\,\mu F$

$\omega = 5 \cdot 10^3 \frac{1}{s}$. $i(t) = ?$ $u_R(t) = ?$ $u_L(t) = ?$ $u_C(t) = ?$ $Z(\omega) = ?$ $\varphi(\omega) = ?$

$u = 20 \cos(\omega t + 15°)$





$U^2 = U_R^2 + (U_L - U_C)^2 / : I^2 \Rightarrow Z^2 = R^2 + (X_L - X_C)^2$

$X_L = \omega L = 5 \cdot 10^3 \cdot 10^{-3} = 5\Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{5 \cdot 10^3 \cdot 10^{-4}} = \frac{10^4}{5 \cdot 10^3} = 2\,\Omega$

$Z^2 = 16 + 9 \Rightarrow \boxed{Z = 5\Omega}$

$I = \frac{U}{Z} = \frac{20/\sqrt{2}}{5} = \frac{20}{5\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$ $I = 2\sqrt{2}$ $\varphi = \operatorname{arc\,tg} \frac{X_L - X_C}{R} = \operatorname{arc\,tg} \frac{3}{4}$

$\varphi = 36{,}86°$

$i = 4 \cos(\omega t + 45° - 36{,}86°) = 4 \cos(\omega t - 22°)$ struja

$u_R = i \cdot R = 16 \cos(\omega t - 22°)$

$u_L = i \cdot X_L = 4 \cdot 5 \cos[(\omega t - 22°) + 90°] = 20 \cos(\omega t + 68°)$ napon

$u_C = i \cdot X_C = 4 \cdot 2 \cos[\omega t - 22° - 90°] = 8 \cos(\omega t - 112°)$ napon

$Z^2 = R^2 + (X_L - X_C)^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = R^2 + \left(\frac{\omega^2 LC - 1}{\omega C}\right)^2$ $Z = \sqrt{R^2 + \left(\frac{\omega^2 LC - 1}{\omega C}\right)^2}$ $\Rightarrow Z = Z(\omega)$

Z_{min} ako je $\frac{\omega^2 LC - 1}{\omega C} = 0$ $\omega^2 LC - 1 = \omega C$ $\omega^2 LC - \omega C - 1 = 0$

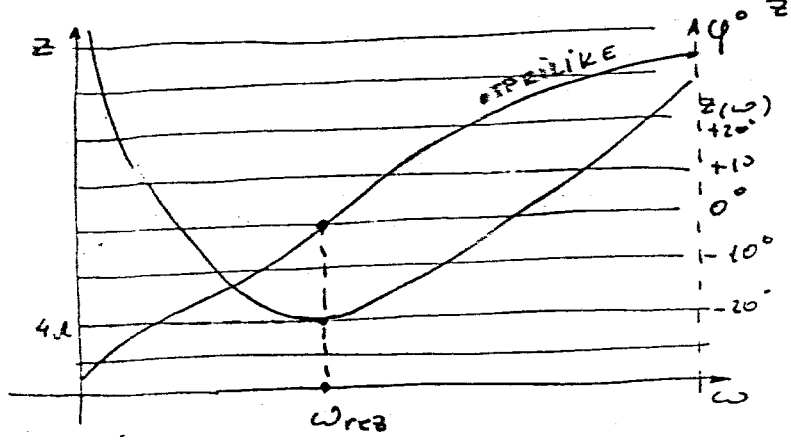
$\omega_{1,2} = \frac{+C \pm \sqrt{C^2 + 4LC}}{2LC} \Rightarrow$ kada je $\omega^2 LC = 1$ $\omega = \sqrt{\frac{1}{LC}}$

za naš slučaj za $\omega = \sqrt{\frac{1}{10^{-3} \cdot 10^{-4}}}$

$= \sqrt{10^7}$

$\omega = \sqrt{10} \cdot 10^3$

$\boxed{Z = R}$



$\varphi = \operatorname{arc\,tg} \frac{X_L - X_C}{R} =$

$\varphi = \operatorname{arc\,tg} \frac{\omega L - \frac{1}{\omega C}}{R} = \operatorname{arc\,tg} \frac{\omega^2 LC - 1}{RC}$

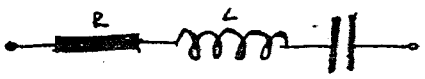
$\varphi = \operatorname{arc\,tg} \frac{\omega^2 LC - 1}{RC\omega}$ za $\omega_{rez} \Rightarrow \boxed{\varphi = 0°}$

$\omega \to 0 \Rightarrow \varphi \to -\frac{\pi}{2}$

za ...

$\omega \to +\infty \Rightarrow \varphi \to +\frac{\pi}{2}$

10. (L.SV.) Zadano je u spoju $u = 20\sin(5000t + 15°)$ $R = 4\,\Omega$ $L = 1\,mH$ $C = 100\,\mu F$

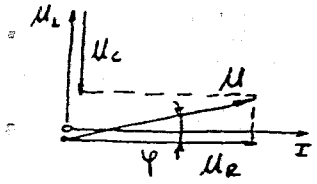


a) Odredi $i(t)$, $p_R(t)$, $p_L(t)$, $p_C(t)$

$w_L(t)$ i $w_C(t)$

$X_C = \frac{1}{\omega C} = \frac{1}{5\cdot10^3\cdot10^{-4}} = \frac{10^4}{5\cdot10^3} = \boxed{2\,\Omega = X_C}$

$X_L = \omega L = 5\cdot10^3\cdot10^{-3} = 5\,\Omega$ $\boxed{X_L = 5\,\Omega}$



$U^2 = U_R^2 + (U_L - U_C)^2 \Rightarrow Z^2 = R^2 + (X_L - X_C)^2$

$Z^2 = 16 + 9 = 25 \Rightarrow \boxed{Z = 5\,\Omega}$ $I = \frac{U}{Z} = \frac{20/\sqrt{2}}{5} = 2\sqrt{2}\,A$

$\varphi = \operatorname{arctg}\frac{X_L - X_C}{R} \Rightarrow \varphi = 36{,}86°$

$i = 4\sin(5000t + 15° - 36{,}86°)$

$i = 4\sin(5000t - 21{,}86°)$ $u_R(t) = 16\sin(5000t - 22°)$

$p_R(t) = u_R(t)\cdot i(t) = 16\cdot\sin(5000t - 22°)\cdot 4\cdot\sin(5000t - 22°)$

$p_R(t) = 64\sin^2(5000t - 22°) = 64\cdot\frac{1}{2}(1 - \cos 2(5000t - 22°))$

$p_R(t) = 32 - 32\cos(10^4t - 44°)$ $\boxed{p_R(t) = 32 - 32\cos(10^4t - 44°)}$ $P_{dijel} = 32\,W$

$P = U\cdot I\cdot\cos\varphi$

$P = U_R\cdot I$

$P = I^2\cdot R$

$p_L(t) = ?$ $p_L(t) = u_L(t)\cdot i(t) \Rightarrow u_L(t) = 4\cdot5\cdot\sin(5000t - 22° + 90°)$ $\Rightarrow X_L$

$u_L = 20\sin(5000t + 68°)\,[V]$

$p_L(t) = 20\sin(5000t + 68°)\cdot 4\sin(5000t - 22°) = 80\sin(5000t + 68°)\sin(5000t - 22°)$

$p_L(t) = 80\cdot\cos(5000t - 22°)\cdot\sin(5000t - 22°) = 40\sin 2(5000t - 22°)$

$p_L(t) = 40\sin(10^4t - 44°) = 40\cos(10^4t + 46°)$ // $P_L = I^2\cdot X_L$ amplituda

$p_C(t) = u_C(t)\cdot i(t) \Rightarrow u_C(t) = 8\sin(5000t - 22° - 90°) = 8\sin(5000t - 112°)$

$p_C(t) = 8\sin(5000t - 112°)\cdot 4\sin(5000t - 22°)$

$= 32\sin(5000t - 112)\cdot\sin(5000t - 22°) = 32\sin(5000t - 112°)\cdot\cos(5000t - 112°)$

$= 32\cdot\frac{1}{2}\sin(10^4t - 224°) = 16\sin(10^4t - 224°)$

$= 16\sin(10^4t + 136°) = 16\sin(10^4t - 224°) = 16\cos(10^4t - 134°)$

$\boxed{p_C(t) = 16\cos(10^4t - 134°)\,VA}$ $P_C = I^2\cdot X_C$

$P_C = \frac{U_C^2}{X_C}$

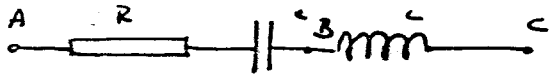
$w_L = \frac{1}{2}Li^2 = \frac{1}{2}\cdot10^{-3}\cdot4\sin^2(5000t - 22°) = 2\cdot10^{-3}\sin^2(5000t - 22°)$

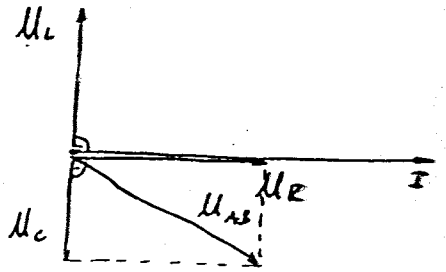
$w_L = 0{,}0024\sin^2(5000t - 22°)$

$w_L = \frac{1}{2}Cu_C^2 = \frac{1}{2}\cdot10^{-4}\cdot8^2\sin^2(5000t - 112°) = 32\cdot10^{-4}\sin^2(5000t - 112°)$ //

2.53

(11.) U spoju u kojem je $U_{ab} = 100V$ $X_L = 6\Omega$ $X_C = 3\Omega$ $R = 4\Omega$ odredi U_L i φ





$U_{AB}^2 = U_R^2 + U_C^2$

$Z_{AB}^2 = R^2 + X_C^2 \quad Z_{AB} = \sqrt{R^2 + X_C^2}$

$Z_{AB} = \sqrt{16 + 9} = 5\Omega$ $\boxed{Z_{AB} = 5\Omega}$

$I = \frac{U_{AB}}{Z_{AB}} = \frac{100V}{5\Omega} = 20A \Rightarrow U_L = I \cdot X_L = 20 \cdot 6 = 120V$

$U_L = 120V \qquad U_R = I \cdot R = 80V \qquad U_C = I \cdot X_C = 60V$

$U^2 = U_R^2 + (U_L - U_C)^2 = 80^2 + 60^2 = 6400 + 3600 \Rightarrow U = 100V$

$\varphi = \operatorname{arc\,tg} \frac{X_L - X_C}{R} = \frac{U_L - U_C}{U_R} = \operatorname{arc\,tg} \frac{U_L - U_C}{U_R} = \operatorname{arc\,tg} \frac{60}{80} = \operatorname{arc\,tg} \frac{3}{4}$

$\Rightarrow \boxed{\varphi = +37^\circ}$ opterećenje je induktivnog karaktera.

2.54

(12.) Zadan je spoj na slici s $U = 100V$ $R = 6\Omega$ $I = 10A$

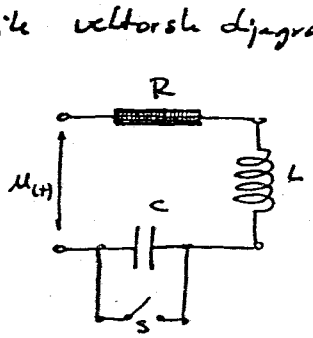
pri zatvorenoj i otvorenoj sklopci S

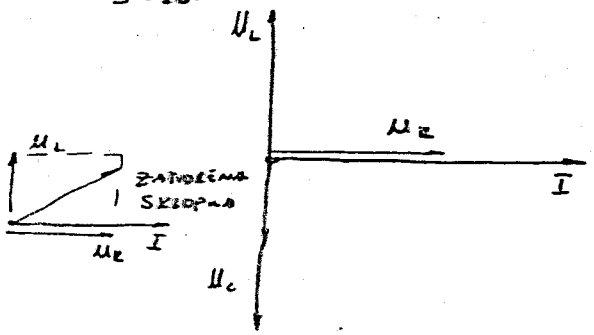
1° Odredite: X_L i X_C. Nacrtajte vektorski dijagram pri otvorenoj i zatvorenoj sklopci.

$U = 100V$

$R = 6\Omega$

$I = 10A$





otvorena sklopka S

$I_1 = \frac{U}{Z_1}$

$I_1 = I_2 = 10A$

$I_2 = \frac{U}{Z_2}$

$Z_L^2 = R^2 + (X_L - X_C)^2$

$Z_L^2 = R^2 + X_L^2$

$\Rightarrow Z_1 = Z_L \quad R^2 + (X_L - X_C)^2 = R^2 + X_L^2 \quad (X_L - X_C)^2 = X_L^2$

pri zatvorenoj sklopci

$I_2 = \frac{U}{Z_2} = 10 \Rightarrow Z_2 = \frac{U}{I} = 10 \quad Z_2 = \sqrt{R^2 + X_L^2} = 10 \,/^2 \quad R^2 + X_L^2 = 100$

$X_L^2 = 64 \quad \boxed{X_L = 8\Omega}$

$(X_L - X_C)^2 = X_L^2$

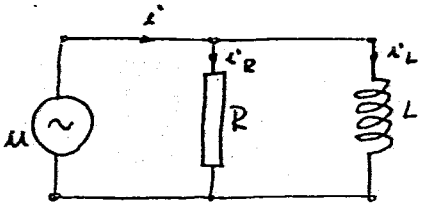
$X_L^2 - 2X_L X_C + X_C^2 = X_L^2 \quad X_C^2 - 2X_L X_C = 0 \quad X_C(X_C - 2X_L) = 0$

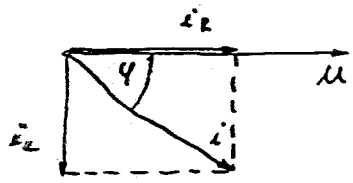
$\boxed{X_C = 16\Omega}$ $\qquad X_C = 0 \qquad X_C = 2X_L = 16\Omega$

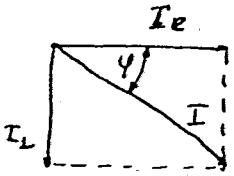
(13) (2,55) U spoju na slici je $i = Re\{5\sqrt{2}\, e^{j1200t}\}$, $U = 48V$ i $R = 16\,\Omega$

Odredi X_L i utvrditi kompleksne ~~brojeve~~ izraze za $i_R(t)$ i $i_L(t)$

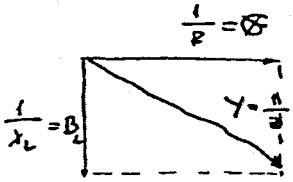
$i = 5\sqrt{2}\cdot\cos 1200t\ [A]$ $\boxed{I = 5\,A}$







/ : U



$$\gamma^2 = G^2 + B_L^2 = \left(\frac{1}{16}\right)^2 + \left(\frac{1}{X_L}\right)^2 = \left(\frac{1}{16}\right)^2 + B_L^2 = \left(\frac{I}{48}\right)^2 \qquad \left(\frac{I}{U}\right)^2 = \left(\frac{1}{R}\right)^2 + B_L^2$$

$$B_L^2 = \left(\frac{I}{U}\right)^2 - \left(\frac{1}{R}\right)^2 = 0{,}01085 - 0{,}00390625 = 0{,}006943$$

$B_L = 0{,}08333$

$X_L = \frac{1}{B_L} = 12$ $\boxed{X_L = 12\,\Omega}$

$\varphi = \text{arc tg}\frac{B_L}{G} = \text{arc tg}\frac{R}{X_L}$

$\varphi = \text{arc tg}\frac{16}{12} = 53{,}13^\circ = \varphi$

$i = 5\sqrt{2}\cos 1200t$

$u = 48\sqrt{2}\cos(1200t + 53{,}13^\circ)$

$i_R = \frac{u}{R} = 3\sqrt{2}\cos(1200t + 53{,}13^\circ) \Rightarrow i_R = Re\{3\sqrt{2}\, e^{j(1200t + 53{,}13^\circ)}\}$

$i_L = \frac{48\sqrt{2}}{12}\cos(1200t + 53{,}13^\circ - 90^\circ) = 4\sqrt{2}\cos(1200t - 36{,}869^\circ)$

$i_L = Re\{4\sqrt{2}\, e^{j(1200t - 36{,}869^\circ)}\}$ $\tilde{I}\ \tilde{I}$

(2,56)

(14) Zadan je spoj prema slici $U = 120V$, $I_R = 15A$ $I_L = 8A$ i $f = 1\,kHz$.

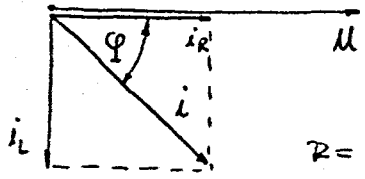
Odredite 1. R i X_L. 2. kolike će biti struje u granama ako se frekvencija napona poveća 2 puta. 3. pri kojoj će frekvenciji napona iznos struja u granama biti jednak

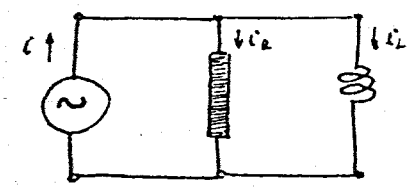
$U = 120V$

$I_R = 15A$

$I_L = 8\,A$

$f = 1\,kHz$





$R = \frac{U}{I_R} = \frac{120}{15} = 8\,\Omega$

$X_L = \frac{U}{I_L} = \frac{120}{8} = 15\,\Omega$

$\Rightarrow X_L = \omega L \Rightarrow \omega = \frac{X_L}{L} \Rightarrow L = \frac{X_L}{\omega} = \frac{15}{2\pi\cdot 1000} \Rightarrow$ $\boxed{L = 0{,}002388\,H}$

akoru frekvencija poveća na dvostruku vrijednost struja će biti

$I_R = \frac{U}{R} = \frac{120}{8} = 15\,A$ $\boxed{I_R = 15A}$ $\quad I_L = \frac{U}{X_L} = \frac{U}{\omega_1 L} = \frac{U}{2\omega L} = \frac{1}{2} I_{L1} = \frac{30}{8} = 4\,A$ $\boxed{I_L = 4A}$

Jinos struja će u granama biti isti ako je $R = X_L$

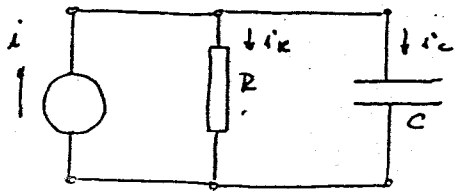
$X_L = \omega L = 8 \quad \omega = \frac{8}{L} \Rightarrow 2\pi f = \frac{8}{L} \quad f = \frac{8}{2\pi L} = \frac{4}{\pi L} = \frac{4}{3{,}14 \cdot 0{,}002388}$

$\boxed{f = 533\,Hz}$

(15.) 2.57

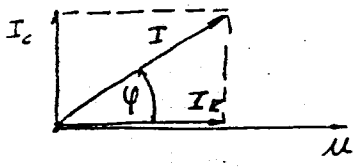
Opoju na slici je $i_c = Im\{10\, e^{j\omega t}\}$, $\omega = 10^4\,s^{-1}$ i $\dot{Y} = 0{,}1 + j\,0{,}1$.

Odredi kompleksne izraze $u(t)$, $i_R(t)$, i $i(t)$. Nacrtajte frekvencijske karakteristike $Y(\omega)$ i $\psi(\omega)$



$\dot{Y} = 0{,}1 + j\,0{,}1 \qquad G = 0{,}1\,S \Rightarrow R = 10\,\Omega$

$B_c = 0{,}1\,S \Rightarrow X_c = 10\,\Omega \qquad C = \frac{1}{X_c \cdot \omega}$



$i_c = Im\{10\, e^{j\omega t}\}$

$i_c = 10\sin\omega t \Rightarrow i_c = \frac{dq}{dt} = C\frac{du}{dt}$

$du = \frac{1}{C} i\,dt \,/\int \qquad u = \frac{1}{C}\int i\,dt = \frac{1}{C}\int 10\sin\omega t\,dt = -\frac{10}{\omega C}\cos\omega t = \frac{10}{\omega C}\sin(\omega t - 9$

$u(t) = \frac{10}{10^4 \cdot \frac{1}{X_c \cdot 10^4}}\sin(\omega t - 90°) = 100\sin(\omega t - 90°) \Rightarrow$

$u(t) = Im\{100\, e^{j(\omega t - 90°)}\}$

$\varphi = \text{arc tg}\frac{B_c}{G} = \text{arc tg}\frac{0{,}1}{0{,}1} \Rightarrow \boxed{\varphi = 45°}$

$i_R = 10\sin(\omega t - 90°) = Im\{10\, e^{j(\omega t - 90°)}\}$

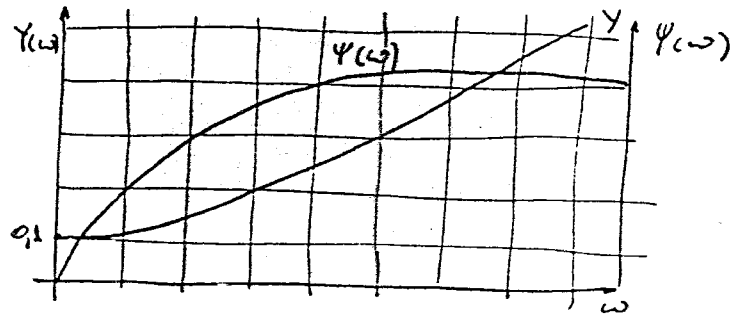
$Y = \sqrt{G^2 + B_c^2} = \sqrt{0{,}01 + 0{,}01}$

$Y = 0{,}14142\,S$

$i_{(\omega)} = 14{,}142\sin(\omega t - 45°) \Rightarrow i = Im\{14{,}142\, e^{j(\omega t - 45°)}\}$

$Y = \sqrt{G^2 + B_c^2} = \sqrt{G^2 + \omega^2 C^2} = \sqrt{0{,}01 + 10^{-8} C^2 \cdot \omega^2}$ $\Rightarrow$ u ∞ u približenje pravcu.

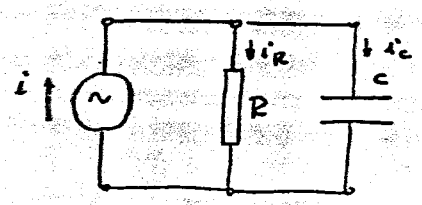
$\psi(\omega) = \text{arc tg}\frac{B_c}{G} = \text{arc tg}\frac{\omega C}{G} = \text{arc tg}\, 10^{-4}\omega$



(2.58)

16. Zadan je spoj na slici s $U = 10V$ i $\dot{y} = 0{,}25\sqrt{2}\, e^{j45^\circ}$. Odredite

1. ukupnu struju
2. snagu spoja



$Y = \frac{1}{4}\sqrt{2}\,(\cos 45^\circ + j \sin 45^\circ)$

$Y = \frac{\sqrt{2}}{4}\left(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right) = \frac{1}{4} + j\frac{1}{4}$ $\quad Y = \frac{1}{4} + j\frac{1}{4}$ $\quad I = \frac{U}{Z} = U \cdot Y = 10\left(\frac{1}{4} + j\frac{1}{4}\right)$

$\boxed{I = \frac{5}{2} + j\frac{5}{2} \ A}$ $\quad I = 2{,}5\sqrt{2}\, e^{j45^\circ}$

b) Snaga spoja

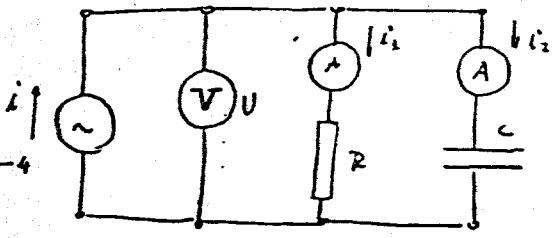
$P = U \cdot I \cdot \cos\varphi = 10 \cdot \frac{5}{2} = 25\,W$ $\quad \boxed{P_{djel} = 25\,W}$ ✓

17. Hjerni instrumenti pokazuju: $U = 60V$ $I_1 = 4A$ $I_2 = 3A$ pri $\omega = 500\,s^{-1}$

(2.59) Pri kojoj će frekvenciji pokazivanja ampermetra biti jednaka.

$\omega = 500\,s^{-1}$

$I_2 = \frac{U}{X_2} \Rightarrow X_2 = \frac{U}{I_2} = \frac{60}{3} = 20\,\Omega$



$\underline{X_C = 20\,\Omega} \Rightarrow C = \frac{1}{\omega X_C} = \frac{1}{500 \cdot 20} = 10^{-4}$

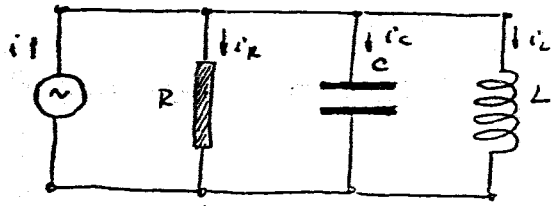
Pokazivanja ampermetara će biti ista

ako je $R = X_C$ $\quad R = \frac{1}{\omega C} \Rightarrow \omega = \frac{1}{RC} = \frac{1}{10^{-4} \cdot 15} = \frac{10^4}{15} = 667\,\frac{rad}{s}$

$\boxed{\omega = 667\,s^{-1}}$ ✓

18. U spoju je $u = Re\left\{20\, e^{j15^\circ} \cdot e^{j(\omega t)}\right\}$, $R = 5\,\Omega$ $L = 1\,mH$ $C = 25\,\mu F$ i $\omega = 10^4\,s^{-1}$

(2.60) $i_L, i_C, i_R = ?$

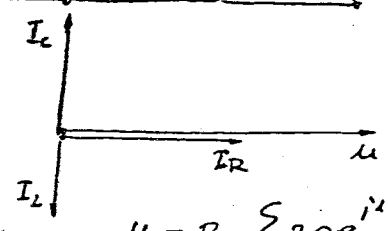


$u = Re\left\{20 \cdot e^{j(\omega t + 15^\circ)}\right\}$

$u = 20\cos(\omega t + 15^\circ)$

$u = 20\cos(\omega t + 15^\circ)$ $\quad i = \frac{u}{R}$

$i = 4\cos(\omega t + 15^\circ)$ $\quad i = Re\left\{4 \cdot e^{j(\omega t + 15^\circ)}\right\}$



$X_L = \omega L = 2\pi f \cdot L = 10^4 \cdot 10^{-3} = 10\,\Omega$ $\quad \boxed{X_L = 10\,\Omega}$

$X_C = \frac{1}{\omega C} = \frac{1}{10^4 \cdot 25 \cdot 10^{-6}} = \frac{100}{25} = 4\,\Omega$

$\boxed{X_C = 4\,\Omega}$

$u = Re\left\{20 e^{j15^\circ} \cdot e^{j(\omega t)}\right\}$

$i_R = Re\left\{4 \cdot e^{j(\omega t + 15^\circ)}\right\}$

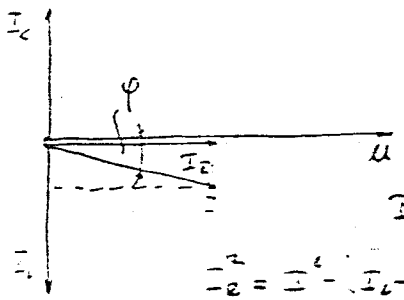
$i_C = 5\cos(\omega t + 15^\circ + 90^\circ) = 5\cos(\omega t + 105^\circ) \Rightarrow i_C = Re\left\{5\, e^{j(\omega t + 105^\circ)}\right\}$

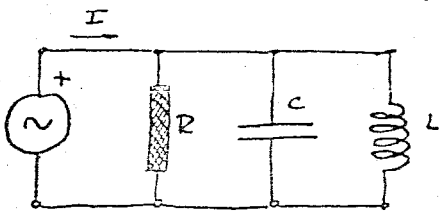
$i_L = 2\cos(\omega t + 15^\circ - 90^\circ) = 2\cos(\omega t - 75^\circ) \Rightarrow i_L = Re\left\{2\, e^{j(\omega t - 75^\circ)}\right\}$

64

(20.) Zadan je strujni krug: $P = 800W$, $I = I_L = 10A$ i $I_c = 1,34A$.

2.64 Odredite Z, X_L, ...





$P = UI\cos\varphi$ $\qquad I^2 = I_R^2 + (I_L - I_c)^2$

$I_R^2 = I^2 - (I_L - I_c)^2$ $\qquad I_R^2 = 100 - (10 - 1,34)^2 = 100 - 75 = 25$

$\boxed{I_R = 5A}$ $\qquad \sin\varphi = \frac{I_L - I_c}{I} = \frac{10 - 1,34}{10} \Rightarrow \boxed{\varphi = 60°}$ $\Rightarrow \cos\varphi = \frac{1}{2}$

$U = \frac{P}{I\cdot\cos\varphi} = \frac{80}{10\cdot\cos 60°} = \frac{80}{10\cdot\frac{1}{2}} = \frac{1600}{10\sqrt{2}} = \frac{160}{\sqrt{2}}$

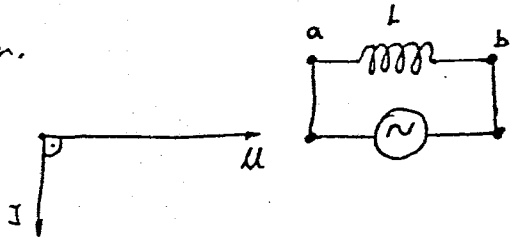
$R = \frac{U}{I_R} = \frac{160}{5} = 32\Omega$ $\qquad \boxed{R = 32\Omega}$ $\qquad X_L = \frac{U}{I_L} = \frac{160}{10} = 16\Omega$ $\qquad \boxed{X_L = 16\Omega}$

$X_c = \frac{U}{I_c} = \frac{160}{1,34}$

$\boxed{X_c = 120\Omega}$

(21.) Na izvor napona $u = 283 \sin 500t$ V priključen je svitak $L = 0,02$ H. Radni otpor

2.66 u može zanemariti. Nacrtaj graf za trenutnu vrijednost struje u svitku i odredi kojom prirodom je ta struja.



$L = 0,02H$

$X_L = \omega L$

$X_L = 500\cdot 0,02 = 10\Omega$ $\qquad \boxed{X_L = 10\Omega}$ $\qquad u = L\frac{di}{dt} \Rightarrow u\,dt = L\,di \Rightarrow di = \frac{u\,dt}{L}$

$i = \frac{1}{L}\int u\,dt = \frac{283}{L}\int \sin 500t\,dt = \frac{-283}{500L}\cos 500t = -\frac{283}{500L}\sin(90 - 500t)$

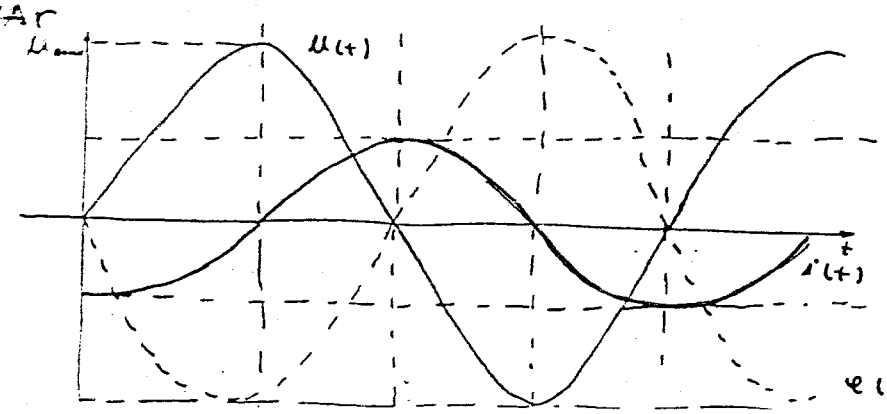
$i = \frac{283}{500\cdot 0,02}\sin(500t - 90°)$

$i = \frac{283}{10}\sin(500t - 90°) = 28,3\sin(500t - 90°)$ A

$S = U\cdot I = \frac{283}{\sqrt{2}}\cdot\frac{28,3}{\sqrt{2}} = \frac{1}{2}\cdot 8008,9 \Rightarrow S = 4004,45$ VA $= 4,00445$ kVA

$P = U\cdot I\cdot\cos\varphi = 4,00\ldots\cdot 0 = 0$ W nema komponente u fazi s naponom

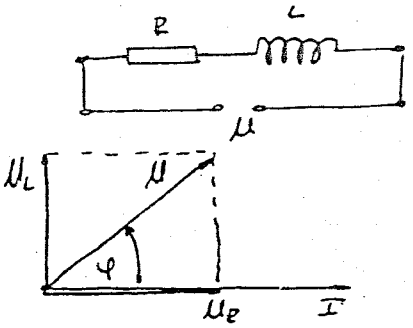
$Q = U\cdot I\cdot\sin\varphi = 4,00\ldots$ kVAr



2.67

(22.) Na izvor kojemu je napon $u = 283 \sin 500t$ [V] priključen serijski spoj kojemu induktivitet zavoja 0,016 H, a radni otpor 6 Ω.

Napiši izraz za trenutnu vrijednost struje u krugu i odredite njegovom pomoću, radni i jalovi snagu.



$u = 283 \sin 500t$ $R = 6\Omega$ $L = 0{,}016$ H

$I = ?$ $i(t) = ?$

$Z^2 = R^2 + X_L^2 \Rightarrow Z = \sqrt{R^2 + (\omega L)^2}$

$Z = \sqrt{36 + 64} = 10\Omega$ $I = \frac{U}{Z} = \frac{283}{\sqrt{2} \cdot 10}$

$I = 20{,}0113$ A $\varphi = \operatorname{arc tg} \frac{X_L}{R} = \operatorname{arc tg} \frac{\omega L}{R} = \operatorname{arc tg} \frac{8}{6} \Rightarrow \varphi = 53{,}13°$

$i = I_{max} \sin(\omega t - \varphi) \Rightarrow i = 28{,}3 \sin(500t - 53{,}13°)$ [A]

$u_R = i \cdot R = 170 \sin(500t - 53{,}13°)$ [V] $u_R + u_L = u$ ∀ trenutak

$u_L = i \cdot X_L = 226{,}4 \sin(500t + \frac{\pi}{2})$ [V]

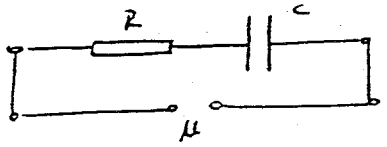
$P = U \cdot I \cdot \cos\varphi = \underbrace{U \cdot \cos\varphi}_{U_R} \cdot I = U_R \cdot I = 2405{,}5$ W $\Rightarrow$ **$P = 2{,}4055$ kW**

$S = U \cdot I = 4004{,}5$ VA $S = 4{,}0045$ kVA $Q = U \cdot I \cdot \sin\varphi = U_L \cdot I = 3203{,}5$ VAr

$Q = 3{,}2035$ kVAr

2.70

(23.) Serijski spoj otpora $R = 1000\Omega$ i kondenzatora kapaciteta $C = 0{,}5\,\mu$F priključenim na električnu mrežu izmjenične struje kojoj je napon $u = 141 \sin 2000t$

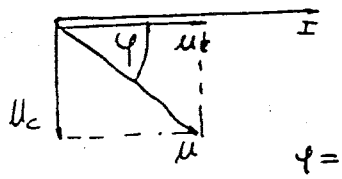


$i(t) = ?$ $u_C = ?$, $u_R = ?$ $P = ?$ $S = ?$ $Q = ?$

$R = 1000\Omega$ $C = 0{,}5\,\mu$F $u = 141 \sin 2000t$

$Z = \sqrt{R^2 + X_C^2} = \sqrt{10^6 + \left(\frac{1}{2000 \cdot 0{,}5 \cdot 10^{-6}}\right)^2}$

$Z = \sqrt{10^6 + 10^6} = \sqrt{2} \cdot 10^3 = \sqrt{2} \cdot 10^3\,\Omega$



$I = \frac{U}{Z} = \frac{U}{\sqrt{2} \cdot \sqrt{2} \cdot 10^3} = \frac{141}{2 \cdot 10^3} = 0{,}0705$ A

$\varphi = \operatorname{arc tg} \frac{X_C}{R} = \operatorname{arc tg} \frac{1000}{1000} = \frac{\pi}{4}$

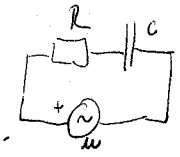
$I = 0{,}0705$ A $i = 0{,}1 \sin(2000t + \frac{\pi}{4})$ [A]

$u_R = 100 \sin(2000t + \frac{\pi}{4})$ [V]

$u_C = 100 \sin(2000t - \frac{\pi}{4})$ [V]

2.71 ISPITNI II PUTA

(24.) Potrošač se sastoji od serijski spojenih otpornika i kondenzatora priključen je na mrežu sinusnog napona $u = U_m \sin 5000t$ V.



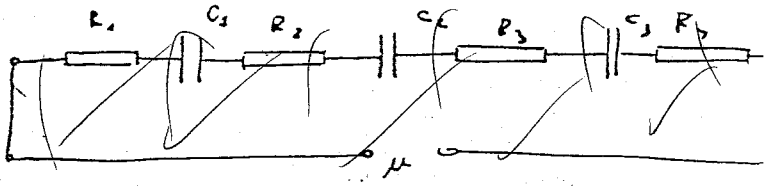
Struja je zadana izrazom $i = 0{,}5 \sin(5000t + \frac{\pi}{6})$ A

Kakav će biti odnos struja ako se frekvencija priključenog napona smanji 2p

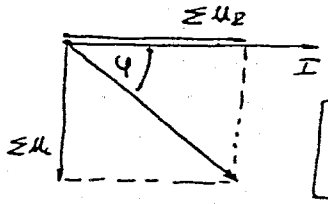
$u = U_m \sin 5000t$ [V]

$i = 0{,}5 \sin(5000t + \frac{\pi}{6})$ [A]

$f_1 = \frac{1}{2} f \quad \omega_1 = \frac{1}{2}\omega$



za slučaj



$\varphi = \text{arc tg} \frac{\Sigma X_C}{\Sigma R} = \text{arc tg} \frac{\frac{1}{\omega}\left(\frac{1}{C_1} + \frac{1}{C_2} + \dots\right.}{\Sigma R}$

$\boxed{\varphi = \text{arc tg} \frac{L}{\omega \cdot C_u \cdot \Sigma R}} \qquad \varphi = \frac{\pi}{6}$

$Z = \sqrt{R^2 + X_C^2} = \sqrt{(\Sigma R)^2 + \left[\Sigma \frac{1}{\omega C}\right]^2} = \sqrt{(\Sigma R)^2 + \frac{1}{\omega^2}\left(\Sigma \frac{1}{C}\right)^2} = \sqrt{(\Sigma R)^2 + \frac{1}{\omega^2} \cdot \frac{L}{C_u^2}}$

ako se frekvencija smanji 2 puta

$\varphi = \text{arc tg} \frac{L}{\omega C_u \Sigma R} = \frac{\pi}{6}$

$\Rightarrow \varphi = \text{arc tg} \frac{2}{\omega C_u \cdot \Sigma R}$

$\Rightarrow \frac{1}{\omega C_u \Sigma R} = 0{,}57735$

$\varphi_1 = \text{arc tg}\, 2 \cdot 0{,}57735 \Rightarrow \boxed{\varphi_1 = 49{,}1^\circ}$

$Z = \sqrt{(\Sigma R)^2 + \frac{L}{\omega^2 C_u^2}}$ iz dijagrama $\Sigma X_C = \Sigma R \cdot \text{tg}\,\varphi$

$\Sigma X_C = \Sigma R \cdot \frac{\sqrt{3}}{3}$

$Z = \sqrt{(\Sigma R)^2 + \left(\frac{\Sigma R \sqrt{3}}{3}\right)^2} = \sqrt{(\Sigma R)^2 + \frac{(\Sigma R)^2}{3}} = \sqrt{\frac{4}{3}}\,\Sigma R$

$Z_1 = \sqrt{(\Sigma R)^2 + 4\frac{(\Sigma R)^2}{3}} = \sqrt{\frac{7(\Sigma R)^2}{3}} = \sqrt{\frac{7}{3}}\,\Sigma R$

$Z = \sqrt{\frac{4}{3}}\,\Sigma R$

$Z_1 = \sqrt{\frac{7}{3}}\,\Sigma R$

$I = \frac{U}{Z} \qquad I_1 = \frac{U}{Z_1}$

$U = I \cdot Z \qquad U = I_1 \cdot Z_1$

$I \cdot Z = I_1 \cdot Z_1 \Rightarrow I_1 = \frac{I \cdot Z}{Z_1} = \frac{0{,}5 \cdot \sqrt{\frac{4}{3}}\,\Sigma R}{\sqrt{\frac{7}{3}}\,\Sigma R} = \frac{0{,}5 \cdot \frac{2}{\sqrt{3}}}{\frac{\sqrt{7}}{\sqrt{3}}} = \frac{1}{\sqrt{7}}$ (MAX)

$I_{1_{max}} = 0{,}37796$ A

$\boxed{i = 0{,}37796 \sin(5000t + 49{,}1^\circ)}$ A

2.72

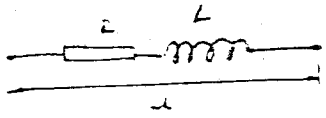
(25.) Pri mrežnom naponu $U = 127$ V i $f_1 = 28{,}7$ Hz struja u svitku iznosila je $I_1 = 2{,}8$ A. Kod toga istog napona, ali pri frekvenciji $f_2 = 51$ Hz struja u svitku iznosi $I_2 = 2{,}1$ A. Nacrtajte nadomjesnu shemu.

$U = 127\,V$ $f_1 = 28{,}7\,Hz$ $I_1 = 2{,}8\,A$

$U = 127\,V$ $f_2 = 51\,Hz$ $I_2 = 2{,}1\,A$

$$\left.\begin{aligned} I &= \frac{U}{X_L} \\ I &= \frac{U}{\omega L} \end{aligned}\right\} \quad X_L = \omega L$$

$L = ?$ zavojnica ima neki omski otpor



$$Z_1 = \sqrt{R^2 + X_{L1}^2} = \frac{U}{I_1} \quad /^2$$

$$Z_2 = \sqrt{R^2 + X_{L2}^2} = \frac{U}{I_2} \quad /^2$$

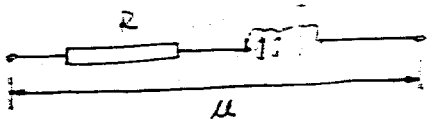
$$\left.\begin{aligned} R^2 + X_{L1}^2 &= 2053{,}27 \\ R^2 + X_{L2}^2 &= 3653{,}363615 \end{aligned}\right| - \qquad \begin{aligned} X_{L1}^2 - X_{L2}^2 &= -1600{,}093615 \\ X_{L2}^2 - X_{L1}^2 &= 1600{,}093615 \end{aligned}$$

$$(\omega_2^2 - \omega_1^2)\,L^2 = 1600{,}093615 \Rightarrow L^2 = \frac{1600{,}093615}{4\pi^2 f_2^2 - 4\pi^2 f_1^2} = \frac{1600{,}1}{4\pi^2 (f_2^2 - f_1^2)} = \frac{1600{,}1}{4\pi^2 (2601 - 823{,}69)}$$

$$L^2 = \frac{1600{,}1}{4\pi^2 \cdot 1777{,}31} = \frac{1600{,}1}{70039{,}2627} = L^2 = 0{,}022827 \Rightarrow \boxed{L = 0{,}15\,H}$$

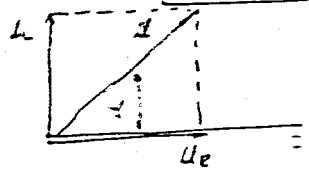
Induktivitet možemo izmjeriti (izračunati) pomoću z mjerenja [illegible] frekvencijama.

(26.) 2.73 Svitak je priključen na mrežu u struju napona $U = 150\,V$. Jakost struje ima iznos $I = 6\,A$, a radna snaga $P = 540\,W$. Odredite induktivitet ako je frekvencija napona $f = 50\,Hz$



$U = 150\,V$ $P = U \cdot I \cdot \cos\varphi \Rightarrow \cos\varphi = \frac{P}{U \cdot I}$

$I = 6\,A$ $\cos\varphi = \frac{540}{150 \cdot 6} = \cos\varphi = 0{,}6 \Rightarrow \varphi = 0{,}6 \Rightarrow \boxed{\varphi = 53{,}13^\circ}$

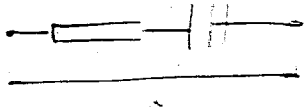


$$X_L = \frac{U_L}{I} = \frac{U \cdot \sin\varphi}{I} = \frac{150 \cdot 0{,}8}{6} = 20\,\Omega$$

$$X_L = \omega L \Rightarrow L = \frac{X_L}{\omega} = \frac{20}{2\pi f} = \frac{20}{314} \Rightarrow L = 0{,}1636\,H$$

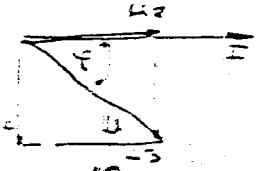
$L = 63{,}7\,mH$ $\boxed{L = 63{,}7\,mH}$

(27.) 2.74 Trošilo sastavljeno od kond. i otpora koji su serijski spojeni priključeno je na mrežu $U = 125\,V$ i $f = 15\,000\,Hz$. Odredite kolika je struja ako je snaga izmjerena [illegible] iznosi $P = 75 \cdot 10^{-3}\,W$.



$U = 125\,V$ $f = 15\,000\,Hz$ $I = 1\,mA$ $P = 75 \cdot 10^{-3}\,W$

$$P = \overset{S}{U} \cdot I \cdot \cos\varphi = U_R \cdot I \Rightarrow \cos\varphi = \frac{P}{U \cdot I} = \frac{75 \cdot 10^{-3}}{125 \cdot 10^{-3}}$$



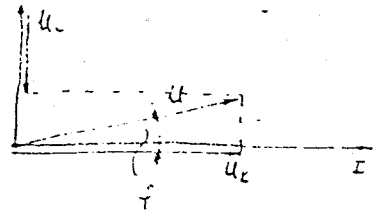
$\cos\varphi = 0{,}6 \Rightarrow \boxed{\varphi = 53{,}13^\circ}$

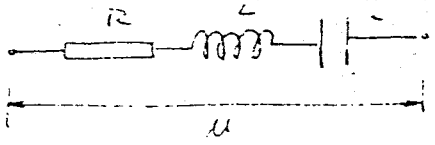
$$X_C = \frac{U_C}{I} = \frac{U \cdot \sin\varphi}{I} = \frac{1}{\omega C} \Rightarrow C = \frac{I}{\omega \cdot U \cdot \sin\varphi} = \frac{10^{-3}}{94252 \cdot 125 \cdot 0{,}8} = 1 \cdot 10^{-7} \cdot 10^{-5} = 10$$

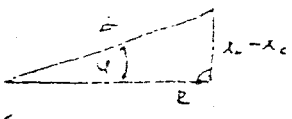
$\boxed{C = 100\,pF}$ $C = 100 \cdot 10^{-12}\,F$

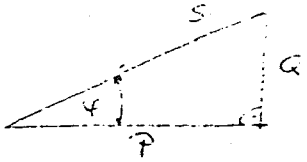
28. $L = 0{,}15\ H$ $R = 30\ \Omega$ $C = 40\ \mu F$ $u = 450 \sin 500t$ (V)

2.75









$Z = \sqrt{R^2 + (X_L - X_C)^2}$ $\quad X_L = \omega L = 30\ \Omega$

$Z = \sqrt{30^2 + (30 - 50)^2}$ $\quad X_C = \frac{1}{\omega C} = \frac{10^6}{500 \cdot 40} = \frac{10^4}{2000} = 50\ \Omega$ $\boxed{X_C = 50\ \Omega}$

$Z = \sqrt{900 + 400} = 50\ \Omega$

$I_{MAX} = \frac{U_{MAX}}{Z} = \frac{450}{50} = 5\ A$ $\quad tg\,\varphi = \frac{X_L - X_C}{R} \Rightarrow \varphi = arc\,tg\,\frac{X_L - X_C}{R}$

$\varphi = arc\,tg\,\frac{40}{30} = 53{,}13°$ $\boxed{\varphi = 53{,}13°}$

$i = 5 \cdot \sin(500t - 53{,}13°)$

$\boxed{i = 5 \sin(500t - 53{,}13°)}$ $\quad \boxed{u_R = 15 \sin(500t - 53{,}13°)}$

$\boxed{u_L = 450 \sin(500t + 36°50')\ [V]}$ $\quad \boxed{u_L = 250 \sin(500t - 143{,}13°)\ [V]}$

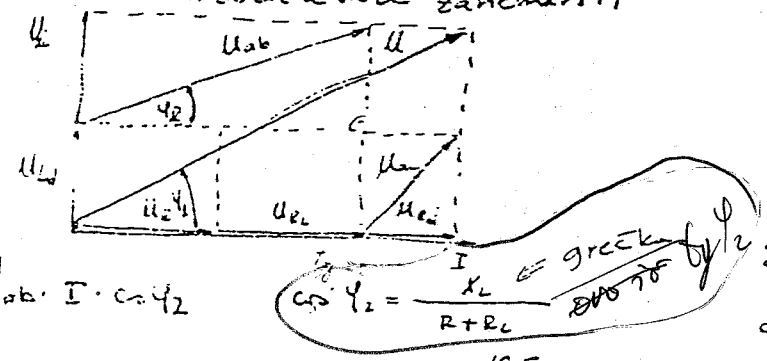
29. Potrošač smanjen je od otpora $R = 13\ \Omega$ i zavojnice s $R_L = 2\ \Omega$ i $X_L = 10{,}5\ \Omega$ koji

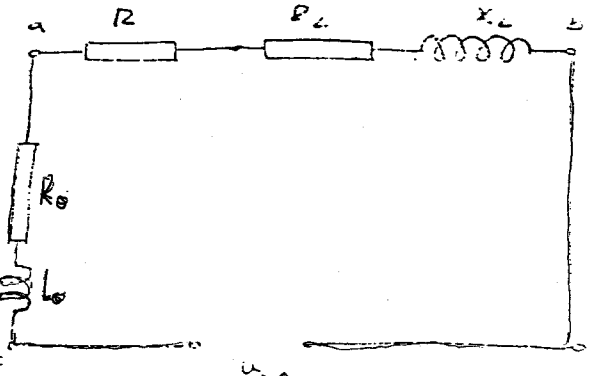
2.79 su u trofaznu spojeni dalekovodom se napaja generatorom s naponom $U = 380\ V$.

Snaga P_L potrošača iznosi $5{,}41\ kW$, a snaga koju daje generator je $5{,}77\ kW$

Odredite napon potrošača, pad napona na dalekovodu, gubitak snage dalekovoda.

Kapacitet dalekovoda se može zanemariti





$P_2 = U_{ab} \cdot I \cdot \cos\varphi_2$ $\quad \cos\varphi_2 = \frac{X_L}{R + R_L}$ ← grčka slova φ_2

$P_2 = U \cdot I \cdot \cos\varphi_2$ $\quad \cos\varphi_2 = \frac{10{,}5}{15} \Rightarrow \varphi_2 = 45{,}57°$ (135°) $\quad \varphi_2 = 35°$

$Z_{ab} = \sqrt{(R + R_L)^2 + X_L^2} = \sqrt{225 + 110{,}25} = 18{,}31\ \Omega$

$P_1 = U \cdot Z_{AB} \cdot I \cdot \cos\varphi_2 = I^2 \cdot Z_{ab} \cdot \cos\varphi_2 = P_L$

$I = \frac{P_L}{Z_{ab} \cdot \cos\varphi_2} = \frac{5{,}41 \cdot 10^3}{18{,}31 \cdot 0{,}7} = \frac{5410}{12{,}817} = 422{,}03 \Rightarrow \boxed{I = 20{,}55\ A}$ $\quad \boxed{I = 19\ A}$

19,2 A

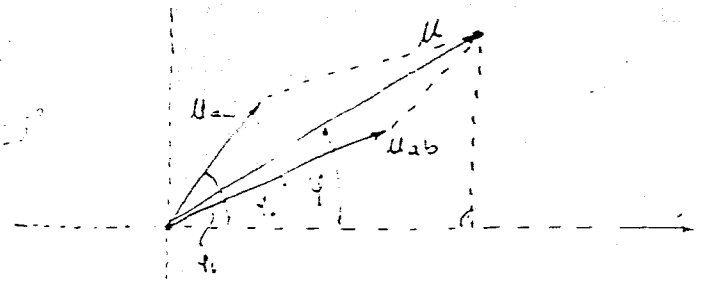
$P_1 = U \cdot I \cdot \cos\varphi_1 \Rightarrow \cos\varphi_1 = \frac{P_1}{U \cdot I} = \frac{5{,}77 \cdot 10^3}{380 \cdot 19} = \frac{5770}{7560} = 0{,}799169 \Rightarrow$

$U_{ab} = I \cdot Z_{AB} = 20{,}55 \cdot 18{,}31 = 376{,}17\ V$ $\quad 347{,}89\ V$ $\quad \varphi_1 = 36{,}949°$

$P_1 = U \cdot \cos \varphi_1$ $\quad \boxed{\varphi_1 = 47{,}67^\circ}$

$\boxed{\varphi_2 = 45{,}57^\circ}$

$\vec{U} = \vec{U}_{ac} + \vec{U}_{ab}$



$U^2 = (U_{ab} \cdot \cos\varphi_2 + U_{ac} \cdot \cos\varphi_1)^2 + (U_{ab} \cdot \sin\varphi_2 + U_{ac} \cdot \sin\varphi_1)^2$

$U^2 = (376{,}17 \cdot \cos 45{,}57^\circ + U_{ac} \cdot \cos 47{,}67^\circ)^2 + (376{,}17 \sin 45{,}57^\circ + U_{ac} \cdot \sin 47{,}67^\circ)^2$

$144400 = (263{,}33 + U_{ac} \cdot 0{,}673)^2 + (268{,}625 + U_{ac} \cdot 0{,}739)^2$

$144400 = 69372{,}68 + 354{,}44\, U_{ac} + 0{,}452\, U_{ac}^2 + 72150{,}39 + 397{,}02 \cdot U_{ac} + 0{,}546\, U_{ac}^2$

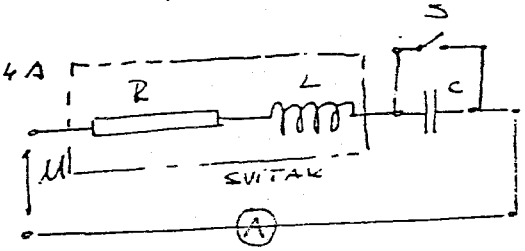
$144400 = 141502{,}07 + 751{,}46\, U_{ac} + U_{ac}^2$

$U_{ac}^2 + 751{,}46\, U_{ac} - 2897{,}93 = 0$ $\qquad U_{ac} = \dfrac{-751{,}46 \pm \sqrt{564692 + 11591{,}72}}{2}$

$U_{ac_{1,2}} = \dfrac{-751{,}46 \pm 759{,}133}{2}$ $\qquad \boxed{U_{ac} = 3{,}83\ V}$

DETOČNO FALIO SAM U POČETKU $\cos\varphi \neq \frac{X_L}{R+R_L} = \operatorname{tg}\varphi$ TREBA

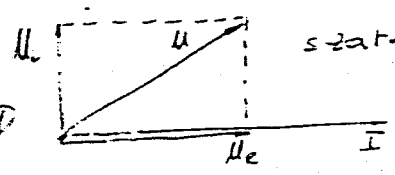
(30.) 2.081 Veličina kondenzatora je odabrana da mu pražnjenje ampermetra iznosi pri otvorenoj i zatvorenoj sklopci. Odredite X_L zavojnice pri mrežnom naponu $U = 120V$ i otporu kondenzatora $X_C = 48\Omega$ struja otklona jednaka 4A



$U = 120V$

$X_C = 48\Omega$

$I = 4A$



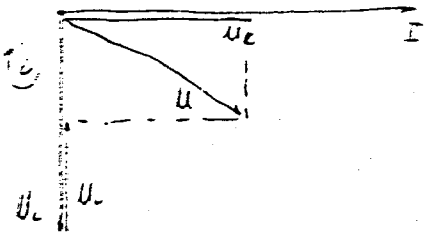
$Z = \dfrac{U}{I}$

$Z_1 = Z_2$ $\qquad Z_1 = \sqrt{R^2 + X_L^2}$ $\quad$ sklopka otvorena

$Z_2 = \sqrt{R^2 + (X_C - X_L)^2}$ $\quad$ sklopka zatvorena

$Z_2^2 = R^2 + (X_C - X_L)^2$

$Z_1^2 = R^2 + X_L^2$



$Z_1^2 = Z_2^2 \Rightarrow R^2 + X_L^2 = R^2 + (X_C - X_L)^2$ $\qquad X_L^2 = (X_C - X_L)^2$ $\qquad X_L^2 - (X_C - X_L)^2 = 0$

$[X_L - (X_C - X_L)][X_L + (X_C - X_L)] = 0$

$[X_L - X_C + X_L][X_L + X_C - X_L] = 0$ $\qquad 2X_L - X_C = 0$ $\qquad 2X_L = X_C \Rightarrow X_L = \frac{1}{2} X_C$ $\qquad X_L = \frac{1}{2} \cdot 48 = 24\Omega$

$\boxed{X_L = 24\Omega}$

$Z_1 = \sqrt{R^2 + X_L^2}\ /^2$ $\qquad Z_1^2 = R^2 + X_L^2$ $\qquad R^2 + X_L^2 = \left(\frac{120}{4}\right)^2$

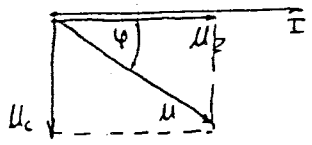
$R^2 + X_L^2 = 900 \Rightarrow R^2 = 900 - X_L^2 = 900 - 576 \Rightarrow \boxed{R = 18\Omega}$

$\boxed{\begin{array}{l} R = 18\Omega \\ X_L = 24\Omega \end{array}}$ spojeno u seriju

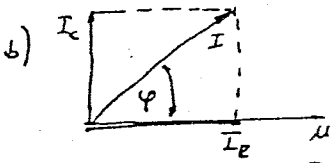
2.82

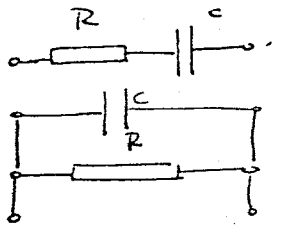
(31.) Koeficijent snage potrošača koji se sastoji od zerijskog spoja otpora i kondenzatora iznosi $\cos\varphi_1 = 0{,}8$. Koliki će biti faktor snage trošila koje se sastoji od tog a istog otpora i kondenzatora u paralelnom spoju.

a)



b)





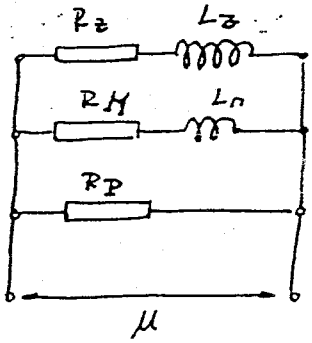
$\operatorname{tg}\varphi_1 = \Rightarrow \quad \varphi_1 = \operatorname{arc\,tg}\frac{X_c}{R} \qquad \varphi_2 = \text{量} = \operatorname{arc\,tg}\frac{B_c}{G} \qquad \varphi_2 = \operatorname{arc\,tg}\frac{B_c}{G}$

$\varphi_2 = \operatorname{arc\,tg}\frac{\frac{1}{X_c}}{\frac{1}{R}} = \operatorname{arc\,tg}\frac{R}{X_c} \qquad \frac{X_c}{R} = \operatorname{tg}\varphi_1 \qquad \frac{X_c}{R} = 0{,}75$

$\varphi_2 = \operatorname{arc\,tg}\frac{1}{0{,}75} \Rightarrow \varphi_2 = 53{,}13^\circ \Rightarrow \boxed{\cos\varphi_2 = 0{,}6}$

(2.) Na mreži su spojeni aparat za zavarivanje, elektromotor i peć za zagrijavanje.

2.85 Aparat za zavarivanje troši snagu $P_z = 6\,kW$ uz $\cos\varphi_1 = 0{,}5$, motorna snaga iznosi $P_z = 12\,kW$ uz $\cos\varphi_2 = 0{,}8$, a peći $P_p = 10\,kW$ uz $\cos\varphi_3 = 1$. Odredite $\cos\varphi$ sistema tereta ako istodobno rade aparat, motor i peć. Koliku treba da bude snaga kondenzatora kojeg valja uključiti paralelno opterećenju da bi faktor snage cijelog kruga povisio na 0,9?



$P_z = 6\,kW \quad \cos\varphi_1 = 0{,}5$

$P_n = 12\,kW \quad \cos\varphi_2 = 0{,}8$

$P_p = 10\,kW \quad \cos\varphi_3 = 1$

$\cos\varphi_{ukupno} = ?$

$P_\mu = P_z + P_n + P_p = 6\,kW + 12\,kW + 10\,kW$

$P_\mu = 28\,kW$

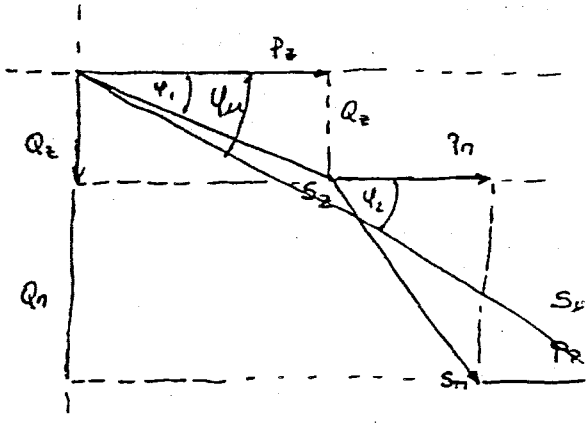
$\operatorname{tg}\varphi_\mu =$

$B_\mu =$

$Q_\mu = P_z \cdot \operatorname{tg}\varphi_1 + P_n \cdot \operatorname{tg}\varphi_2 + 0$

$Q_\mu = P_z \cdot 1{,}732 + P_n \cdot 0{,}75 + 0$

$Q_\mu = 6 \cdot 1{,}73 + 12 \cdot 0{,}75 = 10{,}38 + 9 = 19{,}38\,KVAr$



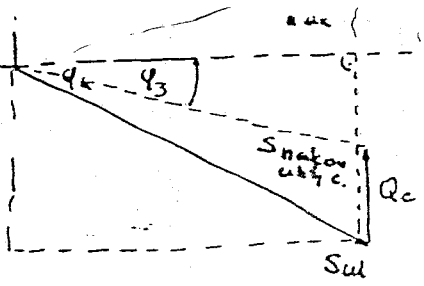
$S_\mu = \sqrt{P_\mu^2 + Q_\mu^2} = \sqrt{784 + 375{,}58} =$

$S_\mu = 34{,}0526\,KVA$

$\Rightarrow \cos\varphi_\mu = \frac{P_\mu}{S_\mu} = 0{,}822256$

$\boxed{\cos\varphi_\mu = 0{,}82}$

Snaga kondenzatora treba da bude ⇒



MAJU DVA RJEŠENJA!!!

$\cos\varphi_3 = 0{,}9$

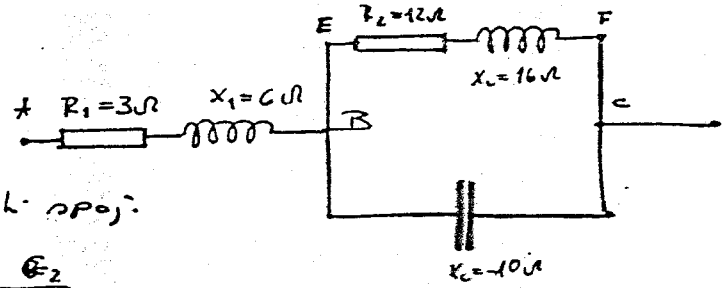
$\text{EES} \Rightarrow \operatorname{tg}\varphi_3 = \dfrac{Q_{uk} - Q_c}{S_{na\ ukl.\ c}}$

$\operatorname{tg}\varphi_3 = \dfrac{Q_{uk} - Q_c}{P_{uk}} \Rightarrow Q_c = Q_{uk} - P_{uk}\cdot \operatorname{tg}\varphi_3 = 19{,}38 - 28\cdot 0{,}48$

$Q_c = 19{,}38 - 13{,}44$ **$Q_c = 5{,}94$ kVAr**

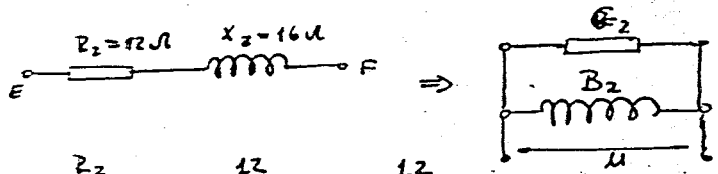
2.86

33. $U = 200$V. Odrediti struje u mreži



I NAČIN

pretvaranje paralelne veze u serijsku vezu:

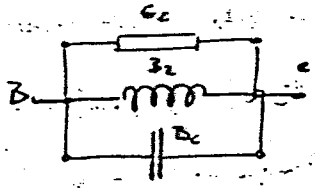


$G_2 = \dfrac{R_2}{R_2^2 + X_2^2} = \dfrac{12}{144+256} = \dfrac{12}{400}$

$G_2 = 0{,}03$ S

$B_2 = \dfrac{X_2}{R_2^2 + X_2^2} = \dfrac{16}{400} = 0{,}04$ S **$B_2 = 0{,}04$ S**

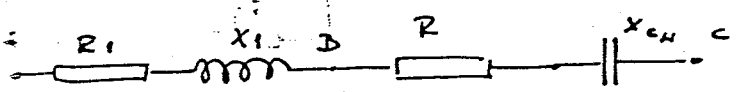
$R = \dfrac{G_2}{G_2^2 + (B_2 - B_C)^2} = \dfrac{0{,}03}{0{,}0009 + 0{,}0036}$ u serijsku vezu



$B_C = 0{,}1$ S

$Z = 6{,}67\ \Omega$

$X_{CH} = \dfrac{-0{,}1 + 0{,}04}{G_2^2 + (B_2 - B_C)^2} = \dfrac{-0{,}06}{0{,}0045} = -13{,}33\ \Omega$ → kap. karaktera



$Z = \sqrt{(R_1+R)^2 + (X_1 - X_{CH})^2}$

$= \sqrt{93{,}51 + 53{,}7289} = 12{,}13\ \Omega$

$I = \dfrac{U}{Z} = \dfrac{200}{12{,}13} = 16{,}48$ A

$U_{BC} = I\cdot Z_{BC} = I\cdot\sqrt{6{,}67^2 + 13{,}33^2} = 16{,}48\cdot\sqrt{44{,}48 + 177{,}6} = 245{,}639$

$I_2 = \dfrac{U_{BC}}{Z_{EF}} = \dfrac{U_{BC}}{\sqrt{R_2^2 + X_2^2}} = \dfrac{245{,}639}{\sqrt{144+256}} = \dfrac{245{,}639}{20} = 12{,}28$ A

$I_2 = 12{,}28$ A

$I_3 = \dfrac{U_{BC}}{X_C} = \dfrac{245{,}639}{10} = 24{,}5639$ A

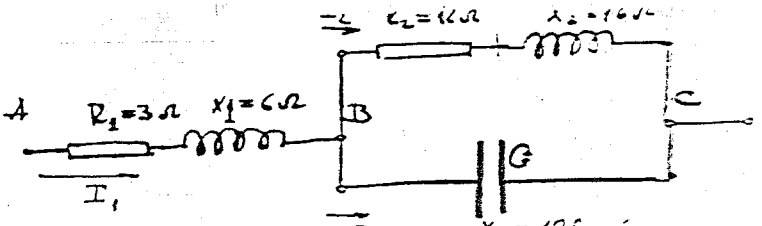
$I = 16{,}48$ A

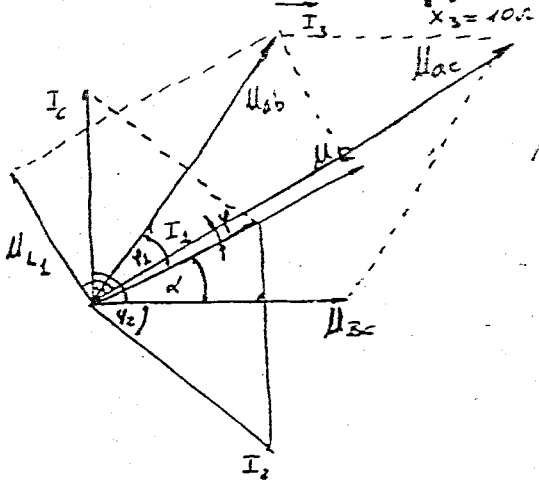
$I_2 = 12{,}28$ A

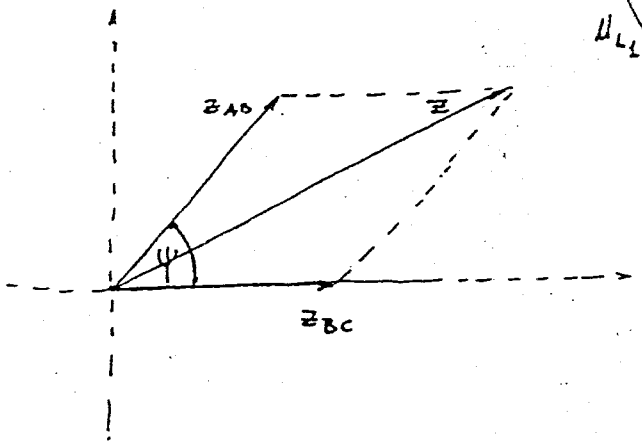
$I_3 = 24{,}5639$ A

Nastavak od (33) zadatka

II NAČIN







$\varphi_2 = ?$ $\quad \varphi_2 = \operatorname{arc\,tg} \frac{X_{L_2}}{R_2} = \operatorname{arc\,tg} \frac{16}{12}$

$\varphi_2 = -53{,}130^\circ$

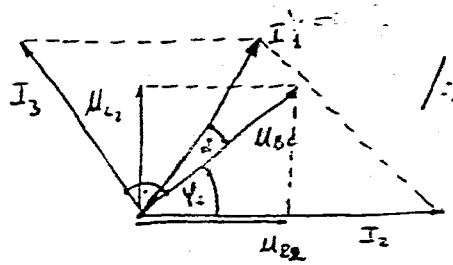
$\operatorname{tg}\alpha = \frac{B_c - B_{L_2}}{G_2} \Rightarrow \alpha = \operatorname{arc\,tg} \frac{B_c - B_{L_2}}{G_2} = \operatorname{arc\,tg} \frac{0{,}1 - 0{,}0625}{0{,}0833} = \operatorname{arc\,tg} \frac{0{,}0375}{0{,}0833}$

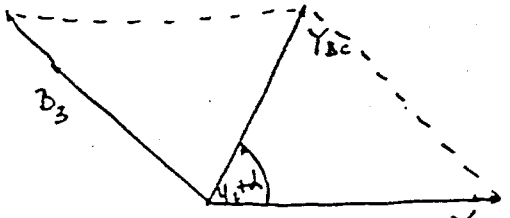
$= \alpha = 24{,}23^\circ$ $\quad \boxed{\alpha = 24{,}23^\circ}$

$\varphi_1 = \operatorname{arc\,tg} \frac{X_{L_1}}{R} = \operatorname{arc\,tg} \frac{6}{3}$

$\boxed{\varphi_1 = 63{,}1343^\circ}$

$\psi = \varphi_1 + \alpha = \boxed{87{,}665^\circ = \phi}$ $\quad$ ## $Z_{Bc} = ?$





$\boxed{B_3 = \frac{1}{X_c} = 0{,}1\ S}$

$Z_{EF} = \sqrt{144 + 256} = 20\ \Omega \Rightarrow \boxed{Y_{EF} = 0{,}05\ S}$

$B_3^2 = Y_{Bc}^2 + Y_{EF}^2 - 2 Y_{Bc} \cdot Y_{EF} \cdot \cos(\varphi_2 + \alpha)$

$0{,}01 = Y_{Bc}^2 + 0{,}0025 - 2 \cdot Y_{Bc} \cdot 0{,}05 \cdot 0{,}2188$

$Y_{Bc}^2 - 0{,}02188\, Y_{Bc} - 0{,}0075 = 0 \quad (Y_{Bc})_{1,2} = \frac{0{,}02188 \pm \sqrt{0{,}000478 + 0{,}03}}{2}$

$(Y_{Bc})_{1,2} = \frac{0{,}02188 \pm 0{,}1745}{2} \quad Y_{Bc} = 0{,}098135\ S \quad \boxed{Z_{Bc} = 10{,}19\ \Omega}$

$Z^2 = Z_{AB}^2 + Z_{Bc}^2 + 2 Z_{AB} \cdot Z_{Bc} \cdot \cos 87{,}665^\circ \quad Z_{AB} = 6{,}708$

$Z^2 = 45 + 103{,}88 + 2 \cdot 6{,}708 \cdot 10{,}19 \cdot 0{,}0407$

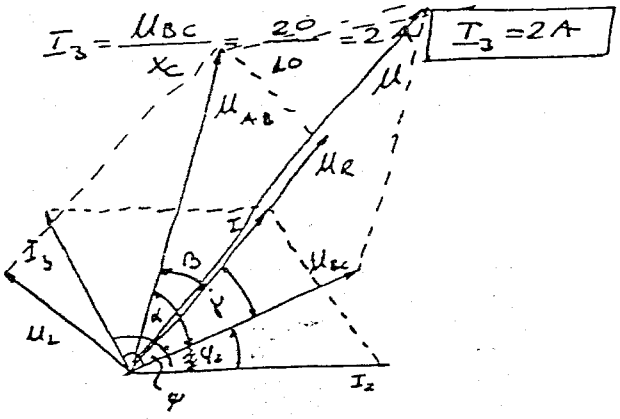
$Z^2 = 154{,}33 \quad \boxed{Z = 12{,}42\ \Omega} \Rightarrow I_1 = \frac{U}{Z} = \frac{200}{12{,}42} \Rightarrow \boxed{I_1 = 16{,}1\ A}$ TT

III NAČIN pretpostavimo da je $I_2 = 1A$

$Z_2 = \sqrt{R_2^2 + X_{L2}^2} = 20\,\Omega$ $\boxed{U_{BC} = I_2 \cdot Z_2 = 20V}$

$\varphi_2 = \operatorname{arc\,tg} \frac{X_{L2}}{R_2} = 53,13^\circ$ $\boxed{\varphi_2 = 53,13^\circ}$

$I_3 = \frac{U_{BC}}{X_C} = \frac{20}{10} = 2A$ $\boxed{I_3 = 2A}$



$\psi = \varphi_2 + 90 = 143,13^\circ$

$I^2 = I_3^2 + I_2^2 + 2I_2 \cdot I_3 \cdot \cos 143,13^\circ$

$I^2 = 4 + 1 + 2 \cdot 1 \cdot 2 \cdot (-0,7999)$

$I^2 = 4 + 1 + (-3,2)$ $I^2 = 1,8$ $\boxed{I = 1,3416\,A}$

$U_R = I \cdot R = 4,0249\,V$ $U_L = I \cdot X_{L1} = 1,3416 \cdot 6 = 8,0496\,V$

$U_{AB} = \sqrt{U_R^2 + U_{XL1}^2} = \sqrt{16,2 + 64,8} = 9V$ $\beta = \operatorname{arc\,tg} \frac{X_{L1}}{R_1} = \operatorname{arc\,tg} 2 \Rightarrow \beta = 63,435$

$\gamma = \operatorname{arc\,tg} \frac{B_C - B_L}{G_2} = \operatorname{arc\,tg} \frac{0,1 - 0,0625}{12 - 1} = \operatorname{arc\,tg} \frac{0,0375}{0,0033} \Rightarrow \boxed{\gamma = 24,227^\circ}$

$\alpha = \beta + \gamma + \varphi_I = 87,662^\circ$

$U^2 = U_{AB}^2 + U_{BC}^2 + 2U_{AB} \cdot U_{BC} \cdot 0,04079 \Rightarrow U^2 = 81 + 400 + 14,68 \Rightarrow U = 22,26V$

$\gamma = \operatorname{arc\,tg} \frac{-I_3 - I_2 \cdot \sin\varphi_2}{I_2 \cdot \cos\varphi_2} = \operatorname{arc\,tg} \frac{-2 - 0,8}{0,6} = \operatorname{arc\,tg} 2 = 63,43^\circ$

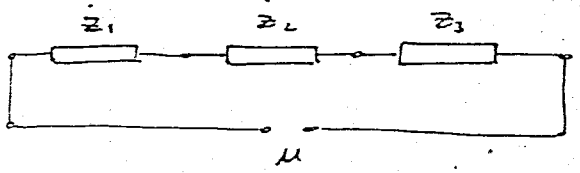
(2.84)

(34.) Mreža sadrži u serijskom spoju tri dvopola koji su karakterizirana sa

$\dot{z}_1 = 10\,e^{j45°}$, $\dot{z}_2 = 6\cdot e^{-j37°}$, $\dot{z}_3 = 8\,e^{j53}$. Napon izvora je $U = 19{,}3\,V$

bb 1) Odredite kolaznu impedanciju 3) napon na stezaljkama pojedinih dvopola

2) struju mreže 4) snagu koju troši pojedini dvopol i cijela mreža



$\dot{z}_1 = 10\,e^{j45} = 10(\cos 45° + j\sin 45°)$

$\dot{z}_1 = \frac{10\sqrt{2}}{2} + j\,10\frac{\sqrt{2}}{2} = 5\sqrt{2} + j5\sqrt{2}$ (Ω) ind.

$\dot{z}_2 = 6\cdot e^{-j37°} = 6(\cos 37° - j\sin 37°) = 4{,}7918 - j3{,}61$ (Ω) kapac.

$\dot{z}_3 = 8\,e^{j53°} = 8(\cos 53° + j\sin 53°) = 4{,}814 + j6{,}39$ (Ω) induk.

$\dot{z} = \dot{z}_1 + \dot{z}_2 + \dot{z}_3 = 7{,}07 + j7{,}07 + 4{,}79 - j3{,}61 + 4{,}814 + j6{,}39 = 16{,}674 + j9{,}85$

$|z| = \sqrt{16{,}6^2 + 9{,}85^2} = \sqrt{275{,}5 + 97{,}02} = 19{,}3\,\Omega$ $\quad \cos\varphi_\mu = \frac{16{,}674}{19{,}3} = 0{,}8639$ $\quad \varphi_\mu = 30,$

1) $\boxed{\dot{Z}_\mu = 19{,}3\cdot e^{j30{,}24°}}$

2) $\dot{I} = \frac{\dot{\mu}}{\dot{z}} = \frac{19{,}3}{19{,}3\,e^{j30{,}24°}} = 1\cdot e^{-j30{,}24°}$ $\quad |\dot{I}| = 1A$ $\quad \dot{I} = 1\cdot e^{-j30{,}24}$

$i = \dot{I}\cdot e^{j\omega t} = \boxed{\mathrm{Im}\left(e^{j(\omega t - 30{,}24°)}\right) = i}$

3) $\dot{\mu}_1 = \dot{I}\cdot\dot{z}_1 = 1\cdot e^{-j30{,}24°}\cdot 10\cdot e^{j45°} = 10\cdot e^{j15°}$ [V]

$\dot{\mu}_2 = \dot{I}\cdot\dot{z}_2 = 1\cdot e^{-j30{,}24°}\cdot 6\cdot e^{-j37°} = 6\cdot e^{-67{,}24°}$ [V]

$\dot{\mu}_3 = \dot{I}\cdot\dot{z}_2 = 1\cdot e^{-j30{,}24°}\cdot 8\cdot e^{j53°} = 8\cdot e^{j23°}$ [V]

EFEKTIVNE VRIJEDNOSTI SU $\mu_1 = 10V$ $\mu_2 = 6V$ $\mu_3 = 8V$

4) $P_1 = \mu_1\cdot I\cdot\cos\varphi_1 = I^2\cdot R_1 = 1^2\cdot 5\sqrt{2} = 7{,}07\,W$

$P_2 = \mu_2\cdot I\cdot\cos\varphi_2 = I^2\cdot R_2 = 1^2\cdot 4{,}79 = 4{,}8W$

$P_3 = I^2\cdot R_3 = 1^2\cdot 4{,}8 = 4{,}8\,W$ $\qquad P = P_1 + P_2 + P_3 = 16{,}7W$

2.88

(35.) Dalekovod je priključen preko otpora sa impedancijom $\dot{z} = 5 + j7$. Impedancija dalekovoda je $\dot{z}_v = 1 + j$. Napon na stezaljkama generatora je $\mu_1 = 380V$.

1) Izračunaj prijenosni koeficijent napona

2) Snagu koju troši prijamnik

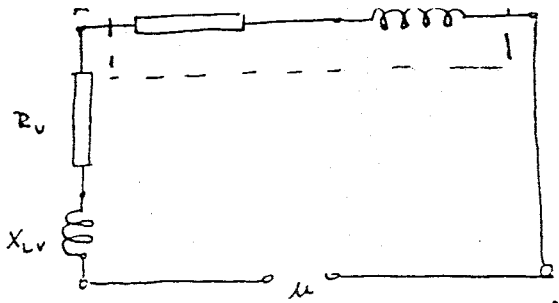
3) Snagu koja se gubi u dalekovodu

4) Korisnost =?

$\dot{z} = 5 + j7$

$\dot{z}_v = 1 + j$

$U = 380V$



$\dot{z} = 6 + 8j$

$\operatorname{tg}\varphi = \frac{8}{6} \Rightarrow \varphi = 53,13^\circ$

$z = \sqrt{36+64} = 10\Omega$

$z_{AB} = \sqrt{25+49} = 8,602\Omega$

$\varphi = 54,46^\circ$

$$\dot{I} = \frac{\dot{U}}{\dot{Z}} = \frac{380 \cdot e^{j0^\circ}}{6+j8} = \frac{380 \cdot e^{j0^\circ}}{10 \cdot e^{j53,13^\circ}} = 38 \cdot e^{-j53,13^\circ} \quad [A]$$

$$\dot{U}_{AB} = \dot{I} \cdot \dot{z}_{AB} = 38 \cdot e^{-j53,13^\circ} \cdot 8,602 \cdot e^{j54,46^\circ} = 326,876 \cdot e^{j1,33^\circ}$$

$k = \frac{U_{AB}}{U} = 0,8602$ $\boxed{k = 0,862}$

$P_2 = I^2 \cdot R_p = 38^2 \cdot 5 = 7,22\,kW$

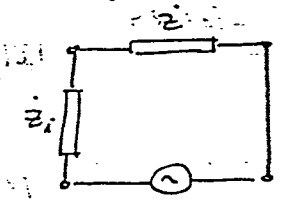
$P_v = I^2 \cdot R_v = 38^2 \cdot 1 = 1,444\,kW$

$P_{izvor} = I^2 \cdot (R_p + R_v) = 38^2 \cdot 6 = 8,664\,kW$ $\quad \eta = \frac{P_2}{P_{izv}} = \frac{7,22}{8,664} = 0,8333$

$\boxed{\eta = 0,833}$

(2.89)

(36.) Potrošač impedancije $\dot{z} = 80 + j40$ napajamo iz jednof. generatora unutrašnje impedancije $\dot{z}_i = 1,5 + j18$. Napon kraćenja priključnih stezaljki iznosi $U_z = 2000V$.



$|z| = \sqrt{6400+1600} = 89,44\,\Omega$

$\operatorname{tg}\varphi = \frac{40}{80} \Rightarrow \varphi = 26,56^\circ$

a) $I = ?$

b) $P = ?$ $\eta = ?$

$$\dot{I} = \frac{\dot{U}_z}{\dot{z}} = \frac{2000e^{j0^\circ}}{89,44 \cdot e^{j26,56^\circ}} = 22,36 \cdot e^{-j26,56^\circ} \quad (A)$$

$\boxed{I = 22,36\,A}$ $\boxed{I = 22,36\,A}$

$P_2 = I^2 \cdot R = 22,36^2 \cdot 80 = 40\,kW$ $\quad P_G = I^2 \cdot (80+1,5) = 40,8\,kW$

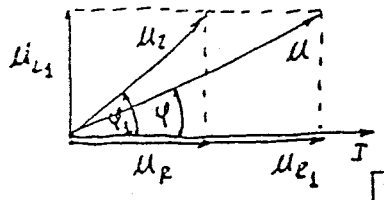
$\eta = \frac{P_2}{P_G} = 0,98165$ $\boxed{\eta = 0,98165}$

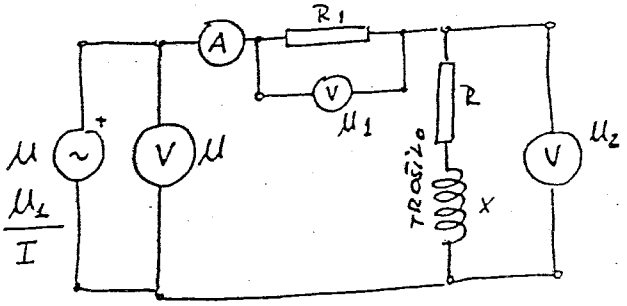
(2.91)

(37.) Instrumenti uključeni u krug pokazuju da je $U = 173V$, $U_1 = U_2 = 100V$ i $I = 10A$.

