ELSEVIER

Microelectronics Journal **28** (1997) 173–182

0026-2692/97/$17.00

PII:S0026-2692(96)00078-X

Research note

A review of SOI transistor models

Małgorzata Jurczak, Andrzej Jakubowski and Lidia Łukasiak

Institute of Microelectronics and Optoelectronics, Warsaw University of Technology, Koszykowa 75, 00–662 Warszawa, Poland. Tel: +48 22 625 73 29. *Fax:* +48 22 628 87 40

Several SOI MOSFET models based on 1-D solution of Poisson's equation are compared in terms of their accuracy. A brief discussion of the simplifying assumptions leads to a conclusion that the methods used for surface potential evaluation strongly influence the accuracy of a model. An analytical approximation for the front surface potential is presented and then introduced into the fully analytical Lim–Fossum model [1]. The accuracy of the modified model is better than that of the original Lim–Fossum model [1].

Notation

α	Variable related interaction between the front and the back gates (V^2/cm^2)
$\beta = kT/q$	Inverse of the thermal voltage
μ_{eff}	Effective carrier mobility (cm^2/Vs)
ϕ	Electrostatic potential (V)
ϕ_F	Fermi potential $\phi_F = kT/q \cdot \ln(N_A/n_i)$
ϕ_{Sb}	Back surface potential (V)
ϕ_{Sf}	Front surface potential (V)
ε_{ox}	Dielectric permittivity (F/cm)
ε_{Si}	Silicon permittivity (F/cm)
C_b	Depletion capacitance per unit area (F/cm^2)
C_{ob}	Back gate oxide capacitance per unit area (F/cm^2)
C_{of}	Front gate oxide capacitance per unit area (F/cm^2)
E	Electric field (V/cm)
E_{Sb}	Back surface electric field (V/cm)
E_{Sf}	Front surface electric field (V/cm)
I_D	Drain current (A)
k	Boltzmann's constant (CV/K)
L	Channel length (cm)
L_B	Extrinsic Debye length (cm)
L_i	Intrinsic Debye length (cm)
n_i	Intrinsic carrier concentration (cm^{-3})
q	Electron charge (C)
Q_b	Body depletion charge (C/cm^2)
Q_{nf}	Front channel charge density per unit area (C/cm^2)
Q_S	Total charge density in the silicon film (C/cm^2)
T	Temperature (K)
t_{of}	Front gate oxide thickness (cm)
t_{Si}	Silicon film thickness (cm)
V_{DS}	Drain–source voltage (V)
V_{FBb}	Back gate flat-band voltage (V)
V_{FBf}	Front gate flat-band voltage (V)
V_{Gb}	Back gate voltage (V)
V_{Gf}	Front gate voltage (V)
W	Channel width (cm)

1. Introduction

Continuous development of SOI technology requires new simulation tools to predict circuits and device behaviour before they

are fabricated. Modern SOI devices due to their small size and charge coupling between the front and back gate require two- and three-dimensional solution of the coupled semiconductor equations which can be solved only by numerical methods. Generally known simulators like MINIMOS or PISCES provide models for SOI devices, however, such simulators need a large amount of computer memory and long computation time, thus they are impractical in CAD.

Another group of models consists of those which use the gradual channel approximation, hence the solution of Poisson's equation can be reduced to a 1-D problem. In this paper we compare the 1-D models of n-channel fully depleted SOI transistors and analyse the influence of such effects as the dependence of the surface potential on the gate voltage on the modelling of I–V characteristics. Although the models describe long-channel transistors, they are useful as the basis for development of improved models for short-channel devices as well as for the extraction of device parameters. Moreover, the long-channel approximation is accurate as long as the channel length L is much higher than λ defined in [2] as:

$$\lambda = \sqrt{\frac{\varepsilon_{Si}}{\varepsilon_{ox}} t_{of} t_{Si}} \tag{1}$$

where t_{of} is the front gate oxide thickness and t_{Si} is the silicon layer thickness. Thus even submicrometre transistors can still be regarded as long-channel devices.

The aim of the review is to verify the validity of the existing long-channel models of an SOI MOSFET and to elucidate the influence of the simplifying assumptions on the accuracy of the resulting models.

2. Models

The models discussed in this paper are based on the following additional assumptions: uniform doping in the silicon film, negligible series resistance of source and drain, constant mobility of the carriers in the channel, negligible influence of the silicon substrate on the device parameters and independence of the effective interface charge of the gate voltage. It is also assumed that the semiconductor is not degenerate, thus the Maxwell–Boltzmann statistics are used. The current of the majority carriers is negligible. Models are based on the quasi-equilibrium approximation across the whole body of the device which assumes that the quasi-Fermi levels are constant across the layer.

In a further analysis an n-channel SOI transistor will be considered.

