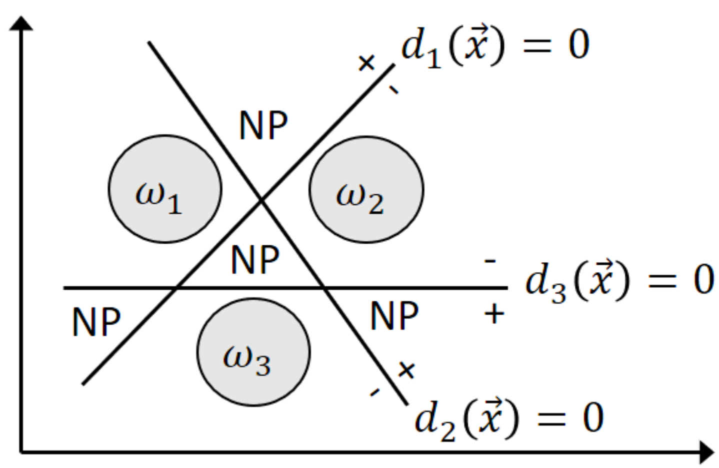
# 1. Auditorne iz Raspoznavanja uzoraka

# Klasifikacija uzoraka u više razreda – 3 slučaja

1. SLUČAJ:
   - Svaki razred se može odvojiti od ostalih razreda decizijskom funkcijom
   - Broj razreda jednak je broju decizijskih funkcija
   - Veliki broj nedefiniranih područja
   - U definiranim područjima samo jedna decizijska funkcija ima pozitivan predznak, sve ostale su negativne

NP – nedefinirano područje



# Klasifikacija uzoraka u više razreda – 3 slučaja

PRIMJER:

$d_1(\vec{x}) = -x_1 + x_2$
$d_2(\vec{x}) = x_1 + x_2 - 5$
$d_3(\vec{x}) = -x_2 + 1$

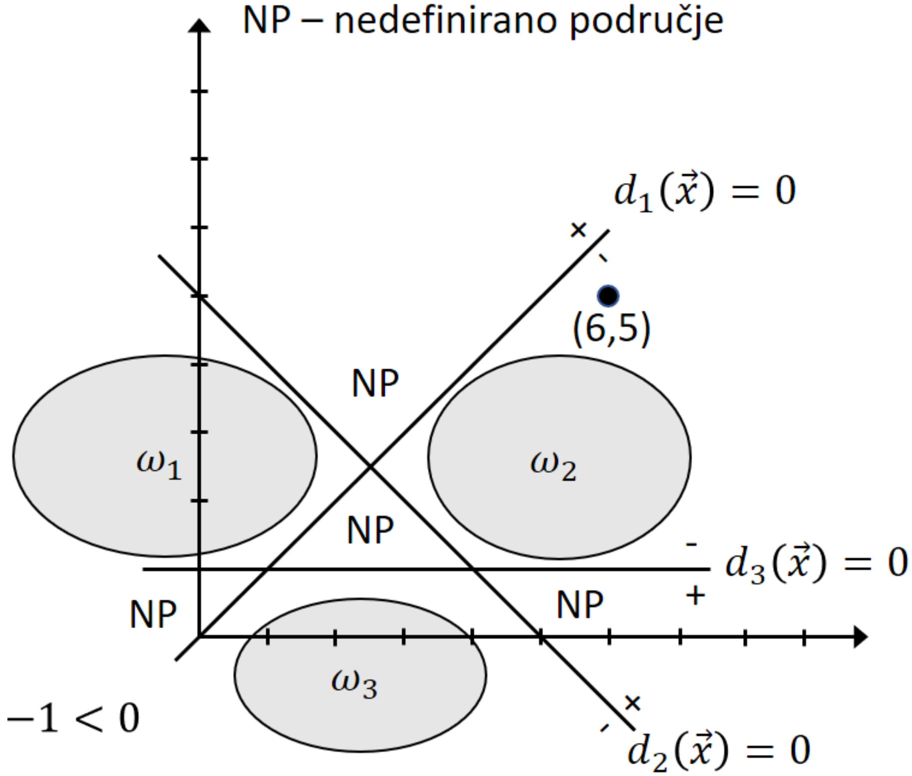
$$\overrightarrow{w_1} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \overrightarrow{w_2} = \begin{bmatrix} 1 \\ 1 \\ -5 \end{bmatrix}, \overrightarrow{w_3} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

a) $\overrightarrow{x_1} = \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix}, \overrightarrow{x_1} \in \omega_2$

$$d_1(\vec{x}) = \overrightarrow{w_1}^T \cdot \vec{x} = [-1 \quad 1 \quad 0] \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix} = -1 < 0$$

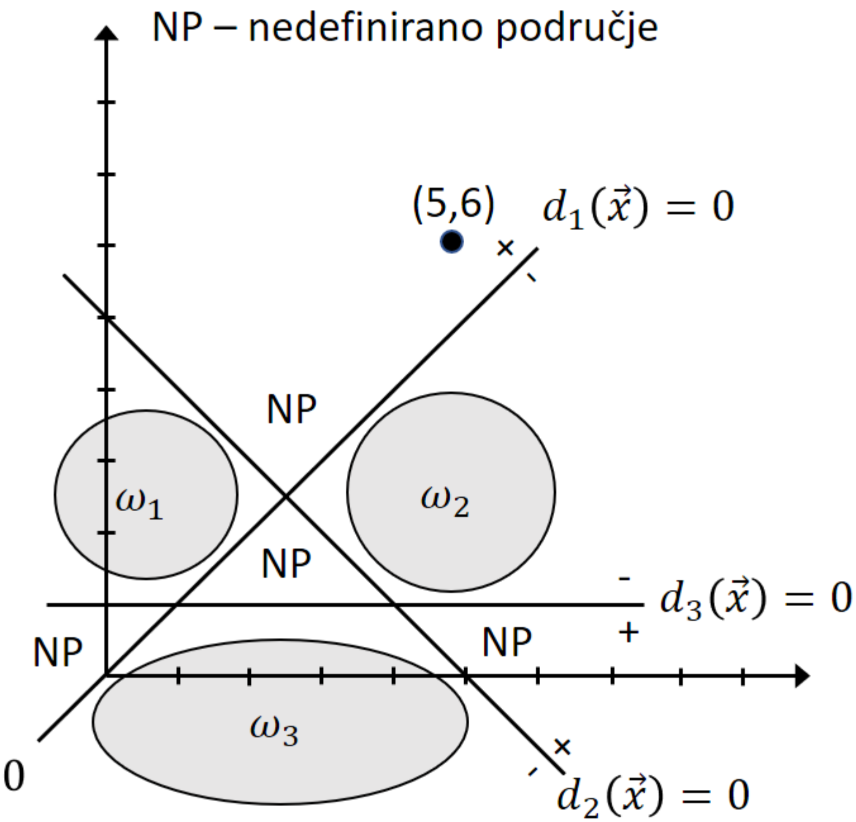
$$d_2(\vec{x}) = \overrightarrow{w_2}^T \cdot \vec{x} = [1 \quad 1 \quad -5] \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix} = 6 > 0$$

