ALGORITAM PERCEPTRONA

ZADATAK

kriterijska fja : $J(\vec{w}, \vec{x}) = \frac{|\vec{w}^T \vec{x}|}{\vec{x}^T \vec{x}} \left[\vec{w}^T \vec{x} - |\vec{w}^T \vec{x}|\right]$

-na temelju ove fje izvesti algoritam učenja za 2 razrede

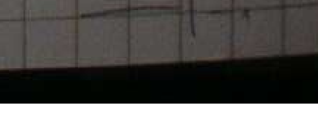
-optimizacija grad. spustom

$$\vec{w}(k+1) = \vec{w}(k) - c \left\{ \frac{\partial J(\vec{w}, \vec{x})}{\partial \vec{w}} \right\}\Big|_{\vec{w} = \vec{w}(k)}$$

-nađi derivaciju
-svaki izraz pojednostavi

$$J(\vec{w}, \vec{x}) = \frac{1}{\vec{x}^T \vec{x}} \left[|\vec{w}^T \vec{x}| (\vec{w}^T \vec{x}) - |\vec{w}^T \vec{x}|^2 \right]$$

$$\frac{d}{dx}(|x|) = \operatorname{sgn} x$$



$$\frac{\partial J}{\partial \vec{w}} = \frac{1}{\vec{x}^T \vec{x}} \Big[sgn(\vec{w}^T \vec{x}) \cdot \vec{x} \cdot (\vec{w}^T \vec{x}) + |\vec{w}^T \vec{x}| \vec{x} - 2 \cdot |\vec{w}^T \vec{x}| \cdot sgn(\vec{w}^T \vec{x}) \vec{x} \Big] =$$

za pojednostavit

$x \cdot sgn(x) = |x|$

$|x| \cdot sgn(x) = x$

$$= \frac{1}{\vec{x}^T \vec{x}} \Big[\vec{x} \cdot |\vec{w}^T \vec{x}| + \vec{x} |\vec{w}^T \vec{x}| - 2 \vec{x} (\vec{w}^T \vec{x}) \Big] =$$

$$= \frac{1}{\vec{x}^T \vec{x}} \Big[2 \vec{x} |\vec{w}^T \vec{x}| - 2 \vec{x} (\vec{w}^T \vec{x}) \Big] =$$

$$\frac{2 \vec{x}}{\vec{x}^T \vec{x}} \Big[|\vec{w}^T \vec{x}| - (\vec{w}^T \vec{x}) \Big]$$

ne može se kratiti !!!

$$\vec{w}(k+1) = \vec{w}(k) - c \cdot \frac{2 \vec{x}}{\vec{x}^T \vec{x}} \Big[|\vec{w}^T \vec{x}| - (\vec{w}^T \vec{x}) \Big]$$

$$\vec{w}(k+1) = \begin{cases} \vec{w}(k) & \text{za } \vec{w}^T \vec{x} > 0 \\ \vec{w}(k) - 4 \frac{c \vec{x}}{\vec{x}^T \vec{x}} \underbrace{(\vec{w}^T \vec{x})}_{\vec{w}(k)} & \vec{w}^T \vec{x} \leq 0 \end{cases}$$

ZADATAK

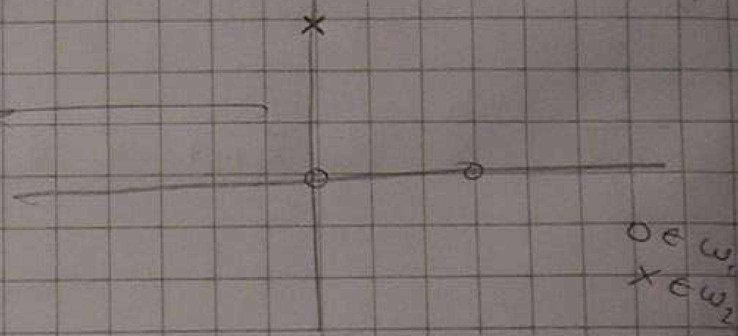
$\omega_1 = \{(0,0)^T, (1,0)^T\}$

$\omega_2 = \{(0,1)^T\}$

- treba naći decizijsku fju koja ispravno

razvrstava uzorke

uzorci su linearno razdvojivi



$$c = 1 \qquad \vec{w}(1) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad x_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad x_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \Rightarrow x_3 = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}$$

1. $\vec{w}(1)^T \cdot \vec{x}(1) = [0\ 0\ 0] \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$ KOREKCIJA

$$\vec{w}(2) = \vec{w}(1)^T + \vec{x}(1) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

