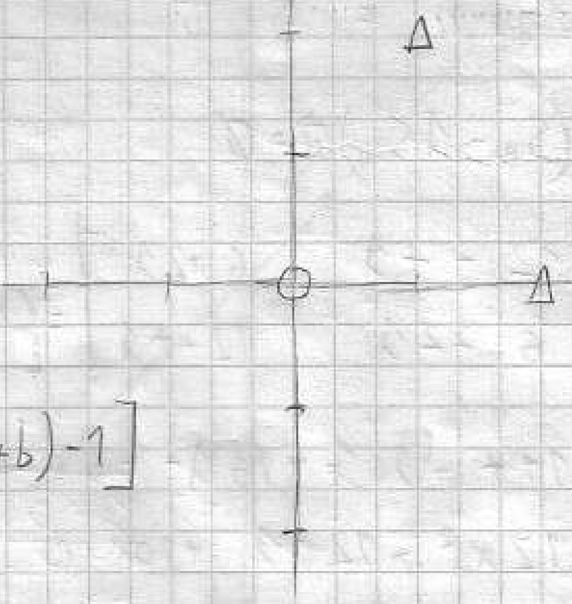
③ SVM - PRIMARNI PROBLEM

$\omega_1 = \{ [0,0]^T \}$ ○

$\omega_2 = \{ [2,0]^T, [1,2]^T \}$ △



<u>KRITERIJSKA FUNKCIJA</u>

$$J(\vec{w}, b, \vec{\lambda}) = \frac{1}{2}\vec{w}^T\vec{w} - \sum_{i=1}^{n} \lambda_i \left[d_i(\vec{w}^T\vec{x} + b) - 1 \right]$$

<u>UVJETI</u>

a) $\lambda_i \left[d_i(\vec{w}^T\vec{x} + b) - 1 \right] = \emptyset$

b) $\lambda_i \geq \emptyset$

c) $(\vec{w}^T\vec{x} + b) - 1 \geq \emptyset$

1: UVRSTITI UZORKE U KRITERIJSKU FUNKCIJU

$$J(\vec{w}, b, \vec{\lambda}) = \frac{1}{2}\vec{w}^T\vec{w} - \lambda_1(b-1) + \lambda$$
$$- \lambda_2(-2w_1 - b - 1)$$
$$- \lambda_3(-w_1 - 2w_2 - b - 1)$$

$$J(\vec{w}, b, \vec{\lambda}) = \frac{w_1^2 + w_2^2}{2} - \lambda_1 b + 1$$
$$+ 2\lambda_2 w_1 + \lambda_2 b + \lambda_2$$
$$+ \lambda_3 w_1 + 2\lambda_3 w_2 + \lambda_3 b + \lambda_3$$

2: PARCIJALNO DERIVIRAJ

$$\frac{\partial J}{\partial w_1} = w_1 + 2\lambda_2 + \lambda_3 = \emptyset \Rightarrow w_1 = -2\lambda_2 - \lambda_3$$

$$\frac{\partial J}{\partial w_2} = w_2 + 2\lambda_3 = \emptyset \Rightarrow w_2 = -2\lambda_3$$

$$\frac{\partial J}{\partial b} = -\lambda_1 + \lambda_2 + \lambda_3 = \emptyset \Rightarrow \lambda_1 = \lambda_2 + \lambda_3$$

3. UVRSTITI UVJETE

$$\lambda_1(b-1)=0 \qquad \Rightarrow \lambda_1 b = 1$$

$$\lambda_2(-2w_1-b-1)=0 \qquad 2\lambda_2 w_1 + \lambda_2 b + \lambda_2 = 0$$

$$\lambda_3(-w_1-2w_2-b-1)=0 \qquad \lambda_3 w_1 + 2\lambda_3 w_2 + \lambda_3 b + 1 = 0$$

(I) $\lambda_1 = \lambda_2 = \lambda_2 = 0 \Rightarrow$ TRIVIJALNO RJEŠENJE, NE GLEDA SE

