1. Auditorne iz Raspoznavanja uzoraka

Zadatak 1.

Derivacija vektorskih funkcija

Izvasti algoritam učenja sustava za klasifikaciju s 2 razredaupotrebom gradijentnog spusta za zadanu kriterijsku funkciju

$$J(\vec{w}, \vec{x}, p) = \frac{1}{8\|\vec{x}\|^2}\left[\left(\vec{w}^T\vec{x} - p\right) - \left|\vec{w}^T\vec{x} - p\right|\right]^2$$

-gradijentni spust

$$\vec{w}(k+1) = \vec{w}(k) - c\left\{\frac{\partial J(\vec{w}, \vec{x}, p)}{\partial \vec{w}}\right\}$$

$$\frac{\partial J(\vec{w}, \vec{x}, p)}{\partial \vec{w}} = \frac{1}{4\|\vec{x}\|^2}\left[\left(\vec{w}^T\vec{x} - p\right) - \left|\vec{w}^T\vec{x} - p\right|\right] \cdot \left[\vec{x} - \operatorname{sgn}\left(\vec{w}^T\vec{x} - p\right) \cdot \vec{x}\right]$$

$$= \frac{1}{4\|\vec{x}\|^2}\left[\left(\vec{w}^T\vec{x} - p\right) \cdot \vec{x} + \left(\vec{w}^T\vec{x} - p\right) \cdot \operatorname{sgn}\left(\vec{w}^T\vec{x} - p\right) \cdot \vec{x} - \left|\vec{w}^T\vec{x} - p\right| \cdot \vec{x} - \left|\vec{w}^T\vec{x} - p\right| \cdot \operatorname{sgn}\left(\vec{w}^T\vec{x} - p\right) \cdot \vec{x}\right]$$

$$= \frac{1}{4\|\vec{x}\|^2}\left[2\cdot\left(\vec{w}^T\vec{x}-p\right)\cdot\vec{x}-2\cdot\left|\vec{w}^T\vec{x}-p\right|\cdot\vec{x}\right]$$

$$= \frac{1}{2\|\vec{x}\|^2}\left[\left(\vec{w}^T\vec{x}-p\right)-\left|\vec{w}^T\vec{x}-p\right|\right]\cdot\vec{x}$$

$$\vec{w}(k+1)=\vec{w}(k)-\frac{c}{2\|\vec{x}\|^2}\left[\left(\vec{w}^T\vec{x}-p\right)-\left|\vec{w}^T\vec{x}-p\right|\right]\cdot\vec{x}$$

$$\vec{w}(k+1)=\begin{cases}\vec{w}(k) & \text{za } \vec{w}^T\vec{x}>p\\ -\frac{c}{\|\vec{x}\|^2}\cdot\left(\vec{w}^T\vec{x}-p\right)\cdot\vec{x} & \text{za } \vec{w}^T\vec{x}\le p\end{cases}$$

Zadatak 2.

Fisher – 2 razreda – dvodimenzionalni uzorci

$$\omega_1=\left\{(1,1)^T,(1,2)^T\right\}$$
$$\omega_2=\left\{(-1,-1)^T,(-2,-1)^T\right\}$$

$$\vec{x}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \vec{x}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \vec{x}_3 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \vec{x}_4 = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

$$\vec{m}_1 = \frac{1}{2}(\vec{x}_1 + \vec{x}_2) = \frac{1}{2}\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1.5 \end{bmatrix}$$

$$\vec{m}_2 = \frac{1}{2}(\vec{x}_3 + \vec{x}_4) = \frac{1}{2}\left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} -1{,}5 \\ -1 \end{bmatrix}$$

$$S_1 = \sum_{\vec{x}_i \in \omega_1} (\vec{x}_i - \vec{m}_1)(\vec{x}_i - \vec{m}_1)^T$$

$$S_1 = \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1.5 \end{bmatrix}\right)\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1.5 \end{bmatrix}\right)^T + \left(\begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1.5 \end{bmatrix}\right)\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1.5 \end{bmatrix}\right)^T$$

