2. Auditorne iz Raspoznavanja uzoraka

Zadatak 1.

Fisher – 2 razreda – dvodimenzionalni uzorci

$$\omega_1 = \{(1,1)^T, (1,2)^T\}$$
$$\omega_2 = \{(-1,-1)^T, (-2,-1)^T\}$$

$$\vec{x}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \vec{x}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \vec{x}_3 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \vec{x}_4 = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

$$\vec{m}_1 = \frac{1}{2}(\vec{x}_1 + \vec{x}_2) = \frac{1}{2}\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1.5 \end{bmatrix}$$

$$\vec{m}_2 = \frac{1}{2}(\vec{x}_3 + \vec{x}_4) = \frac{1}{2}\left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} -1{,}5 \\ -1 \end{bmatrix}$$

$$S_1 = \sum_{\vec{x}_i \in \omega_1} (\vec{x}_i - \vec{m}_1)(\vec{x}_i - \vec{m}_1)^T$$

$$S_1 = \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1.5 \end{bmatrix}\right)\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1.5 \end{bmatrix}\right)^T + \left(\begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1.5 \end{bmatrix}\right)\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1.5 \end{bmatrix}\right)^T$$

$$S_1 = \begin{bmatrix} 0 \\ -0.5 \end{bmatrix}\begin{bmatrix} 0 & -0.5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}\begin{bmatrix} 0 & 0.5 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0.25 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0.25 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 \end{bmatrix}$$

$$S_2 = \sum_{\vec{x}_i \in \omega_2} (\vec{x}_i - \vec{m}_2)(\vec{x}_i - \vec{m}_2)^T$$

$$S_2 = \left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} - \begin{bmatrix} -1.5 \\ -1 \end{bmatrix}\right)\left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} - \begin{bmatrix} -1.5 \\ -1 \end{bmatrix}\right)^T + \left(\begin{bmatrix} -2 \\ -1 \end{bmatrix} - \begin{bmatrix} -1.5 \\ -1 \end{bmatrix}\right)\left(\begin{bmatrix} -2 \\ -1 \end{bmatrix} - \begin{bmatrix} -1.5 \\ -1 \end{bmatrix}\right)^T$$

$$S_2 = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}\begin{bmatrix} 0.5 & 0 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}\begin{bmatrix} 0.5 & 0 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 0.25 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0.25 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix}$$

$$S_w = S_1 + S_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

$$S_w^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$S_B = (\vec{m}_1 - \vec{m}_2)(\vec{m}_1 - \vec{m}_2)^T$$

$$= \left(\begin{bmatrix} 1 \\ 1.5 \end{bmatrix} - \begin{bmatrix} -1.5 \\ -1 \end{bmatrix} \right) \left(\begin{bmatrix} 1 \\ 1.5 \end{bmatrix} - \begin{bmatrix} -1.5 \\ -1 \end{bmatrix} \right)^T$$

$$= \begin{bmatrix} 2.5 \\ 2.5 \end{bmatrix} \begin{bmatrix} 2.5 & 2.5 \end{bmatrix} = \begin{bmatrix} 6.25 & 6.25 \\ 6.25 & 6.25 \end{bmatrix}$$

$$\vec{w} = S_w^{-1} (\vec{m}_1 - \vec{m}_2)$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2.5 \\ 2.5 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$$\vec{y}_1 = \vec{w}^T \vec{x}_1 = \begin{bmatrix} 5 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 10$$

$$\vec{y}_2 = \vec{w}^T \vec{x}_2 = \begin{bmatrix} 5 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 15$$

$$\vec{y}_3 = \vec{w}^T \vec{x}_3 = \begin{bmatrix} 5 & 5 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = -10$$

$$\vec{y}_4 = \vec{w}^T \vec{x}_4 = \begin{bmatrix} 5 & 5 \end{bmatrix} \begin{bmatrix} -2 \\ -1 \end{bmatrix} = -15$$

drugi način

$$\lambda S_w \vec{w} = S_B \vec{w} \qquad / \mathrm{S}_\mathrm{w}^{-1}$$

$$\lambda \vec{w} = \left(\mathrm{S}_\mathrm{w}^{-1} S_B \right) \vec{w}$$

problem svojstvenih vektora

$$\lambda \vec{w} = A \vec{w}$$

$$\mathrm{A} = \mathrm{S}_\mathrm{w}^{-1} S_B$$

$$(\mathrm{A} - \lambda \mathrm{I}) \vec{w} = \vec{0}$$

$$\vec{w} = (\mathrm{A} - \lambda \mathrm{I})^{-1} \vec{0}$$

da bi mogli izračunati $(\mathrm{A} - \lambda\mathrm{I})^{-1}$ mora biti $|\mathrm{A} - \lambda\mathrm{I}| \neq 0$, da bi dobili netrivijalno rješenje uvjet je $|\mathrm{A} - \lambda\mathrm{I}| = 0$, ovo je polinom n-tog stupnja, nul-točke su svojstvene vrijednosti

