RU

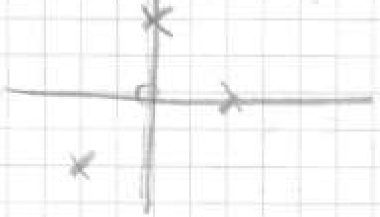
potencijalne funkcije

Bad

$$\omega_1 = \left\{ [0,0]^T \right\}$$

$$\omega_2 = \left\{ [0,1]^T, [1,0]^T, [-1,-1]^T \right\}$$ f-ja potencijala:

$$K(\vec{x}, \vec{x}_k) = \frac{1}{1 + \|x - x_k\|^2}$$



① $d_0(\vec{x}) = 0 \leq 0$

$$d_1(\vec{x}) = d_0(\vec{x}) + K(\vec{x}, \vec{x}_1) = 0 + \frac{1}{1 + \left[(x_1 - x_{11})^2 + (x_2 - x_{12})^2 \right]} =$$

$$= \frac{1}{1 + x_1^2 + x_2^2}$$

② $d_1(\vec{x}_2) = \frac{1}{1+0+1} = \frac{1}{2} \geq 0$

$$d_2(\vec{x}) = d_1(\vec{x}) - K(\vec{x}, \vec{x}_2) = \frac{1}{1 + x_1^2 + x_2^2} - \frac{1}{1 + x_1^2 + (x_2 - 1)^2}$$

③ $d_3 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \geq 0$

$$d_3(\vec{x}) = d_2(\vec{x}) - K(\vec{x}, x_3) = d_2(\vec{x}) - \frac{1}{1 + (x_1 - 1)^2 + x_2^2}$$

④ $d_3(\vec{x}_4) = 0 \geq 0$

$$d_4(\vec{x}) = d_3(\vec{x}) - \frac{1}{1 + (x_1 + 1)^2 + (x_2 + 1)^2}$$

$$d_4(\vec{x}) = \frac{1}{1 + x_1^2 + x_2^2} - \frac{1}{1 + x_1^2 + (x_2 - 1)^2} - \frac{1}{1 + (x_1 - 1)^2 + x_2^2} - \frac{1}{1 + (x_1 + 1)^2 + (x_2 + 1)^2}$$

$d_4(\vec{x}_5) = -\frac{1}{3}$

(5) $d_5(\vec{x}) = d_4(\vec{x}) + K(\vec{x}, \vec{x}_4)$

(6) $d(\vec{x}_6) = -\frac{1}{2}$ ✓

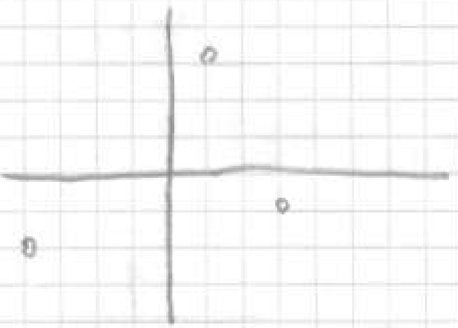
(7) $d(x_7) = -\frac{1}{2}$ ✓

(8) $d(\vec{x}_8) = -\frac{2}{3}$ ✓

(9) $d(\vec{x}_9) = \frac{2}{3}$ ✓

$\boxed{\text{zad}}$ KL - transformacija

$$\{[-4,-2]^T, [1,3]^T, [3,-1]^T\}$$



$$\vec{m} = \frac{1}{N} \sum \vec{x}_i = \frac{1}{3}\left(\begin{bmatrix}-4\\-2\end{bmatrix} + \begin{bmatrix}1\\3\end{bmatrix} + \begin{bmatrix}3\\-1\end{bmatrix}\right) = \begin{bmatrix}0\\0\end{bmatrix}$$

kovarijacijska matrica

$$K = \frac{1}{N-1} \sum (\vec{x}_i - \vec{m})(\vec{x}_i - \vec{m})^T =$$

matrica ne skalar

$$= \frac{1}{2}\left(\begin{bmatrix}-4\\-2\end{bmatrix}[-4\ -2] + \begin{bmatrix}1\\3\end{bmatrix}[1\ 3] + \begin{bmatrix}3\\-1\end{bmatrix}[3\ -1]\right)$$

$$= \frac{1}{2}\left(\begin{bmatrix}16 & 8\\8 & 4\end{bmatrix} + \begin{bmatrix}1 & 3\\3 & 9\end{bmatrix} + \begin{bmatrix}9 & -3\\-3 & 1\end{bmatrix}\right) = \begin{bmatrix}13 & 4\\4 & 7\end{bmatrix}$$

tražimo $\vec{w}$...

