

Osnove - 2. mase instr.

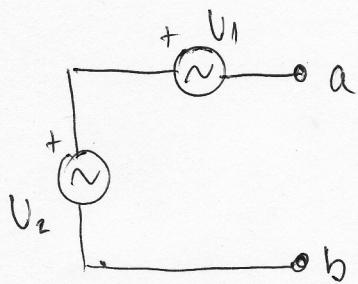
(1)

$$i(t) = I_{\max} \cdot \sin(\omega t + \varphi_i)$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\overset{\circ}{I} = |I| \angle i$$

$$|I| = I_{\text{ef}} = \frac{I_{\max}}{\sqrt{2}}$$



$$U_1(t) = 14.1 \sin(400t + 30^\circ)$$

$$U_2(t) = 10 \cos(400t - 45^\circ)$$

$$U_{ab} = U_2 - U_1$$

$\ominus \rightarrow +$

$+ \rightarrow \ominus$

+

-

$$\begin{aligned} \underline{U}_1 = \dot{U}_1 &= \frac{U_{\max}}{\sqrt{2}} \angle 30^\circ = \frac{14.1}{\sqrt{2}} \angle 30^\circ = 10 \angle 30^\circ \rightarrow \text{polarni oblik} \\ &\quad - \text{pogodan za mu. i dis.,} \\ &\quad \text{ali nepogodan za } \pm \\ &= 10 [\cos 30^\circ - j \sin 30^\circ] \\ &= 8.66 + 5j \end{aligned}$$

\rightarrow algebarski
- pogodan za \pm

$$U_2(t) = 10 \sin(400t - 45^\circ + 90^\circ) \rightarrow \text{prevaranje iz SOS} \rightarrow \sin$$

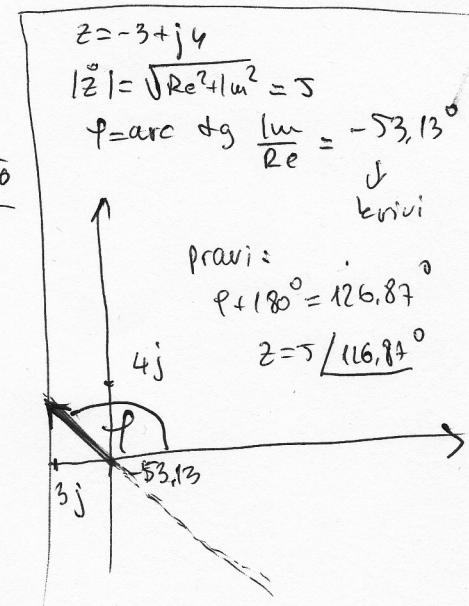
$$\dot{U}_2 = \frac{10}{\sqrt{2}} \angle 45^\circ = 7.07 \angle 45^\circ = 5 + 5j$$

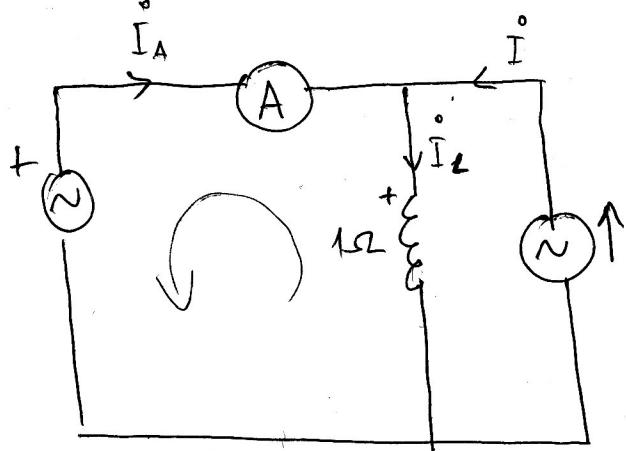
$$+ \frac{\pi}{2} / + 90^\circ$$

$$\dot{U}_{ab} = \dot{U}_2 - \dot{U}_1 = 5 + 5j - 8.66 - 5j = -3.66$$

$$\begin{aligned} &= \sqrt{R^2 + L^2} = \sqrt{3.66^2 + 0^2} = 3.66 \angle \arctg \frac{0}{-3.66} \\ &= 3.66 \angle 180^\circ \end{aligned}$$

