Zbirka riješenih zadataka

iz

Obrade signala u komunikacijama

-

Ivan Sović

2010.

-

Zadatke rješavali:

Ivan Sović i Nepoznati Autori

Napomena:
Rješavani zadatci su bili aktualni ak.god. 2008./2009.
U ovoj zbirci trenutno nedostaju zadatci: 2.1, 2.2, 8.4, 8.9, 11.5, 11.6, 11.7, 11.8 i 14.1.
Poglavlja 1, 3, 4, 5, 7, 12 i 13 nisu imale zadataka, te stoga nema niti rješenja pod tim oznakama.
Ako netko želi pridonijeti izradi zbirke sa zadatcima koji nedostaju ili u nekom drugom pogledu, slobodno me kontaktirajte na mail ivan.sovic@gmail.com .
Autor se odriče svake odgovornosti od neželjenih posljedica nastalih primjenom ovih rješenja (primjerice, ako su pogrešna).

Rješenje ovog zadatka nije isto kao i ono u zadatcima za vježbu, ne sjećam se više točno, ali mislim da je njima bilo krivo... Tko god da ovo čita, neka provjeri sam!

2.3. $x[n] = 2^{-n}\cos\left(\frac{\pi}{4}\cdot n\right)\cdot S[n]$

$X(e^{jw})$

$$a^n f[n] \multimap F\left(\frac{z}{a}\right)$$

$$2^{-n}\cos\left(\frac{\pi}{4}\cdot n\right) = \left(\frac{1}{2}\right)^n \cos\left(\frac{\pi}{4}\cdot n\right)$$

$$X_1(z) = \frac{z^2 - z\cos\frac{\pi}{4}}{z^2 - 2z\cos\frac{\pi}{4} + 1} = \frac{z^2 - z\frac{\sqrt{2}}{2}}{z^2 - \sqrt{2}z + 1}$$

$$X(z) = X_1\left(\frac{z}{\frac{1}{2}}\right) = X_1(2z) = \frac{4z^2 - \sqrt{2}z}{4z^2 - 2\sqrt{2}z + 1}$$

$$X(e^{jw}) = X(z)\Big|_{z=e^{jw}} = \frac{4\cdot e^{2jw} - \sqrt{2}\cdot e^{jw}}{4e^{2jw} - 2\sqrt{2}e^{jw} + 1} =$$

$$= \frac{-\sqrt{2}e^{-jw} + 4}{e^{-2jw} - 2\sqrt{2}e^{-jw} + 4}$$

$$\frac{1}{2 - e^{-j(w-\frac{\pi}{4})}} + \frac{1}{2 - e^{-j(w+\frac{\pi}{4})}} = \frac{4 - e^{-jw}\left(e^{j\frac{\pi}{4}} + e^{-j\frac{\pi}{4}}\right) + e^{-j2w}}{4 - 2\cdot e^{jw}\cdot\underbrace{\left(e^{j\frac{\pi}{4}} + e^{-j\frac{\pi}{4}}\right)}_{\cancel{2}\cdot\frac{\sqrt{2}}{\cancel{2}}}}$$

$$\boxed{\frac{dx(t)}{dt}} \circ\!\!-\!\!\bullet\ (j\omega)^n \cdot X(\omega)$$

$$= (j\omega)^n X(j\omega)$$

2.4. $$x[n] = \sum_{m=0}^{5} \delta[n-m]$$

$$X(k) = X(e^{j\omega})\Big|_{\omega = \frac{2\pi k}{N}}$$

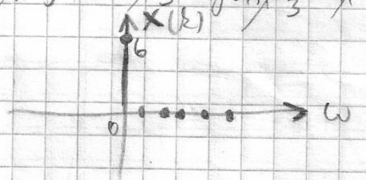
$N = 6$

$$X(k) = \sum_{n=0}^{5} 1 \cdot e^{-j\frac{2\pi k n}{6}} = \underbrace{e^{-j\frac{\pi}{3}k\cdot 0}}_{1} + e^{-j\frac{\pi k}{3}} + e^{-j\frac{2\pi k}{3}} + e^{-j\pi k} + e^{-j\frac{4\pi k}{3}} + e^{-j\frac{5\pi k}{3}}$$

$$= e^{-j\frac{5\pi}{6}k} \cdot \Big(e^{j\frac{5\pi}{6}k}$$

$X(0) = 6$

$$X(1) = \cancel{1} + \cancel{\cos\frac{\pi}{3}} - j\cancel{\sin\frac{\pi}{3}} + \cancel{\cos\frac{2\pi}{3}} - j\cancel{\sin\frac{2\pi}{3}} - \cancel{1} + \cancel{\cos\frac{4\pi}{3}} - j\cancel{\sin\frac{4\pi}{3}} + \cancel{\cos\frac{5\pi}{3}} - j\cancel{\sin\frac{5\pi}{3}}$$
$$= 0$$



$X(2) = 0$

$X(3) = 0$

$X(4) = 0$

$X(5) = 0$

2.5. $x[n] = \sum_{m=0}^{1} 1 \cdot \delta[n-m] + \sum_{m=2}^{5} 0 \cdot \delta[n-m]$

$X(k) = ?$

$N=6$

$$X(k) = 1 + e^{-j\frac{2\pi k}{N}} = 1 + e^{-j\frac{\pi k}{3}}$$

$$X(0) = 2$$

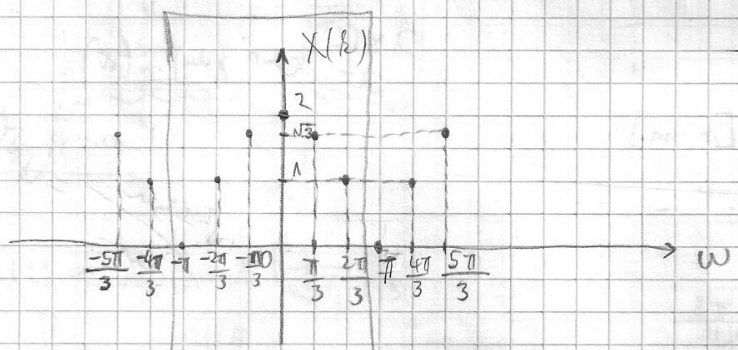
$$X(1) = 1 + \cos\frac{\pi}{3} - j\sin\frac{\pi}{3} \Rightarrow |X(1)| = \sqrt{\left(1+\cos\frac{\pi}{3}\right)^2 + \left(\sin\frac{\pi}{3}\right)^2} = \sqrt{3}$$

$$X(2) = 1 + \cos\frac{2\pi}{3} - j\sin\frac{2\pi}{3} \Rightarrow |X(2)| = 1$$

$$X(3) = 1 + \cos\pi - j\sin\pi \Rightarrow |X(3)| = 0$$

$$X(4) = 1 + \cos\frac{4\pi}{3} - j\sin\frac{4\pi}{3} \Rightarrow |X(4)| = 1$$

$$X(5) = 1 + \cos\frac{5\pi}{3} - j\sin\frac{5\pi}{3} \Rightarrow |X(5)| = \sqrt{3}$$



DFT (PERIODIČKI, MORAM RAZMJESTITI UZO
OKO NULE)!

2.6.

