$$\vec{w} = \sum_{i=1}^{N} \lambda_i d_i \vec{x}_i$$

$$= \cancel{0.5 \cdot 1 \cdot \begin{bmatrix}0\\0\end{bmatrix}} + \cancel{0 \cdot 1 \cdot \begin{bmatrix}-1\\1\end{bmatrix}} + 0.5(-1)\begin{bmatrix}2\\0\end{bmatrix}$$

$$= \begin{bmatrix}-1\\0\end{bmatrix}$$

$$d(\vec{x}) = -x_1 + 0 \cdot x_2 + b$$

$$d(\vec{x}_1) = -0 + 0\,0 + b = 1$$

potporni vektor $\Rightarrow b = 1$

$$\boxed{d(\vec{x}) = -x_1 + 1} \quad \boxed{x_1 = 1}$$

3. cikel

$$\vec{x} = \begin{bmatrix}x_1\\x_2\\1\end{bmatrix} \qquad \vec{x}^* = \begin{bmatrix}x_1^2\\x_1x_2\\x_2^2\\x_1\\x_2\\1\end{bmatrix}$$

$$d = w_1 x_1^2 + w_2 x_1 x_2 + w_3 x_2^2 + \underbrace{w_4 x_1 + w_5 x_2 + w_6}_{\text{linearni}}$$

$$\vec{x} = \begin{bmatrix}x_1\\x_2\\x_3\\1\end{bmatrix} \Rightarrow \vec{x}^* = \begin{bmatrix}x_1^2\\x_2^2\\x_3^2\\x_1x_2\\x_2x_3\\x_1x_3\\x_1\\x_2\\x_3\\1\end{bmatrix}$$

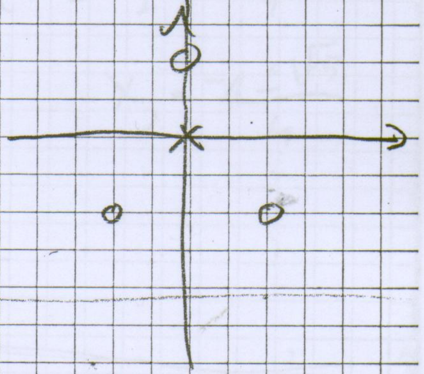
$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix} \qquad \vec{x}^* = \begin{bmatrix} x_1^3 \\ x_2^3 \\ x_1 x_2^2 \\ x_1^2 x_2 \\ x_1^2 \\ x_1 x_2 \\ x_2^2 \\ x_1 \\ x_2 \\ 1 \end{bmatrix}$$

Broj komponenti $\binom{n+r}{r}$

$\boxed{2 \text{ AD } 1}$

$$\omega_1 = \{[0, 2]^T, [-1, -2]^T, [1, -2]^T\}$$

$$\omega_2 = \{[0, 0]^T\}$$



$$\vec{x}^* = \begin{bmatrix} x_1^2 \\ x_1 x_2 \\ x_2^2 \\ x_1 \\ x_2 \\ 1 \end{bmatrix} \qquad c = 1, \quad \vec{w}(1) = \vec{0}$$

$$\vec{x}_1^* = \begin{bmatrix} 0 \\ 0 \\ 4 \\ 0 \\ 2 \\ 1 \end{bmatrix} \qquad \vec{x}_2^* = \begin{bmatrix} 1 \\ 2 \\ 4 \\ -1 \\ -2 \\ 1 \end{bmatrix} \qquad \vec{x}_3^* = \begin{bmatrix} 1 \\ -2 \\ 4 \\ 1 \\ -2 \\ 1 \end{bmatrix}$$

$$\vec{x}_4^* = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

1. $\vec{w}^T(1)\ \vec{x}(1) = 0 \le 0 \qquad \vec{w}(2) = \vec{w}(1) + \vec{x}(1) = [0\ 0\ 4\ 0\ 2\ 1]^T$

