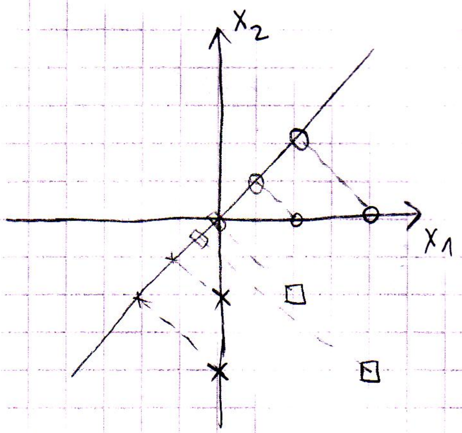
RU | 17.11.2009.

① Fisher s 3 razreda

$\omega_1 = \{(2,0)^T, (4,0)^T\}$

$\omega_2 = \{(0,-2)^T, (0,-4)^T\}$

$\omega_3 = \{(2,-2)^T, (4,-4)^T\}$



$o \in \omega_1$
$x \in \omega_2$
$\square \in \omega_3$

$$\vec{m}_1 = \frac{1}{2}\left(\begin{bmatrix}2\\0\end{bmatrix} + \begin{bmatrix}4\\0\end{bmatrix}\right) = \begin{bmatrix}3\\0\end{bmatrix}$$

$$\vec{m}_2 = \frac{1}{2}\left(\begin{bmatrix}0\\-2\end{bmatrix} + \begin{bmatrix}0\\-4\end{bmatrix}\right) = \begin{bmatrix}0\\-3\end{bmatrix}$$

$$\vec{m}_3 = \frac{1}{2}\left(\begin{bmatrix}2\\-2\end{bmatrix} + \begin{bmatrix}4\\-4\end{bmatrix}\right) = \begin{bmatrix}3\\-3\end{bmatrix}$$

$$\vec{m} = \frac{1}{n}\sum_{i=1}^{c} n_i \vec{m}_i = \frac{1}{6}\left[2\cdot\begin{bmatrix}3\\0\end{bmatrix} + 2\begin{bmatrix}0\\-3\end{bmatrix} + 2\begin{bmatrix}3\\-3\end{bmatrix}\right] = \begin{bmatrix}2\\-2\end{bmatrix}$$

$$S_1 = \sum_{x_1 \in \omega_1} (\vec{x_i} - \vec{m_1})(\vec{x_i} - \vec{m_i})^T = \left(\begin{bmatrix}2\\0\end{bmatrix} - \begin{bmatrix}3\\0\end{bmatrix}\right)\left(\begin{bmatrix}2\\0\end{bmatrix} - \begin{bmatrix}3\\0\end{bmatrix}\right)^T$$

$$+ \left(\begin{bmatrix}4\\0\end{bmatrix} - \begin{bmatrix}3\\0\end{bmatrix}\right)\left(\begin{bmatrix}4\\0\end{bmatrix} - \begin{bmatrix}3\\0\end{bmatrix}\right)^T$$

$$= \begin{bmatrix}-1\\0\end{bmatrix}\begin{bmatrix}-1 & 0\end{bmatrix} + \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1 & 0\end{bmatrix} = \begin{bmatrix}2 & 0\\0 & 0\end{bmatrix}$$

$$S_2 = \begin{bmatrix}0 & 0\\0 & 2\end{bmatrix} \qquad S_3 = \begin{bmatrix}2 & -2\\-2 & 2\end{bmatrix}$$

$$S_B = \sum_{i=1}^{c} n_i (\vec{m_i} - \vec{m})(\vec{m_i} - \vec{m})$$

$$= 2\left(\begin{bmatrix}3\\0\end{bmatrix} - \begin{bmatrix}2\\-2\end{bmatrix}\right)\left(\begin{bmatrix}3\\0\end{bmatrix} - \begin{bmatrix}2\\-2\end{bmatrix}\right)^T + 2\left(\begin{bmatrix}0\\-3\end{bmatrix} - \begin{bmatrix}2\\-2\end{bmatrix}\right)\left(\begin{bmatrix}0\\-3\end{bmatrix} - \begin{bmatrix}2\\-2\end{bmatrix}\right)^T$$

$$+ 2\left(\begin{bmatrix}3\\-3\end{bmatrix} - \begin{bmatrix}2\\-2\end{bmatrix}\right)\left(\begin{bmatrix}3\\-3\end{bmatrix} - \begin{bmatrix}2\\-2\end{bmatrix}\right)^T$$