$$S_1 = \begin{bmatrix} 0 \\ -0.5 \end{bmatrix} \begin{bmatrix} 0 & -0.5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.5 \end{bmatrix} \begin{bmatrix} 0 & 0.5 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0.25 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0.25 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 \end{bmatrix}$$

$$S_2 = \sum_{\vec{x}_i \in \omega_2} (\vec{x}_i - \vec{m}_2)(\vec{x}_i - \vec{m}_2)^T$$

$$S_2 = \left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} - \begin{bmatrix} -1.5 \\ -1 \end{bmatrix}\right)\left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} - \begin{bmatrix} -1.5 \\ -1 \end{bmatrix}\right)^T + \left(\begin{bmatrix} -2 \\ -1 \end{bmatrix} - \begin{bmatrix} -1.5 \\ -1 \end{bmatrix}\right)\left(\begin{bmatrix} -2 \\ -1 \end{bmatrix} - \begin{bmatrix} -1.5 \\ -1 \end{bmatrix}\right)^T$$

$$S_2 = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}\begin{bmatrix} 0.5 & 0 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}\begin{bmatrix} 0.5 & 0 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 0.25 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0.25 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix}$$

$$S_w = S_1 + S_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

$$S_w^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$S_B = (\vec{m}_1 - \vec{m}_2)(\vec{m}_1 - \vec{m}_2)^T$$

$$= \left(\begin{bmatrix} 1 \\ 1.5 \end{bmatrix} - \begin{bmatrix} -1.5 \\ -1 \end{bmatrix} \right) \left(\begin{bmatrix} 1 \\ 1.5 \end{bmatrix} - \begin{bmatrix} -1.5 \\ -1 \end{bmatrix} \right)^T$$

$$= \begin{bmatrix} 2.5 \\ 2.5 \end{bmatrix} \begin{bmatrix} 2.5 & 2.5 \end{bmatrix} = \begin{bmatrix} 6.25 & 6.25 \\ 6.25 & 6.25 \end{bmatrix}$$

$$\vec{w} = S_w^{-1}(\vec{m}_1 - \vec{m}_2)$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2.5 \\ 2.5 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$$\vec{y}_1 = \vec{w}^T \vec{x}_1 = \begin{bmatrix} 5 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 10$$

$$\vec{y}_2 = \vec{w}^T \vec{x}_2 = \begin{bmatrix} 5 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 15$$

$$\vec{y}_3 = \vec{w}^T \vec{x}_3 = \begin{bmatrix} 5 & 5 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = -10$$

$$\vec{y}_4 = \vec{w}^T \vec{x}_4 = \begin{bmatrix} 5 & 5 \end{bmatrix} \begin{bmatrix} -2 \\ -1 \end{bmatrix} = -15$$

drugi način

$$\lambda S_w \vec{w} = S_B \vec{w} \qquad /\mathrm{S}_\mathrm{w}^{-1}$$

$$\lambda \vec{w} = \left(\mathrm{S}_\mathrm{w}^{-1} S_B\right)\vec{w}$$

problem svojstvenih vektora

$$\lambda \vec{w} = A\vec{w}$$

$$\mathrm{A} = \mathrm{S}_\mathrm{w}^{-1} S_B$$

$$(\mathrm{A} - \lambda \mathrm{I})\vec{w} = \vec{0}$$

$$\vec{w} = (\mathrm{A} - \lambda \mathrm{I})^{-1}\vec{0}$$

da bi mogli izračunati $(\mathrm{A} - \lambda \mathrm{I})^{-1}$ mora biti $|\mathrm{A} - \lambda \mathrm{I}| \neq 0$, da bi dobili netrivijalno riješenje uvjet je $|\mathrm{A} - \lambda \mathrm{I}| = 0$, ovo je polinom n-tog stupnja, nul-točke su svojstvene vrijednosti

$$\mathrm{A} = \mathrm{S}_{\mathrm{w}}^{-1} S_B$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 6.25 & 6.25 \\ 6.25 & 6.25 \end{bmatrix} = \begin{bmatrix} 12.5 & 12.5 \\ 12.5 & 12.5 \end{bmatrix}$$

