MI 2014./2015. rješenja

RUS MI. 2015.

1.

$\frac{\omega_s}{2} = 50 \Rightarrow \omega_s = 100 \ \frac{rad}{s}$

$\omega^* = \omega + n\omega_s$

$\omega = \omega^* - 2\omega_s = -4 = 4 \ \frac{rad}{s}$

$|G_1(\omega = 4 \ \frac{rad}{s})| = -5 \ dB = 0.5623$

$y_1(t) = |G_1(\omega - 4)| \cdot x(t) \Rightarrow$ sin $\boxed{A_{y_1} = 0.5623}$

$y_1(t) = |G_1(-4)| \sin(-4t + \angle G_1(-4))$

$G_2(j \cdot 4) = 0.5728 \, e^{j113.63^\circ}$

$y_2(t) = |G_1(-4)| \cdot |G_2(-4)| \cdot \sin(4t + \angle G_1(-4) + \angle G_2)$

$$\boxed{A_{y_2} = A_{y_1} \cdot |G_2(j4)| = 0.322}$$

2. sustav 2. reda

$\varphi(\omega \to \infty) = -90^\circ$ mora imati nulu $\qquad G(s) = \frac{b_0 + b_1 s}{a_0 + a_1 s + s^2}$

$G(s) = \int_0^{+\infty} g(t) e^{-st} dt$

$= \int_0^{+\infty} \left[1 - st + \frac{(st)^2}{2!} - \frac{(st)^3}{3!} + \dots\right] g(t)\, dt$

$= M_0 - sM_1 + \frac{s^2}{2!}M_2 - \frac{s^3}{3!}M_3 + \frac{s^4}{4!}M_4 + \dots = \frac{b_0 + b_1 s}{a_0 + a_1 s + s^2}$

$(a_0 + a_1 s + s^2)(M_0 - sM_1 + \frac{s^2}{2!}M_2 - \frac{s^3}{3!}M_3 + \frac{s^4}{4!}M_4 + \dots) = b_0 + b_1 s$

$a_0 M_0 = b_0$

$-a_0 M_1 + a_1 M_0 = b_1$

$a_0 \frac{M_2}{2} - a_1 M_1 + M_0 = 0$

$-a_0 \frac{M_3}{6} + a_1 \frac{M_2}{2} - M_1 = 0$

$\to \quad a_0 \cdot 2.75 - 2.5 a_1 + 1 = 0$

$-a_0 \cdot 2.875 + a_1 \cdot 2.75 - 2.5 = 0 \Rightarrow a_0 = 2, \ a_1 = +3$

$b_0 = a_0 M_0 = 4$

$b_1 = -a_0 M_1 + a_1 M_0 = 1$

$-a_1 \frac{M_3}{6} + a_0 \cdot \frac{M_4}{24} + \frac{M_2}{2} = 0 \Rightarrow$ NE TREBA

$$\boxed{G(s) = \frac{4 + s}{2 + 3s + s^2}}$$

b)

$g(t) = \frac{dh(t)}{dt}$

$G(s) = \frac{s + 4}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2} \to \begin{cases} A + B = 1 \\ 2A + B = 4 \end{cases} \quad \boxed{\begin{matrix} A = 3 \\ B = -2 \end{matrix}}$

$As + 2A + Bs + B = s + 4$

$G(s) = \frac{3}{s+1} - \frac{2}{s+2}$

$g(t) = (3e^{-t} - 2e^{-2t}) \, S(t)$

$h(t) = \int_0^t (3e^{-t} - 2e^{-2t})\, dt = -3e^{-t} + 3 + e^{-2t} - 1$

$h(t) = (-3e^{-t} + e^{-2t} + 2) \, S(t)$

d) $\frac{dh(t)}{dt} = \left(-3e^{-t} + e^{-2t} + 2\right)S(t) + \left(3e^{-t} - 2e^{-2t}\right)\delta(t)$

Druga derivacija mora postojati i jednaka je nula.

$\ddot{h}(t) = \left(-3e^{-t} + 4e^{-2t}\right)S(t) + \left(3e^{-t} - 2e^{-2t}\right)\delta(t)$

$\ddot{h}(t) = \left(-3e^{-t} + 4e^{-2t}\right)S(t) + \underline{\underline{\delta(t)}}$

→ točka infleksije ne postoji

3. $G(s) = \frac{b(s)}{m(s)}$

a) $e = u - y = -y$

$y = N(A) \cdot \frac{1}{s} \cdot G(s)(-y)$

$N(A) \cdot \frac{1}{s} \cdot G(s) = -1$

$N(A) = \frac{4d}{\pi A^2}\sqrt{A^2 - \varepsilon^2} - j\frac{4d\,\varepsilon}{\pi A^2}$