$$= 2\left(\begin{bmatrix}1\\2\end{bmatrix}\begin{bmatrix}1 & 2\end{bmatrix} + \begin{bmatrix}-2\\-1\end{bmatrix}\begin{bmatrix}-2 & -1\end{bmatrix} + \begin{bmatrix}1\\-1\end{bmatrix}\begin{bmatrix}1 & -1\end{bmatrix}\right)$$

$$= 2\begin{bmatrix}6 & 3\\3 & 6\end{bmatrix} = \begin{bmatrix}12 & 6\\6 & 12\end{bmatrix}$$

$$S_W = S_1 + S_2 + S_3 = \begin{bmatrix}4 & -2\\-2 & 4\end{bmatrix}$$

$S_B \cdot \vec{w} = \lambda \cdot S_W \cdot \vec{w}$ $\rightarrow$ generalizirani svojstveni vektori

$S_W^{-1} \cdot S_B \cdot \vec{w} = \lambda \cdot \vec{w}$

$$\left.\begin{aligned}&\det|S_W^{-1} \cdot S_B - \lambda I| = 0\\&\det|S_B - \lambda S_W| = 0\end{aligned}\right\}$$ ekvivalentne formule (determinante)

$$\det\left|\begin{bmatrix}12 & 6\\6 & 12\end{bmatrix} - \lambda\begin{bmatrix}4 & -2\\-2 & 4\end{bmatrix}\right| = 0$$

$$\det\begin{vmatrix}12-4\lambda & 6+2\lambda\\6+2\lambda & 12-4\lambda\end{vmatrix} = 0$$

$$(12-4\lambda)^2 - (6+2\lambda)^2 = 0$$

$$144 - 96\lambda + 16\lambda - 36 - 24\lambda + 4\lambda^2 = 0$$

$$12\lambda^2 - 120\lambda + 108 = 0$$

$$\lambda^2 - 10\lambda + 9 = 0$$

$$\lambda_{1,2} = \frac{10 \pm 8}{2} \Rightarrow \lambda_1 = 9, \quad \lambda_2 = 1$$

rješenje odgovara najvećim svojstvenim vrijednostima pa se uvijek uzme veći λ

$$(S_B - \lambda S_W) \cdot \vec{w} = \vec{0}$$

$$\begin{bmatrix} -24 & 24 \\ 24 & -24 \end{bmatrix} \cdot \vec{w} = \vec{0}$$

$$24 \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \vec{w} = \vec{0}$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \vec{0}$$

$$\left.\begin{aligned} -w_1 + w_2 &= 0 \\ w_1 - w_2 &= 0 \end{aligned}\right\} \quad w_1 = w_2$$

npr. $w_1 = 1$

$w = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \rightarrow$ pokazuje u smjeru pravca

2) poopćene decizijske funkcije

npr. $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$ $\quad \vec{x}^* = \begin{bmatrix} x_1^2 \\ x_1 x_2 \\ x_2^2 \\ x_1 \\ x_2 \\ 1 \end{bmatrix}$

$$d(\vec{x}) = w_1 x_1^2 + w_2 x_1 x_2 + w_3 x_2^2 + w_4 x_1 + w_5 x_2 + w_6$$

↳ decizijska f-ja oblika polinoma drugog stupnja (dobije se proširenjem vektora značajki x^*)

npr. $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ 1 \end{bmatrix}$ $\quad \vec{x}^* = \begin{bmatrix} x_1^2 \\ x_2^2 \\ x_3^2 \\ x_1 x_2 \\ x_1 x_3 \\ x_2 x_3 \\ x_1 \\ x_2 \\ x_3 \\ 1 \end{bmatrix}$ $\rightarrow$ za polinom trećeg stupnja npr. $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$

$$\vec{x}^* = \begin{bmatrix} x_1^3 \\ x_2^3 \\ x_1^2 x_2 \\ x_1 x_2^2 \\ x_1^2 \\ x_1 x_2 \\ x_2^2 \\ x_1 \\ x_2 \\ 1 \end{bmatrix}$$