$$d_3(\vec{x}) = \overrightarrow{w_3}^T \cdot \vec{x} = [0 \quad -1 \quad 1] \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix} = -4 < 0$$



# Klasifikacija uzoraka u više razreda – 3 slučaja

PRIMJER:




b) $\overrightarrow{x_2} = \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix}, \overrightarrow{x_2} \in \mathrm{NP}$

$$d_1(\vec{x}) = \overrightarrow{w_1}^T \cdot \vec{x} = \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix} = 1 > 0$$

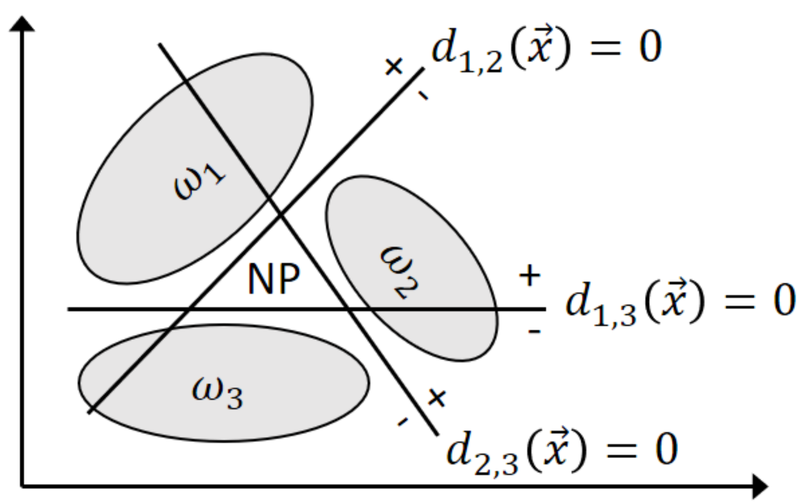
$$d_2(\vec{x}) = \overrightarrow{w_2}^T \cdot \vec{x} = \begin{bmatrix} 1 & 1 & -5 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix} = 6 > 0$$

$$d_3(\vec{x}) = \overrightarrow{w_3}^T \cdot \vec{x} = \begin{bmatrix} 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix} = -5 < 0$$

# Klasifikacija uzoraka u više razreda – 3 slučaja

2. SLUČAJ:
   - Svaki razred se može odvojiti od svakog drugog razreda jednom decizijskom funkcijom,
   - Za M razreda potrebno je $\binom{M}{2} = \frac{M(M-1)}{2}$ decizijskih funkcija,
   - Manji broj nedefiniranih područja
   - Pravilo odlučivanja $\overrightarrow{x_1} \in \omega_i$ $ako\ i\ samo\ ako\ d_{i,j} > 0, \forall\, j \neq i$

NP – nedefinirano područje

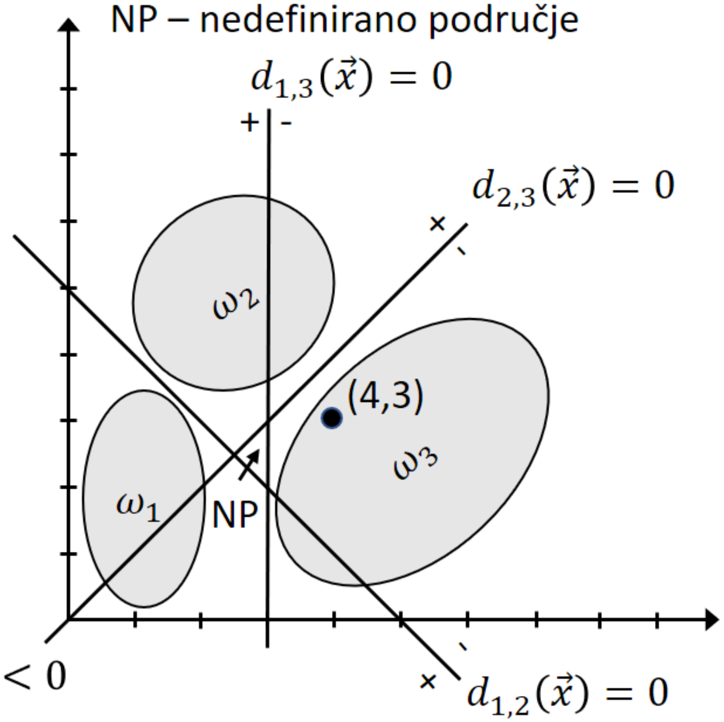


# Klasifikacija uzoraka u više razreda – 3 slučaja

PRIMJER:

$$d_{1,2}(\vec{x}) = -x_1 - x_2 + 5$$
$$d_{1,3}(\vec{x}) = -x_1 + 3$$
$$d_{2,3}(\vec{x}) = -\,x_1 + x_2$$

$$\overrightarrow{w_1} = \begin{bmatrix}-1\\-1\\5\end{bmatrix}, \overrightarrow{w_2} = \begin{bmatrix}-1\\0\\3\end{bmatrix}, \overrightarrow{w_3} = \begin{bmatrix}-1\\1\\0\end{bmatrix}$$