2.1 Ortiz-Conde model [3]

The most accurate among the models based on 1-D solution of Poisson's equation is the Ortiz-Conde model [3] analogous to the Pao–Sah model [4] for bulk MOSFET. The model is based only on gradual channel approximation and the quasi-equilibrium approximation. The drain current, including drift and diffusion components, can be expressed as:

$$I_D = \mu_{eff} \frac{W}{L} \int_0^{V_{DS}} Q_{nf} \, dV \tag{2}$$

where μ_{eff} is the effective carrier mobility, W, L are the width and length of the channel, respectively, Q_{nf} is the total inversion charge density inside the silicon film and V is the electron quasi-Fermi potential. The charge Q_{nf} is calculated as follows:

$$Q_{nf} = \int_{\phi_{Sb}}^{\phi_{Sf}} \frac{q n_i (e^{\beta(\phi - \phi_F - V)} - e^{-\phi_F})}{E} \, d\phi \tag{3}$$

where ϕ_{Sf} and ϕ_{Sb} are the front and back surface potentials, respectively, n_i is the intrinsic concentration, ϕ_F is the Fermi potential, ϕ is the electrostatic potential and E is the electric field obtained from the solution of Poisson's equation:

$$E = \sqrt{\alpha + G^2(\phi, \phi_F, V)} \tag{4}$$

where $G(\phi, \phi_F, V)$ is the Kingston function:

$$G^2(\phi, \phi_F, V) = \frac{1}{\beta^2 L_i^2}\left[e^{\beta\phi_F}(e^{-\beta\phi} + \beta\phi - 1) + e^{-\beta\phi_F}(e^{\beta(\phi - V)} - \beta\phi - e^{-\beta V})\right] \tag{5}$$

$L_i = (\varepsilon_S/2\beta q n_i)^{1/2}$ is the intrinsic Debye length, β is equal to q/kT and α is a variable that describes the interaction between the front and back gates and is defined as:

$$E_{Sf}^2 - G^2(\phi_{Sf}, \phi_F, V) = E_{Sb}^2 - G^2(\phi_{Sb}, \phi_F, V) = \alpha \tag{6}$$

E_{Sf}, E_{Sb} are the front and back electric fields, respectively. The front and back surface potentials and front and back electric fields E_{Sf} and E_{Sb} can be obtained from the numerical solution of the following three equations:

$$V_{Gf} - V_{FBf} = \phi_{Sf} + \frac{\varepsilon_S E_{Sf}}{C_{of}} \tag{7}$$

$$V_{Gb} - V_{FBb} = \phi_{Sb} - \frac{\varepsilon_S E_{Sb}}{C_{ob}} \tag{8}$$

$$t_{Si} = \int_{\phi_{Sb}}^{\phi_{Sf}} \frac{d\phi}{E} \tag{9}$$

where V_{Gf} and V_{Gb} are the front and back gate voltages, V_{FBf}, V_{FBb} are the front and back flat-band voltages and C_{of}, C_{ob} are the front and back gate capacitances, respectively. It is worth noticing that for the case in which the electric field does change the polarity across the silicon film, i.e. when both film surfaces are in strong inversion, the integrand in eq. (9) presents singularity. Therefore, it is convenient to divide the integration region in (9) into two parts:

$$t_{Si} = \int_{\phi_{Sb}}^{\phi_0} \frac{d\phi}{E} + \int_{\phi_0}^{\phi_{Sf}} \frac{d\phi}{E} \tag{10}$$

where ϕ_0 is the electrostatic potential for which the electric field vanishes. This potential can be obtained from (4).

The Ortiz-Conde model [3] takes into account both drift and diffusion currents and is valid for all bias conditions and operation modes including subthreshold and saturation regions. The only disadvantage of the model is long computation time due to numerical evaluation of the double-integral expression (2) and multiple iterative calculations of surface potentials. Therefore, many other simplified models have been proposed. These simplifications can be carried out in several ways:

(1) Transformation of eq. (2).

(2) Simplified estimation of the inversion layer charge density Q_{nf} (3).

(3) Simplification of the Kingston function (5).

(4) Approximation of the front and back surface potentials at the source and drain end of the channel.

(5) Elimination of one of the drain current components (drift or diffusion current).

In the next sections we will present some of these models and show the influence of the individual assumptions on the accuracy of modelling.

2.2 Pierret–Shields type model [6]

Following the idea of Pierret and Shields [6] for bulk devices Ortiz-Conde *et al.* simplified the rigorous model [3] by transforming the double-integral expression into an equivalent single-integral formula. The cumbersome and time

consuming integration along the channel has been eliminated. The front and back surface potentials have to be calculated only at the source and drain ends of the channel. Such simplification of the model [3] reduces the calculation time considerably without discernible loss of accuracy. Nevertheless, the model still has a numerical form and is impractical for circuit simulation.

