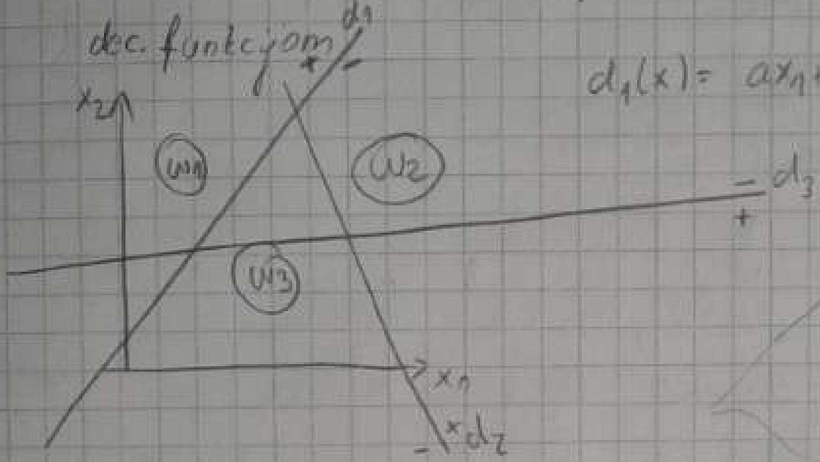
napomena: $D_x = \left| \frac{\vec{w_0}^T \vec{x}}{\|\vec{w_0}\|} + \frac{w_{n+1}}{\|\vec{w_0}\|} \right|$ udaljenost točke od hiperravnine

2) proširi dimenzionalnost i razrede iz ω_2 pomnoži sa -1

3) imamo 3 razreda

1. slučaj

Svaki razred se može odvojiti od ostalih razreda dec. funkcijom



$d_1(x) = ax_1 + bx_2 + c$

=> prvim slučajem ne možemo birati dec. f., tj. dobijemo neodr. područja

$\vec{\omega_1}(\vec{x}) \Rightarrow d_1(\vec{x}) > 0,\ d_2(\vec{x}) < 0,\ d_3(\vec{x}) < 0$

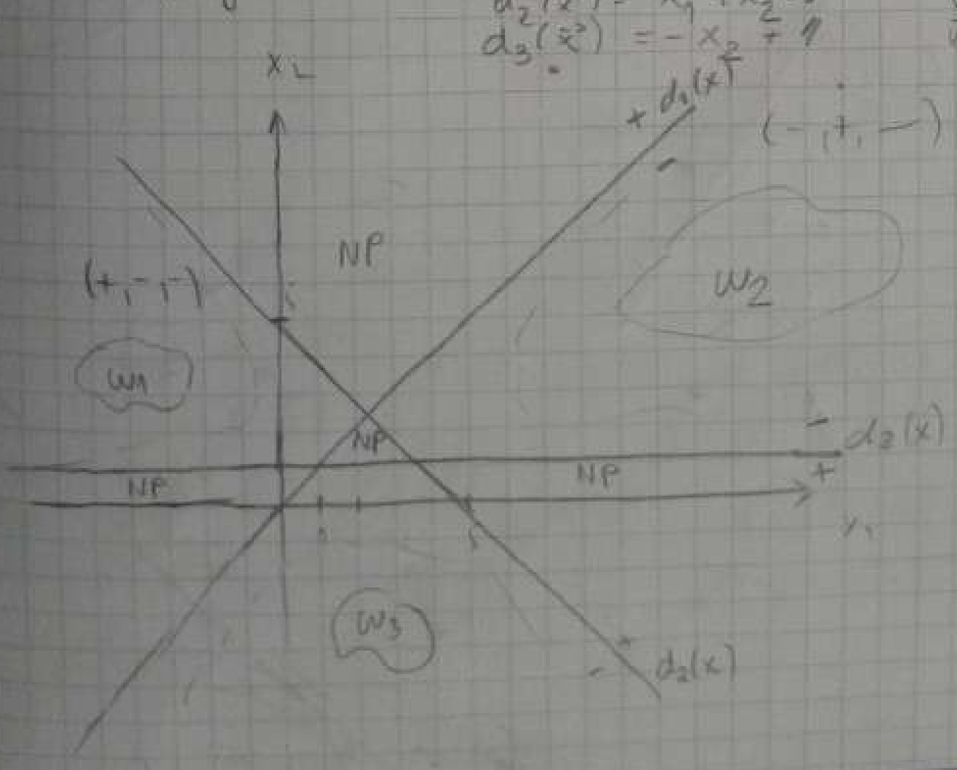
$\vec{\omega_i}(x) \Rightarrow d_i(x) > 0,\ d_j(x) < 0\ \forall j \neq i$

