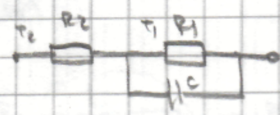
SPUS - D7

9. tjedan

**1.** a) pr. 20.15, str. 270




$$\frac{R_1 \cdot \frac{1}{j\omega C}}{R_1 + \frac{1}{j\omega C}} = \frac{\frac{R_1}{j\omega C}}{\frac{R_1 j\omega C + 1}{j\omega C}} = R_1 \cdot \frac{1}{1 + j\omega R_1 C}$$

1° NADOMJESNI NAPONSKI MODEL ŠUMA

$$\overline{e_{n1}^2} = \frac{2kT_1R_1\,d\omega}{\pi}, \quad \overline{e_{n2}^2} = \frac{2kT_2R_2\,d\omega}{\pi}$$

$$\overline{e_n^2} = \overline{e_{n1}^2} \cdot \left|\frac{1}{1+j\omega R_1 C}\right|^2 + \overline{e_{n2}^2} = \frac{2k\,d\omega}{\pi}\left[T_1R_1 \cdot \frac{1}{1+\omega^2R_1^2C^2} + T_2R_2\right]$$

$$Z_T(\omega) = R_2 + \frac{R_1}{1+j\omega R_1 C} = R_2 + R_1\frac{1-j\omega C}{1+\omega^2R_1^2C^2}$$

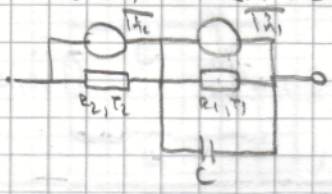
$$Re\{Z_T(\omega)\} = R_2 + \frac{R_1}{1+\omega^2R_1^2C^2}$$

$$\overline{e_n^2} = \frac{2kT_e(\omega) \cdot Re\{Z_T(\omega)\}\,d\omega}{\pi} = \frac{2k\,d\omega}{\pi}\left(\frac{T_1R_1}{1+\omega^2R_1^2C^2} + T_2R_2\right)$$

$$\boxed{T_e(\omega) = \frac{T_1R_1 + T_2R_2(1+\omega^2R_1^2C^2)}{R_1 + R_2(1+\omega^2R_1^2C^2)}}$$


za $T_1 = T_2 \Rightarrow T_e(\omega) = T_1 = T_2$

2° NADOMJESNI STRUJNI MODEL ŠUMA




$$\overline{i_{n1}^2} = \frac{2kT_1\,d\omega}{\pi R_1}$$

$$\overline{i_{n2}^2} = \frac{2kT_2\,d\omega}{\pi R_2}$$

$$\overline{i_n^2} = \overline{i_{n1}^2}\underbrace{\left|\frac{R_1 \| C}{R_2 + R_1 \| C}\right|^2}_{\frac{R_1^2}{(R_1+R_2)^2 + \omega^2 R_1^2 R_2^2 C^2}} + \overline{i_{n2}^2}\underbrace{\left|\frac{R_2}{R_2 + R_1 \| C}\right|^2}_{\frac{R_2^2 + \omega^2 R_1^2 R_2^2 C^2}{(R_1+R_2)^2 + \omega^2 R_1^2 R_2^2 C^2}}$$

$$\boxed{\overline{i_n^2} = \frac{2k\,d\omega}{\pi} \cdot \frac{T_1 R_1 + T_2 R_2 (1 + \omega^2 R_1^2 C^2)}{(R_1+R_2)^2 + \omega^2 R_1^2 R_2^2 C^2}}$$