2. $\vec{w}^T(2)\ \vec{x}(2) = 13 > 0 \qquad \vec{w}(3) = \vec{w}(2)$

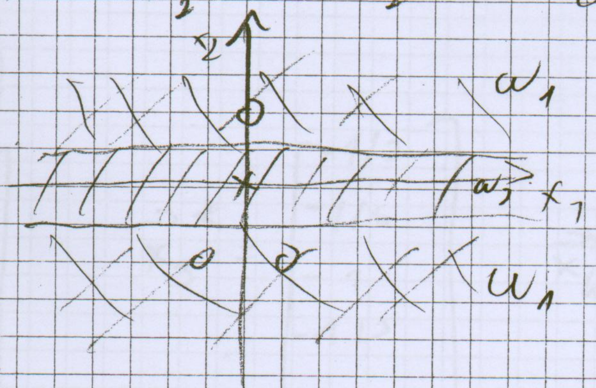
3. $\vec{w}^T(3)\ \vec{x}(3) = 13 > 0 \qquad \vec{w}(4) = \vec{w}(3)$

4. $\vec{w}^T(4)\ \vec{x}(4) = -1 < 0 \qquad \vec{w}(5) = \vec{w}(4) + \vec{x}(4) = [0\ 0\ 4\ 0\ 2\ 0]$

$\vdots$

12. $\vec{w} = [0\ \ 0\ \ 4\ \ 0\ \ 2\ \ -1]^T$

$$d(\vec{x}) = 4x_2^2 + 2x_2 - 1 = 0 \Rightarrow$$



$$\left(2x_2 + \frac{1}{2}\right)^2 - \frac{5}{4} = 0$$

$$x_2 = \frac{-1 \pm \sqrt{5}}{4}$$

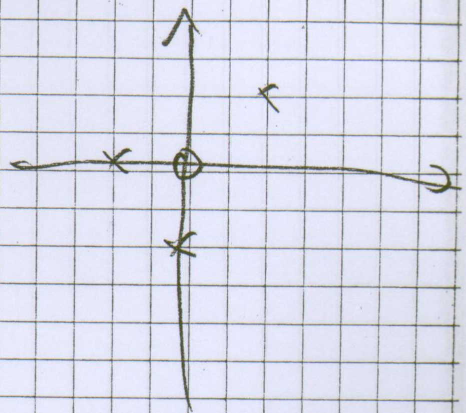
$$\vec{x} = \begin{bmatrix} f(\|\vec{x} - \vec{c}_1\|) \\ \vdots \\ f(\|\vec{x} - \vec{c}_l\|) \\ 1 \end{bmatrix}$$

$$d(\vec{x}) = w_1 f(\|\vec{x} - c_1\|) + \dots + w_l f(\|\vec{x} - \vec{c}_l\|) + w_0$$

ZAD 2

$$\omega_1 = \{[0,0]^T\}$$

$$\omega_2 = \{[1,1]^T, [-1,0]^T, [0,-1]^T\}$$



$$\vec{c}_i = \vec{x}_i \qquad f(\vec{x}) = \frac{1}{1+\|\vec{x}-\vec{c}_1\|^2}$$

$$\vec{x}_1^* = \begin{bmatrix} 1 \\ 1/3 \\ 1/2 \\ 1/2 \\ 1 \end{bmatrix}$$

$$\vec{x}_2^* = \begin{bmatrix} -1/3 \\ -1 \\ -1/6 \\ -1/6 \\ -1 \end{bmatrix} \quad \vec{x}_3^* = \begin{bmatrix} -1/2 \\ -1/6 \\ -1 \\ -1/3 \\ -1 \end{bmatrix} \quad \vec{x}_4^* = \begin{bmatrix} -1/2 \\ -1/6 \\ -1/3 \\ -1 \\ -1 \end{bmatrix}$$

$$\vec{w}(1) = \vec{0} \qquad c = 1$$

1. $\vec{w}^T(1)\ \vec{x}(1) = 0$ $\qquad \vec{w}(2) = \vec{w}(1) + \vec{x}(1) = \left[1\ \ \frac{1}{3}\ \ \frac{1}{2}\ \ \frac{1}{2}\ \ 1\right]^T$
2. $\vec{w}^T(2)\ \vec{x}(2) \leq 0$ $\qquad \vec{w}(3) = \vec{w}(2) + \vec{x}(2) = \left[\frac{2}{3}\ \ -\frac{2}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\right]^T$
3. $\vec{w}^T(3)\ \vec{x}(3) = -\frac{2}{3} \leq 0$ $\qquad \vec{w}(4) = \vec{w}(3) + \vec{x}(3) = \left[\frac{1}{6}\ \ -\frac{5}{6}\ \ -\frac{2}{3}\ \ 0\ \ -1\right]^T$
4. $\vec{w}^T(4)\ \vec{x}(4) = \frac{23}{16} > 0$ $\qquad \vec{w}(5) = \vec{w}(4)$