Odredite 1. otpor trošila

2. otpor trošila R i X





$U_1 = I \cdot R_1 \Rightarrow R_1 = \frac{U_1}{R_1} = \frac{U_1}{I}$

$R_1 = \frac{100V}{10A} = 10\,\Omega$

$\boxed{R_1 = 10\,\Omega}$

$U_2 = \sqrt{U_R^2 + U_L^2} \quad \Rightarrow Z_T^2 = R^2 + X_L^2$

$Z_T = \frac{U_2}{I} = \frac{100}{10} = 10\,\Omega$

$\boxed{R^2 + X^2 = 100}$

(37) $Z^2 = (R+R_1)^2 + X_L^2 = \left(\frac{U}{I}\right)^2$

$R^2 + X_L^2 = 100$ $\Rightarrow R = 5\,\Omega$, $X_L = 8{,}67\,\Omega$ $P = I^2 \cdot R = 500\,W$

19

2.94

(38) $U = 10\,V$ $\dot{Z}_1 = 3 + j4$

$I = 2{,}6\,A$ $\dot{Z}_2 = 10\,e^{-j37} = (0{,}798 - j0{,}6)\cdot 10 = 8 - j6$

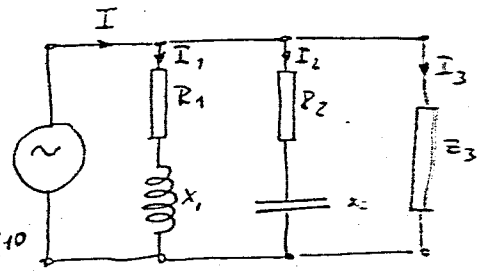
$\varphi = 35°$

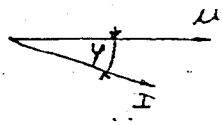
ind. opt. $\dot{I} = 2{,}6(\cos 35° - j\sin 35°)$

$\dot{I} = 2{,}13 - j\,1{,}5$

$\dot{U} = 10 + j0°$ — mogli smo uzeti i da je $\dot{U} = j10$ uvijek isto rezultat

$\dot{I} = \frac{10}{\dot{Z}} \Rightarrow \dot{Z} = \frac{10+0j}{\dot{I}}$





$$\dot{Z} = \frac{10}{2{,}13 - j1{,}5} = \frac{10(2{,}13 + j1{,}5)}{4{,}537 + 2{,}25} = \frac{21{,}3 + j15}{6{,}787} = 3{,}138 + j2{,}21\,\Omega$$

$$\dot{Y} = \frac{1}{\dot{Z}} = \frac{3{,}138 - j2{,}21}{9{,}84 + 4{,}88} = \frac{3{,}138 - j2{,}21}{14{,}72} = 0{,}213 - j0{,}15$$

$$\dot{Y}_1 = \frac{1}{\dot{Z}_1} = \frac{3 - j4}{25} = 0{,}12 - j0{,}16 \qquad \dot{Y}_2 = \frac{8 + j6}{100} = 0{,}08 + j0{,}06$$

$\dot{Y}_3 = \dot{Y} - \dot{Y}_1 - \dot{Y}_2 \Rightarrow \dot{Y}_3 = 0{,}213 - j0{,}15 - 0{,}12 + j0{,}16 - 0{,}08 - j0{,}06$

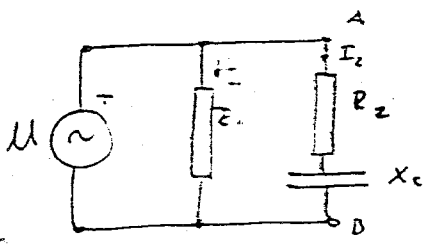
$\dot{Y}_3 = 0{,}013 - j0{,}05 \Rightarrow \dot{Z}_3 = \frac{1}{\dot{Y}_3} = \frac{0{,}013 + j0{,}05}{0{,}000169 + 0{,}0025} = \frac{0{,}013 + 0{,}05}{0{,}002669}$

$\dot{Z}_3 = 4{,}87 + j18{,}73\,\Omega$

$\boxed{\dot{Z}_3 = 4{,}87 - j18{,}73\,\Omega}$ 77

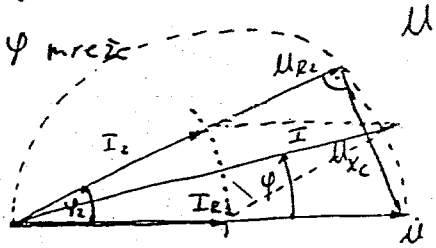
2.98

(39.) U mreži na slici je $R_2 = X_C$



Efektivne vrijednosti struja u granama su jednake

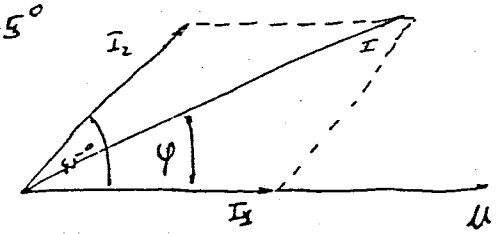
Odredi iz vekt. dijagrama φ mreže



$U_{R_2} = U_{X_C}$

$I_1 = I_2$

$U_{R_1} = U_{XC} \Rightarrow \varphi_2 = 45°$

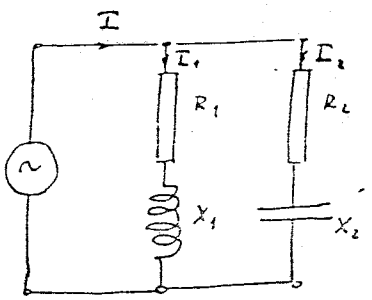


pošto su $I_1 = I_2$

$\Rightarrow \varphi = \frac{1}{2}\varphi_2$

$\boxed{\varphi = 22{,}5°}$ 77

(2.99) Zadana je mreža $\dot{z}_1 = 2+j2$ $\dot{z}_2 = 2-j4$ $I_2 = 5A$. Odredi $\dot{I}$.



Neka je proizvoljno $\dot{I}_2 = 5+j0$

$\dot{U} = \dot{I}_2 \cdot \dot{z}_2 = 5\cdot(2-j4) = 10-20j$

$\dot{I}_1 = \frac{\dot{U}}{\dot{z}_1} = \frac{10-j20}{2+j2} = \frac{(10-j20)(2-j2)}{8}$

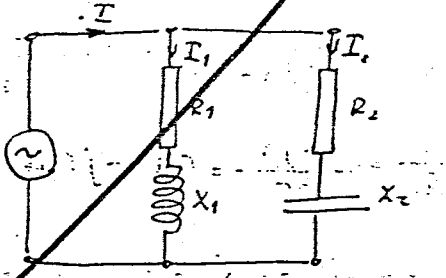
$\dot{I}_1 = \frac{20-j20-j40-40}{8} = \frac{-20-j60}{8}$

$\dot{I}_1 = -2{,}5 - j7{,}5$ $\quad \dot{I} = \dot{I}_1 + \dot{I}_2 = -2{,}5 - j7{,}5 + 5 + j0 =$

$\dot{I} = 2{,}5 - j7{,}5$

$\Rightarrow |\dot{I}| = \sqrt{6{,}25 + 56{,}25} = 7{,}9 \Rightarrow \boxed{I = 7{,}9\ A}$

(41.) Zadana je mreža na slici. $U = 400V$ i $f = 5kHz$. Snaga koju troši prva grana je $P_1 = 2{,}4\ kW$, a kref. poj. grana mreže su međusobno jednaki $\cos\varphi_1 = \cos\varphi_2 = 0{,}6$; $I = 12\ A$. Odredi vrijednosti elemenata u granama mreže



$P_2 = U \cdot I_2 \cdot \cos\varphi \Rightarrow I_2 = \frac{P}{U\cdot\cos\varphi_1} = \frac{2{,}4\cdot10^3}{400\cdot0{,}6}$

$I_1 = \frac{2{,}4\cdot10^3}{240} = 10A$

$P_1 = I_1^2 \cdot R_1 \Rightarrow R_1 = \frac{P_1}{I_1^2} = \frac{2400}{100}$

$\boxed{R_1 = 24\Omega}$ $\quad S_1 = U\cdot I_1 = 400\cdot10 = 4000\ VA$

$\boxed{X_{L1} = 32\Omega}$ $\quad Q_1 = S_1 \cdot \sin\varphi = I^2 X_L \Rightarrow X_L = \frac{S\sin\varphi}{I^2} = \frac{4000\cdot0{,}8}{100} = 32\ \Omega$

$\boxed{\dot{z}_1 = 24+j32}$ $\quad$ uzmimo da je $\dot{U} = 400 + 0j$ (u f.)

$\dot{I}_1 = \frac{400}{24+j32} = \frac{400\cdot(24-j32)}{576+1024} = \frac{9600-j12800}{1600} = 6-j8$

$\boxed{\dot{I}_1 = 6-j8}$

Za 2 granu $\cos\varphi = \frac{U_{R_2}}{U} \Rightarrow U_{R_2} = 400\cdot\cos\varphi = 400\cdot0{,}6 = 240\ V$

$U_{X_2} = 400\cdot\sin\varphi = 400\cdot0{,}8 = 320V$ $\Rightarrow \frac{R_2}{X_2} = tg\varphi$

$\frac{R_2}{X_2} = 1{,}3333$

$R_2 = 1{,}333\cdot X_2$

$\dot{I} = \dot{I}_1 + \dot{I}_2$ $\quad \dot{I} = 6-j8 + \frac{400(1{,}333X_2 + jX_2)}{2{,}777\cdot X_2^2}$

$\dot{I} = 6-j8 + 192\frac{1}{X_2} + j144{,}04\frac{1}{X_2}$ $\quad \dot{I} = \left(6+\frac{192}{X_2}\right) + j\left(\frac{144{,}04}{X_2} - 8\right)$

$\dot{I}_2 = \frac{\dot{U}}{\dot{z}_2}$

$\dot{I}_2 = \frac{400}{R_2 - jX_2}$

$|\dot{I}| = 12A$

$\dot{I}_2 = \frac{400}{1{,}333X_2 - jX_2}$

$144 = \left[6+\frac{192}{X_2}\right]^2 + \left[\frac{144}{X_2} - 8\right]^2$

$\dot{I}_2 = \frac{400(1{,}333X_2 + jX_2)}{2{,}777\cdot X_2^2}$

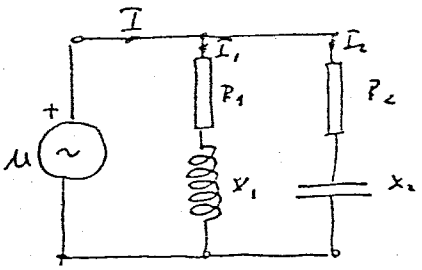
$144 = \left[\frac{6X_2+192}{X_2}\right]^2 + \left[\frac{144-8X_2}{X_2}\right]^2$

$144X_2^2 = 36X_2^2 + 2304X_2 + 36864 + 20736 - 2304X_2 + 64X_2^2$

(2.100)

41. Zadana je mreža na slici $U = 400V$ i $f = 5kHz$. Snaga koju troši prva grana je $P_1 = 2,4kW$ a efektivne snage pojedinih grana mreže su međusobno jednake:

$\cos\varphi_1 = \cos\varphi_2 = 0,6$; $I = 12A$. Odredite vrijednosti elemenata u granama mreže.



$P_2 = U \cdot I_1 \cos\varphi_2$

$\Rightarrow I_1 = \frac{P_1}{U\cos\varphi_2} = \frac{2,4\cdot 10^3}{400\cdot 0,6}$

$I_1 = 10A$ $\quad P_{R_1} = U_{R_1} I_1 = I_1^2 \cdot R$

$R = \frac{P_1}{I_1^2} = \frac{2400}{100} = 24\Omega$

$\boxed{R_1 = 24\Omega}$

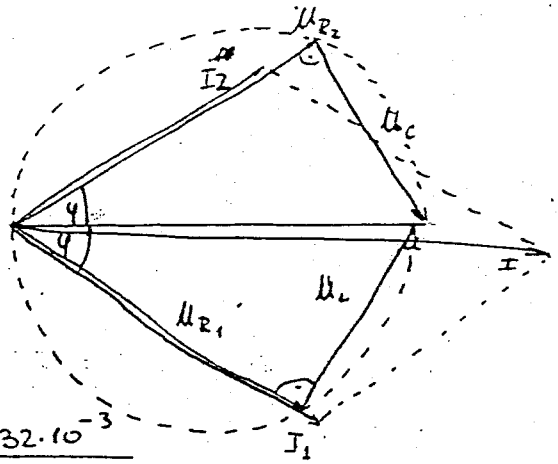
$Z_1 = \frac{U}{I_1} = \frac{400}{10} = 40\Omega$

$Z_1 = \sqrt{R_1^2 + X_L^2}$

$X_L^2 = Z_1^2 - R_1^2 = 1600 - 576 = 1024$

$\boxed{X_L = 32\Omega}$ $\Rightarrow L = \frac{X_L}{\omega} = \frac{32}{2\pi f} = \frac{32}{2\pi\cdot 5\cdot 10^3} = \frac{32\cdot 10^{-3}}{31,4}$

$L = 1,019\cdot 10^{-3} H$ $\quad \boxed{L = 1,019 mH}$



Ukupna impedancija je $Z_u = \frac{U}{I} = \frac{400}{12} = 33,33\Omega$

$U_{R_2}^2 + U_C^2 = U^2$

$\frac{U_C}{U_{R_2}} = \operatorname{tg}\varphi$ $\quad U_{R_2} = U\cdot\cos\varphi = 400\cdot 0,6 = 240V$ $\quad \boxed{U_C = 320V}$ $\quad \boxed{U_{R_2} = 240V}$

$I^2 = I_1^2 + I_2^2 - 2I_1 I_2 \cdot \cos 2\varphi \Rightarrow I_2^2 = I^2 - I_1^2 - 2I_1 I_2 \cdot \cos 2\varphi$

OVO NIJE DOBRO

$I_2^2 = 144 - 100 + 2\cdot 10\cdot I_2 \cdot 0,28$

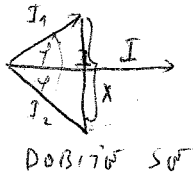
$I_2^2 - 5,6 I_2 - 44 = 0$ $\quad I_2 = \frac{5,6 \pm \sqrt{31,36+176}}{2} = \frac{5,6 \pm 14,4}{2} = 10A$ $\quad \boxed{I_2 = 10A}$

$\Rightarrow I$ je u fazi sa naponom. Spoj je u rezonanciji:

$Z_2^2 = \frac{U^2}{I_2^2} = 1600 = Z_2^2 + X_C^2$ $\quad \frac{X_C}{R_2} = \operatorname{tg}\varphi = 1,3333$ $\quad X_C = 1,3333 R_2$

$1600 = Z_2^2 + 1,7777 Z_2^2 = R_2^2 + 2,7777 = 1600 \Rightarrow \boxed{R = 24\Omega}$

$C = 10^{-6} F$ $\quad \boxed{C = 1\mu F}$



DOBITE SU X A NIJE I

$I^2 = I_1^2 + I_2^2 - 2I_1 I_2 \cos(180-2\varphi)$

ILI DA PROMIJENIMO PREDZNAK

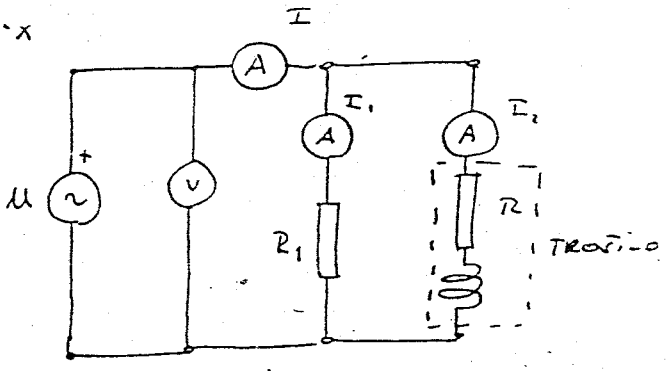
$I^2 = I_1^2 + I_2^2 + 2I_1 I_2 \cos 2\varphi$

2.101

42. Instrumenti uključeni u mrežu pokazuju da je $U = 200V$ $I = 37,9\ A$

$I_1 = I_2 = 20A$. Odredi: 1) snagu trošila

2) otpore trošila R i X



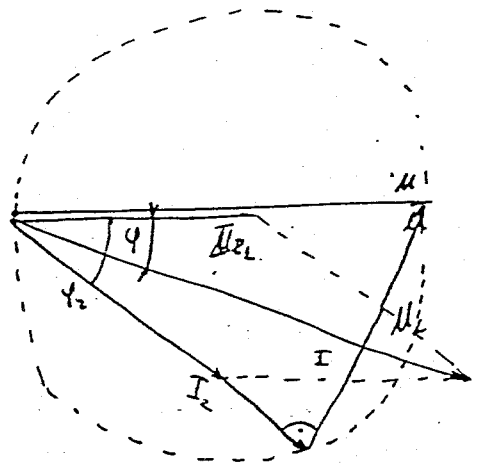
$U = 200V$ $I_1 = I_2 = 20A$

$I = 37,9A$

$R_1 = \frac{U}{I_1} = \frac{200V}{20} = 10\Omega$ $\boxed{R_1 = 10\Omega}$

$I^2 = I_1^2 + I_2^2 + 2I_1 \cdot I_2 \cdot \cos\varphi_2$

$\Rightarrow \cos\varphi_2 = \frac{I^2 - I_1^2 - I_2^2}{2I_1 \cdot I_2} = \frac{1436,41 - 400 - 400}{2 \cdot 20 \cdot 20}$



$\cos\varphi_2 = 0,7955125$

$P_T = U \cdot I_2 \cdot \cos\varphi_2 = 200 \cdot 20 \cdot 0,7955125$

$\boxed{P = 3,182\ kW}$

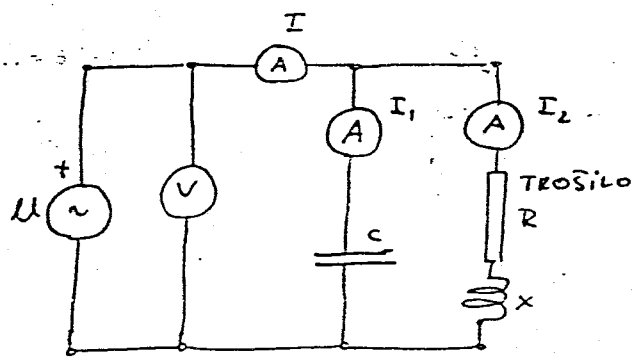
$Z_L = \frac{U}{I_2} = \frac{200}{20} = 10\Omega \Rightarrow R = 10 \cdot 0,7955 \Rightarrow R = 7,955\Omega$

$X_L = 10 \cdot 0,60595 \Rightarrow X_L = 6,0595\Omega$ $\approx$ $\boxed{\begin{array}{l} R = 8\Omega \\ X_L = 6\Omega \\ P = 3,182\ kW \end{array}}$

2.102

43. $U = 100V$ $I = I_1 = I_2 = 10\ A$

1) R i X = ? 2) snaga trošila = ?



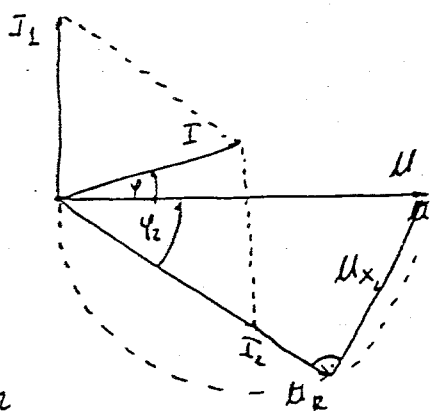
$I^2 = I_1^2 + I_2^2 + 2I_1 \cdot I_2 \cdot \cos(90 + \varphi_2)$

$\cos(90 + \varphi_2) = \frac{I^2 - I_1^2 - I_2^2}{2I_1 \cdot I_2}$

$\cos(90 + \varphi_2) = \frac{100 - 100 - 100}{200}$

$\cos(90 + \varphi_2) = -\frac{1}{2}$

$90 + \varphi_2 = 120° \Rightarrow \boxed{\varphi_2 = 30°}$



$P_T = U \cdot I_2 \cdot \cos 30° = 100 \cdot 10 \cdot \frac{\sqrt{3}}{2} = 865W$

$P_T = 0,865\ kW$

$Z_T = \frac{U}{I_2} = 10\Omega$ $R = Z_T \cdot \cos\varphi = 10 \cdot \frac{\sqrt{3}}{2} = 8,66\Omega$

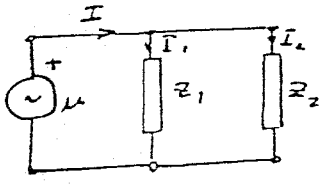
$X_L = Z_T \cdot \sin\varphi = 5\Omega$ $\boxed{\begin{array}{l} R = 8,66\Omega \\ X_L = 5\Omega \end{array}}$

2.103

44. Na krugu priključen je napon $u = 50V$. Sve struje su jednake i iznose

$I = I_1 = I_2 = 5A$; $\varphi_1 = 30°$

Z: i $\dot{Z}_1 = ?$

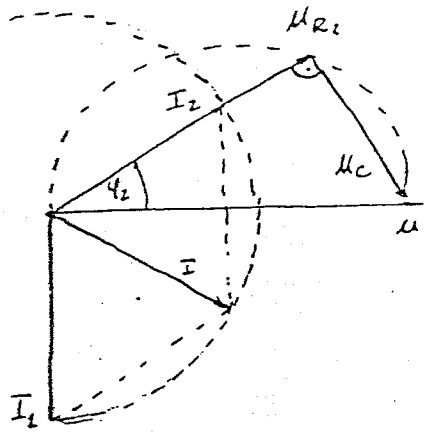


Z_1 je čisto induktivno opterećenje jer je $\varphi_1 = 30° = +\frac{\pi}{2}$.

$\varphi_2 = 30°$ jer će biti I biti $= 5A$ kao i I_1 i I_2.

to je vidi iz dijagrama.

Z_2 je kapacitivnog karaktera



$Z_2 = \frac{U}{I_2} = 10\Omega$ $\quad R_2 = 10 \cdot \cos 30° = 10 \cdot \frac{\sqrt{3}}{2}$

$R_2 = 8{,}66\Omega$

$X_C = 10 \cdot \sin 30° \Rightarrow X_C = 5\Omega$

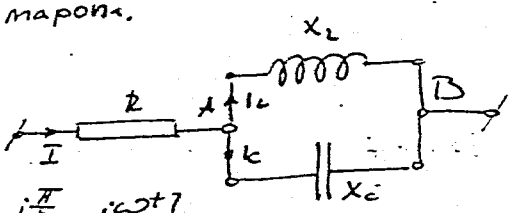
$\Big\} \Rightarrow \boxed{Z_2 = 8{,}66 - j5\ (\Omega)}$ $\quad \boxed{Z_1 = j10\Omega}$

2.108

45. U mreži je $R = X_L = 2\Omega$ $\quad X_C = 1\Omega$ $\quad i$ $i_C = \sin(\omega t - \frac{\pi}{4})$

1. Odredi jednadžbu trenutne vrijednosti priključenog napona.

2. Nacrtati vektorski dijagram



$i_C = 1 \cdot \sin(\omega t - \frac{\pi}{4})$

$i_C = \mathrm{Im}\left\{1 \cdot e^{j(\omega t - \frac{\pi}{4})}\right\} = \mathrm{Im}\left\{1 \cdot e^{-j\frac{\pi}{4}} \cdot e^{j\omega t}\right\} = \mathrm{Im}\left\{e^{-j\frac{\pi}{4}} \cdot e^{j\omega t}\right\}$

$\dot{I}_C = 1 \cdot e^{-j\frac{\pi}{4}} = \cos\frac{\pi}{4} - j\sin\frac{\pi}{4} = \boxed{\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2} = \dot{I}_C}$

$\dot{U}_{AB} = \dot{I}_C \cdot X_C = \left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right)(0 - j) = -j\left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{2}}{2}j + j^2\frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}j - \frac{\sqrt{2}}{2}$

$\dot{U}_{AB} = -\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}$

$\dot{I}_L = \frac{\dot{U}_{AB}}{X_L} = \frac{-\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}}{j2} = \frac{-\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}}{j2} \cdot \frac{-j2}{-j2} = \frac{\sqrt{2}j + j^2\sqrt{2}}{-j^2 \cdot 4} = \frac{-\sqrt{2} + j\sqrt{2}}{4}$

$\dot{I}_L = -\frac{\sqrt{2}}{4} + j\frac{\sqrt{2}}{4}$ $\quad \dot{I} = \dot{I}_L + \dot{I}_C = -\frac{\sqrt{2}}{4} + j\frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2} = \frac{-\sqrt{2} + 2\sqrt{2}}{4} + j\frac{\sqrt{2} - 2\sqrt{2}}{4}$

$\dot{I} = \frac{\sqrt{2}}{4} - j\frac{\sqrt{2}}{4}$ $\quad \dot{U}_2 = \dot{I} \cdot R = \left(\frac{\sqrt{2}}{4} - j\frac{\sqrt{2}}{4}\right) \cdot 2 = \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\ (V)$

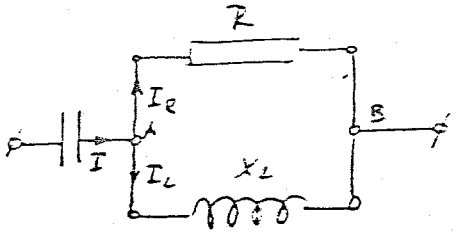
$\dot{U} = \dot{U}_2 + \dot{U}_{AB} = \left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) = -j\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2} = -\sqrt{2}j \Rightarrow \dot{U} = -j\sqrt{2}$

$\dot{U} = -j\sqrt{2} \Rightarrow \boxed{\varphi = -\frac{\pi}{2}}$ $\quad u = \mathrm{Im}\left[\sqrt{2} \cdot e^{-j\frac{\pi}{2}} e^{j\omega t}\right]$

$$\boxed{\hat{u} = \sqrt{2}\sin(\omega t - \tfrac{\pi}{2})}$$

2.109

(46.) Izračunati $R = X_L = 4\,\Omega$, $X_C = 1\,\Omega$ i $i_R = 2\sin\omega t$. Odrediti jednadžbu za trenutnu vrijednost napona (priključenog). Nacrtati vektorski dijagram



$R = 4\,\Omega$ $\quad X_C = 1\,\Omega$

$X_L = 4\,\Omega$ $\quad i_R = 2\sin\omega t \Rightarrow i_R = \mathrm{Im}\{2e^{j\omega t}\}$

$i_R = \mathrm{Im}\{2e^{j\omega t}\}$

Struju u otporniku R ćemo uzeti kao referentnu jer u preostale grane ne znamo struje za $t=0$. $\Rightarrow i_R = 0$

$i_R = 2e^{j\omega t} \Rightarrow \bar{I}_R = 2\,A \qquad \bar{U}_{AB} = \bar{I}_R \cdot R = 2\cdot 4 = 8\,V \qquad \bar{I}_L = \dfrac{\bar{U}_{AB}}{\bar{X}_L} = \dfrac{\mathcal{E}}{j4}$

$\bar{I}_L = -j2\,A$

$\bar{I} = \bar{I}_R + \bar{I}_L = 2A - j2A = 2 - j2\,A$ $\boxed{\bar{I} = 2 - j2\ A}$ $\qquad \bar{U}_C = \bar{I}\cdot X_C = (2-j2)\cdot(-j)$

$\bar{U}_C = -j(2-j2) = -2j-2 = -2-2j$

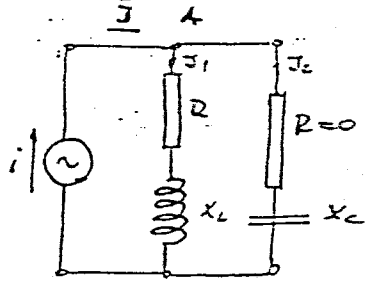
$\bar{U} = \bar{U}_C + \bar{U}_{ab} = -2-2j+8 = 6-2j \qquad U = \sqrt{36+4} = \sqrt{40}\ V$

$\varphi_u = \arctg\dfrac{-2}{6} = -18{,}435^\circ \qquad u = \sqrt{40}\sin(\omega t - 18{,}435^\circ) = \underline{2\sqrt{10}\sin(\omega t - 18{,}435^\circ) = i}$

2.110

(47.) Zadan je spoj kao slici s $R = 1\,\Omega$, $X_L = 1\,\Omega$, $X_C = 0{,}5\,\Omega$ i $u_R = 2\sin\omega t$

Napišite jednadžbu trenutne vrijednosti struje izvora



$R = 1\,\Omega$

$X_L = 1\,\Omega$

$X_C = 0{,}5\,\Omega$

$u_R = 2\sin\omega t = 2\,V$

$I_1 = \dfrac{u_R}{R} = 2A \qquad \bar{U}_{AB} = I_1\cdot Z_1 = 2\cdot(1+j1) = 2+j2\ V$ $\boxed{\bar{U}_{AB} = 2+j2\ V}$

$\bar{I}_2 = \dfrac{\bar{U}_{AB}}{Z_2} = \dfrac{2+j2}{-j0{,}5} = j\cdot\dfrac{2+j2}{0{,}5} = \dfrac{j2-2}{0{,}5} = -4+j4 \qquad I_2 = -4+j4$

$\bar{I} = \bar{I}_1 + \bar{I}_2 = 2-4+j4 = -2+j4\ A \qquad I = \sqrt{4+16} = \sqrt{20}\sin(\omega t - 116^\circ)$

$\varphi = \arctg\dfrac{-4}{2} = \arctg(-2) = 116^\circ$

$i = \sqrt{20}\sin(\omega t - 116^\circ)$

MISLIM DA JE $+116^\circ$

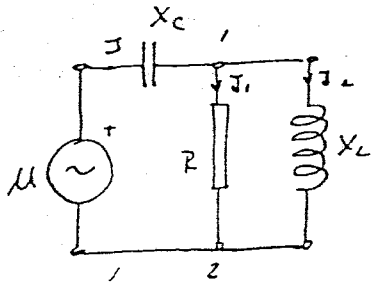
$i = -2+4j$

2.111

(48) U krugu je zadano $U = 80V$, $U_c = 60V$, $U_{12} = 100V$ i $I_2 = 10A$

Odredite: R, X_L, X_c

I_1, I



$\frac{U_{12}}{I_2} = X_L$

$I_2 = 10A \quad \bar{J}_2 = 10 e^{j0^\circ}$

$X_L = \frac{100}{10} = 10\,\Omega$

$\bar{U}_{12} = \bar{J}_2 \cdot jX_L = 10e^{j0^\circ} \cdot jX_L = j10X_L \qquad \bar{J}_1 = \frac{\bar{U}_{12}}{R} = j\frac{10X_L}{R}$

$\bar{J} = \bar{J}_1 + \bar{J}_2 = 10 + j\frac{10X_L}{R} = 10 + j\frac{100}{R}$ $\boxed{\bar{J} = 10 + j\frac{100}{R}}$

$\bar{U}_c = \bar{J} \cdot (-jX_c) = -j\bar{J} \cdot X_c = -jX_c \cdot (10 + j\frac{100}{R}) = -j10X_c + \frac{100X_c}{R}$

$\bar{U}_c = \frac{100X_c}{R} - j10X_c \qquad \bar{U} = \bar{U}_c + \bar{U}_{12} = \frac{100X_c}{R} - j10X_c + j100$

$\bar{U} = \frac{100X_c}{R} + j(100 - 10X_c)$

$U_c = 60V \quad \left(\frac{100X_c}{R}\right)^2 + (10X_c)^2 = 3600$

$U = 80V \quad \left(\frac{100X_c}{R}\right)^2 + (100 - 10X_c)^2 = 6400$ $\Big\} - (10X_c)^2 + (100 - 10X_c)^2 = 2800$

$-(10X_c)^2 + 100(10 - X_c)^2 = 2800 \qquad 100(100 - 20X_c + X_c^2) - 100X_c^2 = 2800$

$10000 - 2000X_c + 100X_c^2 - 100X_c^2 = 2800 \qquad -2000X_c = -7200 \Rightarrow \boxed{X_c = 3,6\,\Omega}$

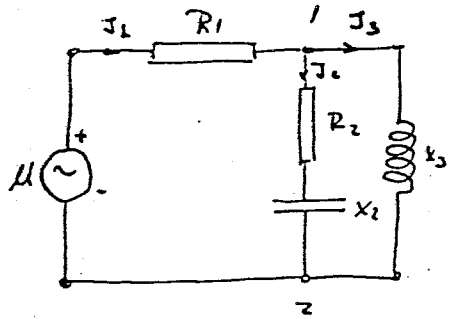
$\Rightarrow \boxed{R = 7,5\,\Omega}$

(49) 2.118

Zadana je mreža $R_1 = 5\Omega$, $R_2 = 6\Omega$ $X_2 = -8\Omega$ $X_3 = 10\Omega$ i $I_3 = 10A$

Odredi 1. I_1, I_2

2. μ; P



Mrežnim dijelom $\bar{J}_3 = 10 e^{j0^\circ}$

$\bar{U}_{12} = \bar{J}_3 \cdot j10 = 10 \cdot j10 = 100j$

$\bar{J}_2 = \frac{100j}{6 - j8} = \frac{100j \cdot (6 + j8)}{36 + 64} = \frac{600j - 800}{100} = -8 + j6$

$\bar{J}_1 = \bar{J}_2 + \bar{J}_3 = -8 + j6 + 10$

$\bar{J}_1 = 2 + j6$

$J_2 = \sqrt{36 + 64} = 10A$ $\boxed{J_2 = 10A}$

$J_1 = \sqrt{4 + 36} \Rightarrow$ $\boxed{J_1 = 6,324\,A}$

$\bar{Z}_u = 5 + \frac{(6 - j8) \cdot j10}{6 - j8 + j10} = 5 + \frac{j60 + 80}{6 + j2} = 5 + \frac{100e^{j36,86}}{6,32 \cdot e^{j18,43}} = 5 + 15,82e^{j18,43}$

$Z_u = 5 + 15,01 + j5 = 20 + j5$ $\boxed{\bar{Z}_u = 20 + j5}$

58

$\bar{U} = \bar{J}_1 \cdot \bar{Z}_{ab} = (2+j6)\cdot(20+j5) = 40+j10+j120-30 = 10+j130$

$U = \sqrt{100 \cdot 16900} = 130{,}38\,V$

$\boxed{U = 130{,}3\,V}$

$S = \bar{U}\cdot\bar{I}_1^* = (10+j130)\cdot(2-j6) = 20-j60+260+780$

$S = 800+j200 \Rightarrow \boxed{P = 500\,W}$

(2.12)

50. Za spoj na slici je zadano

Odredi: I_1, I_2, I_3

U, P

$U_{ab} = 30\,V$

$R_1 = 3\,\Omega$

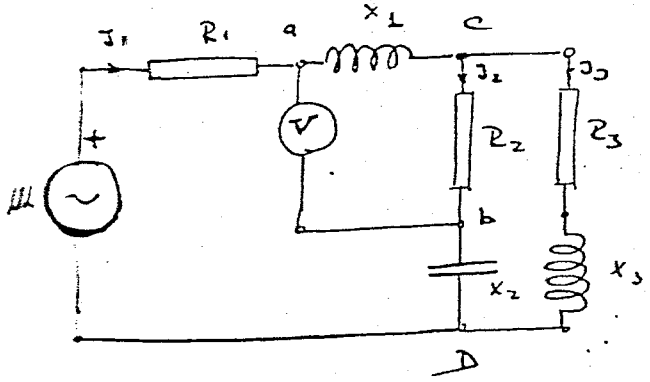
$X_1 = 2\,\Omega$

$R_2 = 1\,\Omega$

$X_2 = -2\,\Omega$

$R_3 = 2\,\Omega$

$X_3 = 6\,\Omega$



$$Z_u = 3+j2+\frac{(1-j2)(2+j6)}{1-j2+2+j6} = 3+j2+\frac{2+j6-j4+12}{3+j4} = 3+j2+\frac{14+j2}{3+j4}$$

$$Z_u = \frac{3(3+j4)+j2(3+j4)+(14+j2)}{3+j4} = \frac{9+j12-j6-8+14+j2}{3+j4} = \frac{15+j20}{3+j4} = \frac{25\cdot e^{j53{,}13^\circ}}{5\cdot e^{j53{,}13^\circ}}$$

$Z_u = 5\cdot e^{j0^\circ}$

Uzmimo $\bar{U} = U\cdot e^{j0^\circ} \Rightarrow \bar{J}_1 = \frac{\bar{U}}{\bar{Z}_u} = \frac{U\cdot e^{j0^\circ}}{5\cdot e^{j0^\circ}} = \boxed{\frac{U}{5}\cdot e^{j0^\circ} = \bar{J}_1}$

$\bar{Z}_{CD} = \frac{14+j2}{3+j4} = \frac{14{,}14\cdot e^{j8{,}13^\circ}}{5\cdot e^{j53{,}13^\circ}} = 2{,}828\cdot e^{-j50^\circ}$

$\bar{Z}_{CD} = 2{,}828\cdot e^{-j50^\circ}$

$\bar{U}_{CD} = \bar{J}_1\cdot\bar{Z}_{CD} = \frac{U}{5}e^{j0^\circ}\cdot 2{,}828\cdot e^{-j50^\circ} = 0{,}5656\,U\cdot e^{-j50^\circ}$

$\bar{J}_2 = \frac{\bar{U}_{CD}}{1-j2} = \frac{0{,}5656\,U\cdot e^{-j50}}{2{,}236\cdot e^{-j63{,}43}} = 0{,}253\,U\cdot e^{j13{,}43^\circ}$ $\boxed{\bar{J}_2 = 0{,}253U\cdot e^{j13{,}43^\circ}}$

$\bar{J}_3 = \bar{J}_1 - \bar{J}_2 = \frac{U}{5}\cdot e^{j0^\circ} - 0{,}253U\cdot e^{j13{,}43^\circ} = 0{,}2U - 0{,}253U\cdot(0{,}9726+j0{,}2322)$

$\bar{J}_3 = 0{,}2U - 0{,}246U - j0{,}058U = -0{,}046U - j0{,}058U$

$\bar{U}_{ab} = \frac{U}{5}\cdot j2 + 0{,}253U\cdot e^{j13{,}43^\circ}\cdot 1 = \frac{2U}{5}j + 0{,}253U(0{,}9726+j0{,}2322)$

$\bar{U}_{ab} = j\frac{2U}{5} + 0{,}246U + j0{,}058U = j0{,}4U - 0{,}246U + j0{,}058U$

$\bar{U}_{ab} = 0{,}246U + j0{,}458U$

$30^2 = 0{,}0605U^2 + 0{,}207936U^2 \Rightarrow 900 = 0{,}268436\,U^2 \Rightarrow \boxed{U = 56\,V}$

$J_1 = \frac{56}{5} = 11{,}2\,A \qquad J_2 = 0{,}253U\cdot e^{j13{,}43} = 0{,}253\cdot 56\,e^{j13{,}43}$

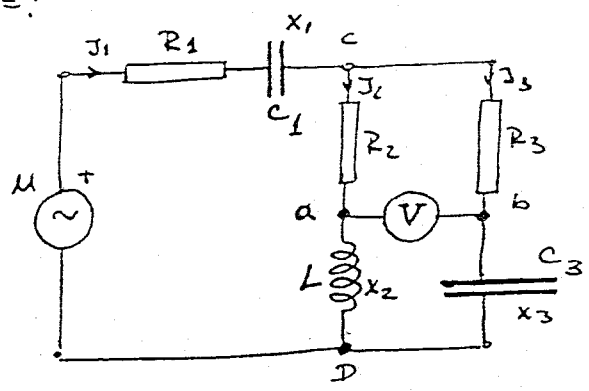
$\boxed{J_2 = 14{,}1\,A} \qquad \boxed{J_3 = 5\,A}$

$P = \bar{U}\cdot J^* = 56\cdot\frac{56}{5} = 627\,W \Rightarrow \boxed{P = 627\,W}$

2.119

(51) W mreži na sliki je $R_1 = 2\,\Omega$ $X_1 = -8\,\Omega$ $X_2 = 5\,\Omega$ $R_2 = 5\,\Omega$ $R_3 = 10\,\Omega$ $X_3 = -10\,\Omega$

$U_{ab} = 20\,V$. Odredite: I_1, I_2, I_3 U, P = ?



$$\bar{Z}_{CD} = \frac{(5+j5)(10-j10)}{5+j5+10-j10} = \frac{50-j50+j50+50}{15-j5}$$

$$\bar{Z}_{CD} = \frac{100}{15-j5} = \frac{100(15+j5)}{250} = \frac{15+j5}{2,5}$$

$\boxed{\bar{Z}_{CD} = 6 + j2\ \Omega}$ $\quad \bar{Z}_u = \underline{2} - j8 + \underline{6} + j2 = 8 - j6\ \Omega$ $\quad \boxed{\bar{Z}_u = 8 - j6}$ $\quad \underline{\bar{Z}_u = 10\,e^{-j36,87^\circ}}$

$U = |U| \cdot e^{j0^\circ}$ proizvoljno $\quad \bar{J}_1 = \frac{\bar{U}}{Z_u} = \frac{U\,e^{j0^\circ}}{10 \cdot e^{-j36,87^\circ}} = \frac{U}{10}\,e^{j36,87^\circ}$

$$J_1 = \frac{U}{10}\,e^{j36,87^\circ}$$

$$\bar{U}_{CD} = \bar{J}_1 \cdot \bar{Z}_{CD} = \frac{U}{10}\,e^{j36,87^\circ} \cdot 6,3245 \cdot e^{j18,435^\circ} = 0,63245\,U \cdot e^{j55,3^\circ}$$

$$\bar{J}_2 = \frac{0,6324\,U \cdot e^{j55,3^\circ}}{5+j5} = \frac{0,6324\,U \cdot e^{j55,3^\circ}}{7,07 \cdot e^{j45^\circ}} = 0,0894 \cdot U \cdot e^{j10,3^\circ}$$

$$\bar{J}_3 = \frac{0,6324\,U \cdot e^{j55,3^\circ}}{10-j10} = \frac{0,6324\,U \cdot e^{j55,3^\circ}}{14,142 \cdot e^{-j45^\circ}} = 0,0447\,U \cdot e^{j100,3^\circ}$$

$$\bar{U}_{ab} = \bar{J}_2 \cdot R_2 + \bar{J}_3 \cdot R_3 = 0,0894\,U \cdot e^{j10,3^\circ} + 0,0447\,U \cdot e^{j100,3} = 0,36\,U + j0,5189\,U$$

$400 = U^2(0,1296 + 0,26925) \Rightarrow \boxed{U = 31,6\,V}$ ww

$$\bar{J}_1 = \frac{U}{10}\,e^{j36,87^\circ} = 3,16 \cdot e^{j36,87^\circ} \Rightarrow \boxed{I_1 = 3,16\,A}$$ ww

$\bar{J}_2 = 0,0894\,U \cdot e^{j10,3^\circ} = 2,825 \cdot e^{j10,3^\circ} \Rightarrow \boxed{I_2 = 2,825\,A}$ ww

$\bar{J}_3 = 0,0447 \cdot U \cdot e^{j100,3^\circ} = 1,41252\,A \Rightarrow \boxed{I_3 = 1,4125\,A}$ ww

$P = ?$ $\quad S = U \cdot I_1^* = U \cdot 3,16 \cdot e^{-j36,87^\circ} = 99,856\,(\cos 36,87^\circ - j\sin 36,87^\circ)$

$\Rightarrow P = 80\,W$ $\boxed{P = 80\,W}$ ww

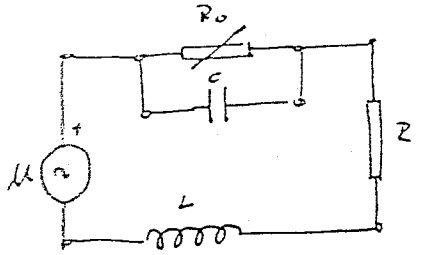
ZADACI IZ REZONANCIJE

3.20

1. $U = 100\,V$ Odredite R_0 uz koji će krug biti u rezonanciji

$Z = 15\,\Omega \quad X_L = 5\,\Omega$

$X_C = 10\,\Omega$



Odredimo ukupnu impedanciju kruga

$$\bar{Z} = R + jX_L + \frac{R_0 \cdot (-jX_C)}{R_0 + jX_C} = 15 + j5 - \frac{R_0 \cdot j10}{R_0 - jX_C} = 15 + j5 - \frac{R_0 \cdot j10}{R_0 - j10} \cdot \frac{R_0 + j10}{R_0 + j10}$$

$$\bar{Z} = 15 + j5 - \frac{R_0^2 \cdot j10 + R_0 \cdot j^2 \cdot 100}{R_0^2 + 100} = 15 + j5 - \frac{R_0^2 \cdot j10 - R_0 \cdot 100}{R_0^2 + 100} = \frac{15(R_0^2+100) + j5(R_0^2+100) - R_0^2 j10 + R_0 \cdot 10}{R_0^2 + 100}$$

$$\bar{Z} = \frac{15R_0^2 + 1500 + 5jR_0^2 + j500 - R_0^2 \cdot j10 + 100R_0}{R_0^2 + 100} \qquad \bar{Z} = \frac{15R_0^2 + 100R_0 + 1500 - 5jR_0^2 + j500}{R_0^2 + 100}$$

$$\bar{Z} = \frac{15R_0^2 + 100R_0 + 1500}{R_0^2 + 100} - j\frac{5R_0^2 - 500}{R_0^2 + 100}$$

Da bi spoj bio u rezonanciji mora da imaginarna komponenta bude jednaka nuli.

$\Rightarrow 5R_0^2 - 500 = 0 \Rightarrow 5R_0^2 - 500 \Rightarrow \boxed{R_0 = 10\,\Omega}$

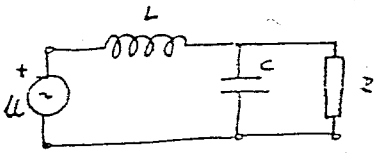
Struja u rezonanciji biti će $\bar{I} = \frac{\bar{U}}{\bar{Z}}$. $\Rightarrow \bar{Z}_{u\,rez} = \frac{15 \cdot 100 + 100 \cdot 10 + 1500}{100 + 100}$

$Z = \frac{1500 + 1000 + 1500}{200} = \frac{4000}{200} = 20\,\Omega \qquad \bar{I} = \frac{\bar{U}}{\bar{Z}} = \frac{100}{20} = 5\,A \qquad \boxed{I = 5\,A}$ u rez

3.21

2. Iz frekvencija $\omega = 0$, $Z_{ul}(\omega = 0) = 5\,\Omega$, a pri rezonantnoj frekvenciji $Z_{ul}(\omega_0) = 2{,}5\,\Omega$. Odredite

R, X_C, X_L,



za $\omega = 0 \Rightarrow$ stacionarna struja $Z = 5\,\Omega = R$

$\Rightarrow \boxed{R = Z = 5\,\Omega}$

Pri rez-frekvenciji $Z_{ul} = 2{,}5\,\Omega$ odredimo operatorsku impedanciju kruga

$$\bar{Z} = +jX_L + \frac{R \cdot (-jX_C)}{R - jX_C} = jX_L - \frac{R \cdot jX_C}{5 - jX_C} = jX_L - \frac{5jX_C}{5 - jX_C} \cdot \frac{5 + jX_C}{5 + jX_C}$$

$$\bar{Z} = jX_L - \frac{25jX_C + 5j^2X_C^2}{25 + X_C^2} = jX_L - \frac{25jX_C - 5X_C^2}{25 + X_C^2} = \frac{25jX_L + jX_L \cdot X_C^2 - 25jX_C + 5X_C^2}{25 + X_C^2}$$

$$\bar{Z} = \frac{5X_C^2}{25 + X_C^2} + j\frac{25X_L + X_L X_C^2 - 25X_C}{25 + X_C^2}$$

pri rez Ruhr im. dio je jednak nuli.

$$\text{pa je } \bar{Z} = \frac{5X_C^2}{25 + X_C^2} = \frac{\frac{1}{(\omega C)^2}}{25 + \frac{1}{(\omega C)^2}} = \frac{5\frac{1}{(\omega C)^2}}{25\frac{(\omega C)^2 + 1}{(\omega C)^2}} = \frac{5}{25(\omega C)^2 + 1}$$

$$\underline{Z} = \frac{5}{25(j\omega C)^2 + 1} = \frac{5}{2} \Rightarrow \frac{5}{2} = \frac{5}{25 - X_C^2} \qquad 125 + 5X_C^2 = 10X_C^2$$

$$125 = 5X_C^2 \qquad 5X_C^2 = 125 \Rightarrow \boxed{X_C = 5\,\Omega}$$

imaginarni dio kompl. broja mora biti jednak nuli:

$$\frac{25X_L^2 + X_L \cdot X_C^2 - 25X_C}{25 + X_C^2} = 0$$

$$25X_L + X_L \cdot X_C^2 - 25X_C = 0 \qquad 25X_L - 25X_L = 125 \qquad 50X_L = 125 \Rightarrow \boxed{X_L = 2{,}5\,\Omega}$$

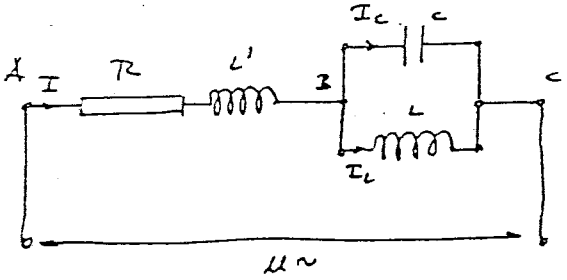
3.23

3.) $U = 50\,V \quad R = 25\,\Omega \quad L' = 2\,mH \quad L = 0{,}1\,H$

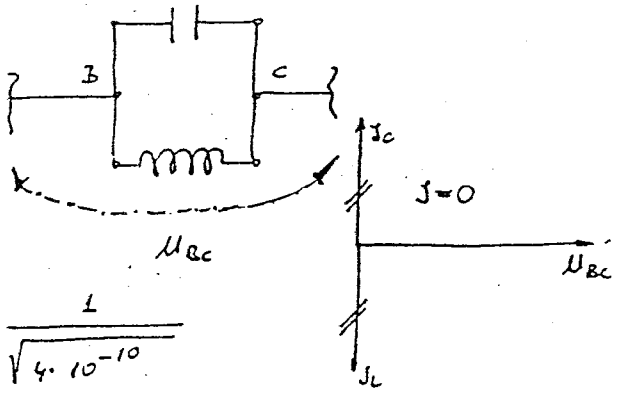
$C = 1\,\mu F$

1. Rez frekv. = ?

3. Struje u granama strujnog kruga za oba rezonantna frekvencija



Krug ima 2 rezonantne frekvencije. Jedna je za paralelu između točaka B i C a jedna za cijeli krug



$\Rightarrow$ tj. za $\boxed{B_L = B_C}$ $\boxed{\frac{1}{\omega L} = \omega C}$

$$\Rightarrow 1 = \omega^2 LC \Rightarrow \omega = \frac{1}{\sqrt{LC}} \qquad \omega = \frac{1}{\sqrt{L \cdot C}}$$

za frekvenciju $\omega_1 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0{,}1 \cdot 10^{-3} \cdot 10^{-6}}} \cdot \frac{1}{\sqrt{0{,}1 \cdot 10^{-9}}} \cdot \frac{1}{\sqrt{4 \cdot 10^{-10}}}$

$$\omega = \frac{1}{2 \cdot 10^{-5}} = \frac{10^5}{2} = 50\,000\ \frac{rad}{s} \Rightarrow \boxed{\omega_{rez_{BC}} = 5 \cdot 10^4\ s^{-1}}$$

OSTALO

$$\underline{Z}_{uk} = R + jX_L' + \frac{jX_L \cdot (-jX_C)}{jX_L + (-jX_C)} = R + jX_L' + \frac{1}{\underline{Y}_{BC}} = R + jX_L' + \frac{1}{j(B_C - B_L)}$$

pri ω_1 tj. kada je div. BC - rezonancija vidimo da je $\boxed{Z = +\infty} \rightarrow \boxed{I = 0\,A}$

$$\underline{Z}_{uk} = R + jX_L' + \frac{1}{j(B_C - B_L)} \cdot \frac{j(B_C - B_L)}{j(B_C - B_L)} = R + jX_L' - j\frac{B_C - B_L}{(B_C - B_L)^2} = R + jX_L' - j\frac{1}{B_C - B_L}$$

$\underline{Z}_{uk} =$ ~~————————~~ $\quad Z_{uk} = R + j\left[X_L' - \frac{1}{B_C - B_L}\right]$ $\quad \omega_2$ ćemo odrediti iz uvjeta $\mathrm{Im}(\underline{Z}) = 0$

$$X_L' = \frac{1}{B_C - B_L} \qquad \omega L' = \frac{1}{-\frac{1}{\omega L} + \omega C} \Rightarrow \omega L' = \frac{\omega L}{-1 + \omega^2 LC} \quad /:\omega \qquad L' = \frac{L}{-1 + \omega^2 LC}$$

$$L'(-1 + \omega^2 LC) = L$$

$$-1 + \omega^2 LC = \frac{L}{L'} \qquad \boxed{\omega^2 LC = 1 + \frac{L}{L'}} \Rightarrow \omega^2 = \frac{1 + \frac{L}{L'}}{LC} \Rightarrow \omega = \sqrt{\frac{1 + \frac{L}{L'}}{LC}}$$

$$\omega = \sqrt{\frac{1 + \frac{10^{-1}}{2 \cdot 10^{-3}}}{2 \cdot 10^{-3} \cdot 10^{-6}}} = \sqrt{\frac{1 + \frac{1}{2} \cdot 10^2}{2 \cdot 10^{-9}}} = \sqrt{\frac{2 + 10^2}{2}} \cdot \sqrt{\frac{10^{-2}}{2 \cdot 10^{-9}}} = \sqrt{\frac{2 \cdot 10^{-2}}{0{,}8 \cdot 10^{-9}}}$$

Krug je u rezonanciji $\quad P = U \cdot I$

$$\bar{Z} = jX_L + \frac{R\cdot(-jX_C)}{R-jX_C} = jX_L - \frac{jRX_C}{R-jX_C}\cdot\frac{R+jX_C}{R+jX_C} = jX_L - \frac{jR^2X_C - RX_C^2}{R^2+X_C^2}$$

$$\bar{Z} = \frac{jR^2X_L + jX_C^2X_L - jR^2X_C + RX_C^2}{R^2+X_C^2} = \frac{RX_C^2 + j\left(R^2X_L + X_C^2X_L - R^2X_C\right)}{R^2+X_C^2} = \frac{RX_C^2}{R^2+X_C^2} + j\frac{R^2X_L + X_C^2\cdot X_L - R^2\cdot X_C}{R^2+X_C^2}$$

Pošto je krug u rezonanciji $Im(\bar{Z}) = 0 \quad R^2X_L + X_C^2\cdot X_L - R^2X_C = 0$

$$\boxed{\bar{Z} = \frac{R\cdot X_C^2}{R^2+X_C^2}} \qquad \bar{I} = \frac{\bar{U}}{\bar{Z}} = \frac{40\cdot(R^2+X_C^2)}{R\cdot X_C^2} \qquad \bar{U}_L = \bar{I}\cdot j\bar{X}_L = 30V$$

$$\frac{40(R^2+X_C^2)}{R\cdot X_C^2}\cdot X_L = 30$$

$$|\bar{U}_R| = \left|\bar{I}\cdot\frac{R\cdot X_C^2 - jR^2\cdot X_C}{R^2+X_C^2}\right| = 50$$

$$\left|\frac{40(R^2+X_C^2)}{R\cdot X_C^2}\cdot\frac{R\cdot X_C(X_C-jR)}{R^2+X_C^2}\right| = 50 \qquad \left|\frac{40}{X_C}(X_C - jR)\right| = 50$$

$$\left|40 - j\,40\frac{R}{X_C}\right| = 50$$

$$\left|40\left(1 - j\frac{R}{X_C}\right)\right| = 50 \qquad \left|\left(1 - j\frac{R}{X_C}\right)\right| = 1{,}25 \qquad 1 + \left[\frac{R}{X_C}\right]^2 = 1{,}5625 \qquad \boxed{\frac{R}{X_C} = 0{,}75}$$

$$P_{pri\ rezonanciji} = \bar{U}\cdot\bar{I}^* = \bar{U}\cdot\left[\frac{\bar{U}}{\bar{Z}}\right]^* = 40\cdot\frac{40(R^2+X_C^2)}{R\cdot X_C^2} = \frac{1600(R^2+X_C^2)}{R\cdot X_C^2} = 200$$

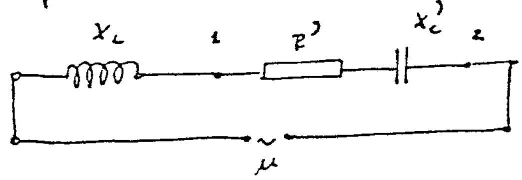
$$\frac{R^2+X_C^2}{R\cdot X_C^2} = 0{,}125 \qquad \frac{X_C^2 + 0{,}75^2\cdot X_C^2}{0{,}75\cdot X_C^3} = 0{,}125 \qquad \frac{X_C^2 + 0{,}75^2\cdot X_C^2\ /:X_C^2}{0{,}75\cdot X_C^3\ /:X_C^2} = 0{,}125$$

$$\frac{1 + 0{,}5625}{0{,}75\cdot X_C} = 0{,}125 \qquad 1{,}5625 = 0{,}09375\,X_C \Rightarrow \boxed{X_C = 16{,}67\,\Omega}$$

$$\boxed{R = 12{,}5\,\Omega}$$

$$\frac{40(R^2+X_C^2)}{R\cdot X_C^2}\cdot X_L = 30 \qquad \frac{40\cdot(156{,}25+277{,}77)}{3472{,}22}\cdot X_L = 30 \qquad 5X_L = 30 \Rightarrow \boxed{X_L = 6\,\Omega}$$

II NAČIN $\quad$ pretvorimo paralelan spoj u serijski (ekvivalentni)



$$R' = \frac{G}{Y^2} = \frac{G}{G^2+B_C^2}$$

$$R' = \frac{\frac{1}{R}}{\frac{1}{R^2}+\frac{1}{X_C^2}} = \frac{1}{R}\cdot\frac{R^2\cdot X_C^2}{R^2+X_C^2}$$

$$\boxed{X_C' = X_C\cdot\frac{R^2}{R^2+X_C^2}} \qquad \boxed{R' = \frac{R\cdot X_C^2}{R^2+X_C^2}}$$

Pri rezonantnoj frekvenciji je otpor u fazi sa naponom $\quad \cdot\ Z = R' = \frac{R\cdot X_C^2}{R^2+X_C^2} \qquad \bar{I} = \frac{\bar{U}}{\bar{Z}} = \frac{U\cdot(R^2+X_C^2)}{R\cdot X_C^2} = \frac{40(R^2+X_C^2)}{R\cdot X_C^2}$

$$\bar{U}_L = \bar{I}\cdot jX_L = j\,\frac{40(R^2+X_C^2)}{R\cdot X_C^2}\cdot X_L \qquad \frac{40(R^2+X_C^2)}{R\cdot X_C^2}\cdot X_L = 30$$

$$|\bar{U}_L| = 30\ V$$

Analogno dalje kao kod prethodnog primjera, itd.

$\omega^2 LC = L + \frac{L^2}{L} \Rightarrow \omega^2 = \frac{1+\frac{L}{L}}{LC} \Rightarrow \omega^2 = \frac{L+L^2}{L^2C \cdot L} \Rightarrow \omega_2 = \sqrt{\frac{L+L}{L^2 \cdot L \cdot C}} = 5{,}48 \cdot 10^4\ s^{-1}$

Struja = ?

$\exists\ \omega_2$ BC je u rezonanciji $\Rightarrow I_L = I_C \quad I = 0 \quad Z_{uk} = \infty$

potpuno $I = 0 \quad U_{BC} = U = 50V \Rightarrow I_L = I_C = \frac{U}{X_L} = \frac{50}{\omega \cdot L} = \frac{50}{5 \cdot 10^4 \cdot 0{,}1 \cdot 10^{-3}} = \frac{50}{5 \cdot 10^4 \cdot 4 \cdot 10^{-4}}$

$I_L = I_C = \frac{50}{20} = \boxed{2{,}5A = I_L = I_C}$

Za $\omega_2 = 5{,}48 \cdot 10^4\ s^{-1} \quad X_{L'} = \omega_2 \cdot L' = 5{,}48 \cdot 10^4 \cdot 2 \cdot 10^{-3} = 10{,}98 \cdot 10 = 109{,}6\ \Omega \quad X_{L'} = 109{,}6\ \Omega$

$X_L = \omega_2 \cdot L = 5{,}48 \cdot 10^4 \cdot 0{,}4 \cdot 10^{-3} = 2{,}192 \cdot 10 = \boxed{21{,}92\ \Omega = X_L} \quad X_C = \frac{1}{\omega_2 C} = \frac{1}{5{,}48 \cdot 10^4 \cdot 10^{-6}}$

$X_C = 18{,}24\ \Omega$

$\bar{Z}_{uk} = R + jX_{L'} - \frac{j^2 X_C \cdot X_L}{j(X_L - X_C)} = R + jX_{L'} + \frac{X_C \cdot X_L}{j(X_L - X_C)} = 25 + j109{,}6 + \frac{18{,}24 \cdot 21{,}92}{j\,3{,}68}$

$\bar{Z}_{uk} = 25 + j109{,}6 + \frac{400}{j3{,}68} = 25 + j109{,}6 - \frac{j400 \cdot 3{,}68}{13{,}54} = 25 + j109{,}6 - j108{,}7113$

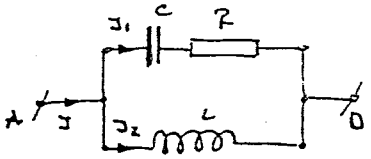
$\boxed{\bar{Z}_{uk} = 25 + j0{,}8885\ \Omega} \quad \boxed{\bar{Z}_{uk} = 25\Omega}$ pri prir. frekv. ostaje samo otpor $\boxed{R = 25\Omega}$

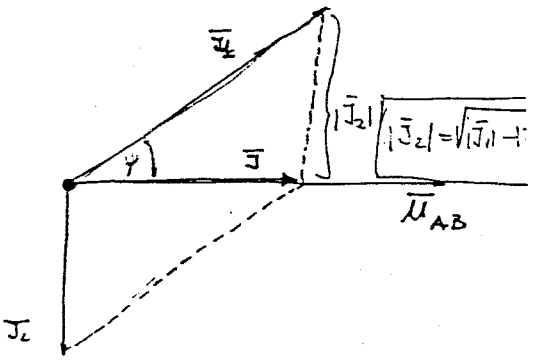
$I = \frac{U}{Z} = \frac{50}{25} = 2A \quad \boxed{I_{(\omega_2)} = 2A}$

3.24

④ Krug na slici je u rezonanciji $I_1 = 7A$ i $I = 3{,}6A$.

Odredite I_2





$J_1 \cdot \cos\psi = J \quad /^2$

$J_1 \cdot \sin\psi = J_2 \quad /^2$ $\Big\}+$

$\boxed{\psi = \text{arc tg}\ \frac{X_C}{R}}$

$J_1^2 \cdot \cos^2\psi = J^2$

$J_1^2 \cdot \sin^2\psi = J_2^2$ $\Big\}+$

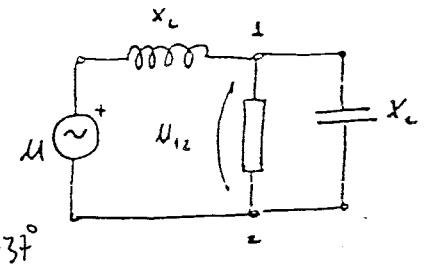
$\boxed{J_1^2 = J^2 + J_2^2} \Rightarrow J_2^2 = J_1^2 - J^2 = 49 - 12{,}96$

$J_2^2 = 36{,}04 \Rightarrow \boxed{J_2 = 6{,}0033\ A}$

3.25

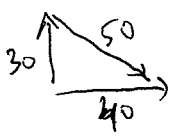
⑤ Krug je u rezonanciji $U = 40V \quad U_L = 30V$ i $U_{12} = 50V$. Snaga koju mjerimo pomoću

je $P = 200W$. Odredi $R, X_L, X_C = ?$



$\cos\varphi = 0$

$I = \frac{P}{U} = \frac{200}{40} = 5A$



$\frac{U_L}{I} = X_L = 6\Omega$

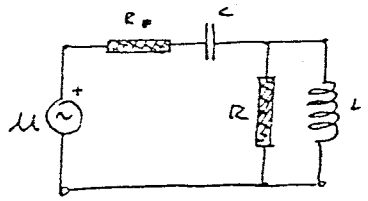
$\underline{Z}_{12} = 10\Omega \angle -37°$

$\underline{Y} = 0{,}08 + 0{,}06j \Rightarrow R = 12{,}5 \quad | \quad X_C = 16{,}667$

3.32.

⑤ Zadan je spoj ma kao. Poznati su R_0, R, C, i f napona. $X_L = ?$ uz koji će krug biti u rezonanciji. Izračunajte pri kojem odnosu $R_0 : X_C$ u promatranom krugu neće nastati rezonancija pri kojem X_L

$R_0, R, C, f \Rightarrow$ poznati $X_L = ?$ u rezonanciji.



67

$$\underline{Z}_u = R_0 - jX_C + \frac{R \cdot jX_L}{R + jX_L} \cdot \frac{R - jX_L}{R - jX_L}$$

$$\underline{Z}_u = R_0 - jX_C + \frac{R^2 \cdot jX_L - j \cdot R \cdot X_L^2}{R^2 + X_L^2} = R_0 + jX_C + \frac{jR^2 \cdot X_L + R \cdot X_L^2}{R^2 + X_L^2} = \frac{R_0 R^2 + R_0 X_L^2 + jX_C(R^2 + X_L^2) + jR^2 X_L + R X_L^2}{R^2 + X_L^2}$$

$$\underline{Z}_u = \frac{R_0 \cdot R^2 + R_0 \cdot X_L^2 + R X_L^2}{R^2 + X_L^2} + j \frac{-X_C(R^2 + X_L^2) + R^2 \cdot X_L}{R^2 + X_L^2} \qquad \mathrm{Im}(\underline{Z}) = 0$$

$$-X_C(R^2 + X_L^2) + R^2 \cdot X_L = 0$$

$$R^2 \cdot X_L = +X_C(X_L^2 + R^2)$$

$$X_L = \frac{+X_C(X_L^2 + R^2)}{R^2} \Rightarrow R^2 \cdot X_L = +X_C \cdot X_L^2 + X_C \cdot R^2 \qquad +X_C \cdot X_L^2 - R^2 \cdot X_L + X_C \cdot R^2 = 0$$

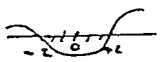
$$X_{L(1,2)} = \frac{R^2 \pm \sqrt{R^4 + 4X_C^2 \cdot R^2}}{2X_C} = \frac{R^2 \pm R\sqrt{R^2 + 4X_C^2}}{2X_C}$$

$$X_{L_{1,2}} = \frac{R^2}{2X_C} \pm \sqrt{\frac{R^4}{4X_C^2} + R^2}$$

$$X_L = \frac{R^2 \pm \sqrt{R^4 - 4X_C^2 \cdot R^2}}{2X_C} \qquad R^4 - 4X_C^2 \cdot R^2 < 0$$

$$R^4 < 4X_C^2 \cdot R^2$$

$$\left[\frac{R}{X_C}\right]^2 - 4 < 0 \qquad \left(\frac{R}{X_C} - 2\right)\left(\frac{R}{X_C} + 2\right) < 0 \qquad R^2 < 4X_C^2 \qquad \left(\frac{R}{X_C}\right)^2 < 4$$

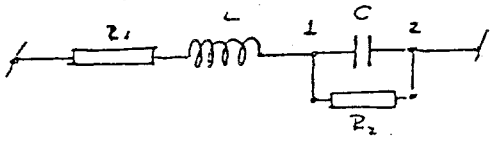


$$\frac{R}{X_C} \in [-2, +2]$$

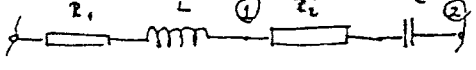
⑥ 3.41

Izračunaj rezonantnu frekvenciju kruga nacrtanog prema slici.

$L = 20\,mH \quad C = 0{,}4\,\mu F \quad R_2 = 500\,\Omega$



pretvorimo paralelni spoj u serijski.



$$R_2' = R_2 \cdot \frac{X_C^2}{R_2^2 + X_C^2} \qquad X_C' = X_C \cdot \frac{R_2^2}{R_2^2 + X_C^2}$$

krug će biti u rezonanciji ako je $X_L = X_C' \Rightarrow X_L = X_C \cdot \frac{R_2^2}{R_2^2 + X_C^2}$

$$\omega L = \frac{1}{\omega C} \cdot \frac{R_2^2}{R_2^2 + \frac{1}{(\omega C)^2}} = \frac{1}{\omega C} \cdot \frac{R_2^2 \cdot (\omega C)^2}{R_2^2 \cdot \omega^2 C^2 + 1} \Rightarrow \omega L$$

$$\frac{R_2^2 \cdot C}{R_2^2 \omega^2 C^2 + 1} = L \qquad R_2^2 \cdot C = L R_2^2 \cdot \omega^2 C^2 + L \qquad \omega^2 C^2 R_2^2 L = R_2^2 C - L \qquad \omega^2 = \frac{R_2^2 C - L}{C^2 R_2^2 \cdot L}$$

$$\omega^2 = \frac{R_2^2 \cdot C}{C^2 R_2^2 \cdot L} - \frac{1}{C^2 \cdot R_2^2} = \frac{1}{CL} - \frac{1}{C^2 R_2^2} \Rightarrow \omega = \sqrt{\frac{1}{LC} - \frac{L}{R_2^2 \cdot C^2}}$$

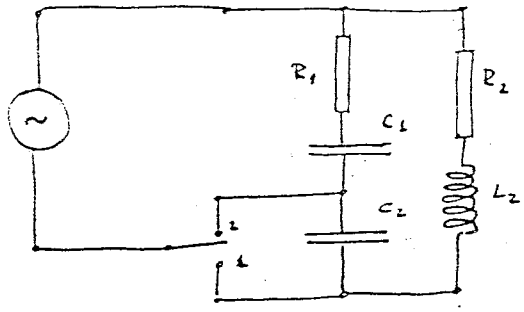
$$\omega = \sqrt{\frac{1}{20 \cdot 10^{-3} \cdot 0{,}4 \cdot 10^{-6}} - \frac{L}{500^2 (0{,}4 \cdot 10^{-6})^2}} = \sqrt{\frac{10^9}{8} - \frac{10^{12}}{40000}} = \sqrt{\frac{10^9}{8} - \frac{10^{12}}{40 \cdot 10^3}} = \sqrt{\frac{10}{8} \cdot 10^8 - \frac{1}{4} \cdot 10^8}$$

$$= 10^4\,s^{-1}$$

$$\omega_{rez} = 10000\,s^{-1}$$

3.54

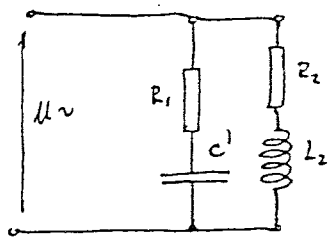
(7.) U krugu na slici odrediti rezonantnu frekvenciju te impedanciju spoja, ukupnu struju i snagu pri rezonantnoj frekvenciji za dva položaja prekidača. Zadano je $L=1\,mH$, $R_1=R_2=20\,\Omega$, $U=4000\,V$ i $C_1=C_2=0{,}2\,\mu F$



$L=1\,mH \quad R_1=R_2=20\,\Omega$

$U=4000\,V \quad C_1=C_2=0{,}2\,\mu F$

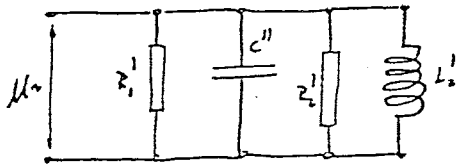
za položaj (1)



$$C' = \frac{C_1 \cdot C_2}{C_1 + C_2} = \frac{\left[0{,}2\cdot 10^{-6}\right]^2}{2\cdot 0{,}2\cdot 10^{-6}} = \frac{0{,}04\cdot 10^{-12}}{0{,}4\cdot 10^{-6}} = 0{,}1\cdot 10^{-6}\,F$$

$$\boxed{C' = 0{,}1\,\mu F}$$

Pretvorimo prvo 1 i 2 granu u paralelni spoj



$$G_1' = \frac{R_1}{R_1^2 + X_{C'}^2} \qquad B_{C'} = \frac{X_{C'}}{R_1^2 + X_{C'}^2}$$

$$G_1' = \frac{20}{400 + X_{C'}^2} \qquad B_{C''} = \frac{X_{C'}}{400 + X_{C'}^2} \qquad G_2' = \frac{R_2}{R_2^2 + X_{L_2}^2} \qquad B_{L_2}' = \frac{X_{L_2}}{R_2^2 + X_{L_2}^2}$$

$$G_2' = \frac{20}{400 + X_{L_2}^2} \qquad B_{L_2}' = \frac{X_{L_2}}{400 + X_{L_2}^2}$$

$$Y^2 = \left[G_1' + G_2'\right]^2 + \left[\frac{X_{L_2}}{R_2^2 + X_{L_2}^2} - \frac{X_{C'}}{R_1^2 + X_{C'}^2}\right]^2 \qquad (B_{L_2}' \text{ nad prvim razlomkom}, \; B_{C'} \text{ nad drugim})$$

$$Y^2 = \left[G_1' + G_2'\right]^2 + \left[B_{L_2}' - B_{C'}\right]^2$$

Rezonancija nastupa kada je $B_{L_2}' = B_{C'}$

$$\frac{X_{L_2}}{R_2^2 + X_{L_2}^2} = \frac{X_{C'}}{R_1^2 + X_{C'}^2}$$

$$\frac{\omega L_2}{400 + \omega^2 L_2^2} = \frac{\frac{1}{\omega C'}}{400 + \left[\frac{1}{\omega C'}\right]^2} \qquad \frac{\omega L_2}{400 - \omega^2 L_2^2} = \frac{\frac{1}{\omega C'}}{\frac{400(\omega C')^2 + 1}{(\omega C')^2}}$$

$$\frac{\omega L_2}{400 + \omega^2 L_2^2} = \frac{\omega C'}{400(\omega C')^2 + 1} \qquad L_2\left(400 + \omega^2 C'^2 + 1\right) = C'\left(400 + \omega^2 L_2^2\right)$$

$$400 L_2 \cdot C'^2 \cdot \omega^2 + L_2 = 400 C' + \omega^2 L_2^2 \cdot C'$$

$$\omega^2\left[400 L_2 \cdot C'^2 - L_2^2 \cdot C'\right] = 400 C' - L_2 \qquad \omega^2 = \frac{400 C' - L_2}{L_2 C'\left[400 \cdot C' - L_2\right]}$$

$$\omega^2 = \frac{400}{L_2\left[400 \cdot C' - L_2\right]} - \frac{1}{C'\left[400 C' - L_2\right]}$$

$$\omega^2 = \frac{400}{10^{-3}\left[400 \cdot 0{,}1\cdot 10^{-6} - 10^{-2}\right]} - \frac{1}{0{,}1\cdot 10^{-6}\left[400 \cdot 0{,}1\cdot 10^{-6} - 10^{-3}\right]} = \frac{400\,000}{40\cdot 10^{-6} - 10^{-3}} - \frac{10^7}{40\cdot 10^{-6} - 10^{-3}}$$

$$\omega^2 = \frac{4\cdot 10^5 - 10^7}{40\cdot 10^{-6} - 10^{-3}} = \frac{10^5\left[4 - 10^2\right]}{10^{-6}\left[40 - 10^3\right]} = \frac{10^{11}\left[4 - 100\right]}{40 - 1000} = \frac{10^{11}\cdot\left[4 - 100\right]}{10\left[4 - 100\right]} = 10^{10}$$

$$\boxed{\omega_r = 10^5\ s^{-1}} \text{ za položaj (1)}$$

p.: $\omega_0 = 10^5\ s^{-1} \Rightarrow Z = R_u = \frac{1}{Y_u} = \frac{1}{G_1' + G_2'} = \frac{1}{\frac{20}{400+X_{C1}^2} + \frac{20}{400+X_{L2}^2}}$

$X_{C1} = \frac{1}{\omega_0 C_1} = \frac{1}{10^5 \cdot 10^{-7}}$

$X_{C1} = 100$

$Z = \frac{1}{\frac{20}{400+10000} + \frac{20}{400+10000}} = \frac{1}{\frac{40}{400+10000}}$

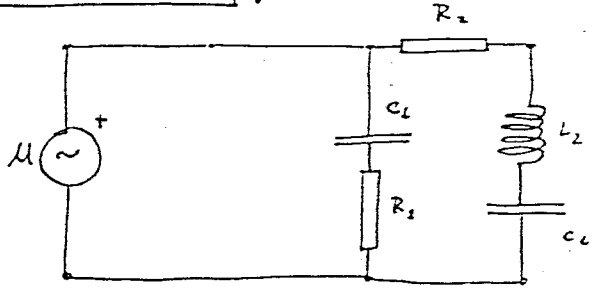
$X_{L2} = \omega_0 \cdot L_2$

$X_{L2} = 10^5 \cdot 10^{-3} = 10^2\ \Omega$

$Z = \frac{400+10000}{40} = 260\ \Omega$ $\quad I = \frac{U}{R_u} = \frac{4000}{260} = 15{,}4\ A \Rightarrow \boxed{I = 15{,}4\ A}$

$\boxed{P = U \cdot I = 61{,}6\ kW}$

Ak je sklopka u drugom položaju. $\Rightarrow$



$$Z_u = \frac{(20 - jX_{C1})(20 + jX_{L2} - jX_{C2})}{(20 - jX_{C1} + 20 + jX_{L2} - jX_{C2})}$$

$$Z_u = \frac{(20 - jX_{C1})[20 + j(X_{L2} - X_{C2})]}{40 + j[X_{L2} - X_{C1} - X_{C2}]}$$

$$Z_u = \frac{400 + j20(X_{L2} - X_{C2}) - j20X_{C1} + X_{C1}(X_{L2} - X_{C2})}{40 + j[X_{L2} - X_{C1} - X_{C2}]} = \frac{400 + X_{C1}(X_{L2} - X_{C2}) + j[20(X_{L2} - X_{C2}) - 20X_{C}}{40 + j[X_{L2} - X_{C1} - X_{C2}]}$$

$$Z_u = \frac{[400 + X_{C1}(X_{L2} - X_{C2})] + j20[(X_{L2} - X_{C2}) - X_{C1}]}{40 + j[X_{L2} - X_{C1} - X_{C2}]} \cdot \frac{40 - j[X_{L2} - X_{C1} - X_{C2}]}{40 - j[X_{L2} - X_{C1} - X_{C2}]}$$

$$Z_u = \frac{40[400 + X_{C1}(X_{L2} - X_{C2})] - j[400 + X_{C1}(X_{L2} - X_{C2})][X_{L2} - X_{C1} - X_{C2}] + j800[X_{L2} - X_{C2} - X_{C1}] + 20[X_{L2} - X_{C2} - X_{C1}]^2}{1600 + [X_{L2} - X_{C1} - X_{C2}]^2}$$

$$Z_u = \frac{40[400 + X_{C1}(X_{L2} - X_{C2})] + 20[X_{L2} - X_{C2} - X_{C1}]^2}{1600 + [X_{L2} - X_{C1} - X_{C2}]^2} + j\frac{800[X_{L2} - X_{C2} - X_{C1}] - [400 + X_{C1}(X_{L2} - X_{C2})] \cdot [X_{L2} - X_{C1}}{1600 + [X_{L2} - X_{C1} - X_{C2}]^2}$$

brunji se rezonancija ako je $\mathrm{Im}(\underline{Z}_u) = 0$

$$800[X_{L2} - X_{C2} - X_{C1}] - [400 + X_{C1}(X_{L2} - X_{C2})][X_{L2} - X_{C1} - X_{C2}] = 0$$

$$[X_{L2} - X_{C2} - X_{C1}]\{800 - 400 - X_{C1}(X_{L2} - X_{C2})\} = 0$$

$$[X_{L2} - X_{C2} - X_{C1}]\{400 - X_{C1}(X_{L2} - X_{C2})\} = 0 \qquad X_{L2} - X_{C2} - X_{C1} = 0$$

$$\omega L_2 = \frac{1}{\omega C_2} + \frac{1}{\omega C_1} \qquad C_1 = C_2$$

$$\omega L_2 = \frac{2}{\omega C_1} \Rightarrow \omega^2 = \frac{2}{C_1 \cdot L_2} = \frac{2}{0{,}2 \cdot 10^{-6} \cdot 10^{-2}}$$

$$\omega^2 = \frac{10}{10^{-9}} = 10^{10} \Rightarrow \omega = 10^5\ s^{-1}$$

$$400 - X_{C1}(X_{L2} - X_{C2}) = 0$$

$$400 = \frac{1}{\omega C_1}\left[\omega \cdot L_2 - \frac{1}{\omega C_2}\right] \qquad 400 = \frac{L_2}{C_1} - \frac{1}{\omega^2 C_1 C_2}$$

$$\frac{1}{\omega^2 C_1 C_2} = \frac{L_2}{C_1} - 400$$

$$\frac{1}{\omega^2} = \left(\frac{L_2}{C_1} - 400\right) \cdot C_1 \cdot C_2$$

$$\frac{1}{\omega^2} = L_2 \cdot C_2 - 400\, C_1 \cdot C_2$$

$$\omega^2 = \frac{1}{L_2^2 \cdot c - 400 c_1 c_2} = \frac{1}{10^{-3} \cdot 0{,}2 \cdot 10^{-6} - 400 \cdot (0{,}2 \cdot 10^{-6})^2} = \frac{1}{0{,}2 \cdot 10^{-9} - 400 \cdot 0{,}04 \cdot 10^{-12}}$$

$$\omega^2 = \frac{1}{0{,}2 \cdot 10^{-9} - 16 \cdot 10^{-12}} = \frac{1}{2 \cdot 10^{-10} - 16 \cdot 10^{-12}} = \frac{1}{2 \cdot 10^{-12}\left[10^2 - 8\right]}$$

$$\omega^2 = \frac{10^{12}}{2 \cdot 92} = \frac{10^{12}}{184} \Rightarrow \frac{10^6}{13{,}56} = 0{,}07372 \cdot 10^6 = 7{,}372 \cdot 10^4 = 73\,720\ s^{-1}$$

$\omega_{rez} = 73720\ s^{-1}$??

$\omega_{rez} = 71638{,}5\ s^{-1}$

$X_{L_2} = \omega_r \cdot L = 7{,}37 \cdot 10^4 \cdot 10^{-3} = 7{,}37 \cdot 10 = 73{,}7\ \Omega$

$X_{C_1} = \frac{1}{\omega_0 C} = \frac{1}{7{,}37 \cdot 10^4 \cdot 0{,}2 \cdot 10^{-6}} = \frac{100}{1{,}474} = 67{,}84\ \Omega$

$$Z_{rez} = R_u = \frac{40\left[400 + X_{C_1}(X_{L_2} - X_{C_2})\right] + 20\left[X_{L_2} - X_{C_2} - X_{C_1}\right]^2}{1600 + \left[X_{L_2} - X_{C_1} - X_{C_2}\right]^2}$$

$$Z_{rez} = R_u = \frac{40(400 + 397{,}51) + 20 \cdot 3841{,}52}{1600 + 3841{,}52} = \frac{76830{,}4 + 31301{,}6}{5441{,}52}$$

$Z_{rez} = R_u = 20\ \Omega$ $\quad I = \frac{400\phi}{2\phi} = 200\ A$ mogućnost regulacije

$Z_{rez} = 72{,}5\ \Omega$

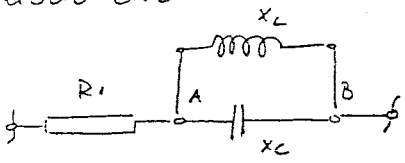
$I = \frac{4000}{72{,}5} = 55{,}2\ A$

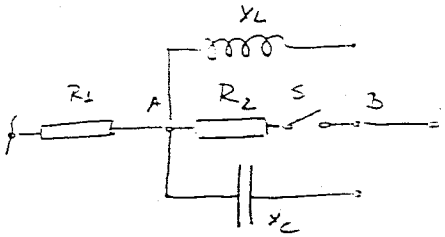
$P = U \cdot I = 4000 \cdot 55{,}2 = 220{,}689\ kW$

3.47

8. Na izvor je priključen napon od 120 V. Odredi struju u granama i napon pri otvorenom i zatvorenom prekidaču P. Zadano je $R_1 = R_2 = X_L = X_C = 25\,\Omega$

Kad je prekidač otvoren



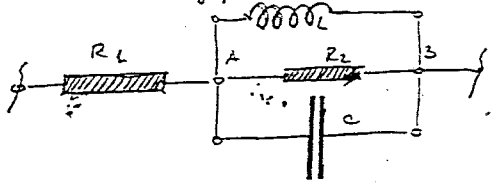


$$\bar{Z} = R_1 + \frac{jX_L \cdot (-jX_C)}{jX_L + (-jX_C)} = R_1 + \frac{X_L \cdot X_C}{j(X_L - X_C)} = R_1 - j\frac{X_L \cdot X_C}{X_L - X_C} \qquad \bar{Z}_u = 25 - j\frac{25 \cdot 25}{j^2 - 25} = 25 - j\infty$$

$\Rightarrow \bar{Z} = \infty \rightarrow \bar{J} = 0\,A$ Spoj je u rezonanciji. $\Rightarrow U_{AB} = 120\,V$ $\Rightarrow I_L = I_C = \frac{120}{25} = 4{,}8\,A$

Vidimo da energija oscilira između X_C i X_L tj. (C i L) nema gubitka energije pa izvor ne unosi nikakvu energiju $\Rightarrow J = 0$ $\quad I_L = I_C = 4{,}8\,A$

Pri zatvorenom prekidaču $\Rightarrow$



$\bar{Z}_u = ?$ $\qquad \bar{Z}_u = R_1 + \bar{Z}_{AB}$

$$\frac{1}{\bar{Z}_{AB}} = \frac{1}{jX_L} + \frac{1}{R_2} + \frac{1}{-jX_C} = \frac{1}{j25} + \frac{1}{25} - \frac{1}{j25} = -j\frac{1}{25} + \frac{1}{25} + j\frac{1}{25} = \frac{1}{25} \Rightarrow \boxed{Z_{AB} = 25\,\Omega}$$

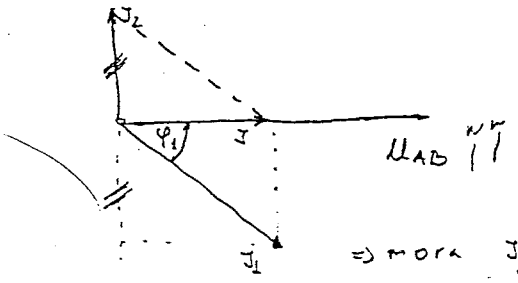
$$\bar{Z}_u = \bar{R}_1 + \bar{Z}_{AB} = 25 + 25 = 50\,\Omega \Rightarrow \bar{J}_u = \frac{120}{50} = 2{,}4\,A$$

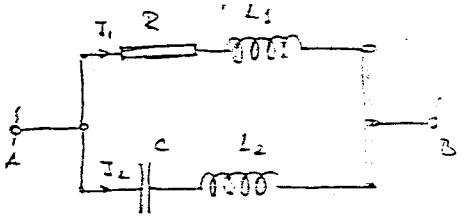
$$\bar{U}_{AB} = \bar{J} \cdot \bar{Z}_{AB} = 2{,}4 \cdot 25 = 120\,V \Rightarrow J_L = \frac{120}{25} = J_C = J_{eL} = 2{,}4\,A$$

$$\boxed{\bar{J}_L = \bar{J}_C - \bar{J}_{L_2} = 2{,}4\,A}$$

3.49

9. Izračunajte induktivitet svitka L_2 ako u kojoj struja rezonancija nastupa pri frekvenciji $\omega = 2000\,s^{-1}$. $R = 20\,\Omega$ $L_1 = 20\,mH$ i $C = 6{,}25\,\mu F$. Otpor (aktivni) svitka L_2 možemo zanemariti





(C)

$\Rightarrow$ mora J_2 prednjačiti naponu za $\frac{\pi}{2}$ $\Rightarrow$ moramo koristiti kapacitivno opterećenje.

$$J_1 \cdot \sin\varphi_1 = I_2 \Rightarrow \frac{U_{AB}}{\sqrt{R^2 + X_{L_1}^2}} \cdot \frac{X_{L_1}}{\sqrt{R^2 + X_{L_1}^2}} = \frac{U_{AB}}{\sqrt{0^2 + (X_C - X_{L_2})^2}}$$

$$\frac{X_{L_1}}{R^2 + X_{L_1}^2} = \frac{1}{X_C - X_{L_2}} \qquad X_C - X_{L_2} = \frac{R^2 + X_{L_1}^2}{X_{L_1}} \Rightarrow X_{L_2} = X_C - \frac{R^2 + X_{L_1}^2}{X_{L_1}}$$

$$X_{L_1} = \frac{1}{\omega C} - \frac{R^2 + (\omega L_1)^2}{\omega L_1} = \frac{1}{2 \cdot 10^3 \cdot 6{,}25 \cdot 10^{-6}} - \frac{400 + (2 \cdot 10^3 \cdot 20 \cdot 10^{-3})^2}{2 \cdot 10^3 \cdot 20 \cdot 10^{-3}} = \frac{10^3}{12{,}5} - \frac{400 + 1600}{40}$$

$$X_{L_2} = 80 - 50 = 30\,\Omega \Rightarrow L_2 = \frac{X_{L_2}}{\omega} = \frac{30}{2\pi f} = \frac{30}{2000} = 15\,mH \Rightarrow \boxed{L_2 = 15\,mH}$$

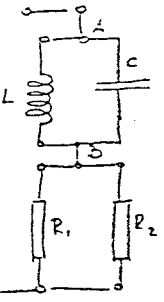
3.50.

(10) Sadrži pokazivanje ampermetra u krugu pri rezonantnoj frekvenciji

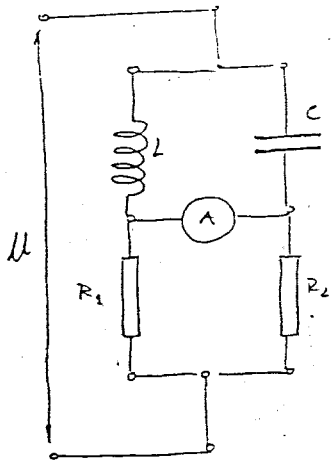
$U = 240\,V$, uz $L = 40\,mH$, $C = 1\,\mu F$. Gubici energije u svitku i kondenzatora i otpor ampermetra su zanemarivo mali.