A
$2T$
$\varphi = 0$

$$X(t) = A \cdot \mathrm{rect}\left(\frac{t}{2T}\right)$$

$$\hat{X}(t) = \frac{1}{\pi} \cdot \int_{-\infty}^{\infty} A \cdot \frac{\mathrm{rect}\left(\frac{\tau}{2T}\right)}{t-\tau}\, d\tau = \frac{A}{\pi} \int_{-T}^{T} \frac{1}{t-\tau}\, d\tau =$$

$$= \left(\begin{array}{c} t - \tau = k \\ dk = -d\tau \\ k(T) = t - T \\ k(-T) = t + T \end{array} \right) = \frac{A}{\pi} \int_{t+T}^{t-T} \frac{-1}{k}\, dk =$$

$$\boxed{\hat{X}(t) = \frac{-A}{\pi} \cdot \ln|k| \Big|_{t+T}^{t-T} = \frac{-A}{\pi} \cdot \ln \frac{t-T}{t+T} = \frac{A}{\pi} \cdot \ln \frac{t+T}{t-T}}$$

$$\boxed{\mathcal{X}(t) = x(t) + j\hat{x}(t) = A\,\mathrm{rect}\left(\frac{t}{2T}\right) + j \cdot \frac{A}{\pi} \cdot \ln \frac{t+T}{t-T}}$$

2.7.

$$x(t) = \delta(t-2) - \frac{3}{\pi} \cdot \mathrm{sinc}\left(\frac{3}{\pi} \cdot (t-2)\right)$$

$$\hat{x}(t) = \frac{1}{\pi(t-2)} - H\left[\frac{3}{\pi} \cdot \frac{\sin(3\cdot(t-2))}{3\cdot(t-2)}\right]$$

$$= \frac{1}{\pi\cdot(t-2)} - \frac{3}{\pi} \cdot \frac{1-\cos(3\cdot(t-2))}{3\cdot(t-2)}$$

$$= \frac{1}{\pi\cdot(t-2)} - \frac{1}{\pi\cdot(t-2)} \cdot (1-\cos(3\cdot(t-2)))$$

$$= \underbrace{\frac{1}{\pi\cdot(t-2)} - \frac{1}{\pi\cdot(t-2)}} + \frac{\cos(3\cdot(t-2))}{\pi\cdot(t-2)}$$

$$\boxed{\hat{x}(t) = \frac{\cos(3\cdot(t-2))}{\pi\cdot(t-2)}}$$

2.8.

1. $x(n) = -\frac{1}{2}\delta(n+2) + \delta(n) - \frac{1}{2}\delta(n-2)$

$$\delta(n) \longrightarrow \boxed{HT} \longrightarrow h_{HT}(t) = \frac{2}{\pi n}\sin^2\left(n\frac{\pi}{2}\right) = \begin{cases} 0, & n \text{ paran} \\ \frac{2\pi}{n}, & n \text{ neparan} \end{cases}$$

$$x_a(n) = x(n) + j\overbrace{\mathcal{H}[x(n)]}^{\tilde{x}(n)}$$

$$= -\frac{1}{2}\delta(n+2) + \delta(n) - \frac{1}{2}\delta(n-2) + j\left\{-\frac{1}{2}\frac{2}{\pi(n+2)}\sin^2\left(\frac{\pi}{2}(n+2)\right) + \frac{2}{\pi n}\sin^2\left(n\frac{\pi}{2}\right) - \frac{1}{2}\frac{2}{\pi(n-2)}\sin^2\left(\frac{\pi}{2}(n-2)\right)\right\}$$

$\sin^2\left(\frac{\pi}{2}n+\pi\right) \to \sin^2\left(\frac{\pi}{2}n\right)$, $\sin^2\left(\frac{\pi}{2}n\right)$

$$= x(n) + j\sin^2\left(\frac{\pi}{2}n\right)\left\{-\frac{1}{(n+2)\pi} + \frac{2}{\pi n} - \frac{1}{\pi(n-2)}\right\}$$

$$\frac{1}{\pi}\,\frac{\cancel{-n^2}\cancel{+4}\cancel{+2n^2}-8\cancel{-n^2}\cancel{-4}}{n(n^2-4)}$$

$$= \underbrace{-\frac{1}{2}\delta(n+2) + \delta(n) - \frac{1}{2}\delta(n-2)}_{} - j\frac{8}{\pi n(n^2-4)}\sin^2\left(\frac{\pi}{2}n\right)$$

samo ću ovo trebati DTFT-ovati kako bih odredio spektar signala

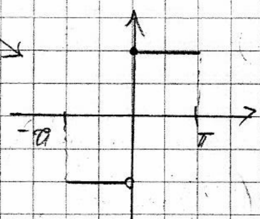
$$DTFT\{x_a(n)\} = DTFT\{x(n) + j\tilde{x}(n)\}$$

$$= X(e^{j\omega}) + j\left[-j\,\mathrm{sgn}(\omega)X(e^{j\omega})\right] \quad \text{za } -\pi < \omega < \pi$$

$$= X(e^{j\omega}) + \mathrm{sgn}(\omega)X(e^{j\omega})$$

$$= X(e^{j\omega})\left[1 + \mathrm{sgn}(\omega)\right]$$

$$= \begin{cases} 0, & -\pi < \omega < 0 \\ X(e^{j\omega}), & \omega = 0 \\ 2X(e^{j\omega}), & 0 < \omega < \pi \end{cases}$$



$$= \left[DTFT(x(n)) = -\frac{1}{2}e^{-j2\omega} + 1 - \frac{1}{2}e^{+j2\omega} = 1 - \cos(2\omega) \right.$$

$$= \begin{cases} 0, & -\pi < \omega < 0 \\ 1 - \cos(2\omega), & \omega = 0 \\ 2 - 2\cos(2\omega), & 0 < \omega < \pi \end{cases}$$

6.1. $G = \frac{|S_1|}{|S_2|} = 10$

$\varphi = \omega t = -5\cdot 10^{-3}\cdot \omega$

$g = ?$

$g = \ln\frac{|S_1|}{|S_2|} + j(0-\varphi) =$

$$\boxed{g = \ln 10 + j\cdot 5\cdot 10^{-3}\,\omega}$$

6.2. (zadaća)

2. – nelinearni sustav

$u(t) = 1{,}5\sin(2t)$

$A, THD, SFDR = ?$

$$y(t) = 0{,}16u^3(t) - 0{,}2u^2(t) + 2u(t)$$
$$= 0{,}16 \cdot 1{,}5^3 \sin^3(2t) - 0{,}2 \cdot 1{,}5^2 \sin^2(2t) + 2 \cdot 1{,}5\sin(2t)$$
$$= 0{,}54\sin(2t)\frac{1-\cos(4t)}{2} - 0{,}45\,\frac{1-\cos(4t)}{2} + 3\sin(2t)$$
$$= 0{,}27\sin(2t) - 0{,}27\sin(2t)\cos(4t) - 0{,}225 + 0{,}225\cos(4t) + 3\sin(2t)$$
$$= 3{,}27\sin(2t) - 0{,}225 + 0{,}225\cos(4t) - 0{,}27\left\{\frac{1}{2}\sin(6t) - \frac{1}{2}\sin(2t)\right\}$$
$$= 3{,}405\sin(2t) - \underbrace{0{,}225 + 0{,}225\cos(4t) - 0{,}135\sin(6t)}_{\text{nove frekvencijske komponente}}$$

