Nastavna jedinica:

Krivulja mjesta korjena

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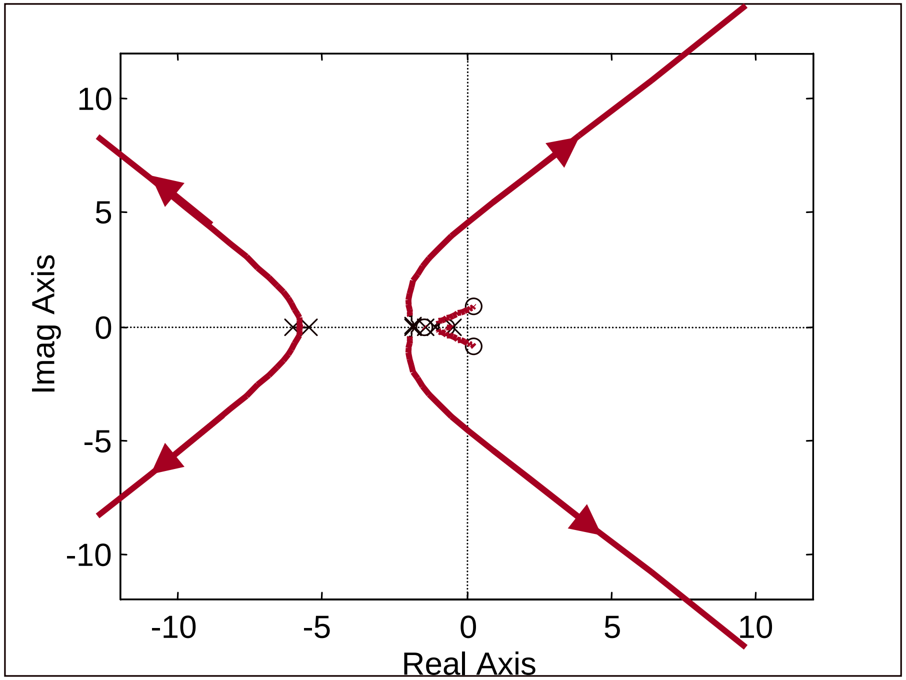
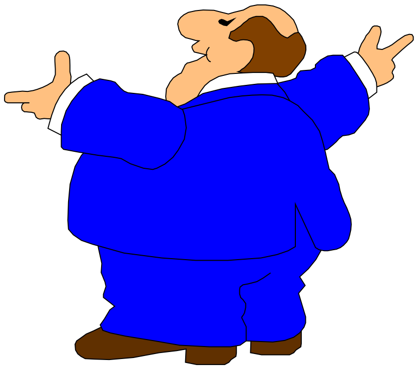
SVEUČILIŠTE U ZAGREBU
FAKULTET
ELEKTROTEHNIKE
I RAČUNARSTVA

Krivulja mjesta korjena

The Root-locus Method

R. Evans (1948): Graphical Analysis of Control Systems. Trans. AIEEE, Vol.67, pp.547-551.

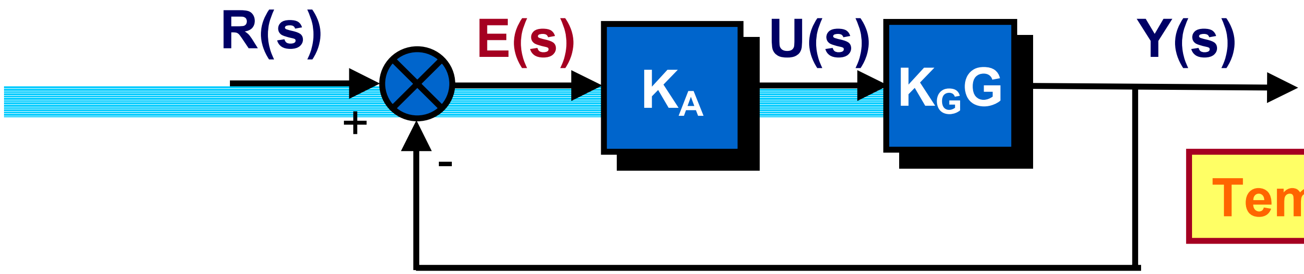




Cilj

- Naučiti pravila za crtanje KMK
- Uporaba KMK
- Kompenzacija s faznim prethođenjem (Lead compensation)
- Kompenzacija s faznim zaostajanjem (Lag compensation)

