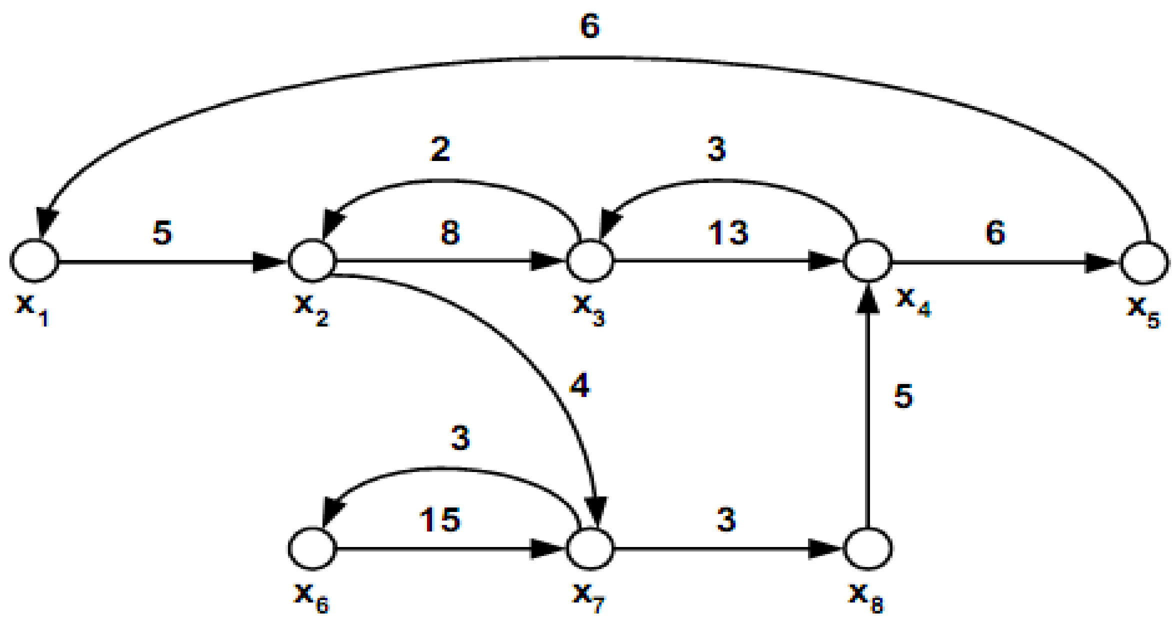
- Kako izgleda max-plus model ako strojevi mogu obrađivati *više* predmeta? Što ako postoji *više* agv-a?
- Kako početni uvjeti utječu na model?

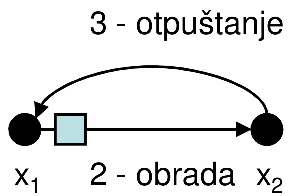
Cilj: odrediti proceduru za određivanje max-plus modela za različite uvjete sustava



Određivanje max-plus modela pomoću paleta

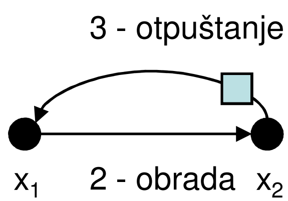
Paleta - u proizvodnim sustavima: stvarno mjesto za smještaj predmeta
- općenito: paket - nosioc informacije

Primjer: stroj koji može obrađivati samo jedan predmet (kapacitet = 1)



Paleta može biti “prazna” i “puna”:

• puna paleta (drži predmet) – nalazi se na luku $x_1 \rightarrow x_2$

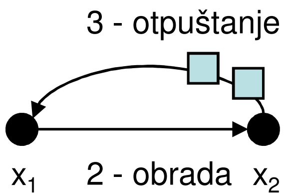


• prazna paleta (nema predmet) – nalazi se na luku $x_2 \rightarrow x_1$

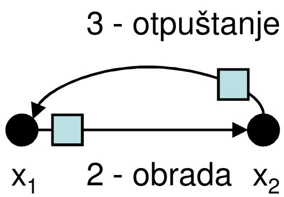
Stroj kapaciteta jedan uvijek drži točno *jednu paletu na svom ciklusu*, koja je puna ili prazna, ovisno o tome da li stroj obrađuje predmet ili se priprema za obradu.