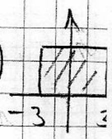
$$THD = 10\log_{10}\frac{\left(\frac{0{,}225}{\sqrt{2}}\right)^2 + \left(\frac{0{,}135}{\sqrt{2}}\right)^2}{\left(\frac{3{,}405}{\sqrt{2}}\right)^2} = -22{,}3\ dB$$

$$SFDR = 10\log_{10}\frac{\left(\frac{0{,}225}{\sqrt{2}}\right)^2}{\left(\frac{3{,}405}{\sqrt{2}}\right)^2} = -23{,}6\ dBc$$

$$A = \frac{3{,}405}{1{,}5} = 2{,}27$$

8.1

3. $u_m(t) = \frac{3}{\pi}\,\mathrm{sinc}\left(\frac{3}{\pi}t\right) \circ\!-\!\circ\ \frac{3}{\pi}\,\frac{\pi}{3}\,\mathrm{rect}\left(\frac{\omega\pi}{2\pi 3}\right) = \mathrm{rect}\left(\frac{\omega}{6}\right)$



$u_0(t) = U_0\cos(\omega_0)$

$u_{USB}(t) = \mathrm{Re}\left[\left(u_m(t) + j\tilde{u}_m(t)\right)e^{j\omega_0 t}\right]$

$=$ (sa predavanja)

$= u_m(t)\cos(\omega_0 t) - \tilde{u}_m(t)\sin(\omega_0 t)$

$\tilde{u}_m(t) = \mathcal{H}\left[\frac{3}{\pi}\,\mathrm{sinc}\left(\frac{3}{\pi}t\right)\right]$

$= \mathcal{H}\left[\frac{3}{\pi}\,\frac{\sin\left(\pi\frac{3}{\pi}t\right)}{\pi\frac{3t}{\pi}}\right]$

$= \mathcal{H}\left[\frac{3}{\pi}\,\frac{\sin(3t)}{3t}\right]$

$= \frac{3}{\pi}\,\frac{1-\cos(3t)}{3t}$ (s predavanja)

$u_{USB}(t) = \frac{3}{\pi}\,\mathrm{sinc}\left(\frac{3}{\pi}t\right)\cos(\omega_0 t) - \frac{1-\cos(3t)}{\pi t}\sin(\omega_0 t)$

$E[u_{USB}(t)] = E[u_m(t)]$

$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}|X(\omega)|^2 d\omega$

$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{rect}^2\left(\frac{\omega}{6}\right)d\omega$

$= \frac{1}{2\pi}\,6$

$= \frac{3}{\pi}$

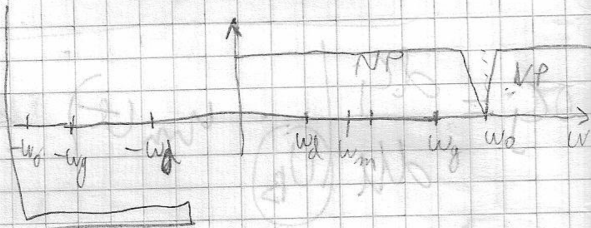
8.2.

$\omega_d \le |\omega| \le \omega_g \Rightarrow u_m(t)$ SPEKTRALNO OGRANIČEN

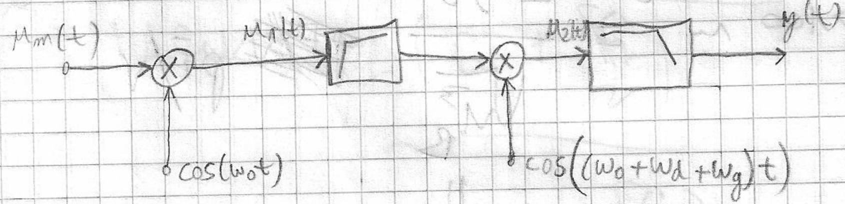
$\omega_{g\,NP} = \omega_{d\,VP} = \omega_0$

$\omega_0 > \omega_g$

$u_m(t) = U_m \cdot \cos(\omega_m t)$



$y(t) = ?$



$$u_1(t) = u_m(t)\cdot\cos(\omega_0 t) = U_m\cos(\omega_m t)\cdot\cos(\omega_0 t) =$$
$$= U_m\cdot\frac{1}{2}\cdot\left(\cos((\omega_m-\omega_0)t) + \cos((\omega_m+\omega_0)t)\right)$$
$$= U_m\cdot\frac{1}{2}\left(\cos((\omega_0-\omega_m)\cdot t) + \cos((\omega_0+\omega_m)t)\right)$$

$$u_2(t) = VP[u_1(t)]\cdot\cos((\omega_0+\omega_d+\omega_g)t) =$$
$$= \frac{U_m}{2}\cdot\cos((\omega_0+\omega_m)t)\cdot\cos((\omega_0+\omega_d+\omega_g)t) =$$
$$= \frac{U_m}{2}\cdot\frac{1}{2}\cdot\left(\cos(\omega_m-\omega_d-\omega_g) + \cos(2\omega_0+\omega_m+\omega_d+\omega_g)\right)$$

$$\boxed{y(t) = NP[u_2(t)] = \frac{U_m}{4}\cdot\cos((\omega_m-\omega_d-\omega_g)t) = \frac{U_m}{4}\cdot\cos((\omega_d+\omega_g-\omega_m)t)}$$

ZOVE SE SCRAMBLER!

(ZZZ)

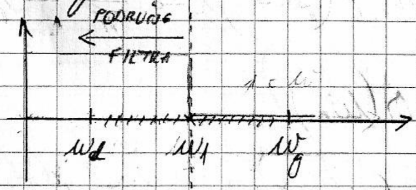
8.3

a) - mod. signal spektralno ograničen $\omega_d \le |\omega| \le \omega_g$

$\omega_1 = \frac{\omega_g + \omega_d}{2}$

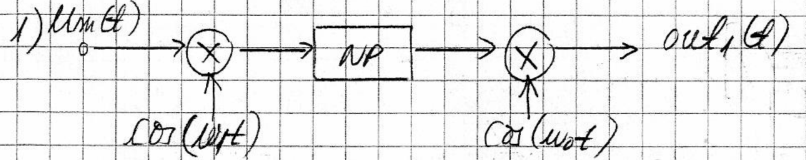
$NP\left(\omega = \frac{\omega_g - \omega_d}{2}\right)$

$\omega_0 > \omega_g$

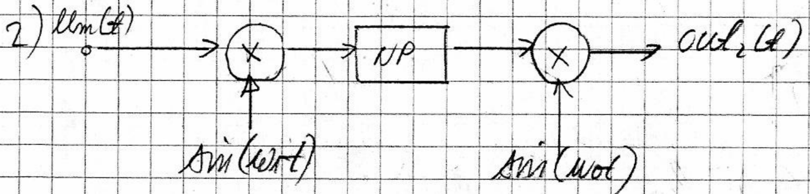


$u_m(t) = U_m \cos(\omega_m t)$, $\omega \in [\omega_d, \omega_g]$

- označimo signal na izlazu s „out(t)"



$$out_1(t) = U_m \cos(\omega_0 t) \cdot NP[\cos(\omega_m t) \cdot \cos(\omega_1 t)]$$
$$= U_m NP\left[\frac{1}{2}\cos(\omega_m + \omega_1) + \frac{1}{2}\cos(\omega_m - \omega_1)\right]\cos(\omega_0 t)$$
$$= \frac{U_m}{2}\cos[(\omega_m - \omega_1)t]\cos(\omega_0 t)$$



$$out_2(t) = U_m NP[\cos(\omega_m t) \cdot \sin(\omega_1 t)]\sin(\omega_0 t)$$
$$= \frac{U_m}{2}\sin[(\omega_1 - \omega_m)t]\sin(\omega_0 t)$$

$$out(t) = out_1(t) + out_2(t)$$

KOSINUS ZBROJA

$$= \frac{U_m}{2}\cos[(\omega_m - \omega_1)t]\cos(\omega_0 t) - \frac{1}{2}\sin[(\omega_m - \omega_1)t]\sin(\omega_0 t)$$
$$= \frac{U_m}{2}\cos[(\omega_m - \omega_1 + \omega_0)t] \quad //$$