R_1 i R_2 nisu zadani

pri rezonantnoj frekvenciji



$\Rightarrow$ općenito



Rez. frekvencija je ona kod koje je $X_L = X_C$ U_{AB} je isto na elemental $\Rightarrow$ struje se poništavaju

pa je ukupna struja $\bar{I}_u = 0\,A$ $\Rightarrow$ rezonancija $\omega_{rez} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{40\cdot10^{-3}\cdot1\cdot10^{-6}}}$

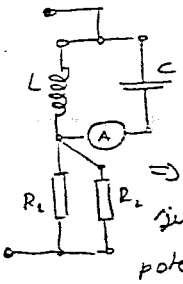
$\omega_{rez} = \frac{1}{\sqrt{40\cdot10^{-9}}} = \frac{1}{\sqrt{4\cdot10^{-8}}} = \frac{1}{2\cdot10^{-4}} = \frac{10^4}{2} = 5000\,s^{-1}$ $\boxed{\omega_{rez} = 5000\,s^{-1}}$

Pri rezonantnoj frekvenciji je $I_u = 0$ $\Rightarrow U_{AB} = 240V = U$ $\Rightarrow I_L = I_C = \frac{240}{\omega\cdot L} = \frac{240}{5\cdot10^3\cdot40\cdot10^{-3}}$

$I_L = I_C = \frac{240}{200} = 1{,}2\,A$ $\boxed{I_L = I_C = 1{,}2\,A}$

$\boxed{I_A = 1{,}2\,A}$

sliku mijenja shematski krug



$\Rightarrow I_A = I_C = 1{,}2A$

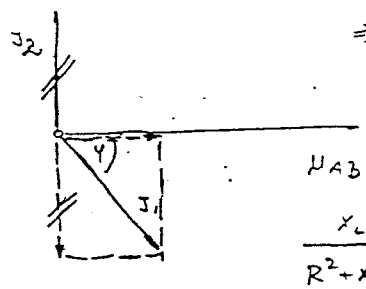
$\Rightarrow$ promjena uredaja jer sjedne točke istog potencijala ($R_A = 0$)

(11) Krug na slici je u rezonanciji. Dobrota svitka

3.51 pri rezonantnoj frekvenciji bilo je $Q = 4$. Nakon

povišenja frekvencije (rezonantne) frekvencije poremećaj za puta. Kako treba izmjeniti kapacitet kondenzatora da bi krug bio ponovo u rezonanciji pri poremećenoj frekvenciji

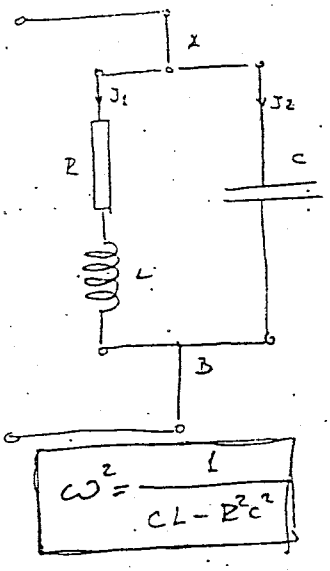
$Q = \frac{I_{L0}}{I_0} = 4$ (0) $\rightarrow$ pri rezonantnoj frekvenciji



$\Rightarrow I_1 \cdot \sin\varphi = I_L$

$\frac{U_{AB}}{\sqrt{R^2 + X_L^2}} \cdot \frac{X_L}{\sqrt{R^2+X_L^2}} = \frac{U_{AB}}{X_C}$

$\frac{X_L}{R^2+X_L^2} = \frac{1}{X_C}$ $\boxed{X_L \cdot X_C = R^2 + X_L^2}$



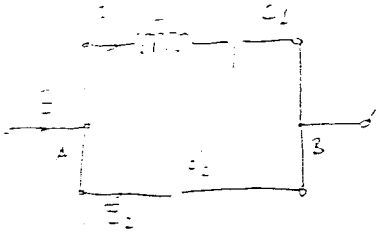
$\omega L \cdot \frac{1}{\omega C} = R^2 + \left[\frac{1}{\omega C}\right]^2$ $\frac{L}{C} = R^2 + \frac{1}{\omega^2 C^2}$

$\frac{1}{\omega^2 C^2} = \frac{L}{C} - R^2$ $\Rightarrow \omega^2 = \frac{1}{C^2\left(\frac{L}{C} - R^2\right)} = \frac{1}{C^2\frac{L - R^2C}{C}}$

$\boxed{\omega^2 = \frac{1}{CL - R^2C^2}}$

Zadatak: Odredi rezonantnu frekvenciju u krugu gdje je $L = 0{,}01\,H$, $C_1 = 0{,}25\,\mu F$ i $C_2 = 0{,}2\,\mu F$

3.57



$$\bar{Z}_1 = j X_L + (-j X_{C_1}) = j X_L - j X_{C_1} = j(X_L - X_{C_1})$$

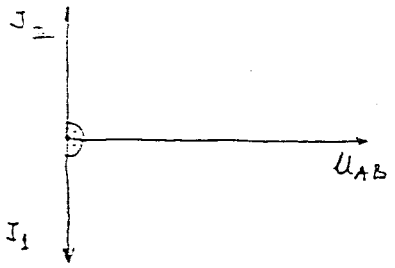
$$\bar{Z}_u = \frac{j(X_L - X_{C_1}) \cdot (-j X_{C_2})}{j(X_L - X_{C_1}) - j X_{C_2}} = \frac{(X_L - X_{C_1}) \cdot X_{C_2}}{j(X_L - X_{C_1} - X_{C_2})} \cdot \frac{j}{j}$$

$$\bar{Z}_u = -j \frac{(X_L - X_{C_1}) \cdot X_{C_2}}{X_L - X_{C_1} - X_{C_2}}$$

Da bi opterećenje bilo u rezonanciji, mora biti $\mathrm{Im}(\bar{Z}) = 0$

$$\frac{(X_L - X_{C_1}) \cdot X_{C_2}}{X_L - X_{C_1} - X_{C_2}} = 0 \qquad \frac{\left(\omega L - \frac{1}{\omega C_1}\right) \cdot \frac{1}{\omega C_2}}{\omega L - \frac{1}{\omega C_1} - \frac{1}{\omega C_2}} = 0$$

$$\omega L = \frac{1}{\omega C_1} \qquad \omega^2 = \frac{1}{L C_1} = \frac{1}{10^{-2} \cdot 0{,}25 \cdot 10^{-6}} = \frac{10^8}{0{,}25} = 4 \cdot 10^8 \qquad \omega = 2 \cdot 10^4 \,\frac{1}{s}$$



$$\omega L = \frac{1}{\omega}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) / \cdot \omega$$

$$\omega^2 L = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\omega^2 \cdot L = \frac{C_1 + C_2}{C_1 \cdot C_2} \qquad \omega^2 = \frac{1}{L} \frac{C_1 + C_2}{C_1 \cdot C_2}$$

$$\omega^2 = \frac{1}{10^{-2}} \cdot \frac{(0{,}25 + 0{,}2) \cdot 10^{-6}}{0{,}25 \cdot 0{,}2 \cdot 10^{-12}} = \frac{1}{10^{-2}} \cdot \frac{0{,}45 \cdot 10^{-6}}{0{,}05 \cdot 10^{-12}}$$

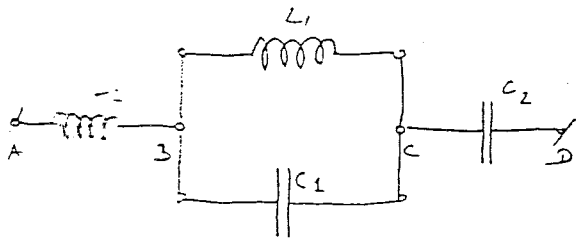
$$\omega^2 = 10^2 \cdot 9 \cdot 10^6 = 9 \cdot 10^8 \Rightarrow \boxed{\omega = 3 \cdot 10^4 \,\frac{1}{s}}$$

$\omega = 2 \cdot 10^4 \,\frac{1}{s}$ $\quad \bar{I}_1 = \infty \Rightarrow$ naponska rezonancija

$\omega = 3 \cdot 10^4 \,\frac{1}{s}$ $\quad I = 0 \Rightarrow$ strujna rezonancija

Zadatak: Odredite frekvenciju prilikom koje nastaje rezonancija

3.58 Zadano je: krug sa $L_1 = L_2 = 0{,}04\,H$ i $C_1 = C_2 = 25\,\mu F$



za $\omega L_1 = \frac{1}{\omega C_1}$

$$\omega^2 = \frac{1}{L_1 \cdot C_1} = \frac{1}{4 \cdot 10^{-2} \cdot 25 \cdot 10^{-6}} = \frac{10^8}{100} = 10^6$$

$$\boxed{\omega = 1000 \,\frac{1}{s}}$$

$$\bar{Z} = \frac{j X_{L_1} \cdot (-j X_{C_1})}{j X_{L_1} - j X_{C_1}} - j X_{L_2} - j X_{C_2}$$

$$\bar{Y} = \frac{j X_{L_1} \cdot j X_{C_1}}{j X_{L_1} - j X_{C_1}} - j X_{L_2} - j X_{C_2} = -\frac{j X_{L_1} \cdot X_{C_1}}{X_{L_1} - X_{C_1}} + j(X_{L_2} - X_{C_2}) = j\left[X_{L_2} - X_{C_2} - \frac{X_{L_1} \cdot X_{C_1}}{X_{L_1} - X_{C_1}}\right] = 0$$

$\omega = 1000 \,\frac{1}{s}$ mora biti strujna rezonancija

$$X_{L2} - X_{C2} - \frac{X_{L1} \cdot X_{C1}}{X_{L1} - X_{C1}} = 0 \qquad \frac{X_{L2}(X_{L1} - X_{C1}) - X_{C2}(X_{L1} - X_{C1}) - X_{L1} \cdot X_{C1}}{X_{L1} - X_{C1}} = 0$$

$$X_{L2}(X_{L1} - X_{C1}) - X_{C2}(X_{L1} - X_{C1}) - X_{L1} \cdot X_{C1} = 0$$

$$\omega L_2 \left[\omega L_1 - \frac{1}{\omega C_1}\right] - \frac{1}{\omega C_2}\left[\omega L_1 - \frac{1}{\omega C_1}\right] - \omega L_1 \cdot \frac{1}{\omega C_1} = 0$$

$$\omega L_2 \frac{\omega^2 C_1 L_1 - 1}{\omega C_1} - \frac{1}{\omega C_2} \frac{\omega^2 C_1 \cdot L_1 - 1}{\omega C_1} - \frac{L_1}{C_1} = 0$$

$$\frac{\omega^2 C_1 \cdot L_1 - 1}{\omega C_1}\left(\omega L_2 - \frac{1}{\omega C_2}\right) - \frac{L_1}{C_1} = 0 \quad \ldots$$

$$\frac{\omega^2 C_1 \cdot L_1 - 1}{\omega C_1} \cdot \frac{\omega^2 L_2 \cdot C_2 - 1}{\omega C_2} = \frac{L_1}{C_1} \qquad \left(\frac{\omega^2 \cancel{C_1} \cdot L_1}{\cancel{\omega} \cdot \cancel{C_1}} - \frac{1}{\omega C_1}\right)\left(\frac{\omega^2 L_2 \cdot \cancel{C_2}}{\cancel{\omega} C_2} - \frac{1}{\omega C_2}\right) = \frac{L_1}{C_1}$$

$$\left(\omega L_1 - \frac{1}{\omega C_1}\right)\left(\omega L_2 - \frac{1}{\omega C_1}\right) = \frac{L_1}{C_1}$$

$$\omega^2 L_1 \cdot L_2 - \cancel{\omega} L_1 \cdot \frac{1}{\cancel{\omega} C_1} - \frac{L_2}{C_1} + \frac{1}{\omega^2 C_1 \cdot C_2} = \frac{L_1}{C_1} \qquad \omega^2 L_1 \cdot L_2 + \frac{1}{\omega^2 C_1 \cdot C_2} = \frac{L_1}{C_1} + \frac{L_2}{C_1} + \frac{L_2}{C_2}$$

$$\frac{\omega^4 L_1 L_2 \cdot C_1 \cdot C_2 + 1}{\omega^2 C_1 \cdot C_2} = 2\frac{L_1}{C_1} + \frac{L_2}{C_1} = \frac{2L_1 + L_2}{C_1}$$

$$\omega^2 = t$$

$$\omega^4 L_1 \cdot L_2 \cdot C_1 \cdot C_2 + 1 = \frac{2L_1 + L_2}{C_1} \cdot \omega^2 \cdot C_1 \cdot C_2$$

$$t^2 L_1 \cdot L_2 \cdot C_1 \cdot C_2 + 1 = \frac{2L_1 + L_2}{\cancel{C_1}} \cdot \cancel{C_1} \cdot C_2 \cdot t \qquad t^2 \cdot L_1 \cdot L_2 \cdot C_1 \cdot C_2 - (2L_1 + L_2) \cdot C_2 \cdot t + 1 = 0$$

$$L_1 \cdot L_2 \cdot C_1 \cdot C_2 = (4 \cdot 10^{-2})^2 \cdot (25 \cdot 10^{-6})^2 = 16 \cdot 10^{-4} \cdot 625 \cdot 10^{-12} = 10000 \cdot 10^{-16} = 10^{-12}$$

$$(2L_1 + L_2) \cdot C_2 = (8 \cdot 10^{-2} + 4 \cdot 10^{-2}) \cdot 25 \cdot 10^{-6} = 12 \cdot 10^{-2} \cdot 25 \cdot 10^{-6} = 300 \cdot 10^{-8} = 3 \cdot 10^{-6}$$

$$10^{-12} \cdot t^2 - 3 \cdot 10^{-6} \cdot t + 1 = 0 \qquad t_{1,2} = \frac{3 \cdot 10^{-6} \pm \sqrt{9 \cdot 10^{-12} - 4 \cdot 10^{-12}}}{2 \cdot 10^{-12}}$$

$$t_{1,2} = \frac{3 \cdot 10^{-6} \pm \sqrt{5} \cdot 10^{-6}}{2 \cdot 10^{-12}} \qquad t_1 = 2{,}618 \cdot 10^6$$

$$t_2 = 0{,}3819 \cdot 10^6$$

$$\omega^2 = 2{,}618 \cdot 10^6 \Rightarrow \boxed{\omega = 1618}$$

$$\omega^2 = 0{,}3819 \cdot 10^6 \Rightarrow \omega = 618$$

rezonantne frekvencije

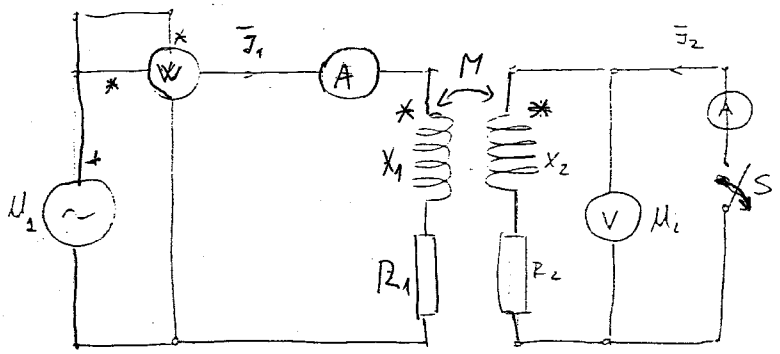
otvojna rezonantna frekvencija $Q = 1000 \frac{1}{s}$

manjša rezonantna frekvencija $\omega_0 = 1618 \frac{1}{s}$

$\omega_0 = 618 \frac{1}{s}$

ZADACI: 4.2.

③ Transformator bez jezgre ima 2 jednaka namota $R_1 = R_2 = R$. U praznom hodu mjerni instrumenti su pokazivali da je $U_1 = 100\,V$, $I_{10} = 4\,A$, $P_{10} = 200\,W$, $U_{20} = 50\,V$. Odredi: R, X, X_M i pokazivanje instrumenata ako su stezaljke transformatora kratko spojene uz napon $U_1 = 100\,V$.



$R_1 = R_2 = R \qquad X_1 = X_2 = X$

Za prazni hod $\quad X_1 = X_2 \Rightarrow L_1 = L_2$

Kirchhoffove jednadžbe

$$\bar{U}_1 = \bar{I}_1 R_1 + \bar{I}_1 j\omega L_1 + \bar{I}_2 j\omega M$$

$$0 = \bar{I}_2 R_2 + \bar{I}_2 j\omega L_2 + \bar{I}_1 j\omega M$$

$$\bar{U}_1 = \bar{I}_1 R_1 + \bar{I}_1 jX + \bar{I}_2 jX_M$$

$$0 = \bar{I}_2 R_2 + \bar{I}_2 jX + \bar{I}_1 jX_M$$

Kod praznog hoda $\Rightarrow P_{10} = U_1 I_{10} \cos\varphi$

$$\Rightarrow \cos\varphi = \frac{P_{10}}{U_1 \cdot I_{10}} = \frac{200}{100 \cdot 4} = 0{,}2 = \cos\varphi \text{ za prazni hod}$$

$$Z = \frac{U_1}{I_{10}} = \frac{100}{4} = 25\,\Omega \qquad \boxed{Z = 25\,\Omega}$$

$$\Rightarrow R = 25 \cdot \cos\varphi = 5\,\Omega$$

$$X = \sqrt{Z^2 - R^2} = \sqrt{25^2 - 5^2} = 24{,}5\,\Omega \qquad \boxed{X = 24{,}5\,\Omega} = 24{,}5\,\Omega$$

$$\boxed{R = 5\,\Omega}$$

Za prazni hod $I_2 = 0 \Rightarrow \bar{U}_{20} = \bar{I}_{10} \cdot j \cdot X_M \Rightarrow X_M = \frac{U_{20}}{I_{10}} = \frac{50}{2{,}5} = 20\,\Omega$ $\qquad \boxed{X_M = 20\,\Omega}$

Kada se kliješta zatvore onda je

$$\bar{U}_1 = \bar{I}_1 R_1 + \bar{I}_1 jX + \bar{I}_2 jX_M \qquad \bar{U}_1 = 5\bar{I}_1 + \bar{I}_1 \cdot 24{,}5j + \bar{I}_2 \cdot 20j$$

$$0 = \bar{I}_2 R_2 + \bar{I}_2 jX + \bar{I}_1 jX_M \qquad 0 = 5\bar{I}_2 + \bar{I}_2 \cdot 24{,}5j + \bar{I}_1 \cdot 20j$$

$$100 = 5\bar{I}_1 + \bar{I}_1 j24{,}5 + \bar{I}_2 j20 \qquad I_1 = a + bj \quad I_2 = c + dj$$

$$0 = 5\bar{I}_2 + \bar{I}_2 j24{,}5 + \bar{I}_1 j20$$

$$100 = 5(a+bj) + j24{,}5(a+bj) + 20j(c+dj)$$

$$0 = 5(c+dj) + j24{,}5(c+dj) + 20j(a+bj)$$

$$100 = 5a + 5bj + 24{,}5aj - 24{,}5b + 20cj - 20d \Rightarrow 5a - 24{,}5b - 20d = 100 \qquad 5b + 24{,}5a + 20c = 0$$

$$0 = 5c + 5dj + 24{,}5cj - 24{,}5d + 20aj - 20c$$

$$5a - 24{,}5b - 20d = 100$$

$$24{,}5a + 5b + 20c = 0$$

$$-20b + 5c - 24{,}5d = 0$$

$$20a + 24{,}5c + 5d = 0$$

$$\left[\begin{array}{cccc|c} 5 & -24,5 & 0 & -20 & 100 \\ 24,5 & 5 & 20 & 0 & 0 \\ 0 & -20 & 5 & -24,5 & 0 \\ 20 & 0 & 24,5 & 5 & 0 \end{array}\right] /\cdot(-4,9) \approx \left[\begin{array}{cccc|c} 5 & -24,5 & 0 & -20 & 100 \\ 0 & 125,05 & 20 & 98 & -490 \\ 0 & -20 & 5 & -24,5 & 0 \\ 0 & 98 & 24,5 & 85 & -400 \end{array}\right] /0,1599 \; /\cdot(-9,7836)$$

$$\left[\begin{array}{cccc|c} 5 & -24,5 & 0 & -20 & 100 \\ 0 & 125,05 & 20 & 98 & -490 \\ 0 & 0 & 8,198 & -8,83 & -78,35 \\ 0 & 0 & 8,822 & +8,2072 & -16,036 \end{array}\right] /(-1,076) \sim \left[\begin{array}{cccc|c} 5 & -24,5 & 0 & -20 & 100 \\ 0 & 125,05 & 20 & 98 & -490 \\ 0 & 0 & 8,198 & -8,83 & -78,35 \\ 0 & 0 & 0 & 17,7 & 68,26 \end{array}\right]$$

$$d = \frac{68,26}{17,7} = 3,85 \qquad 8,198\,c = -78,35 + 34,057$$

$$8,198\,c = -44,3 \Rightarrow c = -5,4028$$

$$\Rightarrow \bar{I}_2 = -5,4028 + 3,85\,j \Rightarrow I_2 = \sqrt{29,19 + 14,82} = 6,63\,A \qquad \boxed{I_2 = 6,63\,A}$$

$$125,05\,b = -490 - 98 \cdot 3,85 + 20 \cdot 5,4028 = -490 - 377,3 + 108,056$$

$$\Rightarrow b = -6,07 \qquad 5a = 100 + 20d + 24,5 \cdot b = 100 + 77 - 148,715$$

$$\Rightarrow a = 5,657 \Rightarrow \bar{I}_1 = 5,657 - j\,6,07 \Rightarrow I_1 = \sqrt{32,0016 + 36,84} = 8,3\,A$$

$$\boxed{I_1 = 8,3\,A}$$

$$P = U \cdot I_1^* = 100\,(5,657 + j\,6,07) = 565,7\,W \qquad \boxed{P = 565,7\,W}$$

$$k = \frac{M}{\sqrt{L_1 \cdot L_2}} = \frac{M \cdot \omega}{L \cdot \omega} = \frac{X_M}{X_L} = \frac{20}{24,5} = 0,816 \Rightarrow \boxed{k = 0,816}$$

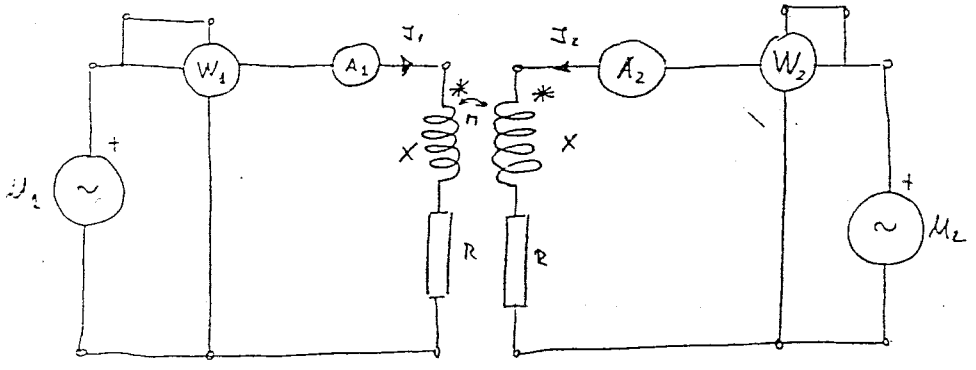
(2.) $U_1 = 10\,V$ $U_2 = 6\,V$ $I_1 = 2\,A$ $P_1 = 12\,W$ $I_2 = 0$ $P_2 = 0$ 4.3

4.3 Odredite R, X i X_M

fazni pomak napona U_1 i U_2

pokazivanje mjernih instrumenata nakon zamjene priključenja jednog od njih

uz $U_1 = 10\,V$



$U_1 = 10\,V$ $U_2 = 6\,V$ $I_1 = 2\,A$ $P_1 = 12\,W$ $I_2 = 0\,A$ i $P_2 = 0\,W$

$\bar{U}_1 = \bar{I}_1 R + \bar{I}_1 jX_L - \bar{I}_2 jX_M$ (1)

$\bar{U}_2 = \bar{I}_2 R - \bar{I}_2 jX_L - \bar{I}_1 jX_M$ (2)

Kako je $I_2 = 0\,A \Rightarrow \bar{U}_1 = \bar{I}_1 R + \bar{I}_1 jX_L$

$\bar{U}_2 = \bar{I}_1 jX_M$

$\Rightarrow X_M = \frac{U_2}{I_1} = \frac{6}{2} = 3\Omega$

$U_1 = I_1 Z_1 \Rightarrow Z_1 = \frac{U_1}{I_1} = \frac{10}{2} = 5\,\Omega$ $\boxed{Z_1 = 5\Omega}$ $P_1 = U_1 I_1 \cos\varphi \Rightarrow \cos\varphi = \frac{P}{U_1 I_1} = \frac{12}{20} = 0{,}6$

$R = Z \cos\varphi = 3\,\Omega$ $\sin\varphi = 0{,}8$ $X_L = 4\,\Omega$

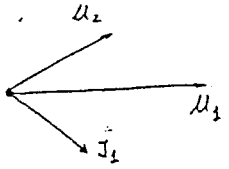
$\boxed{R = 3\Omega \quad X_L = 4\Omega \quad X_M = 3\Omega}$

2-ći način pomak između u_1 i u_2 $\bar{U}_1 = 10\,V$ uz $\bar{I}_2 = 0\,A$

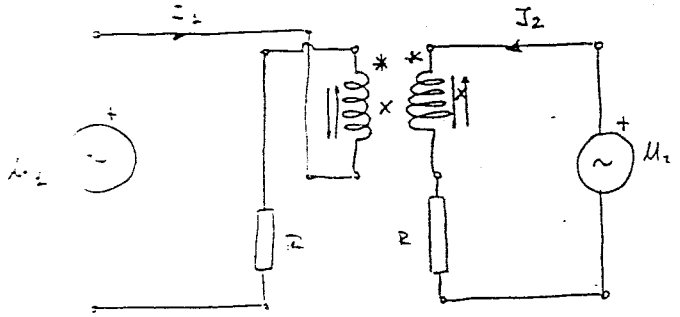
$\bar{I}_1 = \frac{\bar{U}_1}{\bar{Z}_1} = \frac{10}{3+j4} \cdot \frac{3-j4}{3-j4} = \frac{30-j40}{25} = 1{,}2 - j1{,}6\,A$ $\boxed{\bar{I}_1 = 1{,}2 - j1{,}6\,A}$

$\bar{U}_2 = \bar{I}_1 jX_M = (1{,}2 - 1{,}6j)\,j3 = j3{,}6 + 4{,}8 = 4{,}8 + j3{,}6 \Rightarrow \underline{\bar{U}_2 = 4{,}8 + j3{,}6}$

$\bar{U}_2 = 6 \cdot e^{36{,}87^\circ}\,V$ $\boxed{\bar{U}_2 = 6 \cdot e^{36{,}87^\circ}\,V}$



3) pokretanje instrumenata nakon zamjene priključnica jednog od svitaka uz $U_1 = 10V$.



$\bar{U}_1 = \bar{I}_1 \bar{Z}_1 - \bar{I}_2 jX_M$ $\bar{U}_2 = \bar{I}_2 \bar{Z}_2 - \bar{I}_1 jX_M$ $\bar{Z}_1 = \bar{Z}_2 = \bar{Z}$ $\bar{U}_1 = 10\,V$

$\bar{U}_1 = \bar{I}_1 \bar{Z} - \bar{I}_2 jX_M$ $\bar{U}_2 = \bar{I}_2 \bar{Z} - \bar{I}_1 jX_M \Rightarrow \bar{I}_2 \bar{Z} = \bar{U}_2 + \bar{I}_1 jX_M$ $\bar{Z} = 3 + j4$

$\bar{U}_1 = \bar{I}_1 \bar{Z} - jX_M \left[\frac{\bar{U}_2 - \bar{I}_1 jX_M}{\bar{Z}}\right]$ $\bar{I}_1 \bar{Z} = \bar{U}_1 + jX_M \left[\frac{\bar{U}_2 + \bar{I}_1 jX_M}{\bar{Z}}\right]$

$\bar{U}_1 = \bar{I}_1 \bar{Z} - jX_M \frac{\bar{U}_2}{\bar{Z}} - \frac{\bar{I}_1 X_M^2}{\bar{Z}}$ $\bar{U}_1 + jX_M \frac{\bar{U}_2}{\bar{Z}} = \bar{I}_1 \bar{Z} + \bar{I}_1 \frac{X_M^2}{\bar{Z}}$

$\bar{U}_1 + X_M \frac{\bar{U}_2}{\bar{Z}} = \bar{I}_1 \bar{Z} - \frac{X_M^2}{\bar{Z}}$ $\bar{I}_1 = \frac{\bar{U}_1 + jX_M \frac{\bar{U}_2}{\bar{Z}}}{\bar{Z} + \frac{X_M^2}{\bar{Z}}} = \frac{\bar{U}_1 \bar{Z} + jX_M \bar{U}_2}{\bar{Z}\bar{Z} + X_M^2}$

$\bar{I}_1 = \frac{10(3+j4) - j3(-j3,6)}{(3+j4)(3+j4) + 9} = \frac{30 + j40 + 14{,}4 j - 10{,}8}{9 + j12 + j12 - 16 + 9} = \frac{19{,}2 + j54{,}4}{2 + j24} = \frac{57{,}688 \cdot e^{70{,}55^\circ}}{24{,}083 \cdot e^{85{,}23^\circ}}$

$\bar{I}_1 = 2{,}39 \cdot e^{-14{,}68^\circ}\,A$ $I_1 = 2{,}31\,A$ $P_1 = [\bar{U}_1 \cdot \bar{I}_1^*]_{RE} = 10 \cdot 2{,}3110 = 23{,}110\,W$

$I_2 = 2{,}70\,A$

$\boxed{P_1 = 23{,}110\,W \quad P_2 = 12{,}6\,W}$

5.23 DIO

postavimo jednadžbe za konturne struje.

$\bar{U}_1 = \bar{I}_1(R_1+R_0 - j\omega L_1 + j\omega L_1' - j\omega L_0) - \bar{I}_3 \cdot j\omega M_1 - \bar{I}_2(R_0 + j\omega L_0)$

$\bar{U}_2 = \bar{I}_2\left(R_0 + j\omega L_0 + R_2 - j\frac{1}{\omega C} + j\omega L_2' + j\omega L_2'' + 2j\omega M_2\right) - \bar{I}_1(R_0 + j\omega L_0)$

$\bar{U}_1 = 10 \cdot e^{j0^\circ} = 10$

$\bar{U}_2 = 14{,}1 \cdot e^{j45^\circ} = 10 + j10\,V$

$\bar{U}_1 = 10V$

$\bar{U}_2 = 10 + j10\,V$

$L_3 = L_3' = M_1 = 0{,}0636\,mH$

$\omega L_3 = \omega L_3' = \omega M_1 = 2\pi \cdot 5000 \cdot 0{,}0636 \cdot 10^{-3} = 2\,\Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f \cdot 5{,}13 \cdot 10^{-6}} = \frac{10^6}{2\pi \cdot 5000 \cdot 5{,}13} = \frac{10^6}{166507{,}44} = 6\,\Omega$

$L_1 = L_1' = L_0 = L_2' = L_2'' = 0{,}0318\,mH$

$\omega L_1 = \omega L_1' = \omega L_0 = \omega L_2' = \omega L_2'' = 2\pi f \cdot 0{,}0318 \cdot 10^{-3} = 1\,\Omega$

$M_2 = 0{,}0159\,mH \qquad \omega M_2 = 2\pi f \cdot 0{,}0159 \cdot 10^{-3} = 0{,}5\,\Omega$

$\bar{I}_1(2+1+j1+j1+j1) - \bar{I}_3 j2 - \bar{I}_2(1+j1) = \ldots$

$10 + j10 = \bar{I}_2(1 + j1 + 2 - j6 + j1 + j1 + 2j0{,}5) - \bar{I}_1(1+j1)$

$10 = \bar{I}_1(3+j3) - \bar{I}_2(1+j) - \bar{I}_3 \cdot j2$

$10 + j10 = \bar{I}_2(3 - j2) - \bar{I}_1(1+j1)$

$O = \bar{I}_3(1+j4) - \bar{I}_1 \cdot j2 \Rightarrow \bar{I}_1 \cdot j2 = \bar{I}_3(1+j4)$

③ $O = \bar{I}_3(1 - j2 + j2) - \bar{I}_1 j2$

$O = \bar{I}_3(1+j4) - \bar{I}_1 \cdot j2$

$\bar{I}_2(3-j2) = 10 + j10 + \bar{I}_1(1+j)$

$\bar{I}_2 = \frac{10 + j10 + \bar{I}_1(1+j)}{3-j2}$

$\bar{I}_3 = \frac{\bar{I}_1 \cdot j2}{1+j4}$

$10 = \bar{I}_1(3+j3) - \frac{10 + j10 + \bar{I}_1(1+j)}{3-j2}(1+j) - j2 \cdot \frac{\bar{I}_1 \cdot j2}{1+j4}$

$10 = \bar{I}_1(3+j3) - \frac{10(1+j) + \bar{I}_1(1+j)}{3-j2}(1+j) + \frac{\bar{I}_1 \cdot 4}{1+j4} = \bar{I}_1(3+j3) - \frac{(10+\bar{I}_1)(1+j)}{3-j2}(1+j) + \frac{4\bar{I}_1}{1+j4}$

$10 = \bar{I}_1(3+j3) - \frac{(10+\bar{I}_1)[1+j+j-1]}{3-j2} + \frac{4 \cdot \bar{I}_1}{1+j4} = \bar{I}_1(3+j3) - \frac{(10+\bar{I}_1) \cdot j2}{3-j2} + \frac{4 \cdot \bar{I}_1}{1+j4}$

$10 = \bar{I}_1(3+j3) - \frac{(10+\bar{I}_1) \cdot j2 \cdot (3+j2)}{13} + \frac{4 \cdot \bar{I}_1(1-j4)}{17} = \bar{I}_1(3+j3) - \frac{(10+\bar{I}_1)(j6-4)}{13} - \frac{4\bar{I}_1(1-j4)}{17}$

$10 = \bar{I}_1(3+j3) - \frac{10(j6-4) + \bar{I}_1(j6-4)}{13} + \frac{4 \cdot \bar{I}_1(1-j4)}{17}$

$10 + \frac{10}{13}(j6-4) = \bar{I}_1\left[3+j3 - \frac{j6-4}{13} + \frac{4(1-j4)}{17}\right] = \bar{I}_1\left[\frac{221(3+j3) + 17(4-j6) + 52(1-j4)}{221}\right]$

$\frac{130 + 10(j6-4)}{13} = \bar{I}_1\left[\frac{663 + j663 + 68 - j102 + 52 - j208}{221}\right] = \bar{I}_1 \frac{783 + j353}{221} = \frac{130 + j60 - 40}{13}$

$\bar{I}_1 \cdot \frac{783 + j353}{221} = \frac{90 + j60}{13} \Big/ \cdot 221 \Rightarrow \bar{I}_1(783 + j353) = 17(90 + j60)$

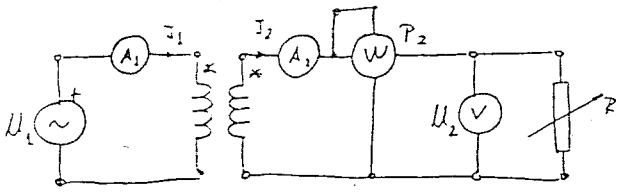
$= \ldots \; 17(90+j60)$

4.4

3.) $R_1 = R_2 = 0$ $\quad r_1 = r_2 = r = 4\,\Omega$ $\quad X_m = 3{,}8\,\Omega$

4. 4. st. 17

Nacrtati I_1, I_2, U_2, P_2 kao funkcije od R $\quad R_1 = R_2 = 0\,\Omega$



$x_1 = x_2 = x = 4\,\Omega$

$X_m = 3{,}8\,\Omega$

Nadomjesna shema transformatora

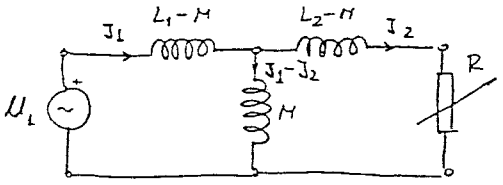
Prvo naponske jednadžbe $\quad \bar{U}_1 = \bar{I}_1 jx - \bar{I}_2 jX_m \quad 0 = \bar{I}_2 jx + \bar{I}_2 \cdot R - \bar{I}_1 jX_m$

dodajmo i odbijmo prvoj jednadžbi $\bar{I}_1 jX_m$ a drugoj $\bar{I}_2 jX_m$ pa će biti:

$$\bar{U}_1 - \bar{I}_1 jx - \bar{I}_2 jX_m - \bar{I}_1 jX_m + \bar{I}_1 jX_m = \bar{I}_1 (jx - jX_m) + jX_m(\bar{I}_1 - \bar{I}_2)$$

$$0 = \bar{I}_2 jx + \bar{I}_2 R - \bar{I}_1 jX_m - \bar{I}_2 jX_m + \bar{I}_2 jX_m = \bar{I}_2 (jx - jX_m) + \bar{I}_2 R + jX_m(\bar{I}_2 - \bar{I}_1) = 0$$

Ove druge jednadžbe iste su za ovaj krug nadalje



$\bar{I}_2 = ?$ pomoću Terminova teorema ćemo to izračunati.

$$\bar{Z}_T = j(X_L - X_m) + \frac{j(X_L - X_m)\cdot jX_m}{j(X_L - X_m) + jX_m} = j(X_L - X_m) + \frac{-(X_L - X_m)\cdot X_m}{jX_L - jX_m + jX_m} = j(X_L - X_m) - \frac{X_m(X_L - X_m)}{jX_L}$$

$$\bar{Z}_T = j(X_L - X_m) + j\,\frac{X_m(X_L - X_m)}{X_L} = j\left[(X_L - X_m) + \frac{X_m(X_L - X_m)}{X_L}\right] = j\,\frac{X_L^2 - X_m X_L + X_m X_L - X_m^2}{X_L} = j\,\frac{X^2 - X_m^2}{X}$$

$\bar{E}_T = ?$ $\quad \bar{E}_T = \dfrac{\bar{U}_1 \cdot jX_m}{j(X_L - X_m) + jX_m} = \dfrac{\bar{U}_1 \cdot jX_m}{jX_L} = \bar{U}_1 \dfrac{X_m}{X_L}$ $\quad$ jer je $\bar{U} = U$

$$\Rightarrow \bar{I}_2 = \frac{\bar{E}_T}{\bar{Z}_T + R} = \frac{\bar{U}\cdot \frac{X_m}{X_L}}{j\,\frac{X_L^2 - X_m^2}{X_L} + R} = \bar{U}\cdot\frac{X_m}{X_L}\cdot\frac{R - j\,\frac{X_L^2 - X_m^2}{X_L}}{R^2 + \left(\frac{X_L^2 - X_m^2}{X_L}\right)^2}$$

$$\bar{I}_2 = \bar{U}\cdot\frac{X_m}{X_L}\cdot\frac{R - j\,\frac{X_L^2 - X_m^2}{X_L}}{\frac{X_L^2 R^2 + (X_L^2 - X_m^2)^2}{X_L^2}} = \bar{U}\cdot\frac{X_m}{X_L}\cdot\frac{X_L^2}{X_L^2 R^2 + (X_L^2 - X_m^2)^2}\cdot\left(R - j\,\frac{X_L^2 - X_m^2}{X_L}\right)$$

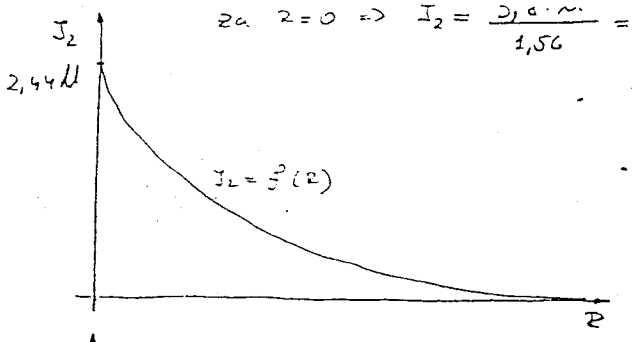
$$\bar{I}_2 = \bar{U}\cdot\frac{X_L X_m}{X_L^2 R^2 + (X_L^2 - X_m^2)^2}\cdot\left(R - j\,\frac{X_L^2 - X_m^2}{X_L}\right) \qquad I_2 = U\cdot\frac{X_L X_m}{X_L^2 R^2 + (X_L^2 - X_m^2)^2}\cdot\sqrt{R^2 + \frac{(X_L^2 - X_m^2)^2}{X_L^2}}$$

$$I_2 = U\cdot\frac{X_L X_m}{X_L^2 R^2 + (X_L^2 - X_m^2)^2}\cdot\frac{\sqrt{R^2 X_L^2 + (X_L^2 - X_m^2)^2}}{X_L} = U\cdot\frac{X_m}{\sqrt{X_L^2 R^2 + (X_L^2 - X_m^2)^2}}$$

$$I_2 = U\cdot\frac{3{,}8}{\sqrt{16R^2 + 2{,}4336}} = \frac{U\cdot 3{,}8}{\sqrt{16R^2 + 2{,}43}}$$

$$I_2 = \frac{U \cdot 3{,}8}{\sqrt{16R^2 + 2{,}434}} \Rightarrow \text{Nacrtati}$$

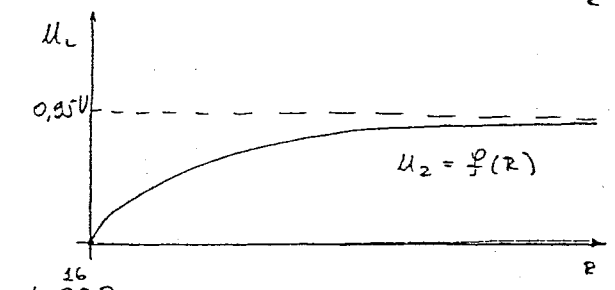
za $R = 0 \Rightarrow I_2 = \frac{3{,}8 \cdot U}{1{,}56} = 2{,}44\,U \Rightarrow I_2 = 2{,}44\,U$



analogno za I_1

$$I_1 = \frac{\sqrt{R^2 + X^2} \cdot U}{\sqrt{R^2 X^2 + (X^2 - X_M^2)^2}}$$

$$U_2 = I_2 \cdot R = \frac{3{,}8 \cdot U \cdot R}{\sqrt{16R^2 + 2{,}434}} \Rightarrow$$



pogledajmo ima li maximum

$$\frac{\partial U_2}{\partial R} = 3{,}8 \cdot U \cdot \frac{\sqrt{16R^2 + 2{,}434} - R \cdot \frac{1}{2} \cdot \frac{1 \cdot 32R}{\sqrt{16R^2 + 2{,}434}}}{16R^2 + 2{,}434} = 0$$

nema maximuma ni minimuma

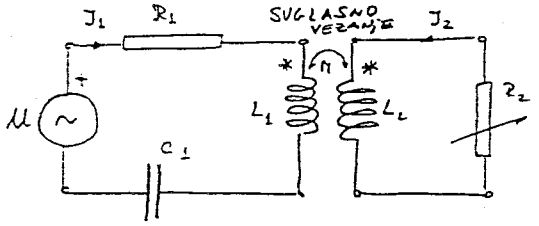
$$\lim_{R \to \infty} \frac{3{,}8\,U}{\sqrt{16R^2 + 2{,}434}} \cdot R \quad /:R \quad /:R \quad R \to \infty \quad = \lim \frac{3{,}8 \cdot U \cdot \frac{R}{R}\frac{1}{R}}{\sqrt{16\frac{R^2}{R^2} + \frac{2{,}434}{R^2}}} = \frac{3{,}8\,U}{4} = 0{,}95\,U$$

(4.) Pri kojoj će međusobnoj otporu R_2 krug biti u rezonanciji. Odredite I_1, I_2, P_1 i P_2

9.24 zadano je $U = 100V$, $R_1 = 2\Omega$, $X_{L_1} = 10\Omega$ $X_{C_1} = 8\Omega$ $X_{L_2} = 9\Omega$ $X_M = 6\Omega$

$U = 100V$ $R_1 = 2\Omega$ $X_{L_1} = 10\Omega$ $X_{C_1} = 8\Omega$

$X_{L_2} = 9\Omega$ $X_M = 6\Omega$



Napišimo naponske jednadžbe za transformator

$$\bar{U} = \bar{I}_1 \cdot R_1 + \bar{I}_1 \cdot jX_{L_1} - \bar{I}_1 \cdot jX_{C_1} + \bar{I}_2 \cdot jX_M \Rightarrow \bar{U} = \bar{I}_1 \cdot \bar{Z}_1 + \bar{I}_2 \cdot jX_M$$

$$0 = \bar{I}_2 R_2 + \bar{I}_2 \cdot jX_{L_2} + \bar{I}_1 \cdot jX_M \Rightarrow 0 = \bar{I}_2 \bar{Z}_2 + \bar{I}_1 \cdot jX_M \Rightarrow \bar{I}_2 \cdot \bar{Z}_2 = -\bar{I}_1 \cdot jX_M$$

$$\bar{I}_2 = -\frac{\bar{I}_1 \cdot jX_M}{\bar{Z}_2}$$

$$\bar{U} = \bar{I}_1 \cdot \bar{Z}_1 + jX_M \cdot \left(-\frac{\bar{I}_1 \cdot jX_M}{\bar{Z}_2}\right) = \bar{I}_1 \cdot \bar{Z}_1 - jX_M \left[\frac{\bar{I}_1 \cdot jX_M}{\bar{Z}_2}\right]$$

$$\bar{U} = \bar{I}_1 \cdot \bar{Z}_1 + \frac{\bar{I}_1 \cdot X_M^2}{\bar{Z}_2} = \bar{I}_1 \cdot (2 + j2) + \bar{I}_1 \cdot X_M^2 \cdot \frac{1}{R + j9} = \bar{I}_1 (2 + j2) + \bar{I}_1 \cdot X_M^2 \cdot \frac{R - j9}{R^2 + 81}$$

$$\bar{U} = \bar{I}_1 (2 + j2) + \bar{I}_1 \cdot 36 \cdot \frac{R - j9}{R^2 + 81} = \bar{I}_1 \left[2 + j2 + 36 \cdot \frac{R - j9}{R^2 + 81}\right] \Rightarrow \bar{Z}_{ul} = \frac{\bar{U}}{\bar{I}_1} = 2 + j2 + 36\frac{R - j9}{R^2 + 81}$$

$$\bar{Z}_{ul} = 2 + j2 + \frac{36R}{R^2 + 81} - j\frac{36 \cdot 9}{R^2 + 81} = 2 + j2 + \frac{36R}{R^2 + 81} - j\frac{324}{R^2 + 81} \qquad \mathrm{Im}(\bar{Z}_{ul}) = 0$$

$$\Rightarrow 2 - \frac{324}{R^2 + 81} = 0 \qquad 2R^2 + 162 = 324$$

$$R^2 = \frac{162}{2} \qquad R^2 = 81 \Rightarrow \boxed{R = 9\,\Omega}$$

Rezonantna frekvencija $f = \frac{1}{2\pi\sqrt{L \cdot C}}$

rezonancija $2+\frac{36}{R^2+81}$ $2+\frac{36\,j}{81+81}$ $2+\frac{54}{162}$ $2+1$

pri REZONANCIJI:

$P_1 = U_1 \cdot I_1 = 100 \cdot 25 = 2{,}5\,kW$

$I_2 = ?$ $\quad \bar{I}_2 = -\frac{\bar{I}_1 \cdot jX_M}{\bar{Z}_2}$ iz ② jednadžbe pri rezonanciji

$\bar{I}_2 = -25 \cdot \frac{j \cdot 6}{9+j9} = -25 \cdot \frac{j6 \cdot (9-j9)}{81+81} = -25 \cdot \frac{j54+54}{162}$

$\bar{I}_2 = -25 \cdot \frac{54+j54}{162}$

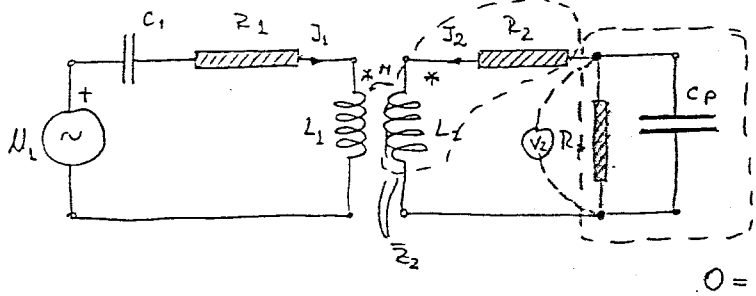
$I_2 = -8{,}333 - j\,8{,}333\,A \Rightarrow \boxed{I_2 = 11{,}78\,A}$

$P_2 = ?$

(5) $R_1 = 10\,\Omega$ $L_1 = 0{,}02\,H$ $C_1 = 100\,\mu F$ $R_p = 100\,\Omega$ $C_p = 10\,\mu F$, $R_2 = 10\,\Omega$ $L_2 = 0{,}04\,H$

4.38 $M = 0{,}03\,H$

Napon na opterećenju otpora R_p je $U_2 = 10\sqrt{2}\sin 1000t$. Odredite izraz za ulazni napon $U_1(t)$.



Z_T naponske jednadžbe

$\bar{U}_2 = \bar{I}_1 \cdot \bar{Z}_1 + \bar{I}_2 \cdot jX_M$

$0 = \bar{I}_2 \cdot \bar{Z}_2 + \bar{I}_2 \cdot \bar{Z}_{T2} + \bar{I}_1 \cdot jX_M$ $\quad \omega = 1000\,\frac{1}{s}$

$\bar{U}_1 = \bar{I}_1 \cdot \bar{Z}_1 + \bar{I}_2 \cdot jX_M$ $\quad 0 = \bar{I}_2 \cdot \bar{Z}_2 + \bar{I}_2 \cdot \bar{Z}_T + \bar{I}_1 \cdot jX_M$ $\quad U_2 = 10\sqrt{2}\sin 1000t = \mathrm{Im}\{10\sqrt{2} \cdot e^{j1000t}\}$

$\boxed{U_2 = 10\sqrt{2} \cdot e^{j0^\circ}\,V}$

mjerimo kao referentni

$X_{L1} = \omega L_1 = 1000 \cdot 0{,}02 = 20\,\Omega$ $\quad R_1 = 10\,\Omega$ $\quad R_2 = 10\,\Omega$

$X_{L2} = \omega L_2 = 1000 \cdot 0{,}04 = 40\,\Omega$ $\quad R_p = 100\,\Omega$

$X_M = \omega \cdot M = 1000 \cdot 0{,}03$

$\boxed{X_M = 30\,\Omega}$

$X_{C1} = \frac{10^6}{1000 \cdot 100} = 10\,\Omega$ $\quad X_{Cp} = \frac{10^6}{1000 \cdot 10} = 100\,\Omega$

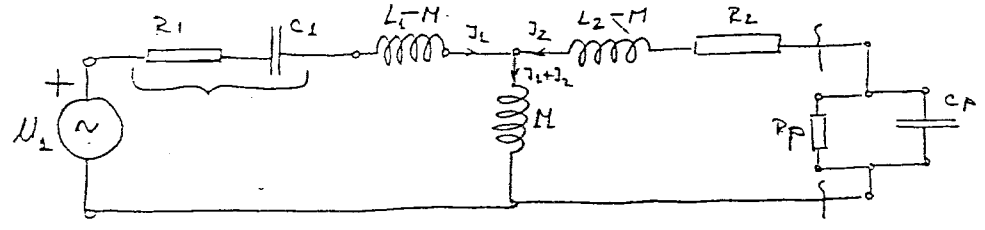
~ nap. jednadžba $\Rightarrow \bar{U}_1 = \bar{I}_1 \cdot (R_1 - jX_{C1}) + \bar{I}_1 \cdot jX_{L1} + \bar{I}_2 \cdot jX_M + \bar{I}_1 \cdot jX_M - \bar{I}_1 \cdot jX_M$

$\bar{U}_1 = \bar{I}_1 \cdot (R_1 - jX_{C1}) + \bar{I}_1 (jX_{L1} - jX_M) + (\bar{I}_1 + \bar{I}_2) jX_M$

② nap. jednadžba $\Rightarrow$ $0 = \bar{I}_2 \bar{Z}_2 + \bar{I}_2 \bar{Z}_T + \bar{I}_1 \cdot jX_M - \bar{I}_2 \cdot jX_M + \bar{I}_2 \cdot jX_M$

$0 = \bar{I}_2 \cdot R_2 + \bar{I}_2 \cdot jX_{L2} + \bar{I}_2 \cdot \bar{Z}_T + \bar{I}_1 \cdot jX_M - \bar{I}_2 \cdot jX_M + \bar{I}_2 \cdot jX_M$

$0 = \bar{I}_2 \cdot R_2 + \bar{I}_2 (jX_{L2} - jX_M) + (\bar{I}_1 + \bar{I}_2) jX_M + \bar{I}_2 \cdot \bar{Z}_T$



THEVENINOVIM TEOREMOM

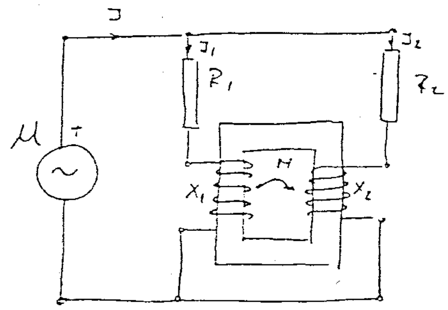
tt

4.20

(6) $U = 90\,V$ $R_1 = 2\,\Omega$ $R_2 = 4\,\Omega$

$X_1 = 3\,\Omega$ $X_2 = 3\,\Omega$ $X_M = 1\,\Omega$

Odredi I_1, I_2 i I 2. Nacrtati vektorski dijagram



Uzmimo da su namotaji vezani suglasno

$\Rightarrow$ $\overline{U} = \overline{I}_1 \cdot R_1 + \overline{I}_1 \cdot jX_1 - \overline{I}_2 \cdot jX_M$ $\overline{U} = 90V$

$\overline{U} = \overline{I}_2 \cdot R_2 + \overline{I}_2 \cdot jX_2 - \overline{I}_1 \cdot jX_M$

$90 = \overline{I}_1 \cdot 2 + \overline{I}_1 \cdot j3 - \overline{I}_2 \cdot j$

$90 = \overline{I}_2 \cdot 4 + \overline{I}_2 \cdot j3 - \overline{I}_1 \cdot j \quad \rightarrow \quad \overline{I}_2(4+j3) - \overline{I}_1 j = 90 \Rightarrow \overline{I}_2 = \dfrac{90 + \overline{I}_1 j}{4+j3}$

$90 = 2 \cdot \overline{I}_1 + \overline{I}_1 \cdot j3 - j \cdot \dfrac{90 + \overline{I}_1 j}{4+j3} = 2\overline{I}_1 + \overline{I}_1 \cdot j3 + \dfrac{-j90 + \overline{I}_1}{4+j3}$

$90 = 2\overline{I}_1 + \overline{I}_1 \cdot j3 + \dfrac{\overline{I}_1 - j90}{4+j3} = \dfrac{\overline{I}_1(2+j3)(4+j3) + \overline{I}_1 - j90}{4+j3} = \dfrac{\overline{I}_1(8+j6+j12-9) + \overline{I}_1 - j90}{4+j3}$

$90 = \dfrac{8\overline{I}_1 + j6 \cdot \overline{I}_1 + j12\,\overline{I}_1 - 9\overline{I}_1 + \overline{I}_1 - j90}{4+j3} = \dfrac{j18\overline{I}_1 - j90}{4+j3}$ $90(4+j3) = j18\overline{I}_1 - j90$

$j18\overline{I}_1 = 90(4+j3) + j90 \Rightarrow \overline{I}_1 = \dfrac{360 + j270 + j90}{j18} = \dfrac{360 + 360j}{j18} \cdot \dfrac{j}{j} = \dfrac{360j - 360}{-18}$

$\overline{I}_1 = \dfrac{360 - 360j}{18} = \boxed{20 - j20\ A}$

$I_1 = \sqrt{400 + 400} = \sqrt{800} = 20\sqrt{2}\ A$

$90 = \overline{I}_1 \cdot 2 + \overline{I}_1 \cdot j3 - \overline{I}_2 \cdot j = \overline{I}_1(2+j3) - j\overline{I}_2 = (20-j20)(2+j3) - j\overline{I}_2 = 90$

$j\overline{I}_2 = (20-j20)(2+j3) - 90 = 40 + j60 - j40 + 60 - 90 = 10 + j20 \Rightarrow \overline{I}_2 = \dfrac{10+j20}{j} \cdot \dfrac{j}{j}$

$\overline{I}_2 = \dfrac{10j - 20}{-1} = \boxed{20 - j10\ A} \rightarrow I_2 = \sqrt{400+100} = \sqrt{500} = 10\sqrt{5}\ A$

$\overline{I} = \overline{I}_1 + \overline{I}_2 = 20 - j20 + 20 - j10 = \boxed{40 - j30}$ $I = \sqrt{1600 + 900} = \sqrt{2500} = 50A$

$\overline{U}_{R_1} = \overline{I}_1 \cdot R_1 = (20-j20) \cdot 2 = 40 - j40\ V$ $\overline{U}_{R_2} = (20-j10) \cdot 4 = 80 - j40\ V$

$\overline{U}_{X_1} = \overline{I}_1 \cdot jX_1 = (20-j20) \cdot j3 = j60 + 60 = 60 + j60\ V$

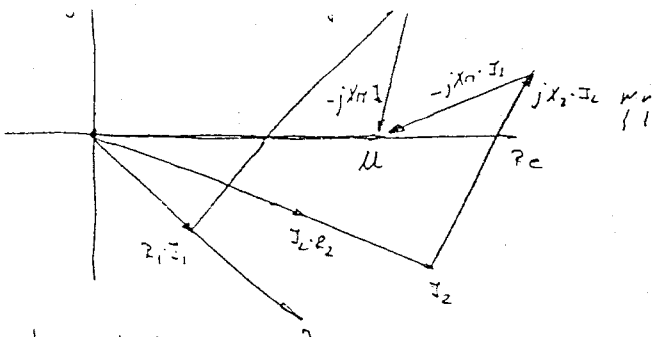
pošto su spojeni namotaji neusglasno $\Rightarrow$ napon međuindukcije djeluje suprotno od napona

samoindukcije $\rightarrow$ $-jX_M \cdot \overline{I}_2 = -j(20-j10) = -j20 - 10 = -10 - j20$

za drugu granu $\Rightarrow$ $\overline{U}_{R_2} = \overline{I}_2 \cdot R_2 = 80 - j80\ V$ $\overline{U}_{X_{L2}} = \overline{I}_2 \cdot jX_{L2} = (20-j10)\,j3 = j60 + 30 = 30 + j60$

napon međuindukcije $\Rightarrow$ $-jX_M \cdot \overline{I}_1 = -j(20-j20) = -j20 - 20 = -20 - j20\ V$

Nacrtaj vektorski dijagram $\Rightarrow$ ocjena!!!



Snaga koja daje izvor $\mathcal{I}$

na lijevoj strani $P_1 = Re\{U \cdot I_1^*\} = 90 \cdot (20 + j20) = 1800 + j1800 = 1800\ W$

djelatna na otporu u toj grani $P_{R_1} = I_1^2 \cdot R_1 = (20\sqrt{2})^2 \cdot 2 = 1600\ W$

razlika $P_1 - R_1 \cdot I^2 = 200\ W$ odredit će snagu koja je prenesena desnoj strani pomoću (preko) magnetskog polja

Snaga u desnoj strani je $P_2 = Re\{U \cdot I_2^*\} = Re\{90 \cdot (20 + j40)\} = 1800 + j900 = 1800\ W$

koja izvor predaje desnoj strani je $P_2 = 1800\ W$

$P \cdot I_2^2 = 2000\ W$ $\quad P_2 - R_2 \cdot I_2^2 = -200\ W$ snaga primljena iz lijeve grane putem međuinduktivne veze.

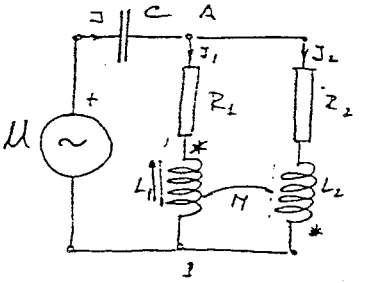
7. Zadan je krug na slici $U = 15V$ $f = 0{,}8\ kHz$ $C = 1\mu F$ $R_1 = R_2 = R = 200\ \Omega$ $L_1 = L_2 = L = 4\ mH$

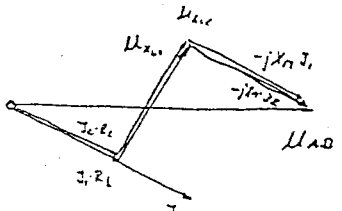
1,2 1) Kako treba rezati oniže uglasno ili neuglasno, da bi krug bio u rezonanciji? Koliki je koeficijent magnetske veze k u rezonanciji?

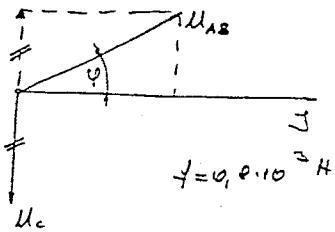
2) Odredite vrijednost otpora u rezonanciji

probajmo za neuglasnim rezanjem

Nacrtajmo vektorski dijagram približno







Odredimo napon U_{AB} prvo za slučaj neuglasnog rezanja oniže $f = 0{,}8 \cdot 10^3\ Hz$

$\bar{I} = \bar{I}_1 - \bar{I}_2$ $\quad \bar{U} = \bar{I} \cdot (-jX_c) + \bar{I}_1 \cdot R_1 + \bar{I}_1 \cdot jX_{L_1} - \bar{I}_2 \cdot jX_M$ $\quad X_{L_1} = \omega \cdot L_1 = 20{,}096\ \Omega$

$0 = \bar{I}_2 \cdot R_1 + \bar{I}_1 \cdot jX_{L_1} - \bar{I}_2 \cdot R_1 - \bar{I}_2 \cdot jX_{L_2} - \bar{I}_2 \cdot jX_M + \bar{I}_1 \cdot jX_M$ $\quad X_{L_2} = \omega \cdot L_2 = 20{,}096\ \Omega$

$0 = \bar{I}_1 \cdot 200 + \bar{I}_1 \cdot j20{,}096 - \bar{I}_2 \cdot 200 - \bar{I}_2 \cdot j20{,}096 - \bar{I}_2 \cdot jX_M + \bar{I}_1 \cdot jX_M$ $\quad X_c = \frac{1}{2\pi f \cdot C} = \frac{10^6}{5024} = 199{,}04\ \Omega$

$0 = \bar{I}_1 (200 + j20{,}096 + jX_M) - \bar{I}_2 (200 + j20{,}096 + jX_M) \Rightarrow \bar{I}_2 = \frac{\bar{I}_1 (200 + j20{,}096 + jX_M)}{200 + j20{,}096 + jX_M}$ $\quad \bar{I}_2 = \bar{I}_1$

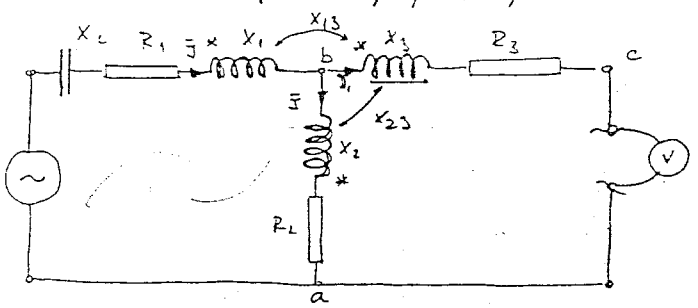
$\bar{U} = \bar{I} \cdot (-jX_c) + \bar{I}_1 \cdot R_1 + \bar{I}_1 \cdot jX_{L_1} - \bar{I}_2 \cdot jX_M \Rightarrow \bar{U} = \underbrace{(\bar{I}_1 + \bar{I}_2)}_{2\bar{I}_1} \cdot (-j199{,}04) + \bar{I}_1 \cdot 200 + \bar{I}_1 \cdot j20{,}096 - \bar{I}_2 \cdot jX_M$

$\bar{U} = -2\bar{I}_1 j199{,}04 + 200\bar{I}_1 + \bar{I}_1 \cdot j20{,}096 - \bar{I}_1 \cdot jX_M = -\bar{I}_1 j398{,}08 + 200\bar{I}_1 + \bar{I}_1 \cdot j20{,}096 - \bar{I}_1 \cdot jX_M$

$\bar{U} = \bar{I}_1 (-j398{,}08 + 200 + j20{,}096 - jX_M)$ $\quad U = 15 e^{j0}\ V$

ZADATAK 8 ⇒ 4.32

$U = 100V$, $X_c = 130\,\Omega$ $R_1 = 30\,\Omega$ $X_1 = 80\,\Omega$ $R_2 = 40\,\Omega$ $X_2 = 50\,\Omega$ $R_3 = 10\,\Omega$ $X_3 = 80\,\Omega$ $X_{13} = 60\,\Omega$ $X_{23} = 40\,\Omega$. Odrediti napon koji je izmjeren voltmetrom



$\bar{I}_1 = 0$

$\varphi_b = \varphi_a + \bar{I} \cdot R_2 + \bar{I} \cdot jX_2$

$\varphi_c = \varphi_b - \bar{I}_1 jX_{13} + \bar{I} jX_{23}$

$\varphi_b = \varphi_c + \bar{I}_1 jX_{13} - \bar{I} jX_{23}$

$\varphi_c + \bar{I}_1 jX_{13} - \bar{I} jX_{23} = \varphi_a + \bar{I} \cdot R_2 + \bar{I} \cdot jX_2$

$\bar{U}_V = \bar{I} \cdot R_2 + \bar{I} \cdot jX_2 - \bar{I} \cdot jX_{13} + \bar{I} jX_{23} = \bar{I}(R_2 + jX_2 - jX_{13} + jX_{23})$

$\bar{U}_V = \bar{I}(40 + j50 - j60 + j40) \Rightarrow \bar{U}_V = \bar{I}(40 + j30)$ $\quad \bar{Z} = R_1 + R_2 + jX_1 + jX_2 - jX_c$

$\bar{Z} = 40 + j80 + j50 - j130 = 40\,\Omega$ $\quad I = \frac{100}{40} = 2{,}5\,A$

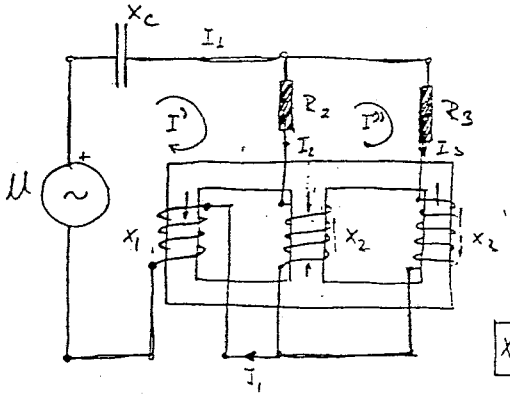
$\bar{U}_V = 2{,}5 \cdot (40 + j30) = 100 + j75$ $\quad U_V = \sqrt{10000 + 5625} = 125\,V$ $\quad = 25\sqrt{10}$

$\boxed{U_V = 125\,V}$ "

4.35

ZADATAK 8 $U = 160\,V$ $R_1 = R_2 = 5\,\Omega$ $X_c = \frac{1}{\omega C} = 2\,\Omega$ $X_1 = X_2 = X_3 = 10\,\Omega$

Svici su na zajedničkoj jezgri. Faktor magnetske veze srednjeg kraka je $k_1 = 0{,}8$, a koeficijent veze krajnjih svitaka je $k_2 = 0{,}6$. Odrediti struje I_1, I_2, I_3.



$M_{12} = k\sqrt{L_1 \cdot L_2}$ $/\cdot\omega \Rightarrow X_{M12} = k\sqrt{X_1 \cdot X_2}$

$X_{M12} = k\sqrt{X_1 \cdot X_2}$ $\quad X_{M13} = k\sqrt{X_1 \cdot X_3}$ $\quad X_{M23} = k\sqrt{X_2 \cdot X_3}$

$X_{M12} = 0{,}8\sqrt{10 \cdot 10}$ $\quad X_{M13} = 0{,}6\sqrt{10 \cdot 10}$ $\quad X_{M23} = 0{,}8\sqrt{10 \cdot 10}$

$\boxed{X_{M12} = 8\,\Omega}$ $\quad X_{M13} = 6\,\Omega$ $\quad X_{M23} = 8\,\Omega$

Metodom konturnih struja

$\bar{U} = \bar{I}'\left(R_2 + jX_2 + jX_1 - jX_c - jX_{12} - jX_{12}\right) + \bar{I}''\left(-R_2 - jX_2 + jX_{12} + jX_{13} + jX_{23}\right)$ $\quad \bar{U} = 160\,V$

$0 = \bar{I}''\left(R_2 + R_3 + jX_2 + jX_3 - jX_{23} - jX_{23}\right) + \bar{I}'\left(-R_2 - jX_2 + jX_{13} + jX_{12} + jX_{23}\right)$

$160 = \bar{I}'\left(5 + j10 + j10 - j2 - j8 - j8\right) + \bar{I}''\left(-5 - j10 + j8 + j6 + j8\right)$

$0 = \bar{I}''\left(5 + 5 + j10 + j10 - j8 - j8\right) + \bar{I}'\left(-5 - j10 + j6 + j8 + j8\right)$

$160 = \bar{I}'(5 + j2) + \bar{I}''(-5 + j12)$

$0 = \bar{I}''(10 + j4) + \bar{I}'(-5 + j12) \Rightarrow \bar{I}'(-5 + j12) = -\bar{I}''(10 + j4)$

$\bar{I}'(5 - j12) = \bar{I}''(10 + j4)$

83

4.35

$160 = \underline{I}'(5+j2) + \underline{I}'\frac{(5-j12)}{10+j4}\cdot(-5+j12)$ $\qquad 160 = \underline{I}'(5+j2) + \underline{I}'\cdot\frac{-25+j60+j60+144}{10+j4}$

$160 = \underline{I}'(5+j2) - \underline{I}'\frac{119+j120}{10+j4}$ $\qquad 160 = \underline{I}'\left[(5+j2) + \frac{119+j120}{10+j4}\right]$

$160 = \underline{I}'\cdot\frac{(5+j2)(10+j4)+119+j120}{10+j4} = \underline{I}'\cdot\frac{50+j20+j20-8+119+j120}{10+j4} = \underline{I}'\cdot\frac{161+j160}{10+j4}$

$160(10+j4) = \underline{I}'(161+j160) \Rightarrow \underline{I}' = \frac{160(10+j4)}{161+j160} = \frac{160\cdot 10{,}77\cdot e^{j\,21{,}805}}{226{,}98\cdot e^{j\,44{,}82}} = 7{,}6\cdot e^{-j23{,}02}$ A

$\boxed{\underline{I}_1 = 7{,}6\ A}$ ww

$\underline{I}'' = \underline{I}'\cdot\frac{5-j12}{10+j4} = 7{,}6\cdot e^{-j23{,}02}\cdot\frac{13\cdot e^{-j\,67{,}38}}{10{,}77\cdot e^{j\,21{,}801}} = 9{,}2\cdot e^{-j112{,}2}$

$\boxed{\underline{I}_3 = 9{,}2\ A}$ ww

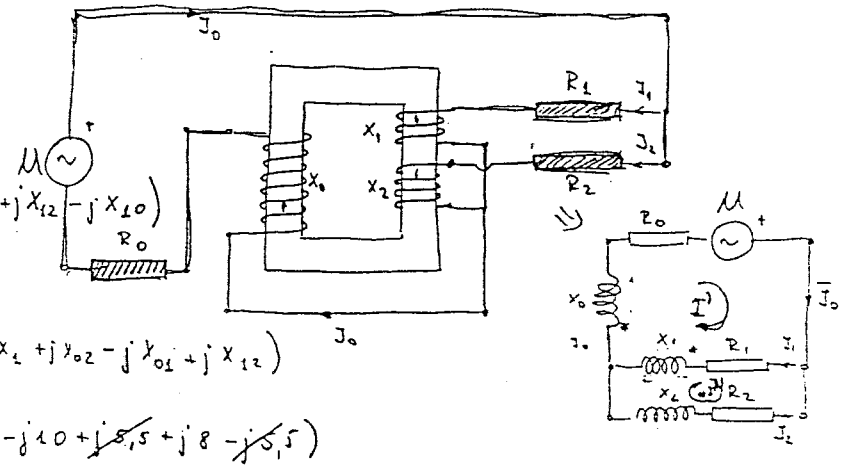
$\underline{I}_2 = \underline{I}' - \underline{I}'' = 7{,}6\cdot e^{-j23{,}02} - 9{,}2\cdot e^{-j112{,}2} = 7{,}6\,(0{,}92 - j\,0{,}391) - 9{,}2\cdot(-0{,}378 - j\,0{,}925)$

$\underline{I}_2 = 7 - j\,2{,}97 + 3{,}4776 + j\,8{,}51 = 10{,}477 + j\,5{,}54 \qquad \underline{I}_2 = 11{,}9\cdot e^{27{,}86}$ A

$\boxed{\underline{I}_2 = 11{,}9\ A}$ ww

4.36.

ZADATAK: $U = 90V$ $\quad R_0 = R_1 = R_2 = 5\Omega$ $\quad X_0 = X_1 = X_2 = 10\Omega$. Faktor veze svitaka koji se nalaze na jednom stupcu (X_1 i X_2) je $k_{12} = 0{,}8$. Veze svitaka X_1 i X_2 sa zavojnicom X_0 odredene su koeficijentima $k_{10} = k_{20} = 0{,}55$. Odredite struje u svim granama mreže.



$\underline{I}_0 = \underline{I}_1 + \underline{I}_2$

$X_{12} = 0{,}8\cdot\sqrt{10\cdot 10} = 8\Omega$

$X_{01} = X_{02} = 0{,}55\cdot\sqrt{10\cdot 10} = 5{,}5\Omega$

$\underline{U} = \underline{I}'\left(R_0 + R_1 + jX_1 + jX_0 + jX_{01}\right) + \underline{I}''\left(-R_1 - jX_1 + jX_{02} + jX_{12} - jX_{10}\right)$

$0 = \underline{I}''\left(R_1 + R_2 + jX_1 + jX_2 - jX_{12} - jX_{12}\right) + \underline{I}'\left(-R_1 - jX_1 + jX_{02} - jX_{01} + jX_{12}\right)$

$\underline{U} = \underline{I}'\left(5+5+j10+j10-j5{,}5+j5{,}5\right) + \underline{I}''\left(-5-j10+j5{,}5+j8-j5{,}5\right)$

$0 = \underline{I}''\left(5+5+j10+j10-j8-j8\right) + \underline{I}'\left(-5-j10+j5{,}5-j5{,}5+j8\right)$

$\underline{U} = \underline{I}'(10+j31) + \underline{I}''(-5-j2)$

$0 = \underline{I}''(10+j4) + \underline{I}'(-5-j2) \Rightarrow \underline{I}''(10+j4) = -\underline{I}'(-5-j2) \quad \underline{I}''(10+j4) = \underline{I}'(5+j2) \Rightarrow \underline{I}' = \frac{10+j4}{5+j2}\underline{I}''$

$\underline{U} = 90V$ $\qquad \underline{I}' = 2\underline{I}''$

$90 = \underline{I}'(10+j31) + \frac{\underline{I}'}{2}(-5-j2) = \underline{I}'(10+j31) - \frac{\underline{I}'}{2}(5+j2) = \underline{I}'(10+j31) - \underline{I}'\left(\frac{5}{2}+j1\right)$

$90 = \underline{I}'\left[10+j31-\frac{5}{2}-j1\right] \Rightarrow \underline{I}' = \frac{90}{7{,}5+j30} = \frac{90}{30{,}92\cdot e^{j75{,}96}} = 2{,}91\cdot e^{-j75{,}96}$ A

$\boxed{\underline{I} = 2{,}91\ A}$ ww

4,36

$I' = 2{,}91 \cdot e^{-j75{,}96}$ $\quad I'' = \frac{1}{2} I' = 1{,}455 \cdot e^{-j75{,}96}$ $\quad \boxed{I_1 = 1{,}455\,A \Rightarrow I_L = 1{,}455\,A}$ W-1

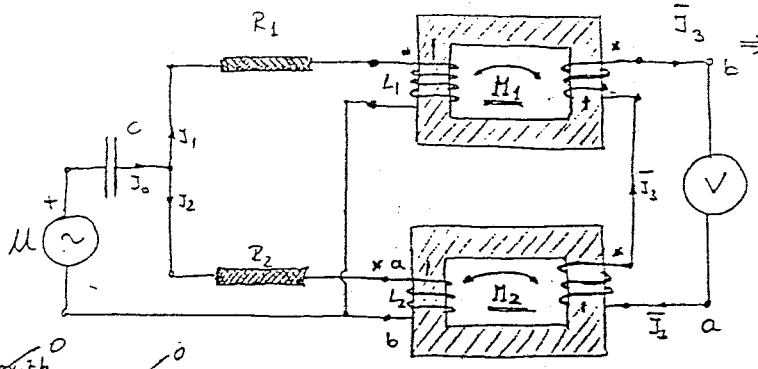
ZADATAK: $U = 130\,V$ $R_1 = 300\,\Omega$ $R_2 = 200\,\Omega$ $X_1 = \omega L_1 = 400\,\Omega$ $X_2 = \omega L_2 = 100\,\Omega$ $X_{M1} = \omega M_1 = 250\,\Omega$

4.37 $X_{M2} = 80\,\Omega$ Krug primara opojen je na izvor i nalazi se u rezonanciji.

ODREDI $\Rightarrow$ pokazivanje voltmetra.

$\Rightarrow$ pokazivanje voltmetra malom zazemljenje priključnica a i b drugog transformatora

58



$\Rightarrow \bar{J}_3 = 0$

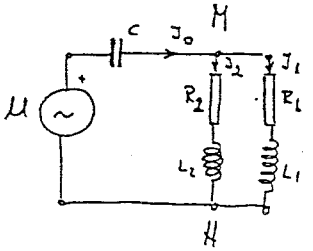
za ukundarni krug

$U_1 = \bar{J}_1 \cdot j M_1 + \bar{J}_2 \cdot j M_2$

$0 = \bar{J}_1 \bar{Z}_s + \bar{J}_2 \bar{Z}_s - \bar{J}_1 \cdot j M_1 - \bar{J}_2 \cdot j M_2 + \bar{J}_3 \cdot Z_{op+} \Rightarrow \boxed{U_V = \bar{J}_1 \cdot j M_1 + \bar{J}_2 \cdot j M_2}$

U_V

Odredimo J_1 i J_2 $\Rightarrow$ primar je u rezonanciji



$R_1 = 300\,\Omega$ $\quad X_1 = \omega L_1 = 400\,\Omega$

$R_2 = 200\,\Omega$ $\quad X_2 = \omega L_2 = 100\,\Omega$

$$\bar{Z}_u = \frac{(R_1 + jX_1)(R_2 + jX_2)}{R_1 + jX_1 + R_2 + jX_2} - jX_C = \frac{(300 + j400)(200 + j100)}{100 + j400 + 200 + j100} - jX_C = \frac{60000 + j30000 + j80000 - 40000}{500 + j500} - jX_C$$

$$\bar{Z}_u = \frac{20000 + j110000}{500 + j500} - jX_C = \frac{200 + j1100}{5 + j5} - jX_C = \frac{5(40 + j220)}{5(1 + j)} - jX_C = \frac{40 + j220}{1 + j} - jX_C$$

$$\bar{Z}_u = \frac{40 + j220 - jX_C(1 + j)}{1 - j} = \frac{40 + j220 - jX_C + X_C}{1 + j} \cdot \frac{1 - j}{1 - j} = \frac{[(40 + X_C) + j(220 - X_C)](1 - j)}{1 + 1}$$

$$\bar{Z}_u = \frac{[(40 + X_C) + j(220 - X_C)](1 - j)}{2} = \frac{1}{2}[(40 + X_C) - j(40 + X_C) + j(220 - X_C) + (220 - X_C)]$$

$\mathrm{Im}\{\bar{Z}_u\} = 0 \quad 220 - X_C - 40 - X_C = 0 \quad 180 = 2X_C \Rightarrow \boxed{X_C = 90\,\Omega}$

$$\bar{J}_0 = \frac{\bar{U}}{\bar{Z}_u} = \frac{130}{\frac{1}{2}[(40 + 90) + (220 - 90)]} = \frac{130}{\frac{1}{2}(130 + 130)} = \frac{130}{130} = 1\,A \qquad \boxed{\bar{J}_0 = 1\,A}$$

$$\bar{U}_{MN} = 1 \cdot \frac{(R_1 + jX_1)(R_2 + jX_2)}{R_1 + jX_1 + R_2 + jX_2} \cdot \frac{40 + j220}{1 + j} \cdot \frac{(1 - j)}{(1 - j)} = \frac{40 - j40 + j220 + 220}{2} = \frac{260 + j180}{2} = 130 + j90\ V$$

$$\bar{J}_1 = \frac{130 + j90}{300 + j400} = \frac{13 + j9}{30 + j40} \cdot \frac{30 - j40}{30 - j40} = \frac{390 - j520 + j270 + 360}{900 + 1600} = \frac{750 - j250}{2500} = 0{,}3 - j0{,}1$$

$$= \frac{75 - j25}{250} = \frac{3 - j}{10}$$

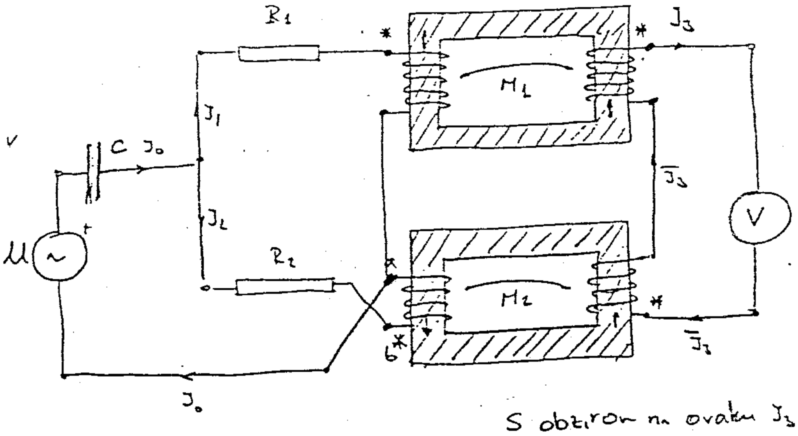
$$\bar{I}_2 = \frac{130+j50}{200-j100} \cdot \frac{(200+j100)}{200-j100} = \frac{26000-j13000+j18000+9000}{40000+10000} = \frac{35000+j5000}{50000} = \frac{35+j5}{50}$$ (4.37)

$\bar{I}_2 = \frac{7+j}{10}$ $\quad \bar{U}_V = \bar{I}_1 \, j\omega M_1 - \bar{I}_2 \, j\omega M_2 = \frac{3-j}{10} j250 + \frac{7+j}{10} \cdot j80 = (3-j) \cdot j25 + (7+j) \cdot j8$

$\bar{U}_V = j75 - 25 - j56 - 8 = j131 - 17 \quad \bar{U}_V = 17 + j131 \quad U_V = \sqrt{131^2 + 17^2} = 132\ V$

$U_V = 132\ V$

pod b – dva zamjena stezaljke a i b => biti će



$0 = \bar{I}_2 Z_3 + \bar{I}_3 Z_3 - \bar{I}_1 \, jX_{M1} - \bar{I}_2 \cdot jX_{M2} + U_V$

$\bar{U}_V = \bar{I}_1 \, jX_{M1} - \bar{I}_2 \cdot jX_{M2}$

$= \frac{3-j}{10} \cdot j250 - \frac{7+j}{10} \cdot j80$

$\bar{U}_V = (3-j) \cdot j25 - (7+j) \cdot j8 = j75 + 25 - j56 + 8$

$\bar{U}_V = j19 + 33 = 33 + j19 \Rightarrow U_V = \sqrt{1089 + 361} = 38V$

$U_V = 38V$

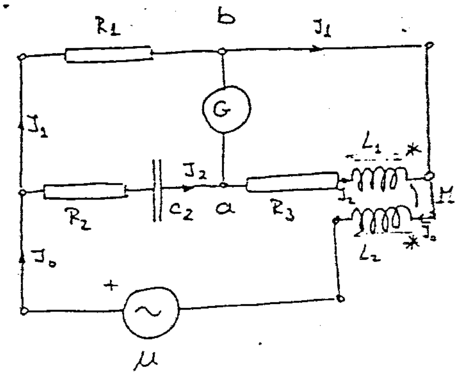
S obzirom na ovakav I_3 kako sam pretpostavio

struja na gornjem transformatoru u neuglasnom spoju

a na 2. u uglasnom spoju.

4.54.

ZADATAK: => Za mjerenje međuinduktiviteta možemo upotrijebiti most kao na slici. Ravnoteža mosta postignuta je za $R_1 = 500\,\Omega$ $R_2 = 300\,\Omega$ $R_3 = 100\,\Omega$ i $C_2 = 0{,}125\,\mu F$. Koliki je međuinduktivitet? Koliki je induktivitet prvog svitka?



Postoje za ove impedancije spora i kapaciteti most u

ravnoteži $\Rightarrow I_G = 0$

$\Rightarrow \varphi_b = \varphi_a \quad \varphi_b = \varphi_a - \bar{I}_2 (R_2 - jX_{C2}) - I_1 \cdot R_1$

$\bar{I}_2 (R_2 - jX_{C2}) = \bar{I}_1 \cdot R_1$

$\varphi_c = \varphi_a - \bar{I}_2 Z_3 - \bar{I}_2 \cdot jX_1 + \bar{I}_0 \cdot jX_M$

$\bar{I}_2 (R_3 - jX_1) = \bar{I}_0 \cdot jX_M \quad \Rightarrow \bar{I}_2 (R_3 + jX_1) = (\bar{I}_1 + \bar{I}_2) \cdot jX_M$

$\bar{I}_2 (R_3 + jX_1) - \bar{I}_2 \cdot jX_M = \bar{I}_1 \cdot jX_M$

$\bar{I}_2 (R_2 - jX_{C2}) = \bar{I}_1 \cdot R_1$

$\bar{I}_2 (R_3 + jX_1 - jX_M) = \bar{I}_1 \cdot jX_M$ / : $\quad \frac{R_3 + jX_1 - jX_M}{R_2 - jX_{C2}} = \frac{jX_M}{R_1}$

$\bar{I}_2 (R_2 - jX_{C2}) = \bar{I}_1 \cdot R_1$

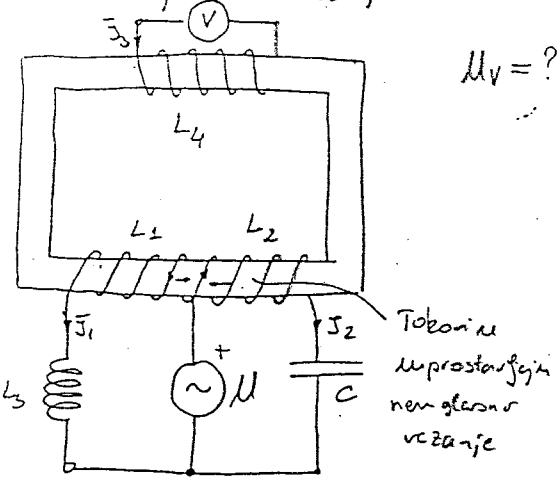
ZADATAK: $U = 120\,V$ na $\omega = 1000\,s^{-1}$. Parametri kruga su $L_1 = 0{,}05\,H$ $L_2 = 0{,}04\,H$

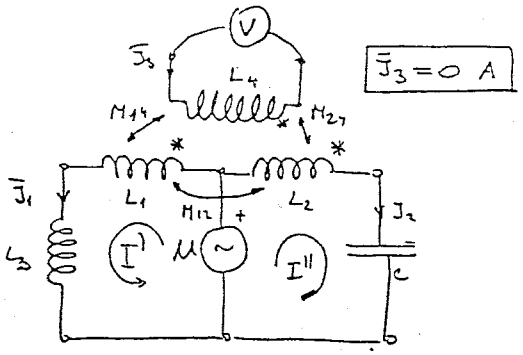
$L_3 = 0{,}01\,H$ $C = 10\,\mu F$ $M_{12} = 0{,}01\,H$ $M_{14} = 0{,}08\,H$ $M_{24} = 0{,}06\,H$

4.47

Koliko je pokazivanje voltmetra.

