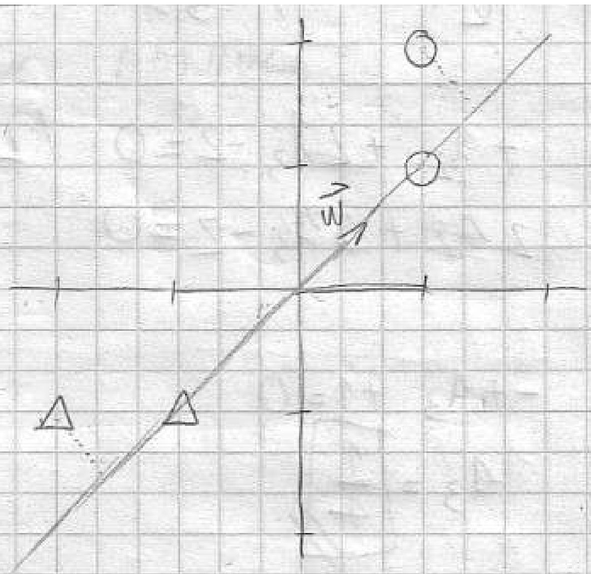
1) FLD SA 2 RAZREDA

$w_1 = \{[1,1]^T, [1,2]^T\} \Rightarrow \bigcirc$

$w_2 = \{[-1,-1]^T, [-2,-1]^T\} \Rightarrow \Delta$



1. KORAK: MATRICE RASPRŠENJA

$$\vec{m}_1 = \frac{1}{n_1}\sum_{i=0}^{n_1} \vec{x}_i = \frac{1}{2}\left(\begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}1\\2\end{bmatrix}\right) = \begin{bmatrix}1\\1.5\end{bmatrix}$$

$$\vec{m}_2 = \frac{1}{n_2}\sum \vec{x}_{1i} = \frac{1}{2}\left(\begin{bmatrix}-1\\-1\end{bmatrix} + \begin{bmatrix}-2\\-1\end{bmatrix}\right) = \begin{bmatrix}-1.5\\-1\end{bmatrix}$$

$$S_1 = \sum_{x \in w_1} (\vec{x}_i - \vec{m}_1)(\vec{x}_i - \vec{m}_1)^T$$

$$= \left(\begin{bmatrix}1\\1\end{bmatrix} - \begin{bmatrix}1\\1.5\end{bmatrix}\right)\left(\begin{bmatrix}1\\1\end{bmatrix} - \begin{bmatrix}1\\1.5\end{bmatrix}\right)^T + \left(\begin{bmatrix}1\\2\end{bmatrix} - \begin{bmatrix}1\\1.5\end{bmatrix}\right)\left(\begin{bmatrix}1\\2\end{bmatrix} - \begin{bmatrix}1\\1.5\end{bmatrix}\right)^T =$$

$$= \begin{bmatrix}0\\-0.5\end{bmatrix}\begin{bmatrix}0 & -0.5\end{bmatrix} + \begin{bmatrix}0\\0.5\end{bmatrix}\begin{bmatrix}0 & 0.5\end{bmatrix} =$$

$$= \begin{bmatrix}0 & 0\\0 & 0.25\end{bmatrix} + \begin{bmatrix}0 & 0\\0 & 0.25\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0.5\end{bmatrix}$$

$$S_2 = \sum_{x \in \omega_2} (x_i - m_2)(x - m_2)^T =$$

$$= \left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} - \begin{bmatrix} -1.5 \\ -1 \end{bmatrix}\right)\left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} - \begin{bmatrix} -1.5 \\ -1 \end{bmatrix}\right)^T + \left(\begin{bmatrix} -2 \\ -1 \end{bmatrix} - \begin{bmatrix} -1.5 \\ -1 \end{bmatrix}\right)\left(\begin{bmatrix} -2 \\ -1 \end{bmatrix} - \begin{bmatrix} -1.5 \\ -1 \end{bmatrix}\right)^T$$

$$= \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}\begin{bmatrix} 0.5 & 0 \end{bmatrix} + \begin{bmatrix} -0.5 \\ 0 \end{bmatrix}\begin{bmatrix} 0.5 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 0.25 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0.25 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix}$$

$$S_W = S_1 + S_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 \end{bmatrix} + \begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

