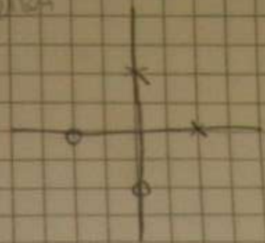
HO – KASHYAP

$c = 1 \quad \vec{b} = [1,1,1,1]^T$ ODLUKA [illegible]

$\omega_1 = \{ [1,0]^T, [0,1]^T \}$

$\omega_2 = \{ [-1,0]^T, [0,-1]^T \}$



$x \in \omega_1$

$o \in \omega_2$

$y_1 = [1,0,1]^T$

$y_2 = [0,1,1]^T$

$y_3 = [1,0,-1]^T$

$y_4 = [0,1,-1]^T$

$$[X] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$[X]^{\#} = \left([X]^T [X]\right)^{-1} [X]^T =$$

$$= \left(\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \right)^{-1} [X]^T =$$

DIJAGONALNA MATRICA, ZA INVERZ SAMO INVERTIRAMO ELEMENTE

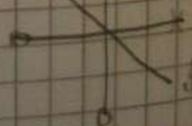
$$= \left(\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix} \right)^{-1} [X]^T = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix} [X]^T =$$

$$= \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{bmatrix}$$

$$\vec{w}(1) = [X]^{\#} \cdot \vec{b}(1) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\vec{e}(1) = [X] \cdot \vec{w}(1) - \vec{b}(1) = \vec{0}$$ ← UZORCI SU ISPRAVNO KLASIFICIRANI

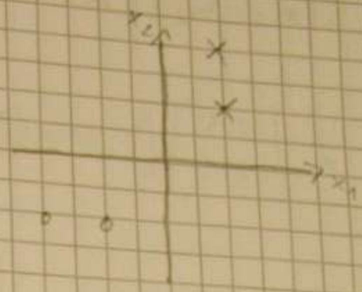
$$d(\vec{x}) = x_1 + x_2$$



FISHEROVA LINEARNA DISKRIMINANTNA ANALIZA (FLD)

$\omega_1 = \{[1,1]^T, [1,2]^T\}$

$\omega_2 = \{[-1,-1]^T, [-2,-1]^T\}$



$\vec{m}_1 = \frac{1}{2}(\vec{x}_1 + \vec{x}_2) = \begin{bmatrix} 1 \\ 1,5 \end{bmatrix}$

BR UZORAKA

$\vec{m}_2 = \frac{1}{2}(\vec{x}_3 + \vec{x}_4) = \begin{bmatrix} -1,5 \\ -1 \end{bmatrix}$

MATRICA RASPRŠENJA

$$S_1 = \sum_{\vec{x}_i \in \omega_1} (\vec{x}_i - \vec{m}_1)(\vec{x}_i - \vec{m}_1)^T =$$

$$= \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1,5 \end{bmatrix}\right)\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1,5 \end{bmatrix}\right)^T +$$

$$+ \left(\begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1,5 \end{bmatrix}\right)\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1,5 \end{bmatrix}\right)^T =$$

$$= \begin{bmatrix} 0 \\ -0,5 \end{bmatrix}\begin{bmatrix} 0 & -0,5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0,5 \end{bmatrix}\begin{bmatrix} 0 & 0,5 \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0,25 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0,25 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0,5 \end{bmatrix}$$

$$S_2 = \sum_{x_i \in \omega_2} (\vec{x}_i - \vec{m}_2)(\vec{x}_i - \vec{m}_2)^T = \begin{bmatrix} 0,5 & 0 \\ 0 & 0 \end{bmatrix}$$

$$S_W = S_1 + S_2 = \begin{bmatrix} 0,5 & 0 \\ 0 & 0,5 \end{bmatrix}$$

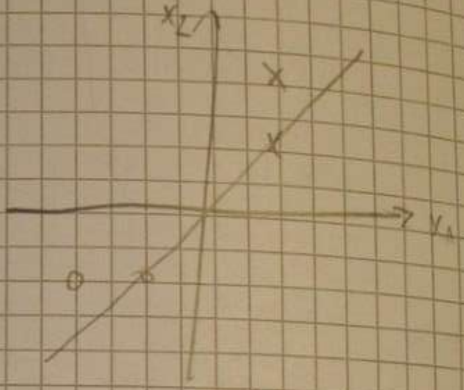
$$S_B = (\vec{m}_1 - \vec{m}_2)(\vec{m}_1 - \vec{m}_2)^T = \left(\begin{bmatrix} 1 \\ 1{,}5 \end{bmatrix} - \begin{bmatrix} -1{,}5 \\ -1 \end{bmatrix}\right)\left(\begin{bmatrix} 1 \\ 1{,}5 \end{bmatrix} - \begin{bmatrix} -1{,}5 \\ -1 \end{bmatrix}\right)^T =$$

$$= \begin{bmatrix} 2.5 \\ 2.5 \end{bmatrix}\begin{bmatrix} 2.5 & 2.5 \end{bmatrix} = \begin{bmatrix} 6{,}25 & 6{,}25 \\ 6{,}25 & 6{,}25 \end{bmatrix}$$

$$\vec{w} = S_w^{-1}(\vec{m}_1 - \vec{m}_2) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 2{,}5 \\ 2{,}5 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