$$\left| \mathrm{A} - \lambda \mathrm{I} \right| = 0$$

$$= \begin{vmatrix} 12.5 - \lambda & 12.5 \\ 12.5 & 12.5 - \lambda \end{vmatrix}$$

$$= (12.5 - \lambda)^2 - 12.5^2$$

$$= \lambda(\lambda - 25) = 0$$

$$\lambda = 25$$

$$(\mathrm{A} - \lambda \mathrm{I})\vec{w} = \vec{0}$$

$$\begin{bmatrix} 12.5 - 25 & 12.5 \\ 12.5 & 12.5 - 25 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \vec{0}$$

$$\begin{bmatrix} -12.5 & 12.5 \\ 12.5 & -12.5 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \vec{0}$$

$$w_1 = w_2$$

$$\vec{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Zadatak 2.

Fisher – 3 razreda – dvodimenzionalni uzorci

$$\omega_1 = \{(2,0)^T, (4,0)^T\}$$
$$\omega_2 = \{(0,\text{-}2)^T, (0,\text{-}4)^T\}$$
$$\omega_3 = \{(2,\text{-}2)^T, (4,\text{-}4)^T\}$$

$$\vec{x}_1 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \vec{x}_2 = \begin{bmatrix} 4 \\ 0 \end{bmatrix}, \vec{x}_3 = \begin{bmatrix} 0 \\ -2 \end{bmatrix}, \vec{x}_4 = \begin{bmatrix} 0 \\ -4 \end{bmatrix}, \vec{x}_5 = \begin{bmatrix} 2 \\ -2 \end{bmatrix}, \vec{x}_6 = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$$

$$\vec{m}_1 = \frac{1}{2}(\vec{x}_1 + \vec{x}_2) = \frac{1}{2}\left(\begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$\vec{m}_2 = \frac{1}{2}(\vec{x}_3 + \vec{x}_4) = \frac{1}{2}\left(\begin{bmatrix} 0 \\ -2 \end{bmatrix} + \begin{bmatrix} 0 \\ -4 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ -3 \end{bmatrix}$$

$$\vec{m}_3 = \frac{1}{2}(\vec{x}_5 + \vec{x}_6) = \frac{1}{2}\left(\begin{bmatrix} 2 \\ -2 \end{bmatrix} + \begin{bmatrix} 4 \\ -4 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$$

$$S_1 = \sum_{\vec{x}_i \in \omega_1} (\vec{x}_i - \vec{m}_1)(\vec{x}_i - \vec{m}_1)^T$$

$$S_1 = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$S_2 = \sum_{\vec{x}_i \in \omega_2} (\vec{x}_i - \vec{m}_2)(\vec{x}_i - \vec{m}_2)^T$$

$$S_2 = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$$

$$S_3 = \sum_{\vec{x}_i \in \omega_3} (\vec{x}_i - \vec{m}_3)(\vec{x}_i - \vec{m}_3)^T$$

$$S_3 = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$S_w = S_1 + S_2 + S_3 = \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$$

$$S_B = \sum_{i=1}^{3} n_i \left(\vec{m}_i - \vec{m}\right)\left(\vec{m}_i - \vec{m}\right)^T$$

$$S_B = \begin{bmatrix} 12 & 6 \\ 6 & 12 \end{bmatrix}$$

drugi način

$$\lambda S_w \vec{w} = S_B \vec{w}$$

$$(S_B - \lambda S_w)\vec{w} = 0$$

$$\left\| \begin{bmatrix} 12 & 6 \\ 6 & 16 \end{bmatrix} - \lambda \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix} \right\| = 0$$

$$\begin{vmatrix} 12 - 4\lambda & 6 + 2\lambda \\ 6 + 2\lambda & 12 - 4\lambda \end{vmatrix} = 0$$

$$\lambda^2 - 10\lambda + 9 = 0$$

$$\lambda_1 = 9$$

$$\lambda_2 = 1$$

$$\lambda_1 = 9$$

$$\begin{bmatrix} -24 & 24 \\ 24 & -24 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = 0$$

$$-w_1 + w_2 = 0$$

$$w_1 - w_2 = 0$$

$$w_1 = w_2$$

$$\vec{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Zadatak 3.