2.3 Mallikarjun–Bhat model [7]

Further simplification of the Ortiz-Conde model [3] has been introduced by Mallikarjun and Bhat. Similarly to the Brews model [8] for the MOS bulk transistor they assumed that the inversion layer is an electrically charged plane of zero thickness (charge-sheet) placed at the oxide–semiconductor interface. Furthermore, they assumed full depletion throughout the silicon film. In that case the inversion layer charge can be estimated as a linear function of the surface potential:

$$Q_{nf} = C_{of}\left[V_{Gf} - V_{FBf} + \frac{Q_b}{2C_{of}} - \phi_{Sf} - \frac{C_b}{C_{of}}(\phi_{Sf} - \phi_{Sb})\right] \tag{11}$$

The depletion approximation results in a simplification of the Kingston function (5):

$$G^2_{(da)}(\phi) = \frac{2}{\beta^2 L_B^2}(\beta\phi - 1) \tag{12}$$

where $L_B = \sqrt{\varepsilon_S/\beta q N_A}$. This modification allows interaction parameter α to be expressed analytically:

$$\alpha_{(da)} = \frac{2}{\beta^2 L_B^2}\left[\left(\frac{\beta\phi_{Sf} - \beta\phi_{Sb} - C^2}{2C}\right)^2 - (\beta\phi_{Sb} - 1)\right] \tag{13}$$

where $C = t_{Si}/\sqrt{2}L_B$. As it is shown in [7], $\alpha_{(da)}$ is a good approximation of α. Therefore, eq. (6) can be rewritten as:

$$E^2_{Sf} - G^2(\phi_{Sf}, \phi_F, V) = E^2_{Sb} - G^2(\phi_{Sb}, \phi_F, V) = \alpha_{(da)} \tag{14}$$

The surface potentials can be computed from (4), (5) and (14). It is worth noticing that the approximation of α saves computation time, as it avoids the use of the integral eq. (9), without loss of accuracy.

In order to obtain the analytical expression for the drain current, Mallikarjun and Bhat considered specific cases of the back channel charge conditions: inversion, accumulation and depletion. As a result they obtained fully analytical expressions for drain current for every case.

The importance of the Mallikarjun–Bhat model [7] based on the charge-sheet model derives from its validity for all bias conditions and its relative simplicity since it does not contain any integral expression. However the calculation of the surface potentials still requires numerical operations.

2.4 Yang–Li model [9]

Following the Van de Wiele model [10] for a conventional MOS transistor, Yang and Li [9] made a simplification which consists of the approximation of the inversion layer charge Q_{nf} as the difference between the total charge in semiconductor film Q_S and the depletion charge:

$$Q_{nf} = Q_S - Q_D \tag{15}$$

where

$$Q_S = \lambda C_{of}\sqrt{\frac{1}{\beta}(e^{-\beta\phi_{Sf}} - e^{-\beta\phi_{Sb}} + e^{\beta(\phi_{Sb} - 2\phi_F - V)}) - \frac{1}{\beta}e^{\beta(\phi_{Sb} - 2\phi_F - V)} + \phi_{Sf} - \phi_{Sb}} \tag{16}$$

is obtained from the solution of Poisson's equation,

$$Q_D = -\lambda C_{of}\sqrt{\phi_{Sf} - \phi_{Sb}} \tag{17}$$

and $\lambda = \sqrt{2\varepsilon_S q N_A}/C_{of}$. Yang and Li assumed that $\exp(-2\phi_F) \approx 0$ which leads to simplification of the Kingston function in (16). The inversion layer charge is calculated in the same way as in the Mallikarjun–Bhat model [7].

The relationship between the front and back surface potentials can be achieved from the numerical solution of (15) and (11) after specifying the conditions at the back gate. For depletion at the back surface the back surface potential is given by:

$$\phi_{Sb} = \frac{C_{ob}}{C_{ob}+C_b}\left(V_{Gb} - V_{FBb} + \frac{C_b}{C_{ob}}\phi_{Sf} + \frac{Q_b}{2C_{ob}}\right) \tag{18}$$

while for accumulation ϕ_{Sb} is assumed to be zero. Similarly to the Van de Wiele model [10], the drain current is given by:

$$I_D = -\frac{W}{L}\mu_{eff}\int_{\phi_{Sf0}}^{\phi_{SfL}} Q_{nf}\frac{dV}{d\phi_{Sf}}d\phi_{Sf} \tag{19}$$

where $dV/d\phi_{Sf}$ can be derived from (11) and (15). The advantage of the transformation of (2) is that the whole function can be analytically integrated. Yang and Li noticed that their model can be further simplified into the charge-sheet model [7] presented above. However, contrary to the Mallikarjun–Bhat model [7], the Yang–Li model [9] is confined to specific charge conditions at the back gate (either accumulation or depletion).

