OSNOVE ELEKTRICITETA

- **Makroskopski pristup** → pojednostavljeni oblik građe atoma

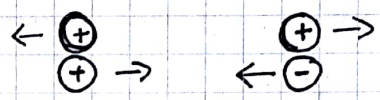
$e = 1{,}6 \cdot 10^{-19}\,[As] = [C]$ elementarni naboj

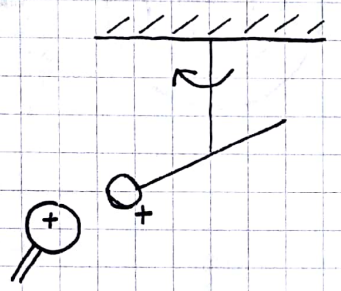
Dobri vodiči - elektroni koji se „slobodno" mogu kretati po kristalnoj rešetci

Izolatori - elektroni su vezani u strukturi materijala

Plinovi - ako su ionski onda mogu

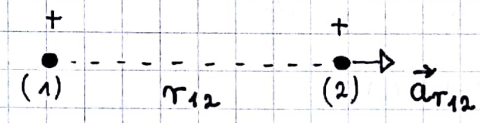
istoimeni naboji se odbijaju
raznoimeni naboji se privlače





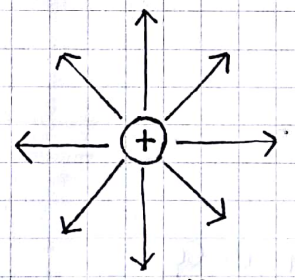
$$F \sim \frac{1}{r^2}$$

$$F = \frac{1}{4\pi\varepsilon_0}\,\frac{Q_1 Q_2}{r^2}$$



$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\,\frac{Q_1 Q_2}{r_{12}^2}\,\vec{a}_{r_{12}}$$

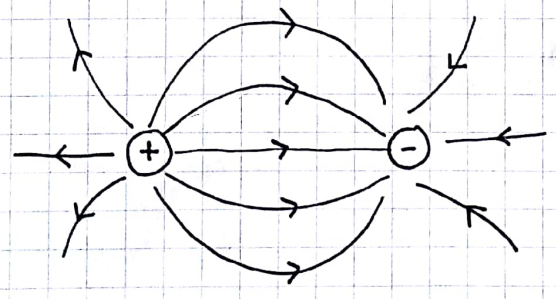
$\vec{a}_{r_{12}}$ jedinični vektor u smjeru spojnice



E - električno polje

(- naboj - silnice radijalno prema naboju)

silnice



\+ izvor polja E

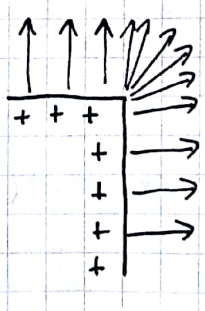
\- ponor polja E

$$\vec{E} = \frac{\vec{F}}{q}$$ ← probni ispitni naboj „MALI"

$$\lim_{q \to 0} \frac{\vec{F}}{q}$$

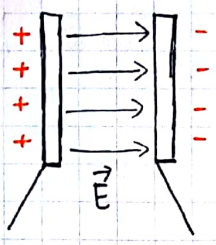
silnice polja jakost polja

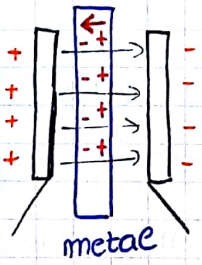
gušće silnice → veći iznos polja



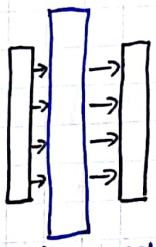
paralelne i ekvidistantne silnice
↳ HOMOGENO POLJE

Maxwellove dvopečće - INFLUENCIJA

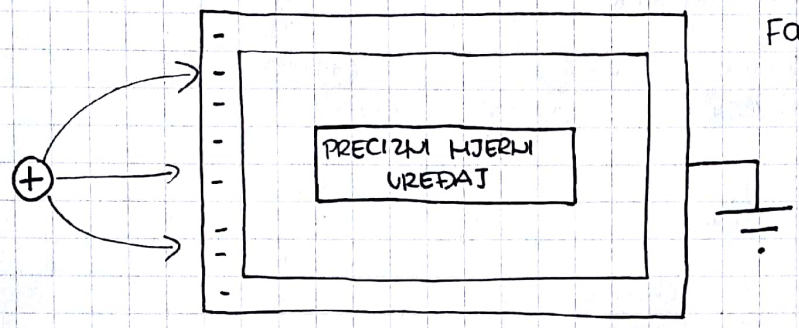




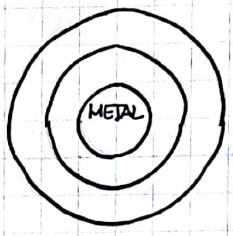
elektroni će ići sve dok ne uspostave svoje električno polje koje će poništavati vanjsko

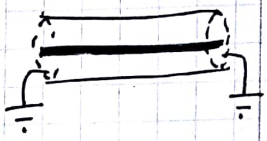


u dobrim vodičima statičko el. polje $E = 0$



Faradejev kavez





Zadaci:

1. $|q| = 20\ \mu C$
   $F = 0{,}1\ N$

a) $F = \frac{q^2}{4\pi\varepsilon_0 d^2}$

$d = \sqrt{\frac{q}{4\pi\varepsilon_0 F}}$ ↑ $8{,}854 \cdot 10^{-12}\ \frac{As}{Vm}$

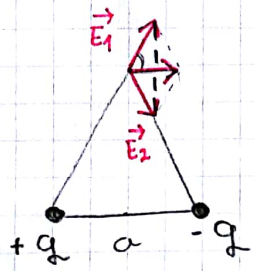
$d = 6\ m$ dielektričnost slobodnog prostora

b) predznaci su različiti

c) $E = \frac{q}{4\pi\varepsilon_0 d^2} = 5\ kV/m$

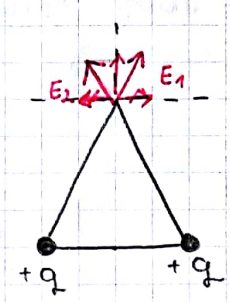
$E = \frac{F}{q} = 5\ kV/m$

2.



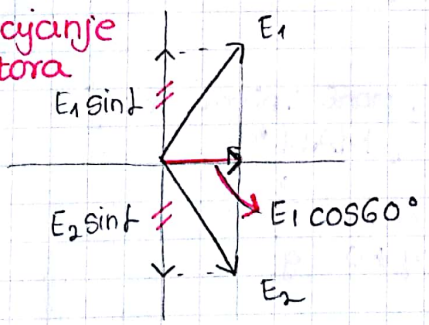
$E = \frac{q}{4\pi\varepsilon_0 r_2}$

$E = 2E_1 \cos 60°$
$= 200\ V/m$



$E = 2E_1 \sin 60°$
$= 346{,}4\ V/m$

Zbrajanje vektora



$|q_1| > |q_2|$

$E_1 = E_1 \cos 60 + E_1 \sin 60°$
$\quad x \qquad\quad y$

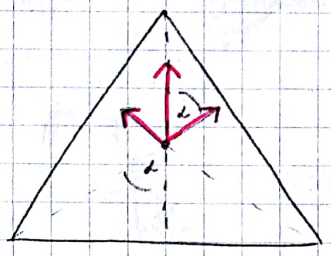
$E_2 = E_2 \cos 60 - E_2 \sin 60°$

$E = [E_1 \cos 60° + E_1 \sin 60°]\ x^S$

$+ [E_1 \sin 60° - E_2 \sin\theta$

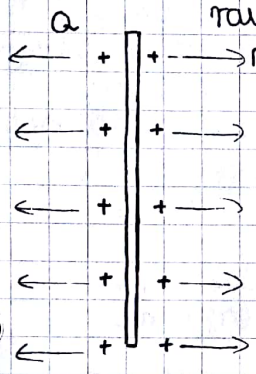
$|\vec{E}| = \sqrt{(\cos 60°(E_1 + E_2))^2 + (\sin 60°(E_1 - E_2))^2}$

d) na visini h iznos E?



$$E = 2 \cdot \frac{q}{4\pi\varepsilon_0 \left(\sqrt{\left(\frac{a}{2}\right)^2 + h^2}\right)^2} \cdot \frac{h}{\sqrt{\left(\frac{a}{2}\right)^2 + h^2}} =$$

$$\frac{2qh}{4\pi\varepsilon_0 \left[\left(\frac{a}{2}\right)^2 + h^2\right] \cdot \frac{3}{2}}$$



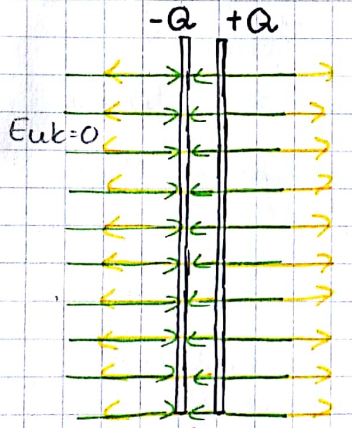
a

ravnomjerno raspoređen naboj Q

$\frac{Q}{S} = \sigma$ površna gustoća naboja $\left[\frac{C}{m^2}\right]$

$E = \frac{\sigma}{2\varepsilon_0}$ homogeno polje i neovisno o udaljenosti (vrijedi samo za male udaljenosti)

* PRINCIP SUPERPOZICIJE (zbrajanje doprinosa)



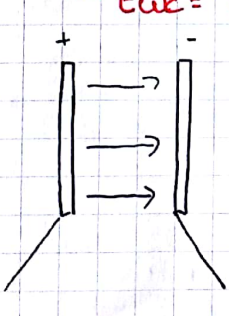
-Q +Q

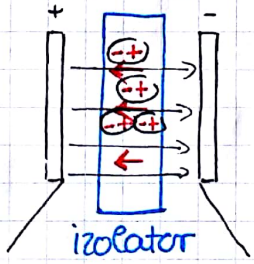
$E_{uk} = 0$

$E_{uk} = 0$

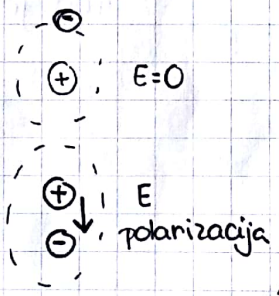
$E_{uk} = E_{+Q} + E_{-Q}$

$E_{uk} = 2E_{+Q} = \frac{2\sigma}{2\varepsilon} = \frac{\sigma}{\varepsilon}$





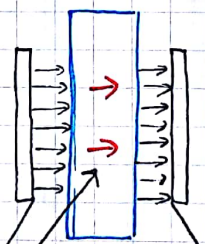
izolator



E=0

E

polarizacija



E u izolatoru $= \vec{E} + \vec{E}_{polarizacija}$

$|E \text{ u izolatoru}| < |\vec{E}|$

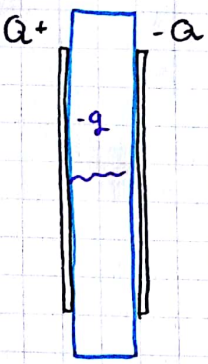
zbog toga smanjen

$\varepsilon = \varepsilon_r \varepsilon$

ε_r relativna dielektrična konst.

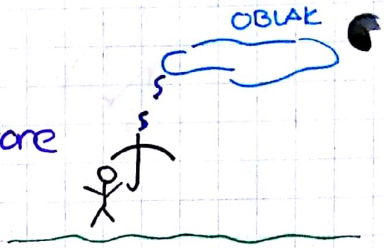
PROBOJNA ČVRSTOĆA

$E_p = \left[\frac{kV}{mm}\right]$ granična vrijednost kod koje izolator prestaje bit izolator



$Q \uparrow \quad \sigma \uparrow \quad E \uparrow$

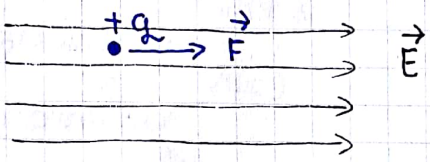
postane toliko jako da počne čupat elektrone uspostavi se vodljiva staza munja



uspostavi se vodljiva staza i proteče velika struja probije se zrak kaže zbog zasićenosti vegom

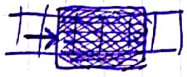
POMICANJE NABOJA U EL. POLJU

C. sila može obaviti rad u svom smjeru

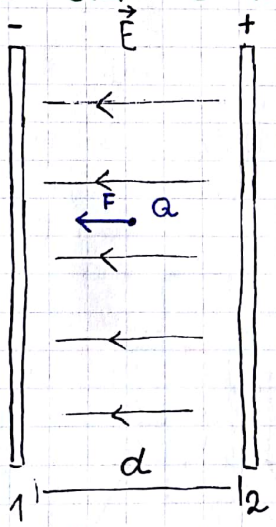


$\vec{F} = q\vec{E}$

$A = \vec{F} \cdot \vec{s} = F \cdot s \cdot \cos(\vec{F}, \vec{s})$



RAD SILA U EL. POLJU



$A_{1,2} = \vec{F}_{12}\vec{s}$
$= F \cdot d \cdot \cos(180°)$
$= -QEd < 0$

↓

rad je utrošen

ELEKTRIČNI POTENCIJAL

$\varphi = \frac{W}{Q}$

$\varphi_1 - \varphi_2 = \frac{W_1 - W_2}{Q} = \frac{A_{12}}{Q} = U_{12}$ el. napon [V]

$A_{12} = W_1 - W_2 = Q(\varphi_1 - \varphi_2) = QU_{12} = -QEd \quad , \quad U_{12} = -Ed$

naboji bi putovali dok postoji razlika φ PA JE ZATO TAKO?

$\varphi_1 - \varphi_2 = U_{12} = -Ed$

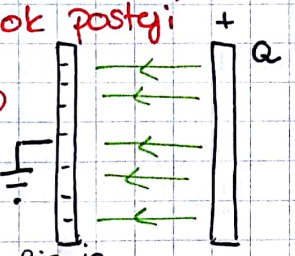
$\varphi_1 = \varphi_2 - Ed$

$\varphi(x) = \varphi_2 - Ex$

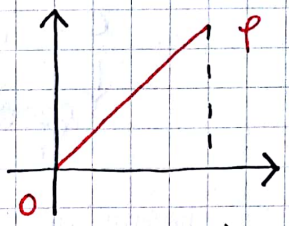
$$\left[\varphi = -\int \vec{E}\cdot d\vec{e} = -\int E \cdot de \right]$$

$$\varphi = Ex + C$$

zbog uzemljenja je potencijal ove ploče 0

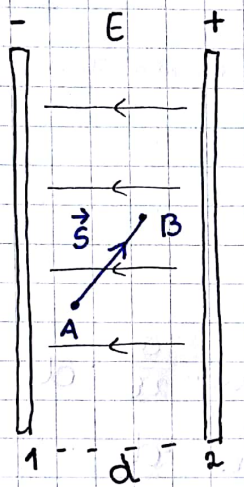


OVA PRIČA VRIJEDI SAMO ZA HOMOGENA POLJA



(polje točkastog naboja nije konst

$E = \frac{F}{q} = \frac{Q}{4\pi\varepsilon_0 r^2}$



$\varphi_B = -\vec{E}\cdot\vec{s} + \varphi_A = -E\cdot s\cdot\cos(\vec{E},\vec{s}) + \varphi_A$

rad se troši samo u vodoravnom smjeru

ako se naboj kreće i vrati u istu točku $W = 0$

$\varphi(x) = -Ex\cos(180°) + (\varphi = 0)$

$\varphi(x) = Ex$

Zadaci:

3. $Q = 20\ nC$

$F = 0.1\ N$

$E = \frac{F}{Q} = 5\ kV/m$

b) $d = 0,1\ m$

$A_{AB} = F\cdot d = 0,01\ J$

rad je poz. => obavilo ga je polje

c) $U_{AB} = \varphi_A - \varphi_B$

$\frac{A_{AB}}{q} = \frac{10\cdot 10^{-3}}{20\cdot 10^{-6}} = 500\ V$

$\varphi_B = \varphi_A - U_{AB}$

$\varphi_B = 0$

d) $W_{PA} = q\cdot\varphi_A = 10\ mJ$

$W_{PB} = q\,\varphi_B = 0$

PAZI NA PREDZNAKE!

4. $\sigma = 7,07\cdot 10^{-9}\ C/m^2$

$d_{AB} = 5\ cm$

$d_{BC} = 3\ cm$

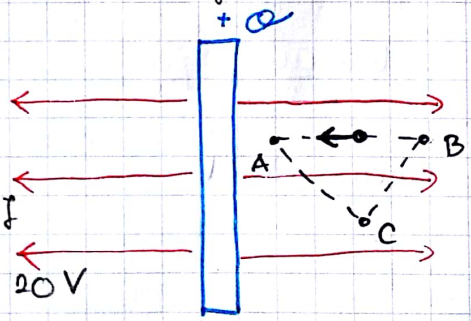
$d_{AC} = 4\ cm$

$q = -2\ mC$

$E = \frac{\sigma}{3\varepsilon} = 400\ V/m$

$A_{AB} = \vec{F}_{el}\cdot\vec{s} = F_{el}\cdot s\cdot\cos(F_{el}s) =$

$= -|q|\,E\,d_{AB} = -0,04\ J$



$A_{BC} = F_{el}\cdot s\cdot\cos\alpha$

$= |q|\cdot E\cdot d_{BC}\cdot\cos\alpha$

$= 0,0144\ J$ rad poz.

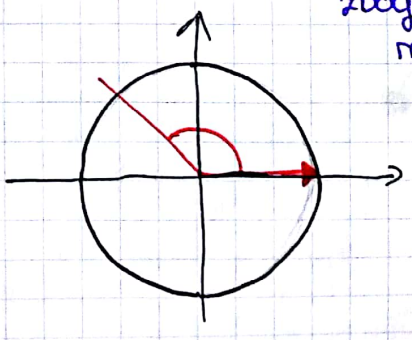
$\cos\alpha = \frac{d_{BC}}{d_{AB}}$

$A_{AC} = A_{AB} + A_{BC} = 0,0256\ J$

b) $U_{AB} = \frac{A_{AB}}{q} = \frac{-0,04}{-2\cdot 10^{-3}} = 20\ V$

$U_{AC} = \frac{A_{AC}}{q} = 12,8\ V$

$U_{BC} = \frac{A_{BC}}{q} = -7,2\ V$



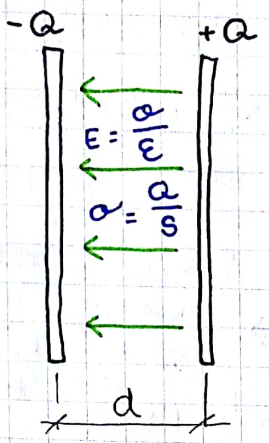
zbog vektora mora biti između F i S

c) $E = \frac{\sigma}{2\varepsilon_0}$

$\sigma_p = 53\ \mu C/m^2$

ELEKTRIČNI KAPACITET

ON NIJE OVISAN NI O NAPONU, NI NABOJU



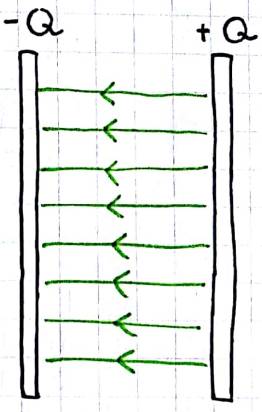
$U = E \cdot d$ $\quad C = \frac{Q}{U}$ [F] farad

$C = \frac{\sigma \cdot S}{E \cdot d} = \frac{\sigma \cdot S}{\frac{\sigma}{\varepsilon} \cdot d} = \varepsilon \cdot \frac{S}{d}$ izraz za kapacitet pločastog kondenzatora

↓ geometrijski ovisan i o svojstima materijala

temeljno svojstvo kond. je njegov kapacitet

ENERGIJA NABIJENOG KONDENZATORA



$F = QE$

$|E_{-Q}| = \frac{\sigma}{2\varepsilon}$

$F = Q\frac{\sigma}{2\varepsilon}$

$W = F \cdot s = Q \cdot \frac{\sigma}{2\varepsilon} \cdot d =$

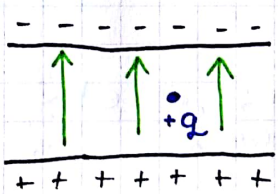
$= Q \cdot \frac{Q/S}{2\varepsilon} \cdot d$

$= \boxed{\frac{Q^2}{2C}} = \left[Q = CU \right] = \frac{CU^2}{2} = \frac{QU}{2}$

odspojen od izvora →

ako razmičem ploče uložem E jer se ploče privlače pa povećavam E kond. (ODSPOJEN!)

Zadatak:



$F_{el} = F_g$

$mg = qE = q \cdot \frac{U}{d}$

$U = \frac{mgd}{q} = 1\ kV$

$W = \frac{QU}{2} = 165\ \mu J$

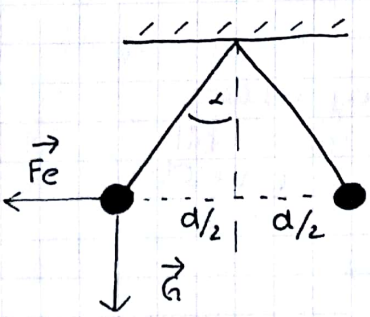
d) $U \uparrow 2x$

C konst

$Q \uparrow 2x$

$W = \frac{QU}{2} \uparrow 4x$

e) $E_p = \frac{U_{maks}}{d}$ $\quad U_{maks} = 30\ kV$

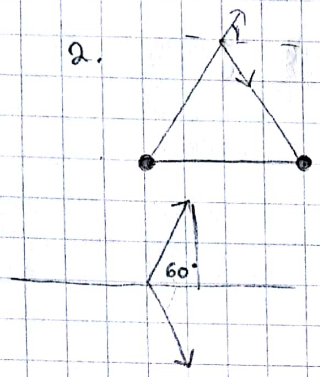


Primjeri iz prezentacije

1. $|q| = 20 \cdot 10^{-6}$ C
   d
   $F = 0,1$ N
   b) raznoimeni
   c) $E = \frac{F}{q} = 5000$ V/m

$$0,1 = 9 \cdot 10^9 \cdot \frac{(20 \cdot 10^{-6})^2}{d^2} \Rightarrow d = 6 \text{ m}$$

2.

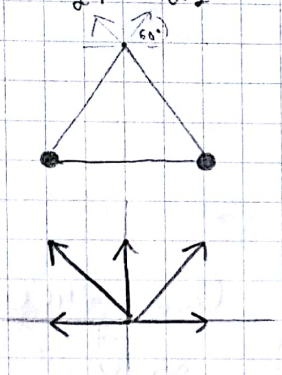


a) $q_1 = -q_2 = 2 \cdot 10^{-9}$ C

$E_1 = E_1 \cos 60° + E_2 \sin 60°$
$E_2 = E_1 \cos 60° - E_2 \sin 60°$ } $2 \cos 60°$

$E = 2E_1 \cos 60° = 200$ V/m

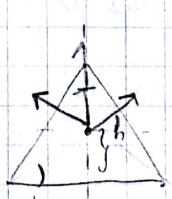
b) $q_1 = q_2 = 2$ mc



$\vec{E_1} = E_1 \cos 60° + E_1 \sin 60°$
$E_2 = -E_2 \cos 60° + E_2 \sin 60°$

$E = 2E_2 \sin 60° = 346.4$ V/m

c)



$E_1 = E_1 \cos \frac{d}{2h} + E_1 \sin \frac{2h}{d}$

$E_2 = -E_2 \cos \frac{d}{2h} + E_2 \sin \frac{2h}{d}$

$E = 2E \sin \frac{2h}{d}$

$\sin \frac{\frac{d}{2}}{h} = \frac{2h}{d} \cos \frac{\frac{d}{2}}{h} = \frac{d}{2h}$

3. $q = 20 \cdot 10^{-6}$ C
   $F = 0,1$ N

a) $E = \frac{F}{q} = 5$ kV/m

b) $\varphi_1 - \varphi_2 = \frac{W}{q} = \frac{\Delta W}{q} \Rightarrow W = q \cdot U = qEd = 0,01$ J

c) $500 - \varphi_B = U$
   $\varphi_B = 0$ V

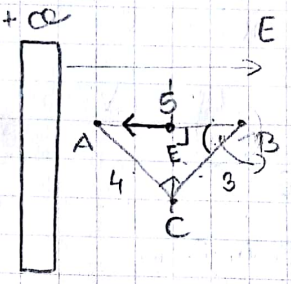
d) $W_A = 0,01$ J
   $W_B = 0$ J

4. $\sigma = 1{,}07 \cdot 10^{-9}\ C/m^2$

$q = -2 \cdot 10^{-3}\ C$

$W_{AB} = -EdQ = -0{,}04\ J$

$U = Ed$



$\cos\alpha = \frac{0{,}03}{0{,}05}$

$\cos\alpha = \frac{V}{0{,}03}$

$W_{BC} = V \cdot \cos\alpha \cdot q \cdot E = 0{,}0144\ J$

$W_{AC} = -0{,}0256\ J$

$E = \frac{\sigma}{2\varepsilon} = 400\ V/m$

b) $U_{AB} = 20V$

$U_{BC} = -7{,}2V$

c) $E = \frac{\sigma}{2\varepsilon} \Rightarrow \sigma = 5{,}3124 \cdot 10^{-5}\ \frac{As}{m^2}$

Primjer 1,

a) $+Q_1$ $\vec{F}_{21}$ $+Q_2$ $\vec{F}_{12}$ $F = 3.595 \cdot 10^{-3}$

b) 4 × veća c) $E = \frac{F}{q} = 3{,}595 \cdot 10^{6}\ \frac{V}{m}$

4. $Q = 50 \cdot 10^{-9}\ C$

$W_A = 2 \cdot 10^{-5}\ Ws$

$W_B = 4 \cdot 10^{-5}\ Ws$

$A_{AB} = W_A - W_B = 2 \cdot 10^{-5}\ Ws$

$U_{AB} = -400V$

$P_A = \frac{W_A}{Q} = 400\ V$

$P_B = \frac{W_B}{Q} = 800\ V$

5. $P_1 = -40V$

$P_2 = 0\ V$

$\Delta W = 0{,}1 \cdot 10^{-3}\ Ws$

$U_{12} = -40 - 0 = -40V$

$U = \frac{W}{Q} \Rightarrow Q = -2{,}5\ 10^{-6}\ C$

naboj ide iz nižeg potencijala u točku višeg

↓

NABOJ JE NEGATIVAN

6. $S = 0{,}5\ m^2$

$d = 1 \cdot 10^{-3}\ m$

$\varepsilon_r = 1$

$E_p = 3\ kV/mm$

$U = 1000\ V$

$C = \varepsilon_0 \varepsilon_r \frac{S}{d} = 4{,}427 \cdot 10^{-9}\ F$

$C = \frac{Q}{U} \Rightarrow Q = CU = 4{,}427 \cdot 10^{-6}\ C$

$W = \frac{CU^2}{2} = 2.213 \cdot 10^{-3}\ J$

$E = \frac{U}{d} \Rightarrow Ed = 3 \cdot 10^{3}$

$\sigma = \frac{Q}{S} = 8{,}854\ \mu As/m^2$

* ako se proboj dogodi u krutom izolatoru dolazi do trajnog uništenja

ZADACI IZ ZBIRKE

1. $|Q_1 / Q_2| = m$

$m = 10$

$Q_1 = +30\,mC$

$d = 0{,}1\ m$

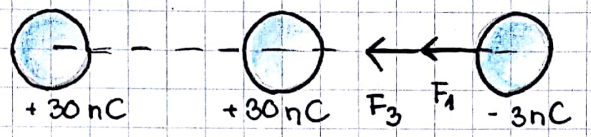
Q_2 je negativan

$$\left|\frac{30\cdot10^{-9}}{Q_2}\right| = 10 \qquad Q_2 = -3\cdot10^{-9}\,C$$

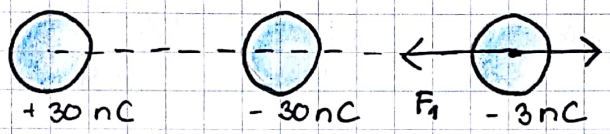
$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 d^2} = 81\,\mu N$$

$F_2 = 324\ \mu F$

2.

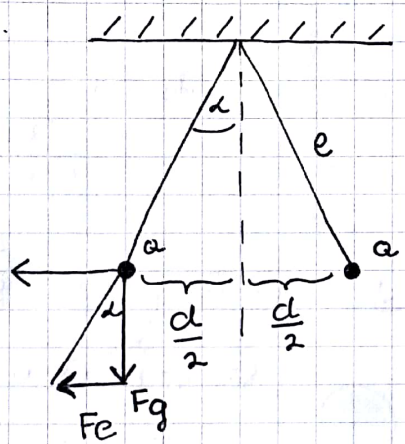


$F = F_1 + F_3 = 405\ \mu C$



$F = |F_1 - F_3| = 243\ \mu C$

3.



$m = 10^{-3}\ kg$

$e = 1\ m$

$d = 0{,}1\ m$

$h = \sqrt{1^2 - 0{,}05^2} = 0.9987$

$$tg\,\alpha = \frac{d}{2\sqrt{e^2 - \frac{d^2}{4}}}$$

$$tg\,\alpha = \frac{Fe}{Fg}$$

$$\frac{d}{2\sqrt{e^2 - \frac{d^2}{4}}} = \frac{\frac{Q^2}{4\pi\varepsilon_0 d^2}}{\frac{mg}{1}} = \frac{Q^2}{4\pi\varepsilon_0 d^2 mg}$$

$$2Q^2\sqrt{e^2 - \frac{d^2}{4}} = 4\pi\varepsilon_0 d^3 mg$$

$$Q = \sqrt{\frac{4\pi\varepsilon_0 d^3 mg}{2\sqrt{e^2 - \frac{d^2}{4}}}}$$

$$Q = 2{,}3447\cdot10^{-8}\,C$$

$$\frac{\frac{d}{2}}{\sqrt{e^2 - \frac{d^2}{4}}} = \frac{Fe}{mg}$$

$e >> d$

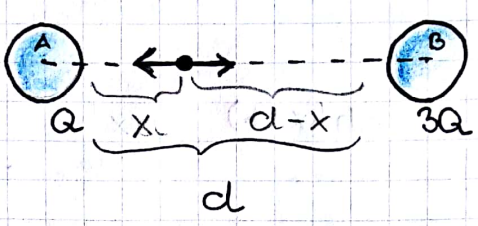
$$\frac{d}{2} = \frac{\frac{3}{16}Q^2}{\frac{4\pi\varepsilon_0 d^2}{mg}} = \frac{3Q^2}{64\pi\varepsilon_0 d^2 mg}$$

$Q_{uk} = 4{,}6894\cdot10^{-8}$

$$64\pi\varepsilon_0 d^3 mg = 6Q^2$$

$$d = \sqrt[3]{\frac{6Q^2}{64\pi\varepsilon_0 mg}} = 0{,}0905\ m$$

4. $Q, 3Q$
A, B, d



$F_{AC} = F_{BC}$

$$\frac{Q \cdot q}{4\pi\varepsilon_0 x^2} = \frac{3Q \cdot q}{4\pi\varepsilon_0 (d-x)^2}$$

$$\frac{1}{x^2} = \frac{3}{d^2 - 2xd + x^2}$$

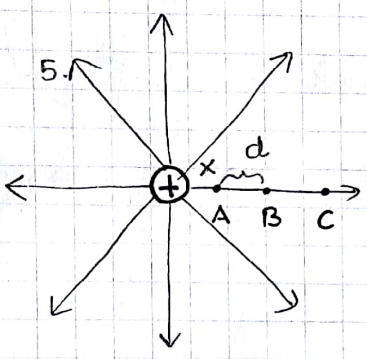
$$d^2 - 2xd + x^2 = 3x^2$$

$$d^2 - 2xd - 2x^2 = 0$$

$$x = \frac{2d \pm \sqrt{4d^2 + 8d^2}}{-4} = \frac{2d \pm 2d\sqrt{3}}{-4} \quad \frac{2d(1-\sqrt{3})}{-4} = \frac{d(1-\sqrt{3})}{-2}$$

$$x = 0{,}366\, d$$

5.



$E_A = 36$ V/m
$E_C = 9$ V/m

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \Rightarrow 9 \cdot 4\pi\varepsilon_0 0{,}01^2 = Q$$

$$Q = 1 \cdot 10^{-3}\ C$$

$$\frac{Q}{4\pi\varepsilon_0 \frac{9}{4} x_2}$$

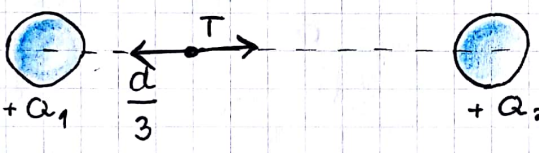
PROVJERI ZA A. ✓

$$E_A = \frac{Q}{4\pi\varepsilon_0 x^2} \quad \frac{4}{9} \frac{Q}{4\pi\varepsilon_0 x^2}$$

$$E_B = \frac{Q}{4\pi\varepsilon_0 (x+d)^2} = \frac{Q}{4\pi\varepsilon_0 (x}$$

$$E_C = \frac{Q}{4\pi\varepsilon_0 (x+2d)^2}$$

6. $Q_1 = 1 \cdot 10^{-9}$ C
$d = 0{,}01$ m



$$\frac{10^{-9}}{4\pi\varepsilon_0 \left(\frac{1}{300}\right)^2} = \frac{Q}{4\pi\varepsilon_0 \left(\frac{1}{150}\right)^2} \qquad \left|\frac{Q_2}{Q_1}\right| = 4$$

$$\frac{E_A}{E_C} = \frac{(x+2d)^2}{x^2} = \frac{36}{9} = 4$$

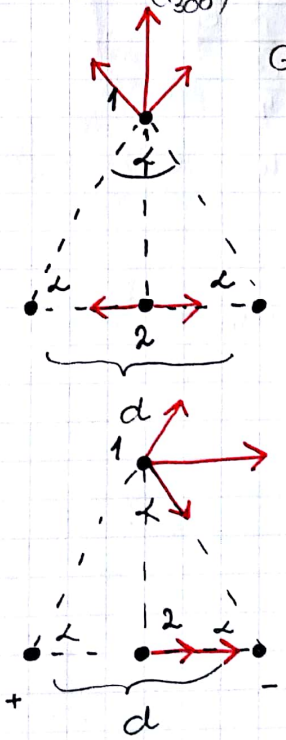
$$x + 2d = 2x$$

$$x = 2d$$

$Q = 4 \cdot 10^{-9}$ C

Kako može imati potencijal ako je el. polje jednako 0 ?

7.



$E_1 = E_2$

$E_{uk_1} = 2E\sin\delta = 6226{,}89$ V/m

$Q = 1 \cdot 10^{-9}$ C
$d = 0{,}05$ m

$E_{uk_2} = 0$

$$\frac{E_{uk_2}}{E_{uk_1}} = 0$$

$E_{uk_1} = 2E\cos\delta = 3595$ V/m

$E_{uk_2} = 28760.7758$ V/m

$$\frac{E_{uk_2}}{E_{uk_1}} = 8$$

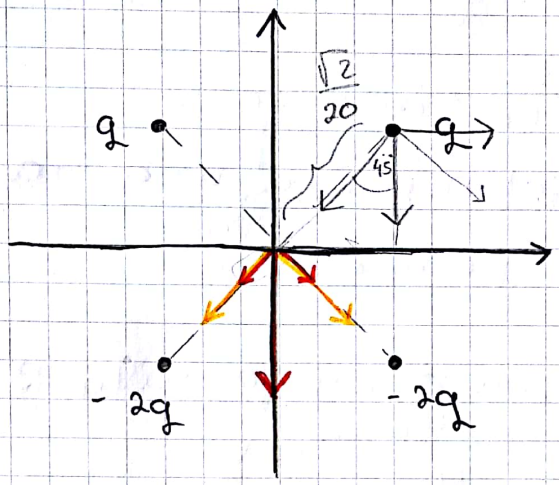
8. $q = 10^{-12}$ C
$W_1 - W_2 = -360 \cdot 10^{-12}$ Ws

* razlika potencijala točaka 1 i 2 iz prethodnog zadatka

$U_{12} = -360$ V

Naboji su pozitivni. Zašto?

9.



$E_{uk} = 2E_q \sin 45° + 2E_{2q} \sin 45°$
$= 2542.12 + 5084,23$
$E_{uk} = 7626$ V/m

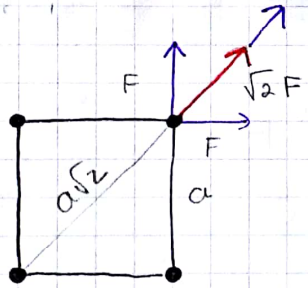
P u polju točkastog naboja

$$P = \frac{Q}{4\pi\varepsilon r}$$

Kako izračunati potencijal u ishodištu???

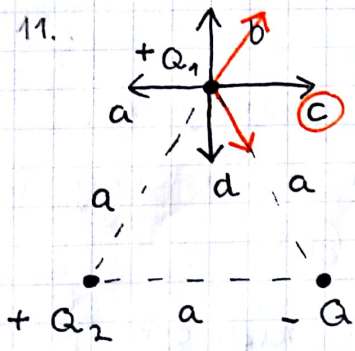
-254

10.



$\sqrt{2}F + \frac{F}{2} = 1.914F$

11.

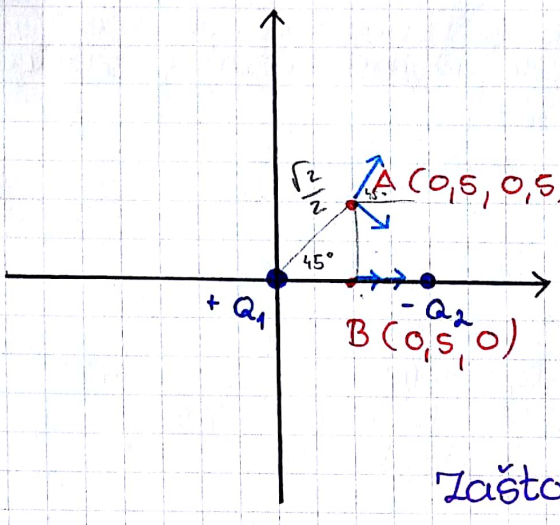


c smjer

$Q_1 = Q_2 = 100 \cdot 10^{-9}$ C
$Q_3 = -100 \cdot 10^{-9}$ C
$a = 0,02$ m

$F_{uk} = 2F\cos 45° = 224,69$ mN

12.



$Q_1 = 10^{-9}\ C$
$Q_2 = -10^{-9}\ C$
$d = 1\ m$

$E_B = E_1 + E_2 = 71,9\ V/m$

$E_A = 2\cos 45 \cdot E = 25.42\ V/m$

$\frac{E_A}{E_B} = 0.35356$

Zašto je potencijal u točkama A i B jednak 0?

13. $q = 10^{-6}\ C$
$W_A - W_B = -20 \cdot 10^{-6}\ Ws$ ma višem potencijalu je točka B

$U = \frac{W}{q} = -20\ V$ $\varphi_A - \varphi_B = \frac{W_A - W_B}{q} = -20V$

14. $W_{PA} = -10^{-4}\ Ws$

$W_{PB} = -2 \cdot 10^{-4}\ Ws$ (ma nižem potencijalu)
↳ netko drugi obavio rad

$q = -2.5 \cdot 10^{-7}\ C$

$W = 1 \cdot 10^{-4}\ Ws$

$A = W_{PA} - W_{PB}$

15. $S = 0.02\ m^2$
$d = 10^{-3}\ m$
$U = 1700\ V$

$C = \varepsilon_0 \varepsilon_r \frac{S}{d} = 1,7708 \cdot 10^{-10}\ F$

$C = \frac{Q}{U} \Rightarrow Q = UC = 301\ mC$

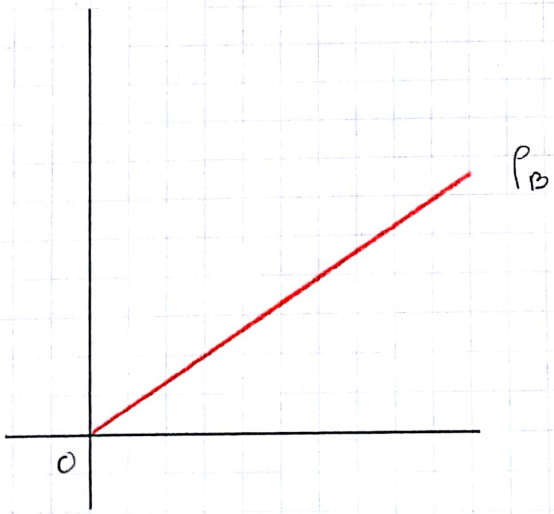
16. $d = 0.04\ m$
$q = 10^{-9} \cdot 10\ C$
$W = 20 \cdot 10^{-6}\ J$

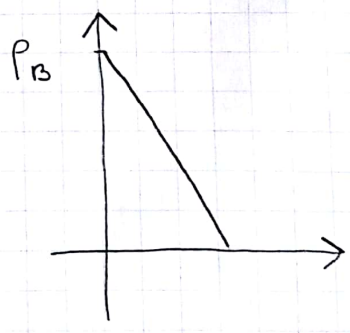
$U = \frac{W}{q} = 2000\ V$

Potencijal negativne ploče je 0 jer je uzemljena.

$E = U/d = 50\,000\ V/m$ * E. polje je gradijent potencijala

$E(\frac{d}{2}) > E(d)$
↑ E za razmak $\frac{d}{2}$??





17. $S = 0.01\ m^2$ $\quad C = \varepsilon_0 \varepsilon_r \frac{S}{d} = 1.3281 \cdot 10^{-9}$

$d = 2 \cdot 10^{-4}\ m$

$E_p = 60\ kV/cm$ $\quad Q = CU = 1{,}59 \cdot 10^{-6}\ C$

$U_{maks} = ?$ $\quad W = \frac{QU}{2} = 9{,}56 \cdot 10^{-4}\ J$

$U = E_p \cdot d = 1200\ V$ $\quad F = \frac{W}{d} = 4{,}78\ N$?