$|N(A)| = \frac{4d}{\pi A^2}\sqrt{A^2 - \cancel{\varepsilon^2} + \cancel{\varepsilon^2}} = \frac{4d}{\pi A}$ $\qquad \varphi_N = \operatorname{arctg}\frac{-\varepsilon}{\sqrt{A^2 - \varepsilon^2}}$

$N(A) \cdot \frac{1}{j\omega_0} \cdot G(j\omega_0) = -1$

$\varphi_N - \frac{\pi}{2} + \arg(G(j\omega_0)) = -\pi$

$\boxed{\arg(G(j\omega_0)) = -\frac{\pi}{2} + \operatorname{arctg}\frac{\varepsilon}{\sqrt{A^2 - \varepsilon^2}}}$

$|N(A)| \cdot \frac{1}{\omega_0} \cdot |G(j\omega_0)| = 1$

$\boxed{|G(j\omega_0)| = \frac{\omega_0}{|N(A)|} = \frac{\omega_0 A \pi}{4d}}$

b) $G_{PID}(s) = K_R\left(1 + \frac{1}{T_I s} + T_D s\right) = \frac{K_R T_I s + K_R + K_R T_I T_D s^2}{T_I s}$

$= \frac{K_R T_I T_D s^2 + K_R T_I s + K_R}{T_I s}$

$G_{PID}(j\omega) = \frac{-K_R T_I T_D \omega^2 + K_R T_I j\omega + K_R}{T_I j\omega} = K_R\left(1 - j\frac{1}{T_I\omega} + jT_D\omega\right)$

$T_I = \alpha T_D$

$\arg[G_{PID}(j\omega)] = \operatorname{arctg}\frac{T_I\omega_0}{1 - T_I T_D \omega_0^2} - \frac{\pi}{2}$

$|G_{PID}(j\omega)| = \frac{K_R}{T_I\omega_0}\sqrt{(T_I\omega_0)^2 + (1 - T_I T_D \omega_0)^2}$

$\boxed{\arg[G_{PID}(j\omega)] = -\frac{\pi}{2} + \operatorname{arctg}\frac{\alpha T_D \omega_0}{1 - \alpha T_D^2 \omega_0^2}}$

$\boxed{|G_{PID}(j\omega)| = K_R\sqrt{1 + \left(T_D\omega - \frac{1}{T_I\omega}\right)^2}}$

$\gamma = \gamma_0 = \pi + \varphi(\omega_c) \qquad \omega_c = \omega_0$

$\gamma_0 = \cancel{\pi} - \varphi_N - \cancel{\frac{\pi}{2}} - \cancel{\frac{\pi}{2}} + \varphi_{PID} = -\varphi_N + \varphi_{PID} = \varphi_N + \mathrm{arctg}\frac{T_I\omega_0}{1 - T_I T_D \omega_0^2}$

$\gamma_0 + \varphi_N = \mathrm{arctg}\frac{T_I\omega_0}{1 - T_I T_D\omega_0^2} \quad / \mathrm{tg}$

$\frac{T_I\omega_0}{1 - T_I T_D\omega_0^2} = \mathrm{tg}(\gamma_0 + \varphi_N)$

$\mathrm{ctg}(\gamma_0 + \varphi_N) = \frac{1}{T_I\omega_0} - T_D\omega_0 \Big|^2$

$\mathrm{ctg}^2(\gamma_0 + \varphi_N) = \left(\frac{1}{T_I\omega_0} - T_D\omega_0\right)^2 = \left(T_D\omega_0 - \frac{1}{T_I\omega_0}\right)^2$

$\mathrm{ctg}(\gamma_0 + \varphi_N) = \frac{1}{\alpha T_D\omega_0} - T_D\omega_0 \quad / \cdot T_D\omega_0 \qquad T_I = \alpha T_D$

$T_D^2\omega_0^2 + T_D\omega_0\,\mathrm{ctg}(\gamma_0 + \varphi_N) - \frac{1}{\alpha} = 0$

$T_D = \frac{-\omega_0\,\mathrm{ctg}(\gamma_0 + \varphi_N) \pm \sqrt{\omega_0^2\,\mathrm{ctg}^2(\gamma_0 + \varphi_N) + \frac{4\omega_0^2}{\alpha}}}{2\omega_0^2}$

$$\boxed{T_D = \frac{-\mathrm{ctg}(\gamma_0 + \varphi_N) + \sqrt{\frac{4}{\alpha} + \mathrm{ctg}^2(\gamma_0 + \varphi_N)}}{2\omega_0}} \qquad \boxed{T_I = \alpha T_D}$$