$U_V = ?$





$\bar{J}_3 = 0\ A$

$\bar{U} = I'(jX_2 + jX_3) - I'' \cdot jX_{M12}$

$\bar{U} = I''(jX_2 - jX_C) - I' \cdot jX_{M12}$

4.1

$\omega L_1 = X_1 = 1000 \cdot 0{,}05 = 50\,\Omega = X_1$

$X_2 = \omega \cdot L_2 = 1000 \cdot 0{,}04 \Rightarrow X_2 = 40\,\Omega$

$X_3 = 1000 \cdot 0{,}01 = 10\,\Omega$

$X_C = \frac{1}{\omega C} = \frac{10^6}{10^3 \cdot 10} = 100\,\Omega$

$X_3 = 10\,\Omega$

$X_{M12} = 1000 \cdot 0{,}01 = 10\,\Omega$ $X_{M12} = 10\,\Omega$

$X_{M24} = 1000 \cdot 0{,}06\,H = 60\,\Omega$

$X_{M14} = 1000 \cdot 0{,}08 = 80\,\Omega$

$120 = I'(j50 + j10) - I'' \cdot j10$ $120 = I' \cdot j60 - I'' \cdot j10$

$120 = I''(j40 - j100) - I' \cdot j10$ $120 = -I' \cdot j10 - I'' \cdot j60$

$12 = I' \cdot j6 - I'' \cdot j$

$12 = -I' \cdot j - I'' \cdot j6 \quad /\cdot 6$ $\Rightarrow$ $72 = -I' \cdot j6 - I'' \cdot j36$

$+$ $84 = -I'' \cdot j(1 + 36)$

$84 = -I'' \cdot j \cdot 37$

$I'' = \frac{-84}{j37} \cdot \frac{j}{j} = \frac{+84 \cdot j}{+37} = j\frac{84}{37}$ $I'' = j2{,}27\ A$ $\bar{J}_2 = j2{,}27\,A$

$12 = I' \cdot j6 - j \cdot j \cdot 2{,}27 = I' \cdot j6 + 2{,}27$

$9{,}73 = I' \cdot j6 \Rightarrow I' = \frac{9{,}73}{j6} = \frac{9{,}73}{j6} \cdot \frac{j}{j} = \frac{j9{,}73}{-6} = -j1{,}62166\ A$

$0 = \bar{J}_3 \cdot X_4 + \bar{U} - \bar{J}_1 \cdot jX_{14} + \bar{J}_2 \cdot jX_{24}$ $\bar{U} = \bar{J}_2 \cdot jX_{24} - \bar{J}_1 \cdot jX_{14}$

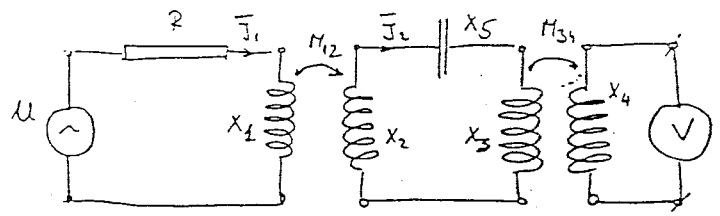
$\bar{U} = j2{,}27 \cdot j60 + j1{,}62166 \cdot j80 = -136{,}2 - 129{,}732 = 266\,V$

$U = 266\,V$ U_4

ZADATAK: Odredite pokazivanje voltmetra. Parametri kruga su: $X_1 = 80\,\Omega$ $X_2 = 180\,\Omega$ $X_3 = 220\,\Omega$

$X_4 = 90\,\Omega$ $X_5 = 220\,\Omega$ $k_{12} = 75\%$ i $k_{34} = 50\%$. Napon izvora je $U = 200\,V$

Otpor voltmetra smatrati beskonačno velikim



$\bar{U} = U \cdot e^{j0^\circ} V \Rightarrow$

$X_1 = \omega L_1$

$X_2 = \omega L_2$

$X_3 = \omega L_3$

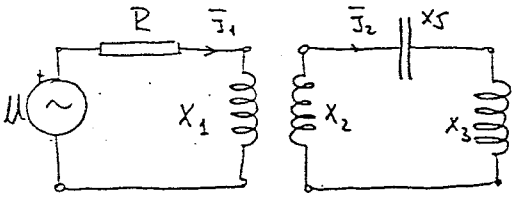
$X_4 = \omega L_4$

$M_{12} = k_{12}\sqrt{L_1 \cdot L_2}$

$M_{12} = k_{12}\sqrt{\frac{X_1}{\omega} \cdot \frac{X_2}{\omega}} = k_{12} \cdot \frac{1}{\omega}\sqrt{X_1 \cdot X_2}$

$\omega \cdot M_{12} = k_{12}\sqrt{X_1 \cdot X_2}$ $\quad X_{M12} = k_{12}\sqrt{X_1 \cdot X_2} = 0{,}75\sqrt{80 \cdot 180} = 90$ $\quad \boxed{X_{M12} = 90\,\Omega}$

$X_{M34} = k_{34}\sqrt{X_3 \cdot X_4} = 0{,}5\sqrt{40 \cdot 90} \approx 30\,\Omega$ $\quad \boxed{X_{M34} = 30\,\Omega}$



$\bar{U} = \bar{J}_1 R + \bar{J}_1 \cdot jX_1 - \bar{J}_2 \cdot jX_{M12}$

$0 = \bar{J}_2 \cdot jX_2 + \bar{J}_2 \cdot jX_3 + \bar{J}_2 \cdot (-jX_5) - \bar{J}_1 \cdot jX_{M12}$

$200 = \bar{J}_1 \cdot R + \bar{J}_1 \cdot j80 - \bar{J}_2 \cdot j90$

$0 = \bar{J}_2 \cdot j180 + \bar{J}_2 \cdot j40 - \bar{J}_2 \cdot j220 - \bar{J}_1 \cdot jX_{M12} \Rightarrow -\bar{J}_1 \cdot jX_{M12} = 0$

$-\bar{J}_1 \cdot j90 = 0 \Rightarrow \bar{J}_1 = 0$

$200 = -\bar{J}_2 \cdot j90 \Rightarrow \bar{J}_2 = -\frac{200}{j90} = -\frac{200}{j90} \cdot \frac{j}{j} = \frac{+200j}{+90} = j2{,}222$ $\quad \bar{J}_2 = 2{,}222 \cdot e^{j90^\circ} A$

$\bar{J}_2 = 2{,}222 \angle 90^\circ A$

$u_V = M_{34} \cdot \frac{di_2}{dt} = M_{34} \cdot \frac{d}{dt}\left[\frac{2{,}222 \cdot e^{j90^\circ} \cdot e^{j\omega t}}{2{,}222 \cdot e^{j(\omega t + 90^\circ)}}\right] = M_{34} \cdot 2{,}222 \cdot e^{j(\omega t + 90^\circ)} \cdot \omega$

$u_V = j\omega M_{34} \cdot 2{,}2222 \cdot e^{j(\omega t + 90^\circ)} = j \cdot X_{M34} \cdot 2{,}2222 \cdot e^{j(\omega t + 90^\circ)}$

$U = 2{,}222 \cdot X_{M34} = 2{,}2222 \cdot 30 = 66{,}7\,V$ $\quad \boxed{U_V = 66{,}7\,V}$

4.45

ZADATAK 8 ⇒ Odrediti pokazivanje voltmetra $U_v = ?$

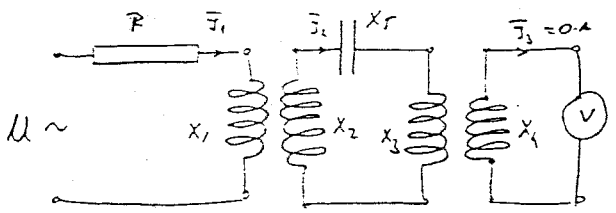
$X_1 = 80\,\Omega$ $k_{12} = 75\%$

$X_2 = 180\,\Omega$ $k_{34} = 50\%$

$X_3 = 40\,\Omega$ $U = 200\,V$

$X_4 = 90\,\Omega$

$X_5 = 220\,\Omega$



$\bar{U} = \bar{J}_1 (R + jX_1) + \bar{J}_2 \cdot jX_{M12}$

$0 = \bar{J}_2 (jX_2 + jX_3 - jX_5) + \bar{J}_1 \cdot jX_{M12}$

$X_{M12} = k\sqrt{X_1 \cdot X_2} = 0{,}7\sqrt{80 \cdot 180}$

$X_{M34} = k\sqrt{X_3 \cdot X_4} = 0{,}5 \cdot \sqrt{40 \cdot 90}$

$200 = \bar{J}_1 \cdot (R + j80) + \bar{J}_2 \cdot j90$

$X_{M12} = 90\,\Omega$ $X_{M34} = 30\,\Omega$

$0 = \bar{J}_2 \cdot 0 + \bar{J}_1 \cdot j90 \Rightarrow 0 = \bar{J}_1 \cdot j90 \Rightarrow \boxed{\bar{J}_1 = 0\,A}$

$200 = J_2 \cdot j90$ $\bar{J}_2 = \frac{200}{j90} = \frac{j}{j} = \frac{-200j}{90} = -2{,}22j$ $\boxed{\bar{J}_2 = -2{,}22j\ A}$

$\bar{U}_V = \bar{J}_2 \cdot j X_{M34} = -j2{,}22 \cdot j30 = 60\,V$

68

$\boxed{U_V = 60\,V}$

5.55

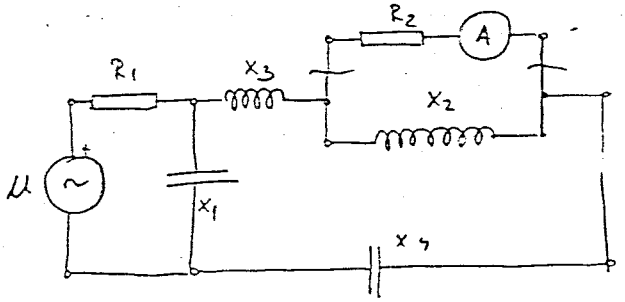
ZADATAK 8 ⇒ Struja koja teče kroz ampermetar uključen u mrežu, kako je pokazano na shemi iznosi: $I = 0{,}2\,A$

Odredite napon priključen na mrežu i kroz pomak između zadane struje i tog napona.

ZADANO: ⇒ $R_1 = 20\,\Omega$ $X_2 = 50\,\Omega$

$R_2 = 40\,\Omega$ $X_3 = 80\,\Omega$

$X_1 = -100\,\Omega$ $X_4 = -65\,\Omega$



- Theveninov teorem

$\bar{E}_T = ?$ $\bar{Z}_T = ?$ $\bar{Z}_{11} = \frac{-20 \cdot j100}{20 - j100} = \frac{-j200\phi}{2\phi - j10\phi} = \frac{-j200}{2 - j10} = \frac{-j100}{1 - j5} \cdot \frac{1 + j5}{1 + j5} = \frac{-j100 + 500}{26}$

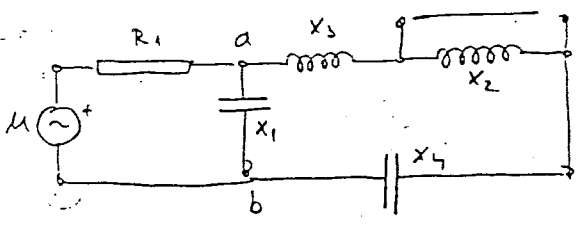
$\bar{Z}_{11} = \frac{500 - j100}{26}$

$\bar{Z}_{1134} = \frac{500 - j100}{26} + j80 - j65 = \frac{500 - j100 + j2340 - j1690}{26} = \frac{500 + j550}{26}$

$\bar{Z}_T = \frac{\frac{500 + j550}{26} \cdot j50}{\frac{500 + j550}{26} + j50} = \frac{\frac{j25000 - 27500}{26}}{\frac{500 + j550 + j1300}{26}} = \frac{j2500\phi - 2750\phi}{50\phi + j185\phi} = \frac{j2500 - 2750}{50 + j185} \cdot \frac{50 - j185}{50 - j185}$

$\bar{Z}_T = \frac{-j125000 + 462500 - 137500 + j508750}{2500 + 34225} = \frac{j633750 + 325000}{36725} = 8{,}85 + j17{,}257\,\Omega$

$\bar{E}_T = ?$



pomoću Millmanova teorema

$\bar{U}_{ab} = \frac{\bar{U} \cdot \frac{1}{R_1} + 0 + 0}{\frac{1}{Z_1} + \frac{1}{-jX_1} + \frac{1}{jX_2 + jX_3 - jX_4}}$

NASTAVAK MPDV 5.10.

$$\frac{500(980+j0{,}45t)}{960400+0{,}2025t^2} - \frac{5760+j144+0{,}4t^2}{57600+t^2} = 0$$

$$500(980+j0{,}45t)(57600+t^2) - (960400+0{,}2025t^2)(5760+j144+0{,}4t^2) = 0$$

$$500\left[56448000 + 980t^2 + j25920t + j0{,}45t^3\right] - \left[5531904000 + j138297600 + 384160t^2 + 1166{,}4t^2 + j29{,}16t^2 + 0{,}081t^4\right] = 0$$

$$500(56448000 + 980t^2) - 5531904000 - 384160t^2 - 1166{,}4t^2 - 0{,}081t^4 = 0$$

$$500(25920t + 0{,}45t^3) - 138297600 - 29{,}16t^2 = 0$$

Rješenje

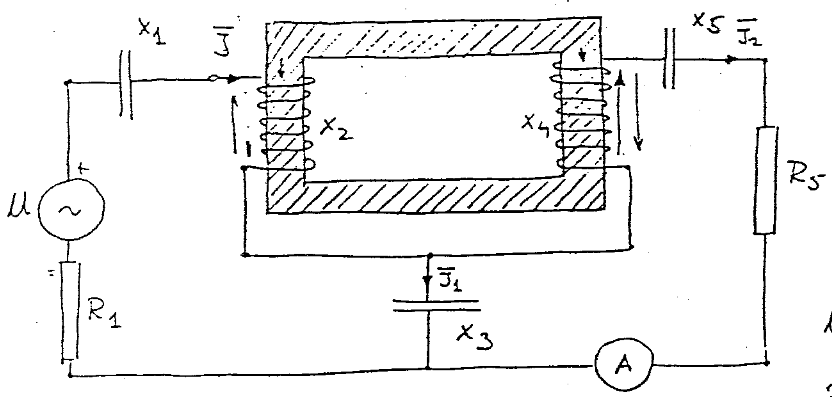
ZADATAK 8 ⇒ Odredite pokazivanje ampermetra priključenog u krug kao na slici.

4.44 B.

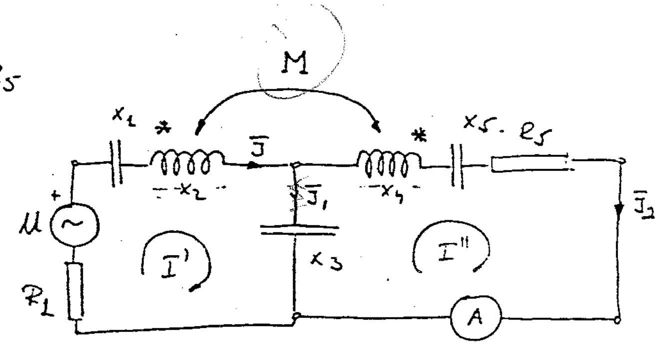
$R_1 = 5\Omega$ $X_1 = -20\Omega$ $X_2 = 80\Omega$ $X_3 = 60\Omega$ $X_4 = 100\Omega$ $X_5 = -40\Omega$

$R_5 = 20\Omega$ $X_{24} = 50\Omega$. Napon generatora je $U = 2kV$ (sinusoidalan)

S(E)



$\bar{U} = U \cdot e^{j0°}V \Rightarrow \bar{U} = 2000V$



$$\bar{U} = I'[5 - j20 + j80 - j60] - I''(-j60) - I''\cdot j50$$

$$0 = I''[j100 - j40 + 20 - j60] - I'(-j60) - I'\cdot j50$$

$$\bar{U} = 5I' + j60\cdot I'' - j50I'' \qquad \bar{U} = 5I' + j10I''$$

$$0 = 20I'' + j60I' - j50I' \Rightarrow 0 = 20I'' + j10I' \Rightarrow j10I' = -20I''$$

$$I' = \frac{-20I''}{j10} = \frac{-20I''}{j10}\cdot\frac{j}{j} = \frac{+j20I''}{+10} = j2I'' \qquad I' = j2I''$$

$$U = 5\cdot j2I'' + j10I'' = 10jI'' + 10jI'' = 20jI'' = 2000 \Rightarrow I'' = \frac{2000}{j20}\cdot\frac{j}{j}$$

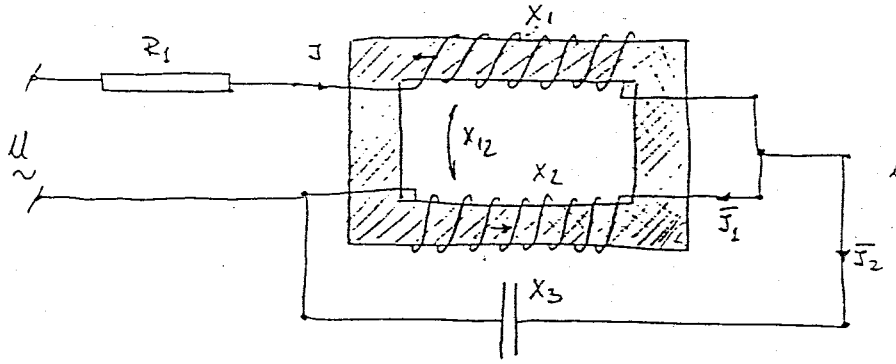
$$I'' = \frac{j2000}{-20} = -j100A \Rightarrow \bar{J}_2 = I'' = -j100A$$

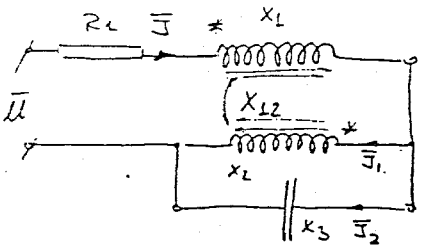
$$I_2 = 100A$$

struja I_2 zaostaje za $\frac{\pi}{2}$ iza napona generatora.

ZADATAK: Odredite ulaznu impedanciju dvoprilaza ako je $Z_1 = 10\,\Omega$ $X_{12} = 10\,\Omega$ $X_1 = 25\,\Omega$

4.49) $X_2 = 40\,\Omega$ i $X_3 = -20\,\Omega$. Odredite pad napona na prvom otporu uz priključen napon na izlaz dvoprilaza $U = 170\,V$ i izračunajte srednju vrijednost snage koja se prenosi iz jednog otpora na drugi međuinduktivnim vezom





Odredimo ulaznu impedanciju sklopa $\bar{Z}_{ul} = \dfrac{\bar{U}}{\bar{J}}$

$\bar{U} = \bar{J}\cdot R_1 + \bar{J}\cdot jX_1 + \bar{J}_1\cdot jX_2 + \bar{J}\cdot jX_{12} + \bar{J}_1 jX_{12}$

$0 = \bar{J}_2\cdot(-jX_3) - \bar{J}_1\cdot jX_2 - \bar{J}\cdot jX_{12}$

$\bar{J} = \bar{J}_1 + \bar{J}_2$

$\bar{U} = \bar{J}(10+j25) + \bar{J}_1\cdot j40 + \bar{J}\cdot j10 + \bar{J}_1\cdot j10 = \bar{J}(10+j35) + j50\bar{J}_1 = \bar{U}$

$0 = \bar{J}_2\cdot(-j20) - \bar{J}_1\cdot j40 - \bar{J}\cdot j10 \quad + \bar{J}_2 j2\phi + \bar{J}_1\cdot j4\phi + \bar{J}\cdot j1\phi = 0 \;/\cdot(-1)$

$\bar{J} = \bar{J}_1 + \bar{J}_2 \qquad \bar{J} = \bar{J}_1 + \bar{J}_2$

$\bar{J}(10+j35) + j50\bar{J}_1 = \bar{U}$

$\bar{J}_2\cdot j2 + \bar{J}_1\cdot j4 + \bar{J}\cdot j = 0 \quad 2\bar{J}_2 + 4\bar{J}_1 + \bar{J} = 0 \Rightarrow 2\cdot(\bar{J}-\bar{J}_1) + 4\bar{J}_1 + \bar{J} = 0$

$\bar{J} = \bar{J}_1 + \bar{J}_2 \quad \bar{J} - \bar{J}_1 + \bar{J}_2 \Rightarrow \bar{J}_2 = \bar{J} - \bar{J}_1 \quad 2\bar{J} - 2\bar{J}_1 + 4\bar{J}_1 + \bar{J} = 0$

$3\bar{J} + 2\bar{J}_1 = 0 \Rightarrow 2\bar{J}_1 = -3\bar{J}$

$\bar{J}_1 = -\frac{3}{2}\bar{J}$

$\bar{J}(10+j35) + j50\left(-\frac{3}{2}\bar{J}\right) = \bar{U}$

$\bar{J}(10+j35) - j75\bar{J} = \bar{U} \qquad \bar{J}(10+j35-j75) = \bar{U} \qquad \dfrac{\bar{U}}{\bar{J}} = 10 - j40$

$\boxed{\bar{Z}_u = 10 - j40}\ \Omega$

$\bar{U} = 170\cdot e^{j0^\circ}\ V$

Pad napona na otporu X_1 = ? $\quad \bar{J} = \dfrac{\bar{U}}{\bar{Z}_u} = \dfrac{170}{10-j40} = \dfrac{170}{41{,}231\cdot e^{-j76^\circ}} = 4{,}123\,e^{j76^\circ}\ A$

$\bar{J}_1$ = ? $\quad \bar{J}_1 = -\frac{3}{2}\bar{J} = -\frac{3}{2}\cdot 4{,}123\cdot e^{j76^\circ} = -6{,}1845\cdot e^{j76^\circ}$

$\bar{J}_1 = -6{,}1845\cdot e^{j76^\circ}$

$\bar{U}_{x_1} = \bar{J}\cdot jX_1 + \bar{J}_1\cdot jX_{12} = 4{,}123\,e^{j76}\cdot 25\cdot e^{j90} + \left(-6{,}1845\cdot e^{j76^\circ}\right)\cdot 10\cdot e^{j90}$

$= 103{,}075\cdot e^{j166} - 61{,}845\cdot e^{j166} = 41{,}23\,e^{j166}\ V$

$\boxed{U_{x_1} = 41{,}23\ V}$

$$\frac{\varphi_3}{\bar{Z}_8}-\frac{\varphi_1}{\bar{Z}_8}-\frac{\bar{E}_7}{\bar{Z}_8}=\frac{\bar{E}_1}{\bar{Z}_1+\bar{Z}_2}+\frac{\varphi_1}{\bar{Z}_1+\bar{Z}_3}-\frac{\varphi_4}{\bar{Z}_1+\bar{Z}_3}+\frac{\varphi_1}{\bar{Z}_6}$$

$$\frac{\bar{E}_1}{\bar{Z}_1+\bar{Z}_2}+\frac{\varphi_1}{\bar{Z}_1+\bar{Z}_3}-\frac{\varphi_4}{\bar{Z}_1+\bar{Z}_3}=\frac{\varphi_4}{\bar{Z}_4+\bar{Z}_5}-\frac{\varphi_3}{\bar{Z}_4+\bar{Z}_5}-\frac{\bar{E}_3}{\bar{Z}_4+\bar{Z}_5}+\frac{\varphi_4}{\bar{Z}_1}+\frac{\bar{E}_2}{\bar{Z}_1}$$

$$\frac{\varphi_1}{\bar{Z}_8}-\frac{\varphi_1}{\bar{Z}_5}-\frac{\bar{E}_4}{\bar{Z}_8}=\frac{\varphi_4}{\bar{Z}_4+\bar{Z}_5}-\frac{\varphi_3}{\bar{Z}_4+\bar{Z}_5}-\frac{\bar{E}_3}{\bar{Z}_4+\bar{Z}_5}-\frac{\varphi_3}{\bar{Z}_7}$$

$$\varphi_4\cdot G_{13}-\varphi_1\left(\bar{G}_8+\bar{G}_6+\bar{G}_{13}\right)+\frac{\varphi_3}{\bar{Z}_8}=\bar{E}_1\bar{G}_{13}+\bar{E}_4\bar{G}_8$$

$$\varphi_1\left(\bar{G}_8+\bar{G}_6+\bar{G}_{13}\right)-\varphi_3\cdot\bar{G}_8-\varphi_4\cdot G_{13}=-\bar{E}_1\bar{G}_{13}-\bar{E}_4\cdot\bar{G}_8 \quad \text{za čvor ①}$$

$$-\varphi_4\left(\bar{G}_{13}+\bar{G}_1+\bar{G}_{25}\right)+\varphi_1\cdot\bar{G}_{13}+\varphi_3\bar{G}_{25}=\bar{E}_2\cdot\bar{G}_4-\bar{E}_3\bar{G}_{25}-\bar{E}_1\bar{G}_{13}$$

$$\varphi_4\left(\bar{G}_{13}+\bar{G}_1+\bar{G}_{25}\right)-\varphi_1\bar{G}_{13}-\varphi_3\bar{G}_{25}=\bar{E}_3\cdot\bar{G}_{25}+\bar{E}_1\cdot\bar{G}_{13}-\bar{E}_2\cdot\bar{G}_4$$

$$\varphi_3\left(\bar{G}_8+\bar{G}_{25}+\bar{G}_7\right)-\varphi_1\cdot\bar{G}_8-\varphi_4\cdot\bar{G}_{25}=\bar{E}_1\cdot\bar{G}_8-\bar{E}_3\cdot\bar{G}_{25} \quad \text{WM}$$

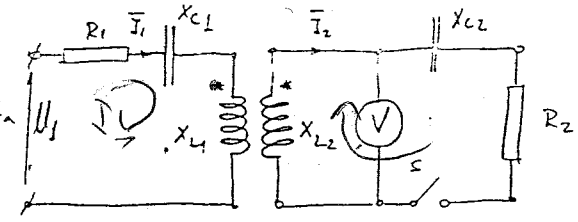
ANALOGNO KAO KOD ISTOSMJERNIH STRUJA

4.59.

ZADATAK: ⇒ Pri otvorenoj sklopci voltmetar je pokazao napon $U=1\,kV$. Odredite pokazivanje voltmetra pri zatvorenoj sklopci. Otpor generatora se može zanemariti, a otpor voltmetra je jako velik u usporedbi s ostalim elementima kruga. Parametri kruga su ovi:

$R_1=5\Omega \quad X_{C1}=50\Omega \quad X_{L1}=40\Omega \quad R_2=24\Omega \quad X_{C2}=92\Omega \quad X_{L2}=30\Omega \quad i \quad k=33{,}33\,\%$

Q01



Upotrebiti ćemo nadomjesnom shemom transformatora zamislimo da je sklopka zatvorena.

$$\bar{U}_1=\bar{I}_1\left(R_1+jX_{L1}-jX_{C1}\right)-\bar{I}_2\cdot jX_M+\bar{I}_1\cdot jX_M-\bar{I}_1\cdot jX_M$$

$$0=\bar{I}_2\left(R_2+jX_{L2}-jX_{C2}\right)-\bar{I}_1\cdot jX_M+\bar{I}_2\cdot jX_M-\bar{I}_2\cdot jX_M$$

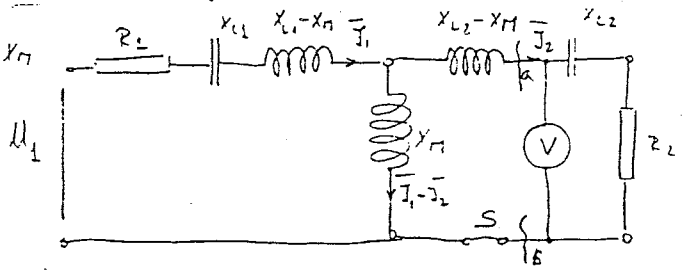
$$X_M=k\sqrt{X_{L1}\cdot X_{L2}}$$

$$=0{,}3333\cdot 60=20\Omega$$

$$\bar{U}_1=\bar{I}_1\cdot R_1+\bar{I}_1\left(jX_{L1}-jX_M\right)-\bar{I}_2 jX_{C1}+\left(\bar{I}_1-\bar{I}_2\right)\cdot jX_M$$

$$0=\bar{I}_2 R_2+\bar{I}_2\left(jX_{L2}-jX_M\right)-\bar{I}_2 jX_{C2}+\left(\bar{I}_2-\bar{I}_1\right)jX_M$$

$$0=\underbrace{\bar{I}_2\left(R_2-jX_{C2}\right)}_{\bar{U}_v}+\bar{I}_2\left(jX_{L2}-jX_M\right)+\left(\bar{I}_2-\bar{I}_1\right)jX_M$$



$$0=\bar{U}_v+\bar{I}_2\left(jX_{L2}-jX_M\right)-\left(\bar{I}_1-\bar{I}_2\right)jX_M$$

$$\bar{U}_v=\left(\bar{I}_1-\bar{I}_2\right)\cdot jX_M-\bar{I}_2\left(jX_{L2}-jX_M\right)$$

Kad je otvorena sklopka voltmetar pokazuje napon od 1 kV. Upotrijebim Theveninov teorem

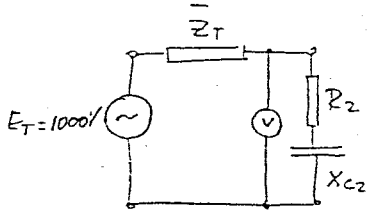
4,59)

$\Rightarrow E_T = 1000V \quad \bar{Z}_T = ?$

$$\bar{Z}_T = \frac{(5-j20)\cdot j20}{5-j30+j20} + j70 \qquad \bar{Z}_T = \frac{(5-j30)\cdot j20}{5-j10} - j70$$

$$\bar{Z}_T = \frac{(5-j30)\cdot j20 + j70(5-j10)}{5-j10} = \frac{j100+600+j350+700}{5-j10} = \frac{1300+j450}{5-j10}\cdot\frac{5+j10}{5+j10} = \frac{6500-j13000+j2250-1}{125}$$

$$\bar{Z}_T = \frac{2000+j15250}{125} = 16+j122\ \Omega$$



$$\bar{Z}_u = \bar{Z}_T + 24 - j92$$

$$\bar{Z}_u = 16+j122+24-j92 = 40+j30\ \Omega \qquad \bar{E}_T = 1000V$$

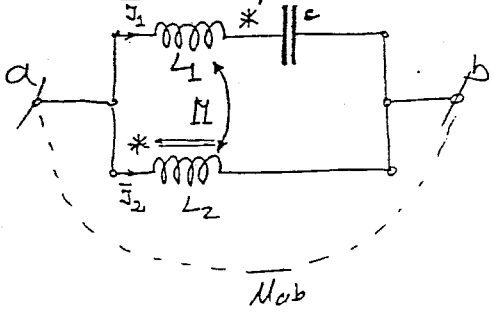
$$\bar{I}_u = \frac{1000}{40+j30} = \frac{1000}{50\,e^{j36,9°}} = 20\cdot e^{-j36,9°}\ A \qquad \bar{Z}_T = 24-j92\ \Omega$$

$$\bar{Z}_T = 95,1\,e^{-j75,37°}\ \Omega$$

$$U_V = 20\cdot 95,1 = 1902\ V \qquad \boxed{U_V = 1902\ V}$$ WL 1

in,57)

ZADATAK: $\Rightarrow$ Odredite koeficijent međuinduktivnosti između svitaka pri kojem će nastati strujna rezonancija. Induktiviteti svitaka je $L_1 = 40mH$ i $L_2 = 20mH$. Kružna frekvencija izvora koji napaja cijeli krug je $\omega = 10^3\ s^{-1}$. Kapacitet kondenzatora je $C = 0,125\mu F$



$$\bar{U}_{ab} = \bar{I}_2\cdot jX_{L_2} - \bar{I}_1\cdot jX_M$$

$$\bar{U}_{ab} = \bar{I}_1\cdot jX_{L_1} + \bar{I}_1\cdot(-jX_{C_1}) - \bar{I}_2\cdot jX_M$$

$$\bar{I}_2\cdot jX_{L_2} - \bar{I}_1\cdot jX_M = \bar{I}_1\cdot jX_{L_1} - \bar{I}_1\cdot jX_{C_1} - \bar{I}_2\cdot jX_M$$

uvjet $\bar{I}_1 = -\bar{I}_2$ $\qquad \bar{I}_2\cdot j\omega L_2 - \bar{I}_1\cdot j\omega M = \bar{I}_1\cdot j\omega L_1 - \bar{I}_1\cdot j\frac{1}{\omega C_1} - \bar{I}_2\cdot j\omega M$

$$\bar{I}_2\cdot j200 - \bar{I}_1\cdot jX_M = \bar{I}_1\cdot j400 - \bar{I}_1\cdot j800 - \bar{I}_2\cdot jX_M \qquad \bar{I}_1 = -\bar{I}_2 \qquad \bar{I}_2 = -\bar{I}_1$$

$$-\bar{I}_1\cdot j200 - \bar{I}_1\cdot jX_M = \bar{I}_1\cdot j400 - \bar{I}_1\cdot j800 + \bar{I}_1\cdot jX_M$$

$$-\bar{I}_1\cdot j200 - \bar{I}_1\cdot j400 + \bar{I}_1\cdot j800 - \bar{I}_1\cdot jX_M + \bar{I}_1\cdot jX_M \qquad \bar{I}_1\cdot j200 = \bar{Z}_1\cdot 2jX_n \qquad 200 = 2X_n$$

$$X_n = 100\ \Omega$$

$$X_n = 100\Omega \Rightarrow M = \frac{100}{10^3} = 0,01H \qquad M = k\sqrt{L_1\cdot L_2} \Rightarrow k = \frac{M}{\sqrt{L_1\cdot L_2}}$$

$$k = \frac{0,01}{\sqrt{40\cdot10^{-3}\cdot20\cdot10^{-3}}} = \frac{0,01}{\sqrt{800\cdot10^{-6}}} = \frac{0,01}{28,28427\cdot10^{-3}} = 0,3535$$

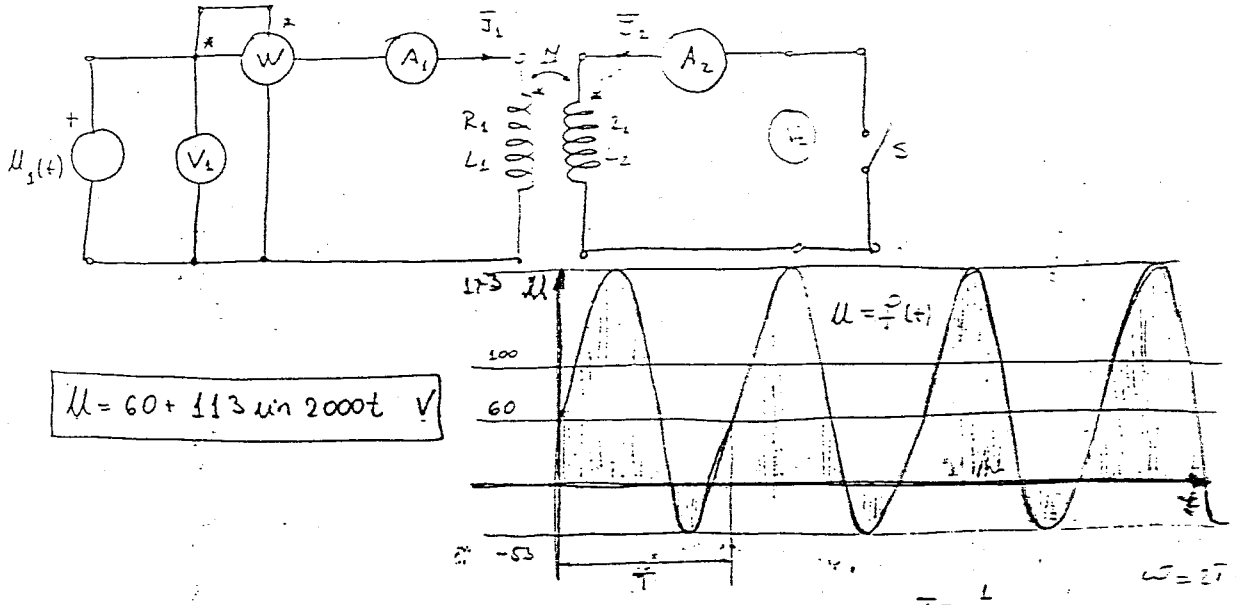
$$\boxed{k = 35,35\%}$$ WL 1

(7.43)

ZADATAK: Zračni transformator priključen je na izvor napona $u_1 = 60 + 113 \sin 2000t$ V.

SLIČNO 4.2 $R_1 = R_2 = 6\,\Omega$ $L_1 = L_2 = 4$ mH, $M = 1$ mH

Nacrtajte pokazivanja instrumenata s nekim idealnim elementom na sekundarnoj strani S.



$u = 60 + 113 \sin 2000t$ V

U_{v_1} = efektivna vrijednost napona izvora

$\omega = 2\pi f$ $\quad T = \frac{1}{f}$ $\quad f = \frac{\omega}{2\pi}$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{2000} = 0{,}003\pi$ s

$U = \sqrt{U_0^2 + U_1^2} = \sqrt{60^2 + \left(\frac{113}{\sqrt{2}}\right)^2} = \sqrt{60^2 + \frac{(113)^2}{2}}$

$U = \sqrt{3600 + 6384{,}5} = 100$ V

$\boxed{U_{v_1} = 100\ V}$

$\bar{I}_1 = \frac{\bar{U}_1}{\bar{Z}_1}$ $\quad \bar{U}_1'$ → istosmjerna komponenta ⇒ $I_1' = \frac{60}{6} = 10$ A

$\bar{U}_1''$ – izmjenična komponenta ⇒

$\omega = 2000 \cdot 4 \cdot 10^{-3} = 8\,\Omega$

$u_2 = 113 \sin 2000t$ ⇒ ef. ⇒ 80 V $\quad \bar{U}_2 = 80 \cdot e^{j0°}$ V

$\bar{I}_1'' = \frac{80 \cdot e^{j0°}}{6 + j8} = \frac{80 \cdot e^{j0°}}{10 \cdot e^{j53{,}13°}} = 8 \cdot e^{-j53{,}13°}$ A $\quad i_1'' = 11{,}314 \sin(\omega t - 53{,}13°)$

$i_1 = 10 + 11{,}314 \sin(\omega t - 53{,}13°)$ $\quad I_1 = \sqrt{100 - 64} = 12{,}8$ A $\quad \boxed{I_1 = 12{,}8\ A}$

$P_W = U_0 \cdot I_0 + U_1 \cdot I_1 \cdot \cos\varphi = 60 \cdot 10 + 80 \cdot 8 \cdot \cos(-53{,}13)$

$P_W = 600 + 640 \cdot 0{,}6 = 600 + 384$

$\boxed{P_W = 984\ W}$ W/W

$u_2 = M \cdot \frac{di_1}{dt} = 1 \cdot 10^{-3} \cdot \frac{d}{dt}\left[10 + 11{,}314 \sin(\omega t - 53{,}13°)\right] = 10^{-3} \cdot 11{,}314 \cdot \omega \cdot \cos(\omega t - 53{,}13°)$

$U_{2M} = 10^{-3} \cdot 11{,}314 \cdot 2000 = 22{,}628 \Rightarrow \boxed{U_2 = 16\ V}$ V/V

Sekundar na osjetljivom instrumentu

Zadatak: 5.22

Mreža se napaja generatorima napona U_1 i U_2, U_2 prednjači naponu U_1 za kut φ $U_1 = 100\,V$, $U_2 = 100\,V$, $\varphi = 90°$ $R_1 = 5\Omega$ $X_1 = 10\Omega$ $R_2 = 5\Omega$ $X_2 = 10\Omega$ $R_0 = 5\Omega$ $X_0 = -5\Omega$ $X_M = 5\Omega$

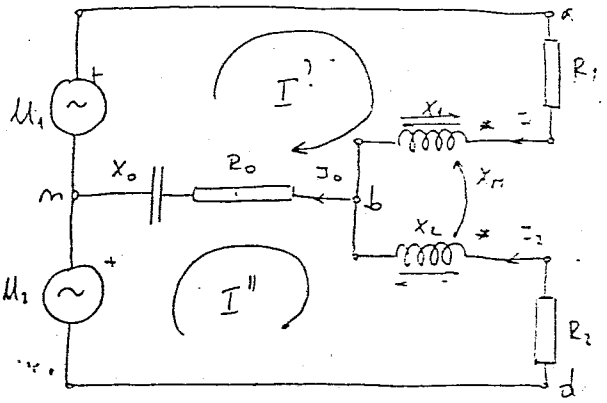
Izračunaj: 1) I_1, I_2, i I_0

2) U_{ab}, U_{bd}, U_{an}

3) snage na pojedinim objektima mreže i cijela mreža, te snage koje oddaje svaki generator

4) struju I_0 pomoću Nortonova teorema.

$\bar{U}_1 = 100\,V$ $\bar{U}_2 = 100e^{j90°}\,V = j100\,V$



1) Napišimo konturne struje

$$\bar{U}_1 = \bar{I}'(R_1 + R_0 + jX_{L1} - jX_0) - \bar{I}''\cdot jX_M - \bar{I}''(R_0 - jX_0)$$

$$\bar{U}_2 = \bar{I}''(R_0 + R_2 + jX_2 - jX_0) - \bar{I}'(R_0 - jX_0) - \bar{I}'\cdot jX_M$$

$$100 = \bar{I}'(10 + j5) - \bar{I}''\cdot j5 - \bar{I}''(5 - j5)$$

$$j100 = \bar{I}''(10 + j5) - \bar{I}'\cdot(5 - j5) - \bar{I}'\cdot j5$$

$$100 = 10\bar{I}' + j5\bar{I}' - \cancel{j5\bar{I}''} - 5\bar{I}'' + \cancel{j\bar{I}''\cdot 5} \qquad 100 = (10 + j5)\bar{I}' - 5\bar{I}''$$

$$j100 = \bar{I}''\cdot 10 + j5\,\bar{I}'' - 5\bar{I}' + \cancel{j\bar{I}'\cdot 5} - \cancel{\bar{I}'\cdot j5} \qquad j100 = -5\bar{I}' + \bar{I}''(10 + j5)$$

$$20 = (2 + j)\bar{I}' - \bar{I}'' \Rightarrow \bar{I}'' = (2 + j)\bar{I}' - 20$$

$$j20 = -\bar{I}' + \bar{I}''(2 + j) \Rightarrow j20 = -\bar{I}' + (2 + j)\left[(2 + j)\bar{I}' - 20\right]$$

$$j20 = -\bar{I}' + (2 + j)(2 + j)\bar{I}' - 20(2 + j)$$

$$j20 = -\bar{I}' + (4 + 2j + 2j - 1)\bar{I}' - 20(2 + j) \qquad j20 + 20(2 + j) = \bar{I}'(4 + 4j - 1 - 1)$$

$$j20 + 40 + j20 = \bar{I}'(2 + 4j) \Rightarrow 40 + j40 = \bar{I}'(2 + 4j) \Rightarrow \bar{I}' = \frac{40 + j40}{2 + j4}$$

$$\bar{I}' = \frac{40\sqrt{2}\,e^{j45°}}{2\sqrt{5}\,e^{j63,43}} = 12,65\cdot e^{-j18,43}\,A \Rightarrow \boxed{I_1 = 12,65\,A} \quad \text{mA} \quad \bar{J}_1 = 12,65\cdot e^{-j18,43}\,A$$

$$\bar{I}'' = (2 + j)\bar{I}' - 20 = (2 + j)(12 - j4) - 20 = 24 - j8 + j12 + 4 - 20 = 8 + j4$$

$$\bar{I}'' = 8,94\cdot e^{j26,56°}\,A \Rightarrow \boxed{I_2 = 8,94\,A} \quad \text{mA} \quad \bar{J}_2 = -8,94\,e^{j26,56°}\,A$$

$$\bar{J}_0 = \bar{I}' - \bar{I}'' = 12,65\cdot e^{-j18,43} - 8,94\cdot e^{j26,56°} =$$

$$(12 - j4) - (8 + j4) = 4 - j8 = 8,94\,e^{-j63,45°}$$

$$\boxed{\bar{J}_0 = 8,94\,e^{-j63,45°}\,A} \quad \text{mA} \quad \bar{J}_0 = 8,94\cdot e^{-j63,45°}\,A$$

- možemo U_{ab}, U_{bd} i U_{bn}

$\bar{U}_{ac} = ?$ $\varphi_a = \varphi_c - \bar{J}_1(R_1 + jX_1) + \bar{J}_2 \cdot jX_m \Rightarrow \bar{U}_{ac} = 12{,}65 \cdot e^{-j12{,}56} \cdot (5+j10) - 8{,}94\, e^{j26{,}56^\circ}$

$\bar{U}_{ac} = 12{,}56 \cdot e^{-j12{,}56} \cdot 11{,}2 \cdot e^{j63{,}43} - 8{,}94 \cdot e^{j26{,}56^\circ}$

$\bar{U}_{ac} = 140{,}672 \cdot e^{j41{,}87} - 8{,}94 \cdot e^{j26{,}56^\circ} = 99{,}7 + j98{,}77 - 8 - j4 = 91{,}7 + j94{,}77$

$\bar{U}_{ac} = 91{,}7 + j94{,}77 = 132 \cdot e^{j46^\circ}$ V

$\bar{U}_{bd} = ?$ $\varphi_b = \varphi_d - \bar{J}_2(R_2 + jX_2) - \bar{J}_1 \cdot jX_m \Rightarrow U_{bd} = -\bar{J}_2(R_2 + jX_2) - \bar{J}_2 jX_m$

$\bar{U}_{bd} = (5+j10) \cdot 8{,}94 \cdot e^{j26{,}56^\circ} - 12{,}65 \cdot e^{-j18{,}56} \cdot 5 \cdot e^{j90} = 99{,}9 \cdot e^{j90^\circ} - 62{,}8 \cdot e^{j71{,}44^\circ}$ V

$\bar{U}_{bd} = j100 - 20 - j59{,}53 = -20 + j40{,}47 \Rightarrow \bar{U}_{bd} = 45 \cdot e^{j116{,}3^\circ}$ V

$\bar{J}_{bn} = ?$ $\varphi_b = \varphi_n + \bar{J}_0(R_0 - jX_0) \Rightarrow \bar{U}_{bn} = \bar{J}_0(R_0 - jX_0) = 8{,}94 \cdot e^{-j63{,}45^\circ} \cdot (5 - j5)$

$\bar{U}_{bn} = 8{,}94 \cdot e^{-j63{,}45^\circ} \cdot 7{,}07 \cdot e^{-j45} = 63{,}2 \cdot e^{-j108{,}2}$ V

aga koja troši izvor na naponom U_1 je = ?

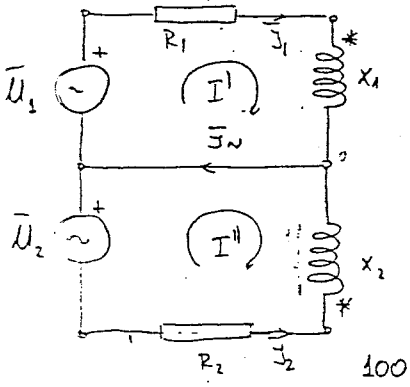
$P_1 = Re\left\{\bar{U}_1 \cdot \bar{J}_1^*\right\} = Re\left\{100 \cdot \left(12{,}56 \cdot e^{+j12{,}56^\circ}\right)\right\} = 1190W$

$P_2 = Re\left\{\bar{U}_2 \cdot \bar{J}_2^*\right\} = Re\left\{100 \cdot e^{j90} \cdot (-8{,}95) \cdot e^{-j26{,}56}\right\} = Re\left\{-895 \cdot e^{j63{,}44^\circ}\right\}$

$P_u = P_1 + P_2 = 1190 + 400 = 1590W$ $\boxed{P = 1590W}$

$P_{bn} = Re\left\{\bar{U}_{bn} \cdot \bar{J}_0^*\right\} = Re\left\{63{,}2 \cdot e^{-j108{,}2} \cdot 8{,}94 \cdot e^{j63{,}45}\right\} = Re\left\{565 \cdot e^{-j44{,}8}\right\} = 400W$

- Struju I_0 pomoću Kirchhoffovog teorema. $\bar{I}_N = ?$



$\bar{U}_1 = I'(R_1 + jX_1) - I'' \cdot jX_m$

$\bar{U}_2 = I''(R_2 + jX_2) - I' \cdot jX_m$

$100 = I'(5+j10) - I'' \cdot j5$

$j100 = I''(5+j10) - I' \cdot j5 \Rightarrow I'' = \dfrac{I' \cdot j5 + j100}{5+j10} =$

$100 = I'(5+j10) - j5 \cdot \dfrac{I' \cdot j5 + j100}{5+j10} = I'(5+j10) - j\dfrac{I' j5 + j100}{1+j2}$

$100 = I'(5+j10) + \dfrac{5I' + 100}{1+j2}$

$100 + j200 = 5I' + j10I' + 5I' + 100$ $I'(10+j10) = j200$ $I' = \dfrac{j200}{10+j10} = I' = \dfrac{200\, e^{j90^\circ}}{10\sqrt{2}\, e^{j45^\circ}}$

$I' = 14{,}14 \cdot e^{j45^\circ}$ A

$I'' = \dfrac{I' \cdot j5 + j100}{5+j10} = \dfrac{j5(10+j10) + j100}{5+j10} = \dfrac{j50 - 50 + j100}{5+j10}$

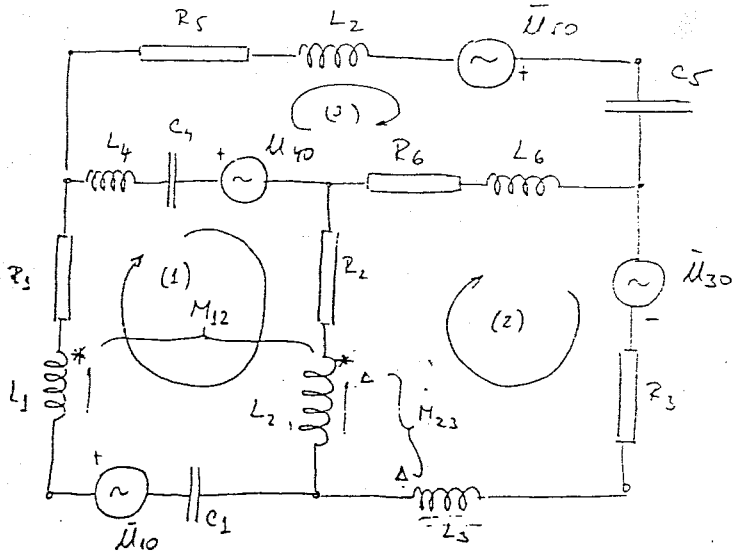
$I'' = \dfrac{-50 + j150}{5+j10} = \dfrac{-10+j30}{1+j2} = \dfrac{31{,}62 \cdot e^{j108{,}43}}{2{,}23 \cdot e^{j63{,}45}} = 14{,}179\, e^{j45^\circ}$

$\bar{I}_N = I' - I'' = (14{,}14 - 14{,}179) = -0{,}04\, e^{j45^\circ}$ A

ZADATAK 8 →

Za mrežu na slici treba napisati jednadžbe Maxwellovih konturnih struja

526



66

ZA KONTURNU STRUJU $I^{(1)}$

$$\bar{U}_{10} - \bar{U}_{40} = I^{(1)}\left(R_1+R_4+j\omega L_1+j\omega L_4+j\omega L_2 - j\frac{1}{\omega C_1} - j\frac{1}{\omega C_4} - 2j\omega M_{12}\right) - I^{(2)}\left(j\omega L_4 - j\frac{1}{\omega C_4}\right)$$
$$+ I^{(2)}\left[-R_4 - j\omega L_2 - j\omega M_{23}\right]$$

ZA KONTURNU STRUJU $I^{(2)}$

$$\bar{U}_{30} = I^{(2)}\left[R_4+R_3+R_6+j\omega L_2+j\omega L_3+j\omega L_6+2j\omega M_{23}\right] + I^{(3)}\left[-R_6-j\omega L_6\right] - I^{(1)}\left[R_4+j\omega L_2\right] - I^{(1)} j\omega M_{23}$$
$$+ I_1^{(1)} j\omega M_{12}$$

ZA KONTURU $I^{(3)}$

$$\bar{U}_{20} + \bar{U}_{10} = I^{(3)}\left[R_5+R_6+j\omega L_2 - j\frac{1}{\omega C_5} + j\omega L_6 - j\frac{1}{\omega C_1} + j\omega L_1\right] - I^{(2)}\left(R_6+j\omega L_6\right) - I^{(1)}\left(j\omega L_4 - j\frac{1}{\omega C_4}\right)$$

TOČNO

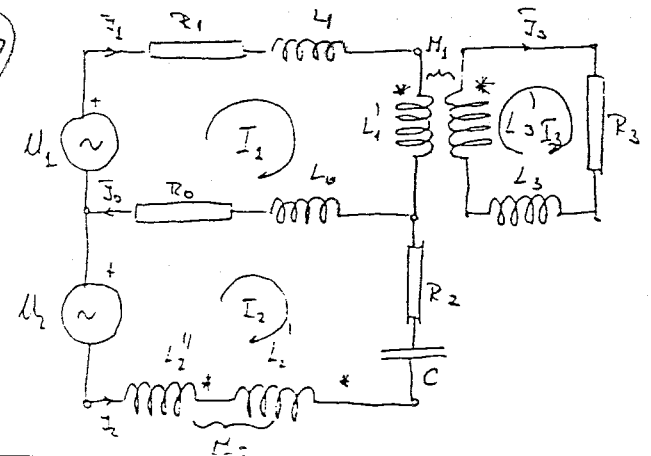
ZADATAK 8 ⇒ $U_1 = 10V$, $U_2 = 14{,}1V$ i $f = 5\,kHz$. Napomena U_1 i U_2 fazno su pomaknuti za kut od 45°.

(U_2 prednjači naponu U_1); $R_1 = R_2 = 2\Omega$ $L_1 = L_1' = L_0 = L_2' = L_2'' = 0{,}0318\,mH$. $R_0 = R_3 = 1\Omega$

$C = 5{,}3\mu F$, $L_3 = L_3' = M_1 = 0{,}0636\,mH$, $M_2 = 0{,}0159\,mH$. Odredite struje I_1, I_2, I_3 i I_0

Snagu koju uzima mreža.

5,2)



DIO ZADATKA MOŽDU OSTVARITI

$\bar{I}_1 = \frac{17 \cdot 10\angle 17 \cdot e^{j\,2,7}}{25\angle 3 \cdot e^{j\,2,160°}} = 2,14 \cdot e^{j\,\dots}$ ÷ $|I_1| = |\bar{I}_1| = 2,14\ A$ W W

$\bar{I}_3 = \frac{\bar{U}_{23}}{-j1} = \frac{2,14 \cdot e^{j\,3,7} \cdot 2 \cdot e^{j90°}}{4,123\ e^{j76°}} = 1,04\, e^{j\,14°}\ A$

$\bar{I}_2 = 4,73\ A$ $\boxed{I_2 = 4,73\ A}$ $I_3 = 1,04\ A$

$\bar{I}_0 = \bar{I}_1 - \bar{I}_2 \Rightarrow \boxed{\bar{I}_0 = 8,95\ A}$ $\underline{\bar{I}_1 = 2,14 \cdot e^{j\,3,44°}}$

J

$\bar{I}_2 = \frac{10 + j10 + (1+j)\cdot 2,11 + j0,35}{3-j2} = \frac{10 + 2,11 + j0,35 + j\,2,11 - 0,35}{3-j2}$

$\bar{I}_2 = \frac{11,76 + j12,46}{3-j2} = \frac{17,13 \cdot e^{j\,0,65°}}{3,605\, e^{-j\,33,7°}} = 4,75\, e^{j\,80,35°}\ A$ $U_1 = 10V$

$U_2 = 14,1 \cdot e^{j45°}$

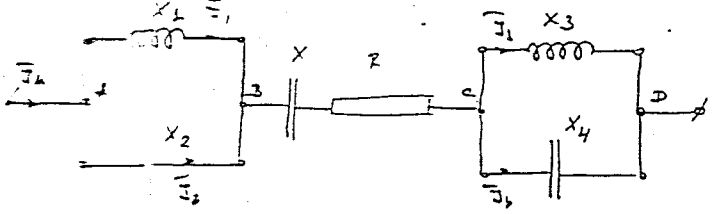
$P = ?$ koju mreža ubima $\bar{S} = U_1 \cdot I_1^* = 10 \cdot 2,14 \cdot e^{-j\,3,44°}$

$\bar{S} = U_2 \cdot I_2^* = 14,1 \cdot e^{j45°} \cdot 4,75 \cdot e^{-j80,35°} = 67 \cdot e^{-j35,35°}$

$P_{1R} = 21,11\ W$ $P_{2R} = 54,59\ W$ $\boxed{P_u = 76\ W}$ W W

(5,5Ω)

ZADATAK 6 ⇒ Na mrežu je priključen napon od 120V. Parametri mreže imaju ove vrijednosti: $X = -20\Omega$ $Z = 50\Omega$ $X_1 = 40\Omega$ $X_2 = -80\Omega$ $X_3 = 60\Omega$ i $X_4 = -30\Omega$. Odredite struje u granama mreže



$\bar{U} = 120V$

$\bar{Z}_u = ?$

$\bar{Z}_u = \frac{-j40 \cdot j80}{j40 - j80} - 50 - j20 - \frac{-j60 \cdot j30}{j60 - j30} = \frac{3200}{-j40} + 50 - j20 + \frac{1800}{j30}$

$\bar{Z}_u = \frac{120\angle \cdot j}{4j} - 50 - j20 - \frac{120 \cdot j}{30} = 30j - 50 - j20 - 60j = 50\ \Omega$

$\bar{I}_u = \frac{\bar{U}}{\bar{Z}_u} = \frac{120}{50} = 2,4A$ $\bar{Z}_{ab} = 20_1$ $\bar{Z}_{cD} = -60j$

$\bar{U}_{ab} = \bar{I}_u \bar{Z}_{ab} = 2,4 \cdot 20j = j\,\dots$ $\bar{U}_{cD} = \bar{I}_u \cdot \bar{Z}_{cD} = -j\,144V$

$\bar{I}_1 = \frac{j\,\dots}{j\,\dots} = 4,\dots A$ $\bar{I}_2 = \frac{\dots}{-\dots} = -2,\dots$ $\bar{I}_3 = \frac{-j144}{j60} = -2,4\ A$ $\bar{I}_4 = \frac{-j144}{-j30} = 4,8A$

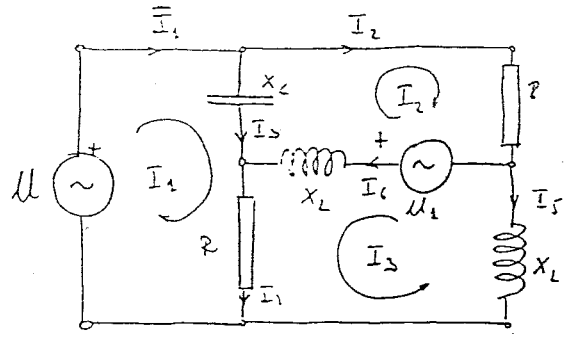
II NAČIN

Pretpostavimo da je $\bar{I}_2 = 1A$ na referentnoj osi.

$\bar{U}_{23} = \bar{I}_2 \cdot jX_2 = 1 \cdot j\,\dots = j40\ V$ $\bar{I}_2 = \frac{j40}{-j80} = -\frac{1}{2}\ A$

$\bar{I}_u = \bar{I}_1 + \bar{I}_2 = 1 - \frac{1}{2} = \frac{1}{2}\ A$

ZADATAK: Zadana je mreža u kojoj je $u = u_1 = 50V$, napon u_1 je za 90° u fazi ispred napona u; $R = X_L = X_C = 5\Omega$. Treba naći struje u mreži.



$u = 50 \cdot e^{j0°} V$

$u_1 = 50 \cdot e^{j90°} V$

$R = X_L = X_C = 5\Omega$

Naći struje u mreži.

$$\bar{u} = \bar{I}_1(-jX_C + R) - \bar{I}_2 \cdot (-jX_C) + \bar{I}_3 \cdot R$$

$$\bar{u}_1 = \bar{I}_2(jX_L - jX_C + R) - \bar{I}_1 \cdot (-jX_C) + \bar{I}_3 \cdot jX_L$$

(2.1) $$\bar{u}_1 = \bar{I}_3(jX_L + jX_L + R) + \bar{I}_1 \cdot R + \bar{I}_2 \cdot jX_L$$

$$50 = \bar{I}_1(5 - j5) + \bar{I}_2 \cdot j5 + 5\bar{I}_3 \quad /:5$$

$$j50 = \bar{I}_2 \cdot 5 + \bar{I}_1 \cdot j5 + \bar{I}_3 \cdot j5 \quad /:5$$

$$j50 = \bar{I}_3(j5 + j5 + 5) + \bar{I}_1 \cdot 5 + \bar{I}_2 \cdot j5 \quad /:5$$

$$10 = \bar{I}_1(1 - j) + \bar{I}_2 \cdot j + \bar{I}_3$$

$$j10 = \bar{I}_2 + j\bar{I}_1 + j\bar{I}_3$$

$$j10 = \bar{I}_3(1 + j2) + \bar{I}_1 + j\bar{I}_2$$

$$D = \begin{vmatrix} (1-j) & j & 1 & (1-j) & j \\ +j & 1 & + & 1 & 1 \\ 1 & + & (1+j2) & 1 & + \end{vmatrix}$$

$$D = (1-j)(1+j2) - 1 - 1 - 1 - j^2(1-j) - j^2(1+j2) = (1-j)(1+j2) - 3 + (1-j) - (1-j)$$

$$D = 1 + j2 - j - j^2 2 - 3 + 1 - j + 1 + j2 = 1 + j2 - j + 2 - 3 + 1 - j + 1 - j2 = 2 + j2$$

$$D_1 = \begin{vmatrix} 10 & j & 1 & 10 & j \\ j10 & 1 & j & j10 & 1 \\ j10 & j & 1+j2 & j10 & j \end{vmatrix}$$

$$= 10(1+j2) - j10 - 10 - j10 - 10 + 10 + j$$

$$= 10 + j20 - j10 - j10 + 10 - 10$$

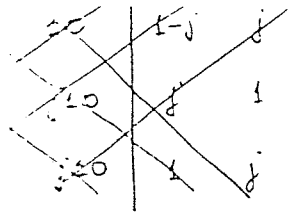
$$= 20 + j20 \qquad I_2 = 20 - j20$$

$$D_2 = \begin{vmatrix} 1-j & 10 & 1 & (1-j) & 10 \\ j & j10 & j & j10 & j10 \\ 1 & j10 & (1+j2) & 1 & j10 \end{vmatrix}$$

$$= (1-j) \cdot j10 \cdot (1+j2) - j10 - 10 - j10 - 10(1-j)$$

$$- j10(1-j2)$$

$$D_2 = j10(1 + j2 - j + 2) + j10 - 10 - j10 + 10 - j10 - j10 + 20$$

$D_2 =$ 

$= j10(1-j) - 10 - 10 - 10 + 10(1-j) + j10$

$= j10 + 10 - 10 - 10 - 10 + 10 - j10 + j10 = -10 + j10$

$\bar{I}_1 = \frac{D_1}{D} = \frac{10+j10}{2+j2} = \frac{10(1+j)}{2(1+j)} = 10\,A$ $\qquad \bar{I}_2 = \frac{D_2}{D} = \frac{10+j10}{2+j2} = \frac{10(1+j)}{2(1+j)} = 5\,A$

$\bar{I}_3 = \frac{D_3}{D} = \frac{-10+j10}{2+j2} = \frac{-10(1-j)}{2(1+j)} = -5 \cdot \frac{1-j}{1+j} \cdot \frac{1-j}{1-j} = -5 \cdot \frac{(1-j)^2}{1+1} =$

$= -5 \cdot \frac{1-2+j^2}{2} = -5\,\frac{-j2}{2} = +5 \cdot \frac{+j2}{2} = j5\,A$ $\qquad \boxed{\bar{I}_3 = j5\,A}$

$\bar{I}_1 = 10\,A \qquad I_3 = I_1 - I_2 = 5\,A$

$I = 5\,A \qquad \bar{I}_4 = \bar{I}_1 + \bar{I}_3 = 10 + j5 \Rightarrow I_4 = \sqrt{100+25} = 11{,}1\,A$

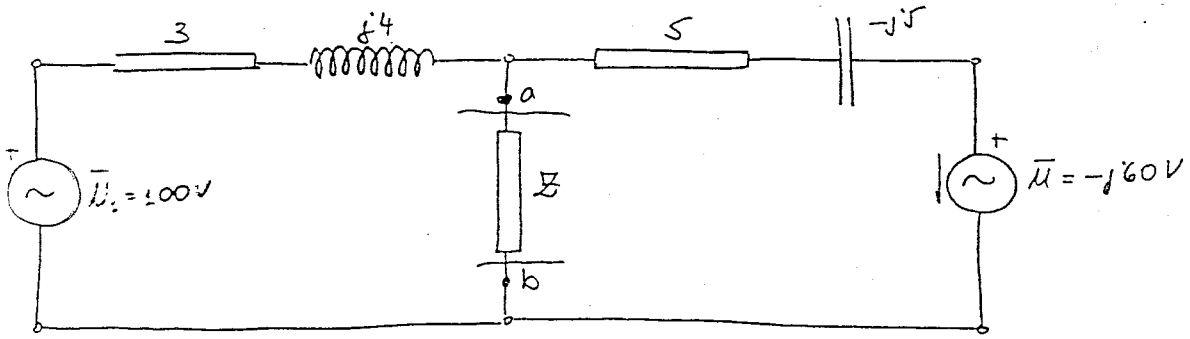
$\bar{I}_5 = -\bar{I}_3 = -j5\,A \Rightarrow \boxed{I_5 = 5\,A}$

$\bar{I}_6 = \bar{I}_2 + \bar{I}_3 = 5 + j5 \Rightarrow \boxed{I_6 = 7{,}071\,A}$

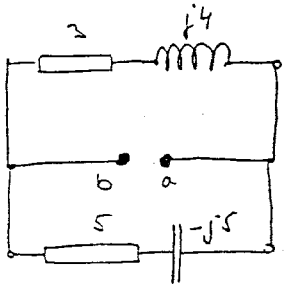
I_1	$= 10\,A$
I_2	$= 5\,A$
I_3	$= 5\,A$
I_4	$= 11{,}1\,A$
I_5	$= 5\,A$
I_6	$= 7{,}071\,A$

ZADATAK: Izračunajte snagu na impedanciji $\bar{Z} = 2 + j4$

5.44. koja je priključena između tačaka a i b



$\bar{E}_T = ?$ $\qquad \bar{E}_T = ?$ $\qquad \bar{Z}_T = ?$



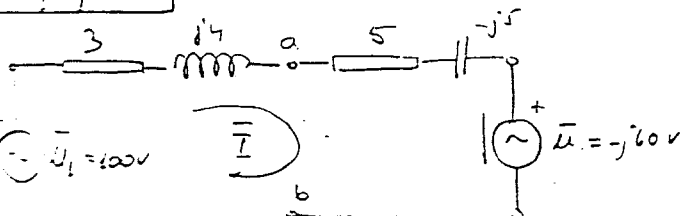
$\bar{Z}_T = \frac{(3+j4)(5-j5)}{3+j4+5-j5} = \frac{15-j15+j20+20}{8-j}$

$\bar{Z}_T = \frac{35+j5}{8-j} = \frac{35{,}355 \cdot e^{j8{,}130^\circ}}{8{,}06225 \cdot e^{-j7{,}125^\circ}}$

$\bar{Z}_T = 4{,}38525 \cdot e^{j15{,}255^\circ}$

$\boxed{\bar{Z}_T = 4{,}23 - j1{,}154}$

$\bar{E}_T = ?$



$\bar{U}_1 - \bar{U} = \bar{I}(3+j4+5-j5)$

$\bar{I} = \frac{100-(-j60)}{8-j}$

$\bar{I} = \frac{100+j60}{8-j} = \frac{116{,}62 \cdot e^{j30{,}96^\circ}}{8{,}06225 \cdot e^{-j7{,}125^\circ}}$

$\overline{I} = 14{,}464 \cdot e^{j38{,}085}$ A

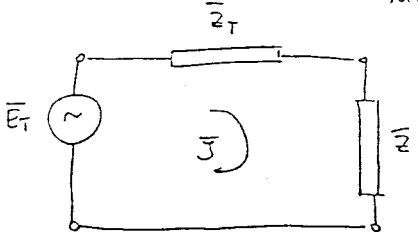
$\varphi_a = \varphi_b + \overline{u} + \overline{I}(5-j5)$

$\overline{u}_{ab} = \overline{u} + \overline{I}(5-j5) = -j60 + 14{,}464 \cdot e^{j38{,}085^\circ} \cdot 5(1-j)$

$\overline{u}_{ab} = -j60 + 5 \cdot 14{,}464 \cdot e^{j38{,}085^\circ} \cdot \sqrt{2} \cdot e^{-j45^\circ} = -j60 + 102{,}275 \cdot e^{-j6{,}96}$

$\overline{u}_{ab} = -j60 + 101{,}53 - j12{,}375$

$\overline{u}_{ab} = 101{,}53 - j72{,}375$ V $= \overline{E}_T$



$$\overline{J} = \frac{\overline{E}_T}{\overline{Z}_T + \overline{Z}} = \frac{101{,}53 - j72{,}375}{4{,}23 + j1{,}154 + 2 + j4} = \frac{101{,}53 - j72{,}375}{6{,}23 + j5{,}154}$$

$$\overline{J} = \frac{121{,}68 \cdot e^{-j35{,}48}}{8{,}0855 \cdot e^{j39{,}6}}$$

$\overline{J} = 15{,}42 \cdot e^{-j75{,}08^\circ}$ A

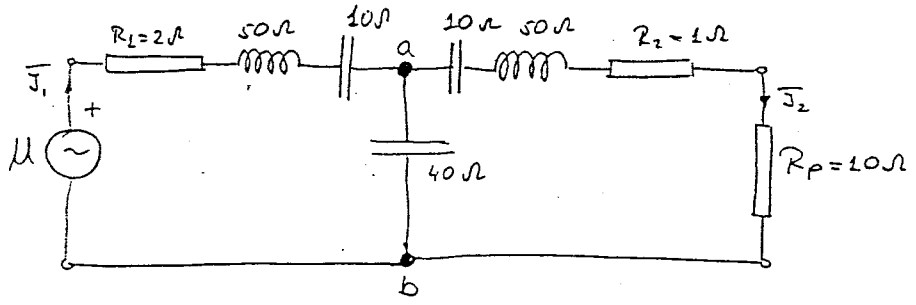
$\overline{u}_z = \overline{J} \cdot \overline{Z} = 15{,}42 \cdot e^{-j75{,}08^\circ} \cdot 4{,}472 \cdot e^{j63{,}435^\circ} = 69 \cdot e^{-j11{,}645}$ V

[2] $P = Re\{\overline{u}_z \cdot \overline{J}^*\} = 68 \cdot e^{-j11{,}645} \cdot 15{,}42 \cdot e^{+j75{,}08^\circ} = Re\{1048{,}56 \cdot e^{j63{,}435}\}$

$P = 469$ W $\quad P \approx 470$ W $\quad \boxed{P = 0{,}470 \text{ kW}}$

[4] $P = Re\{\overline{Z} \cdot I^2\} = (2+j4) \cdot (15{,}416^2) = 2 \cdot 237{,}649 = 475{,}298$ W

ZADATAK: Odredite koeficijent korisnog djelovanja mreže. Korisno opterećenje je
5,57. otpor R_p, a otpori R_1 i R_2 unutrašnji otpori izvora i unutrašnji otpor pojedinih elemenata zadani su na shemi.



$\overline{u} = u \cdot e^{j0^\circ}$

Koristit ćemo Milmanov teorem

$$\overline{U}_{ab} = \frac{\overline{u} \cdot \frac{1}{2+j40}}{\frac{1}{2+j40} + \frac{1}{-j40} + \frac{1}{11+j40}} = \frac{u \cdot \frac{1}{40{,}05 \cdot e^{j87{,}14^\circ}}}{\frac{1}{40{,}05 \cdot e^{j87{,}14^\circ}} + \frac{1}{40 \cdot e^{-90^\circ}} + \frac{1}{41{,}485 \cdot e^{j74{,}623}}}$$

$$\overline{u}_{ab} = \frac{u \cdot 0{,}025 \cdot e^{-j87{,}14^\circ}}{0{,}025 \cdot e^{-j87{,}14^\circ} + 0{,}025 \cdot e^{j90} + 0{,}0241 \cdot e^{-j74{,}623}} = \frac{u \cdot 0{,}025 \cdot e^{-j87{,}14^\circ}}{0{,}00124 - j0{,}0249 + j0{,}025 + 0{,}0064 - j0{,}0232}$$

$$\overline{u}_{ab} = \frac{u \cdot 0{,}025 \cdot e^{-j87{,}14^\circ}}{0{,}00764 - j0{,}02313} = \frac{u \cdot 0{,}025 \cdot e^{-j87{,}14^\circ}}{0{,}024355 \cdot e^{-j71{,}72^\circ}} = 1{,}026\,u \cdot e^{-j15{,}42}$$ V

$$\bar{I}_2 = \frac{\bar{U}_{ab}}{11+j40} = \frac{1,026\,U\,e^{-j15,42}}{41,485\cdot e^{j74,62^\circ}} = 0,0247\,U\cdot e^{-j90^\circ}\ A$$

$$\bar{U}_p = \bar{I}_2 \cdot R_p = 0,0247\,U\cdot e^{-j90^\circ}\cdot 10 = 0,247\,U\cdot e^{-j90^\circ}\ V \qquad \underline{\bar{U}_p = 0,247\,U\cdot e^{-j90^\circ}\ V}$$

$$\bar{I}_1: \quad \varphi_a - \varphi_b = \bar{U} - \bar{I}_1(2+j40) \qquad \bar{U}_{ab} = \bar{U} - \bar{I}_1(2+j40)$$

$$\bar{I}_1(2+j40) = \bar{U} - \bar{U}_{ab} \Rightarrow \bar{I}_1 = \frac{\bar{U}-\bar{U}_{ab}}{2+j40} = \frac{U - 1,026\,U\,e^{-j15,42^\circ}}{40,05\cdot e^{j87,14^\circ}}$$

$$\bar{I}_1 = \frac{U\left(1-1,026\cdot e^{-j15,42^\circ}\right)}{40,05\cdot e^{j87,14^\circ}} = \frac{U\left[1-1,026(0,964-j0,265)\right]}{40,05\cdot e^{j87,14^\circ}}$$

$$\bar{I}_1 = \frac{U\left[1-1+j0,273\right]}{40,05\cdot e^{j87,14^\circ}} = \frac{U\cdot \overbrace{j0,273}^{0,273\cdot e^{j90^\circ}}}{40,05\cdot e^{j87,14^\circ}} = 0,00682\,U\cdot e^{j2,86^\circ}\ A$$

$$P_2 = Re\left\{\bar{U}_T\cdot \bar{I}_2^*\right\} = Re\left\{0,247\cdot U\cdot e^{-j90^\circ}\cdot 0,0247\cdot U\cdot e^{+j90^\circ}\right\} = Re\left\{0,0061\,U^2\cdot e^{j0^\circ}\right\}$$

$$P_2 = 0,0061\,U^2$$

$$P_1 = Re\left\{\bar{U}\cdot \bar{I}_1^*\right\} = Re\left\{U\cdot 0,00682\cdot U\cdot e^{-j2,86^\circ}\right\} = Re\left\{0,00682\cdot U^2\cdot e^{-j2,86^\circ}\right\}$$

$$= 0,00682\cdot U^2\cdot 0,9987$$

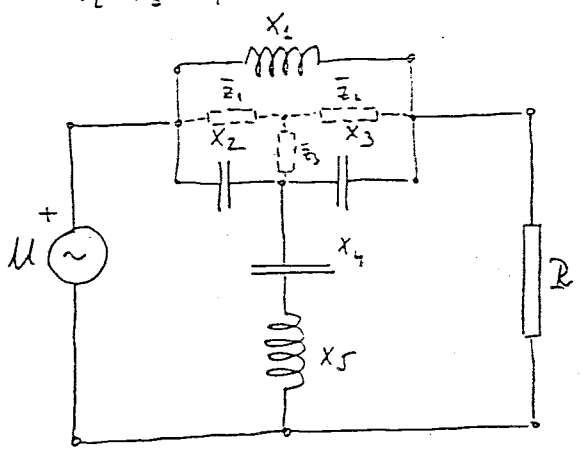
$$= 0,00681\,U^2 = P_1$$

$$\frac{P_2}{P_1}\cdot 100\% = \frac{0,0061\,U^2}{0,00681\,U^2} = 89,57\% \qquad \frac{P_2}{P_1} = 90\%$$

ZADATAK: Izračunajte snagu na otporu R mreže koja je sastavljena prema shemi

5.70 ako napon na ulaznim stezaljkama iznosi $U = 120\,V$, a $X_1 = 120\,\Omega$

$X_2 = X_3 = X_4 = R = 40\,\Omega$ i $X_5 = 80\,\Omega$. Radni otpori grana mogu se zanemariti.



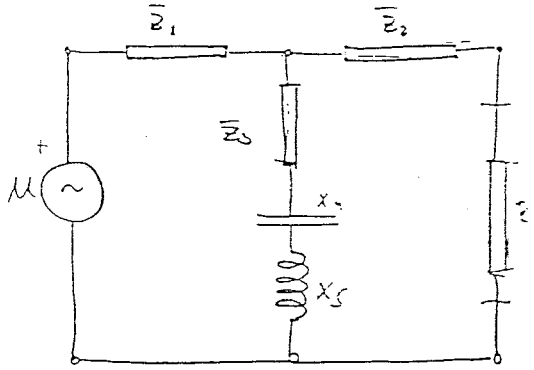
$$\bar{Z}_1 = \frac{j\cdot 120\cdot(-j40)}{j120-j40-j40} = \frac{+4800}{j40}\cdot\frac{j}{j}$$

$$\bar{Z}_1 = \frac{j4800}{-40} = -j120\,\Omega \qquad \boxed{\bar{Z}_1 = -j120\,\Omega}$$

$$\boxed{\bar{Z}_2 = -j120\,\Omega}$$

$$\bar{Z}_3 = \frac{-j40\cdot(-j40)}{j40} = \frac{j40\cdot j40}{j40} = j40\,\Omega$$

$$\boxed{\bar{Z}_3 = j40\,\Omega}$$



Thevenin's theorem

$\bar{E}_T = ?$ $\bar{Z}_T = ?$

$$\bar{Z}_T = \frac{(-j120)\cdot(\cancel{+80} - \cancel{j80} + j80)}{-j120 + \cancel{j80} - \cancel{j80} + j80} - j120$$

$$\bar{Z}_T = \frac{-\cancel{j120}\cdot j80}{-\cancel{j40}} - j120$$

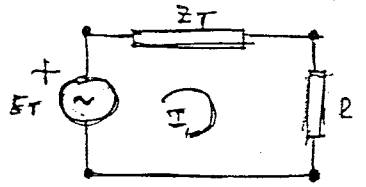
$\bar{Z}_T = j240 - j120 = j120\ \Omega$ $\boxed{\bar{Z}_T = j120\ \Omega}$

$$\bar{E}_T = \frac{\bar{U}}{-j120 + \cancel{j40} - \cancel{j40} + j80}\cdot(\cancel{40} - \cancel{j40} + j80) = \frac{\bar{U}}{-j40}\cdot j80 = -\bar{U}\cdot\frac{j80}{j40} = -\bar{U}\cdot 2$$

$\bar{E}_T = -2\bar{U}$

$$\bar{U} = U\cdot e^{j0^\circ} = 120\cdot e^{j0^\circ}\ V$$

$\bar{E}_T = -240\cdot e^{j0^\circ}\ V$



$$\bar{I} = \frac{\bar{E}_T}{\bar{Z}_T + R} = \frac{-240}{j120 + 40} = \frac{-240}{126{,}49\cdot e^{j71{,}56^\circ}}$$

$\boxed{\bar{I}_1 = -1{,}897\cdot e^{-j71{,}56^\circ}\ A}$

$\bar{U}_R = \bar{I}_1\cdot R = -75{,}88\cdot e^{-j71{,}56^\circ}\ V$

$P_R = Re\{\bar{U}_R\cdot\bar{I}_1^*\} = Re\{-75{,}88\ e^{-j71{,}56}\cdot(-1{,}897)\cdot e^{+j71{,}56}\}$

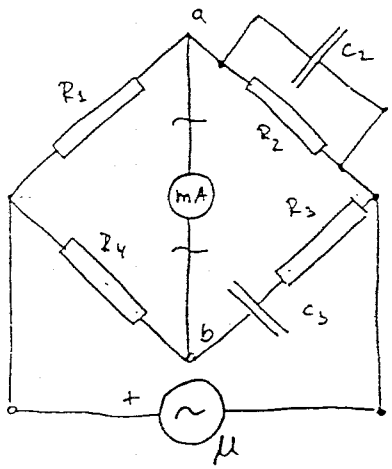
$P_R = Re\{143{,}944\cdot e^{j0^\circ}\}$

$\boxed{P_R = 144\ W}$ $\Rightarrow$ $\boxed{P_R = 0{,}144\ kW}$

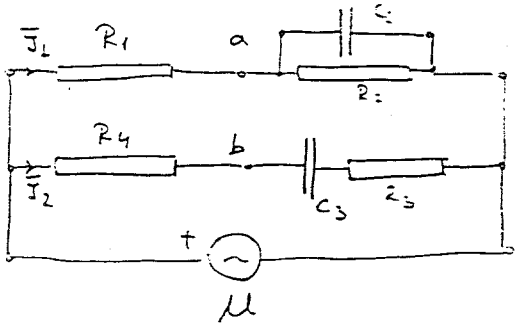
ZADATAK: Kolika je frekvencija napona priključenog na most ako je poznato

5.62 poznata mjerna parametra: $R_1 = 200\ \Omega$ $R_2 = 100\ \Omega$ $C_2 = 2\ \mu F$ $R_4 = 500\ \Omega$

$R_3 = 480\ \Omega$ i $C_3 = 2{,}22\ \mu F$



$\Rightarrow \bar{U}_{ab} = 0$ Primjenimo Thevenin-ov teorem $\bar{U}_{ab} = \bar{E}_T = 0$

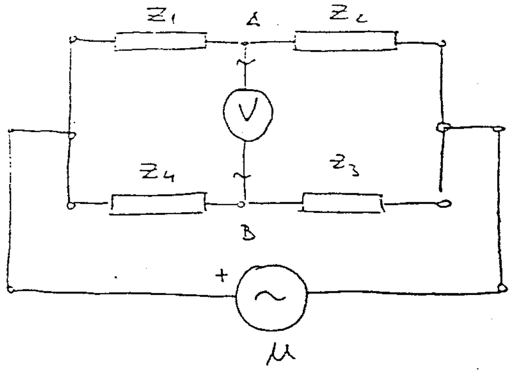


$E_T = 0 = U_{ab} = 0\ V$

ZADATAK: Mreža je priključena na izvor ~ napona. Napon izmjeren elektrostatskim voltmetrom

5.68 između tačaka A i B iznosi je 60V. Kako utjecaj otpora voltmetra možemo zanemariti. Koliko će biti pokazivanje voltmetra sa željezom priključenog na te tačke ako mu je radni otpor $R_V = 3650\Omega$ i induktivni $X_V = 180\Omega$

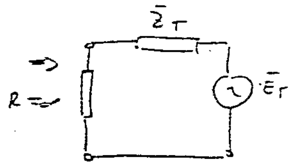
$\bar{Z}_1 = 300 + j800 \quad \bar{Z}_2 = 700 - j800 \quad \bar{Z}_3 = -j1000 \quad i \quad \bar{Z}_4 = 1000\Omega$



Primjenit ćemo Tevenenov teorem.

Za elektrostatski voltmetar ($I=0$) uzimamo kao da je njegov otpor $R=\infty$ pa je

$U_V = \bar{E}_T$



Kad stavimo 2. instrument kroz njega će poteći struja. Primjenimo Tevenenov teorem

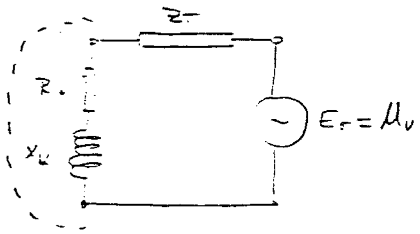
$|\bar{E}_T| = U_{V\,elektrostatsk.}$

$$\bar{Z}_T = \frac{\bar{Z}_1 \cdot \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2} + \frac{\bar{Z}_3 \cdot \bar{Z}_4}{\bar{Z}_3 + \bar{Z}_4} = \frac{(300+j800)(700-j800)}{300+j800+700-j800} - \frac{-j1000 \cdot 1000}{1000 - j1000}$$

$$\bar{Z}_T = \frac{100(3+j8)\cdot 100(7-j8)}{1000} - \frac{j1000 \cdot 1000}{1000(1-j)} = 10(3+j8)(7-j8) - \frac{j1000}{1-j} \cdot \frac{1+j}{1+j}$$

$$\bar{Z}_T = 10[21 - j24 + j56 + 64] - \frac{j1000 - 1000}{2} = 10(85 - j32) - (j500 - 500)$$

$$\bar{Z}_T = 850 - j320 - j500 + 500 = 1350 - j820\ \Omega$$



$$\bar{Z}_u = \bar{Z}_T + R_V + jX_V = 1350 - j820 + 3650 + j180$$

$$\bar{Z}_u = 5000 - j640\ \Omega \qquad \bar{E}_T = U_V = 60\,V$$

$$\bar{I} = \frac{60}{5000 - j640} = \frac{6}{500 - j64} \cdot \frac{500 + j64}{500 + j64} = \frac{3000 + j384}{250000 + 4096} = \frac{3000 + j384}{254096} = 0{,}012 \cdot e^{j7{,}3^\circ}$$

$$\hat{U}_{V_1} = 0{,}012 \cdot 3654{,}43566 = 43{,}8\,V$$

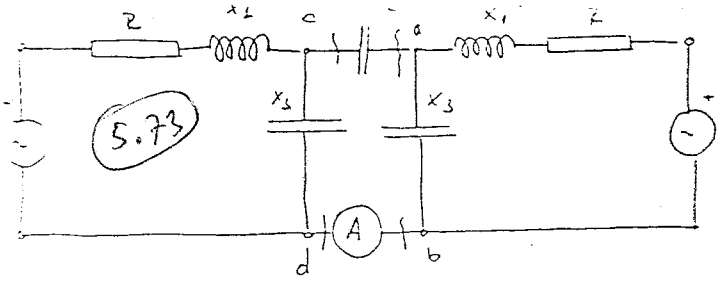
$$\boxed{U_{V_1} = 43{,}8\,V}$$

ZADATAK 8 ⇒ Odrediti pokazivanje ampermetra uključenog u električnu mrežu koja je

5.73 prikazano na slici

ZADANO: ⇒ $E_1 = E_2 = 200\,V$, a E_2 prednjači u fazi E_1 za $\varphi = 36°51'$

$R = 200\,\Omega \quad X_1 = X_3 = 40\,\Omega \quad i \quad X_2 = 20\,\Omega$. Unutrašnji otpor generatora možemo zanemariti.