18. $U = 400\ V$

$Q = 60 \cdot 10^{-6}\ C$ $\quad W = 0.012\ J$ $\quad \Delta W = 0.036\ J$

$C = 1{,}5 \cdot 10^{-7}\ F$ $\quad W_2 = 0.048\ J$

19. $d = 10^{-3}\ m$

$Q = 200 \cdot 10^{-9}\ C$

$F = 0.2\ N$

$W = 2 \cdot 10^{-4} = \frac{Q^2}{2C} \Rightarrow C = 1 \cdot 10^{-10}\ F$

20. $E_p = 3\ kV/mm$

$R = 0.1\ m$

$C = 11.1 \cdot 10^{-12}\ F$

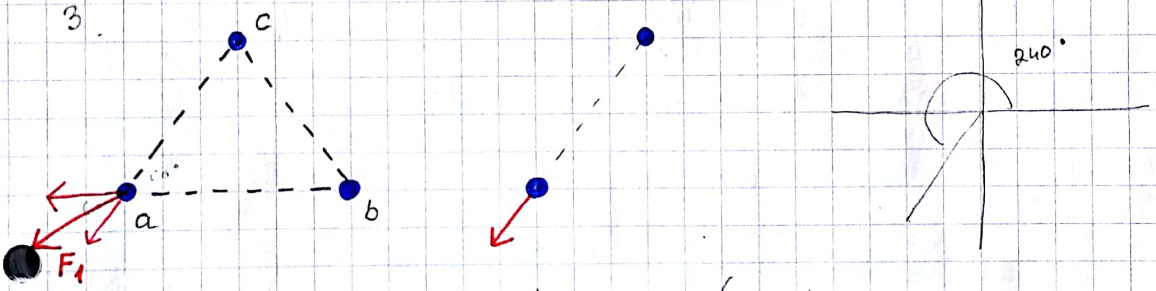
$E = \frac{Q}{4\pi\varepsilon R^2}$

$Q = 3.34 \cdot 10^{-6}\ C$

$C = \frac{Q}{U} \Rightarrow U = \frac{Q}{C} = 300\,709\ V$

TEST PRAVA 1.

3.



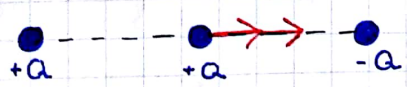
$F = F(\cos 240° + \sin 240°) + F\cos(180°)$

$F(\cos 240° + \cos 180°) + F\sin 240°$

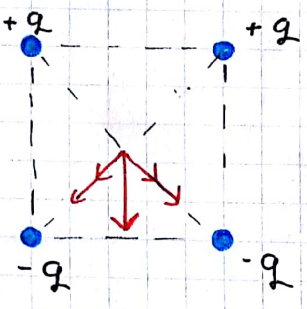
$F = -\frac{3}{2}F - \frac{\sqrt{3}}{2}F$

$F_1 = \sqrt{\frac{9}{4} + \frac{3}{4}} = F\sqrt{3}$ $\quad F = \frac{F_1}{\sqrt{3}} = 0.577\ F_1$

4

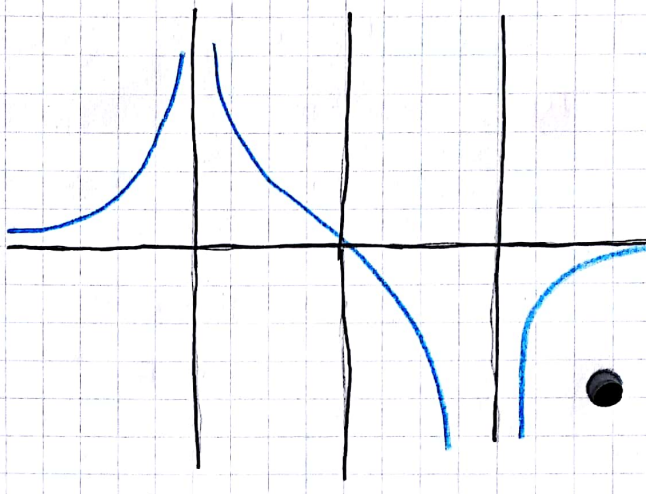


8F

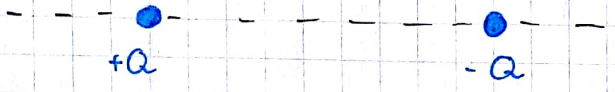
11. 

$4 \sin\alpha F = 2{,}82\,F$

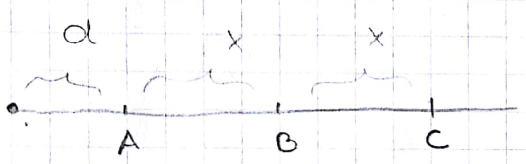
12. $100 = \frac{Q}{4\pi\varepsilon 0{,}1} \Rightarrow Q = 1.1126 \cdot 10^{-9}$



dodatak za 5.



PAZI NA RJEŠENJE KORJENA !!!

5. 

$E_A = 36\ V/m$

$E_C = 9\ V/m$

$$E_A = \frac{Q}{4\pi\varepsilon d^2} = \frac{Q}{4\pi\varepsilon \cdot 4x^2}$$

$$E_C = \frac{Q}{4\pi\varepsilon \cdot (d+2x)^2}$$

$$E_B = \frac{Q}{4\pi\varepsilon (d+x)^2} = \frac{Q}{4\pi\varepsilon\, 9x^2}$$

$$\frac{(d+2x)^2}{d^2} = \frac{36}{9}$$

$$\frac{d+2x}{d} = \frac{4}{3}$$

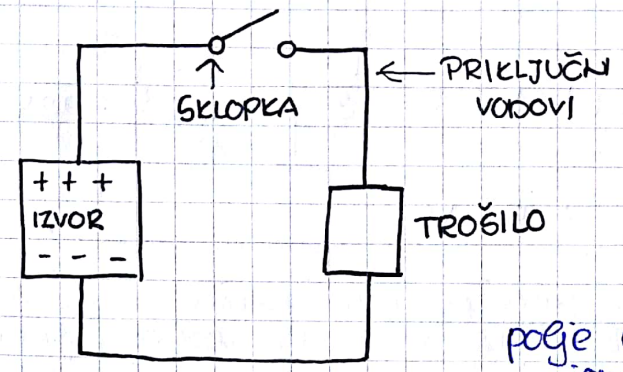
$$4d = 3d + 2x$$

$$d = 2x$$

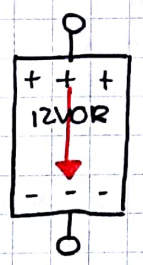
$$\frac{9x^2}{4x^2} = \frac{36}{E_B}$$

$$E_B = 16\ V$$

ELEKTRIČNA STRUJA



PRAZNI HOD



$\vec{E_i}$

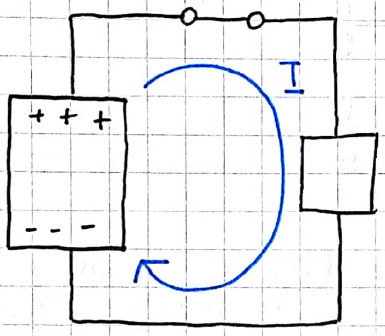
$\vec{F_v} = -\vec{F_{el}}$

vanjska sila

SILE SU U RAVNOTEŽI - PRAZAN HOD

polje unutar izvora je homogeno

UKLAPANJE SKLOPKE



I od + prema -

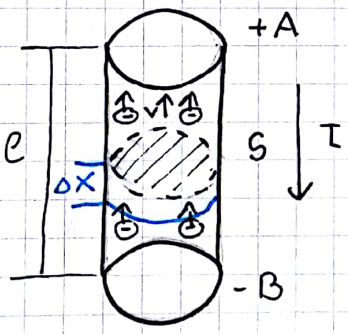
el. struja je usmjereno gibanje nabojа pod djelovanjem stalno prisutnog el. polja

$$I = \frac{\Delta Q}{\Delta t} = N q_e S v = SveN$$

Stalno el. polje uzrokuje vanjski izvor PRAVODNE STRUJE - nastaju pod djelovanjem el. polja u vodičima

$$E = \frac{p_A - p_B}{e} = \frac{U_{AB}}{e}$$

VOĐENJE



srednja brzina elektrona $\vec{v}$ broz presjek S u Δt prođe $\Delta Q = N \cdot q_e \, S \cdot v \, \Delta t$

broj nabojа u jedinici vremena

$$v = \frac{\Delta x}{\Delta t} \qquad \Delta Q = N \cdot q_e \cdot S \cdot \Delta x = N \cdot q_e \cdot S \cdot v \cdot \Delta t$$

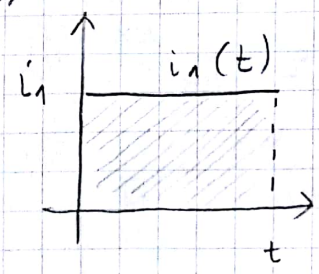
JAKOST EL. STRUJE - količina nabojа koja u jedinici vremena prođe presjek vodiča

Odredite napon U_C na $C = 100\,\mu F$ u trenutku $t = 2\,s$ ako se nabija

a) $i_1 = 8\,mA$

b) $i_2 = 2 \cdot t^2\,mA$

a)



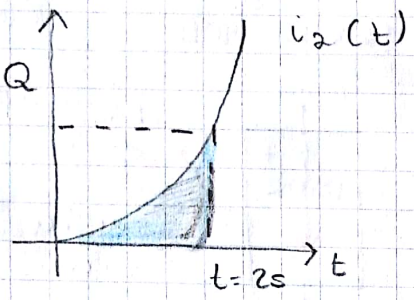
$$i(t) = \frac{dQ}{dt}$$

$$Q = \int i(t)\,dt$$

$$Q = I \cdot t = 8\,mA \cdot 2s = 16\,mAs$$

$$U_{C1} = \frac{Q_1}{C} = 160\,V$$

b) $i_2 = 2t^2$



$$Q = \int i_2(t)\,dt = 2 \cdot \frac{t^3}{3}\Big|_{t=2s} = 5.3\,mC$$

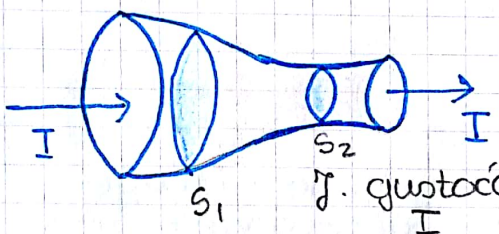
$$U_{C_2} = \frac{Q_2}{C} = 53\,V$$

GUSTOĆA STRUJE

prema principu kontinuiteta el. struje jakost struje unutar vodiča jednaka je na svim presjecima u svakom trenutku

$I = J_1 \cdot S_1$

$I = J_2 \cdot S_2$



J. gustoća struje

$$J = \frac{I}{S} = \left[\frac{A}{m^2}\right]$$

$$J = \frac{\Delta I}{\Delta S}$$

$$J = \frac{dI}{dS}$$

ELEKTRIČNA PROVODNOST

gustoća električne struje koji u različitim materijalima potrene isto električno polje je različita

$$K = \frac{J}{E} = \left[\frac{S}{m}\right]$$ simens

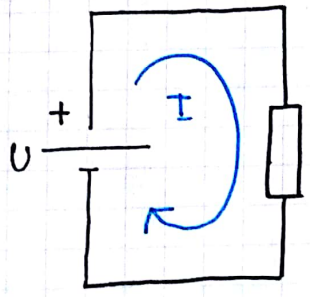
ELEKTRIČNA OTPORNOST

$$\rho = \frac{1}{K} = [\Omega\,m]$$

$$I = J \cdot S = K \cdot E \cdot S = K \cdot \frac{U_{AB}}{e} \cdot S$$

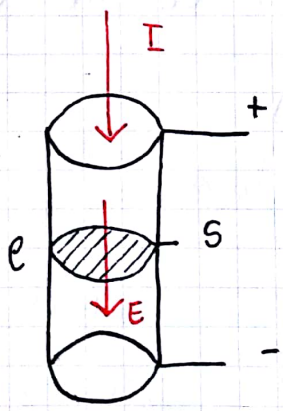
$I = G \cdot U_{AB}$ ← vodljivost

$$I = \frac{U_{AB}}{R}$$ ← el. otpor $\quad R = \frac{1}{G}$



$$I = \frac{U}{R} = \frac{U_R}{R}$$

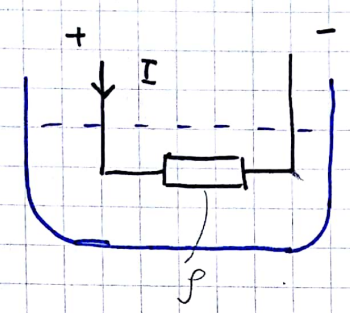
$$R = \rho \frac{e}{S}$$



postoji polje unutar vodiča

ELEKTRODINAMIKA

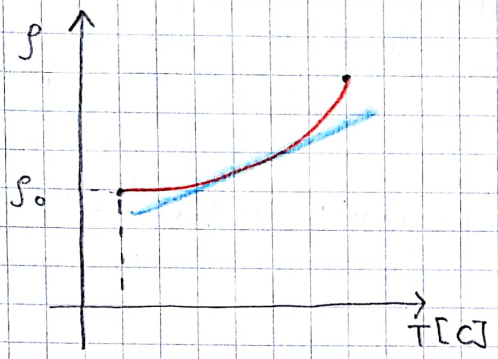
Otpor je zavisan o temperaturi



temperatura promjenjiva

Ohmov zakon - jakost struje u vodiču proporcionalna je naponu na njegovim krajevima

→ struja ulazi na onaj kraj vodiča koji je na višem potencijalu



termička gibanja s porastom temp. postaju jača te raste otpor gibanju slobodnih elektrona

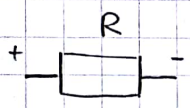
OVA KRIVULJA NIJE LINEARNA ALI SE HOŽE LINEARIZIRATI

ovisnost otpora $\rho(T) = \rho_0(1 + \alpha \Delta t)$ ne ovisio o struji

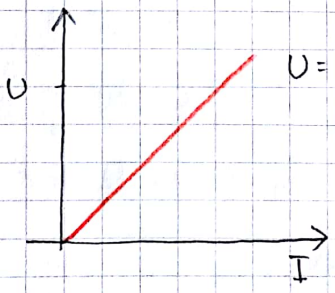
$R(T) = R_{20}(1 + \alpha \Delta t)$ ← temp. koef.

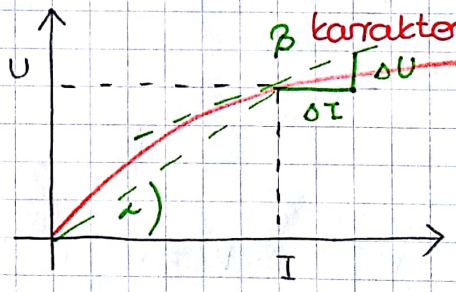
↓

LINEARNI I NELINEARNI OTPORNICI



↑ mijenja se s promjenom radne točke



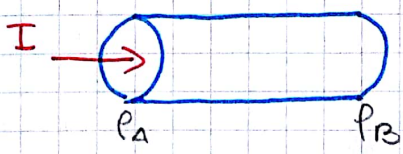


dinamički otpor opisuje u kojoj se mjeri pri promjeni napona mijenja jakost struje

statički otpor: $R_s = \frac{U}{I} = \operatorname{tg} \alpha$

dinamički otpor: $r_d = \frac{\Delta U}{\Delta I} = \frac{du}{dI} = \operatorname{tg} \beta$

UČINAK EL. STRUJE NA OTPORE



$\Delta Q = I \Delta t$

$\Delta W = \Delta Q (\varphi_A - \varphi_B)$

$= I \Delta t \, U_{AB}$

$P = \frac{\Delta W}{\Delta t} = I \cdot U_{AB} = I^2 R$

e^- se sudaraju međusobno i s kristalnom rešetkom materijala

↓

OSLOBAĐA SE TOPLINA

Joulecova toplina

1. $S = 1\ mm^2$
$e = 2\ m$
$U = 5\ V$
$N = 6{,}25 \cdot 10^{11}$
$\Delta t = 250\ ms$

a) $I = \frac{\Delta Q}{\Delta t} = \frac{Ne}{\Delta t} = 4A$

b) $J = \frac{I}{S} = 4\ A/mm^2$

c) $E = \frac{U}{e} = 2{,}5\ \frac{V}{m}$

d) $J' = \frac{I'}{S'} = \frac{4}{0.1} = 40\ A/mm^2$

2. $e = 10\ m$
$S = 1{,}5\ mm^2$
$U = 220\ V$
$I = 10\ A$

$R = \rho \cdot e/S = 0.0169 \cdot \frac{10}{1.5\ mm^2} = 0.1126\ \Omega$

$U = I \cdot R = 1.126\ V$

3. $e_1 = e_2$

$R_{Cu} = \rho_{Cu} \cdot \frac{e}{S_{Cu}} \qquad R_{AE} = \rho_{AE} \cdot \frac{e}{S_{AE}}$

$$\frac{\rho_{Cu} e}{S_{Cu}} = \frac{\rho_{AE} \cdot e}{S_{AL}}$$

$$\frac{S_{AL}}{S_{Cu}} = \frac{\rho_{AL}}{\rho_{Cu}} = \frac{0.0265}{0.0169} = 1.568$$

ZADACI IZ ZBIRKE:

1. $t = 60\ s$
$Q = 30\ C$
$v = konst$
$S = 0.2\ mm^2$

$I = \frac{Q}{t} = 0.5A \qquad J = \frac{I}{S} = 2.5\ A/mm^2$

2. $e = 10\ m$
R, W, U
$\rho = 1.75 \cdot 10^{-8}\ \Omega m$

$R = \rho \cdot \frac{e}{S} = 0.875\ \Omega \qquad R = \frac{U}{I} \Rightarrow U = I \cdot R = 0.44V$

$W = I^2 R t = 13.125\ J$

3. $U = 120V$
$I = 800\ mA$
$R, G, P.$

$R = \frac{U}{I} = 150\ \Omega \qquad G = \frac{1}{R} = 6{,}67 \cdot 10^{-3}\ S$

$P = U \cdot I = 96\ W$

4. $t_0 = 20°C \qquad t_1 = 80°C$
$R = 30\ \Omega \qquad \alpha = 0.0039\ 1/°C$

$R = R_0 (1 + \alpha \Delta t)$
$= 37{,}02\ \Omega$

5. $R = 440\ \Omega$ $t = 20°C$ $R_1 = 510\ \Omega$ $t = ?$

$$550 = 440 \cdot (1 + 0.0039 \cdot \Delta t)$$
$$1.25 = 1 + 0.0039\,\Delta t$$
$$\Delta t = 64.1$$
$$t = 84.1\ °C$$

1. $e = 1\ m$, $U = 1.11\ V$, $S = 1\ mm^2$, $N = 6.24 \cdot 10^{18}\ e^-$, $t = 1\ s$

$$I = \frac{Q}{t} = 1\ A \qquad R = \frac{U}{I} = 1.11\ \Omega$$
$$J = \frac{I}{S} = 1\ A/mm^2$$
$$\rho = \frac{1}{K} \Rightarrow K = \frac{1}{\rho} = 900\,900.9$$
$$R = \rho \cdot \frac{e}{S} \Rightarrow \rho = 1.11 \cdot 10^{-6}$$

← specifičnu el. otpor

$$J = K E$$
$$E = 1.11\ V/m$$

2. $J = konst = 8\ A/mm^2$, $N = 10^{22}\ e/cm^3$,

$$J = \frac{I}{S} = \frac{\Delta Q}{\Delta t} = \frac{1}{\frac{\Delta V}{\Delta e}} = \frac{\Delta Q}{\Delta V} \cdot \frac{\Delta e}{\Delta t} = g \cdot v$$

$$J = N q v$$
$$800\ A/cm^2 = 10^{22} \cdot 1{,}609 \cdot 10^{-19}\ v$$
$$v = 0.5\ cm/s$$

3. $S_1 = 0.5\ mm^2$, $S_2 = 2\ mm^2$, $J_2 = 10^6\ A/m^2$ } $I = J_2 \cdot S_2 = 2\ A$

kroz $S_1 \Rightarrow I = 2\ A$

$S_2 \Rightarrow I = 2\ A$

$S_2 = 4\ mm^2$ $J = \frac{I}{S} = 5 \cdot 10^6$?

4. $U = 220\ V$, $W = 2000\ Ws$, $\rho = 1{,}11\ \Omega\ mm^2/m$

$$P = \frac{W}{t} = \frac{2000\ Ws}{1\,s} = 2000 = \frac{U^2}{R} \Rightarrow R = 24{,}2\ \Omega$$

b) $2000 = 220 I \Rightarrow I = 9.1\ A$

c) $S = 0.23\ mm^2$; $e = ?$

$$24{,}2 = 1.11 \cdot \frac{e}{0.23} \Rightarrow e = 5\ m$$

5. $t = 20°C$ $\quad \alpha = 0.004\ 1/°C$
$R_{20} = 100\ \Omega$ $\quad t_2 = 120°C$

$R_{120} = 140\ \Omega$

6. $S_{Cu} = 1.5\ mm^2$
$\frac{S_{Cu}}{S_{Ae}} = 0.63$

$\rho_{Cu} \cdot \frac{l}{S_{Cu}} = \rho_{Ae} \cdot \frac{l}{S_{Ae}}$

$\frac{\rho_{Cu}}{\rho_{Ae}} \cdot \frac{1}{S_{Cu}} = \frac{1}{S_{Ae}}$

$\rho_1 = \rho_2$
$R_1 = R_2$
$S_{Ae} = ?$

$\frac{\rho_{Cu}}{\rho_{Ae}} = \frac{S_{Cu}}{S_{Ae}} = 0.63 = \frac{1.5}{S_{Ae}} = 2.38\ mm^2$

7. $R = 25\ \Omega$
$U = 110\ V$
$t = 30\ min$

$\frac{W}{t} = \frac{U^2}{R}$

$\frac{W}{30 \cdot 60} = \frac{110^2}{25} \Rightarrow W = 871\ kJ$

$U_2 = 99$

$W_2 = 705\ kJ$

8. R
$R_1 \rightarrow 3R$
$R_2 \rightarrow 2R$
$R_1 = R_2$

$3R = R_0 (1 + \alpha_1 t)$
$2R = R_0 (1 + \alpha_2 t)$
$\frac{3}{2} = \frac{1 + \alpha_1 t}{1 + \alpha_2 t}$
$3 + 3\alpha_2 t = 2 + 2\alpha_1 t$
$1 + 3\alpha_2 t = 2\alpha_1 t$

$3R = \frac{2R}{1 + \alpha_2 t} \cdot (1 + \alpha_1 t)$
$\frac{3}{2} = \frac{1 + \alpha_1 t}{1 + \alpha_2 t}$

$3R = R + R\alpha_1 t$
$2R = R + R\alpha_2 t$

$3 = 1 + \alpha_1 t$
$2 = 1 + \alpha_2 t$

$2 = \alpha_1 t$
$1 = \alpha_2 t$

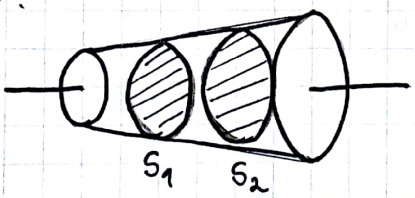
$\frac{\alpha_1 t}{\alpha_2 t} = 2$

$3R = R\alpha_2 t - 2R + R\alpha_1 t$
$5R = R\alpha_2 t + R\alpha_1 t$
$5 = \alpha_2 t + \alpha_1 t \quad / : \alpha_2 t$

9. $\Delta t = 40°C$
$\alpha = 0.004\ 1/°C$

$xR = R(1 + \alpha t)$
$x = 1.16$ $\quad 16\ \%$

10.



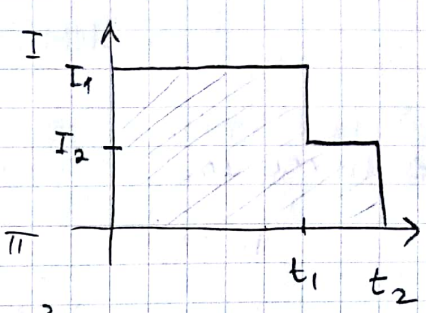
veća je gustoća po presjeku $S_1 \Rightarrow$ veće je i polje

11. ρ
1. S $\quad$ 2. $2S$ $\quad V = S \cdot l$

$R_1 = \rho \cdot \frac{2l}{S}$ $\quad R_2 = \rho \cdot \frac{l}{2S}$

$\frac{R_1}{R_2} = \frac{\rho \cdot \frac{2l}{S}}{\rho \cdot \frac{l}{2S}} = R_1 = 4R_2$

12. $t = 5h$ $t = 2h$
$I = 2A$ $I = 1A$
$Q = 43200\ C$



13. $d = 0{,}4\ mm$
$\ell = 33\ m$
$R = 80\ \Omega$

$80\ \Omega = \rho \cdot \frac{33}{10{,}2\ mm^2 \cdot \pi}$

$K = \frac{1}{\rho} = 3{,}28\ Sm/mm^2$

prezentacija

7. $U = 0.5 \cdot I^2$
$U = 2V$

$I = \sqrt{\frac{2U}{}} = 2A$

$R_S = \frac{U}{I} = \frac{2V}{2A} = 1\ \Omega$

$R_d = \frac{dU}{dI}\Big|_{2V} = \{$ prvo deriviram a onda uvrstim $\} = 0.5 \cdot 2I = I = 2\ \Omega$

8. $R = 1/8\ \Omega$
$P = 2\ kW$

$P = I^2 R \rightarrow I = \sqrt{\frac{P}{R}} = 126\ A$

$W = P \cdot t = 2000 \cdot 2 = 4000\ Wh = 4\ kWh$

9. $\ell = 10\ m$
$S = 1{,}5\ mm^2$
$U = 220V$
$I = 10\ A$

$R_V = 0.1126\ \Omega$

$P = U \cdot I = 2200\ W$ ← ovo je snaga izvora, dolazi do pada napona na trošilu

$P = I^2 \cdot R = 11.26\ W$ vodiča

$P_{trošila} = P_{izvora} - P_{vodiča} = 2200V - 11.26 = 2188{,}74\ V$

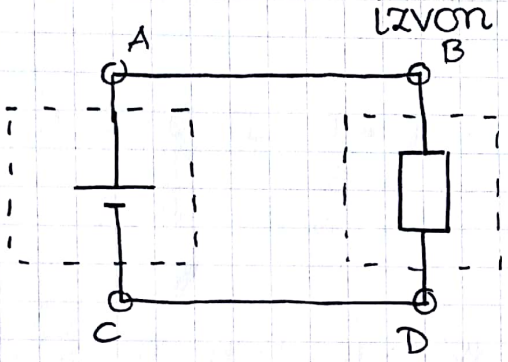
$S_V = 3\ mm^2$
$R_V = 2 \times$ manji
$P_V' = I^2 \cdot R_V' = 5{,}63\ W$

21

KIRCHHOFFOVI ZAKONI

→ objekte koji imaju realne dimenzije modelirati koncentriranim parametrima

R otpor	→	otpornik	PASIVNI ELEMENTI (nisu izvori)
C kapacitet	→	kondenzator	
L induktivitet	→	zavojnica	
izvori			AKTIVNI ELEMENTI



A i B su točke istog potencijala

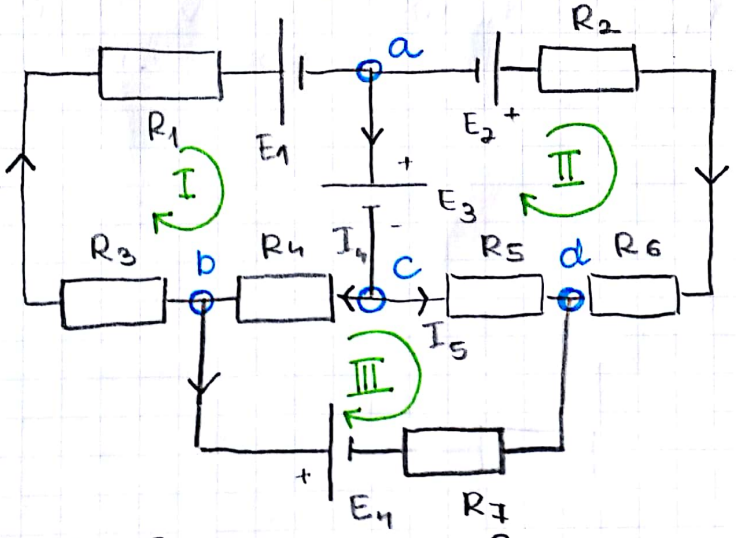
C i D su isto točke istog potencijala

- smatramo da vodovi ne pružaju otpor

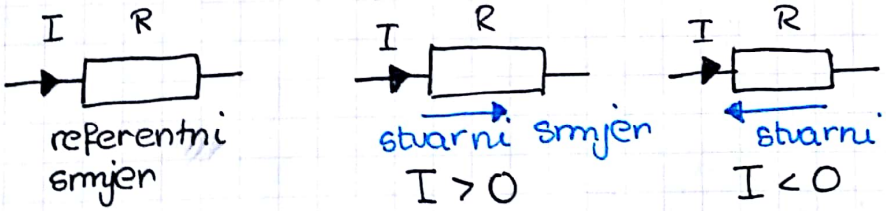
GRANA - dio mreže sastavljen od serijski spojenih elemenata kroz koje teče ista struja

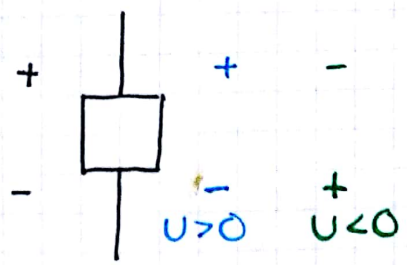
ČVOR - točka u kojoj se sastaju BAREM tri grane

KONTURA - zatvoreni put sastavljen od više grana

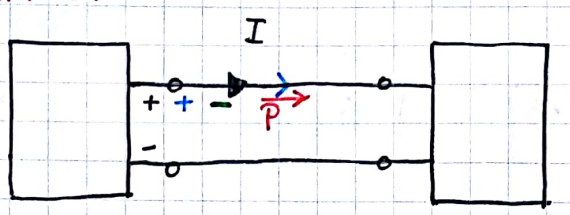


4 čvora
6 grana





SNAGA



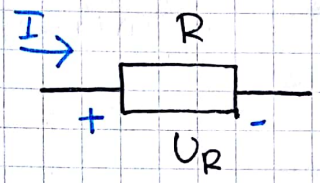
$P > 0$

← I (polaritet isti) $P < 0$

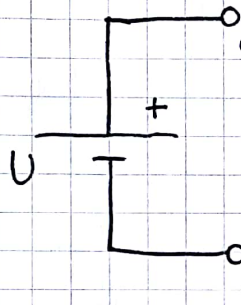
− (smjer isti) $P < 0$

−, ← (oba se promjene) $P > 0$

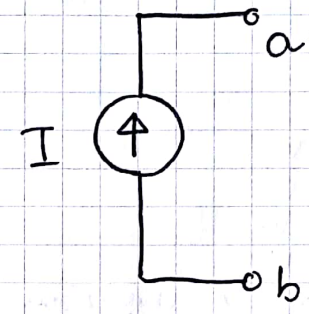
donja priključnica



gdje struja ulazi to je + pada napona



IDEALNI NAPONSKI IZVOR



IDEALNI STRUJNI IZVOR

I KIRCHHOFFOV ZAKON

algebarski $\sum I = 0$ (u čvoru)

$$\sum_{i=1}^{m} I_{ue} = \sum_{j=1}^{n} I_{ize}$$

II KIRCHHOFFOV ZAKON

$$\text{alg} \sum_{i=1}^{m} U_i = 0$$ napon u konturi

$$\text{alg} \sum_{j=1}^{m} E = \text{alg} \sum_{k=1}^{o} U$$

izvori — trošila

I Č − 1 NEZAVISNIH JEDNADŽBI

a) $I_1 = I_2 + I_3$; $I_1 - I_2 - I_3 = 0$

b) $I_4 = I_1 + I_6$; $-I_1 + I_4 - I_6 = 0$

c) $I_2 = I_4 + I_5$; $I_2 - I_4 - I_5 = 0$

$G - (Č - 1) = 6 - (4 - 1) = 3$

I $-E_1 - E_3 = I_1 \cdot R_1 + I_4 \cdot R_4 + I_1 \cdot R_3$

II. $E_2 + E_3 = I_3 R_2 + I_3 R_6 - I_5 R_5$

III. $E_4 = -I_4 R_6 + I_5 R_5 - I_6 R_7$

struju u smjeru obilaska $+IR$

ako je suprotna $-IR$

$$
\begin{array}{c}
\begin{matrix} I_1 & I_2 & I_3 & I_4 & I_5 & I_6 \end{matrix} \\
\begin{bmatrix}
1 & -1 & -1 & 0 & 0 & 0 \\
-1 & 0 & 0 & 1 & 0 & -1 \\
0 & 1 & 0 & -1 & -1 & 0 \\
R_1+R_3 & 0 & 0 & R_4 & 0 & 0 \\
0 & 0 & R_2+R_6 & 0 & -R_5 & 0 \\
0 & 0 & 0 & -R_4 & R_5 & -R_7
\end{bmatrix}
\end{array}
\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ -E_1 - E_3 \\ E_2 + E_3 \\ E_4 \end{bmatrix}
$$

$$R \cdot I = U$$
$$[R] \cdot [I] = [U]$$

$R_1 = 4\ \Omega$
$R_2 = 6\ \Omega$
$R_3 = 5\ \Omega$
$R_4 = 1\ \Omega$
$R_5 = 2\ \Omega$
$R_6 = 10\ \Omega$
$R_7 = 3\ \Omega$

$E_1 = 12\ V$
$E_2 = 4\ V$
$E_4 = 12\ V$
$E_3 = 10\ V$

Zbirka.

14. $2r = 0{,}4\,mm$
$Hj = 1620$
$e = 8{,}4\,cm$
$\rho = 1{,}75\ 10^{-8}\,\Omega$

$R = \rho \cdot \frac{e}{S} = \rho \cdot \frac{1620 \cdot 8.4 \cdot 10^{-2}}{(0.2 \cdot 10^{-3})^2 \pi}$

$= 18.95\,\Omega$

15.

$G = 2.5\,m/mm^2 \quad \rho = 0.5 \quad 2{,}5 = 0.5 \cdot \frac{e}{2}$

$e = 0.35\,m$

16. $2r = d \quad 1.2d$
$e \quad 0.8e$

$R_1 = \rho \cdot \frac{e}{(\frac{d}{2})^2 \pi} \qquad R_2 = \rho \cdot \frac{0.8e}{(0.6d)^2 \pi}$

$\frac{\frac{e}{0.25 d^2 \pi}}{\frac{0.8e}{0.36 d^2 \pi}} = \frac{0.36}{0.25 \cdot 0.8} = 1.8$

$R_2 = 0.55\,R_1$

17. $R_2 = 1.2\,R_1 \qquad 1.21 R_1 = R_1 (1 + 0.0039 \cdot \Delta t)$

$\Delta t = 53{,}84$

$t_2 = 73{,}84\,^\circ C$

18. $t_1 = 20^\circ \quad t_2 = 120^\circ \quad \Delta t = 100$

$XR = R(1 + 0.00005 \cdot 100)$ $\quad$ $XR = R(1 + 0.0046 \cdot 100)$

$0.5\,\%$ $\quad$ $46\,\%$

19. $t = 20 \quad R = 1000\,\Omega$ $\quad$ PTC odpor
$\Delta t = 80 \quad R_2 = 400\,\Omega$

$400 = 1000\,(1 - \alpha \cdot 80)$

$\alpha = -0.0075\ 1/^\circ C$

20. $\alpha_A = 0.0037\ ^\circ C^{-1}$
$\alpha_H = 0.0015\ ^\circ C^{-1}$
$R_{Ae} = R_{Hj} = 2\,\Omega$

$1.3\,R_{HJ} = R_{Ae}$

$R \cdot 1.3_{Hj} = 2(1 + 0.0037 \cdot \Delta t)$
$R_{Hj} = 2(1 + 0.0015\,\Delta t)$

$1.3 \cdot 2 \cdot (1 + 0.0015 \Delta t) = 2 + 7.4 \cdot 10^{-3} \Delta t$

$2.6 + 3.9 \cdot 10^{-3} \Delta t = 2 + 7{,}4 \cdot 10^{-3} \Delta t$

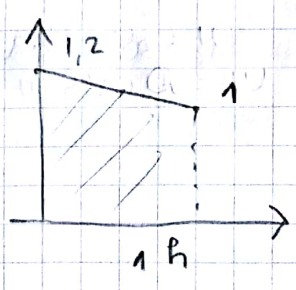
$0.6 = 3.5 \cdot 10^{-3} \Delta t$

$\Delta t = 171{,}4286$

$t = 191{,}43\,^\circ C$
$R_{HJ} = 2.514\,\Omega$
$R_{Ae} = 3.268\,\Omega$

21. $U = 12\,V \qquad I = 1{,}2\,A$
$U_2 = 10\,V$
$t = 1\,h \qquad I_2 = 1\,A$

b) $Q = I \cdot t = 1{,}2\,Ah$



$Q = 1{,}1\,Ah$

Primjeri 11.2.

Struja koji daje izvor stalnog napona ovisi o ukupnom otporu koji se priključuje na njegove stezaljke

1. $U = 12\ V$
$R_1 = 220\ \Omega$ } serijski
$R_2 = 150\ \Omega$ }

$R_{uk} = R_1 + R_2 = 370\ \Omega$

$R = \frac{U}{I} \Rightarrow I = \frac{U}{R} = \frac{12}{370\ \Omega} = 0.0324\ A$ → jednaka u cijelom krugu

$U_1 = 220 \cdot 0.0324 = 7,13\ V$
$U_2 = 4,86\ V$

serijski spoj otpornika nazivamo naponsko djelilo

$P_1 = 0.23\ W$ $P_2 = 0.157\ W$ $P_{uk} = 0.3888\ W$

2. $R_1 = 220\ \Omega$
$R_2 = 150\ \Omega$
$I = 50 \cdot 10^{-3}\ A$ → izvor stalne struje

napon na stezaljkama strujnog izvora ovisi o ukupnom otporu

$R_{uk} = 370\ \Omega$
→ struja je jednaka kroz svaki otpor $I = 50\ mA$
$U_1 = 11\ V$ }
$U_2 = 7,5\ V$ } prema II. KHZ napon izvora je 18,5 V

$P_1 = I^2 R = 0.55\ W$
$P_2 = I^2 R = 0.375\ W$
$P_{izvora} = 0.925\ W$

3. $R_1 = 220\ \Omega$
$R_2 = 150\ \Omega$
$U = 12\ V$ - izvor stalnog napona

ukupan otpor uvijek manji od najmanjeg

$\frac{1}{R} = \frac{1}{220} + \frac{1}{150} \Rightarrow R = 89.2\ \Omega$

paralelni spoj otpornika nazivamo strujno djelilo

$I_1 = \frac{U}{220} = 0.0545\ A$ $I_2 = \frac{U}{150} = 0.08\ A$ $I_{uk} = 0.1345$

naponi na otpornicima su jednaki naponu izvora

$P_1 = I^2 \cdot R = 0.65\ W$ $P_2 = 0.96\ W$ $P_3 = 1,61.$

4. $R_1 = 220\ \Omega$
$R_2 = 150\ \Omega$
$I = 50 \cdot 10^{-3}\ A$
$\frac{1}{R} = \frac{1}{220} + \frac{1}{150} = 89.2\ \Omega$
ukupna struja spoja je struja izvora

$U = I \cdot R_{uk} = 4,46\ V$

$I_1 = U/R_1 = 0.021\ A$ $P_1 = I_1^2 \cdot R_1 = 0.09\ W$ $P_{12} = 0.223\ W$
$I_2 = U/R_2 = 0.0297\ A$ $P_2 = I_2 \cdot R_2 = 0.132\ W$

Zadaci 11.2.

1. $U_1 = 14,4\ V$ $\quad R_2 = 100\ \Omega$
   $U_2 = 9,6\ V$
   $\frac{U_1}{U_2} = \frac{R_1}{R_2} \Rightarrow R_1 = 150\ \Omega$

2. $R = \frac{12\ V}{0.5\ A} = 24\ \Omega$

   $U = 24\ V$ $\quad U_2 = 12\ V$
   $Im = 0.5\ A$ $\quad I = 0.5$ $\quad R = 24\ \Omega$

3. $U_1 : U_2 : U_3 = 1 : 2 : 3$ $\quad U_1 = 60\ V$ $\quad U_2 = 120\ V$ $\quad U_3 = 180\ V$
   $R_1 : R_2 : R_3 = 1 : 2 : 3$ $\quad I = 0.5\ A$
   $R_1 = 120\ \Omega$ $\quad R_2 = 240$ $\quad R_3 = 360\ \Omega$
   $R_{uk} = R_1 + R_2 + R_3 = 720\ \Omega$

4. $R_1 = 12\ \Omega$ $\quad R_2 = 6\ \Omega$ $\quad I_1 = 2A$ $\quad I_{uk} = 6\ A$
   $U = R_1 \cdot I_1 = 24V$ cijelog kruga $\quad R = 4\ \Omega$
   $I_2 = \frac{U}{R_2} = 4\ A$

5. $I_1 : I_2 : I_3 = 2 : 3 : 6$ paralelni spoj

   $R_2 = 33\ \Omega$
   $U = 99V$ $\quad I_2 = \frac{99}{33} = 3\ A$ $\quad R_1 = \frac{99}{2} = 49.5\ \Omega$
   $I_{uk} = 11\ A$

   $I_1 : I_2 = 2 : 3$ $\quad I_2 : I_3 = 3 : 6$ $\quad R_3 = 16.5\ \Omega$
   $I_1 : 3 = 2 : 3$ $\quad 3 : I_3 = 3 : 6$
   $I_1 = 2\ A$ $\quad I_3 = 6\ A$ $\quad R_{uk} = 9\ \Omega$

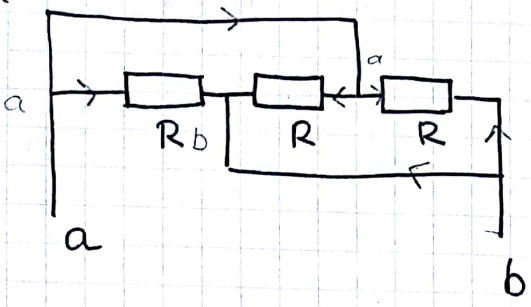
6. $I_p = 8A$ $\quad R$
   paralela $\rightarrow \frac{1}{R_{uk}} = \frac{1}{R} + \frac{1}{R} \Rightarrow R_{uk} = \frac{R}{2}$ $\quad U = I \cdot R = 4R$
   serija $\rightarrow R_{uk} = 2R$ $\quad I_s = \frac{U}{R} \Rightarrow \frac{4R}{2R} = 2A$
   $P = U \cdot I = 4R \cdot 8 = 32R$ $\quad P_s = 4R \cdot 2 = 8R$ $\quad P_p / P_s = 4$

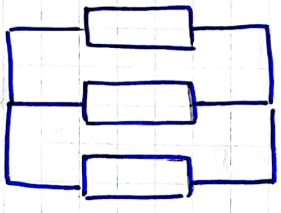
7. $R_{uk} = 2R$ $\quad I = \frac{U}{R} = \frac{U}{2R}$ $\quad I$ je dva puta manja
   $R = \frac{U}{I}$ $\quad P = UI = 0.5\ P_m$

8. $P_1 : P_2 = 3 : 2$ kod paralelnog spoja mapani su jednaki $\frac{U^2}{R} : \frac{U^2}{R} =$

   $I_1^2 R_1 : I_2^2 R_2 = 3 : 2$
   ↳ snage su jednake ⇒ serijski spoj $\quad \frac{1}{R_1} : \frac{1}{R_2} =$
   $R_1 : R_2 = 3 : 2$ $\quad \frac{1}{3} : \frac{1}{2} = / \cdot 6$
   $2 : 3$

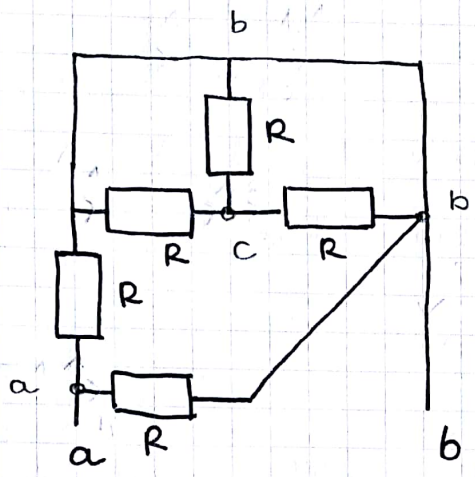
9.