- radi se o SSB(USB) modulatoru //

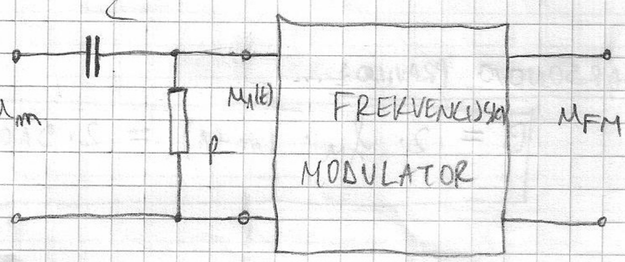
8.5

$\tau = 0.22\ \mu s$

$K_f = 25\ kHz/V$

$f_m \leq 3.1\ kHz$

$U_m = 3V$



frekv. modulator ne utječe na rad filtra

$K_{PM}, B = ?$

$\tau = R \cdot C$

$$K_\omega = \frac{m \cdot \omega_m}{U_m} \Rightarrow K_f = \frac{m \cdot f_m}{U_m} \Rightarrow m = \frac{K_f \cdot U_m}{f_m} = \frac{K_f \cdot U_m}{f_M}$$

$$\boxed{m = 8.2417}$$

$$U_1(s) = U_m(s) \cdot \frac{R}{R + \frac{1}{sC}} = U_m(s) \cdot \frac{sRC}{1 + sRC} = U_m(s) \cdot \frac{s\tau}{1 + s\tau}$$

$$\boxed{\tau << 1}$$

$$U_1(s) \approx U_m(s) \cdot s\tau$$

$$\boxed{u_1(t) = \tau \cdot \frac{du_m(t)}{dt}}$$

8.5 **Nastavak**

$$u_{FM}(t) = U_0 \cdot \cos\left(\omega_0 t + K_\omega \cdot \int_0^t u_1(t)\,dt\right) =$$

$$= U_0 \cdot \cos\left(\omega_0 t + K_\omega \cdot \tau \cdot u_m(t)\right) =$$

$$= U_0 \cdot \cos\left(\omega_0 t + \underbrace{K_\omega \cdot \tau}_{K_{PM}} \cdot U_m \cdot \cos(\omega_m t)\right) =$$

$$\boxed{K_{PM} = K_\omega \cdot \tau = 2\pi \cdot K_f \cdot \tau = 0.034557519 \text{ rad/V}}$$

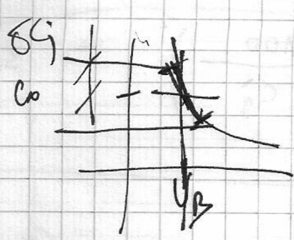
$$m = K_{PM} \cdot U_m = 0.10367$$

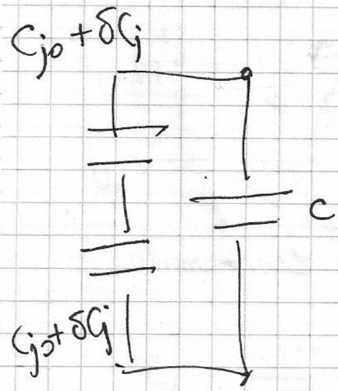
CARSONOVO PRAVILO:

$$\boxed{B = 2 \cdot f_M \cdot (m+1) = 2 \cdot 9100 \cdot 1.10367 = 20086.84 \approx 20 \text{ kHz}}$$

5. iz DZ1 (8.6) - Molnarov način

$$C_{uk}(t) = C_0 + \frac{\delta C_j}{2}$$





$$C_{uk} = C + \frac{C_{j0} + \delta C_j}{2}$$

$$= \underset{\substack{\downarrow \\ 1\,\mathrm{MHz}}}{C_0} + \frac{\delta C_j}{2}$$

$$\boxed{\delta C_j \ll C_0}$$

$$\omega_i = \frac{1}{\sqrt{L C_{uk}}} = \frac{1}{\sqrt{\underbrace{L\left(C_0 + \frac{\delta C_j}{2}\right)}_{L C_0\left(1 + \frac{\delta C_j}{2 C_0}\right)}}} = \frac{\omega_0}{\sqrt{1 + \frac{\delta C_j}{2 C_0}}}$$

$$\frac{1}{\sqrt{1+x}} \underset{x \ll}{=} 1 - \frac{x}{2}$$

$$= \omega_0 \left(1 - \frac{\delta C_j}{4 C_0}\right)$$

$$\omega_i = \omega_0 - \underbrace{\frac{\omega_0 \delta C_j}{4 C_0}}_{\delta \omega_i}$$

8.6 - Molnarov način - nastavak

$$\boxed{\delta\omega_i = -\frac{\omega_0\,\delta C_j}{4C_0}} \Rightarrow \boxed{\delta\omega_i \sim \delta C_j}$$

Želim $\delta\omega_i = K_\omega \cdot u_m(t) \Rightarrow \delta C_j \sim u_m(t)$

$$\boxed{K_\omega \cdot u_m(t) = -\frac{\omega_0\,\delta C_j}{4C_0}}$$

$$\delta C_j = \left.\frac{dC_j}{du}\right|_{U_B} \cdot u_m(t)$$

$$C_j = \frac{100}{\sqrt{u}} \Rightarrow 100 \cdot u^{-\frac{1}{2}} \Rightarrow \frac{-50}{\sqrt{U_B^3}}\ \mathrm{pF/V}$$

$$\| \quad 3{,}5$$

$$K_\omega = -\frac{\omega_0\,\delta C_j}{4C_0\,u_m(t)} = -\frac{\omega_0 \cdot \left.\frac{dC_j}{du}\right|_{U_B} \cdot \cancel{u_m(t)}}{4C_0 \cdot \cancel{u_m(t)}}$$

$$K_f = \cancel{-} f_0 \frac{\frac{\cancel{-}50}{\sqrt{U_B^3}}}{4C_0} = 15\,\mathrm{kHz/V}$$

