Sveučilište u Zagrebu
Fakultet elektrotehnike i računarstva
Zavod za elektrostrojarstvo i automatizaciju

Projektiranje i konstruiranje u elektrostrojarstvu

METODA KONAČNIH ELEMENATA

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1. OPIS MAGNETSKOG POLJA U STACIONARNOM STANJU MAXWELLOVIM JEDNADŽBAMA

Ampèreov kružni zakon (zakon protjecanja)

$$\oint_C \vec{H}d\vec{l} = \int_S \vec{J}\vec{n}dS \quad \Rightarrow \quad \nabla \times \vec{H} = \vec{J}$$

Gaussov zakon

$$\int_S \vec{B}\vec{n}dS = 0 \quad \Rightarrow \quad \nabla \cdot \vec{B} = 0$$

$$\vec{B} = \mu_0 \mu_r \vec{H} = \mu \vec{H}$$

Budući da je rotor divergencije vektorske funkcije uvijek jednak nuli, može se definirati vektorska funkcija $\vec{A}$ koja će biti jednaka rotoru vektorske funkcije magnetske indukcije, tj.

$$\vec{B} = \nabla \times \vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

Funkcija $\vec{A}$ predstavlja vektorski magnetski potencijal.

Pretpostavimo da struja teče samo u z osi. Onda iz $\vec{A} = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}}{R}dV$ slijedi da je jedino z komponenta vektorskog magnetskog potencijala različita od nule.

$$\vec{J} = \nabla \times \frac{1}{\mu}\vec{B} = \nabla \times \left(\frac{1}{\mu}\nabla \times \vec{A}\right) = \nabla \times \left(\frac{1}{\mu}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & A_z \end{vmatrix}\right) = \nabla \times \left[\frac{1}{\mu}\left(\frac{\partial A_z}{\partial y}\vec{i} - \frac{\partial A_z}{\partial x}\vec{j}\right)\right]$$

$$\vec{J} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{1}{\mu}\frac{\partial A_z}{\partial y} & -\frac{1}{\mu}\frac{\partial A_z}{\partial x} & 0 \end{vmatrix} = -\left[\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial A_z}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{1}{\mu}\frac{\partial A_z}{\partial y}\right)\right]\vec{k} + \frac{\partial}{\partial z}\left(\frac{1}{\mu}\frac{\partial A_z}{\partial x}\right)\vec{i} + \frac{\partial}{\partial z}\left(\frac{1}{\mu}\frac{\partial A_z}{\partial y}\right)\vec{j}$$

$$J_z = -\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial A_z}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{1}{\mu}\frac{\partial A_z}{\partial y}\right)$$

Pišemo pojednostavljeno bez indeksa z

$$J = -\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial A}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{1}{\mu}\frac{\partial A}{\partial y}\right)$$

Poissonova jednadžba

$\Delta A = -\mu J$

Područje I : područje kojim teče struja, permeabilnost zraka

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = -\mu_0 J$$

Područje II : područje kojim ne teče struja (zrak)

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = 0$$

Područje III : područje kojim ne teče struja (željezo)

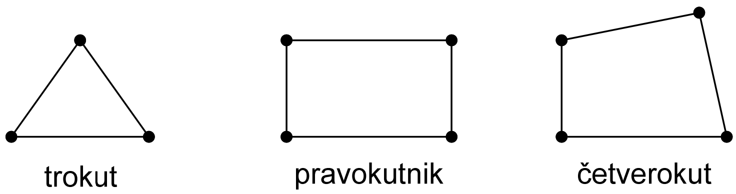
$$\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial A}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{1}{\mu}\frac{\partial A}{\partial y}\right) = 0$$