$$K\vec{w} = \lambda \vec{w}$$

$$(K - \lambda I)\vec{w} = 0$$

$$|K - \lambda I| = 0$$

$$\left|\begin{bmatrix}13 & 4\\4 & 7\end{bmatrix} - \begin{bmatrix}\lambda & 0\\0 & \lambda\end{bmatrix}\right| = 0$$

$$\begin{vmatrix}13-\lambda & 4\\4 & 7-\lambda\end{vmatrix} = 0$$

$$(13-\lambda)(7-\lambda)-16=0$$

$$\lambda^2 - 20\lambda + 75 = 0$$

$$\lambda_{1,2} =$$

$$(\lambda-5)(\lambda-15)=0$$

$$\underline{\underline{\lambda_1 = 5}} \qquad \boxed{\underline{\underline{\lambda_2 = 15}}}$$

vect

$$\begin{bmatrix} 13-15 & 4 \\ 4 & 7-15 \end{bmatrix} \vec{w} = 0$$

$$\begin{bmatrix} -2 & 4 \\ 4 & -8 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = 0$$

$$\left.\begin{array}{l} 2w_1 + 4w_2 = 0 \\ 4w_1 - 8w_2 = 0 \end{array}\right\}$$

$$w_1 = 2w_2$$

$$\vec{w} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$\boxed{\text{2nd}}$ $P(A|B) = \dfrac{P(B|A)\cdot P(A)}{P(B)}$

$$P(\omega_1|\vec{x}) = \frac{P(\vec{x}|\omega_1)\, P(\omega_1)}{P(\vec{x})}$$

$\vec{x} \in \omega_1$ ako $P(\omega_1|\vec{x}) > P(\omega_j|\vec{x}), \ \forall j \neq 1$

$x \in \omega_1$ ako $\dfrac{p(\vec{x}|\omega_i)\cdot p(\omega_i)}{\cancel{\ }} > \dfrac{P(\vec{x}|\omega_j)\cdot p(\omega_j)}{\cancel{\ }}, \ \forall j \neq i$

$$P(x) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$$

$$p(x) = \frac{1}{(2\pi)^{n/2}\, |C_i|^{1/2}}\, e^{-\frac{1}{2}(\vec{x}-\vec{m}_i)^T \cdot C_i^{-1}(\vec{x}-\vec{m}_i)} = p(\vec{x}|\omega_i)$$

$$C_i = \frac{1}{N_i} \sum_{x \in \omega_i} (\vec{x}-\vec{m}_i)(\vec{x}-\vec{m}_i)^T$$

$$\ln\left[p(\vec{x}|\omega_i)\cdot p(\omega_i)\right] = \ln\left(p(\vec{x}|\omega_i)\right) + \ln p(\omega_i) =$$

$$= \ln(p(\omega_i)) - \cancel{\frac{1}{2}\ln(2\pi)} - \frac{1}{2}\ln|C_i| - \frac{1}{2}\left[(\vec{x}-\vec{m}_i)^T C_i^{-1}(\vec{x}-\vec{m}_i)\right]$$

ne bitno jer je konst

$$d_i = \ln(p(\omega_i)) - \frac{1}{2}\ln|C_i| - \frac{1}{2}\left[(\vec{x}-\vec{m}_i)^T C_i^{-1}(\vec{x}-\vec{m}_i)\right]$$

-slučajevi:

(1) $C_i = C$

$$d_i = \ln(p(\omega_i)) - \frac{1}{2}\left[\cancel{\vec{x}^T C \vec{x}} - 2\vec{x}^T C(\vec{m}_i + \vec{m}_i^T C \ldots\right.$$

$$d_i = \ln p(\omega_i) + \vec{x}^T C^{-1}\vec{m}_i - \frac{1}{2}\vec{m}_i^T \cdot C^{-1}\vec{m}_i$$