Određivanje max-plus modela pomoću paleta

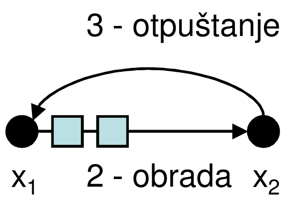
Primjer: stroj koji može obrađivati *dva* predmeta (kapacitet = 2) – dvije palete na ciklusu



• stroj prazan, priprema se za prihvat predmeta



• stroj obrađuje jedan predmet, može primiti još jedan



• stroj obrađuje dva predmeta

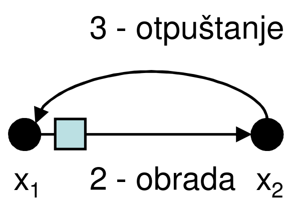
Na ciklusu ovakvog resursa su uvijek dvije palete!

Veza palete – max-plus model:

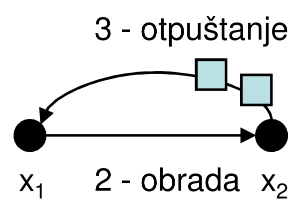
PRAVILO: ako veza $x_1 \rightarrow x_2$ koja je težine a, drži p $(p \geq 0)$ paleta, vrijedi:

$$x_2 = ax_1(k - p)$$

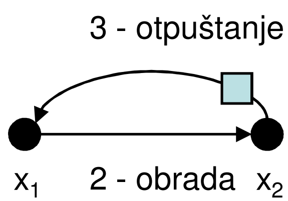
- uz poznatih onoliko početnih uvjeta koliki je p – početni uvjeti postavljaju se prema fizikalnom stanju sustava



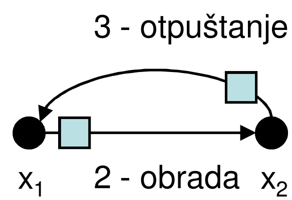
$x_1(k) = 3x_2(k)$
$x_2(k) = 2x_1(k - 1)$
-poznat $\mathbf{x}(0) = [e\ \varepsilon]^T$



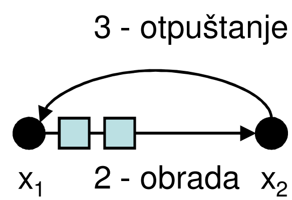
$x_1(k) = 3x_2(k - 2)$
$x_2(k) = 2x_1(k)$
- poznat $\mathbf{x}(0)$, $\mathbf{x}(-1)$



$x_1(k) = 3x_2(k - 1)$
$x_2(k) = 2x_1(k)$
- poznat $\mathbf{x}(0) = [\varepsilon\ e]^T$

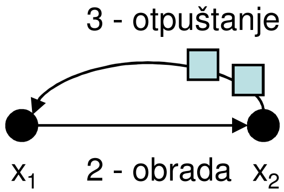


$x_1(k) = 3x_2(k - 1)$
$x_2(k) = 2x_1(k - 1)$
- poznat $\mathbf{x}(0)$



$x_1(k) = 3x_2(k)$
$x_2(k) = 2x_1(k - 2)$
- poznat $\mathbf{x}(0)$, $\mathbf{x}(-1)$

Matrice sustava



$x_1(k) = 3x_2(k - 2)$
$x_2(k) = 2x_1(k)$
- poznat $\mathbf{x}(0)$, $\mathbf{x}(-1)$

1. način

$$\mathbf{A}_0 = \begin{bmatrix} \varepsilon & \varepsilon \\ 2 & \varepsilon \end{bmatrix}, \mathbf{A}_1 = \begin{bmatrix} \varepsilon & \varepsilon \\ \varepsilon & \varepsilon \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} \varepsilon & 3 \\ \varepsilon & \varepsilon \end{bmatrix}$$

$$\mathbf{A}_0^* = \begin{bmatrix} e & \varepsilon \\ 2 & e \end{bmatrix}, \mathbf{A}_0^* \otimes \mathbf{A}_1 = \begin{bmatrix} \varepsilon & \varepsilon \\ \varepsilon & \varepsilon \end{bmatrix}, \mathbf{A}_0^* \otimes \mathbf{A}_2 = \begin{bmatrix} \varepsilon & 3 \\ \varepsilon & 5 \end{bmatrix}$$

$$\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = \begin{bmatrix} \varepsilon & 3 \\ \varepsilon & 5 \end{bmatrix} \begin{bmatrix} x_1(k-2) \\ x_2(k-2) \end{bmatrix}$$

