Četiri razreda dvodimenzionalnih uzoraka zadana su svojim matricama raspršenja, središtima i brojem uzoraka u razredu

$S_1 = \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix}$ $m_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$S_2 = \begin{bmatrix} 20 & 20 \\ 20 & 20 \end{bmatrix}$ $m_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$S_3 = \begin{bmatrix} 20 & 20 \\ 20 & 20 \end{bmatrix}$ $m_3 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$S_4 = \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix}$ $m_4 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

Pronaći vektor **w** koji daje optimalnu projekciju ovakvih uzoraka u smislu Fisherovog kriterija.

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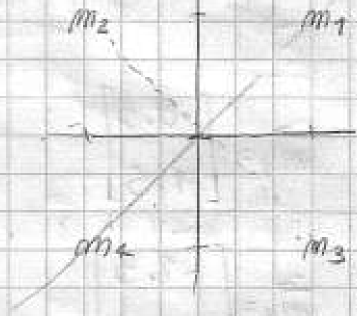
$$S_1=\begin{bmatrix}10 & 10\\ 10 & 10\end{bmatrix}\quad S_2=\begin{bmatrix}20 & 20\\ 20 & 20\end{bmatrix}\quad S_3=\begin{bmatrix}20 & 20\\ 20 & 20\end{bmatrix}\quad S_4=\begin{bmatrix}10 & 10\\ 10 & 10\end{bmatrix}$$

$$\vec{m}_1=\begin{bmatrix}1\\ 1\end{bmatrix}\quad \vec{m}_2=\begin{bmatrix}-1\\ 1\end{bmatrix}\quad \vec{m}_3=\begin{bmatrix}1\\ -1\end{bmatrix}\quad \vec{m}_4=\begin{bmatrix}-1\\ -1\end{bmatrix}$$



$n_{1234}=5$

1)
$$S_W=\sum_{i=1}^{4} S_i=\begin{bmatrix}60 & 60\\ 60 & 60\end{bmatrix}$$

$$\vec{m}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$S_B=\sum_{i=1}^{N} n_i(\vec{m}_i-\vec{m})(\vec{m}_i-\vec{m})^T$$

$$S_B=5\cdot\left(\begin{bmatrix}1\\ 1\end{bmatrix}\begin{bmatrix}1 & 1\end{bmatrix}+\begin{bmatrix}-1\\ 1\end{bmatrix}\begin{bmatrix}-1 & 1\end{bmatrix}+\begin{bmatrix}1\\ -1\end{bmatrix}\begin{bmatrix}1 & -1\end{bmatrix}+\begin{bmatrix}-1\\ -1\end{bmatrix}\begin{bmatrix}-1 & -1\end{bmatrix}\right)$$

$$=5\cdot\left(\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}+\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}+\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}+\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\right)$$

$$=5\cdot\begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix}=\begin{bmatrix}20 & 0\\ 0 & 20\end{bmatrix}$$

$$(S_B-\lambda S_W)\cdot\vec{w}=\emptyset$$

$$\det(S_B-\lambda S_W)=0$$

$$S_B-\lambda S_W=\begin{bmatrix}20 & 0\\ 0 & 20\end{bmatrix}-\begin{bmatrix}60\lambda & 60\lambda\\ 60\lambda & 60\lambda\end{bmatrix}=\begin{bmatrix}20-60\lambda & -60\lambda\\ -60\lambda & 20-60\lambda\end{bmatrix}$$

$$\det=(20-60\lambda)^2-(-60\lambda)^2=0$$

$$400-2400\lambda+\cancel{3600\lambda^2}-\cancel{3600\lambda^2}=0$$

$$2400\lambda=400 \Rightarrow \lambda=\frac{4}{24}=\frac{1}{6}$$

$$\begin{bmatrix}10 & -10\\ -10 & 10\end{bmatrix}\cdot\begin{bmatrix}w_1\\ w_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}\qquad w_1=w_2 \Rightarrow \vec{w}=\begin{bmatrix}\frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2}\end{bmatrix}$$

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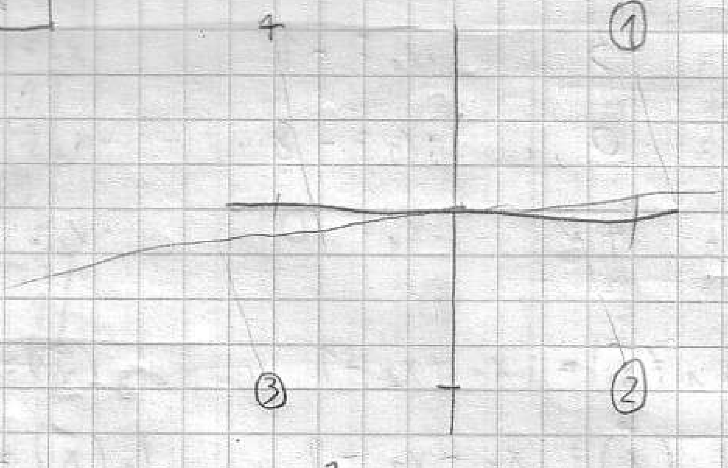
MOJ ZADATAK



1: [1 1]

2: [1 -1]

3: [-1 -1]

4: [-1 1]

$$S_B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix} \qquad S_W = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 8-\lambda & -\lambda \\ -\lambda & 8-\lambda \end{bmatrix}$$

$$(8-\lambda)^2 - \lambda^2 = 0$$

$$8 - 16\lambda = 0 \Rightarrow \lambda = \frac{1}{2}$$

$$\begin{bmatrix} 7.5 & -0.5 \\ -0.5 & 7.5 \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = 0$$

$7.5 w_1 = 0.5 w_2$ $\quad 15 w_1 = w_2$

$$w = \begin{bmatrix} 15 \\ 1 \end{bmatrix}$$