a) $\overrightarrow{x_1} = \begin{bmatrix}4\\3\\1\end{bmatrix}, \overrightarrow{x_1} \in \omega_3$

$$d_{1,2}(\vec{x}) = \overrightarrow{w_1}^T \cdot \vec{x} = \begin{bmatrix}-1 & -1 & 5\end{bmatrix}\begin{bmatrix}4\\3\\1\end{bmatrix} = -2 < 0$$

$$d_{1,3}(\vec{x}) = \overrightarrow{w_2}^T \cdot \vec{x} = \begin{bmatrix}1 & 1 & -5\end{bmatrix}\begin{bmatrix}4\\3\\1\end{bmatrix} = -1 > 0$$

$$d_{2,3}(\vec{x}) = \overrightarrow{w_3}^T \cdot \vec{x} = \begin{bmatrix}0 & -1 & 1\end{bmatrix}\begin{bmatrix}4\\3\\1\end{bmatrix} = -1 < 0$$

$$\left.\begin{aligned} d_{3,1}(\vec{x}) &= 1 > 0 \\ d_{3,2}(\vec{x}) &= 1 > 0 \end{aligned}\right\}, \overrightarrow{x_1} \in \omega_3$$

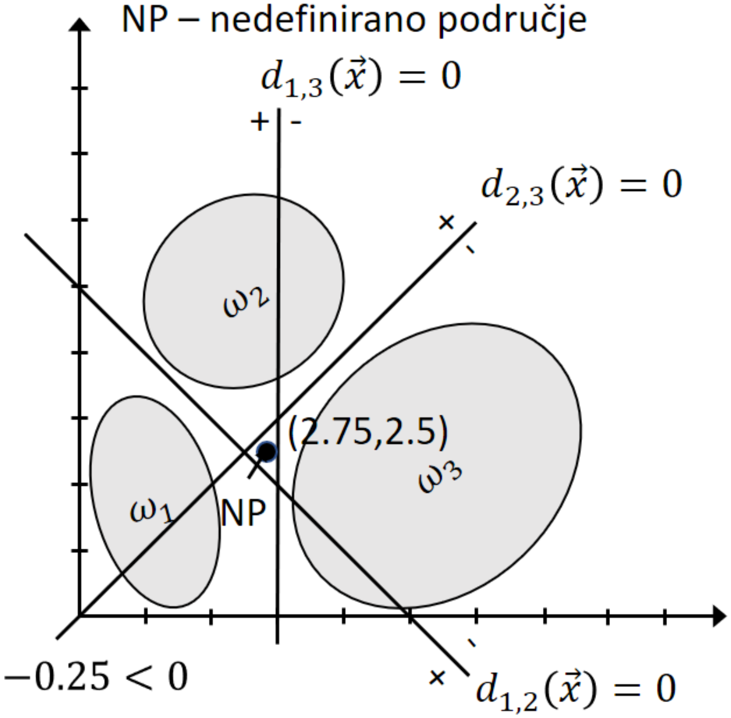
# Klasifikacija uzoraka u više razreda – 3 slučaja

PRIMJER:

$$d_{1,2}(\vec{x}) = -x_1 - x_2 + 5$$
$$d_{1,3}(\vec{x}) = -x_1 + 3$$
$$d_{2,3}(\vec{x}) = -\,x_1 + x_2$$

$$\overrightarrow{w_1} = \begin{bmatrix} -1 \\ -1 \\ 5 \end{bmatrix}, \overrightarrow{w_2} = \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}, \overrightarrow{w_3} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

b) $\overrightarrow{x_1} = \begin{bmatrix} 2.75 \\ 2.5 \\ 1 \end{bmatrix}, \overrightarrow{x_1} \in NP$



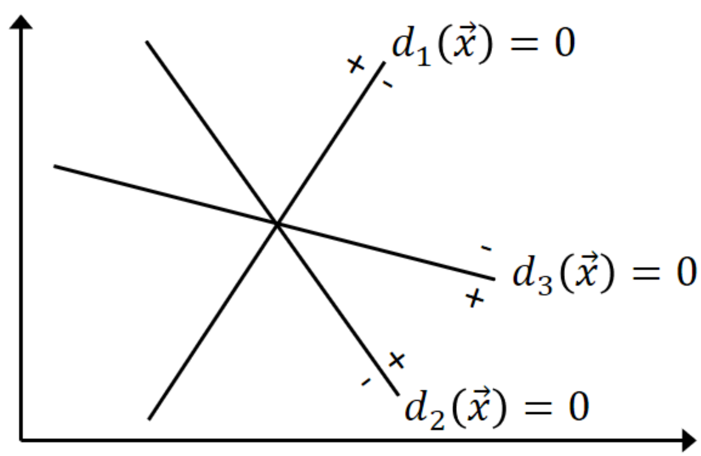

$$d_{1,2}(\vec{x}) = \overrightarrow{w_1}^T \cdot \vec{x} = [-1 \quad -1 \quad 5]\begin{bmatrix} 2.75 \\ 2.5 \\ 1 \end{bmatrix} = -0.25 < 0$$

$$d_{1,3}(\vec{x}) = \overrightarrow{w_2}^T \cdot \vec{x} = [1 \quad 1 \quad -5]\begin{bmatrix} 2.75 \\ 2.5 \\ 1 \end{bmatrix} = 0.25 > 0$$

$$d_{2,3}(\vec{x}) = \overrightarrow{w_3}^T \cdot \vec{x} = [0 \quad -1 \quad 1]\begin{bmatrix} 2.75 \\ 2.5 \\ 1 \end{bmatrix} = -0.25 < 0$$

## Klasifikacija uzoraka u više razreda – 3 slučaja

3. SLUČAJ:
   - Svaki razred ima svoju decizijsku funkciju,
   - Nedefinirano područje jedna točka u kojoj se sijeku sve decizijske funkcije
   - Pravilo odlučivanja $\overrightarrow{x_1} \in \omega_i \; ako \; i \; samo \; ako \; d_i > d_j, \forall \; j \neq i$