primjer za doma:

$d_1(\vec{x}) = -x_1 + x_2 \Rightarrow \vec{w_1} = [-1\ \ 1\ \ 0]^T$

$d_2(\vec{x}) = x_1 + x_2 - 5 \quad \vec{w_2} = [1\ \ 1\ \ -5]^T$

$d_3(\vec{x}) = -x_2 + 1 \quad \vec{w_3} = [0\ \ -1\ \ 1]^T$



2. slučaj:

Svaka dec f-ja odvaja par razreda
(relativno manje respektivan slučaj od 1.)

$$\binom{M}{2} = \frac{M(M-1)}{1\cdot 2} \Leftarrow \text{broj dec. f-ja}$$

za $M=3$ $\binom{3}{2}=3$

$$\vec{x} \in \omega_i : \; d_{ij}(\vec{x}) > 0 \quad \forall j \neq i$$

pravilo $d_{ij} = -d_{ji}$

d_{12} +

d_{13} -

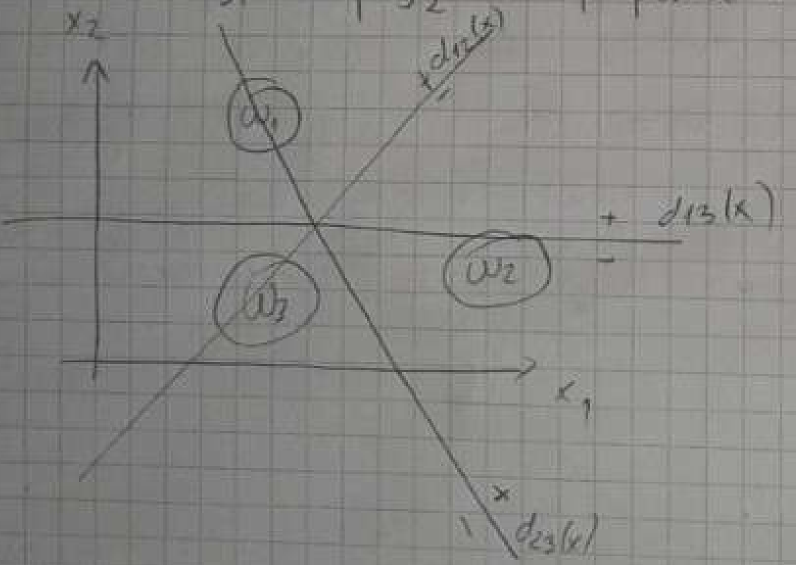
d_{23} -

$\Downarrow$

$d_{12} > 0$, $d_{13} < 0$ nije ω_1

$d_{21} < 0$, $d_{23} < 0$ nije ω_2

$d_{31} > 0$, $d_{32} > 0$ pripada ω_3



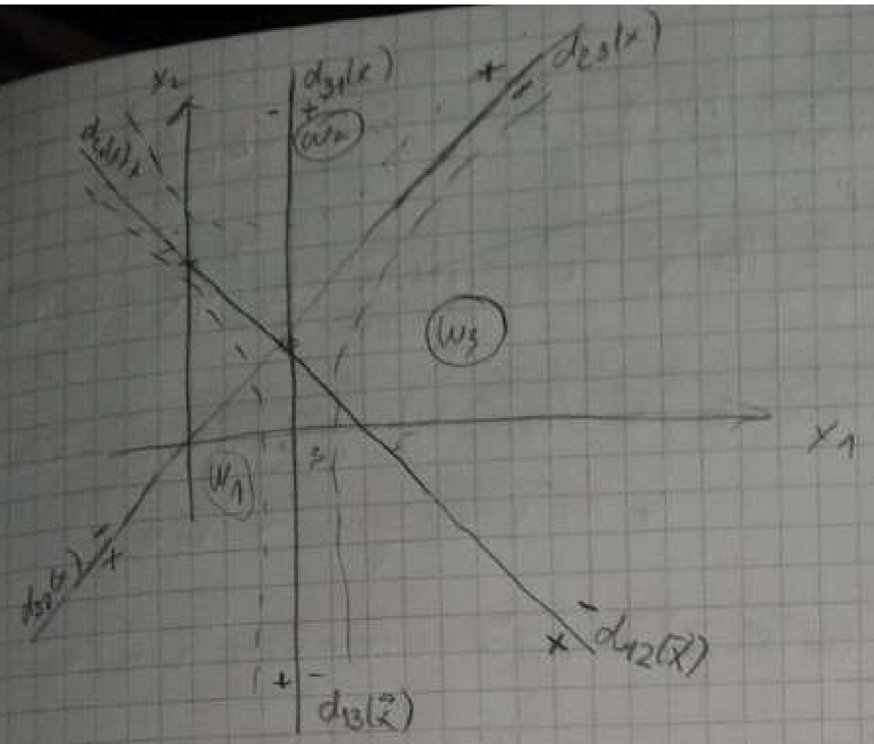
graf. prikaz nije potpun bez tih - na pravima

→ ovaj primjer vjerojatniji da dođe na ispitu

$d_{12}(\vec{x}) = -x_1 - x_2 + 5$ $\quad \vec{w}_1 = \begin{bmatrix} -1 \\ -1 \\ 5 \end{bmatrix}$

$d_{13}(\vec{x}) = -x_1 + 3$ $\quad \vec{w}_2 = \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}$

