KL TRANSFORMACIJA - PRIMJER IZ SKRIPTE

$$\omega_1 = \{ [0\ 0\ 0]^T, [1\ 0\ 0]^T, [1\ 0\ 1]^T, [1\ 1\ 0]^T \}$$

$$\omega_2 = \{ [0\ 0\ 1]^T, [0\ 1\ 0]^T, [0\ 1\ 1]^T, [1\ 1\ 1]^T \}$$

$$P(\omega_1) = \frac{4}{8} \qquad P(\omega_2) = \frac{4}{8}$$

$$R = \sum_{i=1}^{K} P(\omega_i) \cdot E\{x_i \cdot x_i^T\} = \frac{1}{2} E\{\vec{x}_1, \vec{x}_1^T\} + \frac{1}{2} E\{\vec{x}_2, \vec{x}_2^T\}$$

- E JE OČEKIVANJE, IZRAČUNA SE KAO SREDNJA VRIJEDNOST SVIH $\vec{x_i} \cdot \vec{x_i}^T$ UZORAKA

$$R = \frac{1}{2} \cdot \frac{1}{4} \cdot \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \cdot [0\ 0\ 0] + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot [1\ 0\ 0] + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \cdot [1\ 0\ 1] + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \cdot [1\ 1\ 0] \right) +$$

$$\frac{1}{2} \cdot \frac{1}{4} \left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot [0\ 0\ 1] + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} [0\ 1\ 0] + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} [0\ 1\ 1] + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} [1\ 1\ 1] \right)$$

$$= \frac{1}{8} \cdot \left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right) +$$

$$\frac{1}{8} \left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right)$$

$$= \frac{1}{8} \begin{bmatrix} 3 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} + \frac{1}{8} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} //$$

- RAČUNAMO SADA LAMBDE PO FORMULI $(R - \lambda I)\vec{e} = 0$ TAKO DA PRETPOSTAVIMO DA JE $(R - \lambda I)$ REGULARNA MATRICA ŠTO ZNAČI DA JE DETERMINANTA TOGA $== \emptyset$

$$(R - \lambda I)\vec{e} = \emptyset$$

$$- \left(\frac{1}{4} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right) \cdot \vec{e} = \emptyset \Rightarrow \begin{vmatrix} 0.5-\lambda & 0.25 & 0.25 \\ 0.25 & 0.5-\lambda & 0.25 \\ 0.25 & 0.25 & 0.5-\lambda \end{vmatrix} = \emptyset$$

$$(0.5-\lambda)\begin{vmatrix}0.5-\lambda & 0.25\\ 0.25 & 0.5-\lambda\end{vmatrix} - 0.25\begin{vmatrix}0.25 & 0.25\\ 0.25 & 0.5-\lambda\end{vmatrix} + 0.25\begin{vmatrix}0.25 & 0.5-\lambda\\ 0.25 & 0.25\end{vmatrix}$$

$(0.5-\lambda)((0.5-\lambda)^2-0.25^2)-0.25(0.125-0.25\lambda-0.0625)$

$+0.25(0.0625-0.125+0.25\lambda)=0\ /\cdot 4$

$(2-4\lambda)(0.25-\lambda+\lambda^2-0.0625)-0.0625+0.25\lambda$

$-0.0625+0.25\lambda=0$

$0.5-2\lambda+2\lambda^2-0.125-\lambda+4\lambda^2-4\lambda^3+0.25\lambda-0.125+0.5\lambda=$

$-4\lambda^3+6\lambda^2-2.25\lambda+0.25=0\ /\cdot 4$

$-16\lambda^3+24\lambda^2-9\lambda+1=0$

$-\lambda(16\lambda^2-24\lambda+9)+1=0$

$-\lambda((4\lambda-3)^2)+1=0$

$\lambda(4\lambda-3)^2=1$

– NE ZNAM DALJE, ALI $\lambda=[1\ \frac{1}{4}\ \frac{1}{4}]^T$

TRAŽIMO SVOJSTVENE VEKTORE

$$R\vec{e}=\lambda\vec{e}$$

1) $\frac{1}{4}\begin{bmatrix}2&1&1\\1&2&1\\1&1&2\end{bmatrix}\begin{bmatrix}e_1\\e_2\\e_3\end{bmatrix}=\lambda\begin{bmatrix}e_1\\e_2\\e_2\end{bmatrix}\ /\cdot 4$

