Sustavi s diskretnim događajima

Sadržaj

Osnovne strukture sustava
- sustavi vođeni vremenom
- sustavi vođeni događajima

Matematički opis sustava
- statička analiza (logički model; KAKO ?)
- dinamička analiza (vremenski model; KADA ?)

Algoritmi upravljanja

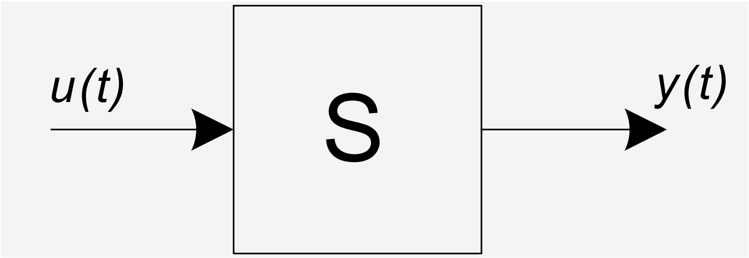
Alati

Petrijeve mreže
Max-plus algebra
Matrična algebra

Sustav

Sustav => skup entiteta koji zajedno djeluju na način koji ne može biti ostvaren njihovim samostalnim djelovanjem

=> ulazi, izlazi, parametri, varijable, podsustavi



Model => “predviđanje budućnosti”

Tehnički sustavi => vođeni vremenom

=> vođeni događajima

Sustavi vođeni vremenom

time driven systems

Vrijeme => nezavisna varijabla

$$\mathbf{y}(t) = \mathrm{G}\left[\mathbf{u}(t), t\right].$$

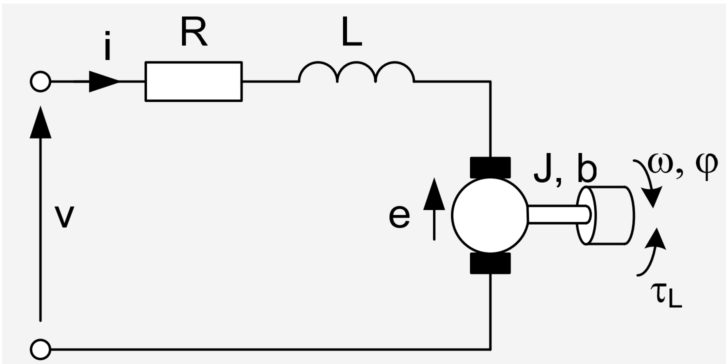
izlazi <= ulazi

=> “unutrašnja” stanja sustava

$$\dot{\mathbf{x}}(t) = \mathrm{f}\left[\mathbf{x}(t), \mathbf{u}(t), t\right], \ \mathbf{x}(t_0) = \mathbf{x}_0 ,$$

$$\mathbf{y}(t) = \mathrm{g}\left[\mathbf{x}(t), \mathbf{u}(t), t\right],$$

Sustavi vođeni vremenom



$$R \cdot i(t) + L\frac{di(t)}{dt} + e(t) = v(t)\,,\quad e(t) = K \cdot \omega(t)\,,$$

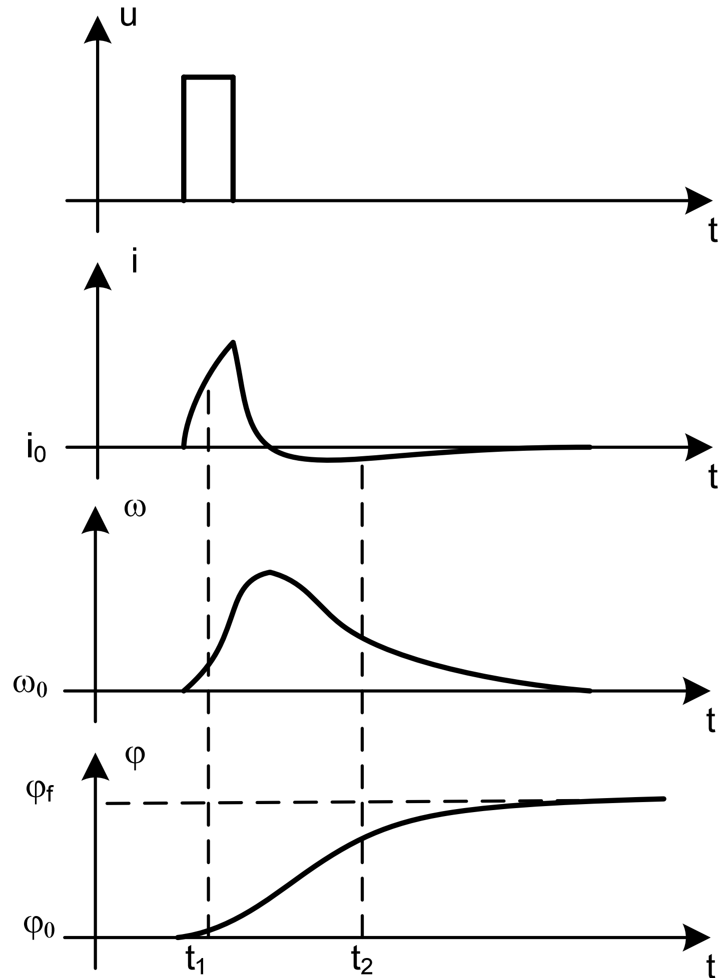
$$J\frac{d\omega(t)}{dt} + b \cdot \omega(t) = \tau_M(t) - \tau_L(t)\,,\quad \tau_M(t) = K \cdot i(t)\,,$$

$$\frac{di(t)}{dt} = \frac{1}{L}\Big[-R\cdot i(t) - K\cdot\omega(t) + v(t)\Big],$$

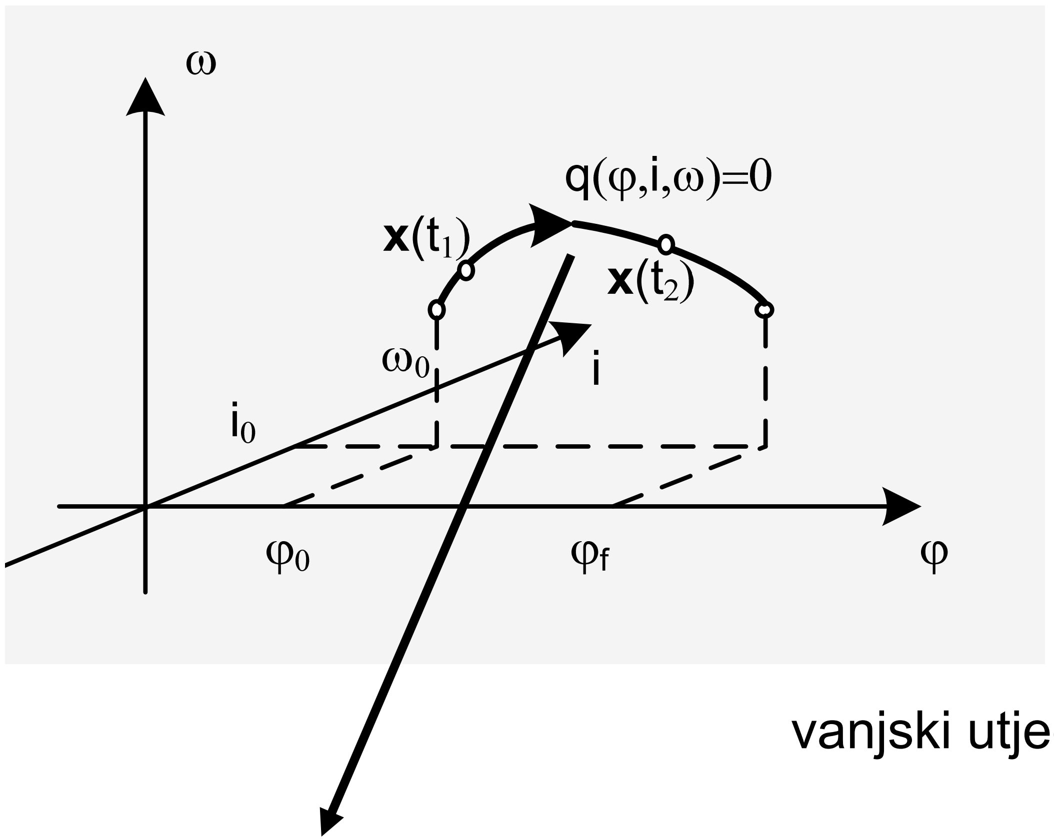
$$\frac{d\omega(t)}{dt} = \frac{1}{J}\Big[K\cdot i(t) - b\cdot\omega(t) - \tau_L(t)\Big],$$

$$\frac{d\varphi(t)}{dt} = \omega(t)\,,$$

$$y(t) = \varphi(t)\,.$$



Sustavi vođeni vremenom



Kako sustav iz stanja A dovesti u stanje B?

Kako sustav održati u stanju A?

"predviđanje budućnosti"

upravljanje

$$\mathbf{u}(t) = h\left[\mathbf{u_r}(t)\right] \ ,$$

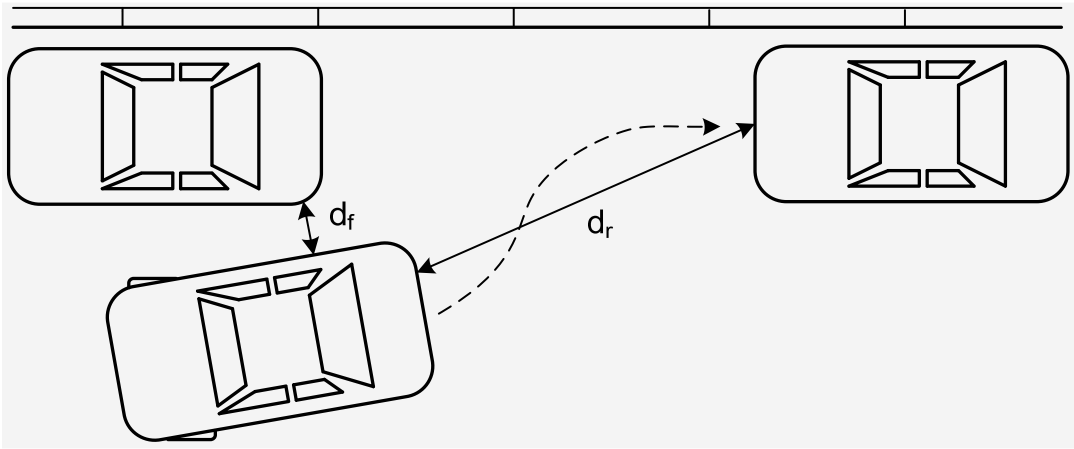
vanjski utjecaji => mjerenja => povratna veza