(II) $\lambda_1 > 0 \quad \lambda_2 = 0 \quad \lambda_3 = \lambda_1$

$w_1 = -\lambda_1$

$w_2 = -2\lambda_1$

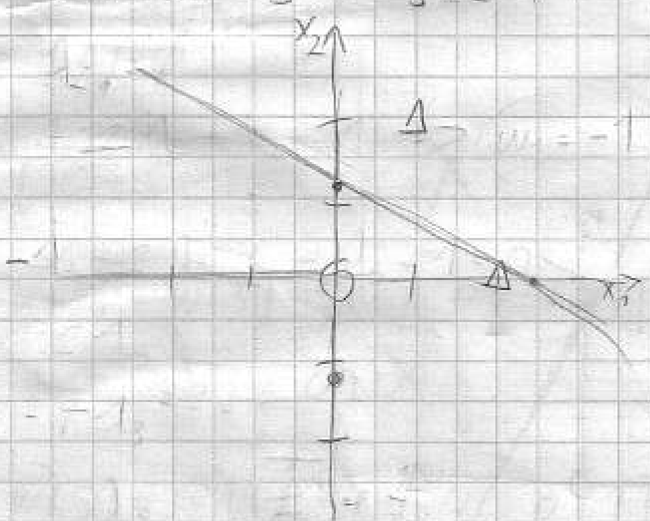
$b-1=0 \Rightarrow b=1$

$-w_1-2w_2-b-1=0$

$\lambda_1 + 4\lambda_1 - 1 - 1 = 0$

$$\lambda_1 = \frac{2}{5} \Rightarrow w = \begin{bmatrix} -\frac{2}{5} \\ -\frac{4}{5} \end{bmatrix} \quad b=1$$

$$d(\bar{x}) = -\frac{2}{5}x_1 - \frac{4}{5}x_2 + 1 = 0$$



RJEŠENJE NIJE DOBRO JER NE UZIMA JEDAN UZORAK U OBZIR KOD IZGRADNJE SVM-A, VRTIMO DRUGE VARIJANTE

(III) $\lambda_1 \geq 0, \lambda_2 > 0, \lambda_3 > 0$, DOBIT ĆEMO SLIČNU STVAR KO (II)

