PERCEPTRON

$\omega_1 = \{ [0,0]^T, [1,0]^T \}$

$\omega_2 = \{ [0,1]^T \}$ $\qquad c = 1 \qquad \vec{w}^T(1) = \vec{0}$

množimo s -1

$\vec{x}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad \vec{x}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad \vec{x}_3 = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}$

(1) $\vec{w}^T(1) \cdot \vec{x}(1) = [0\ 0\ 0] \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$ $\quad \leq 0$, radimo korekciju

$\vec{w}(2) = \vec{w}(1) + \vec{x}(1) =$

$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(2) $\vec{w}^T(2) \cdot \vec{x}(2) = [0\ 0\ 1] \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 1 > 0$ ok

$\vec{w}(3) = \vec{w}(2)$

(3) $\vec{w}^T(3) \cdot \vec{x}(3) = [0\ 0\ 1] \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} = -1 \leq 0$, korekcija

$\vec{w}(4) = \vec{w}(3) + \vec{x}(3) = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$

(4) $\vec{w}^T(4) \cdot \vec{x}(4) = [0\ -1\ 0] \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0 \leq 0$ kor

$\vec{w}(5) = \vec{w}(4) + \vec{x}(4) = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

(5) $\vec{w}^T(5) \cdot \vec{x}(5) = [0\ -1\ 1] \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 1 > 0$

$\vec{w}(6) = \vec{w}(5)$

(6) $\vec{w}^T(6) \cdot \vec{x}(6) = [0\ -1\ 1] \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} = 0 \leq 0$

(6) $\vec{w}^T(6) \cdot \vec{x}(6) = \begin{bmatrix} 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} = 0 \leq 0$

$\vec{w}(7) = \vec{w}(6) + \vec{x}(6) = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$

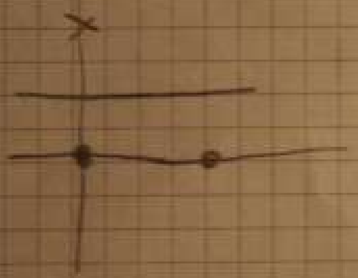
(7) $\vec{w}^T(7) \cdot \vec{x}(7) = \begin{bmatrix} 0 & -2 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$

$\vec{w}(8) = \vec{w}(7) + \vec{x}(7) = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$

(8) $\vec{w}^T(8) \cdot \vec{x}(8) = 1$

(9) $\vec{w}^T(9) \cdot \vec{x}(9) = 1$

(10) $\vec{w}^T(10) \cdot \vec{x}(10) = 1$ $\quad \vec{w} = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$



$\bullet \in \omega_1$

$\times \in \omega_2$

$0x_1 - 2x_2 + 1 = 0$

$x_2 = \frac{1}{2}$

PERCEPTRON S APSOLUTNIM [illegible]

$$\omega_1 = \{ [0,0]^T, [1, 0,5]^T \}$$

$$\omega_2 = \{ [0,1]^T, [1,2]^T \}$$

$$\vec{w}(1) = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

$$\vec{x_1} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \vec{x_2} = \begin{bmatrix} 1 \\ 0,5 \\ 1 \end{bmatrix}$$

$$\vec{x_3} = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} \quad \vec{x_4} = \begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix}$$

$$c > \frac{|\vec{w}^T(k) \cdot \vec{x}(k)|}{\vec{x}(k) \cdot \vec{x}(k)}$$

$$w(1)^T x(1) = [-1 \; 0 \; 0] \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0 \leq 0$$

$$c > \frac{0}{2} = 0 \Rightarrow c = 1$$

$$w(2) = w(1) + x(1) = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$w(2)\, x(2) = [-1 \; 0 \; 1] \begin{bmatrix} 1 \\ 0,5 \\ 1 \end{bmatrix} = 0 \leq 0$$

$$c > \frac{0}{} \Rightarrow c = 1$$

$$w(3) = w(2) + x(2) = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0,5 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0,5 \\ 2 \end{bmatrix}$$

$$w(3) \cdot x(3) = [0 \; 0,5 \; 2] \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} = -2,5 \leq 0$$

$$c > \frac{|-2{,}5|}{2} = 1{,}25 \Rightarrow c = 2$$

$$w(4) = w(3) + 2x(3) = \begin{bmatrix} 0 \\ 0{,}5 \\ 2 \end{bmatrix} + 2\begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1{,}5 \\ 0 \end{bmatrix}$$

$$w(4) \cdot x(4) = \begin{bmatrix} 0 & -1{,}5 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix} = 3 > 0 \quad \text{ok}$$