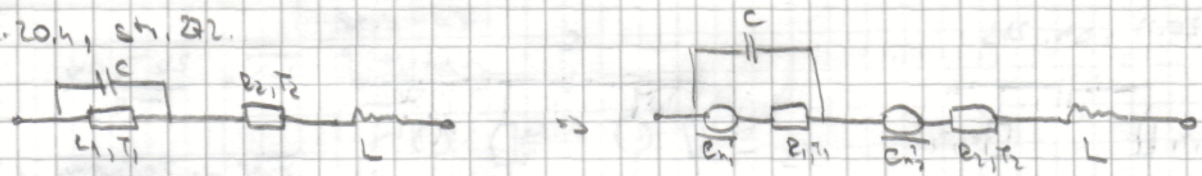
$$Y_N(\omega) = \frac{1}{Z_e(\omega)} = \frac{1 + j\omega R_1 C}{R_1 + R_2 + j\omega R_1 R_2 C}$$

$$Re\{Y_N(\omega)\} = G_N(\omega) = \frac{R_1 + R_2(1 + \omega^2 R_1^2 C^2)}{(R_1+R_2)^2 + \omega^2 R_1^2 R_2^2 C^2}$$

$$\frac{2k\,T_e(\omega)\,Re\{Y_N\}\,d\omega}{\pi} = \frac{2k\,d\omega}{\pi} \cdot \frac{T_1 R_1 + T_2 R_2 (1 + \omega^2 R_1^2 C^2)}{(R_1+R_2)^2 + \omega^2 R_1^2 R_2^2 C^2}$$

$$\boxed{T_e(\omega) = \frac{T_1 R_1 + T_2 R_2 (1 + \omega^2 R_1^2 C^2)}{R_1 + R_2 (1 + \omega^2 R_1^2 C^2)}}$$

b) zd. 20.4, sl. 272.




$T_e(\omega) = ?$

$$\overline{e_{n1}^2} = \frac{2kT_1R_1\,d\omega}{\pi}, \quad \overline{e_{n2}^2} = \frac{2kT_2R_2\,d\omega}{\pi}$$

$$Z_T = R_1 \frac{1}{1+j\omega R_1 C} + R_2 \cdot 1 + j\omega L$$ ⇒ SAMO OTPORI UNOSE ŠUM

$$\overline{e_n^2} = \left|\frac{1}{1+j\omega R_1 C}\right|^2 \cdot \overline{e_{n1}^2} + \overline{e_{n2}^2} \cdot 1$$

$$= \frac{1}{1+j\omega R_1 C} \cdot \frac{1}{1-j\omega R_1 C} \cdot \frac{2kT_1R_1\,d\omega}{\pi} + \frac{2kT_2R_2\,d\omega}{\pi}$$

$$= \frac{2k\,d\omega}{\pi}\left[T_1R_1 \cdot \frac{1}{1+\omega^2R_1^2C^2} + T_2R_2\right]$$

$$= \frac{2kT_e(\omega)\,Re(Z_T)\,d\omega}{\pi}$$

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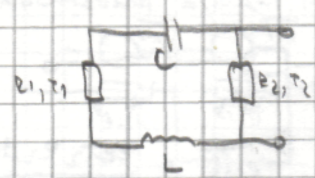
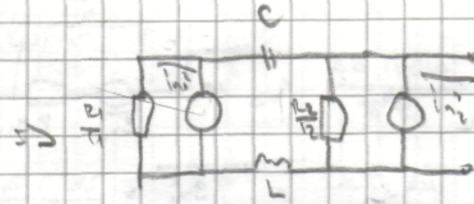
$$T_e(\omega) \cdot Re\{Z_T\} = T_1 \cdot R_1 \frac{1}{1+\omega^2R_1C^2} + T_2R_2$$

$$Re\{Z_T\} = Re\left\{\frac{1-j\omega R_1 C}{1+\omega^2R_1^2C^2}R_1 + R_2 + j\omega L\right\} = \frac{R_1}{1+\omega^2R_1^2C^2} + R_2$$

$$T_e(\omega) = \frac{\frac{T_1R_1}{1+\omega^2R_1^2C^2} + T_2R_2}{\frac{R_1}{1+\omega^2R_1^2C^2} + R_2} = \frac{T_1R_1 + T_2R_2 + \omega^2R_1^2C^2T_2R_2}{R_1 + R_2 + \omega^2R_1^2R_2C^2}$$