2.) METODA ZA 2 RAZREDA

$$\vec{w} = S_W^{-1}(\vec{m}_1 - \vec{m}_2)$$

$$S_W^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\vec{w} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2.5 \\ 2.5 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

3) NORMALIZACIJA RJESENJA

$$\vec{w}' = \frac{\vec{w}}{|\vec{w}|} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

ALT. METODA $\Rightarrow$

1. METODA ZA C RAZREDA ZA ZADATAK S 2 RAZREDA

$$\lambda S_W \vec{w} = S_B \vec{w} \quad / \cdot S_W^{-1}$$

$$\lambda \cdot \vec{w} = S_W^{-1} \cdot S_B \cdot \vec{w}$$

$$\left(S_W^{-1} \cdot S_B - \lambda I\right) \cdot \vec{w} = \emptyset$$

$$S_W^{-1} \cdot S_B - \lambda I = \emptyset$$

$$S_B = \sum_{i=1}^{2} n_i (\vec{m}_i - \vec{m})(\vec{m}_i - \vec{m})^T$$

$$\vec{m} = \frac{1}{\cancel{4}_2}(\cancel{2}\vec{m}_1 + \cancel{2}\vec{m}_2) = \frac{1}{2}\left(\begin{bmatrix}1\\1.5\end{bmatrix} + \begin{bmatrix}-1.5\\-1\end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix}-0.5\\0.5\end{bmatrix} = \begin{bmatrix}-0.25\\0.25\end{bmatrix}$$

$$S_B = 2 \cdot \left(\begin{bmatrix}1\\1.5\end{bmatrix} - \begin{bmatrix}-0.25\\0.25\end{bmatrix}\right)\left(\begin{bmatrix}1\\1.5\end{bmatrix} - \begin{bmatrix}-0.25\\0.25\end{bmatrix}\right)^T + 2\left(\begin{bmatrix}-1.5\\-1\end{bmatrix} - \begin{bmatrix}-0.25\\0.25\end{bmatrix}\right)\left(\cdots\right)^T$$

$$= 2 \cdot \begin{bmatrix}1.25\\1.25\end{bmatrix}\begin{bmatrix}1.25 & 1.25\end{bmatrix} + 2\begin{bmatrix}-1.25\\-1.25\end{bmatrix}\begin{bmatrix}-1.25 & -1.25\end{bmatrix}$$

$$= 2\begin{bmatrix}1.25^2 & 1.25^2\\1.25^2 & 1.25^2\end{bmatrix} + 2\begin{bmatrix}1.25^2 & 1.25^2\\1.25^2 & 1.25^2\end{bmatrix}$$

$$S_B = \begin{bmatrix}6.25 & 6.25\\6.25 & 6.25\end{bmatrix}$$

$$\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}\begin{bmatrix}6.25 & 6.25\\6.25 & 6.25\end{bmatrix} - \begin{bmatrix}\lambda & 0\\0 & \lambda\end{bmatrix} = 0$$

$$\begin{bmatrix}12.5-\lambda & 12.5\\12.5 & 12.5-\lambda\end{bmatrix} = 0$$

$$12.5 - \lambda + 12.5 = 0$$
$$12.5 + 12.5 - \lambda = 0 \quad \Rightarrow \boxed{\lambda = 25}$$

$$\left(\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 6.25 & 6.25 \\ 6.25 & 6.25 \end{bmatrix} - \begin{bmatrix} 25 & 0 \\ 0 & 25 \end{bmatrix}\right)\cdot \vec{w} = 0$$

$$\begin{bmatrix} -12.5 & 12.5 \\ 12.5 & -12.5 \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = 0$$