...

$$\vec{w} = \begin{bmatrix} 0 & -1{,}5 & 1 \end{bmatrix}^T$$

$$w_1 = \{[0,0]^T, [0,1]^T, [0{,}5, 0{,}5]^T\}$$

$$w_2 = \{[1,0]^T, [1,1]^T\}$$

$$\vec{w}(1) = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \qquad \lambda = 1{,}5$$

$$w_1 = \{[0,0,1]^T, [0,1,1]^T, [0{,}5, 0{,}5, 1]^T\}$$

$$w_2 = \{[-1,0,-1]^T, [-1,-1,-1]^T\}$$

$$\begin{bmatrix} -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0 \leq 0$$

$$c = \lambda \cdot 0 = 0$$

$$\vec{w}(2) = \vec{w}(1) + 0\,\vec{x} = \vec{w}(1)$$

$$\begin{bmatrix} -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\vec{w}(3) = \vec{w}(2)$$

3) $$\begin{bmatrix} -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0,5 \\ 0,5 \\ 1 \end{bmatrix} = -0,5 < 0$$

$$c = \frac{0,5}{0,5^2 + 0,5^2 + 1} \cdot \frac{3}{2} = \frac{0,5}{1,5} \cdot \frac{3}{2} = 0,5$$

$$\vec{w}(4) = \vec{w}(3) + 0,5 \begin{bmatrix} 0,5 \\ 0,5 \\ 1 \end{bmatrix} = \begin{bmatrix} -0,75 \\ 0,25 \\ 0,5 \end{bmatrix}$$

...

$$J(\vec{w}, \vec{x}) = \frac{1}{8\|x\|^2} \left[(w^T x - b) - |\vec{w}^T \vec{x} - b| \right]^2$$

$$\vec{w}(k+1) = \vec{w}(k) - c \left. \frac{\partial J(\vec{w}, \vec{x})}{\partial \vec{w}} \right|_{\vec{w} = \vec{w}(k)}$$

$$\frac{\partial J}{\partial \vec{w}} = \frac{1}{8 \cdot \|x\|^2} 2 \left[(w^T x - b) - |\vec{w}^T \vec{x} - b| \right] \cdot \left[\vec{x} - \mathrm{sgn}(\vec{w}^T \vec{x} - b) \cdot \vec{x} \right] =$$

$$= \frac{1}{4\|x\|^2} \Big[(\vec{w}^T \vec{x} - b) \cdot \vec{x} - \underbrace{(\vec{w}^T \vec{x} - b) \cdot \mathrm{sgn}(\vec{w}^T \vec{x} - b)}_{|\vec{w}^T \vec{x} - b|} \cdot \vec{x} -$$

$$- |\vec{w}^T \vec{x} - b| \cdot \vec{x} + \underbrace{|\vec{w}^T \vec{x} - b| \cdot \mathrm{sgn}(\vec{w}^T \vec{x} - b)}_{w^T \vec{x} - b} \vec{x} \Big] =$$

$$= \frac{1}{4\|x\|^2} \left[2 (\vec{w}^T \vec{x} - b) \cdot \vec{x} - 2 |\vec{w}^T \vec{x} - b| \vec{x} \right] =$$

$$= \frac{1}{2\|x\|^2} \left[(\vec{w}^T \vec{x} - b) - |\vec{w}^T \vec{x} - b| \right] \cdot \vec{x}$$

$$\vec{w}(k+1) = \begin{cases} \vec{w}(k) & \text{ZA } \vec{w}^T\vec{x} > b \\ \vec{w}(k) - \frac{c}{\|x\|^2}\left(\vec{w}^T\vec{x} - b\right)\cdot\vec{x} & \text{ZA } \vec{w}^T x \le b \end{cases}$$