2. METODA KONAČNIH ELEMENATA

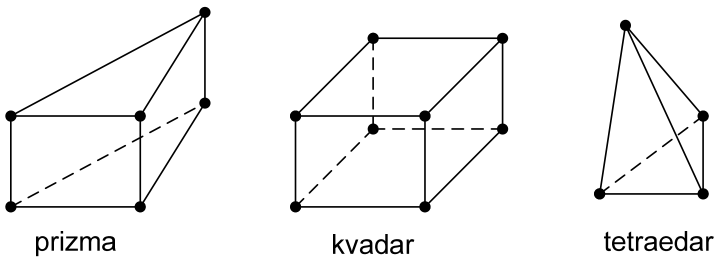
1. Definirati domenu na kojoj se vrši proračun
2. Rastaviti domenu na manja područja koristeći jednostavne oblike (trokut, četverokut...)
3. Kreirati mrežu
4. Definirati interpolacijske funkcije
5. Aproksimirati jednadžbu polja na svakom elementu (kreirati matricu za svaki element)
6. Sastaviti matričnu jednadžbu za cijelu domenu koristeći elementarne matrice
7. Riješiti sustav linearnih jednadžbi

<u>*Diskretizacija domene*</u>

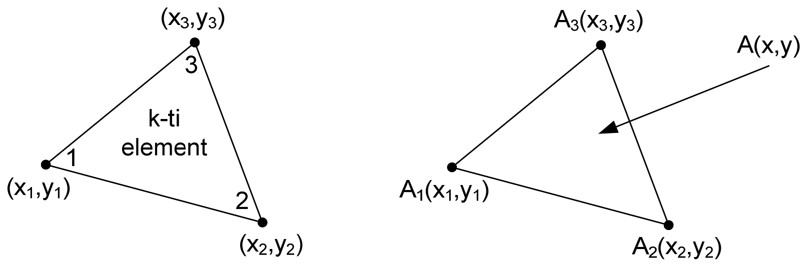
2-D



3-D



Primjer diskretizacije 2-D domene pomoću trokuta



Uz poznavanje iznosa vektorskog magnetskog potencijala u vrhovima trokuta $A_1(x,y)$, $A_2(x,y)$ i $A_3(x,y)$, potencijal $A(x,y)$ u bilo kojoj točki na površini trokuta se može interpolirati polinomom prvog, drugog ili n-tog reda. Primjena metode konačnih elemenata će biti prikazana na primjeru linearne interpolacijske funkcije.

Linearna interpolacijska funkcija

$$A = a + bx + cy$$

$$A_1 = a + bx_1 + cy_1$$
$$A_2 = a + bx_2 + cy_2$$
$$A_3 = a + bx_3 + cy_3$$

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}}_{B} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$A(x,y) = \begin{bmatrix} 1 & x & y \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 & x & y \end{bmatrix} \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}^{-1} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}$$

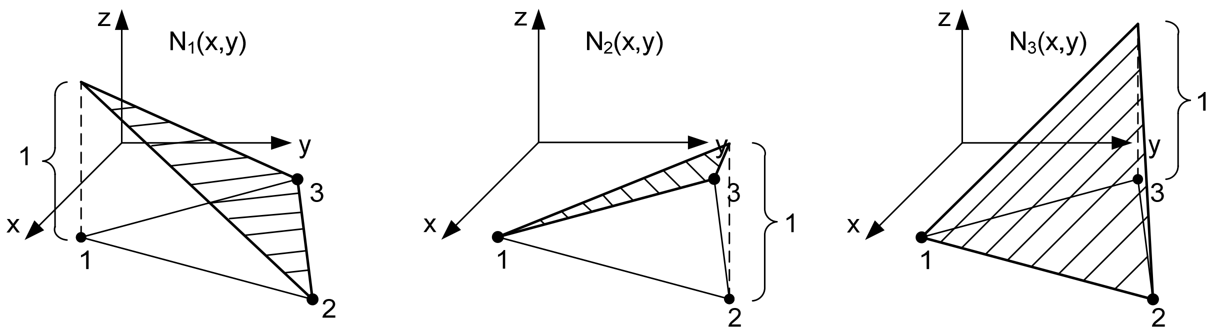
$$B^{-1} = \frac{1}{\det B} \begin{bmatrix} x_2 y_3 - x_3 y_2 & x_3 y_1 - x_1 y_3 & x_1 y_2 - x_2 y_1 \\ y_2 - y_3 & y_3 - y_1 & y_1 - y_2 \\ x_3 - x_2 & x_1 - x_3 & x_2 - x_1 \end{bmatrix} \quad , \quad \det B = \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} = 2S_\Delta$$