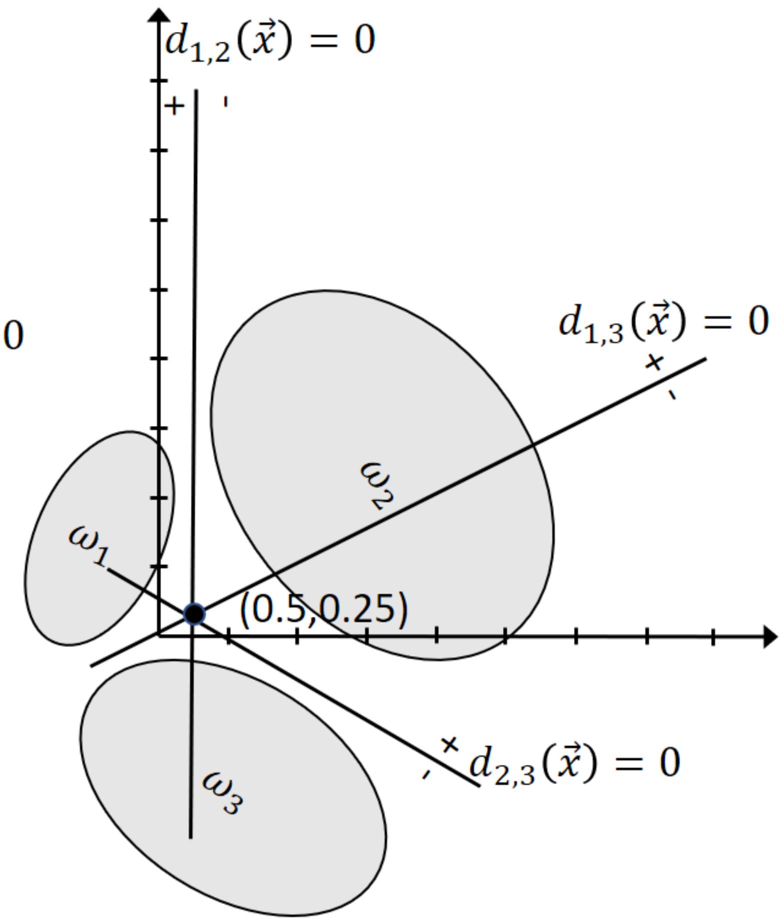
# Klasifikacija uzoraka u više razreda – 3 slučaja

PRIMJER:

$$d_1(\vec{x}) = -x_1 + x_2$$
$$d_2(\vec{x}) = x_1 + x_2\text{-}1$$
$$d_3(\vec{x}) = -x_2$$

$$d_{1,2}(\vec{x}) = d_1(\vec{x}) - d_2(\vec{x}) = -2x_1 + 1 = 0$$
$$d_{2,3}(\vec{x}) = d_2(\vec{x}) - d_3(\vec{x}) = x_1 + 2x_2 - 1 = 0$$
$$d_{1,3}(\vec{x}) = d_1(\vec{x}) - d_3(\vec{x}) = -x_1 + 2x_2 = 0$$

a) $\overrightarrow{x_1} = \begin{bmatrix}2\\3\\1\end{bmatrix}, \overrightarrow{x_2} = \begin{bmatrix}0\\2\\1\end{bmatrix}, \overrightarrow{x_3} = \begin{bmatrix}-3\\1\\1\end{bmatrix},$



## Zadatak 1. – Gradijentni spust

Pravila deriviranja za skalarne funkcije, vektore i matrice

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}[cf(x)] = cf'(x)$$

$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$

$$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$$

$$(fg)'(x) = f(x)g'(x) + g(x)f'(x)$$

$$\left(\frac{f}{g}\right)'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x = \frac{1}{\cos^2 x}$$

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

$$\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1}f'(x)$$

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x = -\frac{1}{\sin^2 x}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(\arccos x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{d\vec{x}}(\vec{x}^T A) = A \qquad \frac{d}{dX}(\vec{a}^T X \vec{b}) = \vec{a}\vec{b}^T$$

$$\frac{d}{d\vec{x}}(\vec{x}^T) = I \qquad \frac{d}{dX}(\vec{a}^T X \vec{a}) = \frac{d}{dX}(\vec{a}^T X^T \vec{a}) = \vec{a}\vec{a}^T$$

$$\frac{d}{d\vec{x}}(\vec{x}^T \vec{a}) = \frac{d}{dx}(\vec{a}^T \vec{x}) = \vec{a} \qquad \frac{d}{d\vec{x}}(\vec{x}^T \vec{x}) = 2\vec{x}$$

$$\frac{d}{d\vec{x}}(\vec{x}^T C \vec{x}) = (C + C^T)\vec{x} \qquad \frac{d}{d\vec{x}}(A\vec{x} + \vec{b})^T (D\vec{x} + \vec{e}) = A^T (D\vec{x} + \vec{e}) + D^T (A\vec{x} + \vec{b})$$

- Osnovna ideja: Smjer gradijenta u nekoj točki pokazuje smjer najvećeg rasta funkcije
- Gradijent skalarne funkcije vektora se računa kao
- Povećanje argumenta u smjeru negativnog gradijenta vodi nas do minimuma funkcije f
- Postupak nas dovodi do lokalnog minimuma

Primjer: Izvođenje algoritma učenja sustava za klasifikaciju s 2 razreda upotrebom gradijentnog spusta za zadanu kriterijsku funkciju

$$J(\vec{w}, \vec{x}, p) = \frac{1}{8\|\vec{x}\|}\left[\left(\vec{w}^T \vec{x} - p\right) - \left|\vec{w}^T \vec{x} - p\right|\right]^2$$

$$\vec{w}(k+1) = \vec{w}(k) - c \cdot \left\{\frac{\partial J(\vec{w}, \vec{x}, p)}{\partial w}\right\}$$

$$\frac{\partial J(\vec{w}, \vec{x}, p)}{\partial \vec{w}} = \frac{1}{4\|\vec{x}\|}\left[\left(\vec{w}^T \vec{x} - p\right) - \left|\vec{w}^T \vec{x} - p\right|\right] \cdot \left[\vec{x} - \operatorname{sgn}\left(\vec{w}^T \vec{x} - p\right) \cdot \vec{x}\right] =$$