2. $\vec{w}(2) \cdot \vec{x}(2) = [0\ 0\ 1] \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 1 > 0 \Rightarrow$ ISPRAVNO

$\vec{w}(3) = w(2)$

3) $\vec{w}(3) \cdot \vec{x}(3) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} = -1$

$\vec{w}(4) = \vec{w}(3) + \vec{x}(3) = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$

4) $\vec{w}(4)^T \cdot \vec{x}(1) = \begin{bmatrix} 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$

$\vec{w}(5) = \vec{w}(4) + \vec{x}(1) = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

5) $\vec{w}(5)^T \cdot \vec{x}(2) = \begin{bmatrix} 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 1$

$\vec{w}(6) = \vec{w}(5)$

6) $\vec{w}(6)^T \vec{x}(3) = \begin{bmatrix} 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} = 0$

$w(7) = \vec{w}(6) + \vec{x}(3) = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$

7) $w(7)^T \cdot \vec{x}(1) = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} = 0$

$w(8) = w(7) + x(7) = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$

8 $\quad w(8)^T \cdot x(2) = \begin{bmatrix} 0 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 1$ //

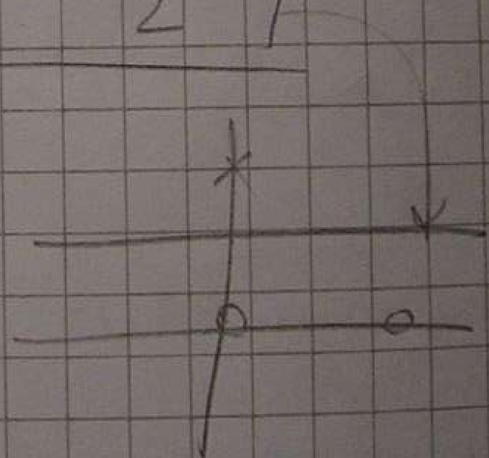
9 $\quad w(9)^T \cdot x(3) = \begin{bmatrix} 0 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} = 1$ //

10 $\quad w(10)^T \cdot x(1) = \begin{bmatrix} 0 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 1$ //

AAAA AGH!

$$0x_1 - 2x_2 + 1 = 0$$

$$\boxed{x_2 = \frac{1}{2}}$$

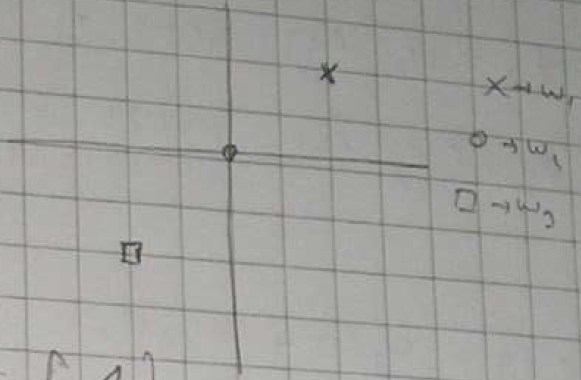


3 razreda, za svaki imamo po jedan uzorak

$\omega_1 = \{ (1, 1)^T \}$

$\omega_2 = \{ (0, 0)^T \}$

$\omega_3 = \{ (-1, -1)^T \}$



$$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad x_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad x_3 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

$$\vec{w}_1(1) = \vec{w}_2(1) = \vec{w}_3(1) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad c = 1$$

(1) $\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$d_1 = \vec{w}_1^T(1) \cdot \vec{x}(1) = 0$ → ovaj bi trebao biti > 0

$d_2 = \vec{w}_2^T(1) \cdot \vec{x}(1) = 0$ } ove isto treba korigirati

$d_3 = \vec{w}_3^T(1) \cdot \vec{x}(1) = 0$

$\vec{w}_1(2) = \vec{w}_1(1) + \vec{x}(1) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

→ jer uzorak pripada razredu koji se korigira

$\vec{w}_2(2) = \vec{w}_2(1) - \vec{x}(1) = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$

$\vec{w}_3(2) = \vec{w}_3(1) - \vec{x}(1) = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$

② $\vec{x}(2) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$d_1 = \vec{w}_1^T(2) \cdot \vec{x}(2) = 1$

$d_2 = w_2^T(2)\ x(2) = -1$

$d_3 = w_3^T(2)\ x(2) = -1$

} sve treba korigirati

$w_1(3) = w_1(2) - x(2) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$w_2(3) = w_2(2) + x(2) = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$

$w_2(3) = w_3(2) - x(2) = \begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix}$

③ $x_3 = x(3) = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

$d_1 = \vec{w}_1^T(3) \cdot \vec{x}(3) = -2$ → pošto d_1 manji od d_3 ne treba kor.