5.73

$\bar{E}_1 = E_2 = 200\,V$

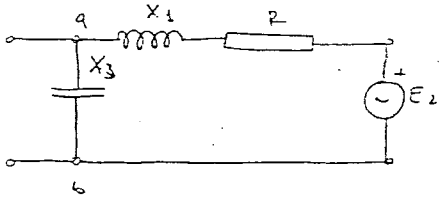
E_2 prednjoda u fazi $\bar{E}_1$ za $\varphi = 36°51'$

$R = 200\,\Omega$

$X_1 = X_3 = 40\,\Omega$

$X_2 = 20\,\Omega$

Zamjenimo dio desno od a i b ekvivalentnim Thevenineovim izvorom i otporom



$\bar{E}_1 = 200\,e^{j0°}$

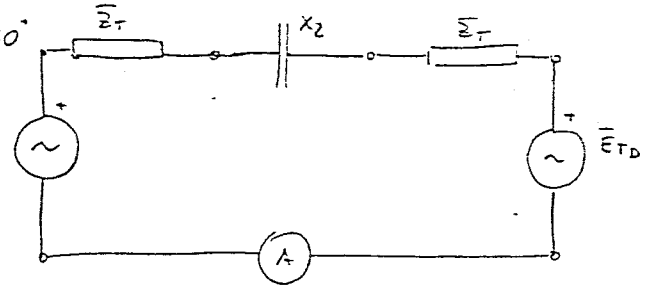
$\bar{E}_2 = 200\,e^{j36°51'} = 200\,e^{j36,85°}$

$$\bar{Z}_T = \frac{-j40\cdot(200+j40)}{-j40+200+j40} = \frac{-j8000+1600}{200} = 8-j40 \qquad \bar{Z}_T = 8-j40\,\Omega$$

$$\bar{E}_T = \frac{200\cdot e^{j36,85°}}{200+j40-j40}\cdot(-j40) = e^{j36,85°}\cdot 40\cdot e^{-j90°} = 40\cdot e^{-j53,15°}$$

Mrežu lijevo od otezaljki zamijenimo $\bar{E}_T$ i $\bar{Z}_T$ $\quad \bar{Z}_T = 8-j40\,\Omega$

$$\bar{E}_{TL} = \frac{200\,e^{j0°}}{200+j40-j40}\cdot(-j40) = e^{j0°}\cdot 40e^{-j90°} = 40\cdot e^{-j90°}$$



$\bar{E}_u = 2\cdot\bar{Z}_T - jX_2 = 2\cdot(8-j40)-j20$

$\bar{E}_u = 16-j80-j20 = 16-j100\,\Omega$

$Z_u = 101,27\,\Omega$

$\bar{E}_u = 40\,e^{-j53,15} - 40\,e^{-j90}$

$\bar{E}_u = 24-j32-40(0-j1)$

$\bar{E}_u = 24-j30+j40 = 24+j10$

$I = \frac{E_u}{Z_u} = 0,2567\,A$

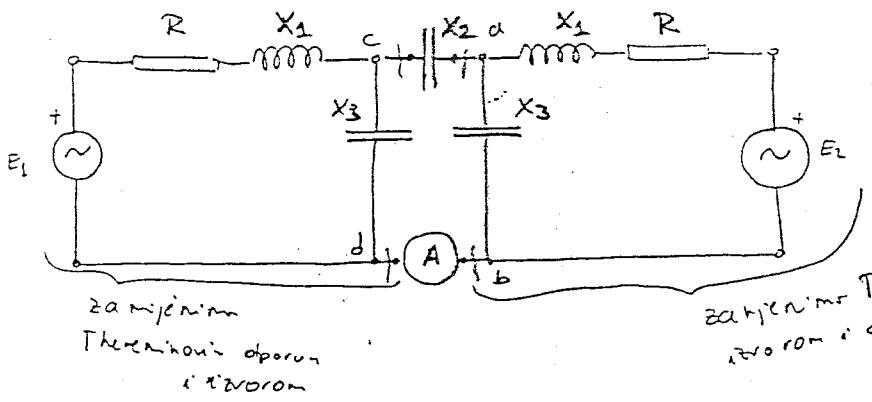
$E_u = 26\,V$

$I = 0,2567\,A$

~~ZADATAK~~: Odrediti pokazivanje ampermetra

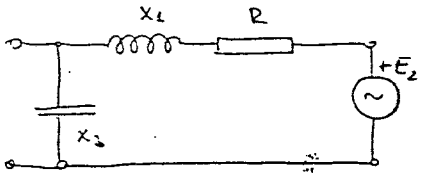
ZADANO: $E_1 = E_2 = 200\ V$ $\bar{E}_2$ prethodi u fazi E_1 za $\varphi = 36^\circ 51'$ $R = 200\ \Omega$

5.73 $X_1 = X_3 = 40\ \Omega$ i $X_2 = 20\ \Omega$. Unutrašnji otpori generatora se mogu zanemariti



$E_1 = 200\, e^{j0^\circ}$

$E_2 = 200\, e^{j36,85^\circ}$



$$\bar{Z}_T = \frac{(R + jX_1)\cdot(-jX_3)}{R + jX_1 - jX_3} = \frac{(200 + j40)\cdot(-j40)}{200 + j40 - j40}$$

$$\bar{Z}_T = \frac{-j40(200 + j40)}{200} = \frac{-j8000 + 1600}{200}$$

$\boxed{\bar{Z}_T = 8 - j40\ \Omega}$

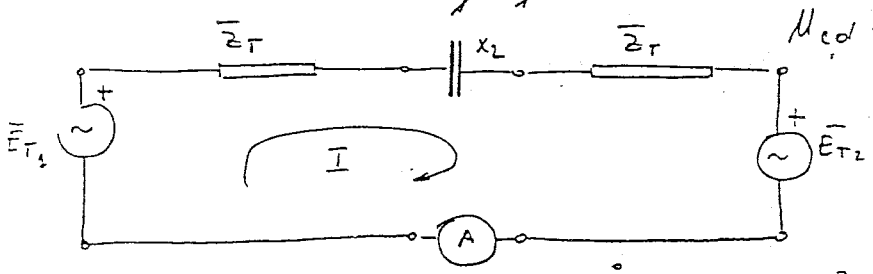
za desni dio $\bar{U}_{ab} = \bar{E}_{Tab} = \dfrac{\bar{E}_2}{200 + j40 - j40}\cdot(-j40)$

$$\bar{E}_{Tab} = \frac{\bar{E}_2}{200}\cdot(-j40)$$

$$\bar{E}_{Tab} = -j\frac{\bar{E}_2}{5} = \frac{-j}{5}\cdot 200\cdot e^{j36,85^\circ} = -j\cdot 40\cdot e^{j36,85^\circ} = 40\cdot e^{-53,15^\circ}\ V$$

za lijevi dio $\bar{U}_{cd} = \dfrac{\bar{E}_1}{200 + j40 - j40}\cdot(-j40) = -\frac{j}{5}\bar{E}_1 = -\frac{j}{5}\cdot 200\cdot e^{j0^\circ} = -j\cdot 40\cdot e^{j0^\circ} =$

$$\bar{U}_{cd} = 40\cdot e^{-j90^\circ}\ V$$



$$\bar{I} = \frac{\bar{E}_{T1} - \bar{E}_{T2}}{2\bar{Z}_T - j20} = \frac{40\cdot e^{-j90^\circ} - 40\cdot e^{-53,15^\circ}}{16 - j80 - j20} = \frac{40\left(e^{-j90^\circ} - e^{-53,15^\circ}\right)}{16 - j100}$$

$$\bar{I} = \frac{40\left[(0 - j1) - (0,6 - j0,8)\right]}{101,272\cdot e^{-j81^\circ}} = \frac{40(-j - 0,6 + j0,8)}{101,272\cdot e^{-j81}} = \frac{40\cdot(-0,6 - j0,2)}{101,272\cdot e^{-j81}}$$

$$\bar{I} = \frac{-40\cdot 0,632\cdot e^{j18,44^\circ}}{101,272\cdot e^{-j81^\circ}} = -0,25\cdot e^{j99,45^\circ} \qquad I_A = 0,25\ A$$

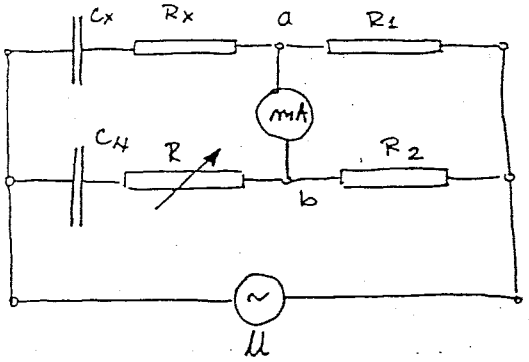
$\boxed{I_A = 0,25\ A}$

5.39.

ZADATAK: Za mjerenje kapaciteta s malim gubicima u dielektriku upotrebljava se most ravnoteže u kojem je C_N mjerni kapacitet (normala) C_x kapacitet koji želimo mjeriti i R_x otpor gubitaka kondenzatora

1) Izrazite C_x i R_x pomoću ostalih parametara mosta u ravnoteži

2) Izračunajte te veličine ako je $C_N = 15\mu F$, $R_1 = 500\Omega$ $R_2 = 1000\Omega$ i $R = 2\Omega$



$$\frac{Z_x}{R_1} = \frac{Z_N}{R_2}$$

$$R_2 \cdot Z_x = R_1 \cdot Z_N$$

$$R_2 \cdot \left(R_x - j\frac{1}{\omega C_x}\right) = R_1 \cdot \left(R - j\frac{1}{\omega C_N}\right) \qquad R_2 R_x - j\frac{R_2}{\omega C_x} = R_1 R - j\frac{R_1}{\omega C_N}$$

a) $R_2 R_x = R_1 R$ $\qquad \frac{R_2}{\omega C_x} = \frac{R}{\omega C_N}$

$$\boxed{R_x = \frac{R_1 \cdot R}{R_2}}$$

$R_2 \cdot C_N = R \cdot C_x$

$C_x = R_2 \cdot \frac{C_N}{R}$

$$\boxed{C_x = \frac{R_2}{R_1} C_N}$$

b) $R_x = \frac{500 \cdot 2}{1000} = 1\Omega$ $\qquad R_x = 1\Omega$ $\qquad C_x = \frac{1000}{500} \cdot 15\mu F = 30\mu F$

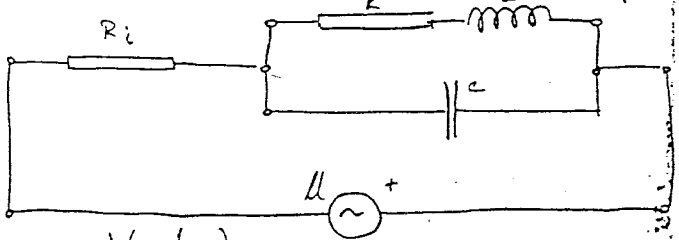
$$\boxed{R_x = 1\Omega}$$

$$\boxed{C_x = 30\mu F}$$

ZADATAK: $R_i = 40\,k\Omega$ $U_m = 6\,kV$ $C = 1000\,nF$ $L = 40\mu H$ $R = 1\Omega$

3.55

Odredite pojas propuštanja, snagu utrošenu u krugu pri rezonantnoj frekvenciji i graničnim frekvencijama pojasa propuštanja



$$\underline{Z} = R_i + \frac{(R + j\omega L)\left(-\frac{1}{\omega C}\right)}{R + j\omega L - \frac{1}{\omega C}} =$$

$$= 40\,000 + \frac{\left(1 + j\omega \cdot 4 \cdot 10^{-5}\right)\left(-j \cdot \frac{10^6}{\omega \cdot}\right)}{}$$

i.t.d

$(Z_2 - jX_1 - jX_M)R_1 = (R_2 - jX_{C2}) \cdot jX_M$

$Z_2 R_2 + jX_1 R_2 - jX_M R_2 = R_2 jX_M + X_M X_{C2}$ $\quad / \; Z_2 = -X_1 R_1 + X_M R_1 + X_M X_{C2} - R_1 R_2 = 0$

$Z_2 X_M - X_1 R_1 + X_M R_1 = 0$ $\quad X_M = X_{C2} = \frac{1}{\omega C_2}$ $\quad \omega M \cdot \frac{1}{\omega C_2} = R_1 R_2$ $\quad \boxed{M = R_1 R_2 C_2}$

$M = 500 \cdot 100 \cdot 0{,}125 \cdot 10^{-6} = 50000 \cdot 0{,}125 \cdot 10^{-6}$

$M = 6250 \cdot 10^{-6} = 0{,}00625$ H $\quad \boxed{M = 6{,}25 \text{ mH}}$ $\quad X_2 - X_M - X_1 R_1 + X_M R_2 = 0$

$X_1 R_1 = R_2 X_M + X_M R_2$

$X_1 = \frac{R_2 X_M + X_M R_1}{R_1} \Rightarrow L_1 = \frac{M(R_1 + R_2)}{R_2} = 6{,}25 \text{ mH} \cdot \frac{800}{500} = 10 \text{ mH}$

$\boxed{L_1 = 10 \text{ mH}}$

18

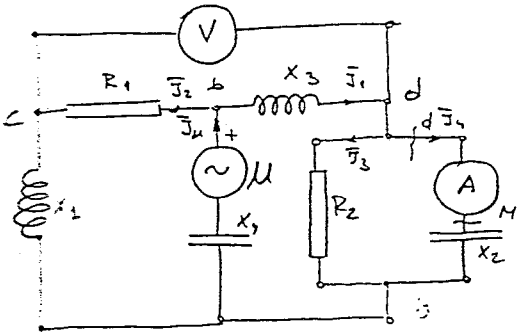
ZADATAK 8 $\Rightarrow$ S.54.

Odredite pokazivanje instrumenata u mreži u kojoj je $X_3 = 40\,\Omega$ $R_2 = 200\,\Omega$ $X_2 = -100\,\Omega$

$R_1 = 40\,\Omega$ $X_1 = 40\,\Omega$ $X_1 = -20\,\Omega$ $U = 300$ V

Odredite fazni pomak između struje koja prolazi kroz ampermetar i napona priključenog na mrežu

Odredimo struju koju mjeri ampermetar i napon koji mjeri voltmetar.



Izračunajmo ukupnu impedanciju

za R_2 i X_2 $\Rightarrow$ $\bar{Z} = \frac{200 \cdot (-j100)}{200 - j100} = \frac{-j20000}{200 - j100} = \frac{-j100}{2 - j}$ $\quad Z = \frac{-j200}{2-j} \cdot \frac{2+j}{2+j} = \frac{-j400 + 200}{4+1}$

$\bar{Z} = \frac{200 - j400}{5} = 40 - j80$

$\bar{Z}_{23X_2} = 40 - j80 + j40 = 40 - j40\,\Omega$ $\quad \bar{Z}_{AB} = \frac{(40 - j40)(40 + j40)}{40 - j40 + 40 + j40} = \frac{1600 + 1600}{80} = \frac{3200}{80} = 40\,\Omega$

$\bar{Z}_\mu = 40 - j20\,\Omega$

$\bar{I}_\mu = \frac{\bar{U}_\mu}{\bar{Z}_\mu} = \frac{200}{40 - j20} = \frac{20}{4 - j2} = \frac{10}{2 - j} \cdot \frac{2+j}{2+j} = \frac{20 + j10}{5} = 4 + j2$ $\quad \bar{J}_\mu = 4 + j2$

$\bar{U}_{ab} = \bar{J}_\mu \cdot \bar{Z}_{AB} = (4 + j2) \cdot 40 = 160 + j80$ $\quad \boxed{\bar{U}_{ab} = 160 + j80 \text{ V}}$

$\bar{J}_1 = \frac{\bar{U}_{ab}}{\bar{Z}_{23X_2}} = \frac{160 + j80}{40 - j40} = \frac{80(2 + j)}{40(1 - j)} = \frac{2(2 - j)}{1 - j} = \frac{4 + 2j}{1 - j} \cdot \frac{1 + j}{1 + j} = \frac{4 + 4j + 2j - 2}{1 + 1} = \frac{2 + 6j}{2} = 1 + j3$ A

$\bar{U}_{da} = \bar{J}_1 \cdot \bar{Z}_{dA} = [1 + j3] \cdot [40 - j80] = 40 - j80 + j120 + 240 = 280 + j40$ V

$\bar{J}_a = \frac{280 - j40}{-j100} \cdot \frac{j}{j} = \frac{280j + 40}{100} = 2{,}8j - 0{,}4 = -0{,}4 + 2{,}8j$ $\quad \bar{J}_a = 0{,}4 + 2{,}8j \Rightarrow \boxed{J_a = 2{,}828 \text{ A}}$

$\boxed{\bar{J}_A = 2{,}83 \text{ A}}$

$\bar{J}_2 = \frac{160 + j80}{40 + j40} = \frac{80(2 + j)}{40(1 + j)} = \frac{4 + 2j}{1 + j} \cdot \frac{1 - j}{1 - j} = \frac{4 - 4j + 2j + 2}{2} = \frac{6 - j2}{2} = 3 - j$ A

(5.54)

$\varphi_c = \varphi_d - \bar{I}_1 \cdot X_3 - \bar{I}_2 \cdot R_2$ $\quad \bar{U}_V = \bar{I}_1 \cdot X_3 - \bar{I}_2 \cdot R_2 = (1+j3)\cdot j40 - (3-j)\cdot 40$

$\bar{U}_1 = j40 - 120 - 120 + j40 = -240 + j80$ $\quad U_V = \sqrt{240^2 + 80^2} = \sqrt{57600 + 6400} = \sqrt{64000} = 253V$

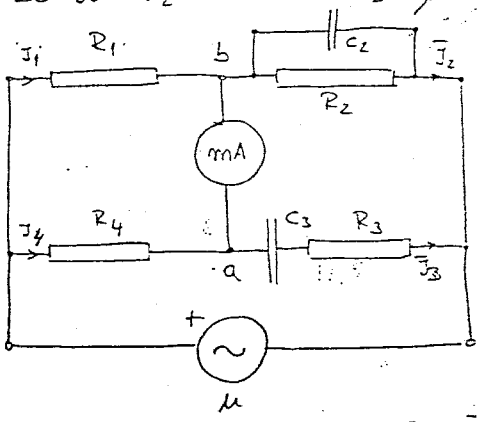
$\boxed{U_V = 253V}$ ✓

(5.62)

ZADATAK: Kolika je frekvencija napona priključenog mamont dok je ravnoteža postignuta

uz ove parametre: $R_1 = 200\Omega$ $R_2 = 300\Omega$ $C_2 = 2\mu F$ $R_4 = 500\Omega$ $R_3 = 480\Omega$

i $C_3 = 2{,}22\mu F$



$\varphi_D = \varphi_a + \bar{I}_4 \cdot R_4 - \bar{I}_1 \cdot R_1$

$\varphi_D = \varphi_a - \bar{I}_3 \cdot \bar{Z}_3 + \bar{I}_2 \cdot \bar{Z}_2$

$\bar{I}_4 \cdot R_4 - \bar{I}_1 \cdot R_1 = 0$

$\bar{I}_3 \cdot \bar{Z}_2 - \bar{I}_3 \cdot Z_3 = 0$

$\bar{I}_1 \cdot R_1 = \bar{I}_4 \cdot R_4$ / : $\bar{I}_2 \bar{Z}_2 = \bar{I}_3 \bar{Z}_3$

$\frac{R_1}{\bar{Z}_2} = \frac{R_4}{\bar{Z}_3}$ $\quad R_1 \cdot \bar{Z}_3 = \bar{Z}_2 \cdot R_4$

$R_1 \cdot \bar{Z}_3 - \bar{Z}_2 \cdot R_4 = 0$

kako je most u ravnoteži

onda je $I_1 = \bar{I}_2$

$\bar{I}_3 = \bar{I}_4$

$\bar{Z}_2 = \frac{-300 \cdot j\frac{1}{\omega C_2}}{300 - j\frac{1}{\omega C_2}} = \frac{-j\frac{300}{\omega C_2}}{300 - j\frac{1}{\omega C_2}}$

$\bar{Z}_2 = \frac{-300 \cdot j X_{C2}}{300 - j X_{C2}} \cdot \frac{300 + j X_{C2}}{300 + j X_{C2}}$

$\bar{Z}_3 = 480 - j X_{C3}$

$200 \cdot (480 - j X_{C3}) - \frac{300 \cdot X_{C2}^2 - 90000 j X_{C2}}{90000 + X_{C2}^2} \cdot 500 = 0$ $\quad \bar{Z}_2 = \frac{-90000 j X_{C2} + 300 \cdot X_{C2}^2}{90000 + X_{C2}^2}$

$(96000 - j200 X_{C3}) - \frac{300 X_{C2}^2 - j90000 X_{C2}}{90000 + X_{C2}^2} \cdot 500 = 0$ $\quad (90000 + X_{C2}^2)(96000 - j200 X_{C3}) - (300 X_{C2}^2 - j90000 X_{C2})$

$9216000000 - j18000000 X_{C3} + 96000 X_{C2}^2 - j200 X_{C3} \cdot X_{C2}^2 - 150000 X_{C2}^2 + j45000000 X_{C2} = 0$

$-500 X_{C2}^2 + 9216000000 = 0$ $\quad -18000000 X_{C3} - 200 X_{C3} \cdot X_{C2}^2 + 45000000 X_{C2} = 0$

$-180000 \cdot \frac{10^6}{\omega \cdot 2{,}22} - 2 \cdot \frac{10^6}{\omega \cdot 2{,}22} \cdot \frac{10^{12}}{\omega^2 \cdot 4} + 450000 \cdot \frac{10^6}{\omega \cdot 2} = 0$

$\frac{-180000}{2{,}22} - \frac{2 \cdot 10^{12}}{8{,}88 \omega^2} + 225000 = 0$ $\quad -81081{,}08 - 0{,}225 \cdot \frac{10^{12}}{\omega^2} + 225000 = 0$

$0{,}225 \cdot \frac{10^{12}}{\omega^2} = 143919$ $\quad \omega^2 = \frac{0{,}225 \cdot 10^{12}}{143919} = 1563371{,}4$

$\boxed{\omega = 1250{,}35\ \#z}$ ✓

$X_{C2}^2 = 493{,}118$ $\quad X_C = \frac{1}{\omega C}$

$\omega = \frac{1}{X_C \cdot C} = \frac{10^6}{493{,}118 \cdot 2} =$

$$\bar{I}''\left[\underbrace{6{,}65\cdot e^{j63{,}437^\circ} - 2{,}237 e^{j84{,}478^\circ} - 1{,}488\cdot e^{j162^\circ}}\right] = 72{,}743\cdot e^{j12{,}915^\circ}$$

$$6{,}65\left(0{,}432+j\,0{,}903\right) - 2{,}237\left(0{,}096+j\,0{,}995\right) - 1{,}488\left(0{,}334+j\,0{,}943\right)$$

$$= 2{,}873+j\,5{,}988 - 0{,}221 - j\,2{,}286 - 0{,}497 - j\,1{,}404 = 2{,}155+j\,2{,}308$$

$$\bar{I}''\left(2{,}155+j\,2{,}308\right) = 72{,}743\cdot e^{j12{,}915^\circ} \qquad \bar{I}'' = \frac{72{,}743\cdot e^{j12{,}915^\circ}}{3{,}158\cdot e^{j46{,}963}} = 23{,}035\,e^{-j34{,}048}\ A$$

$$\bar{I}''' = 0{,}4212\cdot e^{j12{,}436}\cdot \bar{I}'' = 0{,}4212\cdot e^{j12{,}436}\cdot 23{,}035\,e^{-j34{,}048} = 9{,}702\,e^{-j21{,}612}\ A$$

$$\bar{J}_2 = \bar{I}'' - \bar{I}''' = 23{,}035\,e^{-j34{,}048} - 9{,}702\,e^{-j21{,}612}\ A$$

$$\bar{J}_2 = 23{,}035\left(0{,}829-j\,0{,}560\right) - 9{,}702\left(0{,}930-j\cdot 0{,}368\right)$$

$$\bar{J}_2 = 10{,}07 - j\,9{,}329\ A$$

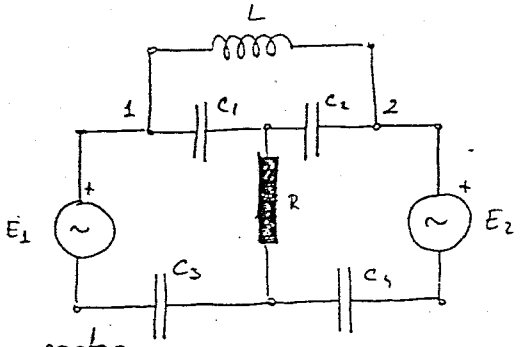
5.74 ZADATAK: ⇒ U mreži na slici snagama otpora $R=10\,\Omega$ iznosi $P=40\,W$.

Pri tome je $E_1=E_2=E$ i kod odabranih pozitivnih smjerova prema slici E_1 prethodi u fazi E_2 za 60°. Odredite veličinu E ako je $X_{C1}=20\,\Omega$ $X_{C2}=10\,\Omega$ $X_{C3}=40\,\Omega$

$X_{C1}=20\,\Omega$ i $X_L=30\,\Omega$

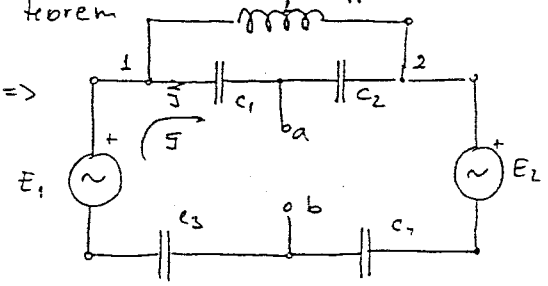
$\bar{E}_2 = E\cdot e^{j0} = E\ V$

$\bar{E}_1 = E\cdot e^{j60^\circ}\ [V]$



Koristit ćemo Theveninov teorem

$\bar{E}_T = ?$ $\bar{Z}_T = ?$ $\bar{E}_T \Rightarrow$



otpor paralelne grane je

$$\bar{Z}_{12} = \frac{-j30\cdot 30}{j30-j30} = \infty\ \Omega$$

$J = ?$ $J = \dfrac{\bar{E}_1-\bar{E}_2}{\infty} = 0\ A$

Pošto struja neteče u krugu nejavlja se pad napona na C_3 i C_4 i Energija oscilira u paralelnom spoju

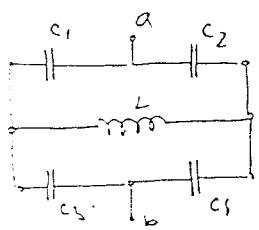
$$\bar{J}_c = \frac{\bar{E}_1-\bar{E}_2}{-j20-j10} = \frac{\bar{E}_1-\bar{E}_2}{-j30} = \frac{E\cdot e^{j60}-E}{-j30} = \frac{E\left(e^{j60}-1\right)}{-j30} = \frac{E\left(\frac{1}{2}+j\frac{\sqrt{3}}{2}-1\right)}{-j30}$$

$$\bar{J}_c = \frac{E\cdot\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)}{-j30} = \frac{\frac{1}{2}E\cdot e^{-j60^\circ}}{30\,e^{j90^\circ}} = \frac{E}{30}\cdot e^{-j150^\circ} \qquad \varphi_a = \varphi_b + \bar{E}_2 + \bar{J}_c\,(-j\,10)$$

$$\bar{U}_{ab} = \bar{E}_T = \bar{E}_2 - j\,10\,\bar{J}_c = E - j10\cdot\frac{E}{30}\cdot e^{-j150^\circ} = E - j\cdot\frac{E}{3}\,e^{-j150^\circ} = E - \frac{E}{3}\cdot e^{j90}\cdot e^{-j150^\circ}$$

$$\bar{E}_T = E - \frac{E}{2}\cdot e^{-j60} =$$

$\bar{Z}_T = ?$ (5.74)



$X_{C_1} = 20\,\Omega$

$X_{C_2} = 10\,\Omega$

$X_{C_3} = 40\,\Omega$

$X_{C_4} = 20\,\Omega$

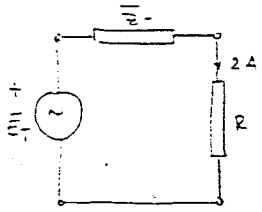
$X_L = 30\,\Omega$

$20 \cdot 20 = 10 \cdot 40$ — most je uravnotežen, pa X_L crta možemo maknuti

$$\frac{X_{C_2}}{X_{C_1}} = \frac{X_{C_4}}{X_{C_3}} = \frac{1}{2}$$

$$\bar{Z}_T = \frac{-j30 \cdot (-j60)}{-j20} = \frac{-1200}{-j20} = -j20\,\Omega$$

$$\boxed{\bar{Z}_T = -20\,\Omega}$$



$R = 10\,\Omega$, $P = 40\,W \Rightarrow I^2 = \frac{P}{R} \Rightarrow \boxed{I = 2\,A}$

$$\bar{E}_T = E - \frac{E}{3} e^{-j60^\circ}$$

$$\bar{I} = \frac{\bar{E}_T}{\bar{Z}_u} = \frac{E - \frac{E}{3} e^{-j60^\circ}}{10 - j20}$$

$$\bar{I} = \frac{E - \frac{E}{3}\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)}{10 - j20} = \frac{E - \frac{E}{6} + j\frac{E\sqrt{3}}{6}}{10 - j20} = \frac{\frac{5E}{6} - \frac{\sqrt{3}}{6}Ej}{10 - j20} = \frac{5E - \sqrt{3}Ej}{60 - j120}$$

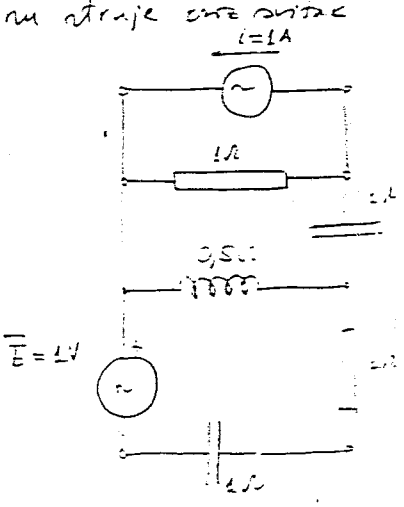
$$\bar{I} = \frac{5E + \sqrt{3}Ej}{60 - j120} = \frac{E(5 + \sqrt{3}j)}{60 - j120} = \frac{E \cdot 5{,}29 \cdot e^{j19{,}1}}{134{,}16 \cdot e^{-j63{,}43}} \Rightarrow \bar{I} = 0{,}03943 \cdot E \cdot e^{j82{,}53^\circ}$$

$I = 2 \Rightarrow E = \frac{2}{0{,}03943} = 50{,}723\,V$

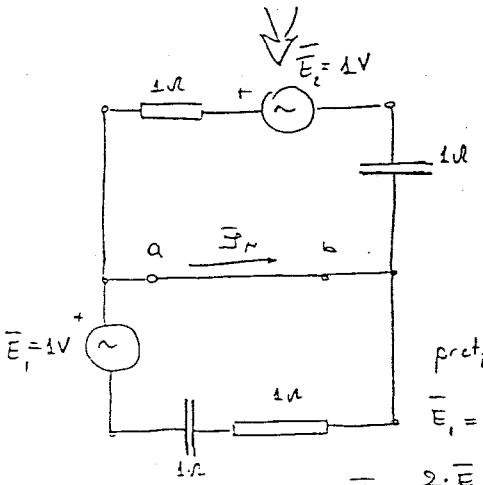
$$\boxed{E = 50{,}723\,V}$$ 77

5.77

ZADATAK 8 => U spoju na slici izračunajte veličinu struje kroz otpor



pretvorimo strujni izvor u naponski izvor

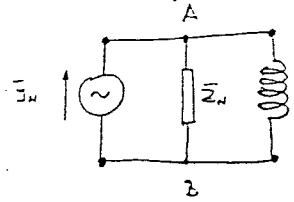


pretpostavimo da su

$\bar{E}_1 = \bar{E}_L = 1\,V$

$$\bar{I}_N = \frac{2 \cdot \bar{E}}{1 - j} = \frac{2}{1 - j} = \frac{2(1 + j)}{2} = 1 + j$$

$$\bar{Z}_N = \frac{(1 - j)(1 - j)}{1 - j + 1 - j} = \frac{1 - j - j - 1}{2 - j2} = \frac{-j2}{2 - j2} \cdot \frac{2 + j2}{2 + j2} = \frac{-j4 + 4}{4 - 4} = \frac{4 - j4}{?} = \frac{?}{?} = 0{,}5 - 0{,}5j$$



$$\bar{Z}_u = \frac{j\frac{1}{2} \cdot \left(\frac{1}{2} - j\frac{1}{2}\right)}{j\frac{1}{2} + \frac{1}{2} - \frac{1}{2}} = \frac{j\frac{1}{4} + \frac{1}{4}}{\frac{1}{2}} = \frac{\frac{1+j}{4}}{\frac{1}{2}} = \frac{1+j}{2} = 0{,}5 + 0{,}5j$$

$$\bar{U}_{ab} = \bar{I}_N \cdot \frac{1+j}{2} = (1+j)(1+j) \cdot \frac{1}{2} = \frac{1}{2}(1 + j + j - 1) = \frac{1}{2} \cdot 2j = j$$

$$\bar{I}_L = \frac{\bar{U}_{ab}}{j0{,}5\,\Omega} = \frac{j}{j0{,}5} = \frac{+1}{+0{,}5} = 2\,A \qquad \boxed{I_L = 2\,A}$$

S.55

$$\bar{U}_{20} = \frac{\bar{U}\ \frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{-jX_1} + \frac{1}{jX_2 + jX_3 - jX_c}} = \frac{\bar{U}\ \frac{1}{20}}{\frac{1}{20} + \frac{1}{100} + \frac{1}{j50 + j90 - j65}} = \frac{\bar{U}\cdot\frac{1}{20}}{\frac{1}{20} + j\frac{1}{100} + \frac{1}{j75}}$$

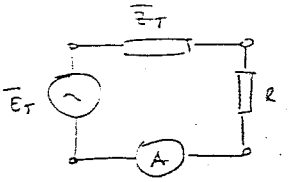
$$\bar{U}_{20} = \frac{\bar{U}\cdot 0{,}05}{0{,}05 + j0{,}01 - j0{,}013333} = \frac{\bar{U}\cdot 0{,}05}{0{,}05 - j0{,}003333}\cdot\frac{0{,}05 + j0{,}003333}{0{,}05 + j0{,}003333} = \frac{\bar{U}\,(0{,}0025 + j0{,}0001667)}{0{,}0025111}$$

$$\bar{U}_{20} = \bar{U}\,(0{,}9955 + j0{,}06638) \qquad \bar{E}_T = \frac{\bar{U}\,(0{,}9955 + j0{,}06638)}{j75}\cdot j50$$

POČETAK MREŽU 4. ZADATKA ČETVRTICATTA – MREŽA INDUKTIVNOSTI

$$\bar{E}_T = \bar{U}\,(0{,}9955 + j0{,}06638)\cdot 0{,}666666$$

$$\bar{E}_T = \bar{U}\,(0{,}663667 - j0{,}04425)$$



$$\bar{E}_T = \bar{I}\cdot\bar{Z}_u$$

$$\bar{U}\,(0{,}663667 + j0{,}04425) = 0{,}2\cdot(48{,}85 + j17{,}257)$$

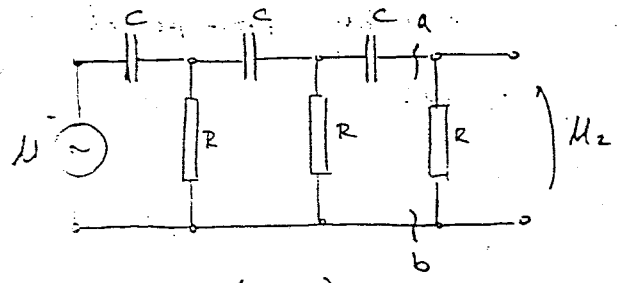
$$\bar{U} = \frac{0{,}2\,(48{,}85 + j17{,}257)}{0{,}6636 + j0{,}04425} = \frac{0{,}2\cdot 51{,}8\cdot e^{j19{,}45^\circ}}{0{,}665\cdot e^{j3{,}81}} = 15{,}6\cdot e^{j15{,}6^\circ}$$

$$\boxed{\bar{U} = 15{,}6\cdot e^{j15{,}6^\circ}\ V}$$

S.61

ZADATAK: ⇒ Pomoću spoja zvijezda pronađi da li se pomak između izlaznog i ulaznog napona može biti 180°. Kolika mora biti vrijednost kapacitet C uz zadane vrijednosti R i f?

Primijenit ćemo Theveninov teorem



$$\bar{E}_T = ? \qquad \bar{Z}_T = ?$$

$$\bar{E}_1 = \frac{-R\cdot jX_c}{R - jX_c}$$

$$\bar{Z}_1 = -\frac{R\,jX_c}{R - jX_c} - jX_c = \frac{-R\cdot jX_c - jX_c(R - jX_c)}{R - jX_c} = \frac{-R\,jX_c - jRX_c - X_c^2}{R - jX_c}$$

$$\bar{Z}_1 = \frac{-2RX_c\,j}{R - jX_c} = \frac{-j2R\cdot X_c - X_c^2}{R - jX_c} = \frac{-X_c\,(X_c + j2R)}{R - jX_c}$$

$$\bar{Z}_2 = \frac{\frac{-X_c\,(X_c + j2R)}{R - jX_c}\cdot R}{\frac{-X_c\,(X_c + j2R)}{R - jX_c} + R} = \frac{\frac{-X_c R\,(X_c + j2R)}{R - jX_c}}{\frac{-X_c\,(X_c + j2R) + R^2 - jRX_c}{R - jX_c}} = \frac{-X_c\cdot R\,(X_c + j2R)}{-X_c^2 - j2RX_c + R^2 - jRX_c}$$

$$\bar{Z}_3 = \frac{-X_c\cdot R\,(X_c + j2R)}{R^2 - X_c^2 - j3RX_c} \qquad \bar{Z}_T = \frac{-X_c\,R\,(X_c + j2R)}{R^2 - X_c^2 - j3\cdot R\cdot X_c} - jX_c$$

$$\bar{E}_T = \frac{-X_c\cdot R\,(X_c + j2R) - jX_c\,(R^2 - X_c^2 - j3R\cdot X_c)}{R^2 - X_c^2 - j3R\cdot X_c} = \frac{-X_c R^2 - j2RX_c^2 - jX_c R^2 + jX_c^3 - 3R\cdot X_c^2}{R^2 - X_c^2 - j3R\cdot X_c}$$

$$\bar{Z}_T = \frac{-4X_c^2\cdot R - j3R^2 X_c + jX_c^3}{R^2 - X_c^2 - j3R\cdot X_c}$$

5.61

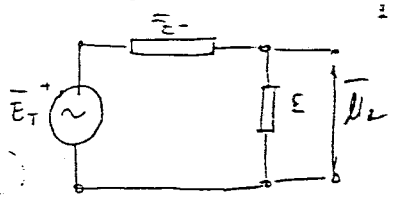
$\bar{E}_T = ?$ $\quad \bar{Z}_s = R - jX_c$ $\quad \bar{Z}_p = \dfrac{R\cdot(R-jX_c)}{R+R-jX_c} = \dfrac{R(R-jX_c)}{2R-jX_c}$ $\quad \bar{Z}_u = \dfrac{R(R-jX_c)}{2R-jX_c} - jX_c$

$$\bar{Z}_u = \frac{R(R-jX_c) - jX_c(2R-jX_c)}{2R-jX_c} = \frac{R^2 - jR\cdot X_c - j2RX_c - X_c^2}{2R-jX_c} = \frac{R^2 - X_c^2 - j3RX_c}{2R-jX_c}$$

$$J = \frac{U_1}{Z_u} = \left\{ \bar{U}_1 = U_1 \angle 0^\circ \right\} = U_1 \cdot \frac{2R-jX_c}{R^2-X_c^2-j3RX_c} = \bar{J}$$

$$\bar{U}_{ab} = U_1 \cdot \frac{R-jX_c}{R^2-X_c^2-j3RX_c} \cdot \frac{R(R-jX_c)}{2R-jX_c} = U_1 \cdot \frac{R(R-jX_c)}{R^2-X_c^2-j3RX_c}$$

$$\bar{E}_T = \frac{\bar{U}_{ab}}{R-jX_c}\cdot R = \frac{U_1 \cdot \dfrac{R(R-jX_c)}{R^2-X_c^2-j3RX_c}}{\dfrac{R-jX_c}{1}} \cdot R = \boxed{U_1 \cdot \frac{R^2}{R^2-X_c^2-j3RX_c} = E_T}$$



$$\bar{U}_2 = \frac{\bar{E}_T}{R+\bar{Z}_T}\cdot R = \frac{\dfrac{U_1\cdot R^2}{R^2-X_c^2-j3RX_c}}{R + \dfrac{-4X_c^2R - j3R^2X_c + jX_c^3}{R^2-X_c^2-j3RX_c}} \cdot R$$

$$\bar{U}_2 = \frac{\dfrac{U_1\cdot R^2}{R^2-X_c^2-j3RX_c}}{\dfrac{R^3 - RX_c^2 - j3R^2X_c - 4X_c^2R - j3R^2X_c + jX_c^3}{R^2-X_c^2-j3RX_c}} \cdot R = \frac{U_1\cdot R^3}{R^3 + jX_c^3 - 5X_c^2R - j6R^2X_c}$$

$$\bar{U}_2\left[(R^3 - 5X_c^2R) + j(X_c^3 - 6R^2X_c)\right] = R^3U_1 \qquad \bar{U}_2 = |U_2|\cdot e^{+j180^\circ} = -U_2$$

$$U_2\left[-(R^3 - 5X_c^2R) - j(X_c^3 - 6R^2X_c)\right] = R^3U_1 \Rightarrow X_c^3 - 6R^2X_c = 0$$

$$X_c^2 - 6R^2 = 0 \qquad X_c^2 = 6R^2$$

$$\frac{1}{\omega C} = \sqrt{6}\cdot R \Rightarrow C = \frac{1}{\omega\sqrt{6}\cdot R} \qquad X_c = \sqrt{6}\,R$$

$$\boxed{C = \frac{\sqrt{6}}{\omega 6\cdot R}\ F}$$

Kolik je odnos U_2/U_1

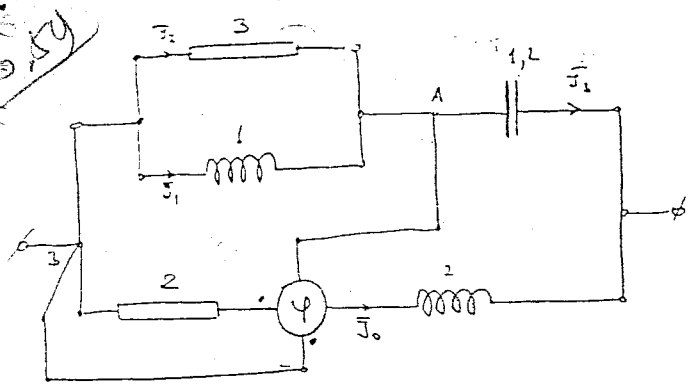
$$\frac{U_2}{U_1} = \frac{R^3}{5X_c^2R - R^3} = \frac{R^3}{5\cdot 6R^3 - R^3} = \frac{1}{30-1} = \frac{1}{29} \qquad \boxed{\frac{U_2}{U_1} = \frac{1}{29}}$$

ZADATAK 8 ⇒ 5.50

Odredi pokaz ranga ohmetra koji je uključen u mrežu prema slici.

Otpori pojedinih dijelova mreže dani su na slici.

5.84



Fazometar pokazuje razliku u fazi između napona $\bar{U}_{AB}$ i $\bar{I}_0$

Neka je narinuti napon $\bar{U} = U$ [V]

$$\bar{I}_0 = \frac{\bar{U}}{2+j2} = \frac{U}{2+j2} \cdot \frac{2-j2}{2-j2} = \frac{U(2-j2)}{4+4} = \frac{U}{8}(2-j2) = \frac{U}{4}(1-j) \qquad \bar{I}_0 = \frac{U}{4}(1-j)$$

$$\bar{Z}_{12} = \frac{3 \cdot j}{3+j} = \frac{j3}{3+j} \cdot \frac{3-j}{3-j} = \frac{9j+3}{9+1} = \frac{3+9j}{10} = \frac{3+j9}{10} \qquad \bar{Z}_{123} = \frac{3+j9}{10} - j\frac{12}{10}$$

$$\bar{Z}_{123} = \frac{3}{10} + j\frac{9}{10} - j\frac{12}{10} = \frac{3}{10} - j\frac{3}{10} = \frac{3}{10}(1-j) \qquad \bar{I}_1 = \frac{\bar{U}}{\bar{Z}_{123}} = \frac{U}{\frac{3}{10}(1-j)} = \frac{10U}{3(1-j)}$$

$$\bar{U}_{AB} = \bar{I}_0 \cdot \bar{Z}_{12} = \frac{10U}{3(1-j)} \cdot \frac{3+j9}{10} = \frac{U}{3(1-j)} \cdot 3(1+j3) = \frac{U(1+j3)}{1-j} \cdot \frac{1+j}{1+j} = \frac{U(1+j+j3-3)}{2}$$

$$\bar{U}_{AB} = \frac{U}{2}(-2+j4) = \frac{U}{2} \cdot 2(-1+j2) = U(-1+j2) \qquad \bar{I}_0 = \frac{U}{4}(1-j)$$

$\varphi_u = \operatorname{arctg} -2$

$\varphi_i = \operatorname{arctg} -1$

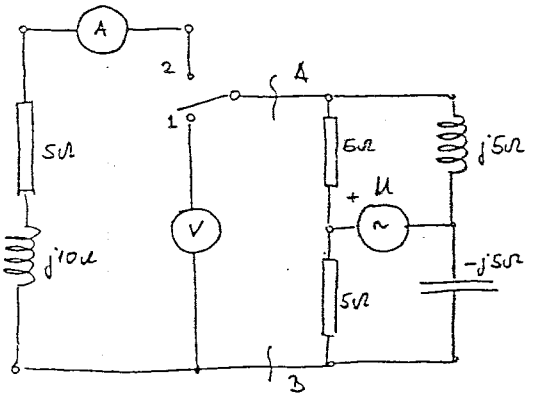
NAĐINO FAZNI POMAK $\quad \varphi = \varphi_u - \varphi_i = 116{,}56 - (-45) = 161{,}56^\circ$

$$\boxed{\varphi = 161{,}56^\circ}$$

5.85

ZADATAK 8 ⇒ U spoju na slici uz sklopku u položaju 1 idealni voltmetar pokazuje napon od 10V. Izračunajte veličinu struje koju mjeri idealni ampermetar ako je sklopka u položaju 2.

Instrumenti mjere efektivne vrijednosti

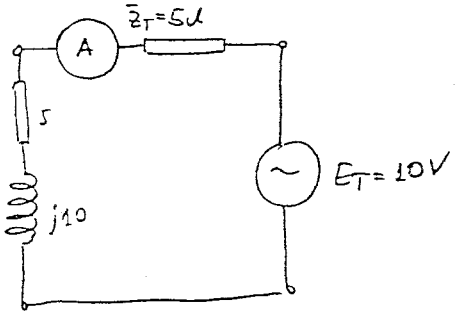


Pošto znamo U_v a on je jednak E_T kod mrežnih presječenih u A i B ⇒ onda sam odredim $\bar{Z}_T$ =?

$$\bar{Z}_T = \frac{5 \cdot j5}{5+j5} + \frac{-5 \cdot j5}{5-j5} = \frac{j25}{5+j5} - \frac{j25}{5-j5}$$

$$\bar{Z}_T = \frac{j25(5-j5)}{50} - \frac{j25(5+j5)}{50}$$

$$\bar{Z}_T = \frac{j125 + 125 - (j125 - 125)}{50} = \frac{j125 + 125 - j125 + 125}{50} = \frac{250}{50} =$$



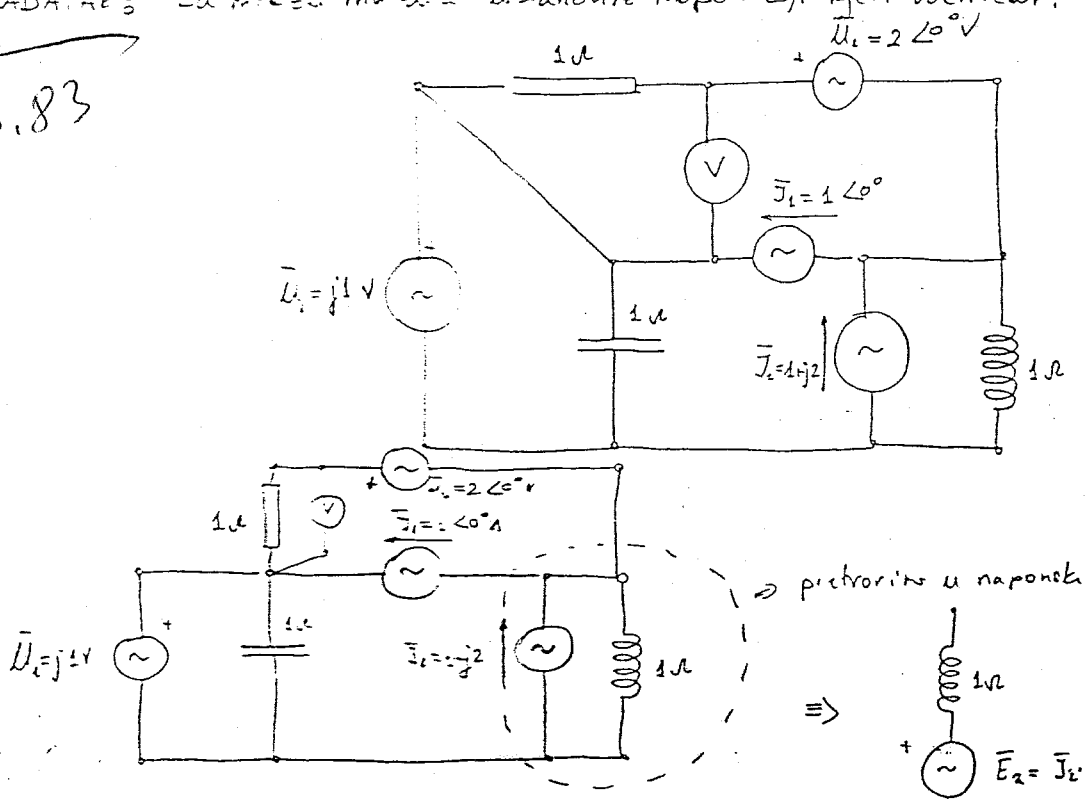
I_A =? $\qquad Z_u = \sqrt{10^2 + 10^2} = \sqrt{200} = 10\sqrt{2}\ \Omega$

$$I_A = \frac{10}{10\sqrt{2}} = \frac{\sqrt{2}}{2} = 0{,}707\ A$$

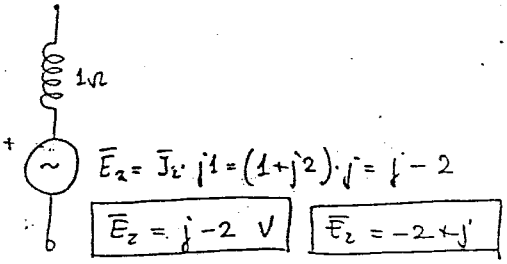
$$\boxed{I_A = 0{,}707\ A}$$

ZADATAK 9 Za mrežu na slici odrediti napon koji mjeri voltmetar.

5.83

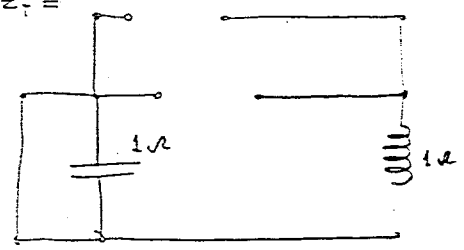


⇒ pretvorimo u naponski izvor



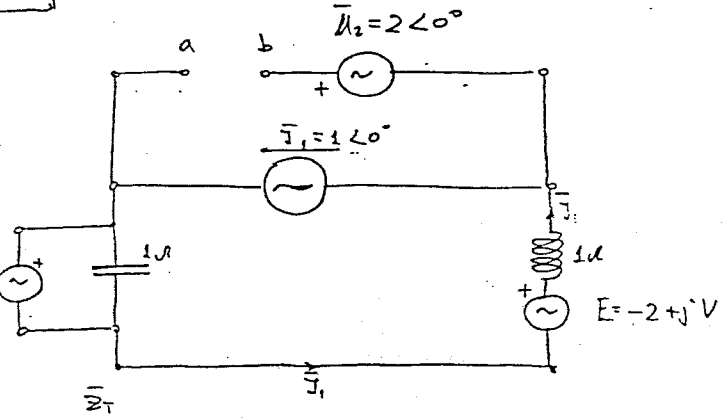
Koristit ćemo Theveninov teorem

$\bar{Z}_T =$



$\Rightarrow \boxed{\bar{Z}_T = j1\,\Omega}$

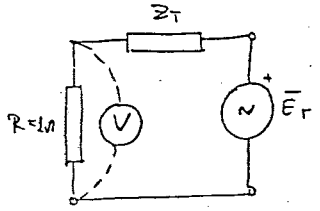
$\bar{E}_T = ?$



$\bar{E}_T = \varphi_b - \varphi_a$

$\varphi_b = \varphi_a - j1 + (-2-j) - 1\cdot j1 + 2$

$\varphi_b - \varphi_a = -j - 2 - j - j + 2 \rightarrow \boxed{\bar{E}_T = -j\,V}$



$\bar{Z}_u = 1+j \quad Z = \sqrt{1+1} = \sqrt{2}\,\Omega$

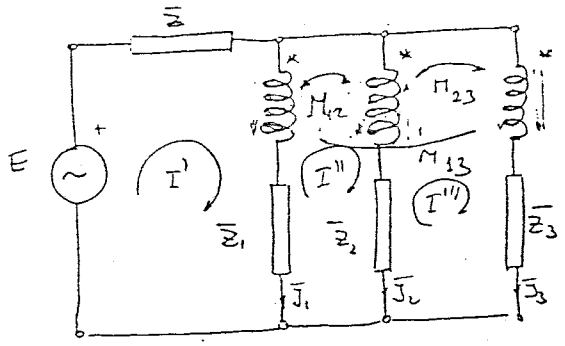
$U_v = \frac{1}{\sqrt{2}} \cdot 1 = \frac{\sqrt{2}}{2} = 0{,}707$ V $\quad \boxed{U_v = 0{,}707\,V}$

5.10

ZADATAK: ⇒ zadano je $\bar{Z}_M = j\Omega$ $E = 100\,V$ $\bar{Z} = 1{,}26 - j\,0{,}29\,\Omega$ $\bar{Z}_1 = 1 + \dots$ $\bar{Z}_2 = \dots - j\dots$

$\bar{Z}_3 = 4 + j4\,\Omega$. Odredite struje u mreži

Upotrebimo metodu kontura struja -



$$\bar{E} = \bar{I}'(\bar{Z} + \bar{Z}_1) - I''\cdot\bar{Z}_1 + I''\cdot j1 + I'''\cdot j1 - I'''j1$$

$$0 = \bar{I}''(\bar{Z}_1 + \bar{Z}_2) - I''\cdot j1 - I''\cdot j1 - I'\cdot\bar{Z}_1 + I'\cdot j1 - I'''\cdot\bar{Z}_2 + I''j1 - I'''j1 + I'''j1$$

$$0 = I'''\cdot(\bar{Z}_2 + \bar{Z}_3) - I'''\cdot j1 - I''\cdot j1 - I''\cdot\bar{Z}_2 + I'j1 - I'j1 - I''\cdot j1 + I''j1 + I''\cdot j1$$

$$\bar{E} = \bar{I}'(\bar{Z} + \bar{Z}_1) + \bar{I}''(-\bar{Z}_1 + j1)$$

$$0 = I''(\bar{Z}_1 + \bar{Z}_2) - 2I''j1 - I'\cdot\bar{Z}_1 + I'\cdot j1 - I'''\bar{Z}_2 + I''\cdot j1$$

$$0 = I'''(\bar{Z}_2 + \bar{Z}_3) - 2I'''j1 - I''\cdot\bar{Z}_2 + I''\cdot j1$$

$$100 = \bar{I}'(1{,}26 - j0{,}29 + 1 + j4) + \bar{I}''(-1 - j4 + j1)$$

$$0 = I''(1 + j4 + 1{,}87 + j4) - 2\cdot I''j1 + I'(j1 - 1 - j4) + I'''(j1 - 1{,}87 - j4)$$

$$0 = I'''(1{,}87 + j4 + 4 + j4) - 2j\cdot I''' + I''(j1 - 1{,}87 - j4)$$

$$100 = \bar{I}'(2{,}26 + j3{,}71) + \bar{I}''(-1 - j3)$$

$$0 = \bar{I}''(2{,}87 + j8) - j2I'' + I'(-1 - j3) + I''(-1{,}87 - j3)$$

$$0 = \bar{I}''(5{,}87 + j8) - j2I''' + I''(-1{,}87 - j3)$$

$$100 = I'(2{,}26 + j3{,}71) - I''(1 + j3) \Rightarrow I_1'(2{,}26 + j3{,}71) = 100 + I''(1 + j3) \Rightarrow \bar{I}_1' = \frac{100 - \bar{I}''(1+j3)}{2{,}26 - j3{,}71}$$

$$0 = I''(2{,}87 + j6) - I'(1 + j3) - I'''(1{,}87 + j3)$$

$$0 = I^N(5{,}87 + j6) - I''(1{,}87 + j3) \Rightarrow I''(1{,}87 + j3) = I'''(5{,}87 + j6) \Rightarrow \bar{I}'' = \frac{1{,}87 - j3}{5{,}87 - j6}\,\bar{I}'''$$

$$I_1' = \frac{100}{2{,}26 + j3{,}71} + I''\cdot\frac{1 + j3}{2{,}26 + j3{,}71} = \frac{100}{4{,}344\cdot e^{j58{,}652}} + I''\cdot\frac{3{,}16\cdot e^{j71{,}565}}{4{,}344\cdot e^{j58{,}65}}$$

$$I_1' = 23{,}02\cdot e^{-j58{,}65} + I''\cdot 0{,}727\cdot e^{j12{,}913}$$

$$\Rightarrow I''(2{,}87 + j6) - (1 + j3)\cdot\left[23{,}02\cdot e^{-j58{,}65} + I''0{,}727\cdot e^{j12{,}913}\right] - (1{,}87 + j3)\cdot 0{,}4212\cdot e^{j12{,}436^\circ}\bar{I}''' = 0$$

$$I''\cdot 6{,}65\cdot e^{j64{,}427} - 3{,}16\cdot e^{j71{,}565}\cdot\left[23{,}02\cdot e^{-j58{,}65} + I''0{,}727\cdot e^{j12{,}913}\right] - 3{,}535\cdot e^{j58{,}063^\circ}\cdot 0{,}4212\cdot e^{j12{,}436^\circ}\bar{I}''' = 0$$

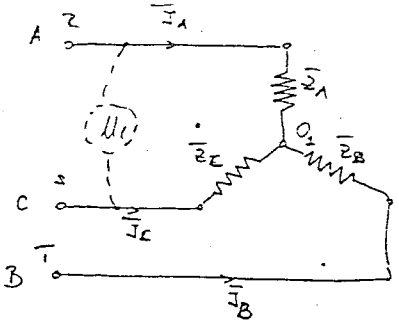
$$I''\cdot 6{,}65\cdot e^{j64{,}427} - 72{,}743\cdot e^{j12{,}915} - I''\cdot 2{,}297\cdot e^{j84{,}478} - 1{,}419\cdot e^{j70{,}499^\circ}\bar{I}''' = 0$$

$$I'' = \frac{3{,}535\cdot e^{j58{,}063^\circ}}{8{,}293\cdot e^{j45{,}627^\circ}}\,\bar{I}''' \qquad \bar{I}'' = 0{,}4262\cdot e^{j12{,}436^\circ}\bar{I}'''$$

TROFAZNI SUSTAVI

6.1

DATAK: ⇒ Potrošač je spojen u zvijezdu $\bar{Z}_A = \bar{Z}_B = \bar{Z}_C = 10e^{j37°}$. Linijski napon izvora je $U_L = 220V$



a) Odredite struje i snagu trošila

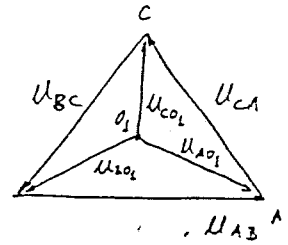
Linijski napon - napon između dvije faze.

$\bar{U}_{AB} = 220e^{j0°}$ V proizvoljno

$\bar{U}_{CA} = 220e^{j120°}$ V

$\bar{U}_{BC} = 220e^{j240°}$ V

$U_f = \frac{U_L}{\sqrt{3}} = 127V$



Fazni naponi će biti:

$\bar{U}_{AO_1} = 127e^{-j30°}$

$\bar{U}_{CO_1} = 127 \cdot e^{j90°}$

$\bar{U}_{BO_1} = 127 \cdot e^{-j150°}$

Fazne struje jednake su linijskim strujama

$\bar{I}_A = \frac{127e^{-j30°}}{10e^{j37°}} = 12,7 \cdot e^{-j67°}$ A

$\bar{I}_B = \frac{127e^{-j150°}}{10e^{j37°}} = 12,7 \cdot e^{-j187°}$ A $\quad \bar{I}_C = \frac{127e^{-j90°}}{10 \cdot e^{j37°}} = 12,7 \cdot e^{-j127°}$ A

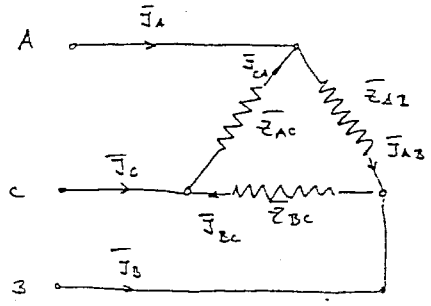
Snaga - ? $\quad P = 3 \cdot U_f \cdot I_f \cdot \cos\varphi = \sqrt{3}\, U_L \cdot I_f \cdot \cos\varphi = \sqrt{3} \cdot 220 \cdot 12,7 \cdot \cos 37°$

$= 3860,3$ W $\quad \boxed{P = 3,86\ kW}$ W

6.2

DATAK: ⇒ Trošilo je spojeno u trokut $\bar{Z}_{AB} = \bar{Z}_{BC} = \bar{Z}_{CA} = 10e^{j37°}$. Linijski napon izvora je $U_L = 220V$

1) odredite fazne i linijske struje.



Odabiremo da je ⇒ $\bar{U}_{AB} = 220e^{j0°}$ V

$\bar{U}_{CA} = 220e^{j120°}$ V

$\bar{U}_{BC} = 220e^{j240°}$ V

FAZNE STRUJE

$\bar{I}_{CA} = \frac{\bar{U}_{CA}}{\bar{Z}} = \frac{220 \cdot e^{j120°}}{10e^{j37°}} = 22 \cdot e^{j83°}$ A

$\bar{I}_{AB} = \frac{220 \cdot e^{j0°}}{10 \cdot e^{j37°}} = 22 \cdot e^{-j37°}$ A

$\bar{I}_{BC} = \frac{220 \cdot e^{j240°}}{10 \cdot e^{j37°}} = 22 \cdot e^{j203°}$

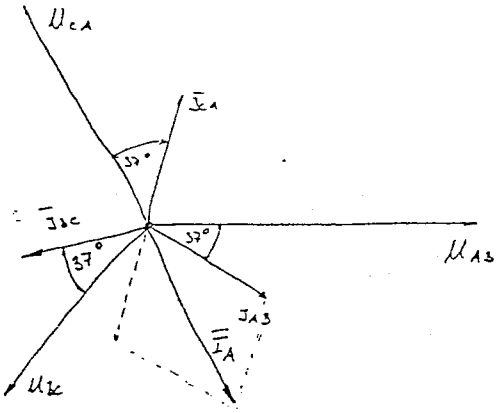
Linijske struje = ? $\bar{I}_A = \bar{I}_{AB} - \bar{I}_{CA} = 22 \cdot e^{-j37^\circ} - 22\, e^{j83^\circ} = 22\left(e^{-j37^\circ} - e^{j83^\circ}\right)$

$= 22\left[0{,}8 - j\,0{,}6 - 0{,}12 - j\,0{,}99\right] = 22\left[0{,}68 - j\,1{,}59\right]$

$= 22 \cdot 1{,}73 \cdot e^{-j67^\circ} = 38 \cdot e^{-j67^\circ}\ A$

$\bar{I}_B = \bar{I}_{BC} - \bar{I}_{AB} = 22\, e^{j203^\circ} - 22\, e^{-j37^\circ} = 22\left(e^{j203^\circ} - e^{-j37^\circ}\right) = 22\left[-0{,}92 - j\,0{,}391 - 0{,}8 + j\,0{,}6\right]$

$= 22\left[-1{,}72 + j\,0{,}209\right] = 38 \cdot e^{j173^\circ}\ A$



linijski napon čine zrijezda.

$P = 3 \cdot U_f \cdot I_f \cdot \cos\varphi$

$P = 3 \cdot \frac{1}{\sqrt{3}} U_L \cdot I_L \cdot \cos\varphi = \sqrt{3}\, U_L \cdot I_L \cdot \cos\varphi$

I_f $= \sqrt{3} \cdot 220 \cdot 38 \cdot 0{,}8 = 11570{,}24$

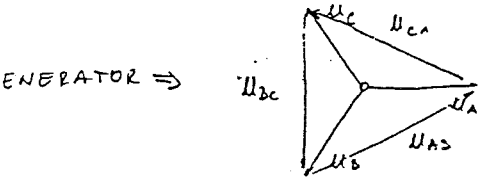
$P = 11{,}57\ kW$

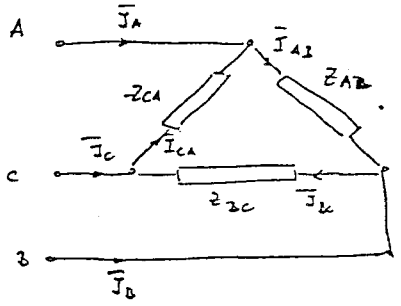
6.4

ZADATAK: Na trofaznu mrežu linijskog napona $U_L = 240\,V$ priključen je potrošač spojen u trokut: $\bar{Z}_{AB} = 10\, e^{j0^\circ}$ $\bar{Z}_{BC} = 10\, e^{j30^\circ}$ i $\bar{Z}_{CA} = 15\, e^{-j30^\circ}$. Izračunajte fazne i linijske struje, snagu trošila.

Uzeti ćemo da je

$\bar{U}_{BC} = 240\, e^{j0^\circ}$





$\bar{U}_{BC} = 240\, e^{j0^\circ}\,V$ $\bar{U}_{AB} = 240 \cdot e^{j120^\circ}\,V$ $\bar{U}_{CA} = 240 \cdot e^{j240^\circ}\,V$

$U_L = U_f$

$\bar{I}_{BC} = \frac{\bar{U}_{BC}}{\bar{Z}_{BC}} = \frac{240 \cdot e^{j0^\circ}}{10\, e^{j30^\circ}} = 24 \cdot e^{-j30^\circ}\ A$

$\bar{I}_{CA} = \frac{\bar{U}_{CA}}{\bar{Z}_{CA}} = \frac{240 \cdot e^{j240^\circ}}{15 \cdot e^{-j30^\circ}} = 16 \cdot e^{j270^\circ}\ A$

$\bar{I}_{AB} = \frac{240\, e^{j120^\circ}}{10\, e^{j0^\circ}} = 24 \cdot e^{j120^\circ}\ A$

Linijske struje = ?

$\bar{I}_A = \bar{I}_{AB} - \bar{I}_{CA} = 24 \cdot e^{j120^\circ} - 16 \cdot e^{j270^\circ} = -12 + j\,20{,}78 - 0 + j\,16 = -12 + j\,36{,}78 = 41\, e^{j108{,}1^\circ}\ A$

$\bar{I}_C = \bar{I}_{CA} - \bar{I}_{BC} = 16\, e^{j270^\circ} - 24 \cdot e^{-j30^\circ} = -j\,16 - 20{,}78 + j\,12 = -20{,}78 - j\,4 = 21{,}16\, e^{j190^\circ}\ A$

$\bar{I}_B = \bar{I}_{BC} - \bar{I}_{AB} = 46{,}1\, e^{-j15^\circ}$

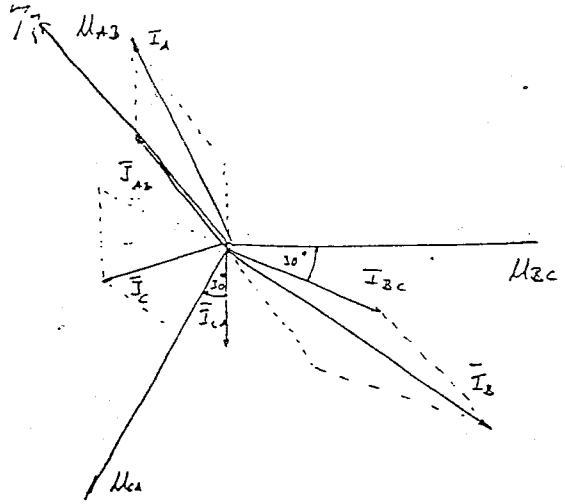
Izračunaj snagu $P = P_{AB} + P_{Bc} + P_{cA} \Rightarrow$

$P = U_{AB} \cdot I_{AB} \cdot \cos\varphi_{AB} + U_{Bc} \cdot I_{Bc} \cdot \cos\varphi_{Bc} + U_{cA} \cdot I_{cA} \cdot \cos\varphi_{cA}$

$= 240 \cdot 24 \cdot \cos 0^\circ + 240 \cdot 24 \cdot \cos(+30^\circ) + 240 \cdot 16 \cdot \cos(-30^\circ)$

$= 5760 + 4988,3 + 3325,53 = 14,073\ kW$

VEKTORSKI DIJAGRAM



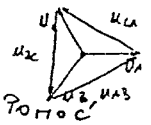
6.5

ZADATAK: → Na trofaznu mrežu linijskog napona $U_L = 208\ V$, s nulčim vodičem priključen je potrošač spojen u zvijezdu. $\bar{Z}_A = 10\angle 0^\circ$ $\bar{Z}_B = 15\angle 30^\circ$ i $\bar{Z}_c = 10\angle -30^\circ$

Odredite: 1) linijske struje, struju kroz nulti vodič i snagu potrošača

2) -II- ab pukne nulti vodič

Uzeti da je $\bar{U}_{Bc} = 208\angle 0^\circ$



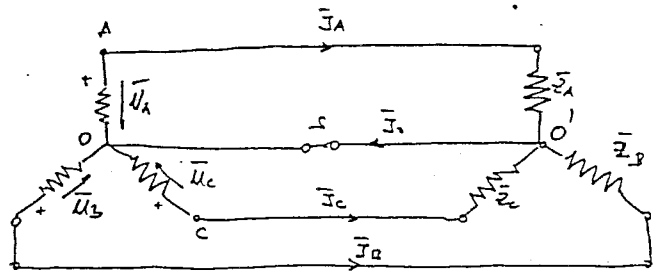
a) Ak je sklopka zatvorena

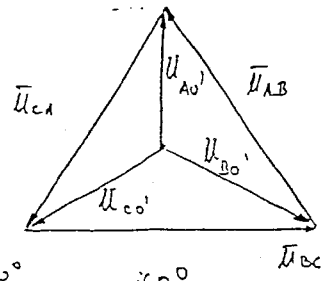
$\bar{U}_{Bc} = 208 \cdot e^{j0^\circ}\ [V]$

$\bar{U}_{AB} = 208 \cdot e^{j120^\circ}\ [V]$

$\bar{U}_{cA} = 208 \cdot e^{j240^\circ}\ [V]$ $\quad U_{o'o} = 0\ V$

$U_f = \frac{208}{\sqrt{3}} = 120\ V$





$\bar{I}_A = \frac{\bar{U}_{Ao'}}{\bar{Z}_A} = \frac{120\, e^{j0^\circ}}{10} = 12\, e^{j0^\circ}\ A$ $\quad \bar{I}_B = \frac{120 \cdot e^{-j30^\circ}}{15 \cdot e^{j30}} = 8 \cdot e^{-j60^\circ}\ A$

$\bar{I}_c = \frac{120 \cdot e^{j210^\circ}}{10 \cdot e^{-j30^\circ}}$

$\bar{I}_c = 12\, e^{j240^\circ}\ A$

$P = P_A + P_B + P_c = 120 \cdot 12 \cdot 1 + 120 \cdot 8 \cdot \cos 30^\circ + 120 \cdot 12 \cdot \cos 30^\circ$

$= 1440 + 831,38 + 1247,07 = 3518,5\ W$

$P = 3,5185\ kW$

$\bar{I}_o = \bar{I}_A + \bar{I}_B + \bar{I}_c = 5,69 \cdot e^{j69,4^\circ}\ A$

(6,5)

ako se sklopka otvori biti će $\Longrightarrow$

$$U_{0'0} = \frac{\overline{U}_{A0}\overline{Y}_A + \overline{U}_{B0}\overline{Y}_B + \overline{U}_{C0}\overline{Y}_C}{\overline{Y}_A + \overline{Y}_B + \overline{Y}_C} = \frac{120 \cdot e^{j90} \cdot 0{,}1 + 120 \cdot e^{-j30^\circ} \cdot 0{,}0667 \cdot e^{-j30^\circ} + 120 \cdot e^{210^\circ} \cdot 0{,}1 \cdot e^{j30^\circ}}{0{,}1 + 0{,}0667 \cdot e^{-j30^\circ} + 0{,}1 \cdot e^{j30^\circ}}$$

$$U_{0'0} = \frac{12(0+j) + 8 \cdot e^{-j60^\circ} + 12 \cdot e^{240^\circ}}{0{,}1 + 0{,}0667(0{,}867 - j0{,}5) + 0{,}1(0{,}867 + j0{,}5)}$$

$$U_{0'0} = \frac{j12 + 8(0{,}5 - j0{,}867) + 12(-0{,}5 - j0{,}867)}{0{,}1 + 0{,}0578 - j0{,}033 + 0{,}0867 + j0{,}05} = \frac{j12 + 4 - j6{,}936 - 6 - j10{,}404}{0{,}2445 + j0{,}017}$$

$$U_{0'0} = \frac{-2 - j5{,}34}{0{,}2445 + j0{,}017} = \frac{5{,}7 \cdot e^{j250^\circ}}{0{,}245 \cdot e^{j4}} = 23{,}26 \cdot e^{j246^\circ}\ V$$

$$\overline{U}_{0'0} = \overline{U}_{A0} - \overline{U}_{A0'} \Rightarrow \overline{U}_{A0'} = \overline{U}_{A0} - \overline{U}_{0'0} = 120e^{j90} - 23{,}26e^{j246^\circ}$$

$$= +j120 + (0{,}4 + j0{,}913) \cdot 23{,}26 = j120 + 9{,}3 + j21{,}23$$

$$= 9{,}3 + j141{,}23 = 141{,}5 \cdot e^{j86{,}23^\circ}$$ ✓

$$\overline{I}_A = \frac{\overline{U}_{A0'}}{\overline{Z}_A} = \frac{141{,}5 \cdot e^{j86{,}23^\circ}}{10\ e^{j0}} = 14{,}15e^{j86{,}23^\circ}\ A$$

analogno $\overline{I}_B = 8\angle -48{,}5^\circ$

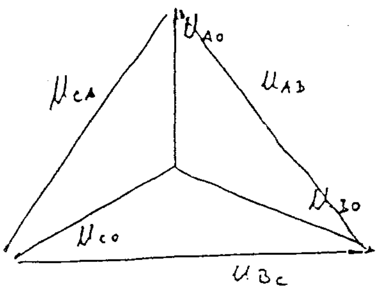
$\overline{I}_C = 10{,}2\angle 125^\circ$

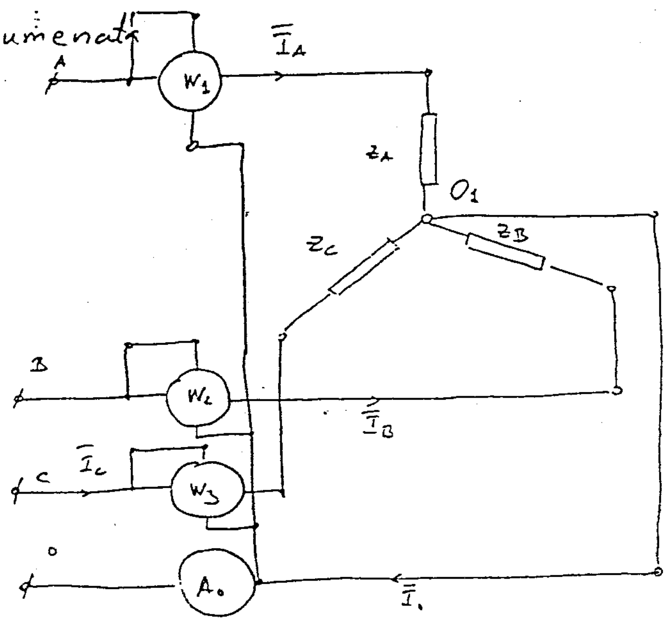
ZADATAK: 6.14 Zadana je mreža $U_L = 173\ V$, $\overline{Z}_A = 3 + j4$, $\overline{Z}_B = 6 + j8$, $\overline{Z}_C = 12 + j16$

Odredi pokazivanje instrumenata!

$$U_f = \frac{173}{\sqrt{3}} = 100\ V$$

za generator





Neka je $U_{A0} = 100\ e^{j90}\ V$

$U_{C0} = 100\ e^{j210}\ V$

$U_{B0} = 100 \cdot e^{-j30^\circ}\ V$

$$\overline{I}_A = \frac{100\ e^{j90^\circ}}{3 + j4} = \frac{100\ e^{j90^\circ}}{5 \cdot e^{j53^\circ}} = 20\ e^{j37^\circ}\ A$$

$$P_{W_1} = Re\{\overline{U}_A \cdot \overline{I}_A^*\} = Re\{100 \cdot e^{j90} \cdot 20 \cdot e^{-j37}\} =$$

$$P_{W_1} = Re\{2000 \cdot e^{j53}\} = 1203{,}6\ W$$

$$\boxed{P_{W_1} = 1{,}2036\ kW}$$

$$\bar{I}_3 = \frac{100\,e^{-j30^\circ}}{6-j8} = \frac{100\cdot e^{-j30^\circ}}{10\cdot e^{-j53^\circ}} = 10\cdot e^{-j83^\circ}\ A$$

$$P_{W2} = Re\{\bar{U}_{20}\cdot \bar{I}_2^*\} = Re\{100\,e^{-j30}\cdot 10\cdot e^{+j53^\circ}\}$$

$$P_{W2} = 1000\cdot e^{-j53^\circ} = 600\,W \qquad \boxed{P_2 = 0{,}6\,kW}$$

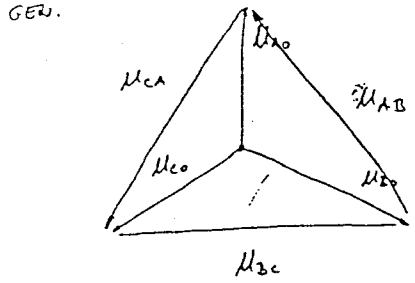
$$\bar{I}_c = \frac{100\cdot e^{j210}}{12-j16} = \frac{100\cdot e^{j210}}{20\cdot e^{j53^\circ}} = 5\cdot e^{j157^\circ}$$

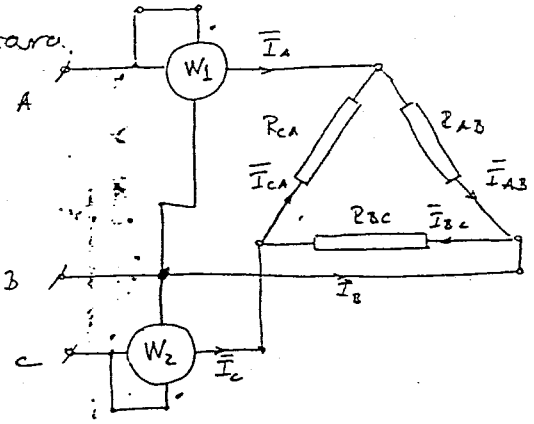
$$P_{W3} = Re\{100\cdot e^{j210}\cdot 5\cdot e^{-j157^\circ}\} = 300\,W \qquad \boxed{P_3 = 0{,}3\,kW}$$

6.15

ZADATAK: $U_L = 380V$ $Z_{AB} = Z_{BC} = Z_{CA} = 38\Omega$

Odredi snagu na osnovu pokazivanja vatmetara.





$$\bar{U}_{BC} = 380\,e^{j0^\circ}\,V \qquad \bar{U}_{AB} = 380\,e^{j120^\circ}\,V \qquad \bar{U}_{CA} = 380\cdot e^{j240^\circ}\,V$$

$$\bar{I}_{CA} = \frac{\bar{U}_{CA}}{R_{CA}} = \frac{380\cdot e^{j240^\circ}}{38} = 10\,e^{j240^\circ}\,A \qquad \bar{I}_{BC} = 10\cdot e^{j0^\circ}\,A \qquad \bar{I}_{AB} = 10\cdot e^{j120^\circ}\,A$$

$$\bar{I}_A = \bar{I}_{AB} - \bar{I}_{CA} = 10\,e^{j120^\circ} - 10\cdot e^{j240^\circ}$$

$$\bar{I}_A = 10\left(e^{j120^\circ} - e^{j240^\circ}\right)$$

$$\bar{I}_A = 10\left[-0{,}5 + j\frac{\sqrt{3}}{2} + 0{,}5 + j\frac{\sqrt{3}}{2}\right]$$

$$\bar{I}_A = 10\cdot\sqrt{3}\,j = j17{,}3\,A$$

$$P_{W1} = Re\{\bar{U}_{AB}\cdot I_A^*\} =$$

$$P_{W1} = Re\{380\cdot e^{j120^\circ}\cdot 17{,}3\,e^{-j90^\circ}\}$$

$$= Re\{6574\cdot e^{j30^\circ}\} = \boxed{5693{,}25\,W = P_{W1}}$$

$$P = 3\cdot U_f\cdot I_f\cdot\cos\varphi = 3\cdot U_l\cdot\frac{1}{\sqrt{3}}I_l\cdot\cos 0 = \sqrt{3}\cdot U_l\cdot I_l = \sqrt{3}\cdot 380\cdot 17{,}3 = 11{,}374\,kW$$

$$\boxed{P = 11{,}374\,kW}$$

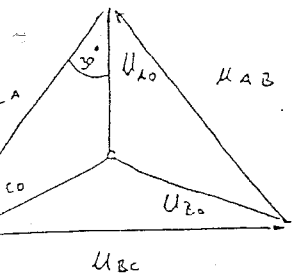
6.18

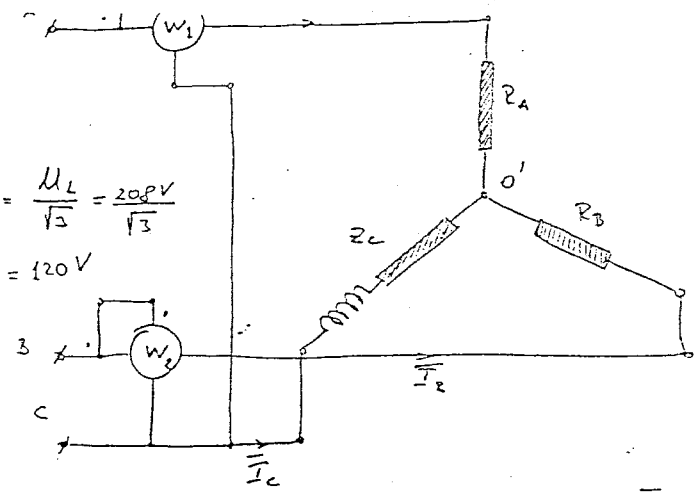
ZADATAK: Zadana je mreža $U_L = 208V$, $R_a = 6\Omega$ $R_b = 3\Omega$ $\bar{Z}_c = 2\sqrt{3}\,e^{j30^\circ}\,\Omega$

Izračunajte 1) struje potrošača

2) snagu koju troše potrošači na osnovi pokazivanja vatmetara

Generator (6.18)





$U_f = \frac{U_L}{\sqrt{3}} = \frac{208\,V}{\sqrt{3}}$

$U_f = 120\,V$

$\bar{U}_{CA} = 208 \cdot e^{+j240°}$

$\bar{U}_{AO} = 120 \cdot e^{j90°}$ V

$\bar{U}_{BO} = 120 \cdot e^{-j30°}$ V

$\bar{U}_{CO} = 120 \cdot e^{j210}$ V

$\bar{U}_{O'O} = \dfrac{\bar{U}_{AO}\cdot\bar{Y}_A + \bar{U}_{BO}\cdot\bar{Y}_B + \bar{U}_{CO}\cdot\bar{Y}_C}{\bar{Y}_A + \bar{Y}_B + \bar{Y}_C}$

$\bar{Y}_C = \dfrac{1}{\bar{Z}_C} = \dfrac{1}{2\sqrt{3}\,e^{j30°}}$

$\bar{Y}_C = 0{,}288 \cdot e^{-j30°}$

$$\bar{U}_{O'O} = \frac{120\cdot e^{j90°}\cdot 0{,}1667 + 120\cdot e^{-j30°}\cdot 0{,}3333 + 120\cdot e^{j210}\cdot 0{,}288\cdot e^{-j30°}}{0{,}1667 + 0{,}3333 + 0{,}288\cdot e^{-j30}}$$

$$\bar{U}_{O'O} = \frac{20\cdot e^{j90°} + 40\cdot e^{-j30°} + 34{,}56\cdot e^{j180}}{0{,}1667 + 0{,}3333 + 0{,}25 - j0{,}144} = \frac{j20 + 40(0{,}866 - j0{,}5) + 34{,}56(-1 + j\cdot 0)}{0{,}75 - j0{,}144}$$

$$\bar{U}_{O'O} = \frac{\cancel{j20} + 34{,}64 - \cancel{j20} - 34{,}56}{0{,}75 - j0{,}144} = 0\,V$$

$$\Rightarrow \bar{I}_A = \frac{120\cdot e^{j90°}}{6} = 20\cdot e^{j90°}\,A$$

$$\bar{I}_B = \frac{120\cdot e^{-j30}}{3} = 40\cdot e^{-j30}\,A$$

$$\bar{I}_C = \frac{120\cdot e^{j210°}}{2\sqrt{3}\,e^{j30°}} = \frac{60}{\sqrt{3}}\cdot e^{j180°} = 20\sqrt{3}\,e^{j180°}\,A$$

$P = ?$ $\quad P_{W_1} = Re\left\{\bar{U}_{AC}\cdot\bar{I}_A^*\right\} = Re\left\{208\cdot e^{j240°}\cdot 20\cdot e^{-j90°}\right\} = Re\left\{4160\cdot e^{+j150}\right\}$

$P_{W_2} = Re\left\{\bar{U}_{BC}\cdot\bar{I}_B^*\right\} = Re\left\{208\cdot e^{j0°}\cdot 40\cdot e^{-j30}\right\} = 3602{,}6\,W$

$P_{W_2} = Re\left\{8320\cdot e^{-j30}\right\}$

$= 7205{,}33\,W$

$\boxed{P = 10{,}8\,kW}$

ili $\quad P = P_1 + P_2 + P_3 = 120\cdot 20\cdot 1 + 120\cdot 40\cdot 1 + 120\cdot 20\sqrt{3}\cdot\cos 30°$

$= 2400 + 4800 + 120\cdot 20\sqrt{3}\cdot\frac{\sqrt{3}}{2} = \underline{10{,}8\,kW}$

ZADATAK: Potrošač vezan u trokut spojen je na trofaznu mrežu s naponom $U_l = 220\,V$

6.19 $\underline{Z}_{AB} = \underline{Z}_{BC} = \underline{Z}_{CA} = 22 \cdot e^{j60^\circ}\ \Omega$

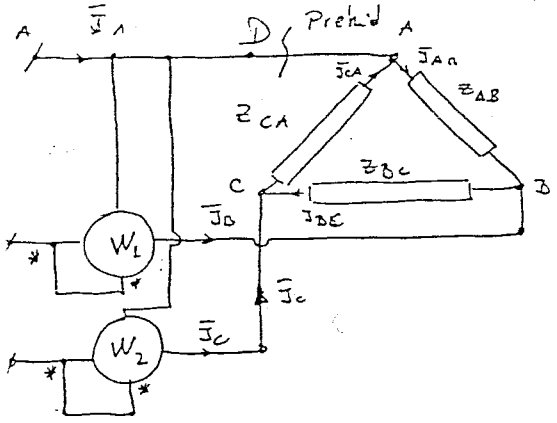
Linijski vod je prekinut u točki D.