(IV) $\lambda_1 > 0 \quad \lambda_2 > 0 \quad \lambda_3 > 0$

$b - 1 = 0 \Rightarrow b = 1$

$2w_1 + b + 1 = 0 \Rightarrow -4\lambda_2 - 2\lambda_3 + 2 = 0 \quad /:2$

$w_1 + 2w_2 + b + 1 = 0 \qquad -2\lambda_2 - \lambda_3 - 4\lambda_3 + 2 = 0$

$-4\lambda_3 = -1 \Rightarrow \lambda_3 = \frac{1}{4}$

$-4\lambda_2 - \frac{1}{2} + 2 = 0$

$-4\lambda_2 = -\frac{3}{2} \quad /:-4$

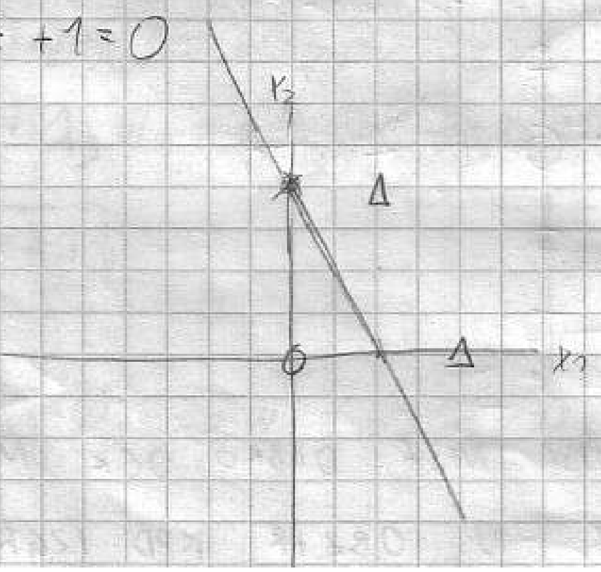
$\boxed{\lambda_2 = \frac{3}{8}}$

$\lambda_1 = \lambda_2 + \lambda_3 = \frac{5}{8}$

$w_1 = -2\lambda_2 - \lambda_3 = -\frac{6}{8} - \frac{2}{8} = -1$

$w_2 = -2\lambda_3 = -\frac{1}{2}$

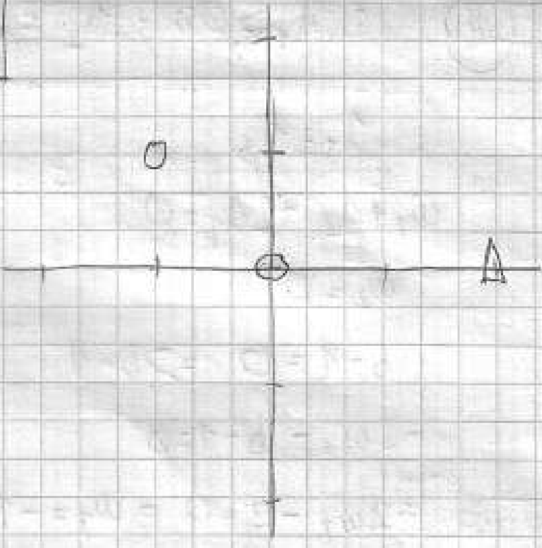
$d(\vec{x}) = -x_1 - \frac{x_2}{2} + 1 = 0$



SVM AUDITORNE fer2.net @mala

$\omega_1 = \{[0\ 0]^T, [-1\ 1]^T\}$ O

$\omega_2 = \{[2\ 0]^T\}$ Δ



RJEŠAVAMO KAO PRIMARNI PROBLEM

1)
$$J(\vec{w}, b, \vec{\lambda}) = \frac{1}{2}\vec{w}^T\vec{w} - \lambda_1(b-1) - \lambda_2(-w_1+w_2+b-1) - \lambda_3(-2w_1-b-1)$$

$$J(\vec{w}, b, \vec{\lambda}) = \frac{w_1^2+w_2^2}{2} - \lambda_1 b + \lambda_1 + \lambda_2 w_1 - \lambda_2 w_2 - \lambda_2 b + \lambda_2 + 2\lambda_3 w_1 + \lambda_3 b + \lambda_3$$

2)
$$\frac{\partial J}{\partial w_1} = w_1 + \lambda_2 + 2\lambda_3 = 0$$

$$\frac{\partial J}{\partial w_2} = w_2 - \lambda_2 = 0$$

$$\frac{\partial J}{\partial b} = -\lambda_1 - \lambda_2 + \lambda_3 = 0 \Rightarrow \lambda_1 = -\lambda_2 + \lambda_3$$

3)
$$\lambda_1(b-1) = 0$$
$$\lambda_2(-w_1+w_2+b-1) = 0$$
$$\lambda_3(-2w_1-b-1) = 0$$

4) VRTIT SLUČAJEVE λ

I $\lambda_1 = \lambda_2 = \lambda_3 = 0 \Rightarrow$ TRIVIJALNO RJEŠENJE, ODBACUJEMO