Although expressions for drain current are very complicated and contain many parameters, numerical integration has been eliminated completely, which considerably reduces computation time. However, surface potentials still have to be solved numerically.

2.5 McKitterick–Caviglia model [11]

Additional simplification has been introduced by McKitterick and Caviglia [11]. They noticed that if the back surface is in neither the accumulation state nor in the inversion state and the silicon layer is lightly doped or thin enough compared to the buried oxide thickness, then, applying the charge-sheet approximation, the silicon layer can be replaced by a sheet of charge with zero thickness. Thus the potential is constant across the silicon film.

The McKitterick–Caviglia model [11] is based on drift and diffusion components so it is valid for subthreshold and saturation regions. Although the drain current expression is in analytical form, potential is still evaluated numerically.

2.6 Lim–Fossum model [1]

Further simplification can be made if the diffusion current is neglected. Such approximation leads to the boundary condition given by:

$$\phi_{Sf}(L) = \phi_{Sf}(0) + V_{DS} \tag{20}$$

which is incorrect in the weak inversion and saturation region. Therefore, models based only on the drift component are not valid either in the subthreshold or in the saturation regime. The value of drain current in saturation is determined from the condition $\delta I_d/\delta V_{DS} = 0$, however, there is no natural transition into the saturation.

Lim and Fossum assumed that in strong inversion the surface potential at the source is constant and equal to double the Fermi potential. As it will be shown later this assumption will have strong influence on the I–V characteristics of the SOI transistor.

The relationship between the front and back surface potentials is described as in (18). Similarly to the Yang–Li model [9] the Lim–Fossum model [1] is applicable only in the case of such bias conditions where there is either accumulation or depletion at the back gate.

The Lim–Fossum model [1] is fully analytical in form, therefore it is simple and does not require long computation time. The model may be useful for CAD.

2.7 Modification of the Lim–Fossum model

The assumption of the constant front surface potential leads to significant errors in current calculation. The actual dependence of surface potential on the gate voltages is shown in Fig. 1 (solid line). For the considered front gate oxide thickness (50 nm) the real value is about 150 mV higher than the assumed one. The discrepancies increase for thinner gate oxides.

The accuracy of modelling of I–V curves improves considerably when the dependence of the surface potential on gate voltages is introduced. In [13] Jurczak and Jakubowski proposed an analytical expression for the front surface potential as a function of gate voltages:

$$\phi_{Sf}(0) = 2\phi_F + \frac{1}{\beta}\ln\left|\frac{L_B^2\beta^2}{2}\left[\frac{C_{of}^2}{\varepsilon_S^2}(V_{Gf} - V_{FBf} - 2\phi_F)^2 - \alpha(\phi_{Sf}, \phi_{Sb})\right] - 2\phi_F + \frac{1}{\beta}\right| \quad (21)$$

where analytical approximation of α is taken from the charge-sheet model [7] and back