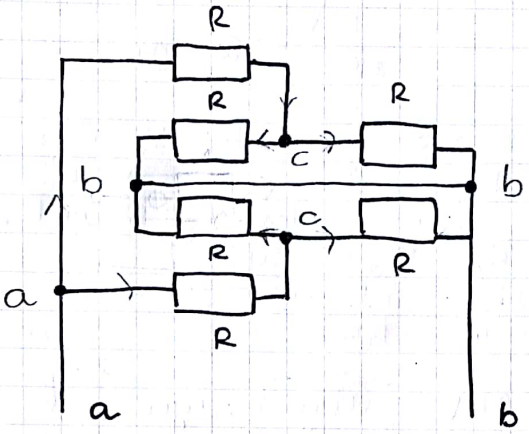


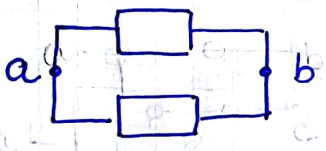


$\frac{1}{R_{uk}} = \frac{3}{R}$

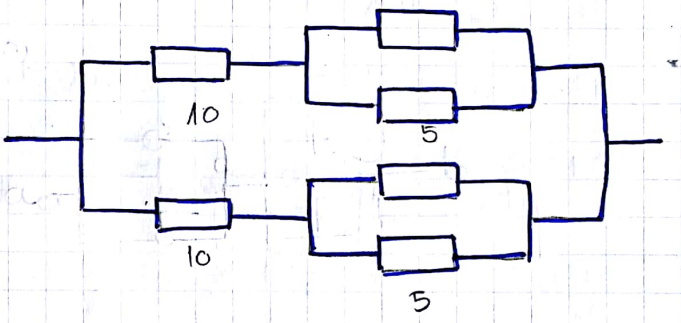
$R_{uk} = 10/3\ \Omega$



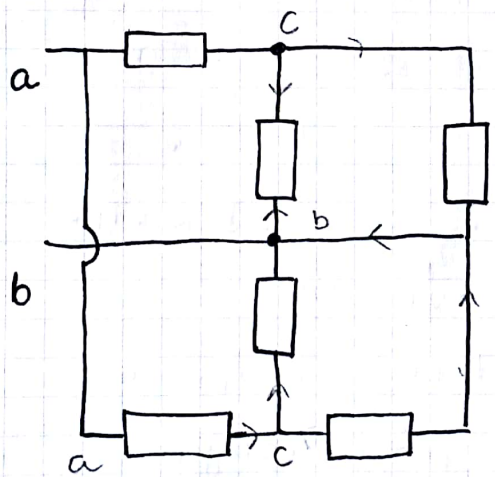


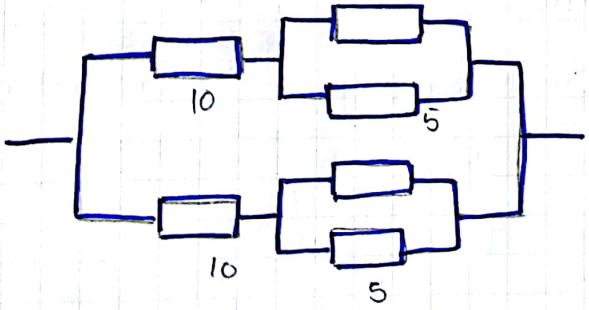


$5\ \Omega$

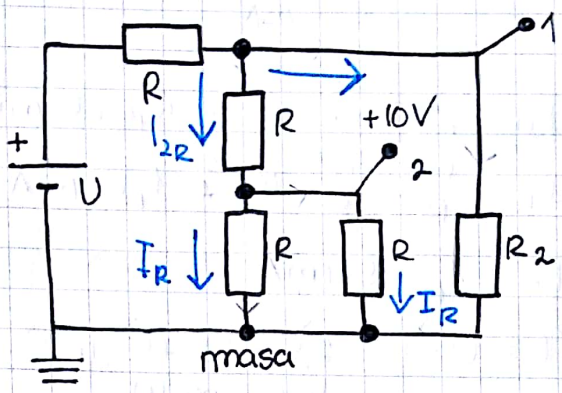


$\frac{15}{2} = 7.5$





10.



$P_2 = 10\,V$
$R = 10\,\Omega$

ukupna snaga

R_2 - ako je om 10 Ω
$I = 3\,A$

$I_{uk} = I_{2R} + \frac{P_1}{R_2}$
$= 2 + \frac{30}{10} = 5A$

$P_1 =$ $\quad I_R = \frac{P_2}{R} = 1A$

$U = P_1 + I \cdot R$
$= 30 + 50 = 80\,V$

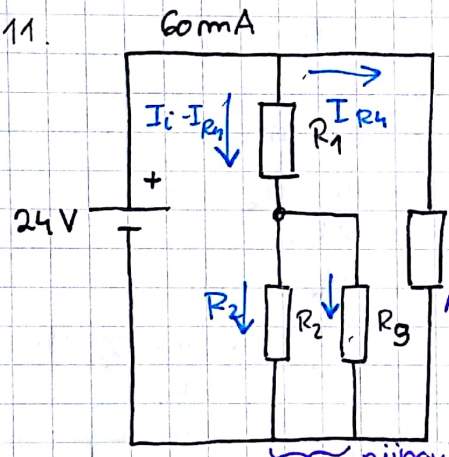
$I_{2R} = 2A$ $\quad P_1 = P_2 + I_{2R} \cdot R = 10 + 20 = 30\,V$

=

ako odspojim R_2

$R_{uk}' = R + R + R \| R = 25\,\Omega$
$I_{uk}' = U / R_{uk} = 80/25 = 3{,}2\,A$
$P_2' = \frac{I_{uk}'}{2} \cdot R = 1{,}6 \cdot R = 16\,\Omega$

11.



$I_i = 60\,mA$
$U_{R3} = 9\,V$
$P_{R3} = 45\,mV$
$R_{R4} = 480\,mW$

na njemu je U = naponu izvora zbog paralele

njihovi U isti

$P_{R4} = \frac{U_i^2}{R_4} \Rightarrow R_4 = \frac{U_i^2}{R_4} = 1200\,\Omega$

$I_4 = U/R_4 = 20\,mA$

$I_{R1} = I_i - I_4 = 40\,mA$

$P_{R3} = \frac{U_{R3}^2}{R_3} = R_3 = 1800\,\Omega$

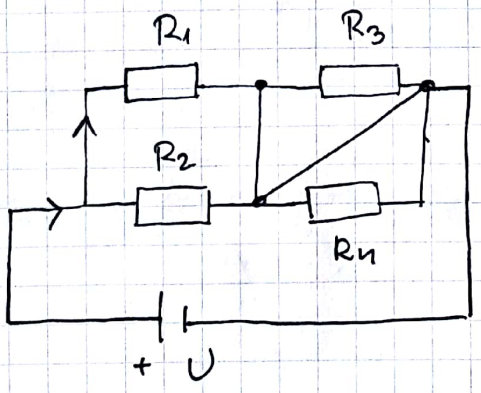
$I_{R3} = \frac{U_{R3}}{R_3} = \frac{9}{1800} = 5mA$

$I_{R2} = I_{R1} - I_{R3} = 35\,mA$

$R_2 = \frac{U_{R3}}{I_{R2}} = \frac{9}{35 \cdot 10^{-3}} = 257{,}14\,\Omega$

$U_{R1} = U_i - U_{R3} = 15\,V$

$R_1 = \frac{U_{R1}}{I_{R1}} = \frac{15}{40 \cdot 10^{-3}} = 375\,\Omega$



kroz R_3 i R_4 ne teče struja, oni su u kratkom spoju, pa nema pada napona

$R_1 = 6\,\Omega$
$I_1 = 1A$

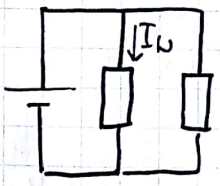
$U_{R1} = 6V$
$I_2 = \frac{U_{R2}}{R_2} = 2A$

$I_i = I_1 + I_2 = 3A$

17. $I = 0.1 \cdot 10^{-3} \cdot U^2$

$R = 2\,k\Omega$

$U = 2,5\,V$



$I_N = 0,1 \cdot 10^{-3} \cdot 2,5^2 = 0,625\ mA$

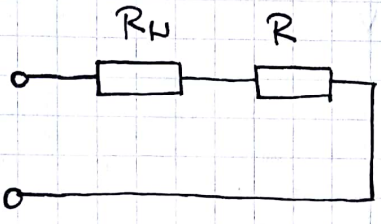
$I_R = \frac{U}{R} = \frac{2,5}{2} \cdot 10^{-3} = 1,25\ mA$

$I_i = 1,875\ mA$

$P = U \cdot I = 2,5 \cdot 1,875 \cdot 10^{-3}$

$= 4,68\ mW$

18. $U = k \cdot I^{0.5}$ u seriju spajamo $R = 1\,\Omega$, tada je $I = 2A$



$U = \sqrt{I}$ Umelinearnog $= \sqrt{2}\ V$

$U_R = 2V$

$U = U_m + U_R = 2 + 1,41 = 3,41$

$R = \frac{3,41}{2} = 1,705\,\Omega$

13. Um - jednak za oba otpornika

$P_{m_1} = 50\,W$ $P_1 = 1 \cdot U_m$

$P_{m_2} = 100\,W$ $P_2 = 1 \cdot U_m$

serijski su spojeni

ukupna snaga spoja

2 Um je napon izvora

$U \cdot I =$

$P = \frac{U^2}{R}$

$\sqrt{50R_1} = \sqrt{100R_2}\ /^2$

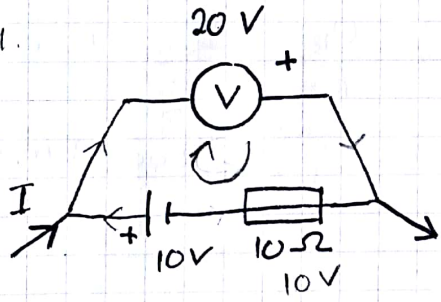
$50R_1 = 100R_2$

$R_1 = 2R_2$

$R_{uk} = 3R_2$

$P = \frac{U^2}{R} \frac{\sqrt{100R_2}^2}{3R_2} = \frac{100}{3} = 33,3$

14.



$-10 = -20 + 1 \cdot 10$

$I = 3A$

$-10 = -20 + 10I$

$I = 1\,A$

15. serijski spoj nelinearnog elemenata i linearnog otpornika

$I = 2A$

otpornik $1 = 1 \cdot U$

$2 = U$

nelinearni $U = 1 \cdot 1^2$

$U = 4$

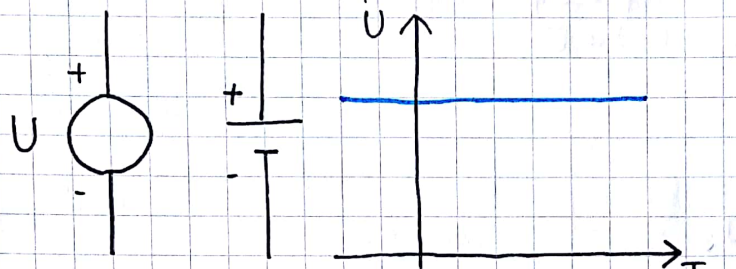
Ukupno $= 6\,V$

16. paralelno spojeni omski i nelinearni element

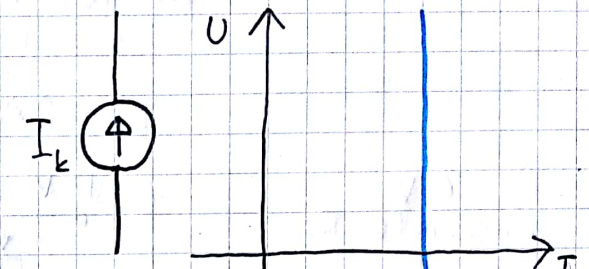
$U = 4\,V$ $I_1 = 4\,A$ $I_2 = 2A$ $I_{uk} = 6\,A$

JEDNOSTAVNI KRUGOVI ISTOSMJERNE STRUJE

Idealni izvori

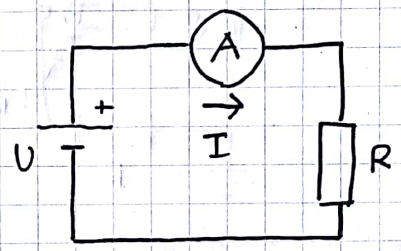


naponski izvor



strujni izvor

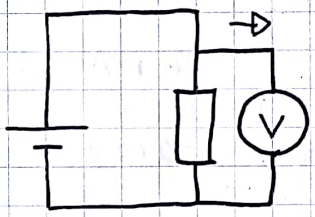
amperrmetarr
- -> mjeri struju
- -> spaja se serijski
- -> idealni $R_A = 0$
- -> Realni R_A <<



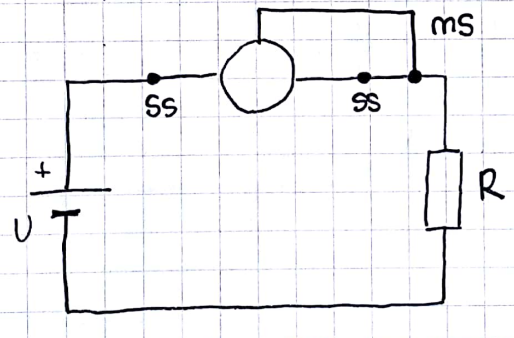
bez A:
$U = I \cdot R$

S A:
$U = I \cdot R_A + I \cdot R$

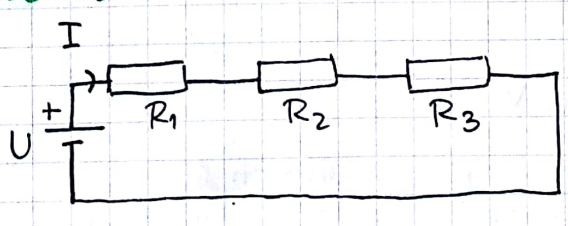
voltmetarr
- -> mjeri naponm
- -> spaja se paralelno
- -> idealni $R_V = \infty$
- -> realni $R_V >>$



Vatmetarr



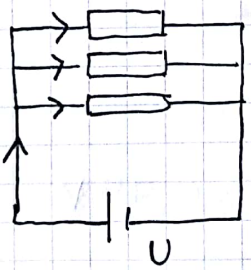
SERIJSKI SPOJ OTPORNIKA



$$\text{II KI} \Rightarrow U = I \cdot (R_1 + R_2 + R_3)$$

$$\frac{U}{I} = R_{uk} = R_1 + R_2 + R_3$$

PARALELNI SPOJ OTPORNIKA

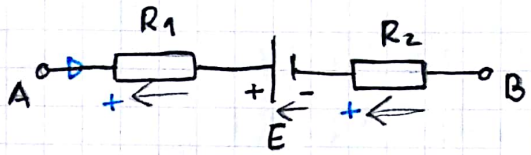


$$I_{uk} = I_1 + I_2 + I_3$$

$$= \frac{U}{R_1} + \frac{U}{R_2} + \frac{U}{R_3} \quad / : U$$

$$\frac{I_{uk}}{U} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{R_{uk}}$$

1. $U_{AB} = 0$
$E = 16\ V$
$R_1 = 5\ \Omega$
$R_2 = 3\ \Omega$
$I = ?$



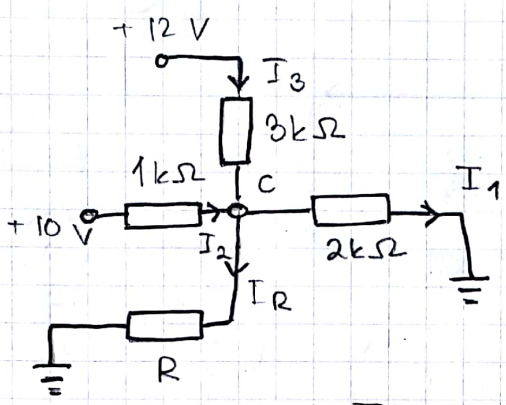
$$P_A - I \cdot R_1 - E - I \cdot R_2 = P_B$$

$$P_A - P_B = E + I \cdot R_1 + I R_2$$

$$0 = I(R_1 + R_2) + E$$

$$I = -2\ A$$

2. $P_C = 6\ V$
odrediti R



$$I_1 = \frac{P_C - 0}{2000\ \Omega} = 3\ mA$$

$$I_2 = \frac{10 - 6\ V}{1000\ \Omega} = 4\ mA$$

$$I_3 = \frac{12 - 6\ V}{3000\ \Omega} = 2\ mA$$

I KI. za C

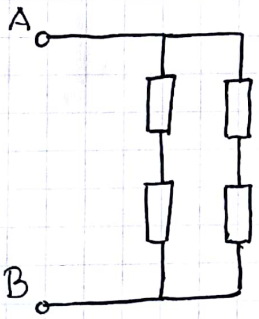
$$I_2 + I_3 = I_1 + I_R$$

$$I_R = 3\ mA$$

$$R = \frac{\varphi_C - 0}{I_R}$$

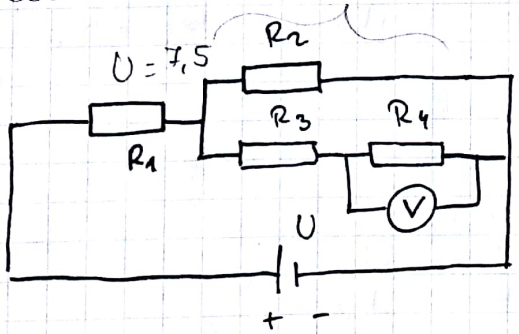
$$R = 2\ k\Omega$$

3. Zadani su otpori $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 3\ \Omega$ i $R_4 = 4\ \Omega$.
Kako ih povezati da se dobije $R_{uk} = 2{,}5\ \Omega$



$$R_{AB} = \frac{R_{14} \circ R_{23}}{R_{14} + R_{23}}$$

4. Odrediti Uv



$U = 12\ V$

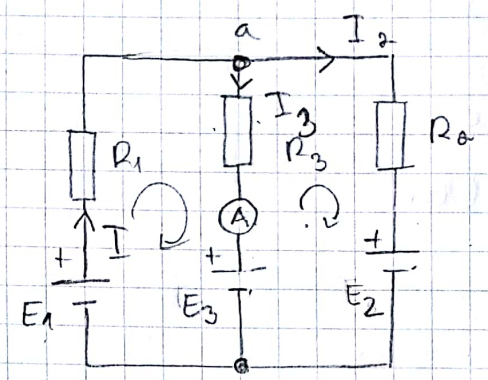
$R_1 = 15\ \Omega$
$R_2 = 36\ \Omega$
$R_3 = 10\ \Omega$
$R_4 = 2\ \Omega$

$R_{uk} = 24$

$I = 0{,}5\ A$

$$I = \frac{4{,}5}{12} = 0{,}375 \qquad U = 0{,}75\ V$$

5. Struja ampermetra u grani jednaka je 0.



$E_1 = 24\,V$
$E_2 = 40\,V$
$R_1 = 24\,\Omega$
$R_2 = 40\,\Omega$
$R_3 = 60\,\Omega$

$I = I_2$

$E_1 - E_3 = I R_1$
$24 - E_3 = 24 I$

$E_3 - E_2 = I \cdot 40$
$E_3 - 40 = 40 I$

$24 - 24I - 40 = 40I$
$-64I = 16 \quad I = \frac{-1}{4}$

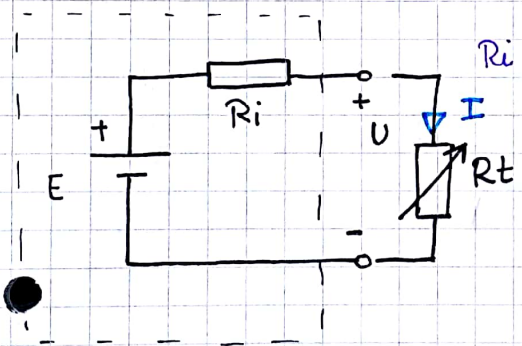
$E_1 - E_3 = 24 I$
$24 - E_3 = 24 I$
$24 - 24 I = E_3$

$E_3 - 40 = 40 \cdot I$
$E_3 = 40I + 40$

$24 - 24I = 40I + 40$
$-16I = 64I$
$I = \frac{-1}{4}$

$E_3 = 30\,V$

REALNI IZVORI

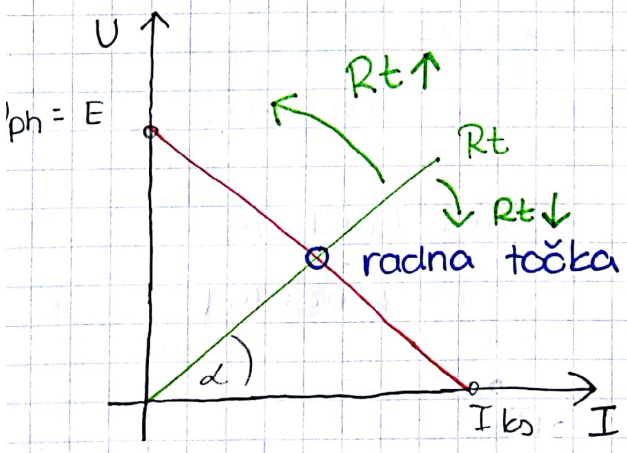


Ri je karakteristika izvora (elektronički, žice...)

$$I = \frac{E}{Ri + Rt}$$

$$\underbrace{U = E - I \cdot R_i}_{\text{ona određuje kar. izvora}} = I \cdot R_t$$

ovisnost napona o struji za realni naponski izvor



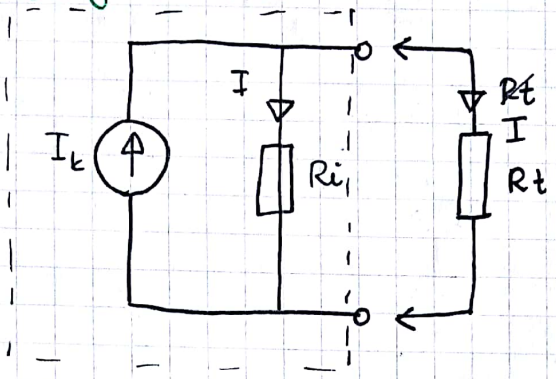
prva točka $I = 0$, $U = E_{PH}$

druga točka $U = 0$, $I = I_{ks} = \frac{E}{Ri}$

karakteristika izvora
karakteristika trošila

BITNO ZA USMENI!

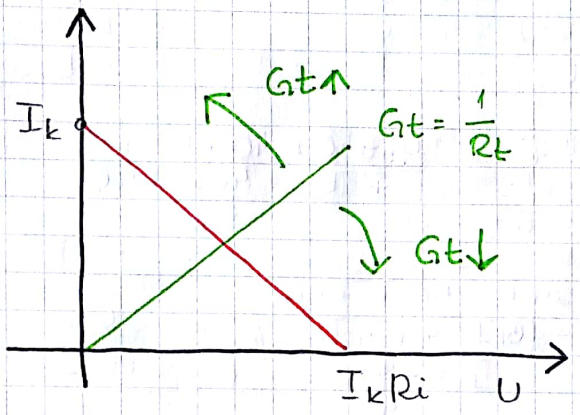
→ strujni izvor



$I = I_k - I_i$

$= I_k - \frac{U}{R_i}$

$I = I_k - UG_i$



prva točka: $U = 0$

$I = I_k$

druga točka $I = 0$

$U = I_k / G_i = I_k \cdot R_i$

karakteristika strujnog izvora

1. Na izvor su priključena dva različita otpora i izmjerene vrijednosti U i I bez izvore. Odredite E i Ri.

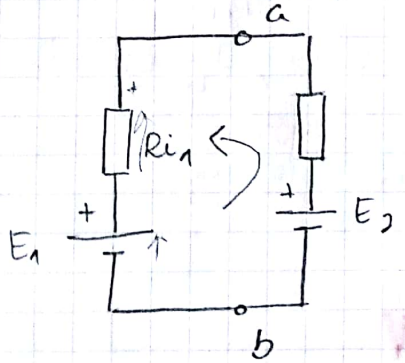
$U_1 = 40V$
$I_1 = 2A$
$U_2 = 21V$
$I_2 = 21A$

$U_1 = E - I_1 \cdot R_i$
$U_2 = E - I_2 \cdot R_i$

$40 = E - 2R_i$
$-21 = E - 21R_i$
$19 = 19R_i$
$R_i = 1\,\Omega$

$E = 42\,V$

2. Automobilski akumulator $E_1 = 12\,V$ i $R_{i1} = 1\,\Omega$ puni se iz izvora $E_2 = 16\,V$ i $R_{i2} = 1\,\Omega$. Odredite napon na stezaljkama akumulatora.



II KZ.

$E_2 - E_1 = I(R_1 + R_2)$

$I = 2A$

$U_{ab} = E_1 + IR_1$

$= E_2 - IR_2$

$= 12 + 2 = 14\,V$

TEST PITANJA II.

4. $P = 1500\ W$
$t = 10 \cdot 60\ s$
$E = P \cdot t$

Snaga je brzina pretvorbe energije

5. $R = 5\ \Omega$
$t = 1\ h$ $\quad E = P \cdot t$
$I = 5A$
$U = 25$
$P = 125\ W$

7. $U = 100\ V$
$R = 100\ \Omega$

8. $4R = R(1 + \alpha \cdot t)$
$2R = R(1 + \alpha_2 t)$
$4 = 1 + \alpha_1$
$2 = 1 + \alpha_2$
$3 = \alpha_1$
$1 = \alpha_2$

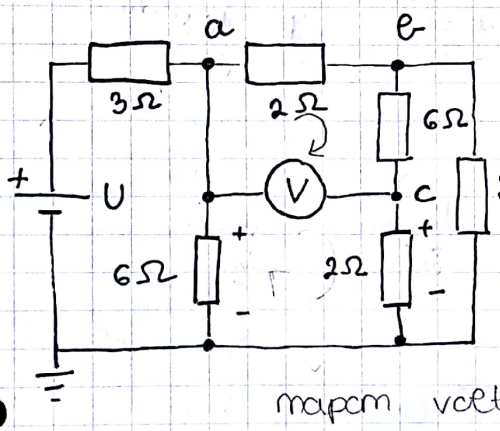
9. $1{,}1 = 1(1 + \alpha \cdot 1)$
$0.1 = \alpha$

10. $2 = 1(1 + 0.01t)$
$t = 100$

11. $I = 1\ A$
$t = 60\ s$
$E = 300\ Ws$ } $P = 5$
$R = ?$

$P = I^2 \cdot R$
$R = 5\ \Omega$

ZADACI - JEDNOSTAVNI KRUGOVI ISTOSMJERNE STRUJE



voltmetar mjeri + 10 V

ukupan otpor, mapon izvora U, mapon U_{AB}.

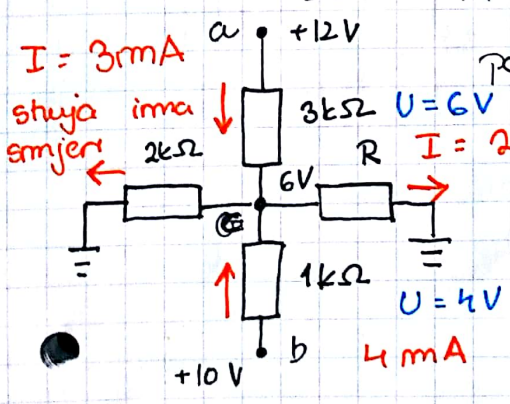
$R_{uk} = 6\ \Omega$

mapon voltmetra je algebarska suma mapona na otpornicima

$\frac{1}{2} \cdot 2 + \frac{1}{4} \cdot 6 = 10$

$1 + \frac{3}{2} I = 10$ $\qquad U = 24\ V$

$I_{uk} = 4\ A$



potencijal u točki c = 6 V

U (razlika potencijala točaka a i b

$2 + 4 = 3 + x$
$x = 3$

$R = \frac{U}{I} = \frac{6}{3\,mA} = 2000\ \Omega$

1. $R_{t_1} = 10\,\Omega$, $I = 1\,A$ $\quad R_{t_2} = 2{,}5\,\Omega$, $I_2 = 2\,A$ $\quad$ struja kratkog spoja, napon praznog hoda, unutarnji otpor

$E = R_i \cdot 1 + 10 \cdot 1$
$E = R_i + 10$

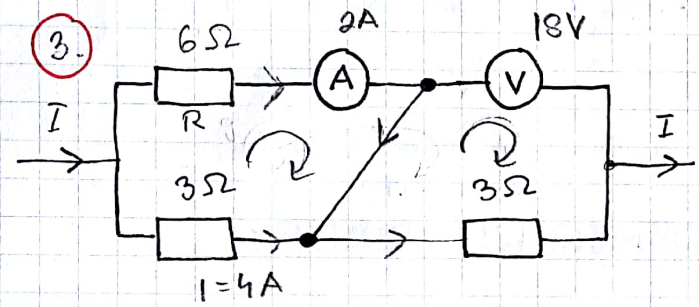
$E = R_i \cdot 2 + 5$
$R_i + 10 = R_i \cdot 2 + 5$
$R_i = 5\,\Omega$
$E = 15\,V$
$I_{ks} = \frac{E}{R_i} = 3A$

$R = 2{,}5\,\Omega$
$15 = 5 \cdot 1 + 2{,}5 \cdot I$
$I = 2\,A$
$P = I^2 \cdot R = 10\,W$

$R = 10\,\Omega$
$15 = 5I + 10I$
$I = 1\,A$
$P = I^2 R = 10\,W$

$R = 5\,\Omega$
$15 = 5I + 5I$
$I = 1{,}5\,A$
$P = I^2 \cdot R = 11{,}25\,W$

3.



otpor R, struja I, ukupna snaga spoja

$18 - 3 \cdot I = 0$
$I = 6A$

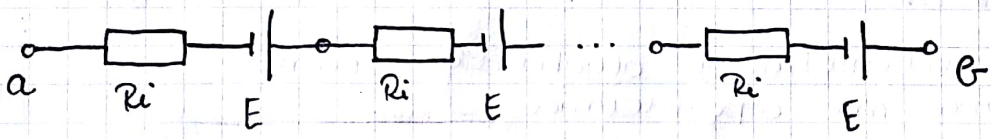
$R \cdot 2 - 3 \cdot 4 = 0$
$R = 6\,\Omega$

$R_{uk} = 3$ $\quad I_{uk} = 6$ $\quad P = 108$

=

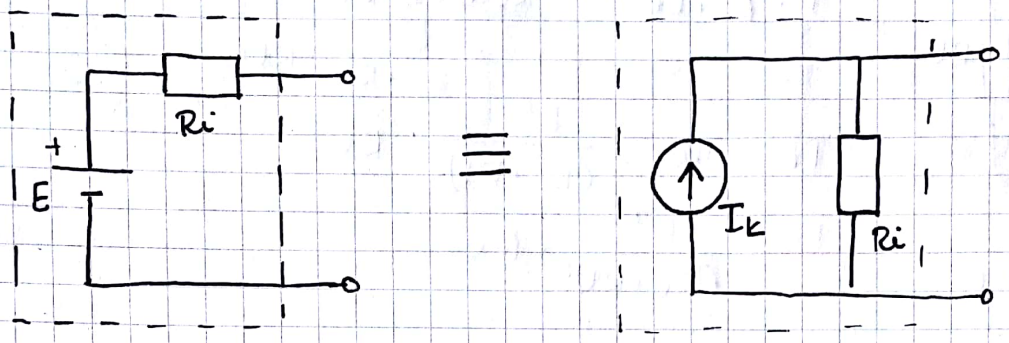
$E = 1{,}25\,V$
$R_i = 0{,}004\,\Omega$
$U = 115\,V$

a) ako je neopterećen spoj 92
b) spoj daje $I = 25\,A$ 100



$U_{ab} = N(E - I \cdot R_i)$

PRETVORBA MODELA REALNOG NAPONSKOG IZVORA U REALNI STRUJNI I OBRATNO



→ prazni hod
realni naponski izvor

$E = I_k \cdot R_{is}$

napon na priključnicama $E = U_{ab}$
realni strujni izvor $I_k \cdot R_i = U_{ab}$

$I_k = \frac{E}{R_{is}}$

→ kratki spoj

$R_{in} = R_{is}$

realni naponski izvor

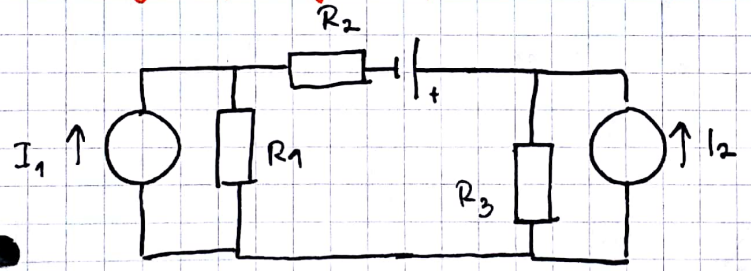
$I = \frac{E}{R_i}$

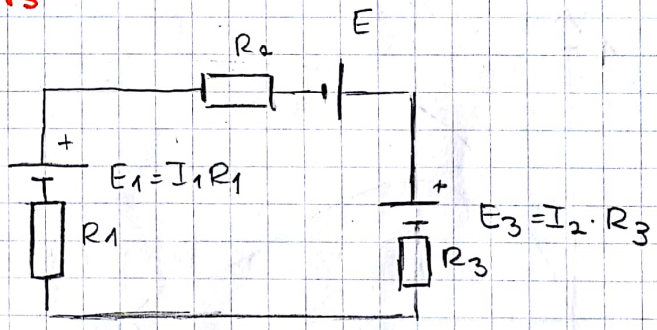
realni strujni izvor

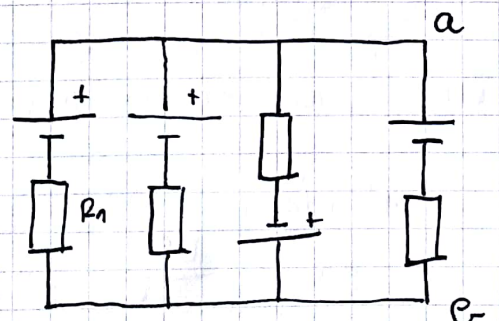
$I = I_k$

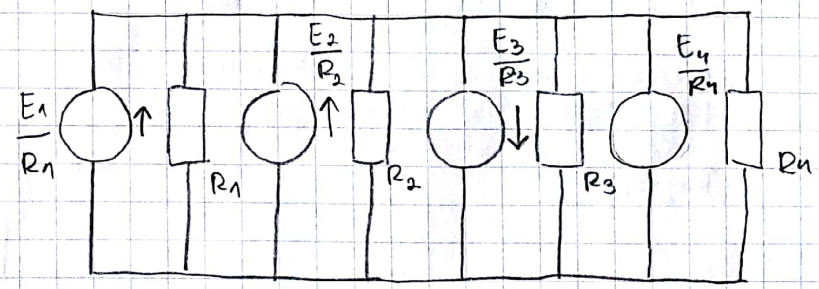
Pretvorba vrijedi ako je $R_{in} = R_{is} = R_i$ i $E = I_k \cdot R_i$

Primjer primjene pretvorbe $r_n <=> r_s$





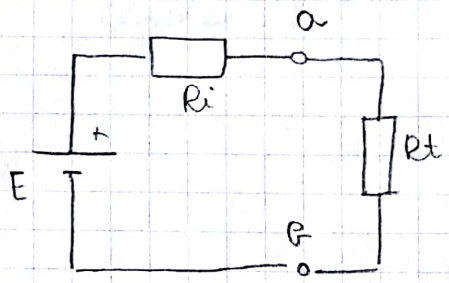




$U_{ab} = I \cdot R$

$= \left(\frac{E_1}{R_1} + \frac{E_2}{R_2} + \frac{E_3}{R_3} + \frac{E_4}{R_4} \right)$

Teorem o prijenosu maksimalne snage



$$I = \frac{E}{Ri + Rt}, \quad Uab = I \cdot Rt = \frac{E}{Ri + Rt} \cdot Rt$$

$$Pt = I^2 \cdot Rt = \frac{E^2}{(Ri + Rt)^2} \cdot Rt$$

$$Pi = E \cdot I = \frac{E^2}{Ri + Rt}$$

$$P_{iz} = \frac{E^2}{(Ri + Rt)^2} \cdot Ri$$

$Pt = f(Rt)$ pri kojem otporu Rt je snaga trošila maksimalna

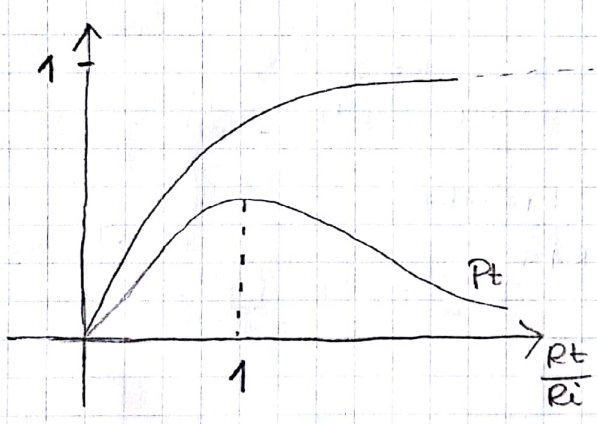
$$\frac{dPt}{dRt} = \frac{1 \cdot (Ri + Rt)^2 - 2(Ri + Rt) \cdot Rt}{(Ri + Rt)^4} \cdot E^2 = 0$$

brojnik = 0

$$Ri^2 - Rt^2 = 0$$

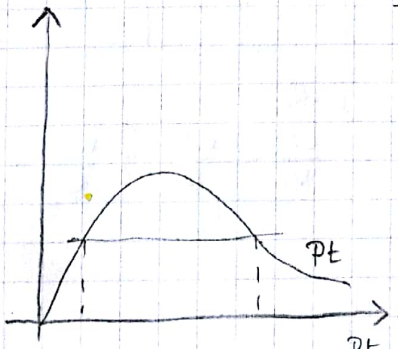
$Ri = Rt$ za + prohvat

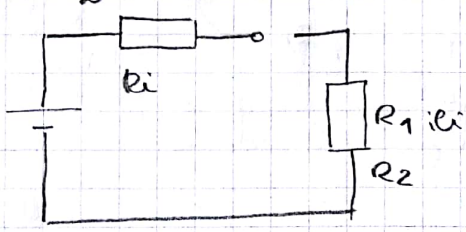
$$\eta = \frac{Pt}{Pi} = \frac{\frac{E^2}{(Ri + Rt)^2} \cdot Rt}{\frac{E^2}{(Ri + Rt)}} = \frac{Rt}{Ri + Rt}$$



Naponski izvor $E = 20\ V$; $Ri = 4\ \Omega$; R_1, P uz stupanj iskorištenja $\eta_1 = 0{,}2$
Ishi izvor R_2, P uz η_2

R_1, R_2, P





1) $Rt = R_1$

$$Pt = P = \frac{E^2}{(Ri + R_1)^2} \cdot R_1 = 16\ W$$

$$\eta = \frac{Rt}{Ri + Rt} \Rightarrow (4 + R_1) \cdot 0{,}2 = R_1$$

$$R_1 = 1\ \Omega$$

b)

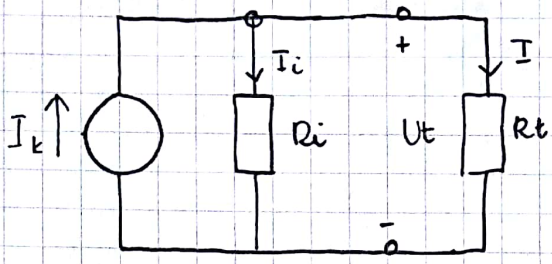
$$P = \frac{E^2}{(Ri + R_2)^2} \cdot R_2$$

$$16 = \frac{400}{R_2^2 + 8 \cdot R_2 + 16} \cdot R_2$$

$$16 + R_2^2 + 8R_2 = 25R_2 \qquad R_2 = 16\ \Omega$$

$$\eta_2 = \frac{R_2}{Ri + R_2} = 0{,}8 \qquad Pt\,max = \frac{E^2}{4Ri} = 25\ W$$

→ strujni izvor



$I_k = I_i + I$

$U_t = I \cdot R_t = I_i \cdot R_i$

$P_t = I^2 R_t = I_k^2 \cdot \frac{R_i^2}{(R_i+R_t)^2} \cdot R_t$

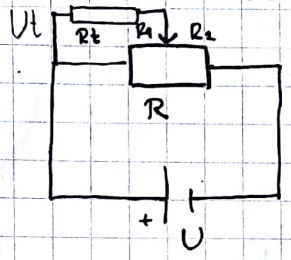
$I \cdot R_t = (I_k - I) \cdot R_i$

$I(R_t + R_i) = I_t \cdot R_i$

$I = I_k \cdot \frac{R_i}{R_i + R_t}$

$\frac{dP_t}{dR_t} = 0 \Rightarrow R_i = R_t \Rightarrow P_{max}$

Potenciometarski spoj



$R = R_1 + R_2$

$R_1 = \lambda \cdot R$

$R_2 = (1-\lambda) \cdot R$

$\lambda = 0$ klizač je savrm lijevo

$\lambda = 1$ klizač u krajnjem desnom položaju

$U_t = I \cdot R_t \| R_1 = I \cdot \frac{R_t \cdot R_1}{R_t + R_1}$ *

$I = \frac{U}{R_{ut}} = \frac{U}{R_2 + R_t \| R_1} = \frac{U}{R_2 + \frac{R_t \cdot R_1}{R_t + R_1}}$