$$\begin{aligned} A(x,y) = {} & \frac{1}{2S_\Delta}\left[\left(x_2 y_3 - x_3 y_2\right) + \left(y_2 - y_3\right)x + \left(x_3 - x_2\right)y\right]A_1 + \\ & \frac{1}{2S_\Delta}\left[\left(x_3 y_1 - x_1 y_3\right) + \left(y_3 - y_1\right)x + \left(x_1 - x_3\right)y\right]A_2 + \\ & \frac{1}{2S_\Delta}\left[\left(x_1 y_2 - x_2 y_1\right) + \left(y_1 - y_2\right)x + \left(x_2 - x_1\right)y\right]A_3 \\ = {} & N_1(x,y)A_1 + N_2(x,y)A_2 + N_3(x,y)A_3 \end{aligned}$$

$$A(x,y) = \sum_{i=1}^{3} N_i(x,y) A_i \quad , \quad N_i(x,y) \text{ - interpolacijske funkcije}$$

$$N_i(x_j, y_j) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$

$$\sum_{i=1}^{3} N_i(x, y) = 1$$



Na domeni Ω je moguće naći samo približno rješenje Poissonove jednadžbe $\tilde{A}$ pa će u jednadžbi postojati ostatak R

$$R(x, y) = \frac{1}{\mu}\frac{\partial^2 \tilde{A}}{\partial x^2} + \frac{1}{\mu}\frac{\partial^2 \tilde{A}}{\partial y^2} + J$$

Cilj je minimizirati integral tog ostatka pomnoženog nekom težinskom funkcijom W (METODA TEŽINSKIH OSTATAKA) na cijeloj domeni, tj.

$$\int_\Omega W(x, y) R(x, y)\, dxdy = 0$$

$$-\int_\Omega W\left(\frac{1}{\mu}\frac{\partial^2 \tilde{A}}{\partial x^2} + \frac{1}{\mu}\frac{\partial^2 \tilde{A}}{\partial y^2}\right) dxdy = \int_\Omega WJ\, dxdy$$

$$\frac{\partial}{\partial x}\left(W\frac{\partial \tilde{A}}{\partial x}\right) = \frac{\partial W}{\partial x}\frac{\partial \tilde{A}}{\partial x} + W\frac{\partial^2 \tilde{A}}{\partial x^2}$$

$$\frac{\partial}{\partial y}\left(W\frac{\partial \tilde{A}}{\partial y}\right) = \frac{\partial W}{\partial y}\frac{\partial \tilde{A}}{\partial y} + W\frac{\partial^2 \tilde{A}}{\partial y^2}$$

$$-\int_\Omega W\left(\frac{1}{\mu}\frac{\partial^2 \tilde{A}}{\partial x^2} + \frac{1}{\mu}\frac{\partial^2 \tilde{A}}{\partial y^2}\right) dxdy = \int_\Omega \frac{1}{\mu}\left(\frac{\partial W}{\partial x}\frac{\partial \tilde{A}}{\partial x} + \frac{\partial W}{\partial y}\frac{\partial \tilde{A}}{\partial y}\right) dxdy - \int_\Omega \frac{1}{\mu}\left[\frac{\partial}{\partial x}\left(W\frac{\partial \tilde{A}}{\partial x}\right) + \frac{\partial}{\partial y}\left(W\frac{\partial \tilde{A}}{\partial y}\right)\right] dxdy$$