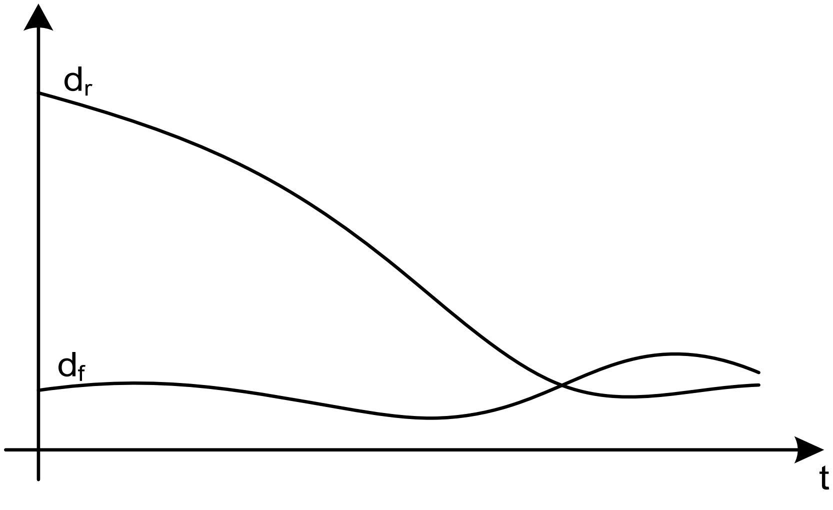
$$\mathbf{u}(t) = h\left[\mathbf{u_r}(t), \mathbf{x}(t)\right] \ .$$

stanje sustava mijenja se s tijekom vremena

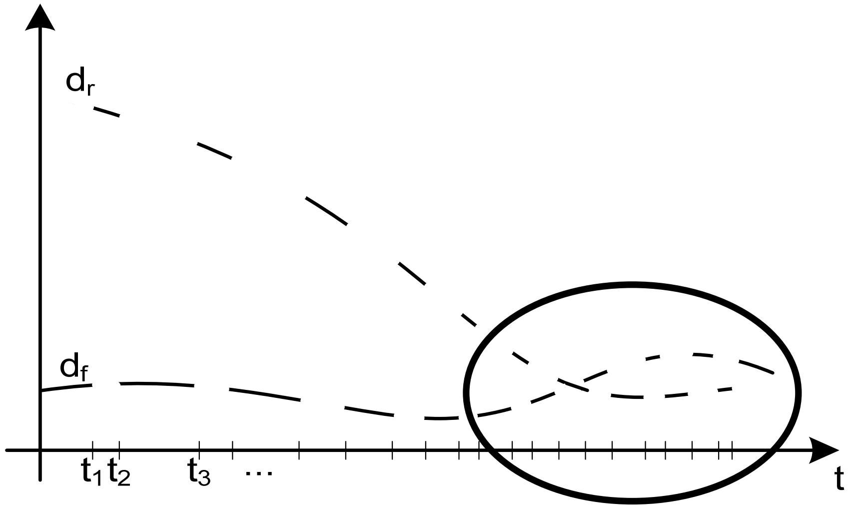
prostor stanja je kontinuiran i neprebrojiv

Sustavi vođeni vremenom





stvarne vrijednosti

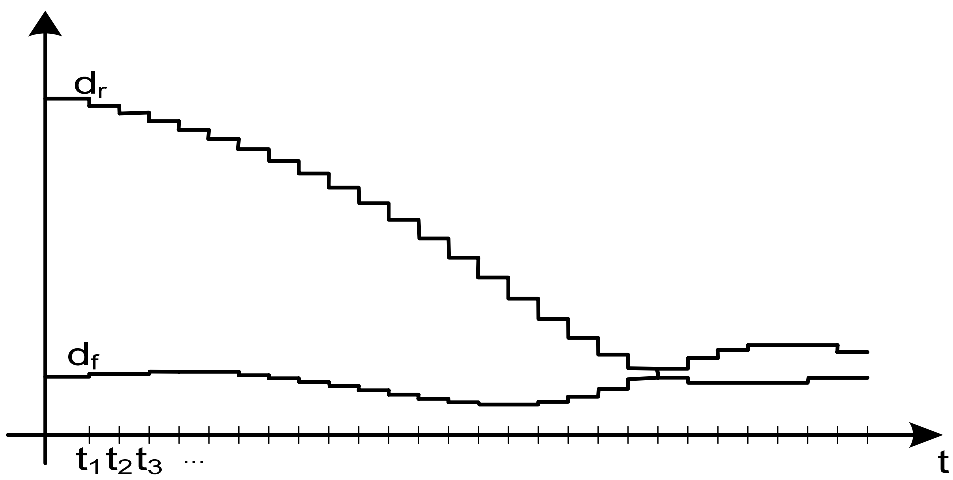


“vozačeve” vrijednosti

Sustavi vođeni vremenom

jednaki vremenski intervali $T_d = t_k - t_{k-1}$

$$\mathbf{x}(t_{k+1}) = \Phi\left[\mathbf{x}(t_k), \mathbf{u}(t_k), t_k\right], \ \mathbf{x}(t_0) = \mathbf{x}_0 ,$$
$$\mathbf{y}(t_k) = \Gamma\left[\mathbf{x}(t_k), \mathbf{u}(t_k), t_k\right].$$



Kako opisati sustave čije se stanje ne mijenja s tijekom vremena ?

Primjer: ventilacija cestovnog tunela

Stanje sustava => koncetracija ugljičnog monoksida, *CO* (L, M, H)

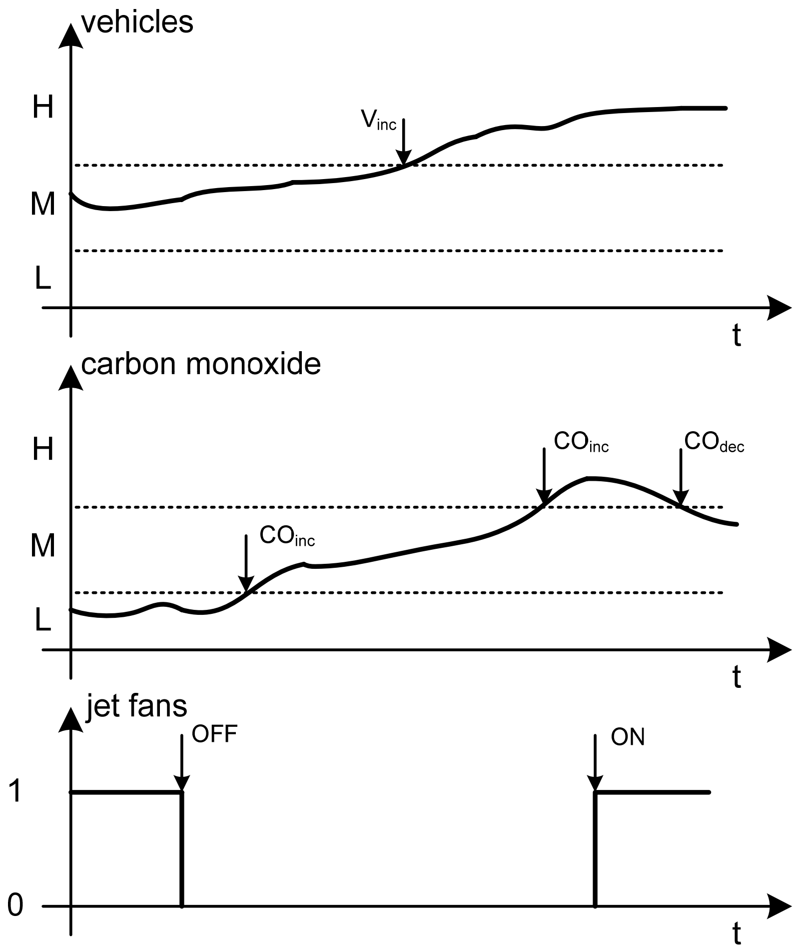
=> broj vozila u tunelu, *Nv* (L, M, H)

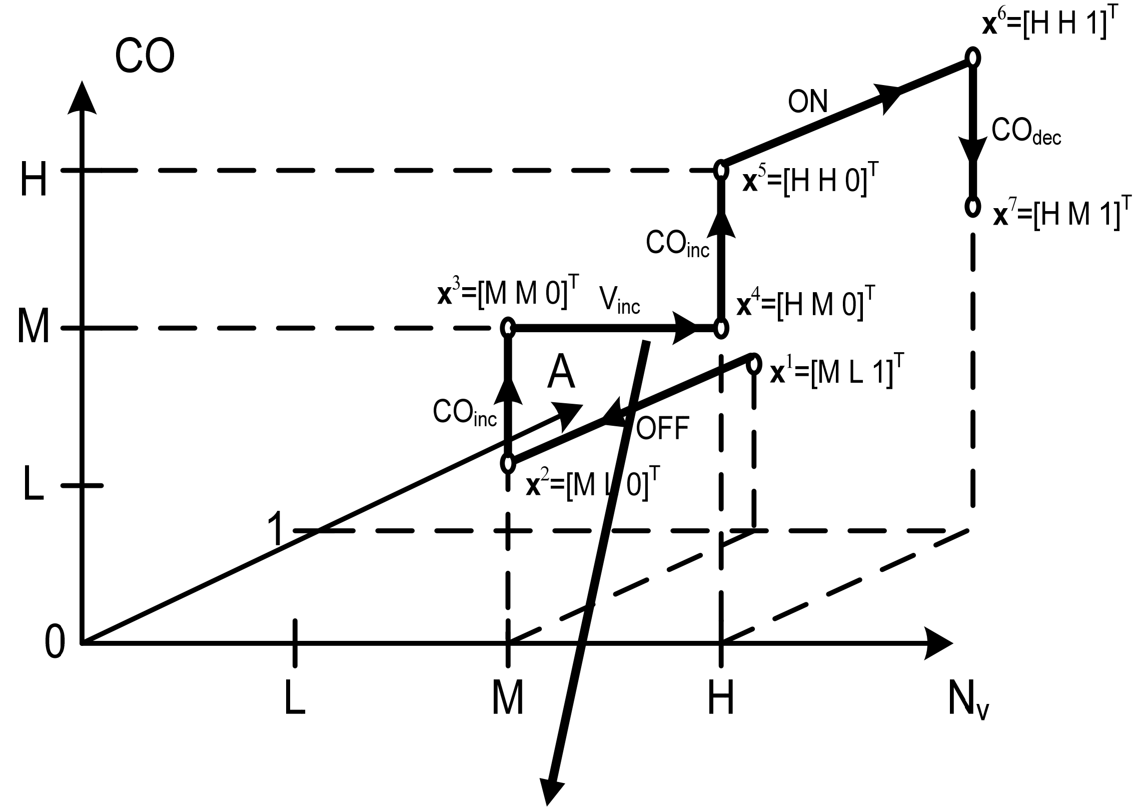
=> stanje ventilatora, *A* (0, 1)

