14.4. MASK modulacija s 4 amplitude

$(-3V, -1V, 1V, 3V)$

W

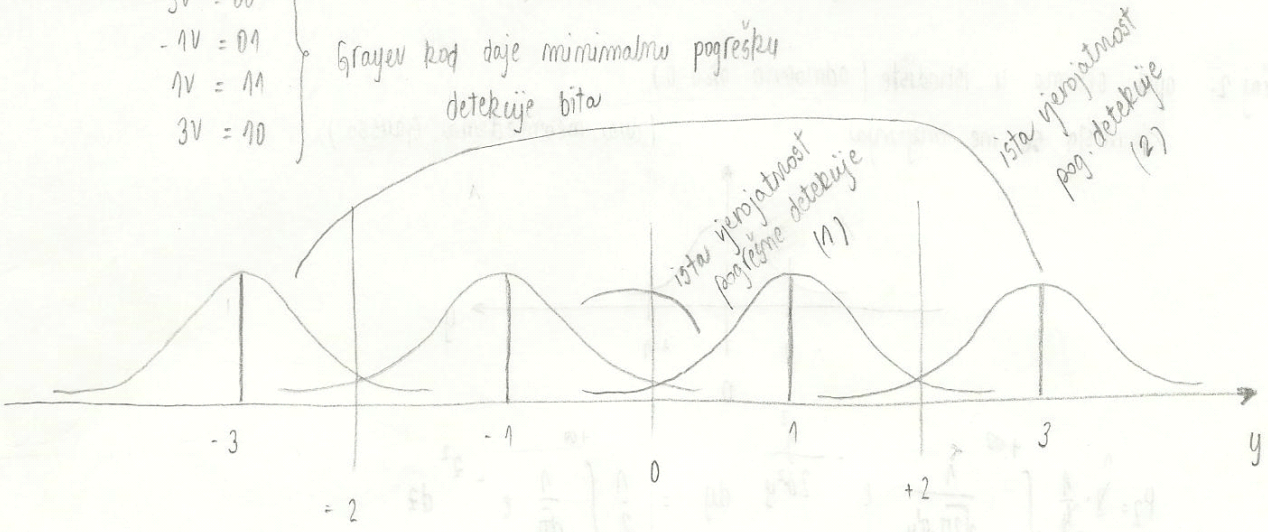
Gaussov šum srednje vrijednosti 0V i varijance $0.07\ V^2$

odrediti vjerojatnost pogrešne detekcije simbola u prijamniku

vjerojatnost pojava simbola su jednake

$-3V = 00$
$-1V = 01$
$1V = 11$
$3V = 10$

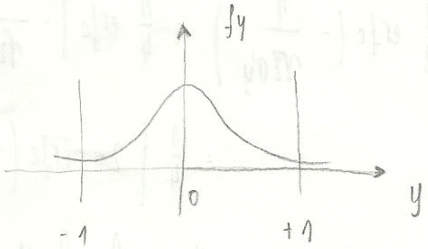
} Grayev kod daje minimalnu pogrešku detekcije bita



$P_{00} = P_{01} = P_{11} = P_{10} = \frac{1}{4}$

(zbroj vjerojatnosti mora biti 1)

slučaj 1. - možemo obje funkcije gustoće vjerojatnosti (PDF) preseliti u ishodište, a da se ništa ne promijeni (dva omeđena Gaussa)



- selimo u ishodište kako bi srednja vrijednost bila 0, pa je lakše računati integral, tj. pdf

$$P_1 = 2 \cdot \frac{1}{4} \int_{-1}^{1} \frac{1}{\sqrt{2\pi}\,\sigma_y} e^{-\frac{y^2}{2\sigma_y^2}} dy$$