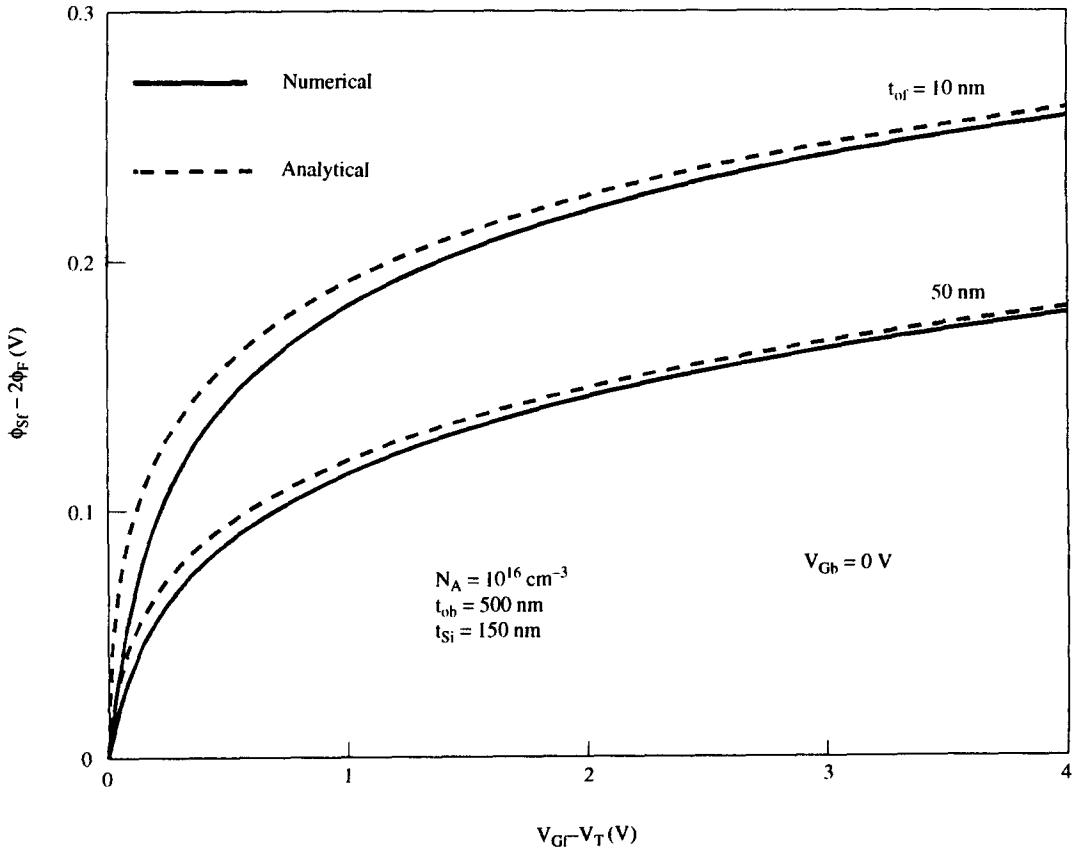

Fig. 1. Front surface potential at the source end of the channel as a function of front gate voltage.

surface potential is obtained as in the Yang–Li [9] or the Lim–Fossum model [1], i.e. from (18). In order to preserve the analytical form of the model the front surface potential in (13) and (18) is equal to $2\phi_F$. The analytical approximation of the front surface potential (21) (dotted line in Fig. 1) is in agreement with numerical calculations in strong inversion. The proposed formula has been introduced into the Lim–Fossum model [1]. The modified model [13] retains a fully analytical form, which guarantees short computation time. Hence the model is suitable for SOI circuits simulation.

3. Comparison of the models

In order to estimate the accuracy of the models described above they were compared to the Ortiz-Conde model [3] (regarded as the most accurate 1-D long-channel SOI MOSFET model). In the calculations we assumed that $V_{FBf} = V_{FBb} = 0\,V$. Figure 2 shows the relative error of the drain current in the above-threshold region according to different models with respect to the Ortiz-Conde model [3] calculated for zero bias at the back gate. The relative error is defined as $(I_D - I_{OC})/I_{OC} \cdot 100\%$, where I_{OC} is the drain current obtained from