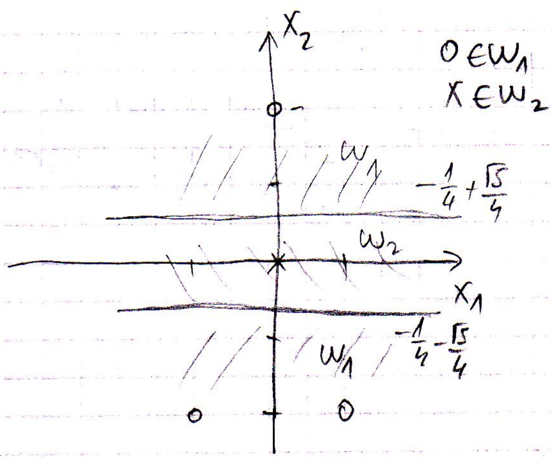
→ redoslijed komponenti nije bitan, samo da je konzistentan

→ isto trećeg stupnja

③ poopćene decizijske funkcije

$\omega_1 = \{ (0,2)^T, (-1,-2)^T, (1,-2)^T \}$

$\omega_2 = \{ (0,0)^T \}$



postupak perceptrona sa stalnim prirastom

$c = 1 \quad \vec{w}(1) = \vec{0}$

$$\vec{x}^* = \begin{bmatrix} x_1^2 \\ x_1 x_2 \\ x_2^2 \\ x_1 \\ x_2 \\ 1 \end{bmatrix}$$

$\vec{x}_1 = [0 \ 0 \ 4 \ 0 \ 2 \ 1]^T$

$\vec{x}_2 = [1 \ 2 \ 4 \ -1 \ -2 \ 1]^T$

$\vec{x}_3 = [1 \ -2 \ 4 \ 1 \ -2 \ 1]^T$

$\vec{x}_4 = [0 \ 0 \ 0 \ 0 \ 0 \ 1]^T \quad / \cdot (-1)$

$\vec{x}_4 = [0 \ 0 \ 0 \ 0 \ 0 \ -1]^T$

$\vec{w} \cdot \vec{x} > 0$

$\vec{w}(1) = [0 \ 0 \ 0 \ 0 \ 0 \ 0]^T$

1) $\vec{w}(1)^T \cdot \vec{x}_1 = [0\ 0\ 0\ 0\ 0\ 0] \cdot \begin{bmatrix} 0 \\ 0 \\ 4 \\ 0 \\ 2 \\ 1 \end{bmatrix} = 0 < 0$

$\vec{w}(2) = \vec{w}(1) + c \cdot \vec{x}_1 = [0\ 0\ 4\ 0\ 2\ 1]^T$

2) $\vec{w}(2)^T \cdot \vec{x}_2 = [0\ 0\ 4\ 0\ 2\ 1] \cdot \begin{bmatrix} 1 \\ 2 \\ 4 \\ -1 \\ -2 \\ 1 \end{bmatrix} = 13 > 0$

$\vec{w}(3) = \vec{w}(2)$

3) $\vec{w}(3)^T \cdot \vec{x}(3) = [0\ 0\ 4\ 0\ 2\ 1] \cdot \begin{bmatrix} 1 \\ -2 \\ 4 \\ 1 \\ -2 \\ 1 \end{bmatrix} = 13 > 0$

$\vec{w}(4) = \vec{w}(3)$

4) $\vec{w}(4)^T \cdot \vec{x}(4) = [0\ 0\ 4\ 0\ 2\ 1] \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} = -1 < 0$

$\vec{w}(5) = \vec{w}(4) + c \cdot \vec{x}(4) = [0\ 0\ 4\ 0\ 2\ 0]^T$

5) $\vec{w}^T(5) \cdot \vec{x}(5) = 20 > 0$

6) $\vec{w}^T(6) \cdot \vec{x}(6) = 12 > 0$

7) $\vec{w}^T(7) \cdot \vec{x}(7) = 12 > 0$

8) $\vec{w}^T(8) \cdot \vec{x}(8) = 0 < 0$

9) $\vec{w}(9) = [0\ 0\ 4\ 0\ 2\ -1]^T \Rightarrow$ rješenje

$\vec{w}(9)^T \cdot \vec{x}(9) = 19 > 0$

10) $\vec{w}(10)^T \cdot \vec{x}(10) = 11 > 0$

11) $\vec{w}(11)^T \cdot \vec{x}(11) = 11 > 0$

12) $\vec{w}(12)^T \cdot \vec{x}(12) = 1 > 0$

$4x_2^2 + 2x_2 - 1 = 0$

$4x_2^2 + 2x_2 + \frac{1}{4} - \frac{5}{4} = 0$

$\left(2x_2 + \frac{1}{2}\right)^2 = \frac{5}{4}$

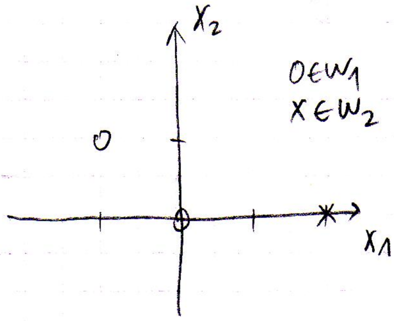
$2x_2 + \frac{1}{2} = \pm \frac{\sqrt{5}}{2}$