Krivulja mjesta korjena - KMK



Temeljna struktura SAU

Funkcija prijenosa zatvorenog kruga

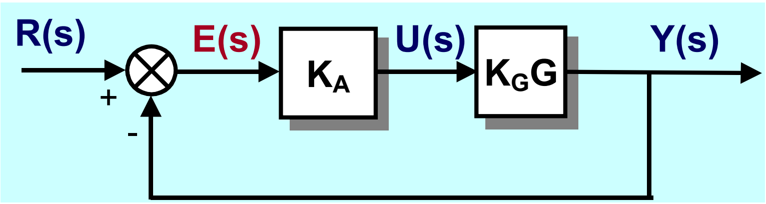
$$\frac{Y(s)}{R(s)} = \frac{K_A K_G G(s)}{1 + K_A K_G G(s)}$$

Karakteristična jednadžba

$$1 + K_A K_G G(s) = 0$$

KMK je mjesto položaja polova zatvorenog kruga kada se mijenja jedan od parametara karakt. jedn. zatv. kruga

KMK



KMK parametar

$$K \equiv K_A K_G$$

$$G(s) = \frac{B(s)}{A(s)} = \frac{s^m + b_{m-1}s^{m-1} + \cdots + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_0} = \frac{\prod_{i=1}^{m}(s - z_i)}{\prod_{i=1}^{n}(s - p_i)}$$

$$G_{cl}(s) = \frac{KG(s)}{1 + KG(s)} = \frac{K\dfrac{B(s)}{A(s)}}{1 + K\dfrac{B(s)}{A(s)}} = K\frac{B(s)}{A(s) + KB(s)}$$

Nule: $B(s) = 0$

Polovi: $A(s) + KB(s) = 0$

A(s) i B(s) su monički polinomi

KMK

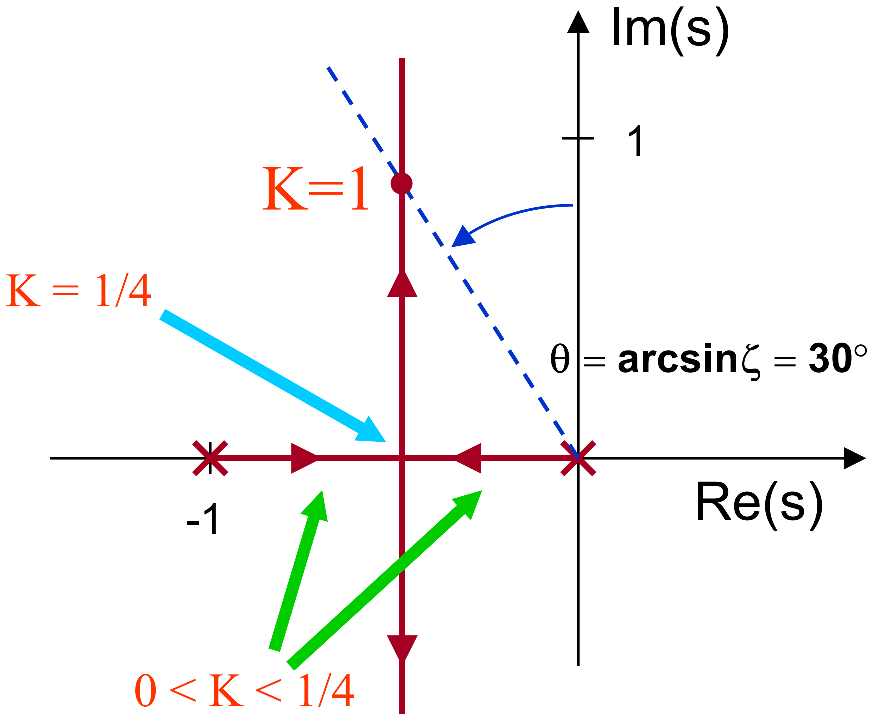
Primjer: DC – motor mijenja se pojačanje

$$\frac{\theta_m(s)}{v_a(s)} = K_G G(s) = \frac{1}{s(s+1)}$$

KMK uz promjenjiv K:

$$s^2 + s + K = 0$$

Karakt. jedn. zatv. kruga



$$s_{(1,2)cl} = -\tfrac{1}{2} \pm \frac{\sqrt{1-4K}}{2}$$

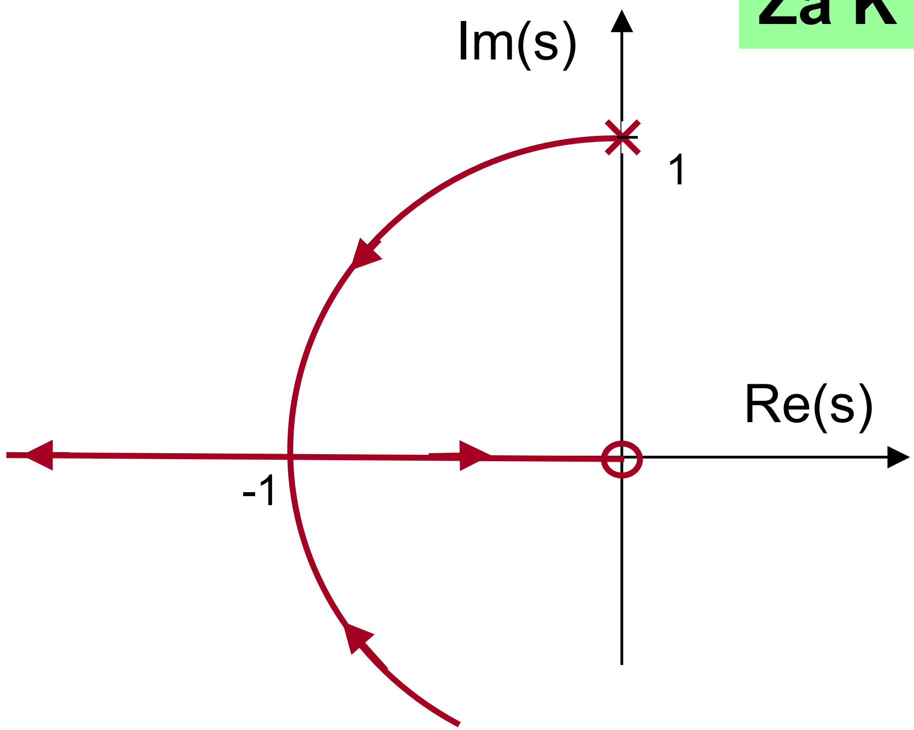
$$K = 1$$

$$\Downarrow$$

$$\zeta = \frac{1}{2}$$

Primjer: Moguće je mijenjati i neki drugi parametar procesa na pr. c

$$G(s) = \frac{1}{s(s+c)}$$



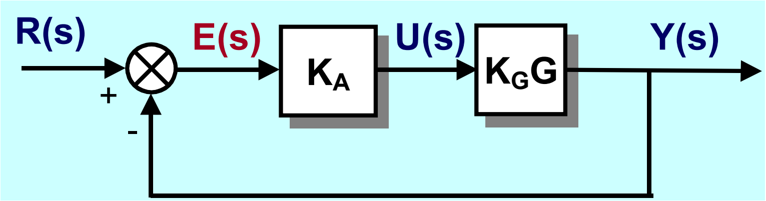
Za K = 1

Karakteristična jednadžba

$$s^2 + cs + 1 = 0$$

$$s_{(1,2)cl} = -\frac{c}{2} \pm \frac{\sqrt{c^2-4}}{2}$$

KMK



$$1 + \mathbf{KG}(\mathbf{s}) = \mathbf{0}$$

$$1 + K\frac{B(s)}{A(s)} = 0$$

KMK oblik

Polovi zatvorenog kruga: $A(s) + KB(s) = 0$

Za $K = 0$ polovi otvorenog kruga: $A(s) = 0$

KMK polazi (K = 0) s polova otvorenog kruga
KMK završava (K = ∞) u nulama otvorenog kruga
ili u ∞ !!

KMK je uvijek simetrična u odnosu na realnu os !!

KMK se dobije zadovoljenjem dvaju uvjeta:

1. uvjet kuteva
2. uvjet amplituda

relacije dane sa 1 + KG(s) = 0, odnosno:

$$K\frac{\prod_{i=1}^{m}(s-z_i)}{\prod_{i=1}^{n}(s-p_i)}=-1\Rightarrow G(s)=\frac{\prod_{i=1}^{m}(s-z_i)}{\prod_{i=1}^{n}(s-p_i)}=-\frac{1}{K}$$

Pozicija polova duž KMK dobiti će se zadovoljenjem oba uvjeta.