Fisher – 3 razreda – trodimenzionalni uzorci

$$\omega_1 = \left\{(2,0,-1)^T, (4,0,1)^T\right\}$$
$$\omega_2 = \left\{(0,-2,1)^T, (0,-4,0)^T\right\}$$
$$\omega_3 = \left\{(2,-2,1)^T, (4,-4,-1)^T\right\}$$

$$\vec{m}_1 = \frac{1}{2}\left(\vec{x}_1 + \vec{x}_2\right) = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

$$\vec{m}_2 = \frac{1}{2}\left(\vec{x}_3 + \vec{x}_4\right) = \begin{bmatrix} 0 \\ -3 \\ 0.5 \end{bmatrix}$$

$$\vec{m}_3 = \frac{1}{2}\left(\vec{x}_5 + \vec{x}_6\right) = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}$$

$$S_1 = \sum_{\vec{x}_i \in \omega_1} (\vec{x}_i - \vec{m}_1)(\vec{x}_i - \vec{m}_1)^T$$

$$S_1 = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

$$S_2 = \sum_{\vec{x}_i \in \omega_2} (\vec{x}_i - \vec{m}_2)(\vec{x}_i - \vec{m}_2)^T$$

$$S_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 0.5 \end{bmatrix}$$

$$S_3 = \sum_{\vec{x}_i \in \omega_3} (\vec{x}_i - \vec{m}_3)(\vec{x}_i - \vec{m}_3)^T$$

$$S_3 = \begin{bmatrix} 2 & -2 & -2 \\ -2 & 2 & 2 \\ -2 & 2 & 2 \end{bmatrix}$$

$$S_w = S_1 + S_2 + S_3 = \begin{bmatrix} 4 & -2 & 0 \\ -2 & 4 & 3 \\ 0 & 3 & 4.5 \end{bmatrix}$$

$$S_B = \sum_{i=1}^{3} n_i (\vec{m}_i - \vec{m})(\vec{m}_i - \vec{m})^T$$

$$S_B = \begin{bmatrix} 12 & 6 & -2 \\ 6 & 12 & -1 \\ -2 & -1 & 1/3 \end{bmatrix}$$

drugi način

$$\lambda S_w \vec{w} = S_B \vec{w}$$

$$\left(S_B - \lambda S_w\right)\vec{w} = 0$$

$$\left\| \begin{bmatrix} 12 & 6 & -2 \\ 6 & 12 & -1 \\ -2 & -1 & 1/3 \end{bmatrix} - \lambda \begin{bmatrix} 4 & -2 & 0 \\ -2 & 4 & 3 \\ 0 & 3 & 4.5 \end{bmatrix} \right\| = 0$$

$$\lambda_1 = 25.8173$$

$$\lambda_2 = 1.0716$$

$$\lambda_3 = 0$$

$$\lambda_1 = 25.8173 \qquad \vec{w} = \begin{bmatrix} -0.62 \\ -0.9587 \\ 0.66 \end{bmatrix}$$

Zadatak 4.

Fisher – 3 razreda – trodimenzionalni uzorci

$$\omega_1 = \left\{(2,0,1)^T, (4,0,3)^T\right\}$$
$$\omega_2 = \left\{(0,-2,2)^T, (0,-4,4)^T\right\}$$
$$\omega_3 = \left\{(2,-2,3)^T, (4,-4,1)^T\right\}$$

$$\vec{m}_1 = \frac{1}{2}\left(\vec{x}_1 + \vec{x}_2\right) = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$$

$$\vec{m}_2 = \frac{1}{2}\left(\vec{x}_3 + \vec{x}_4\right) = \begin{bmatrix} 0 \\ -3 \\ 3 \end{bmatrix}$$

$$\vec{m}_3 = \frac{1}{2}\left(\vec{x}_5 + \vec{x}_6\right) = \begin{bmatrix} 3 \\ -3 \\ 2 \end{bmatrix}$$