$$\mathrm{A} = \mathrm{S}_{\mathrm{w}}^{-1} S_B$$
$$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 6.25 & 6.25 \\ 6.25 & 6.25 \end{bmatrix} = \begin{bmatrix} 12.5 & 12.5 \\ 12.5 & 12.5 \end{bmatrix}$$
$$|\mathrm{A} - \lambda\mathrm{I}| = 0$$
$$= \begin{vmatrix} 12.5-\lambda & 12.5 \\ 12.5 & 12.5-\lambda \end{vmatrix}$$
$$= (12.5-\lambda)^2 - 12.5^2$$
$$= \lambda(\lambda - 25) = 0$$
$$\lambda = 25$$
$$(\mathrm{A} - \lambda\mathrm{I})\vec{w} = \vec{0}$$
$$\begin{bmatrix} 12.5-25 & 12.5 \\ 12.5 & 12.5-25 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \vec{0}$$
$$\begin{bmatrix} -12.5 & 12.5 \\ 12.5 & -12.5 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \vec{0}$$
$$w_1 = w_2$$
$$\vec{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Zadatak 2.

Fisher – 3 razreda – dvodimenzionalni uzorci

$$\omega_1 = \{(2,0)^T, (4,0)^T\}$$
$$\omega_2 = \{(0,-2)^T, (0,-4)^T\}$$
$$\omega_3 = \{(2,-2)^T, (4,-4)^T\}$$

$$\vec{x}_1 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \vec{x}_2 = \begin{bmatrix} 4 \\ 0 \end{bmatrix}, \vec{x}_3 = \begin{bmatrix} 0 \\ -2 \end{bmatrix}, \vec{x}_4 = \begin{bmatrix} 0 \\ -4 \end{bmatrix}, \vec{x}_5 = \begin{bmatrix} 2 \\ -2 \end{bmatrix}, \vec{x}_6 = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$$

$$\vec{m}_1 = \frac{1}{2}(\vec{x}_1 + \vec{x}_2) = \frac{1}{2}\left(\begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$\vec{m}_2 = \frac{1}{2}(\vec{x}_3 + \vec{x}_4) = \frac{1}{2}\left(\begin{bmatrix} 0 \\ -2 \end{bmatrix} + \begin{bmatrix} 0 \\ -4 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ -3 \end{bmatrix}$$

$$\vec{m}_3 = \frac{1}{2}(\vec{x}_5 + \vec{x}_6) = \frac{1}{2}\left(\begin{bmatrix} 2 \\ -2 \end{bmatrix} + \begin{bmatrix} 4 \\ -4 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$$

$$S_1 = \sum_{\vec{x}_i \in \omega_1} (\vec{x}_i - \vec{m}_1)(\vec{x}_i - \vec{m}_1)^T$$

$$S_1 = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$S_2 = \sum_{\vec{x}_i \in \omega_2} (\vec{x}_i - \vec{m}_2)(\vec{x}_i - \vec{m}_2)^T$$

$$S_2 = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$$

$$S_3 = \sum_{\vec{x}_i \in \omega_3} (\vec{x}_i - \vec{m}_3)(\vec{x}_i - \vec{m}_3)^T$$

$$S_3 = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$S_w = S_1 + S_2 + S_3 = \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$$

$$S_B = \sum_{i=1}^{3} n_i (\vec{m}_i - \vec{m})(\vec{m}_i - \vec{m})^T$$

$$S_B = \begin{bmatrix} 12 & 6 \\ 6 & 12 \end{bmatrix}$$

drugi način

$$\lambda S_w \vec{w} = S_B \vec{w}$$

$$(S_B - \lambda S_w)\vec{w} = 0$$

$$\left| \begin{bmatrix} 12 & 6 \\ 6 & 16 \end{bmatrix} - \lambda \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 12 - 4\lambda & 6 + 2\lambda \\ 6 + 2\lambda & 12 - 4\lambda \end{vmatrix} = 0$$

$$\lambda^2 - 10\lambda + 9 = 0$$

$$\lambda_1 = 9$$

$$\lambda_2 = 1$$

$$\lambda_1 = 9$$

$$\begin{bmatrix} -24 & 24 \\ 24 & -24 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = 0$$

$$-w_1 + w_2 = 0$$

$$w_1 - w_2 = 0$$

$$w_1 = w_2$$

$$\vec{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Zadatak 3.