(2) $C_i = \sigma^2 I$

$$C_i^{-1} = \frac{1}{\sigma^2} I$$

$$d_i(\vec{x}) = \ln p(\omega_i) - \frac{1}{2\sigma^2}\|\vec{x} - \vec{m}_i\|^2 = \ln p(\omega_i) + \frac{1}{2\sigma^2}\Big(\cancel{\vec{x}^T\vec{x}} +$$

$$+ 2\vec{x}^T\vec{m}_i + \vec{m}_i^T\vec{m}_i\Big) =$$

$$= \ln p(\omega_i) + \frac{1}{\sigma^2}\vec{x}^T\vec{m}_i - \frac{1}{2\sigma^2}\vec{m}_i^T \cdot \vec{m}_i$$

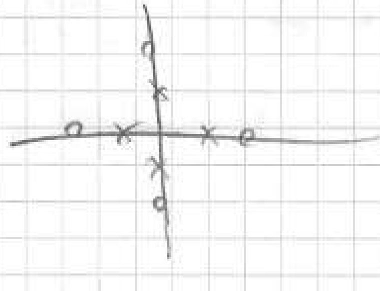
(3) $C = \sigma^2 I = p(\omega_i)\ \forall i, j$

$$d_i(\vec{x}) = -\|\vec{x} - \vec{m}_i\|^2 = \vec{x}^T\vec{m}_i - \frac{1}{2}\vec{m}_i^T\vec{m}_i$$

prim.

$$\omega_1 = \left\{ (-1,0)^T, (0,-1)^T, (1,0)^T, (0,1)^T \right\}$$

$$\omega_2 = \left\{ (-2,0)^T, (0,-2)^T, (2,0)^T, (0,2)^T \right\}$$



$$p(\omega_1) = \frac{N_1}{N} = \frac{N_1}{N_1+N_2} = \frac{1}{2}$$

$$p(\omega_2) = \frac{1}{2}$$

$$\vec{m}_1 = \frac{1}{N_1} \sum x_i = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\vec{m}_2 = \vec{0}$$

$$C_1 = \frac{1}{N_1} \sum_{\vec{x} \in \omega_1} (\vec{x} - \vec{m}_1)(\vec{x} - \vec{m}_1)^T = \frac{1}{4}\left(\begin{bmatrix} -1 \\ 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \end{bmatrix} + \right.$$

$$\left. + \begin{bmatrix} 0 \\ -1 \end{bmatrix} \begin{bmatrix} 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} \right) =$$

$$= \frac{1}{4}\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right) =$$

$$= \frac{1}{2} I$$

$$C_2 = 2I$$

$$d_i = \cancel{\ln(p(\omega_i))} - \frac{1}{2} \ln|C_i| - \frac{1}{2}\left[(\vec{x} - \vec{m}_i)^T C^{-1} (\vec{x} - \vec{m}_i)^T \right]$$

$$|C_1| = \begin{vmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{vmatrix} = \frac{1}{4} \qquad C_1^{-1} = 2I$$

$$|C_2| = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4 \qquad C_2^{-1} = \frac{1}{2} I$$

$$d_1 - d_2 = 0$$

$$-\frac{1}{2}\ln|C_1| - \frac{1}{2}(\vec{x}-\vec{m_1})^T C_1^{-1}(\vec{x}-\vec{m_1})$$

$$+\frac{1}{2}\ln|C_2| + \frac{1}{2}(\vec{x}-\vec{m_2})^T C_2^{-1}(\vec{x}-\vec{m_2}) = 0$$

$$0.693 - \vec{x}^T\vec{x} + 0.693 + \frac{1}{4}\vec{x}^T\vec{x} = 0$$

$$1.386 - \frac{3}{4}\vec{x}^T\vec{x} = 0$$

$$1.386 - \frac{3}{4}(x_1^2 + x_2^2) = 0$$

$$x_1^2 + x_2^2 = 1.848$$

$$r = 1.36$$