Uvodimo supstituciju:

$$\frac{y}{\sqrt{2}\sigma_y} = z \Rightarrow dy = \sqrt{2}\,\sigma_y\, dz$$

nove granice integracije: $y = -1 \Rightarrow z = -\frac{1}{\sqrt{2}\sigma_y}$

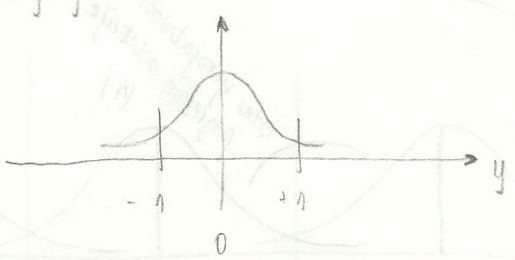
$y = +1 \Rightarrow z = \frac{1}{\sqrt{2}\sigma_y}$

$$P_1 = \frac{1}{2}\int_{-\frac{1}{\sqrt{2}\sigma_y}}^{\frac{1}{\sqrt{2}\sigma_y}} \frac{1}{\sqrt{\pi}} e^{-z^2}\,dz = \frac{1}{2}\cdot 2\int_{0}^{\frac{1}{\sqrt{2}\sigma_y}} \frac{1}{\sqrt{\pi}} e^{-z^2}\,dz$$

$$\frac{2}{\sqrt{\pi}}\int_{u}^{\infty} e^{-z^2}\,dz = \operatorname{erfc}(u)$$

$$P_1 = \frac{1}{2}\operatorname{erf}\left(\frac{1}{\sqrt{2}\sigma_y}\right) = \frac{1}{2}\left[1-\operatorname{erfc}\left(\frac{1}{\sqrt{2}\sigma_y}\right)\right] = \frac{1}{2} - \frac{1}{2}\operatorname{erfc}\left(\frac{1}{\sqrt{2}\sigma_y}\right)$$

Slučaj 2. opet gledamo u ishodište (odnosno oko 0)
i ništa se ne mijenja (dva neomeđena Gaussa)



$$P_2 = \frac{1}{2}\cdot\frac{1}{2}\int_{-1}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_y} e^{-\frac{y^2}{2\sigma_y^2}}\,dy = \frac{1}{2}\int_{-\frac{1}{\sqrt{2}\sigma_y}}^{+\infty} \frac{1}{\sqrt{\pi}} e^{-z^2}\,dz$$

nove granice

$$\left.\begin{aligned} y = -1 &\Rightarrow z = \frac{-1}{\sqrt{2}\sigma_y} \\ y = +\infty &\Rightarrow z = +\infty \end{aligned}\right\} \quad z = \frac{y}{\sqrt{2}\sigma_y}$$

$$P_2 = \frac{1}{2}\cdot\frac{1}{2}\operatorname{erfc}\left(-\frac{1}{\sqrt{2}\sigma_y}\right) = \frac{1}{4}\operatorname{erfc}\left(-\frac{1}{\sqrt{2}\sigma_y}\right)$$

$$= \frac{1}{4}\left[2-\operatorname{erfc}\left(\frac{1}{\sqrt{2}\sigma_y}\right)\right]$$

$$P_2 = \frac{1}{2} - \frac{1}{4}\operatorname{erfc}\left(\frac{1}{\sqrt{2}\sigma_y}\right)$$

$P = P_1 + P_2$

$$P = \frac{1}{2} - \frac{1}{2}\operatorname{erfc}\left(\frac{1}{\sqrt{2}\sigma_y}\right) + \frac{1}{2} - \frac{1}{4}\operatorname{erfc}\left(\frac{1}{\sqrt{2}\sigma_y}\right)$$

$$P = 1 - \frac{3}{4}\operatorname{erfc}\left(\frac{1}{\sqrt{2}\sigma_y}\right)$$

$$\boxed{P_e = 1 - P} = 1 - 1 + \frac{3}{4}\operatorname{erfc}\left(\frac{1}{\sqrt{2}\sigma_y}\right) = \frac{3}{4}\operatorname{erfc}\left(\frac{1}{\sqrt{2}\sigma_y}\right) = 1.98\cdot 10^{-4}$$

gdje je P vjerojatnost
da nije došlo do greške