$\mathrm{ctg} = \frac{\cos}{\sin}$

$|G_0(j\omega_c)| = 1 = |G_0(j\omega_0)|$

$\frac{\omega_0 A\pi}{4d} \cdot K_R \cdot \sqrt{1 + \mathrm{ctg}^2(\gamma_0 + \varphi_N)} = 1$

$$\boxed{K_R = \frac{4d}{A\pi\omega_0} \cdot \sin(\gamma_0 + \varphi_N)}$$

1. a) $G_R(s) = \frac{1+Ts}{1+\alpha Ts}$

$G_R(j\omega) = \frac{1+j\omega T}{1+j\omega\alpha T} \cdot \frac{1-j\omega\alpha T}{1-j\omega\alpha T} = \frac{1 - j\omega\alpha T + j\omega T + \omega^2\alpha T^2}{1+\alpha^2\omega^2T^2}$

$Re(\omega) = \frac{1+\alpha\omega^2T^2}{1+\alpha^2\omega^2T^2}$ $\qquad Im(\omega) = \frac{\omega T(1-\alpha)}{1+\alpha^2\omega^2T^2}$

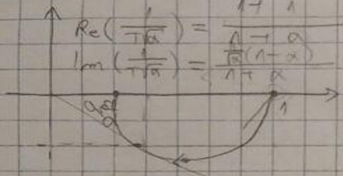
$\omega = 0 \quad Re = 1 \quad Im = 0$

$\omega = \infty \quad Re = \frac{1}{\alpha} \quad Im = 0$

$\omega_m = \frac{1}{T\sqrt{\alpha}}$

$Re\left(\frac{1}{T\sqrt{\alpha}}\right) = \frac{1+1}{1+\alpha}$

$Im\left(\frac{1}{T\sqrt{\alpha}}\right) = \frac{\frac{1}{\sqrt{\alpha}}(1-\alpha)}{1+\alpha}$



$\alpha > 1$

$\sin\varphi_m = \frac{\frac{1}{\sqrt{\alpha}}(1-\alpha)}{\sqrt{1+\frac{1}{\alpha}+\frac{1}{\alpha}+2}}$

$\sin\varphi_m = \frac{\frac{1}{\sqrt{\alpha}}(1-\alpha)}{\sqrt{\left(\frac{1}{\sqrt{\alpha}}+\sqrt{\alpha}\right)^2}}$

$\sin\varphi_m = \frac{1-\alpha}{1+\alpha}$ $\rightarrow$ faza

$G_R(s) \quad \lim_{s\to\infty} \frac{1+Ts}{1+\alpha Ts} = \lim_{s\to\infty} \frac{\frac{1}{s}+T}{\frac{1}{s}+\alpha T} = \frac{1}{\alpha}$ $\rightarrow$ amplitude

$\omega = T$ ili $\omega = \alpha T$

b) $\varphi_m = 35$ $\rightarrow$ $\boxed{\omega_c = 1 \frac{rad}{s}}$ $\qquad G_R(s) = K_R\alpha \cdot \frac{1+Ts}{1+\alpha Ts}$, $\alpha > 1$

$|G(j\omega_c)| = 1 = \frac{0.1 K_R}{\omega_c\sqrt{1+0.1^2\omega_c^2}\cdot\sqrt{1+\omega_c^2}}$ $\Rightarrow$ $K_R = 10\,\omega_c\sqrt{(1+0.1^2\omega_c^2)(1+\omega_c^2)} = \sqrt{2.02}$

$e_\infty = \lim_{s\to 0} \frac{1}{s} \cdot \frac{1}{1+\frac{0.1K_R}{s(1+0.1s)(1+s)}\cdot\alpha\cdot\frac{1+Ts}{1+\alpha Ts}} = \frac{1}{0.1K_R\alpha} < 0.05$

$\boxed{K_R\alpha = 200}$ $\rightarrow$ $\boxed{\alpha = 14.072}$, $\frac{1}{T} = \frac{\omega_c}{10}$ $\Rightarrow$ $\boxed{T = 10s}$

$$\boxed{G_R(s) = 200 \cdot \frac{1+10s}{1+140.72s}}$$

c) $T_s = \frac{0.17+0.34}{2} = 0.255$

$G_{zoh}(s) = e^{-s\frac{T_s}{2}}$ utječe na fazu

$\arg_{zoh} = -\omega_c\frac{T_s}{2}$

$\arg = -0.1275 = -7.31°$ $\rightarrow$ pušta fazno osiguranje što znači da % nadvišenje poveća za 7.31% ne utječe na ω_c pa brzina sustava ostaje ista