$\frac{1}{2}e_1+\frac{1}{4}e_2+\frac{1}{4}e_3=e_1$

$-2e_1+e_2+e_3=0 \Rightarrow e_1=[1\ 1\ 1]^T$

2) $e_1+2e_2+e_3=e_2$

$e_1+e_2+e_3=0 \Rightarrow e_2=[-2\ 1\ 1]^T$

3) $e_1+e_2+2e_3=e_3=[0\ -1\ 1]$

TREBA TAKO IZABRAT VEKTORE $\vec{e}$ DA ZADOVOLJAVA

$e_i^T e_j=0$, I SVAKAKO NORMALIZIRATI $\hat{e}_i=\frac{1}{\|\vec{e}_i\|}e_i$

DALJE SE IZRAČUNA MATRICA A I ODABERU PRVE 2 DIMENZIJE I POMNOŽE SE PRIMJERI S A

KL TRANSFORMACIJA - AUDITORNE 2010/2011

$$w = \{[-4\ \ -2]^T, [1, 3]^T, [3\ \ -1]^T\}$$

$$K = \frac{1}{N-1}\sum_{i=1}^{N}(\vec{x}_i - \vec{m})(\vec{x}_i - \vec{m})^T$$

$$m = \frac{1}{3}\left(\begin{bmatrix}-4\\-2\end{bmatrix} + \begin{bmatrix}1\\3\end{bmatrix} + \begin{bmatrix}3\\-1\end{bmatrix}\right) = \begin{bmatrix}0\\0\end{bmatrix}$$

$$K = \frac{1}{2}\left(\begin{bmatrix}-4\\-2\end{bmatrix}[-4\ -2] + \begin{bmatrix}1\\3\end{bmatrix}[1\ \ 3] + \begin{bmatrix}3\\-1\end{bmatrix}[3\ \ -1]\right)$$

$$= \frac{1}{2}\left(\begin{bmatrix}16 & 8\\8 & 4\end{bmatrix} + \begin{bmatrix}1 & 3\\3 & 9\end{bmatrix} + \begin{bmatrix}9 & -3\\-3 & 1\end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix}26 & 8\\8 & 14\end{bmatrix} = \begin{bmatrix}13 & 4\\4 & 7\end{bmatrix}$$

$$(K - \lambda I)\vec{e} = \emptyset$$

$$|K - \lambda I| = 0$$

$$\begin{vmatrix}13-\lambda & 4\\4 & 7-\lambda\end{vmatrix} = 0 \Rightarrow (13-\lambda)(7-\lambda) - 16 = 0$$

$$91 - 13\lambda - 7\lambda + \lambda^2 - 16 = 0$$

$$\lambda^2 - 20\lambda + 75 = 0$$

$$\lambda_{1/2} = \frac{20 \pm \sqrt{20^2 - 4\cdot 75}}{2} = \frac{20 \pm 10}{2} = \begin{bmatrix}15\\5\end{bmatrix}$$

$$(K - \lambda I)\vec{e} = \emptyset$$

$$\begin{bmatrix}-2 & 4\\4 & -8\end{bmatrix}\cdot\begin{bmatrix}e_1\\e_2\end{bmatrix} = \emptyset \Rightarrow -2e_1 + 4e_2 = 0 \quad e_1 = 2e_2 \quad e = \begin{bmatrix}2\\1\end{bmatrix}$$

$$\boxed{e = \frac{1}{\sqrt{5}}\begin{bmatrix}2\\1\end{bmatrix}}$$

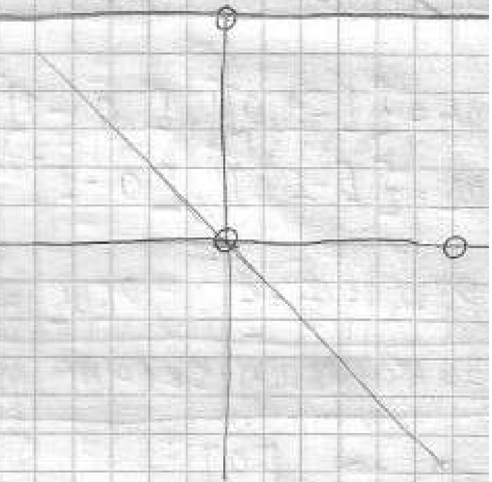
2 *(6 bodova)* **Skup uzorka**

$$\{ [0, 0]^T, [6, 0]^T [0, 6]^T \}$$

transformirajte iz dvodimenzionalnog u jednodimenzionalni prostor uporabom *KL transformacije*. Uputa: koristiti **kovarijacijsku** matricu.