Uvjet modula:

$$K\frac{\prod_{i=1}^{m}\left|(s-z_i)\right|}{\prod_{i=1}^{n}\left|(s-p_i)\right|}=1\Rightarrow\frac{\prod_{i=1}^{m}\left|(s-z_i)\right|}{\prod_{i=1}^{n}\left|(s-p_i)\right|}=\frac{1}{K}$$

Uvjet kuteva:

$$\sum_{i=1}^{m}\arg(s-z_i)-\sum_{i=1}^{n}\arg(s-p_i)=-180^0+360^0\nu$$

Uvjet modula može se zadovoljiti za svaki iznos od s_0 u s-ravnini prikladnim izborom pojačanja K.
Zbog toga je uvjet kuteva bitan za traženje da li neka točka s_0 pripada KMK.

Svaka KMK ima ukupno n grana koje započinju na polovima otvorenog kruga !!

Za velika pojačanja će se KMK približavati nulama otvorenog kruga ili ići u ∞

$$K\frac{\prod_{i=1}^{m}\left|\left(s-z_i\right)\right|}{\prod_{i=1}^{n}\left|\left(s-p_i\right)\right|}=1\Rightarrow\frac{\prod_{i=1}^{m}\left|\left(s-z_i\right)\right|}{\prod_{i=1}^{n}\left|\left(s-p_i\right)\right|}=\frac{1}{K}$$

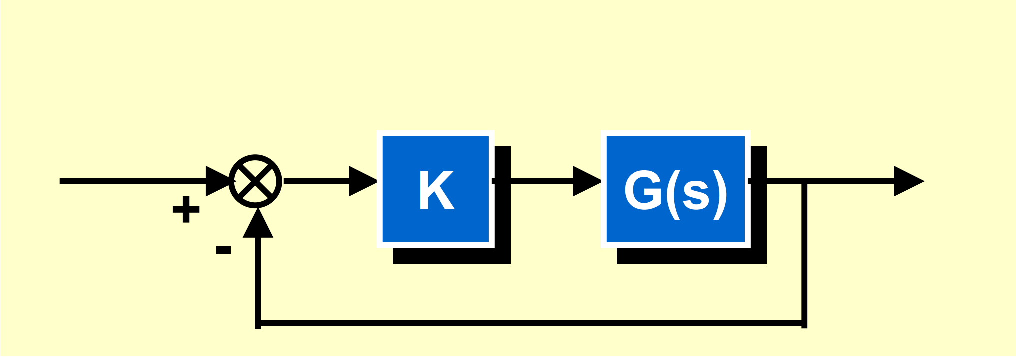
Za K = ∞ će biti 1/K = 0 što je moguće jedino ako $s = z_i$.
Preostalih n-m grana KMK će završiti u ∞