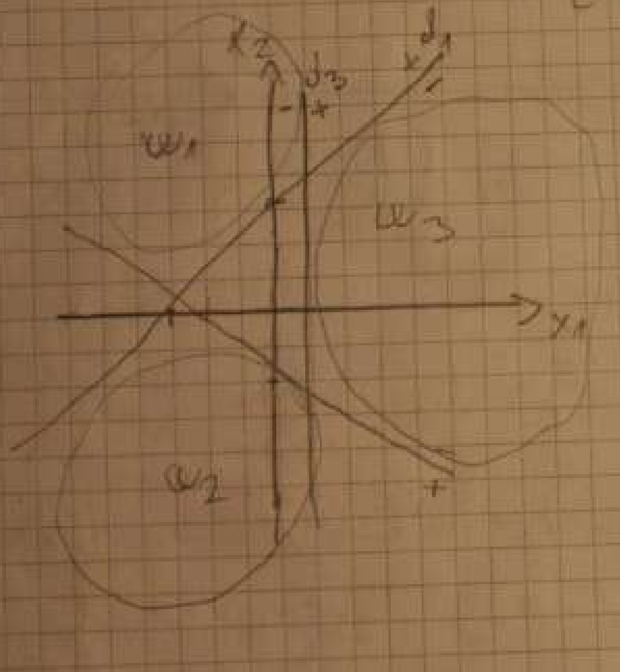
PROBLEM VIŠE OD DVA RAZREDA

$x \in \omega_i$ AKO $d_i > 0$

$d_1 = -x_1 + x_2 - 3$ $\qquad w_1 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}$

$d_2 = -x_1 - 2x_2 - 2$ $\qquad w_2 = \begin{bmatrix} -1 \\ -2 \\ -2 \end{bmatrix}$

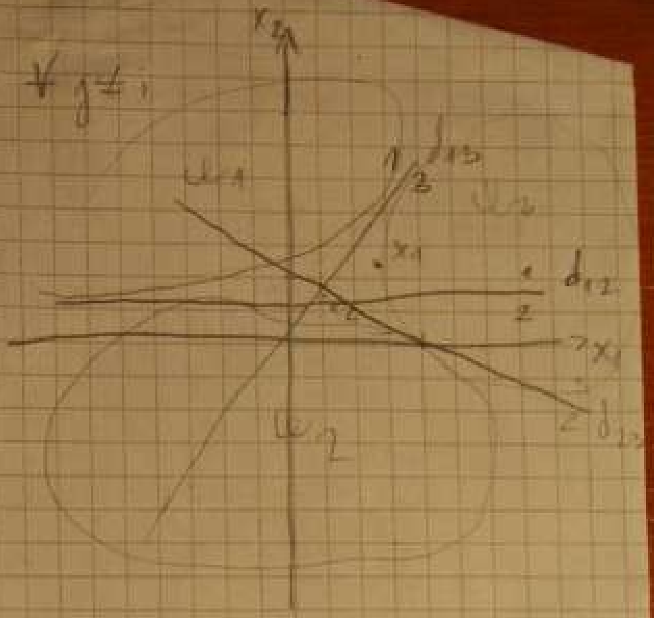
$d_3 = x_1 - 1$ $\qquad w_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$



I

$x \in \omega_i$ AKO $d_{ij} > 0 \ \forall j \neq i$

$d_{ij}(\vec{x}) = -d_{ji}(\vec{x})$



$d_{12} = x_2 - 1$

$d_{13} = -2x_1 + x_2$

$d_{23} = -x_1 - 2x_2 + 4$

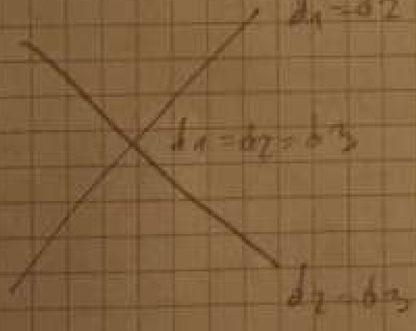
x_1		x_2	
$d_{12} > 0$	$d_{21} < 0$	$d_{12} > 0$	$d_{21} < 0$
$d_{13} < 0$	$d_{31} > 0$	$d_{13} < 0$	$d_{31} > 0$
$d_{23} < 0$	$d_{32} > 0$	$d_{23} > 0$	$d_{32} < 0$