Vektor stanja $\longrightarrow$ $\mathbf{x} = [N\text{v} \;\; CO \;\; A]^T$

Skup događaja

$$E = \{\mathrm{V}_{\mathrm{inc}}, \mathrm{V}_{\mathrm{dec}}, \mathrm{CO}_{\mathrm{inc}}, \mathrm{CO}_{\mathrm{dec}}, \mathrm{ON}, \mathrm{OFF}\}$$

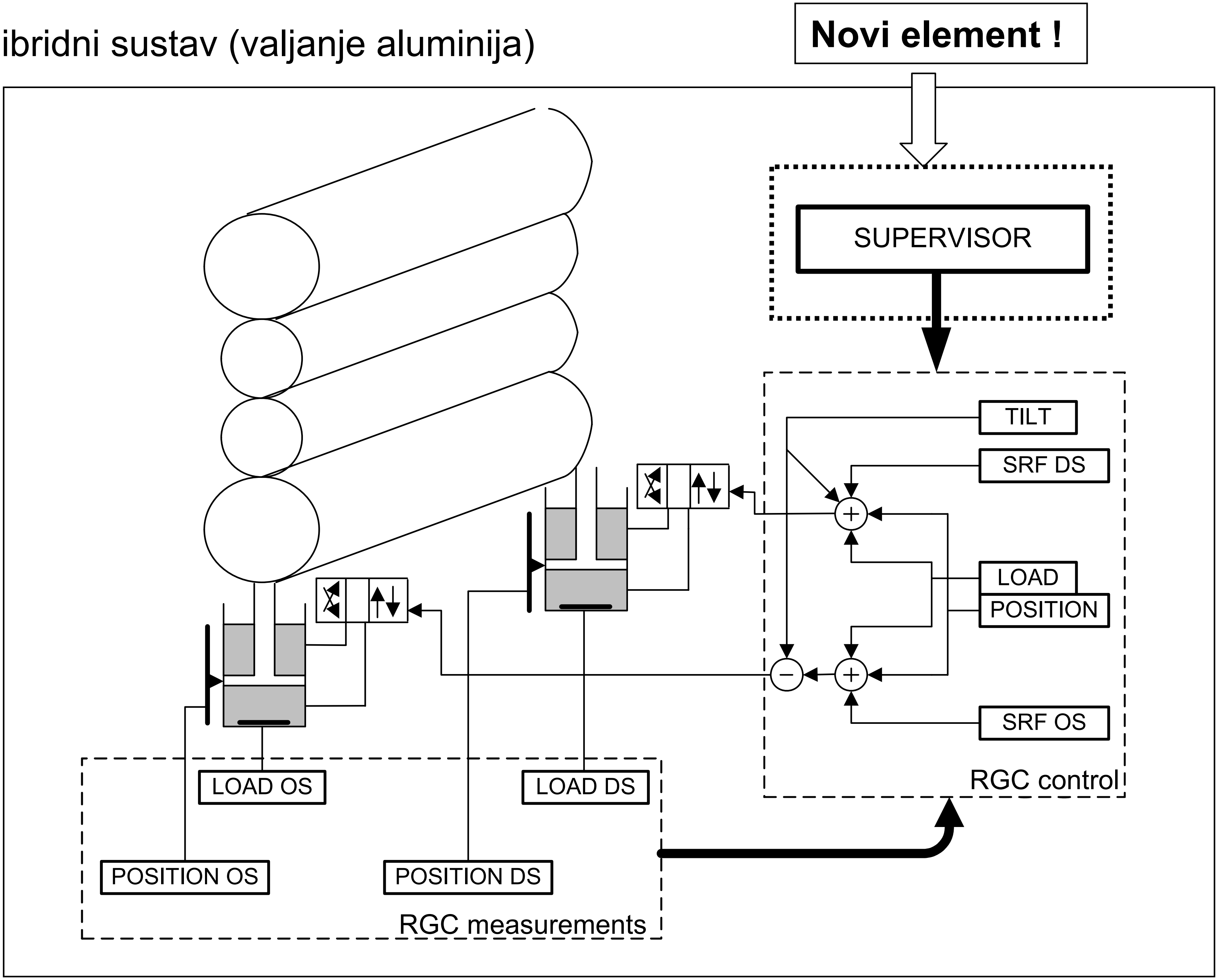


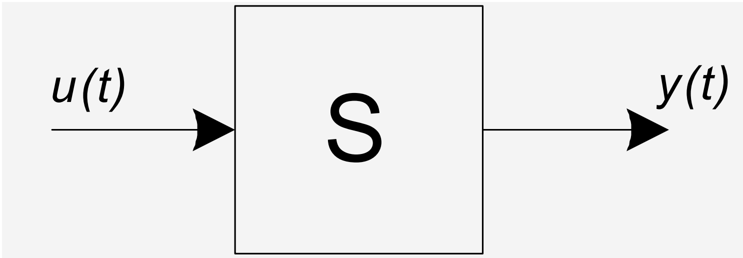


stanje sustava mijenja se s ostvarenjem pojedinih događaja

prostor stanja je diskretan i prebrojiv

hibridni sustav (valjanje aluminija)





Prostor stanja je kontinuiran i neprebrojiv

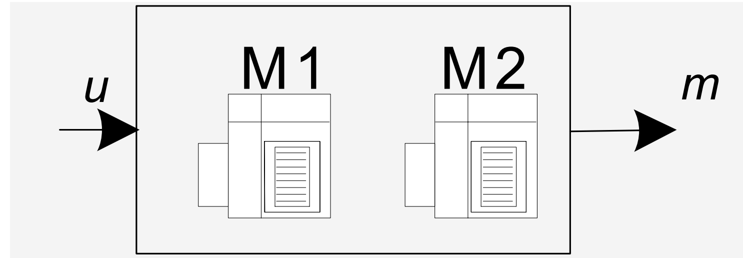
Diferencijalne jednadžbe

Kemijski procesi

Elektromagnetski procesi

Mehanički procesi

Toplinski procesi



Prostor stanja je diskretan i prebrojiv

Petrijeve mreže

Max/plus algebra

Matrična algebra

Grafcet

Automati ...

Komunikacijski protokoli

Fleksibilni proizvodni sustavi

Računalni programi

...

Hibridni sustavi

Sustav => skup resursa (ljudi i strojevi) koji pretvorbom materijala, energije i informacija stvaraju “proizvode” (materijalna dobra, usluge, informacije).

Djelovanje na veličinu, oblik ili strukturu gotovog “proizvoda”

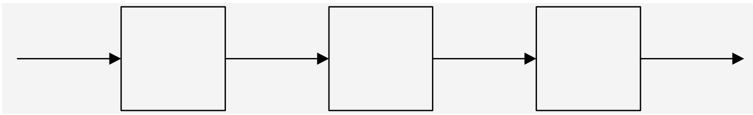
=> djelovanje na njegovu konačnu namjenu.

Koji uvjeti moraju biti ispunjeni da bi neki događaj počeo, odnosno što se događa kada on završi

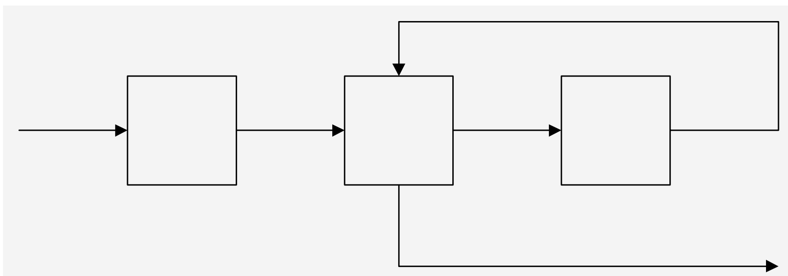
=> stanja resursa + stanje “predmeta” obrade.

Ostvarenje cilja procesa (tj. gotov “proizvod”) uz zadovoljenje postavljenih kriterija.

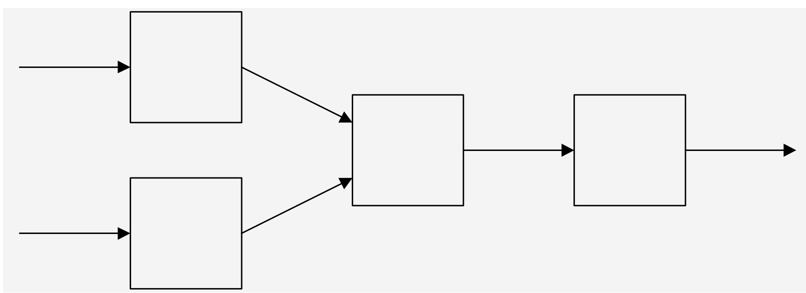
Osnovne strukture sustava



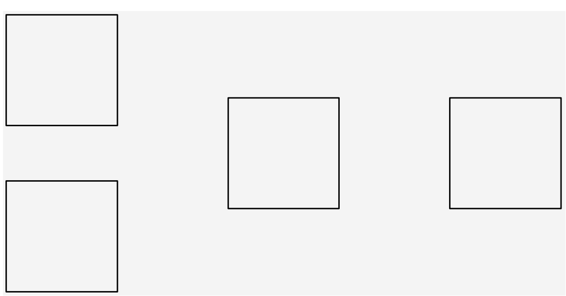
Slijed zadataka



“Višeulazni” slijed zadataka



Slijed zadataka s operacijom “sastavljanja”



Slobodan odabir zadataka

Elementi sustava

Resursi => $R = \{r_i\}$ (strojevi, ljudi, spremnici, ...)

višeradni resursi => R_{vr}

jednoradni resursi => R_{jr}

$$R = R_{vr} \cup R_{jr}$$

Zadaci => $J^n = \{j^n_1, j^n_2, \dots j^n_l\}$

=> $J = \{J^n\}$

$J(r_i)$, $C \subset R$, $J(C) = \cup_{r \in C} J(r)$

$$|J(r_{vr})| > 1$$

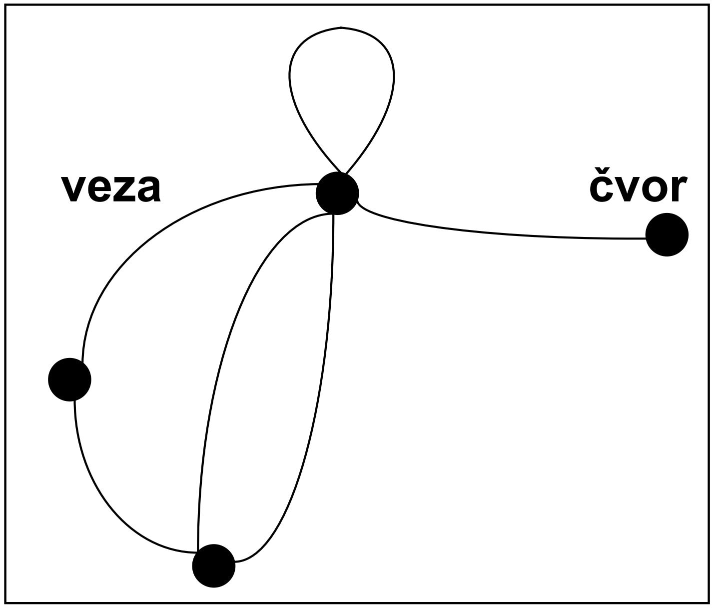
$$|J(r_{jr})| = 1$$

Veze među elementima ?