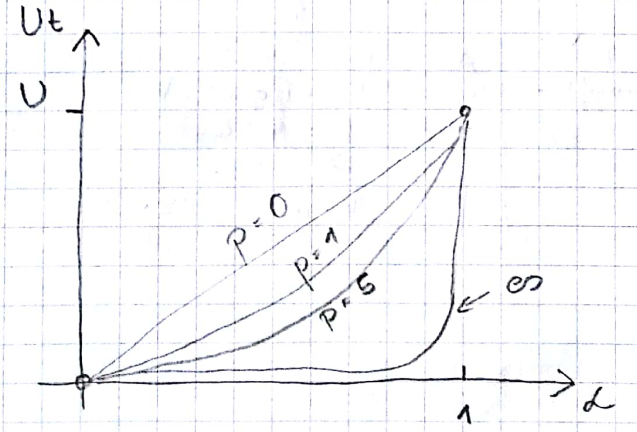
$* \frac{U}{R_2 + \frac{R_t R_1}{R_t + R_1}} \cdot \frac{R_t R_1}{R_t + R_1} =$

$= \frac{U(R_t + R_1)}{R_2(R_t + R_1) + R_t R_1} \cdot \frac{R_t \cdot R_1}{R_t + R_1} = \frac{U \cdot R_t \cdot R_1}{R_2 \cdot R_t + R_1 \cdot R_2 + R_1 \cdot R_t} = \frac{U \cdot R_t \cdot \lambda R}{R \cdot R_t + R^2 \lambda (1-\lambda)}$

$\frac{U \cdot \lambda \cdot R_t}{R_t + \lambda R (1-\lambda)}$

$U_t = U \cdot \frac{1}{\frac{1}{\lambda} + (1-\lambda)\frac{R}{R_t}}$; $P = \frac{R}{R_t}$

$U_t = U \cdot \frac{1}{\frac{1}{\lambda} + p \cdot (1-\lambda)}$



a) $\lambda = 0$; $U_t = 0$

b) $\lambda = 1$; $U_t = U$

1. Na potenciometar $R = 100\,\Omega$ priključero je trošilo R_t. Na kojem dijelu potenciometra treba postaviti klizač da bi na trošilu imali $1/2$ U izvora

a) $R_t = 10\,\Omega$
b) $R_t = 50\,\Omega$

$p = 10$

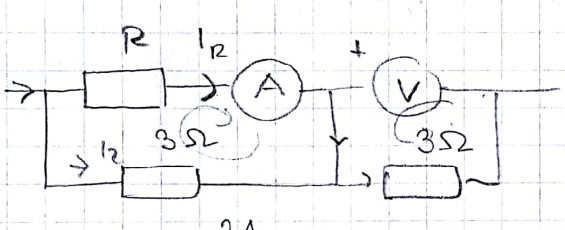
$$\frac{1}{2}U = \frac{1}{\frac{1}{\lambda} + (1-\lambda)\cdot 10} \cdot U$$

$$\frac{1}{2} = \frac{1}{\frac{1 + \lambda(1-\lambda)\cdot 10}{\lambda}} = \frac{\lambda}{1 + (\lambda - \lambda^2)10}$$

$$1 + 10\lambda - 10\lambda^2 = 2\lambda$$
$$1 + 8\lambda - 10\lambda^2 = 0$$
$$\lambda = 0{,}91$$

$$1 + (\lambda - \lambda^2)\cdot 2 = 2\lambda$$
$$1 + 2\lambda - 2\lambda^2 = 2\lambda$$
$$1 - 2\lambda^2 = 0$$
$$\lambda = 0{,}71$$

2. Instrumenti pokazuju $I_A = 2A$, $U_V = 18.V$ Odredite R.



$I = I_R + I_2$

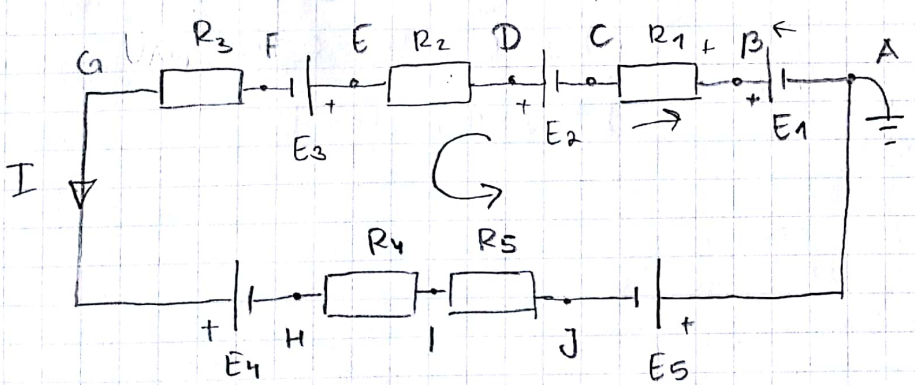
$-3\cdot I_2 + I_R \cdot R$

$-3\cdot(I - 2) + 2\cdot R = 0$

$-3\cdot 4 + 2\cdot R = 0$
$-12 = -2R$
$R = 6\,\Omega$

$-18\,V + I\cdot 3 = 0$
$I = 6\,A$

3. Nacrtajte potencijalni dijagram za krug prema slici te odredite napone U_{AG}, U_{FI}, U_{HE}



$E_1 = 10V$
$E_2 = E_4 = 5V$
$E_3 = E_5 = 15V$
$R_1 = 1\,\Omega$
$R_2 = 3\,\Omega$
$R_2 = R_4 = R_5 = 2\,\Omega$

$E_1 + E_2 - E_3 - E_4 + E_5 = I(R_1 + R_2 + R_3 + R_4 + R_5)$

$10\ V = I \cdot 10$

$I = 1\ A$

$\varphi_A = 0$

$\varphi_B = \varphi_A + E_1 = 10\ V$

$\varphi_C = \varphi_B - I \cdot R_1 = 9V$

$\varphi_D = \varphi_C + E_2 = 14V$

$\varphi_E = \varphi_D - I \cdot R_2 = 12V$

$\varphi_F = \varphi_E - E_3 = -3V$

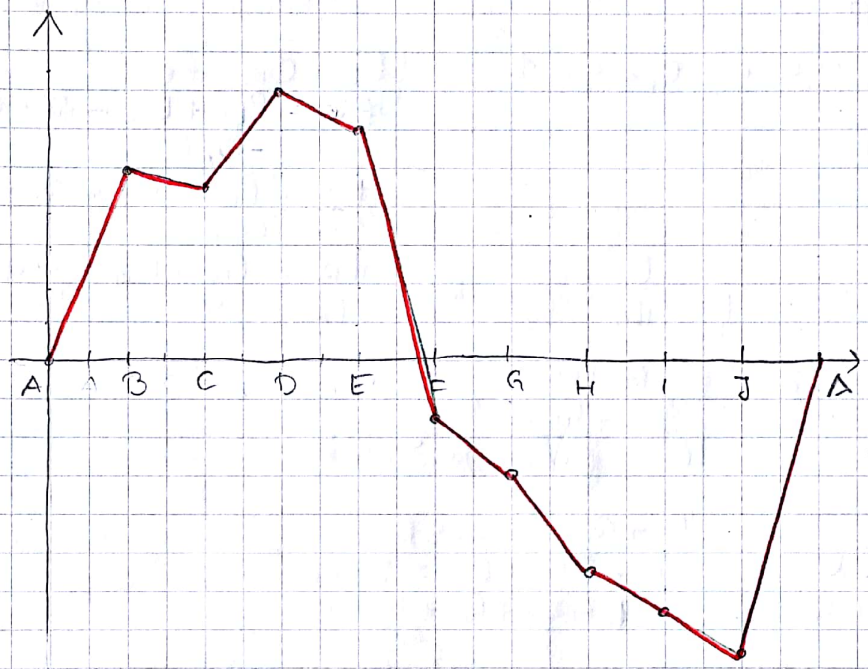
$\varphi_G = \varphi_F - I \cdot R_3 = -6V$

$\varphi_H = \varphi_G - E_4 = -11V$

$\varphi_I = \varphi_H - I \cdot R_4 = -13V$

$\varphi_J = \varphi_I - I R_5 = -15V$

$\varphi_A = \varphi_J + E_5 = 0$



$U_{AG} = 6V$

$U_{FI} = 10V$

$U_{HE} = -23V$

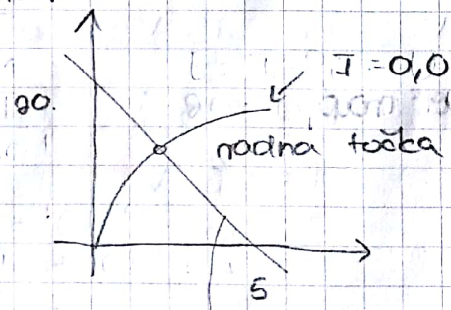
4. Na naponski izvor $E = 20V$; $R_i = 4\Omega$ priključeno je nelinearno trošilo $I_N = 0.01 U_1^2$. Odredite I u krugu i U na nelinearnom elementu.

$E = 20V$

$R_i = 4\Omega$

$E = U$; $I = 0$

$U = 0$; $I_{ks} = \frac{E}{R_i} = 5A$



$20 - 4I = 20 - 4\frac{U^2}{100}$

$U = 13,12\ V$

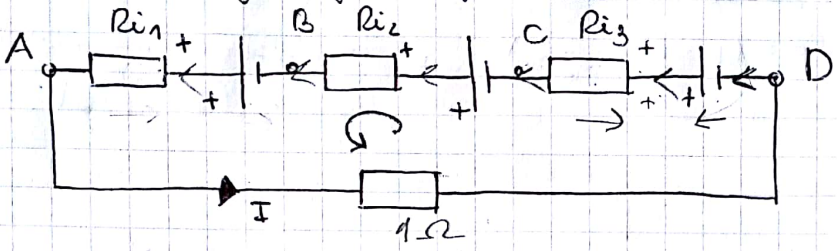
$I = 1,72\ A$

$U = 4 \cdot (5 - I)$

$U = 20 - 4I$

Prezentacija

3. $E = 1{,}25\ V$
$R_i = 0{,}004\ \Omega$
$U = 115\ V$
a) kad neopterećenog spoja 92
b) kad spoj daje struju $I = 25\ A$ = 100



$R = 1\ \Omega$ $R_{i1} = 0{,}2\ \Omega$
$E_1 = 2{,}2\ V$ $R_{i2} = 0{,}4\ \Omega$
$E_2 = 1{,}1\ V$ $R_{i3} = 0{,}5\ \Omega$
$E_3 = 0{,}9\ V$ U_A 0, I, P_R

$-4{,}2 = 1 \cdot 0{,}5 + 1 \cdot 0{,}4 + 1 \cdot 0{,}2 + 1 \cdot 1$
$I = -2\ A$

$P = I^2 \cdot R = 4$

$\varphi_D = 0$
$\varphi_C = \varphi_D + E_3 - 2 \cdot 0{,}5$
$= -0{,}1$
$\varphi_B = \varphi_C + E_2 - 2 \cdot 0{,}4$
$= 0{,}2$
$\varphi_A = \varphi_B + E_1 - 2 \cdot 0{,}2$
$\varphi_A = 2V$

3.6. $E = 20V$ R_i $P = \frac{E^2}{(R_i + R_t)^2} \cdot R_t$
$R_i = 4\ \Omega$ P
$m = 0{,}2$ $= 16\ W$

$m = \frac{R_t}{R_i + R_t}$ $16 = \frac{400}{(4+R)^2} \cdot R_2$

$0{,}2 = \frac{R_t}{4 + R_t}$ $(4+R)^2 = 25R_2$

$0{,}8 + 0{,}2R_t = R_t$ $16 + 8R + R^2 = 25R_2$

$R_t = 1\ \Omega$ $-R^2 - 17R + 16 = 0$

$R = 16$ $m_2 = 0{,}8$

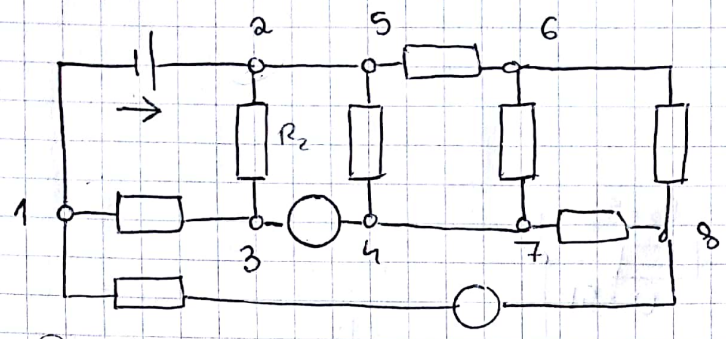
$E_1 = 20V$ $R_1 = 1\ \Omega$
$E_2 = E_4 = 5V$ $R_3 = 3\ \Omega$
$E_3 = E_5 = 15V$ $R_2 = R_4 = R_5 = 8\ 2\ \Omega$

$E_1 - E_2 - E_3 - E_4 + E_5 = I(R_1 + R_2 + R_3 + R_4 + R_5)$
$I = 1\ A$

$\varphi_B = 20\ V$ $\varphi_C = 19\ V$ $\varphi_D = 14\ V$ $\varphi_E = 12\ V$

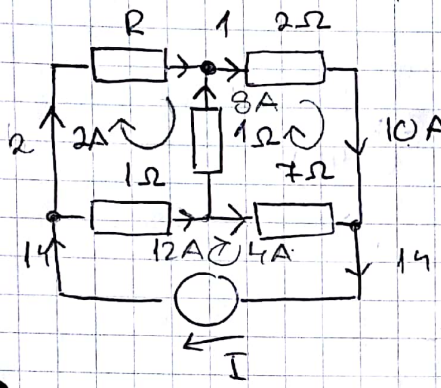
$\varphi_F = -3\ V$ $\varphi_G = -6\ V$ $\varphi_H = -11\ V$ $\varphi_I = -13\ V$

$\varphi_J = -15\ V$ $\varphi_A = 0\ V$



$U = P_2 - P_1 = 24\ V$

5. R, smjer izvora, napon na stezaljkama smjernog izvora



$I_1 + I_2 = I_3$

$2 + I_2 = 10$

$I_2 = 8A$

$10 \cdot 2 + 7 \cdot X + 1 \cdot 8 = 0$

$7Ix = -28$

$I = -4A$

$I = 8 + 4$

$I = 12\ A$

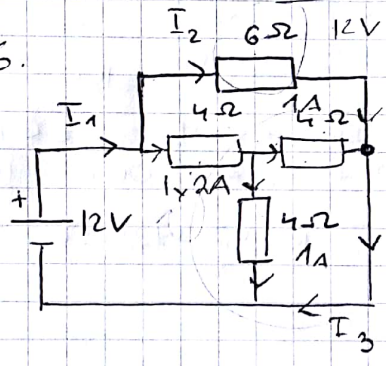
$2R - 8 - 12 = 0$

$2R = 20$

$R = 10\ \Omega$

$\underline{I = 14\ A}$ $\quad U = 12 + 28 = \underline{40\ V}$

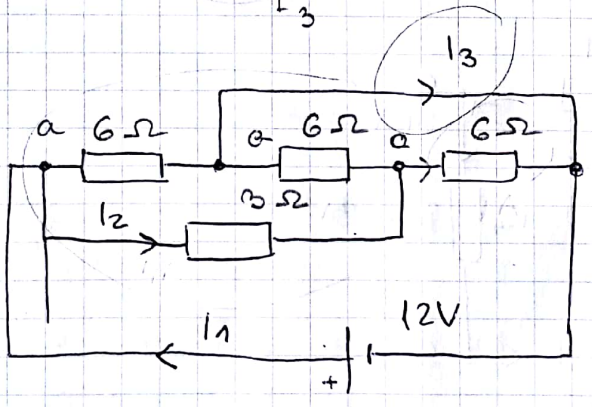
6.



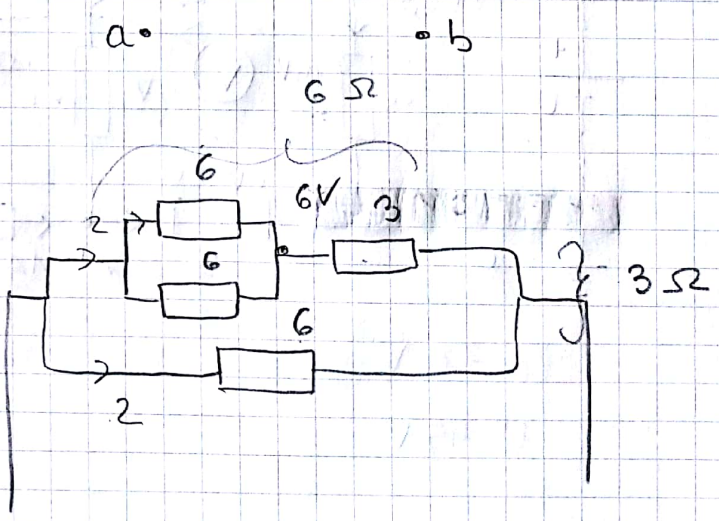
$I_2 = 2A$

6Ω } 3Ω

$I_{uk} = 4A$ $\quad I_1 = 4A$

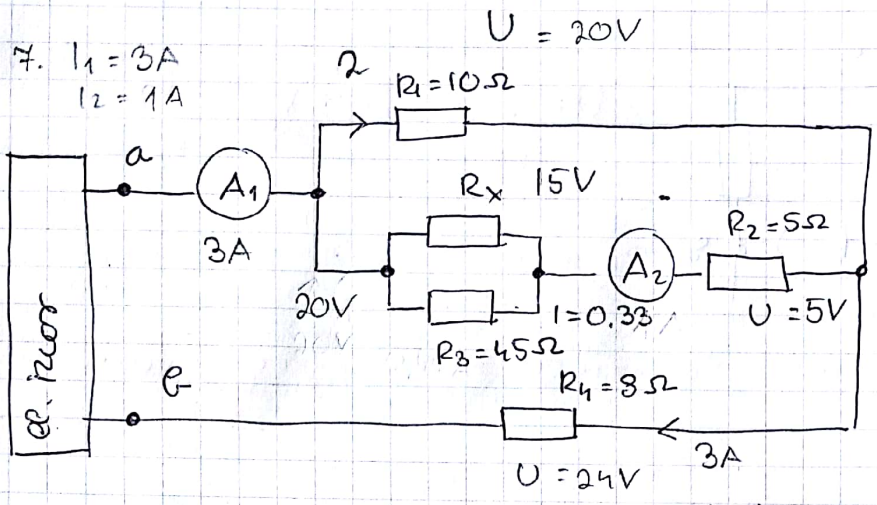


kako može ići struja ako nema razlike potencijala



$I = 4A$

7. $I_1 = 3A$
$I_2 = 1A$



$I = \frac{U}{R}$

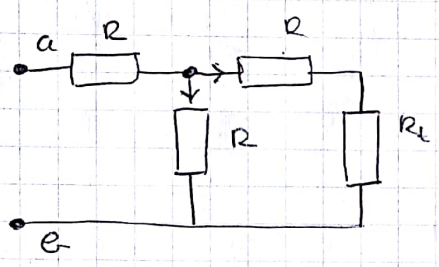
$R = \frac{U}{I}$

$R_x = \frac{15}{0.67} = 22,5\Omega$

$U_{wk} = 44V \quad I_{wk} = 3A$

Strujni izvor $I = 3A$
naponski izvor $U = 44V$

8.



$R_t = 100\Omega$

$\frac{1}{100+R} + \frac{1}{R} = \frac{1}{R_p}$

$\frac{R + 100 + R}{R(100+R)} = \frac{1}{R_p}$

$\frac{R(100+R)}{2R+100} + R = 100$

$\frac{R(100+R) + R(2R+100)}{2R+100} = 100$

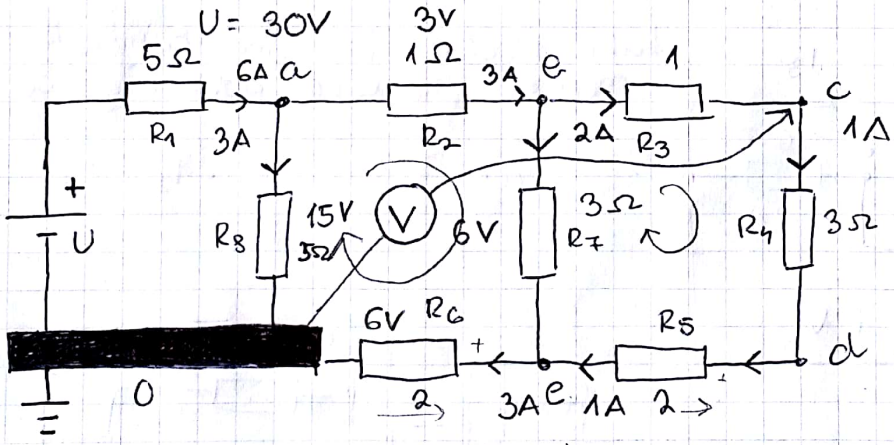
$100R + R^2 + 2R^2 + 100R = 200R + 100^2$

$200R + 3R^2 = 200R + 100^2$

9. bratlci spoj $R = 86,6\Omega$
beskonačan otpor $R = 115,47\Omega$

$R = 57,735\Omega$

10.

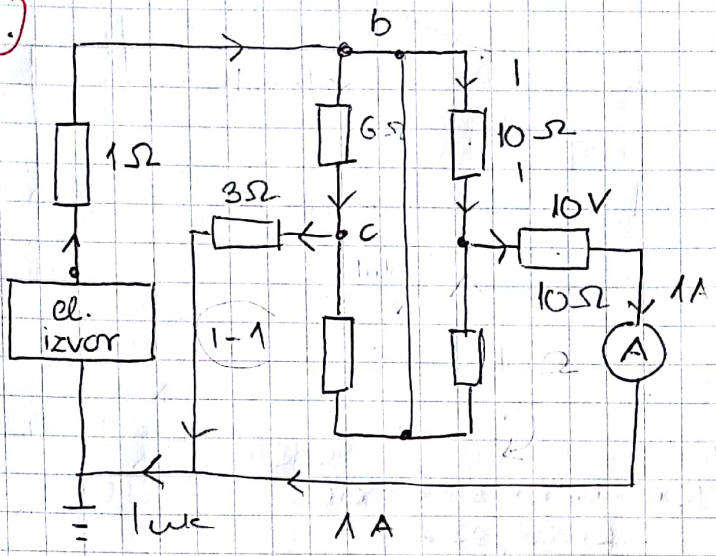


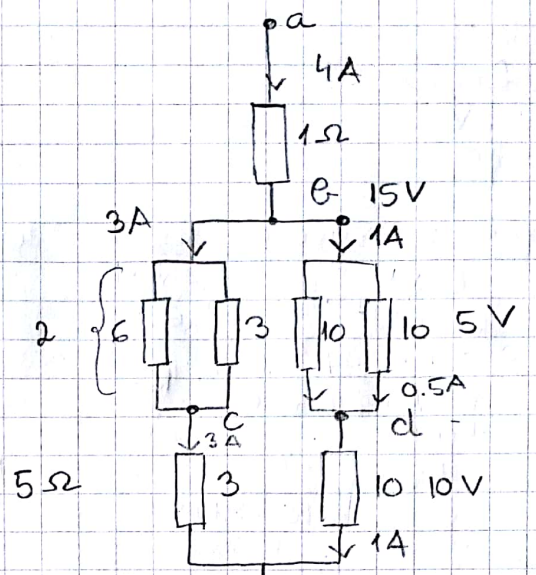
$3 + 6 + 6 = V$

$\varphi_e = 6V \quad \varphi_d = 8V \quad \varphi_c = 11V \quad \varphi_b = 12V \quad \varphi_a = 15V$

$U = 45V$

11.

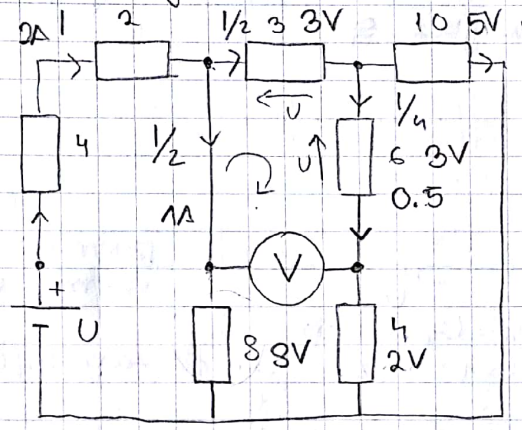




$\varphi_d = 10V \quad \varphi_b = 15V \quad \varphi_c = 9V \quad \varphi_a = 19V \quad 0$

Struja od 4A ide naponski 19V

15.



$U_V = 6V$

$I/2 \cdot 3 + I/4 \cdot 6 = 6$

$\frac{3}{2}I + \frac{3}{2}I = 6$

$3I = 6$

$I = 2A$

$I = 2A$

$R_{uk} = 10\Omega$

$U = 20V \quad P = U \cdot I = 40W$

$16 - 4 = 12V$

17. $d = 30\ cm$

$R_t = 55\Omega$

$R_{uk} = 55\Omega$

a) $0.25\,U$

$U_t = U \cdot \frac{1}{\frac{1}{\lambda} + \rho \cdot (1-\lambda)} \qquad \rho = \frac{R}{R_t}$

$0.25 = \frac{1}{\frac{1}{\lambda} + 1 - \lambda} = \frac{1}{\frac{1 + \lambda - \lambda^2}{\lambda}} = \frac{\lambda}{1 + \lambda - \lambda^2}$

$0.25 + 0.25\lambda - 0.25\lambda^2 = \lambda$

$0.25 - 0.75\lambda - 0.25\lambda^2 = 0$

18. $R_1 = 10\,\Omega$

$R_2 = [0, 100\,\Omega]$

$U = 12\,V$

$I = 2A$

$I =$

Snaga je najveća $R_2 = R_1 = 10\,\Omega$

$$P = \frac{E^2}{(R_i + R_t)^2} \cdot R_i = 3,6\,W$$

$$1,8 = \frac{12^2}{(10 + R_t)^2} \cdot R_t$$

$$1,8 = \frac{1440\,R_t}{(10 + R_t)^2}$$

$$100 + 20R_t + R_t^2 = 80R_t$$

$$100 - 60R_t + R_t^2 = 0$$

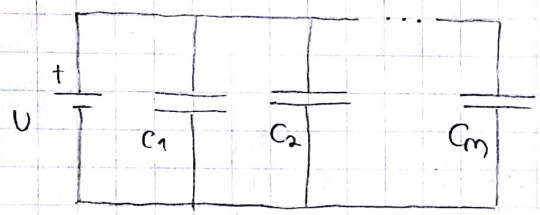
$$R_t = 58,28\,\Omega$$

$$R_t = 1,72\,\Omega$$

1.

KONDENZATORSKI SPOJEVI

paralelni spoj



$U = U_1 = U_2 \dots U_m$

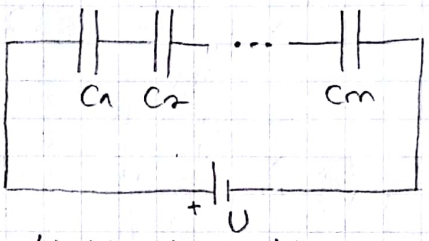
$Q = Q_1 + Q_2 \dots Q_m$

$Q = CU$

$CU = C_1U_1 + C_2U_2 \dots C_mU_{m_m}$

$C = C_1 + C_2 \dots C_m \quad \sum_{i=1} C_i$

serijski spoj



Izvor će nabiti samo vanjske

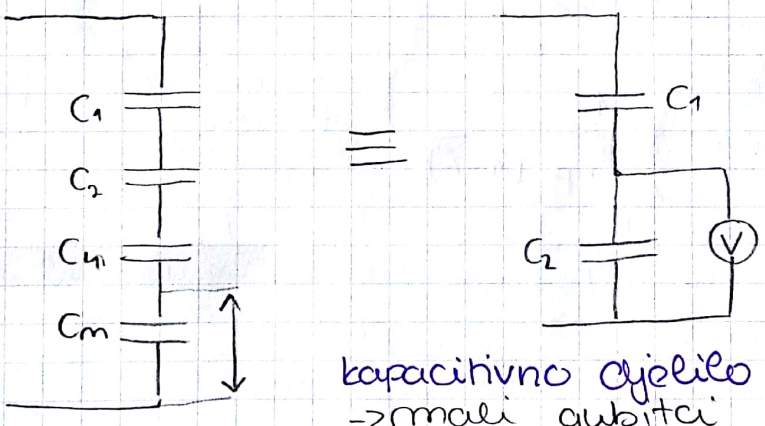
$U = U_1 + U_2 \dots U_m$

$Q = Q_1 = Q_2 \dots Q_n$ ← zbog influencije

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q_2}{C_2} \dots \frac{Q}{C_m}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \dots \frac{1}{C_m}$$

VISOKI NAPON



kapacitivno djelilo
→mali gubitci snage

$Q_1 = Q_2$

$C_1U_1 = C_2U_2$

$U = U_1 + U_2$

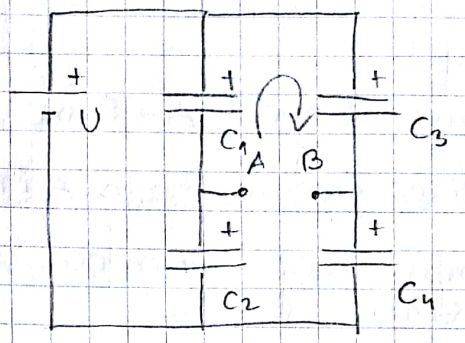
$$U_2 = \frac{C_1}{C_2}U_1 = \frac{C_1}{C_2}(U - U_2)$$

$$U_2\left(1 + \frac{C_1}{C_2}\right) = \frac{C_1}{C_2}U$$

$$U_2 = \frac{C_1}{C_2} \cdot U \cdot \frac{1}{1 + \frac{C_1}{C_2}}$$

$$= U \frac{C_1}{C_2\left(1 + \frac{C_1}{C_2}\right)}$$

$$= U \cdot \frac{C_1}{C_1 + C_2}$$



$U = 100\ V$
$U_{AB} = 55\ V$
$C_1 = 15\ mF$
$C_2 = C_3 = 5\ mF$
$Q_4 = ?$

$Q_1 = Q_2$
$C_1 U_1 = C_2 U_2$
$\frac{U_1}{U_2} = \frac{C_2}{C_1} = \frac{1}{3}$

$U_2 = 3U_1$
$U_2 = 75\ V$

II KI $\quad U = U_1 + U_2$
$= U_1 + 3U_1$
$100 = 4U_1$
$U_1 = 25\ V$

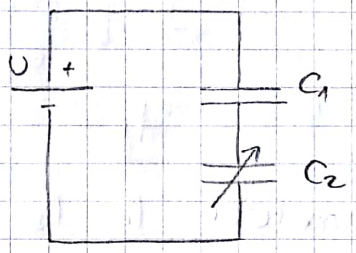
$\varphi_A + U_1 - U_3 = \varphi_B$
$\varphi_A - \varphi_B = U_3 - U_1$
$U_{AB} = U_3 - U_1$
$U_3 = U_{AB} + U_1$
$= 80\ V$

$Q_3 = Q_4 = C_3 \cdot U_3 = 80 \cdot 5 \cdot 10^{-6} F$
$= 400\ mC$

$U_4 = U - U_3 = 20V$

$C_4 = \frac{400mC}{20V} = 20mF$

2.9.



$C_2 \rightarrow$ mijenjam $\rightarrow$ maks W_2

$W = f(C_2)$ on se mijenja po izrazi $f(U)$

$W_2 = \frac{C_2 \cdot U_2^2}{2}$

$= \frac{U^2}{2} \cdot \frac{C_2}{\left(1 + \frac{C_2}{C_1}\right)^2} = \frac{U^2}{2} \cdot \frac{C_2 C_1^2}{(C_1 + C_2)^2}$

$U_2 = f(U)$
$U = U_1 + U_2$
$C_1 U_1 = C_2 U_2$

$U = \frac{C_2}{C_1} U_2 + U_2 = U_2 \left(\frac{C_2}{C_1} + 1\right)$

$U_2 = U \cdot \frac{1}{\frac{C_2}{C_1} + 1}$

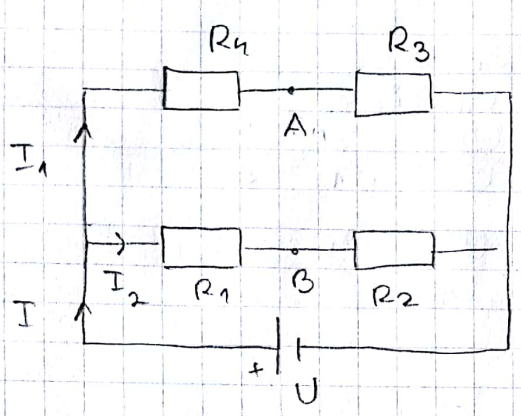
mak $W_2 \rightarrow$ derivacija $\frac{dW}{dC_2} = 0$

$W_2 = \frac{C_2 U_2^2}{2}$
$= 125\ nJ$

$\frac{U^2}{2} \cdot \frac{C_1^2 (C_1 + C_2)^2 - 2(C_1 + C_2) C_2 \cdot C_1^2}{(C_1 + C_2)^4} = 0$

$(C_1 + C_2)^2 - 2C_2(C_1 + C_2) = 0$
$C_1^2 + 2C_1C_2 + C_2^2 - 2C_1C_2 - 2C_2^2 = 0$
$C_1^2 - C_2^2 = 0$
$C_1 = C_2$
$C_2 = 10\ mF$
$U_2 = \frac{U}{2} = 5V$

MOSNI SPOJ



Analiziram pojavu između A i B ne teče struja
→ kao na slici (nema spoja A i B)

→ kroz element spojen između A i B ne teče struja $P_A = P_B$

$U = I_1 \cdot R_4 + I_1 \cdot R_3$

$U = I_2 \cdot R_1 + I_2 \cdot R_2$

$U_{AB} = I_1 \cdot R_3 - I_2 \cdot R_2$

$U_{AB} = I_2 \cdot R_1 - I_1 \cdot R_4$

Ako je $P_a = P_b \rightarrow U_{AB} = 0$

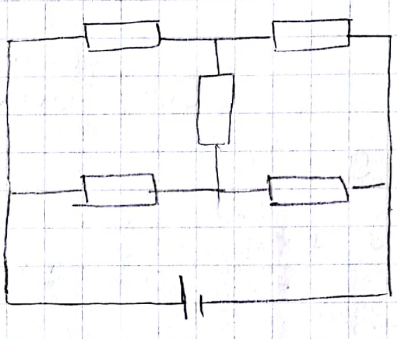
$I_1 \cdot R_3 = I_2 \cdot R_2$

$I_1 \cdot R_4 = I_2 \cdot R_1 \quad / :$

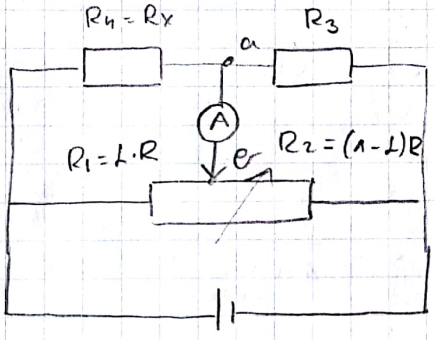
$\frac{R_3}{R_4} = \frac{R_2}{R_1} \quad | \quad R_1 R_3 = R_2 R_4$

Ako je umnožak dijagonalnih elemenata u mostu jednak, onda mosnom granom ne teče struja.

$U_{AB} = 0$



Primjena:



$I_A = 0$

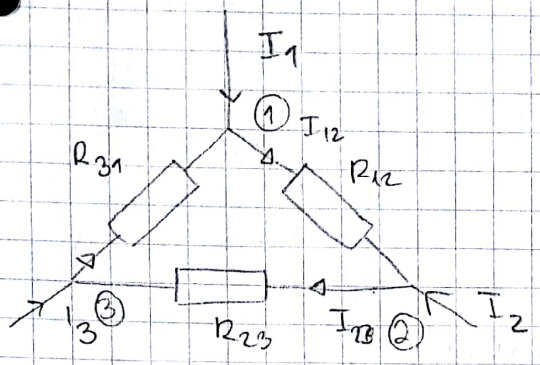
most u ravnoteži $P_a = P_b$

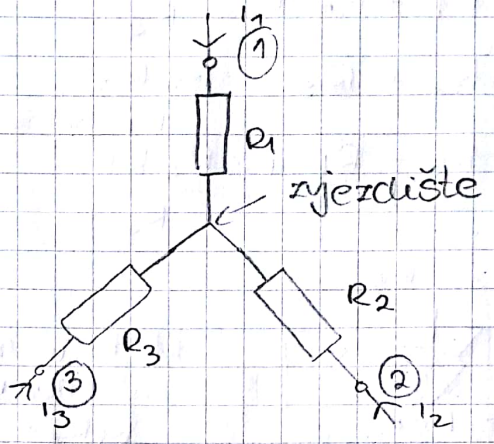
$R_2 R_4 = R_1 \cdot R_3$

$(1-l) R \cdot R_x = l \cdot R \cdot R_3$

$R_x = \frac{l}{1-l} \cdot R_3$

TRANSFORMACIJA TROKUT ZVIJEZDA Δ $\curlyvee$





pri transformaciji naponi i struje prema van moraju biti ne promijenjene

Δ $\curlyvee$

$$U_{12} = I_{12} \cdot R_{12} = I_1 R_1 - I_2 R_2 \quad \text{I}$$
$$U_{23} = I_{23} R_{23} = I_2 R_2 - I_3 R_3 \quad \text{II}$$
$$U_{31} = I_{31} R_{31} = I_3 R_3 - I_1 R_1 \quad \text{III}$$

$$I_1 = I_{12} - I_{31} \quad \text{IV}$$
$$I_2 = I_{23} - I_{12} \quad \text{V}$$
$$I_3 = I_{31} - I_{23} \quad \text{VI}$$

$$\text{I} + \text{II} + \text{III} = I_{12} R_{12} + I_{23} R_{23} + I_{31} R_{31} = 0$$

$$\text{IV} + \text{V} + \text{VI} = I_1 + I_2 + I_3 = 0$$

$\Delta \rightarrow \curlyvee$

znam: R_{12}, R_{23}, R_{31}
ne znam: R_1, R_2, R_3

iz IV $I_{31} = I_{12} - I_1$ u III
iz V $I_{23} = I_{12} + I_2$ u II

$$I_{12} R_{12} = I_1 R_1 - I_2 R_2$$
$$I_{12} R_{23} + I_2 \cdot R_{23} = I_2 R_2 - I_3 R_3$$
$$+ \quad I_{12} R_{31} - I_1 R_{31} = I_3 R_3 - I_1 R_1$$
$$I_{12}(\underbrace{R_{12} + R_{23} + R_{31}}_{R\Delta}) = -I_2 R_{23} + I_1 R_{31}$$

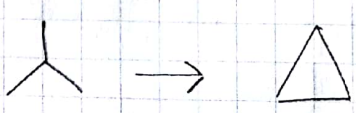
$$I_{12} = \frac{-I_2 R_{23} + I_1 R_{31}}{\Delta R} \quad \text{VII}$$

VII u I

$$\frac{I_1 \cdot R_{31}}{R\Delta} \cdot R_{12} - I_2 \cdot \frac{R_{23}}{R\Delta} R_{12} = I_1 R_1 - I_2 R_2$$

$$R_1 = \frac{R_{12} R_{31}}{R\Delta} \qquad R_2 = \frac{R_{23} R_{12}}{R\Delta} \qquad R_3 = \frac{R_{31} \cdot R_{23}}{R\Delta}$$

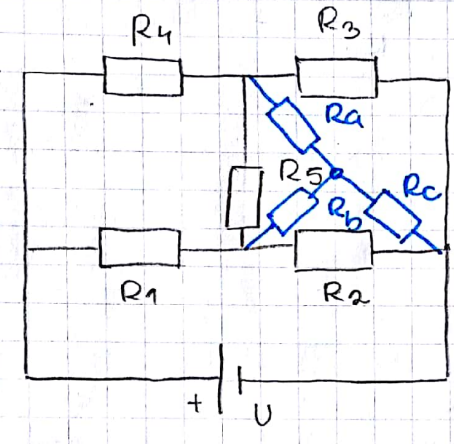
$$R_\Delta = R_{12} + R_{23} + R_{31}$$



$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

$$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

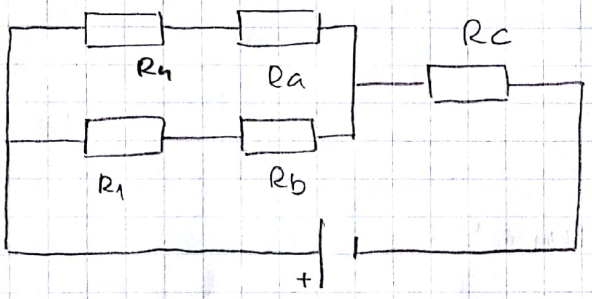
$$R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2}$$



Most u ravnoteži?
umnožak dijagonalnih
$R_1 R_3 = R_2 R_4$

Da → zanemarujem R5

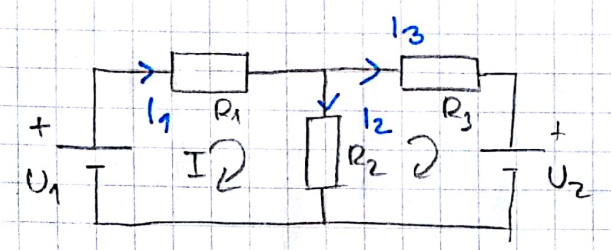
Ne → Δ → Y



$$R_a = \frac{R_3 R_5}{R_\Delta} \quad ; \quad R_\Delta = R_2 + R_3 + R_5$$

$$R_b = \frac{R_2 R_5}{R_\Delta} \quad ; \quad R_C = \frac{R_2 R_3}{R_\Delta}$$

METODA SUPERPOZICIJE



II KZ.