$x_2 = -\frac{1}{4} \pm \frac{\sqrt{5}}{4}$

④ dualni problem SVM-a s kvadratnim programiranjem – quadprog funkcija u MATLAB-u

$w_1 = \{ (0,0)^T, (-1,1)^T \}$

$w_2 = \{ (2,0)^T \}$



· kvadratno programiranje:

$$\min_{\{x\}} \frac{1}{2} \vec{x}^T Q \vec{x} + \vec{c}^T \cdot \vec{x}$$

uz uvjete $A\vec{x} \leq \vec{b}$

$E\vec{x} = \vec{d}$

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naš zadatak

$$\max_{\{\lambda\}} \sum_{i=1}^{n} \lambda_i - \frac{1}{2} \sum_{i,j=1}^{n} \lambda_i \lambda_j d_i d_j \vec{x_i}^T \vec{x_j}$$

uz uvjete $\sum_{i=1}^{n} \lambda_i d_i = 0$

$\lambda_i \geq 0$

(treba svesti na kvad. programiranje)

$$\vec{x} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}$$

$$\min_{\{\lambda\}} \underbrace{\frac{1}{2} \sum_{i,j=1}^{n} \lambda_i \lambda_j d_i d_j \vec{x_i}^T \vec{x_j}}_{\frac{1}{2}\vec{x}^T Q \vec{x}} \underbrace{- \sum_{i=1}^{n} \lambda_i}_{+\vec{c}^T \cdot \vec{x}}$$

$$\frac{1}{2}\vec{x}^T Q \vec{x} = \frac{1}{2} \sum_{i,j=1}^{n} \vec{x_i} \vec{x_j} Q_{i,j}$$

$$Q_{i,j} = d_i d_j \vec{x_i}^T \vec{x_j}$$

$$Q_{2,2} = 1 \cdot 1 \cdot [-1 \;\; 1] \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix} = 2$$

$$Q_{2,3} = Q_{3,2} = 1 \cdot (-1) \cdot [-1 \;\; 1] \begin{bmatrix} 2 \\ 0 \end{bmatrix} = 2$$

$$Q_{3,3} = (-1) \cdot (-1) \cdot [2 \;\; 0] \cdot \begin{bmatrix} 2 \\ 0 \end{bmatrix} = 4$$

$$Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 2 & 4 \end{bmatrix}$$

$$\vec{c} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

$$\vec{c}^T \cdot \vec{x} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}^T \cdot \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = -\lambda_1 - \lambda_2 - \lambda_3 = -\sum_{i=1}^{3} \lambda_i$$

– uvjeti isto moraju biti jednaki:

$\lambda_i \geq 0 \quad / \cdot (-1)$

$A\vec{x} \leq \vec{b}$

$-\lambda_i \leq 0$

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sum_{i=1}^{n} \lambda_i d_i = 0 \qquad \Leftrightarrow \qquad E\vec{x} = \vec{d}$$

$$E = [d_1 \ d_2 \ d_3] \qquad \vec{d} = [0]$$

$$[d_1 \ d_2 \ d_3] \cdot \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = 0$$

$\left.\begin{matrix} d_1 = 1 \\ d_2 = 1 \end{matrix}\right\} \omega_1$

$d_3 = -1 \Rightarrow$ treći uzorak iz ω_2

$$\lambda = \begin{bmatrix} 0{,}5 \\ 0 \\ 0{,}5 \end{bmatrix}$$ → to dobije MATLAB

$\vec{x}_1$ i $\vec{x}_3$ su potporni vektori

$$\vec{w} = \sum_{i=1}^{n} \lambda_i d_i \vec{x}_i = 0{,}5 \cdot 1 \cdot \begin{bmatrix} 0 \\ 0 \end{bmatrix} + 0 \cdot 1 \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix} + 0.5 \cdot (-1) \cdot \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\vec{w} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$\cancel{-1 \cdot x_1 = 0}$ – nije granica između razreda, nedostaje mu $\vec{b}$