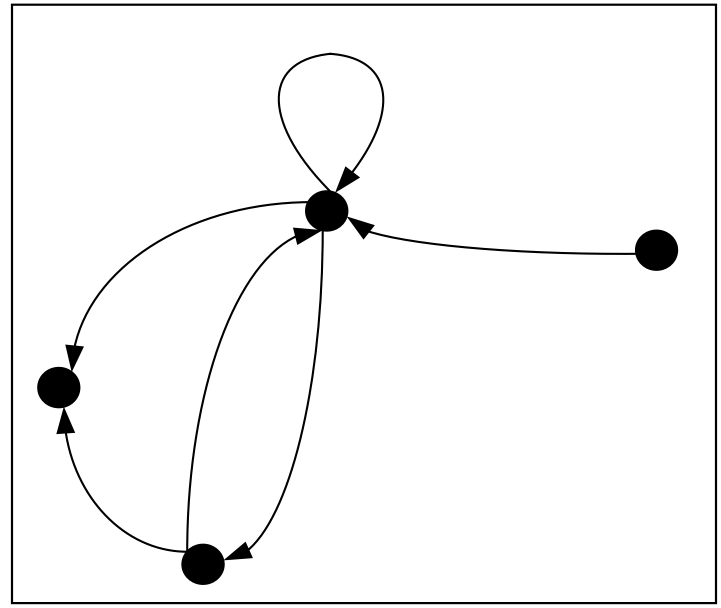
=> matematički alati

Grafovi

Graf (*graph)* – struktura sastavljena od **čvorova** (*nodes)* i **veza** (*arcs)*



Graf



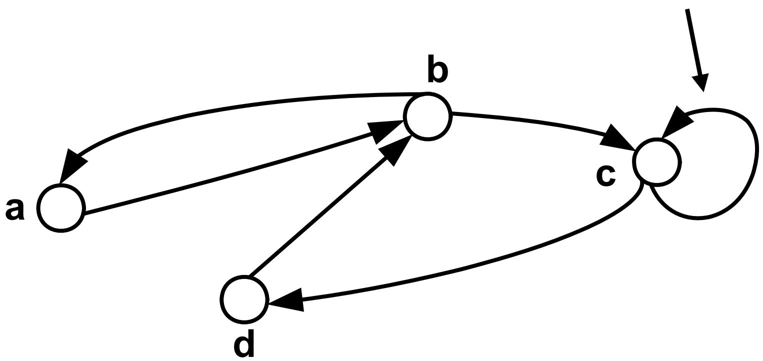
Usmjereni graf

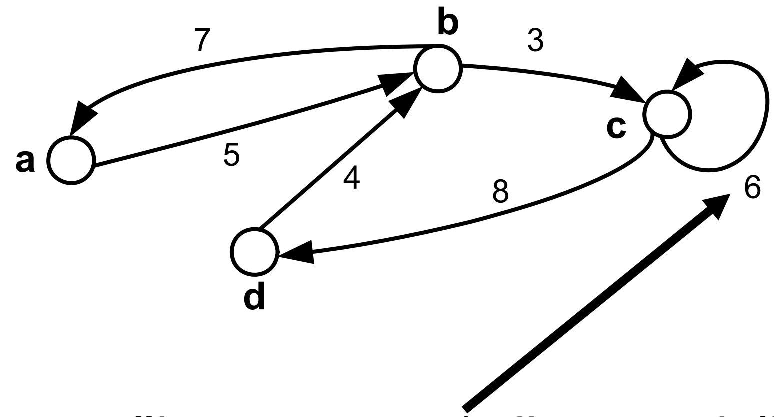
čvorovi (*nodes*, *vertices, vrhovi*) predstavljaju određeni oblik aktivnosti – skup *V*

veze (*arcs*, *edges, bridovi*) predstavljaju međusobnu povezanost čvorova – skup *E*

Grafovi

petlja (*loop*)





V = {a, b, c, d}
E = {(a,b), (b,a), (b,c), (c,c), (c,d), (d,b)}

težina veze - *w* (vrijeme, udaljenost, cijena, ...) (*weight*)

staza (*path*) - $\sigma = (n1, n2, n3, \ldots, nj)$,

$\boxed{\text{težina staze}}$ $$\sigma_w = \sum_{i=1}^{j} w_i$$

$\boxed{\text{duljina staze = broj veza koje čine stazu}}$

$\sigma_1=(a, b, c)$, $\sigma_2=(a, b, c, d)$, $\sigma_3=(b, c, d, b)$ → kružna staza (*cycle*)

$\sigma_{1\ell} = 2$, $\sigma_{2\ell} = 3$, $\sigma_{3\ell} = 3$, $\sigma_{1w} = 8$, $\sigma_{2w} = 16$, $\sigma_{3w} = 17$

Grafovi

srednja težina kružne staze => ciklus

srednja težina staze

$$\overline{\sigma}_w = \frac{\sigma_w}{\sigma_\ell}$$

maksimalni ciklus grafa

$$\lambda = \max_c(\overline{\sigma}_w)$$

$$\Downarrow$$

kritična kružna staza

Matrični opis grafa

matrica susjedstva (veza) (*adjacency matrix)* – **G**=[g_{ij}] – binarni elementi, 0 i 1

Red(**G**) = broj čvorova

$\boxed{g_{ij}=1 \text{ ako postoji veza od čvora } j \text{ prema čvoru } i}$ $g_{ii}=1$ => petlja

potencije matrice veza $G^r = G^{r-1} \cdot G$,

$$g_{ij}^r = \sum_k g_{ik}^{r-1} \cdot g_{kj} \ , \quad i, j, k = 1,2,...,n \quad ,$$

$g_{ij}^r = m > 0$ => postoji *m* različitih staza od *j* prema *i* duljine *r*

Matrični opis grafa

matrica incidencije (*incidence matrix)* – $\mathbf{W}=[w_{ij}]$

broj redaka = broj čvorova

broj stupaca = broj veza

za stupac *l* koji predstavlja vezu (n_i,n_j), $i \neq j$, vrijedi $w_{il}=1$ i $w_{jl}=-1$ dok su ostali elementi stupca 0

za stupac *l* koji predstavlja petlju (n_i,n_i), svi su elementi 0

Matrični opis grafa

težinska matrica susjedstva (*weighted adjacency matrix*) – $\mathbf{A}=[a_{ij}]$

a_{ij}= težina veze od čvora *j* prema čvoru *i*

$a_{ij}=\varepsilon$ ako ne postoji veze od čvora *j* prema čvoru *i*

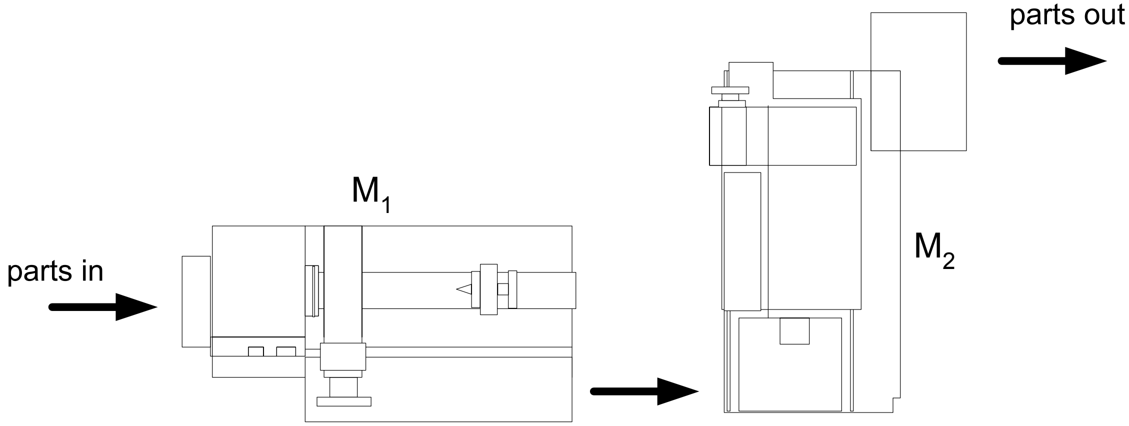
$a_{ij}=e$ ako je težina veze 0

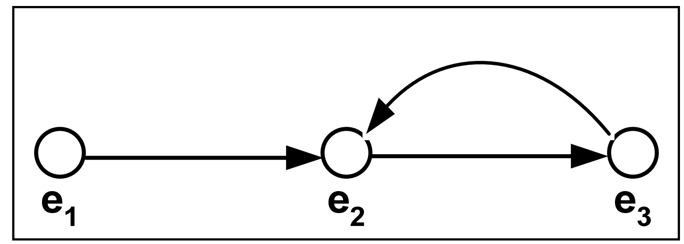
novi elementi =>

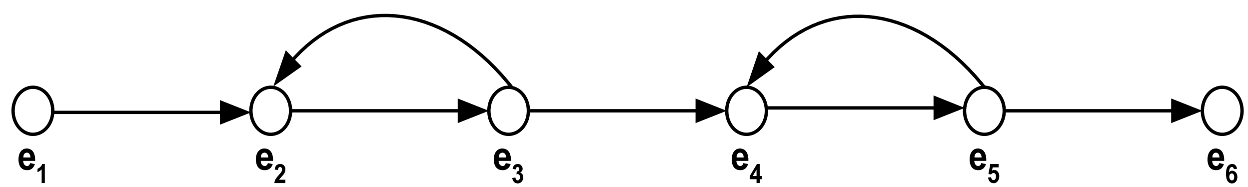
ε => $-\infty$

e => 0

Primjer: modeliranje proizvodnog sustava grafom







e_1 - predmet prisutan na ulazu,

e_2 – početak obrade u M_1,

e_3 – kraj obrade u M_1,

e_4 – početak obrade u M_2,

e_5 – kraj obrade u M_2,

e_6- predmet napustio sustav.