$$U_{ab} = 3.66 \cdot \sqrt{2} \cdot \sin(400t + 180^\circ)$$





$$U_{AB} = 1 \sin(\omega t)$$

$$i(t) = 1 \sin(\omega t)$$

(2)

$$X_L = \omega L$$

$$\dot{X}_L = \omega L \angle 90^\circ = j\omega L$$

$$\dot{U}_L = \dot{I}_L \cdot \dot{X}_L$$

$$\dot{U}_L = 10 \angle 0^\circ V$$

$$X_L = 5 \Omega$$

$$\dot{I}_L = \frac{\dot{U}_L}{\dot{X}_L} = \frac{10 \angle 0^\circ}{5 \angle 90^\circ} = 2 \angle 0^\circ - 90^\circ \rightarrow \text{kao dijeljena se održavaju}$$

$$= 2 \angle -90^\circ A \rightarrow \text{struja kasnije za naponom } 90^\circ$$

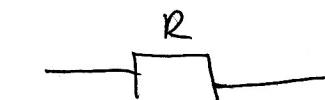
$$X_C = \frac{1}{\omega C}$$

$$\dot{X}_C = \frac{1}{\omega C} \angle -90^\circ = -\frac{j}{\omega C}$$

$$\dot{U}_C = 10 \angle 0^\circ V$$

$$X_C = 10 \Omega$$

$$\dot{I} = \frac{\dot{U}_C}{\dot{X}_C} = 1 \angle 0^\circ A$$



- na otporniku R su struja i napon u fazi

$$\dot{I} = R \angle 0^\circ = R$$

$$\dot{U} = \frac{1}{\sqrt{2}} \angle 0^\circ$$

$$\dot{I} = \frac{1}{\sqrt{2}} \angle 0^\circ$$

$$\dot{I}_A + \dot{I} = \dot{I}_L$$

$$\dot{I}_A = \dot{I}_L - \dot{I}$$

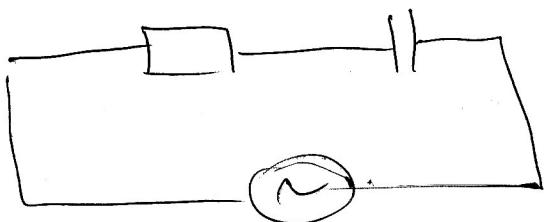
$$\dot{I}_L \cdot \dot{x}_L - \dot{U} = 0$$

$$\dot{I}_L = \frac{\dot{U}}{\dot{x}_L} = \frac{\frac{1}{\sqrt{2}} \angle 0^\circ}{1 \angle 90^\circ} = \frac{1}{\sqrt{2}} \angle 90^\circ = -j \cdot \frac{1}{\sqrt{2}}$$

$$\dot{I}_A = -j \cdot \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$$

$$|\dot{I}_A| = \sqrt{R^2 + \omega^2} = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1 A \rightarrow \text{effekt. vrije radn.}$$

(3)



$$U(t) = 100 \sin(400t)$$

$$R = 50 \Omega$$

$$C = 50 \mu F = 50 \cdot 10^{-6} F$$

$$U_C = ? \quad t = 5 \text{ ms}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{400 \cdot 5 \cdot 10^{-5}} = 50 \Omega$$

$$\dot{Z}_{eq} = R + jX_C = 50 - j50$$

$$\text{impedanca} = 70,7 \angle -45^\circ$$

$$\dot{I} = \frac{\dot{U}}{\dot{Z}_{eq}}$$

$$\dot{U} = \frac{100}{\sqrt{2}} \angle 0^\circ = 70,7 \angle 0^\circ$$

$$\dot{I} = \frac{70,7 \angle 0^\circ}{70,7 \angle -45^\circ} = 1 \angle 45^\circ$$

$$\dot{U}_C = \dot{I} \cdot \dot{x}_C = 1 \angle 45^\circ \cdot 50 \angle 90^\circ$$