$$\frac{1}{4\|\vec{x}\|}\left[\left(\vec{w}^T \vec{x} - p\right) \cdot \vec{x} - \left(\vec{w}^T \vec{x} - p\right) \cdot \operatorname{sgn}\left(\vec{w}^T \vec{x} - p\right) \cdot \vec{x} - \left|\vec{w}^T \vec{x} - p\right| \cdot \vec{x} + \left|\vec{w}^T \vec{x} - p\right| \cdot \operatorname{sgn}\left(\vec{w}^T \vec{x} - p\right) \cdot \vec{x}\right]$$

$$\left(\vec{w}^{T}\vec{x}-p\right)\cdot \operatorname{sgn}\left(\vec{w}^{T}\vec{x}-p\right)=\left|\vec{w}^{T}\vec{x}-p\right|$$

$$\left|\vec{w}^{T}\vec{x}-p\right|\cdot \operatorname{sgn}\left(\vec{w}^{T}\vec{x}-p\right)=\left(\vec{w}^{T}\vec{x}-p\right)$$

$$\frac{\partial J\left(\vec{w},\vec{x},p\right)}{\partial \vec{w}}=\frac{1}{4\left\|\vec{x}\right\|}\left[2\cdot\left(\vec{w}^{T}\vec{x}-p\right)\cdot\vec{x}-2\cdot\left|\vec{w}^{T}\vec{x}-p\right|\cdot\vec{x}\right]$$

$$\frac{\partial J\left(\vec{w},\vec{x},p\right)}{\partial \vec{w}}=\frac{1}{2\left\|\vec{x}\right\|}\left[\left(\vec{w}^{T}\vec{x}-p\right)\cdot\vec{x}-\left|\vec{w}^{T}\vec{x}-p\right|\cdot\vec{x}\right]$$

$$\vec{w}(k+1)=\vec{w}(k)-\frac{c}{2\left\|\vec{x}\right\|}\left[\left(\vec{w}^{T}\vec{x}-p\right)-\left|\vec{w}^{T}\vec{x}-p\right|\right]\cdot\vec{x}$$

$$\vec{w}(k+1)=\begin{cases}\vec{w}(k) \quad \text{za} \quad \vec{w}^{T}\vec{x}\geq p\\ \vec{w}(k)-\dfrac{c}{\left\|\vec{x}\right\|}\left(\vec{w}^{T}\vec{x}-p\right)\cdot\vec{x} \quad \text{za} \quad \vec{w}^{T}\vec{x}<p\end{cases}$$

## Zadatak 2. - Algoritam perceptrona s apsolutnom korekcijom

Želimo u svakom koraku odabrati c tako da se nakon korekcije težinskog vektora pravilno razvrsta uzorak koji je uzrokovao korekciju

$$\vec{w}^T(k+1)\cdot\vec{x}(k)=[\vec{w}(k)+c\cdot\vec{x}(k)]^T\cdot\vec{x}(k)>0$$

$$c>\frac{\left|\vec{w}^T(k)\cdot\vec{x}(k)\right|}{\vec{x}^T(k)\cdot\vec{x}(k)}$$

c se izabire kao najmanji cijeli broj veći od $\frac{\left|\vec{w}^T(k)\cdot\vec{x}(k)\right|}{\vec{x}^T(k)\cdot\vec{x}(k)}$

Primjer:

$$\omega_1=\{(0),(1)\}$$
$$\omega_2=\{(2)\}$$
$$\vec{w}(1)=\begin{bmatrix}2\\3\end{bmatrix}$$

$$\vec{x}(1)=\begin{bmatrix}0\\1\end{bmatrix},\ \vec{x}(2)=\begin{bmatrix}1\\1\end{bmatrix},\ \vec{x}(3)=\begin{bmatrix}-2\\-1\end{bmatrix}$$

**Korak 1.**

$$\vec{w}^T(1)\cdot\vec{x}(1)=\begin{bmatrix}2 & 3\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}=3$$
$$\vec{w}(2)=\vec{w}(1)$$

**Korak 2.**

$$\vec{w}^T(2)\cdot\vec{x}(2)=\begin{bmatrix}2 & 3\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}=5$$

$$\vec{w}(3)=\vec{w}(2)$$

**Korak 3.**

$$\vec{w}^T(3)\cdot\vec{x}(3)=\begin{bmatrix}2 & 3\end{bmatrix}\begin{bmatrix}-2\\-1\end{bmatrix}=-7$$

$$\frac{\left|\vec{w}^T(3)\cdot\vec{x}(3)\right|}{\vec{x}^T(3)\cdot\vec{x}(3)}=\frac{7}{5},c=2$$

$$\vec{w}(4)=\vec{w}(3)+2\cdot\vec{x}(3)=\begin{bmatrix}2\\3\end{bmatrix}+\begin{bmatrix}-4\\-2\end{bmatrix}=\begin{bmatrix}-2\\1\end{bmatrix}$$

**Korak 4.**

$$\vec{w}^T(4)\cdot\vec{x}(1)=\begin{bmatrix}-2 & 1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}=1$$

$$\vec{w}(5)=\vec{w}(4)$$

**Korak 5.**

$$\vec{w}^T(5)\cdot\vec{x}(2)=\begin{bmatrix}-2 & 1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}=-1$$

$$\frac{\left|\vec{w}^T(5)\cdot\vec{x}(2)\right|}{\vec{x}^T(2)\cdot\vec{x}(2)}=\frac{1}{2},c=1$$

$$\vec{w}(6)=\vec{w}(5)+\vec{x}(2)=\begin{bmatrix}-2\\1\end{bmatrix}+\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}-1\\2\end{bmatrix}$$

**Korak 6.**

$$\vec{w}^T(6)\cdot\vec{x}(3)=\begin{bmatrix}-1 & 2\end{bmatrix}\begin{bmatrix}-2\\-1\end{bmatrix}=0$$

$$\frac{\left|\vec{w}^T(6)\cdot\vec{x}(3)\right|}{\vec{x}^T(3)\cdot\vec{x}(3)}=0,c=1$$

$$\vec{w}(7)=\vec{w}(6)+\vec{x}(3)=\begin{bmatrix}-1\\2\end{bmatrix}+\begin{bmatrix}-2\\-1\end{bmatrix}=\begin{bmatrix}-3\\1\end{bmatrix}$$