(II) $\lambda_1 > 0$, $\lambda_2 = 0$ $\lambda_3 > 0$,

$\lambda_1 = \lambda_3$

$w_1 + \lambda_2 + 2\lambda_1 = 0$

$w_2 = 0$

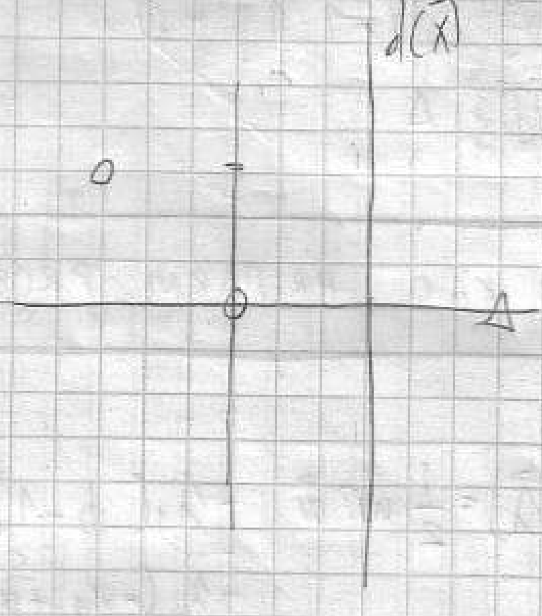
$b - 1 = 0 \Rightarrow b = 1$

$-2w_1 - b - 1 = 0$

$-2w_1 - 2 = 0 \Rightarrow w_1 = -1$

$\vec{w} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ $b = 1$

$d(\vec{x}) = -x_1 + 1$



ISPRAVNA GRANICA RAZREDA

ALI NE I OPTIMALNA

(III) $\lambda_1 > 0$ $\lambda_2 > 0$ $\lambda_3 = 0$

$\lambda_1 = \lambda_2$

$w_1 + \lambda_2 = 0$

$w_2 - \lambda_2 = 0$ $\Big\} + \Rightarrow w_1 = -w_2$

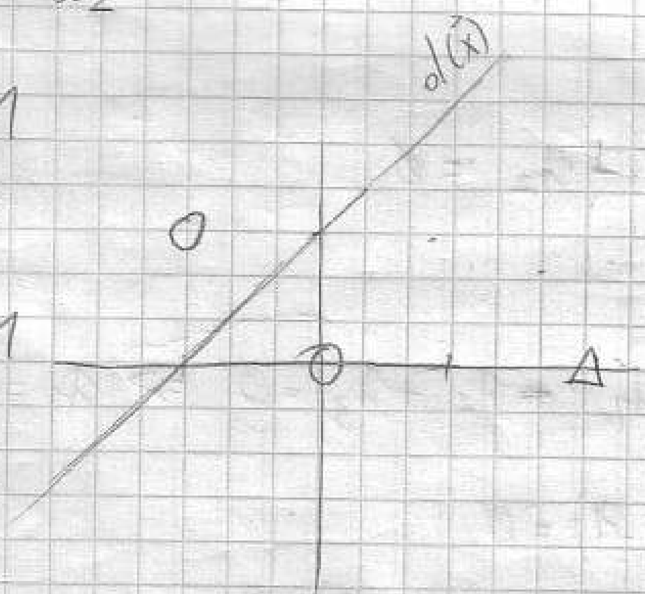
$b - 1 = 0$ $\Rightarrow b = 1$

$-w_1 + w_2 + b - 1 = 0$

$-w_1 - w_1 - 2 = 0 \Rightarrow w_1 = 1$

$w = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $b = 1$

$d(\vec{x}) = x_1 - x_2 + 1$



NE KLASIFICIRA DOBRO

RAZREDE

IV $\quad \lambda_1 > 0, \; \lambda_2 > 0 \quad \lambda_3 > 0$

$$w_1 + \lambda_2 + 2\lambda_3 = 0$$

$$w_2 - \lambda_2 = 0$$

$$\lambda_1 = \lambda_2 + \lambda_3$$

$$b - 1 = 0 \quad \Rightarrow b = 1$$

$$-w_1 + w_2 + b - 1 = 0$$

$$-2w_1 - b - 1 = 0$$

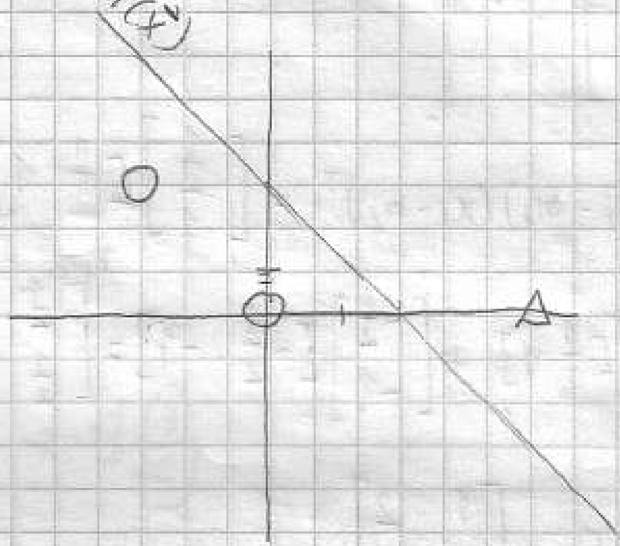
$$-2w_1 - 2 = 0$$

$$w_1 = -1$$

$$-1 + w_2 = 0$$

$$w_2 = -1$$

$$\vec{w} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \quad b = 1$$



$$d(\vec{x}) = -x_1 - x_2 + 1$$

PROVJERA UVJETA:

$$\vec{w}^T [x] + b \geq 1$$

$$\begin{bmatrix} -1 \\ -1 \end{bmatrix} [0 \; 0] + 1 \geq 1 \checkmark$$

$$\begin{bmatrix} -1 \\ -1 \end{bmatrix} [-1 \; -1] + 1 \geq 1 \checkmark$$

$$\begin{bmatrix} -1 \\ -1 \end{bmatrix} [2 \; 0] + 1 \geq 1 \checkmark$$

DAKLE GRANICA JE OPTIMALNA!