$$S_1 = \sum_{\vec{x}_i \in \omega_1} (\vec{x}_i - \vec{m}_1)(\vec{x}_i - \vec{m}_1)^T$$

$$S_1 = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

$$S_2 = \sum_{\vec{x}_i \in \omega_2} (\vec{x}_i - \vec{m}_2)(\vec{x}_i - \vec{m}_2)^T$$

$$S_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & -2 \\ 0 & -2 & 2 \end{bmatrix}$$

$$S_3 = \sum_{\vec{x}_i \in \omega_3} (\vec{x}_i - \vec{m}_3)(\vec{x}_i - \vec{m}_3)^T$$

$$S_3 = \begin{bmatrix} 2 & -2 & -2 \\ -2 & 2 & 2 \\ -2 & 2 & 2 \end{bmatrix}$$

$$S_w = S_1 + S_2 + S_3 = \begin{bmatrix} 4 & -2 & 0 \\ -2 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$S_B = \sum_{i=1}^{3} n_i (\vec{m}_i - \vec{m})(\vec{m}_i - \vec{m})^T$$

$$S_B = \begin{bmatrix} 12 & 6 & -4 \\ 6 & 12 & -2 \\ -4 & -2 & 1.3\dot{3} \end{bmatrix}$$

drugi način

$$\lambda S_w \vec{w} = S_B \vec{w}$$

$$\left(S_B - \lambda S_w\right)\vec{w} = 0$$

$$\left| \begin{bmatrix} 12 & 6 & -4 \\ 6 & 12 & -2 \\ -4 & -2 & 1.3\dot{3} \end{bmatrix} - \lambda \begin{bmatrix} 4 & -2 & 0 \\ -2 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix} \right| = 0$$

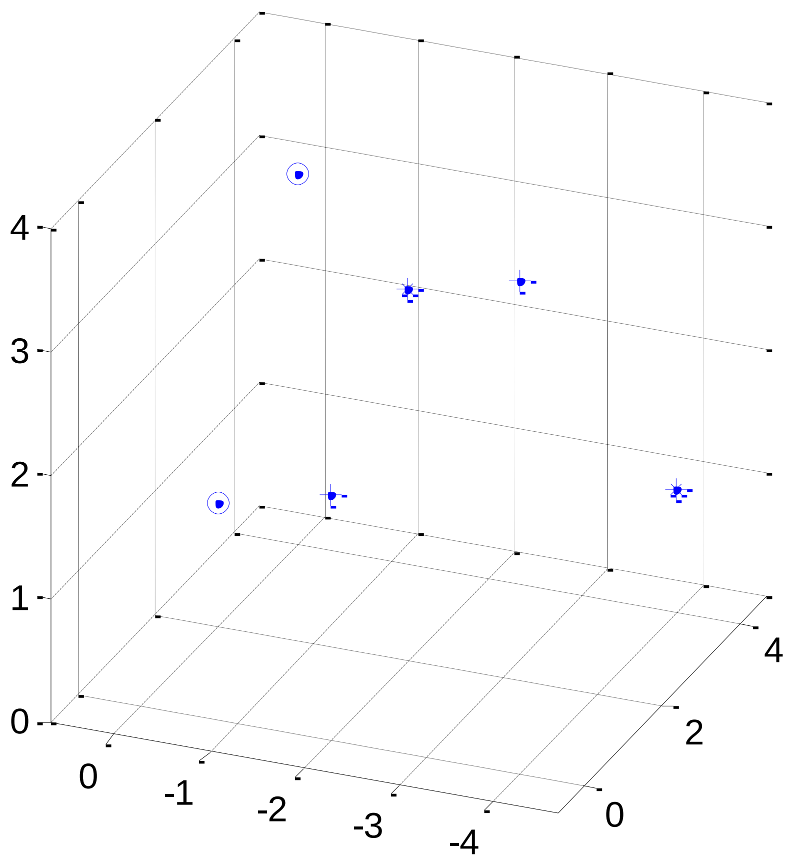
$$\lambda_1 = 9.1678$$

$$\lambda_2 = 1.0544$$

$$\lambda_3 = \quad 0$$

$$\lambda_1 = 9.1678 \qquad \vec{w}_1 = \begin{bmatrix} 0.4965 \\ 0.4942 \\ -0.0554 \end{bmatrix}$$

$$\lambda_2 = 1.0544 \qquad \vec{w}_2 = \begin{bmatrix} 0.2638 \\ -0.2984 \\ -0.0918 \end{bmatrix}$$