II

$d_i(\vec{x}) > d_j(\vec{x}) \ \forall j \neq i \Rightarrow \vec{x} \in \omega_i$

GRANICA RAZREDA i, j $\quad d_i(\vec{x}) = d_j(\vec{x})$

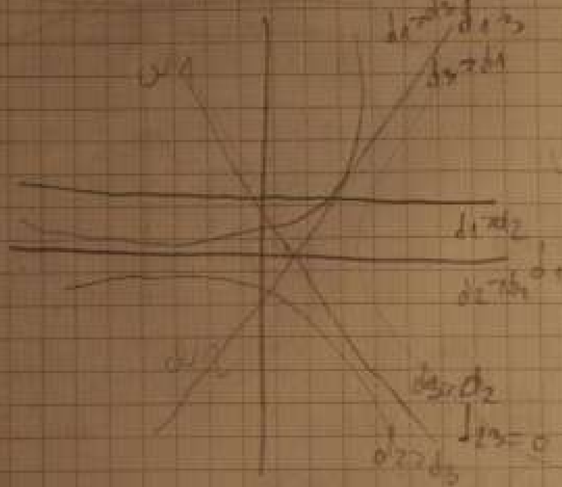
$d_i(\vec{x}) - d_j(\vec{x}) = 0$



$d_1(\vec{x}) = -x_1 + x_2 - 1$ $\qquad d_1 - d_2 = 0 \Rightarrow 2x_2 + 1 = 0$

$d_2(\vec{x}) = -x_1 - x_2$ $\qquad d_1 - d_3 = 0 \Rightarrow -2x_1 + x_2 + 1 = 0$

$d_3(\vec{x}) = x_1$ $\qquad d_2 - d_3 = 0 \Rightarrow -2x_1 - x_2 = 0$



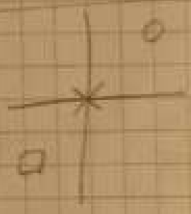
NEMA NEDEFINIRANIH PODRUČJA

ω_3

4 S. TEORIJA
2 S. ZADACI

SLIČNO KAO SVE

$\omega_1 = \{[1, 1]^T\}$ $\quad \omega_2 = \{[0, 0]^T\}$ $\quad \omega_3 = \{[-1, -1]^T\}$



$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad x_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $\quad x_3 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

AKO X PRIPADA RAZREDU ω_1

$\vec{w}_1(1) = \vec{w}_2(1) = \vec{w}_3(1) = \vec{0}$ $\quad c = 1$

(1) $\vec{x}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad d_1 = \vec{w}_1^T \vec{x}_1 = 0$

$d_2 = \vec{w}_2 \vec{x}_1 = 0$

$d_3 = \vec{w}_3 \vec{x}_1 = 0$

$\vec{w}_1(2) = \vec{w}_1(1) + \vec{x}(1) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\vec{w}_2(2) = \vec{w}_2(1) - \vec{x}(1) = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$ $\quad$ – AKO NE PRIPADA

$\vec{w}_3(2) = \vec{w}_3(1) - \vec{x}(1) = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$

(2) $\vec{x}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $\quad d_1 = \vec{w}_1^T \vec{x}_2 = [1\ 1\ 1] \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 1$ $\quad \vec{w}_1(3) = w_1(2) - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$d_2 = [-1\ -1\ -1] \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = -1$ $\quad \vec{w}_2(3) = w_2(2) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$

$d_3 = [-1\ -1\ -1] \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = -1$ $\quad \vec{w}_3(3) = w_3(2) - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix}$

(3) $\vec{x}_3 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$ $\quad d_1 = \vec{w}_1^T x_3 = [1\ 1\ 0] \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = -2$ $\qquad d_1 < d_3$ PA JE OK

$d_2 = \vec{w}_2 x_3 = [-1\ -1\ 0] \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = 2$ $\qquad d_2$ TREBA SMANJITI

$d_3 = \vec{w}_3 x_3 = [-1\ -1\ -2] \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = 0$ $\qquad d_3$ TREBA POVEĆATI