Greenov teorem izražava krivuljni integral po zatvorenoj krivulji C plošnim integralom plošnim integralom po plohi Ω omeđenoj krivuljom C.

$$\int_\Omega \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial x}\right) dxdy = \oint_C -Q dx + P dy$$

$$-\int_\Omega \frac{1}{\mu}\left[\frac{\partial}{\partial x}\underbrace{\left(W\frac{\partial \tilde{A}}{\partial x}\right)}_{P} + \frac{\partial}{\partial y}\underbrace{\left(W\frac{\partial \tilde{A}}{\partial y}\right)}_{Q}\right] dxdy = -\oint_C \frac{1}{\mu}\left[-W\frac{\partial \tilde{A}}{\partial y} dx + W\frac{\partial \tilde{A}}{\partial x} dy\right]$$

Vektor normale na krivulju C je jedinični vektor koji se definira kao

$$\vec{n} = \frac{1}{\sqrt{dx^2 + dy^2}} \left(dy\vec{i} - dx\vec{j} \right)$$

Usmjerena derivacija funkcije $\tilde{A}$ u smjeru vektora $\vec{n}$ je jednaka

$$\frac{\partial \tilde{A}}{\partial \vec{n}} = \vec{n} \cdot \nabla \tilde{A} = \frac{1}{\sqrt{dx^2 + dy^2}} \left(\frac{\partial \tilde{A}}{\partial x} dy - \frac{\partial \tilde{A}}{\partial y} dx \right)$$

Duljina infinitenzimalnog dijela krivulje C iznosi $dl = \sqrt{dx^2 + dy^2}$.
Odatle slijedi da je

$$-\oint_C \frac{1}{\mu} \left[-W \frac{\partial \tilde{A}}{\partial y} dx + W \frac{\partial \tilde{A}}{\partial x} dy \right] = -\oint_C \frac{1}{\mu} W \frac{\partial \tilde{A}}{\partial \vec{n}} dl$$

Na kraju se dobiva

$$\int_\Omega \frac{1}{\mu} \left(\frac{\partial W}{\partial x} \frac{\partial \tilde{A}}{\partial x} + \frac{\partial W}{\partial y} \frac{\partial \tilde{A}}{\partial y} \right) dxdy - \oint_C \frac{1}{\mu} W \frac{\partial \tilde{A}}{\partial \vec{n}} dl = \int_\Omega WJ dxdy$$

Integral na cijeloj domeni se prikazuje kao suma integrala na pojedinačnim elementima

$$\sum_{e=1}^{N} \left\{ \frac{1}{\mu^e} \int_{\Omega^e} \left(\frac{\partial W^e}{\partial x} \frac{\partial \tilde{A}^e}{\partial x} + \frac{\partial W^e}{\partial y} \frac{\partial \tilde{A}^e}{\partial y} \right) dxdy - \frac{1}{\mu^e} \frac{\partial \tilde{A}^e}{\partial \vec{n}} \oint_C W^e dl \right\} = J \int_{\Omega^e} W^e dxdy$$

Član $\frac{1}{\mu^e} \frac{\partial \tilde{A}^e}{\partial \vec{n}} \oint_C W^e dl$ definira uvjete na granici. Ako se postavi $\frac{\partial \tilde{A}^e}{\partial \vec{n}} = 0$, onda se nameću tzv. prirodnu rubni uvjeti, tj. polje je okomito na granicu domene.

$$\tilde{A}^e = \begin{bmatrix} N_1^e & N_2^e & N_3^e \end{bmatrix} \begin{bmatrix} \tilde{A}_1^e \\ \tilde{A}_2^e \\ \tilde{A}_3^e \end{bmatrix}$$

$$N_1^e = \frac{1}{2S_\Delta} \left(a_1^e + b_1^e x + c_1^e y \right)$$

$$N_2^e = \frac{1}{2S_\Delta} \left(a_2^e + b_2^e x + c_2^e y \right)$$

$$N_3^e = \frac{1}{2S_\Delta} \left(a_3^e + b_3^e x + c_3^e y \right)$$

Potrebno je definirati težinsku funkciju W^e. Najčešće se koristi Galerkinova metoda kod koje su težinske funkcije jednake interpolacijskim funkcijama.

