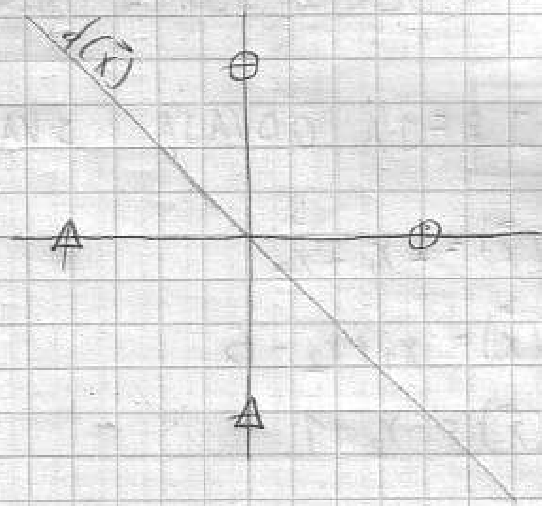
HO-KASHYAP / AUDITORNE



$\omega_1 = \{ [1\ 0]^T, [0\ 1]^T \}$ $\oplus$

$\omega_2 = \{ [-1\ 0]^T, [0\ -1]^T \}$ Δ

$c = 1$ $\quad \vec{b} = [1\ 1\ 1\ 1]^T$ → ONOLKO '1' KOLKO IMA UZORAKA

1. KORAK: PROŠIRI I INVERTIRAJ UZORKE

$x_1 = [1\ 0\ 1]^T$

$x_2 = [0\ 1\ 1]^T$

$x_3 = [1\ 0\ -1]^T$

$x_4 = [0\ 1\ -1]^T$

$$X = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

2. KORAK: IZRAČUNAJ $X^\#$

$$X^\# = (X^T X)^{-1} \cdot X^T = \left(\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \right)^{-1} \cdot X^T =$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}^{-1} \cdot X = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 1/2 & 0 \\ 0 & 1/2 & 0 & 1/2 \\ 1/4 & 1/4 & -1/4 & -1/4 \end{bmatrix}$$

3. KORAK: ITERIRANJE

$$\vec{w}(1) = [X]^\# \cdot \vec{b}(1) = [1\ 1\ 0]^T$$

$$\vec{e}(1) = [X] \cdot \vec{w}(1) - \vec{b}(1) = [1\ 1\ 1\ 1] - \vec{b}(1) = [\emptyset]$$ ✓

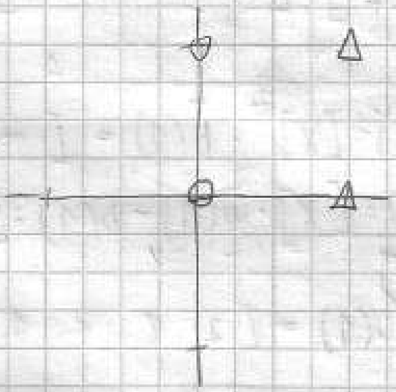
KRAJ LOL!)

$$\boxed{d(\vec{x}) = x_1 + x_2}$$

HO-KASHYAP / PREDAVANJA

$\omega_1 = \{[0\ 0]^T, [[0\ 1]^T\} \circ \qquad b(1) = [1\ 1\ 1\ 1]^T$

$\omega_2 = \{[1\ 0]^T, [1\ 1]^T\} \Delta \qquad c = 1$



1)

$x_1 = [0\ 0\ 1]^T$

$x_2 = [0\ 1\ 1]^T \qquad X = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & 0 & -1 \\ -1 & -1 & -1 \end{bmatrix}$

$x_2 = [-1\ 0\ -1]^T$

$x_3 = [-1\ -1\ -1]^T$

2)

$$X^{\#} = (X^T X)^{-1} X^T = \begin{bmatrix} 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & 0 & -1 \\ -1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 4 \end{bmatrix}^{-1} \cdot X^T$$

ALGORITAM INVERTIRANJA - ŠKOLSKI

1. PODJELI 2x ŠIRU MATRICU I ZDESNA DODAJ I

$$X^{-1} = \left[\begin{array}{ccc|ccc} 2 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 2 & 2 & 4 & 0 & 0 & 1 \end{array}\right]$$

2) ZBRAJANJEM, ODUZIMANJEM, MNOŽENJEM REDOVA DOĐI DO I S LIJEVE STRANE NAKON TOGA, DESNO JE INVERZ

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 2 & 2 & 4 & 0 & 0 & 1 \end{array}\right] - = \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & -1 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 2 & 2 & 4 & 0 & 0 & 1 \end{array}\right] \cdot 0{,}5 \; -$$

$$= \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -\frac{1}{2} \\ 1 & 1 & 2 & 0 & 0 & \frac{1}{2} \end{array}\right] - = \begin{bmatrix} 1 & -1 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 2 & 2 & -1 & 1 & \frac{1}{2} \end{bmatrix} \cdot \frac{1}{2} \; + \; - =$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} & -\frac{1}{2} & \frac{3}{4} \end{array}\right]$$