⋮

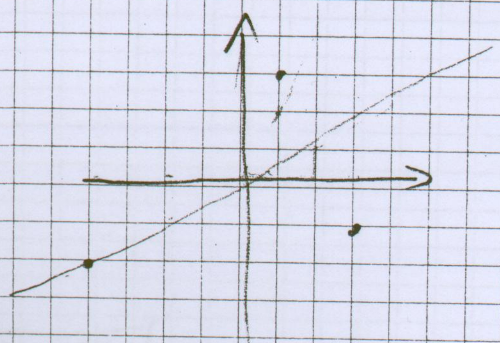
16. $\vec{w} = \left[\frac{11}{6}\ \ -\frac{7}{6}\ \ -\frac{4}{3}\ \ \frac{2}{3}\ \ -1\right]^T$

$$d(\vec{x}) = \frac{11}{6}\frac{1}{1+\|\vec{x}-\vec{c}_1\|^2} - \frac{7}{6}\frac{1}{1+\|\vec{x}-\vec{c}_2\|^2} - \frac{4}{3}\frac{1}{1+\|\vec{x}-\vec{c}_3\|^2} + \frac{2}{3}\frac{1}{1+\|\vec{x}-\vec{c}_4\|^2} - 1$$

PCA - KL Transformacja

ZAD 3

$\{ [-4, -2]^T, [1, 3]^T, [3, -1]^T \}$



$$K = E\left[(\vec{x} - E[\vec{x}]) (\vec{x} - E[\vec{x}])^T \right]$$

$$K = \frac{1}{N-1} \sum_{i=1}^{N} (\vec{x}_i - \vec{m}) (\vec{x}_i - \vec{m})^T$$

$$\vec{m} = \frac{1}{N} \sum_i \vec{x}_i = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$K = \frac{1}{2} \left(\begin{bmatrix} -4 \\ -2 \end{bmatrix} [-4 \ \ -2] + \begin{bmatrix} 1 \\ 3 \end{bmatrix} [1 \ \ 3] + \begin{bmatrix} 3 \\ -1 \end{bmatrix} [3 \ \ -1] \right)$$

$$= \frac{1}{2} \begin{bmatrix} 26 & 8 \\ 8 & 14 \end{bmatrix} = \begin{bmatrix} 13 & 4 \\ 4 & 7 \end{bmatrix}$$

$$K \vec{w} = \lambda \vec{w}$$

$$(K - \lambda I) \vec{w} = 0$$

$$|K - \lambda I| = 0$$

$$\begin{vmatrix} 13 - \lambda & 4 \\ 4 & 7 - \lambda \end{vmatrix} = 0$$

$$\lambda^2 - 20\lambda + 75 = 0$$

$$\boxed{\lambda_1 = 15} \quad \lambda_2 = 5$$

$$(K - \lambda I)\vec{w} = 0$$

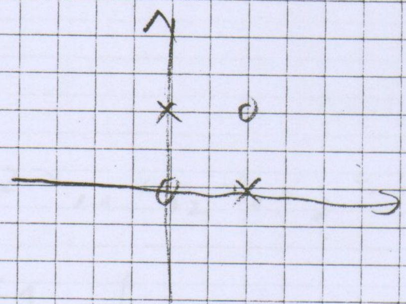
$$\begin{bmatrix} -2 & 4 \\ 4 & -8 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$w_1 = 2 w_2$$

$$\vec{w} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad \vec{w} = \begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$$

$\boxed{\text{ZAD 4}}$

$$\omega_1 = \{ [0, 0]^T, [1, 1]^T \}$$
$$\omega_2 = \{ [0, 1]^T, [1, 0]^T \}$$



$$K(\vec{x}_i, \vec{x}_j) = (\vec{x}_i^T \vec{x}_j + 1)^2$$

$$K = \begin{bmatrix} K(x_1, x_1) & \cdots & K(x_1, x_4) \\ \vdots & \ddots & \vdots \\ K(x_4, x_1) & \cdots & K(x_4, x_4) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 9 & 4 & 4 \\ 1 & 4 & 4 & 1 \\ 1 & 4 & 1 & 4 \end{bmatrix}$$