$$W^e = \begin{bmatrix} N_1^e \\ N_2^e \\ N_3^e \end{bmatrix}$$

$$\frac{\partial \tilde{A}^e}{\partial x}=\frac{1}{2S_\Delta}\begin{bmatrix} b_1^e & b_2^e & b_3^e \end{bmatrix}\begin{bmatrix} \tilde{A}_1^e \\ \tilde{A}_2^e \\ \tilde{A}_3^e \end{bmatrix} \quad , \quad \frac{\partial \tilde{A}^e}{\partial y}=\frac{1}{2S_\Delta}\begin{bmatrix} c_1^e & c_2^e & c_3^e \end{bmatrix}\begin{bmatrix} \tilde{A}_1^e \\ \tilde{A}_2^e \\ \tilde{A}_3^e \end{bmatrix}$$

$$\frac{\partial W^e}{\partial x}=\frac{1}{2S_\Delta}\begin{bmatrix} b_1^e \\ b_2^e \\ b_3^e \end{bmatrix} \quad , \quad \frac{\partial W^e}{\partial y}=\frac{1}{2S_\Delta}\begin{bmatrix} c_1^e \\ c_2^e \\ c_3^e \end{bmatrix}$$

$$\frac{1}{\mu^e}\left(\frac{\partial W^e}{\partial x}\frac{\partial \tilde{A}^e}{\partial x}+\frac{\partial W^e}{\partial y}\frac{\partial \tilde{A}^e}{\partial y}\right)\int_{\Omega^e} dxdy = J\int_{\Omega^e} W^e dxdy$$

$$\int_{\Omega^e} dxdy = S_\Delta$$

Uvrštavanjem parcijalnih derivacija u integral dobiva se

$$\frac{1}{\mu^e}\left(\frac{\partial W^e}{\partial x}\frac{\partial \tilde{A}^e}{\partial x}+\frac{\partial W^e}{\partial y}\frac{\partial \tilde{A}^e}{\partial y}\right)\int_{\Omega^e} dxdy = \frac{1}{4\mu^e S_\Delta}\begin{bmatrix} \left(b_1^e\right)^2+\left(c_1^e\right)^2 & b_1^e b_2^e + c_1^e c_2^e & b_1^e b_3^e + c_1^e c_3^e \\ b_1^e b_2^e + c_1^e c_2^e & \left(b_2^e\right)^2+\left(c_2^e\right)^2 & b_2^e b_3^e + c_2^e c_3^e \\ b_1^e b_3^e + c_1^e c_3^e & b_2^e b_3^e + c_2^e c_3^e & \left(b_3^e\right)^2+\left(c_3^e\right)^2 \end{bmatrix}\begin{bmatrix} \tilde{A}_1^e \\ \tilde{A}_2^e \\ \tilde{A}_3^e \end{bmatrix}$$

$$=\begin{bmatrix} S_{11}^e & S_{12}^e & S_{13}^e \\ S_{21}^e & S_{22}^e & S_{23}^e \\ S_{31}^e & S_{32}^e & S_{33}^e \end{bmatrix}\begin{bmatrix} \tilde{A}_1^e \\ \tilde{A}_2^e \\ \tilde{A}_3^e \end{bmatrix}$$

$$J\int_{\Omega^e} W^e dxdy = \frac{J}{2S_\Delta}\begin{bmatrix} \int_{\Omega^e} N_1^e dxdy \\ \int_{\Omega^e} N_2^e dxdy \\ \int_{\Omega^e} N_3^e dxdy \end{bmatrix} = \frac{J}{2S_\Delta}\begin{bmatrix} \int_{\Omega^e}\left(a_1^e+b_1^e x + c_1^e y\right)dxdy \\ \int_{\Omega^e}\left(a_2^e+b_2^e x + c_2^e y\right)dxdy \\ \int_{\Omega^e}\left(a_3^e+b_3^e x + c_3^e y\right)dxdy \end{bmatrix}$$