$d_2 = \vec{w}_2^T(3)\ \vec{x}(3) = 2$

$d_3 = \vec{w}_3^T(3)\ \vec{x}(3) = 0$

} samo ov treba korigirati

→ trebao bi biti najveći

$$w_1(4) = w_1(3) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$w_2(4) = w_2(3) - x(3) = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$w_3(4) = w_2(3) + x(3) = \begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix}$$

(4)

$$x(4) = x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$d_1 = \vec{w}_1(4)\, x(4) = 2$$ ✓ → najveći → ne treba uopće korekcije

$$d_2 = \vec{w}_2(4)\, x(4) = -1$$

$$d_3 = \vec{w}_3(4)\, x(4) = -3$$

(5) $\vec{x}(5) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $w_1(5) = w_1(4)$

$$d_1 = w_1(5) \cdot x(5) = 0$$

$$-1$$

$$-1$$

sve treba korigirati

$$w_1(6) = w_1(5) - x(5) = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$w_2(6) = w_2(5) + x(5) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$w_3(6) = w_3(5) - x(5) = \begin{bmatrix} -2 \\ -2 \\ -2 \end{bmatrix}$$

⑥ $x(6) = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$

$d_1 = -3$

$d_2 = 0$

$d_3 = 2$ ← želimo da ji najveći pa ne treb. konj.

⑦ $x(7) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$d_1 = 1$ → treba biti najveći → ne treb. konj.

$d_2 = 0$

$d_3 = -6$

⑧ $x(8) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$d_1 = -1$

$d_2 = 0$ → najveći → u treba uvrstit → kraj

$d_3 = -2$

$d_1 = x_1 + x_2 - 1$

$d_2 = 0$

$d_3 = -2x_1 - 2x_2 + 2$

Klasifikator se vede kot odločit kod ij

$d_1 = d_3$

$$x_1 + x_2 - 1 = -2x_1 - 2x_2 - 2$$

$$3x_1 + 3x_2 + 1 = 0 \quad // \rightarrow \text{granica}$$

FLD

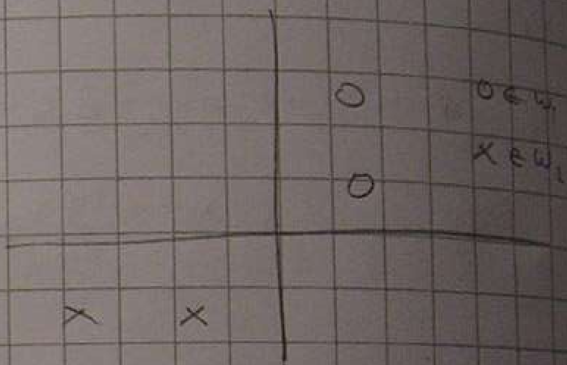
$$J(\vec{w}) = \frac{(\tilde{m}_1 - \tilde{m}_2)^2}{\tilde{\sigma}_1^2 + \tilde{\sigma}_2^2}$$

kriterijska f-ja koju želimo maksimizirati

ZADATAK

$\omega_1 = \{ (1,1)^T, (1,2)^T \}$

$\omega_2 = \{ (-1,-1)^T, (-2,-1)^T \}$



$\vec{x}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \vec{x}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

ne treba proširivati

$\vec{x}_3 = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \qquad \vec{x}_4 = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$

$$\vec{m}_1 = \frac{1}{2}(\vec{x}_1 + \vec{x}_2) = \frac{1}{2}\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1{,}5 \end{bmatrix}$$

$$\vec{m}_2 = \frac{1}{2}(\vec{x}_3 + \vec{x}_4) = \frac{1}{2}\left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} -1{,}5 \\ -1 \end{bmatrix}$$