$Q_{ij} = d_i d_j \vec{x}_i^T \vec{x}_j$ za linearni

$Q_{ij} = d_i d_j K(\vec{x}_i, \vec{x}_j)$ za nelinearni

$$Q = \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 9 & -4 & -4 \\ -1 & -4 & 4 & 1 \\ -1 & -4 & 1 & 4 \end{bmatrix}$$

Ostale matrice i vektori se zadrži isto i kod linearnog

$$\vec{\lambda} = \begin{bmatrix} 10/3 \\ 2 \\ 8/3 \\ 8/3 \end{bmatrix} \qquad d(\vec{x}) = \sum_{i=1}^{n} \lambda_i d_i K(\vec{x}, \vec{x}_i) + b$$

$$d(\vec{x}_1) = 1$$

$$\Rightarrow \frac{10}{3} \cdot 1 \cdot 1 + 2 \cdot 1 \cdot 1 + \frac{8}{3}(-1) \cdot 1 + \frac{8}{3}(-1) \cdot 1 + b =$$

$$\boxed{b = 1}$$

$$d(\vec{x}) = \sum_{i=1}^{n} \lambda_i d_i (\vec{x}^T \vec{x}_i + 1)^2 + 1$$

$$\Rightarrow K(\vec{x}_i, \vec{x}) = x_{i1}^2 x_1^2 + x_{i2}^2 x_2^2 + 2 x_{i1} x_{i2} x_1 x_2 + 2 x_{i1} x_1 + 2 x_{i2} x_2 + 1$$

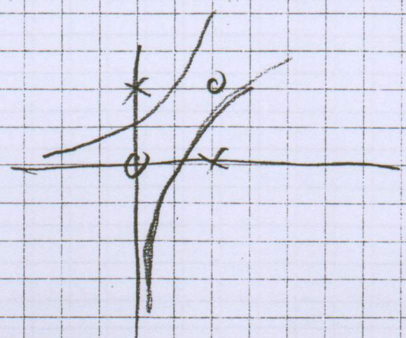
$$\Rightarrow d(\vec{x}) = \frac{10}{3}(0 \cdot x_1^2 + 0 x_2^2 + 0 x_1 x_2 + 0 x_1 + 0 x_2 + 1)$$

$$+ 2 \cdot 1 \cdot (x_1^2 + x_2^2 + 2 x_1 x_2 + 2 x_1 - 2 x_2 + 1)$$

$$+ \frac{8}{3}(-1)(0 x_1^2 + x_2^2 + 0 x_1 x_2 + 0 x_1 + 2 x_2 + 1)$$

$$+ \frac{8}{3}(-1)(x_1^2 + 0 x_2^2 - 0 x_1 x_2 + 2 x_1 + 0 x_2 - 1) + 1$$

$$= -\frac{2}{3} x_1^2 - \frac{2}{3} x_2^2 + 4 x_1 x_2 - \frac{4}{3} x_1 - \frac{4}{3} x_2 + 1$$



Bayesov klasifikator

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

$$P(\omega_1|\vec{x}) = \frac{p(\vec{x}|\omega_1)\,P(\omega_1)}{p(\vec{x})}$$

$$\vec{x} \in \omega_i \qquad P(\omega_i|\vec{x}) > P(\omega_j|\vec{x}) \quad \forall\, i \neq j$$

$$d_i = P(\omega_i|\vec{x})$$

$$\vec{x} \in \omega_i \qquad \frac{p(\vec{x}|\omega_i)\,P(\omega_i)}{\cancel{p(\vec{x})}} > \frac{p(\vec{x}|\omega_j)\,P(\omega_j)}{\cancel{p(\vec{x})}} \quad \forall_j$$

$$p(\vec{x}|\omega_i) = \frac{1}{(2\pi)^{\frac{n}{2}}|C_i|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(\vec{x}-\vec{m}_i)^T C_i^{-1}(\vec{x}-\vec{m}_i)\right)$$

$$x \in \omega_i \qquad \ln p(\vec{x}|\omega_i) + \ln P(\omega_i) > \ln p(\vec{x}|\omega_j) + \ln P(\omega_j) \quad \forall\, i \neq j$$