8.6 - Molnarov način - nastavak

$$m = \frac{\Delta f}{f_m} = \frac{K_f U_m}{f_m}$$

$$U_m = \frac{m f_m}{K_f} = \frac{0{,}09 \cdot 10k}{15k} = 60\,mV$$

8.7

$$\frac{\Delta f}{\Delta U} = \frac{10\ kHz}{V}$$

$f_{mc} = 8\ kHz$

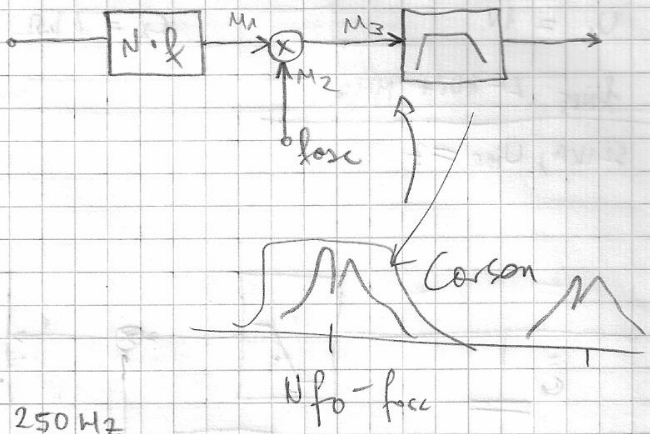
$U_m = \pm 25\ mV$

$f_{nosioc} = 0{,}5\ MHz$

$f_{izl} = 90\ MHz$

$\Delta f_{izl} = 80\ kHz$

$N,\ f_{osc},\ f_1, f_2 = ?$



$\Delta f_{mc} = 0.025V \cdot 10\,kHz/V = 250\ Hz$

$$\boxed{N = \frac{\Delta f_{izl}}{\Delta f_{mc}} = 320}$$

$$\boxed{f'_{nosioc} = f_{nosioc} \cdot N = 160\ MHz}$$

$f_{izl} = f'_{nosioc} - f_{osc} = 90 \cdot 10^6\ Hz$

$$\boxed{f_{osc} = f'_{nosioc} - 90 \cdot 10^6 = 70\ MHz}$$

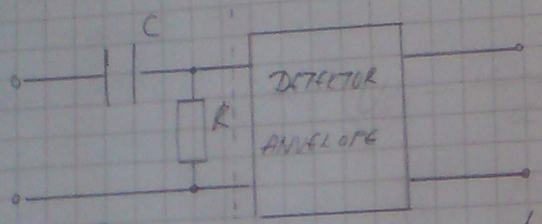
PARAMETRI FILTRA:

$f_c = f_{izl} = 90\ MHz$

Verzija 1:

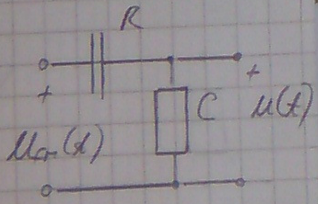
8.8

6. $u_{FM}(t) = U_0 \cos\left\{\omega_0 t + k_\omega \int_0^t u_m(t)\,dt\right\}$



- na frekvencijama ω: $R << \frac{1}{\omega C}$
- detektor ne utječe na rad RC člana

SHEMA



$$H(s) = \frac{R}{R+\frac{1}{sC}} = \frac{sRC}{sRC+1} \xrightarrow{s=j\omega} \frac{j\omega RC}{j\omega RC+1} = \left[\begin{matrix}R << \frac{1}{\omega C}\\ \omega RC << 1\end{matrix}\right] \approx j\omega RC$$

$$\Rightarrow H(s) = sRC \Rightarrow u(t) = RC\frac{du_{FM}(t)}{dt}$$

$$u(t) = RC\frac{du_{FM}(t)}{dt}$$

$$= -RCU_0 \sin(\omega \quad)\cdot\left[\omega_0 + k_\omega \cdot u_m(t)\right]$$

$=$ (nakon prolaska kroz detektor anvelope)

$$= RC(\omega_0 + k_\omega u_m(t))$$

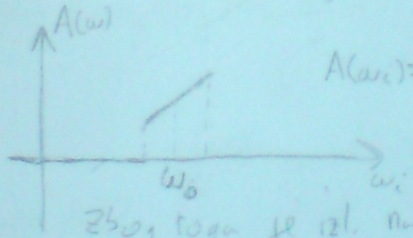
$$= RC\omega_0 + RC k_\omega \cdot u_m(t)_{//}$$

Verzija 2, loše slikana:

8.8 loše slikani

6.

$R \ll Z_c$ Pa je karakteristika u tom frekv. području približno linearno rastuća s koeficijentom RC



$A(\omega_i) = RC \cdot \omega_i$

Zbog toga je izl. napon na RC članu

$$u_{RC} = A(\omega_i) U_0 \cos\left(\int_0^t \omega_i \, dt\right)$$

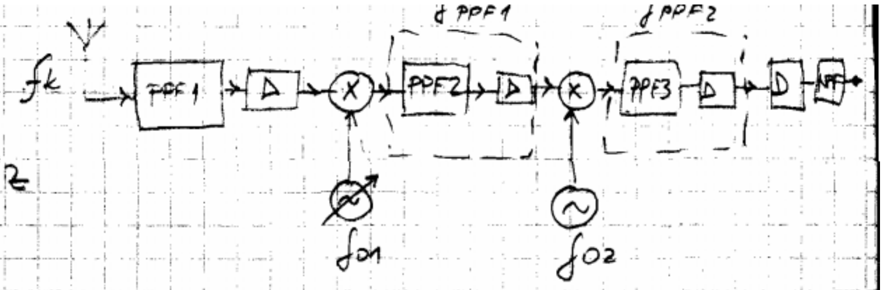
$$\omega_i = \omega_0 + K_f u_m(t)$$

tada je $u_{RC} = U_0 \left[RC(\omega_0 + K_f u_m(t))\right] \cdot \cos\left(\int_0^t [\omega_0 + k_\omega u_m(t)] \, dt\right)$

$$u_{RC} = U_0 \left[RC\omega_0 + RCk_f u_m\right] \cos\left(\omega_0 t + k_\omega \int_0^t u_m(t) \, dt\right)$$

u_{RC} je zapravo amplitudno modulirani nosilac promjenjive frekvencije. Detektor

Zadaci OSK



9.1.

$f_k = 42\,MHz - 43\,MHz$

$B = 10\,kHz$

a) $f_{PPF2} = 10.7\,MHz$

$f_{PPF3} = 455\,kHz$

$f_{o1} = f_k - f_{PPF1} = \{33.1,\ 31.31,\ 31.32,\ 31.33,\ \dots\ 32.3\}\,MHz$

$f_{o2} = f_{PPF2} - f_{PPF3} = 10.245\,MHz$

b) $f'_{1k} = f_{o1} - f_{PPF2}$

zrcalna komponenta koja je najbliža željenom frekv. području:

$f_{z1} = \max(f'_{1k}) = \max(f_{1k}) - 2 f_{PPF2} = 43 - 2 \cdot 10.7 = 21.6$

$f_{PPF1} = \sqrt{\min(f_k) \cdot \max(f_k)} = \sqrt{42\,MHz \cdot 43\,MHz} = 42.497\,MHz$

$B_{PPF1} = 1\,MHz$

Butterworth

$$|H_{PPF1}(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega^2 - \omega^2_{PPF1}}{B_{PPF1}\,\omega}\right)^{2N}}}$$

$$20 \log_{10} |H_{PPF1}(\omega_{z1})| \leq -80$$

$$1 + \left(\frac{\omega_{z1}^2 - \omega^2_{PPF1}}{B_{PPF1}\,\omega_{z1}}\right)^{2N} \geq 10^8$$

$$N \geq \frac{4}{\log_{10}\left|\frac{f_{z1}^2 - f_{PPF1}^2}{B_{PPF1} f_{z1}}\right|} = \frac{4}{1.79246} = 2.231 \Rightarrow 3$$