2. način

Nova varijabla stanja:
$x_3(k) = x_2(k-1)$

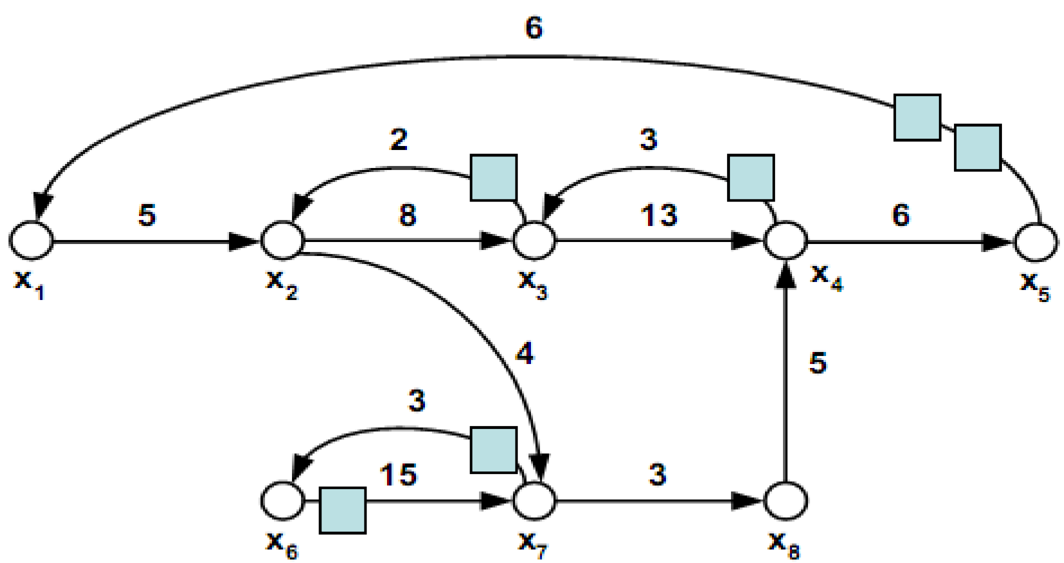
Jednažbe:
$x_1(k) = 3x_3(k - 1)$
$x_2(k) = 2x_1(k)$
$x_3(k) = x_2(k - 1)$

$$\mathbf{A}_0 = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon \\ 2 & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon \end{bmatrix}, \mathbf{A}_1 = \begin{bmatrix} \varepsilon & \varepsilon & 3 \\ \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & e & \varepsilon \end{bmatrix}, \mathbf{A}_0^* = \begin{bmatrix} e & \varepsilon & \varepsilon \\ 2 & e & \varepsilon \\ \varepsilon & \varepsilon & e \end{bmatrix}$$

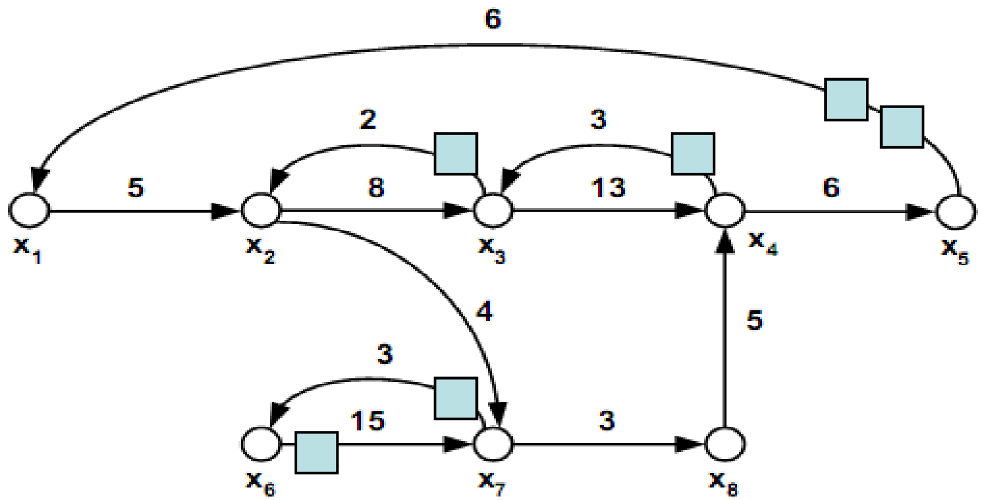
$$\begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} = \begin{bmatrix} \varepsilon & \varepsilon & 3 \\ \varepsilon & \varepsilon & 5 \\ \varepsilon & e & \varepsilon \end{bmatrix} \begin{bmatrix} x_1(k-1) \\ x_2(k-1) \\ x_3(k-1) \end{bmatrix}$$