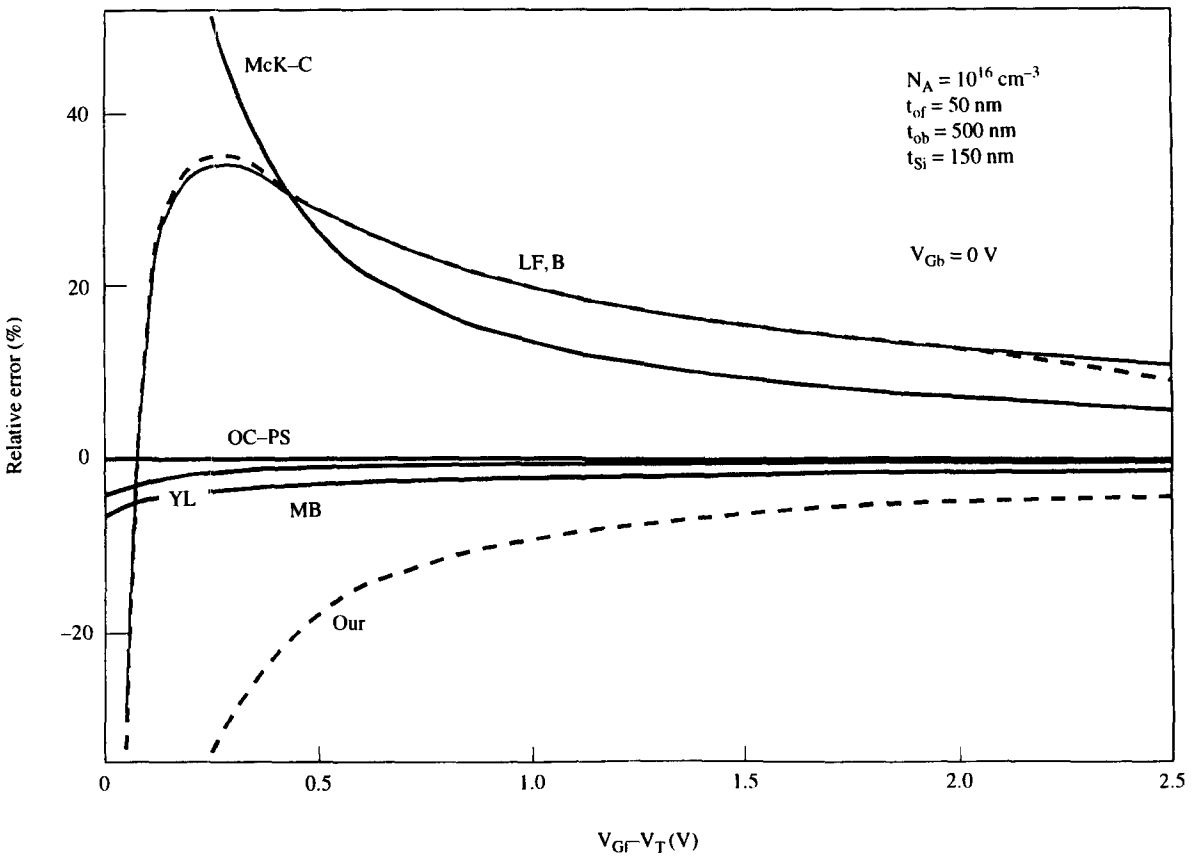

Fig. 2. Relative error of SOI transistor saturation current models with respect to the Ortiz-Conde model [3] vs $V_{GS} - V_T$ voltage for depletion at the back gate. LF: Lim–Fossum [1], B: Barth [12] YL: Yang–Li [9], OC-PS: Pierret–Shields' type [5], MB: Malikarjun–Bhat [7], McK–C: McKitterick–Caviglia [11], our: Jurczak–Jakubowski [13].

the Ortiz-Conde model [3]. As it can be seen the Pierret–Shield type model [5] cannot be distinguished from the reference model. The error of the charge-sheet model [7] and Yang–Li model [9] does not exceed 5% over the whole range of device operation. The error of both models increases around the threshold voltage. When a transistor operates in the subthreshold region or near threshold voltage the electron concentration in the channel is rather low and consequently, the accuracy of the inversion layer charge, calculated as the difference between the total charge and the depletion layer charge, is much worse. Both models also lose their accuracy for the case of accumulation at the back gate (Fig. 3) because the depletion approximation in silicon film is not valid for such conditions. The error increases more rapidly (than in the case of depletion) around the threshold voltage as the contribution of the accumulation charge at the back surface becomes significant and cannot be ignored in the analysis (it is in the Yang–Li model [9]). Although for depletion at the back interface the Yang–Li model [9] seems to be more accurate than the Mallikarjun–Bhat model [7], in accumulation the model is worse because of different approximation of the surface potentials (e.g. the back surface potential is equal to zero).