$$\dot{U}_C = 50 \angle 45^\circ$$

$$U_C(t) = 50\sqrt{2} \sin(400t - 45^\circ)$$

$$U_C = 70,7 \sin(2 - 45^\circ)$$

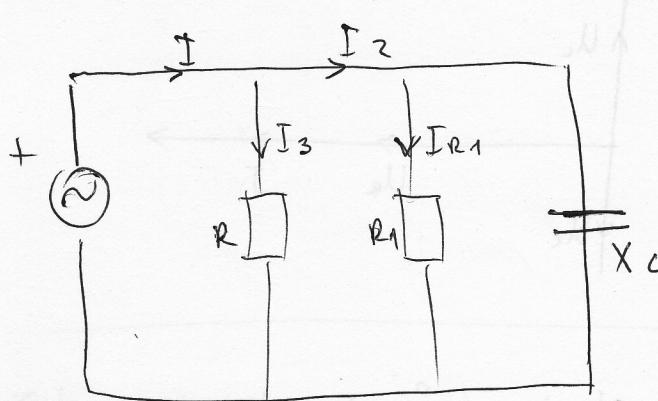
$$= 70,7 \sin\left(2 \cdot \frac{180^\circ}{\pi} - 45^\circ\right)$$

↓

predvaranje je ravnost.

RLC krugovi

(4)



$$I = 3A$$

$$I_{R1} = 2A$$

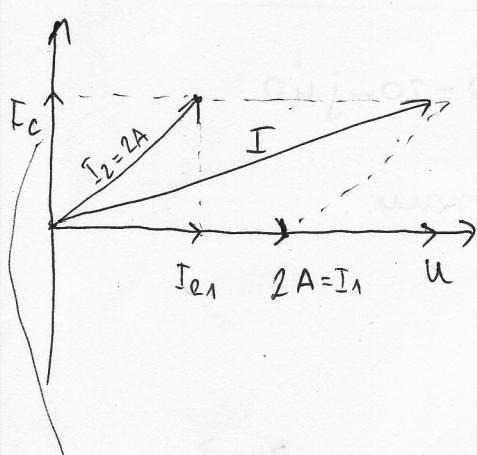
$$I_3 = 2A$$

$$R_1 = 4\Omega$$

I_3 - u fazi s naponom

I_c - prethodi naponu za 90° (iz grafu)

I_2 - prethodi naponu za 90°



$$I_2^2 = I_c^2 + I_{R1}^2 \quad (-1)$$

$$I^2 = (I_{R1} + I_1)^2 + I_c^2 \rightarrow \begin{array}{l} I_{R1} + I_1 \text{ zbrojimo} \\ \text{jer su u fazi f} \\ \text{na istoj osi} \end{array}$$

$$-I_2^2 = -I_c^2 - I_{R1}^2$$

$$I^2 = I_{R1}^2 + 2I_{R1} \cdot I_1 + I_1^2 + I_c^2 \quad /+$$

$$I^2 - I_2^2 = 2I_{R1}I_1 + I_1^2$$

$$\frac{I^2 - I_2^2 - I_1^2}{2I_1} = I_{R1}$$

$$I_{R1} = \frac{1}{4} A$$

$$U_{R1} = U_{VR} = R_1 \cdot I_{R1} = 4 \cdot \frac{1}{4} = 1V.$$

→ kako crtati graf:

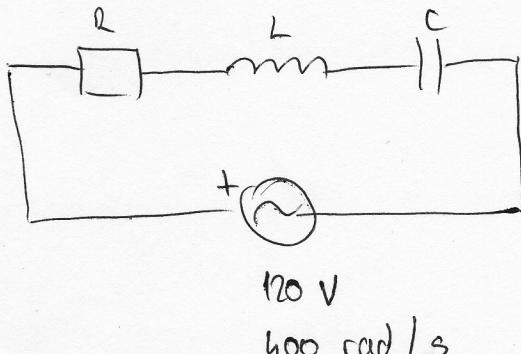
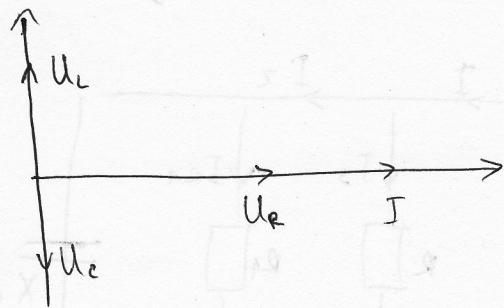
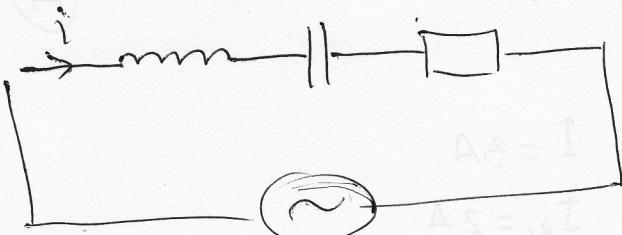
struja kroz kond.