**Korak 7.**

$$\vec{w}^T(7)\cdot\vec{x}(1)=\begin{bmatrix}-3 & 1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}=1$$

$$\vec{w}(8)=\vec{w}(7)$$

**Korak 8.**

$$\vec{w}^T(8)\cdot\vec{x}(2)=\begin{bmatrix}-3 & 1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}=-2$$

$$\frac{\left|\vec{w}^T(8)\cdot\vec{x}(2)\right|}{\vec{x}^T(2)\cdot\vec{x}(2)}=1,c=2$$

$$\vec{w}(9)=\vec{w}(8)+2\cdot\vec{x}(2)=\begin{bmatrix}-3\\1\end{bmatrix}+\begin{bmatrix}2\\2\end{bmatrix}=\begin{bmatrix}-1\\3\end{bmatrix}$$

**Korak 9.**

$$\vec{w}^{T}(9)\cdot\vec{x}(3)=\begin{bmatrix}-1 & 3\end{bmatrix}\begin{bmatrix}-2\\-1\end{bmatrix}=-1$$

$$\frac{\left|\vec{w}^{T}(9)\cdot\vec{x}(3)\right|}{\vec{x}^{T}(3)\cdot\vec{x}(3)}=\frac{1}{5},c=1$$

$$\vec{w}(10)=\vec{w}(9)+\vec{x}(3)=\begin{bmatrix}-1\\3\end{bmatrix}+\begin{bmatrix}-2\\-1\end{bmatrix}=\begin{bmatrix}-3\\2\end{bmatrix}$$

**Korak 10.**

$$\vec{w}^{T}(10)\cdot\vec{x}(1)=\begin{bmatrix}-3 & 2\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}=2$$

$$\vec{w}(11)=\vec{w}(10)$$

**Korak 11.**

$$\vec{w}^{T}(11)\cdot\vec{x}(2)=\begin{bmatrix}-3 & 2\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}=-1$$

$$\frac{\left|\vec{w}^{T}(11)\cdot\vec{x}(2)\right|}{\vec{x}^{T}(2)\cdot\vec{x}(2)}=\frac{1}{2},c=1$$

$$\vec{w}(12)=\vec{w}(11)+\vec{x}(2)=\begin{bmatrix}-3\\2\end{bmatrix}+\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}-2\\3\end{bmatrix}$$

**Korak 12.**

$$\vec{w}^{T}(12)\cdot\vec{x}(3)=\begin{bmatrix}-2 & 3\end{bmatrix}\begin{bmatrix}-2\\-1\end{bmatrix}=1$$

$$\vec{w}(13)=\vec{w}(12)$$

**Korak 13.**

$$\vec{w}^{T}(13)\cdot\vec{x}(1)=\begin{bmatrix}-2 & 3\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}=3$$

$$\vec{w}(14)=\vec{w}(13)$$

**Korak 14.**

$$\vec{w}^{T}(14)\cdot\vec{x}(2)=\begin{bmatrix}-2 & 3\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}=1$$

$$\vec{w}(15)=\vec{w}(14)$$

**Zadatak 3. -** Algoritam perceptrona sa stalnim prirastom

$$\omega_1 = \left\{(0,0)^T, (2,2)^T\right\}$$
$$\omega_2 = \left\{(1,1)^T\right\}$$
$$\omega_3 = \left\{(-1,-1)^T\right\}$$

c=1

$$\vec{x}_i = \begin{bmatrix} x_1^2 \\ x_2^2 \\ x_1 \cdot x_2 \\ x_1 \\ x_2 \\ 1 \end{bmatrix}, \vec{x}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \vec{x}_2 = \begin{bmatrix} 4 \\ 4 \\ 4 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \vec{x}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \vec{x}_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}, \vec{w}_1(1) = \vec{w}_2(1) = \vec{w}_3(1) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, c = 1$$

**Korak 1.**

$$d_1(1) = \vec{w}_1^T(1) \cdot \vec{x}(1) = 0$$
$$d_2(1) = \vec{w}_2^T(1) \cdot \vec{x}(1) = 0$$
$$d_3(1) = \vec{w}_3^T(1) \cdot \vec{x}(1) = 0$$

$$\vec{w}_1(2)=\vec{w}_1(1)+\vec{x}(1)=\begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix},\qquad \vec{w}_2(2)=\vec{w}_2(1)-\vec{x}(1)=\begin{bmatrix}0\\0\\0\\0\\0\\-1\end{bmatrix},\qquad \vec{w}_3(2)=\vec{w}_3(1)-\vec{x}(1)=\begin{bmatrix}0\\0\\0\\0\\0\\-1\end{bmatrix}$$

**Korak 2.**

$$d_1(2)=\vec{w}_1^T(2)\cdot\vec{x}(2)=1$$
$$d_2(2)=\vec{w}_2^T(2)\cdot\vec{x}(2)=-1$$
$$d_3(2)=\vec{w}_3^T(2)\cdot\vec{x}(2)=-1$$

$$\vec{w}_1(3)=\vec{w}_1(2)$$
$$\vec{w}_2(3)=\vec{w}_2(2)$$
$$\vec{w}_3(3)=\vec{w}_3(2)$$

**Korak 3.**

$$d_1(3)=\vec{w}_1^T(3)\cdot\vec{x}(3)=1$$
$$d_2(3)=\vec{w}_2^T(3)\cdot\vec{x}(3)=-1$$
$$d_3(3)=\vec{w}_3^T(3)\cdot\vec{x}(3)=-1$$

$$\vec{w}_1(4)=\vec{w}_1(3)-\vec{x}(3)=\begin{bmatrix}-1\\-1\\-1\\-1\\-1\\0\end{bmatrix},\qquad \vec{w}_2(4)=\vec{w}_2(3)+\vec{x}(3)=\begin{bmatrix}1\\1\\1\\1\\1\\0\end{bmatrix},\qquad \vec{w}_3(4)=\vec{w}_3(3)-\vec{x}(3)=\begin{bmatrix}-1\\-1\\-1\\-1\\-1\\-2\end{bmatrix}$$