9.1. nastavak

c) $N=3$

$$\frac{f_{PPF1}}{B_{PPF1}} = \frac{f_{PPF2}}{B_{PPF2}} \Rightarrow B_{PPF2} = \frac{f_{PPF2}}{f_{PPF1}} B_{PPF1} = \frac{10.7}{42.497} \cdot 1\,MHz = 0.2517\,MHz$$

$$|H_{PPF1}(\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega_{z}^2-\omega_{PPF2}^2}{B_{PPF1}\,\omega_{z1}}\right)^{2N}}} =$$

$$f_{z2} = 10.7 - 2 \cdot 0.455 = 9.79\,MHz$$

$$20 \cdot \log_{10} \frac{1}{\sqrt{1+\left(\frac{9.79^2-10.7^2}{0.2517 \cdot 9.79}\right)^6}} = -10 \log_{10}(1+187717.456) =$$

$$= -52.735\ dB$$

d)

$$f_{z3} = (455+10)\,kHz = 445\,kHz$$

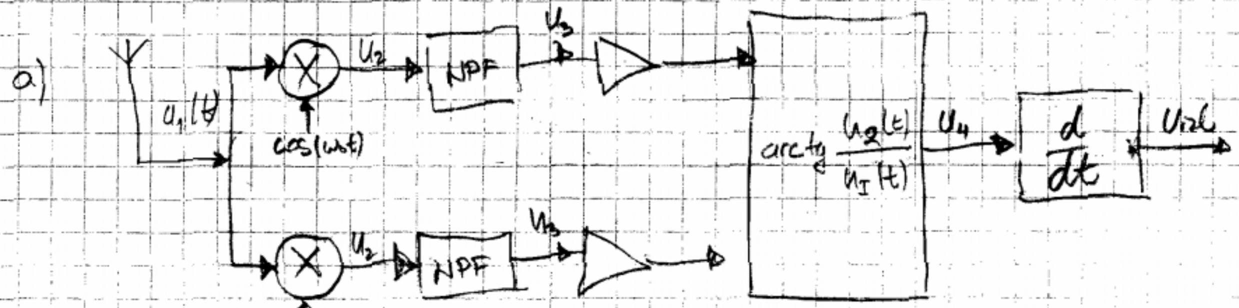
$$f_{PPF3} = 455\,kHz$$
$$B_{PPF3} = 10\,kHz$$

$$N = \frac{\frac{-30}{2 \cdot (-10)}}{\log_{10}\left|\frac{f_{z3}^2 - f_{PPF3}^2}{B_{PPF3} \cdot f_{z3}}\right|} = \frac{1.5}{0.3058} = 4.90384 = \underline{\underline{5}}$$

9.2 f_t 2MHz do 49MHz

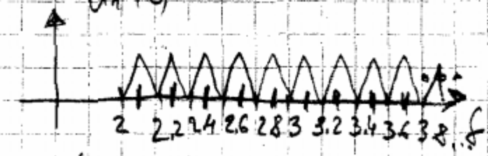
17MHz $B_t = 200kHz$

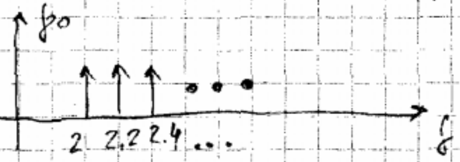
a)

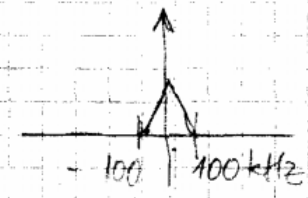


isto s druge ←

b)







$$f_0 = \{2, 2.2, \dots 48.8, 49\}$$

$$f_{NPF} = 100kHz$$

c) $u_{FM}(t) = u_{PAM}(t)\cos\left[\omega_0 t + K_\omega \int_0^t u_m(t)\,dt\right]$

$$u_2 = u_{FM}\cdot u_0 = u_{PAM}(t)\cos\left[\omega_0 t + K_\omega \int_0^t u_m(t)\,dt\right]\cdot e^{j\omega_0 t}$$

$$= u_{PAM}(t)\cdot\left[\cos\left(\omega_0 t + K_\omega \int_0^t u_m(t)\,dt\right)\cdot\cos(\omega_0 t) + j\cos\left(\omega_0 t + K_\omega \int_0^t u_m(t)\,dt\right)\sin(\omega_0 t)\right]$$

$$= u_{PAM}(t)\left[\frac{1}{2}\left(\cos\left(K_\omega \int_0^t u_m(t)\,dt\right) + \cos\left[2\omega_0 t + K_\omega \int_0^t u_m(t)\,dt\right]\right) + j\frac{1}{2}\left(\sin\left(K_\omega \int_0^t u_m(t)\,dt\right) + \sin\left(2\omega_0 t + K_\omega \int_0^t u_m(t)\,dt\right)\right)\right]$$

uklanja NPF

$$u_3 = \left|\begin{array}{c}\text{poslije}\\ \text{NPF}\end{array}\right| = u_{PAM}(t)\,\frac{1}{2}\left(\underbrace{\cos\left(K_\omega \int_0^t u_m(t)\,dt\right)}_{u_I(t)} + j\underbrace{\sin\left(K_\omega \int_0^t u_m(t)\,dt\right)}_{u_Q(t)}\right)$$

$$u_4 = \text{arctg}\,\frac{u_Q(t)}{u_I(t)} = \text{arctg}\left(\text{tg}\left(K_\omega \int_0^t u_m(t)\,dt\right)\right) = K_\omega \int_0^t u_m(t)\,dt$$

$$u_{izl} = \frac{du_4(t)}{dt} = \frac{d}{dt}\left(K_\omega \int_0^t u_m(t)\,dt\right) = K_\omega u_m(t)$$

(10.1.) PODUZORKOVANJE

$f_{s\,max} = 70\,MHz$

$f_g = 500\,MHz$

$f_1 = 300\,MHz$

$f_2 = 320\,MHz$

$B = 20\,MHz$

$f_s,\ f_{AA1},\ f_{AA2},$

$$f_c = \frac{f_1+f_2}{2} = 310\ MHz$$

- ako je $\frac{f_s}{2}$ max. frekv., centralna nakon „transponiranja" je na $\frac{f_s}{4}$

$$f_c \rightarrow \frac{f_s}{4} \quad \Rightarrow \quad f_c - k\cdot f_s = \frac{f_s}{4}$$

$$k = \frac{f_c - \frac{f_s}{4}}{f_s} = \frac{f_c}{f_s} - \frac{1}{4}$$

$$f_s = \frac{f_c}{\frac{1}{4}+k} = \frac{4}{1+4k}\cdot f_c$$

$$k_{min} = \left\lceil \frac{f_c}{f_{s\,max}} - \frac{1}{4} \right\rceil = \lceil 4.17857 \rceil = 5$$

$$\boxed{f_s = \frac{4}{4k_{min}+1}\cdot f_c = 59.048\ MHz}$$

$B_{pass} = f_2 - f_1 = 20\ MHz$

$B_{tran} = f_1 - k\cdot f_s = 300 - 5\cdot 70 = 4.76\ MHz$

- područje propuštanja: $[300\,MHz, 320\,MHz]$
- područje gušenja: $[f_1 - B_{tran}, f_1] = [295.24\ MHz, 300\,MHz]$

$\cup\ [f_2, f_2 + B_{tran}] = [320\,MHz, 324.76\ MHz]$

M.1.