c) pr. 20.17, str. 243





$$\overline{i_{n1}^2} = \frac{2kT_1\,d\omega}{\pi R_1}$$

$$\overline{i_{n2}^2} = \frac{2kT_2\,d\omega}{\pi R_2}$$

$T_e(\omega)$, $\omega$ + )

$$Y_N = \frac{1}{R_2} + \frac{1}{R_1 + j\omega L + \frac{1}{j\omega C}} = \frac{1}{R_2} + \frac{j\omega C}{(1-\omega^2 LC) + j\omega R_1 C}$$

$$= \frac{1}{R_2} + \frac{j\omega C(1-\omega^2 LC) + \omega^2 R_1 C^2}{(1-\omega^2 LC) + \omega^2 R_1^2 C^2}$$

$$Re\{Y_N\} = \frac{1}{R_2} + \frac{R_1\omega^2 C^2}{(1-\omega^2 LC)^2 + \omega^2 R_1^2 C^2} \cdot \frac{R_1}{R_1}$$

$$= \frac{1}{R_2} + \frac{1}{R_1} \cdot \frac{\omega^2 R_1^2 C^2}{(1-\omega^2 LC)^2 + \omega^2 R_1^2 C^2}$$

$$= \frac{(1-\omega^2 LC)^2 + \omega^2 R_1^2 C^2 + \omega^2 R_1 R_2 C^2}{[(1-\omega^2 LC)^2 + \omega^2 R_1^2 C^2) \cdot R_2}$$

$$\overline{i_n^2} = \overline{i_{n1}^2}\,\frac{\omega^2 R_1^2 C^2}{(1-\omega^2 LC)^2 + \omega^2 R_1^2 C^2} + \overline{i_{n2}^2} = \frac{2k\,T_e(\omega)\,Re\{Y_N\}\,d\omega}{\pi}$$

$$T_e(\omega)\,\frac{(1-\omega^2 LC)^2 + \omega^2 R_1^2 C^2 + \omega^2 R_1 R_2 C^2}{[(1-\omega^2 LC)^2 + \omega^2 R_1^2 C^2]\,R_2} = \frac{T_2}{R_2} + \frac{T_1}{R_1}\,\frac{R_1^2\omega^2 C^2}{(1-\omega^2 LC)^2 + \omega^2 R_1^2 C^2}$$

$$\boxed{T_e(\omega) = \frac{T_1 R_1 R_2 \omega^2 C^2 + T_2[(1-\omega^2 LC)^2 + \omega^2 R_1 C^2]}{(1-\omega^2 LC)^2 + \omega^2 C^2 R_1 (R_1 + R_2)}}$$

za $\omega = 0$ $\wedge$ $\omega = \frac{1}{\sqrt{LC}}$ $T_e(\omega)$ NE ovisi o L i C

d) zd. 20.5, str. 258


$$\overline{i_{n1}^2} = \frac{2kT_1\,d\omega}{\pi R_1}$$

$$\overline{i_{n2}^2} = \frac{2kT_2\,d\omega}{\pi R_2}$$

$$\overline{i_{n3}^2} = \frac{2kT_3\,d\omega}{\pi R_3}$$

$T_e(\omega) = ?$

$$Y_N(\omega) = \frac{1}{R_3} + \frac{1}{\frac{1}{j\omega C} + R_1\|R_2} = \frac{1}{R_3} + \frac{1}{\frac{1+j\omega(R_1\|R_2)C}{j\omega C}} = \frac{1}{R_3} + \frac{j\omega C}{1+j\omega(R_1\|R_2)C}\cdot\frac{1-j\omega(R_1\|R_2)C}{1-j\omega(R_1\|R_2)C}$$

$$= \frac{1}{R_3} + \frac{j\omega C + \omega^2(R_1\|R_2)C^2}{1+\omega^2(R_1\|R_2)^2C^2}$$

$$\mathrm{Re}\{Y_N(\omega)\} = \frac{1}{R_3} + \frac{\omega^2(R_1\|R_2)C^2}{1+\omega^2(R_1\|R_2)^2C^2}$$

