Modeliranje sustava s diskretnim događajima matričnom algebrom

- Petrijeve mreže - dobar alat za “vizualizaciju” zbivanja u FPS-u,
- problem pri algoritmizaciji određivanja strukturnih svojstava => baratanje velikim matricama I i O,
- ideja: pronaći matrice manjih dimenzija koje će opisati sustav, a da istovremeno omoguće jednostavnu analizu,

Matrična algebra

- FPS se promatra kao skup pravila AKO – ONDA,
- za razliku od Petrijevih mreža operacije i resursi tretiraju se odvojeno,
- operacije nad matricama uključuju i logičke operacije,
- rezultat modeliranja je “hibridni sustav”.

Matrica incidencije stroj-posao *(Machine-job incidence matrix - MJI)*

- Stewardova matrica slijeda *(Steward sequencing matrix* or *design structure matrix* (DSM) *)*

$$\mathbf{\Gamma} = \begin{array}{cc} & J\;o\;b\;s \\ \begin{array}{c} J \\ o \\ b \\ s \end{array} & \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \\ & & \end{bmatrix} \end{array}$$

If task j is an immediate predecessor of task i, then DSM element (i,j) is equal to ‘1’, otherwise is ‘0’.

- Matrica zahtjeva za resursima *(resource requirements matrix* or *machine-part incidence matrix - MPI)*

$$\mathbf{\Theta} = \begin{array}{cc} & Resources \\ \begin{array}{c} P \\ a \\ r \\ ts \end{array} & \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \\ & & \end{bmatrix} \end{array}$$

If resource j executes an operation in the processing sequence of part i, then MPI element (i, j) is equal to ‘1’, otherwise is ‘0’

Ideja: povezati te dvije matrice u jednu.

- matrica incidencije stroj-posao *(Machine-job incidence matrix - MJI)*

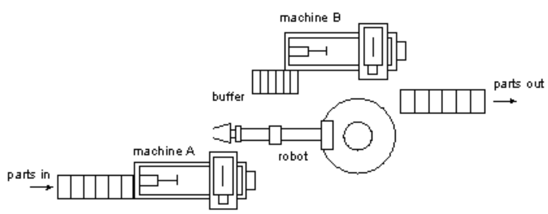
$$\begin{array}{cc} & \textit{Resources} \\ \boldsymbol{\Lambda} = \begin{array}{c} J \\ o \\ b \\ s \end{array} & \left[\begin{array}{ccc} \quad & \quad & \quad \\ & & \\ & & \\ & & \end{array} \right] \end{array}$$

In case job *i* is performed by resource *j*, matrix element (*i*, *j*) is equal to ‘1’, otherwise is ‘0’.

- ako u sustavu postoji više paralelnih procesa

$$\boldsymbol{\Lambda} = \left[{}^{1}\boldsymbol{\Lambda}^{\mathbf{T}} \quad {}^{2}\boldsymbol{\Lambda}^{\mathbf{T}} \quad \ldots \quad {}^{m}\boldsymbol{\Lambda}^{\mathbf{T}} \right]^{\mathrm{T}}$$

Primjer



		MA	MB	B	R
Θ =	P1	**1**	**1**	**1**	**1**

		MAP	RP1	BP	MBP	RP2
	MAP	**0**	**0**	**0**	**0**	**0**
	RP1	**1**	**0**	**0**	**0**	**0**
Γ =	BP	**0**	**1**	**0**	**0**	**0**
	MBP	**0**	**0**	**1**	**0**	**0**
	RP2	**0**	**0**	**0**	**1**	**0**

slijed operacija

		MA	MB	B	R
	MAP	**1**	**0**	**0**	**0**
	RP1	**0**	**0**	**0**	**1**
Λ =	BP	**0**	**0**	**1**	**0**
	MBP	**0**	**1**	**0**	**0**
	RP2	**0**	**0**	**0**	**1**

Rekurzivni model sustava s MJI matricom

- vektor stanja **m** razlaže se na dva dijela:

$$\mathbf{m}(k) = [\,\mathbf{v}(k)^T \; \mathbf{r}(k)^T\,]^T$$

operacije, poslovi, zadaci, ... *resursi*

$$\mathbf{v}(k) = \mathbf{v}(k\text{-}1) + \mathbf{d}(k)$$

$$\mathrm{d}_i(k) = \mathrm{d}_i^+(k) - \mathrm{d}_i^-(k)$$

- razlika u broju dijelova (klijenata) nad kojima je zadatak *i* započeo (+), odnosno završio (-) u koraku *k*

$$\mathrm{d}_i^+(k) = \mathrm{v}_{i-1}(k\text{-}1) \cdot \left[\Lambda_i \,\Delta\, \mathbf{r}(k\text{-}1)\right]$$

i-ti redak MJI matrice

$$\mathrm{d}_i^-(k) = \mathrm{v}_i(k\text{-}1) \cdot \left[\Lambda_{i+1} \,\Delta\, \mathbf{r}(k\text{-}1)\right]$$

Δ and/or algebra – standardno množenje zamjenjuje se s logičkim "*and*", a standardno zbrajanje s logičkim "*or*"

- uvodi se operacija pomaka vektora

$$\mathbf{b} = \uparrow_x \mathbf{a} \longrightarrow b_j = \begin{cases} x, \text{ if } & j = n \\ a_{j+1} & \text{, otherwise} \end{cases}$$

$$\mathbf{b} = \downarrow^x \mathbf{a} \longrightarrow b_j = \begin{cases} x, \text{ if } & j = 1 \\ a_{j\text{-}1} & \text{, otherwise} \end{cases}$$

MJI matrica sadrži informaciju o slijedu zadataka pa se ovom operacijom pomiču indeksi vektora **v** (**r**)

$$\mathbf{d}(k) = \mathbf{d}^{+}(k) - \mathbf{d}^{-}(k) = (\downarrow^{1} \mathbf{v}(k-1)) \cdot \left[\mathbf{\Lambda} \, \Delta \, \mathbf{r}(k-1) \right] - \mathbf{v}(k-1) \cdot \uparrow_{1} \left[\mathbf{\Lambda} \, \Delta \, \mathbf{r}(k-1) \right]$$

- konačni rekurzivni model

$$\mathbf{v}(k) = \mathbf{v}(k-1) + \left\{ (\downarrow^{1} \mathbf{v}(k-1)) \cdot \left[\mathbf{\Lambda} \, \Delta \, \mathbf{r}(k-1) \right] - \mathbf{v}(k-1) \cdot \uparrow_{1} \left[\mathbf{\Lambda} \, \Delta \, \mathbf{r}(k-1) \right] \right\}$$

$$\mathbf{r}(k) = \overline{\mathbf{\Lambda}^{\mathrm{T}} \Delta \mathbf{v}(k)}$$

Osim dinamičkog modeliranja MJI matrica može poslužiti i za određivanje strukturalnih svojstava sustava.

		MA	MB	B	R
	MAP	**1**	**0**	**0**	**0**
	RP1	**0**	**0**	**0**	**1**
Λ =	BP	**0**	**0**	**1**	**0**
	MBP	**0**	**1**	**0**	**0**
	RP2	**0**	**0**	**0**	**1**

kružno čekanje