$S_1 =$ matrica raspršenja razreda ω_1

$$= \sum_{\vec{x}_i \in \omega_1} (\vec{x}_i - \vec{m}_1)(\vec{x}_i - \vec{m}_1)^T =$$

$$= \left(\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1,5 \end{pmatrix} \right) \left(\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1,5 \end{pmatrix} \right)^{-1} +$$

$$\left(\begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 1,5 \end{pmatrix} \right) \left(\begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 1,5 \end{pmatrix} \right)^{-1} =$$

$$= \begin{bmatrix} 0 \\ -0,5 \end{bmatrix} \begin{bmatrix} 0 & -0,5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0,5 \end{bmatrix} \begin{bmatrix} 0, & 0,5 \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0,25 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0,25 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0,5 \end{bmatrix}$$

$$S_2 = \sum_{x_i \in \omega_2} (\vec{x}_i - \vec{m}_2)(\vec{x}_i - \vec{m}_2)^T =$$

$$= \begin{bmatrix} 0,5 & 0 \\ 0 & 0 \end{bmatrix}$$

matrica raspršenja razreda ω_2

$$S_W = S_1 + S_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0,5 \end{bmatrix} + \begin{bmatrix} 0,5 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0,5 & 0 \\ 0 & 0,5 \end{bmatrix}$$

$S_B \rightarrow$ matrica raspršenja između razreda =

$$(\vec{m}_1 - \vec{m}_2)(\vec{m}_1 - \vec{m}_2)^T = \begin{bmatrix} 6,25 & 6,25 \\ 6,25 & 6,25 \end{bmatrix}$$

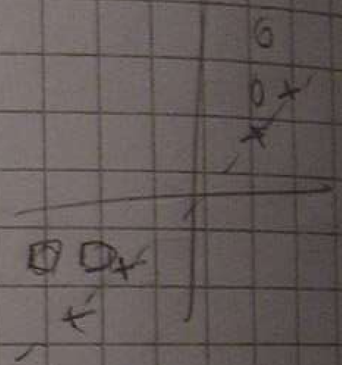
$\vec{w} = S_w^{-1}(\vec{m}_1 - \vec{m}_2)$ → pokazuje nam smjer pravca

$S_w^{-1} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ za dijagonalnu matricu

$$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2.5 \\ 2.5 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

smjer pravca na koji projiciramo uzorke

nije vektor: ~~$5x_1 + 5x_2 = 0$~~ !



$y_1 = \vec{w}^T \cdot x_1 = [5\ 5] \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 10$

$y_2 = 15$

$y_3 = -10$

$y_4 = -15$

vidimo da se razdvajaju

projekcije na pravac

2. NAČIN RJEŠAVANJA (ovo ima više od

λ razredi ... može biti ovaj postupak)

$$\lambda \cdot S_W \vec{w} = S_B \vec{w}$$

$$\lambda \vec{w} = (S_W^{-1} S_B) \vec{w}$$

$$A = (S_W^{-1} S_B) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{pmatrix} 6,25 & 6,25 \\ 6,25 & 6,25 \end{pmatrix}$$

$$= \begin{bmatrix} 12,5 & 12,5 \\ 12,5 & 12,5 \end{bmatrix}$$

$$\begin{vmatrix} 12,5 - \lambda & 12,5 \\ 12,5 & 12,5 - \lambda \end{vmatrix} = 0$$

$$\cancel{12,5^2} - 25\lambda + \lambda^2 - \cancel{12,5^2} = 0$$

$$\lambda \cdot (\lambda - 25) = 0$$

$$\lambda = 0$$

$$\lambda = 25$$

uzima se rješenje koje ima svoj. vrijednost veću

svojstveni vektor matrice A

$$A\vec{x} = \lambda \cdot \vec{x}$$

svojstvena vrijednost λ

$$(A - \lambda I)\vec{x} = \vec{0}$$

$$\vec{x} = (A - \lambda I)^{-1} \cdot \vec{0}$$

trivijalno rj. $\vec{x} = \vec{0}$

$$|A - \lambda I| \neq 0$$

$$\boxed{|A - \lambda I| = 0}$$

determ. mora biti 0 onda neće ... i onda se dobivaju netrivijalni svojstveni vektori

polinom n-tog stupnja po λ

$$(A - \lambda I) \cdot \vec{x} = \vec{0}$$

$$\left(\begin{pmatrix} 12,5 & 12,5 \\ 12,5 & 12,5 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right) \cdot \vec{x} = \vec{0}$$

$$\begin{pmatrix} -12,5 & 12,5 \\ 12,5 & -12,5 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \vec{0}$$

$$\left. \begin{aligned} -12,5\, x_1 + 12,5\, x_2 &= 0 \\ 12,5\, x_1 - 12,5\, x_2 &= 0 \end{aligned} \right\} \text{2 puta identične jdž.}$$

– iz njih dobijemo

$\underline{\underline{x_1 = x_2}}$ → uzmemo bilo koji jer nas zanima samo smjer pravca

npr. $x_1 = 1$

$\vec{w} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ← smjer pravca

isto smo dobili drugim postupkom

CLAPACK