$$\overline{i_n^2} = \overline{i_{n1}^2}\left|\frac{R_1\|R_2}{R_1\|R_2+\frac{1}{j\omega C}}\right|^2 + \overline{i_{n2}^2}\left|\frac{R_1\|R_2}{R_1\|R_2+\frac{1}{j\omega C}}\right|^2 + \overline{i_{n3}^2}$$

$$= \frac{R_1\|R_2}{R_1\|R_2+\frac{1}{j\omega C}}\cdot\frac{R_1\|R_2}{R_1\|R_2-\frac{1}{j\omega C}} = \frac{(R_1\|R_2)^2}{(R_1\|R_2)^2+\frac{1}{\omega^2C^2}} = \frac{(R_1\|R_2)^2\omega^2C^2}{1+(R_1\|R_2)^2\omega^2C^2}$$

$$= \frac{\left(\frac{R_1R_2}{R_1+R_2}\right)^2\omega^2C^2}{1+\left(\frac{R_1R_2}{R_1+R_2}\right)^2\omega^2C^2} = \frac{R_1^2R_2^2\omega^2C^2}{(R_1+R_2)^2+R_1^2R_2^2\omega^2C^2}$$

$$\overline{i_n^2} = \frac{2k\,d\omega}{\pi}\left(\frac{T_1}{R_1}\cdot\frac{R_1^2R_2^2\omega^2C^2}{(R_1+R_2)^2+R_1^2R_2^2\omega^2C^2} + \frac{T_2}{R_2}\cdot\frac{R_1^2R_2^2\omega^2C^2}{(R_1+R_2)^2+R_1^2R_2^2\omega^2C^2} + \frac{T_3}{R_3}\right)$$

$$= \frac{2k\,T_e(\omega)\cdot\mathrm{Re}\{Y_N\}\,d\omega}{\pi}$$

$$T_e(\omega)\cdot\mathrm{Re}\{Y_N\} = \frac{T_1\cdot R_1R_2^2\omega^2C^2}{(R_1+R_2)^2+R_1^2R_2^2\omega^2C^2} + \frac{T_2R_1^2R_2\cdot\omega^2C^2}{(R_1+R_2)^2+R_1^2R_2^2\omega^2C^2} + \frac{T_3}{R_3}$$

$$\mathrm{Re}\{Y_N\} = \frac{1}{R_3} + \frac{\omega^2(R_1\|R_2)C^2}{1+\omega^2(R_1\|R_2)^2C^2} = \frac{1}{R_3} + \frac{\omega^2C^2\frac{R_1R_2}{R_1+R_2}}{1+\omega^2C^2\left(\frac{R_1R_2}{R_1+R_2}\right)^2} = \frac{1}{R_3} + \frac{\omega^2C^2(R_1+R_2)R_1R_2}{(R_1+R_2)^2+\omega^2C^2R_1^2R_2^2}$$

$$= \frac{(R_1+R_2)^2+\omega^2C^2R_1^2R_2^2+\omega^2C^2(R_1+R_2)R_1R_2R_3}{R_3\left[(R_1+R_2)^2+\omega^2C^2R_1^2R_2^2\right]}$$

$$\boxed{T_e(\omega) = \frac{T_1R_1R_2^2R_3\omega^2C^2 + T_2R_1^2R_2R_3\omega^2C^2 + T_3\left[(R_1+R_2)^2+R_1^2R_2^2\omega^2C^2\right]}{(R_1+R_2)^2+\omega^2C^2R_1^2R_2^2+\omega^2C^2(R_1+R_2)R_1R_2R_3}}$$