prethodi naponu za 90° ,

a posle su ovo rotirajući

vektori → ćemo ju

ispred naponu (na x osi je u)



→ struja 10° , tj referentna veličina

$$L = 25 \text{ mH}$$

$$C = 50 \mu\text{F}$$

$$\varphi = 63.4^\circ$$

$$X_L = \omega L = j10 \Omega$$

$$X_C = \frac{1}{\omega C} = -j50 \Omega$$

Trenut otpora:

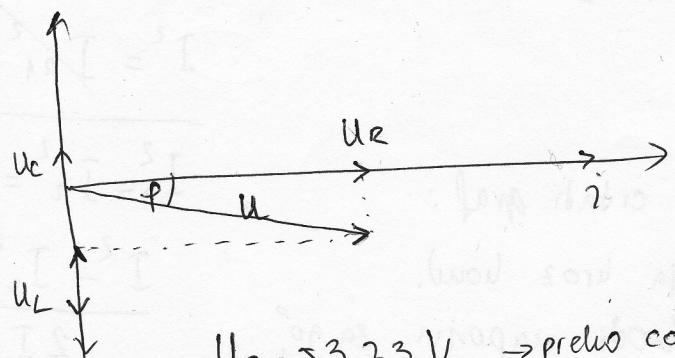
imped. $\leftarrow Z_{\text{uk}}$

$$(X_L - X_C) = 140 \Omega$$

$$\tan \varphi = \frac{40}{R} \rightarrow R = \frac{40}{\tan \varphi} = 20 \Omega$$

$$Z_{\text{uk}} = 20 + j10 - j50 = 20 - j40$$

fazorski dijagram:



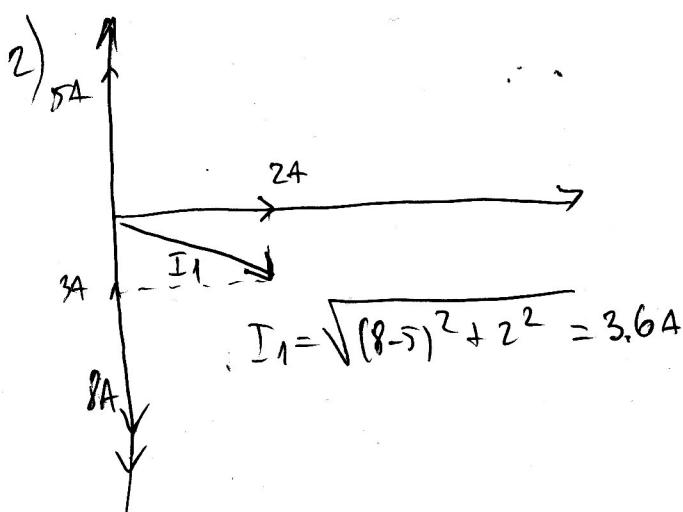
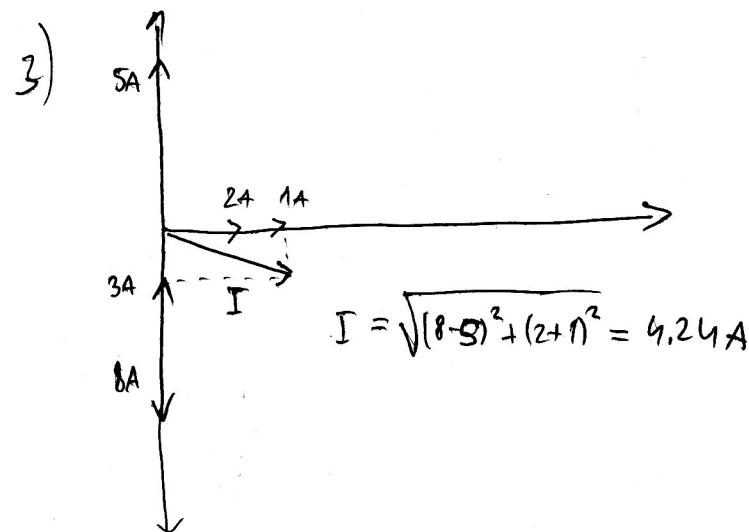
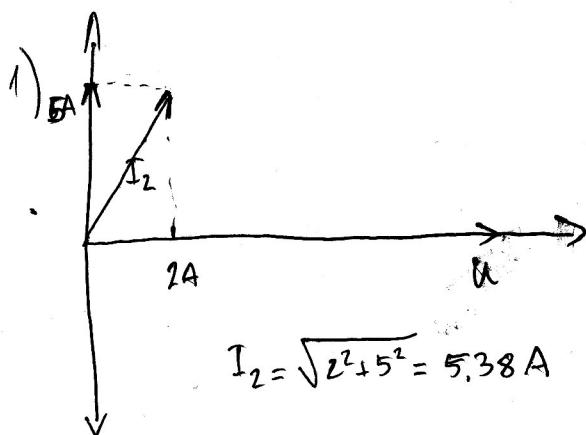
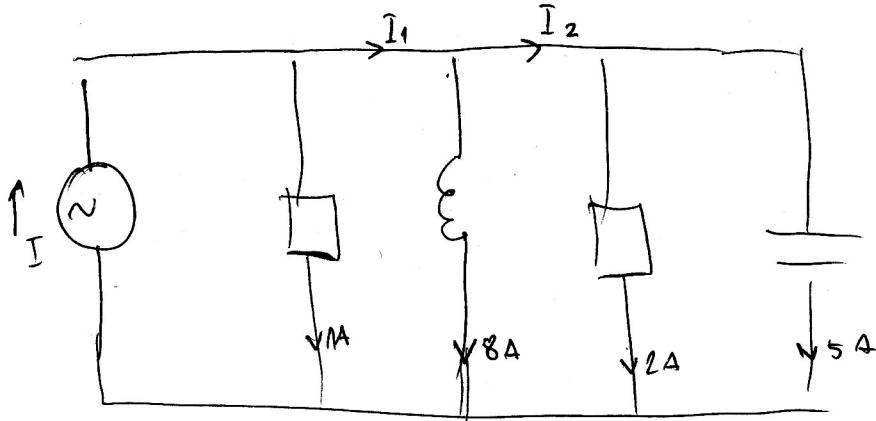
$$U_R = 53.73 \text{ V} \rightarrow \text{prelaz cos}$$

$$I = \frac{U_R}{R} = 2.68 \text{ A}$$

$$U_L = I \cdot X_L = 26.8 \text{ V}$$

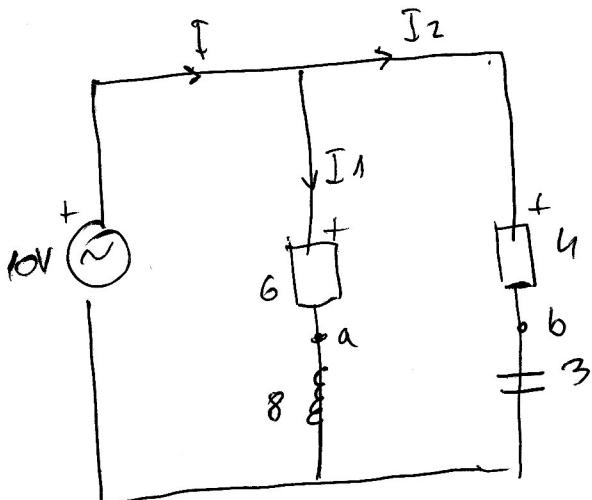
$$U_C = I \cdot X_C = 134 \text{ V}$$

(6)



Komentar: struje je lakše crtaći kada ih ponovo
rastavimo na komponente!