$d_{23}(\vec{x}) = -x_1 + x_2$ $\quad \vec{w}_3 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$



$\vec{x}_1(4,3)$ $\quad d_{12}(\vec{x}_1) = -2$
$d_{13}(\vec{x}_1) = -1$
$d_{23}(\vec{x}_1) = -1$
} -> potvrđeno da je u w_3

$\vec{x}_2(2{,}75\ ,\ 2{,}5) \rightarrow d_{12}(\vec{x}_2) = -0.25$
$d_{13}(\vec{x}_2) = 0.25$
$d_{23}(\vec{x}_2) = -0.25$

~~w_1~~
~~w_2~~
~~w_3~~

NEODREĐENO

3. slučaj:

$$d_k(\vec{x}) = \vec{w}_k^T \vec{x} \quad , k = 1, \dots, M$$

P.O. $\quad \vec{x} \in \omega_i \quad$ akko $\quad d_i(\vec{x}) > d_j(x) \ \forall j \neq i$

stvarna granica $\quad d_i(\vec{x}) - d_j(\vec{x}) > 0$

$$d_{ij}(\vec{x}) = d_i(\vec{x}) - d_j(\vec{x}) = 0$$

primjer: $d_1(\vec{x}) = -x_1 + x_2$

$$d_2(\vec{x}) = x_1 + x_2 - 1$$

$$d_3(\vec{x}) = -x_2$$

$$d_{12}(\vec{x}) = d_1(\vec{x}) - d_2(\vec{x}) = -2x_1 + 1$$

$$d_{13}(\vec{x}) = -2x_1 - 1$$

$$d_{23}(\vec{x}) = x_1 + 2x_2 - 1$$

izračunati vrij. od d-bve i uzet najveći

d_1	7
d_2	1
d_3	-3

nedef. pod. degeneriraju u točku

d-ovi je jako u točki

ZAD (derivacija)