**Korak 4.**

$d_1(4)=\vec{w}_1^T(4)\cdot\vec{x}(4)=-1$

$d_2(4)=\vec{w}_2^T(4)\cdot\vec{x}(4)=1$

$d_3(4)=\vec{w}_3^T(4)\cdot\vec{x}(4)=-3$

$$\vec{w}_1(5)=\vec{w}_1(4)-\vec{x}(4)=\begin{bmatrix}-2\\-2\\-2\\0\\0\\-1\end{bmatrix},\qquad \vec{w}_2(5)=\vec{w}_2(4)-\vec{x}(4)=\begin{bmatrix}0\\0\\0\\2\\2\\-1\end{bmatrix},\qquad \vec{w}_3(5)=\vec{w}_3(4)+\vec{x}(4)=\begin{bmatrix}0\\0\\0\\-2\\-2\\-1\end{bmatrix}$$

**Korak 5.**

$d_1(5)=\vec{w}_1^T(5)\cdot\vec{x}(5)=-1$

$d_2(5)=\vec{w}_2^T(5)\cdot\vec{x}(5)=-1$

$d_3(5)=\vec{w}_3^T(5)\cdot\vec{x}(5)=-1$

$$\vec{w}_1(5) = \vec{w}_1(4) + \vec{x}(5) = \begin{bmatrix} -2 \\ -2 \\ -2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad \vec{w}_2(5) = \vec{w}_2(4) - \vec{x}(5) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \\ 2 \\ -2 \end{bmatrix}, \qquad \vec{w}_3(5) = \vec{w}_3(4) - \vec{x}(5) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -2 \\ -2 \\ -2 \end{bmatrix}$$

## **Zadatak 4. –** Algoritam Ho-Kashyap

Inicijaliziraj $c, \vec{b}(1) > 0, k = 0$

$$[X]^{\#} = ([X]^T \cdot [X])^{-1} \cdot [X]^T$$

$$\vec{w}(1) = [X]^{\#} \vec{b}(1)$$

ponavljaj{

$$k = k + 1$$

$$\vec{e}(k) = [X] \cdot \vec{w}(k) - \vec{b}(k)$$

$$\vec{b}(k+1) = \vec{b}(k) + c \cdot [\vec{e}(k) + |\vec{e}(k)|]$$

$$\vec{w}(k+1) = c \cdot [X]^{\#} \cdot \vec{b}(k+1)$$

} dok nije ispunjen jedan od uvjeta za zaustavljanja

ako je $\vec{e}(k) = \vec{0}$ našli smo rješenje jer tada vrijedi $[X] \cdot \vec{w} = \vec{b}$

ako su sve komponente vektora $\vec{e}(k)$ negativne (ali ne sve jednake nuli) znaci da razredi nisu separabilni

Primjer:

$$\omega_1 = \left\{(1,0)^T, (0,1)^T\right\}$$
$$\omega_2 = \left\{(-1,0)^T, (0,-1)^T\right\}$$

$$\vec{x}(1) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \ \vec{x}(2) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \ \vec{x}(3) = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \ \vec{x}(4) = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$$c = 1 \quad \vec{\mathrm{b}}(1) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix},$$ treba imati n komponenti

$$[\mathrm{X}] = \begin{bmatrix} \vec{x}_1^T \\ \vec{x}_2^T \\ \vec{x}_3^T \\ \vec{x}_4^T \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$[\mathrm{X}]^T [\mathrm{X}] = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\left([\mathrm{X}]^T [\mathrm{X}]\right)^{-1} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.25 \end{bmatrix}$$

$$[\mathrm{X}]^{\#} = \left([\mathrm{X}]^T [\mathrm{X}]\right)^{-1} [\mathrm{X}]^T = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.25 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0.5 \\ 0.25 & 0.25 & -0.25 & -0.25 \end{bmatrix}$$

$$\vec{w}(1) = [\mathrm{X}]^{\#} \vec{\mathrm{b}}(1) = \begin{bmatrix} 0.5 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0.5 \\ 0.25 & 0.25 & -0.25 & -0.25 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\vec{e}(1) = [X] \cdot \vec{w}(1) - \vec{b}(1)$$

$$\vec{e}(k) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

**Zadatak 5. –** Algoritam Ho-Kashyap

$$\omega_1 = \{(0,0)^T, (1,0)^T, (0,1)^T\}$$
$$\omega_2 = \{(1,1)^T\}$$

$$\vec{x}(1) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},\ \vec{x}(2) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},\ \vec{x}(3) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix},\ \vec{x}(4) = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

$c = 1 \quad \vec{b}(1) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, treba imati n komponenti

$$[X] = \begin{bmatrix} \vec{x}_1^T \\ \vec{x}_2^T \\ \vec{x}_3^T \\ \vec{x}_4^T \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix}$$

$$[X]^T[X] = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$

$$\left([X]^T[X]\right)^{-1} = \begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & -0.5 \\ -0.5 & -0.5 & 0.75 \end{bmatrix}$$

$$[\mathrm{X}]^{\#} = \left([\mathrm{X}]^T[\mathrm{X}]\right)^{-1}[\mathrm{X}]^T = \begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & -0.5 \\ -0.5 & -0.5 & 0.75 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} -0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & -0.5 \\ 0.75 & 0.25 & 0.25 & 0.25 \end{bmatrix}$$

$$\vec{w}(1) = [\mathrm{X}]^{\#}\vec{\mathrm{b}}(1) = \begin{bmatrix} -0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & -0.5 \\ 0.75 & 0.25 & 0.25 & 0.25 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1.5 \end{bmatrix}$$

$$\vec{\mathrm{e}}(1) = [\mathrm{X}] \cdot \vec{\mathrm{w}}(1) - \vec{\mathrm{b}}(1)$$

$$\vec{\mathrm{e}}(1) = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ -1 \\ -1.5 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5 \\ -0.5 \\ -0.5 \\ -0.5 \end{bmatrix}$$