(7)



$$u_{ab} = ? \rightarrow od b prema a$$

$$\dot{u}_{ab} = 4 \cdot I_2 - 6 \cdot \dot{I}_1$$

$$Z_1 = 6 + j8$$

$$\dot{I}_1 = \frac{\dot{u}}{Z_1} = \frac{10}{6+j8} = 1 \angle -53.13^\circ \\ = 0.6 - j 0.8$$

$$\dot{Z}_2 = 1.6 - j 1.2 \rightarrow \\ \dot{I}_2 = \frac{\dot{u}}{\dot{Z}_2} = 0.4 + j 0.3$$

$$\dot{u}_{ab} = 6 \cdot 4 + j 4 \cdot 8 - 3 \cdot 6 + j 4 \cdot 8 \\ = 2.8 + j 9.6 \\ = \sqrt{2.8^2 + 9.6^2} \quad \boxed{\text{arc tg } \frac{9.6}{2.8}}$$

Frekv. ovisnost

Rezonancija:

$$\operatorname{Im}\{\dot{Z}_{ur}\} = 0$$

$$\operatorname{Im}\{\dot{Y}_{ur}\} = 0$$

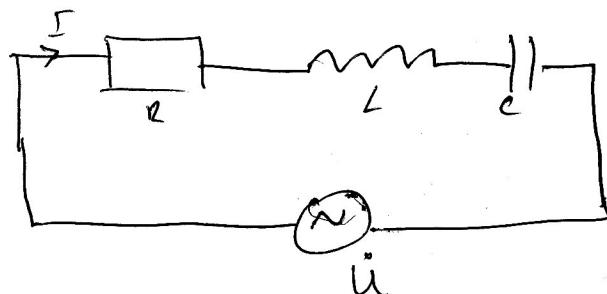
$$\dot{I} = \frac{\dot{U}}{Z} = \frac{\dot{U}}{R}$$

↓

ukupna \dot{I} i ukupan
otpor - u fazu

↓

rezonancija



$$\dot{Z}_{ur} = R + jX_L + jX_C = R$$

$$\operatorname{Im}\{\dot{Z}_{ur}\} = X_L - X_C = 0 \Rightarrow \text{rezon.}$$

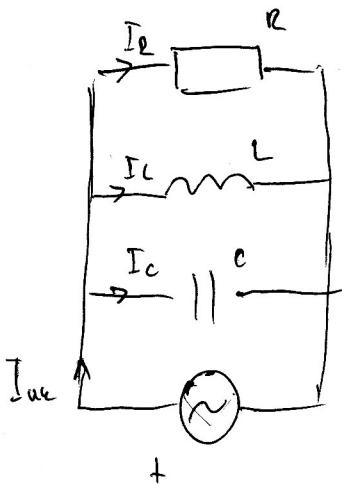
$$X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C}$$

$$\omega_R = \frac{1}{\sqrt{LC}}$$

$$\dot{U}_L = \dot{I} \cdot jX_L \quad \dot{U}_C = \dot{I} \cdot (-jX_C)$$

(8)

Paralelni RLC kružnici



↓
struja na L i C
postoji, ali one
se ponište!

$$\frac{1}{Z_{uu}} = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C}$$

$$Y_{uu} = G - jB_L + jB_C = 0$$

Y-uvodimo vodljivost

$$B_C = B_L$$

$$W_C = \frac{1}{WL}$$

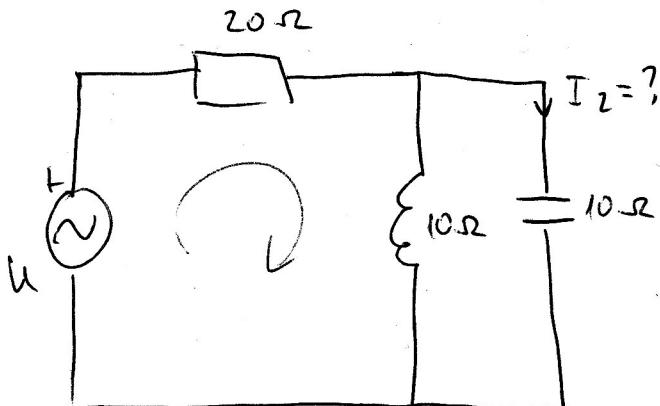
$$\dot{I} = \frac{\dot{U}}{Z_{uu}} = \frac{\dot{U}}{R}$$