Odredite a) fazne i linijske struje

b) pokazivanje wattmetar

$U_l = 220\,V$

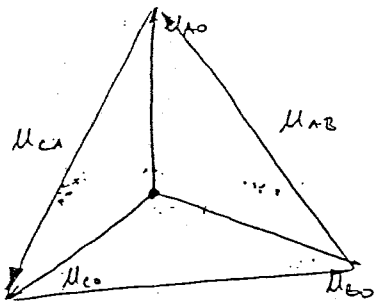
$U_f = \frac{220}{\sqrt{3}} =$



$\underline{U}_{AO} = 155{,}56 \cdot e^{j90^\circ}\ V$

$\underline{U}_{BO} = 155{,}56 \cdot e^{-j30^\circ}\ V$

$\underline{U}_{CO} = 155{,}56 \cdot e^{j210^\circ}\ V$



$U_{BC} = 220 \cdot e^{j0^\circ}\ V$

$U_{AB} = 220 \cdot e^{j120^\circ}\ V$

$U_{CA} = 220 \cdot e^{j240^\circ}\ V$

$$\underline{I}_{CA} = \underline{I}_{AB} = \frac{\underline{U}_{BD}}{\underline{Z}_{CA} + \underline{Z}_{AB}} = \frac{-220 \cdot e^{j0^\circ}}{2 \cdot 22 \cdot e^{j60}} = -5 \cdot e^{-j60^\circ}\ A \qquad \underline{I}_{BC} = \frac{\underline{U}_{BC}}{Z_{BC}} = \frac{220 \cdot e^{j0^\circ}}{22 \cdot e^{j60}} = 10 \cdot e^{-j60^\circ}\ A$$

$$\underline{I}_B = \underline{I}_{BC} - \underline{I}_{AB} = 10 \cdot e^{-j60} - (-5 \cdot e^{-j60^\circ}) = 15 \cdot e^{-j60^\circ}\ A$$

$$\underline{I}_C = \underline{I}_{CA} - \underline{I}_{BC} = -5 \cdot e^{-j60^\circ} - 10 \cdot e^{-j60^\circ} = -15 \cdot e^{-j60^\circ}\ A$$

$\boxed{\underline{I}_B = 15 \cdot e^{-j60^\circ}\ A}$

$\boxed{\underline{I}_C = -15 \cdot e^{-j60^\circ}\ A}$

$$P_{W1} = Re\left\{\underline{U}_{BA} \cdot \underline{I}_B^*\right\} = Re\left\{-220 \cdot e^{j120^\circ} \cdot 15 \cdot e^{j60^\circ}\right\} = Re\left\{-3300 \cdot e^{j180^\circ}\right\} \Rightarrow P_{W1} = 3{,}3\,kW$$

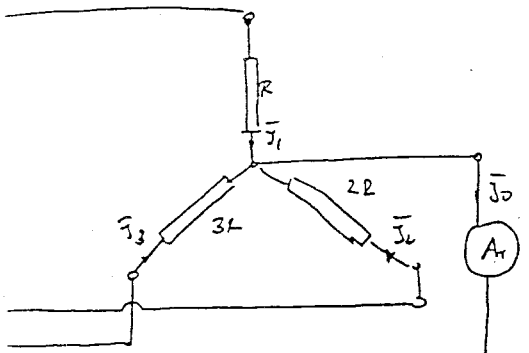
$$P_{W2} = Re\left\{\underline{U}_{CA} \cdot \underline{I}_C^*\right\} = Re\left\{220 \cdot e^{j240^\circ} \cdot (-15 \cdot e^{+j60^\circ})\right\} = Re\left\{-3300 \cdot e^{j300^\circ}\right\} = Re\left\{-3300 \cdot e^{j300^\circ}\right\}$$

$$= -1650\,W$$

$\boxed{P_{W2} = -1650\,W}$

ZADATAK: ZADANO: U_f i otpori u žilama $R, 2R, 3R$,

6.20 Odredi struju u nultom vodu



$\underline{U}_{AO} = U_f \cdot e^{j90^\circ}\ V$ $\qquad \underline{I}_1 = \frac{U_f}{R} \cdot e^{j90^\circ}\ A$

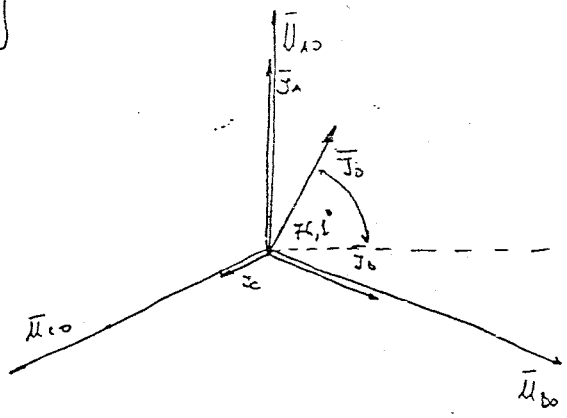
$\underline{U}_{BO} = U_f \cdot e^{-j30^\circ}\ V$ $\qquad \underline{I}_2 = \frac{U_f}{2R} \cdot e^{-j30^\circ}\ A$

$\underline{U}_{CO} = U_f \cdot e^{j210^\circ}\ V$ $\qquad \underline{I}_3 = \frac{U_f}{3R} \cdot e^{j210^\circ}\ A$

$$= \bar{J}_1 + \bar{J}_2 + \bar{J}_3 = \frac{U_f}{R}\left[e^{j90^\circ} + \frac{1}{2}e^{-j30^\circ} + \frac{1}{3}e^{j210^\circ}\right] = \frac{U_f}{R}\left[j + \frac{1}{2}\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right) + \frac{1}{3}\left(-\frac{\sqrt{3}}{2} - j\frac{1}{2}\right)\right]$$

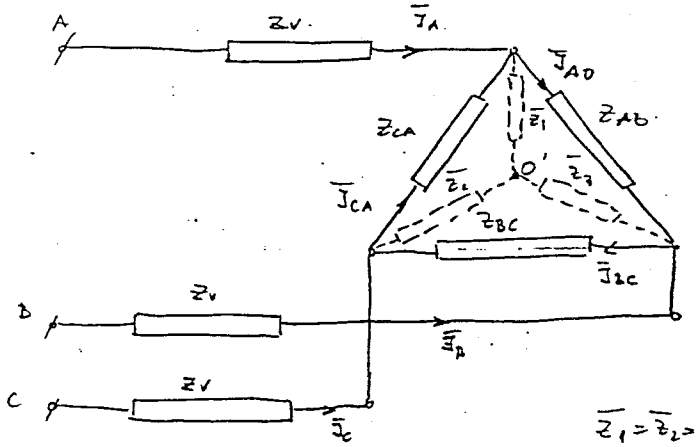
$$= \frac{U_f}{R}\left[j + \frac{\sqrt{3}}{4} - j\frac{1}{4} - \frac{\sqrt{3}}{6} - j\frac{1}{6}\right] = \frac{U_f}{R}\left[j\frac{7}{12} + \frac{\sqrt{3}}{12}\right] = \frac{U_f}{R}\cdot\frac{\sqrt{3}+j7}{12} = \frac{U_f}{R}\left(0,144338 + j\,0,583333\right)$$

$$= \frac{U_f}{R}\,0,6\cdot e^{j76,103^\circ}\ A$$



ZATAK 8: Zadano je mreža gdje je $U_l = 127\ V$, $\bar{Z}_{AD} = \bar{Z}_{DC} = \bar{Z}_{CA} = 10e^{j37^\circ}$ i $\bar{Z}_v = e^{j53^\circ}\ \Omega$

6.30 Odredi linijske i fazne struje, napone trošila, pad napona na vodovima, snagu potrošača i snagu gubitaka u vodičima.



$U_l = 127\ V$

$\bar{Z}_{AB} = \bar{Z}_{BC} = \bar{Z}_{CA} = 10e^{j37^\circ}\ \Omega$

$\bar{Z}_V = e^{j53^\circ}\ \Omega$

$$\bar{Z}_1 = \bar{Z}_2 = \bar{Z}_3 = \frac{\bar{Z}_{AB}\cdot\bar{Z}_{CA}}{Z_{CA}+Z_{BC}+Z_{AB}} = \frac{100\cdot e^{j74^\circ}}{30\,e^{j37^\circ}}$$

$$\bar{Z}_1 = \bar{Z}_2 = \bar{Z}_3 = 3,3333\cdot e^{j37^\circ}\ \Omega$$

$\bar{U}_{00} = 0$ simetrično opterećenje

$\bar{U}_{AO} = 73,4\,e^{j30^\circ}\ V$

$\bar{U}_{BO} = 73,4\,e^{-j30^\circ}\ V$

$\bar{U}_{CO} = 73,4\,e^{j210^\circ}\ V$

$$\bar{J}_A = \frac{73,4\,e^{j30^\circ}}{\bar{Z}_V + \bar{Z}_1} = \frac{73,4\,e^{j30^\circ}}{3,3333\cdot e^{j37^\circ} + e^{j53^\circ}} = \frac{73,4\cdot e^{j30^\circ}}{2,6621 + j2 + 0,6018 + j\,0,7986}$$

$$\bar{J}_A = \frac{73,4\cdot e^{j30^\circ}}{3,264 + j\,2,7986} = \frac{73,4\,e^{j30^\circ}}{4,3\cdot e^{j40,6^\circ}} = 17,06\,e^{j49,4}\ A$$

$$J_A = J_B = J_C = 17,06\ A$$

$$J_{CA} = J_{AB} = J_{BC} = \frac{17,04}{1,73} = 9,85\ A$$

$$U_T = 98,5\ V$$

$U_V = J_A \cdot R_V = 17,06\ V$

$P = 3U_f \cdot I_f \cdot \cos\varphi = 2,4\ kW$

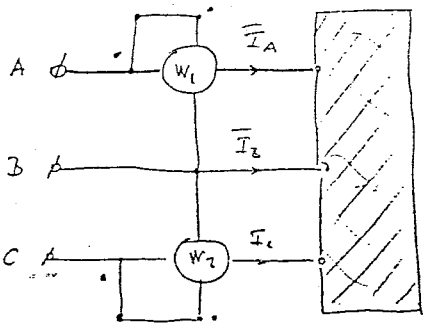
6.22

ZADATAK: ⇒ Snaga koju uzima trofazni simetrični potrošač mjerena sa dva wadmetra

$P_1 = -100\,W$ i $P_2 = 500\,W$ Izračunajte snagu koju uzima potrošač

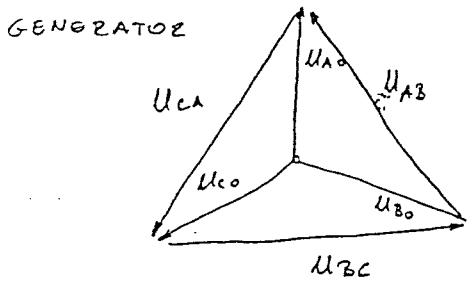
$P_{POTROŠAČA} = -100 + 500 = 400\,W$

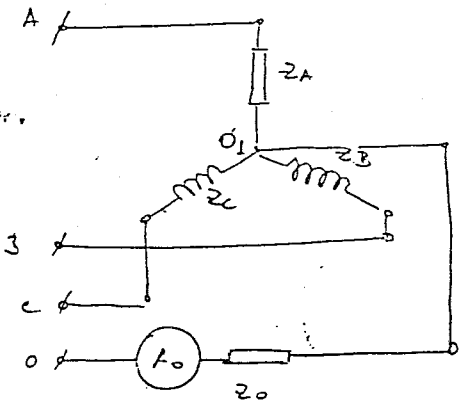
$P_{POTROŠAČA} = 400\,W$



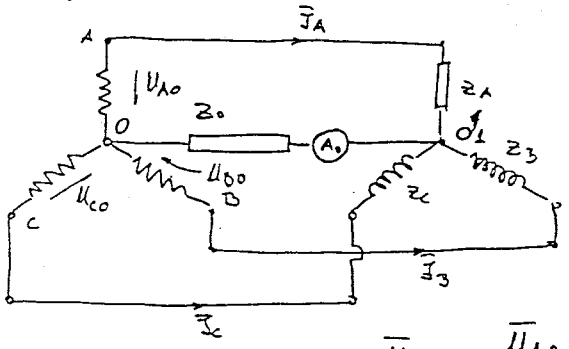
6.25

ZADATAK: $Z_A = Z_B = Z_C = Z_0 = 20\Omega$ U_f generatora $= 220\,V$ Kolika je struja u nul-vodiču.





$U_{f\,gen} = 220\,V \Rightarrow U_l = 220 \cdot \sqrt{3} = 380\,V$



$\bar{U}_{AO} = 220 \cdot e^{j90^\circ}$

$\bar{U}_{BO} = 220 \cdot e^{-j30^\circ}$

$\bar{U}_{CO} = 220 \cdot e^{j210^\circ}$

$$\bar{U}_{O'O} = \frac{\bar{U}_{AO}\cdot\bar{Y}_A + \bar{U}_{BO}\cdot\bar{Y}_B + \bar{U}_{CO}\cdot\bar{Y}_C}{\bar{Y}_A + \bar{Y}_B + \bar{Y}_C}$$

$$\bar{U}_{O'O} = \frac{220\cdot e^{j90^\circ}\cdot 0{,}05 + 220\cdot e^{-j30^\circ}\cdot\frac{1}{j20} + 220\cdot e^{j210^\circ}\cdot\frac{1}{j20}}{0{,}05 + \frac{1}{j20} + \frac{1}{j20} + 0{,}05}$$

$$\bar{U}_{O'O} = \frac{11\cdot e^{j90} - 220\cdot e^{-j30^\circ}\cdot j\,0{,}05 - 220\cdot e^{j210^\circ}\cdot j\,0{,}05}{0{,}05 - j0{,}05 - j0{,}05 + 0{,}05}$$

$$\bar{U}_{O'O} = \frac{11\cdot e^{j90^\circ} - 11\cdot e^{j90}\cdot e^{-j30^\circ} - 11\cdot e^{j90^\circ}\cdot e^{j210^\circ}}{0{,}05 - j0{,}1 + 0{,}05} = \frac{11\cdot e^{j90^\circ} - 11\,e^{j60^\circ} - 11\cdot e^{j300^\circ}}{0{,}05 - j0{,}1 + 0{,}05}$$

$$\bar{U}_{O'O} = \frac{11(0+j1) - 11\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) - 11\left(0{,}5 - j\frac{\sqrt{3}}{2}\right)}{0{,}05 - j0{,}1 + 0{,}05}$$

$$\bar{U}_{O'O} = \frac{j11 - 5{,}5 - j\frac{11\sqrt{3}}{2} - 5{,}5 + j\frac{11\sqrt{3}}{2}}{0{,}1 - j0{,}1} = \frac{j11 - 11}{0{,}1 - j0{,}1} = \frac{-11 + j11}{1 - j1} = \frac{\sqrt{2}\cdot 11\cdot e^{j135^\circ}}{\sqrt{2}\cdot e^{-j45^\circ}}$$

6.25)

$\bar{U}_{o'o} = 110 \cdot e^{j180°}\ V$ $\quad \bar{I}_{AO} = \dfrac{\bar{U}_{o'o}}{\bar{Z}_o} = \dfrac{110 \cdot e^{j180°}\ V}{20} \Rightarrow$

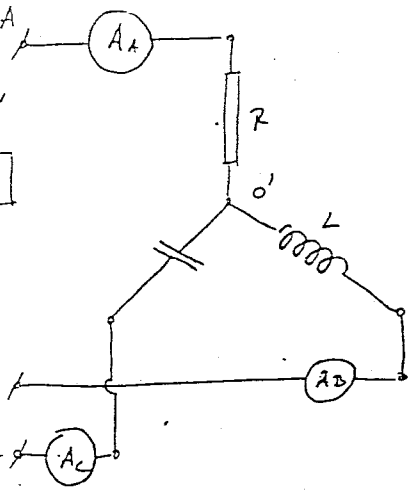
$\boxed{I_o = 5{,}5\ A}$ ✓

6,38

ADATAK 8 $U_L = 220\ V$ $\quad R = \omega L = \dfrac{1}{\omega C} = 100\Omega$

1) Ustanovite pokazivanje ampermetra

2) Nacrtajte vektorski dijagram napona i struja



$\bar{U}_{o'o} = \dfrac{\bar{U}_{AO}\bar{Y}_A + \bar{U}_{BO}\bar{Y}_B + \bar{U}_{CO}\cdot\bar{Y}_C}{\bar{Y}_A + \bar{Y}_B + \bar{Y}_C}$ $\quad U_f = \dfrac{220}{\sqrt{3}} = 127\ V$

$\boxed{U_f = 127\ V}$

$\bar{U}_{AO} = 127\, e^{j90°}\ V$

$\bar{U}_{BO} = 127 \cdot e^{-j30°}\ V$

$\bar{U}_{CO} = 127 \cdot e^{j210°}\ V$

$\bar{U}_{o'o} = \dfrac{127\, e^{j90°} \cdot 0{,}01 + 127 \cdot e^{-j30°} \cdot (-j0{,}01) + 127 \cdot e^{j210°} \cdot j0{,}01}{0{,}01 - j0{,}01 + j0{,}01}$

$\bar{U}_{o'o} = \dfrac{1{,}27 \cdot e^{j90°} - 1{,}27 \cdot e^{j60°} + 1{,}27 \cdot e^{j300°}}{0{,}01} = j1{,}27 - 1{,}27\left(0{,}5 + j\frac{\sqrt{3}}{2}\right) + 1{,}27\left(0{,}5 - j\frac{\sqrt{3}}{2}\right)$

$\bar{U}_{o'o} = \dfrac{j1{,}27 - 0{,}635 - j1{,}0998 + 0{,}635 - j1{,}0998}{0{,}01} = \dfrac{-j0{,}9206}{0{,}01} = -j92{,}06$

$\boxed{\bar{U}_{o'o} = 93 \cdot e^{-j90°}\ V}$

$\bar{U}_{Ao'} = 127\, e^{j90°} - 93 \cdot e^{-j90°} = j127 + 93 \cdot j = j220\ V$

$\bar{U}_{Bo'} = 127 \cdot e^{-j30°} - 93 \cdot e^{-j90°} = 127\left(\frac{\sqrt{3}}{2} - j0{,}5\right) - 93(0 - j)$

$= -j63{,}5 + 109{,}98 + j93 = 109{,}98 + j30$

$\bar{U}_{Bo'} = 114{,}0175 \cdot e^{j15°}$

$\bar{U}_{Co'} = 127 \cdot e^{j210°} - 93 \cdot e^{-j90°} \Rightarrow$

$= 127(-0{,}866 - j0{,}5) - 93(0 - j) = -110 - j63{,}5 + j93$

$\bar{U}_{Co'} = 113{,}88 \cdot e^{j165°}\ V$

$I_A = \dfrac{U_{Ao'}}{R} = \dfrac{220 \cdot e^{j90°}}{100} = 2{,}2\, e^{j90°}\ A$ $\quad \bar{I}_B = \dfrac{114 \cdot e^{j15°}}{100\, e^{j90°}} = 1{,}14\ e^{-j75°}$

$= -110 + 29{,}5$

$\bar{I}_C = \dfrac{114 \cdot e^{j165°}}{100 \cdot e^{-j90°}} = 1{,}14 \cdot e^{j255°}\ A$

$P = ?$ Snaga u razvija u fazi A $\quad P = Re\{\bar{U}_{Ao'} \cdot \bar{I}_A^*\}$

$= Re\{j220 \cdot 2{,}2 \cdot e^{-j90°}\}$

$= Re\{220 \cdot e^{j90°} \cdot 2{,}2\, e^{-j90°}\} = 484\ W$

$\boxed{P = 484\ W}$

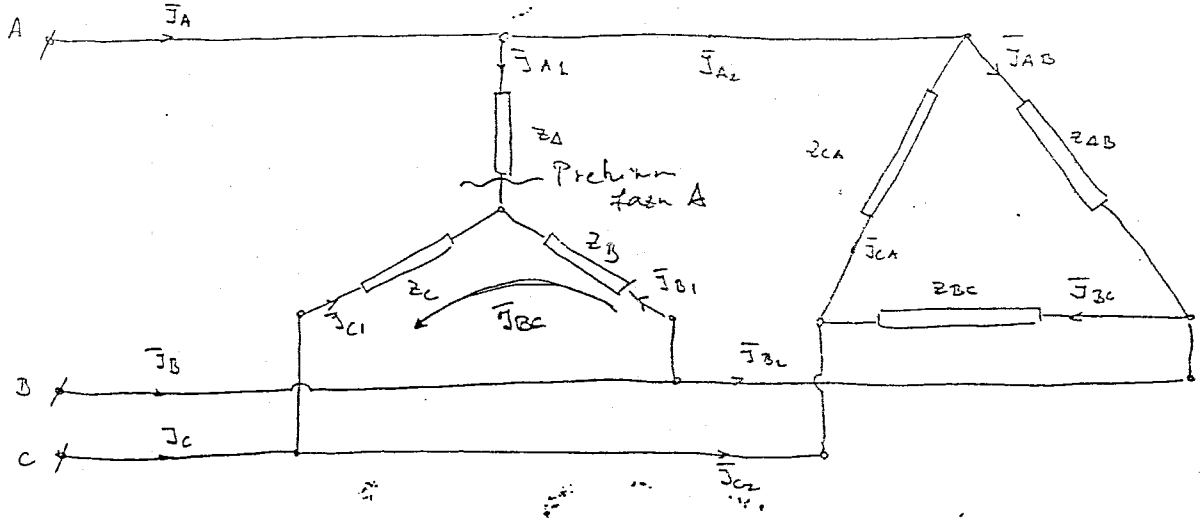
6.36

ZADATAK 8 Potrošač koji je spojen u zvijezdu priključen je na trofazni mrežu paralelno potrošaču

SLIČNO 6.46 spojenome u trokut. U normalnome radu je $\underline{I_{A1} = I_{B1} = I_{C1} = 2A}$ $\varphi_1 = 0$

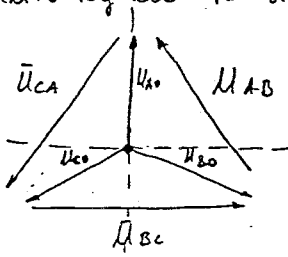
$\underline{I_{AB} = I_{BC} = I_{CA} = 1A}$ i $\varphi_2 = 30°$

Treba pronaći struje I_A, I_B, I_C, kada u zvijezdi prekine fazu A



Prvo trošilo je posve radno $R_A = R_B = R_C \Rightarrow$ simetrično

drugo trošilo je induktivnog karaktera $\varphi = +30°$



$\bar{U}_{A0} = U_f \cdot e^{j90°}$ $\quad \bar{J}_{A1} = \frac{U_f}{R} \cdot e^{j90°}$ $\quad \frac{U_f}{R} = 2A$ $\quad \bar{J}_{A1} = 2\,e^{j90°}\ A$

$\bar{U}_{B0} = U_f \cdot e^{-j30°}$ $\quad \bar{J}_{B1} = \frac{U_f}{R} \cdot e^{-j30°}$ $\quad \frac{U_f}{R} = 2A$ $\quad \bar{J}_{B1} = 2\,e^{-j30°}\ A$

$\bar{U}_{C0} = U_f \cdot e^{j210°}$ $\quad \bar{J}_{C1} = \frac{U_f}{R} \cdot e^{j210°}$ $\quad \frac{U_f}{R} = 2A$ $\quad \bar{J}_{C1} = 2\,e^{j210°}\ A$

$$\bar{J}_{AB} = \frac{\bar{U}_{AB}}{\bar{Z}_{AB}} = \frac{\sqrt{3}U_f \cdot e^{j120°}}{\bar{Z}_{AB}} = \frac{\sqrt{3}U_f \cdot e^{j120°}}{Z_{AB} \cdot e^{j30°}} = \frac{\sqrt{3}U_f}{Z_{AB}} \cdot e^{j90°} = 1 \cdot e^{j90°}\ A$$

$$\bar{J}_{CA} = \frac{\bar{U}_{CA}}{\bar{Z}_{CA}} = \frac{\sqrt{3}U_f \cdot e^{j240°}}{\bar{Z}_{CA}} = \frac{\sqrt{3}U_f}{Z_{CA}} \cdot \frac{e^{j240°}}{e^{j30°}} = \frac{\sqrt{3}U_f}{Z_{CA}} \cdot e^{j210°} = 1 \cdot e^{j210°}\ A$$

$$\bar{J}_{BC} = \frac{\bar{U}_{BC}}{\bar{Z}_{BC}} = \frac{\sqrt{3}U_f \cdot e^{j0°}}{Z_{BC} \cdot e^{j30°}} = \frac{\sqrt{3}U_f}{Z_{BC}} e^{-j30°} \qquad J_{BC} = \frac{\sqrt{3}U_f}{Z_{BC}} \cdot e^{-j30°} = 1 \cdot e^{-j30°}\ A$$

Kad prekinem fazu A u trošilu ① struje u drugom trošilu neće se promijeniti

$$\bar{J}_{BC_1} = \frac{\bar{U}_{BC}}{2R} = \frac{\sqrt{3}U_f \cdot e^{j0°}}{2R} = \frac{\sqrt{3}}{2}\frac{U_f}{R} \cdot e^{j0°} = \frac{\sqrt{3}}{\cancel{2}} \cdot \cancel{2} \cdot e^{j0°} = \sqrt{3} \cdot e^{j0°}\ A \qquad \boxed{\bar{J}_{BC} = \sqrt{3}\,e^{j0°}\ A}$$

$$\Rightarrow \bar{J}_A = \bar{J}_{AB} - \bar{J}_{CA} \Rightarrow \bar{J}_A = e^{j90°} - e^{j210°} = [0 + j] - \left[-\frac{\sqrt{3}}{2} - j\frac{1}{2}\right] = j + \frac{\sqrt{3}}{2} + j\frac{1}{2}$$

$$\bar{J}_A = \frac{\sqrt{3}}{2} + j\frac{3}{2} \Rightarrow \quad I_A = \sqrt{\frac{3}{4} + \frac{9}{4}} = \sqrt{\frac{12}{4}} = \sqrt{3} \qquad \boxed{\bar{J}_A = \sqrt{3} \cdot e^{j60°}\ A}$$

$$\operatorname{tg}\gamma = \frac{\frac{3}{2}}{\frac{\sqrt{3}}{2}} = \sqrt{3}$$

6,36

$\bar{I}_{B2} = \bar{I}_{BC} - \bar{I}_{AB} = e^{-j30^\circ} - e^{j90^\circ} = \left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right) - (0 + j) = \frac{\sqrt{3}}{2} - j\frac{1}{2} - j = \boxed{\frac{\sqrt{3}}{2} - j\frac{3}{2}\ A = \bar{I}_{B2}}$

$\bar{I}_B = \bar{I}_{BC1} + \bar{I}_{B2} = \sqrt{3} + \frac{\sqrt{3}}{2} - j\frac{3}{2} = \frac{3\sqrt{3}}{2} - j\frac{3}{2} = \frac{3}{2}(\sqrt{3} - j) = \frac{3}{2}\cdot 2\, e^{-j30^\circ} = 3\cdot e^{-j30^\circ}$

$\Rightarrow \boxed{I_B = 3\ A}$

$\bar{I}_{C2} = \bar{I}_{CA} - \bar{I}_{BC} = e^{j210^\circ} - e^{-j30^\circ} = -\frac{\sqrt{3}}{2} - j\frac{1}{2} - \left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right) = -\frac{\sqrt{3}}{2} - j\frac{1}{2} - \frac{\sqrt{3}}{2} + j\frac{1}{2}$

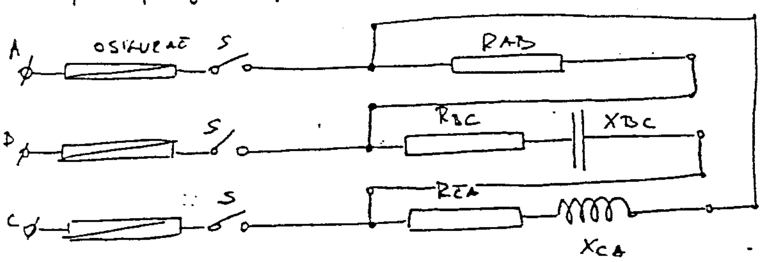
$\bar{I}_{C2} = -\frac{2\sqrt{3}}{2} = -\sqrt{3}\ A \Rightarrow \boxed{\bar{I}_{C2} = -\sqrt{3}\ A}$

$\bar{I}_C = \bar{I}_{C1} - \bar{I}_{BC} = -\sqrt{3} - \sqrt{3} = -2\sqrt{3}\ A \Rightarrow \boxed{I_C = 2\sqrt{3}\ A}$

ZADATAK 8 Na trofaznu mrežu priključen je potrošač prema slici, uz $U_L = 120\ V$

6.86 i $R_{AB} = 6{,}67\ \Omega$ a $R_{BC} = R_{CA} = 10\ \Omega$ $X_{BC} = X_{CA} = 5{,}77\ \Omega$ i $X_{AB} = 0$

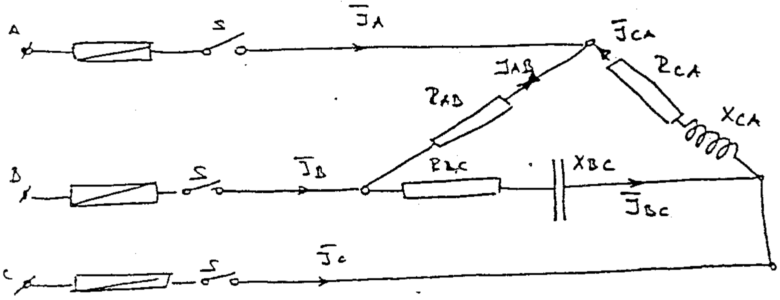
Odredite fazne i linijske struje te snagu pri nominalnom režimu rada i pri pregaranju osigurača u vodu C. Nacrtajte vektorske dijagrame.



$\bar{U}_{A0} = U_f \cdot e^{j90^\circ}\ V$

$\bar{U}_{B0} = U_f \cdot e^{-j30^\circ}\ V$

$\bar{U}_{C0} = U_f \cdot e^{j210^\circ}\ V$



$\bar{U}_{AB} = 120 \cdot e^{j120^\circ}\ V$

$\bar{U}_{CA} = 120 \cdot e^{j240^\circ}\ V$

$\bar{U}_{BC} = 120 \cdot e^{j360^\circ} = 120 \cdot e^{j0}\ V$

$\bar{I}_{AB} = \frac{\bar{U}_{AB}}{6{,}67} = \frac{120 \cdot e^{j120^\circ}}{6{,}67} = 18 \cdot e^{j120^\circ}$ $\bar{I}_{BC} = \frac{\bar{U}_{BC}}{10 - j5{,}77} = \frac{120 \cdot e^{j0^\circ}}{11{,}545 \cdot e^{-j30^\circ}} = 10{,}4 \cdot e^{j30^\circ}\ A$

$\bar{I}_{CA} = \frac{\bar{U}_{CA}}{10 + j5{,}77} = \frac{120 \cdot e^{j240^\circ}}{11{,}545 \cdot e^{+j30^\circ}} = 10{,}4 \cdot e^{j210^\circ}\ A$

$\bar{I}_A = \bar{I}_{AB} - \bar{I}_{CA} = 18 \cdot e^{j120^\circ} - 10{,}4 \cdot e^{j210^\circ} = 18(-0{,}5 + j0{,}8660) - 10{,}4(-0{,}866 - j0{,}5)$

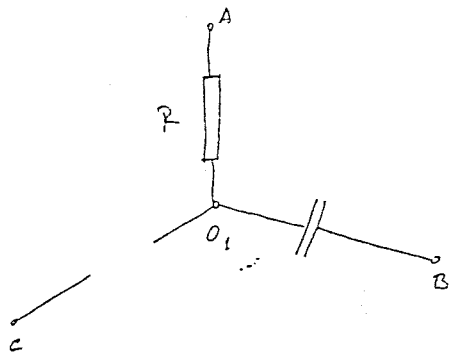
$\bar{I}_A = -9 + j15{,}588 + 9 + j5{,}2 \Rightarrow \boxed{\bar{I}_A = j20{,}788\ A}$ $\boxed{I_A = 20{,}788\ A}$

$\bar{I}_B = \bar{I}_{BC} - \bar{I}_{AB} = 10{,}4 \cdot e^{j30^\circ} - 18 \cdot e^{j120^\circ} = 10{,}4\left(\frac{\sqrt{3}}{2} + j0{,}5\right) - 18(-0{,}5 + j0{,}866)$

$\bar{I}_B = 9 + j5{,}2 + 9 - j15{,}588 = 18 - j10{,}388 = 20{,}78 \cdot e^{-j30^\circ}$ $\boxed{\bar{I}_B = 20{,}78 \cdot e^{-j30^\circ}\ A}$

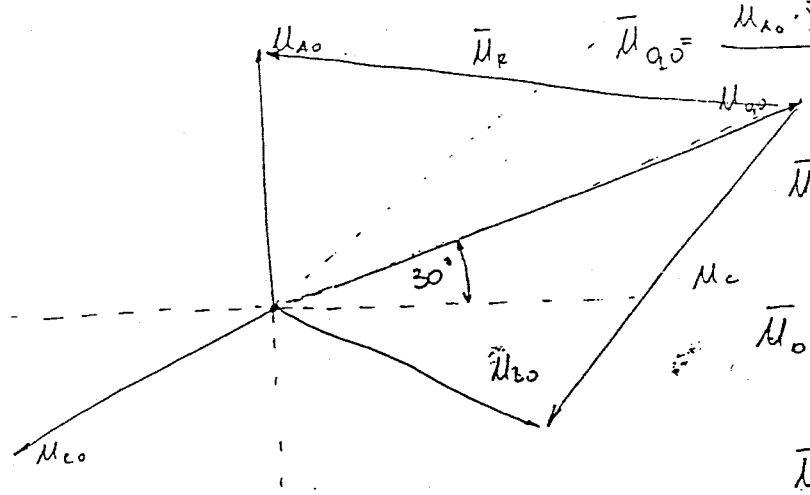
6.44.

ZADATAK ⇒ Zadano je $R = \omega L = \frac{1}{\omega C}$. Nacrtajte (kvalitativno) vektorske dijagrame napona i struja



$$\bar{U}_{AO} = U_f \cdot e^{j90^\circ} \; [V]$$

$$\bar{U}_{BO} = U_f \cdot e^{-j30^\circ} \; [V]$$



$$\bar{U}_{O1O} = \frac{\bar{U}_{AO}\cdot\bar{Y}_A + \bar{U}_{BO}\cdot\bar{Y}_R}{\bar{Y}_A + \bar{Y}_R} = \frac{U_f\cdot e^{j90^\circ}\cdot\frac{1}{R} + U_f\cdot e^{-j30^\circ}\cdot\frac{1}{-jR}}{\left.\frac{1}{R} + \frac{1}{-jR}\right\}\frac{j}{R}}$$

$$\bar{U}_{O1O} = \frac{U_f\cdot e^{j90^\circ}\cdot\frac{1}{R} + U_f\cdot e^{-j30^\circ}\cdot j\cdot\frac{1}{R}}{\frac{1}{R}\left(1+j\right)}$$

$$\bar{U}_{O1O} = \frac{U_f\cdot e^{j90^\circ} + U_f\cdot e^{j60^\circ}}{(1+j)}$$

$$\bar{U}_{O1O} = \frac{U_f\left(e^{j90^\circ} + e^{j60^\circ}\right)}{1+j} = \frac{U_f\cdot\left[+j + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right]}{1+j}$$

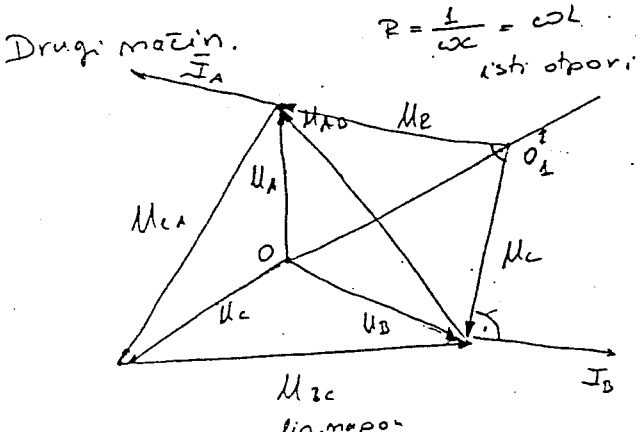
$$\bar{U}_{O1O} = \frac{U_f\cdot\left[\frac{1}{2} + j\left(\frac{\sqrt{3}}{2}+1\right)\right]}{1+j} = \frac{U_f\left[\frac{1}{2} + j1{,}866\right]^{1+j}}{\sqrt{2}\cdot e^{j45^\circ}}$$

$$\bar{U}_{O1O} = U_f\cdot\frac{1{,}932\cdot e^{j75^\circ}}{\sqrt{2}\cdot e^{j45^\circ}}$$

$$\bar{U}_{O1O} = U_f\cdot 1{,}365\cdot e^{j30^\circ}$$

$$\boxed{\bar{U}_{O1O} = U_f\cdot 1{,}365\cdot e^{j30^\circ}\,V}$$

Drugi način. $R = \frac{1}{\omega C} = \omega L$ ... isti otpori



Fazni napon

Pošto je teret nesimetričan točka O_1 biti će pomaknuta od točke O. Pošto su otpori jednaki $R = \frac{1}{\omega C} \Rightarrow |\bar{U}|_{AO_1} = |\bar{U}|_{BO_1}$

$\bar{I}_A + \bar{I}_R + \bar{I}_C = 0$ ... $\bar{I}_C = 0 \Rightarrow \bar{I}_A + \bar{I}_C = 0$

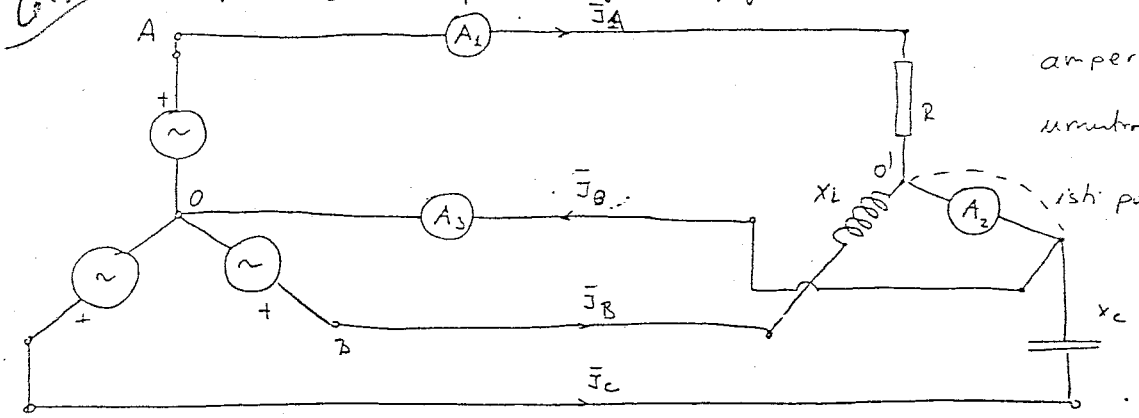
$\Rightarrow \bar{I}_A = -\bar{I}_R$ struje su suprotne u fazi.

napon na R je u fazi s $\bar{I}_B$ a napon na C zaostaje za $\bar{I}_B$

vektori napona U_R i U_C su međusobno okomiti.

ZADATAK 8 ⇒ Tri trosila napaja trofazni generator spojen u zvijezdu. Otpori imaju jednake otpore. Odredite pokazivanja drugog i treceg ampermetra ako prvi pokazuje 2A

6.55



ampermetri mjerenja unutrašnjeg otpora.

Visi potencijal mjerenja ih spojiti

prvi pokazuje 2A $\quad \bar{U}_{AO} = \bar{J}_A \cdot R \qquad \bar{U}_{AO} = U_f \cdot e^{j90^\circ}$ V

$\bar{J}_A = \dfrac{\bar{U}_{AO}}{R}$ $\qquad \bar{U}_{BO} = U_f \cdot e^{-j30^\circ}$ V

$\bar{J}_B = \dfrac{\bar{U}_{BO}}{jX_L} \qquad \bar{J}_C = \dfrac{\bar{U}_{CO}}{-jX_C} \qquad \bar{U}_{CO} = U_f \cdot e^{+j210^\circ}$ V

$$\bar{J}_O = \bar{J}_A + \bar{J}_B + \bar{J}_C = \frac{\bar{U}_{AO}}{R} + \frac{\bar{U}_{BO}}{jX_L} + \frac{\bar{U}_{CO}}{-jX_C}$$

$R = X_L = X_C = R$

$$\bar{J}_O = \frac{\bar{U}_{AO}}{R} + \frac{\bar{U}_{BO}}{jR} + \frac{\bar{U}_{CO}}{-jR} = \frac{\bar{U}_{AO}}{R} - \frac{\bar{U}_{BO}\cdot j}{R} + \frac{\bar{U}_{CO}\cdot j}{R} = \frac{1}{R}\left[\bar{U}_{AO} - j\bar{U}_{BO} + j\bar{U}_{CO}\right]$$

$$\bar{J}_O = \frac{1}{R} U_f \left[e^{j90^\circ} - e^{j60^\circ} + e^{j300^\circ}\right] = \frac{U_f}{R}\left[0 + j - \left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)\right]$$

$$\bar{J}_O = \frac{U_f}{R}\left[j - \frac{1}{2} - j\frac{\sqrt{3}}{2} + \frac{1}{2} - j\frac{\sqrt{3}}{2}\right] = \frac{U_f}{R}\left[j - j\sqrt{3}\right] = -j0{,}73 \cdot \frac{U_f}{R}$$

$\bar{J}_O = -j\,1{,}464$ A $\qquad \boxed{J_{A2} = J_O = 1{,}464 \text{ A}}$ ✓

$$\bar{J}_B = \frac{\bar{U}_{BO}}{jX_L} = \frac{\bar{U}_f \cdot e^{-j30}}{R \cdot j} = \frac{U_f}{R} \cdot e^{-j120^\circ} = 2e^{-j120^\circ}$$

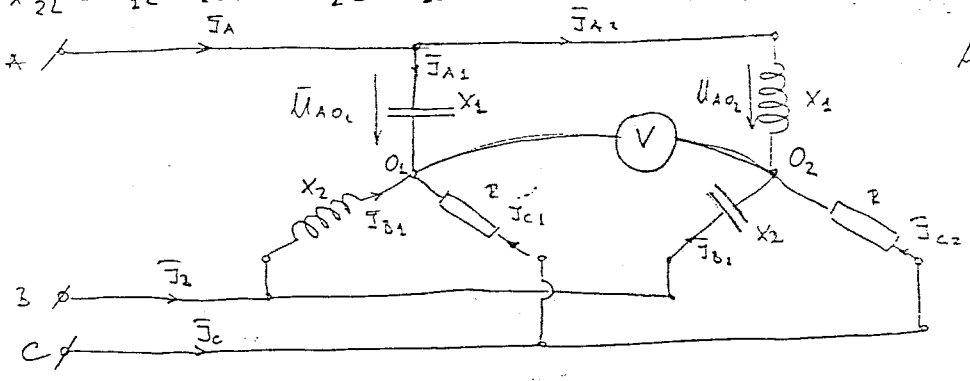
$$\bar{J}_{A2} = \bar{J}_A + \bar{J}_B = 2 \cdot e^{j90^\circ} + 2 \cdot e^{-j120^\circ} = 2\left[e^{j90^\circ} + e^{-j120^\circ}\right] = 2\left[j + \left(-0{,}5 - j\frac{\sqrt{3}}{2}\right)\right]$$

$\bar{J}_{A2} = 2\left[-0{,}5 + j\,0{,}131\right] = -1 + j\,0{,}268 \qquad \boxed{J_{A2} = 1{,}0353 \text{ A}}$ ✓

6.6.11

ZADATAK: $\Rightarrow U_L = 346\,V$ odredi pokazivanje voltmetra

$X_{2L} = X_{1C} = 100\,\Omega \quad X_{2C} = X_{1L} = 50\,\Omega \quad R = 100\,\Omega \quad R_V = \infty$



$U_L = 346\,V$

$\Rightarrow U_f = \frac{346}{\sqrt{3}}$

$U_f = 200\,V$

$\bar{I}_A = \bar{I}_{A1} + \bar{I}_{A2} \quad \bar{I}_B = \bar{I}_{B1} + \bar{I}_{B2} \quad \bar{I}_C = \bar{I}_{C1} + \bar{I}_{C2}$

9) $\bar{U}_{O_2O} = \bar{U}_{AO} - \bar{I}_{A2} \cdot \bar{Z}_{2A} \Rightarrow \bar{I}_{A2} \cdot \bar{Z}_{2A} = \bar{U}_{AO} - \bar{U}_{O_2O} \Rightarrow \bar{I}_{A2} = \frac{\bar{U}_{AO} - \bar{U}_{O2O}}{\bar{Z}_{2A}}$

općenito

$\sum \bar{I}_{X2} = 0 \quad \therefore \bar{I}_{X2} = \frac{\bar{U}_{XO} - \bar{U}_{O_2O}}{\bar{Z}_{X2}} \quad X = A, B, C$

$\sum (\bar{U}_{XO} - \bar{U}_{O_2O}) \cdot \bar{G}_{X2} = 0$

$\sum_{X=A}^{B} \bar{U}_{XO} \cdot \bar{G}_{X2} = \sum_{X=A}^{B} \bar{U}_{O_2O} \cdot \bar{G}_{X2} \Rightarrow \bar{U}_{O_2O} = \frac{\sum_{X=A}^{B} \bar{U}_{XO} \cdot \bar{G}_{X2}}{\bar{G}_{X2}}$

Računamo Millmanovim teoremom zasebno .... $\Rightarrow$

$\bar{U}_{AO} = 200\,e^{j90^\circ}\,V$

$\bar{U}_{BO} = 200\,e^{-j30^\circ}\,V$ Za čvor $O_1 \Rightarrow$

$\bar{U}_{CO} = 200\,e^{j210^\circ}\,V$ $\bar{U}_{O_1O} = \frac{\bar{U}_{AO} \cdot \bar{Y}_1 + \bar{U}_{BO} \cdot \bar{Y}_2 + \bar{U}_{CO} \cdot \bar{Y}_3}{\bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3}$

$$\bar{U}_{O_1O} = \frac{200\,e^{j90} \cdot \frac{1}{-j100} + 200\,e^{-j30^\circ} \cdot \frac{1}{j100} + 200\,e^{j210^\circ} \cdot \frac{1}{100}}{\frac{1}{-j100} + \frac{1}{j100} + \frac{1}{100}}$$

$$\bar{U}_{O_1O} = \frac{200\,e^{j90^\circ} \cdot j0{,}01 - 200\,e^{-j30^\circ} \cdot j0{,}01 + 200 \cdot e^{j210^\circ} \cdot 0{,}01}{j0{,}01 - j0{,}01 + 0{,}01} = \frac{2 \cdot e^{j180^\circ} - 2 \cdot e^{j60^\circ} + 2 \cdot e^{j210^\circ}}{0{,}01}$$

$$\bar{U}_{O_1O} = \frac{2(-1 + j0) - 2\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + 2\left(-\frac{\sqrt{3}}{2} - j\frac{1}{2}\right)}{0{,}01} = \frac{-2 - 1 - j\sqrt{3} - \sqrt{3} - j}{0{,}01}$$

$$\bar{U}_{O_1O} = \frac{-3 - \sqrt{3} - j(\sqrt{3} + 1)}{0{,}01} = \frac{-4{,}73 - j2{,}73}{0{,}01} = -473 - j273 \qquad \bar{U}_{O_1O} = 546{,}13 \cdot e^{j210^\circ}\,V$$

$$\bar{U}_{O_2O} = \frac{200\,e^{j90^\circ} \cdot \frac{1}{j50} + 200\,e^{-j30} \cdot \frac{1}{-j50} + 200 \cdot e^{j210^\circ} \cdot \frac{1}{100}}{\frac{1}{j50} + \frac{1}{-j50} + \frac{1}{100}}$$

$$\bar{U}_{O_2O} = \frac{\frac{\bar{E}}{-j0{,}02} \cdot j0{,}02 + 200 \cdot j0{,}02 + 200 \cdot 0{,}02}{-j0{,}02 + j0{,}02 + 0{,}01}$$

$$\bar{U}_{O_2O} = \frac{-j4 \cdot e^{j60^\circ} + j4\, e^{-j30^\circ} + 2\, e^{j210}}{0{,}01} = \frac{4 \cdot e^{-j60} \cdot e^{j60} + 4 \cdot e^{j90^\circ} \cdot e^{-j30^\circ} + 2 \cdot e^{j210}}{0{,}01}$$

$$\bar{U}_{O_2O} = \frac{4 + 4 \cdot e^{j60^\circ} + 2 \cdot e^{j210}}{0{,}01} = \frac{4 + 4\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + 2\left(-\frac{\sqrt{3}}{2} - j\frac{1}{2}\right)}{0{,}01}$$

$$\bar{U}_{O_2O} = \frac{4 + 2 + j2\sqrt{3} - \sqrt{3} - j}{0{,}01} = \frac{4{,}27 + j3{,}46}{0{,}01} = 427 + j246 = 432{,}8 \cdot e^{j30^\circ}$$

$$\bar{U}_{AO_1} = \bar{U}_{AO} - \bar{U}_{O_1O} = 200 e^{j90} - 546{,}13 \cdot e^{j210} = 200(0 + j) - 546{,}13(-0{,}866 - j0{,}5)$$

$$= j200 + 473 + j273 = 473 + j473 \text{ V}$$

$$\bar{U}_{AO_2} = \bar{U}_{AO} - \bar{U}_{O_2O} = j200 - 427 - j246 = -427 - j46$$

$$\bar{U}_V = \bar{U}_{O_2O_1} = \bar{U}_{AO_1} - \bar{U}_{AO_2} = 473 + j473 + 427 + j46 = 900 + j520$$

$$\boxed{U_V = 1039 \text{ V}}$$

6.62

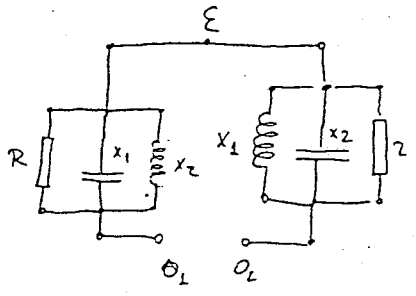
ZADATAK 8 ⇒ U prethodnom zadatku voltmetar je zamijenjen ampermetrom

6.63

Odredi pokazivanje ampermetra

Thevenin-ov teorem $U_T = 1020 \text{ V}$

$\bar{Z}_T = ?$



$$\frac{1}{\bar{Z}_{O_1\mathcal{E}}} = \frac{1}{R} + \frac{1}{-jX_1} + \frac{1}{jX_2} = \frac{1}{100} + \frac{j}{100} - \frac{j}{100} = 0{,}01 \Rightarrow \bar{Z}_{O_1\mathcal{E}} = 100\,\Omega$$

$$\frac{1}{\bar{Z}_{O_2\mathcal{E}}} = \frac{1}{R} + \frac{1}{-jX_2} + \frac{1}{jX_1} = \frac{1}{100} + \frac{j}{50} - \frac{j}{50} \Rightarrow \bar{Z}_{O_2\mathcal{E}} = 100\,\Omega$$

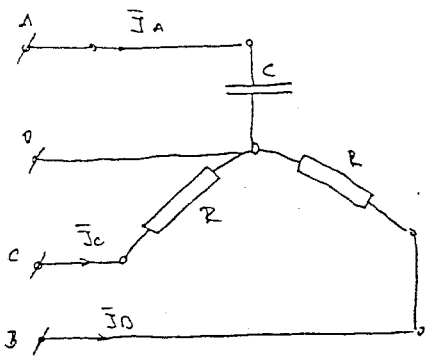
$$\bar{Z}_u = 200\,\Omega$$

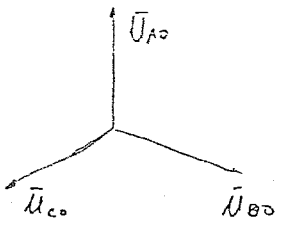
$$\Rightarrow I_A = \frac{1039}{200} = 5{,}2 \text{ A}$$

$$\boxed{I_A = 5{,}2 \text{ A}}$$

ZADATAK 8 Tri trošila spojena u zvijezdu priključena su na trofazni simetrični ...

6.65 Prvo trošilo je kondenzator, a dva druga su jednaki otpori. Pri redu s nultim vodičem struje u trošilima su iznosile $I_A = 1A$ $I_B = I_C = 2A$. Kolike će struje biti ako se prekine nultivodič.





$$\bar{U}_{AO} = U_f \cdot e^{j90^\circ}$$

$$\bar{U}_{BO} = U_f \cdot e^{-j30^\circ}$$

$$\bar{U}_{CO} = U_f \cdot e^{j210^\circ}$$

$$\bar{I}_A = \frac{\bar{U}_{AO}}{-jX_C} = \frac{U_f \cdot e^{j90^\circ}}{X_C \cdot e^{-j90}} = \frac{U_f}{X_C} \cdot e^{j180^\circ}$$

$$\bar{I}_B = \frac{U_f}{R} e^{-j30^\circ}$$

$$\bar{I}_C = \frac{U_f}{R} e^{j210^\circ}$$

$$\frac{U_f}{X_C} = 1A = I_A \Rightarrow U_f = X_C$$

$$\frac{U_f}{R} = 2A = I_B \Rightarrow U_f = 2R$$

$$\frac{U_f}{R} = 2A = I_C \Rightarrow \boxed{X_C = 2R}$$

Ako se prekine nultivodič bit će:

$$\bar{U}_{O'O} = \frac{U_f e^{j90}\frac{1}{-jX_C} + U_f e^{-j30}\cdot\frac{1}{R} + U_f e^{j210}\cdot\frac{1}{R}}{\frac{1}{-jX_C} + \frac{1}{R} + \frac{1}{R}} = \frac{U_f \cdot e^{j90}\cdot j\frac{1}{X_C} + U_f \cdot e^{-j30^\circ}\cdot\frac{1}{R} + U_f \cdot e^{j210^\circ}\cdot\frac{1}{R}}{\frac{2}{R} + j\frac{1}{X_C}}$$

$$\bar{U}_{O'O} = \frac{U_f\left[e^{j90}\cdot j\frac{1}{2R} + e^{-j30^\circ}\cdot\frac{1}{R} + e^{j210^\circ}\cdot\frac{1}{R}\right]}{\frac{2}{R} + j\frac{1}{2R}}$$

$$\bar{U}_{O'O} = \frac{U_f\left[e^{j180^\circ}\cdot\frac{1}{2R} + e^{-j30^\circ}\cdot\frac{1}{R} + e^{j210^\circ}\cdot\frac{1}{R}\right]}{\frac{4+j}{2R}} = \frac{U_f \cdot \frac{1}{2R}\left[e^{j180^\circ} + 2e^{-j30^\circ} + 2\cdot e^{j210^\circ}\right]}{\frac{4+j}{2R}}$$

$$\bar{U}_{O'O} = \frac{U_f}{4+j}\left[-1 + 0j^\circ + 2\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right) + 2\left(-\frac{\sqrt{3}}{2} - j\frac{1}{2}\right)\right]$$

$$\bar{U}_{O'O} = \frac{U_f}{4+j}\left[-1 + \sqrt{3} - j - \sqrt{3} - j\right] = \frac{U_f}{4+j}\left[-1 - j^2\right] = -\frac{U_f(1+j2)}{(4+j)}$$

$$\bar{U}_{O'O} = -\frac{U_f(1+j2)}{(4+j)}\cdot\frac{(4-j)}{(4-j)} = -\frac{U_f\left[4 - j + j^2 + 2\right]}{16+1} = -\frac{U_f\left[6+j7\right]}{17}$$

$$\bar{U}_{AO'} = \bar{U}_{AO} - \bar{U}_{O'O} = U_f \cdot e^{j90} + \frac{U_f}{17}\cdot(6+j7) = U_f \cdot e^{j90} + \frac{U_f}{17}(6+j7)$$

$$\bar{U}_{AO'} = U_f\left[j + \frac{6+j7}{17}\right] = U_f\left[\frac{17j+6+j7}{17}\right] = \frac{U_f}{17}\left[6+j24\right]$$

$$\bar{I}_A = \frac{\bar{U}_{AO'}}{\bar{Z}_A} = \frac{\frac{U_f}{17}(6+j24)}{-jX_C} = \frac{U_f(6+j24)}{-jX_C\cdot 17} = \frac{U_f\,6\,(1+j4)}{17\,X_C\cdot e^{-j90}} = \frac{6}{17}\cdot\frac{(1+j4)}{e^{-30^\circ}}$$

($U_f = X_C = 1A$)

$$= \frac{6}{17}\cdot\frac{\sqrt{17}\cdot e^{j76^\circ}}{e^{-30^\circ}} = 1{,}45\, e^{j166^\circ}\ A$$

G.65

$\bar{J}_A = 1{,}45 \cdot e^{j166^\circ}$ $\boxed{I_A = 1{,}45\,A}$ pri iskopčanom nul-vodu.

$$\bar{U}_{BO'} = \bar{U}_{BO} - \bar{U}_{O'O} = U_f \cdot e^{-j30^\circ} + \frac{U_f(6+j7)}{17} = U_f\left[e^{-j30^\circ} + \frac{6+j7}{17}\right]$$

$$= U_f\left[\frac{\sqrt{3}}{2} - j\frac{1}{2} + \frac{6+j7}{17}\right] = U_f\left[\frac{\sqrt{3}}{2} - j\frac{1}{2} + \frac{6}{17} + j\frac{7}{17}\right] = U_f\left[0{,}866 + 0{,}353 + j(0{,}41176 - 0{,}5)\right]$$

$$\bar{U}_{BO'} = U_f\left[1{,}22 - j\,0{,}088\right]$$

$$\bar{J}_B = \frac{\bar{U}_{BO'}}{R} = \left(\frac{U_f}{R} = 2A\right)\left[1{,}22 - j0{,}088\right] = 2\,(1{,}22 - j0{,}088) = 2 \cdot 1{,}223 \cdot e^{-j4{,}125^\circ} \Rightarrow I_B = 2{,}446$$

$$\bar{U}_{CO'} = \bar{U}_{CO} - \bar{U}_{O'O} = U_f \cdot e^{j210^\circ} + \frac{U_f(6+j7)}{17} = U_f\left[e^{j210^\circ} + \frac{6+j7}{17}\right] = U_f\left[-0{,}866 - j0{,}5 + 0{,}353 + j0{,}41176\right]$$

$$\bar{U}_{CO'} = U_f\left[-0{,}513 - j\,0{,}089\right] = -U_f \cdot 0{,}5205 \cdot e^{j9{,}81^\circ} \Rightarrow$$

$$\bar{J}_C = \frac{\bar{U}_{CO}}{R} = -\left(\frac{U_f}{R} = 2\right) \cdot 0{,}5205 \cdot e^{+j9{,}81^\circ}$$

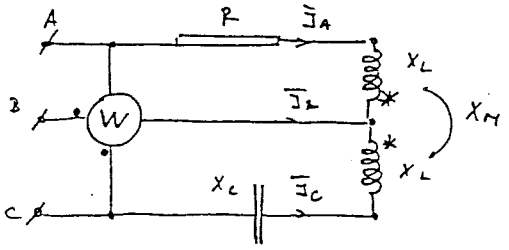
$$\bar{J}_C = -1{,}041 \cdot e^{+j9{,}81^\circ}$$

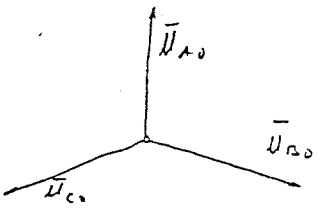
$$\Rightarrow \boxed{I_C = 1{,}041\,A}$$

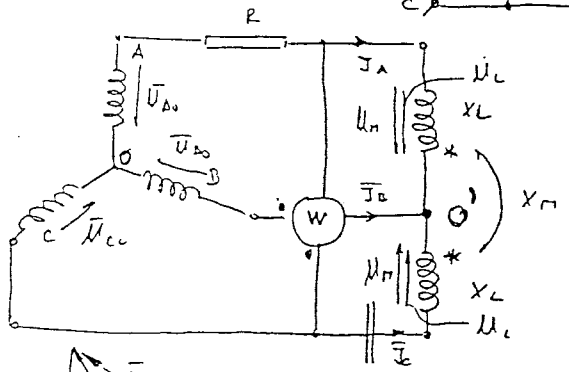
$\boxed{\begin{aligned} I_A &= 1{,}45\,A \\ I_B &= 2{,}446\,A \\ I_C &= 1{,}041\,A \end{aligned}}$

ZADATAK: Odrediti pokazivanje vatmetra na slici

6.78 ZADANO: $U_A = 220\,e^{j0^\circ}$ $Z = 22\,\Omega$ $X_L = X_C = X_M = 38\,\Omega$







Postavimo jednačžbe prvog i drugog Kirchhoffovog zakona

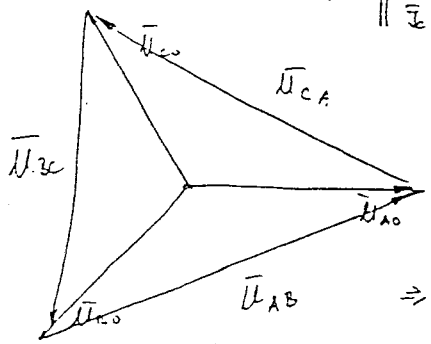
$$\bar{U}_{AO} = \bar{J}_A \cdot R + \bar{J}_A \cdot jX_L + \bar{J}_C \cdot jX_M + \bar{U}_{BO}$$

$$\boxed{\bar{U}_{AO} - \bar{U}_{BO} = \bar{J}_A \cdot R + \bar{J}_A \cdot jX_L + \bar{J}_C \cdot jX_M}$$

$$\bar{U}_{CO} = \bar{J}_C \cdot (-jX_C) + \bar{J}_C \cdot jX_L + \bar{J}_A \cdot jX_M - \bar{U}_{BO}$$

$$\boxed{\bar{U}_{CO} - \bar{U}_{BO} = -\bar{J}_C jX_C + \bar{J}_C \cdot jX_L + \bar{J}_A \cdot jX_M}$$

$\boxed{\bar{J}_A + \bar{J}_B + \bar{J}_C = 0}$



$$\Rightarrow \bar{U}_{AB} = \sqrt{3}\,U_{AO} \cdot e^{j30^\circ}\ V$$

$$\bar{U}_{CB} = \sqrt{3}\,U_{AO} \cdot e^{j90^\circ}\ V$$

$$\bar{U}_{\quad} = \sqrt{3}\,U_{\quad} \cdot e^{j150^\circ}\ V$$

6.7

$\bar{U}_{AB} = \bar{J}_A \cdot R + \bar{J}_A \cdot jX_L + \bar{J}_C \cdot jX_M$ $\quad \bar{J}_A + \bar{J}_B + \bar{J}_C = 0$ $\quad \bar{U}_{AB} = 380 \cdot e^{j120}\ V$

$\bar{U}_{CD} = -\bar{J}_C \cdot jX_C + \bar{J}_C \cdot jX_L + \bar{J}_A \cdot jX_M$ $\quad \bar{U}_{CB} = 380 \cdot e^{j90^\circ}\ V.$

$380 \cdot e^{j30^\circ} = \bar{J}_A \cdot 22 + \bar{J}_A \cdot j38 + \bar{J}_C \cdot j38$

$380 \cdot e^{j90^\circ} = -\bar{J}_C j38 + \bar{J}_C \cdot j38 + \bar{J}_A \cdot j38$

$380 \cdot e^{j30^\circ} = \bar{J}_A \cdot 22 + \bar{J}_A \cdot j38 + \bar{J}_C \cdot j38$

$380 \cdot e^{j90^\circ} = j38(-\bar{J}_C + \bar{J}_C + \bar{J}_A)$

$330 \cdot e^{j30^\circ} = \bar{J}_A \cdot 22 + \bar{J}_A \cdot j38 + \bar{J}_C \cdot j38$

$380 \cdot e^{j90^\circ} = \bar{J}_A \cdot j38$ $\quad 380 \cdot e^{j90^\circ} = \bar{J}_A \cdot 38 \cdot e^{j90^\circ}$ $\quad \bar{J}_A = \frac{10 \cdot 380 \cdot e^{j90^\circ}}{38 \cdot e^{j90^\circ}} = \boxed{\bar{J}_A = 10\ A}$

) $380 \cdot e^{j30^\circ} = 220 + j380 + \bar{J}_C \cdot j38$

$380 \cdot e^{j30^\circ} = 220 + j380 + \bar{J}_C \cdot j38$ $\quad \bar{J}_C \cdot j38 = 380 \cdot e^{j30^\circ} - 220 - j380$

$\bar{J}_C = \frac{380 \cdot e^{j30}}{38 \cdot e^{j90}} - \frac{220}{38 \cdot e^{j90}} = \frac{j380}{j38} = 10 \cdot e^{-j60^\circ} - 5,79 \cdot e^{-j90^\circ} - 10$

$\bar{J}_C = 5 - j8,66 - 5,79 \cdot (0 - j) - 10 = 5 - j8,66 - 0 + j5,79 - 10$

$\bar{J}_C = -5 - j2,87\ A$

$\bar{J}_D = -\bar{J}_A - \bar{J}_C = -10 + 5 + j2,87 = -5 + j2,87\ A \Rightarrow \bar{J}_D = 5,765 \cdot e^{j150,144^\circ}$

$\boxed{\bar{U}_{CA} = \sqrt{3}\, U_{A0} \cdot e^{j150^\circ}\ V}$

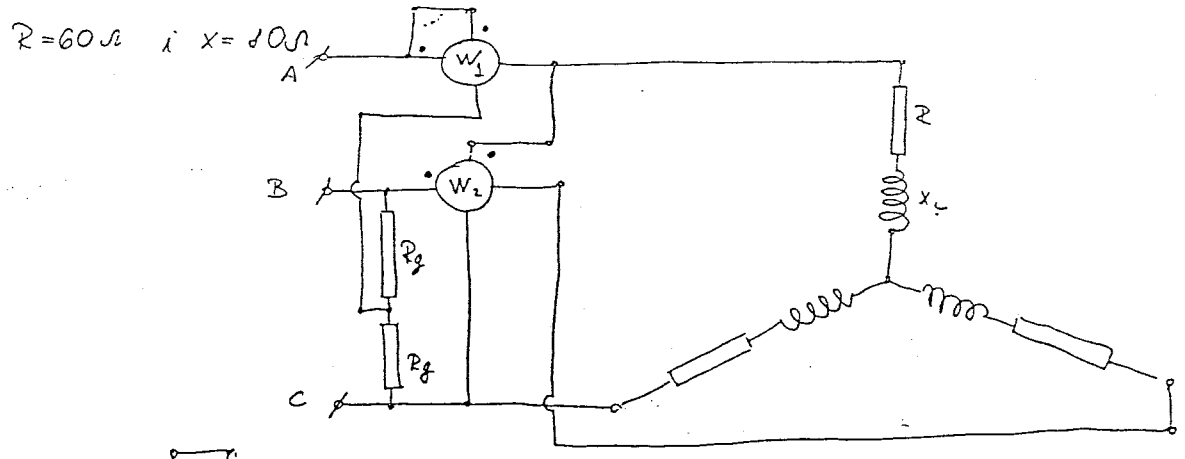
$P_W = Re\left\{\sqrt{3}\, U_{A0} e^{j150} \cdot 5,765 \cdot e^{-j150,144}\right\}$

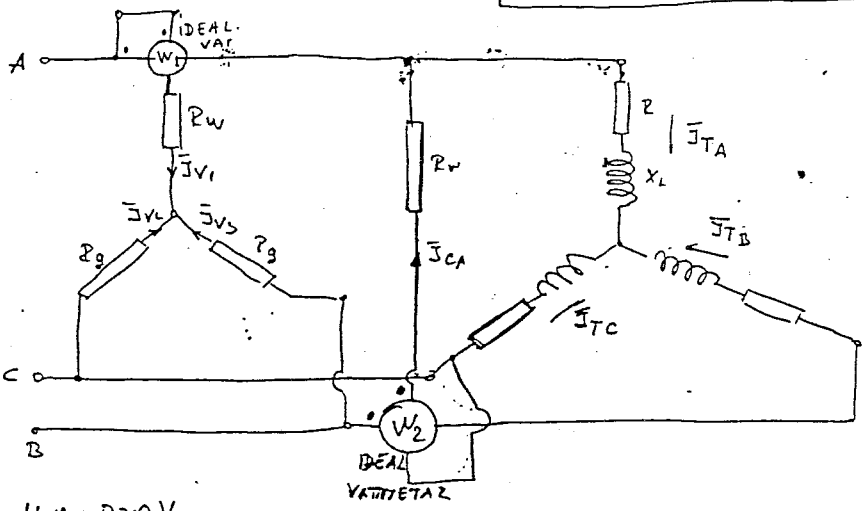
$P_W = Re\left\{2190,7 \cdot e^{j0^\circ}\right\} \Rightarrow P_W = 2190,7\ W$

$\boxed{P_W = 2,191\ kW}$ W

DATAK 8 ⇒ Trijednaka trošila spojena su u zvijezdu. Linijski napon $U_L = 380\,V$

6.71 Izračunajte radnu P i jalovu snagu Q koje u trošenje unosi i pokazivanja P_1 i P_2 jednofaznih vatmetara. Nadite odnose $\frac{P}{P_1}$ i $\frac{Q}{P_1}$. Otpori naponskih (paralelnih ogranaka) vatmetra su $R_w = R_g$, a parametri trošila iznose $R = 60\,\Omega$ i $X = 80\,\Omega$





$U_L = 380\,V \quad U_f = 220\,V$

$\bar{U}_{Ao} = 220 \cdot e^{j90^\circ}\,V$

$\bar{U}_{Bo} = 220 \cdot e^{-j30^\circ}\,V$

$\bar{U}_{Co} = 220 \cdot e^{j210^\circ}\,V$

$R_w = R_g = R_g \Rightarrow \quad \bar{I}_{V_1} = \frac{220 \cdot e^{j90^\circ}}{R_w} \quad \bar{I}_{V_2} = \frac{220 \cdot e^{-j30^\circ}}{R_w} \quad \bar{I}_{V_3} = \frac{220 \cdot e^{j210^\circ}}{R_w}$

$\bar{I}_{CA} = \frac{\bar{U}_{CA}}{R_w} = \frac{380 \cdot e^{j240^\circ}}{R_w}$

$\bar{I}_{TA} = \frac{220 \cdot e^{j90^\circ}}{100 \cdot e^{j53,13^\circ}} = 2,2 \cdot e^{j36,87^\circ}\,A \qquad \bar{I}_{TB} = \frac{220 \cdot e^{-j30^\circ}}{100 \cdot e^{j53,13^\circ}} = 2,2 \cdot e^{-j83,13^\circ}\,A$

$\bar{I}_{TC} = \frac{220 \cdot e^{j210}}{100 \cdot e^{j53,13}} = 2,2 \cdot e^{j156,87}\,A$

$P_T = 3 \cdot Re\{\bar{U}_{Ao} \cdot \bar{I}_{TA}^*\} = 3 \cdot Re\{220 \cdot e^{j90^\circ} \cdot 2,2 \cdot e^{-j36,87}\} = 3484 \cdot 0,6 = 271,2\,W$

$\boxed{P_T = 271,2\,W}$

$P_{W_2} = Re\{\bar{U}_{oc} \cdot \bar{I}_B^*\} = Re\{-\bar{U}_{CA} \cdot \bar{I}_B^*\} = Re\{-380 \cdot e^{+j240^\circ} \cdot 2,2 \cdot e^{+j83,13^\circ}\}$

$= -836 \cdot 0,8 = -668,8\,W$

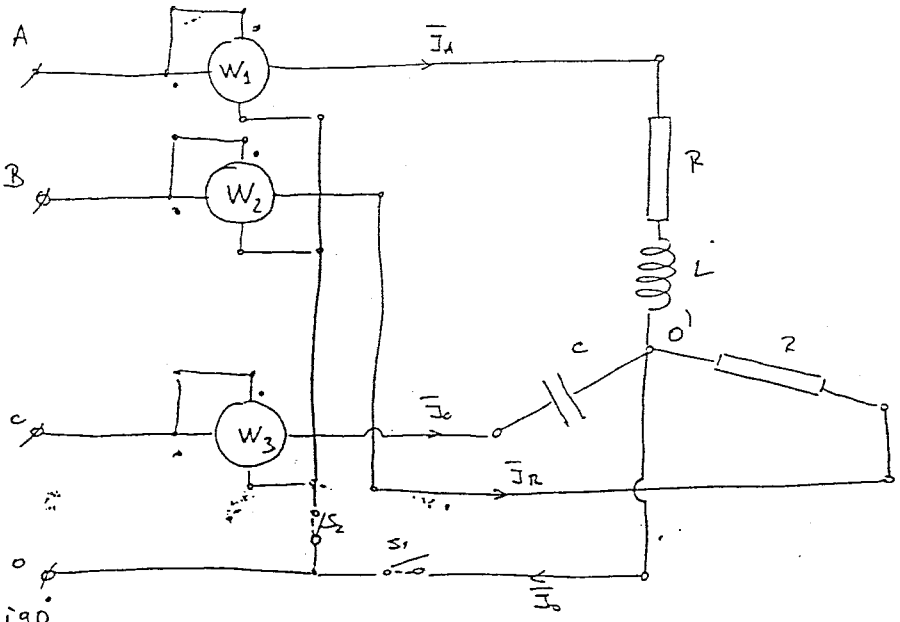
$P_{W_1} = Re\{\bar{U}_{AO} \cdot \bar{I}_A^*\} = Re\{220 \cdot e^{j90^\circ} \cdot 2,2 \cdot e^{j36,87^\circ}\} = \underline{-290\,W}$.

ZADATAK : => Tri trošila spojena u zvijezdu napajaju se iz trofazne mreže s linijskim naponom

6.73 $U_L = 346\,V$.

Parametri trošila su $\bar{Z}_A = 20 + j20\,\Omega$ $\bar{Z}_R = 50\,\Omega$ $\bar{Z}_C = -j40\,\Omega$

Treba naći pokazivanja wattmetara i snagu utrošenu u trošilima pri zatvorenim sklopkama S_1 i S_2



$U_f = \frac{U_L}{\sqrt{3}} = 200\,V$

$U_f \Rightarrow \quad \bar{U}_{AO} = 200 \cdot e^{j90^\circ}\,V$

$\bar{U}_{BO} = 200 \cdot e^{-j30^\circ}\,V$

$\bar{U}_{CO} = 200 \cdot e^{+j210^\circ}\,V$

$\bar{I}_A = \frac{\bar{U}_{AO}}{\bar{Z}_A} = \frac{200 \cdot e^{j90^\circ}}{20\sqrt{2}\,e^{j45^\circ}} = 5\sqrt{2} \cdot e^{j45^\circ}\,A$

$\bar{I}_B = \frac{\bar{U}_{BO}}{\bar{Z}_B} = \frac{200 \cdot e^{-j30^\circ}\,V}{50\,\Omega} = 4 \cdot e^{-j30^\circ}\,A$

$\bar{I}_C = \frac{\bar{U}_{CO}}{\bar{Z}_C} = \frac{200 \cdot e^{+j210^\circ}}{40 \cdot e^{-j90^\circ}} = 5 \cdot e^{j300^\circ}\,A$

$P_{W_1} = Re\left\{\bar{U}_{AO} \cdot \bar{I}_A^*\right\} = Re\left\{200\,e^{j90^\circ} \cdot 5\sqrt{2}\,e^{-j45^\circ}\right\} = Re\left\{1000\sqrt{2} \cdot e^{j45^\circ}\right\}$

$P_{W_1} = 1000\sqrt{2} \cdot \frac{\sqrt{2}}{2} = 1\,kW$ $\boxed{P_{W_1} = 1\,kW}$

$P_{W_2} = Re\left\{\bar{U}_{BO} \cdot \bar{I}_B^*\right\} = Re\left\{200\,e^{-j30^\circ} \cdot 4 \cdot e^{+j30^\circ}\right\} = 800\,W$ $\boxed{P_{W_2} = 0{,}8\,kW}$

$P_{W_3} = Re\left\{\bar{U}_{CO} \cdot \bar{I}_C^*\right\} = Re\left\{200 \cdot e^{j210^\circ} \cdot 5 \cdot e^{-j300^\circ}\right\} = 1000 \cdot 0 = 0\,W$

$P_T = 1\,kW + 0{,}8\,kW = 1{,}8\,kW$

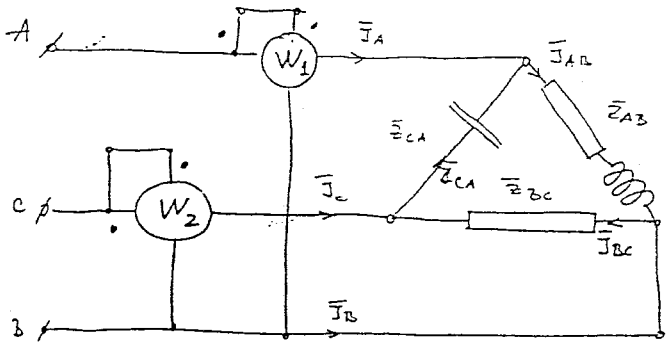
$\boxed{P_T = 1{,}8\,kW}$

ZADATAK 8 ⇒ $U_L = 100V$. Odredite pokazivanja vatmetara uključenih na trofazni spoj za mjerenje snage trošila. Usporedite pokazivanja vatmetra s izračunatom radnom snagom koja se troši u trošilima. Parametri trošila su

$\bar{Z}_{AB} = 20 + j20$ $\bar{Z}_{BC} = 50\Omega$ $\bar{Z}_{CA} = -j40\Omega$, $U_L = 100V$

8.74

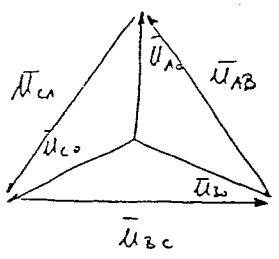
$U_f = \frac{U_L}{\sqrt{3}} =$



$\bar{U}_{A0} = 57,8 \cdot e^{+j90°}$ V

$\bar{U}_{B0} = 57,8 \cdot e^{-j30°}$ V

$\bar{U}_{C0} = 57,8 \cdot e^{+j210°}$ V



$\bar{U}_{BC} = 100 \cdot e^{j0°}$ V

$\bar{U}_{AB} = 100 \cdot e^{j120°}$ V

$\bar{U}_{CA} = 100 \cdot e^{j240°}$ V

$\bar{J}_{AB} = \frac{\bar{U}_{AB}}{\bar{Z}_{AB}} = \frac{100 \cdot e^{j120°}}{20\sqrt{2} \cdot e^{j45°}} = \frac{5}{\sqrt{2}} \cdot e^{j75°}$ A

$\bar{J}_{CA} = \frac{\bar{U}_{CA}}{\bar{Z}_{CA}} = \frac{160 \cdot e^{j240°}}{2 \cdot 40 \cdot e^{-j90°}} = \frac{5}{2} \cdot e^{j330°}$ A

$P_{W_L} = Re\{\bar{U}_{AB} \cdot \bar{J}_A^*\} = Re\{100 \cdot e^{j120°} \cdot 4,83 \cdot e^{-j105°}\}$

$P_{W_L} = 466,5 W$??