$$\vec{\mathrm{b}}(2) = \vec{\mathrm{b}}(1) + \mathrm{c} \cdot \left(\vec{\mathrm{e}}(1) + |\vec{\mathrm{e}}(1)|\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, + \begin{bmatrix} 0.5 \\ -0.5 \\ -0.5 \\ -0.5 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix},$$

$$\vec{w}(2) = [\mathrm{X}]^{\#}\vec{\mathrm{b}}(2) = \begin{bmatrix} -0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & -0.5 \\ 0.75 & 0.25 & 0.25 & 0.25 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.5 \\ -1.5 \\ 2.25 \end{bmatrix}$$

$$\vec{e}(2) = [X] \cdot \vec{w}(2) - \vec{b}(2)$$

$$\vec{e}(2) = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} -1.5 \\ -1.5 \\ 2.25 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.25 \\ 0.75 \\ 0.75 \\ 0.75 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.25 \\ -0.25 \\ -0.25 \\ -0.25 \end{bmatrix}$$

$$\vec{b}(3) = \vec{b}(2) + c \cdot \left(\vec{e}(2) + |\vec{e}(2)|\right) = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix}, + \begin{bmatrix} 0.5 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 1 \\ 1 \\ 1 \end{bmatrix},$$

$$\vec{w}(3) = [X]^{\#} \vec{b}(3) = \begin{bmatrix} -0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & -0.5 \\ 0.75 & 0.25 & 0.25 & 0.25 \end{bmatrix} \begin{bmatrix} 2.5 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.75 \\ -1.75 \\ 2.425 \end{bmatrix}$$

$$\vec{e}(3) = [X] \cdot \vec{w}(3) - \vec{b}(3)$$

$$\vec{e}(3) = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} -1.75 \\ -1.75 \\ 2.425 \end{bmatrix} - \begin{bmatrix} 2.5 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.125 \\ -0.125 \\ -0.125 \\ -0.125 \end{bmatrix}$$

konvergira

$$\vec{w}(k) = \begin{bmatrix} -2 \\ -2 \\ 3 \end{bmatrix}, \vec{\mathrm{b}}(3) = \begin{bmatrix} 3 \\ 1 \\ 1 \\ 1 \end{bmatrix},$$

**Zadatak 6. –** Polinomna decizijska funkcija zapis u rekurzivnom obliku linearne decizijske funkcije:

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ 1 \end{bmatrix}$$

$$d(\vec{x}) = w_1 x_1 + w_2 x_2 + \ldots + w_n x_n + w_{n+1}$$

Poopćene linearne decizijske funkcije:

$$\vec{x}^* = \begin{bmatrix} f_1(\vec{x}) \\ f_2(\vec{x}) \\ \vdots \\ f_k(\vec{x}) \\ 1 \end{bmatrix}$$

$$d(\vec{x}^*) = w_1 \cdot f_1(\vec{x}) + w_2 \cdot f_2(\vec{x}) + \ldots + w_k \cdot f_k(\vec{x}) + w_{k+1}$$

Polinomna decizijska funkcija zapis u rekurzivnom obliku

$$g^r(\vec{x}) = \left( \sum_{p_1=1}^{l} \sum_{p_2=p_1}^{l} \ldots \sum_{p_r=p_{r-1}}^{l} w_{p_1 p_2 \ldots p_r} x_{p_1} x_{p_2} \ldots x_{p_r} \right) + g^{r-1}(\vec{x})$$

$$g^0(\vec{x}) = w_{l+1}$$

$r$ = 3 i $l$ = 2

$$g^3(\vec{x}) = \left( \sum_{p_1=1}^{2} \sum_{p_2=p_1}^{2} \sum_{p_3=p_2}^{2} w_{p_1 p_2 p_3} x_{p_1} x_{p_2} x_{p_3} \right) + g^2(\vec{x})$$

$$g^2(\vec{x}) = \left( \sum_{p_1=1}^{2} \sum_{p_2=p_1}^{2} w_{p_1 p_2} x_{p_1} x_{p_2} \right) + g^1(\vec{x})$$

$$g^1(\vec{x}) = \left( \sum_{p_1=1}^{2} w_{p_1} x_{p_1} \right)$$

$$g^0(\vec{x}) = w_3$$

$$\begin{aligned} g^3(\vec{x}) = {} & w_{111} x_1^3 + w_{112} x_1^2 x_2 + w_{122} x_1 x_2^2 + \\ & w_{222} x_2^3 + w_{11} x_1^2 + w_{12} x_1 x_2 + w_{22} x_2^2 + \\ & w_1 x_1 + w_2 x_2 + w_3 \end{aligned}$$

$r$ = 3 i $l$ = 3

$$g^3(\vec{x}) = \left( \sum_{p_1=1}^{3} \sum_{p_2=p_1}^{3} \sum_{p_3=p_2}^{3} w_{p_1 p_2 p_3} x_{p_1} x_{p_2} x_{p_3} \right) + g^2(\vec{x})$$

$$g^2(\vec{x}) = \left( \sum_{p_1=1}^{3} \sum_{p_2=p_1}^{3} w_{p_1 p_2} x_{p_1} x_{p_2} \right) + g^1(\vec{x})$$

$$g^1(\vec{x}) = \left( \sum_{p_1=1}^{3} w_{p_1} x_{p_1} \right)$$

$$g^0(\vec{x}) = w_3$$

$$
\begin{aligned}
g^{3}(\vec{x}) = {} & w_{111}x_1^3 + w_{112}x_1^2 x_2 + w_{113}x_1^2 x_3 + w_{122}x_1 x_2^2 \\
& + w_{123}x_1 x_2 x_3 + w_{133}x_1 x_3^2 + w_{222}x_2^3 + w_{223}x_2^2 x_3 + \\
& + w_{233}x_2 x_3^2 + w_{333}x_3^3 + w_{11}x_1^2 + w_{12}x_1 x_2 + w_{13}x_1 x_3 + \\
& + w_{22}x_2^2 + w_{23}x_2 x_3 + w_{33}x_3^2 + w_1 x_1 + w_2 x_2 + w_3 x_3 + w_3
\end{aligned}
$$