$f_{ul} = 200\,\text{MHz}$

$f_s = 48\,\text{MHz}$

$t_{aRMS} = 0.1\,\text{ps}$

$SNR \geq 70\,\text{dB}$

---

t_{jRMS}, $\frac{t_{jRMS}}{T_s} = ?$

t_{jRMS} → APSOLUTNO PODRHTAVANJE

$\frac{t_{jRMS}}{T_s}$ → RELATIVNO PODRHTAVANJE

$$SNR_{sin} = -20 \log_{10} \sqrt{\left(2\sqrt{2}\,\pi f\, t_{jRMS}\right)^2 +}$$

$$SNR = -20 \log_{10} \left(2\sqrt{2}\,\pi f\, t_{jRMS}\right)$$

$$-20 \log_{10} \left(2\sqrt{2}\,\pi \cdot f \cdot t_{jRMS}\right) \leq 70$$

$$2\sqrt{2}\,\pi \cdot f \cdot t_{jRMS} \leq 3162.27766$$

$$f_{ul}\, t_{jRMS} \leq 3.558812717 \cdot 10^{-5}$$

$$t_{jRMS} = \sqrt{t_{aRMS}^2 + t_{cRMS}^2}$$

$$t_{aRMS}^2 + t_{cRMS}^2 \leq \left(\frac{3.558812717 \cdot 10^{-5}}{f_{ul}}\right)^2$$

$$t_{jRMS} \leq 1.779 \cdot 10^{-13}$$

$$t_{aRMS}^2 + t_{cRMS}^2 \leq 3.166287 \cdot 10^{-26}$$

$$\boxed{t_{cRMS} \leq 0.147183\,\text{ps}}$$ => APSOLUTNO

$$\boxed{\frac{t_{cRMS}}{T_s} = t_{cRMS} \cdot f_s \leq 7.06479\,\mu\text{s}}$$ ⇒ RELATIVNO

(11.2.) $N = 12$

$U_m = \pm 1.7\,V$

$f_s = 60\,MHz$

$f_g = 500\,MHz$

$U_{min} = 3.5\ mV/\sqrt{Hz}$

$P_m,\ U_{minLSB} = ?$

– broj kanala u frekv. području (širine $\frac{f_s}{2}$):

$$N_a = \frac{f_g}{\frac{f_s}{2}} = 16.67$$ ⇒ NE ZAOKRUŽUJE SE NA VEĆE ILI MANJE – OVAKO ISPADA TOČNO RJEŠENJE

– za 1 kanal:

$$P_{m1} = U_{min}^2 \cdot \frac{f_s}{2} = 3.675 \cdot 10^{-10}\ W$$

– sve ukupno:

$$\boxed{P_m = N_a \cdot P_{m1} = 16.67 \cdot 3.675 \cdot 10^{-10} = 6.125\ mW}$$

$$U_m = \sqrt{P_m} = 7.826 \cdot 10^{-5}\ V$$

$$U_{LSB} = \frac{U_m}{2^N} = 3.6621 \cdot 10^{-4}\ V$$

$$\boxed{U_{m,LSB} = \frac{U_m}{U_{LSB}} = 0.2137\ LSB}$$

11.3.

$f_{s_{max}} = 50\,MHz$

$N = 12$

$U_{m,LSB\,RMS} = 0.2\,LSB$

$\varepsilon = 0.5\,LSB$

$t_{a_{RMS}} = 1.2\,ps$

$t_{c_{RMS}} = 0.8\,ps$

$f_{ul} = 20\,MHz$

$SNR_{sin},\ SNR_{square} = ?$

$$t_{j_{RMS}} = \sqrt{t_{a_{RMS}}^2 + t_{c_{RMS}}^2} = 1.44222051\,ps$$

$$SNR_{sin} = -20\log_{10}\sqrt{\left(2\sqrt{2}\pi f \cdot t_{j_{RMS}}\right)^2 + \left(\frac{\sqrt{2}\cdot(1+\varepsilon)}{2^N\sqrt{3}}\right)^2 + \left(\frac{\sqrt{2}\,U_{m,N,LSB}}{2^N}\right)^2}$$

$$= -20\log_{10} 3.998342477\cdot 10^{-4} =$$

$$\boxed{SNR_{sin} = 67.9624\ dB}$$

$$SNR_{square} = -20\log_{10}\sqrt{\left(2\pi f t_{j_{RMS}}\right)^2 + \left(\frac{1+\varepsilon}{2^N}\right)^2 + \left(\frac{U_{m,N,LSB}}{2^N}\right)^2}$$

$$= -20\log_{10} 4.115102423\cdot 10^{-4}$$

$$\boxed{SNR_{square} = 67.7124\ dB}$$

✓

11.4.

$N = 14$

$SNR_{ADsin} = 72\ dB$

$U_m = 1.5\ V$

$f_s = 100\ MHz$

$B_{kanala} = 3\ kHz$

$U_{ueRMS}, N_1 = ?$

a) $SNR_{min} = 10\ dB$

$U_{ueRMS} = ?$

$$N_a = \frac{\frac{f_s}{2}}{B_{kanala}} = 16.67 \cdot 10^3$$

$$PG = 10 \log_{10} N_a = 42.2184875\ dB$$

$$SNR = SNR_{ADsin} + PG = 117.2288\ dB$$

$$SNR = -20 \log \frac{\sqrt{2} \cdot U_{MIN}}{U_m} \Rightarrow \boxed{U_{MIN} = 2.0637\ \mu V}$$

$$SNR_{min} = -20 \log \frac{\sqrt{2}\, U_{MIN}}{U_{m\,min}} = -20 \log \frac{U_{MIN}}{U_s} \Rightarrow$$

$$\Rightarrow U_s = U_{MIN} \cdot 10^{\frac{SNR_{min}}{20}} =$$

$$\boxed{U_s = 6.526\ \mu V}$$

(4.2) ASK

$A_1 = 1V$

$A_0 = -1V$

$E[n] = 0V$

$\sigma_n^2 = 0.15 V^2$

$p_1 = 0.28 \Rightarrow p_0 = 0.72$

$p_e = ?$

$$p_0 = f_{y_0}(y=\lambda) = \frac{1}{\sqrt{2\pi\sigma_y^2}} \cdot e^{-\frac{(\lambda+A_0)^2}{2\sigma_y^2}}$$

$$p_1 = f_{y_1}(y=\lambda) = \frac{1}{\sqrt{2\cdot\pi\cdot\sigma_y^2}} \cdot e^{-\frac{(\lambda+A_1)^2}{2\sigma_y^2}}$$

$$\frac{p_1}{p_0} = \frac{e^{-\frac{(\lambda+A_1)^2}{2\sigma_y^2}}}{e^{-\frac{(\lambda+A_0)^2}{2\sigma_y^2}}} \Rightarrow$$

$$\Rightarrow \frac{p_1}{p_0} = e^{-\frac{(\lambda+A_1)^2}{2\sigma_y^2} + \frac{(\lambda+A_0)^2}{2\sigma_y^2}} \Big/ \ln$$

$$\frac{-(\lambda+A_1)^2 + (\lambda+A_0)^2}{2\sigma_y^2} = \ln\frac{p_1}{p_0}$$

$$-\cancel{\lambda^2} - 2A_1\lambda - A_1^2 + \cancel{\lambda^2} + 2A_0\lambda + A_0^2 = 2\sigma_y^2 \cdot \ln\frac{p_1}{p_0}$$

$$\lambda(2A_0 - 2A_1) + \underbrace{A_0^2 + A_1^2}_{(A_1+A_0)(A_1-A_0)} = 2\sigma_y^2 \cdot \ln\frac{p_1}{p_0}$$

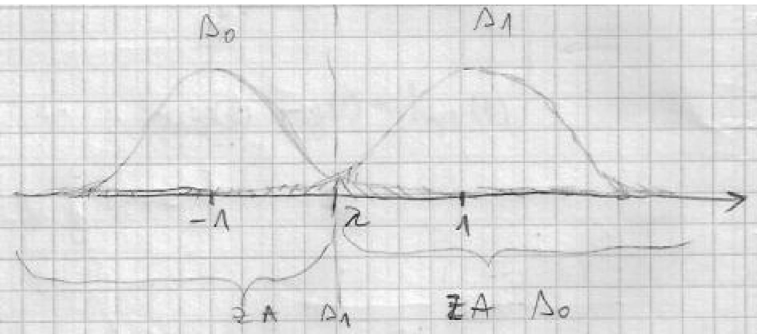
$$\lambda = \frac{+A_1^2 - A_0^2}{2\cdot(A_1 - A_0)} + \frac{\cancel{2}\sigma_y^2}{\cancel{2}\cdot(A_0 - A_1)} \ln\frac{p_1}{p_0}$$

$$\lambda = \frac{A_1 + A_0}{2} + \frac{\sigma_y^2}{A_0 - A_1} \cdot \ln\frac{p_1}{p_0}$$

$$\boxed{\lambda = 0.07083462066\ V}$$

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \cdot \int_x^{\infty} e^{-x^2} dx$$