KL TRANSFORMACIJA - ZAVRŠNI 2010/2011

$\omega = \{ [0 \quad 0]^T, [6 \quad 0]^T, [0 \quad 6]^T \}$



BITNA INFORMACIJA:

KORELACIJSKA MATRICA: R
KOVARIJACIJSKA ILI KOVARIJANTNA JE: K
I ODUZIMA SE SREDNJA VRIJEDNOST:

$$\vec{m} = \frac{1}{3}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 6 \end{bmatrix} \right) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$K = \frac{1}{3-1}\left(\begin{bmatrix} -2 \\ -2 \end{bmatrix}[-2 \quad -2] + \begin{bmatrix} 4 \\ -2 \end{bmatrix}[4 \quad -2] + \begin{bmatrix} -2 \\ 4 \end{bmatrix}[-2 \quad 4] \right)$$

$$= \frac{1}{2}\left(\begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} + \begin{bmatrix} 16 & -8 \\ -8 & 4 \end{bmatrix} + \begin{bmatrix} 4 & -8 \\ -8 & 16 \end{bmatrix} \right) = \frac{1}{2}\begin{bmatrix} 24 & -12 \\ -12 & 24 \end{bmatrix} = \begin{bmatrix} 12 & -6 \\ -6 & 12 \end{bmatrix}$$

$$|K - \lambda I| = 0$$

$$\begin{vmatrix} 12-\lambda & -6 \\ -6 & 12-\lambda \end{vmatrix} = 0$$

$$(12-\lambda)^2 - 36 = 0$$

$$\lambda^2 + 144 - 24\lambda - 36 = 0$$

$$\lambda^2 - 24\lambda + 108 = 0$$

$$\lambda_{1/2} = \frac{24 \pm \sqrt{576 - 4 \cdot 108}}{2} = \frac{24 \pm 12}{2} = \begin{bmatrix} 18 \\ 6 \end{bmatrix}$$

a) $(K - 18I) \cdot \vec{w} = \vec{0}$

$$\begin{bmatrix} -6 & -6 \\ -6 & -6 \end{bmatrix} \cdot \vec{w} = \vec{0}$$

$-6w_1 - 6w_2 = 0 \Rightarrow w_1 = -w_2 \Rightarrow \vec{w}_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

b) $\begin{bmatrix} 6 & -6 \\ -6 & 6 \end{bmatrix} \vec{w} = \vec{0}$

$6 \cdot w_1 - 6w_2 = 0 \Rightarrow \vec{w}_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$A = \left[\frac{\sqrt{2}}{2} \quad -\frac{\sqrt{2}}{2} \right]$, $\vec{y}_i = A \cdot \vec{x}_i$;

$y = \{0, 3\sqrt{2}, -3\sqrt{2}\}$

KL TRANSFORMACIJA - 2. ZADATAK IZ SKRIPTE

$$E = \begin{bmatrix} 1{,}9 \\ 2 \end{bmatrix}$$

$$K = \begin{bmatrix} 0.3889 & 0.3456 \\ 0{,}3456 & 0.4044 \end{bmatrix}$$

$$(K - \lambda I)\vec{w} = \vec{\emptyset}$$

$$|K - \lambda I| = \emptyset$$

$$(0.3889 - \lambda)(0.4044 - \lambda) - 0.3456^2 = \emptyset$$

$$0.3889 \cdot 0.4044 - 0.3889\lambda - 0.4044\lambda + \lambda^2 - 0.3456^2 = \emptyset$$

$$0.03783 - 0.7933\lambda + \lambda_2 = 0$$

$$\lambda_{1/2} = \frac{0.7933 \pm \sqrt{0.62932489 - 0.15132}}{2}$$

$$\lambda_{1/2} = \frac{0.7933 \pm 0.691379}{2}$$

$$\lambda_1 = 0.7423 \quad , \quad \lambda_2 = 0.051$$

$$(K - \lambda_1 I)\vec{w} = \emptyset$$

$\vec{w_1}$)
$$\begin{bmatrix} -0.3534 & 0.3456 \\ 0.3456 & -0.3379 \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} \emptyset \\ \emptyset \end{bmatrix}$$

$$w_2 = \frac{0.3534}{0.3456} w_1 \Rightarrow w_2 = 1.02912\, w_1 \qquad \vec{w_1} = \begin{bmatrix} 0.697 \\ 0.717 \end{bmatrix}$$

$\vec{w_2}$)
$$\begin{bmatrix} 0.3379 & 0.3456 \\ 0.3456 & 0.3534 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} \emptyset \\ \emptyset \end{bmatrix}$$

$$w_1 = \frac{-0.3456}{0.3379} w_2 \Rightarrow w_2 = -1.0227 \qquad w_2 = \begin{bmatrix} -0.717 \\ 0.6991 \end{bmatrix}$$