Primjer: a) postoje 2 AGV-a, u početnom stanju oba se pripremaju za x_1
b) stroj M3 može obrađivati 2 premeta, u početnom stanju drži (obrađuje) jedan predmet

1. Postaviti palete na ciklus svakog resursa da odgovaraju početnim uvjetima



2. Odrediti jednadžbe max-plus modela prema pravilima



$x_1(k) = 6x_5(k - 2)$
$x_2(k) = 5x_1(k) + 2x_3(k - 1)$
$x_3(k) = 8x_2(k) + 3x_4(k - 1)$
$x_4(k) = 13x_3(k) + 5x_8(k)$
$x_5(k) = 6x_4(k)$
$x_6(k) = 3x_7(k - 1)$
$x_7(k) = 15x_6(k - 1) + 4x_2(k)$
$x_8(k) = 3x_7(k)$

Vektor(i) početnih stanja = ?
- zbog $(k - 2) \rightarrow \mathbf{x}(0)$ i $\mathbf{x}(-1)$

$\mathbf{x}(-1) = [\varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ e\ \varepsilon\ \varepsilon\ \varepsilon]^T$
$\mathbf{x}(0) = [\varepsilon\ \varepsilon\ e\ e\ e\ e\ e\ \varepsilon]^T$

Početno stanje $\rightarrow$ palete $\rightarrow$ max-plus model $\rightarrow$ matrice $\mathbf{A}_0$, $\mathbf{A}_1$, ..., $\mathbf{A}$ $\rightarrow$ ciklus sustava

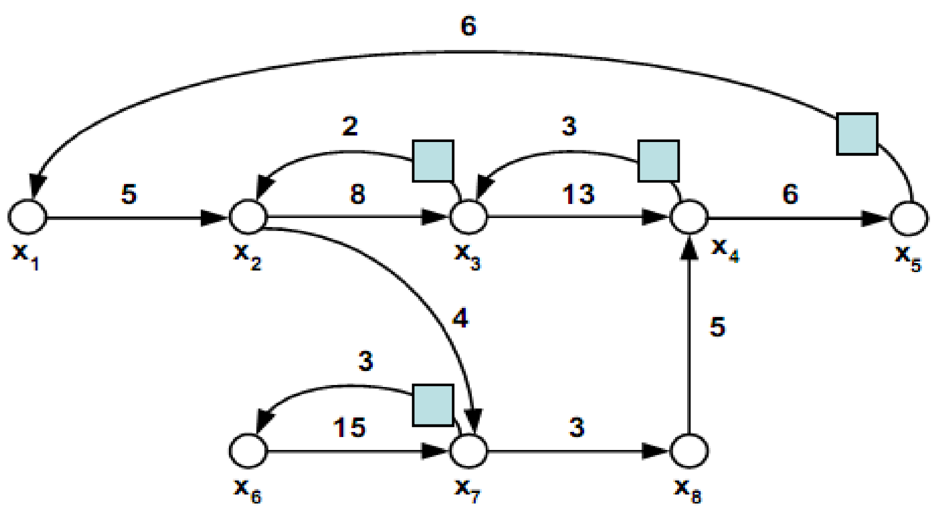
Pitanje: Kako postaviti početno stanje da ciklus sustava bude što manji?