$$\frac{1}{R_{uu}} = \frac{1}{R}$$

$$\dot{Y}_{uu} = G$$

$$\dot{I}_R = \frac{\dot{U}}{R}$$

$$\dot{I}_L = \frac{\dot{U}}{jX_L}$$



$$\dot{Z}_{uu} = 20 + \frac{j10(-j10)}{j10 + (-j10)} \rightarrow = 0$$

$$= 20 + \infty = \infty$$

$$\dot{I} = \frac{\dot{U}}{Z_{uu}} = \frac{20}{\infty} = 0A$$

$$\dot{U}_R = \dot{I} \cdot R = 0V$$

$$\dot{U}_P = \dot{I} \cdot \frac{100}{0} = \frac{0}{0} \rightarrow \text{nedef. obl.}$$

$$\dot{U}_P = \dot{U} = 20 \angle 0^\circ$$

$$\dot{I}_C = \frac{\dot{U}_P}{-jX_C} = \frac{20}{-j10} = j2A$$

$$\dot{I}_L = \frac{\dot{U}_P}{jX_L} = \frac{20}{j10} = -j2A$$

$$\dot{I} = \dot{I}_L + \dot{I}_C$$

$$0 = -j2 + j2$$

Realni svitak

(9)

zad: odredi frekv. da su I_{UR} i U u fazi

→ znači da su pod istim kutem

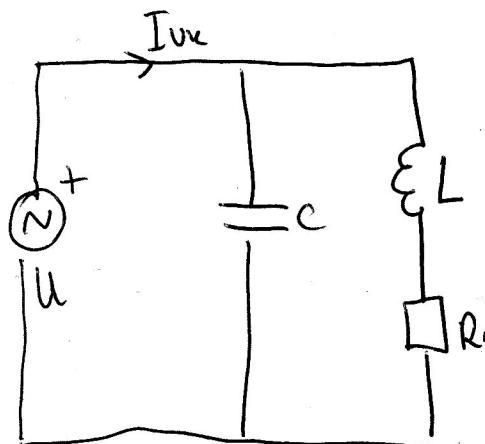
→ znači da su u rezon.

→ znači da je $\text{Im}(Z_{UR}) = 0$

$$R_L = 5 \Omega$$

$$L = 20 \text{ mH}$$

$$C = 300 \text{ nF}$$



$$\dot{Z}_{UR} = \frac{(R + jX_L)(-jX_C)}{R + jX_L - jX_C}$$

$$= \frac{X_L X_C - jR X_C}{R + j(X_L - X_C)} \quad / \cdot \frac{R - j(X_L - X_C)}{R - j(X_L - X_C)}$$

$$= \frac{R X_L X_C - R X_C (X_L - X_C) - j[X_L X_C (X_L - X_C) + R^2 X_C]}{R^2 + (X_L - X_C)^2}$$

$$\text{Im}(Z_{UR}) = - \frac{X_L X_C (X_L - X_C) + R^2 X_C}{R^2 + (X_L - X_C)^2} = 0$$