Integral linearne funkcije oblika $a_1^e + b_1^e x + c_1^e y$ po površini trokuta se može izračunati iz koordinata težišta trokuta na sljedeći način:

$$\int_{\Omega^e}\left(a_1^e+b_1^e x + c_1^e y\right)dxdy = \int_{\Omega^e}\left(a_1^e+b_1^e \overline{x} + c_1^e \overline{y}\right)dxdy$$

$$\overline{x}=\frac{x_1+x_2+x_3}{3} \quad , \quad \overline{y}=\frac{y_1+y_2+y_3}{3}$$ - koordinate težišta trokuta

$$\frac{J}{2S_\Delta}\int_{\Omega^e}\left(a_1^e+b_1^e x+c_1^e y\right)dxdy=\frac{J}{2S_\Delta}\int_{\Omega^e}\left[\left(x_2y_3-x_3y_2\right)+\left(y_2-y_3\right)\frac{x_1+x_2+x_3}{3}+\left(x_3-x_2\right)\frac{y_1+y_2+y_3}{3}\right]dxdy$$

$$=\frac{J}{2S_\Delta}\frac{1}{3}\left[x_2y_3-x_3y_2+x_1(y_2-y_3)+y_1(x_3-x_2)\right]\int_{\Omega^e}dxdy$$

$$=\frac{J}{2S_\Delta}\frac{1}{3}2S_\Delta S_\Delta=\frac{JS_\Delta}{3}$$

$$\frac{J}{2S_\Delta}\int_{\Omega^e}\left(a_2^e+b_2^e x+c_2^e y\right)dxdy=\frac{JS_\Delta}{3}$$

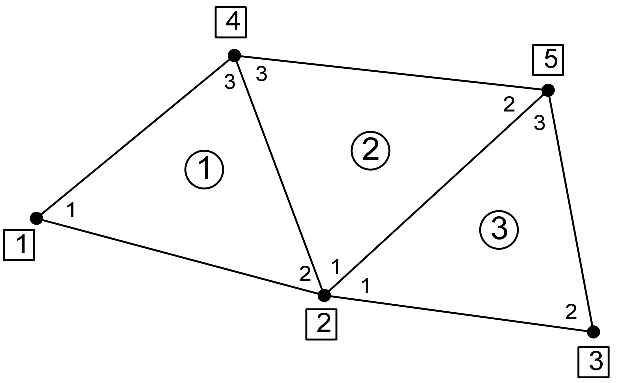
$$\frac{J}{2S_\Delta}\int_{\Omega^e}\left(a_2^e+b_2^e x+c_2^e y\right)dxdy=\frac{JS_\Delta}{3}$$

$$J\int_{\Omega^e}W^e dxdy=\frac{JS_\Delta}{3}\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\begin{bmatrix}S_{11}^e & S_{12}^e & S_{13}^e\\ S_{21}^e & S_{22}^e & S_{23}^e\\ S_{31}^e & S_{32}^e & S_{33}^e\end{bmatrix}\begin{bmatrix}\tilde{A}_1^e\\ \tilde{A}_2^e\\ \tilde{A}_3^e\end{bmatrix}=\frac{JS_\Delta}{3}\begin{bmatrix}1\\1\\1\end{bmatrix}$$

Kreiranje globalne „Stiffness“ matrice

Primjer: N=3 (broj elemenata), n=5 (broj čvorova), dimenzije matrice n×n
Element 1 je vodič s gustoćom struje J_0. Na čvorovima 3 i 5 su postavljeni Dirichletovi rubni uvjeti (A=0)