4
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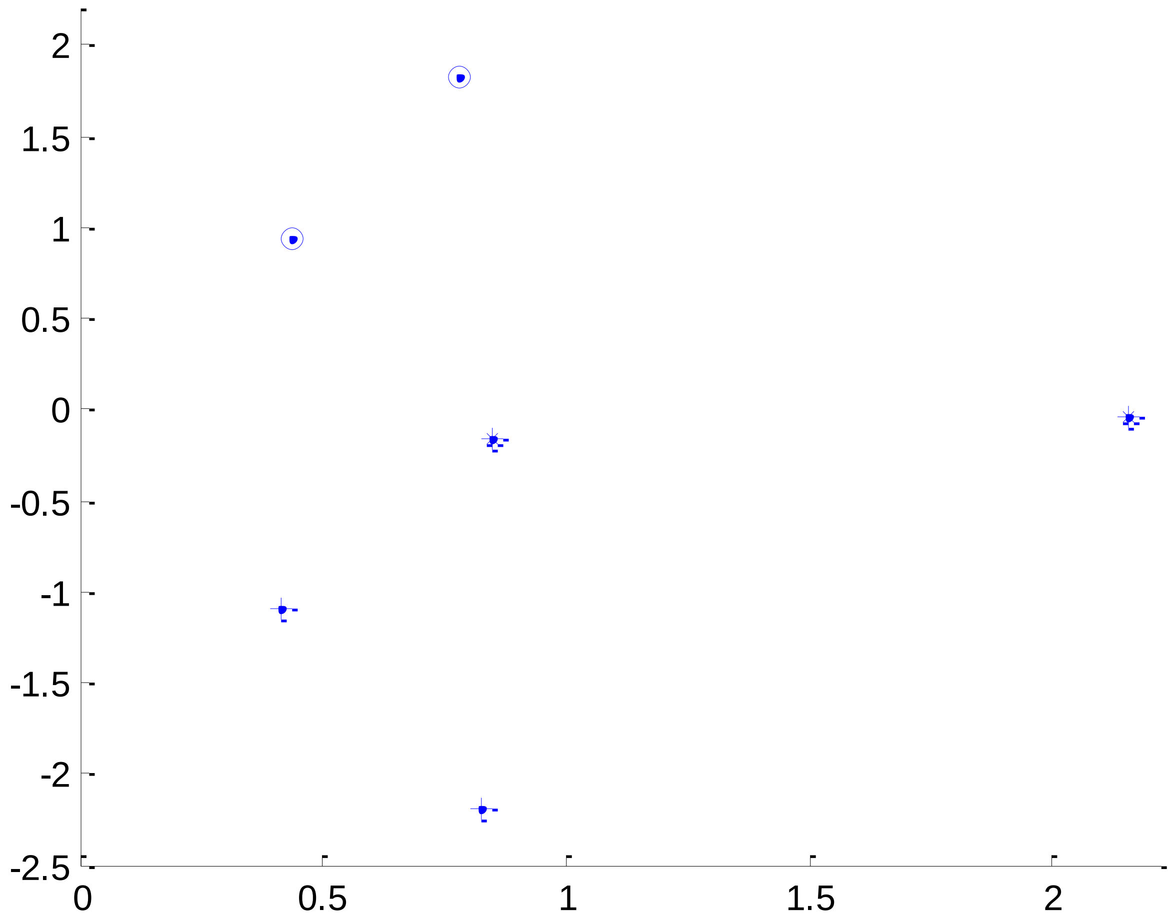
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