Fisher – 3 razreda – trodimenzionalni uzorci

$$\omega_1 = \left\{(2,0,-1)^{\mathrm{T}}, (4,0,1)^{\mathrm{T}}\right\}$$
$$\omega_2 = \left\{(0,-2,1)^{\mathrm{T}}, (0,-4,0)^{\mathrm{T}}\right\}$$
$$\omega_3 = \left\{(2,-2,1)^{\mathrm{T}}, (4,-4,-1)^{\mathrm{T}}\right\}$$

$$\vec{m}_1 = \frac{1}{2}\left(\vec{x}_1 + \vec{x}_2\right) = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

$$\vec{m}_2 = \frac{1}{2}\left(\vec{x}_3 + \vec{x}_4\right) = \begin{bmatrix} 0 \\ -3 \\ 0.5 \end{bmatrix}$$

$$\vec{m}_3 = \frac{1}{2}\left(\vec{x}_5 + \vec{x}_6\right) = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}$$

$$S_1 = \sum_{\vec{x}_i \in \omega_1} \left(\vec{x}_i - \vec{m}_1\right)\left(\vec{x}_i - \vec{m}_1\right)^T$$

$$S_1 = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

$$S_2 = \sum_{\vec{x}_i \in \omega_2} \left(\vec{x}_i - \vec{m}_2\right)\left(\vec{x}_i - \vec{m}_2\right)^T$$

$$S_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 0.5 \end{bmatrix}$$

$$S_3 = \sum_{\vec{x}_i \in \omega_3} (\vec{x}_i - \vec{m}_3)(\vec{x}_i - \vec{m}_3)^T$$

$$S_3 = \begin{bmatrix} 2 & -2 & -2 \\ -2 & 2 & 2 \\ -2 & 2 & 2 \end{bmatrix}$$

$$S_w = S_1 + S_2 + S_3 = \begin{bmatrix} 4 & -2 & 0 \\ -2 & 4 & 3 \\ 0 & 3 & 4.5 \end{bmatrix}$$

$$S_B = \sum_{i=1}^{3} n_i (\vec{m}_i - \vec{m})(\vec{m}_i - \vec{m})^T$$

$$S_B = \begin{bmatrix} 12 & 6 & -2 \\ 6 & 12 & -1 \\ -2 & -1 & 1/3 \end{bmatrix}$$

drugi način

$$\lambda S_w \vec{w} = S_B \vec{w}$$

$$(S_B - \lambda S_w)\vec{w} = \vec{0}$$

$$\left\| \begin{bmatrix} 12 & 6 & -2 \\ 6 & 12 & -1 \\ -2 & -1 & 1/3 \end{bmatrix} - \lambda \begin{bmatrix} 4 & -2 & 0 \\ -2 & 4 & 3 \\ 0 & 3 & 4.5 \end{bmatrix} \right\| = 0$$

$$\lambda_1 = 25.8173$$

$$\lambda_2 = 1.0716$$

$$\lambda_3 = 0$$

$$\lambda_1 = 25.8173 \qquad \vec{w} = \begin{bmatrix} -0.62 \\ -0.9587 \\ 0.66 \end{bmatrix}$$

Zadatak 4.

PCA – 3 – razreda – dvodimenzionalni uzorci

$$\omega_1 = \{(2,0)^T, (4,0)^T\}$$
$$\omega_2 = \{(0,-2)^T, (0,-4)^T\}$$
$$\omega_3 = \{(2,-2)^T, (4,-4)^T\}$$

$$\vec{m} = \frac{1}{n}\sum_{i=1}^{n} \vec{x}_i = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

