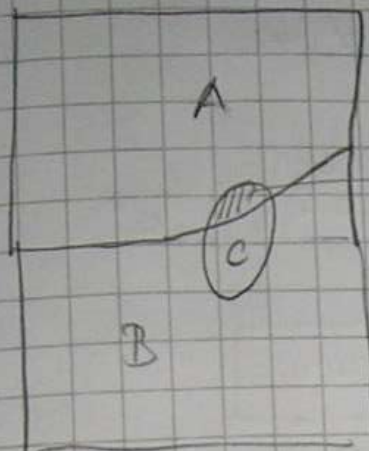


PROSTOR

DOGAĐAJA

BAYESOV TEOREM

13.1.2009.



manje poznato događaj je
1 (uijek u kvadrat)

$P(AC)$ - da su +
događaji A i C

$P(A|C)$ - vjer. A ako
znemo da se dogodio C

$$P(A|C) = \frac{P(AC)}{P(C)}$$

$$P(C|A) = \frac{P(AC)}{P(A)}$$

$$P(A|C) \cdot P(C) = P(AC)$$

$$P(C|A) \cdot P(A) = P(AC)$$

$$P(A|C) \cdot P(C) = P(C|A) \cdot P(A)$$

$$P(A|C) = \frac{P(C|A) P(A)}{P(C)}$$

BAYESOV
TEOREM (izvedeno
iz geom.
razmjeranja)

A razmatramo se w_1 , a C se X

$$P(w_1 | x) = \frac{P(x|w_1) \cdot P(w_1)}{P(x)}$$

Manji de ako imamo
da on pripada w_1 uzorak

- x želimo svrstati u ovaj razred za koji vjerujemo da pripada tom razredu ujedno

$$\bar{x} \in W_i \quad P(W_i | \bar{x}) > P(W_j | \bar{x}) \quad \forall j \neq i$$

$$\frac{P(x|W_i) \cdot P(W_i)}{P(x)} > \frac{P(x|W_j) \cdot P(W_j)}{P(x)} \quad \forall j \neq i$$

$$P(x|W_i) \cdot P(W_i) > P(x|W_j) \cdot P(W_j)$$

minimiziraj
gor. pogrešku

Ophvatio pravilo u
vjerojatnost. pogrešno

- uvide u izbita ta vjerojatnost ugo da u
- u dojadi ude odredac greske
- definiraj se rizik i nekakvu fun. gubitka

gubitak koji
uistoj ali je uvek
iz tog a mi smo
se svrstali u W_i

Očelivati gubitak $\Rightarrow R(x) = \sum_{j=1}^c \lambda(x_i|W_j) P(W_j|x)$

- Smanjivati rizik

$$x \in W_i \quad R(x_i | \bar{x}) < R(x_j | \bar{x}) \quad \forall j \neq i$$

gubitak vjerojatnost
da pripada
tom razredu

Gaussian D.

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

σ^2 .. varijanca
w.. sr. vrijed.
uzorak

- kod i varijable skalar

- višedimenzionalno:

$$p(\vec{x} | w_i) = \frac{1}{(2\pi)^{d/2} |\Sigma_i|^{1/2}} \exp\left(-\frac{1}{2} (\vec{x} - \mu_i)^T \Sigma_i^{-1} (\vec{x} - \mu_i)\right)$$

determinant
od

kovar. matrice

kovarijantna
matrica uzoraka →
očelivuje

$$\Sigma_i = \frac{1}{N_i} \sum (\vec{x} - \mu_i)(\vec{x} - \mu_i)^T$$

$p(w_i | x)$

- da odaberemo elemente logaritmiramo tj
(to možemo jer $a > b \Rightarrow \log a > \log b$
tako da će i dalje vrijediti pravilo)

$$\ln(p(w_i | x)) = \ln(p(\vec{x} | w_i) \cdot p(w_i)) =$$

$$\ln(p(w_i)) + \ln(p(\vec{x} | w_i))$$

Ovo se pomoću formule pretvara

$$= \ln(p(w_i)) - \frac{n}{2} \ln(2\pi) - \frac{1}{2} \ln |\Sigma_i| - \frac{1}{2} [(\vec{x} - \mu_i)^T \Sigma_i^{-1} (\vec{x} - \mu_i)]$$

$g_i(x) \rightarrow$ kvadratne $\eta^2 \rightarrow$ najvišje potence
 \rightarrow pomoč Gaussove h.c. najvišje + moir
 dobi kvadratne η^2 (kod Bayesove te inaci
 moir dobi proizvodne dozvol)

Pojednostavljen

$$c=2 \quad \Sigma_i = \begin{pmatrix} \sigma_i^2 & 0 \\ 0 & \sigma_i^2 \end{pmatrix}$$

dec. funkcije \rightarrow gornja izvedba razreda

$$p(w_i) = p(w_j) \quad \ln \left(\frac{p(w_i)}{p(w_j)} \right) = \ln 1 = 0 \quad \rightarrow x_0 = \frac{1}{2} (\vec{\mu}_i + \vec{\mu}_j)$$

$$(\vec{x} - \vec{\mu}_i)^T \Phi \Lambda^{-1} \Phi^T (\vec{x} - \vec{\mu}_i) =$$

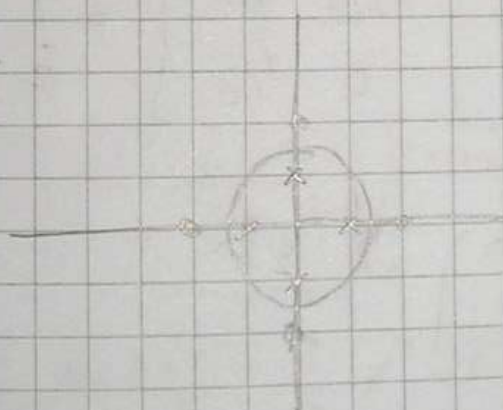
$$(\Phi^T \vec{x} - \Phi^T \vec{\mu}_i)^T \Lambda^{-1} (\Phi^T \vec{x} - \Phi^T \vec{\mu}_i)$$