I. $U_1 = I_1 R_1 + I_2 R_2$

II. $-U_2 = -I_2 R_2 + I_3 R_3$

$0 = I_1 - I_2 - I_3$

$$\begin{bmatrix} U_1 \\ -U_2 \\ 0 \end{bmatrix} = \begin{bmatrix} R_1 & R_2 & 0 \\ 0 & -R_2 & R_3 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$\begin{bmatrix} U_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -U_2 \\ 0 \end{bmatrix} =$$

$\Downarrow$

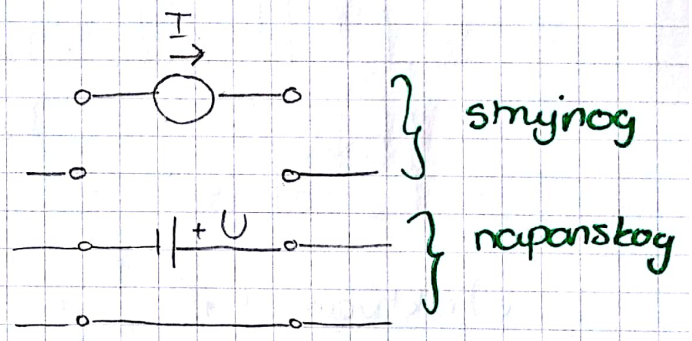
$\leftarrow$ doprinos izvora U_2

$\uparrow$ doprinos izvora U_1

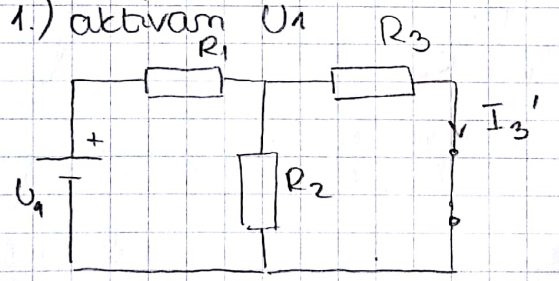
$$[U]_1 + [U_2] = [R][I]$$

$$[R]^{-1}[U]_1 + [R]^{-1}[U]_2 = [I]$$

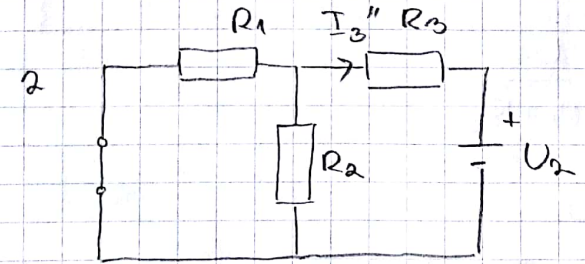
ISKLJUČENJE



1.) aktivam U_1



2



$$I_1' = \frac{U_1}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$$

$$I_3' = I_1' \cdot \frac{R_2}{R_2 + R_3}$$

$$I_3'' = \frac{-U_2}{R_3 + \frac{R_1 R_2}{R_1 + R_2}}$$

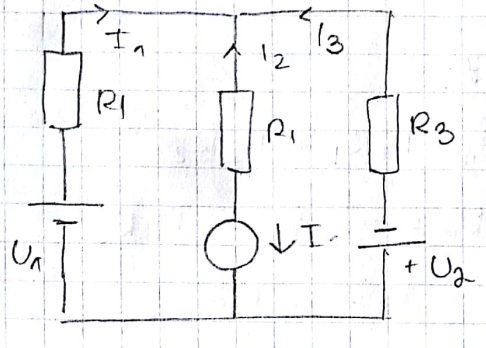
$$I_3 = I_3' + I_3''$$

KADA SUPERPOZICIJA

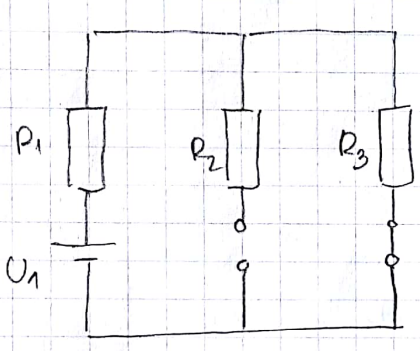
1. isključenje smjnih izvora kada pojednostavi mrežu
2. kad se izvori mijenjaju u zadatku

mpr. (Izračunajte prilike kada U_2 smanjimo na $U_2/2$)

I_1, I_2, I_3 u mreži prema slici



a) aktivan U_1

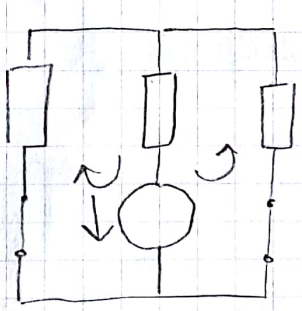


$$I_1' = \frac{U_1}{R_1 + R_3}$$

$$I_2' = 0$$

$$I_3' = \frac{-U_1}{R_1 + R_3}$$

b) aktivan I

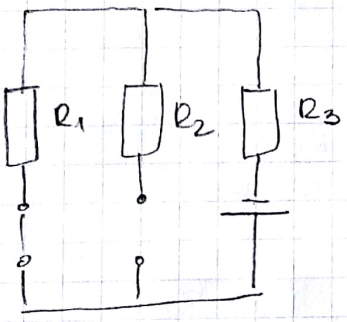


$$I_1'' = I \cdot \frac{R_3}{R_1 + R_3}$$

$$I_2'' = -I$$

$$I_3'' = I \cdot \frac{R_1}{R_1 + R_3}$$

c) aktivan U_2



$$I_1''' = \frac{U_2}{R_1 + R_3}$$

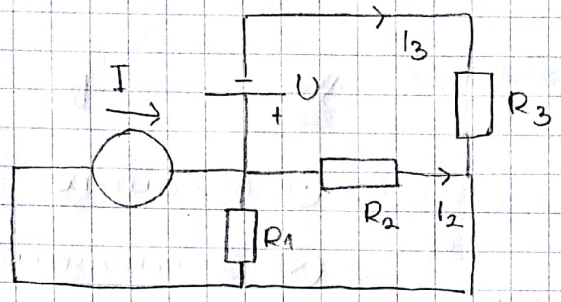
$$I_2''' = 0$$

$$I_3''' = -\frac{U_2}{R_1 + R_3}$$

$$I_1 = I_1' + I_1'' + I_1'''$$

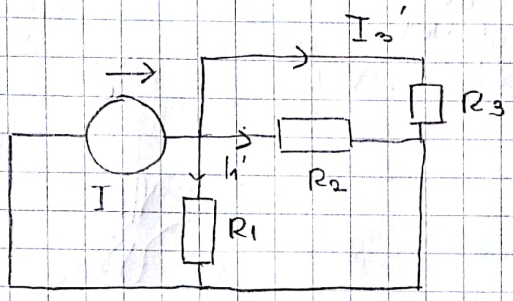
$$I_2 = I_2' + I_2'' + I_2'''$$

$$I_3 = I_3' + I_3'' + I_3'''$$



$I = 4A$
$U = 8V$
$R_1 = 2\,\Omega$
$R_2 = 2\,\Omega$
$R_3 = 1\,\Omega$

a)



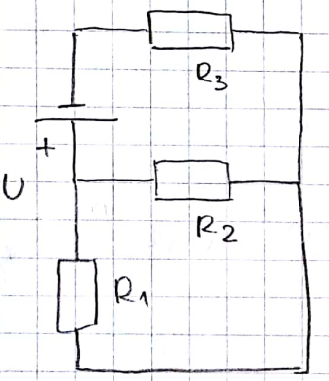
$$\frac{1}{R_{uk}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = 2$$

$$R_{uk} = \frac{1}{2}\,\Omega$$

$$U_{ab} = I \cdot R = 2V$$

$$I_2' = \frac{U_{ab}}{R_2} = 1\,A$$

b) aktivam U



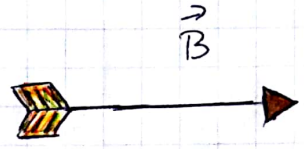
$$I_u = \frac{U}{\frac{R_1 \cdot R_2}{R_1 + R_2}} = \frac{8}{1+1} = 4\,A$$

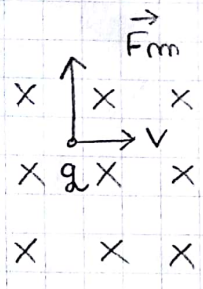
$$I_2' = I_u \cdot \frac{R_1}{R_1 + R_2} = 4 \cdot \frac{2}{2 \cdot 2} = 2\,A$$

$$I_2 = I_2' + I_1' = 3A$$

OSNOVE MAGNETIZMA

Magnetska sila na naboj u gibanju:





$\vec{F}m = q\,\vec{v} \times \vec{B}$ (vektorski produkt)

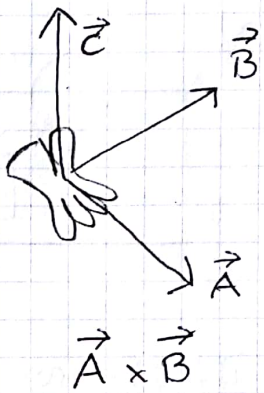
$= q\,|\vec{v}|\,|\vec{B}|\sin \vec{v}\,\vec{B}$

⊙ iz ravnine

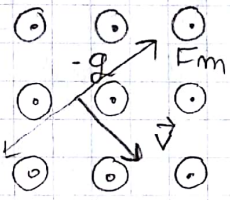
⊗ u ravninu

$\vec{v} \perp \vec{B} \qquad |Fm| = qvB$

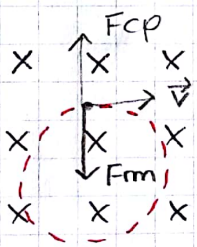
$\vec{A} \times \vec{B} = |\vec{A}| \cdot |\vec{B}| \cdot \sin \vec{A}\,\vec{B}$



$\vec{v} \parallel \vec{B} \qquad Fm = 0$



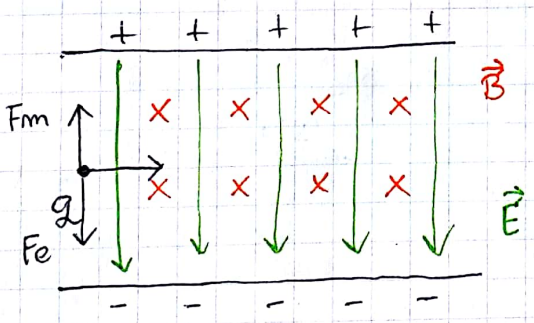
Elektron $m = 9{,}11 \cdot 10^{-31}$ kg, $e = 1{,}602 \cdot 10^{-19}$ C $\quad v = 2$ m/s, $B = 10^{-5}$ T



$$\frac{mv^2}{r} = qvB$$

$$r = \frac{mv}{qB} = 1{,}14 \cdot 10^{-6}\ \text{m}$$

Mlaz pozitivno nabijenih čestica, različitih m, a, v ulaze u prostor u kojem postoje magnetsko polje E i B. Odredite v čestica koje će se gibati pravocrtno.



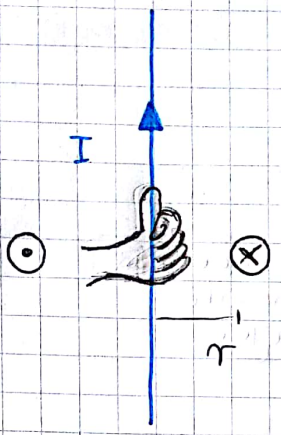
čestice se gibaju pravocrtno za slučaj

$|\vec{F}e| = |\vec{F}m|$

$qE = qvB$

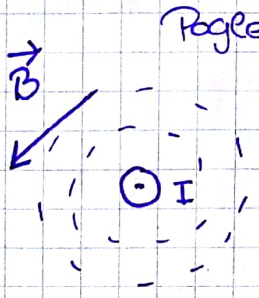
$v = \frac{E}{B}$

Protjecanje struje kroz vodiče



$$\vec{B} = \mu_0 \frac{I}{2\pi r}$$

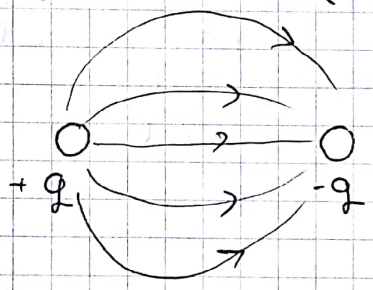
$$\mu_0 = 4\pi \cdot 10^{-7} \frac{H}{m}$$



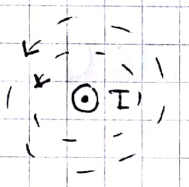
B svugdje tangencijalan jednakog iznosa na istoj kružnici

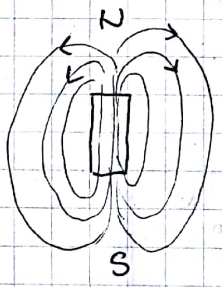
SILNICE MAGNETSKOG POLJA su zatvorene, kod el. polja nisu.

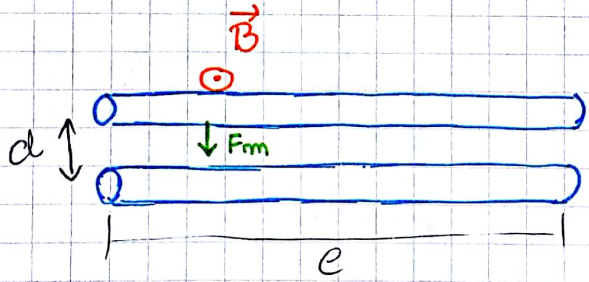
Elektrostatika



Magnetizam







$$F_1 = B_2 \cdot I_1 \cdot e$$

$$= \mu_0 \frac{I_2}{2\pi a} I \cdot e$$

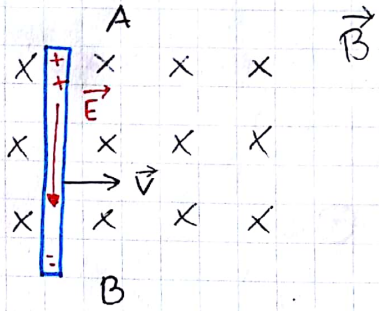
$$F = \mu_0 \frac{I_1 I_2}{2\pi a} \cdot e$$

Ako u vremenu Δt brzinom $\vec{v}$ prođe N naboja od kraja do kraja vodiča struja je $I = \frac{\Delta Q}{\Delta t} = \frac{\Delta N q}{\Delta t}$ Sila na svaki naboj $F = qvB$ $F = NqvB = N q \cdot \frac{e}{\Delta t} B = BIe$

$$\vec{F} = I(\vec{e} \times \vec{B})$$

↑ smjer prema smjeru struje

ELEKTROMAGNETSKA INDUKCIJA



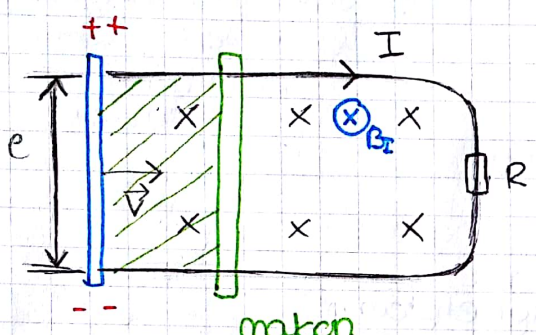
E - inducirano el. polje

rad sile koja razdvaja naboje

$$A = qvBe$$

$$U_{AB} = \frac{A}{q} = v \cdot B \cdot e$$

ravnoteža se uspostavlja kada je inducirano el. polje jednako polju razdvojenih naboja



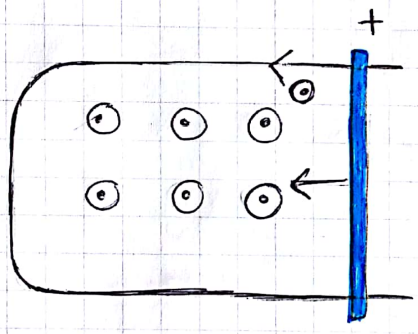
u Δt put vodiča

$$\Delta S = v \Delta t$$

$$\Delta S = \Delta S \cdot e = e \cdot v \cdot \Delta t$$

$$|U_{ind}| = v \cdot B \cdot e = \frac{v \cdot B \cdot e \cdot \Delta t}{\Delta t} = \frac{B \Delta S}{\Delta t}$$

$$\Delta \Phi = B \cdot \Delta S \qquad = \frac{\Delta \Phi}{\Delta t}$$



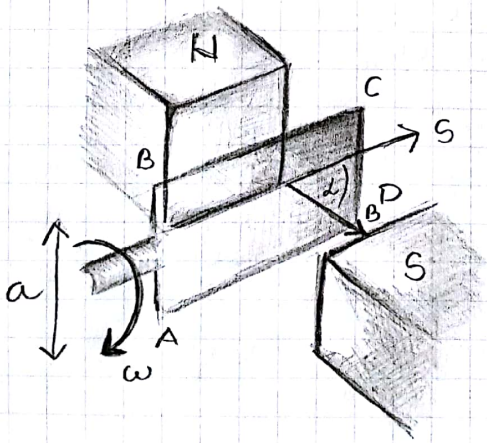
$$\Phi = B \cdot \Delta S$$
$$\uparrow \quad \downarrow$$

LENZOVO PRAVILO

$$U_{ind} = - \frac{\Delta \Phi}{\Delta t}$$

- zbog opiranja promjeni

PRINCIP RADA GENERATORA



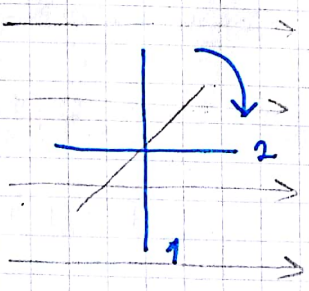
1) $\Phi = S \cdot B \cdot \cos\alpha$

$S = ab$

$\alpha = \omega t$

$$U_{ind} = \frac{d\Phi}{dt} =$$

$$\omega S B \sin(\omega t)$$



2) $v_\perp = v \sin\alpha$

$U_{BC} = BCv_\perp = BCv\sin\alpha$

$U_{AD} = BCv\sin\alpha$

$U = 2BCv\sin\alpha$

$v = r \cdot \omega = \frac{a}{2} \cdot \omega$

1) Φ maks

2) $\Phi = 0$

$U_{ind} = BCa\omega\sin\alpha$

zbirka; kondenzatorski krugovi

1. $C_1 = 3{,}3 \cdot 10^{-6}\ F$ $U_{m1} = 63\ V$
$C_2 = 4{,}7 \cdot 10^{-6}\ F$ $U_{m2} = 100\ V$

} Ako je napon veći od maksimalnog dolazi do povećanja el. polja i proboja

u paraleli - 63 V

$Q_1 = C_1 \cdot 63\ V = 207\ nC$ $Q_2 = C_2 \cdot 63\ V = 296\ nC$

serijski spoj $\frac{C_1}{C_2} = \frac{U_2}{U_1}$ manji kondenzator dobiva veći napon

$\frac{C_1}{C_2} \times \frac{U_2}{U_1}$

$C_1 U_1 = C_2 U_2$

$3{,}3 \cdot 10^{-6} \cdot 63 = 4{,}7 \cdot 10^{-6} \cdot U_2$

$U_2 = 44{,}23\ V$ $U_{uk} = 107{,}23\ V$

$Q_1 = Q_2 = 207\ nC$

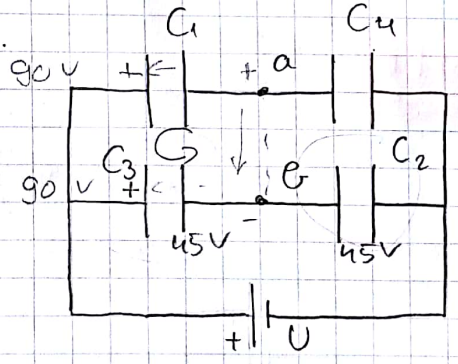
2. $C_1 = C$
U
$Q = U \cdot C$
$= 1000\ nC$

$U_1 = U_2$
$\frac{Q_1}{C_1} = \frac{Q_2}{3C_2}$
$3Q_1 = Q_2$

$Q_1 = 250\ nC$
$Q_2 = 750\ nC$
$U = 25\ V$

↓ naboji prestaju prelaziti kada se naponi izjednače

3.



$C_1 = 30\ nF$ $C_3 = C_2 = C_4 = 60\ nF$
$U = 90\ V$

$Q_3 = Q_2 = 45\ V \cdot 60 \cdot 10^{-9}\ F =$

$C_{uk} = 50\ nF$

$\frac{C_1}{C_4} = \frac{U_4}{U_1}$ $U_1 = 60\ V$ $U_{ab} = 45 - 60 = -15\ V$
$\frac{1}{2} = \frac{U_4}{U_1}$ $U_4 = 30\ V$
$U_1 = 2U_4$

$Q_{uk} = U \cdot C_{uk} = 4500\ C$

zašto idem od b do a da bih dobila U

2.1. serija $C_1 : C_2$

$U/U_2 = 2$ $\quad \frac{U}{U_2} = 2$

$\frac{C_2}{C_1} = \frac{U_1}{U_2}$ $\quad U = 2U_2$

$U_2 = \frac{1}{2} U$

$\frac{C_2}{C_1} = 1$

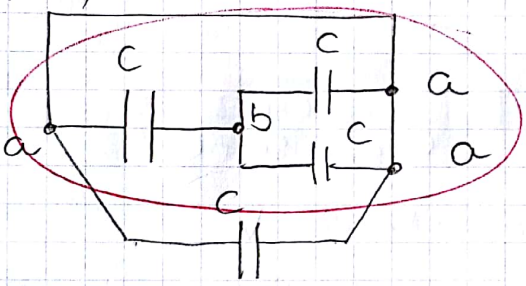
$\frac{U}{U_2} = 9$ $\quad \frac{C_2}{C_1} = \frac{\frac{8}{9}U}{\frac{1}{9}U} = 8$

$U = 9U_2$

$U_2 = \frac{1}{9} U$

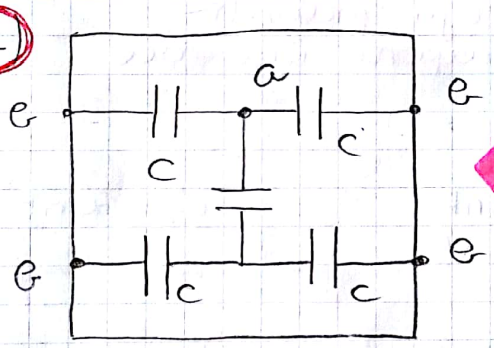
$U_1 = \frac{8}{9} U$

2.2. a)



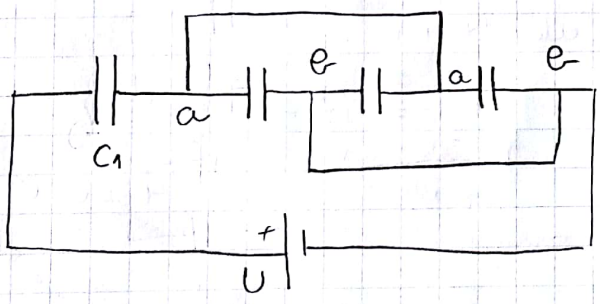
3 paralelno 3C

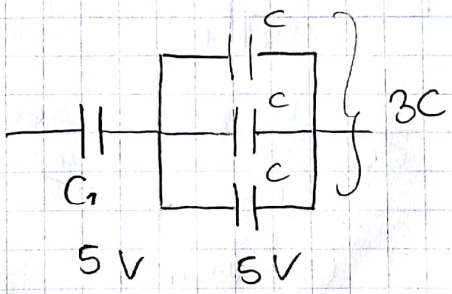
e)



NE ZNAM!

2.3.

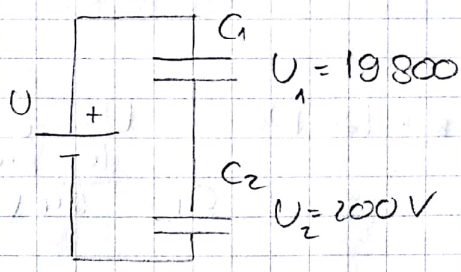




$U = 10\,V$ $\quad \frac{1}{3 \cdot 10} + \frac{1}{30} = \frac{1}{C_{uk}}$

$C_{uk} = 15\,nF$

2.5. $U = 20\ kV$
$n = 100$
$W_{uk} = 0{,}1\ J$

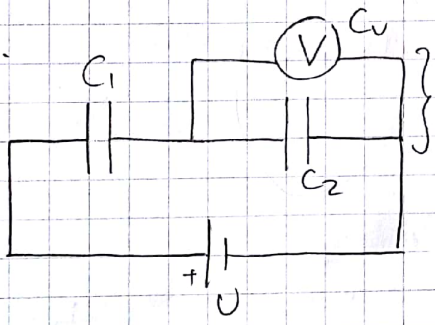


$$\frac{C_1}{C_2} = \frac{U_2}{U_1}$$

$$W_{uk} = \frac{C_{uk} \cdot U^2}{2} \Rightarrow 0{,}1 = \frac{20000^2 \cdot C_{uk}}{2} \Rightarrow C_{uk} = 5 \cdot 10^{-10}\ C$$

$$Q_1 = Q_2$$
$$19800\ C_1 = 200\ C_2$$
$$19800 \cdot C_1 = 200 \cdot (5 \cdot 10^{-9} - C_1)$$
$$19800\ C_1 = 1 \cdot 10^{-7} - 200\ C_1$$
$$20000\ C_1 = 1 \cdot 10^{-7}$$

2.6.



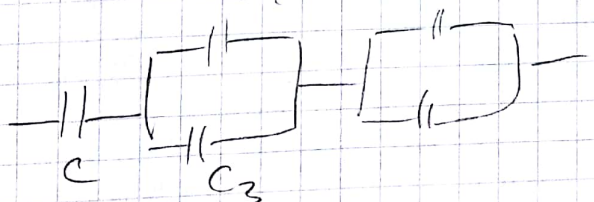
$C_1 = 2{,}2 \cdot 10^{-9}\ F$
$C_2 = 330\ pF$
$C_v = 150\ pF$
$U = 100\ V$

$$\frac{1}{2{,}2 \cdot 10^{-9}} + \frac{1}{480 \cdot 10^{-12}} = \frac{1}{C_{uk}} \Rightarrow C_{uk} = 3{,}94 \cdot 10^{-10}\ F$$

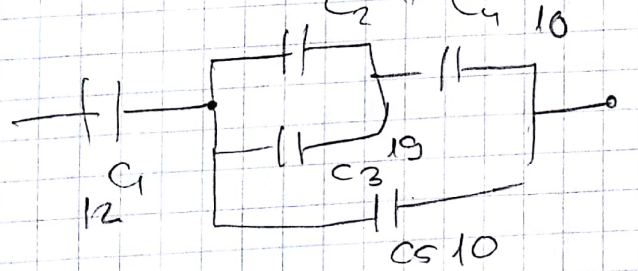
$$Q = C_{uk} \cdot 100 = 3{,}94 \cdot 10^{-8}\ C$$

$$U_2 = 100 - \frac{Q_1}{C_1} = 98{,}2$$

bez Cv



$$\frac{1}{11} + \frac{1}{19}$$



$Q_4 =$

$C_{uk} =$

$$C = \frac{Q}{U}$$

2.7. $C_3 = 11\,nF$, $U_{C_3} = 20\,V$ } $Q = C \cdot U = 2{,}2 \cdot 10^{-4}\,C = 220\,nC$

$C_2 = 5\,nF$, $Q_{C_2} = 120\,nC$, $U = \frac{Q}{C} = 24\,V$

$Q_{C_1} = 100\,nC$, $C_1 = \frac{100\,nC}{24\,V} = 4{,}16\,nF$

$U = 44\,V$

$C_{12} = 9.16\,nC$

$C_{uk} = 5\,nF$

2.8. $Q_1 = 10 \cdot 10^{-9}\,C$, $C_1 = 1 \cdot 10^{-9}\,F$ } $U = 10\,V$

$\frac{C_1}{C_{23}} = \frac{U_{23}}{U_1}$

$2 = \frac{U_{23}}{10}$

$U_{23} = 20\,V$

$C_2 = 2 \cdot 10^{-9}\,F$

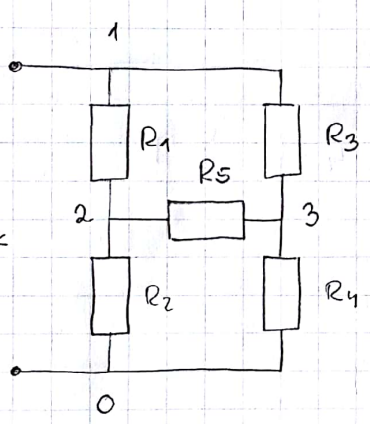
$C_3 = 3 \cdot 10^{-9}\,F$

$C_4 = 4 \cdot 10^{-9}\,F$

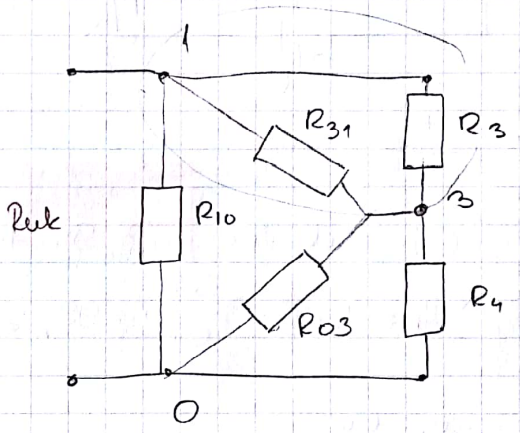
$C_{23} = 5 \cdot 10^{-9}\,F$, $U = 30\,V$, $Q = 150\,nC$ } $30.5\,10^{-9}$

SLOŽENI KRUGOVI ISTOSMJERNE STRUJE

P. 1.



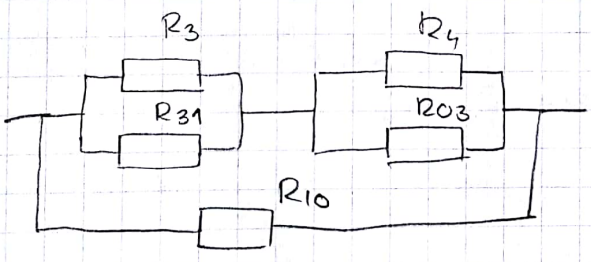
$R_1 = 4\,\Omega$
$R_2 = 1{,}6\,\Omega$
$R_3 = 6\,\Omega$
$R_4 = 12\,\Omega$
$R_5 = 4\,\Omega$



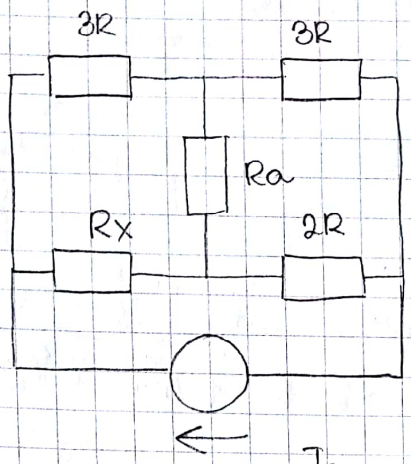
$R_{10} = R_1 + R_2 + \frac{R_1 R_2}{R_5} = 7.2\,\Omega$

$R_{03} = R_2 + R_5 + \frac{R_2 R_5}{R_1} = 7{,}2\,\Omega$

$R_{31} = R_5 + R_1 + \frac{R_5 \cdot R_1}{R_2} = 18\,\Omega$



P.2.



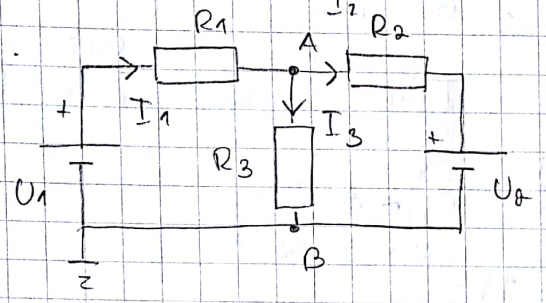
$R_x = 2R = 6\ \Omega$

$R_{uk} = 11,2\ \Omega$

$I = 1,5\ A$

$U = R_{uk} \cdot I = 10,8\ V$

P.3.



$R_1 = 3\ \Omega$
$R_2 = 3\Omega$
$R_3 = 6\ \Omega$
$U_1 = 24\ V$
$U_2 = 12\ V$

superpozicija

aktivan U_1

$R_{23}' = 2\ \Omega$
$R_{uk}' = 5\ \Omega$
$I' = U/R = 4,8\ A$

$U_1 = U_2$

$I_3 \cdot R_3 = I_2 \cdot R_2$

$I_3 \cdot 6 = I_2 \cdot 3$

$2I_3 = I_2$

$I_1' = 4,8\ A$
$I_3' = 1,6\ A$
$I_2' = 3,2\ A$

aktivan U_2

$R_{13} = 2\ \Omega$
$R_{uk} = 5\ \Omega$
$I' = 2,4\ A$

$6I_3 = 3I_1$
$2I_3 = I_1$

$I_1'' = -1,6\ A$
$I_2'' = -2,4\ A$
$I_3'' = 0,8\ A$

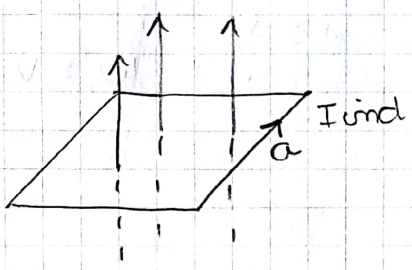
$I_1 = 3,2\ A$
$I_2 = 0,8\ A$
$I_3 = 2,4\ A$

Primjer 4.

$v = 900\ km/h$
$\ell = 48\ m$
$B = 5\ 10^{-5} T$

$$U_{ind} = B\ell v = 5 \cdot 10^{-5} \cdot 48 \cdot \frac{900 \cdot 10^3}{3600} = 0.6\ V$$

5. $B = 1\ T$
$a = 4\ cm$
$R = 0,1\ \Omega$
$\Delta t = 2\ ms$



$$|U_{ind}| = \frac{\Delta\Phi}{\Delta t} = \frac{16 \cdot 10^{-4}}{2 \cdot 10^{-3}} = 0,8\ V$$

$\Phi_1 = B \cdot S = 1 \cdot (4 \cdot 10^{-2})^2 = 16 \cdot 10^{-4}\ Vs$

$\Phi_2 = 0$

$$I_{ind} = \frac{|U_{ind}|}{R} = \frac{0,8}{0,1} = 8A$$

6. $\ell = 2\ m$
$B = 0,1\ T$
$v = 5\ m/s$
$R = 5\ \Omega$

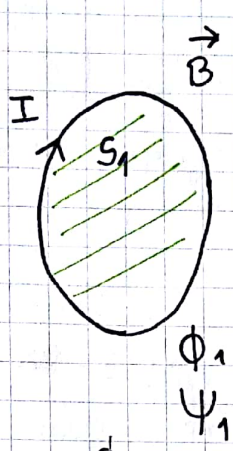
a) $$I = \frac{|U_{ind}|}{R} = \frac{1\ V}{5\ \Omega} = 0,2\ A$$

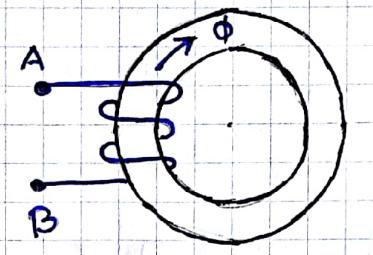
$|U_{ind}| = B\ell v = 1\ V$

b) $F = BI\ell = 0,1 \cdot 0,2 \cdot 2 = 0.04\ N$

c) $P = F \cdot v = 0.04 \cdot 5 = 0.2\ W$ $\quad P = I^2 R = 0.2\ W$

INDUKTIVITET

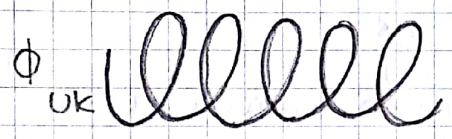




$\Psi = N \cdot \Phi$
$= NBS$
$= N(\mu_0 \frac{NI}{l})S$
$= \boxed{k} I$

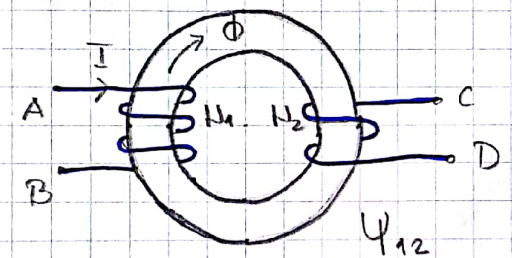
koeficijent samoindukcije

$L = \frac{\Phi}{I}$ $\quad U_{ind} = -\frac{d\Phi}{dt} = -\frac{d}{dt}(L \cdot i) = -L\frac{dI}{dt}$ $\quad L\,[1\,H]$



$U_{ind1} = \frac{d\Phi_{uk}}{dt}$

$U_{ind} = N \cdot U_{ind1} = -N\frac{d\Phi_{uk}}{dt}$



$e_m = -\frac{d\Psi}{dt}$ $\quad U_{ind} = -\frac{\Psi_{12}}{dt}$

$\Psi_{12} = N_2\Phi = N_2BS$
$= N_2\mu_0\frac{N_1 e_1}{l} \cdot S$
$= k \cdot i_1$

$\frac{d(M_{12} i_1)}{dt} = -M_{12}\frac{di_1}{dt}$

$M = \frac{\Psi_{21}}{I_1} = \frac{N_2 \cdot \Phi_{21}}{I_1} = \frac{\Psi_{21}}{I_2} = \frac{N_1\Phi_{12}}{I_2}$

$M = k\sqrt{L_1 L_2}$ $\quad 0 \le k \le 1$

$N_1 = 50$
$N_2 = 100$

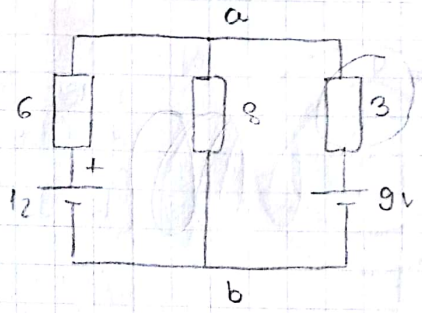
$I_1 = 1\,A$
$\Phi_1 = 2 \cdot 10^{-2}\,Vs$
$\Phi_1 = 10^{-2}\,Vs$

$L_1 = \frac{\Psi_1}{I_1} = 1\,H$

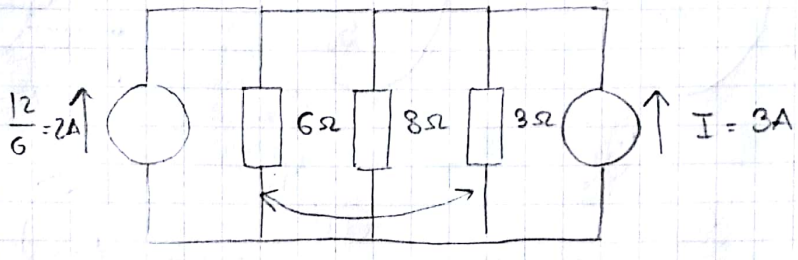
$M = \frac{\Psi_{21}}{I_1}$

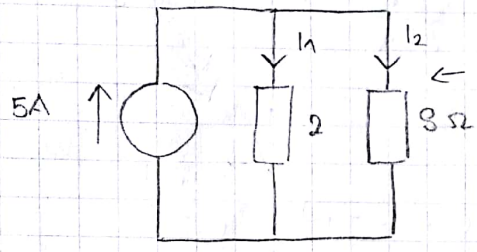
Složeni krugovi istosmjerne struje - zadaci

P. H.



njih trebamo kao unutarnji otpor izvora





← djelilo struje

$\frac{R_1}{R_2} = \frac{(5 - I_1)}{I_1}$

$40 - 8I_1 = 2I_1$

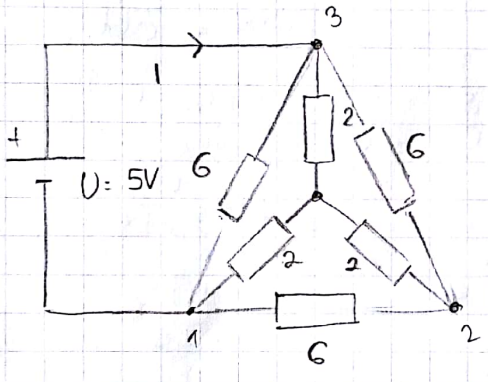
$40 = 10 I_1$

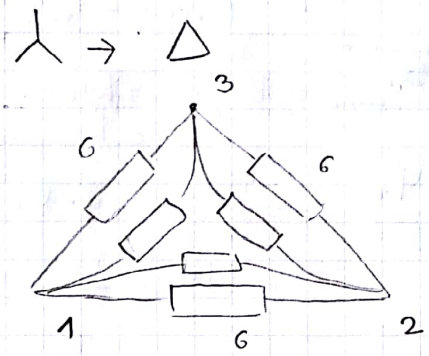
$I_1 = 4A$

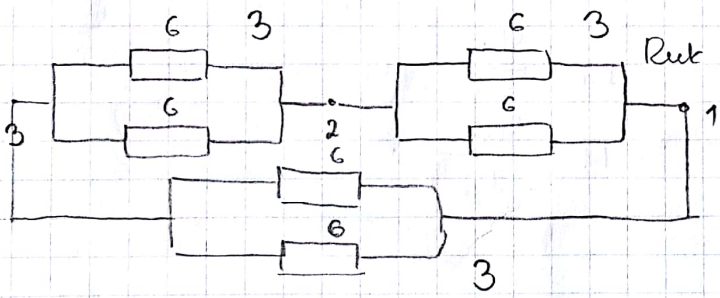
$I_2 = 1\ A$

napon između a i b jednak je padu napona na otporniku $U = I \cdot R = 8V$

1.