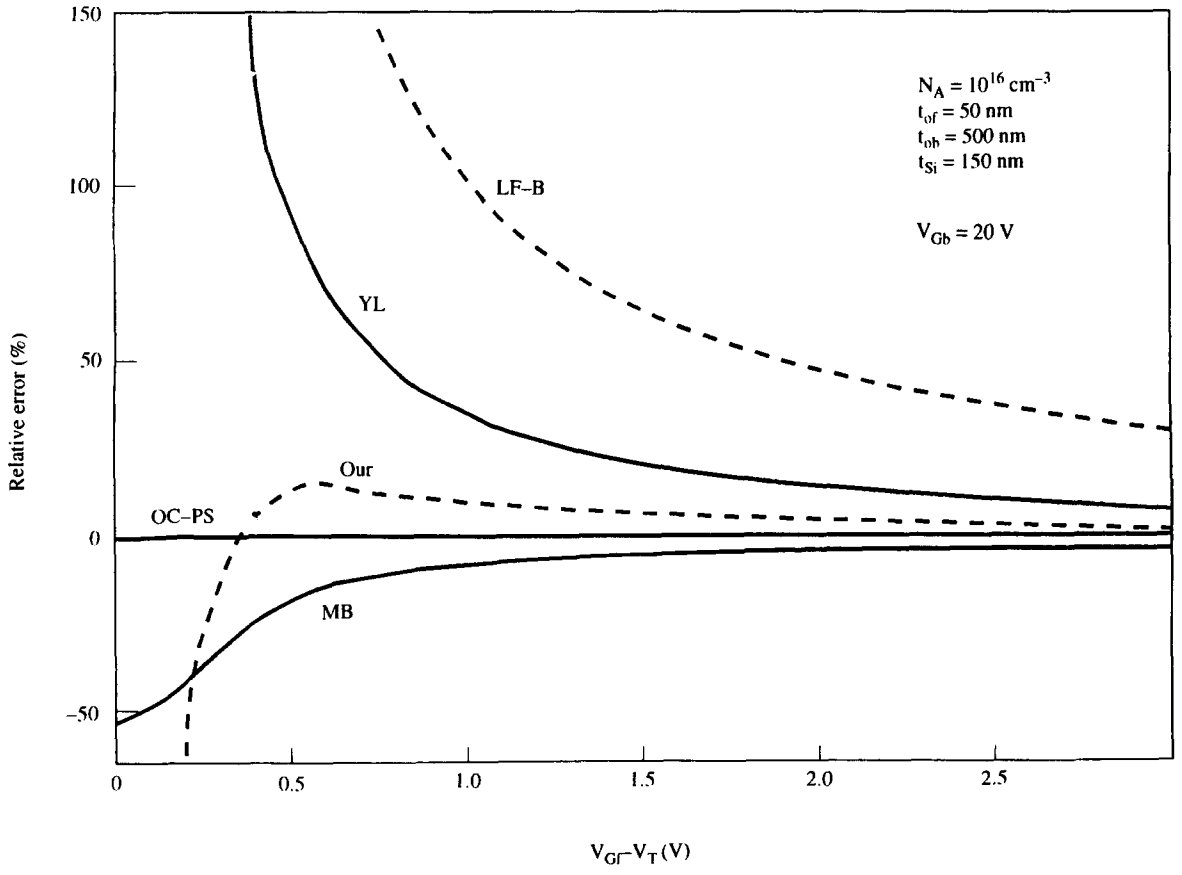

Fig. 3. Relative error of saturation current models with respect to the Ortiz-Conde model [3] vs $V_{GS} - V_T$ voltage for accumulation at the back gate. Symbols as in Fig. 2.

Figure 4 shows the relative error of the numerical models as a function of silicon layer thickness. The accuracy of the Pierret–Shields' type model [5] and Yang–Li model [9] is almost insensitive to silicon thickness. The accuracy of the Mallikarjun–Bhat model [7] aggravates with the silicon thickness reduction. For an ultra-thin film the charge-sheet approximation cannot be used because the inversion layer thickness is comparable with silicon film thickness and the model loses its accuracy. Contrary to the charge-sheet model [7], the McKitterick–Caviglia [11] model improves together with silicon layer thickness reduction. For thicker silicon film or higher doping density the depletion charge is comparable with the inversion layer charge and cannot be ignored in the analysis. A similar situation is observed for the front gate voltage in the subthreshold and near threshold region, where depletion layer charge is even higher than Q_{nf}.

Another group of the models consists of the fully analytical ones which are based only on the drift component: Lim–Fossum [1], Barth *et al.* [12] (not discussed here since it is based on the same assumption as the Lim–Fossum model [1]) and the modified Lim–Fossum model [13] (denoted in the figures as 'our' model). For these models the determination of surface potential strongly