srednja vrijednost se oduzima od svih uzoraka

$$X = [(\vec{x}_1 - \vec{m}) \ \ (\vec{x}_1 - \vec{m}) \ \ ...]$$
$$X = \begin{bmatrix} 0 & 2 & -2 & -2 & 0 & 2 \\ 2 & 2 & 0 & -2 & 0 & -2 \end{bmatrix}$$
$$\vec{y} = \vec{w}^T X$$
$$J^{PCA}(\vec{w}) = \frac{1}{n}\vec{y}\vec{y}^T = \frac{1}{n}\vec{w}^T XX^T \vec{w} = \vec{w}^T C\vec{w} \ \text{ uvjet } \vec{w}^T\vec{w} = 1$$
$$K = \frac{1}{N-1} XX^T$$
$$J^{PCA}(\vec{w}) = \vec{w}^T K\vec{w} - \lambda(\vec{w}^T\vec{w} - 1)$$
$$(K - \lambda I)\vec{w} = \vec{0}$$
$$K\vec{w} = \lambda\vec{w}$$

$$K = \frac{1}{6-1}\begin{bmatrix} 0 & 2 & -2 & -2 & 0 & 2 \\ 2 & 2 & 0 & -2 & 0 & -2 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 2 & 2 \\ -2 & 0 \\ -2 & -2 \\ 0 & 0 \\ 2 & -2 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 16 & 4 \\ 4 & 16 \end{bmatrix}$$

$$\left|\begin{bmatrix} 16/5 & 4/5 \\ 4/5 & 16/5 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right| = 0$$

$$\begin{vmatrix} 16/5 - \lambda & 4/5 \\ 4/5 & 16/5 - \lambda \end{vmatrix} = 0$$

$$\lambda_1 = 4$$
$$\lambda_2 = 2.4$$

$$\lambda_1 = 4$$
$$\begin{bmatrix} -4/5 & 4/5 \\ 4/5 & -4/5 \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = 0$$
$$-w_1 + w_2 = 0$$
$$w_1 - w_2 = 0$$
$$w_1 = w_2$$
$$\vec{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Zadatak 5.

SVM – 2 razreda – dvodimenzionalni uzorci – dualni problem

$$\omega_1 = \{(0,0)^T\}$$
$$\omega_2 = \{(2,0)^T, (1,2)^T\}$$

Rješavanje dualnog problema

$$J(\lambda) = \sum_{i=1}^{N} \lambda_i - \frac{1}{2} \sum_{i,j=1}^{N} \lambda_i \lambda_j d_i d_j \vec{x}_i^T \vec{x}_j \ \text{ uvjet } \sum_{i=1}^{N} \lambda_i d_i = 0 \ \text{ i } \ \lambda_i \geq 0, i = 1,\ldots,3$$

$d_i = 1$ za $x_i \in \omega_1$ i $d_i = -1$ za $x_i \in \omega_2$

Langrangeovi multiplikator α

$$F = \sum_{i=1}^{N} \lambda_i - \frac{1}{2} \sum_{i,j=1}^{N} \lambda_i \lambda_j d_i d_j \vec{x}_i^T \vec{x}_j + \alpha \sum_{i=1}^{N} \lambda_i d_i$$

$$F = \sum_{i=1}^{N} \lambda_i - \frac{1}{2} \sum_{i,j=1}^{N} \lambda_i \lambda_j d_i d_j \vec{x}_i^T \vec{x}_j + \alpha \sum_{i=1}^{N} \lambda_i d_i$$

$$F = \lambda_1 + \lambda_2 + \lambda_3 - \frac{1}{2}\lambda_1\lambda_1\vec{x}_1^T\vec{x}_1 + \frac{1}{2}\lambda_1\lambda_2\vec{x}_1^T\vec{x}_2 + \frac{1}{2}\lambda_1\lambda_3\vec{x}_1^T\vec{x}_3$$
$$+\frac{1}{2}\lambda_2\lambda_1\vec{x}_2^T\vec{x}_1 - \frac{1}{2}\lambda_2\lambda_2\vec{x}_2^T\vec{x}_2 - \frac{1}{2}\lambda_2\lambda_3\vec{x}_2^T\vec{x}_3$$
$$+\frac{1}{2}\lambda_3\lambda_1\vec{x}_3^T\vec{x}_1 - \frac{1}{2}\lambda_3\lambda_2\vec{x}_3^T\vec{x}_2 - \frac{1}{2}\lambda_3\lambda_3\vec{x}_3^T\vec{x}_3$$
$$+\alpha(\lambda_1 - \lambda_2 - \lambda_3)$$

$$F = \lambda_1 + \lambda_2 + \lambda_3 - 2\lambda_2^2 - \frac{5}{2}\lambda_3^2 - 2\lambda_2\lambda_3 + \alpha(\lambda_1 - \lambda_2 - \lambda_3)$$