$\bar{J}_{BC} = \frac{\bar{U}_{BC}}{\bar{Z}_{BC}} = \frac{100 \cdot e^{j0°}}{50} = 2 \cdot e^{j0°}$ A

$\bar{J}_A = \bar{J}_{AB} - \bar{J}_{CA} = \frac{5\sqrt{2}}{2} \cdot e^{j75°} - \frac{5}{2} e^{j330°}$

$\bar{J}_A = 0,915 + j3,415 - (2,165 - j1,25)$

$\bar{J}_A = 0,915 + j3,415 - 2,165 + j1,25$

$\bar{J}_A = -1,25 + j4,665 = 4,83 \cdot e^{j105}$

$P_{N_2} = Re\{\bar{U}_{CB} \cdot \bar{J}_C^*\}$

$\bar{J}_C = \bar{J}_{CA} - \bar{J}_{BC}$ → $\boxed{\bar{J}_C = \bar{J}_{CA} - \bar{J}_{BC}}$

$\bar{J}_C = \frac{5}{2} e^{j330°} - 2 \cdot e^{j0°} = \frac{5}{2}(0,866 - j0,5) - 2(1 + j0)$

$\bar{J}_C = 2,165 - j1,25 - 2 = 0,165 - j1,25$

$\bar{J}_C = 1,261 \cdot e^{-j83,5°}$ $\qquad \bar{U}_{CB} = -\bar{U}_{BC}$

$P_{W_2} = Re\{-\bar{U}_{BC} \cdot \bar{J}_C^*\} = Re\{-100 \cdot e^{j0°} \cdot 1,261 \cdot e^{j83,5°}\} = -126,1 \cdot 0,13 = -17W$

$\boxed{P_{W_2} = -17W}$

4
1

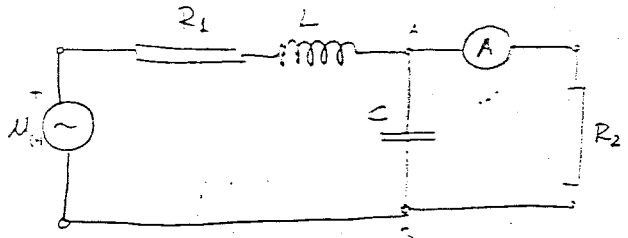
6 91

7.42

zadatak 8 ⇒ U mreži je $u = 100 \sin \omega t + 20° - 50 \sin(3\omega t + 60°)$

$R_1 = 6\,\Omega \quad \omega L = 8\,\Omega \quad \frac{1}{\omega C} = 10\,\Omega \quad R_2 = 10\,\Omega$

Kolika je pokazivanje ampermetra: metoda superpozicije



$u = 100 \sin(\omega t + 30°) + 50 \sin(3\omega t + 60°)$ $\quad u' = 100 \sin(\omega t + 30°)$

$u = u' + u''$ $\quad u'' = 50 \sin(3\omega t + 60°)$

Za prvi harmonik napona $\quad \omega L = 8\,\Omega \quad \frac{1}{\omega C} = 10\,\Omega$

$R_1 = 6\,\Omega \quad R_2 = 10\,\Omega$

$$\underline{Z}_u = \frac{10 \cdot (-j10)}{10 - j10} + 6 - j8 = \frac{-j100}{10 - j10} + 6 + j8 = \frac{-j10 \cdot 10}{10(1-j)} + 6 + j8 = \frac{-j10}{1-j} + 6 + j8$$

$$\underline{Z}_u = \frac{-j10 + 6(1-j) + j8(1-j)}{1-j} = \frac{-j10 - 6 - j6 + 8 + 8}{1-j} = \frac{-8j + 14}{1-j} = \frac{14 - 8j}{1-j}$$

$$\underline{Z}'_u = \frac{16,1245 \cdot e^{-j29,7°}}{1,4142 \cdot e^{-j45°}} = 11,4 \cdot e^{j15,3°}$$

$$\underline{I}' = \frac{\underline{U}'}{\underline{Z}'_u} = \frac{\frac{100}{\sqrt{2}} \cdot e^{j30°}}{11,4 \cdot e^{j15,3°}} = 6,203\, e^{j14,7°}\ A$$

$$\underline{U}'_{AO} = 6,203 \cdot e^{j14,7°} \cdot \frac{-j10}{1-j} = 6,203 \cdot e^{j14,7°} \cdot \frac{10\, e^{-j90°}}{\sqrt{2} \cdot e^{-j45°}} = 43,86 \cdot e^{-j30,3°}$$

$$\boxed{\underline{I}'_2 = 4,386 \cdot e^{-j30,3°}\ A}$$

Za u'' $\quad 3\omega L = 24\,\Omega \quad \frac{1}{3\omega C} = \frac{10}{3}$

$u'' = 50 \sin(3\omega t + 60°)$ $\quad \underline{Z}_u = \frac{10 \cdot (-j\frac{10}{3})}{10 - j\frac{10}{3}} + 6 + j24$

$$\underline{Z}_u = \frac{-j\frac{100}{3}}{30 - j10} + 6 + j24 = \frac{-j100}{3(3 - j1)} + 6 + j24 = \frac{-j10}{3-j} + 6 + j24 = \frac{-j10 + 6(3-j) + j24(3-j)}{3-j}$$

$$\underline{Z}_u = \frac{-j10 + 18 - j6 + j72 + 24}{3-j} = \frac{156 + j42}{3-j} = \frac{42 + j156}{3-j} = \frac{70 \cdot e^{j53,13}}{3,162 \cdot e^{-j18,44}} = 22,14 \cdot e^{j71,57°}\ \Omega$$

$$\underline{I}'' = \frac{50 \cdot e^{j60°}}{22,14 \cdot e^{j71,57°}} = 2,26 \cdot e^{-j11,6°}\ A \qquad \underline{U}_{AO} = 2,26 \cdot e^{-j11,6°} \cdot \frac{10\, e^{-j90°}}{\sqrt{2} \cdot e^{-j45°}} = 16 \cdot e^{-j56,6°}\ V$$

$$\underline{I}''_R = \frac{16 \cdot e^{-j56,6°}}{10} = 1,6\, e^{-j56,6°} = \underline{I}''_2$$

$$i_2 = 6,202 \sin(\omega t - 30,3°) - 1,6 \sin(3\omega t - 56,6°)\ A$$

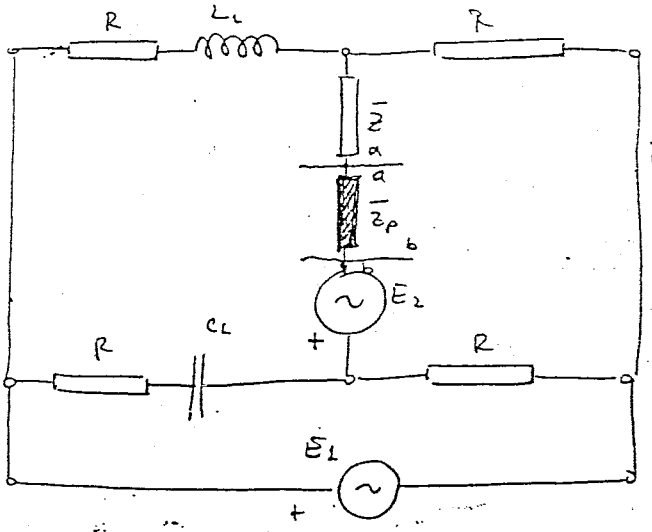
$$I_2 = \sqrt{I'^2_2 - I''^2_2} = \sqrt{38,48 - 2,56} = \sqrt{35,92} = 5,99$$

$$\boxed{I_2 = 6,0\ A}$$

ZADATAK. Odredite vrijednost impedancije $\bar{Z}$ da bi na impedanciji $\bar{Z}_p$ razvio maksimalnu korisnu snagu. Kolika je tada učinska snaga ako je $e_1(t) = 200/\sqrt{2}\ \sin(\omega t + \frac{\pi}{6})$

$e_2(t) = 30/\sqrt{2}\ \sin \omega t \quad R = 10\,\Omega \quad L_1 = 20/\omega\ H \quad C_1 = \frac{1}{10\omega}\ F$

$\bar{Z}_p = 15{,}5 - j40{,}1\ \Omega$



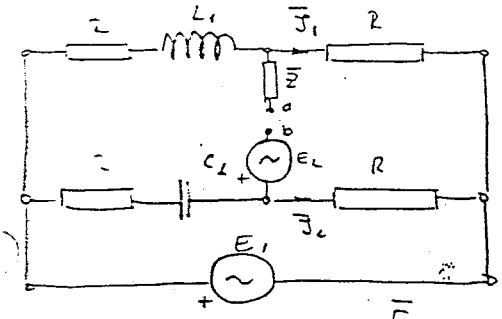
$\bar{E}_1 = 141{,}42\, e^{j90°}\ V$

$\bar{E}_2 = 22{,}213 \cdot e^{j30°}\ V$

b)

$\bar{E}_T = ?\quad \bar{Z}_T = ?$

$\bar{E}_T = \varphi_a - \varphi_b$



$$\bar{J}_1 = \frac{\bar{E}_1}{2R + j\omega \cdot \frac{20}{\omega}} = \frac{\bar{E}_1}{2\cdot 10 + j20} = \frac{141{,}42 \cdot e^{j90°}}{20 + j20} = \frac{141{,}42 \cdot e^{j90°}}{28{,}28 \cdot e^{j45°}} = 5 \cdot e^{j45°}$$

$\bar{J}_1 = 5 \cdot e^{j45°}$

$$\bar{J}_2 = \frac{\bar{E}_1}{20 - j\frac{1}{\omega \cdot \frac{1}{10\omega}}} = \frac{141{,}42 \cdot e^{j90°}}{20 - j10} = \frac{141{,}42 \cdot e^{j90}}{22{,}3606 \cdot e^{-j26{,}56}} = 6{,}3246 \cdot e^{j116{,}56}\ A$$

$\bar{J}_2 = 6{,}3246 \cdot e^{j116{,}57°}\ A$

$\varphi_a = \varphi_c + \bar{E}_2 + \bar{J}_2 \cdot (R - jX_c) - \bar{J}_1 \cdot (R + jX_L) \qquad \bar{E}_T = \bar{E}_2 + \bar{J}_2(R - jX_c) - \bar{J}_1(R + jX_L)$

$\bar{E}_T = 21{,}21\, e^{j90} + 6{,}325 \cdot e^{j116{,}56°}(10 - j10) - 5 \cdot e^{j45°} \cdot (10 + j20)$

$\bar{E}_T = j21{,}21 + 63{,}25 \cdot e^{j116{,}56°} - j63{,}25 \cdot e^{j116{,}56°} - 50\, e^{j45°} - 5 \cdot e^{j45°} \cdot j20$

$\bar{E}_T = j21{,}21 - 28{,}28 + j56{,}57 - 63{,}25 \cdot e^{j206{,}56°} - 50\, e^{j45°} - 100 \cdot e^{j135°}$

$\bar{E}_T = j21{,}21 - 28{,}28 + j56{,}57 - (-56{,}57 - j28{,}28) - 31{,}35 - j31{,}35 - 100(-0{,}707 + j0{,}707)$

$\bar{E}_T = j21{,}21 - 28{,}28 + j56{,}57 + 56{,}57 + j28{,}28 - 35{,}35 - j35{,}35 + 70{,}7 - j70{,}7$

$\bar{E}_T = 63{,}64 + j0 \qquad \boxed{E_T = 63{,}67\ V}$

$\bar{Z}_T = ? \qquad \bar{Z}_T = \bar{Z} + \frac{(R + jX_L) \cdot R}{R + R + jX_L} + \frac{(R - jX_c) \cdot R}{R + R - jX_c} = \bar{Z} + \frac{(10 + j20) \cdot 10}{20 + j20} + \frac{(10 - j10) \cdot 10}{20 - j10}$

$\bar{Z}_T = \bar{Z} + \frac{10 - j20}{2 - j2} + \frac{10 - j10}{2 - j} = \bar{Z} + \frac{10(1 + j2)}{2(1 + j)} + 10\,\frac{1 - j}{2 - j}$

$\bar{Z}_T = \bar{Z} + 5 \cdot \frac{1 + j2}{1 + j} + 10\,\frac{1 - j}{2 - j} = \bar{Z} + 5 \cdot \frac{(1 + j2)(1 - j)}{2} + 10\,\frac{1 - j}{2 - j} \cdot \frac{2 + j}{2 + j}$

$\bar{Z}_T = \bar{Z} + \frac{5}{2}(1 - j + j2 + 2) + \frac{10}{5}(2 + j - j2 + 1) = \bar{Z} + \frac{5}{2}(3 + j) + 2(3 - j)$

$\bar{Z}_T = \left(a + \frac{27}{2}\right) + j\left(b + \frac{1}{2}\right)$ $\quad \boxed{\bar{Z}_p = 13,5 - j10,5\ \Omega}$

da bi u mr $\bar{Z}_r$ ostarion max učin morao bith

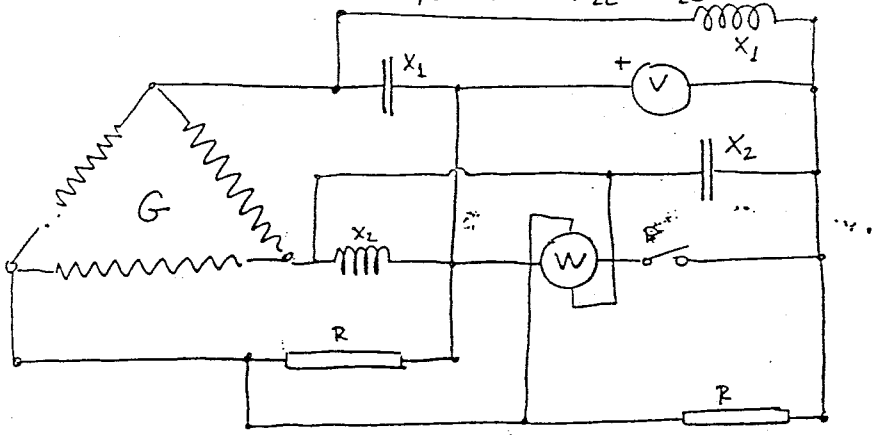
$a + \frac{27}{2} = 13,5 \qquad a = 13,5 - 13,5 = 0$

$b + \frac{1}{2} = 10,5 \qquad b = 10,5 - 0,5 = 10$

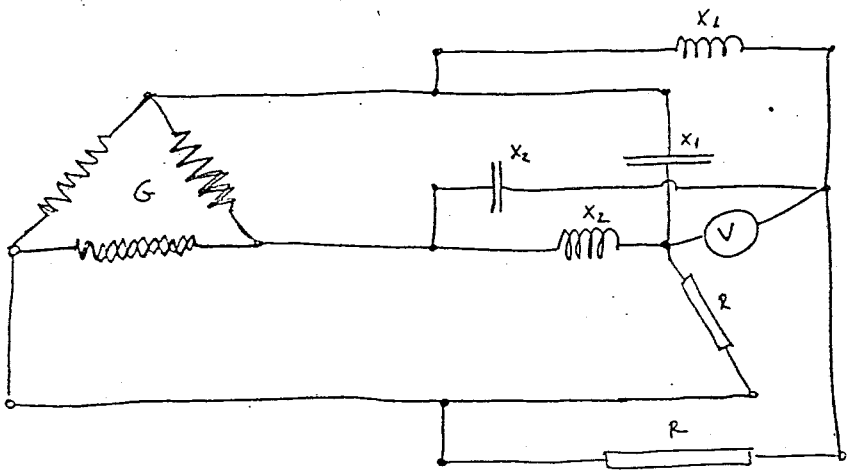
$\bar{Z}_T = 13,5 + j10,5$

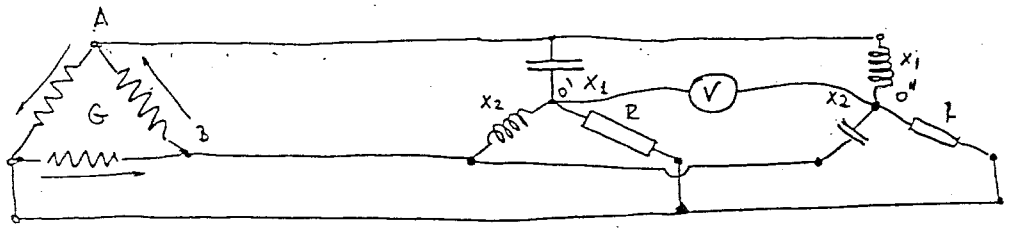
$= \bar{Z} = 0 + j10 \Rightarrow \boxed{\bar{Z} = j10\Omega}$

ZADATAK: Obrediti razliku pokazivanja instrumenata za sklopku zatvorenu i otvorenu oblopku ako je fazni napon generatora 346 V te otpor strujne grane vatmetra $r_w = 0,5\Omega$ i Z $X_{2L} = X_{1C} = 200\Omega$ $X_{2C} = X_{1L} = 50\Omega$ $R = 100\Omega$



ak je sklopka P → otvorena



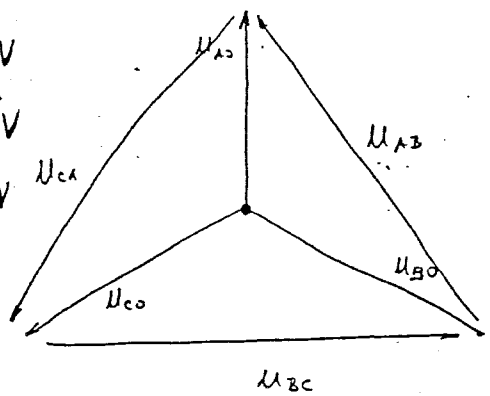


$P_w = 0$

$\bar{U}_{AO} = 200 \cdot e^{j90°}\ V$

$\bar{U}_{BO} = 200 \cdot e^{-j30°}\ V$

$\bar{U}_{CO} = 200 \cdot e^{j210°}\ V$



$$U_{ab} = \dots + \bar{I}_2 \cdot R_4 - \bar{I}_1 \cdot R_1$$

$$\bar{I}_2 = \frac{\bar{U}}{(R_3 + R_4) - j\frac{1}{\omega C_3}}$$

$$\bar{I}_2 = \frac{\bar{U}}{(R_3 + R_4) - j\frac{1}{\omega C_3}}$$

$$\bar{I}_1 = \frac{\bar{U}}{R_1 + \frac{-j\frac{1}{\omega C_1} \cdot R_2}{R_2 - j\frac{1}{\omega C_2}}}$$

$$\bar{I}_1 = \frac{\bar{U}\left(R_2 - j\frac{1}{\omega C_2}\right)}{R_1\left(R_2 - j\frac{1}{\omega C_2}\right) - j\frac{1}{\omega C_2} \cdot R_2} = \frac{\bar{U}\left(R_2 - j\frac{1}{\omega C_2}\right)}{R_1 \cdot R_2 + j\frac{1}{\omega C_2} R_1 - j\frac{1}{\omega C_2} R_2} = \frac{\bar{U}\left(R_2 - j\frac{1}{\omega C_2}\right)}{R_1 \cdot R_2 - j\frac{R_1 + R_2}{\omega C_2}}$$

$$\bar{U}_{ak} = \bar{E}_T = \bar{I}_2 \cdot R_4 - \bar{I}_1 \cdot R_1$$

$$\bar{U}_{ak} = \bar{E}_T = \frac{\cancel{\bar{U}} \cdot R_4}{(R_3 + R_4) - j\frac{1}{\omega C_3}} - \frac{\cancel{\bar{U}} \cdot R_1\left(R_2 - j\frac{1}{\omega C_2}\right)}{R_1 \cdot R_2 - j\frac{R_1 + R_2}{\omega C_2}} = 0$$

$$\frac{Z_1}{(R_3 - Z_4) - j\frac{1}{\omega C_3}} = \frac{R_1\left(R_2 - j\frac{1}{\omega C_2}\right)}{R_1 \cdot R_2 - j\frac{R_1 + R_2}{\omega C_2}}$$

$$\frac{500}{320 - j\frac{10^6}{\omega \cdot 2{,}22}} - \frac{200\left(300 - j\frac{10^6}{\omega 2}\right)}{60000 - j\frac{500 \cdot 10^6}{\omega \cdot 2}} = 0$$

$$\frac{500}{320 - j\left(0{,}45 \cdot \frac{10^6}{\omega}\right)} - \frac{200\left(300 - j\,0{,}5 \cdot \frac{10^6}{\omega}\right)}{60000 - j\,250 \cdot \frac{10^6}{\omega}} = 0$$

$$\frac{500}{320 - j\,0{,}45 \cdot \frac{10^6}{\omega}} - \frac{200\left(300 - j\,0{,}5 \cdot \frac{10^6}{\omega}\right)}{250\left(240 - j\frac{10^6}{\omega}\right)} = 0 \qquad \boxed{\frac{10^6}{\omega} = t}$$

$$\frac{500}{320 - j\,0{,}45t} - \frac{4(300 - j\,0{,}5t)}{5(240 - j\,t)} = 0$$

$$\frac{500\,(320 - j\,0{,}45t)}{102400 + 0{,}2025t^2} - 0{,}8 \cdot \frac{(300 - j\,0{,}5t)(240 + j\,t)}{57600 + t^2} = 0$$

$$\frac{500(320 - j\,0{,}45t)}{102400 + 0{,}2025t^2} - 0{,}8 \cdot \frac{72000 + j\,300t - j\,120t + 0{,}5t^2}{57600 + t^2} = 0$$

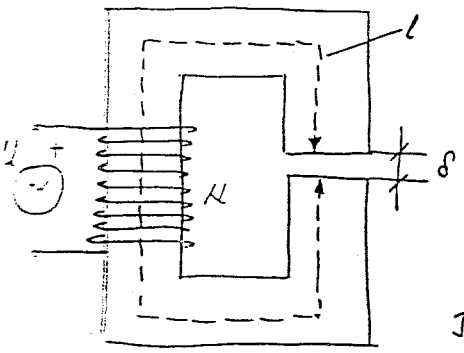
$$\frac{500\,(320 - j\,0{,}45t)}{102400 + 0{,}2025t^2} - 0{,}8 \cdot \frac{72000 + j\,180t + 0{,}5t^2}{57600 + t^2} = 0$$

SVITAK S FEROMAGNETSKOM JEZGROM I TRANSFORMATOR

ZADATAK 3 Na feromag. jezgru namotan je svitak. $U = 220\,V$ $f = 50\,Hz$. Otpor namota je zanemariv. Izračunajte 1.) broj namota N ako je $B_m = 1{,}4\,T$, a $S = 50\,cm^2$

2.1 2.) Efektivna vrijednost struje magnetiziranja, ako se zna da je struja sinusoidalna; srednja duljina silnice je $l = 60\,cm$, a zračni raspor $\delta = 1\,mm$

Napomena: kod $B = 1{,}4\,T$ je jakost magnetskog polja iznosi $H = 1410\,\frac{A}{m}$



$U = 220\,V$ $B_m = 1{,}4\,T$

$f = 50\,Hz$ $S = 50\,cm^2$ $\Phi_{MAX} = B_m \cdot S = 1{,}4 \cdot 50 \cdot 10^{-4}$

$N = ?$ $l = 60\,cm$ $\Phi_{MAX} = 70 \cdot 10^{-4}\,Vs$

$I = ?$ $\delta = 1\,mm$ $\Phi_{MAX} = 7 \cdot 10^{-3}\,Vs$

Tok mora biti takav da odražava ravnotežu narinutom naponu

$$u_L = N \cdot \frac{d\Phi}{dt}$$

$$\boxed{U = 4{,}44 \cdot N \cdot f \cdot \Phi_{MAX}}$$

$$N = \frac{220}{1554 \cdot 10^{-3}} = 141{,}5 \text{ zavoja}$$

$$N = \frac{U}{4{,}44 \cdot f \cdot \Phi_{MAX}} = \frac{220}{4{,}44 \cdot 50 \cdot 7 \cdot 10^{-3}}$$

$\boxed{N = 142 \text{ ZAVOJA}}$ ✓✓

$d\Phi = \frac{1}{N} \cdot U \cdot dt \,/\int$

$\Phi = \frac{1}{N} \int U_M \sin\omega t\, dt$

$\Phi = \frac{1}{N} \cdot U_m \cdot \omega \cdot \cos\omega t$

$\Phi = \frac{U}{\sqrt{2}} \cdot \frac{1}{N} \cdot 2\pi f \cos\omega t$

$\Phi = \underbrace{\frac{\sqrt{2}\pi \cdot f \cdot U}{N}}_{\Phi_{MAX}} \cos\omega t$ $\boxed{U = N \cdot 4{,}44 \cdot f \cdot \Phi_{MAX}}$

$I_\mu = ?$ $\oint \vec{H} \cdot d\vec{l} = \Sigma I = N \cdot I_\mu$ za zrak

$$H_0 = \frac{B}{\mu_0} = \frac{1{,}4}{4\pi \cdot 10^{-7}}$$

$$H_0 = \frac{1{,}4 \cdot 10^7}{4\pi}\ \frac{A}{m}$$

$$1410\,\frac{A}{m} \cdot 0{,}6m + \frac{1{,}4 \cdot 10^7}{4\pi} \cdot 10^{-3} = 142 \cdot I_\mu$$

$$846 + 1114{,}65 = 142 \cdot I_\mu \Rightarrow I_{\mu_{MAX}} = 13{,}81\,A$$

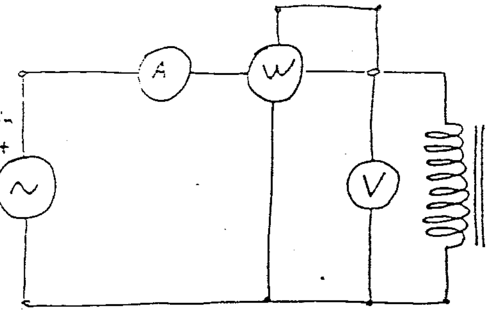
$$I_{\mu ef} = \frac{13{,}81}{\sqrt{2}}$$

$\boxed{I_{\mu ef} = 9{,}8\,A}$ ✓✓

ZADATAK 8: Svitak s jezgrom od transformatorskih limova debljine 0,35 mm priključen je na $U = 120\,V$ i $f = 50\,Hz$. Ampermetar, voltmetar i vatmetar su idealni instrumenti, uključeni u krug prema slici. Vrijednosti koje pokazuju instrumenti za svitak s jezgrom i bez nje su dani u tablici

8.4.

U	I [A]	P [W]	
120	10	150	s jezgrom od transf. lim.
120	12	72	bez jezgre (zrak)



Odrediti → elektromot. silu samoindukcije, I_μ, I_g, zanemariti rasipanje i pretpostaviti da nije nastupilo zasićenje. Parametre nadomjesne sheme.

za svitak bez jezgre

$$Z = \frac{U}{I} = \frac{120}{12} = 10\,\Omega \qquad \boxed{Z = 10\,\Omega} \qquad \boxed{P = 72\,W} \qquad P = I^2 \cdot R \Rightarrow R = \frac{P}{I^2} = \frac{72}{144} = 0{,}5\,\Omega$$

$$\boxed{R = 0{,}5\,\Omega}$$

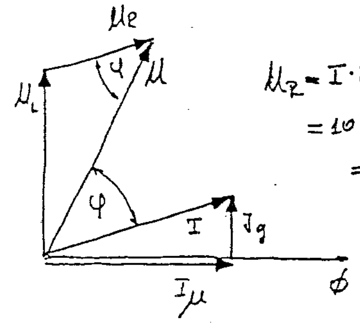
$$X_L = \sqrt{100 - 0{,}25} = 9{,}987\,\Omega$$

$$\boxed{X_L = 9{,}987\,\Omega} \Rightarrow L = \frac{9{,}987}{314{,}159} \Rightarrow \boxed{L = 0{,}03179\,H}$$

za svitak sa željeznom jezgrom

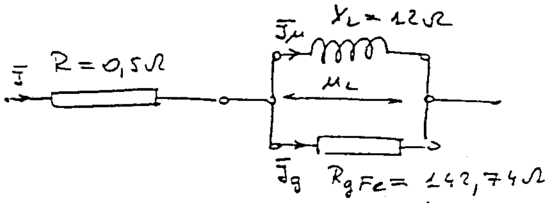
$$Z = \frac{U}{I} = 12\,\Omega \qquad Z = 12\,\Omega \qquad P = I^2 \cdot R + P_{željezu}$$

$$P_{ŽELJEZU} = P - I^2 \cdot R = 150 - 100 \cdot 0{,}5 = 150 - 50 = 100\,W$$



$$U_R = I \cdot R = 10 \cdot 0{,}5 = 5\,V$$

$$\boxed{P_{FE} = 100\,W}$$



$$S = U \cdot I = 120 \cdot 10 = 1200\,VA$$

$$P = 150\,W \Rightarrow \cos\varphi = \frac{P}{S} = 0{,}125 \qquad \cos\varphi = 0{,}125$$

$$E = U_L = \sqrt{U_R^2 + U^2 - 2U_R \cdot U \cdot \cos\varphi} = \sqrt{25 + 14400 - 2 \cdot 5 \cdot 120 \cdot 0{,}125} = \sqrt{14425 - 150} = 119{,}478\,V$$

$$\boxed{U_L = 119{,}478\,V}$$

$$P_{FE} = U_L \cdot I_g \Rightarrow I_g = \frac{100}{119{,}478} = 0{,}837\,A \qquad \boxed{I_g = 0{,}837\,A}$$

$$I_\mu = \sqrt{I^2 - I_g^2} = \sqrt{100 - 0{,}7} = 9{,}965\,A$$

$$R_{gFe} = \frac{119{,}478}{0{,}837} = 142{,}75\,\Omega \qquad X_L = \frac{119{,}478}{9{,}965} = 12\,\Omega$$

$$\boxed{X_L = 12\,\Omega}$$

5 41

ZADATAK: Za regulaciju jačine svjetla žarulja upotrebljena je prigušnica sa željeznom jezgrom

8.7 u spoju prema slici. Odredite potrebnu veličinu zračnog raspora δ uz koji će napon na žarulji iznositi $U_Z = 120\,V$, a ukupna struja $I = 10\,A$.

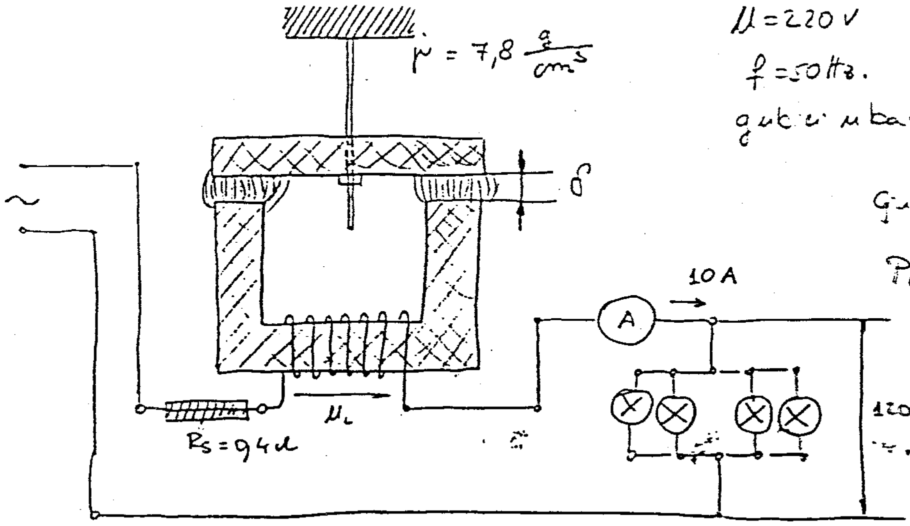
masa jezgre $m = 10\,kg$ — otpor žice prigušnice $R = 0,4\,\Omega$

$l_{sr} = 100\,cm$ — $N = 1000$ zavoja

$\rho = 7,8\,\frac{g}{cm^3}$ — $U = 220\,V$

$f = 50\,Hz$.

gubici u bakru jednaki su gubicima u željezu



Gubici u bakru su $P_{Cu} = I^2 \cdot R = 10^2 \cdot 0,4 = 40\,W$

$P_{Cu} = 40\,W$

$P_{Cu} = P_{Fe} = 40\,W$

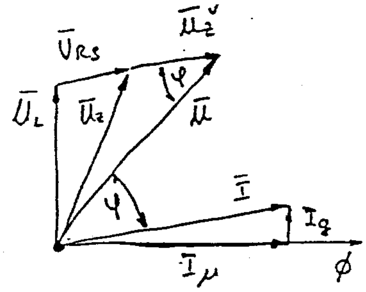
Ukupna snaga je

$P = P_{Cu} + P_{Fe} + P_Z$

$= 40 + 40 + 120 \cdot 10$

$S = 220 \cdot 10 = 2200$ $\quad \cos\varphi = \frac{P}{S} = \frac{1280}{2200} = 0,581818$ $\quad = 80 + 1200 = 1280\,W$

$\boxed{\cos\varphi = 0,582}$



$U_Z + U_{RS} = 4 + 120 = 124\,V$

$U_L = \sqrt{(U_Z + U_{RS})^2 + U^2 - 2(U_Z - U_{RS}) \cdot U \cdot \cos\varphi}$

$U_L = \sqrt{15376 + 48400 - 2 \cdot 124 \cdot 220 \cdot 0,582}$

$U_L = \sqrt{15376 + 48400 - 31753,92} = 179\,V$ $\quad U_L = 179\,V$

$U_L = N \frac{d\phi}{dt}$ $\quad U_L = N \cdot f \cdot 4,44 \cdot \phi_{max} \Rightarrow \phi_{max} = \frac{U_L}{4,44 \cdot N \cdot f} = \frac{179}{4,44 \cdot 1000 \cdot 50} = \frac{179}{4,44 \cdot 50000}$

$\phi_{max} = 8,063 \cdot 10^{-4}\,Vs$

$S = ?$ $\quad S \cdot l \cdot \rho = m \Rightarrow S = \frac{m}{l\,\rho} = \frac{10\,kg}{100\,cm \cdot 7,8 \cdot 10^{-3}}$

$S = \frac{10}{100 \cdot 7,8 \cdot 10^{-3}} = \frac{10^2}{7,8} = 12,82\,cm^2$ $\quad \boxed{S = 12,82\,cm^2}$

$I = 10\,A$

$I_g = \frac{P_{Fe}}{U_L} = \frac{40}{179} = 0,224$

$B_{max} = \frac{\phi_{max}}{S} = \frac{8,063 \cdot 10^{-4}\,Wb}{12,82 \cdot 10^{-4}} = 0,63\,T$ $\quad$ iz dijagrama određujemo H budući $\boxed{I_\mu \approx 10\,A}$

$I_{\mu m} = 14,142\,A$

$B = 0,63\,T \Rightarrow H = 125\,\frac{A}{m}$

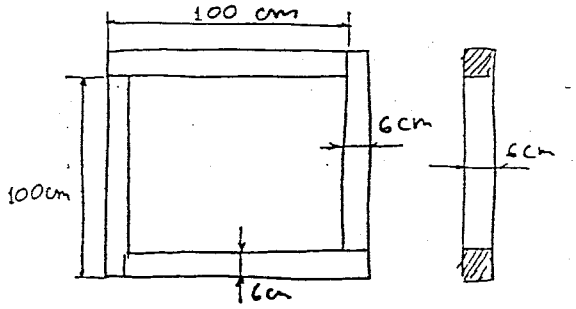
$H_0 = \frac{B}{\mu_0} = \frac{0,63}{4\pi \cdot 10^{-7}} = 501352,357\,\frac{A}{m}$

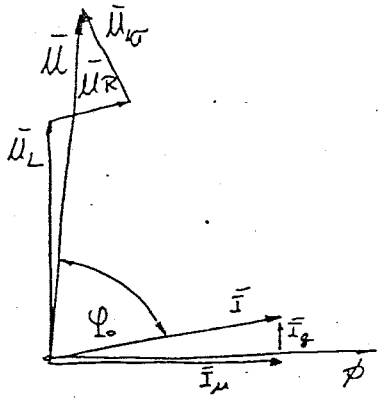
$H_Z \cdot l_Z + 2 \cdot \delta \cdot H_0 = 1000 \cdot 14,142$

$\Rightarrow \delta = \frac{1}{2H_0}\left[14142 - 125 \cdot 1\right] = \frac{14017}{2 \cdot 501352,357} = 0,01397\,m$ $\quad \delta = 13,97\,mm$ $\quad \boxed{\delta = 1,397\,cm}$

ZADATAK: Jednofazni transformator od 220/50V ima N₁=250 zavoja. Jezgra mu je

8.29 načinjena od transformatorskih limova debljine d=0,5 mm, ρ=7,6 g/cm³ i dimenzija prema slici. Na izolaciju između limova odpada 10% pop. presjeka. Otpor primarnog namota iznosi 1Ω, a na rasipne gubitke 10% ukupnog toka. Odredite struju praznog hoda i cosφ₀.





Pretpostavimo da je inducirani napon na primaru jednak naponu izvora. Zapravo tj. će napon biti nešto manji zbog pada napona na otporu i na rasipnom induktivitetu.

$U_1 = E_1 = 4{,}44 \cdot f \cdot N \cdot \Phi_{max} \qquad \Phi_{max} = \frac{220}{4{,}44 \cdot 50 \cdot 200} = 39{,}7 \cdot 10^{-4}\,Vs \qquad B_M = \frac{\Phi_{max}}{0{,}9 \cdot S} = \frac{39{,}7 \cdot 10^{-4}}{0{,}9 \cdot 36 \cdot 10^{-4}}$

$\boxed{B_M = 1{,}225\ T}$

Iz krivulje magnetiziranja dobijemo $H = 430\ A/m$

$I_{\mu max} \cdot N_1 = H_m \cdot l_{sr} \rightarrow I_{\mu m} = \frac{H_m \cdot l_{sr}}{N_1} = \frac{430 \cdot 4{,}00}{250} = 6{,}9\,A$

$I_\mu = \frac{I_{\mu m}}{\sqrt{2}} = 4{,}9\,A \qquad \boxed{I_\mu = 4{,}9\,A}$

Iz krivulje gubitaka i ukupne mase jezgre izračunavamo struju I_g

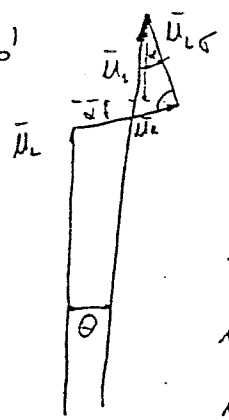
$m = \rho \cdot V = 7{,}6\,\frac{g}{cm^3} \cdot 4 \cdot 100\,cm \cdot 0{,}9 \cdot 36\,cm^2 = 7{,}6 \cdot 4 \cdot 100 \cdot 0{,}9 \cdot 36\,g = 101\,kg$

Iz krivulje očitamo snagu gubitaka

$P_{Fe} = 4{,}5 \cdot 101 = 454\,W \qquad I_g = \frac{P_{Fe}}{U_1} = \frac{454}{220} = 2{,}06\,A$

$I_0 = \sqrt{I_\mu^2 + I_g^2} = \sqrt{24{,}01 + 4{,}2436} = 5{,}315\,A \qquad \boxed{I_0 = 5{,}315\,A} \qquad \boxed{I_g = 2{,}06\,A}$

$\alpha = \operatorname{arctg}\frac{I_1}{I_\mu} = 22°50'$



pošto je Θ obično maličak možemo pisati da je

$U_1 = U_2 + U_L \cdot \sin\alpha + U_1 \cdot \cos\alpha \qquad U_R = I \cdot R = 5{,}315\,A$

$U_{L\sigma} = 0{,}1 \cdot 220\,V = 22\,V$

$U_1 = 220 + 5{,}315 \cdot \sin 22°50' + 22 \cdot \cos 22°50'$

$U_1 = 220 + 2{,}059 + 20{,}28 = 242\,V$

Očitan je napon U_L pred in po vrši korekcija $U_L = 220 \cdot \frac{220}{242,0} = 200\,V$ $\boxed{U_L = 200\,V}$

$\Phi_m = \frac{200}{250 \cdot 4,44 \cdot 50} = 3,6 \cdot 10^{-3}\,Vs = 36 \cdot 10^{-4}\,Vs \Rightarrow B_M = \frac{36 \cdot 10^{-4}}{36 \cdot 0,9 \cdot 10^{-4}} = 1,11\,T$

Iz krivulje magnetiziranja očitamo $H = 275\,\frac{A}{m}$

$\Rightarrow I_{\mu m} = \frac{275 \cdot 4}{250} = 4,4\,A \quad I_\mu = \frac{4,4}{\sqrt{2}} = 3,11\,A$ $\boxed{I_\mu = 3,11\,A}$

$P_{Fe} = 101 \cdot 4,1 = 414,1\,W \Rightarrow I_g = \frac{414,1}{200} = 2\,A \quad \alpha = \operatorname{arc\,tg} \frac{2}{3,11} = 33,65^\circ$

$U_R = I \cdot R = 3,69\,V$ $\boxed{U_L = 3,69\,V}$ $\quad I = \sqrt{9,672 + 4} = 3,69\,A$ $\boxed{I = 3,69\,A}$

$\boxed{U_g = 20\,V} \quad \cos\varphi = \cos(90 - \alpha - \Theta) \Rightarrow \Theta = ?$

$\Theta = \operatorname{arc}\cos \frac{U_L + U_R \cdot \sin\alpha + U_g \cdot \cos\alpha}{U_1} = \frac{200 + 3,69 \cdot 0,554 + 20 \cdot 0,832}{220} = \frac{200 + 2,04 + 16,64}{220}$

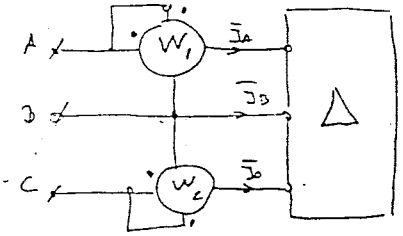
$= 7,65^\circ$

$\cos\varphi = \cos(90 - 33 - 7,65) = 0,5$

$\bar{I}_c = \bar{I}_{CO} - \bar{I}_{BC} = 10,4 \cdot e^{j210^\circ} - 10,4 \cdot e^{j30^\circ} = 10,4\left[\left(-\frac{\sqrt{3}}{2} - j\frac{1}{2}\right) - \left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right)\right]$

$\bar{I}_c = 10,4\left[-\sqrt{3} - j\right] = -10,4\left(\sqrt{3} + j\right) = -10,4 \cdot 2 \cdot e^{j30^\circ}\ A \Rightarrow \boxed{\bar{I}_c = -20,8\, e^{j30^\circ}\ A}$

Snagu odredimo pomoću 2 wattmetra tako da umnožimo napon i struju na svakoj fazi, potrošača.



$\bar{U}_{AC} = 120 \cdot e^{j120^\circ}\ V$

$\bar{U}_{CB} = -120 \cdot e^{j0^\circ}\ V$

$P = Re\left\{\bar{U}_{AB} \cdot \bar{I}_A^*\right\} + Re\left\{\bar{U}_{CB} \cdot \bar{I}_c^*\right\} = Re\left\{120 \cdot e^{j120} \cdot 20,788 \cdot e^{-j\varphi}\right\} + Re\left\{-120 \cdot e^{j0^\circ} \cdot (-20,8)\, e^{-j\varphi}\right\}$

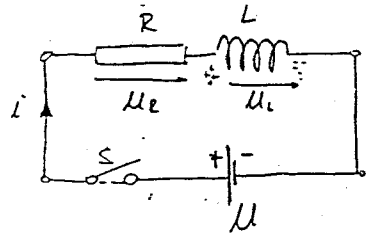
$P = 2160,288 + 2161,536 = 4321,824\ W \qquad P_T = 4,321824\ kW$

$\boxed{P_T = 4,32\ kW}$ ww

PRELAZNE POJAVE

① $L = 0,1\,H \quad R = 10\,\Omega \quad U = 100\,V$: Odredite vrijeme u trenutku kojega struja dosegne 90% svoje stacionarne vrijednosti, kolika je brzina porasta struje u trenutku uključivanja i ako ima 90% stacionarne vrijednosti.

9.1



tt1

$u = u_R + u_L \qquad u = i \cdot R + L \cdot \frac{di}{dt} \;/:R$

$\frac{u}{R} = i + \frac{L}{R}\frac{di}{dt} \qquad \frac{L}{R}\frac{di}{dt} + i = \frac{u}{R} \qquad \frac{L}{R} = \tau$

$\tau\frac{di}{dt} + i = \underbrace{\frac{u}{R}}_{I} \qquad \tau\frac{di}{dt} = I - i \qquad \frac{\tau\, di}{I - i} = dt \qquad \tau = \frac{L}{R} = \frac{0,1}{10}$

$\tau = 0,01$

$\frac{di}{I-i} = \frac{dt}{\tau} \;/\int \qquad \int\frac{di}{I-i} = \frac{1}{\tau}\cdot t + c \qquad -\ln(I-i) = \frac{t}{\tau} + c$ iz poč. uvjeta $t=0$, $i=0$

$-\ln I = c \qquad -\ln(I-i) = \frac{t}{\tau} - \ln I \;/(-1) \qquad \ln(I-i) = -\frac{t}{\tau} + \ln I$

$\ln\frac{I-i}{I} = -\frac{t}{\tau} \qquad e^{-\frac{t}{\tau}} = \frac{I-i}{I} \qquad \boxed{i = I\left(1 - e^{-\frac{t}{\tau}}\right)} \qquad i = 10\left(1 - e^{-\frac{t}{0,01}}\right)\ A$

$\left.\frac{di}{dt}\right|_{(t=0)} = +10 \cdot e^{-\frac{t}{0,01}} \cdot \frac{1}{0,01} = 1000 \cdot e^{-0} = 1000\ A/s$

struja dosegne 90% svoje vrijednosti u trenutku $= ?$

$9 = 10\left(1 - e^{-\frac{t}{0,01}}\right) \Rightarrow 0,9 = 1 - e^{-t/0,01} \qquad e^{-t/0,01} = 0,1 \;/\ln \qquad +\frac{t}{0,01} = +2,303$

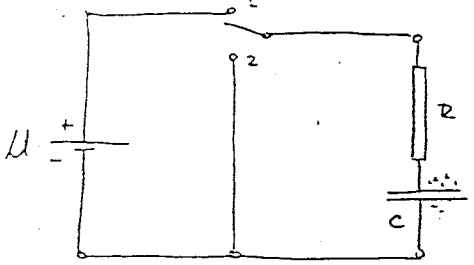
brzina porasta u trenutku kad struja ima 99% svoje vrijednosti

$$\left.\frac{di}{dt}\right|_{t=23ms} = 10 \cdot e^{-\frac{t}{0,01}} \cdot \frac{1}{0,01} = 1000 \cdot e^{-\frac{0,023}{0,01}} = 100,25\ \frac{A}{s}$$

~~ZADATAK~~: Skicirajte kvalitativno struju napona i snage kao funkcije vremena za slučaj

9.3 a) kad sklopka dolazi u položaj 1.

b) kad nakon završenog nabijanja sklopka prebacimo u položaj 2.



a) sklopka je u položaju 1)

$$U = i \cdot R + U_C \Rightarrow U = i \cdot R + \frac{Q}{C} = i \cdot R - \frac{Q}{C}$$

$$U = \frac{dQ}{dt} \cdot R + \frac{Q}{C} \quad / \cdot C \qquad U \cdot C = \frac{dQ}{dt} RC + Q$$

$$U \cdot C = \frac{dQ}{dt}\tau + Q \qquad Q - Q = \frac{dQ}{dt}\tau \qquad \frac{dt}{\tau} = \frac{dQ}{Q - Q} \quad / \int$$

$$\frac{1}{\tau} t + C = -\ln(Q - Q) \qquad \text{za } t = 0 \quad Q = 0$$

$$\frac{1}{\tau} 0 + C = -\ln(Q - 0) \qquad C = -\ln Q \qquad \frac{t}{\tau} - \ln Q = -\ln(Q - Q)$$

$$\frac{t}{\tau} = -\ln(Q - Q) + \ln Q \qquad -\frac{t}{\tau} = \ln\left(\frac{Q - Q}{Q}\right)$$

$$\frac{Q - Q}{Q} = e^{-\frac{t}{\tau}} \Rightarrow \boxed{Q = Q\left(1 - e^{-\frac{t}{\tau}}\right)} \qquad U_C = \frac{Q}{C} \qquad \boxed{U_C = U\left(1 - e^{-\frac{t}{\tau}}\right)}$$

$$i = \frac{dQ}{dt} = Q\left[0 + e^{-\frac{t}{\tau}} \cdot \frac{1}{\tau}\right] = \left(e^{-\frac{t}{\tau}} \cdot \frac{1}{\tau}\right) \cdot Q = \frac{Q}{\tau} \cdot e^{-\frac{t}{\tau}} = \frac{U \cdot C}{RC} \cdot e^{-\frac{t}{\tau}} = I\, e^{-\frac{t}{\tau}}$$

$$\boxed{U_C = \frac{1}{C}\int i\, dt} \text{ il.} \qquad \boxed{i = \frac{U}{R} e^{-\frac{t}{\tau}}}$$

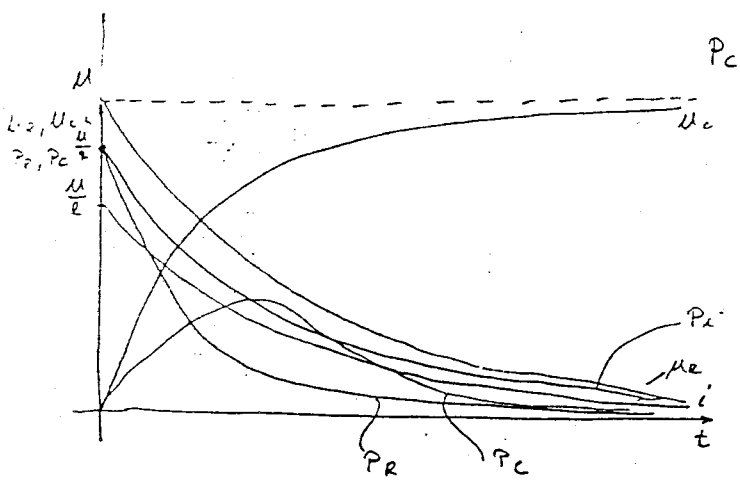
$$\boxed{U_R = i \cdot R = U \cdot e^{-\frac{t}{\tau}}}$$

$$\boxed{P_R = \frac{U^2}{R} e^{-\frac{2t}{\tau}}}$$

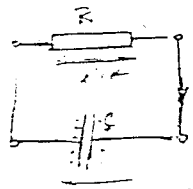
$$P_R = i^2 \cdot R = \frac{U^2}{R^2} R\, e^{-\frac{2t}{\tau}} = \frac{U^2}{R} e^{-\frac{2t}{\tau}}$$

$$P_C = i \cdot U_C = \frac{U}{R} e^{-\frac{t}{\tau}} \cdot U \cdot \left(1 - e^{-\frac{t}{\tau}}\right) = \frac{U^2}{R} e^{-\frac{t}{\tau}}\left(1 - e^{-\frac{t}{\tau}}\right) = \boxed{\frac{U^2}{R}\left[e^{-\frac{t}{\tau}} - e^{-\frac{2t}{\tau}}\right] = P_C}$$

$$P_C = \underbrace{\frac{U^2}{R} e^{-\frac{t}{\tau}}}_{\text{snaga izvora}} - \underbrace{\frac{U^2}{2} e^{-\frac{2t}{\tau}}}_{\text{snaga na otporu } 2}$$



2. Izbijanje kondenzatora.



$U_C + U_R = 0$

$\frac{Q}{C} + i \cdot R = 0 \qquad \frac{Q}{C} + \frac{dQ}{dt} \cdot R = 0 \;/\cdot C \qquad Q + \frac{dQ}{dt} \cdot R \cdot C = 0$

U_C = vanjski napon

$\pm \frac{dQ}{dt} \tau = 0 \qquad \frac{dQ}{dt} \tau = -Q \qquad \frac{dQ}{Q} = -\frac{dt}{\tau} \;/\int \qquad \ln Q = -\frac{1}{\tau} t + C \qquad$ za $t=0$ $Q = Q$

$\ln Q = -\frac{0}{\tau} + C \Rightarrow C = \ln Q \qquad \ln Q = -\frac{t}{\tau} + \ln Q \qquad \ln \frac{Q}{Q} = -\frac{t}{\tau}$

$\frac{Q}{Q} = e^{-\frac{t}{\tau}} \Rightarrow Q = Q \cdot e^{-\frac{t}{\tau}} \rightarrow \boxed{U_C = U e^{-\frac{t}{\tau}}}$

$i = \frac{dQ}{dt} = Q \cdot e^{-\frac{t}{\tau}} \cdot \left(-\frac{1}{\tau}\right) = -\frac{Q}{\tau} \cdot e^{-\frac{t}{\tau}} = \frac{-U \cdot \not{C}}{R \not{C}} \cdot e^{-\frac{t}{\tau}} = -I \cdot e^{-\frac{t}{\tau}} \qquad \boxed{i = -I \cdot e^{-\frac{t}{\tau}}}$

i je suprotnog smera.

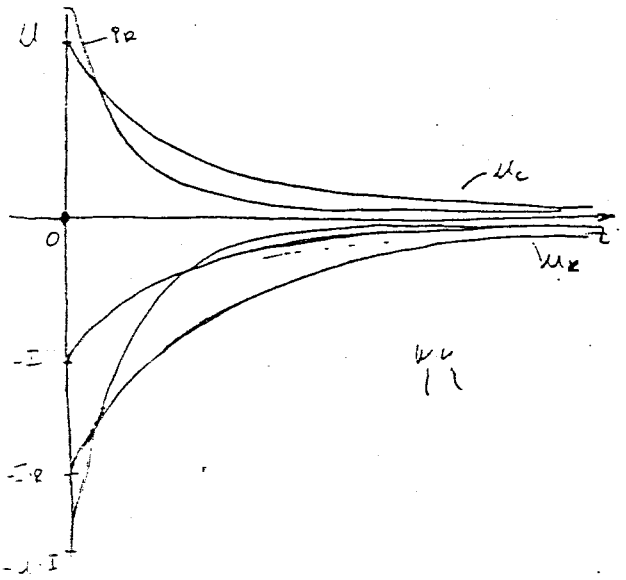
$\boxed{U_R = i \cdot R = -I \cdot R \, e^{-\frac{t}{\tau}}}$

$P_R = i^2 \cdot R = +I^2 R \cdot e^{-\frac{2t}{\tau}}$

$P_C = U_C \cdot i = -I \cdot e^{-\frac{t}{\tau}} \cdot U \cdot e^{-\frac{t}{\tau}} = \boxed{-U \cdot I \cdot e^{-\frac{2t}{\tau}} = P_C}$

$P_C = -U \cdot I \cdot e^{-\frac{2t}{\tau}}$

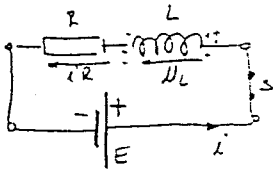
$P_C = -I^2 \cdot R \, e^{-\frac{2t}{\tau}} = +P_R$



PRAVILO AKO ZAČUNAM SA VANJSKIM NAPONIMA ONDA UZIMAM $\sum_{i=1}^{n} U_{vanj_i} = 0$

ZADATAK: $R = 50\,\Omega \quad L = 10\,H \quad E = 100\,V$

9.4



$E = iR + U_L$

$\tau = \frac{L}{R} \qquad \tau = \frac{10}{50} = 0{,}2$

$\Rightarrow E = iR + L \cdot \frac{di}{dt}$

$= I \left(1 - e^{-\frac{t}{\tau}}\right)$

$= \frac{E}{R}\left(1 - e^{-\frac{t}{\tau}}\right) \qquad U_R = i \cdot R = E \cdot \left(1 - e^{-\frac{t}{\tau}}\right) \qquad U_L = L \frac{di}{dt} = L \cdot \frac{E}{R} \cdot \left(0 + e^{-\frac{t}{\tau}} \frac{1}{\tau}\right)$

$U_L = \not{L} \cdot \frac{E}{\not{R}} \cdot \frac{\not{R}}{\not{L}} \cdot e^{-\frac{t}{\tau}} = E \cdot e^{-\frac{t}{\tau}}$

$\boxed{U_L = E \cdot e^{-\frac{t}{\tau}}}$

$\boxed{i = 2\left(1 - e^{-\frac{t}{\tau}}\right)}$

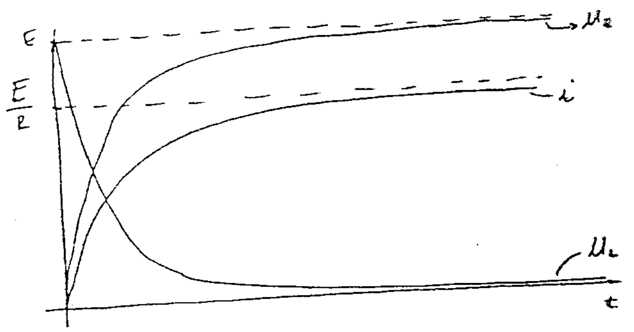
$\boxed{U_R = 100\left(1 - e^{-\frac{t}{\tau}}\right)}$

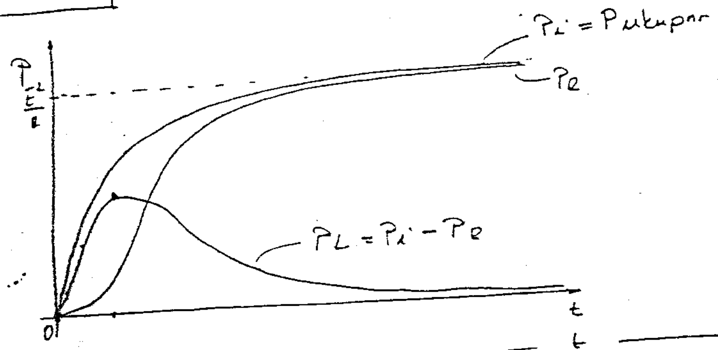
$P_R = i^2 \cdot R = \frac{E^2}{R}\left(1 - e^{-\frac{t}{\tau}}\right)^2 = \boxed{\frac{E^2}{R}\left(1 - e^{-\frac{t}{\tau}}\right)^2 = P_R}$

$P_L = U_L \cdot i = E \cdot e^{-\frac{t}{\tau}} \cdot 2 \cdot \left(1 - e^{-\frac{t}{\tau}}\right)$

$\boxed{P_L = 200\left[e^{-\frac{t}{\tau}} - e^{-\frac{2t}{\tau}}\right]}$

$P_i = E \cdot i = 100 \cdot 2 \cdot \left(1 - e^{-\frac{t}{\tau}}\right) = \boxed{200\left(1 - e^{-\frac{t}{\tau}}\right) = P_i}$





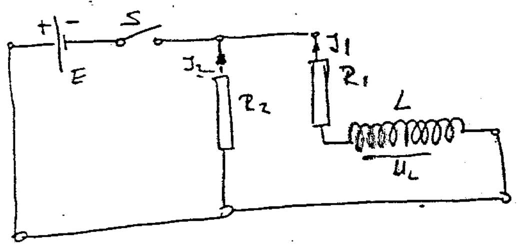
$\boxed{W = \int_0^t P(t)\,dt}$

$\boxed{W_{top} = \infty}$ $\boxed{W_{MAG.POLJA} = \frac{1}{2} L \cdot I^2}$ ⇒ zavisi o krajnjem toku procesa a ne o obliku procesa.

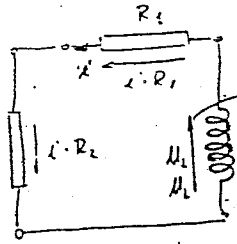
ZADATAK: 9.6 $E = 120V$ $R_1 = 20\Omega$ $R_2 = 30\Omega$ $L_1 = 1H$. U $t=0$ otvaramo sklopku s nakon čega je uspostavljeno stacionarno stanje.

Izračunajte energiju mag. polja nakon $t = 40\,ms$. Za inducirani napon na zavojnici u trenutku isklapanja

za stacionarno stanje $I_1 = \frac{E}{R_1}$ $I_2 = \frac{E}{R_2}$



$i_2 = -i_1$



pretpostavljeni smjer u_L vidimo u rezultatu da je u suprotnom smjeru.

$i(R_1 + R_2) + u_L = 0$

$u_L = i(R_1 + R_2)$ $\quad L\frac{di}{dt} + (R_1 + R_2)\, i = 0 \,/ (R_1 + R_2)$ $\quad \underbrace{\frac{L}{R_1 + R_2}}_{\tau} \frac{di}{dt} + i = 0$

$-\tau \frac{di}{dt} = i$ $\quad \frac{di}{i} = -\frac{dt}{\tau} \,/ \int$

$\ln i = \frac{1}{\tau} t + c$ $\quad$ za $t = 0$ $\quad I = \frac{E}{R_1}$

$\ln i = \frac{-1}{\tau} t + c$ $\quad \ln \frac{E}{R_1} = \frac{-1}{\tau} \cdot 0 + c$ $\quad c = \ln \frac{E}{R_1}$ $\quad \ln i - \ln \frac{E}{R_1} = -\frac{1}{\tau} t$ $\quad \tau = \frac{L}{R_1 + R_2}$

$\ln \frac{i}{I_1} = \frac{-t}{\tau}$ $\quad \ln \frac{i}{I_1} = -\frac{t}{\tau}$ $\quad i = I_1 e^{-\frac{t}{\tau}}$ $\quad \boxed{i = \frac{E}{R_1} e^{-t/\tau}}$

$t = 40\,ms$ $\quad W = \frac{1}{2} L i^2 = \frac{1}{2} L \cdot \left[\frac{E}{R_1}\right]^2 \cdot e^{-\frac{2t}{\tau}} = \frac{1}{2} \cdot 1 \cdot 36 \cdot e^{-\frac{2 \cdot 40 \cdot 10^{-3}}{\frac{1}{50}}} = 18 \cdot e^{-4000 \cdot 10^{-3}}$

$W = 18 \cdot e^{-4} = 0{,}329\ Ws$ $\quad \boxed{W = 0{,}329\ Ws}$

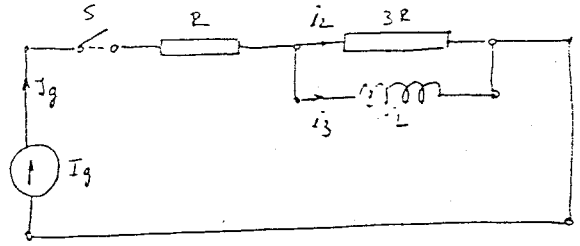
$u_L = L \cdot \frac{di}{dt} = L \cdot \frac{E}{R_1} e^{-\frac{t}{\tau}} \cdot -\frac{1}{\tau} = -\frac{E}{R_1} \frac{L}{\frac{L}{(R_1 + R_2)}} = -\frac{R_1 + R_2}{R_1} E \cdot e^{-\frac{t}{\tau}}$ $\quad \boxed{u_L = -\frac{R_1 + R_2}{R_1} E\, e^{-\frac{t}{\tau}}}$

za $t = 0$ $\quad u_L = -\frac{R_1 + R_2}{R_1} \cdot E = -\frac{5}{2} \cdot E = -\frac{5}{2} \cdot 120 = 300V$

ZADATAK 8

Električni krug napaja se strujnim generatorom $i_g = I_0$. Nacrtajte vremenske dijagrame i_2 i i_3 odredite izraz za struju i napon na ulaznim stezaljkama kruga.

9.8



$t < 0 \Rightarrow i_g = I_g = i_2 + i_3$

$3R\,i_2 = L\,\frac{di_3}{dt}$

$i_2 + i_3 = I_g \Rightarrow i_2 = I_g - i_3$

$3R\,i_2 = L\cdot\frac{di_3}{dt} \Rightarrow 3R\cdot(I_g - i_3) = L\cdot\frac{di_3}{dt}$ $\quad 3R\,I_g - 3R\,i_3 = L\frac{di_3}{dt}$

$3R\,I_g = L\cdot\frac{di_3}{dt} + 3R\,i_3 \;/:3R$ $\quad I_g = \frac{L}{3R}\frac{di_3}{dt} + i_3$ $\quad I_g - i_3 = \frac{di_3}{dt}\cdot\tau$

$\frac{dt}{\tau} = \frac{di_3}{I_g - i_3}\;/\int$ $\quad \frac{1}{\tau}t + c = -\ln(I_g - i_3)$ $\quad c = -\ln I_g$

$\frac{t}{\tau} - \ln I_g = -\ln(I_g - i_3)$ $\quad \ln(I_g - i_3) - \ln I_g = -\frac{t}{\tau}$

$\ln\frac{I_g - i_3}{I_g} = -\frac{t}{\tau}$ $\quad \frac{I_g - i_3}{I_g} = e^{-\frac{t}{\tau}}$ $\quad \boxed{i_3 = I_g\left(1 - e^{-\frac{t}{\tau}}\right)}$

$i_2 = I_g - i_3 = I_g - I_g\left(1 - e^{-\frac{t}{\tau}}\right) = I_g\left[1 - \left(1 - e^{-\frac{t}{\tau}}\right)\right] = I_g\left[1 - 1 + e^{-\frac{t}{\tau}}\right] = I_g\cdot e^{-t/\tau}$

$\boxed{i_2 = I_g\cdot e^{-\frac{t}{\tau}}}$ NIJ

$u_{NA} = I_g\cdot R + i_2 3R = 2\left(I_g + 3i_2\right) = 2\left[I_0 + 3\cdot I_g e^{-\frac{t}{\tau}}\right]$

SIETAČKATA

$u = 2\cdot I_g\left(1 + 3e^{-\frac{t}{\tau}}\right)$

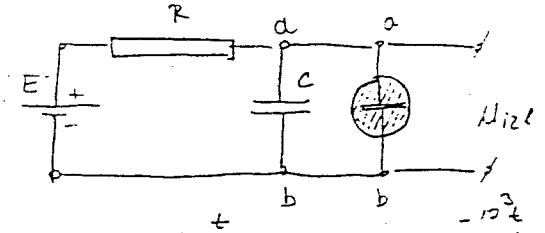
$\boxed{u = 2\cdot I_g\left(1 + 3\cdot e^{-\frac{3t}{\tau}}\right)}$ NN

I NAČIN THEVENINOV TEOREM

$R_T = 3R$ $\quad E_T = I_g\cdot 3R$

ZADATAK 8 Odredi frekvenciju pilastog napona? $E = 120$ V. Neonska lampa (idealna) pali se pri naponu od $U_p = 80$ V a gasi se pri naponu $U_g = 40$ V. Otpor $R = 2000\ \Omega$, a $C = 0{,}5\ \mu$F. Otpor uspostavljanja lampe prema zemlji zanemarite a ugašenu možemo smatrati kao prekid.

9.13



$u_c = E\left(1 - e^{-\frac{t}{\tau}}\right)$ nabijanje kondenzatora

= pražnjenje kondenzatora

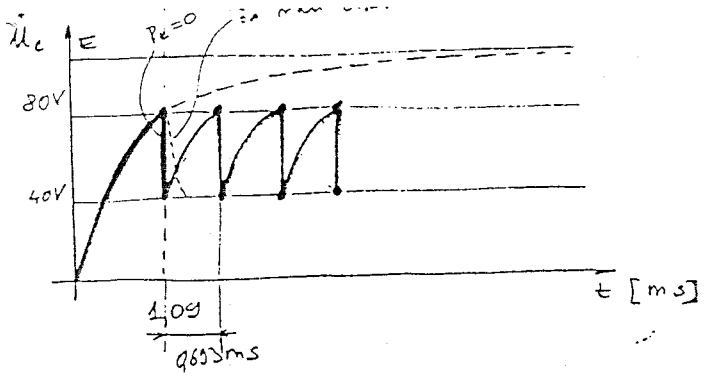
$u_c = u_t$

$80 = 120\left(1 - e^{-\frac{t}{10^{-3}}}\right) = 120\left(1 - e^{-10^3 t}\right)$

$\frac{80}{120} = \left(1 - e^{-10^3 t}\right)$ $\quad 1 - e^{-10^3 t} = 0{,}66667$

$e^{-10^3 t} = 0{,}333333$

$+10^3 t = +1{,}0986 \Rightarrow t = 0{,}001098\ \text{s} =$ vrijeme nabijanja od 0!



Kad napon dosegne vrijednost od 80V otpor tiristora postaje jednak nuli pa napon između katode i anode pada na nulu $u_c \to 0$

Kad napon u_c padne na nulu $R_t \to \infty$ pa ponovo započinje proces nabijanja kondenzatora

$$E = i \cdot R + \frac{Q}{C} = \frac{dQ}{dt} R + \frac{Q}{C} \quad / \cdot C \qquad \tau = RC$$

$$E \cdot C = \frac{dQ}{dt} \tau + Q \qquad Q = \tau \frac{dQ}{dt} + Q \qquad Q - Q = \tau \frac{dQ}{dt} \qquad \frac{dQ}{Q - Q_0} = \frac{dt}{\tau} \Big/ \int$$

$$-\ln(Q - Q_0) = \frac{t}{\tau} + c \qquad \text{za } t = 0 \Rightarrow \text{pomjeramo koordinatni sustav u } \underline{t = 1{,}09\,ms}$$

$$c = -\ln(Q - Q_0) = -\ln(Q - 40 \cdot C) = -\ln(E \cdot C - 40 \cdot C) = -\ln(80C)$$

$$-\ln(Q - Q_0) = \frac{t}{\tau} - \ln(80C) \qquad \ln \frac{Q - Q_0}{80C} = -\frac{t}{\tau} \qquad \frac{Q - Q_0}{80C} = e^{-\frac{t}{\tau}}$$

$$\frac{E \cdot C - Q_0}{80C} = e^{-\frac{t}{\tau}}$$

$$Q_0 = EC - 80Ce^{-\frac{t}{\tau}} = 120C - 80Ce^{-\frac{t}{\tau}} = \dots$$

$$u_c = \frac{Q}{C} = 120 - 80e^{-\frac{t}{\tau}} = 40 + 80\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$u_c = 80V = 40 + 80\left(1 - e^{-\frac{t}{\tau}}\right)$$

$g(t+T) = g(t)\ V(t)$

$\Rightarrow$ T = period funkcije

$$40 = 80\left(1 - e^{-\frac{t}{\tau}}\right) \Rightarrow 0{,}5 = \left(1 - e^{-\frac{t}{\tau}}\right)$$

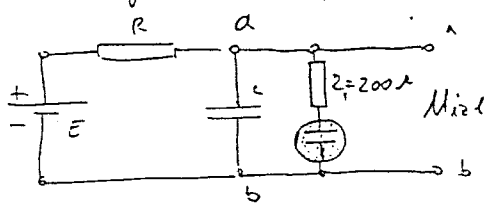
$$e^{-\frac{t}{\tau}} = +0{,}5 \qquad -\frac{t}{\tau} = -0{,}693 \qquad t = \tau \cdot 0{,}693 = 2000 \cdot \frac{1}{2} \cdot 10^{-6} \cdot 0{,}693$$

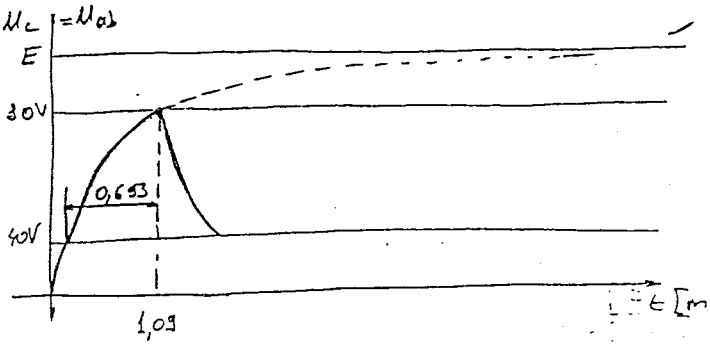
$t = 0{,}693\,ms \Rightarrow$ će ponovo skočiti sa 40V na 80V

$$f = \frac{1}{T} = \frac{1}{0{,}693 \cdot 10^{-3}\,s} = \frac{10^3}{0{,}693} = 1443\,Hz$$

$$\boxed{f = 1443\,Hz}$$

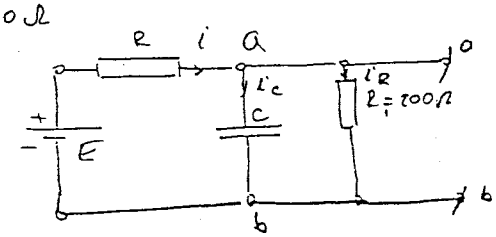
ZADATAK: Kojа je frekvencija oscilacija na izlazu iz sklopa prikazanog otpor $R_L = 200\,\Omega$





za $u_c = 80$ tj. za $t = 1{,}09\,ms$ otpor tiristora postane 200Ω

za $0 \le t < x \Rightarrow$



$$i = i_c + i_R \qquad E = i \cdot R + u_c = i \cdot R + \frac{Q_c}{C}$$

$$E = i \cdot R + R_L \cdot i_R$$

za $1{,}09 \le t < \infty$ $\quad i = i_C + i_2$

$E = i_1 R + u_C = (i_C + i_R) R + u_C = E$

$E = i_2 R + i_2 R_1 = (i_C + i_R) R + i_2 R_1 = E$ } –

$u_C = \frac{Q}{C} = \frac{1}{C} \cdot Q = \frac{1}{C} \int i_C dt$ $\quad u_C = i_2 R_1$ $\quad \frac{1}{C}\int i_C dt = i_R \cdot R_1 \;/\frac{d}{dt}$

$\frac{1}{C} \cdot i_C = R_1 \cdot \frac{di_2}{dt} \Rightarrow \boxed{C = R_1 \cdot \frac{di_2}{dt}}$

$E = (i_C + i_2) R + i_2 R_1 = \left[R_1 \cdot \frac{di_2}{dt} + i_2\right] R + i_R \cdot R_1 = E$

$R_1 R C \cdot \frac{di_2}{dt} + i_R (R + R_1) = E \quad (R + R_1)$ $\quad \underbrace{\frac{R_1 \cdot R \cdot C}{R + R_1}}_{\tau} \frac{di_2}{dt} + i_R = \frac{E}{R + R_1}$

$\tau \frac{di_R}{dt} + i_R = \frac{E}{R + R_1}$ $\quad \tau \frac{di_R}{dt} = \frac{E}{R + R_1} - i_R$ $\quad \frac{di_R}{\frac{E}{R+R_1} - i_R} = \frac{dt}{\tau}$

$-\ln\left(\frac{E}{R+R_1} - i_2\right) = \frac{t}{\tau} + C$ $\quad$ za $t =$ prejšnem koordinata smatramo $t = 1{,}09$ ms

$i_2 = \frac{30\,V}{200\,\Omega} = 0{,}4\,A$

$C = -\ln\left(\frac{E}{R+R_1} - 0{,}4\right)$

$+ \ln\left[\frac{E}{R+R_1} - i_R\right] = -\frac{t}{\tau} + \ln\left[\frac{E}{R+R_1} - 0{,}4\right] = \dots$

$\ln \frac{\frac{E}{R+R_1} - i_R}{\frac{E}{R+R_1} - 0{,}4} = -\frac{t}{\tau}$ $\quad \ln \frac{i_2 - \frac{E}{R+R_1}}{0{,}4 - \frac{E}{R+R_1}} = -\frac{t}{\tau}$

$\frac{i_R - 0{,}0545}{0{,}34545} = e^{-\frac{t}{\tau}}$ $\quad i_R = 0{,}34545 \cdot e^{-\frac{t}{\tau}} + 0{,}0545$

$u_R = u_C - u_{ab} = 69{,}1 \cdot e^{-\frac{t}{\tau}} + 10{,}9$

$u_C = 69{,}1 \cdot e^{-\frac{t}{\tau}} + 10{,}9$ $\quad$ za $t = 0$ $\quad u_C = 80\,V$

Izračunajmo za kaj se napetost u_C zniža na $40\,V$

$\tau = \frac{R_1 \cdot R \cdot C}{R + R_1} =$

$40 = 69{,}1 \cdot e^{-\frac{t}{\tau}} + 10{,}9$

$29{,}1 = 69{,}1 \cdot e^{-\frac{t}{\tau}}$ $\quad e^{-\frac{t}{\tau}} = 0{,}2113 \Rightarrow -\frac{t}{\tau} = -0{,}8648$

$t = 0{,}8648 \cdot \frac{200 \cdot 2000 \cdot 5 \cdot 10^{-6}}{2200} = 78{,}618 \cdot 10^{-6}\,s = 0{,}000078618\,s$

$T = 0{,}000078618 + 0{,}0006 = 0{,}000678618\,s$

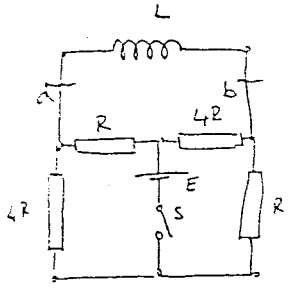
$f = \frac{1}{T} = \frac{1}{0{,}000771618} = 1295{,}9\,Hz$

$f = 1296\,Hz$

$\boxed{f = 1300\,Hz}$ ww

ZADATAK 8. U krugu radiju zatvaranu sklopka S. Napišite izraze za struje kroz svitak i izvor

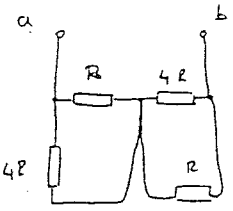
9.9.



Odredimo struju kroz svitak L.

Theveninov teorem

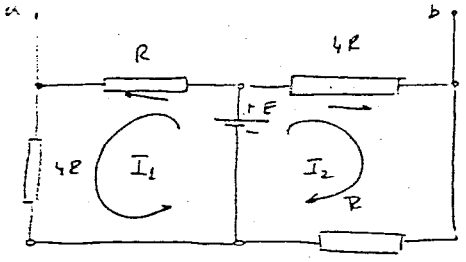
$R_T = ?$ $E_T = ?$



$R_T = \frac{4R \cdot R}{5R} + \frac{4R \cdot R}{5R} = 2 \cdot \frac{4R}{5} = \frac{2R}{5}$

$\boxed{R_T = \frac{8}{5} R}$

$E_T = ?$



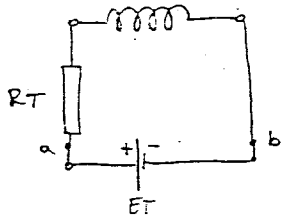
$I_1 = \frac{E}{5R}$

$I_2 = \frac{E}{5R}$

$\varphi_a = \varphi_b + 4R \cdot I_2 - R \cdot I_1$

$U_{ab} = 4R \cdot \frac{E}{5R} - R \cdot \frac{E}{5R} = \frac{4E}{5} - \frac{E}{5} = \frac{3E}{5}$

$U_{ab} = E_T = \frac{3E}{5}$



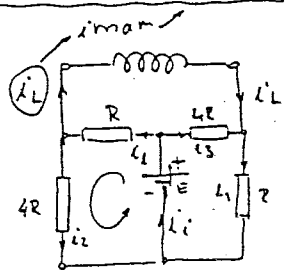
$i = \frac{E_T}{R_T}\left(1 - e^{-\frac{t}{\tau}}\right)$

$i = \frac{\frac{3E}{5}}{\frac{8R}{5}}\left(1 - e^{-\frac{t}{\tau}}\right)$ $\tau = \frac{L}{R_T} = \frac{L}{\frac{8}{5}R} = \frac{5}{8}\frac{L}{R}$

$i = \frac{3}{8}\frac{E}{R}\left(1 - e^{-\frac{t}{\frac{5L}{8R}}}\right) = \frac{3E}{8R}\left[1 - e^{-\frac{8Rt}{5L}}\right]$

$\boxed{i_L = 0{,}375 \frac{E}{R}\left[1 - e^{-1{,}6\frac{R}{L}t}\right]}$

za $i_L = ?$



$i_1 R + i_2 4R = E$

$i_1 = i_2 + i_L \Rightarrow i_2 = i_1 - i_L$

$i_1 \cdot R + i_1 4R - i_L 4R = E$

$5 i_1 \cdot R = E + i_L 4R$

$i_1 = \frac{1}{5R}\left[E + i_L 4R\right] = \frac{1}{5R}\left[E + 4R \cdot 0{,}375 \frac{E}{R}\left(1 - e^{-1{,}6\frac{R}{L}t}\right)\right]$

$i_L = \frac{1}{5R}\left[E + 0{,}375E\left(1 - e^{-1{,}6\frac{R}{L}t}\right)\right]$

$\boxed{i_1 = \frac{E}{5R}\left[1 + 1{,}5\left(1 - e^{-1{,}6\frac{R}{L}t}\right)\right]}$

$E = i_3 \cdot 2R - i_4 \cdot R$ $\quad i_3 + i_L = i_4 \Rightarrow \boxed{i_4 = i_3 + i_L}$

$E = i_3 \cdot 2R - 2\left[i_2 + i_L\right] = 4R i_3 + R i_2 + R i_L = 5R i_2 + R i_L$

$5R i_2 = E - R \cdot i_L \Rightarrow i_2 = \frac{1}{5R}\left[E - R \cdot i_L\right]$

$i_3 = \frac{1}{5R}\left[E - R \cdot 0{,}375 \frac{E}{R}\left(1 - e^{-1{,}6\frac{R}{L}t}\right)\right]$

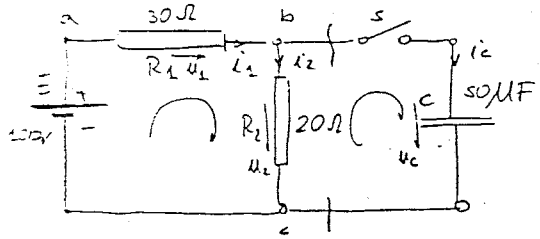
$i_3 = \frac{1 \cdot E}{5R}\left[1 - 0{,}375\left(1 - e^{-1{,}6\frac{R}{L}t}\right)\right]$

$i_1 - i_3 = \frac{E}{5R}\left\{\left[1 + 1{,}5\left(1 - e^{-1{,}6\frac{R}{L}t}\right) + 1 - 0{,}375\left(1 - e^{-1{,}6\frac{R}{L}t}\right)\right]\right.$

$\left. \frac{E}{5R}\left\{2 + 1{,}125\left(1 - e^{-1{,}6\frac{R}{L}t}\right)\right\}\right.$ 1/4

Zadatak 3. U trenutku t=0 zatvara se sklopka S. Kako se mijenja napon $u_{ab}(t)$

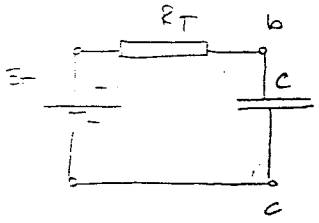
9.24 Izračunajte $u_{ab}(t)$ na dijagramu



Primjenimo Theveninov teorem

$$R_T = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

$$E_T = \frac{E}{R_1 + R_2} \cdot R_2$$



$$u_c = E_T\left(1 - e^{-\frac{t}{\tau}}\right) = \frac{E \cdot R_2}{R_1 + R_2}\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$\tau = \frac{R_1 \cdot R_2}{R_1 + R_2} \cdot C$$

$$= \frac{30 \cdot 20}{50} \cdot 50 \cdot 10^{-6}$$

$$= 600 \cdot 10^{-6}$$

$$= 6 \cdot 10^{-4}$$

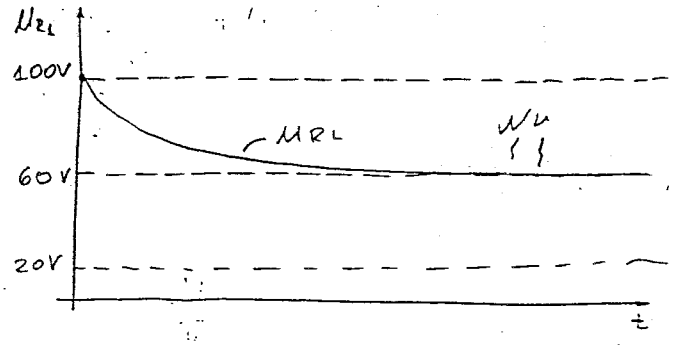
$$u_c = \frac{100 \cdot 20}{50}\left[1 - e^{-\frac{t}{6 \cdot 10^{-4}}}\right]$$

$$u_c = 40\left[1 - e^{-\frac{10^4 t}{6}}\right]$$

$$E = i_1 \cdot R_1 + u_c = u_{R_1} + u_c \Rightarrow u_{R_1} = 100 - 40\left[1 - e^{-\frac{10^4 t}{6}}\right]$$

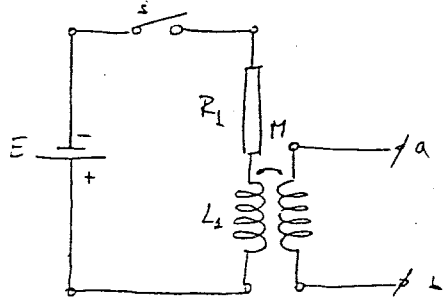
$$\boxed{u_{R_1} = 60 + 40\, e^{-\frac{10^4 t}{6}}\ V}$$

$u_{R_1} = 60 + 40\left(1 + e^{-\frac{t}{\tau}}\right)$



Zadatak 2: Odredite zakon promjene induciranog napona u sekundarnom svitku transformatora pri zatvaranju sklopke S.

