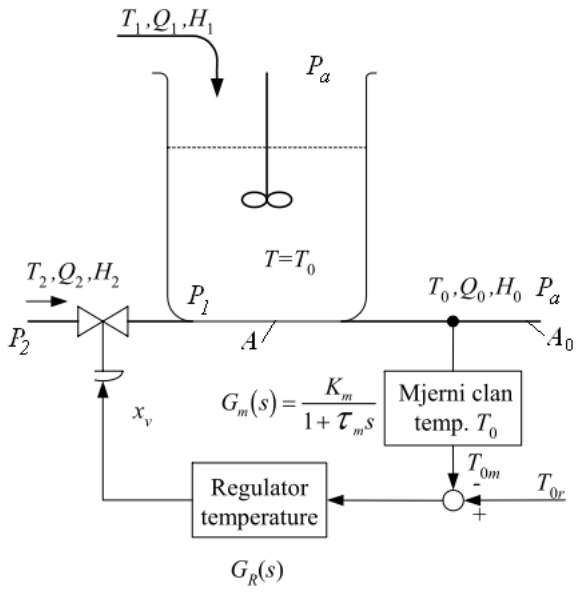
Auditorne vježbe

Regulacija temperature miješanjem

Zadatak



Slika 1: Regulacija temperature miješanjem

Za proces sa slike 1 potrebno je:

- odrediti nelinearni matematički model i
- naći prijenosnu funkciju $G_s(s) = \dfrac{T_0(s)}{X_v(s)}$ u okolini radne točke $P_2, X_{v0}, Q_{10}, T_{10}, T_{20}, T_{00}, H_0$. Ulazni tok Q_2 jednak je:

$$Q_2(t) = k_q x_v \sqrt{P_2 - P_1}.$$

Pretpostavite savršeno miješanje tekućina i $A_0 \ll A$.

Rješenje:
Jednadžba toplinske ravnoteže:

$$\begin{aligned} \frac{d}{dt}(\rho V C_p T_0) &= \sum H = H_1 + H_2 - H_0 \\ &= \rho Q_1 C_p T_1 + \rho Q_2 C_p T_2 - \rho Q_0 C_p T_0. \end{aligned} \tag{1}$$

Budući da su i volumen V i temperatura tekućine T_0 promjenjive veličine iz (1) proizlazi:

$$\rho C_p T_0 \frac{dV}{dt} + \rho C_p V \frac{T_0}{dt} = \rho Q_1 C_p T_1 + \rho Q_2 C_p T_2 - \rho Q_0 C_p T_0. \tag{2}$$

Jednadžba ravnoteže mase (volumena) za spremnik glasi:

$$\frac{dV}{dt} = Q_1 + Q_2 - Q_0. \tag{3}$$

Nakon uvrštavanja (3) u (2) i sređivanja dobiva se:

$$\frac{dT_0}{dt} = \frac{Q_1}{V}\left(T_1 - T_0\right) + \frac{Q_2}{V}\left(T_2 - T_0\right). \tag{4}$$

Ulazni protok Q_2 jednak je:

$$Q_2 = k_q x_v \sqrt{P_2 - P_a - \rho g h}. \tag{5}$$

Izlazni protok Q_0 jednak je:

$$Q_0 = A_0 \sqrt{2gh}. \tag{6}$$

Uvrštavanjem (5) i (6) u (4) i (3), uz činjenicu $V = Ah$, nakon sređivanja dobiva se nelinearni matematički model:

$$\begin{aligned} \frac{dT_0}{dt} &= \underbrace{\frac{Q_1}{Ah}\left(T_1 - T_0\right)}_{f_1} + \underbrace{\frac{k_q x_v \sqrt{P_2 - P_a - \rho g h}}{Ah}\left(T_2 - T_0\right)}_{f_2} \\ \frac{dh}{dt} &= \underbrace{\frac{1}{A}\left(Q_1 + k_q x_v \sqrt{P_2 - P_a - \rho g h} - A_0\sqrt{2gh}\right)}_{f_3} \end{aligned} \tag{7}$$

Uz pretpostavku da su P_2,Q_1,T_1 i T_2 konstantni, linearizacijom (7) dobiva se:

$$\begin{aligned} \Delta\dot{T}_0 &= k_1\Delta h + k_2\Delta T_0 + k_3\Delta h + k_4\Delta T_0 + k_5\Delta x_v, \\ \Delta\dot{h} &= k_6\Delta x_v + k_7\Delta h, \end{aligned} \tag{8}$$

gdje je:

$$k_1 = \left.\frac{\partial f_1}{\partial h}\right|_0 = -\frac{Q_{10}\left(T_{10} - T_{00}\right)}{AH_0^2},$$

$$k_2 = \left.\frac{\partial f_1}{\partial T_0}\right|_0 = -\frac{Q_{10}}{AH_0},$$

$$k_3 = \left.\frac{\partial f_2}{\partial h}\right|_0 = -\frac{k_q X_{v0}\left(2P_2 - 2P_a - \rho g H_0\right)\left(T_{20} - T_{00}\right)}{2AH_0^2\sqrt{P_2 - P_a - \rho g H_0}},$$

$$k_4 = \left.\frac{\partial f_2}{\partial T_0}\right|_0 = -\frac{k_q X_{v0}\sqrt{P_2 - P_a - \rho g H_0}}{AH_0},$$

$$k_5 = \left.\frac{\partial f_2}{\partial x_v}\right|_0 = \frac{k_q\sqrt{P_2 - P_a - \rho g H_0}\left(T_{20} - T_{00}\right)}{AH_0},$$

$$k_6 = \left.\frac{\partial f_3}{\partial x_v}\right|_0 = \frac{k_q\sqrt{P_2 - P_a - \rho g H_0}}{A},$$

$$k_7 = \left.\frac{\partial f_3}{\partial h}\right|_0 = -\frac{A_0\sqrt{\dfrac{2g}{H_0}} + \dfrac{k_q X_{v0}\rho g}{\sqrt{P_2 - P_a - \rho g H_0}}}{2A}.$$

Tražena prijenosna funkcija dobiva se iz (8):

$$G_s\left(s\right) = \frac{T_0\left(s\right)}{X_v\left(s\right)} = \frac{k_5 s + \left(k_1 + k_3\right)k_6 - k_5 k_7}{s^2 - \left(k_2 + k_4 + k_7\right)s + k_7\left(k_2 + k_4\right)}. \tag{9}$$