$$J(\vec w, \vec x, b) = \frac{1}{8\|\vec x\|^2}\left[(\vec w^T\vec x - b) - |\vec w^T\vec x - b|\right]^2$$

$$\frac{\partial J}{\partial \vec w} = \frac{1}{8\|\vec x\|^2}\, 2\left[(\vec w^T\vec x - b) - |\vec w^T\vec x - b|\right]\cdot\left(\vec x - \mathrm{sgn}(\vec w^T\vec x - b)\cdot\vec x\right)$$

$$= \frac{1}{4\|\vec x\|^2}\Big[(\vec w^T\vec x - \vec b)\vec x - (\vec w^T\vec x - b)\,\mathrm{sgn}(\vec w^T\vec x - b)\cdot\vec x - \vec x|\vec w^T\vec x - b|$$
$$+ |\vec w^T\vec x - b|\,\mathrm{sgn}(\vec w^T\vec x - b)\vec x\Big]$$

$$= \frac{1}{4\|\vec x\|^2}\left[(\vec w^T\vec x - \vec b)\vec x - |\vec w^T\vec x - \vec b|\vec x - \vec x|\vec w^T\vec x - \vec b| + (\vec w^T\vec x - \vec b)\vec x\right]$$

$$= \frac{1}{\underset{2}{\cancel{4}}\|\vec x\|^2}\left[\cancel{2}(\vec w^T\vec x - b)\vec x - \cancel{2}|\vec w^T\vec x - \vec b|\vec x\right]$$

$$= \frac{1}{2\|\vec x\|^2}\left[\vec x\left[(\vec w^T\vec x - \vec b) - |\vec w^T\vec x - \vec b|\right]\right]$$

$$\vec w(k+1) = \vec w(k) - \frac{c}{2\|\vec x\|^2}\cdot\left[(\vec w^T\vec x - b) - |\vec w^T\vec x - b|\right]\vec x$$

$$\vec w(k+1) = \begin{cases} \vec w(k) & \text{za } \vec w^T\vec x - b \geq 0 \\ \vec w(k) - \frac{c\cdot\vec x}{\|\vec x\|^2}\cdot(\vec w^T\vec x - \vec b) & \text{za } \vec w^T\vec x - b < 0 \end{cases}$$

ZAD (PERC. SA STALNOM KOR.)

$b = 1$
$c = 1$

$\omega_1 = \{(1,0)^T, (0,-1)^T\}$

$\omega_2 = \{(0,1)^T, (-1,0)^T\}$

$x_1 = \begin{bmatrix}1\\0\\1\end{bmatrix} \quad x_2 = \begin{bmatrix}0\\-1\\1\end{bmatrix} \quad x_3 = \begin{bmatrix}0\\-1\\-1\end{bmatrix} \quad x_4 = \begin{bmatrix}1\\0\\-1\end{bmatrix}$

$\vec w(1) = [1\ 0\ 0]^T$

$\vec w^T(1)\cdot\vec x(1) = [1\ 0\ 0]\begin{bmatrix}1\\0\\1\end{bmatrix} = 1$ ✓ $\geq b \qquad \vec w(2) = \vec w(1)$

$\vec w^T(2)\cdot\vec x(2) = [1\ 0\ 0]\begin{bmatrix}0\\-1\\1\end{bmatrix} = 0$ ✗ < 1

$$\vec w(3) = \vec w(1) - \frac{1}{2}\cdot(0-1)\begin{bmatrix}0\\-1\\1\end{bmatrix} =$$

$$= \begin{bmatrix}1\\0\\0\end{bmatrix} - \begin{bmatrix}0\\+\frac{1}{2}\\-\frac{1}{2}\end{bmatrix} = \begin{bmatrix}1\\-0.5\\0.5\end{bmatrix}$$