SMJER PRAVCA
(U SMJERU (5,5))

VRIJEDI SAMO
ZA 2 RAZREDA



$$\lambda S_w \vec{w} = S_B \vec{w} \quad / S_w^{-1}$$

$$\lambda \vec{w} = (S_w^{-1} S_B)\vec{w}$$

$$A\vec{x} = \lambda \vec{x}$$

$$(A - \lambda I)\vec{x} = \vec{0}$$

$$\vec{x} = (A - \lambda I)^{-1}\vec{0}$$

$$|A - \lambda I| = 0$$

NE POSTOJI INVERZ

$$A = S_w^{-1} S_B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 6{,}25 & 6{,}25 \\ 6{,}25 & 6{,}25 \end{bmatrix}$$

$$= \begin{bmatrix} 12{,}5 & 12{,}5 \\ 12{,}5 & 12{,}5 \end{bmatrix}$$

$$\begin{vmatrix} 12{,}5-\lambda & 12{,}5 \\ 12{,}5 & 12{,}5-\lambda \end{vmatrix} = 0$$

karakteristični polinom

$$(12{,}5-\lambda)(12{,}5-\lambda) - (12{,}5 \cdot 12{,}5) = 0$$

$$\cancel{12{,}5^2} - 25\lambda + \lambda^2 - \cancel{12{,}5^2} = 0$$

$$\lambda(\lambda - 25) = 0$$

$$\lambda_1 = 0, \quad \lambda_2 = 25$$

$$(A - \lambda I)\vec{x} = 0$$

$$\left(\begin{bmatrix} 12{,}5 & 12{,}5 \\ 12{,}5 & 12{,}5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)\vec{x} = 0$$

$\lambda = 25$

$$\begin{bmatrix} -12{,}5 & 12{,}5 \\ 12{,}5 & -12{,}5 \end{bmatrix} \cdot \vec{x} = 0$$

$$-12{,}5x_1 + 12{,}5x_2 = 0$$

$$12{,}5x_1 - 12{,}5x_2 = 0$$

- ZANIMA NAS SAMO SMJER

ODABIREMO NPR $x_1 = 1$

$$\vec{x} = \vec{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\omega_1 = \left\{ [2,0]^T, [4,0]^T \right\}$ •

$\omega_2 = \left\{ [0,-2]^T, [0,-4]^T \right\}$ ×

$\omega_3 = \left\{ [2,-2]^T, [4,-4]^T \right\}$ o

$$\vec{m}_1 = \frac{1}{2}\left(\begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$\vec{m}_2 = \begin{bmatrix} 0 \\ -3 \end{bmatrix} \qquad \vec{m}_3 = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$$

$$\vec{m} = \frac{1}{n} \sum_{i=1}^{c} n_i \vec{m}_i = \frac{1}{6}\left(2 \begin{bmatrix} 3 \\ 0 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ -3 \end{bmatrix} + 2 \begin{bmatrix} 3 \\ -3 \end{bmatrix} \right) = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$S_1 = \sum_{x \in \omega_1} (\vec{x}_i - \vec{m}_1)(\vec{x}_i - \vec{m}_1)^T = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \qquad S_3 = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$S_W = S_1 + S_2 + S_3 = \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$$

$$S_B = \sum_{i=1}^{c} n_i (\vec{m}_i - \vec{m})(\vec{m}_i - \vec{m})^T =$$

$$= 2 \cdot \left(\begin{bmatrix} 3 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 \\ -2 \end{bmatrix} \right) \left(\begin{bmatrix} 3 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 \\ -2 \end{bmatrix} \right)^T +$$

$$+ 2 \cdot \left(\begin{bmatrix} 0 \\ -3 \end{bmatrix} - \begin{bmatrix} 2 \\ -2 \end{bmatrix} \right) \left(\begin{bmatrix} 0 \\ -3 \end{bmatrix} - \begin{bmatrix} 2 \\ -2 \end{bmatrix} \right)^T +$$

$$+ 2 \cdot \left(\begin{bmatrix} 3 \\ -3 \end{bmatrix} - \begin{bmatrix} 2 \\ -2 \end{bmatrix} \right) \left(\begin{bmatrix} 3 \\ -3 \end{bmatrix} - \begin{bmatrix} 2 \\ -2 \end{bmatrix} \right)^T = \begin{bmatrix} 12 & 6 \\ 6 & 12 \end{bmatrix}$$

