Određivanje strukturnih svojstava sustava korištenjem matričnog modela

Matrice modela sustava s predavanja:

$\mathbf{v} = \begin{bmatrix} M1P & RU1 & BS & M2P & RU2 \end{bmatrix}^T$

$\mathbf{r} = \begin{bmatrix} M1 & M2 & B & R \end{bmatrix}^T$

$$\mathbf{F}_v = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{F}_r = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{S}_v = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{S}_r = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\hat{\mathbf{F}}_v = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\hat{\mathbf{F}}_r = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\hat{\mathbf{S}}_v = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\hat{\mathbf{S}}_r = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Određivanje p-invarijanti

$$\mathbf{P} = \begin{bmatrix} -(\hat{\mathbf{S}}_v^T - \hat{\mathbf{F}}_v)^{-1} \cdot (\hat{\mathbf{S}}_r^T - \hat{\mathbf{F}}_r) \\ \mathbf{I} \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} \mathbf{v} \\ \mathbf{r} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Postoje četiri p-invarijante, svaka odgovara jednom stupcu matrice $\mathbf{P}$: $p_1 = sup(\mathbf{P}(:,1)) = \{M1, M1P\}, p_2 = sup(\mathbf{P}(:,2)) = \{M2, M2P\}, p_3 = sup(\mathbf{P}(:,3)) = \{B, BS\}, p_4 = sup(\mathbf{P}(:,4)) = \{R, RU1, RU2\}$.

Određivanje kritičnog sifona

Kružno čekanje: $\mathbf{c} = \begin{bmatrix} 0 & 1 & 1 & 1 \end{bmatrix}^T$

Višeradni resursi u kružnom čekanju: $\mathbf{c}_s = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$

$$\mathbf{s}_c = \begin{bmatrix} \mathbf{v}_{sc} \\ \mathbf{c} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_v^T \Delta \mathbf{S}_r^T \Delta \mathbf{c}_s \wedge \overline{\mathbf{F}_v^T \Delta \mathbf{F}_r \Delta \mathbf{c}} \\ \mathbf{c} \end{bmatrix}$$

$\mathbf{F}_v^T \Delta \mathbf{S}_r^T \Delta \mathbf{c}_s = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \end{bmatrix}^T$

$\overline{\mathbf{F}_v^T \Delta \mathbf{F}_r \Delta \mathbf{c}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$

$\mathbf{v}_{sc} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$

$\mathbf{s}_c = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}^T$

Kritični sifon jednak je $S_c = sup(\mathbf{s}_c) = \{M2, B, R, RU2\}$.

Određivanje kritičnih podsustava

$$\begin{bmatrix} \mathbf{v}_{0c} \\ \mathbf{0}_n \end{bmatrix}^T = \mathbf{P}\Delta\mathbf{c} \wedge \overline{\mathbf{s}_c} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \Rightarrow \mathbf{v}_{0c} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \end{bmatrix}^T$$

Kritični podsustav jednak je $J_0 = sup(\mathbf{v}_{0c}) = \{RU2, BS, M2P\}$.