**2.**

$R_T = 10\ k\Omega$

$A = 100$

$R_{ue} = 100\ k\Omega$

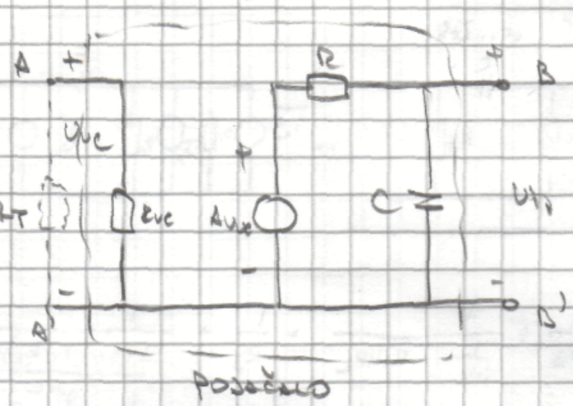
$R = 100\ k\Omega$ } ideal

$C$

$\frac{1}{RC} = 2\pi \cdot 10^3\ Hz \Rightarrow C = 1.59\ nF$

$T = 293.15\ K$

$k = 1.380662 \cdot 10^{-23}\ J K^{-1}$




$|H(j\omega)|^2,\ T_e(\omega) = ?$

$$A \cdot u_{ue} \cdot \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = U_{iz} \qquad H(j\omega) = \frac{u_{iz}}{u_{ue}} = A \cdot \frac{1}{1 + j\omega RC}$$

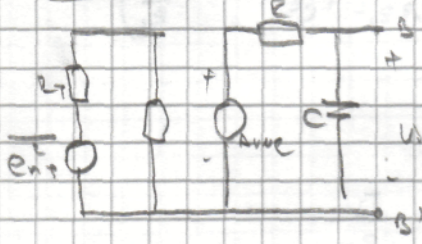
$$u_{iz} = A \cdot u_{ue} \cdot \frac{1}{1 + j\omega RC}$$

$$|H(j\omega)|^2 = H(j\omega) \cdot H(-j\omega) = A\,\frac{1}{1 + j\omega RC} \cdot A \cdot \frac{1}{1 - j\omega RC}$$

$$\boxed{|H(j\omega)|^2 = \frac{A^2}{1 + \omega^2 R^2 C^2}} = \frac{10^4}{1 + 2.533 \cdot 10^{-8}\omega^2}$$

$$Z_T = \frac{R \cdot \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{R}{1 + j\omega RC} \cdot \frac{1 - j\omega RC}{1 - j\omega RC} = \frac{R - j\omega R^2 C}{1 + \omega^2 R^2 C^2}$$

$$\mathrm{Re}\{Z_T\} = \frac{R}{1 + \omega^2 R^2 C^2} = \frac{10^5}{1 + 2.533 \cdot 10^{-8}\omega^2}$$




$$\overline{e_{ue}^2} = \overline{e_{nT}^2}\left(\frac{R_{ue}}{R_T + R_{ue}}\right)^2 = \frac{2kT R_T\, d\omega}{\pi} \cdot \left(\frac{R_{ue}}{R_T + R_{ue}}\right)^2$$

$$\overline{e_n^2} = \frac{2kT_e(\omega)\,\mathrm{Re}\{Z_T\}\, d\omega}{\pi} = \frac{2k\, d\omega}{\pi} \cdot T_e(\omega) \cdot \mathrm{Re}\{Z_T\} = \overline{e_{ue}^2} \cdot |H(j\omega)|$$

$$= \frac{2k\, d\omega}{\pi} \cdot T \cdot R_T \left(\frac{R_{ue}}{R_T + R_{ue}}\right)^2 \cdot \frac{A}{1 + \omega^2 R^2 C^2}$$

$$T_e(\omega) = \frac{T \cdot R_T \left(\frac{R_{ue}}{R_T + R_{ue}}\right)^2 \cdot \frac{A^2}{1 + \omega^2 R^2 C^2}}{\frac{R}{1 + \omega^2 R^2 C^2}} = \frac{T \cdot R_T}{R} \cdot A^2 \left(\frac{R_{ue}}{R_T + R_{ue}}\right)^2$$

$$\boxed{T_e(\omega) = 242\,272.2723\ K = 242.27 \cdot 10^3\ K}$$