Fizičko ograničenje sustava = kapacitet resursa tj. broj paleta na svakom resursu

Prosječan ciklus sustava – λ

λ = max {duljina ciklusa / broj paleta na ciklusu} - kritični ciklus

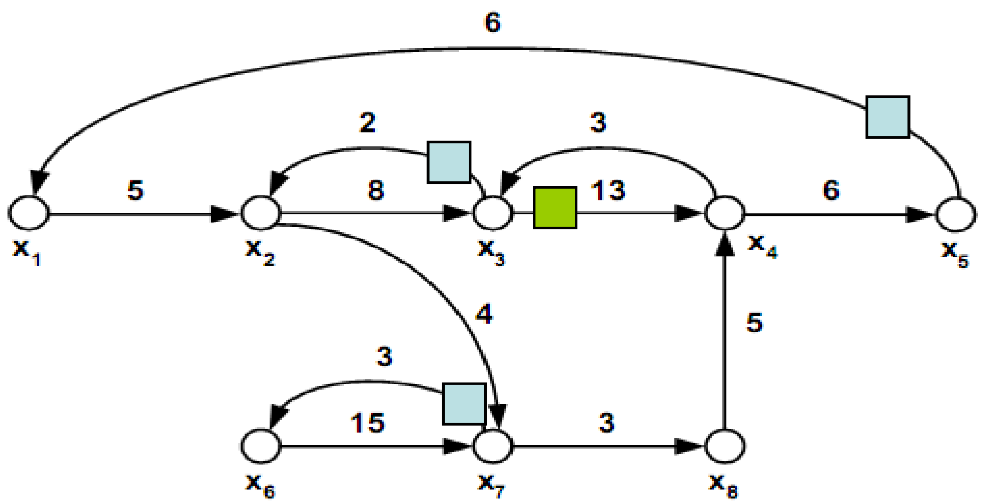
- predstavlja maksimalnu srednju težinu ciklusa "matrice" sustava **A**
- palete se mogu "klizati " po *vlastitom ciklusu resursa* kako bi λ bio što manji
- n_p – broj paleta na ciklusu



Ciklusi:
(x_1,x_2,x_3,x_4,x_5), $n_p = 1 \rightarrow \lambda_1 = 38$
(x_2,x_3), $n_p=1 \rightarrow \lambda_2 = 10$
(x_3,x_4), $n_p=1 \rightarrow \lambda_3 = 16$
(x_6,x_7), $n_p=2 \rightarrow \lambda_4 = 9$
$(x_1,x_2,x_7,x_8,x_4,x_5)$, $n_p=1 \rightarrow \lambda_5 = 29$
(x_2,x_7,x_8,x_4,x_3), $n_p=2 \rightarrow \lambda_6 = 8.5$
...

$\lambda = 38$

Da li se može smanjiti ciklus?



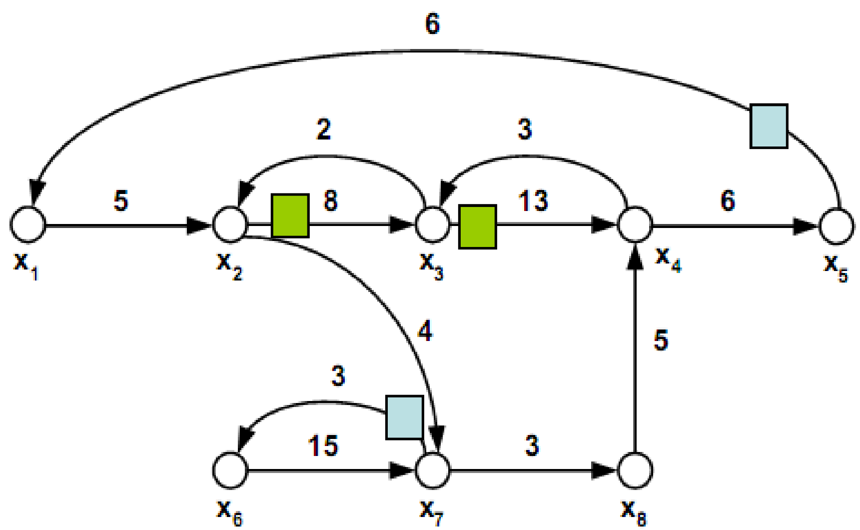
$x_1(k) = 6x_5(k - 1)$
$x_2(k) = 5x_1(k) + 2x_3(k - 1)$
$x_3(k) = 8x_2(k) + 3x_4(k)$
$x_4(k) = 13x_3(k - 1) + 5x_8(k)$
$x_5(k) = 6x_4(k)$
$x_6(k) = 3x_7(k - 1)$
$x_7(k) = 15x_6(k) + 4x_2(k)$
$x_8(k) = 3x_7(k)$