$$\mathbf{G} = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

matrica susjedstva

Primjer: modeliranje proizvodnog sustava grafom

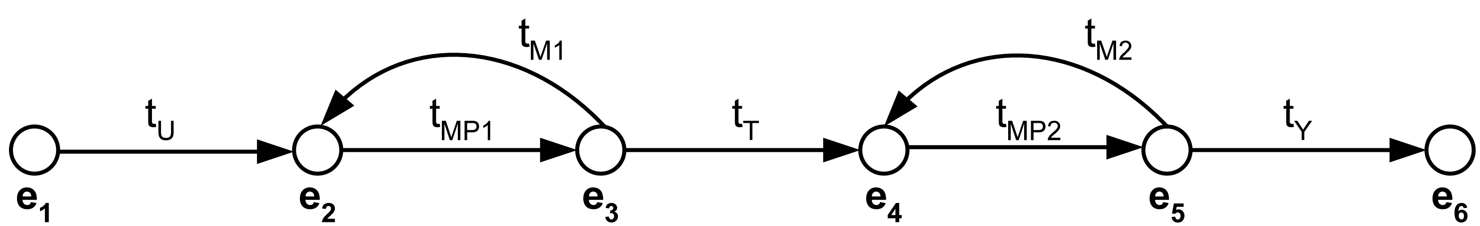
uključenje dinamike u model

vremena obrade predmeta i pripreme strojeva => odgovaraju vezama

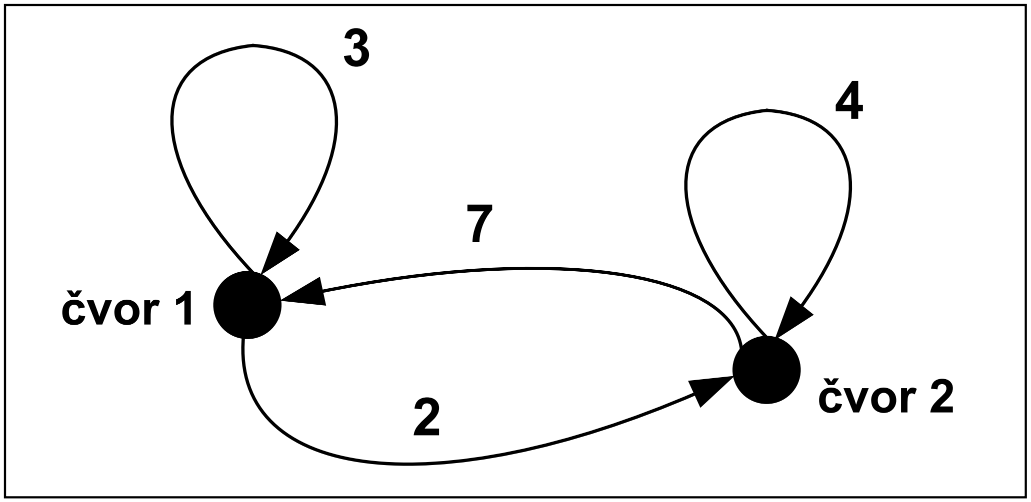
- (e_1, e_2) - predmet ulazi u M_1 (t_u),
- (e_2, e_3) – predmet se obrađuje u M_1 (t_{MP1}),
- (e_3, e_2) – priprema stroja M_1 (t_{M1}),
- (e_3, e_4) – predmet putuje od M_1 prema M_2 (t_T),
- (e_4, e_5) – predmet se obrađuje u M_2 (t_{MP2}),
- (e_5, e_4) – priprema stroja M_2 (t_{M2}),
- (e_5, e_6) - predmet napušta sustav (t_y).

$$\mathbf{A} = \begin{array}{cc} & \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{array}{c} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{array} & \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ t_U & \varepsilon & t_{M1} & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & t_{MP1} & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & t_T & \varepsilon & t_{M2} & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & t_{MP2} & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_Y & \varepsilon \end{bmatrix} \end{array}$$

težinska matrica susjedstva



Stanje grafa



x_1(k) - stanje čvor 1

x_2(k) - stanje čvor 2

značenje

x_1(5) = 1; čvor 1 u koraku 5 je aktivan (0 => neaktivan) ← matrice

x_1(5) = 23; čvor 1 je postao 5. put aktivan u 23. koraku ← Max-plus

Pitanje: kako opisati dinamiku grafa ?

Jednadžba stanja

$$\boxed{\mathbf{x(k+1) = A\ x(k)\ ,\ \ x(0) = x_0}}$$

A – kvadratna matrica (struktura i svojstva grafa) => težinska matrica susjedstva

$\mathbf{a_{ij}}$ – elementi matrice **A** ; zbroj vremena trajanja aktivnosti (*activity time)* i vremena "putovanja" (*traveling time)* od čvora **j** do čvora **i**

OPERACIJE U JEDNADŽBI STANJA OBAVLJAJU SE max-plus ALGEBROM

+ => max

*** => +**

$$\mathbf{x}(\mathbf{k}+\mathbf{1}) = \mathbf{A} \otimes \mathbf{x}(\mathbf{k}), \mathbf{x}(\mathbf{0}) = \mathbf{x}_0$$

Max-plus algebra

$\boldsymbol{\varepsilon} \cong -\infty$ neutralan element s obzirom na **max**

$$\max(x,\varepsilon) = x \oplus \varepsilon = x$$

$a_{ij} = \varepsilon$ => ne postoji veza od čvora j prema čvoru i

$e \cong 0$ neutralan element s obzirom na **+**

$$x \otimes e = x$$

u(k) – ulazni vektor - čvorovi koji nemaju ulazne veze

y(k) – izlazni vektor - čvorovi koji nemaju izlazne veze

$$\begin{aligned} x(k+1) &= A \otimes x(k) \oplus B \otimes u(k) \\ y(k) &= C \otimes x(k), \qquad x(0) = x_0 \end{aligned}$$

$\oplus$ **max** po komponentama

Max-plus algebra

matrice

$$\mathbf{C} = \mathbf{A} \oplus \mathbf{B} \qquad c_{ij} = a_{ij} \oplus b_{ij} = \max(a_{ij}, b_{ij}).$$

$$\mathbf{C} = \mathbf{A} \otimes \mathbf{B} \qquad c_{ij} = \bigoplus_{k} a_{ik} \otimes b_{kj} = \max_{k}(a_{ik} + b_{kj}) \ .$$

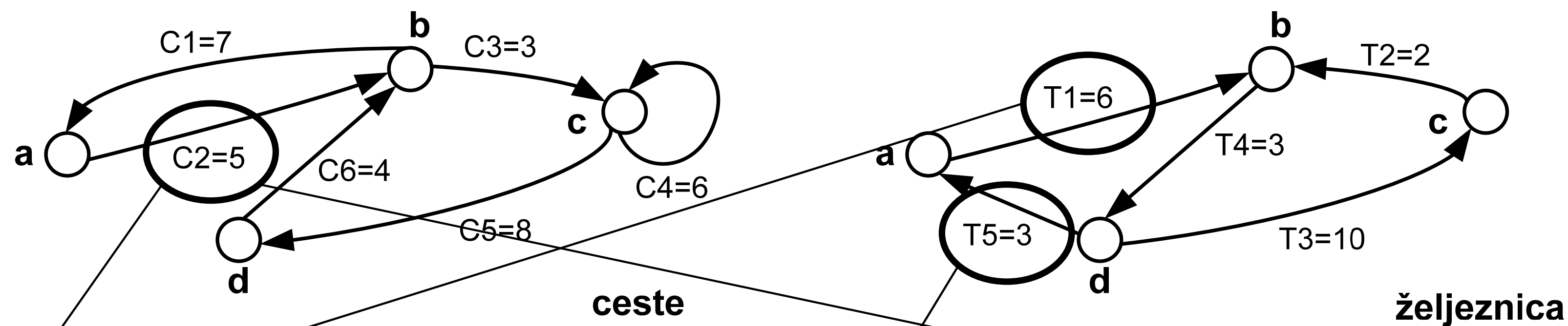
$$\mathbf{A} \otimes \mathbf{B} \neq \mathbf{B} \otimes \mathbf{A}$$

jedinična matrica E => dijagonalni elementi = *e*, ostali = ε

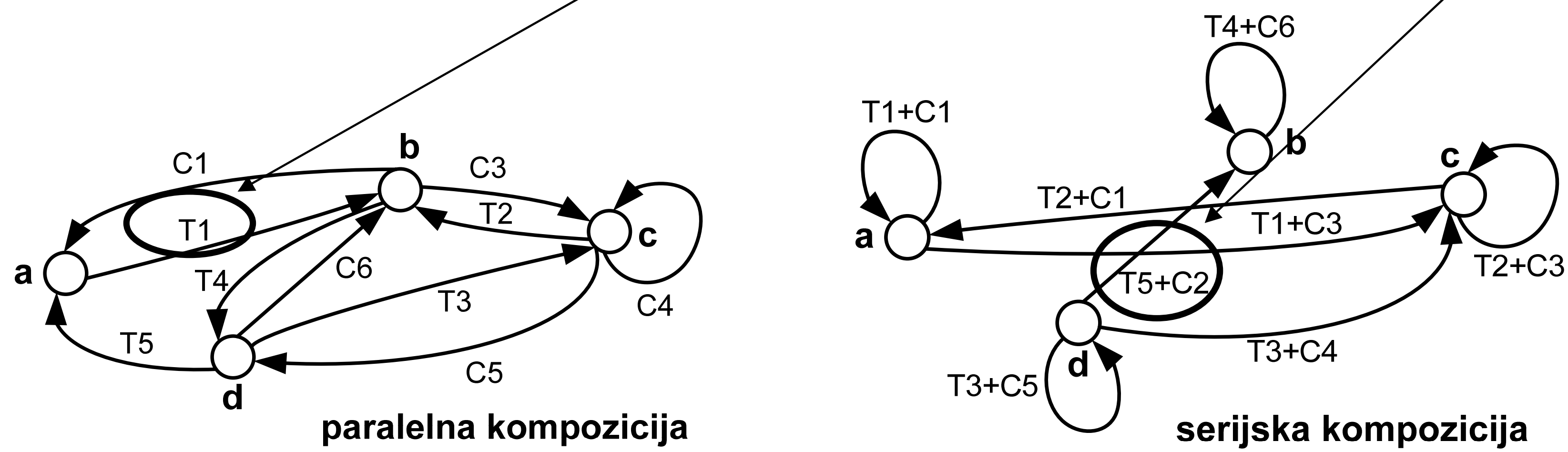
max operacija nad matricama => paralelna kompozicija

plus operacija nad matricama => serijska kompozicija

Max-plus algebra

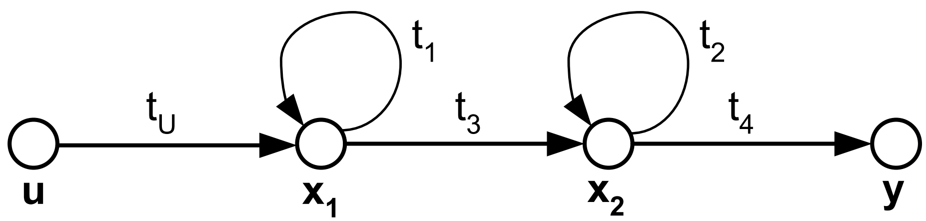


$$C2 \oplus T1 = \max(5,6) = 6 = T1 \qquad T5 \otimes C2 = 3 + 5 = 8 = T5 + C2$$



Max-plus model sustava

Napomena: vrijedi samo za sustave u kojima je redoslijed zadataka unaprijed definiran!