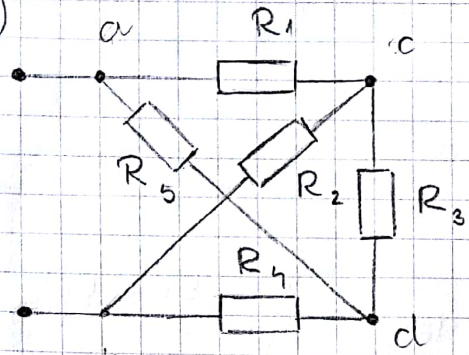


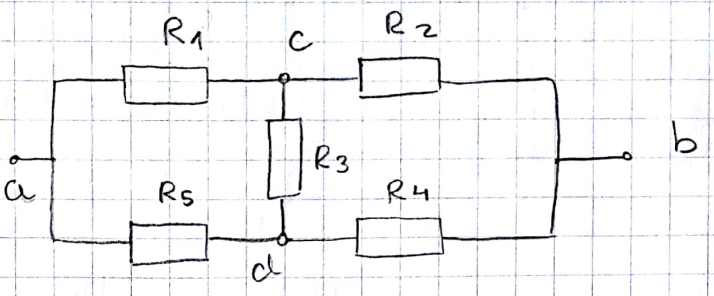


Rut $6 \| 3 = 2\ \Omega$

$I = \frac{U}{R} = 2.5A$

2. a)



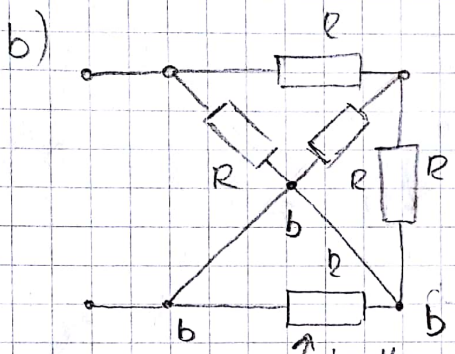


$R_1 \cdot R_4 = R_2 \cdot R_5$

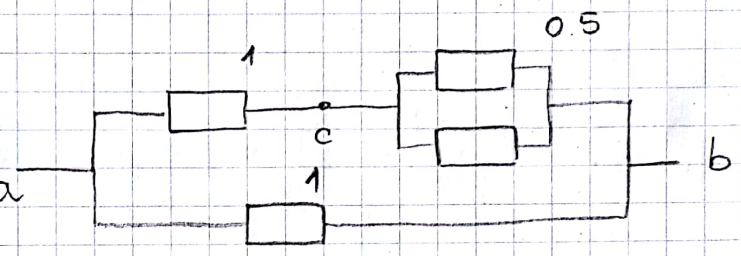
$1 = 1$ ⇒ spoj je el. most

$R_{uk} = \frac{1}{\frac{1}{2} + \frac{1}{2}} = 1\ \Omega$

b)

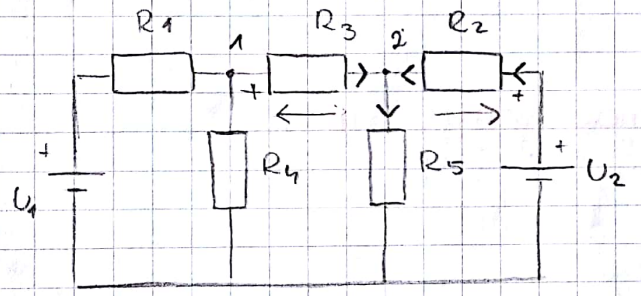


↑ kratko spojen



$R_{uk} = 0.6\ \Omega$

3.



$R_3 = 4\ \Omega$

$R_2 = 6\ \Omega$

$U_2 = 12\ V$

$P_1 = 12\ V$

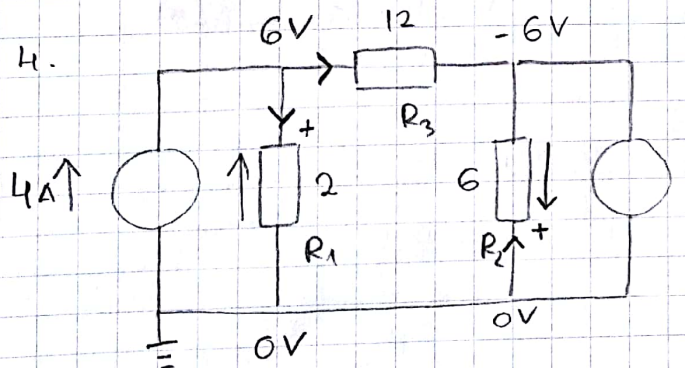
$P_2 = 9.96\ V$

pad napona na $R_2 = U_{R2} - P_2 = 2.04V$ $\quad I_{R_2} = \frac{U}{R} = \frac{2,04}{6} = 0.34A$

pad napona na $R_3 = P_1 - U_{R3} = P_2$ $\quad U_{R3} = 2,04V$ $\quad I_{R3} = 0.51\ A$

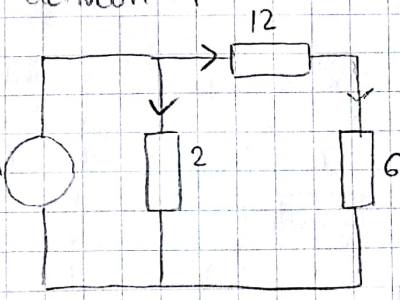
$I_{R5} = I_{R2} + I_{R3} = 0.85\ A$

4.



Superpozicija

aktivan I_1



$R_{uk} = 1.8\ \Omega$

$\frac{R_1}{R_{11}} = \frac{4 - I_{R_1}}{I_{R_1}}$

$\frac{2}{18} = \frac{4 - 1}{1}$

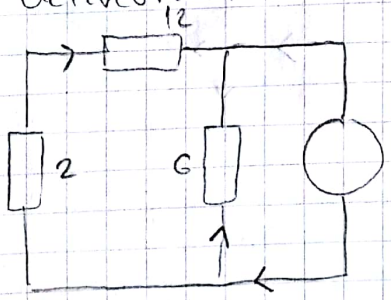
$21 = 72 - 18I$

$20I = 72$

$I_{R_1} = 3,6$

$I_{R_{23}} = 0.4A$

aktivan I_2



$\frac{6}{14} = \frac{I_{R_{31}}}{I_{R_2}} = \frac{2 - I_{R_2}}{}$

$28 - 14I_{R_2} = 6I_{R_2}$

$28 = 20I_{R_2}$

$I_{R_2} = 1.4A$

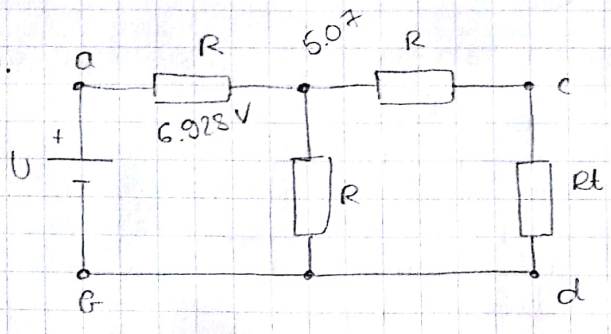
$I_{R_{31}} = 0.6A$

$I_{R_3} = 1\ A$

$I_{R_2} = 1A$

$I_{R_1} = 3A$

5.



$U = 12\ V$
$Rt = 100\ \Omega$
$R = 57.735\ \Omega$

$R_{uk} = 100\ \Omega$ $\quad I = 0.12\ A$ $\quad U_R = 6.928$

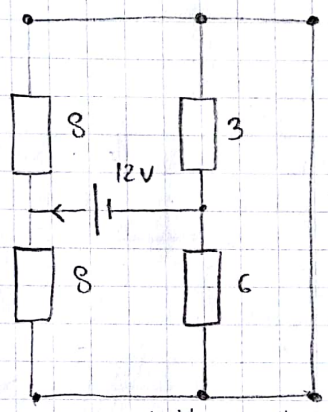
$\frac{R}{R_t} = \frac{5,07 - U_t}{U_t}$

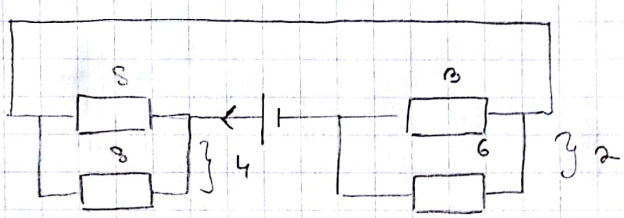
$57.735\ U_t = 507 - 100\ U_t$

$R_t = 3.21\ V$

$\frac{U_{Rt}}{U} = \frac{3.21}{12} = 0.268$

6.



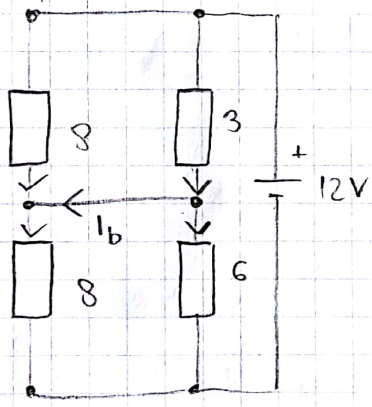


$R_{uk} = 6\ \Omega$

$I = \frac{U}{R} = 2\ A$

točke istog potencijala

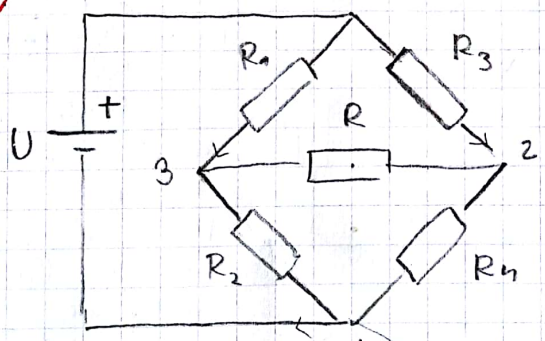
b)



Kako izračunati struju Ib ?

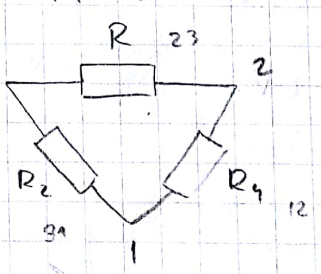
c) superpozicijom iz a i b se dobije $I = I_b + I_a = 2,33\ A$

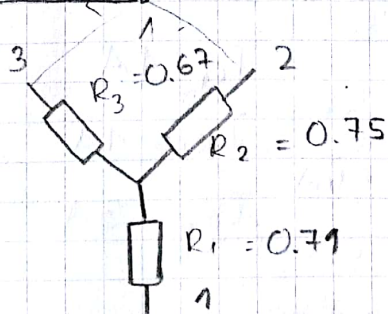
7



$U = 12\ V$
$R = 5\ \Omega$
$R_1 = 6\ \Omega$
$R_2 = 3\ \Omega$
$R_3 = 12\ \Omega$
$R_4 = 4\ \Omega$

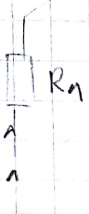
$R_1 \cdot R_4 = R_2 \cdot R_3$
$R_1 \cdot 4 = 3 \cdot 12$
$R_1 = 9\ \Omega$





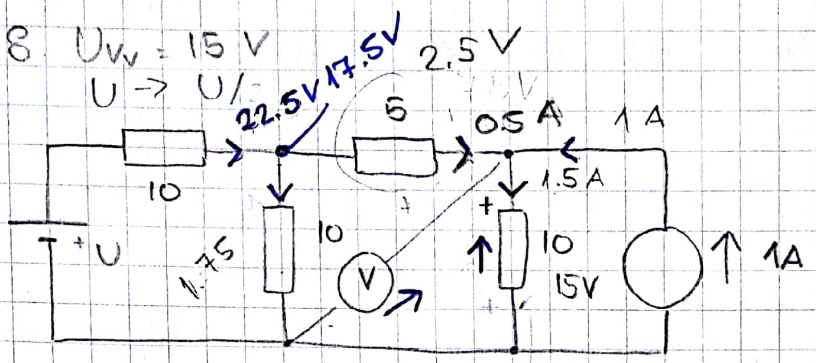
$R_{uk} = 5.089$

$I_{uk} = 14.1478\ A$



8 $U_V = 15\ V$

$U \to U/$

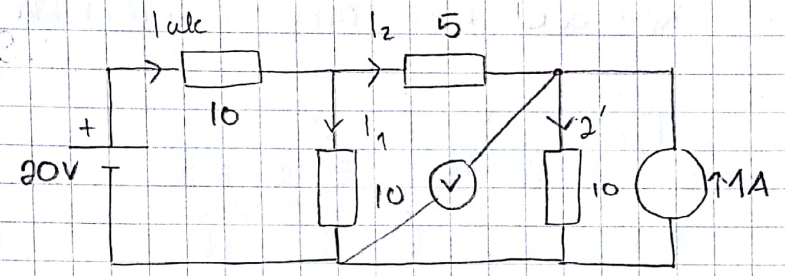


Superpozicija

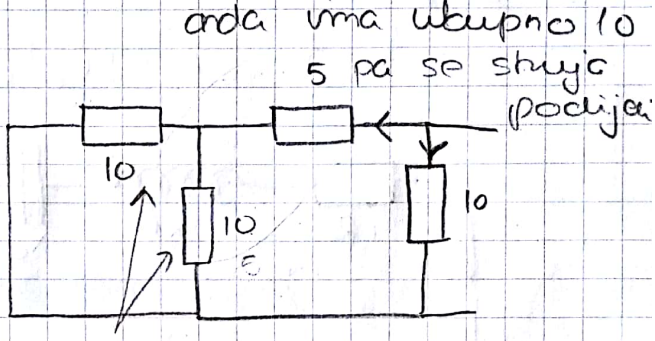
$U = 15\ V$
$R = 10$
$I = 1.5\ A$

$U = 40V$

$U = 20\ V$



Superpozicija



onda ima ukupno 10
5 pa se struje podije

možemo zamijeniti s 5

a) aktivan naponski

$R_{uk} = 16\ \Omega$

$I_{uk} = 1{,}25\ A$

$\frac{10}{15} = \frac{I_2}{1.25 - I_2}$

$12.5 - 10I_2 = 15I_2$

$I_2 = 0.5A$

b) aktivan strujni

$I = 1\ A$

$I_{2'} = 0.5\ A$

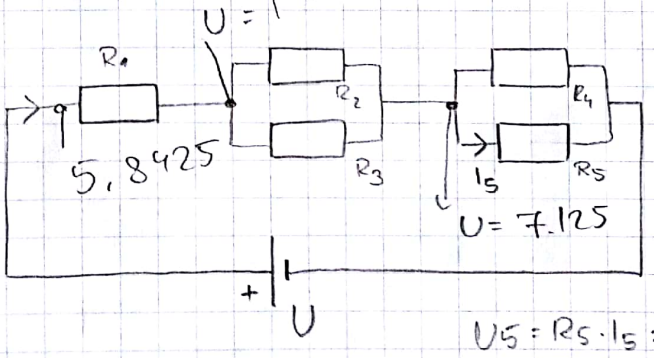
53.268

$I_R = 1A$

$R = 10\ \Omega \quad U = 10\ V$

DZ.

$U = I \cdot R = 2.98$



$U = 15.9475\ V$

$I_5 = 95 \cdot 10^{-3} A$

$R_1 = 24\Omega$

$R_2 = 24\Omega$

$R_3 = 25\Omega$

$R_4 = 2R_2 = 48\Omega$

$R_5 = 3R_3 = 75\Omega$

$U_5 = R_5 \cdot I_5 = 7.125$

$I_4 = \frac{7.125}{48\Omega} = 0.1484\ A$

$U_{R_1} = 5.8425\ V$

$I_{uk} = 0.2434\ A$

$\frac{R_2}{R_3} = \frac{0.2434 - I_2}{I_2}$

$P_2 = I^2R = 0.37\ W = 370\ mW$

$24I_2 = 25 \cdot 0.2434 - 25I_2$

$49I_2 = 0.1241836$

$I_3 = 0.1192\ A$

IMPEDANCIJA → kompleksan br. ali nije fazor

$$Z = \frac{\dot{U}}{\dot{I}} = \frac{U \angle \varphi_u}{I \angle \varphi_i} = \frac{U}{I} \angle \varphi_u - \varphi_i$$

* mjerimo u omima

$$|Z| = \frac{U}{I}$$

$$\varphi = \varphi_u - \varphi_i$$

R $\quad \dot{U} = \dot{I} \cdot R$

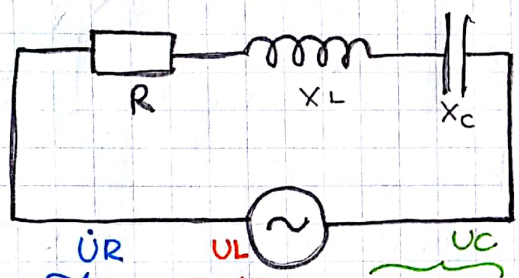
L $\quad \dot{U} = \dot{I}(j\omega L) \qquad X_L = \omega L$

C $\quad \dot{U} = \dot{I} \frac{1}{j\omega C} = \dot{I}(-j)\frac{1}{\omega C}$

isto Ω

INDUKTIVNA REAKTANCIJA

$$X_C = \frac{1}{\omega C}$$ KAPACITIVNA REAKTANCIJA



I. KZ $\quad \sum_{i=1}^{m} \dot{I}_i = \emptyset$

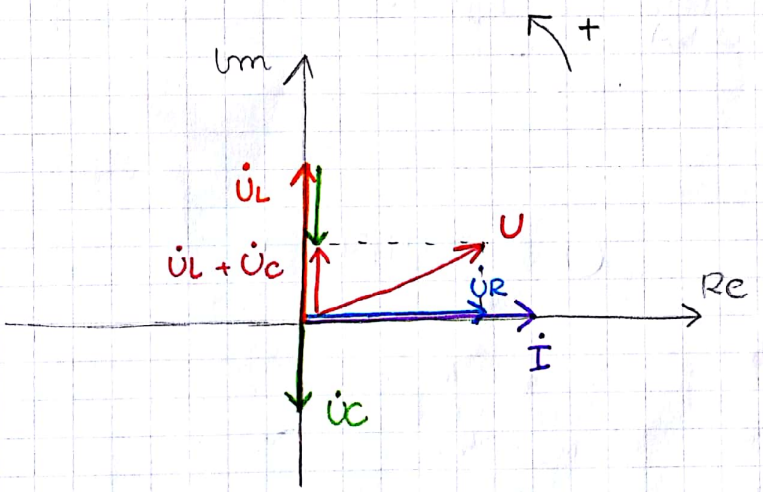
II. KZ $\quad \sum_{i=1}^{m} \dot{E}_i = \sum_{j}^{m} \dot{U}_k$

$$\dot{U} = \dot{I}R + \dot{I}jX_L + \dot{I}(-jX_C)$$
$$= \dot{I}(R + j(X_L - X_C))$$

$$Z = \frac{\dot{U}}{\dot{I}} = R + j(X_L - X_C)$$

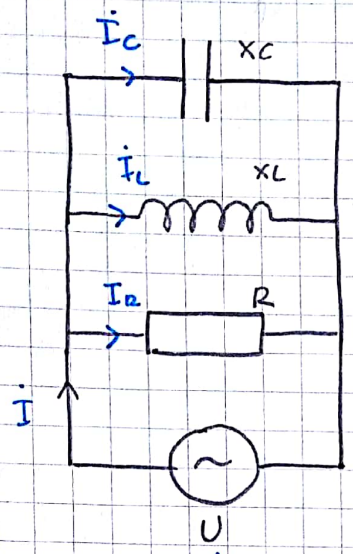
$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\operatorname{tg}\varphi = \frac{X_L - X_C}{R}$$



ono što je jednako u svim elementima crtam pod kutem 0

(ADMITANCIJA) ADMITANCIJA



$$\dot{I} = \dot{I}_R + \dot{I}_L + \dot{I}_C$$

$$\dot{I}_R = \frac{\dot{U}}{R} = \dot{U} \cdot G$$

$$\dot{I}_L = \frac{\dot{U}}{jX_L} = \dot{U}(-jB_L)$$

$$\dot{I}_C = \frac{\dot{U}}{-jX_C} = \dot{U}(jB_C)$$

$G = \frac{1}{R}$ omska vodljivost (KONDUKTANCIJA)

$B_L = \frac{1}{X_L}$ induktivna vodljivost (SUSCEPTANCIJA)

$B_C = \frac{\omega C}{1} = \omega C$ kapacitivna vodljivost

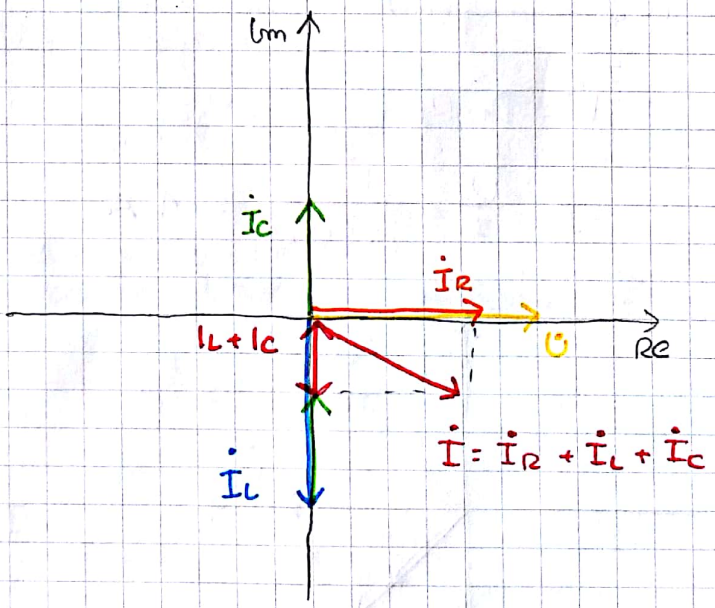
$$I = \dot{U}G + \dot{U}(-jB_L) + \dot{U}(jB_C) = \dot{U}(G + j(B_C - B_L)) = Y\dot{U}$$

$$Y = \frac{\dot{I}}{U} = G + j(B_C - B_L)$$

$$Y = |Y| e^{i\Psi} \qquad |Y| = \sqrt{G^2 + (B_C - B_L)^2} \qquad tg\,\Psi = \frac{B_C - B_L}{G}$$

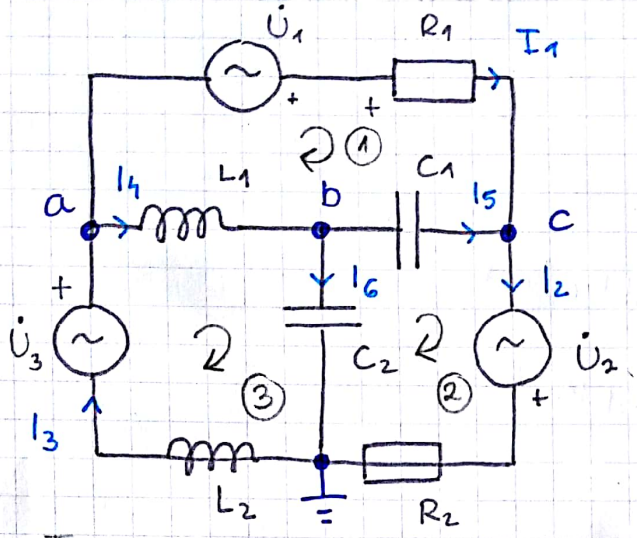
$$Y = \frac{1}{Z} \qquad \Psi = -\varphi$$

- biram $\dot{U}\ \angle 0°$



sljedeći puta ispit iz fazora

Zadaci:



3 I KI
3 II KI

I KI

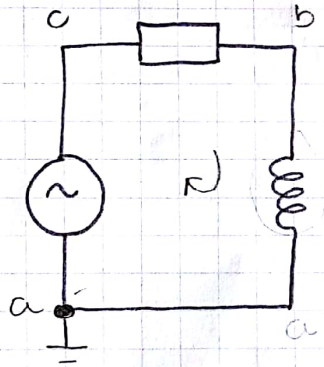
a) $\dot{I}_3 = \dot{I}_4 + \dot{I}_1$

b) $\dot{I}_4 = \dot{I}_5 + \dot{I}_6$

c) $\dot{I}_1 + \dot{I}_5 = \dot{I}_2$

II KI

1. $U_1 = \dot{I}_1 \cdot R_1 - \dot{I}_5(-jxC_1) - \dot{I}_4 (j x L_1)$
2. $\dot{U}_2 = \dot{I}_2 R_2 - \dot{I}_6 (-jxC_2) + \dot{I}_5 (-jxC_1)$
3. $\dot{U}_3 = \dot{I}_4(jxL_1) + \dot{I}_6 (-jxC_2) + \dot{I}_3(jxL_2)$



$R = X_L = 10\ \Omega$

$\dot{U} = 10\sqrt{2}\ \angle 0°$

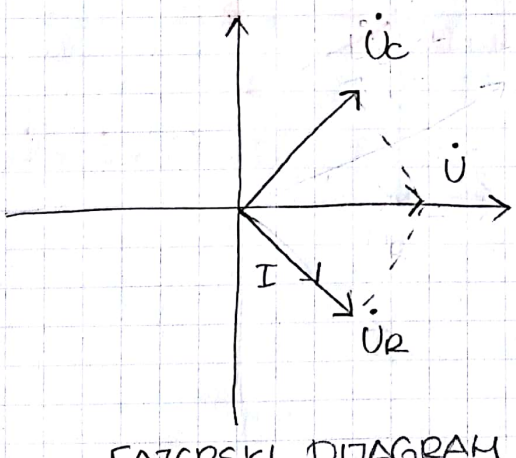
$\dot{U} = \dot{I}_1 \cdot R_1 + \dot{I}_1 \cdot (jXL_2)$

$tg\,\varphi = \frac{XL}{R} \Rightarrow 45° = \varphi$

$\dot{I} = \frac{\dot{U}}{Z} = \frac{\dot{U}}{\sqrt{R^2 + XL^2}} = 1\ A$

$Z = R + jX_2 = 10 + j10$

$= 10\sqrt{2}\ \angle 45°$



FAZORSKI DIJAGRAM

$U_L = 10$

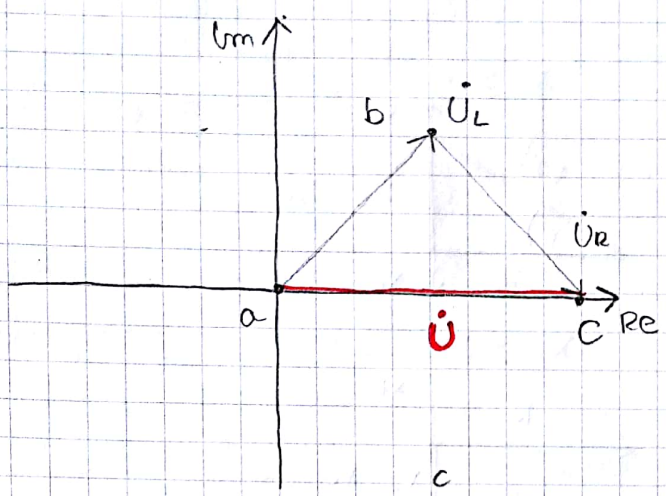
$U_R = 10$

$\dot{I} = \frac{\dot{U}}{Z} = \frac{10\sqrt{2}\ \angle 0°}{10\sqrt{2}\ \angle 45°} = 1\ \angle -45°\ A$

$U_R = \dot{I} \cdot R = 1\ \angle -45 = 10\ \angle -45°$

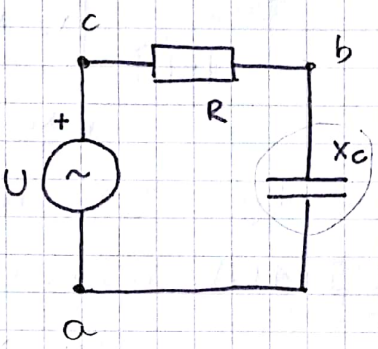
$U_L = \dot{I} \cdot jXL = 1\ \angle -45 \cdot 10\ \angle 90° = 10\ \angle 45°$

TOPOGRAFSKI DIJAGRAM



$R = X_C = 10\,\Omega$

$\dot{U} = 10\sqrt{2}\,\angle 0°$



$$\underline{Z} = R - jX_C$$

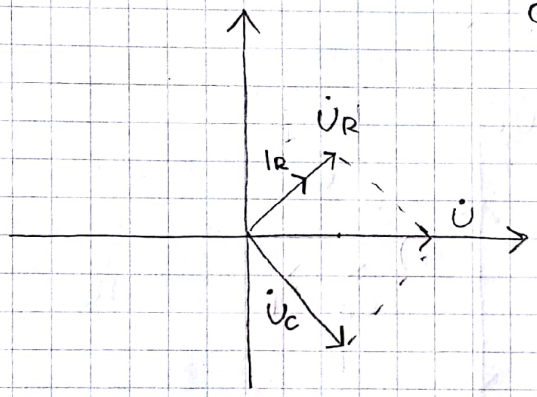
$$\underline{Z} = 10\sqrt{2}\,\angle -45°$$

$$\underline{I} = \frac{\dot{U}}{R} = \frac{10\sqrt{2}\,\angle 0°}{10\sqrt{2}\,\angle 45°} = 1\,\angle 45°\,A$$

$$U_R = \underline{I} \cdot R = 10\,\angle 45°\,V$$

$$U_C = \underline{I} \cdot (-jX_C) = 10\,\angle 45° - 90$$

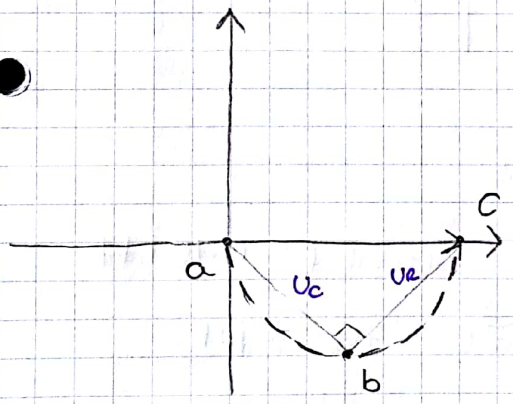
$$= 10\,\angle -45°$$



Ako postoji serijski spoj R i C

$\dot{U}_R \perp \dot{U}_C$

$$U = \sqrt{U_R^2 + U_C^2}$$



točka kliže po polukružnici
to se događa kada se mijenja f

TROKUT IMPEDANCIJE

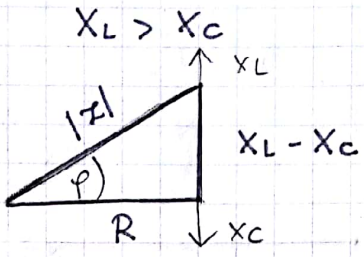
Serijski R, L, C spoj

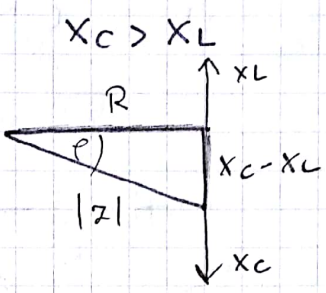
$$\underline{Z} = R + j(X_L - X_C)$$

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

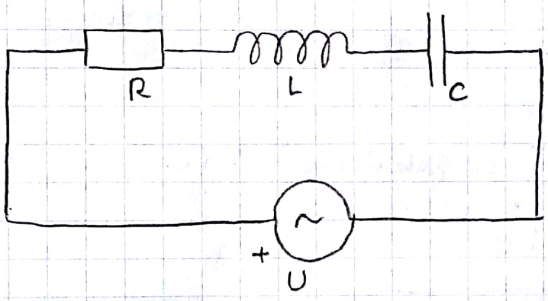
$$R = |Z| \cos\varphi$$

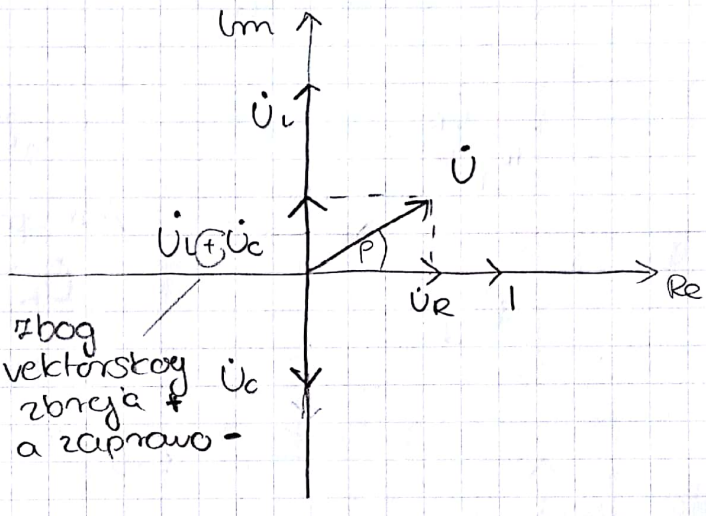
$$X_L - X_C = |Z| \sin\varphi$$





U serijskom R, L, C krugu priključenom na U od f = 50H i U_{ef} = 220 V.
U_L = 660 V ; U_C = 500 V , I = 11 A. odredit iznose R, L, C te fazni pomak U, I





$$U^2 = \dot{U}_R^2 + (U_L - U_C)^2$$

$$U_R = \sqrt{U^2 - (U_L - U_C)^2}$$

$$U_R = 151\ V$$

$$U_R = I \cdot R \Rightarrow R = \frac{U_R}{I} = \frac{151\ V}{11\ A} = 13.73\ \Omega$$

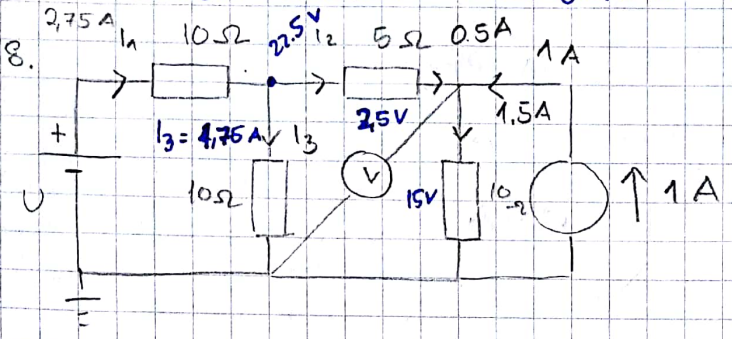
$$U_L = I \cdot jX_L \qquad U_L = I \cdot \omega L \Rightarrow L = \frac{U_L}{\omega I} = \frac{660}{11 \cdot 2\pi \cdot f} = 0.191\ H$$

$$U_C = I\ (-jX_C) \qquad U_C = I \cdot \frac{1}{\omega C} \Rightarrow C = \frac{I}{\omega U_C} = 70{,}03\ \mu F$$

$$\operatorname{tg}\varphi = \frac{U_L - U_C}{U_R} \Rightarrow \varphi = 46{,}66° \qquad \varphi = \alpha_u - \alpha_i > 0$$

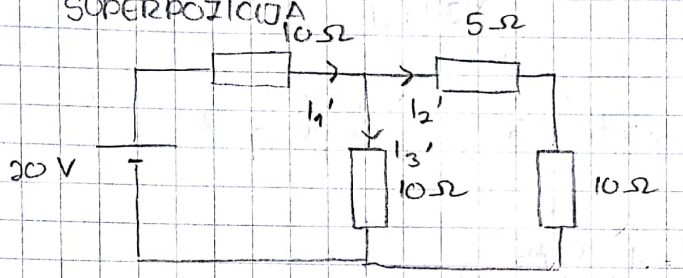
Zbirka. SLOŽENI KRUGOVI ISTOSMJERNE STRUJE

8.



$U = 40V$

SUPERPOZICIJA



$R_{uk} = 16\,\Omega$

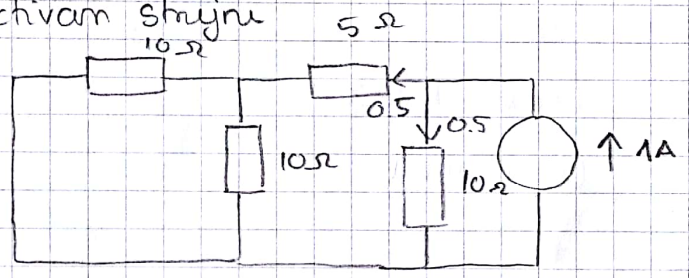
$I_{uk} = 1,25$ A

$$\frac{10\,\Omega}{15\,\Omega} = \frac{I_2}{1,25 - I_2}$$

$$12.5 - 10I_2 = 15I_2$$

$$I_2' = 0.5\,A$$

aktivam struju



$I_{2uk} = 1$ A

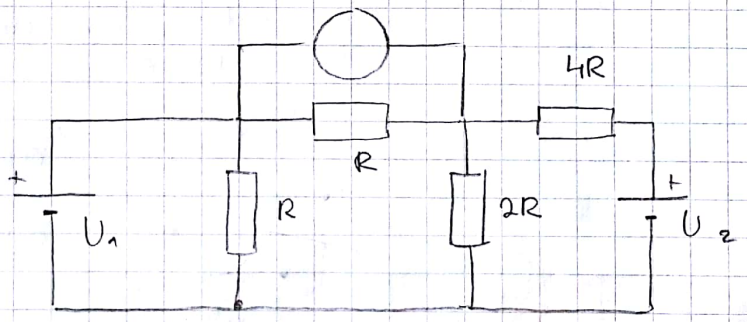
$U = I_2 \cdot R = 10$ V

$I_2'' = 0.5$A

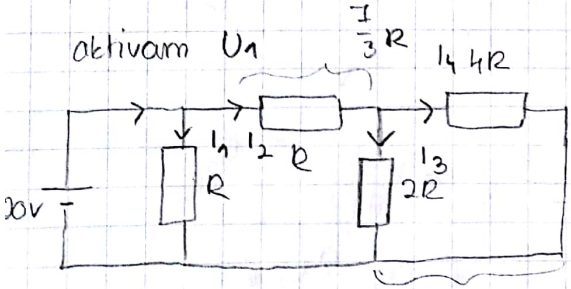
9. $R = 5\,\Omega$

$I = 2$ A

$U = 20$V



aktivam U_1



$R_{uk} = 0.7R = 3.5\,\Omega$

$I_{uk} = 5,71$ A

$\frac{4}{3}R$

$$\frac{\frac{R}{1}}{\frac{7R}{3}} = \frac{I_2}{5,71 - I_2}$$

$$\frac{3}{7} = \frac{I_2}{5,71 - I_2}$$

$$17,13 - 3I_2 = 7I_2$$

$$I_2 = 1,713$$

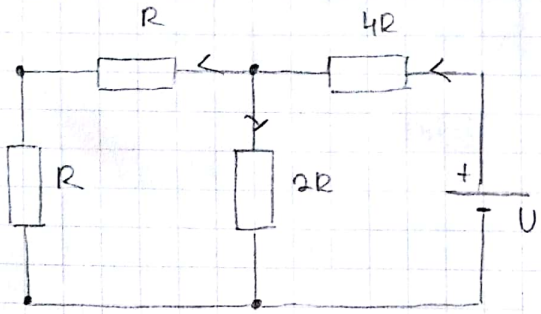
$$\frac{1}{2}\frac{2R}{4R} = \frac{I_4}{1,713 - I_4}$$

$$1,713 - I_4 = 2I_4$$

$$I_4' = 0.571\ A$$

$$I_3' = 1,142\ A$$

b) aktivam U_2

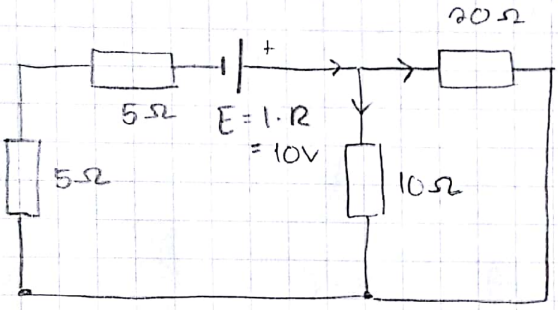


$R_{ek} = 5 \cdot 5 = 25\ \Omega$

$I_{ek} = 0.8\ A$

$I_3' = 0.4\ A$

c) aktivam strujni

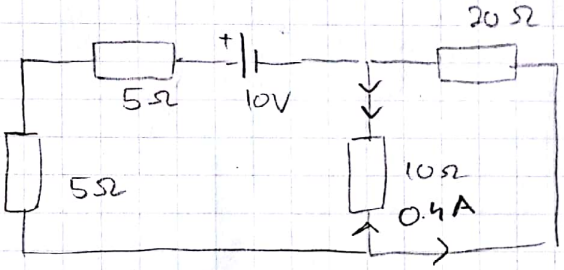


$R_{ek} = 16,67$ $I_{ek} = 0.6\ A$

$\frac{20}{10} = \frac{I_2}{0.6}$

$I_3' = 0.4\ A$

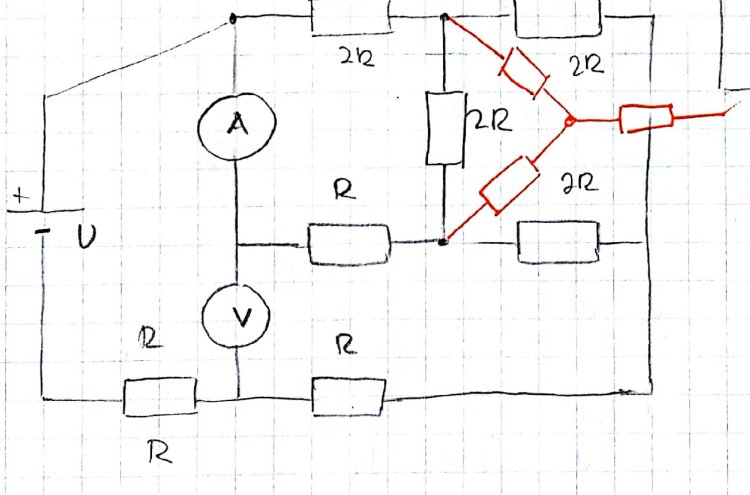
$I_3 = 1,942$ $U_3 = 19,42\ V$



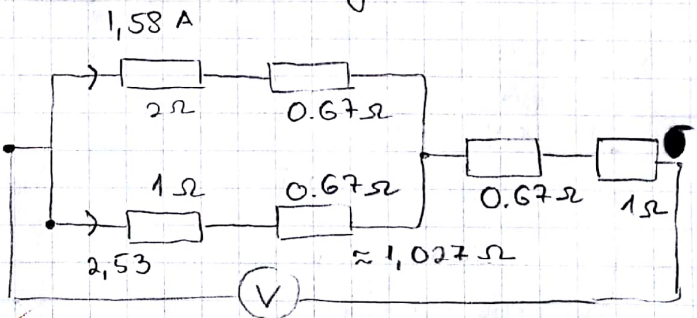
$I_{3ek} = 1,142$

$U = 11,42\ V$

10.



transformacija $\Delta \rightarrow \curlywedge$



$U = 7V$

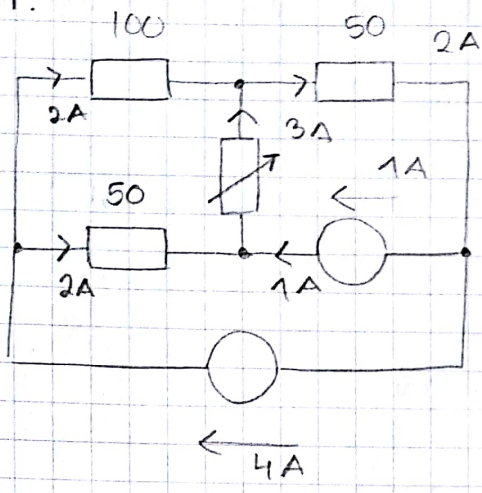
$\frac{1,027\Omega}{1,67\Omega} = \frac{U_p}{7 - U_p}$

$0.62 \cdot (7 - U_p) = U_p$

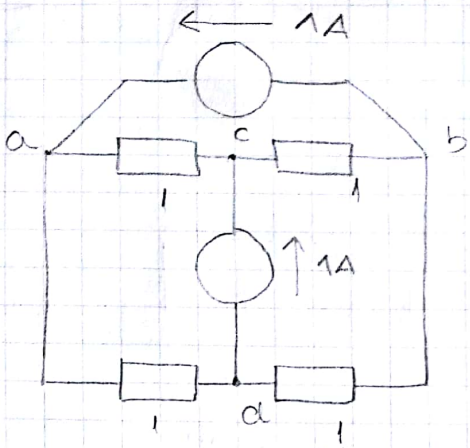
$U_p = 2,68$

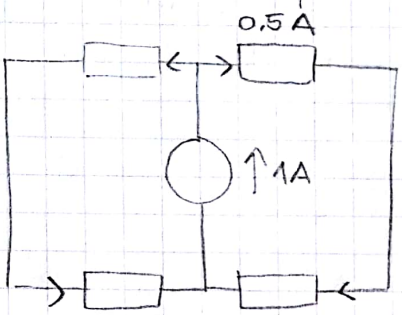
$I = \frac{2,68}{1,67} = 1,6\ A$

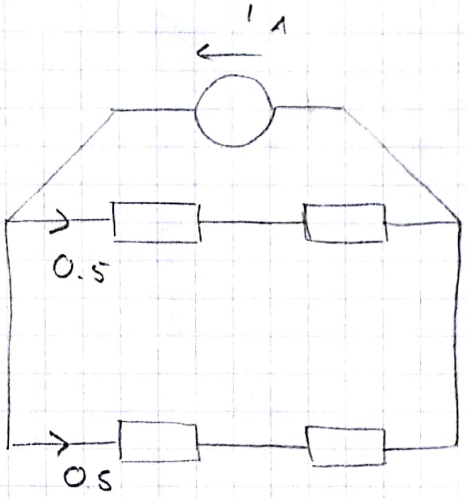
11.