$$-12.5\, w_1 + 12.5\, w_2 = 0$$

$$12.5\, w_1 - 12.5\, w_2 = 0$$

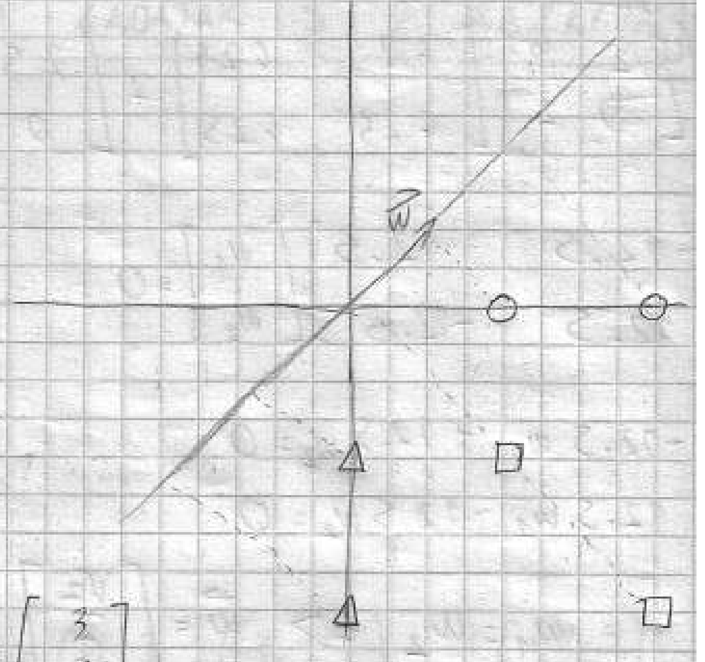
$$w_1 = w_2 \Rightarrow \vec{w} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

② FLD ZA C RAZREDA

$\omega_1 = \{ [2,0]^T, [4,0]^T \}$ O

$\omega_2 = \{ [0,-2]^T, [0,-4]^T \}$ △

$\omega_3 = \{ [2,-2]^T, [4,-4]^T \}$ □



1. KORAK: MATRICE S_W i S_B

$$\vec{m}_1 = \begin{bmatrix} 3 \\ 0 \end{bmatrix} \quad \vec{m}_2 = \begin{bmatrix} 0 \\ -3 \end{bmatrix} \quad \vec{m}_3 = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$$

$$\vec{m} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} \begin{bmatrix} 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$$

$$S_3 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$S_W = S_1 + S_2 + S_3 = \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix} //$$

$$S_B = 2 \cdot \left(\begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} + \begin{bmatrix} -2 \\ -1 \end{bmatrix} \begin{bmatrix} -2 & -1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix} \right)$$

$$= 2 \cdot \left(\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \right) = 2 \cdot \begin{bmatrix} 6 & -3 \\ -3 & 6 \end{bmatrix} =$$

$$S_B = \begin{bmatrix} 12 & +6 \\ +6 & 12 \end{bmatrix} //$$

$$S_B \vec{w} = \lambda S_w \vec{w}$$

$$(S_B - \lambda S_w)\vec{w} = \emptyset$$

$$\begin{vmatrix} 12 - 4\lambda & 6 + 2\lambda \\ 6 + 2\lambda & 12 - 4\lambda \end{vmatrix} = 0$$

$$144 - 96\lambda + 16\lambda^2 - 36 - 24\lambda + 4\lambda^2 = 0$$

$$12x^2 - 120\lambda + 108 = 0 \Rightarrow \lambda_1 = 1, \lambda_2 = 9$$

$$(S_B - 9 S_w)\vec{w} = \emptyset$$

$$\begin{bmatrix} 12 - 36 & 6 + 18 \\ 6 + 18 & 12 - 36 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = 0$$

$$\begin{matrix} -24 w_1 + 24 w_2 = 0 \\ 24 w_1 - 24 w_2 = 0 \end{matrix} = w_1 = w_2 \Rightarrow \vec{w} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