$$x_1(k) = \max\left(x_1(k-1) + \mathrm{t}_1, u(k) + \mathrm{t}_U\right)$$

$$x_2(k) = \max\left(x_1(k) + \mathrm{t}_3, x_2(k-1) + \mathrm{t}_2\right)$$
$$= \max\left(x_1(k-1) + \mathrm{t}_1 + \mathrm{t}_3, u(k) + \mathrm{t}_U + \mathrm{t}_3, x_2(k-1) + \mathrm{t}_2\right)$$

$$y(k) = x_2(k) + \mathrm{t}_4$$

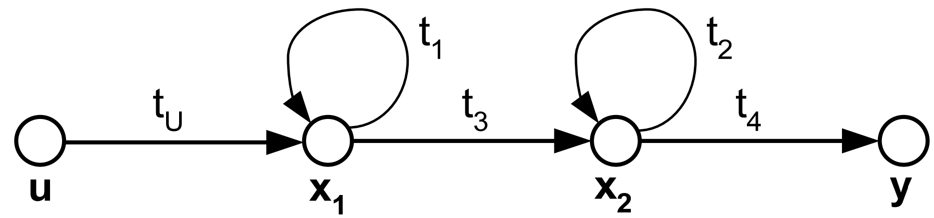
max-plus zapis

$$x_1(k) = x_1(k-1) \otimes \mathrm{t}_1 \oplus u(k) \otimes \mathrm{t}_U$$

$$x_2(k) = x_1(k-1) \otimes (\mathrm{t}_1 + \mathrm{t}_3) \oplus x_2(k-1) \otimes \mathrm{t}_2 \oplus u(k) \otimes (\mathrm{t}_U + \mathrm{t}_3)$$

$$y(k) = x_2(k) \otimes \mathrm{t}_4$$

Max-plus model sustava



$$x(k) = \begin{bmatrix} t_1 & \varepsilon \\ t_1 + t_3 & t_2 \end{bmatrix} \otimes x(k-1) \oplus \begin{bmatrix} t_U \\ t_U + t_3 \end{bmatrix} \otimes u(k) \ ,$$

$$y(k) = \begin{bmatrix} \varepsilon & t_4 \end{bmatrix} \otimes x(k) \ .$$

$$\Downarrow$$

$$x(k) = A \otimes x(k-1) \oplus B \otimes u(k), \ x(0) = x_0$$

$$y(k) = C \otimes x(k).$$

Napomena: matrica **A** nije nužno težinska matrica susjedstva grafa koji opisuje sustav!

Max-plus model sustava

pretpostavka – novi “predmet” ulazi u sustav tek kada prethodni napusti sustav

$$u(k) = y(k-1)$$

$$u(k) = G \otimes y(k-1)$$

$$\overline{A} = A \oplus B \otimes G \otimes C$$

$$x(k) = A \otimes x(k-1) \oplus B \otimes y(k-1)\ ,$$
$$y(k-1) = C \otimes x(k-1).$$

alternativa – “regulator po varijablama stanja”

$$u(k) = K \otimes x(k-1)$$

$$x(k) = \overline{A} \otimes x(k-1)\ ,\ \ x(0) = x_0\ \ ,$$
$$y(k) = C \otimes x(k)\ ,$$
$$\overline{A} = A \oplus B \otimes C$$

Rezultat: zatvoreni sustav s jediničnom povratnom vezom !

Periodičko ponašanje sustava

Period (*cycle*) – vrijeme između dvije uzastopne pojave događaja.

$$x_i(k+1) - x_i(k) = \lambda \ ,$$
$$x_i(k+2) - x_i(k+1) = \lambda \ ,$$
$$\ldots$$
$$x_i(k+r) - x_i(k+(r-1)) = \lambda \ ,$$
$$\ldots$$

$$x_i(k+1) = \lambda + x_i(k),$$
$$x_i(k+2) = 2\lambda + x_i(k),$$
$$x_i(k+3) = 3\lambda + x_i(k),$$
$$\ldots$$

$$x_i(k+r) = r\lambda + x_i(k), \ \ i = 1,2,\ldots,n \ , \ \ k \geq k_0$$

Problem – potrebno je poznavati $x_i(1)$, $x_i(2)$, ..., k_0 takodjer nepoznat

$$\lambda = \frac{x_i(k+r) - x_i(k)}{r} \ , \ \ i = 1,2,\ldots,n \ , \ \ k \geq k_0$$

Period

periodičko ponašanje započinje nakon k_0

propusnost = 1/λ

Periodičko ponašanje sustava

određivanje ciklusa u max-plus algebri

podsjetnik

maksimalni ciklus grafa

srednja težina kružne staze => ciklus

$$\lambda = \max_{c}(\overline{\sigma}_w)$$

srednje težine kružnih staza duljine *r* => dijagonalni elementi matrice $\mathbf{A}^r$

Koja kružna staza ima maksimalnu srednju težinu i koliko ona iznosi?

Vrijedi za čvrsto povezane grafove

$$\lambda = \bigoplus_{i=1}^{n}\left(\frac{trace(A^i)}{i}\right)$$

standardno dijeljenje

$$g_{ij} + g_{ij}^2 + g_{ij}^3 + \ldots + g_{ij}^{n-1} \neq 0,\ \forall i, j$$

elementi potencija matrice veza **G**

$$trace(A) = \bigoplus_{j=1}^{n} a_{jj}$$

Periodičko ponašanje FPS-a

Iskoristivost resursa $\eta_i = \dfrac{T_{Mi}}{\lambda}$. T_{Mi} – vrijeme obrade + vrijeme pripreme

Spremnici (buffers) – pohranjivanje rezultata između dva zadatka

Spremnik s N mjesta smješten između strojeva M_{i} i $M_{\mathrm{i+1}}$,
obrada u M_{i} predhodi obradi u $M_{\mathrm{i+1}}$

$$x_i(k) = \max\left(x_i(k-1) + t_i, x_{i+1}(k-(N+1))\right)$$

$$\Downarrow$$

$$x(k) = A_0 \otimes x(k) \oplus A_1 \otimes x(k-1) \oplus \ldots$$
$$\ldots \oplus A_p \otimes x(k-p) \oplus B \otimes u(k),\ x(0) = x_0 ,$$

$$y(k) = C \otimes x(k).$$

x(k) se ne može odrediti eksplicitno,

Cilj – eliminirati *x*(k) s desne strane izraza

Spremnici

$$x(k) = A_0 \otimes x(k) \oplus A_1 \otimes x(k-1) \oplus \dots$$
$$\dots \oplus A_p \otimes x(k-p) \oplus B \otimes u(k),\ x(0) = x_0$$

$$x(k) = A_0 \otimes \left[A_0 \otimes x(k) \oplus A_1 \otimes x(k-1) \oplus \dots \oplus A_p \otimes x(k-p) \oplus B \otimes u(k)\right] \oplus$$
$$\oplus A_1 \otimes x(k-1) \oplus \dots \oplus A_p \otimes x(k-p) \oplus B \otimes u(k) =$$
$$A_0^2 \otimes x(k) \oplus \left[A_0 \oplus E\right] \otimes \left[A_1 \otimes x(k-1) \oplus \dots \oplus A_p \otimes x(k-p) \oplus B \otimes u(k)\right]$$

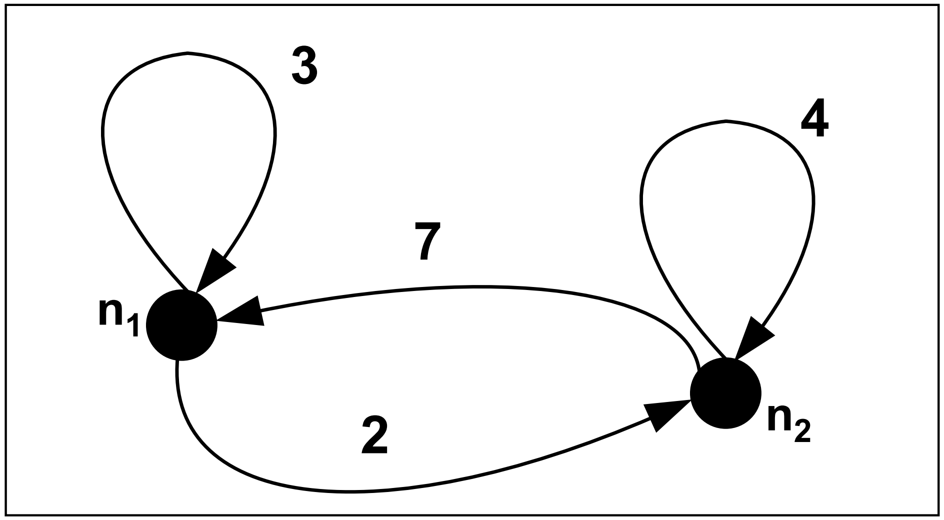
ponoviti *n* puta

$$x(k) = A_0^{n+1} \otimes x(k) \oplus \left[A_0^n \oplus A_0^{n-1} \oplus \dots \oplus A_0 \oplus E\right] \otimes$$
$$\left[A_1 \otimes x(k-1) \oplus \dots \oplus A_m \otimes x(k-m) \oplus B \otimes u(k)\right]$$

$$\mathbf{A_0}^{n+1} = [\varepsilon],$$

$$x(k) = A_0^* \otimes \left[A_1 \otimes x(k-1) \oplus \dots \oplus A_m \otimes x(k-m) \oplus B \otimes u(k)\right]$$

Primjer: max-plus algebra



trajanje aktivnosti:	$n_1 = 1$	$n_2 = 4$
trajanje pripreme:	$n_1 = 2$	$n_2 = 0$
trajanje putovanja:	$n_1 => n_2 = 1$	$n_2 => n_1 = 3$

"trajanje putovanja" čvora samog prema sebi

$a_{11} = 1+2 = \mathbf{3}$, $a_{12} = 4+3 = \mathbf{7}$

$a_{21} = 1+1 = 2$, $a_{22} = 4+0 = 4$

Primjer: max-plus algebra

$a_{11} = 1+2 = 3$, $a_{12} = 4+3 = 7$

$a_{21} = 1+1 = 2$, $a_{22} = 4+0 = 4$ $\longrightarrow$ $A = \begin{pmatrix} 3 & 7 \\ 2 & 4 \end{pmatrix}$

$x(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$\mathbf{x(1)} = \begin{bmatrix} \mathbf{3} & \mathbf{7} \\ \mathbf{2} & \mathbf{4} \end{bmatrix} \otimes \begin{bmatrix} \mathbf{1} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{max(3+1,7+0)} \\ \mathbf{max(2+1,4+0)} \end{bmatrix} = \begin{bmatrix} \mathbf{7} \\ \mathbf{4} \end{bmatrix}$$