Upute za crtanje KMK

1. KMK korjeni od $1 + KG(s) = 0$

$K > 0$ mijenja se od 0 do ∞

Polovi zatvorenog kruga



Upute za crtanje KMK

KMK $G(s) = -\frac{1}{K}$

2. KMK s sa fazom G(s) = -180^0

$$\arg(G) = Faza\ G(s) = -180^\circ + 360^\circ \nu$$

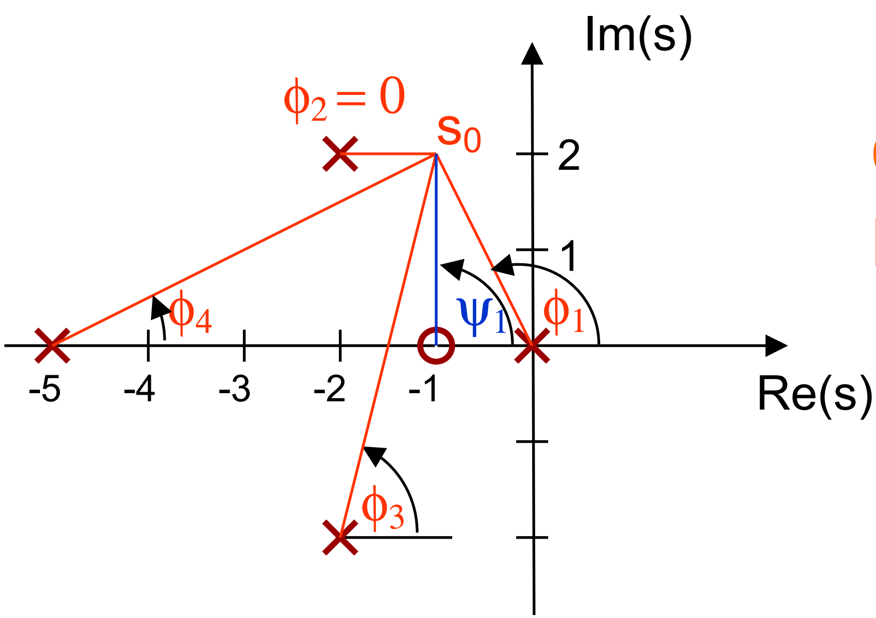
ν **cijeli broj**

Pozitivni K ili -180^0 mjesto korjena

Negativni K ili 0^0 mjesto korjena

Primjer za fazni uvjet

$$G(s) = \frac{s+1}{s\left[(s+2)^2+4\right](s+5)}$$



Crtanje KMK obavlja se postupkom pokušaja i pogreške.

Da li je točka s_0 na KMK?

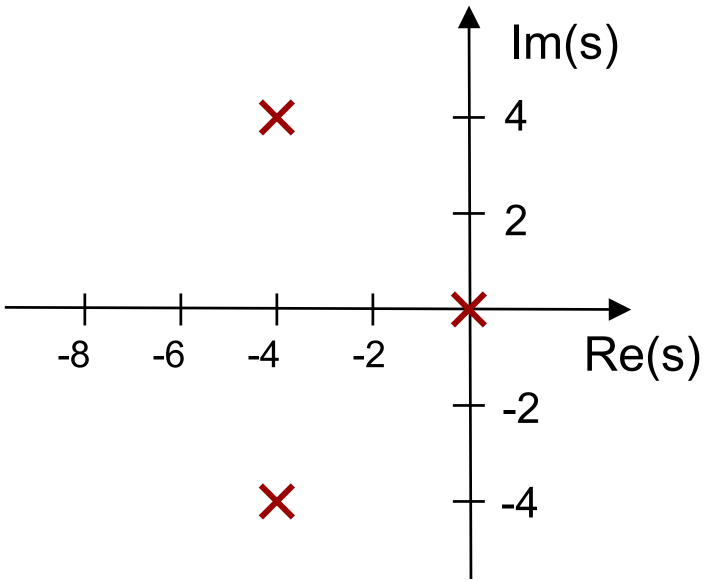
$$\arg\left[G(s_0)\right] = \psi_1 - \phi_1 - \phi_2 - \phi_3 - \phi_4 = -129^\circ \neq -180^0$$

s_0 nije na KMK !

7 koraka za crtanje KMK

Primjer:

$$G(s) = \frac{1}{s\left[(s+4)^2+16\right]}$$

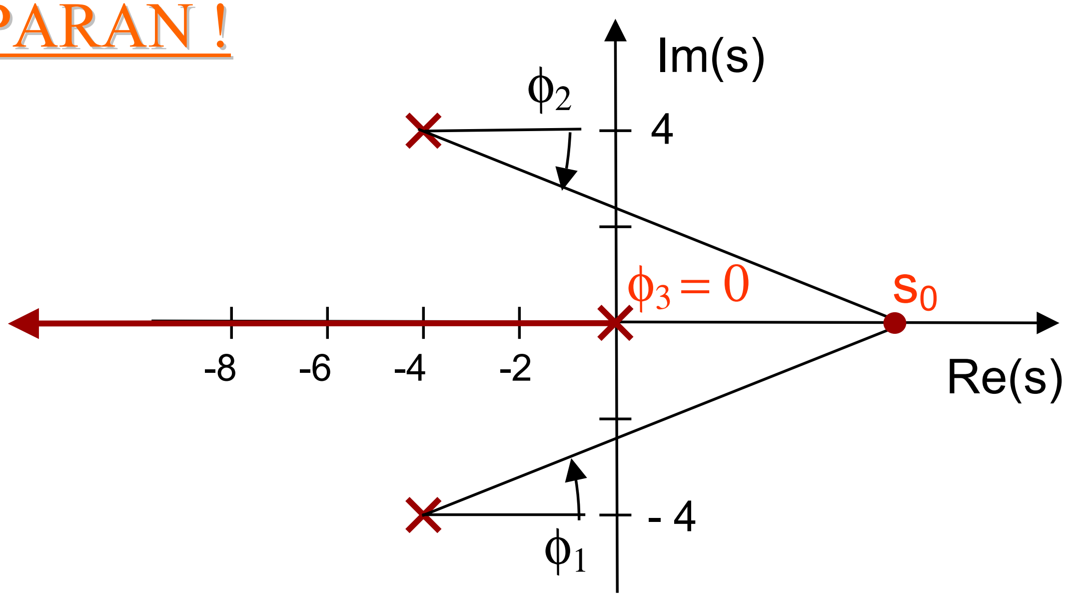


1. korak

Označi polove x nule 0 od G(s)

2. korak
Dio KMK na realnoj osi

KMK na određenom sektoru realne osi (za $K > 0$) postoji akko je ukupan broj <u>realnih</u> polova i nula $G_0(s)$ <u>na desno od sektora NEPARAN !</u>



u s_0: $\phi_1 = -\phi_2$

$$\phi_3 = 180^\circ$$

3. korak
Asimptote za velike K

7 koraka

Grane koje teže ∞ težiti će ∞ duž pravaca – asimptota. Asimptote formiraju kut s pozitivnom realnom osi.

$$G(s) = -\frac{1}{K}$$

$$K \to \infty \quad \Rightarrow \quad G(s) \to 0$$

nule

$$1 + K\frac{s^m + b_{m-1}s^{m-1} + \cdots + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 + a_0} = 0$$

$$n > m \quad \Rightarrow \quad G(s) \to 0 \quad za \quad s \to \infty$$

$$1+K\frac{s^m+b_{m-1}s^{m-1}+\cdots+b_0}{s^n+a_{n-1}s^{n-1}+\cdots+a_1+a_0}=0$$

$za \quad s\to\infty$

$$\mathbf{1+K\frac{1}{(s-\alpha)^{n-m}}=0}$$

$$\alpha=\mathbf{Re}^{\mathbf{j}\phi} \quad \Rightarrow \quad (n-m)\phi_\nu=-180^\circ+360^\circ\nu$$

$$\boxed{\phi_\nu=\frac{180^\circ+360^\circ(\nu-1)}{(n-m)} \qquad \nu=1,2,\cdots,n-m}$$

7 koraka

$$s^n + a_{n-1}s^{n-1} + \cdots + a_0 = (s - p_1)(s - p_2)\cdots(s - p_n)$$

$$a_{n-1} = -p_1 - p_2 - \cdots - p_n = -\sum p_i$$

također: $b_{m-1} = -\sum z_i$

$$s^n + a_{n-1}s^{n-1} + \cdots + a_0 + K\left(s^m + b_{m-1}s^{m-1} + \cdots + b_0\right) = 0$$

$za \quad m < n-1$

Polovi zatvorenog kruga s_{icl}

$$\boxed{-\sum s_{icl} = -\sum p_i}$$

7 koraka

Za velike K:

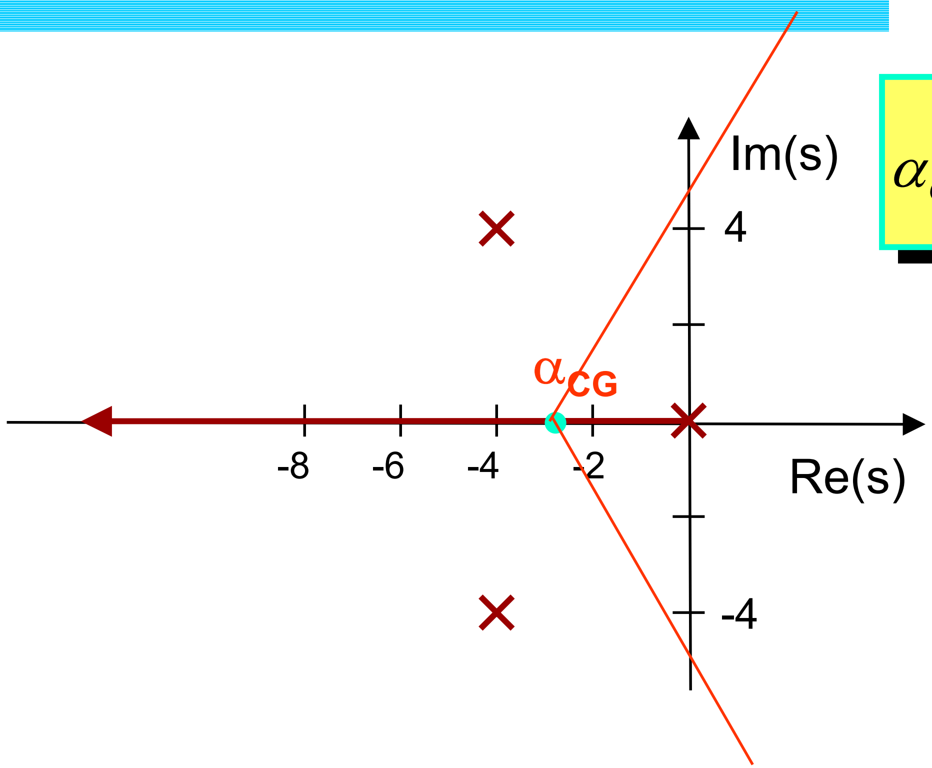
- **m korjena** → **teži nulama**
- **n-m korjena** → **α_{CG} po asimptotama u ∞**

$$\boxed{\sum s_{icl} = (n-m)\alpha_{CG} + \sum z_i = \sum p_i}$$

Centar gravitacije asimptota na negativnoj realnoj osi:

$$\boxed{\alpha_{CG} = \frac{\sum p_i - \sum z_i}{n-m} = \frac{b_{m-1} - a_{n-1}}{n-m}}$$

$$G(s)=\frac{1}{s\left[(s+4)^2+16\right]}=\frac{1}{s^3+8s^2+32s}$$



$$\alpha_{CG}=\frac{\sum p_i-\sum z_i}{n-m}=\frac{b_{m-1}-a_{n-1}}{3-0}=\frac{-8}{3}$$

$$\alpha_{CG}=-\tfrac{8}{3}=-2\tfrac{2}{3}$$

$$\phi_v=\frac{180°+360°(v-1)}{(n-m)} \qquad v=1,2,3$$

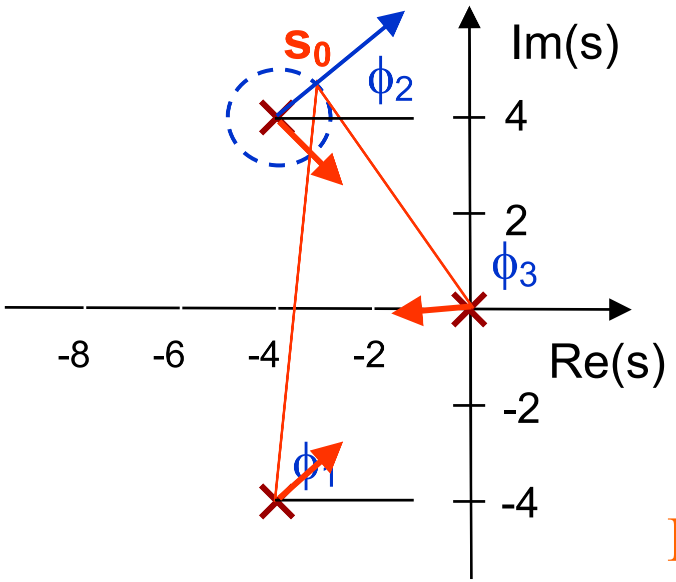
$\phi_1 =$ **60°**

$\phi_2 =$ **180°**

$\phi_3 =$ **300°**

4. korak
Kutevi odlaska i dolaska

s_0 blizu pola



Odlazak

Uvjet kuta:

$$\sum_{i=1}^{m} \arg(s - z_i) - \sum_{i=1}^{n} \arg(s - p_i) = -180^0 + 360^0 \nu$$

$$-90° - \phi_2 - 135° = -180° + 360° \nu$$

Potreban je kut:

$$\phi_2 = -45°$$

Uvjet kuta za višestruke polove:

Odlazak

$$q\phi_{dep} = \sum \psi_i - \sum \phi_i - 180° - 360° v$$

pol q puta **nule** **polovi**

Dolazak

$$q\psi_{arr} = \sum \phi_i - \sum \psi_i + 180° + 360° v$$

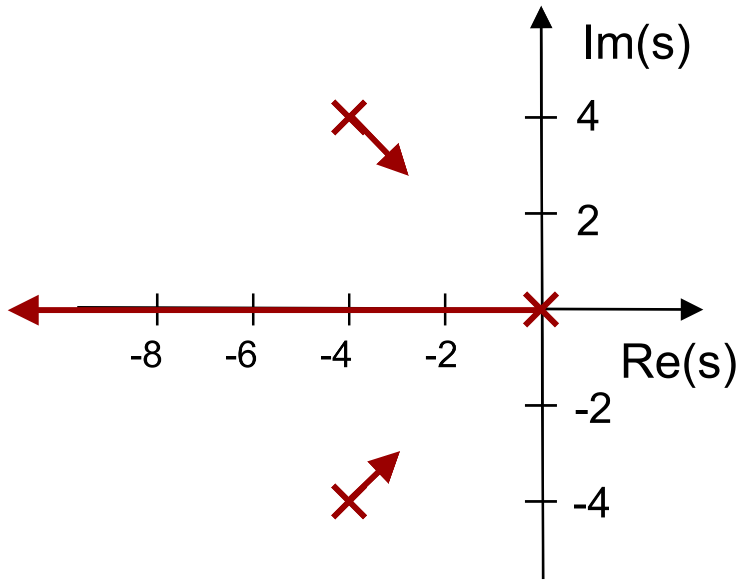
nula q puta **polovi** **nule**

5. korak
Točka presjeka imaginarne osi

$$G(s) = \frac{1}{s\left[(s+4)^2+16\right]}$$

Karakteristična jednadžba

$$s\left[(s+4)^2+16\right]+K=0$$



Routhova tablica:

s^3:	1	32
s^2:	8	K
s^1:	$\frac{256-K}{8}$	
s^0:	K	

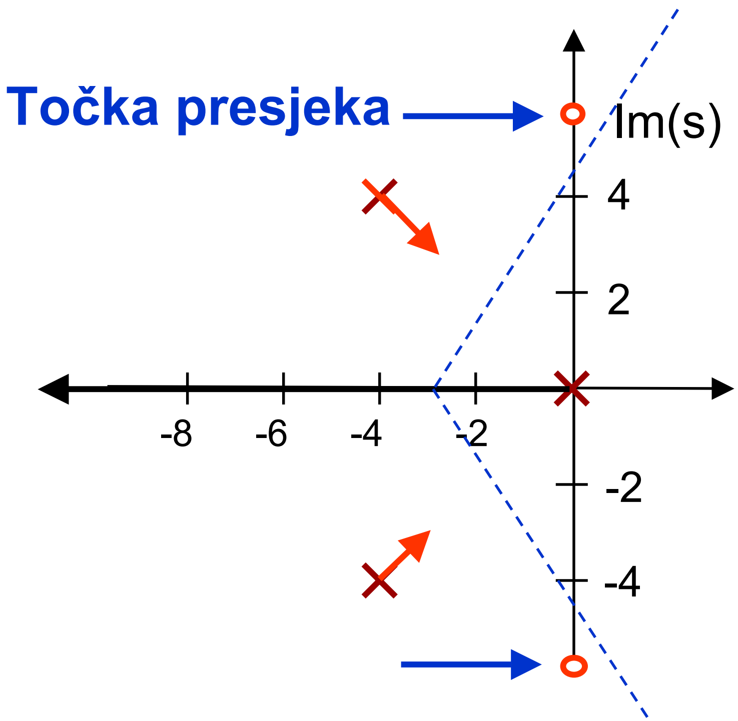
$$0 < K < 256$$

7 koraka

Karakteristična jednadžba $s^3 + 8s^2 + 32s + K = 0$

za K = 256 → točka presjeka → $s = j\omega_0$

$$(j\omega_0)^3 + 8(j\omega_0)^2 + 32(j\omega_0) + 256 = 0$$



Realni dio:

$$-8\omega_0^2 + 256 = 0$$

Imaginarni dio:

$$-\omega_0^3 + 32\omega_0 = 0$$

$$\boxed{\omega_0 = \pm\sqrt{32} = \pm 5.66}$$