S_{mn} – suma svih S_{ij}^e za sve elemente *e* koji imaju zajedničke čvorove *m* i *n*

$S_{11}=S_{11}^1$ - zato što globalni čvor $\boxed{1}$ pripada samo elementu 1

$S_{22}=S_{22}^1+S_{11}^2+S_{11}^3$

$S_{33}=S_{22}^3$

$S_{44} = S_{33}^1 + S_{33}^2$

$S_{55} = S_{22}^2 + S_{33}^3$

$S_{12} = S_{21} = S_{12}^1$

$S_{13} = S_{31} = 0$

$S_{14} = S_{41} = S_{13}^1$

$S_{15} = S_{51} = 0$

$S_{23} = S_{32} = S_{12}^3$

$S_{24} = S_{42} = S_{23}^1 + S_{13}^2$

$S_{25} = S_{52} = S_{12}^2 + S_{13}^3$

$S_{35} = S_{53} = S_{23}^3$

$S_{34} = S_{43} = 0$

$S_{45} = S_{54} = S_{32}^2$

Matrica S je simetrična, singularna i rijetko popunjena.

$$\begin{bmatrix} S_{11}^1 & S_{12}^1 & 0 & S_{13}^1 & 0 \\ S_{12}^1 & S_{22}^1 + S_{11}^2 + S_{11}^3 & S_{12}^3 & S_{23}^1 + S_{13}^2 & S_{12}^2 + S_{13}^3 \\ 0 & S_{12}^3 & S_{22}^3 & 0 & S_{23}^3 \\ S_{13}^1 & S_{23}^1 + S_{13}^2 & 0 & S_{33}^1 + S_{33}^2 & S_{32}^2 \\ 0 & S_{12}^2 + S_{13}^3 & S_{23}^3 & S_{32}^2 & S_{22}^2 + S_{33}^3 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{bmatrix} = \begin{bmatrix} \frac{J_0 S_\Delta^1}{3} \\ \frac{J_0 S_\Delta^1}{3} \\ 0 \\ \frac{J_0 S_\Delta^1}{3} \\ 0 \end{bmatrix}$$

Nakon primjene Dirichletovih rubnih uvjeta dobiva se

$$\begin{bmatrix} S_{11}^1 & S_{12}^1 & 0 & S_{13}^1 & 0 \\ S_{12}^1 & S_{22}^1 + S_{11}^2 + S_{11}^3 & S_{12}^3 & S_{23}^1 + S_{13}^2 & S_{12}^2 + S_{13}^3 \\ 0 & 0 & 1 & 0 & 0 \\ S_{13}^1 & S_{23}^1 + S_{13}^2 & 0 & S_{33}^1 + S_{33}^2 & S_{32}^2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{bmatrix} = \begin{bmatrix} \frac{J_0 S_\Delta^1}{3} \\ \frac{J_0 S_\Delta^1}{3} \\ 0 \\ \frac{J_0 S_\Delta^1}{3} \\ 0 \end{bmatrix}$$

Proračun magnetske indukcije

$$\vec{B} = \nabla \times \vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 0 & 0 & A_z \end{vmatrix} = \vec{i}\,\frac{\partial A_z}{\partial y} - \vec{j}\,\frac{\partial A_z}{\partial x} \quad \Rightarrow \quad B_x = \frac{\partial A_z}{\partial y} \;,\quad B_y = -\frac{\partial A_z}{\partial x}$$

$$B_x = \frac{1}{2S_\Delta}\left(c_1^e \tilde{A}_1^e + c_2^e \tilde{A}_2^e + c_3^e \tilde{A}_3^e\right)$$

$$B_y = -\frac{1}{2S_\Delta}\left(b_1^e \tilde{A}_1^e + b_2^e \tilde{A}_2^e + b_3^e \tilde{A}_3^e\right)$$

Magnetska indukcija je konstantna na cijeloj površini elementa.

3. LITERATURA

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