$\mathbf{x}(0) = [\varepsilon\ \varepsilon\ e\ \varepsilon\ e\ \varepsilon\ e\ \varepsilon]^T$

Ciklusi:
(x_1,x_2,x_3,x_4,x_5), $n_p = 2 \rightarrow \lambda_1 = 19$
(x_2,x_3), $n_p=1 \rightarrow \lambda_2 = 10$
(x_3,x_4), $n_p=1 \rightarrow \lambda_3 = 16$
(x_6,x_7), $n_p=1 \rightarrow \lambda_4 = 18$
$(x_1,x_2,x_7,x_8,x_4,x_5)$, $n_p=1 \rightarrow \lambda_5 = 29$
(x_2,x_7,x_8,x_4,x_3), $n_p=1 \rightarrow \lambda_6 = 17$
...

$\lambda = 29$

Da li se može ovako smanjiti ciklus?



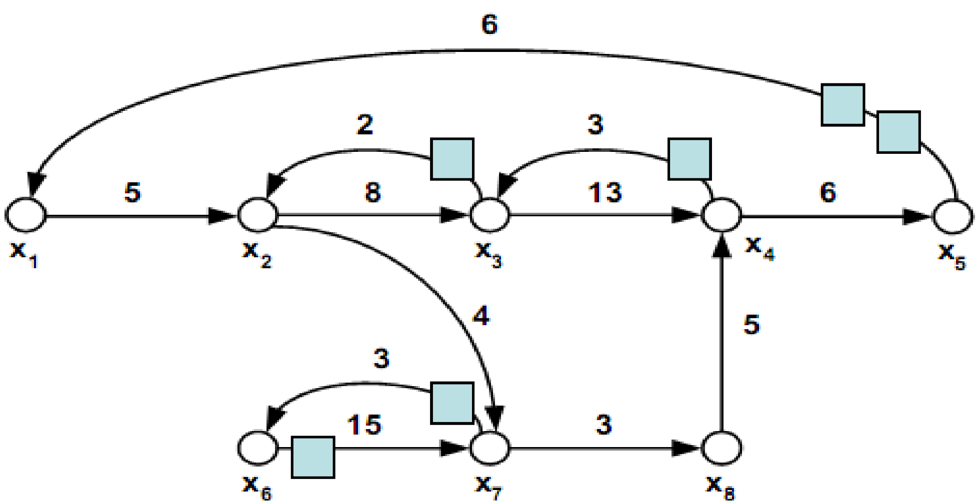
$x_1(k) = 6x_5(k - 1)$
$x_2(k) = 5x_1(k) + 2x_3(k)$
$x_3(k) = 8x_2(k - 1) + 3x_4(k)$
$x_4(k) = 13x_3(k - 1) + 5x_8(k)$
$x_5(k) = 6x_4(k)$
$x_6(k) = 3x_7(k - 1)$
$x_7(k) = 15x_6(k) + 4x_2(k)$
$x_8(k) = 3x_7(k)$

$\mathbf{x}(0) = [\varepsilon\ e\ e\ \varepsilon\ e\ \varepsilon\ e\ \varepsilon]^T$

Zaglavljenje – ciklus bez paleta (x_2,x_7,x_8,x_4,x_3) – $n_p = 0 \rightarrow \lambda = \infty$

Matematički – nekauzalnost, ne vrijedi $A_0^{n+1} = [\varepsilon]$

Drugi primjer:



$x_1(k) = 6x_5(k - 2)$
$x_2(k) = 5x_1(k) + 2x_3(k - 1)$
$x_3(k) = 8x_2(k) + 3x_4(k - 1)$
$x_4(k) = 13x_3(k) + 5x_8(k)$
$x_5(k) = 6x_4(k)$
$x_6(k) = 3x_7(k - 1)$
$x_7(k) = 15x_6(k - 1) + 4x_2(k)$
$x_8(k) = 3x_7(k)$