PROVJERA:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{3}{4} \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

$$X^{\#}=\begin{bmatrix}1 & 0 & -\frac{1}{2}\\ 0 & 1 & -\frac{1}{2}\\ -\frac{1}{2} & -\frac{1}{2} & \frac{3}{4}\end{bmatrix}\begin{bmatrix}0 & 0 & -1 & -1\\ 0 & 1 & 0 & -1\\ 1 & 1 & -1 & -1\end{bmatrix}=\begin{bmatrix}-\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2}\\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2}\\ \frac{3}{4} & \frac{1}{4} & -\frac{1}{4} & +\frac{1}{4}\end{bmatrix}\qquad X=\begin{bmatrix}0 & 0 & 1\\ 0 & 1 & 1\\ -1 & 0 & -1\\ -1 & -1 & -1\end{bmatrix}$$

3) $\vec{w}(1)=[X]^{\#}\cdot\vec{b}(1)=[-2\quad 0\quad 1]^T$

$\vec{e}(1)=[X]\cdot\vec{w}(1)-b(1)=[1\quad 1\quad 1\quad 1]^T-[1\quad 1\quad 1\quad 1]^T=\emptyset$ ✓

ZA VJEŽBU
NPR: $b(1)=[2\quad 1\quad 2\quad 1]$

$\vec{w}(1)=[X]^{\#}\cdot\vec{b}(1)=[-3\quad 0\quad \frac{3}{2}]^T$

$\vec{e}(1)=[X]\cdot\vec{w}(1)-b=[\frac{3}{2}\quad\frac{3}{2}\quad\frac{3}{2}\quad\frac{3}{2}]-[2\quad 1\quad 2\quad 1]^T=[-\frac{1}{2}\quad\frac{1}{2}\quad-\frac{1}{2}\quad\frac{1}{2}]^T$

$\vec{b}(2)=[2\quad 2\quad 2\quad 2]^T$ /// $b_i \mathrel{+}= e_i$ AKO $e_i>0$

$\vec{w}(2)=[X]^{\#}\cdot\vec{b}(2)=[-4\quad 0\quad 2]^T$

$\vec{e}(2)=[X]\cdot\vec{w}(2)-\vec{b}(2)=[2\quad 2\quad 2\quad 2]-[2\quad 2\quad 2]=\emptyset$ ✓

1. **(6 bodova)** Postupkom Ho-Kashyapa želimo naći linearnu decizijsku funkciju za skup dvodimenzionalnih uzoraka. Zadana je matrica uzoraka, X, i njezin generalizirani inverz, $X^{\#}$

$$X = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 1 & -1 \\ 2 & -1 & -1 \\ -1 & -2 & 1 \end{bmatrix} \qquad X^{\#} = \begin{bmatrix} 0.1 & -0.2 & 0.2 & -0.1 \\ 0.2 & 0.1 & -0.1 & -0.2 \\ 0.25 & -0.25 & -0.25 & 0.25 \end{bmatrix}$$

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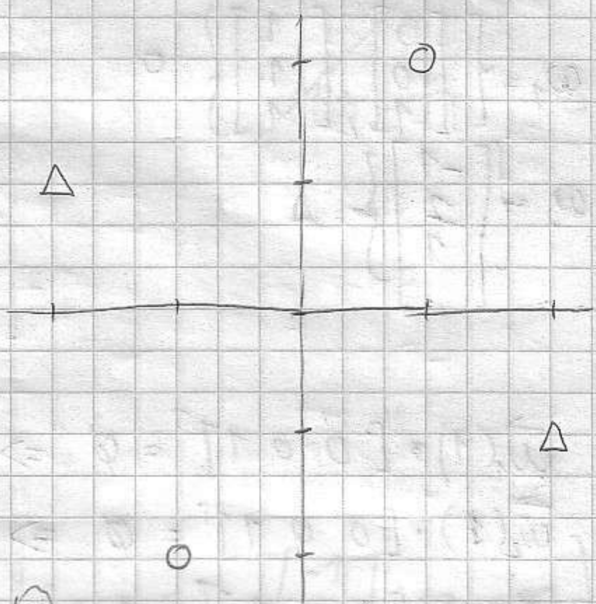
$$X=\begin{bmatrix}1 & 2 & -1\\ -2 & 1 & -1\\ 2 & -1 & -1\\ -1 & -2 & 1\end{bmatrix} \qquad X^{\#}=\begin{bmatrix}0.1 & -0.2 & 0.2 & -0.1\\ 0.2 & 0.1 & -0.1 & -0.2\\ 0.25 & -0.25 & -0.25 & 0.25\end{bmatrix}$$

$$\vec{b}(1)=[1\ 1\ 1\ 1]^T \qquad c=1$$



$\boxed{\text{ZADATAK}}$, RASPISAT ALGORITAM U 1. KORAKU

a) $\vec{\omega}(1)=[X]^{\#}\cdot\vec{b}(1)=[0\quad 0\quad 0]^T$

$\vec{e}(1)=[X]\cdot\vec{\omega}(1)-\vec{b}(1)=[-1\quad -1\quad -1]^T$

c) $J(1)=\frac{1}{2}\|[X]\cdot\vec{\omega}(1)-\vec{b}(1)\|^2=\frac{1}{2}\begin{bmatrix}-1\\-1\\-1\end{bmatrix}[-1\ -1\ -1]=\frac{3}{2}$

d) POŠTO SU SVI ELEMENTI $\vec{e}<0 \Rightarrow$ RAZREDI LINEARNO NEODVOJIVI