14.2 NASTAVAK



$$P_{e0} = \int_{\lambda}^{\infty} \frac{1}{\sqrt{2\pi\sigma_y^2}} \cdot e^{-\frac{(y+A)^2}{2\sigma_y^2}} dy = \frac{1}{2}\,\mathrm{erfc}\left(\frac{\lambda + A}{\sqrt{2\sigma^2}}\right) = \left| A = -A_0 \right| = \frac{1}{2}\,\mathrm{erfc}\left(\frac{\lambda - A_0}{\sqrt{2\sigma^2}}\right)$$

$$P_{e1} = \int_{-\infty}^{\lambda} \frac{1}{\sqrt{2\pi\sigma_y^2}} \cdot e^{-\frac{(y+A)^2}{2\sigma_y^2}} dy = \left| \begin{matrix} x = \frac{y+A}{\sqrt{2\sigma_y^2}} & x(-\infty) = -\infty \\ dx = \frac{1}{\sqrt{2\sigma_y^2}} dy & x(\lambda) = \frac{\lambda + A}{\sqrt{2\sigma_y^2}} \end{matrix} \right) = \frac{1}{2} \int_{-\frac{\lambda+A}{\sqrt{2\sigma_y^2}}}^{\infty} e^{-(-x)^2} dx$$

$$= \frac{1}{2}\,\mathrm{erfc}\left(-\frac{\lambda + A}{\sqrt{2\sigma_y^2}}\right) = \left| A = -A_1 \right| = \frac{1}{2}\,\mathrm{erfc}\left(-\frac{\lambda - A_1}{\sqrt{2\sigma_y^2}}\right)$$

$$\boxed{P_e = P_{e0} \cdot p_0 + P_{e1} \cdot p_1 = 0.0043509480584}$$

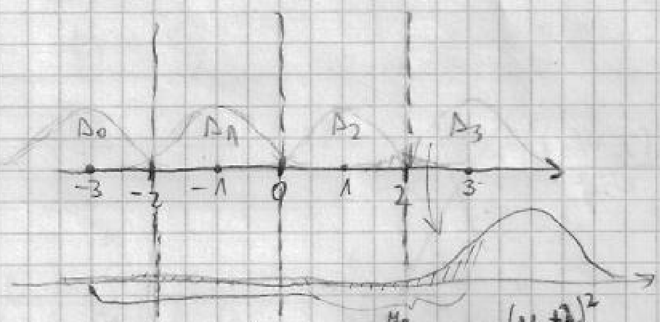
14.3. 4-ASK $\quad P_{s1}=P_{s2}=P_{s3}=P_{s4}$

$-3V,\ -1V,\ 1V,\ 3V$

$E=0$

$\sigma^2 = 0{,}07\,V^2$

$P_e = ?$



$$p(e|A_3) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \int_{-\infty}^{y_0} e^{-\frac{(y+\lambda)^2}{2\sigma^2}}\, dy = \left| \begin{array}{l} \alpha = \frac{y+\lambda}{\sqrt{2\sigma^2}} \\ d\alpha = \frac{1}{\sqrt{2\sigma^2}}\,dy \end{array} \right. \quad \begin{array}{l} \alpha(-\infty) = -\infty \\ \alpha(2) = \frac{y_0+\lambda}{\sqrt{2\sigma^2}} \end{array}$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \int_{-\infty}^{\frac{y_0+\lambda}{\sqrt{2\sigma^2}}} \sqrt{2\sigma^2}\, e^{-\alpha^2}\, d\alpha = \frac{1}{\sqrt{\pi}} \cdot \int_{-\left(\frac{y_0+\lambda}{\sqrt{2\sigma^2}}\right)}^{\infty} e^{-(-\alpha)^2}\, d\alpha$$

$$= \frac{1}{2}\, \mathrm{erfc}\left(-\left(\frac{y_0+\lambda}{\sqrt{2\sigma^2}}\right)\right)$$

MORA BITI NEGATIVAN PREDZNAK, MAKAR JE ZA A_3 (valjda zbog pomaka po X-osi u desno)

$$\boxed{p(e|A_3) = \left(\begin{array}{l} y_0 = 2 \\ \lambda = -3 \end{array}\right) = \frac{1}{2}\, \mathrm{erfc}\left(\frac{1}{\sqrt{2\sigma^2}}\right)}$$

$p(e|A_0)$ je simetrično sa $p(e|A_3)$ $\Rightarrow$

$$\Rightarrow \boxed{p(e|A_0) = \frac{1}{2}\, \mathrm{erfc}\left(\frac{1}{\sqrt{2\sigma^2}}\right)}$$

$$p(e|A_2) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{y_0} e^{-\frac{(y+\lambda)^2}{2\sigma^2}}\, dy + \frac{1}{\sqrt{2\pi\sigma^2}} \int_{y_1}^{\infty} e^{-\frac{(y+\lambda)^2}{2\sigma^2}}\, dy$$

$$= \frac{1}{2}\, \mathrm{erfc}\left(-\frac{y_0+\lambda}{\sqrt{2\sigma^2}}\right) + \frac{1}{2}\, \mathrm{erfc}\left(\frac{y_1+\lambda}{\sqrt{2\sigma^2}}\right) = \left| \begin{array}{l} y_0 = 0 \\ y_1 = 2 \\ \lambda = -1 \end{array} \right|$$

$$= \frac{1}{2}\, \mathrm{erfc}\left(+\frac{1}{\sqrt{2\sigma^2}}\right) + \frac{1}{2}\, \mathrm{erfc}\left(\frac{1}{\sqrt{2\sigma^2}}\right) =$$

$$\boxed{p(e|A_2) = \mathrm{erfc}\left(\frac{1}{\sqrt{2\sigma^2}}\right)}$$

NASTAVAK

$p(e|\Delta_1)$ je simetrično sa $p(e|\Delta_2)$ $\Rightarrow$

$$\Rightarrow \boxed{p(e|\Delta_1) = \text{erfc}\left(\frac{1}{\sqrt{2\sigma^2}}\right)}$$

$$P_e = p(e|\Delta_0)\cdot p_0 + p(e|\Delta_1)\cdot p_1 + p(e|\Delta_2)\cdot p_2 + p(e|\Delta_3)\cdot p_3 =$$

$$\left.\begin{aligned} p_0 = p_1 = p_2 = p_3 \\ p_0 + p_1 + p_2 + p_3 = 1 \end{aligned}\right\} p_0 = p_1 = p_2 = p_3 = \frac{1}{4}$$

$$P_e = \frac{1}{4}\cdot\left(p(e|\Delta_0) + p(e|\Delta_1) + p(e|\Delta_2) + p(e|\Delta_3)\right) =$$

$$\boxed{P_e = \frac{3}{4}\cdot \text{erfc}\left(\frac{1}{\sqrt{2\sigma^2}}\right)}$$

$$\boxed{P_e = 1.17789 \cdot 10^{-4}}$$