$\mathbf{x}(-1) = [\varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ e\ \varepsilon\ \varepsilon\ \varepsilon]^T$
$\mathbf{x}(0) = [\varepsilon\ \varepsilon\ e\ e\ e\ e\ e\ \varepsilon]^T$

Ciklusi:
(x_1,x_2,x_3,x_4,x_5), $n_p = 2 \rightarrow \lambda_1 = 19$
(x_2,x_3), $n_p=1 \rightarrow \lambda_2 = 10$
(x_3,x_4), $n_p=1 \rightarrow \lambda_3 = 16$
(x_6,x_7), $n_p=2 \rightarrow \lambda_4 = 9$
$(x_1,x_2,x_7,x_8,x_4,x_5)$, $n_p=2 \rightarrow \lambda_5 = 14.5$
(x_2,x_7,x_8,x_4,x_3), $n_p=2 \rightarrow \lambda_6 = 8.5$
...

$\lambda = 19$

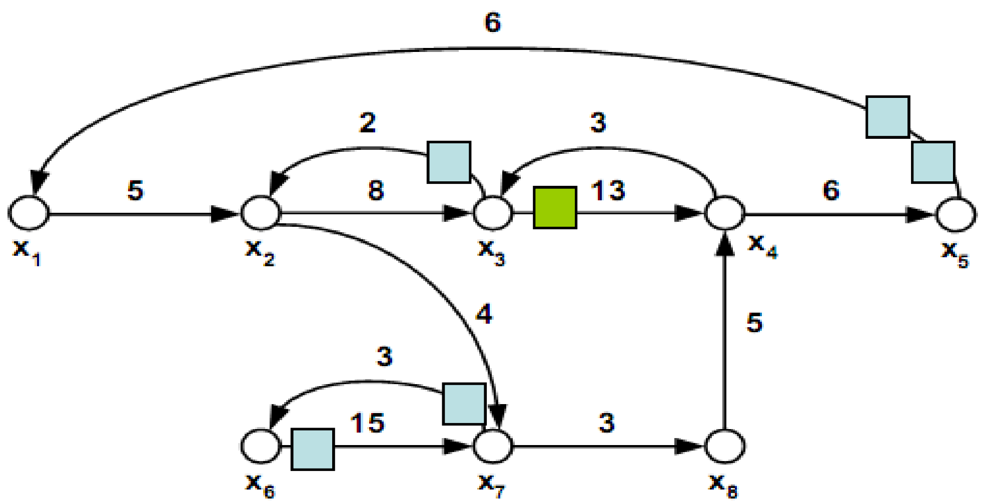
Stvarna vremena:

k	1	2	3	4	5	6
x_1	6	6	44	60	82	
x_2	11	21	49	65	87	
x_3	19	35	57	73	95	
x_4	32	48	70	86	108	
x_5	38	54	76	92	114	
x_6	3	18	28	56	72	
x_7	15	25	53	69	91	
x8	18	28	56	72	94	

Δ							
x_1	0	38	16	22	16	22	16
x_2	10	28	16	22	16	22	16
x_3	16	22	16	22	16	22	16
x_4	16	22	16	22	16	22	16
x_5	16	22	16	22	16	22	16
x_6	15	10	28	16	22	16	22
x_7	10	28	16	22	16	22	16
x8	10	28	16	22	16	22	16

Prosječan ciklus = 19

Da li se može smanjiti ciklus?



$x_1(k) = 6x_5(k - 2)$
$x_2(k) = 5x_1(k) + 2x_3(k - 1)$
$x_3(k) = 8x_2(k) + 3x_4(k)$
$x_4(k) = 13x_3(k - 1) + 5x_8(k)$
$x_5(k) = 6x_4(k)$
$x_6(k) = 3x_7(k - 1)$
$x_7(k) = 15x_6(k - 1) + 4x_2(k)$
$x_8(k) = 3x_7(k)$

$\mathbf{x}(-1) = [\varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ e\ \varepsilon\ \varepsilon\ \varepsilon]^T$
$\mathbf{x}(0) = [\varepsilon\ \varepsilon\ e\ \varepsilon\ e\ e\ e\ \varepsilon]^T$