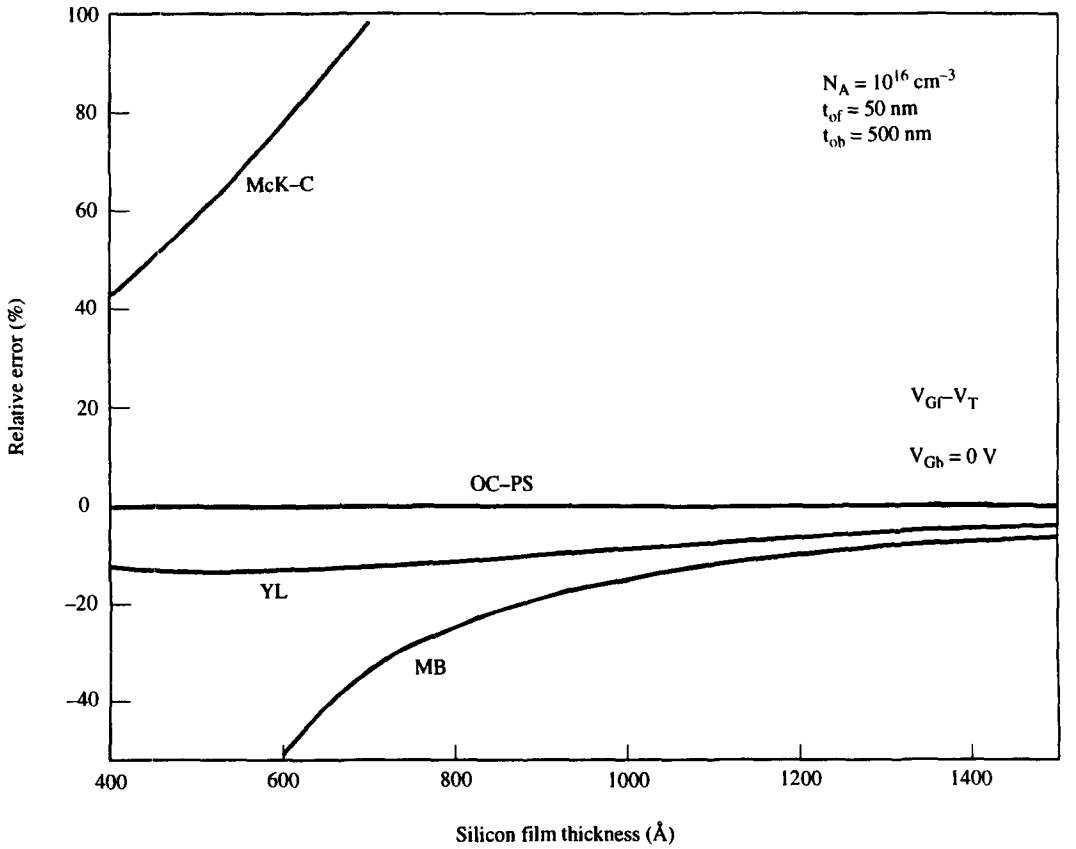

Fig. 4. Relative error of the saturation current with respect to the Ortiz-Conde model [3] vs silicon film thickness. YL: Yang–Li [9], OC–PS: Pierret–Shields' type [5], MB: Mallikarjun–Bhat [7].

influences accuracy. As shown in Fig. 2 the accuracy of the Lim–Fossum [1] and Barth *et al.* [12] models is poor. The models assume that front surface potential is constant and equal to double Fermi potential. This assumption leads to very high densities of inversion layer charge and, consequently, to a high value of drain current. The modified Lim–Fossum model [13], which contains the dependence of the front surface potential on the gate voltages, is more accurate than the original one and the error does not exceed 10% for the front gate voltages $V_{Gf} - V_T > 1$ V. It is worth mentioning that the accuracy of the model [13] is even better than those of some numerical models as long as accumulation at the back gate is considered (Fig. 3). At the threshold voltage all analytical models lose their accuracy due to the fact that they do not take into account the diffusion current.

4. Conclusions

Several long-channel models of SOI transistor have been reviewed and compared in terms of their accuracy and assumptions. It was shown that the charge-sheet model cannot be used for modelling of ultra-thin SOI MOSFETs due to the fact that the charge-sheet approximation is not valid in this case. For such structures the Yang–Li [9] or McKitterick–Caviglia [11] models seem to be sufficiently accurate.

The assumption that the surface potential in the strong inversion is independent of the gate voltages leads to significant errors in modelling of I–V curves. The determination of surface potential values strongly affects the accuracy of SOI models. To avoid complicated formulae and time-consuming numerical calculations, without significant loss of accuracy, a simple analytical formula for surface potentials may be applied.

References

[1] H. Lim and J.G. Fossum, Current–voltage characteristics of thin-film SOI MOSFETs in strong inversion, *IEEE Trans. Electron. Devices*, ED-31(4) (1984) 401–407.

[2] R.H. Yan, A. Ourmazd and K.F. Lee, Scaling the Si MOSFET: from bulk to SOI to bulk, *IEEE Trans. Electron. Devices*, 39(7) (1992) 1704–1710.

[3] A. Ortiz-Conde *et al.*, The nonequilibrium inversion layer charge of thin-film SOI MOSFET, *IEEE Trans. Electron. Devices*, 36(9) (1989) 1651–1656.

[4] H.C. Pao and C.T. Sah, Effects of diffusion current on characteristics of metal-oxide-semiconductor transistors, *Solid State Electron.*, 9 (1966) 927–937.

[5] A. Ortiz-Conde *et al.*, Long-channel silicon-on-insulator MOSFET theory, *Solid State Electron.*, 35(9) (1992) 1291–1298.

[6] R.F. Pierret and J.A. Shields, Simplified long channel MOSFET theory, *Solid State Electron.*, 26 (1983) 143–147.

[7] C. Mallikarjun and K.N. Bhat, Numerical and charge-sheet models for thin-film SOI MOSFETs, *IEEE Trans. Electron. Devices*, 37(9) (1990) 2039–2051.

[8] J.R. Brews, Charge-sheet model of the MOSFET, *Solid State Electron.*, 21 (1978) 345–355.

[9] P.C. Yang and S.S. Li, Analysis of current–voltage characteristics of fully depleted SOI MOSFETs, *Solid State Electron.*, 36(5) (1993) 685–692.

[10] F. Van de Wiele, A long-channel MOSFET model, *Solid State Electron.*, 22 (1979) 991–997.

[11] J.B. McKitterick and A.L. Caviglia, An analytic model for thin SOI transistors, *IEEE Trans. Electron. Devices*, 36(6) (1989) 1133–1138.

[12] P.W. Barth, P.R. Apte and J.B. Angell, The MISIMFET in the thin semiconductor layers: depletion-approximation model of I–V characteristics, *IEEE Trans. Electron. Devices*, ED-30(12) (1983) 1717–1725.

[13] M. Jurczak and A. Jakubowski, An improved analytical description of thin-film SOI MOSFET in the above-threshold region, *Microelectronics Engineering*, 22 (1993) 395–398.