- PCA (Kolumnen - Loewe Transf.) \rightarrow trebe
sami

PRIMER BAYESOV KLASIFIKATOR

$$\omega_1 = \{(-1, 0)^T, (0, -1)^T, (1, 0)^T, (0, 1)^T\}$$

$$\omega_2 = \{(-2, 0)^T, (0, -2)^T, (2, 0)^T, (0, 2)^T\}$$



$x \in \omega_1$
 $o \in \omega_2$

- izjava hoda populacije uzorke \rightarrow ako nije

Zadamo \Rightarrow $N_1 = 4$ $N = N_1 + N_2$
 $N_2 = 4$

$$P(\omega_1) = \frac{N_1}{N} = \frac{4}{8} = \frac{1}{2}$$

$$P(\omega_2) = \frac{1}{2}$$

\vec{m} (inacé μ) \Rightarrow středisk vzorku (rozrod)

$$\vec{m}_1 = \frac{1}{N_1} \sum_{x_i \in w_1} x_i = \frac{1}{4} \left(\begin{bmatrix} -1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\vec{m}_2 =$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

kovarijancijne matrica C (inacé Σ)

$$C_1 = \frac{1}{N_1} \sum_{x_i \in w_1} (\vec{x}_i - \vec{m}_1) (\vec{x}_i - \vec{m}_1)^T =$$

$$= \frac{1}{4} \left(\begin{bmatrix} -1 \\ 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \end{bmatrix} + \dots + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} \right) =$$

$$= \frac{1}{4} \left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \dots + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right) =$$

$$= \frac{1}{4} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} = \frac{1}{2} I$$

$$C_2 = \dots = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 2I$$

$$d_m(\vec{x}) = \ln(p(w_i)) - \frac{1}{2} \ln |C_i| -$$

$$\frac{1}{2} [(\vec{x} - \vec{w}_i)^T C_i^{-1} (\vec{x} - \vec{w}_i)]$$

$$|C_1| = \begin{vmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{vmatrix} = \frac{1}{4}$$

$$|C_2| = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4$$

$$C_1^{-1} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2I$$

$$C_2^{-1} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} = \frac{1}{2}I$$

$$d_1 - d_2 = 0$$

$$-\frac{1}{2} \ln |C_1| - \frac{1}{2} (\vec{x} - \vec{w}_1)^T C_1^{-1} (\vec{x} - \vec{w}_1) +$$

$$\frac{1}{2} \ln |C_2| + \frac{1}{2} (\vec{x} - \vec{w}_2)^T C_2^{-1} (\vec{x} - \vec{w}_2) = 0$$

$$-\frac{1}{2} \ln \left(\frac{1}{4}\right) - \frac{1}{2} \vec{x}^T \cdot 2I \vec{x} + \frac{1}{2} \ln 4 + \frac{1}{2} \vec{x}^T \frac{1}{2}I \vec{x} = 0$$

$$0,693 - \vec{x}^T \vec{x} + 0,693 + \frac{1}{4} \vec{x}^T \vec{x} = 0$$

$$1,386 - \frac{3}{4} \vec{x}^T \vec{x} = 0$$

2. Zusammenhang
der
zu
produzierte
vier. rain,
i'nerre ne

$$1,386 - \frac{3}{4} [x_1 \ x_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$1,386 - \frac{3}{4} (x_1^2 + x_2^2) = 0 \quad | \cdot \frac{4}{3}$$

$$1,848 - x_1^2 - x_2^2 = 0$$

$$\boxed{x_1^2 + x_2^2 = 1,848}$$

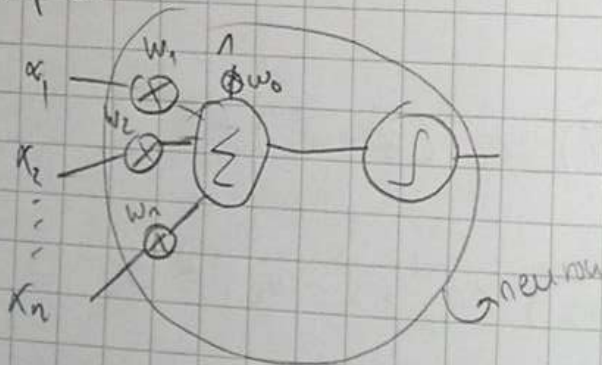
↓
Granice
↓
Jedn. kr. $r = 1,36$

PRIMER

NEURON SUE

PREZE

- perceptron



BACKPROPAGATION ALG.

- gradient do

izlazi do

$$\delta_0 = y_0 (1 - y_0) (d_0 - y_0)$$

sluveni sloj:

$$\delta_n = y_n (1 - y_n) \sum_{l \in (\text{downstream } n)} w_{ln} \delta_l$$

PRIMER → ključari (dodati) (iterativni alg.)

M → Obavezno dijeliti!!!!

$$\Delta w = \eta \cdot \delta \cdot w$$

- alg. se zamjenjuje kada se produkt

izlazi za ulazni i sledeći posrednik,

celo je da sledeći se zamjenjuje, a

ovo se idejno dalje

- bolje je da imamo jedan skup, a

projeckuju na drugi (skup za validaciju)