$$\frac{\partial F}{\partial \lambda_1} = 1 + \alpha = 0 \quad \alpha = -1$$

$$\frac{\partial F}{\partial \lambda_2} = 1 - 4\lambda_2 - 2\lambda_3 - \alpha = 0$$

$$\frac{\partial F}{\partial \lambda_3} = 1 - 5\lambda_3 - 2\lambda_2 - \alpha = 0$$

$$\frac{\partial F}{\partial \alpha} = \lambda_1 - \lambda_2 - \lambda_3 = 0 \quad \alpha = -1$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & -4 & -2 & -1 \\ 0 & -2 & -5 & -1 \\ 1 & -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \alpha \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & -4 & -2 & -1 \\ 0 & -2 & -5 & -1 \\ 1 & -1 & -1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ -1 \\ -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \alpha \end{bmatrix} = \frac{1}{16} \begin{bmatrix} -5 & -3 & -2 & 16 \\ -3 & -5 & 2 & 0 \\ -2 & 2 & -4 & 0 \\ 16 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ -1 \\ 0 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 5 \\ 3 \\ 2 \\ -8 \end{bmatrix}$$

$$2-4\lambda_2-2\lambda_3=0$$
$$2-5\lambda_3-2\lambda_2=0/2$$
$$2-4\lambda_2-2\lambda_3=0$$
$$4-10\lambda_3-4\lambda_2=0$$
$$8\lambda_3-2=0$$
$$\lambda_3=\frac{1}{4}$$
$$\lambda_2=\frac{2-2\lambda_3}{4}=\frac{3}{8}$$
$$\lambda_1-\lambda_2-\lambda_3=0$$
$$\lambda_1=\frac{5}{8}$$

$$\vec{w}=\sum_{i=1}^{N}\lambda_i d_i \vec{x}_i=\frac{5}{16}\begin{bmatrix}0\\0\end{bmatrix}-\frac{3}{8}\begin{bmatrix}2\\0\end{bmatrix}-\frac{1}{4}\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}-1\\-1/2\end{bmatrix}$$
$$b=\pm1-\vec{w}^T\vec{x}=1$$
$$-x_1-\frac{1}{2}x_2+1=0$$

Zadatak 6.

SVM – 2 razreda – dvodimenzionalni uzorci – dualni problem

Analitički naći rješenje dualnog problema za skup uzoraka

$$\omega_1 = \left\{(0,0)^{\mathrm{T}}\right\}$$
$$\omega_2 = \left\{(2,0)^{\mathrm{T}}, (1,2)^{\mathrm{T}}, (2,2)^{\mathrm{T}}\right\}$$

Rješavanje dualnog problema

$$J(\lambda) = \sum_{i=1}^{N} \lambda_i - \frac{1}{2}\sum_{i,j=1}^{N} \lambda_i \lambda_j d_i d_j \vec{x}_i^T \vec{x}_j \quad \text{uvjet} \quad \sum_{i=1}^{N} \lambda_i d_i = 0 \ \text{ i } \ \lambda_i \geq 0, i = 1,\ldots,4$$

$$d_i = 1 \text{za } x_i \in \omega_1 \text{ i } \ d_i = -1 \text{ za } x_i \in \omega_2$$

Langrangeovi multiplikator α

$$F = \sum_{i=1}^{N} \lambda_i - \frac{1}{2}\sum_{i,j=1}^{N} \lambda_i \lambda_j d_i d_j \vec{x}_i^T \vec{x}_j + \alpha \sum_{i=1}^{N} \lambda_i d_i$$

$$\begin{aligned}
F = {} & \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + -\frac{1}{2}\lambda_1\lambda_1\vec{x}_1^T\vec{x}_1 + \frac{1}{2}\lambda_1\lambda_2\vec{x}_1^T\vec{x}_2 + \frac{1}{2}\lambda_1\lambda_3\vec{x}_1^T\vec{x}_3 + \frac{1}{2}\lambda_1\lambda_4\vec{x}_1^T\vec{x}_4 \\
& + \frac{1}{2}\lambda_2\lambda_1\vec{x}_2^T\vec{x}_1 - \frac{1}{2}\lambda_2\lambda_2\vec{x}_2^T\vec{x}_2 - \frac{1}{2}\lambda_2\lambda_3\vec{x}_2^T\vec{x}_3 - \frac{1}{2}\lambda_2\lambda_4\vec{x}_2^T\vec{x}_4 \\
& + \frac{1}{2}\lambda_3\lambda_1\vec{x}_3^T\vec{x}_1 - \frac{1}{2}\lambda_3\lambda_2\vec{x}_3^T\vec{x}_2 - \frac{1}{2}\lambda_3\lambda_3\vec{x}_3^T\vec{x}_3 - \frac{1}{2}\lambda_3\lambda_4\vec{x}_3^T\vec{x}_4 \\
& + \frac{1}{2}\lambda_4\lambda_1\vec{x}_4^T\vec{x}_1 - \frac{1}{2}\lambda_4\lambda_2\vec{x}_4^T\vec{x}_2 - \frac{1}{2}\lambda_4\lambda_3\vec{x}_4^T\vec{x}_3 - \frac{1}{2}\lambda_4\lambda_4\vec{x}_4^T\vec{x}_4 \\
& + \alpha(\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)
\end{aligned}$$