$\vec{w}^T(3)\cdot\vec{x}(3) = [1 \ -0.5 \ 0.5]\begin{bmatrix}0\\-1\\-1\end{bmatrix} = 0 \quad < 1$

$\vec{w}(4) = \begin{bmatrix}1\\-0.5\\0.5\end{bmatrix} - \frac{1}{2}\cdot(0-1)\begin{bmatrix}0\\-1\\-1\end{bmatrix} = \begin{bmatrix}1\\-1\\0\end{bmatrix}$

$\vec{w}^T(4)\cdot\vec{x}(4) = [1 \ -1 \ 0]\cdot\begin{bmatrix}1\\0\\-1\end{bmatrix} = 1 \checkmark \quad \geq 1$

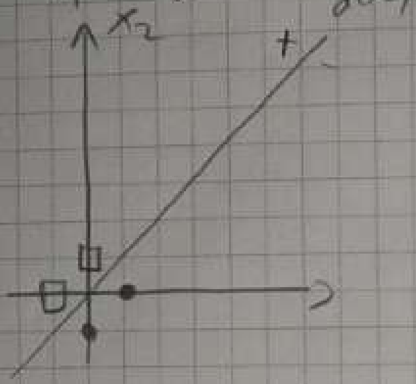
$\vec{w}^T(5)\cdot\vec{x}(1) = [1 \ -1 \ 0]\begin{bmatrix}1\\0\\1\end{bmatrix} = 1 \checkmark \quad \geq 1$

$\vec{w}^T(6)\cdot\vec{x}(2) = [1 \ -1 \ 0]\begin{bmatrix}0\\-1\\1\end{bmatrix} = 1 \checkmark \quad \geq 1$

$\vec{w}^T(7)\cdot\vec{x}(3) = [1 \ -1 \ 0]\begin{bmatrix}0\\-1\\-1\end{bmatrix} = 1 \checkmark \quad \geq 1$

$\vec{w} = [1 \ -1 \ 0]^T$

$d(x) = x_1 - x_2$



x_1

$\bullet \in \omega_1$

$\square \in \omega_2$

(naučiti izvode...)

ZAD (Ho-Kashyap)

$\omega_1 = \{(1,0), (0,1)\}$

$\omega_2 = \{(-1,0), (0,-1)\}$

$\vec{b}(1) = [1 \ 1 \ 1 \ 1]$

$$[X] = \begin{bmatrix}1 & 0 & 1\\0 & 1 & 1\\1 & 0 & -1\\0 & 1 & -1\end{bmatrix}$$

$$[X]^\# = \left[[X]^T[X]\right]^{-1}[X]^T$$

$$[X]^T[X] = \begin{bmatrix}1 & 0 & 1 & 0\\0 & 1 & 0 & 1\\1 & 1 & -1 & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 1\\0 & 1 & 1\\1 & 0 & -1\\0 & 1 & -1\end{bmatrix} = \begin{bmatrix}2 & 0 & 0\\0 & 2 & 0\\0 & 0 & 4\end{bmatrix}$$

$$\left([X]^T[X]\right)^{-1} = \begin{bmatrix}1/2 & 0 & 0\\0 & 1/2 & 0\\0 & 0 & 1/4\end{bmatrix}$$

$$[X]^\# = \begin{bmatrix}1/2 & 0 & 1/2 & 0\\0 & 1/2 & 0 & 1/2\\1/4 & 1/4 & -1/4 & -1/4\end{bmatrix}$$

$$\vec{w}(1) = [X]^{\#} \underline{b}(1)$$

$$= [1 \quad 1 \quad 0]^T$$

$$\vec{e}(1) = [X] \cdot \vec{w}(1) - \vec{b}(1) = [0 \quad 0 \quad 0 \quad 0]$$

→ uzorci pravilno razdvojeni