$$S_w = \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$$

$$S_B = \begin{bmatrix} 12 & 6 \\ 6 & 12 \end{bmatrix}$$

$$S_B \vec{w} = \lambda S_w \vec{w}$$

$$|S_B - \lambda S_w| = 0$$

$$\left| \begin{bmatrix} 12 & 6 \\ 6 & 12 \end{bmatrix} - \lambda \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 12 - 4\lambda & 6 + 2\lambda \\ 6 + 2\lambda & 12 - 4\lambda \end{vmatrix} = 0$$

$$(12 - 4\lambda)^2 - (6 + 2\lambda)^2 = 0$$

$$12\lambda^2 - 120\lambda + 108 = 0$$

$$\lambda^2 - 10\lambda + 9 = 0$$

$$\lambda_1 = 9 \quad \lambda_2 = 1$$

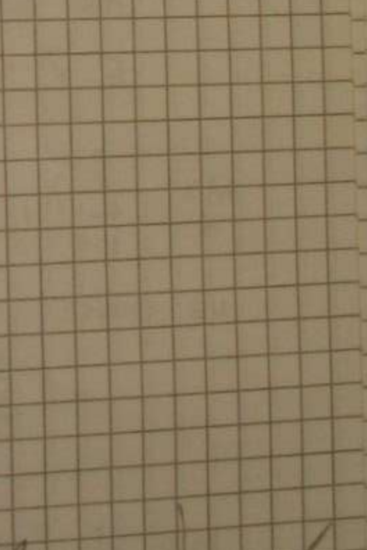
$$(S_B - \lambda S_w)\vec{w} = 0$$

$$\begin{bmatrix} -24 & 24 \\ 24 & -24 \end{bmatrix} \vec{w} = 0$$

$$\cancel{24} \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = 0 \quad \Rightarrow \quad w_2 \quad w_1 = 1 \quad \vec{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$-w_1 + w_2 = 0$$

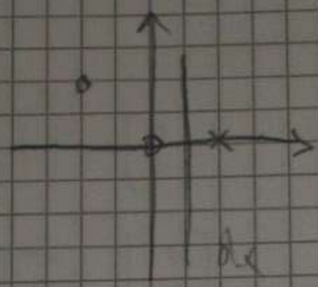
$$w_1 - w_2 = 0$$



SVM

$$\omega_1 = \{[0,0]^T, [-1,1]^T\}$$

$$\omega_2 = \{[2,0]^T\}$$



$$\min_{x} \quad \frac{1}{2}\vec{x}^T Q \vec{x} + \vec{c}^T \cdot \vec{x}$$

UZ UVJETE

$$A\vec{x} \leq \vec{b}$$

$$E\vec{x} = \vec{d}$$

$$\max_{x} \quad \sum_{i=1}^{N} \lambda_i - \frac{1}{2}\sum_{i,j}^{N} \lambda_i \lambda_j d_i d_j \vec{x_i}^T \vec{x_j}$$

UZ UVJETE

$$\sum_{i=1}^{N} \lambda_i d_i = 0 \qquad \lambda_i \geq 0$$

$$\min_{\lambda} \quad \underbrace{\frac{1}{2}\sum_{i,j}^{N} \lambda_i d_i d_j \vec{x_i}^T \vec{x_j}}_{\frac{1}{2}\vec{x}^T Q \vec{x}} - \underbrace{\sum_{i=1}^{N} \lambda_i}_{+\vec{c}^T \vec{x}}$$

$$Q_{i,j} = d_i d_j \vec{x_i}^T \vec{x_j}$$

$$Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 2 & 4 \end{bmatrix}$$

$$a_{22} = 1 \cdot 1 \quad \begin{bmatrix} -1 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = 2$$

$$\vec{c} = \begin{bmatrix} -1 \end{bmatrix}$$

$$\vec{c}^{\,T} \vec{x} = \begin{bmatrix} -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = -\lambda_1 - \lambda_2 - \lambda_3$$

$$\sum_{i=1}^{N} \lambda_i d_i = 0$$

$$E \vec{x} = \vec{d} \qquad E = \begin{bmatrix} d_1 & d_2 & d_3 \end{bmatrix}$$

$$\vec{d} = \begin{bmatrix} 0 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix}$$

[illegible]

$\lambda_i \geq 0$ [illegible] $A\vec{x} \leq b$

$$-\lambda_i \leq 0 \qquad A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\vec{w} = \sum_{i=1}^{N} \lambda_i d_i \vec{x}_i = 0{,}5 \cdot 1 \begin{bmatrix} 0 \\ 0 \end{bmatrix} + 0 \cdot 1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} + 0{,}5(-1) \begin{bmatrix} 2 \\ 0 \end{bmatrix} =$$

$$= \begin{bmatrix} -1 \\ 0 \end{bmatrix} \qquad d(x^{+}) = -x_1 + 0 x_2 + b$$

$d\left(\vec{y}_1\right)$ [illegible] 1

$\Rightarrow b = 1$

$d\left(\vec{x}\right) = -x_1 + 1$