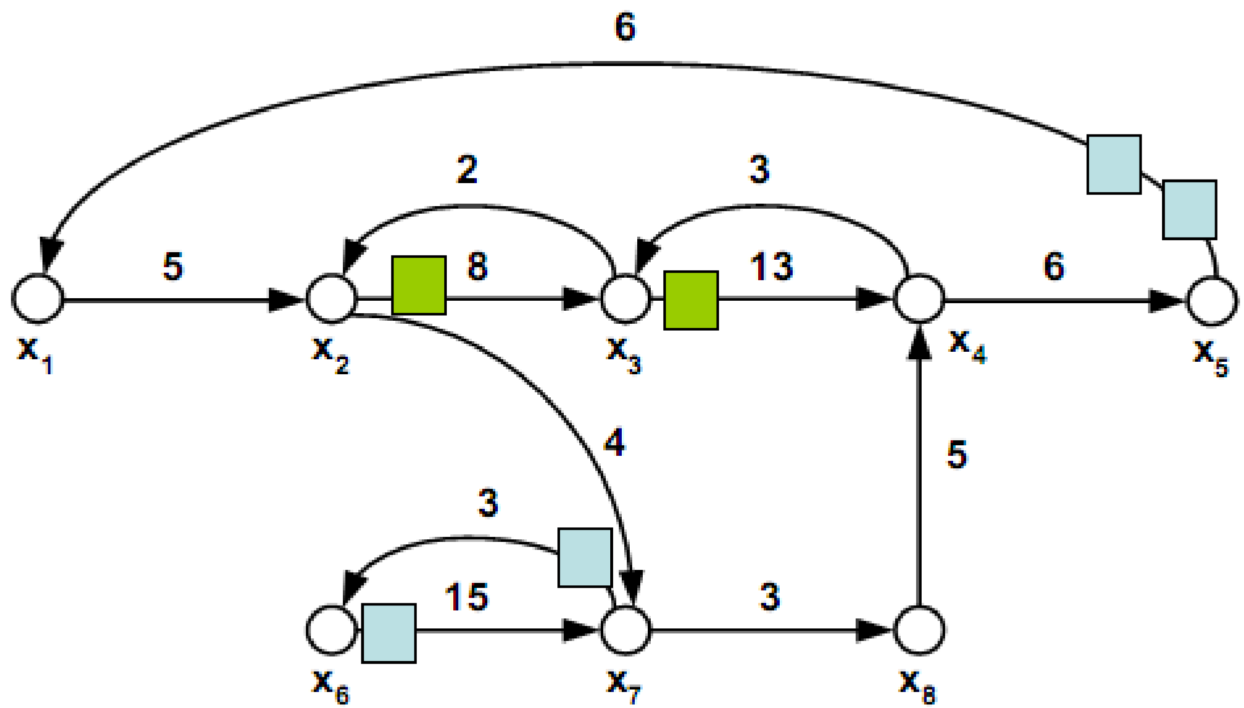
Ciklusi:
(x_1,x_2,x_3,x_4,x_5), $n_p = 3 \rightarrow \lambda_1 = 38/3$
(x_2,x_3), $n_p=1 \rightarrow \lambda_2 = 10$
(x_3,x_4), $n_p=1 \rightarrow \lambda_3 = 16$
(x_6,x_7), $n_p=2 \rightarrow \lambda_4 = 9$
$(x_1,x_2,x_7,x_8,x_4,x_5)$, $n_p=2 \rightarrow \lambda_5 = 14$
(x_2,x_7,x_8,x_4,x_3), $n_p=1 \rightarrow \lambda_6 = 17$
...

$\lambda = 17$

Stvarna vremena:

k	1	2	3	4	5	6
x_1	6	6	35	52	69	
x_2	11	28	45	62	79	
x_3	26	43	60	77	94	
x_4	23	40	57	74	91	
x_5	29	46	63	80	97	
x_6	3	18	35	52	69	
x_7	15	32	49	66	83	
x8	18	35	52	69	86	

Δ					
x_1	0	29	17	17	
x_2	17	17	17	17	
x_3	17	17	17	17	
x_4	17	17	17	17	
x_5	17	17	17	17	
x_6	15	17	17	17	
x_7	17	17	17	17	
x8	17	17	17	17	



Zaglavljenje – ciklus bez paleta – $n_p = 0 \rightarrow$ ciklus = ∞

Kako se može povećati ciklus?

λ = 38