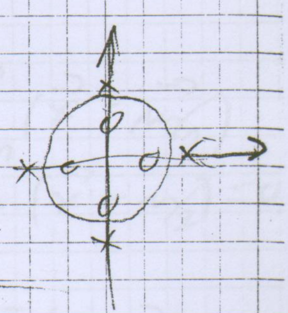
$$d_i = \ln P(\omega_i) + \ln p(\vec{x}|\omega_i)$$

$$= \ln P(\omega_i) \cancel{- \frac{n}{2}\ln 2\pi} - \frac{1}{2}\ln|C_i| - \frac{1}{2}(\vec{x}-\vec{m}_i)^T C_i^{-1}(\vec{x}+\vec{m}_i)$$

ZAD 5

$$\omega_1 = \{[-1,0]^T, [0,-1]^T, [1,0]^T, [0,1]^T\}$$

$$\omega_2 = \{[-2,0]^T, [0,-2]^T, [2,0]^T, [0,2]^T\}$$



$N_1 = 4 \qquad P(\omega_1) = \frac{\omega_1}{N} = \frac{N_1}{N_1+N_2} = \frac{1}{2}$

$N_2 = 4 \qquad P(\omega_2) = \frac{1}{2}$

$$\vec{m}_1 = \frac{1}{N_1}\sum_{x_i\in\omega_1} \vec{x}_i = \frac{1}{4}\left(\begin{bmatrix}-1\\0\end{bmatrix} + \begin{bmatrix}0\\-1\end{bmatrix} + \begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}0\\0\end{bmatrix}$$

$$\vec{m}_2 = \frac{1}{N_2}\sum_{x_i\in\omega_2} \vec{x}_i = \frac{1}{4}\left(\begin{bmatrix}-2\\0\end{bmatrix} + \begin{bmatrix}0\\-2\end{bmatrix} + \begin{bmatrix}2\\0\end{bmatrix} + \begin{bmatrix}0\\2\end{bmatrix}\right) = \begin{bmatrix}0\\0\end{bmatrix}$$

$$C_1 = \frac{1}{N_1}\sum_{x_i\in\omega_1} (\vec{x}_i - \vec{m}_1)(\vec{x}_i - \vec{m}_1)^T$$

$$= \frac{1}{4}\left(\begin{bmatrix}-1\\0\end{bmatrix}[-1 \ \ 0] + \begin{bmatrix}0\\-1\end{bmatrix}[0 \ \ -1] + \begin{bmatrix}1\\0\end{bmatrix}[1 \ \ 0] + \begin{bmatrix}0\\1\end{bmatrix}[0 \ \ 1]\right)$$

$$= \frac{1}{2} I$$

$$C_2 = \frac{1}{N_2}\sum_{x_i\in\omega_2} (\vec{x}_i - \vec{m}_2)(\vec{x}_i - \vec{m}_2)^T = 2I$$

$$|C_1| = \frac{1}{4} \qquad |C_2| = 4$$

$$C_1^{-1} = 2I \qquad C_2^{-1} = \frac{1}{2} I$$

$d_1 - d_2 = 0$

$$\cancel{\ln P(\omega_1)} - \frac{1}{2}\ln|C_1| - \frac{1}{2}(\vec{x}-\vec{m}_1)^T C_1^{-1}(\vec{x}-\vec{m}_1)$$
$$\cancel{-\ln P(\omega_2)} + \frac{1}{2}\ln|C_2| + \frac{1}{2}(\vec{x}-\vec{m}_2)^T C_2^{-1}(\vec{x}-\vec{m}_2) = 0$$

$$-\frac{1}{2}\ln\frac{1}{4} - \frac{1}{\cancel{2}}\vec{x}^T(\cancel{2}I)\vec{x} + \frac{1}{2}\ln 4 + \frac{1}{2}\vec{x}^T\left(\frac{1}{2}I\right)\vec{x} = 0$$

$$0.693 - \vec{x}^T\vec{x} + 0.693 + \frac{1}{4}\vec{x}^T\vec{x} = 0$$

$$1.386 - \frac{3}{4}\vec{x}^T\vec{x} = 0$$

$$1.386 - \frac{3}{4}x_1^2 - \frac{3}{4}x_2^2 = 0 \quad / \cdot \frac{4}{3}$$

$$x_1^2 + x_2^2 = 1.848$$