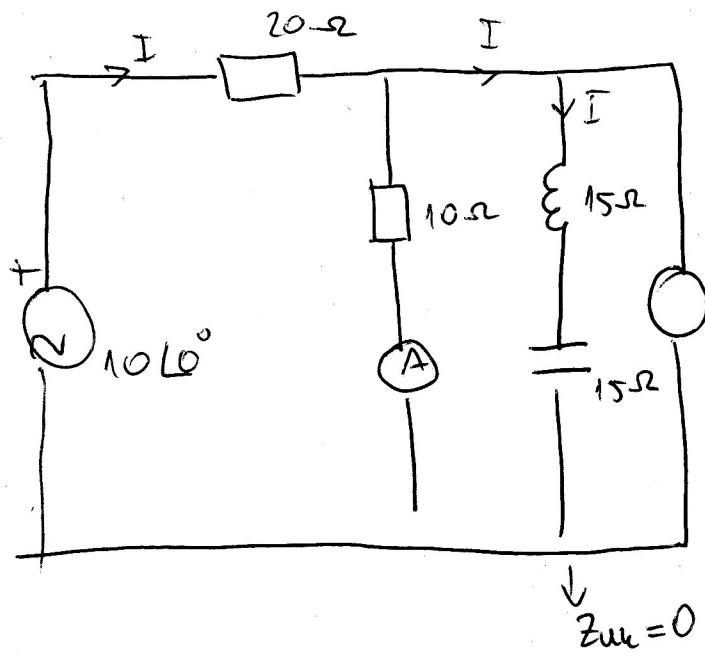
$$-X_L^2 + X_L X_C = R^2$$

$$X_L^2 = X_L X_C - R^2$$

$$(WL)^2 = WL \cdot \frac{1}{WC} - R^2$$

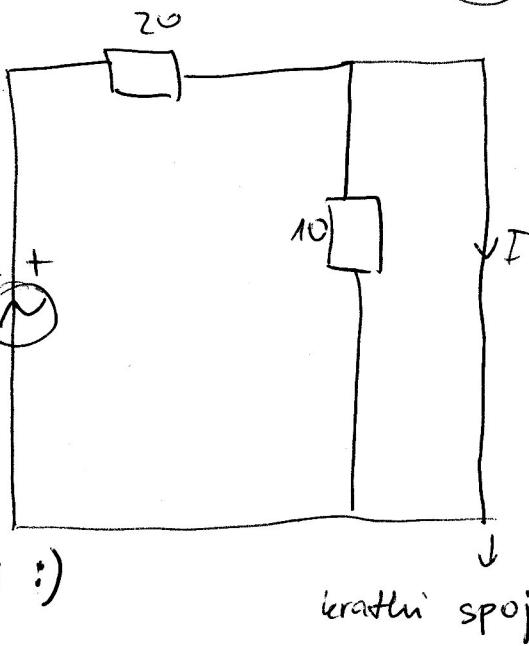
$$W^2 = \frac{1}{LC} - \left(\frac{R}{L}\right)^2$$

10



matnemosi
instrum.
jer ih
Luka
ne voli :)

$$Z_{\text{ur}} = 0!$$

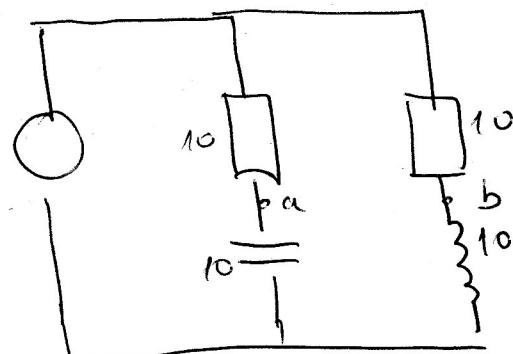
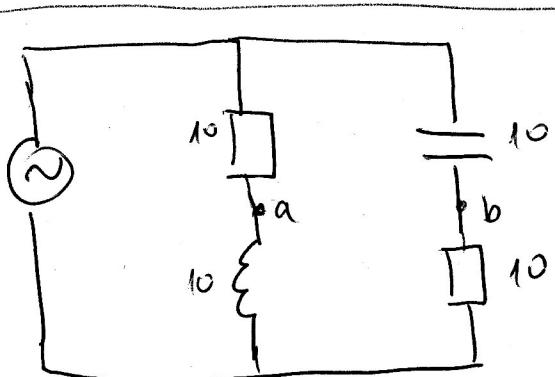


$$Z_{\text{ur}} = 20\Omega$$

$$U = 10V \angle 0^\circ$$

$$I = \frac{10 \angle 0^\circ}{20} = 0.5 \angle 0^\circ$$

$$\dot{U}_L = I \cdot jX_L = 0.5 \angle 0^\circ \cdot 15 \angle 90^\circ \\ = 7.5 \angle 90^\circ = j 7.5 V$$



ako je umnožak nasuprotnih
isti, onda je napon između
točaka :

$$10 \cdot 10 = j 10 (-j 10)$$

$$100 = 100$$



$$U_{ab} = ?$$

$$10 \cdot j 10 = 10 (-j 10)$$

$$j 100 \neq -j 100$$

~~$$U_{ab} = 0$$~~