9.22



Nakon zatvaranja sklopke S bit će prinar ić će biti struja oblika

$$i_p = \frac{E}{R_1}\left(1 - e^{-\frac{t}{\tau}}\right) \qquad \tau = \frac{L_1}{R_1}$$

$$u_{ab} = M \cdot \frac{di_p}{dt}$$

$$u_{ab} = M \cdot \frac{E}{R_1}\left(0 + e^{-\frac{t}{\tau}}\right)$$

$$= M \cdot \frac{E}{R_1} \cdot e^{-\frac{t}{\tau}}$$

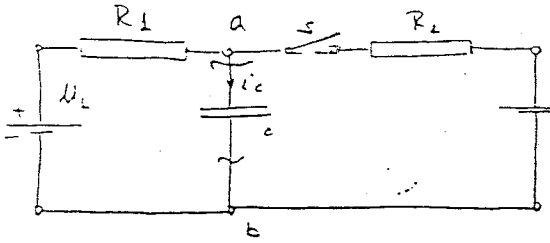
$$\boxed{u_{ab} = \frac{ME}{R_1} \cdot e^{-\frac{R_1 t}{L_1}}\ (V)}$$

NIJE DOBRO JER V_{AB} NEMA JEDINICU VOLT [V]

RJEŠENJE: $e = -E\,\frac{M}{L_1}\,e^{-\frac{R_1}{L_1} \cdot t}$

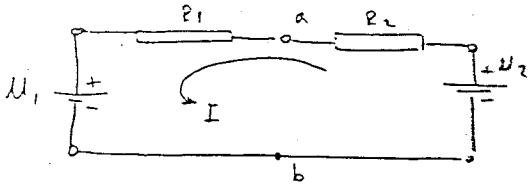
ZADATAK 8 U t=0s otvara se sklopka S i zatvara. Odrediti $u_c = f(t)$ $i_c = f(t)$

9.28 ZADANO: $U_1 = 100V$ $U_2 = 400V$ $R_1 = 100\Omega$ $R_2 = 50\Omega$ $C = 30\mu F$



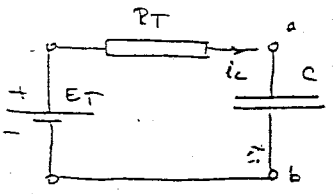
Thevenin-ovim teoremom ćemo se koristiti

$E_T = ?$ $R_T = ?$ $R_T = \frac{R_1 \cdot R_2}{R_1 + R_2}$ $E_T = ?$ $\varphi_a = \varphi_b + U_1 + I \cdot R_1$



$I = \frac{400-100}{150} = \frac{300}{150} = 2A$ $E_T = U_{ab} = U_1 + I \cdot R_1$

$E_T = U_1 + I \cdot R_1 = 100 + 2 \cdot 100 = 300V$ $\boxed{E_T = 300\,V}$



$R_T = \frac{100 \cdot 5\Omega}{15\Omega} = \frac{500}{15} = \frac{100}{3}\Omega$ $\boxed{R_T = \frac{100}{3}\Omega}$

$\tau = R_T \cdot C$

$\tau = \frac{100}{3} \cdot 30 \cdot 10^{-6}$

$\tau = \frac{100}{3} \cdot 3 \cdot 10^{-5} = 100 \cdot 10^{-5}$

$\tau = 10^2 \cdot 10^{-5}$ $\boxed{\tau = 10^{-3}\,s}$

$E_T = u_c + i_c \cdot R_T = \frac{q_c}{C} + i_c \cdot R_T = \frac{q_c}{C} + R_T \cdot \frac{dq}{dt} = E_T$

$\frac{dq}{dt} \cdot R_t + \frac{q}{C} = E_T \,/\cdot C$ $\frac{dq}{dt}\tau + q = E_T \cdot C$

$\frac{dq}{dt}\tau = E_T \cdot C - q$ $\frac{dq}{E_T \cdot C - q} = \frac{dt}{\tau}$ $-\ln(E_T \cdot C - q) = \frac{t}{\tau} + c$ za $t=0$ s

$\Rightarrow q = U_1 \cdot C$

$-\ln(E_T \cdot C - U_1 \cdot C) = c$ $+\ln(E_T \cdot C - q) = -\frac{t}{\tau} + \ln(E_T \cdot C - U_1 \cdot C)$

$\ln \frac{E_T \cdot C - q}{E_T \cdot C - U_1 \cdot C} = -\frac{t}{\tau}$

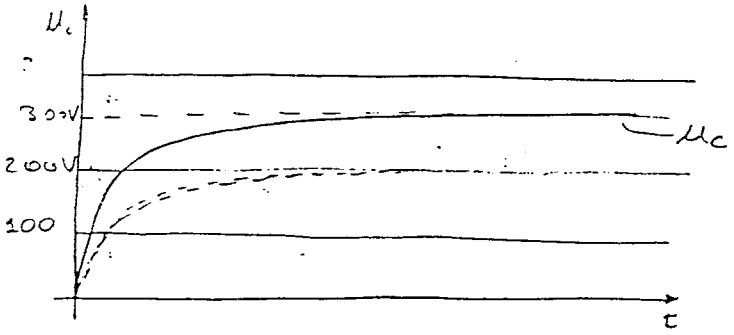
$c:\backslash \quad \frac{E_T \cdot C - q}{E_T \cdot C - U_1 \cdot C} = e^{-\frac{t}{\tau}}$ $\frac{E_T - u_c}{E_T - U_1} = e^{-\frac{t}{\tau}}$

$E_T - u_c = [E_T - U_1] e^{-t/\tau}$

$u_c = E_T - [E_T - U_1] e^{-\frac{t}{\tau}} = E_T - E_T \cdot e^{-\frac{t}{\tau}} + U_1 \cdot e^{-\frac{t}{\tau}} = E_T\left(1 - e^{-\frac{t}{\tau}}\right) + U_1 \cdot e^{-\frac{t}{\tau}}$

$u_c = 300\left(1 - e^{-\frac{t}{\tau}}\right) + 100 \cdot e^{-\frac{t}{\tau}} = 300 - 300\,e^{-\frac{t}{\tau}} + 100\,e^{-\frac{t}{\tau}}$

$\boxed{u_c = 300 - 200\,e^{-\frac{t}{\tau}}}$ $= 100 + 200 - 200\,e^{-\frac{t}{\tau}} = 100 + 200\left(1 - e^{-\frac{t}{\tau}}\right)$



$i_c = \frac{dq}{dt}$

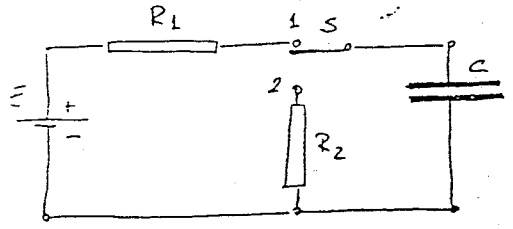
$i_c = \frac{6000}{1000} e^{-t/\tau}$

$i_c = 6 \cdot e^{-\frac{t}{\tau}}$

$\boxed{i_c = 6 \cdot e^{-\frac{t}{\tau}}}$

ZADATAK: U sklopu na slici sklopka S u ritmičnim vremenskim razmacima prebacuje iz položaja 1 u položaj 2. U svakom položaju ostaje 10 ms. Vrijeme prebacivanja je zanemarivo kratko. Zadano: $R_1 = 50\,\Omega$ $C = 1\,\mu F$ $R_2 = 2\,k\Omega$ $E = U = 100\,V$. Izračunajte napon na izlazu ako struja kroz otpornik R_2

9.31/2



za S u 1 (položaju 1)

$u_c = E\left(1 - e^{-\frac{t}{\tau}}\right)$ $\tau = R_1 \cdot C$

$\tau = 50 \cdot 10^{-6}$

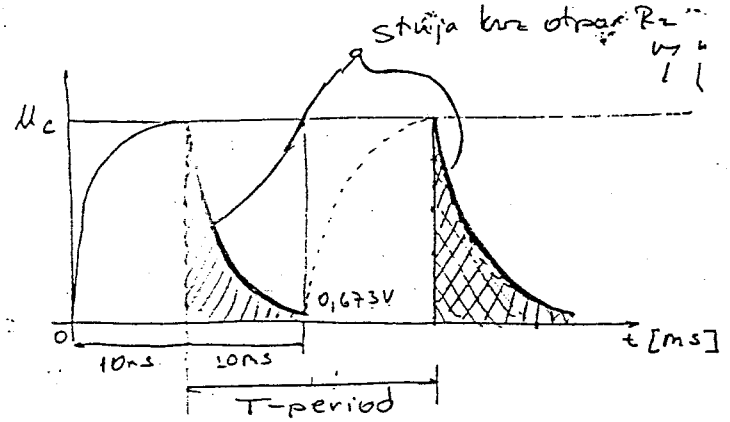
$u_c = 100\left[1 - e^{-\frac{t}{50\cdot 10^{-6}}}\right]$

$u_c = 100\left[1 - e^{-\frac{10^6 t}{50}}\right]$

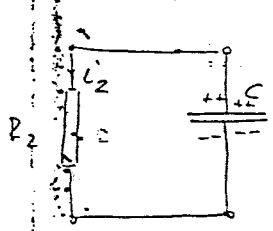
za $t = 10\,ms$ $u_c = 100\left[1 - e^{-\frac{10^6 \cdot 10^{-2}}{50}}\right]$

$t = 10 \cdot 10^{-3}\,s$

$\tau = 10^{-2}$ $u_c = 100\left[1 - e^{-\frac{10^4}{50}}\right] = 100\left[1 - 1{,}38 \cdot 10^{-87}\right] = 100\,V$



Kad nema t=10 ms sklopka u prebacuje u položaj 2. i tu ostaje 10 ms



$u_C + i_2 \cdot R_2 = 0$ $\frac{Q}{C} + i_2 \cdot R_2 = 0$ $\frac{Q}{C} + \frac{dQ}{dt} R_2 = 0 \;/\cdot C$ $\frac{dQ}{dt} \cdot R_2 \cdot C + Q = 0$

$\tau = R_2 \cdot C$

$\tau = 2 \cdot 10^3 \cdot 10^{-6}$

$\tau = 2 \cdot 10^{-3}$

$\frac{dQ}{dt} R_2 C = -Q$ $\frac{dQ}{Q} = -\frac{dt}{\tau} \;/\int$ $\ln Q = -\frac{1}{\tau} t + C$ za $t=0$ $Q = Q$

$c = \ln Q$

$\ln \frac{Q}{Q} = -\frac{t}{\tau}$ $\Rightarrow$ $\boxed{Q = Q \cdot e^{-\frac{t}{\tau}}}$ $\Rightarrow$ $u_c = \frac{Q}{C} = \frac{Q}{C} \cdot e^{-\frac{t}{\tau}} = 100 \cdot e^{-\frac{t}{\tau}}$

$u_c = u_{e_2} = 100 \cdot e^{-\frac{t}{2\cdot 10^{-3}}}$

$\boxed{u_c = u_{e_2} = 100 \cdot e^{-500t}\,V}$

$u_c = 100\, e^{-500 \cdot 10^{-2}} = 100 \cdot e^{-5}$ za $t = 10\,ms$

$u_c = 0{,}673\,V$

$i_2 = \frac{u_c}{R_2} \Rightarrow u_c = \frac{100\, e^{-500t}}{2000} = 0{,}05\, e^{-500t}$

$i_s = \frac{1}{2 \cdot 10^{-3}} \int_0^{10^{-2}} 0{,}05\, e^{-500t}\, dt$

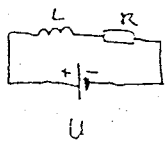
$i_s = \frac{1}{2 \cdot 10^{-2}} \int_0^{10^{-2}} 0{,}05\, e^{-500t}\, dt = \frac{0{,}05}{2 \cdot 10^{-2}} \int_0^{10^{-2}} e^{-500t}\, dt = 10^2 \cdot 0{,}05 \cdot \frac{1}{2 \cdot (-500)} \int_0^{10^{-2}} e^{-500t} \cdot d(-500t)$

$= 5 \cdot \frac{1}{-1000} e^{-500t} \Big|_0^{10^{-2}} = -\frac{0{,}01}{2}\left(e^{-5} - e^{0}\right) = -\frac{0{,}01}{2}\left(e^{-5} - 1\right) = \frac{0{,}01}{2}\left(1 - e^{-5}\right) = \frac{0{,}0099}{2}\,A \approx 5\,mA$

INDUKCJA ZWROTNA

9.1 $L = 0{,}1\ H$, $R = 10\ \Omega$, $U = 100\ V$, $I_x = 0{,}9\, I_0$

$\frac{di}{dt} = ?$



$i = I_{max}\left[1 - e^{-\frac{t}{\tau}}\right]$ $\quad I_{max} = \frac{U}{R} = 10\ A$ $\quad \tau = \frac{L}{R} = 0{,}01\ s$

$i = 0{,}9\, I_{max}$

$0{,}9 \cancel{I_{max}} = \cancel{I_{max}}\left[1 - e^{-\frac{t}{\tau}}\right]$

$-0{,}1 = -e^{-\frac{t}{\tau}}$

$-\frac{t}{\tau} = \ln 0{,}1$

$t = -\tau \ln 0{,}1$

$\boxed{t = 0{,}023\ s}$

$\left.\frac{di}{dt}\right|_{t=0} = \frac{d}{dt}\left[I_{max}\left(1 - e^{-\frac{t}{\tau}}\right)\right] = I_{max}\left[-e^{-\frac{t}{\tau}}\right]\left[-\frac{1}{\tau}\right]$

$\left.\frac{di}{dt}\right|_{t=0} = 1000\, e^{-0} = 1000\ \frac{A}{s}$ //

$\left.\frac{di}{dt}\right|_{t=0{,}023\,s} = 1000\, e^{-\frac{0{,}023}{0{,}01}} = 100{,}25\ \frac{A}{s}$ //

9.2. $t_1 = \tau$, $t_2 = 2\tau$, $\ldots$, $t_5 = 5\tau$

$\frac{i}{I_0} = ?$

$i = I_0\left[1 - e^{-\frac{t}{\tau}}\right]$

$\left.\frac{i}{I_0}\right|_{t_1} = 1 - e^{-1} = 0{,}63$

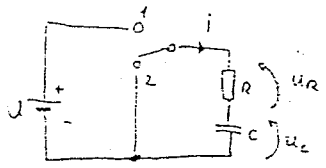
$\left.\frac{i}{I_0}\right|_{t_2} = 1 - e^{-2} = 0{,}864$

$\left.\frac{i}{I_0}\right|_{t_3} = 1 - e^{-3} = 0{,}95$

$\left.\frac{i}{I_0}\right|_{t_4} = 1 - e^{-4} = 0{,}981$

$\left.\frac{i}{I_0}\right|_{t_5} = 1 - e^{-5} = 0{,}993$

9.3. $[i, U_R, U_C, P_z, P_R, P_t = f(t)] = ?$



$i = \frac{dq}{dt} = \frac{d}{dt}\left[Q\left(1 - e^{-\frac{t}{\tau}}\right)\right] = Q\left[0 - e^{-\frac{t}{\tau}}\right]\left(-\frac{1}{\tau}\right)$

$i = \frac{Q}{\tau} e^{-\frac{t}{\tau}} = \frac{Q}{RC} e^{-\frac{t}{RC}}$

$\boxed{i = I\, e^{-\frac{t}{\tau}}}$

① $U = U_R + U_C$

$U = R\frac{dq}{dt} + \frac{q}{C}$

$U = \frac{dq}{dt}R + \frac{q}{C}$

$U C = \frac{dq}{dt} RC + q$

$Q = \tau \frac{dq}{dt} + q$

$Q - q = \tau \frac{dq}{dt}$

$\frac{dt}{\tau} = \frac{dq}{Q - q}$ //

$\frac{t}{\tau} = -\ln(Q - q) + \ln K \quad /(-1)$

$-\frac{t}{\tau} = \ln\frac{Q - q}{K}$

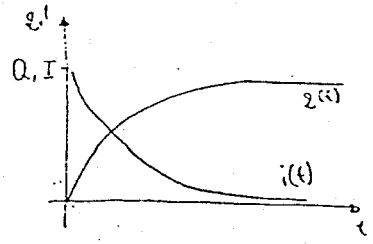
$Q - q = K e^{-\frac{t}{\tau}}$

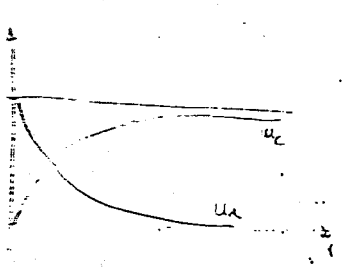
- ich dla $t = 0 \Rightarrow q = 0 : Q = K$

$q = Q\left[1 - e^{-\frac{t}{\tau}}\right]$

$\boxed{u_R = iR = \ldots e^{-\frac{t}{\tau}}}$

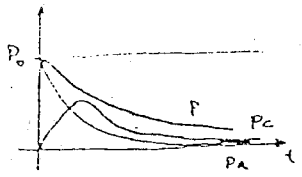
$\boxed{u_C = \frac{q}{C} = \ldots\left[1 - e^{-\frac{t}{\tau}}\right]}$





$u_R i = U e^{-\frac{t}{\tau}} \cdot I e^{-\frac{t}{\tau}} = P_0 e^{-\frac{t}{\tau}}$

$u_C i = U\left[1-e^{-\frac{t}{\tau}}\right] I e^{-\frac{t}{\tau}} = P_0 e^{-\frac{t}{\tau}}\left[1-e^{-\frac{t}{\tau}}\right]$



$P + P_C = P_0 e^{-2\frac{t}{\tau}} + P_0 e^{-\frac{t}{\tau}}\left[1-e^{-\frac{t}{\tau}}\right] = P_0 e^{-\frac{t}{\tau}}$

$W = \int_0^\infty P_C\,dt = P_0 \int_0^\infty e^{-\frac{t}{\tau}}\left[1-e^{-\frac{t}{\tau}}\right]dt = UI\left[\frac{e^{-\frac{t}{\tau}}}{-\frac{1}{\tau}} - \frac{e^{-\frac{2t}{\tau}}}{-\frac{2}{\tau}}\right]_0^\infty$

$W = UI\tau\left[-e^{-\frac{t}{\tau}} + \frac{1}{2}e^{-\frac{2t}{\tau}}\right]_0^\infty = -UIRC\left[-1+\frac{1}{2}\right] = \frac{U^2C}{2}$

$0 = u_R + u_C$

$0 = iR + \frac{q}{C}$

$0 = \frac{dq}{dt}R + \frac{q}{C} \quad /\cdot C$

$0 = RC\frac{dq}{dt} + q$

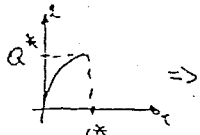
$-q = dq\frac{\tau}{dt}$

$\frac{dt}{\tau} = -\frac{dq}{q}$

$\frac{t}{\tau} = -\ln q + \ln K$

$-\frac{t}{\tau} = \ln\frac{q}{K}$

$q = Ke^{-\frac{t}{\tau}}$



$\Rightarrow Q^* = K$

$\boxed{q = Q^* e^{-\frac{t}{\tau}}}$

$i = \frac{dq}{dt} = \frac{d}{dt}\left[Q^* e^{-\frac{t}{\tau}}\right] = Q^* e^{-\frac{t}{\tau}}\left[-\frac{1}{\tau}\right] = -\frac{Q^*}{\tau}e^{-\frac{t}{\tau}}$

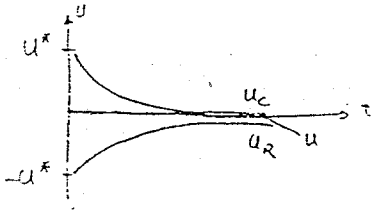
$i = -\frac{Q^*}{RC}e^{-\frac{t}{\tau}} = -\frac{U^*C}{R\tau}e^{-\frac{t}{\tau}} = -I^* e^{-\frac{t}{\tau}}$

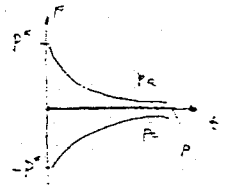
$\boxed{i = -I^* e^{-\frac{t}{\tau}}}$

$u_R + u_C = 0$

$u_R = iR = -U^* e^{-\frac{t}{\tau}}$

$u_C = \frac{q}{C} = \frac{Q^*}{C}e^{-\frac{t}{\tau}} = \frac{CU^*}{C}e^{-\frac{t}{\tau}} = U^* e^{-\frac{t}{\tau}}$





$P_R = u_R i = U^* I^* e^{-2\frac{t}{\tau}} = P^* e^{-2\frac{t}{\tau}}$

$P_C = u_C i = -U^* I^* e^{-2\frac{t}{\tau}} = -P^* e^{-2\frac{t}{\tau}}$

7/ $R = 50\,\Omega$

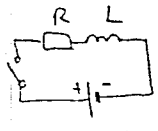
$L = 10\,H$

$t_0 = 0$

$U = 100\,V$

$u_R, i = ?$

$P = ?$



$U = U_R + U_L$

$U = iR + L\frac{di}{dt} \quad /:R$

$I = i + \tau\frac{di}{dt}$

$\tau\frac{di}{dt} = I - i$

$\frac{dt}{\tau} = \frac{di}{I-i} \quad /\int$

$\frac{t}{\tau} = -\ln(I-i) + \ln K$

$-\frac{t}{\tau} = \ln\frac{I-i}{K}$

$\frac{I-i}{K} = e^{-\frac{t}{\tau}}$

$I - i = Ke^{-\frac{t}{\tau}}$

dla $t=0 \Rightarrow i=0 : I=K$

$\boxed{i = I\left[1-e^{-\frac{t}{\tau}}\right]}$

$i = 2\left[1-e^{-5t}\right]$

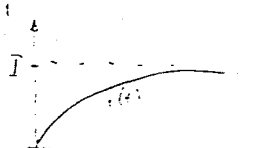
$u_R = iR = 100\left[1-e^{-5t}\right]$

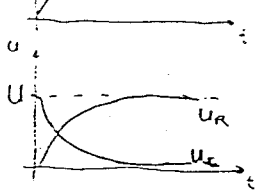
$u_L = L\frac{di}{dt} = L\frac{d}{dt}\left[I(1-e^{-\frac{t}{\tau}})\right] = LI\left(0-e^{-\frac{t}{\tau}}\right)\left(-\frac{1}{\tau}\right) = \frac{L}{\tau}I e^{-\frac{t}{\tau}} = 100e^{-5t}$

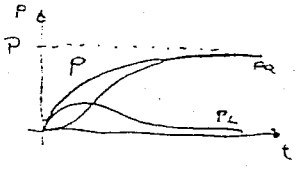
$P_R = iU_R = 200\left[1-2e^{-6t}+e^{-10t}\right]$

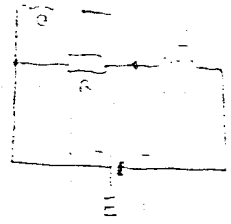
$P_L = iU_L = 200\left[e^{-5t}-e^{-10t}\right]$

$P = Ui = P_R + P_L = 200\left[1-e^{-5t}\right]$









$I = \frac{E}{R} = 2\frac{E}{R} = 2I_1$

$i = I\left[1 - e^{-\frac{t}{\tau}}\right]$

$\frac{1}{2}\frac{dt}{\tau} = \frac{di}{2I - i}$

$\frac{1}{2}\frac{t}{\tau} = -\ln(2I - i) + \ln K$

$-\frac{1}{2}\frac{t}{\tau} = \ln\frac{2I - i}{K}$

$K = C$

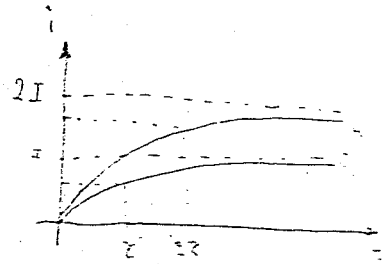
$2I - i = K e^{-\frac{t}{2\tau}}$ — struja i kad se prebaci u ②, a struja je bila =

$t=0 \Rightarrow i = \frac{E}{R} : K = I$

$2I - i = I e^{-\frac{t}{2\tau}}$

$i = 2I - I e^{-\frac{t}{2\tau}}$

$i = I\left[2 - e^{-\frac{t}{2\tau}}\right]$ //



9.6. $E = 120\,V$

$R_1 = 20\,\Omega$

$R_2 = 50\,\Omega$

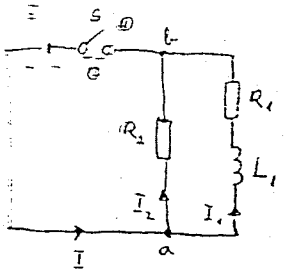
$L_1 = 1\,H$

$t_1 = 0$

$t = \epsilon\,\Omega$

$W_L = ?$

$U_{Ro} = ?$



① $I = \frac{E}{R_1 R_2}(R_1 + R_2) = \frac{120}{60\Omega} \cdot 5\Omega = 10A$

$I_1 = \frac{E}{R_1} = 6A$

$I_2 = \frac{E}{R_2} = 4A$

$\tau = \frac{L}{R_1 + R_L} = \frac{1}{50} = 0{,}02\,s$

$U_R + U_L = 0$

$i[R_1 + R_2] + L\frac{di}{dt} = 0$

$i + \frac{L}{R_1 + R_2}\frac{di}{dt} = 0$

$i + \tau\frac{di}{dt} = 0$

$\frac{dt}{\tau} = -\frac{di}{i}$

$\frac{t}{\tau} = -\ln i + \ln K$

$-\frac{t}{\tau} = \ln\frac{i}{K}$

$\frac{i}{K} = e^{-\frac{t}{\tau}}$

$i = K e^{-\frac{t}{\tau}}$

- u trenutku $t=0 \Rightarrow i = 6A : K = 6$

$\boxed{i = 6e^{-\frac{t}{\tau}} = 6e^{-50\,t}}$

$u_L = L\frac{di}{dt} = L\frac{d}{dt}\left[6e^{-50t}\right]$

$u_L = L \cdot 6 \cdot e^{-50t}\,[-50]$

$u_L = -300\,e^{-50t}$

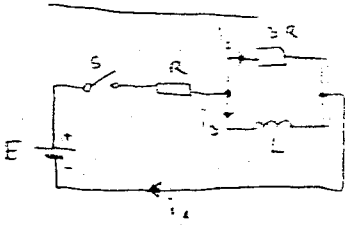
$p_L = u_L i_L = -300e^{-50t} \cdot 6e^{-50t}$

$p_L = -1800\,e^{-100t}$

$W = \int_0^t p_L\,dt = -1800\int_0^t e^{-100t}\,dt = \frac{1800}{100}e^{-100t}\Big|_0^{\infty} = 18\,e^{-4} = 0{,}329\,Ws$ //

$u_L(t=0) = -300\,\epsilon^0 = -300\,V$ //

9.7. $i_1, i_2, i_3(t) = ?$



$i_1 = i_2 + i_3 \qquad U_L = i_2 \cdot 3R$

$i_1 = \frac{E - U_L}{R} \qquad U_L = (i_1 - i_3)3R$

$U_L = \left[\frac{E - U_L}{R} - i_3\right]3R$

$U_L = 3E - 3U_L - i_3 3R$

$4U_L = 3E - i_3 3R$

$4\frac{di_3}{dt} = 3E - i_3 3R \;/:3R$

$\frac{4}{3}\frac{di_3}{dt} = \frac{E}{R} - i_3$

$\frac{di_3}{dt} = \frac{E}{R} - i_3$

$\frac{di_3}{\frac{E}{R} - i_3} = \frac{dt}{\tau}$

$\frac{1}{\frac{E}{R} - i_3}\,di_3 = \frac{1}{\tau}\,dt$

$-\ln\left(\frac{E}{R} - i_3\right) = \frac{t}{\tau} + \ln K$

$\ln\frac{\frac{E}{R} - i_3}{K} = -\frac{t}{\tau}$

$\frac{\frac{E}{R} - i_3}{K} = e^{-\frac{t}{\tau}}$

$\frac{E}{R} - i_3 = Ke^{-\frac{t}{\tau}}$

$t=0 \Rightarrow i_3 = 0 : K = \frac{E}{R}$

$\boxed{i_3 = \frac{E}{R}\left[1 - e^{-\frac{t}{\tau}}\right]}$

$\tau = \frac{4L}{3R}$

$U_L = i_2 3R$

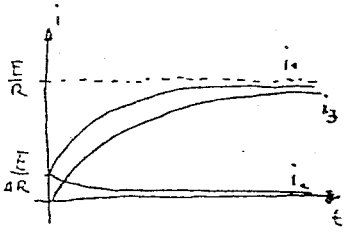
$L\frac{di_2}{dt} = i_2 3R$

$i_2 = \frac{L}{3R} \cdot \frac{E}{R}\left(-e^{-\frac{t}{\tau}}\right) \cdot \left(-\frac{1}{\tau}\right)$

$i_2 = \frac{LE}{3R^2\tau}e^{-\frac{t}{\tau}}$

$i_2 = \frac{\cancel{L}E}{\cancel{3}R^2} \cdot \frac{\cancel{3}R}{4\cancel{L}}e^{-\frac{t}{\tau}}$

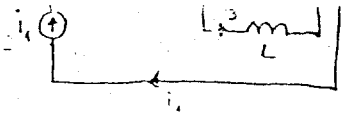
$\boxed{i_2 = \frac{1}{4}\frac{E}{R}e^{-\frac{t}{\tau}}}$



$i_1 = i_2 + i_3$

$i_1 = \frac{E}{R}\left[\frac{1}{4}e^{-\frac{t}{\tau}} + 1 - e^{-\frac{t}{\tau}}\right]$

$\boxed{i_1 = \frac{E}{R}\left[1 - \frac{3}{4}e^{-\frac{t}{\tau}}\right]}$



$= i_2 + i_3$

$= i_2 + I_0\left[1 - e^{-\frac{t}{\tau}}\right]$

$= I_0 e^{-\frac{t}{\tau}}$

$\tau = \frac{L}{3R}$

$I_0 = \frac{U_L}{3R} + i_3$

$I_0 = \frac{L}{3R}\frac{di_3}{dt} + i_3$

$I_0 = \tau \frac{di_3}{dt} + i_3$

$\tau \frac{di_3}{dt} = I_0 - i_3$

$\frac{\tau}{dt} = \frac{I_0 - i_3}{di_3}$

$\frac{\tau}{\tau} = \frac{-}{I_0 - i_3}$

$\frac{t}{\tau} = -\ln\left[I_0 - i_3\right] + \ln K$

$-\frac{t}{\tau} = \ln\frac{I_0 - i_3}{K}$

$I_0 - i_3 = K e^{-\frac{t}{\tau}}$

$t=0 \Rightarrow i_3 = 0 \Rightarrow K = I_0$

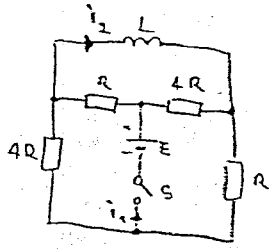
$i_3 = I_0\left[1 - e^{-\frac{t}{\tau}}\right]$

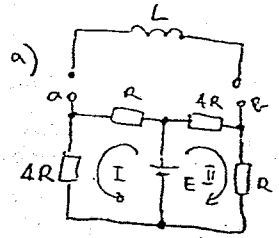
$= I_0 R + i_2 \cdot 3R$

$= I_0 R + 3R I_0 e^{-\frac{t}{\tau}}$

$= I_0 R\left[1 + 3e^{-\frac{t}{\tau}}\right]$

$i_1, i_2 = ?$



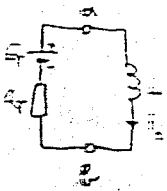


$R_T = 2 \cdot \frac{4R}{5} = \frac{8}{5}R$

$I_{(1)} = \frac{E}{5R}$ $I_{(2)} = \frac{E}{5R}$

$\varphi_a + I_{(2)}R - I_{(1)}4R = \varphi_b$

$E_T = E\left[\frac{4}{5} - \frac{1}{5}\right] = \frac{3}{5}E$



$i_2 R_T + L\frac{di_2}{dt}$

$= i_2 \cdot \frac{8}{5}R + L\frac{di_2}{dt} \quad /\cdot\frac{5}{8R}$

$= i_2 + \frac{5L}{8R}\frac{di_2}{dt}$

$= i_2 + \tau\frac{di_2}{dt}$

$\frac{di_2}{dt} = \frac{\frac{3E}{8R} - i_2}{\tau}$

$\frac{\tau}{dt} = \frac{\frac{3E}{8R} - i_2}{di_2}$

$\frac{dt}{\tau} = \frac{di_2}{\frac{3E}{8R} - i_2} \quad /\int$

$\frac{t}{\tau} = -\ln\left[\frac{3E}{8R} - i_2\right] + \ln K$

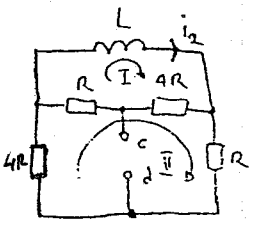
$\frac{3E}{8R} - i_2 = K e^{-\frac{t}{\tau}}$

$t=0 \Rightarrow i_2 = 0 : K = \frac{3E}{8R}$

$i_2 = \frac{3E}{8R}\left[1 - e^{-\frac{t}{\tau}}\right]$

$U_L = L\frac{di_2}{dt} = L\frac{d}{dt}\left[\frac{3E}{8R}\left(1 - e^{-\frac{t}{\tau}}\right)\right] = \frac{3LE}{8R}\left(-e^{-\frac{t}{\tau}}\right)\cdot\left(-\frac{1}{\tau}\right)$

$U_L = \frac{3LE}{8R}\cdot\frac{8R}{5L}e^{-\frac{t}{\tau}} = \frac{3}{5}Ee^{-\frac{t}{\tau}}$



$R_T = 2 \cdot \frac{4R^2}{5R} = \frac{8}{5}R$

$\varphi_c + i_{(2)}R - i_{(1)}R + i_{(2)}4R = \varphi_d$

$U_{cd} = i_{(2)}R - i_{(1)}3R$

$E_T = \frac{8}{25}Ee^{-\frac{t}{\tau}} - \frac{3}{25}Ee^{-\frac{t}{\tau}}$

$E_T = \frac{5}{25}Ee^{-\frac{t}{\tau}}$

$I_{(1)}5R - I_{(2)}5R = U_L$

$-I_1 25 + I_{(2)}10R = 0 \quad /\cdot\frac{1}{5}$

$(I_{(1)} - I_{(2)})5R = U_L$

$(I_1 - I_2)\frac{U_L}{5R} = \frac{L}{5R}\frac{di_2}{dt}$

$\frac{L}{5R}\frac{d}{dt}\left[\frac{3E}{8R}\left(1 - e^{-\frac{t}{\tau}}\right)\right]$

$= \frac{L}{5R}\cdot\frac{3E}{8R}\left(-e^{-\frac{t}{\tau}}\right)\left(-\frac{1}{\tau}\right)$

$= \frac{3LE}{40R^2}\cdot\frac{8R}{5L}e^{-\frac{t}{\tau}}$

$i_{(2)} = \frac{3}{25}\frac{E}{R}e^{-\frac{t}{\tau}}$

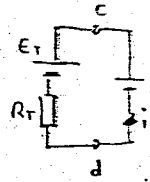
$i_{(1)}5R = i_{(2)}(-)U_L$

$i_{(1)}5R = \frac{3}{5}Ee^{-\frac{t}{\tau}} + \frac{3}{25}\frac{E}{R}e^{-\frac{t}{\tau}} \cdot \frac{5}{R}$

$i_{(1)}5R = \frac{3}{5}Ee^{-\frac{t}{\tau}} + \frac{3}{5}Ee^{-\frac{t}{\tau}}$

$i_{(1)}5R = \frac{6}{5}Ee^{-\frac{t}{\tau}}$

$i_{(1)} = \frac{6}{25}\frac{E}{R}e^{-\frac{t}{\tau}}$

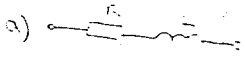


$i_1 = \frac{E - E_T}{R_T} = \frac{E + \frac{3}{25}Ee^{-\frac{t}{\tau}}}{\frac{8}{5}R} = \frac{5}{8}\frac{E}{R}\left[1 + \frac{3}{25}e^{-\frac{t}{\tau}}\right]$

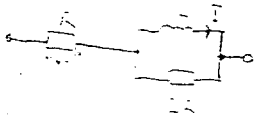
$i_1 = \frac{E}{R}\left[0{,}625 + 0{,}225\,e^{-\frac{t}{\tau}}\right]$

$\tau_1 = T_1 = T_2 = \frac{L}{R}$ [s]

$[\tau_1] =$

a) 

$u_L = L \frac{di}{dt}$

b) 

c) 

9.11. $C = 2\ \mu F$

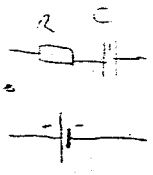
$R = 1000\ \Omega$

$t_1 = 0,002\ s$

$U_1 = 200\ V$

$U_2 = 300\ V$

$t_2 = ?$



$E = u_R + u_c$

$E = iR + \frac{q}{C}$

$E = \frac{dq}{dt} R + \frac{q}{C} \quad / \cdot C$

$EC = \tau \frac{dq}{dt} + q$

$\tau \frac{dq}{dt} = EC - q$

$\frac{dt}{\tau} = \frac{dq}{EC - q}$

$\frac{t}{\tau} = -\ln [EC - q] + \ln K$

$-\frac{t}{\tau} = \ln \frac{EC - q}{K}$

$EC - q = K e^{-\frac{t}{\tau}}$

$t = 0 \Rightarrow q = 0 : \quad K = EC = Q$

$Q - q = Q e^{-\frac{t}{\tau}}$

$\boxed{q = Q \left[1 - e^{-\frac{t}{\tau}}\right]}$

$i = \frac{dq}{dt} = \frac{d}{dt} \left[Q \left(1 - e^{-\frac{t}{\tau}}\right) \right] = \frac{U}{R} e^{-\frac{t}{\tau}}$

$i = \frac{q}{RC} e^{-\frac{t}{\tau}} = \frac{U}{R} e^{-\frac{t}{\tau}}$

$\boxed{\tau = RC}$

$u_c = \frac{1}{C} \int_0^t i\, dt = \int_0^t \frac{U}{R} e^{-\frac{t}{\tau}} dt = \frac{U}{CR} \int_0^t e^{-\frac{t}{\tau}} dt = \frac{U}{CR} (-\tau) \left| e^{-\frac{t}{\tau}} \right|_0^t = -U \left| e^{-\frac{t}{\tau}} \right|_0^t = -U \left[e^{-\frac{t}{\tau}} - 1 \right]$

$200 = -U \left[e^{-\frac{0,002}{0,002}} - 1 \right] \Rightarrow U = 316,38\ V$

$e^{-\frac{t}{\tau}} = -\frac{u_c}{U} + 1$

$-\frac{t}{\tau} = \ln \left[-\frac{u_c}{U} + 1 \right]$

$\boxed{t = 0,00312\ s}$

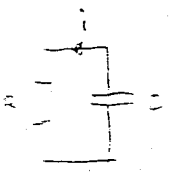
9.12. $U_1 = 400\ V$

$U_2 = 200\ V$

$C = 2\ \mu F$

$R = 1\ k\Omega$

$\Delta t = ?$



$u_c - u_R = 0 \Rightarrow \frac{t}{\tau} = -\ln q + \ln K$

$u_c = -iR$

$\frac{q}{C} = -\frac{dq}{dt} R$

$q = -\frac{dq}{dt} \tau$

$\frac{dt}{\tau} = -\frac{dq}{q}$

$-\frac{t}{\tau} = \ln \frac{q}{K}$

$q = K e^{-\frac{t}{\tau}}$

$t = 0 \Rightarrow q = CU : \quad K = CU$

$q = CU e^{-\frac{t}{\tau}} \quad / : C$

$\boxed{u_c = U^* e^{-\frac{t}{\tau}}}$

$400 = U^* e^{-\frac{t_1}{0,002}}$

$-\frac{t_1}{0,002} = \ln \frac{400}{U^*}$

$t_1 = -0,002 \ln \frac{400}{U^*}$

$t_2 = -0,002 \ln \frac{200}{U^*}$

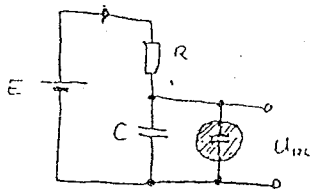
$\Delta t = t_2 - t_1 = -0,002 \left[\ln \frac{200}{U^*} - \ln \frac{400}{U^*} \right]$

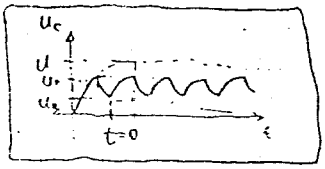
$\Delta t = -0,002 \ln \frac{\frac{200}{U^*}}{\frac{400}{U^*}}$

$\Delta t = -0,002 \ln \frac{1}{2} \Rightarrow \boxed{\Delta t = 1,386\ ms}$

$U_p = 80\,V$
$U_g = 40\,V$
$R = 2\,k\Omega$
$C = 0{,}5\,\mu F$

$f = ?$





$C = IK + \frac{Q}{C}$

$E = \frac{dQ}{dt} R + \frac{Q}{C} \quad /\cdot C$

$EC = \frac{dQ}{dt} RC + Q$

$\frac{dQ}{dt} RC = EC - Q$

$\frac{dt}{RC} = \frac{dQ}{EC - Q} \quad /\int$

$\frac{t}{RC} = -\ln[EC - Q] + \ln K$

$\ln \frac{EC - Q}{K} = -\frac{t}{RC}$

$EC - Q = K e^{-\frac{t}{RC}}$

- kond. la $t=0 \Rightarrow Q = U_g C \;:\; K = (U - U_g)C$

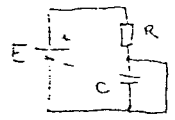
$UC - Q = (UC - U_g C) e^{-\frac{t}{RC}}$

$Q = UC - [UC - U_g C] e^{-\frac{t}{RC}}$

$\boxed{Q = Q - [UC - U_g C] e^{-\frac{t}{RC}}}$

$I_c = \frac{Q}{C} = U - [U - U_g] e^{-\frac{t}{RC}}$

$I_p = U - [U - U_g] e^{-\frac{t_1}{RC}} \Rightarrow t_1 = -RC \ln \frac{U - U_p}{U - U_g} = 0{,}69\,ms$

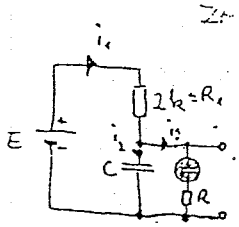


- kond. la inițierea treimii, t_2. $t_2 = 0$

$f = \frac{1}{t_1 + t_2} = 1442\,Hz$

$R = 200\,\Omega$

$f = ?$



$i_1 = i_2 + i_3$

$i_3 R = \frac{Q}{C}$

$(i_1 - i_2) R = \frac{Q}{C}$

$\left[\frac{E - U_c}{R_1} - i_2\right] R = \frac{Q}{C}$

$-U_c \frac{R}{R_1} - i_2 R = U_c$

$\left(1 + \frac{R}{R_1}\right) U_c + i_2 R = -E \frac{R}{R_1}$

$\left(1 + \frac{R}{R_1}\right) U_c + i_2 R = E \frac{R}{R_1}$

$\frac{R}{R_1} \frac{Q_2}{C} + \frac{dQ_2}{dt} R = U \frac{R}{R_1}$

$\frac{Q_2}{C} + R_1 \frac{dQ_2}{dt} = 0{,}1\,U$

$\frac{R_1 C}{1{,}1} \frac{dQ_2}{dt} = \frac{0{,}1}{1{,}1} UC$

$\frac{dQ_2}{dt} = \frac{0{,}1}{1{,}1} UC - Q_2$

$\frac{dt}{RC} = \frac{dQ_2}{\frac{0{,}1}{1{,}1} UC - Q_2} \quad /\int$

$\frac{t}{RC} = -\ln\left[\frac{0{,}1}{1{,}1} UC - Q_2\right] + \ln K$

$\frac{0{,}1}{1{,}1} UC - Q_2 = K e^{-\frac{t}{RC}}$

- kond. la $t=0 \Rightarrow Q_2 = U_p C$:

$K = \frac{0{,}1}{1{,}1} U - U_p C$

$\boxed{Q_2 = \frac{0{,}1}{1{,}1} UC - \left[\frac{0{,}1}{1{,}1} UC - U_p C\right] e^{-\frac{t}{RC}}}$

$\boxed{U_c = \frac{Q_2}{C} = \frac{0{,}1}{1{,}1} U - \left[\frac{0{,}1}{1{,}1} U - U_p\right] e^{-\frac{t}{RC}}}$

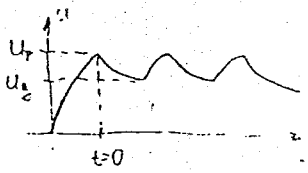
$U_g = \frac{0{,}1}{1{,}1} U - \left[\frac{0{,}1}{1{,}1} U - U_p\right] e^{-\frac{t}{RC}}$

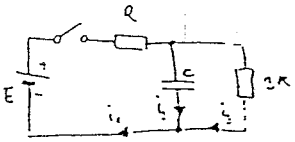
$\Downarrow$

$t = -RC \ln \frac{-\frac{0{,}1}{1{,}1} U + U_g}{-\frac{0{,}1}{1{,}1} U + U_p}$

$\boxed{t = 786\,\mu s}$

$\boxed{f = 1271\,Hz}$





$i_1 = i_2 + i_3$

$(i_1 - i_2) 3R$

$\left[\frac{E - U_c}{R} - i_2\right] 3R$

$\left[\frac{E - U_c}{R} - \frac{dQ_2}{dt}\right] 3R$

$3E - 3U_c - 3R \frac{dQ_2}{dt}$

$3E - 3R \frac{dQ_2}{dt} \quad / \frac{C}{4}$

$\frac{3}{4} CE - \frac{3}{4} RC \frac{dQ_2}{dt}$

$\varepsilon = -\frac{3}{4} CE = -\tau \frac{dQ_2}{dt}$

$\frac{dt}{\tau} = \frac{dQ_2}{\frac{3}{4} CE - Q_2} \quad /\int$

$\frac{t}{\tau} = -\ln\left[\frac{3}{4} CE - Q_2\right] + \ln K$

$\frac{3}{4} CE - Q_2 = K e^{-\frac{t}{\tau}}$

- kond. la $t=0 \Rightarrow Q_2 = 0$:

$K = \frac{3}{4} CE$

$\boxed{Q_2 = \frac{3}{4} CE \left[1 - e^{-\frac{t}{\tau}}\right]}$

$i_2 = \frac{dQ_2}{dt} = \frac{3}{4} CE \left[-e^{-\frac{t}{\tau}}\right] \left[-\frac{1}{\tau}\right]$

$\boxed{i_2 = \frac{E}{R} e^{-\frac{t}{\tau}}}$

$\boxed{i_3 = \frac{U_c}{3R} = \frac{Q_2}{3RC} = \frac{E}{4R} \left[1 - e^{-\frac{t}{\tau}}\right]}$

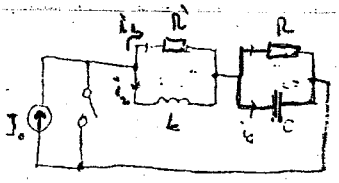
$i_1 = i_2 + i_3 = \frac{E}{R} \left[e^{-\frac{t}{\tau}} + \frac{1}{4} - \frac{1}{4} e^{-\frac{t}{\tau}}\right]$

$\boxed{i_1 = \frac{E}{R} \left[\frac{1}{4} + \frac{3}{4} e^{-\frac{t}{\tau}}\right]}$

9.16)

$C = C'$

$U = ?$



$U = U_1 + U_2$

$U_1 = i_1 R$

$L \frac{di_2}{dt} = i_1 R$

$i_1 + i_2 = I_0$

$L \frac{di_2}{dt} = [I_0 - i_2] R$

$L \frac{di_2}{dt} = I_0 R - i_2 R \quad / : R$

$\frac{L}{R} \frac{di_2}{dt} = I_0 - i_2$

$\frac{dt}{\tau} = \frac{di_2}{I_0 - i_2} \quad / \int$

$\frac{t}{\tau} = -\ln [I_0 - i_2] + \ln K$

$I_0 - i_2 = K e^{\frac{t}{\tau}}$

- kad je $t = 0 \Rightarrow i_2 = 0$:

$K = I_0$

$i_2 = I_0 \left[1 - e^{-\frac{t}{\tau}}\right]$

$U_1 = L \frac{di_2}{dt} = L \frac{d}{dt} \left[I_0 (1 - e^{-\frac{t}{\tau}})\right]$

$U_1 = L I_0 \left[-e^{-\frac{t}{\tau}}\right] \left[-\frac{1}{\tau}\right]$

$\boxed{U_1 = I_0 R e^{-\frac{t}{\tau}}}$

$I_0 = i_3 + i_4$

$U_2 = i_3 R$

$\frac{q_4}{C} = i_3 R$

$\frac{q_4}{C} = [I_0 - i_4] R$

$\frac{q_4}{C} = I_0 R - \frac{dq_4}{dt} R$

$q_4 = I_0 RC - \frac{dq_4}{dt} RC$

$\frac{dq_4}{dt} \tau = I_0 RC - q_4$

$\frac{dt}{\tau} = \frac{dq_4}{I_0 RC - q_4}$

$\frac{t}{\tau} = -\ln [I_0 RC - q_4] + \ln K$

$I_0 RC - q_4 = K e^{-\frac{t}{\tau}}$

- kad je $t > 0 \Rightarrow q_4 = 0$:

$K = I_0 RC$

$q_4 = I_0 RC \left(1 - e^{-\frac{t}{\tau}}\right)$

$U_2 = \frac{q_4}{C}$

$\boxed{U_2 = I_0 R \left[1 - e^{-\frac{t}{\tau}}\right]}$

$U = U_1 + U_2$

$U = I_0 R e^{-\frac{t}{\tau}} + I_0 R - I_0 R e^{-\frac{t}{\tau}}$

$\boxed{U = I_0 R}$

9.17. $t = 0 \Rightarrow ①$

$t' = \tau \Rightarrow ②$

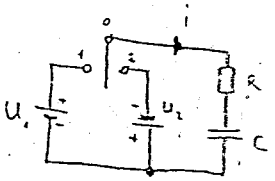
$R = 500 \Omega$

$C = 0{,}5 \mu F$

$U_1 = 20 V$

$U_2 = 40 V$

$i(t) = ?$



① $U_1 = U_R + U_C$

$U_1 = iR = \frac{q_1}{C}$

$U_1 = \frac{dq_1}{dt} R + \frac{q_1}{C}$

$U_1 C = RC \frac{dq_1}{dt} + q_1$

$U_1 C - q_1 = \tau \frac{dq_1}{dt}$

$\frac{dt}{\tau} = \frac{dq_1}{U_1 C - q_1}$

$\frac{t}{\tau} = -\ln [U_1 C - q_1] + \ln K$

$U_1 C - q_1 = K e^{-\frac{t}{\tau}}$

- kad je $t = 0 \Rightarrow q_1 = 0$: $K = U_1 C$

$q_1 = U_1 C \left[1 - e^{-\frac{t}{\tau}}\right]$

$\boxed{i_1 = \frac{U_1}{R} e^{-\frac{t}{\tau}}}$

② $U_2 = U_R - U_C$

$U_2 = iR - \frac{q_2}{C}$

$U_2 C = \frac{dq_2}{dt} RC - q_2$

$\frac{dt}{\tau} = \frac{dq_2}{U_2 C + q_2}$

$\frac{t}{\tau} = -\ln [U_2 C + q_2] + \ln K$

?

$U_2 C + q_2 = K e^{-\frac{t}{\tau}}$

- kad je $t = 0 \Rightarrow q_2 = U_1 C \left[1 - e^{-\frac{t}{\tau}}\right] = -q_1$

$K = U_2 C - U_1 C \left[1 - e^{-\frac{t}{\tau}}\right]$

$q_2 = -U_2 C + \left[U_1 C - U_1 C \left(1 - \frac{1}{e}\right)\right] e^{-\frac{t}{\tau}}$

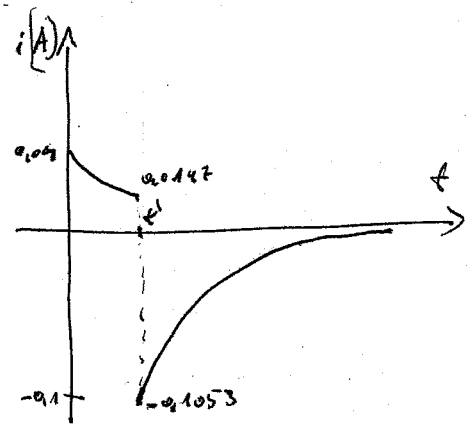
$i_2 = \frac{dq_2}{dt} = \frac{d}{dt} \left[-U_2 C + U_1 C e^{-\frac{t}{\tau}} - U_1 C e^{-\frac{t}{\tau}} \left(1 - \frac{1}{e}\right)\right]$

$i_2 = 0 + U_1 C e^{-\frac{t}{\tau}} \left[-\frac{1}{\tau}\right] + U_1 C e^{-\frac{t}{\tau}} \frac{1}{\tau} \left[0 - e^{-1}\right]$

$i_2 = -\frac{U_2}{R} e^{-\frac{t}{\tau}} - \frac{U_1}{R} e^{-\frac{t}{\tau} - 1}$

$i_2 = -\frac{e^{-\frac{t}{\tau}}}{R} \left[U_2 - \frac{U_1}{e}\right]$

ZADATAK NIJE ISTI KAO i ONAJ PRAVILNI NEGO ZA 6. PONAVLJAN



TRAŽA POSTAVITI OVAKO

$-U_2 = iR + U_C$

$t' = 500\,ms \Rightarrow$ ③

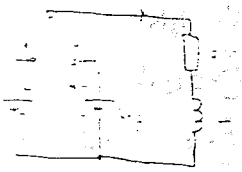
$U_1 = 100\,V$

$U_2 = 50\,V$

$R = 10\,\Omega$

$L = 0{,}1\,H$

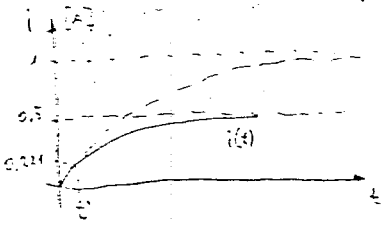
$i(t) = ?$



$I_1 = \frac{U_1}{R} = 10\,A$

$I_2 = \frac{U_2}{R} = 5\,A$

$I_2 = \frac{U_2}{R} = 5\,A$



$U_1 = U_R + U_L$

$U_1 = i_1 R + L\frac{di_1}{dt}$

$\frac{U_1}{R} = i_1 + \frac{L}{R}\frac{di_1}{dt}$

$\tau\frac{di_1}{dt} = \frac{U_1}{R} - i_1$

$\frac{dt}{\tau} = \frac{di_1}{\frac{U_1}{R} - i_1}$

$\frac{t}{\tau} = -\ln\left[I_1 - i_1\right] + \ln K$

$I_1 - i_1 = K e^{-\frac{t}{\tau}}$

$t=0 \Rightarrow i=0 \Rightarrow K = I_1$

$$\boxed{i_1 = I_1\left[1 - e^{-\frac{t}{\tau}}\right]}$$

$U_2 = U_R + U_L$

$U_2 = i_2 R + L\frac{di_2}{dt}$

$I_2 = i_2 + \frac{L}{R}\frac{di_2}{dt}$

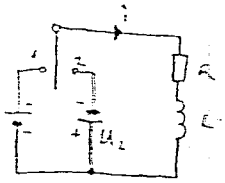
$\frac{t'}{\tau} = -\ln\left[I_2 - i_2\right] + \ln K$

$I_2 - i_2 = K e^{-\frac{t'}{\tau}}$

$t=0 \Rightarrow i_2 = 0{,}221\,A \Rightarrow K = 0{,}278\,A$

$i_2 = I_2 - K e^{-\frac{t'}{\tau}}$

$$\boxed{i_2 = I_2 - 0{,}278\, e^{-\frac{t'}{\tau}}}$$



① $i_1 = I_1\left[1 - e^{-\frac{t}{\tau}}\right]$

② $U_2 = U_R - U_L$

$U_2 = -i_2 R - L\frac{di_2}{dt}$

$I_2 = -i_2 - \tau\frac{di_2}{dt}$

$\frac{\tau}{dt} = \frac{-i_2 - I_2}{di_2}$

$\frac{dt}{\tau} = \frac{di_2}{-i_2 - I_2}$

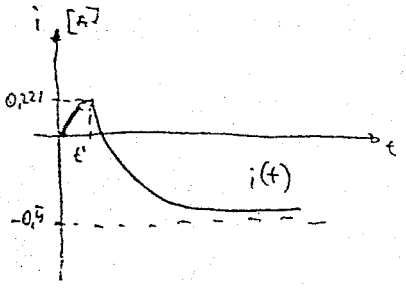
$\frac{t'}{\tau} = -\ln\left[-i_2 - I_2\right] + \ln K$

$-I_2 - i_2 = K e^{-\frac{t'}{\tau}}$

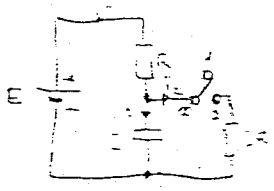
$t=0 \Rightarrow i_2 = 0{,}221\,A \Rightarrow K = -0{,}72\,A$

$i_2 = -I_2 - K e^{-\frac{t'}{\tau}}$

$$\boxed{i_2 = -0{,}5 + 0{,}72\, e^{-\frac{t'}{\tau}}}$$



20. $i_c, U_c = ?$



$E = U_R + U_c$

$E = i_c R + \frac{q}{C}$

$E = \frac{dq}{dt}R + \frac{q}{C}\ /\cdot C$

$EC = \tau\frac{dq}{dt} + q$

$\frac{dt}{\tau} = \frac{dq}{EC - q}$

$\frac{t}{\tau} = -\ln\left[EC - q\right] + \ln K$

$EC - q = K e^{-\frac{t}{\tau}}$

$t=0 \Rightarrow q=0 \Rightarrow K = EC$

$q = EC\left[1 - e^{-\frac{t}{\tau}}\right]$

$i_c = \frac{dq}{dt} = \frac{d}{dt}\left[EC\left(1 - e^{-\frac{t}{\tau}}\right)\right]$

$i_c = EC\left[-e^{-\frac{t}{\tau}}\cdot\left(-\frac{1}{RC}\right)\right]$

$$\boxed{i_c = \frac{E}{R}e^{-\frac{t}{\tau}}}$$

$t=0 \Rightarrow e^{-\frac{t}{\tau}} = 0$

$q = EC\left[1 - 0\right]$

$q = EC$

② $U_c = i_2 \cdot 3R$

$\frac{q}{C} = \left[i_1 - i_c\right]3R$

$\frac{q}{C} = \left[\frac{E - U_c}{R} - i_c\right]3R$

$U_c = 3E - 3U_c - i_c \cdot 3R$

$4U_c = 3E - \frac{dq}{dt}3R$

$4\frac{q}{C} = 3E - \frac{dq}{dt}3R$

$q = \frac{3}{4}EC - \frac{3}{4}RC\frac{dq}{dt}$

$q = \frac{3}{4}EC - \tau\frac{dq}{dt}$

$\frac{dt}{\tau} = \frac{dq}{\frac{3}{4}EC - q}$

$\frac{t}{\tau} = -\ln\left[\frac{3}{4}EC - q\right] + \ln K$

$\frac{3}{4}EC - q = K e^{-\frac{t}{\tau}}$

$t=0 \Rightarrow q = EC \Rightarrow K = \frac{3}{4}EC - EC$

$K = -\frac{1}{4}EC$

$\frac{3}{4}EC - q = -\frac{1}{4}EC\, e^{-\frac{t}{\tau}}$

$q = \frac{3}{4}EC + \frac{1}{4}EC\, e^{-\frac{t}{\tau}}$

$$\boxed{q = \frac{EC}{4}\left[3 + e^{-\frac{t}{\tau}}\right]}$$

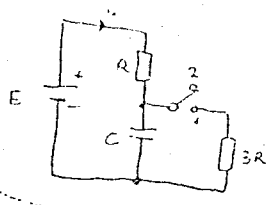
$U_c = \frac{q}{C}$ $\quad$ $\boxed{U_c = \frac{E}{4}\left[3 + e^{-\frac{t}{\tau}}\right]}$

$i_1 = \frac{E - U_c}{R} = \frac{E}{R} - \frac{E}{4R}\left[3 + e^{-\frac{t}{\tau}}\right]$

$i_1 = \frac{E}{R}\left[1 - \frac{3}{4} - \frac{1}{4}e^{-\frac{t}{\tau}}\right]$

$$\boxed{i_1 = \frac{E}{4R}\left[1 - e^{-\frac{t}{\tau}}\right]}$$

9.21. $i_R, U_C = ?$



$\tau = \frac{3}{4}RC$

① $q = \frac{3}{4}EC\left[1 - e^{-\frac{t}{\tau}}\right]$

$i = \frac{E}{R}e^{-\frac{t}{\tau}}$

② $E = U_R + U_C$

$E = iR + \frac{q}{C}$

$E = \frac{dq}{dt}R + \frac{q}{C}$

$EC = \frac{dq}{dt}\tau + q$

$\frac{dt}{\tau} = \frac{dq}{EC - q}$

$\frac{t}{\tau} = -\ln[EC - q] + \ln K$

$EC - q = Ke^{-\frac{t}{\tau}}$

$t = 0 \Rightarrow q = \frac{3}{4}EC \Rightarrow K = \frac{1}{4}EC$

$EC - q = \frac{1}{4}EC\,e^{-\frac{t}{\tau}}$

$q = EC - \frac{1}{4}EC\,e^{-\frac{t}{\tau}}$

$q = EC\left[1 - \frac{1}{4}e^{-\frac{t}{\tau}}\right]$

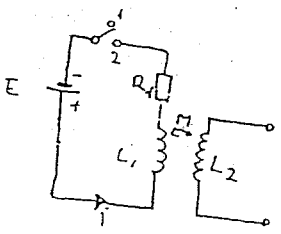
$U_C = \frac{q}{C} = E\left[1 - \frac{1}{4}e^{-\frac{t}{\tau}}\right]$

$\tau = RC$

$U_C = E\left[1 - \frac{1}{4}e^{-\frac{t}{\tau}}\right]$

$= \frac{E}{4\tau}e^{-\frac{t}{\tau}}$

9.22. $U_{IND} = ?$



② $E = U_{R_1} + U_{L_1}$

$E = iR_1 + L\frac{di}{dt}$

$\frac{E}{R_1} = i + \frac{L}{R_1}\frac{di}{dt}$

$I = i + \tau\frac{di}{dt}$

$\frac{dt}{\tau} = \frac{di}{I - i}$

$\frac{t}{\tau} = -\ln[I - i] + \ln K$

$I - i = Ke^{-\frac{t}{\tau}}$

$t = 0 \Rightarrow i = 0 \Rightarrow K = I$

$i = I\left[1 - e^{-\frac{t}{\tau}}\right]$

$U_{L_1} = L_1\frac{di}{dt} = L_1\frac{I}{\tau}\left[I(1 - e^{-\frac{t}{\tau}})\right]$

$U_{L_1} = K_1 I\left(-e^{-\frac{t}{\tau}}\right)\left(-\frac{1}{\tau}\right)$

$U_{L_1} = R_1 I e^{-\frac{t}{\tau}}$

$U_{L_1} = E \cdot e^{-\frac{t}{\tau}}$

$\mathcal{E}_2 = -M\frac{di}{dt} = -M\frac{E}{R_1}\left[1 - e^{-\frac{t}{\tau}}\right]$

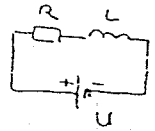
$\mathcal{E}_2 = -\frac{E}{R_1}\left(-e^{-\frac{t}{\tau}}\right)\left(-\frac{1}{\tau}\right)$

$\mathcal{E}_2 = -E\frac{\tau}{\tau}e^{-\frac{t}{\tau}}$

9.23. $U = 600V$

grot

$R, L = ?$



$i = I\left[1 - e^{-\frac{t}{\tau}}\right]$ $\quad \tau = \frac{L}{R}$ $\quad$ -iegenek $I = i_{\infty}$

$\frac{i}{I} = 1 - e^{-\frac{t}{\tau}}$

$e^{-\frac{t}{\tau}} = 1 - \frac{i}{I}$

$-\frac{t}{\tau} = \ln\left[1 - \frac{i}{I}\right]$

$\tau = -\frac{t}{\ln\left[1 - \frac{i}{I}\right]}$

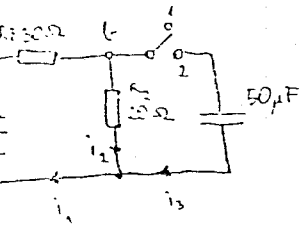
$I = \frac{U}{R}$

$R = \frac{U}{I} = 2{,}05\,\Omega$

$t = 80\,ms \Rightarrow i = 0{,}6A$

$L = 2{,}1\,mH$

$U_{ab}(\tau) = ?$



$U_c = i_2 \cdot 20\Omega$

$U_c = [i_1 - i_3] \cdot R_2$

$U_c = \left[\frac{E - U_c}{R_1} - i_3\right] \cdot R_2$

$U_C = \frac{R_2}{R_1} E - \frac{R_2}{R_1} U_c - i_3 R_2$

$\frac{5}{3}\frac{q}{C} = \frac{2}{3}E - \frac{dq}{dt} R_2$

$q = \frac{2}{5}EC - \frac{3}{5}CR_2 \frac{dq}{dt}$

$\tau \frac{dq}{dt} = \frac{2}{5}EC - q$

$\frac{dt}{\tau} = \frac{dq}{\frac{2}{5}EC - q}$

$\frac{t}{\tau} = -\ln\left[\frac{2}{5}EC - q\right] + \ln K$

$\frac{2}{5}EC - q = K e^{-\frac{t}{\tau}}$

$t=0 \Rightarrow q=0 \Rightarrow K = \frac{2}{5}EC$

$q = \frac{2}{5}EC\left[1 - e^{-\frac{t}{\tau}}\right]$

$U_c = \frac{q}{C} = \frac{2}{5}E\left[1 - e^{-\frac{t}{\tau}}\right]$

$\boxed{i_2 = \frac{U_c}{R_2} = \frac{2}{5}\frac{E}{R_2}\left[1 - e^{-\frac{t}{\tau}}\right]}$

$i_3 = \frac{d}{dt}\left[\frac{2}{5}EC\left(1 - e^{-\frac{t}{\tau}}\right)\right] = \frac{2}{5}EC\left(-e^{-\frac{t}{\tau}}\right)\left(-\frac{1}{\tau}\right)$

$\boxed{i_3 = \frac{2}{3}\frac{E}{R_2} e^{-\frac{t}{\tau}}}$

i_3

$\left[\frac{1}{5} - \frac{1}{5}e^{-\frac{t}{\tau}} - \frac{1}{5}e^{-\frac{t}{\tau}}\right]$

$\left[\frac{1}{5} + \frac{2}{15}e^{-\frac{t}{\tau}}\right]$

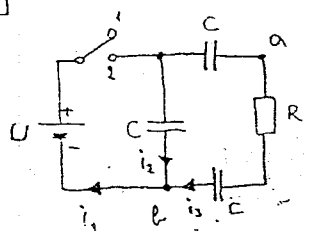
$= i_1 \cdot R_1$

$= 2E\frac{R_1}{R_2}\left[\frac{1}{5} + \frac{2}{15}e^{-\frac{t}{\tau}}\right]$

$= 60\left[1 + \frac{2}{3}e^{-\frac{t}{\tau}}\right]$

5. $U = 4V$
$C = 2\mu F$
$R = 1k\Omega$

$U_{ab} = ?$



② $i_2 = 0 \Rightarrow i_1 = i_3 = i$

$U = U_R + 2U_c$

$U = iR + 2\frac{q}{C}$

$U = \frac{dq}{dt}R + 2\frac{q}{C} \quad / \cdot \frac{C}{2}$

$\frac{UC}{2} = \frac{RC}{2}\frac{dq}{dt} + q$

$\frac{UC}{2} - q = \tau \frac{dq}{dt}$

$\frac{dt}{\tau} = \frac{dq}{\frac{UC}{2} - q}$

$\frac{t}{\tau} = -\ln\left[\frac{UC}{2} - q\right] + \ln K$

$\frac{UC}{2} - q = K e^{-\frac{t}{\tau}}$

$t=0 \Rightarrow q=0 \Rightarrow K = \frac{UC}{2}$

$\boxed{q = \frac{UC}{2}\left[1 - e^{-\frac{t}{\tau}}\right]}$

$i = \frac{d}{dt}\left[\frac{UC}{2}\left(1 - e^{-\frac{t}{\tau}}\right)\right] = \frac{UC}{2}\left(-e^{-\frac{t}{\tau}}\right)\left(-\frac{1}{\tau}\right)$

$\boxed{i = \frac{U}{R} e^{-\frac{t}{\tau}}}$

$\boxed{U_c = \frac{U}{2}\left[1 - e^{-\frac{t}{\tau}}\right]}$

$- iR - U_c = 0$

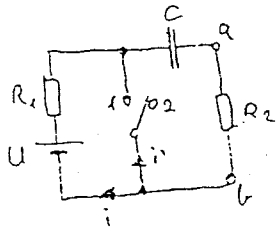
$U_c = iR + U_c = U e^{-\frac{t}{\tau}} + \frac{U}{2} - \frac{U}{2} e^{-\frac{t}{\tau}}$

$U_c = \frac{U}{2} + \frac{U}{2} e^{-\frac{t}{\tau}}$

$\boxed{U_c = \frac{U}{2}\left[1 - e^{-\frac{t}{\tau}}\right]}$

26. $R_1 = 2k\Omega$
$R_2 = 2k\Omega$
$C = 10\mu F$
$U = 10V$
$\Delta t = 1s$

$U_{ab} = ?$



$\tau = (R_1 + R_2)C$

$\tau = 100ms$

$\int i^2 R_2 dt$

$\int U^2 \frac{R_2}{(R_1+R_2)^2} e^{-\frac{2t}{\tau}} dt$

$\frac{U^2 R_2}{(R_1+R_2)^2} e^{-\frac{2t}{\tau}} \left(-\frac{\tau}{2}\right)$

$\tau = 2\frac{1}{3}$

② $U = U_R + U_c$

$U = i(R_1 + R_2) + \frac{q}{C}$

$U = \frac{dq}{dt}R + \frac{q}{C}$

$UC = \tau \frac{dq}{dt} + q$

$\frac{dt}{\tau} = \frac{dq}{UC - q}$

$\frac{t}{\tau} = -\ln\left[UC - q\right] + \ln K$

$UC - q = K e^{-\frac{t}{\tau}}$

$t=0 \Rightarrow q=0 \Rightarrow K = UC$

$\boxed{q = UC\left[1 - e^{-\frac{t}{\tau}}\right]}$

$t = 1s \Rightarrow q = 10^{-4} As$

$i = \frac{d}{dt}\left[UC\left(1 - e^{-\frac{t}{\tau}}\right)\right]$

$i = UC\left(-e^{-\frac{t}{\tau}}\right)\left(-\frac{1}{R_2 C}\right)$

$i = \frac{U}{R_1 + R_2} e^{-\frac{t}{\tau}}$

$U_{ab} = iR_2 = U\frac{R_2}{R_1 + R_2} e^{-\frac{t}{\tau}}$

$\boxed{U_{ab} = \mathcal{E} e^{-\frac{t}{\tau}}}$

④ $U_{q'} + U_c = 0$

$i' R_2 = -\frac{q'}{C}$

$\frac{dq'}{dt} R_2 C = -q'$

$\frac{dq'}{dt}\tau' = -q'$

$\frac{dt'}{\tau'} = \frac{dq'}{-q'}$

$\frac{t'}{\tau'} = -\ln\left[-q'\right] + \ln K'$

$-q' = K' e^{-\frac{t'}{\tau'}}$

$t'=0 \Rightarrow q' = 10^{-4} \Rightarrow K' = -10^{-4}$

$\boxed{q' = 10^{-4} e^{-\frac{t'}{\tau'}}}$

$i' = \frac{d}{dt'}\left[10^{-4} e^{-\frac{t'}{\tau'}}\right]$

$i' = 10^{-4} e^{-\frac{t'}{\tau'}}\left(-\frac{1}{R_2 C}\right)$

$i' = -\frac{10^{-4}}{R_2 C} e^{-\frac{t'}{\tau'}}$

$U_{ab} = i' R_2$

$U_{ab} = -\frac{10^{-4}}{C} e^{-\frac{t'}{\tau'}}$

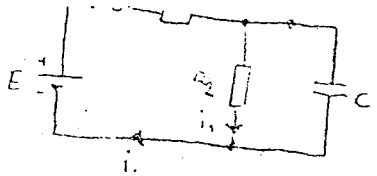
$\boxed{U'_{ab} = 10 e^{-\frac{t'}{\tau'}}}$

$R_1 = R_2 = 2{,}5\,k\Omega$

$E = 100\,V$

$C = 0{,}1\,mF$

$i(t_1) = ?$



$t_1 = 0{,}25\,s$

$\Downarrow$

$i = 22{,}7\,mA$

$i_1 = \frac{U_c}{R_2}$

$i_1 = \frac{E}{2R_2}\left[1 - e^{-\frac{t}{\tau}}\right]$

$i = i_1 + i_2$

$i = \frac{1}{2}\frac{E}{R_2} - \frac{1}{2}\frac{E}{R_2}e^{-\frac{t}{\tau}} + \frac{E}{R_2}e^{-\frac{t}{\tau}}$

$i = \frac{1}{2}\frac{E}{R_2} + \frac{1}{2}\frac{E}{R_2}e^{-\frac{t}{\tau}}$

$i = \frac{E}{2R_2}\left[1 + e^{-\frac{t}{\tau}}\right]$

$U_c = [i - i_2]R_2$

$U_c = \left[\frac{E - U_c}{R_2} - i_2\right]R_2$

$U_c = E - U_c - i_2 R_2$

$2\frac{q}{C} = E - R_2\frac{dq}{dt}$

$q = \frac{CE}{2} - \frac{R_2 C}{2}\frac{dq}{dt}$

$\frac{dt}{\tau} = \frac{dq}{\frac{CE}{2} - q}$

$\frac{t}{\tau} = -\ln\left[\frac{CE}{2} - q\right] + \ln K$

$t = 0 \Rightarrow q = 0 \Rightarrow K = \frac{CE}{2}$

$q = \frac{CE}{2}\left[1 - e^{-\frac{t}{\tau}}\right]$

$i_2 = \frac{d}{dt}\left[\frac{CE}{2}\left(1 - e^{-\frac{t}{\tau}}\right)\right]$

$i_2 = \frac{CE}{2}\left(-e^{-\frac{t}{\tau}}\right)\left(-\frac{2}{R_2 C}\right)$

$i_2 = \frac{E}{R_2}e^{-\frac{t}{\tau}}$

$U_c = \frac{q_2}{C} = \frac{E}{2}\left[1 - e^{-\frac{t}{\tau}}\right]$

9.28. $U_1 = 100\,V$

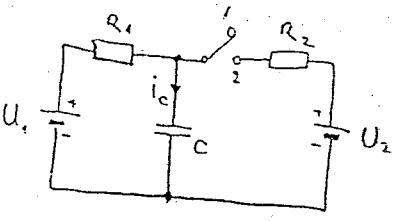
$U_2 = 400\,V$

$R_1 = 100\,\Omega$

$R_2 = 50\,\Omega$

$C = 30\,\mu F$

$i_c, U_c = ?$



$E_T = U_{R_T} + U_c$

$E_T = i_c R_T + \frac{q}{C}$

$E_T C = \frac{dq'}{dt'}R_T C + q$

$\frac{dt'}{\tau'} = \frac{dq'}{E_T C - q}$

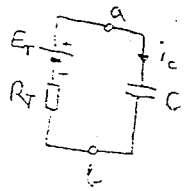
$\frac{t'}{\tau'} = -\ln[E_T C - q] + \ln K$

$E_T C - q = K e^{-\frac{t'}{\tau'}}$

$t' = 0 \Rightarrow q = U_1 C \Rightarrow K = E_T C - U_1 C$

$q' = E_T C - [E_T C - U_1 C]e^{-\frac{t'}{\tau'}}$

$q' = 0{,}003 - 0{,}006\,e^{-\frac{t'}{\tau'}}$



① $U_1 = U_{R_1} + U_c$

$U_1 = iR_1 + \frac{q}{C}$

$U_1 = \frac{dq}{dt}R_1 + \frac{q}{C}$

$U_1 C = R_1 C\frac{dq}{dt} + q$

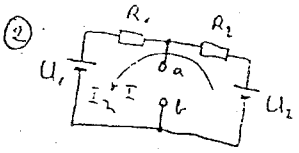
$\frac{dt}{\tau} = \frac{dq}{U_1 C - q}$

$\frac{t}{\tau} = -\ln[U_1 C - q] + \ln K$

$U_1 C - q = K e^{-\frac{t}{\tau}}$

$t = 0 \Rightarrow q = 0 \Rightarrow K = U_1 C$

$q = U_1 C\left[1 - e^{-\frac{t}{\tau}}\right]$

② 

$I = \frac{U_2 - U_1}{R_1 + R_2} = 2\,A$

$\varphi_a - IR_1 - U_1 = \varphi_b$

$U_{ab} = IR_1 + U_1$

$E_T = 300\,V$

$R_T = \frac{R_1 R_2}{R_1 + R_2} = 33{,}\overline{3}\,\Omega$

$i_c = \frac{d}{dt}\left[0{,}003 - 0{,}006\,e^{-\frac{t'}{\tau'}}\right] = -0{,}006\,e^{-\frac{t'}{\tau'}}\left[-\frac{1}{R_T C}\right]$

$i_c = 6\,e^{-\frac{t'}{\tau'}}$

$U_c = \frac{q}{C} \Rightarrow U_c = 300 - 100\,e^{-\frac{t'}{\tau'}}$

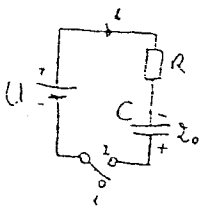
9.29. $C = 20\,\mu F$

$I_0 = 500\,\mu A$

$R = 1\,k\Omega$

$U = 50\,V$

$i_2 = ?$



① $U = U_R - U_c$

$U = iR + \frac{q}{C}$

$UC = \frac{dq}{dt}RC + q$

$\frac{dt}{\tau} = \frac{dq}{UC - q}$

$UC - q = K e^{-\frac{t}{\tau}}$

$t = 0 \Rightarrow i = -0{,}5\,mA \Rightarrow K = UC - q_0$

$q = UC - [UC - q_0]e^{-\frac{t}{\tau}}$

$i = 10^{-3} - 1{,}5\cdot 10^{-3}\,e^{-\frac{t}{\tau}}$

$i = \frac{d}{dt}\left[10^{-3} - 1{,}5\cdot 10^{-3}\,e^{-\frac{t}{\tau}}\right]$

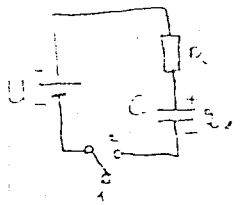
$i = 0{,}075\,e^{-\frac{t}{\tau}}$

$C = 10\,\mu F$

$q_0 = 0{,}5\,mA\cdot s$

$R = 1\,k\Omega$

$U = 50\,V$



$i, q = ?$

$U = U_R + U_C$

$U = \frac{dq}{dt} R - \frac{q}{C}$

$UC - q = RC\frac{dq}{dt}$

$t = 0 \Rightarrow q = 0{,}5\,mA\cdot s = 0{,}5\cdot 10^{-3}\,C$

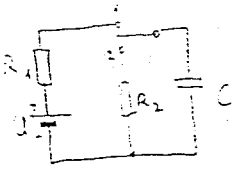
$q = UC - Ke^{-\frac{t}{\tau}} = 0{,}5\,mA\cdot s - Ke^{-\frac{t}{\tau}}$

$q = 10^{-3} - C = 10^{-3} e^{-\frac{t}{\tau}}$

$i = \frac{d}{dt}\left[10^{-3} - 0{,}5\cdot 10^{-3} e^{-\frac{t}{\tau}}\right]$

$= -0{,}5\cdot 10^{-3} e^{-\frac{t}{\tau}}\left(-\frac{1}{\tau}\right)$

$i = 0{,}025\, e^{-\frac{t}{\tau}}$



a) $\Delta t = 10\,ms$
$R_1 = 7\,k\Omega$
$R_2 = 2\,k\Omega$
$C = 1\,\mu F$
$U = 100\,V$

b) $R_1 = 50\,\Omega$
$C = 1\,\mu F$
$R_2 = 2\,k\Omega$
$U = 100\,V$

a) $i, u_c = ?$
b) $I_{2a} = ?$

a) $U = U_{R_1} + U_C$

$U = iR_1 + \frac{q}{C}$

$UC = \frac{dq}{dt} R_1 C + q$

$Q = \frac{dq}{dt}\tau + q$

$\frac{dt}{\tau} = \frac{dq}{Q - q}$

$Q - q = Ke^{-\frac{t}{\tau}}$

proc. ujęć: $t = 0$, $q = 0$: $K = Q$

$q = Q\left[1 - e^{-\frac{t}{\tau}}\right]$

$q(t = 10\,ms) = 76\,\mu As$

$i = \frac{dq}{dt} = Q\left[-e^{-\frac{t}{\tau}}\left(-\frac{1}{\tau}\right)\right] = \frac{Q}{\tau} e^{-\frac{t}{\tau}} = I e^{-\frac{t}{\tau}}$

$i(t=0) = \frac{U}{R_1} = 14{,}28\,mA$

$i(t = 10\,ms) = 2{,}42\,mA$

$U_c = \frac{q}{C} = U\left[1 - e^{-\frac{t}{\tau}}\right]$

$U_c(t=0) = 0$

$U_c(t = 10\,ms) = 76\,V$

b) $i = \frac{U}{R_1} e^{-\frac{t}{\tau}} = 2e^{-2000t}$

$i_2 = -0{,}05\, e^{-500t}$

$\bar{I}_2 = \frac{1}{10\,ms}\int_0^{10\,ms} -0{,}05 e^{-500t}\,dt$

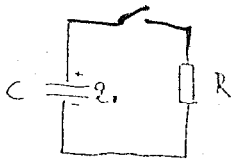
$\bar{I}_{2_1} = \frac{5}{-500}\left[e^{-5} - 1\right] = 10\,mA$

$t < 10\,ms \Rightarrow \bar{I}_{2_2} = 0$

$\bar{I}_2 = \frac{\bar{I}_{2_1} - \bar{I}_{2_2}}{2} = 5\,mA$

$R = 10\,\Omega$
$C = 2\,\mu F$
$p_R(t) = 360 e^{-10^5 t}$

$Q_0 = ?$



$Q = CU$

$t=0,\ Q=Q_0 : K = Q_0$

$Q = Q_0 e^{-50000t}$

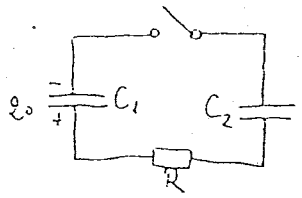
$i = \frac{dQ}{dt}$

$p_R = i^2 R = [50000 Q_0]^2 e^{-100000t} \cdot 10 \Rightarrow 360 = [50000 Q_0]^2 \cdot 10 \Rightarrow$ **$Q_0 = 120\,\mu A\,s$**

9.33. $Q_0 = 3 \cdot 10^{-4}\,As$

$C_1 = 6\,\mu F$
$R = 20\,\Omega$
$C_2 = 3\,\mu F$

$i, u_C = ?$



$U_0 = \frac{Q_0}{C_1} = 50\,V$

ZAPAMTI

$U_0 = U_R + U_{C_1} + U_{C_2}$

$U_0 = iR + Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$

$U_0 = \frac{dQ}{dt} R + Q \frac{1}{\frac{C_1 C_2}{C_1 + C_2}}$

$U_0 C = \frac{dQ}{dt} RC + Q$

$Q = \frac{dQ}{dt} \tau + Q$

$Q - Q = K e^{-\frac{t}{\tau}}$

- poč. uvj.: $t=0,\ Q=0 : K = Q$

$Q = Q\left[1 - e^{-\frac{t}{\tau}}\right]$

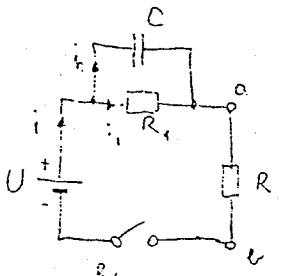
$i = \frac{dQ}{dt} = \frac{Q}{\tau} e^{-\frac{t}{\tau}}$

$i = 2{,}5 e^{-15000t}$

$U_{C_1} = \frac{1}{C_1}\int_0^\infty i\,dt = \frac{2{,}5}{-25000 C_1}\left[0 - 1\right] = \frac{2{,}5}{25000 C_1} = 16{,}\overline{6}\,V$

$U_{C_2} = \frac{1}{C_2}\int_0^\infty i\,dt = \frac{2{,}5}{-25000 C_2}\left[0 - 1\right] = 33{,}\overline{3}\,V$

9.34. $U_{ab}(t) = ?$



- poč. uvjet: $t=0,\ Q=0 : K = UC\frac{R_1}{R+R_1}$

$Q_C = UC\frac{R_1}{R+R_1}\left[1 - e^{-\frac{t}{\tau}}\right]$

$i_C = \frac{dQ_C}{dt} = \frac{UC}{\frac{RR_1C}{R+R_1}} \cdot \frac{R_1}{R+R_1} e^{-\frac{t}{\tau}} = \frac{U}{R} e^{-\frac{t}{\tau}}$

$u_C = \frac{Q_C}{C} = U\frac{R_1}{R+R_1}\left[1 - e^{-\frac{t}{\tau}}\right]$

$i_3 = \frac{u_C}{R_1} = \frac{U}{R+R_1}\left[1 - e^{-\frac{t}{\tau}}\right]$

$i = i_C + i_3 = Ue^{-\frac{t}{\tau}}\left[\frac{1}{R} - \frac{1}{R+R_1}\right] + \frac{U}{R+R_1}$

$i = Ue^{-\frac{t}{\tau}}\frac{R+R_1+R}{R(R+R_1)} + \frac{U}{R+R_1} = U\frac{R_1}{R(R+R_1)} e^{-\frac{t}{\tau}} + \frac{U}{R+R_1}$

$U_{ab} = iR = U\frac{R_1}{R+R_1} e^{-\frac{t}{\tau}} + \frac{U}{R+R_1} R$

$U_C = i_1 R_1$

$U_C = [i - i_C] R_1$

$U_C = \left[\frac{U - U_C}{R} - i_C\right] R_1$

$U_C = U\frac{R_1}{R} - U_C\frac{R_1}{R} - i_C R_1$

$U_C\left[1 + \frac{R_1}{R}\right] = U\frac{R_1}{R} - i_C R_1$

$\frac{Q}{C}\frac{R+R_1}{R} = U\frac{R_1}{R} - \frac{dQ}{dt} R_1$

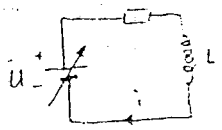
$\frac{Q}{C} = UC\frac{R_1}{R+R_1} - \frac{dQ}{dt} R_1 C$

$Q = UC\frac{R_1}{R+R_1} - \frac{dQ}{dt}\frac{RR_1C}{R+R_1}$

$Q = UC\frac{R_1}{R+R_1} - \tau\frac{dQ}{dt}$

$\frac{dt}{\tau} = \frac{dQ}{UC\frac{R_1}{R+R_1} - Q}$

$UC\frac{R_1}{R+R_1} - Q = Ke^{-\frac{t}{\tau}}$



$u_L = ?$

t [s]

$u(t) = \omega t$

$U = U_R + U_L$

$U = iR + L\frac{di}{dt}$

$\frac{U}{R} = i + \frac{L}{R}\frac{di}{dt}$

$I = i + \tau\frac{di}{dt}$

$\frac{dt}{\tau} = \frac{di}{I - i}$

$I - i = Ke^{-\frac{t}{\tau}}$

$t=0,\ i=0 : K = I$

$i = I\left[1 - e^{-\frac{t}{\tau}}\right]$

$i = \frac{\omega\tau}{R} - \frac{\omega\tau}{R}e^{-\frac{t}{\tau}}$

$i = \frac{\omega t}{R} - \frac{\omega\tau}{R}e^{-\frac{t}{\tau}}$?

936

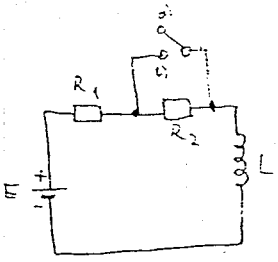
$t_1 = 0{,}08$ s

$t_2 = 0{,}12$ s

$R_1 = R_2 = 20\,\Omega$

$L = 2$ H

$E = 60$ V



$P_{R_2} = ?$

a) $E = U_{R_1} + U_L$

$E = iR_1 + L\frac{di}{dt}$

$\frac{E}{R_1} = i + \frac{L}{R_1}\frac{di}{dt}$

$I' = i + \tau'\frac{di}{dt}$

$I' - i = K e^{-\frac{t}{\tau'}}$

-poč. uvjeti: $t=0,\ i=1{,}652$ A : $K = -0{,}152$ A

$i = I' + 0{,}152\,e^{-\frac{t}{\tau'}}$

$i(0{,}12) = 1{,}515$

a) $E = U_{R_1} + U_2$

$E = iR_1 + L\frac{di}{dt}$

$\frac{E}{R_1} = i + \frac{L}{R_1}\frac{di}{dt}$

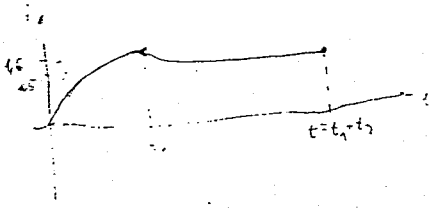
$I - i = Ke^{-\frac{t}{\tau}}$

-poč. uvj.: $t=0,\ i=0$:

$K = I$

$i = I\left[1 - e^{-\frac{t}{\tau}}\right]$

$i(0{,}12) = 1{,}652$ A



$I_{sr} = \frac{1}{t_1}\int_0^{t_1} i\,dt = \frac{I}{t_1}\left[\int_0^{t_1} dt - \int_0^{t_1} e^{-\frac{t}{\tau}}\,dt\right] = \frac{I}{t_1}\left[t_1 - \tau\left(e^{-\frac{t_1}{\tau}} - 1\right)\right] = 0{,}935$ A

$I_{sr} = \frac{1}{t_2}\int_0^{t_2} i\,dt = \frac{I'}{t_2}\int_0^{t_2} dt + \frac{0{,}152}{t_2}\int_0^{t_2} e^{-\frac{t}{\tau}} = I' - 0{,}152\frac{\tau'}{t_2}\left[e^{-\frac{t_2}{\tau'}} - 1\right] = 1{,}5575$ A

$P_{R_1} = I_{sr_1}^2 R_1 + I_{sr_2}^2 R_1 = 66$ W

$P_{R_2} = I_{sr_2}^2 R_2 = 49{,}3$ W

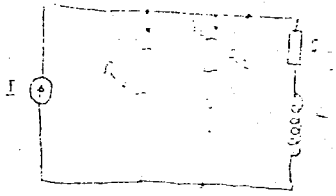
9.37

$R_{1,2} = 1\,\Omega$

$R_3 = 0{,}5\,\Omega$

$L = 1\,H$

$i, u = ?$



a) $IR_1 = U_2 + U_L$

$IR_1 = i(R_1+R_3) + L\frac{di}{dt}$

$I\frac{R_1}{R_1+R_3} = i + \frac{L}{R_1+R_3}\frac{di}{dt}$

$I\frac{R_1}{R_1+R_3} = i + \tau\frac{di}{dt}$

$\frac{dt}{\tau} = \frac{di}{I\frac{R_1}{R_1+R_2} - i}$

$I\frac{R_1}{R_1+R_2} - i = K e^{-\frac{t}{\tau}}$

- poč. uvjeti: $t=0,\ i=0 : K = I\frac{R_1}{R_1+R_2}$

$i_3 = I\frac{R_1}{R_1+R_3}\left[1 - e^{-\frac{t}{\tau}}\right]$

$i_3 = 2\left[1 - e^{-2t}\right]$ $\quad t=\infty \Rightarrow \boxed{i_3 = 2\,A}$

$i_1 = I - i_3 = 3 - 2 + 2e^{-2t} = 1 + 2e^{-2t}$

$t=\infty \Rightarrow \boxed{i_1 = 1\,A}$

9.38. $P_2(t) = 250\,W$

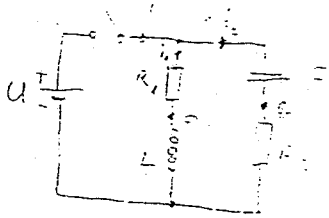
$P_2(\tau) = 25\,W$

$U = 100\,V$

$L = 0{,}1\,H$

$C = 500\,\mu F$

$i, u_{2L} = ?$



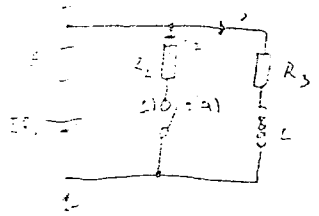
$250 = \frac{100^2}{R_1}\left[e^{-0{,}69} - e^{-1{,}38}\right] \Rightarrow R_1 = 10\,\Omega$

$25 = \frac{100^2}{R_1}\left[e^{-0{,}69} - e^{-1{,}38}\right] \Rightarrow R_2 = 100\,\Omega$

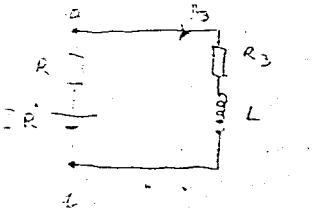
$\psi_0 + i_1 R_1 = u_C + \psi_L$

$u_{2L} = 100\left[1 - e^{-20t}\right] - 100\left[1 - e^{-100t}\right]$

$\boxed{u_{2L} = 100\left[e^{-100t} - e^{-20t}\right]}$



$i_1 = I - i_2 - i_3$



$R' = R_1 \| R_2 = \frac{1}{2}\,\Omega$

$i_1 = i_2 = \frac{1}{2}\left[I - i_3\right]$

$IR' = u_1 - u_L$

$IR' = i(R'+R_3) + L\frac{di}{dt}$

$I\frac{R'}{R'+R_3} = i + \frac{L}{R'+R_3}\frac{di}{dt}$

$I\frac{R'}{R'+R_3} - i = \tau'\frac{di}{dt}$

$I\frac{R'}{R'+R_3} - i = K e^{-\frac{t}{\tau'}}$

- poč. uvjeti: $t_0 = 0,\ i_3 = 2$

$K = -0{,}5\,A$

$\boxed{i_3 = 1{,}5 + 0{,}5\,e^{-t}}$

$i_1 = i_2 = 1{,}5 - 0{,}75 - 0{,}25\,e^{-t}$

$\boxed{i_1 = i_2 = 0{,}75 - 0{,}25\,e^{-t}}$

$\psi_a = i_3 \cdot - IR' = \psi_L$

$\boxed{u_{2L} = 0 - 0{,}5\,e^{-t}}$

$u_L = L\frac{di_1}{dt} = \Delta\frac{U}{R_1}\left[\frac{-e^{-\frac{R_1}{L}t}}{-\frac{U}{R_1}}\right]$

$u_L = U e^{-\frac{R_1}{L}t}$

$P_L = \frac{U^2}{R_1}\left[e^{-\frac{R_1}{L}t} - e^{-2\frac{R_1}{L}t}\right]$

$\frac{dP_L}{dt} = 0 \Rightarrow e^{-\frac{t}{\tau}} = \frac{1}{2} \Rightarrow t = 0{,}693\tau$

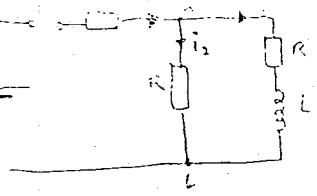
$\boxed{i_1 = 10\left[1 - e^{-100t}\right]}$

$P_C = \frac{U^2}{R_1}\left[e^{-\frac{t}{R_1 C}} - e^{-2\frac{t}{R_1 C}}\right]$

$\frac{dP_C}{dt} = 0 \Rightarrow e^{-\frac{t}{\tau}} = \frac{1}{2} \Rightarrow t = 0{,}693\tau$

$\boxed{i_2 = e^{-20t}}$

$R = 10\Omega$
$L = 0{,}1\,H$
$i_{(t=0)} = ?$



$i_1 = i_2 + i_3$

- prąd. wyj.: $t=0,\ i_3=0:\ E=I$

$$E = i_3 R + L\frac{di_3}{dt}$$

$$\frac{E - L\frac{di_3}{dt} - i_3}{R} = i_3 R + L\frac{di_3}{dt}$$

$$E - L\frac{di_3}{dt} - i_3 R = i_3 R + L\frac{di_3}{dt}$$

$$E = i_3 \cdot 3R + 2L\frac{di_3}{dt}$$

$$\frac{E}{3R} = i_3 + \frac{2L}{3R}\frac{di_3}{dt}$$

$$\frac{E}{3R} = i_3 + \tau\frac{di_3}{dt}$$

$$i_3 = K e^{-\frac{t}{\tau}}$$

$$i_3 = \frac{U}{3R}\left[1 - e^{-\frac{3R}{2L}t}\right]$$

$$\boxed{i_3 = 0{,}5\left[1 - e^{-150t}\right]}$$

$$i_2 = i_3 + \frac{L}{R}\frac{di_3}{dt}$$

$$i_2 = i_3 + \frac{L}{R}\cdot\frac{1}{2}\left[+e^{-150t}\cdot(-150)\right]$$

$$i_2 = \frac{1}{2} - \frac{1}{2}e^{-150t} + \frac{1}{4}e^{-150t}$$

$$\boxed{i_2 = \frac{1}{2} + \frac{1}{4}e^{-150t}}$$

$$i_1 = i_2 + i_3$$

$$i_1 = \frac{1}{2} + \frac{1}{4}e^{-150t} + \frac{1}{2} - \frac{1}{2}e^{-150t}$$

$$\boxed{i_1 = 1 - \frac{1}{4}e^{-150t}}$$