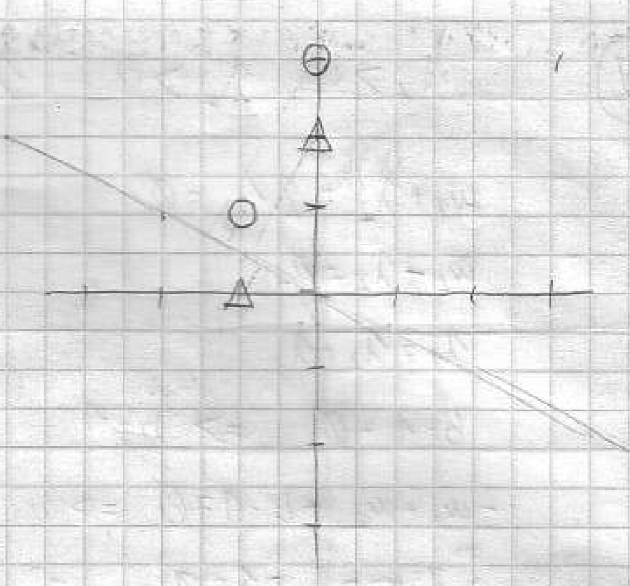
3. (3 boda) Za skup uzoraka $\omega_2 = \{[0, 2]^T, [1, 0]^T\}$

$\omega_1 = \{ [0, 3]^T, [-1, 1]^T \}$,

$\omega_2 = \{ [0, 2]^T, [-1, 0]^T \}$,

naći pravac koji daje optimalnu projekciju tih uzoraka u smislu maksimizacije raspršenja između razreda i minimizacije raspršenja unutar razreda.
Nacrtati pravac, uzorke i njihove projekcije.

FLD 1.MI 2008/2009



$\omega_1 = \{[0\ 3]^T, [-1\ 1]^T\}$ $\circ$

$\omega_2 = \{[0\ 2]^T, [-1\ 0]^T\}$ Δ

1)

$$\vec{m}_1 = \begin{bmatrix} -\frac{1}{2} \\ 2 \end{bmatrix}$$

$$\vec{m}_2 = \begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix}$$

$$S_1 = \sum_{x \in \omega_1} (x_i - m_i)(x_i - m_i)^T = \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 1 \end{bmatrix} + \begin{bmatrix} -\frac{1}{2} \\ -1 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix} + \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 1 \\ 0 & 2 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 1 \end{bmatrix} + \begin{bmatrix} -\frac{1}{2} \\ -1 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix} + \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 1 \\ 1 & 2 \end{bmatrix}$$

$$S_W = S_1 + S_2 = \begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$$

$$\vec{w} = S_W^{-1}(\vec{m}_1 - \vec{m}_2)$$

$$S_W^{-1} = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 1 & 4 & 0 & 1 \end{array}\right] = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 2 & -1 & 1 \end{array}\right] = \left[\begin{array}{cc|cc} 1 & 0 & 2 & -1 \\ 0 & 2 & -1 & 1 \end{array}\right] \cdot \frac{1}{2} =$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 2 & -1 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \end{array}\right]; \quad S_W^{-1} = \begin{bmatrix} 2 & -1 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\vec{w} = \begin{bmatrix} 2 & -1 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ \frac{1}{2} \end{bmatrix}$$

(7 bodova) Za skup uzoraka

$\omega_1 = \{ [0, 0]^T, [1, 3]^T \}$,

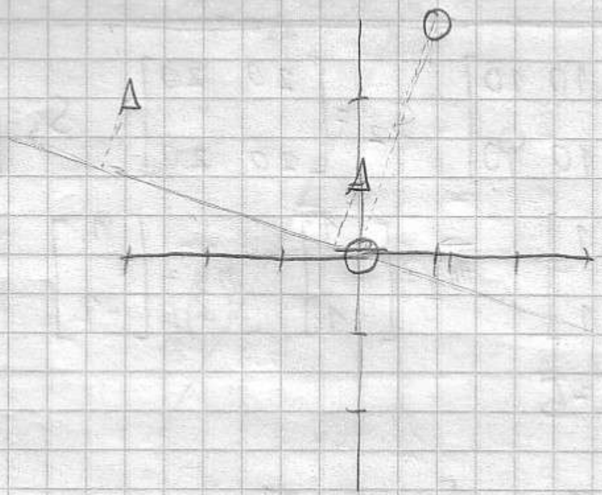
$\omega_2 = \{ [0, 1]^T, [-3, 2]^T \}$,

naći pravac koji daje optimalnu projekciju tih uzoraka u smislu maksimizacije raspršenja između razreda i minimizacije raspršenja unutar razreda.
Nacrtati pravac, uzorke i njihove projekcije.