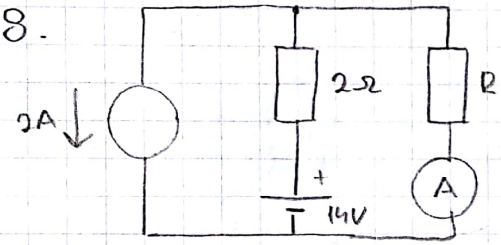
12.



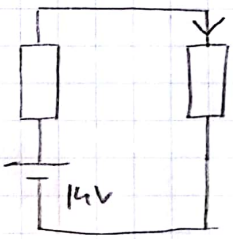




8.



aktivan iracionalski



$$I = \frac{14}{R+2}$$

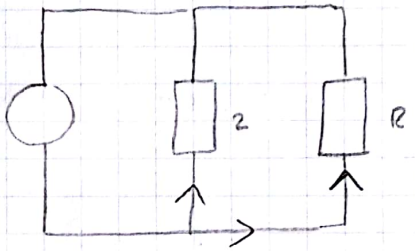
$$\frac{14}{R+2} - \frac{4}{R+2} = 1$$

$$\frac{10}{R+2} = 1$$

$$10 = R+2$$

$$R = 8$$

aktivan smjni



2

$$\frac{2}{R} = \frac{I}{2-I}$$

$$4 - 2I = RI$$

$$I(R+2) = 4$$

$$I = \frac{4}{R+2}$$

Zbirka. OSNOVE MAGNETIZMA

U trenutcima kada struja broz induktivitet raste, u mjernu se povećava magnetska energija (električna energija pretvara se u magnetsku) i induktivitet se ponaša kao trošilo $U_{AB} > 0$

U trenutcima kada struja broz induktivitet opada, u mjernu se smanjuje magnetska energija (magnetska energija se pretvara u el.) i induktivitet se ponaša kao izvor $U_{AB} < 0$

Kada je struja konst. $U_{AB} = 0$, nema promjene magnetske energije.

tamo gdje je točkica je el. pozitivnije

V-P1.
$l_s = 20\,cm$
$I = 5\,A$
$F = 1\,N$
$F = BIl$
$B = \frac{F}{Il} = 1\,T$

V-P2. $v = 500\ km/s$
$B = 2\,T$
$F = qvB = 1{,}6 \cdot 10^{-13}\,N$

V-P5. $S = 2\,cm^2$
$r = 3{,}5\,cm$
$N = 1000$
$I = 5\,mA$
Φ

magnetsko polje za toroidni oblik je zarobljeno u području jezgre

V-P6. $l = 0{,}1\,m$
$N = 360$
$I = 0{,}1\,A$
$B = \mu_0 \frac{NI}{l}$

1. ravni vodič
$B = 0.5\,T$
$l = 0{,}1\,m$
$I = 20\,A$
$\alpha = 30°$
$F = BIl\sin\alpha = 0.5\,N$

2. dva paralelna vodiča
$l = 1\,m$
$d = 1\,m$
$I_1 = I_2 = 2\,A$
suprotnog smjera $\Rightarrow$ ODBIJAJU
$F = \mu_0 \frac{I_2 I_1 \cdot l}{2\pi d} = 8 \cdot 10^{-7}$
smanji se na 1/4

3. $l = 2m$
$d = 0{,}01\,m$
$F = 4\ 10^{-3} N$
$F = \mu_0 \cdot \frac{I^2 l}{2\pi d}$
$\sqrt{\frac{F 2\pi d}{\mu_0 l}} = I = 10\,A$

4. zavojnica N

za duge zavojnice vrijedi da je polje na krajevima duplo manje nego u sredini

$B = ?$
$\ell = 0.12\ m$
$N = 300$
$r = 0,01\ m$
$I = 0,8\ A$

$$B = \mu_0 \cdot \frac{NI}{\ell} = 2,5\ mT$$

5. $\ell = 0,2\ m$
$v = 15\ m/s$
$\alpha = 90°$
$B = 1,2\ T$

$U_{ind} = B\ell v = 3,6\ V$

6. $\ell = 80\ m$
$v = 900\ km/h = 250\ m/s$
$B = 20 \cdot 10^{-6}\ T$

$U_{ind} = B\ell v = 0,4\ V$

7. $L = 1\ H$
$I = 1\ A$
$\Delta t = 100 \cdot 10^{-3}\ s$
$N = 100$

$$U_{ind} = -L\frac{dI}{dt} = 10\ V$$

$U = 10/100 = 0,1\ V$

8. ako se struja smanjuje $\Rightarrow$ induktivitet izvor $\Rightarrow U_{ab} > 0$

9. $B = 0.3\ T$
$R_v = 1\ \Omega$
$R = 2\ \Omega$
$v = 10\ m/s$
$\ell = 0,1\ m$
$\Delta t = 0.1\ s$

$U_{ind} = B\ell v = 0.3\ V$

$$I = \frac{U}{1+2} = 0.1\ A$$

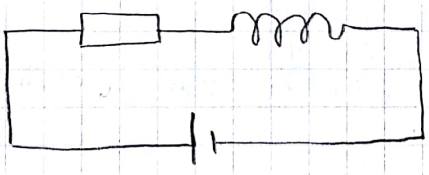
$W = P \cdot t$
$= I^2 R \cdot t$
$= 2 \cdot 10^{-3}\ J$

10. $a = 0.1\ m$
$N = 500$
$T = \frac{60}{1200} = 0.05$
$B = 40 \cdot 10^{-6}\ T$

$\omega = \frac{2\pi}{T} = 40\pi$

$f = 20\ Hz$ $U_m = N \cdot B \cdot S \cdot \omega = 0.025\ V$

11. $L = 1\ H$
$U = 10\ V$
$R_1 = 2\ \Omega$



zanemarivo mali otpor

$I = \frac{U}{R} = \frac{10}{2} = 5\ A$

$W = \frac{LI^2}{2} = \frac{25}{2} = 12,5\ J$

12. $\Delta t = 1\ s$
$\Delta\Phi = 1\ Vs$
$R = 1\ \Omega$

$U_{ind} = 1\ V$ $I = 1\ A = \frac{Q}{t} \Rightarrow Q = 1\ C$

13. $N = 20$
$S = 4\,cm^2$
$B = 1\ 10^{-3}\,T$
$\Delta t = 2\ 10^{-3}\,s$
$U_{ind} = \frac{\Phi \cdot N}{\Delta t} = 4 \cdot 10^{-3}\,V$

14. $L = 100 \cdot 10^{-3}\,H$

$U_{ind} = -L \cdot \frac{di}{dt} = 0.5\,V$ - struja raste -> induktivitet se ponaša kao trošilo -> negativan

$U_{ind} = 0.33$. - pozitivan

$U = 0$ u intervalu od 2ms do 3ms

15. $N = 1000$
$a = 1,5\,ms$
$\Phi = 1,5$
$U_{ind} = N \cdot \frac{\Delta \Phi}{\Delta t} = 5000\,V$

16. $L_1 = 5 \cdot 10^{-3}\,H$
$L_2 = 10 \cdot 10^{-3}\,H$ } senja

faktor magnetske veze $k = 0,75$

$M = k\sqrt{L_1 \cdot L_2} = 5,3 \cdot 10^{-3}\,H$

$U = -L \cdot \frac{dI}{dt} + M \cdot \frac{dI}{dt} = 0.103\,V$

17. $\Delta I = 10\,A$ $U_{ind} = 30\,V$ $k = ?$
$t = 100 \cdot 10^{-3}\,s$

$U_m = M \frac{di(t)}{dt}$

$30V = M \cdot \frac{10}{100 \cdot 10^{-3}}$

$M = 0,3$ => koef međusobne indukcije

18. $L = 1\,H$
$R = 10\,\Omega$ $I = \frac{U}{R} = 1\,A$
$U = 10\,V$

U trenutku uključenja struja je 0, pa postepeno raste, te na kraju iznosi 1 A.

$W = \frac{LI^2}{2} = 0.5\,J$

19. $U_{34} = M \cdot \frac{di}{dt} = -0.5\,V$

Zbirka: SINUSOIDNO PROMJENJIVE VELIČINE

1. $u(t) = 155.5 \sin(377t + \pi/6)$ V

$U_{ef} = 109,6$ V $\varphi_0 = \frac{\pi}{6}$ $\frac{2\pi}{T} = 377 \Rightarrow T = 0.0167$ $f = 60$ Hz

2. $i = I_m \sin(\omega t + \pi/4)$ A

$t = 0$

$I = 1$ A

$1 = I_m \sin(\frac{\pi}{4})$

$I_m = \sqrt{2}$

b) $I = 1,41$

$I_m = 2$ A

3. $A = 1,41$ A

φ

$t = 0$, $I = 1$ A

$I = 1,41 \sin(\omega t + \varphi)$

$1 = 1,41 \sin\varphi$

$\varphi = \frac{\pi}{4}$ za trend porasta

$\varphi = \frac{3\pi}{4}$ za trend pada

4. $i_1 = \sin\omega t$

$i_2 = \cos\omega t$

1) $i_1 + i_2 = \sin\omega t + \cos\omega t$

2) $i_1 = \frac{1}{\sqrt{2}} \angle 0°$

$i_2 = \frac{1}{\sqrt{2}} \angle 90°$

$\Big\} \; i_1 + i_2 = 1 \angle 45°$

3) $i_1 + i_2 = \frac{1}{\sqrt{2}} + 0 \cdot j + 0 + \frac{1}{\sqrt{2}} j = \frac{1}{\sqrt{2}} [1 + j] = \frac{\sqrt{2} \angle 45°}{\sqrt{2}} = 1 \angle 45°$

$i_3 = 1,41 \sin(\omega t + \frac{\pi}{4})$ A

5. $i = -5$ A

$t = \frac{7\pi}{8}$ ms $\omega = \frac{2\pi}{T} = 2000$

$T = \pi$ ms

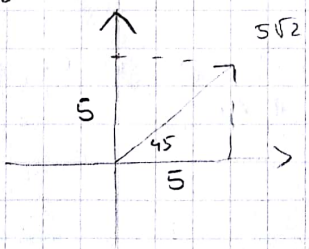
$-5 = 5 \sin(\frac{7\pi}{8} \cdot 2000 + \varphi)$

$\frac{3\pi}{2} =$

6. $u(t) = 10 \sin(\omega t + \frac{\pi}{4})$

$5\sqrt{2} \angle 45°$

$5 \angle 0 + 5 \angle \frac{\pi}{2}$



$I_{0f} \sin\omega t + I_{0f} \cos\omega t$

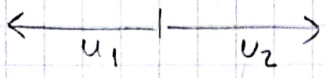
7. $i(t) = 5\cos(10t - 3\pi/4)$

$i(t) = 5\sin(10t - \frac{\pi}{4})$ $\quad i(0) = -3,5\ A$

$i(t) = 5\cos(10t - \frac{\pi}{4}) \cdot 10$ $\quad i'(0) = 35,35\ A/s$

8. $U_1 = 5\sin(\omega t + \pi)$

$U_2 = 5\sin(\omega t)$



$U_{ab} = -U_1 + U_2$

$U_2 - U_1$

$U_{ab} = \frac{5}{\Gamma_2} \cdot \frac{5}{\Gamma_2}$

$U = 10\sin\omega t$

$U_{ba} = -U_{ab} = 10\sin(\omega t + \pi)\ V$

9. $\dot{I} = 12 + j12\ A$

$\dot{I} = 16.97\ \angle 45°$ $\quad i = 24\sin(\omega t + \frac{\pi}{4})$

$Re = 12$ $\quad Im = 12$

10. $U = 173 + j100$

$U = 282,59\sin(\omega t + 30°1') \Rightarrow U(0) = 141,36\ V$

$U = 173 - j100\ V$

$U = 282,59\sin(\omega t + 329°58') = -141,44\ V$

11. $I_1 = 5,2\ \angle 37°$ $\quad I_2 = 6,6\ \angle 108°$

$I = 5.2(\cos 37° + j\sin 37°) + 6.6(\cos 108° + j\sin 108°)$

$= 4.15 + j\cdot 3.13 - 2,04 + j6.28$

$= 2.11 + 9.41j$

$I = 13,64\sin(\omega t + 77°21')$

Zbirka: NAČELA RJEŠAVANJA KRUGOVA IZMJENIČNE VELIČINE

P3. $U_m = 311\,V$
$C = 100 \cdot 10^{-6}\,F$
$f = 60\,Hz$

veća $f \Rightarrow$ manji otpor

$$X_C = \frac{1}{2\pi f C} = 31{,}831\,\Omega$$

$$X_C = \frac{U}{I} \Rightarrow I = 6{,}909\,A$$

2.1. $R = 100\,\Omega$
$u(t) = 310 \sin(314t)\,V$

$$R = \frac{U}{I} \Rightarrow I = 3{,}1\,A$$

napon i struja su u fazi

$I_{ef} = 2{,}192\,A$

2.2.

$X_L = \omega L$ 2x veća $f \Rightarrow$ 2 x veći otpor

$X_L = \frac{U}{I}$ 2x veći otpor $\Rightarrow$ 2 x manja struja

2.3. $L = 0{,}1\,H$
$U = 220\,V$
$f = 50\,Hz$

struja kasni za naponom za $\frac{\pi}{2}$

$U = 220$

$X_L = 2\pi f \cdot L$

$I = -7i$

$$I = \frac{U}{X_L} = 7\,A$$

Fazori: $U = 220\angle 0°$ $I = 7\angle -90°$

$u(t) = 220 \sin(314t)$
$i(t) = 7 \sin\left(314t - \frac{\pi}{2}\right)$

2.4. $u(t) = 100\sqrt{2} \sin(\omega t + 0.5\pi)$

$i(t) = 1\sqrt{2} \cos(\omega t + 0.5\pi)$
$= \sqrt{2} \sin(\omega t + \pi)$

struja predhodi naponu

KONDENZATOR

$$X_C = \frac{U}{I} = 100 \qquad X_C = \frac{1}{\omega C} \qquad X_C = -j\,100\,\Omega$$

$$C = \frac{1}{\omega X_C} = 10 \cdot 10^{-6}\,F$$

2.5. $U = 100\angle 90°$
$I = 1\angle 180°$

$$Z = \frac{U}{I} = 100\angle 90 - 180° = 100\angle -90°$$

2.6. $X_L = 314\,\Omega$
$X_L = \omega L \Rightarrow L = 1\,H$

$u = 220\sqrt{2} \sin(314t)$

$X_L = \frac{U}{I} \Rightarrow I = 0.7$ (efektivna

$i = 1 \sin\left(314t - \frac{\pi}{2}\right)$

$X_L = j\,314\,\Omega$

2.7. $I = \frac{U}{X_L} = \frac{220^\circ \angle 0}{314^\circ \angle 90^\circ} = 0.7 \angle -90$

2.8. $C = 10 \cdot 10^{-6}$ F

$\underline{I} = 1 \angle 0^\circ$

$f = 50$ Hz

$X_C = \frac{1}{2\pi f C} = 318,309\ \Omega$

$X_C = \frac{U}{I} \Rightarrow U = 318,309 \angle -90 \cdot 1 \angle 0^\circ$

$= 318,309 \angle -90^\circ$

$U = 450 \cdot \sin\left(100\pi t - \frac{\pi}{2}\right)$

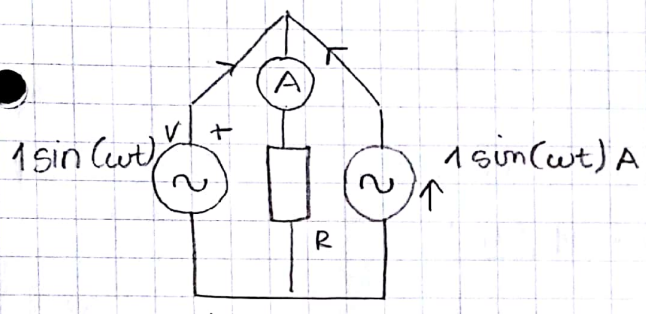
$U(5 \cdot 10^{-3}) = 0$ V

$U(10 \cdot 10^{-3}) = 450$ V $\quad W = 1,01$ J

2.9. $R = 1\ \Omega$, $X_C = 1\ \Omega$

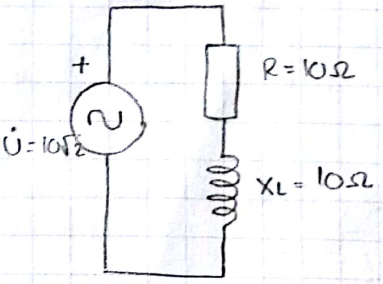
a) b)

lef (napanskog)



lef $\frac{1}{\sqrt{2}} = 0.7$ A

RLC krugovi

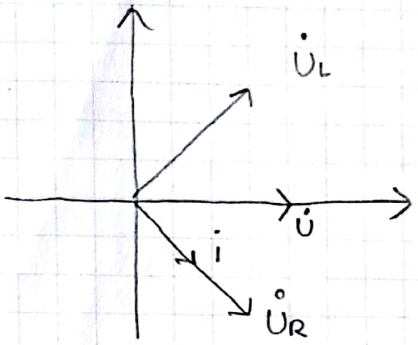


$$\underline{Z} = R + jX_L = 10 + 10j = 10\sqrt{2}\ \angle 45°$$

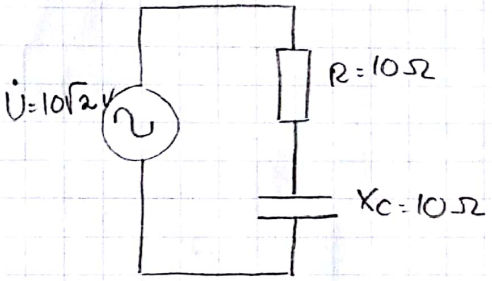
$$\dot{I} = \frac{\dot{U}}{\underline{Z}} = \frac{10\sqrt{2}\ \angle 0°}{10\sqrt{2}\ \angle 45°} = 1A\ \angle -45°$$

$$\dot{U}_R = I \cdot R = 1A\ \angle -45° \cdot 10 = 10\ \angle -45°\ V$$

$$\dot{U}_L = \dot{I} \cdot X_L = 1\ \angle -45 \cdot 10\ \angle 90° = 10\ \angle 45°\ V$$



Primjer 2.

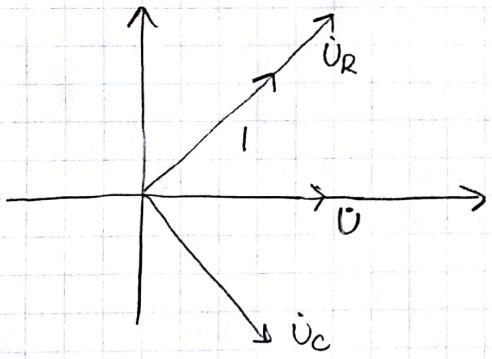


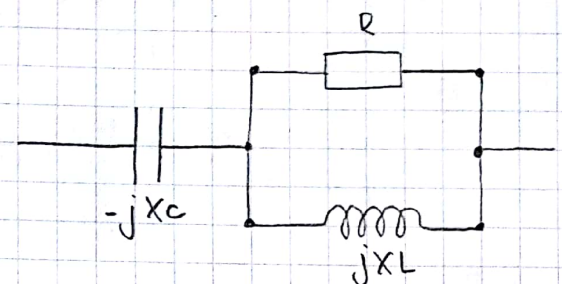
$$\underline{Z} = R - jX_C = 10 - 10j = 10\sqrt{2}\ \angle 315°$$

$$\dot{I} = \frac{\dot{U}}{\underline{Z}} = \frac{10\sqrt{2}\ \angle 0°}{10\sqrt{2}\ \angle 315°} = 1\ \angle 45°$$

$$\dot{U}_R = R \cdot I = 10 \cdot 1\ \angle 45° = 10\ \angle 45°\ V$$

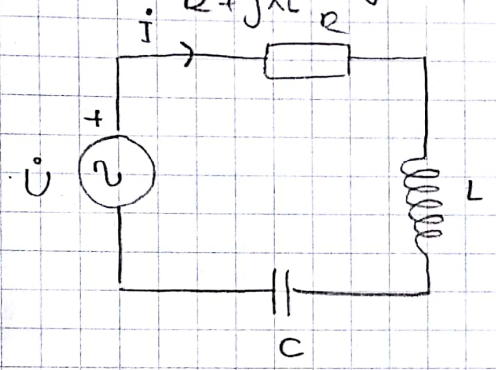
$$\dot{U}_C = -jX_C \cdot I = 10\ \angle -90° \cdot 1\ \angle 45° = 10\ \angle -45°\ V$$





$$\underline{Z}_p = \frac{R \cdot jX_L}{R + jX_L}$$

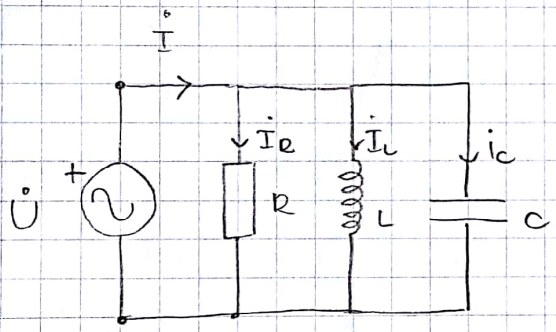
$$Z_{uk} = \frac{R \cdot jX_L}{R + jX_L} - jX_C$$



$$\underline{Z} = R + j\omega L - j \cdot \frac{1}{\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\varphi = \operatorname{arctg} \frac{\omega L - \frac{1}{\omega C}}{R}$$



$$\dot{I}_R = \frac{\dot{U}}{R} = \dot{U} \cdot G$$

$$\dot{I}_L = \frac{\dot{U}}{jX_L} = \dot{U} \cdot (-jB_L)$$ - induktivna vodljivost

$$\dot{I}_C = \frac{\dot{U}}{-jX_C} = \dot{U} \cdot (jB_C)$$ - kapacitivna vodljivost

$$\dot{I} = \frac{\dot{U}}{\underline{Z}} = \dot{U} \cdot \underline{Y}$$

RLC krugai; zbirka

1. $R = 75\,\Omega$, $X_L = 25\,\Omega$ } serijski
$U = 220\,V$
$\underline{Z} = ?$ $\dot{U}_L = ?$

$$\underline{Z} = R + jX_L = 75 + 25j = 25\sqrt{10}\ \angle 18°26'\ \Omega$$
$$\dot{I} = \frac{\dot{U}}{\underline{Z}} = \frac{220\ \angle 0°}{25\sqrt{10}\ \angle 18°26'} = 2{,}783\ \angle 341°34'\ A$$
$$\dot{U}_L = jX_L \cdot \dot{I} = 25\ \angle 90° \cdot 2{,}783\ \angle 341°34 = 69{,}575\ \angle 71°34'$$

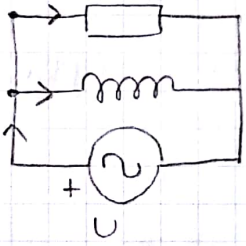
2. $R = 75\,\Omega$
$L = 79{,}6 \cdot 10^{-3}\,H$ $\quad X_L = 25{,}007\,j\,\Omega$
$U = 220\,V$
$f = 50\,Hz$ $\quad \underline{Z} = 75 + 25{,}007\,j = 79{,}05$

3. $f = 50\,Hz$
$I = 4{,}5\,A$

$R = 47\,\Omega$ $\quad \underline{Z} = R + jX_L - jX_C$
$X_C = 27\,\Omega$
$X_L = 13{,}5\,\Omega$ $\quad Z = \sqrt{R^2 + (X_L - X_C)^2} = 43{,}9$

$U = Z \cdot I = 220.052\ V$

4. $R = 15\,\Omega$
$X_L = 20\,\Omega$
$P_R = 9{,}6\,W$



$$9{,}6 = \frac{U^2}{R}$$
$$U = \sqrt{PR} = 12\ V\ \angle 0°$$
$$I_R = \frac{U}{R} = 0.8A\ \angle 0°$$
$$I_{XL} = 0.6A\ \angle -90°$$
$$I_{uk} = 0.8\ \angle 0° + 0.6\ \angle -90° = 0.8 - 0.6j = 1\ A\ \angle -36°52'$$

5. $Z_1 = 2 - 4j$
$Z_2 = 4 + 2j$

$$Z_{uk} = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2} = \frac{(2-4j)(4+2j)}{6-2j} = \frac{8 + 4j - 16j + 8}{6-2j} = \frac{16 - 12j}{6-2j} \cdot \frac{6+2j}{6+2j}$$
$$= \frac{96 + 32j - 72j + 24}{36 + 4} = 3 - j = \sqrt{10}\ \angle -18°26$$

6. $I = 2A$

$$\frac{Z_1}{Z_2} = \frac{I_2}{I - I_2}$$

$$Z_1 \cdot (I - I_1) = I_1 \cdot Z_2$$

$$Z_1 \cdot I = I_1 \cdot (Z_2 + Z_1)$$

$$\frac{Z_1 \cdot I}{Z_2 + Z_1} = I_1$$

$$\frac{(2 - 4j) \cdot 2}{2 - 4j + 4 + 2j} = I_1$$

$$\frac{4 - 8j}{6 - 2j} \cdot \frac{6 + 2j}{6 + 2j} = \frac{24 - 48j + 8j + 16}{40} = 1 - j = \sqrt{2} \angle -45°$$

Zadaci VII.1

1. $R = 7{,}5\ \Omega$
$U = 125\ V$
$f = 50\ Hz$
$U_L = 100$

$$I = \frac{125}{Z} = \frac{125}{\sqrt{7{,}5^2 + X_L^2}}$$

$$U_L = X_L \cdot I$$

$$100 = 2\pi f \cdot L \cdot \frac{125}{\sqrt{7{,}5^2 + (2\pi f \cdot L)^2}}$$

$$4\sqrt{7{,}5^2 + (2\pi f L)^2} = 10\pi f \cdot L \quad / ^2$$

$$16\,(56{,}25 + 4\pi^2 f^2 L^2 = 100\pi^2 f^2 L^2$$

$$900 + 64\pi^2 f^2 L^2 = 100\pi^2 f^2 L^2$$

$$900 = 36\pi^2 f^2 L^2$$

$$25 = \pi^2 f^2 L^2$$

$$L = 0.032\ H$$

$$Z = \sqrt{7{,}5^2 + (2\pi \cdot 50 \cdot 0.032)^2}$$

$$Z = 12.54\ \Omega$$

$$I = \frac{U}{Z} = 9{,}96\ A$$

2. $R = 100\ \Omega$
$C = 43 \cdot 10^{-6}\ F$
$U = 220\ V \angle 0°$
$f = 50\ Hz$

$$I = \frac{U}{Z} = \frac{220 \angle 0°}{124{,}4 \angle -36{,}5°} = 1{,}768 \angle 36{,}5°$$

$$Z = R - jX_C = 100 - 74j = 124{,}4 \angle -36{,}5$$

$$U_C = I \cdot -jX_C = 1{,}768 \angle 36{,}5° \cdot 74 \angle -90° = 130{,}832 \angle -53{,}5°\ A$$

3. $U_m = 160\,V$
$P_m = 40\,W$
$U = 220\,V$
$f = 50\,Hz$

$P = \frac{U^2}{R} \quad 40 = \frac{160^2}{R}$

$R = 640\,\Omega \qquad P = I^2 R$

$I = 0{,}25\,A$

$U_c = I \cdot X_C = I \cdot \frac{1}{2\pi f \cdot C}$

$220\angle 0 = 160\angle 0 + 20\sqrt{57}\angle -90°$

$220 = 160 - X_j$

$\sqrt{160^2 + X^2} = 220$

$X = 20\sqrt{57}$

$20\sqrt{57} \cdot 2\pi \cdot 50 \cdot C = 0{,}25$

$C = 5{,}27 \cdot 10^{-6}\,F$

4. $R = 5\,\Omega$
$L = 0{,}03\,H$

$Z = R + jX_L$

$\rho = 60°$

$\sqrt{3} = \frac{2\pi f \cdot 0{,}03}{5} \Rightarrow f = 45{,}94\,Hz$

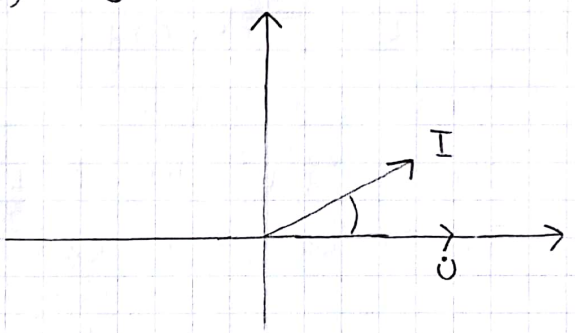
5. $R = 4\,\Omega$
$X_L = 3\,\Omega$
$X_C = 6\,\Omega$
$U = 100\,V$

$Z = R + jX_L - jX_C = 5\angle -36{,}9° = 4 - 3j$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = 5$

$\rho = -36{,}9°$

$I = \frac{100\angle 0°}{5\angle -36{,}9°} = 20\angle 36{,}9°$



6. $R_1 = 1\,\Omega$
$R_2 = 2\,\Omega$
$X_L = 1\,\Omega$
$X_{C_1} = 0{,}5\,\Omega$
$X_{C_2} = 1\,\Omega$
$U = 2\,V$

$I_1 = \frac{2V}{1\Omega} = 2A\angle 0°$

$I_2 = \frac{2V}{2\Omega} = 1A\angle 0°$

$I_L = \frac{2V}{1\Omega\angle 90°} = 2A\angle -90$

$I_{C_1} = \frac{2V}{0.5\Omega\angle -90°} = 4A\angle 90°$

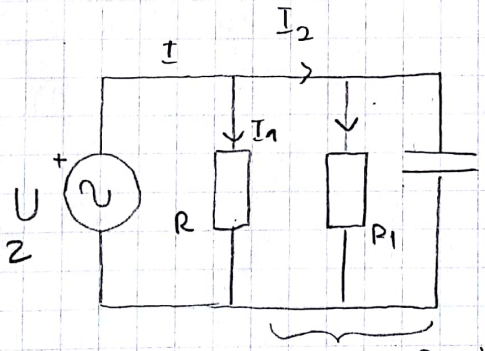
$I_{C_2} = \frac{2V}{1\Omega\angle -90°} = 2A\angle 90°$

$I_{uk} = 2 + 1 - 2j + 4j + 4$

$= 3 + 4j$

$= 5\angle 53{,}13°$

7. $I = 3A$
$I_1 = 2A$
$I_2 = 2A$
$R_1 = 4\,\Omega$



imaju jednake Z

$Z_{uk} = \frac{1}{R} \cdot 3 = U$

$3 = UR$

$Z_p = \frac{R_1 \cdot X_C}{R_1 + X_C} = \frac{4X_C}{4 + X_C}$

$Z_{uk} = \frac{R \cdot \frac{4X_C}{4+X_C}}{R + \frac{4X_C}{4+X_C}}$

$\left(\frac{\frac{4X_C}{4+X_C}}{\frac{R}{1}}\right) = 1$

$4X_C = 4R + RX_C$

$\frac{4X_C}{4 + X_C} = R$

$\left(\frac{\frac{16X_C^2}{(4+X_C)\cancel{X}}}{\frac{8X_C}{(4+X_C)}}\right) = Z_{uk}$

$\frac{16X_C^2}{8X_C(4+X_C)} = Z_{uk}$

$$\frac{\frac{3}{R}}{\frac{4X_C}{4+X_C}} = 2 \qquad \frac{12+3X_C}{4RX_C} = 2$$

$$\frac{12+3X_C}{4\cdot\frac{4X_C}{4+X_C}\cdot X_C} = 2$$

$$\frac{(12+3X_C)(4+X_C)}{16X_C^2} = 2$$

$$48 + 12X_C + 12X + 3X_C^2 = 32X_C^2$$

$$48 + 24X_C - 29X_C^2 = 0$$

$$X_C = 1{,}76$$

1,8. $R = 5\,\Omega$

$X_L = j8\,\Omega$

$U = 50\angle 45°\ V$

$I = 2{,}5\angle -15°\ A$

$$I = \frac{U}{Z} \Rightarrow \underline{Z} = \frac{\dot{U}}{\dot{I}} = \frac{50\angle 45°}{2{,}5\angle -15°} = 20\angle 60°\ \Omega$$

$$10 + 10\sqrt{3}j = 5 + j8 + Z$$

$$Z = 5 + 9{,}32j\ \Omega$$

9. $L = 25\cdot 10^{-3}\ H$

$C = 50\cdot 10^{-6}\ F$

R,

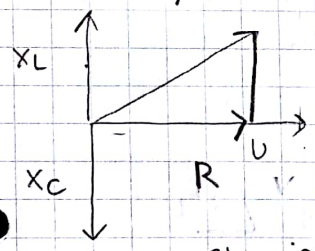
$U = 120\ V$

$\omega = 400\ rad/s$

$\varphi = 63{,}4°$

$X_L = 2\pi f \cdot 25\cdot 10^{-3} = 10j$

$X_C = -j50$

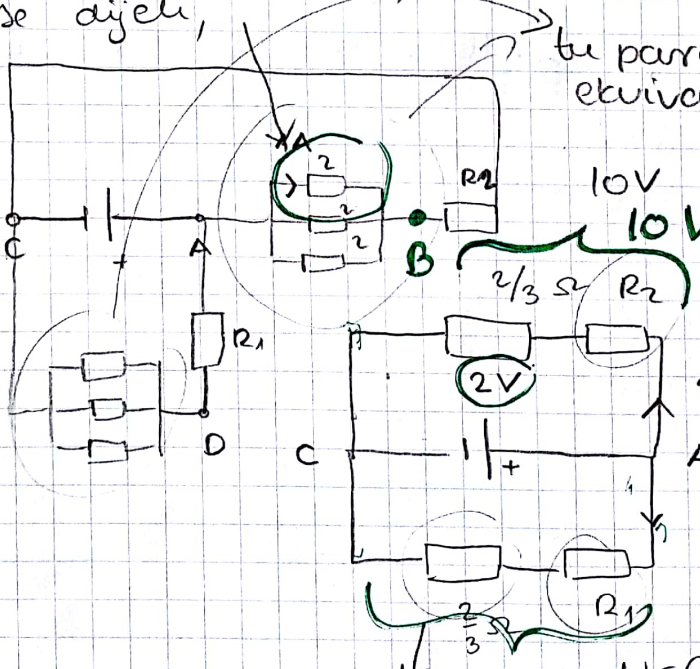


$40j$

$$1{,}997 = \frac{40}{R}$$

$$R = 20\,\Omega$$

jednaki su otpori, struja se dijeli, ako je u jednom 1 A onda je i u ostala dva pa je ukupna 3A



tu paralelu zamjenim ekvivalentnim otporom $\frac{2}{3}\,\Omega$

10V

10V

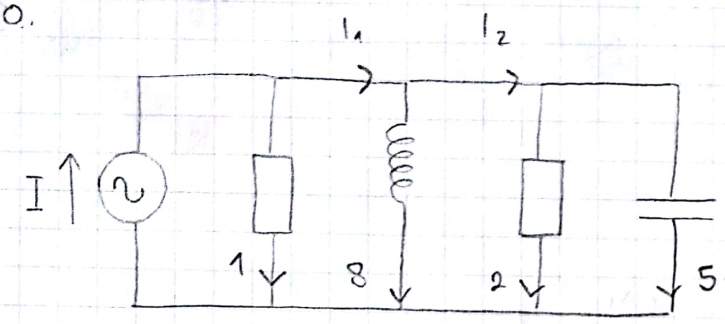
8V

$2/3\,\Omega$ R_2

$$R_2 = \frac{8}{3} = 2{,}67$$

$$R_1 = \frac{6}{6} = 1\,\Omega$$

10.