$$F = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 - 2\lambda_2^2 - \frac{5}{2}\lambda_3^2 - 2\lambda_2\lambda_3 - 4\lambda_2\lambda_4 - 6\lambda_3\lambda_4 - 4\lambda_4^2 + \alpha(\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

$$\frac{\partial F}{\partial \lambda_1} = 1 + \alpha = 0 \quad \alpha = -1$$

$$\frac{\partial F}{\partial \lambda_2} = 1 - 4\lambda_2 - 2\lambda_3 - 4\lambda_4 - \alpha = 0$$

$$\frac{\partial F}{\partial \lambda_3} = 1 - 5\lambda_3 - 2\lambda_2 - 6\lambda_4 - \alpha = 0$$

$$\frac{\partial F}{\partial \lambda_4} = 1 - 6\lambda_3 - 4\lambda_2 - 8\lambda_4 - \alpha = 0$$

$$\frac{\partial F}{\partial \alpha} = \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4 = 0$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & -4 & -2 & -4 & -1 \\ 0 & -2 & -5 & -6 & -1 \\ 0 & -4 & -6 & -8 & -1 \\ 1 & -1 & -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \alpha \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ 0 \end{bmatrix}$$

$$\begin{vmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & -4 & -2 & -4 & -1 \\ 0 & -2 & -5 & -6 & -1 \\ 0 & -4 & -6 & -8 & -1 \\ 1 & -1 & -1 & -1 & 0 \end{vmatrix} = 0$$

Pretpostavka $\lambda_2 = 0$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & -5 & -6 & -1 \\ 0 & -6 & -8 & -1 \\ 1 & -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_3 \\ \lambda_4 \\ \alpha \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \lambda_1 \\ \lambda_3 \\ \lambda_4 \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & -5 & -6 & -1 \\ 0 & -6 & -8 & -1 \\ 1 & -1 & -1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ -1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 1 \\ -0.5 \\ -1 \end{bmatrix}$$

Pretpostavka $\lambda_4 = 0$

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & -4 & -2 & -1 \\ 0 & -2 & -5 & -1 \\ 1 & -1 & -1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ -1 \\ -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & -4 & -2 & -1 \\ 0 & -2 & -5 & -1 \\ 1 & -1 & -1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ -1 \\ -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \alpha \end{bmatrix} = \frac{1}{16} \begin{bmatrix} -5 & -3 & -2 & 16 \\ -3 & -5 & 2 & 0 \\ -2 & 2 & -4 & 0 \\ 16 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ -1 \\ 0 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 5 \\ 3 \\ 2 \\ -8 \end{bmatrix}$$

$$2 - 4\lambda_2 - 2\lambda_3 = 0$$
$$2 - 5\lambda_3 - 2\lambda_2 = 0 / 2$$
$$2 - 4\lambda_2 - 2\lambda_3 = 0$$
$$4 - 10\lambda_3 - 4\lambda_2 = 0$$
$$8\lambda_3 - 2 = 0$$
$$\lambda_3 = \frac{1}{4}$$
$$\lambda_2 = \frac{2 - 2\lambda_3}{4} = \frac{3}{8}$$
$$\lambda_1 - \lambda_2 - \lambda_3 = 0$$
$$\lambda_1 = \frac{5}{8}$$

$$\vec{w} = \sum_{i=1}^{N} \lambda_i d_i \vec{x}_i = \frac{5}{16}\begin{bmatrix} 0 \\ 0 \end{bmatrix} - \frac{3}{8}\begin{bmatrix} 2 \\ 0 \end{bmatrix} - \frac{1}{4}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -1/2 \end{bmatrix}$$

$$b = \pm 1 - \vec{w}^T \vec{x} = 1$$

$$-x_1 - \frac{1}{2} x_2 + 1 = 0$$