FLD ZA 2 RAZREDA ZAVRŠNI 2009/2010

$\omega_1 = \{[0\ 0]^T, [1\ 3]^T\}$ o

$\omega_2 = \{[0\ 1]^T, [-3\ 2]^T\}$ Δ



1) $\vec{m}_1 = \begin{bmatrix} 0.5 \\ 1.5 \end{bmatrix}$

$\vec{m}_2 = \begin{bmatrix} -1.5 \\ 1.5 \end{bmatrix}$

$S_1 = \begin{bmatrix} -0.5 \\ -1.5 \end{bmatrix} [-0.5\ \ -1.5] + \begin{bmatrix} 0.5 \\ 1.5 \end{bmatrix} [0.5\ \ 1.5] = \begin{bmatrix} 0.25 & 0.75 \\ 0.75 & 2.25 \end{bmatrix} \cdot 2 = \begin{bmatrix} 0.5 & 1.5 \\ 1.5 & 4.5 \end{bmatrix}$

$S_2 = \begin{bmatrix} 1.5 \\ -0.5 \end{bmatrix} [1.5\ \ -0.5] + \begin{bmatrix} -1.5 \\ 0.5 \end{bmatrix} [-1.5\ \ 0.5] = \begin{bmatrix} 2.25 & -0.75 \\ -0.75 & 0.25 \end{bmatrix} + \begin{bmatrix} 2.25 & -0.75 \\ -0.75 & 0.25 \end{bmatrix} =$

$S_2 = \begin{bmatrix} 4.5 & -1.5 \\ -1.5 & 0.5 \end{bmatrix}$

$S_W = S_1 + S_2 = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

2) $\vec{w} = S_W^{-1} (\vec{m}_1 - \vec{m}_2) = \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{5} \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} \\ 0 \end{bmatrix}$

2. METODA

$$\vec{m} = \frac{\vec{m}_1 + \vec{m}_2}{2} = \begin{bmatrix} -0.5 \\ 1.5 \end{bmatrix}$$

$$S_B = 2\cdot\left(\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1 & 0\end{bmatrix} + \begin{bmatrix}-1\\0\end{bmatrix}\begin{bmatrix}-1 & 0\end{bmatrix}\right) = 2\cdot\left(\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} + \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\right) = \begin{bmatrix}4 & 0\\0 & 0\end{bmatrix}$$

$$S_W^{-1}\cdot S_B - \lambda I = 0$$

$$\begin{bmatrix}\frac{1}{5} & 0\\0 & \frac{1}{5}\end{bmatrix}\cdot\begin{bmatrix}4 & 0\\0 & 0\end{bmatrix} - \begin{bmatrix}\lambda & 0\\0 & \lambda\end{bmatrix} = 0$$

$$\begin{bmatrix}\frac{4}{5} & 0\\0 & 0\end{bmatrix} - \begin{bmatrix}\lambda & 0\\0 & \lambda\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix}\frac{4}{5}-\lambda & 0\\0 & -\lambda\end{bmatrix}\cdot w = \begin{bmatrix}0\\0\end{bmatrix}$$

$$\frac{4}{5} - \lambda = 0$$

$$-\lambda = 0$$

$$\begin{bmatrix}\frac{4}{5} & 0\\0 & 0\end{bmatrix}\cdot\begin{matrix}w_1\\w_2\end{matrix} = 0$$

$$\left(\frac{4}{5}-\lambda\right)\cdot(-\lambda) = 0$$

$$-\frac{4}{5}\lambda + \lambda^2 = 0$$

$$\lambda\left(\lambda - \frac{4}{5}\right) = 0$$

$$\lambda_1 = 0$$

$$\lambda_2 = \frac{4}{5}$$