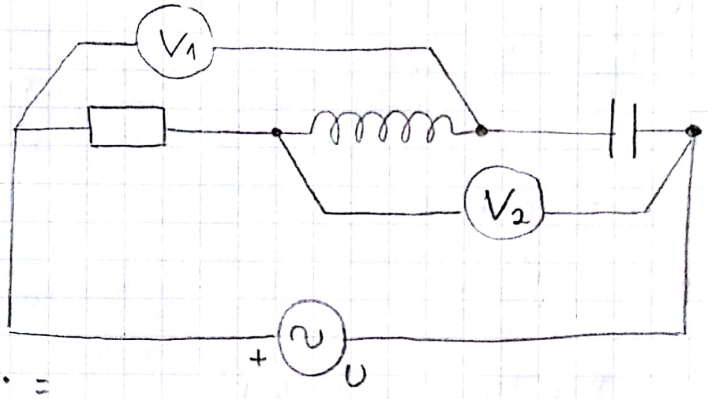


$I_2 = 5 \angle 90° + 2 = 2 + 5j = 5{,}39 \angle 68{,}2°$

$I_1 = 8 \angle -90 + 5{,}39 \angle 68{,}2° = -8j + 2 + 5j = 2 - 3j = 3{,}6 \angle -56{,}3°$

$I_{wk} = 3 - 3j = 4{,}24 \angle -45°$

11. $U = 100\ V$
$R = 20\ \Omega$
$X_L = 20\ \Omega$
$X_C = 5\ \Omega$



$\underline{Z} = 25 \angle 36{,}86°$

$I = \frac{U}{\underline{Z}} = \frac{100 \angle 0}{25 \angle 36{,}86°} =$

$4 \angle -36{,}86°\ A$

$\dot{U}_R = 4 \angle -36{,}86 \cdot 20 = 80 \angle -36{,}86\ V$

$\dot{U}_L = 4 \angle -36{,}86 \cdot 20 \angle 90° = 80 \angle 53{,}14°\ V$

$\dot{U}_C = 4 \angle -36{,}86 \cdot 5 \angle -90 = 20 \angle -126{,}86\ V$

$V_1 = -48j + 64{,}2 + 48 + 64j = 112 + 16j = 113{,}137 \angle 8{,}13°$

$V_2 = 48 + 64j - 12 - 16j = 36 + 48j = 60 \angle 53{,}13°$

12. (220 V / 50 Hz)

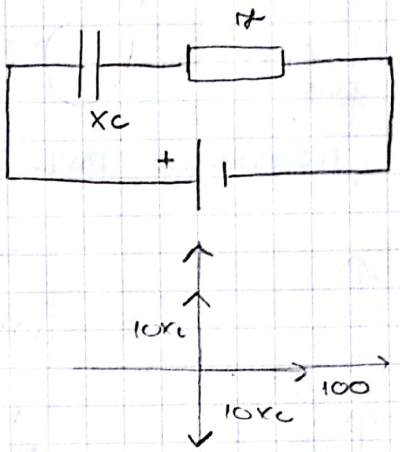
13. $U = 25\ V$
$\omega = 100\ rad/s$
$R = 12,5\ \Omega$
$C = 200 \cdot 10^{-6}\ F$ } paralela
$L = 100 \cdot 10^{-3}\ H$

$$\frac{1}{\underline{Z}} = \frac{1}{R} + \frac{1}{jxL} + j\omega C$$

$$= 0.08 - j \cdot 0.1 + j \cdot 0.02$$

$$= 0.08$$

14. C, Z } serijski spojeni



$$\underline{Z} = \underline{Z} + Xc$$

$$Z = \sqrt{Z^2 + Xc^2}$$

$U = 100\ V$
$I = 10\ A$

$U = 100\ \angle -90°$
$I = 10\ \angle 0°$

$$Xc = \frac{U}{I} = 10\ \angle -90°$$

$$jXC = 10\ \angle -90°$$

$$Xc = 10\ \angle 0°$$

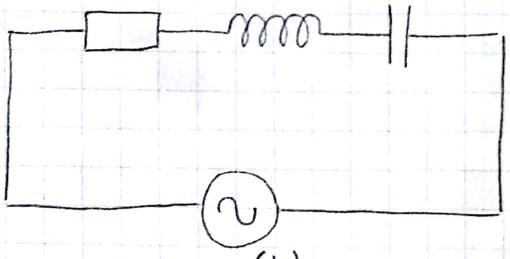
$100 = 100\ \angle -90 + 100\ \angle$

$100 = 100 - j + 100\ \angle$

$100\ \angle 0° - 100\ \angle -90° = 100\ \angle$

$100 + 100j = 100\ \angle$

FREKVENCIJSKE KARAKTERISTIKE



$$Z = R + j(X_L - X_C)$$

$$Z(\omega) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

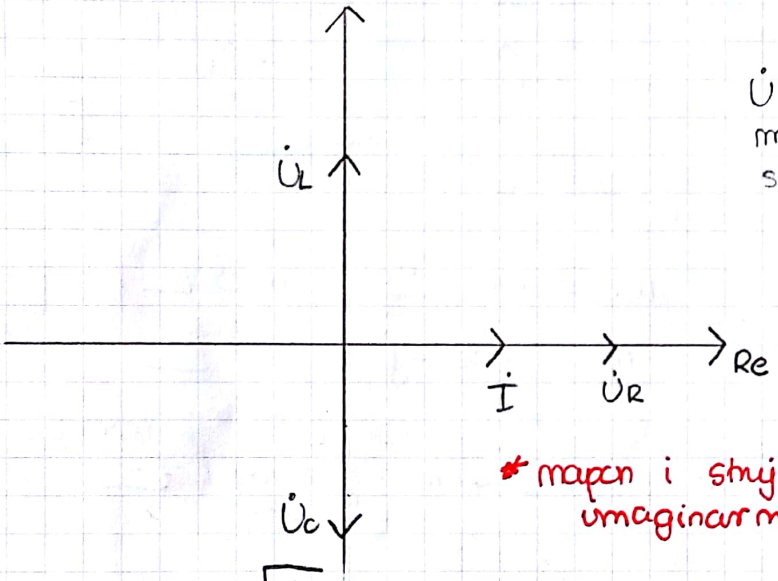
$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\operatorname{tg}\varphi = \frac{\omega L - \frac{1}{\omega C}}{R} \Rightarrow \varphi = \operatorname{arctg}\frac{\omega L - \frac{1}{\omega C}}{R}$$

$$X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C}$$

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$ rezonantna frekvencija $$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$X_L = X_C$



$$\dot{U} = \dot{U}_R + \underbrace{\dot{U}_L + \dot{U}_C}_{0} = \dot{U}_R$$

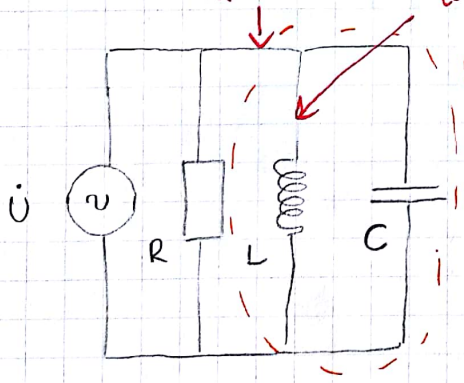
maksimalna struja u fazi s naponom izvora

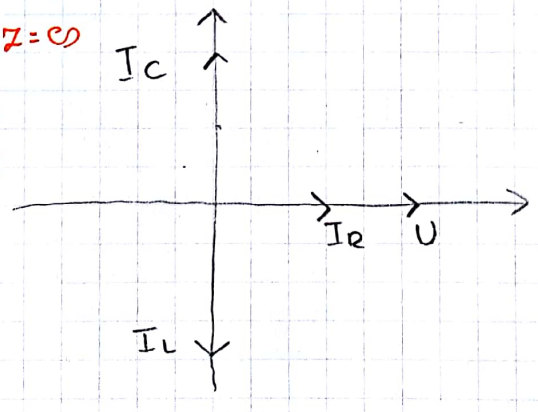
$$\dot{I}_0 = \frac{U}{R}$$

* napon i struja su u fazi $\Rightarrow$ REZOLVANCIJA imaginarni dio od $Z = 0$

valni otpor $\sqrt{\frac{L}{C}} = \rho$

Q_S faktor dobrote $Q_S = \frac{\rho}{R}$ $\Rightarrow$ sposobnost kruga da proizvede titraje





$$\dot{I} = \dot{I}_R + \dot{I}_L + \dot{I}_C = \dot{I}_R$$

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C}$$

$$Z = R$$

$$Z_{LC} = \frac{jX_L \cdot jX_C}{jX_L - jX_C} = \frac{\infty}{\infty} = \infty$$

Pri rezonanciji

$$X_{L0} = \omega_0 L = \frac{1}{\sqrt{LC}} \cdot L = \sqrt{\frac{L}{C}} = \rho$$

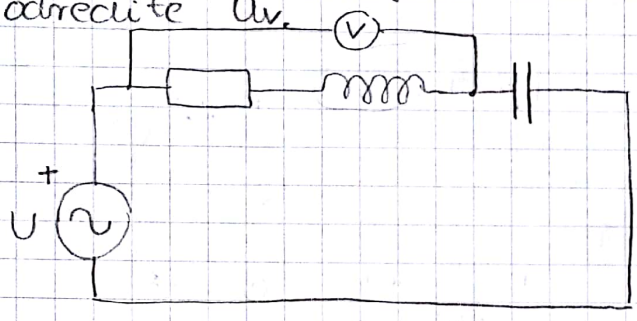
$$X_{C0} = \frac{1}{\omega_0 C} = \frac{\sqrt{LC}}{C} = \sqrt{\frac{L}{C}} = \rho$$

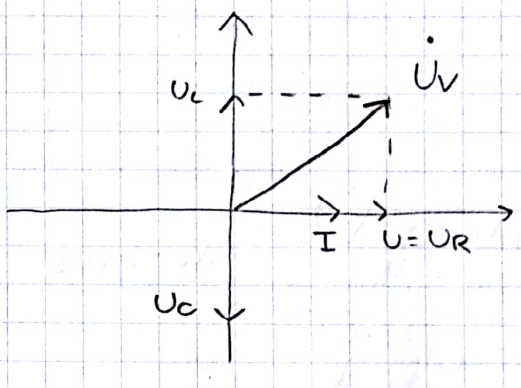
$$U_{R0} = I_0 \cdot R = U_i$$

$$U_{L0} = I_0 X_{L0} = U_i \frac{\rho}{R}; \quad U_{C0} = I \cdot X_{C0} = U_i \frac{\rho}{R}$$

$$U_{L0} = U_i \cdot Q_S \qquad U_{C0} = U_i Q_S$$

1. Serijski R,L,C krug je u rezonanciji ako je $Q_S = 2$, a $U_i = 5$ V odredite U_V.



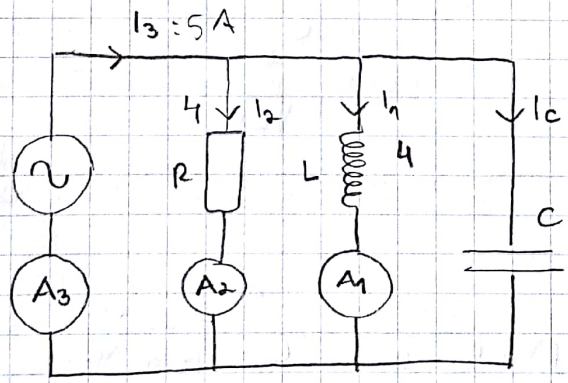


U rezonanciji $\dot{U} = \dot{U}_R$

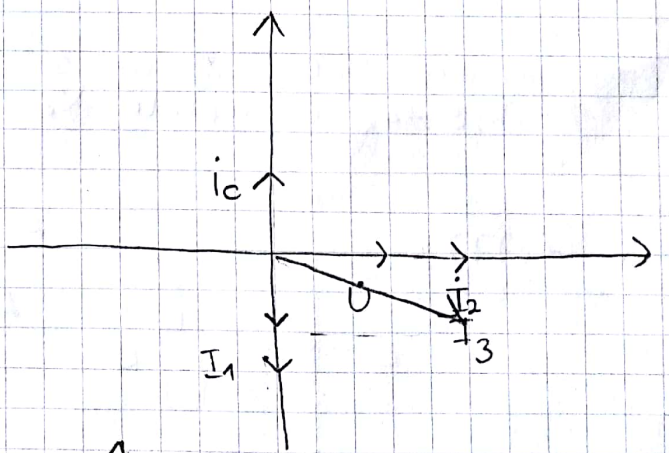
$$U_{C0} = U_{L0} = Q_S \cdot U_i = 10\ V$$

$$U_V = \sqrt{U_R^2 + U_L^2} = \sqrt{5^2 + 10^2} = 11.18\ V$$

Pri frekvenciji ω instrumenti pokazuju $I_1 = 4A$, $I_2 = 4A$, $I_3 = 5\ A$ uz $X_L < X_C$. Odredite I_3 ako ω postane 2ω



$$I_3 = I_2 + I_1 + I_C$$



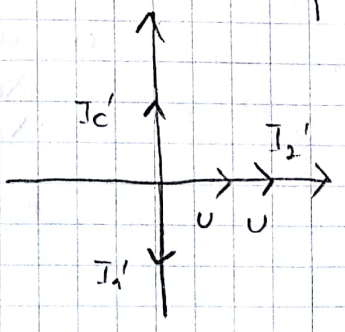
$$(\dot{I}_1 - \dot{I}_C)^2 + \dot{I}_2^{\,2} = I_3^{\,2}$$

$$I_C = 1\ A$$

$$I_2' = I_2 = 4A$$

$I_C' = 2A$ } struje su ist

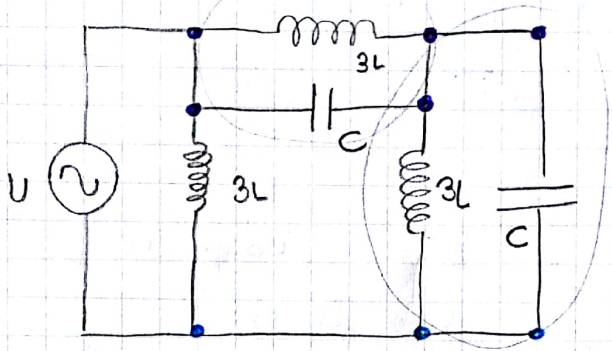
$I_1' = 2A$ } REZONANCIJA

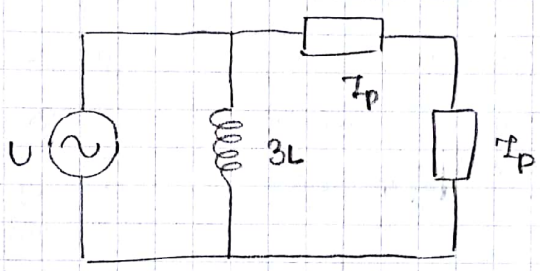


$$I_3' = I_2'$$

Odredite f pri kojoj će struja izvora biti jednaka 0

$L = 1{,}6$ mH
$C = 100$ μF





$$Z_p = \frac{jX_L(-jX_C)}{jX_L - jX_C}, \quad X_L = 3\omega L$$

$$= -j\frac{X_L X_C}{X_L - X_C}$$

$$Z = jX_2 \,||\, (Z_p + I_p) = \frac{jX_L\left(-Ij\frac{X_L X_C}{X_L - X_C}\right)}{jX_L - 2j\frac{X_L X_C}{X_L - X_C}}$$

$$Z = \frac{-2jX_L^2 X_C}{X_L(X_L - X_C) - 2X_L X_C}$$

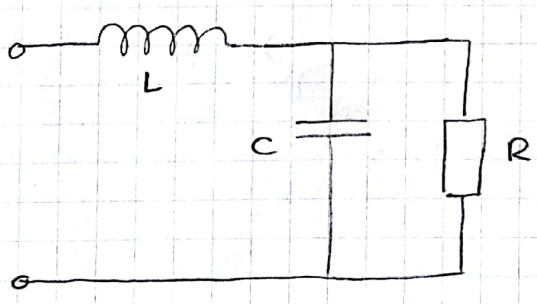
$$= \frac{-2jX_L X_C}{X_L - 3X_C} = \infty \Rightarrow \quad X_L - 3X_C = 0$$

$$X_L = 3X_C$$

$$3\omega L = \frac{3}{\omega C}$$

$$\omega = \frac{1}{\sqrt{LC}} = 250$$

Za spoj prema slici Z pri $\omega = 0$ $Z = 5\,\Omega$ a pri rezonantnoj f $Z = 2{,}5\,\Omega$. Odredite X_C.



imaginarni dio $Z = 0$

$$Z = jX_L + \frac{R(-jX_C)}{R - jX_L}\cdot\frac{R + jX_C}{R + jX_C}$$

$$Z = jX_L + \frac{RX_C^2 - jR^2X_C}{R^2 + X_C^2}$$

=

a) $\omega = 0$ istosmjerna struja
$X_L = 0$ (zavojnica se ponaša kao žica)
$X_C = \infty$

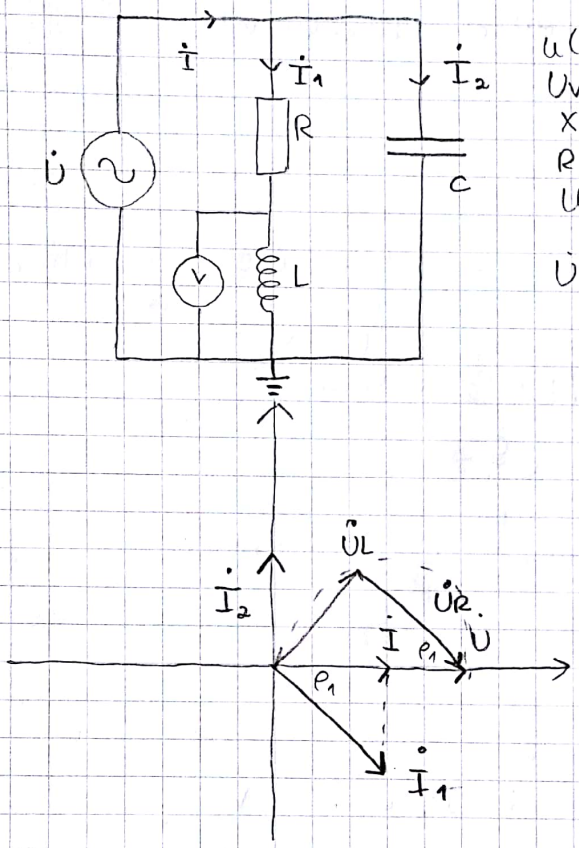
$R = 5\,\Omega$

b) $\frac{-R^2 X_C}{R^2 + X_C^2} + X_L = 0$

$\frac{R X_C^2}{R^2 + X_C^2} = 2{,}5 \Rightarrow \frac{5 X_C^2}{25 + X_C^2} = 2{,}5 \Rightarrow 5X_C^2 = 2{,}5 \cdot 25 + 2{,}5 X_C^2$

$2{,}5 X_C^2 = 2{,}5 \cdot 25$

$X_C = 5\,\Omega$



$u(t) = 100\sqrt{2}\sin(\omega t)$

$U_V = 60\,V$

$X_C = 10\,\Omega$

$R, X_L, = ?$

Ui u fazi Ii

$\dot{U} = 100\,\angle 0°$

$\dot{U}_R = \sqrt{U^2 - U_L^2} = 80\,\angle$

$\sin\varphi_1 = \frac{U_L}{U} = 0.6$

$\sin\varphi_1 = \frac{I_2}{I_1}$

$I_2 = \frac{\dot{U}}{-jX_C} = \frac{100}{10}\angle 90°$

$= 10\,\angle 90°$

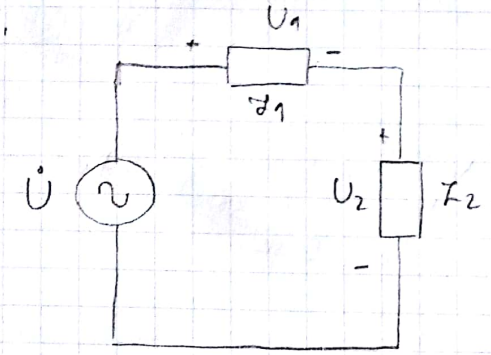
$I_1 = \frac{I_2}{\sin\varphi_1} = \frac{10}{0{,}6}$

$= \frac{50}{3}\,A$

$R = \frac{U_R}{I_1} = 4{,}8\,\Omega$

$X_L = \frac{U_L}{I_1} = 3{,}6\,\Omega$

5.



$U = U_1 = U_2 = 100\,V$ ⇒ jednakostranični trokut

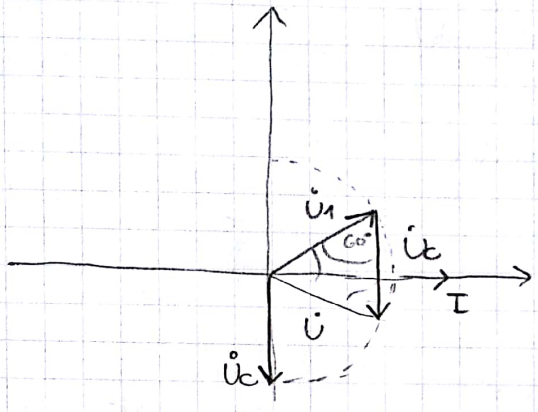
$I = 10\,A$

$Z_1 \rightarrow \varphi_1 = -90°$ = kondenzator

$Z_1, Z_2 = ?$

$$Z_1 = \frac{U_1}{I} = 10\,\Omega$$

$Z_1 = 10\,\angle -90\,\Omega$ kapacitet



$\varphi_2 = 30°$

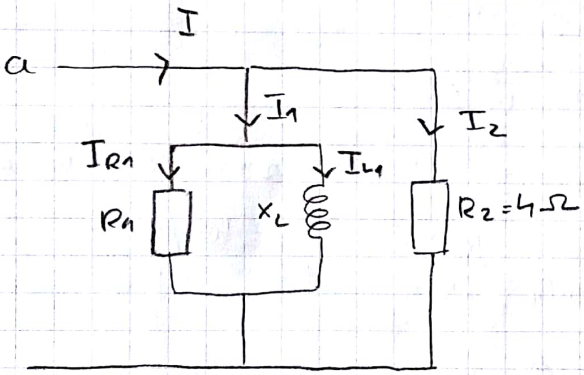
$Z_2 = R_2 + jX_L$ ⇒ nije čisto induktivno jer nije 90°

$R_2 = |Z_2| \cos\varphi_2 = 5\sqrt{3}\,\Omega$

$X_{L2} = |Z_2| \sin\varphi_2 = 5\,\Omega$

$Z_2 = 5\sqrt{3} + j5\,\Omega$

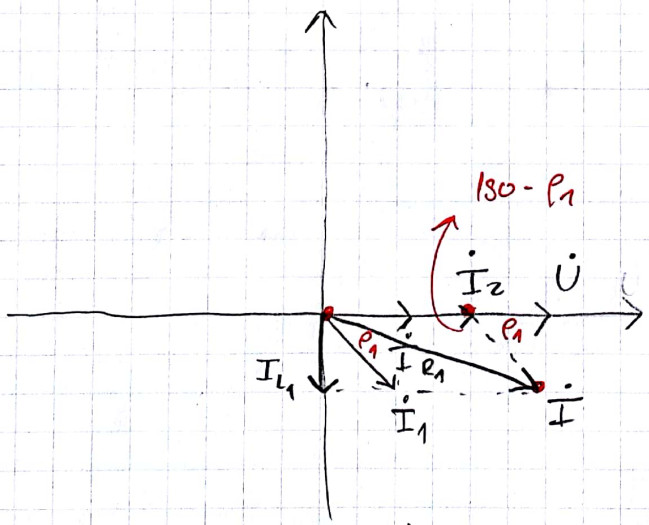
6.



$I_1 = 18\,A$
$I_2 = 15\,A$
$I = 30\,A$

$R_1 = ?$
$X_L = ?$

$U = I_2 \cdot R_2 = 60\,V = 60\,\angle 0°$



$$I^2 = I_1^2 + I_2^2 - 2I_1 I_2 \cos(180 - \varphi_1)$$
$$= I_1^2 + I_2^2 + 2I_1 I_2 \cos(\varphi_1)$$
$$\cos(\varphi_1) = \frac{I^2 - I_1^2 - I_2^2}{2I_1 I_2}$$

$\varphi_1 = 49{,}46°$

$I_1 = 18\,\angle -49{,}46$

$$Y_1 = \frac{I_1}{U} = \frac{18\,\angle -49{,}46°}{60\,\angle 0°} = 0{,}3\,\angle -49{,}46$$

$= G - jB_C$

$G_1 = 0.3 \cos(-49{,}46°) = 0.195\,S$

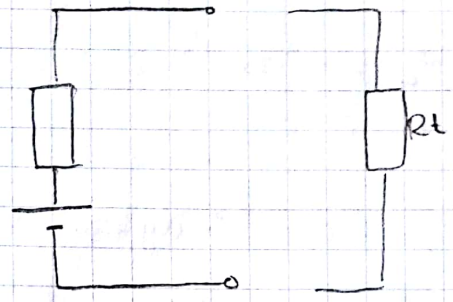
$R_1 = \frac{1}{G_1} = 5{,}128\,\Omega$

$-B_C = 0{,}3 \sin(-49{,}46°) = 0{,}228$

$X_C = \frac{1}{B_C} = 4{,}386\,\Omega$

2012.

1.



$E_{i1} = I_1 \cdot R_{i1} + I_1 \cdot Rt$

$E_{i2} = I_2 \cdot R_{i2} + I_2 \cdot Rt$

sa slike

$E_{i2} = 16\ V \quad I_{k0} = 4A \quad R_{i2} = 4\Omega$

$E_{i1} = 4\ V \quad I_{ks} = 4A \quad R_{i1} = 1\Omega$

$4 = I_1 \cdot 1 + I_1 \cdot Rt \qquad I_2 = 2I_1$

$16 = I_2 \cdot 4 + I_2 \cdot Rt$

$4 = I_1 + I_1 \cdot Rt \quad /-2$

$+\ 16 = 8I_1 + 2I_1 Rt$

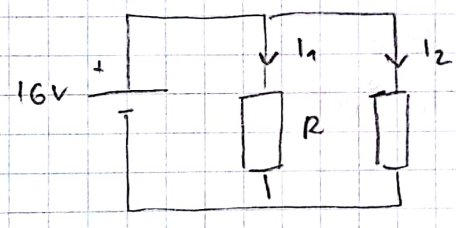
$8 = 6I_1$

$I_1 = \frac{4}{3} \Rightarrow I_1 Rt = 4 - \frac{4}{3} \Rightarrow Rt = \frac{\frac{8}{3}}{\frac{4}{3}} = 2$

2. $U = 0{,}25\, I^2$

$R = 16\Omega$

$U = 16V$



$I = I_1 + I_2 = 9\ A$

$I_1 = \frac{U}{R} = 1A$

$U = 0.25\, I_2^2$

$16 = 0.25\, I_2^2 \Rightarrow I_2 = 8$

3. $C_1 = C_2 = 2mF$

$C_3 = 4\,mF$

$C_4 = 6mF$

$U = 12\ V$

$U = U_2 + U_4$

$C_2 U_2 = C_4 U_4$

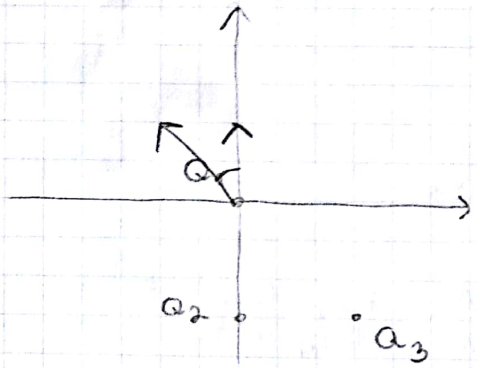
$U_2 = 3U_4$

$12 = 3U_4 + U_4$

$U_4 = 3\ V$

$\frac{P_1}{P_2} = \frac{I^2 R_1}{I^2 R_2} \qquad \frac{R_1}{R_2} = \frac{1}{2}$

$\frac{P_1}{P_2} = \frac{\frac{U^2}{R_1}}{\frac{U^2}{R_2}} = \frac{R_2}{R_1} = ?$ (2)



$$\vec{F_1} = \vec{F_{12}} + \vec{F_{31}}$$

$$\vec{F_{21}} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r_{12}^2} = (-\vec{a_x})$$

$$= 9\cdot10^{-9}\,N\,(-\vec{a_x})$$

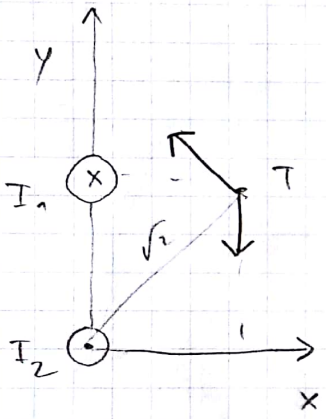
$$\vec{F_{31}} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_3}{(r\sqrt{2})^2}\,(\cos 45°\,\vec{a_y} + \sin 45°\,(\vec{a_x}))$$

$$= 9\cdot10^{-9}\left(\frac{\sqrt{2}}{2}(-\vec{a_y}) - \frac{\sqrt{2}}{2}(\vec{a_x})\right)$$

$$\vec{F_1} = 9\cdot10^{-9}\left[\left(-1-\frac{\sqrt{2}}{2}\right)\vec{a_x} - \frac{\sqrt{2}}{2}\right]$$

$$|F| = 9\cdot10^{-9}\cdot\sqrt{\left(-1-\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2}$$

$$= 16{,}6\ mN$$



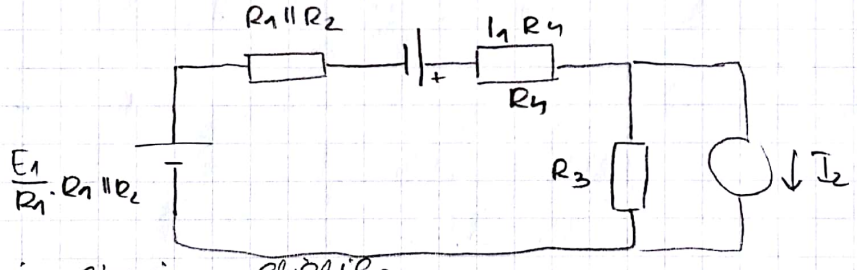
$$\vec{B_1} = \frac{\mu_0 I}{2\pi r}\vec{a_n} = \frac{4\pi\cdot10^{-7}\cdot\cancel{1}}{\cancel{2\pi}\cdot 1}\,(-\vec{a_y})$$

$$\vec{B_2} = \frac{4\pi\cdot10^{-7}\cdot 4}{2\pi\cdot\sqrt{2}}\left(\frac{-\sqrt{2}}{2}\vec{a_x} + \frac{\sqrt{2}}{2}\vec{a_y}\right)$$

$$= 4\pi\cdot10^{-7}\,(-\vec{a_x} + \vec{a_y})$$

$$\vec{B} = \vec{B_1} + \vec{B_2} = -4\cdot10^{-7}\vec{a_x}$$

$E_1 = 20\ V$
$I_1 = 1\ A$
$I_2 = 0.5\ A$
$R_1 = R_3 = 20\Omega$
$R_2 = R_4 = 40\Omega$



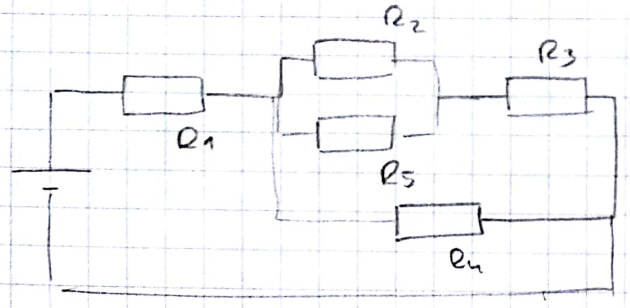
Superpozicija i smjino djelic

$$I_3' = \frac{\frac{E_1}{R_1}\cdot\frac{R_1\cdot R_2}{R_1+R_2} + I_1 R_4}{\frac{R_1 R_2}{R_1+R_2} + R_3 + R_4} = \frac{8}{11} = 0{,}727$$

$$I_3'' = -I_2\,\frac{\frac{R_1 R_2}{R_1+R_2+R_4}}{\frac{R_1 R_2}{R_1+R_2} + R_3 + R_4} = -0.364$$

$$I_3 = I_3' + I_3'' = 0.3636\ A$$

8.



$$R_{ek} = R_1 + (R_{25} + R_3) \| R_4 = 1{,}6\,\Omega$$

$$I_{ek} = \frac{U}{R_{ek}} = 7{,}5\ A$$

$$I_3 = I_{ek} \frac{R_4}{R_3 + R_4 + R_{25}} = 3A$$

$$U_{ab} = I_3 \cdot R_{25} = 1{,}5\ V$$

$$I_{R2} = \frac{U_{ab}}{R_2} = 1{,}5\ A$$

9. $I_1 = \frac{I}{2} = 3 - j4\ A$

$U_{AB} = I_1 \cdot 3 + j4 \cdot 5$

10. $U_C = 60\ V$

$U_L = 40V$

$$\frac{X_L}{X_C} = \frac{\frac{U_L}{I_L}}{\frac{U_C}{I_C}}$$

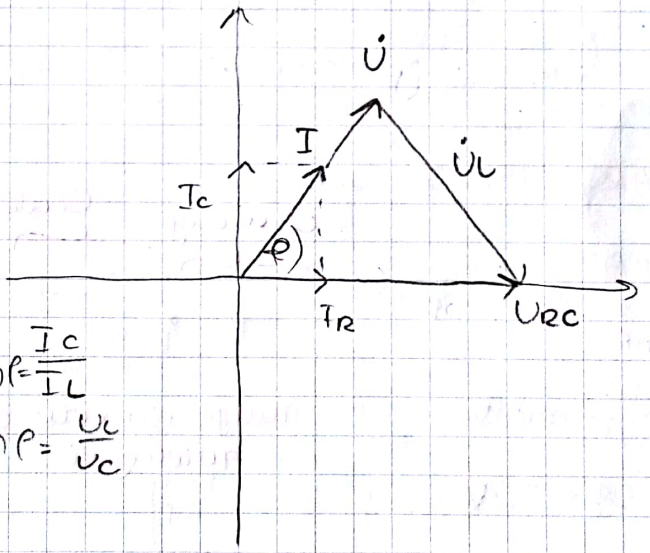
$$\frac{U_L}{U_C} \cdot \frac{I_C}{I_L}$$

$$\sin\varphi = \frac{I_C}{I_L}$$

$$\frac{U_L}{U_C} \cdot \frac{I_C}{I_L} = \frac{U_L}{U_C} \cdot \frac{U_L}{U_C}$$

$$\sin\varphi = \frac{U_L}{U_C}$$

$$\frac{U_L^2}{U_C^2} = 0{,}44$$



2014.

1. $\Delta W = -20\,\mu J$
$\varphi_A = -50\,V$
$\varphi_B = 50\,V$
$Q = ?$

$\Delta W = Q \cdot U = Q(\varphi_B - \varphi_A)$

$Q = \frac{-20\,\mu J}{50-(-50)} = -0{,}2\,\mu C$

2. $U_N = 2 \cdot I_m^{0.5}$
$R = 2\,\Omega$
$P_R = 2\,W$

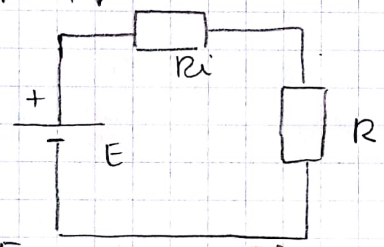
$I_R = \sqrt{\frac{P}{R}} = 1\,A$

$U_R = R I_R = 2\,V$
$U_N = 2 \cdot 1^{0.5} = 2$
$E = U_m + U_r = 4\,V$

$I_2 = E/2\Omega = 2\,A$

$I_{uk} = I_R + I_2 = 3\,A$

3. $R = 2\,\Omega$
$P = 50\,W$



$P = P_{maks} \Rightarrow$
$R = R_i$

$P = I^2 \cdot R = \left(\frac{E}{R+R_i}\right)^2 \cdot R = \frac{E^2}{4R^2} \cdot R = \frac{E^2}{4R}$

$E = \sqrt{P \cdot 4R} = \sqrt{50 \cdot 8} = 20\,V$

4. $Q_3 = 10\,mAs$
$C_1 = C_2 = C_3 = 1\,mF$
$C_4 = 0{,}5\,mF$

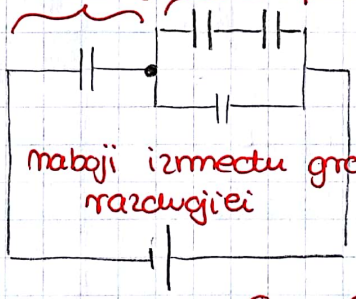
$U_2 = U_3 = \frac{Q_3}{C_3} = 10\,V$

$U_p = U_2 + U_3 = 20\,V$

$Q_4 = C_4 \cdot U_p = 10\,mAs$

$Q_1 = Q_3 + Q_4$

Q u ovoj = Quk



naboji izmedu grana se razvojiei

$Q_1 = Q_3 + Q_4$

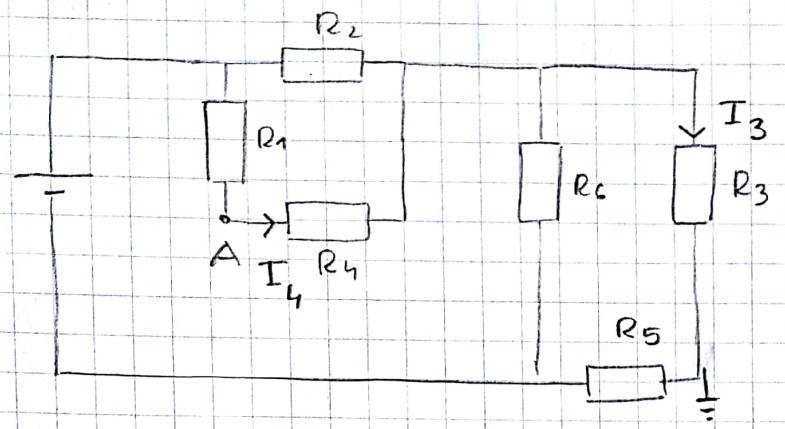
5. $E_1 - E_2 + E_4 - E_3 = (R_1 + R_2 + R_3 + R_4)\,I$
$-10 = I\,(30 + R_2)$

$E_1 - E_3 = I\,(R_1 + R_3)$

$I = \frac{E_1 - E_3}{R_1 + R_3} = -0{,}2\,A$

$-10 = -0{,}2\,(30 + R_2)$
$R_2 = 20\,\Omega$

6.



$$U = I\left((R_1 + R_4) \parallel R_2 + (R_3 + R_5) \parallel R_6\right)$$

$$I = \frac{36}{24 \parallel 24 + 16 \parallel 16} = \frac{36}{12 + 8} = 1{,}8\ mA$$

$$P_A = I_3 R_3 + I_4 R_4 = 0.9\ mA \cdot 8k\Omega + 0.9mA \cdot 12k\Omega = 18\ V$$

$$I_4 = I \cdot \frac{R_2}{R_2 + R_1 + R_4} = I \cdot \frac{24}{48} = 0{,}9\ mA$$

$$I_3 = I \cdot \frac{R_6}{R_3 + R_5 + R_6} = I \cdot \frac{16}{16 + 8 + 8} = 0.9\ mA$$

7. za poziciju 2:

$$I_A = I \cdot \frac{R_1}{R_1 + R_2} \Rightarrow$$

$$0.2\,A = I \cdot \frac{100}{200}$$

$$I = 0.4\ A$$

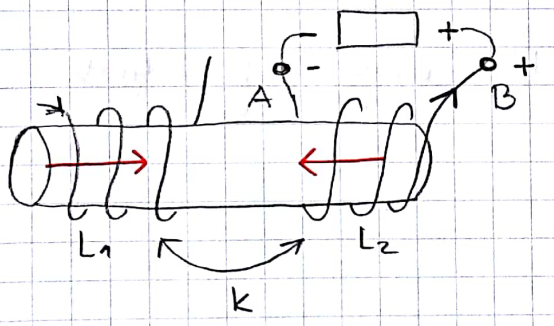
za poziciju 1:

$I_A = 0.3A$

doprinos naponskog izvora $I_{A1} - I_{A2} = 0{,}1$

$$I_E = \frac{E}{R_1 + R_2} \Rightarrow E = I_E (R_1 + R_2) = 20\ V$$

8.



$$M = k\sqrt{L_1 L_2} = 1\ mH$$

$$|U_{AB}| = \left|M \frac{di}{dt}\right| = \left|10^{-3} \cdot \frac{20}{2 \cdot 10^{-3}}\right|$$

$$= 10$$

-10 zato što je A na manjem potencijalu

9. $i(t) = \sqrt{2}\sin(10^4 t + 90°)\ [A]$

$u_C(t) = 10\sqrt{2}\sin(10^4 t)\ [V]$

$$i_C = C\frac{du}{dt} = 10 \cdot 10^{-6} \cdot 10^4 \cos(-10^4 t) \cdot 10\sqrt{2} \quad \left| \begin{array}{l} \cos x = \sin(90 - x) \\ = -\sin(x - 90) \end{array} \right.$$

$$= -\sqrt{2}\sin(10^4 t - 90)$$

$$\dot{I} = 1\ \angle 90°$$

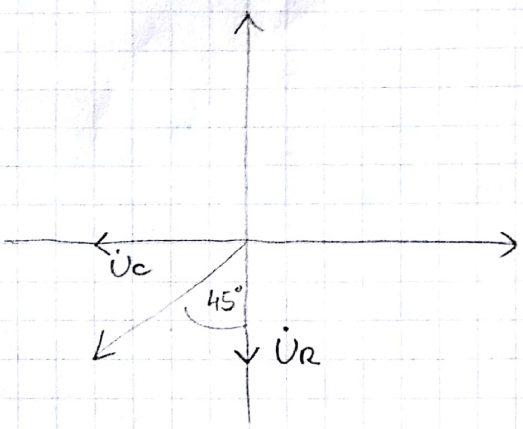
$$\dot{I}_C = -1\ \angle -90°$$

$$\dot{I}_R = \dot{I} + \dot{I}_C = 2\ \angle 90°$$

$$\dot{U}_R = R \cdot \dot{I}_R = 10\ \angle 90°$$

$$\dot{U}_{21} = \dot{U}_R + \dot{U}_C$$

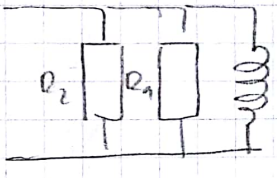
$$\dot{U}_{12} = -\dot{U}_R - \dot{U}_C$$

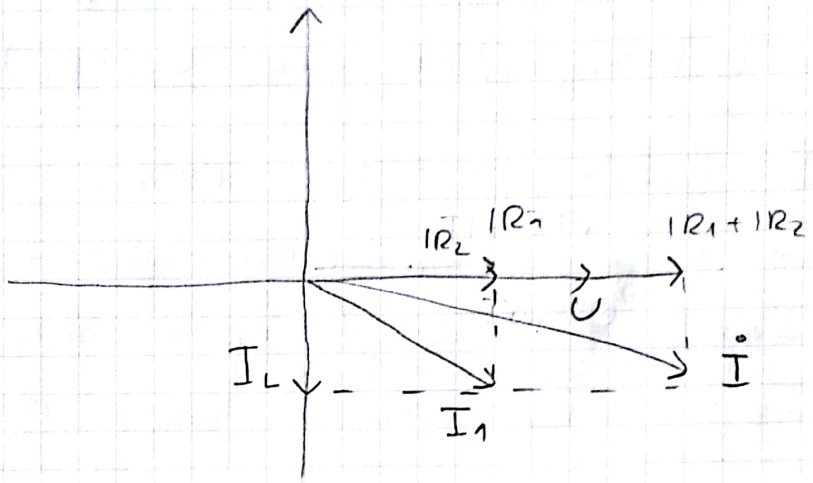


$\dot{U}_{12} = 10\sqrt{2}\ \angle -135°$

$U_{12} = 20 \sin(\omega t - 135°)$

10. $I = 4\sqrt{2}\ A$
$I_1 = \sqrt{20}\ A$
$R = 20\ \Omega$





$U = I_{R_1} \cdot R$

$I_1^2 = I_L^2 + I_{R_1}^2$ I (iz manjeg pravokutnog trokuta)

$I^2 = I_L^2 + (I_{R_1} + I_{R_2})^2$ II

II - I

$I^2 - I_1^2 = 4I_{R_1}^2 - I_{R_1}^2$

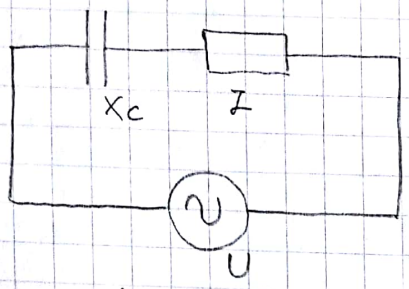
$32 - 20 = 3I_{R_1}^2$

$I_{R_1} = 2$

$U = I_{R_1} \cdot R = 2 \cdot 20 = 40\ V$

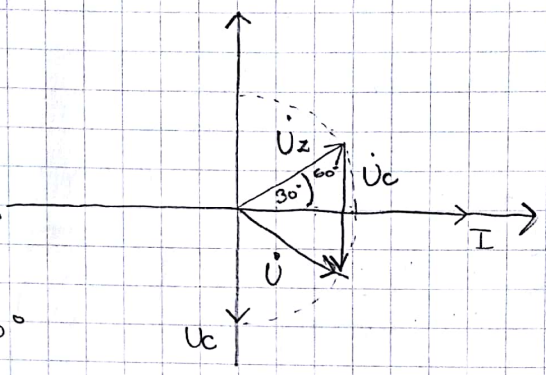
Zbirka: RLC krugovi

14.



$I = 10\ A$

$U_c = U_I = U = 100\ V$ jednakostranični trokut

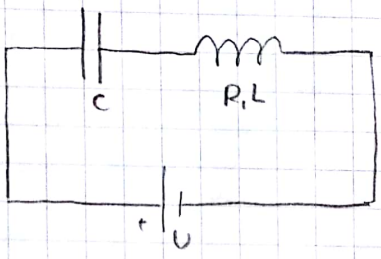


$$Z_c = \frac{\dot{U}}{\dot{I}} = \frac{100\ \angle -90^\circ}{10\ \angle 0^\circ} = 10\ \angle -90^\circ$$

$$Z_1 = \frac{\dot{U}}{\dot{I}} = \frac{100\ \angle 30^\circ}{10} = 10\ \angle 30^\circ$$

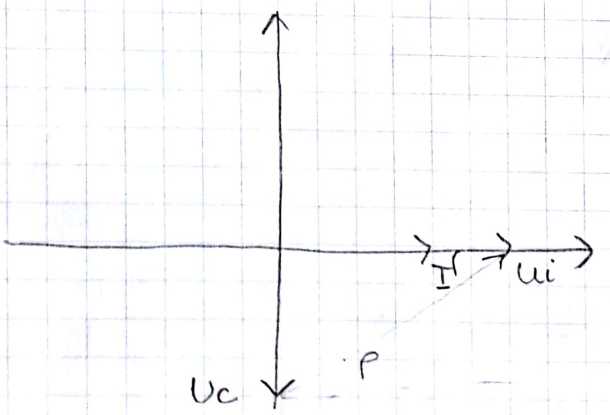
$$= 5\sqrt{3} + 5j =$$

15.



$U = 120\ V$

$U_c = 160\ V\ \angle -90^\circ$



$$\operatorname{tg}\varphi = \frac{U_c}{u_i}$$

$$\varphi = 53.13^\circ$$

$$\sin\varphi = \frac{U_c}{U_{RL}}$$

$$U_{RL} = \frac{U_c}{\sin\varphi}$$

$$U_{RL} = 200\ V$$

16. $U = 110\ V$
$f = 50\ Hz$
$R = 10\ \Omega$
$X = 60\ \Omega$

$G =$
$B = ?$

$Z =$