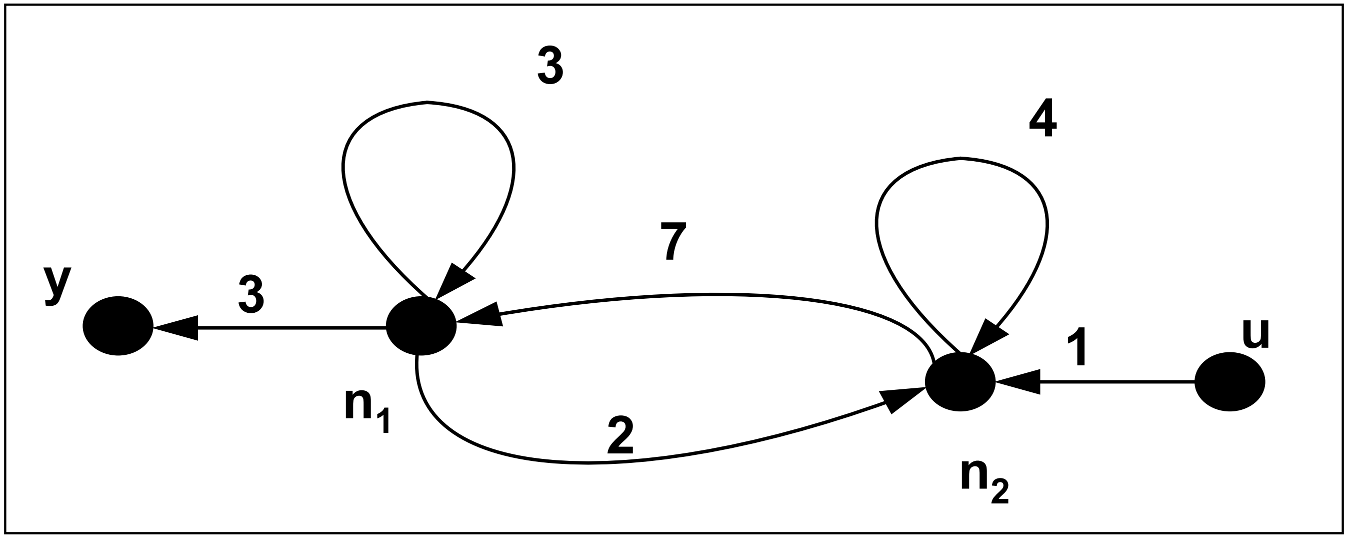
$$\mathbf{x(2)} = \begin{bmatrix} \mathbf{3} & \mathbf{7} \\ \mathbf{2} & \mathbf{4} \end{bmatrix} \otimes \begin{bmatrix} \mathbf{7} \\ \mathbf{4} \end{bmatrix} = \begin{bmatrix} \mathbf{max(3+7,7+4)} \\ \mathbf{max(2+7,4+4)} \end{bmatrix} = \begin{bmatrix} \mathbf{11} \\ \mathbf{9} \end{bmatrix}$$

$$\mathbf{x(2)} = \mathbf{A} \otimes \mathbf{x(1)} = \mathbf{A} \otimes [\mathbf{A} \otimes \mathbf{x(0)}] = \mathbf{A}^2 \otimes \mathbf{x(0)}$$

$$\mathbf{A}^2 = \begin{bmatrix} \mathbf{3} & \mathbf{7} \\ \mathbf{2} & \mathbf{4} \end{bmatrix} \otimes \begin{bmatrix} \mathbf{3} & \mathbf{7} \\ \mathbf{2} & \mathbf{4} \end{bmatrix} = \begin{bmatrix} \mathbf{max(3+3,7+2)} & \mathbf{max(3+7,7+4)} \\ \mathbf{max(2+3,4+2)} & \mathbf{max(2+7,4+4)} \end{bmatrix} = \begin{bmatrix} \mathbf{9} & \mathbf{11} \\ \mathbf{6} & \mathbf{9} \end{bmatrix}$$

$$\mathbf{A}^k = \underbrace{\mathbf{A} \otimes \mathbf{A} \otimes \mathbf{A} \otimes \ldots \otimes \mathbf{A}}_{\textbf{k puta}}$$

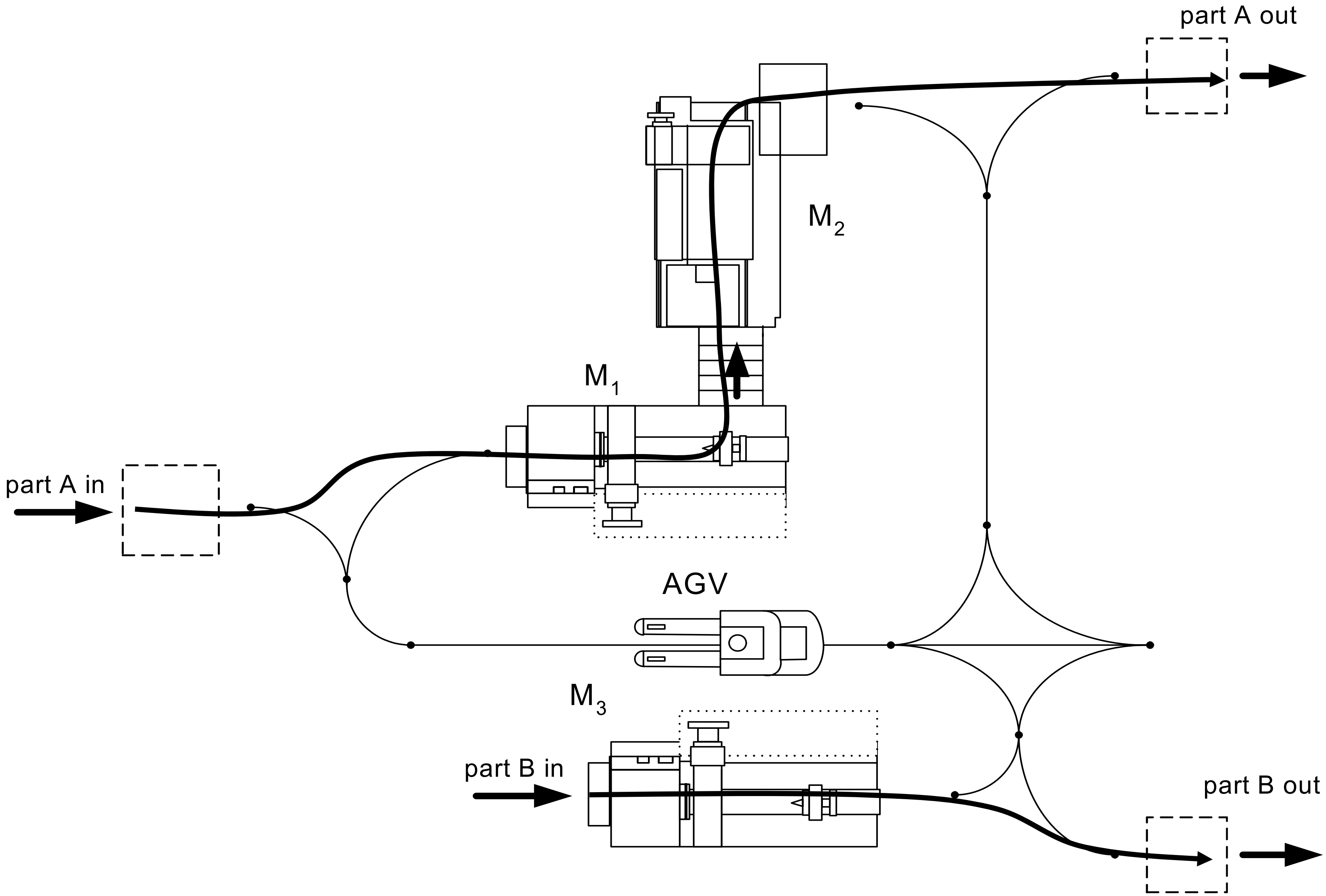
Primjer: max-plus algebra



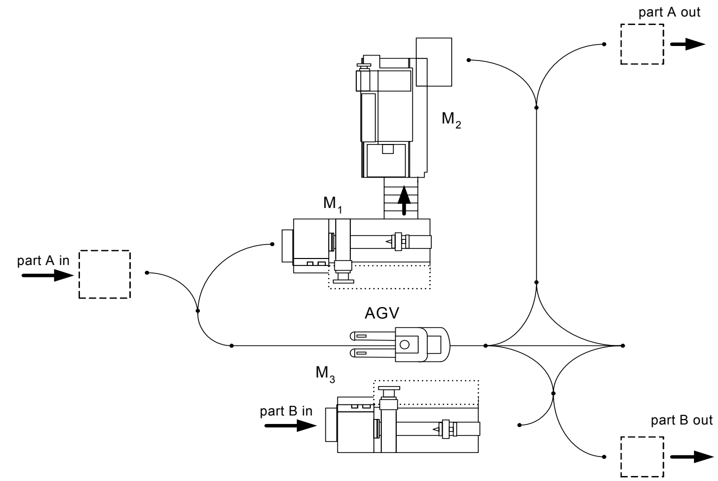
$$\mathbf{x}(k+1) = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix} \otimes \mathbf{x}(k) \oplus \begin{bmatrix} \varepsilon \\ 1 \end{bmatrix} \otimes \mathbf{u}(k)$$

$$\mathbf{y}(k) = \begin{bmatrix} 3 & \varepsilon \end{bmatrix} \otimes \mathbf{x}(k)$$

Primjer: analiza FPS-a max-plus algebrom



Primjer: analiza FPS-a max-plus algebrom



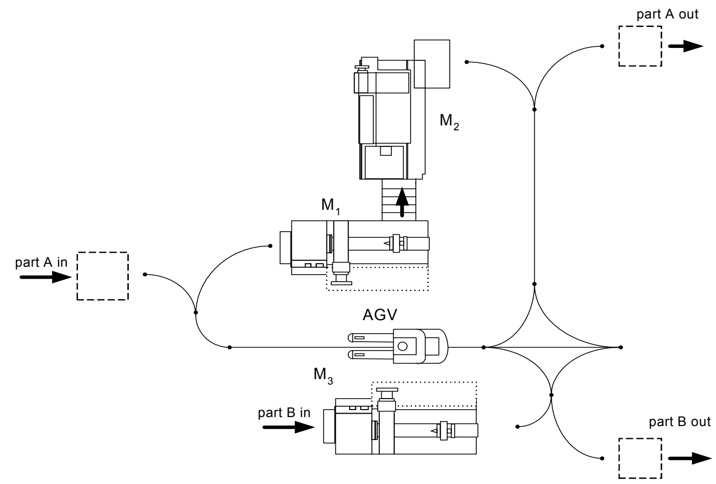
Trajanja obrada predmeta i priprema strojeva

stroj	M_1	M_2	M_3
trajanje obrade	8	13	15
priprema	2	3	3

Trajanje transporta i pripreme autonomnog vozila.