$\vec{w}^T \cdot \vec{x}^{(s)} + \vec{b} = \pm 1 \qquad \vec{x}^{(s)}$ – potporni vektor

$[-1 \ 0] \begin{bmatrix} 0 \\ 0 \end{bmatrix} + b = 1$ → zato jer $\vec{x}_1$ pripada razredu ω_1

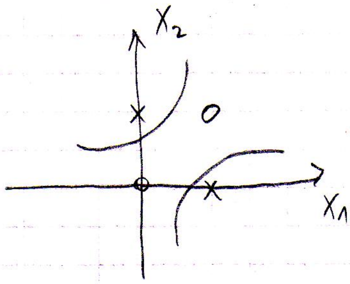
$b = 1$

$-x_1 + 1 = 0$ → prava granica između razreda

(5) problem: XOR sa SVM-om (granica u obliku polinoma drugog stupnja)

$w_1 = \{ (0,0)^T, (1,1)^T \}$

$w_2 = \{ (0,1)^T, (1,0)^T \}$



kernel funkcija:

$K(\vec{x_i}, \vec{x_j}) = (\vec{x_i}^T \vec{x_j} + 1)^2$ → simetrična f-ja
simetrična matrica K

$K_{i,j} = K(\vec{x_i}, \vec{x_j})$

$$K = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 9 & 4 & 4 \\ 1 & 4 & 4 & 1 \\ 1 & 4 & 1 & 4 \end{bmatrix}$$

$K(\vec{x_1}, \vec{x_1}) = \left([0\ 0] \cdot \begin{bmatrix} 0 \\ 0 \end{bmatrix} + 1 \right)^2 = 1$

$K(\vec{x_2}, \vec{x_2}) = \left([1\ 1] \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 1 \right)^2 = 9$

$K(\vec{x_2}, \vec{x_3}) = \left([1\ 1] \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 1 \right)^2 = 4$

$Q_{i,j} = d_i d_j (\vec{x_i}^T x_j) = d_i d_j \cdot K(\vec{x_i}, \vec{x_j})$

$$Q = \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 9 & -4 & -4 \\ -1 & -4 & 4 & 1 \\ -1 & -4 & 1 & 4 \end{bmatrix} \quad c = \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} \quad A = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$E = [d_1\ d_2\ d_3\ d_4] = [1\ \ 1\ \ -1\ \ -1] \qquad d = [0]$

$$\lambda = \begin{bmatrix} 3.33 \\ 2.00 \\ 2.66 \\ 2.66 \end{bmatrix} \rightarrow \text{opet taj MATLAB} \Rightarrow \lambda = \begin{bmatrix} 10/3 \\ 2 \\ 8/3 \\ 8/3 \end{bmatrix}$$

$$d(\vec{x}) = \sum_{i=1}^{n} \lambda_i d_i K(\vec{x_i}, \vec{x}) + \vec{b}$$

$\vec{x_i}$ – potporni vektori
$\vec{x}$ – nepoznati uzorak

$$K(\vec{x_i}, \vec{x}) = \left([x_{i1}\ x_{i2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 1 \right)^2$$

$$= x_{i1}^2 x_1^2 + x_{i2}^2 x_2^2 + 2x_{i1} x_{i2} x_1 x_2 + 2x_{i1} x_1 + 2x_{i2} x_2 + 1$$

– svi uzorci iz skupa za učenje su potporni vektori

$$d(\vec{x}) = \frac{10}{3} \cdot 1 \, (0 \cdot x_1^2 + 0x_2^2 + 0x_1x_2 + 0x_1 + 0x_2 + 1)$$
$$+ 2 \cdot 1 \, (x_1^2 + x_2^2 + 2x_1x_2 + 2x_1 + 2x_2 + 1)$$
$$+ \frac{8}{3} (-1) (0x_1^2 + x_2^2 + 0x_1x_2 + 0x_1 + 2x_2 + 1)$$

$$+\ \frac{8}{3}(-1)\left(x_1^2+0x_2^2+0\cdot x_1x_2+2x_1+0x_2+1\right)+\vec{b}$$

$$d(\vec{x})=-\frac{2}{3}x_1^2-\frac{2}{3}x_2^2+4x_1x_2-\frac{4}{3}x_1-\frac{4}{3}x_2+\vec{b}$$

neki od potpornih vektora uvrstiti u $d(\vec{x})$ i postaviti da sve bude jednako nula – dobije se $\vec{b}$

$$d(\vec{x}_1)=0+\vec{b}=1\Rightarrow\quad\vec{b}=1$$