operacija	transport A u M_1	transport A iz M_2	transport B iz M_3
trajanje	5	6	3
priprema	4	6	5

Primjer: analiza FPS-a max-plus algebrom

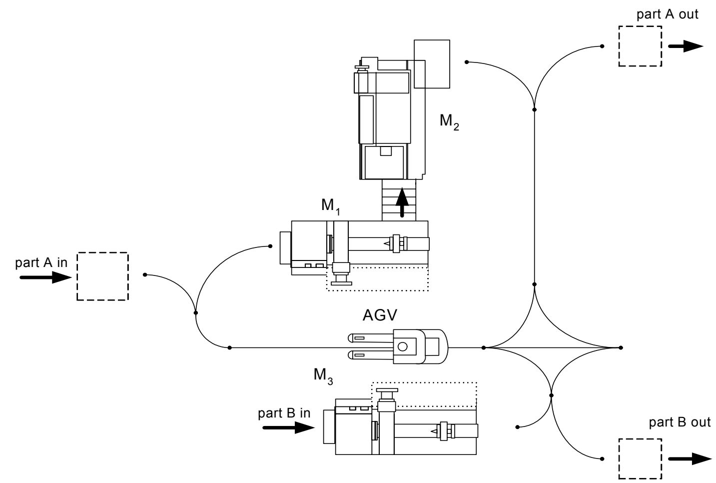


Događaji od interesa

event	*Description*
x_1	**Početak transporta predmeta A u stroj M_1**
x_2	**Transport predmeta A u M_1 je završen; otpuštanje AGV-a i početak obrade u stroju M_1**
x_3	**Obrada u M_1 je završila; otpuštanje M_1 i početak obrade u M_2**
x_4	**AGV preuzima predmet iz M_2; otpuštanje M_2**
x_5	**Transport predmeta A prema izlazu je završio; otpuštanje AGV-a**
x_6	**Početak obrade predmeta B u M_3**
x_7	**AGV preuzima predmet iz M_3; otpuštanje M_3**
x_8	**Transport predmeta B prema izlazu je završio; otpuštanje AGV-a**

AGV je višeradni resurs!

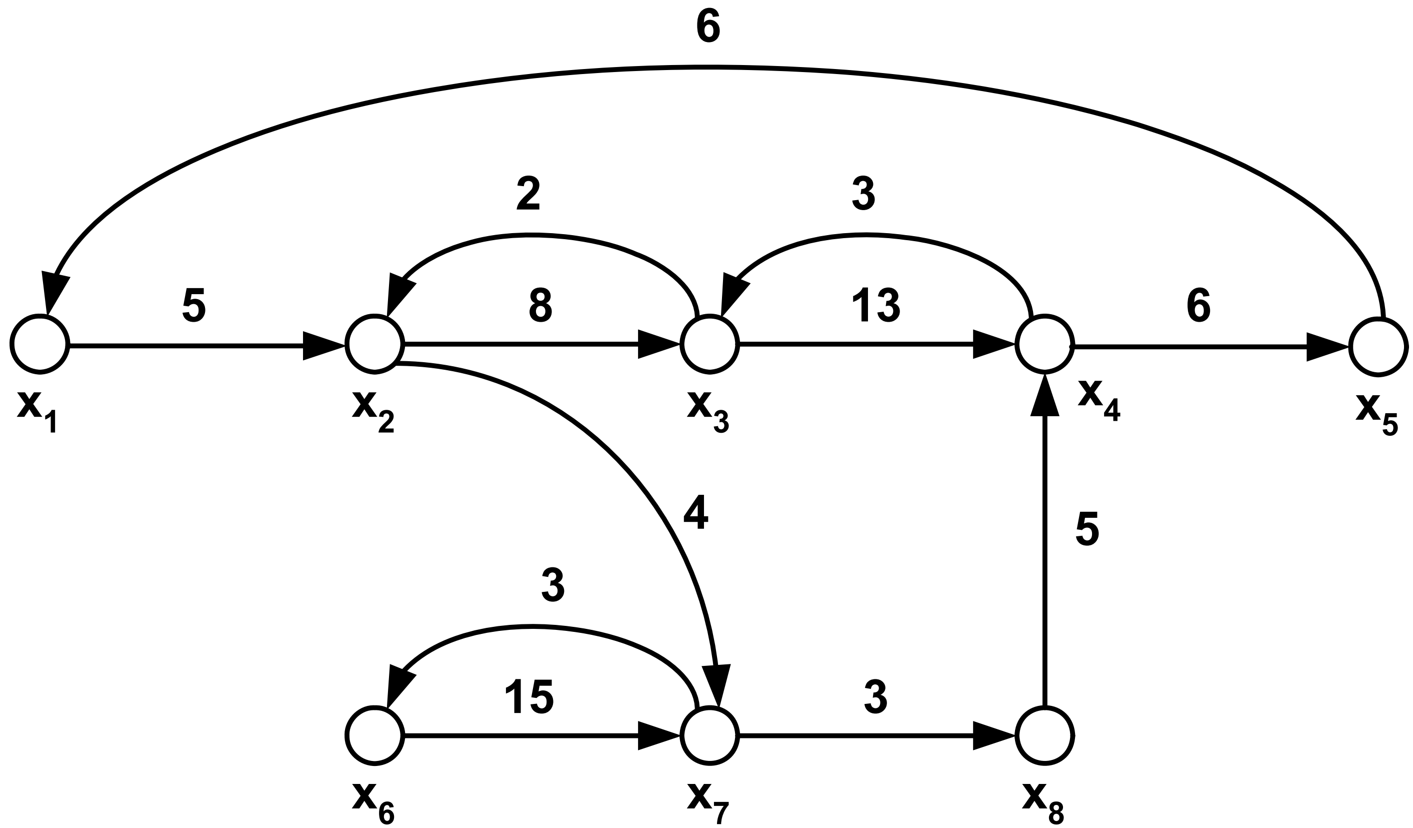
Primjer: analiza FPS-a max-plus algebrom



dvije moguće sekvence resursa AGV:

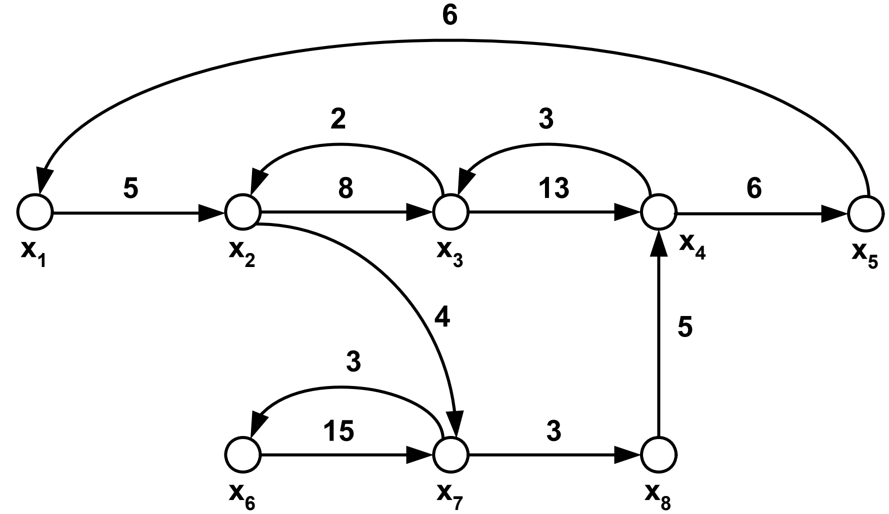
a) punjenje M_1 – pražnjenje M_2 – pražnjenje M_3,
b) punjenje M_1 – pražnjenje M_3 – pražnjenje M_2.

sekvenca b)



Graf je čvrsto povezan!

Primjer: analiza FPS-a max-plus algebrom



težinska matrica veza

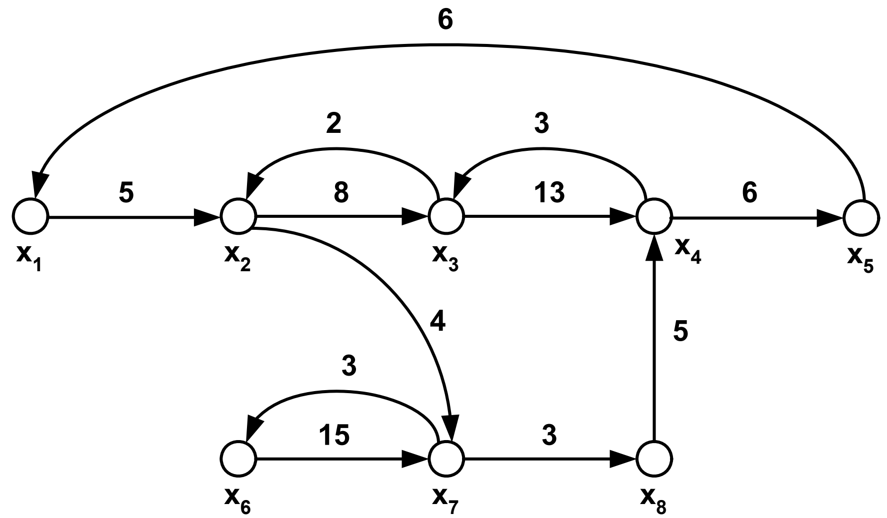
$$A = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & 6 & \varepsilon & \varepsilon & \varepsilon \\ 5 & \varepsilon & 2 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 8 & \varepsilon & 3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & 13 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 5 \\ \varepsilon & \varepsilon & \varepsilon & 6 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 3 & \varepsilon \\ \varepsilon & 4 & \varepsilon & \varepsilon & \varepsilon & 15 & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 3 & \varepsilon \end{bmatrix}$$

jednadžbe sustava

$$x_1(k) = 6x_5(k-1)$$
$$x_2(k) = 5x_1(k) \oplus 2x_3(k-1)$$
$$x_3(k) = 8x_2(k) \oplus 3x_4(k-1)$$
$$x_4(k) = 13x_3(k) \oplus 5x_8(k)$$
$$x_5(k) = 6x_4(k)$$
$$x_6(k) = 3x_7(k-1)$$
$$x_7(k) = 4x_2(k) \oplus 15x_6(k)$$
$$x_8(k) = 3x_7(k)$$

Implicitne jednadžbe stanja => potrebna transformacija

Primjer: analiza FPS-a max-plus algebrom



$$x_1(k) = 6x_5(k-1)$$

$$x_2(k) = 5x_1(k) \oplus 2x_3(k-1)$$

$$x_3(k) = 8x_2(k) \oplus 3x_4(k-1)$$

$$x_4(k) = 13x_3(k) \oplus 5x_8(k)$$

$$x_5(k) = 6x_4(k)$$

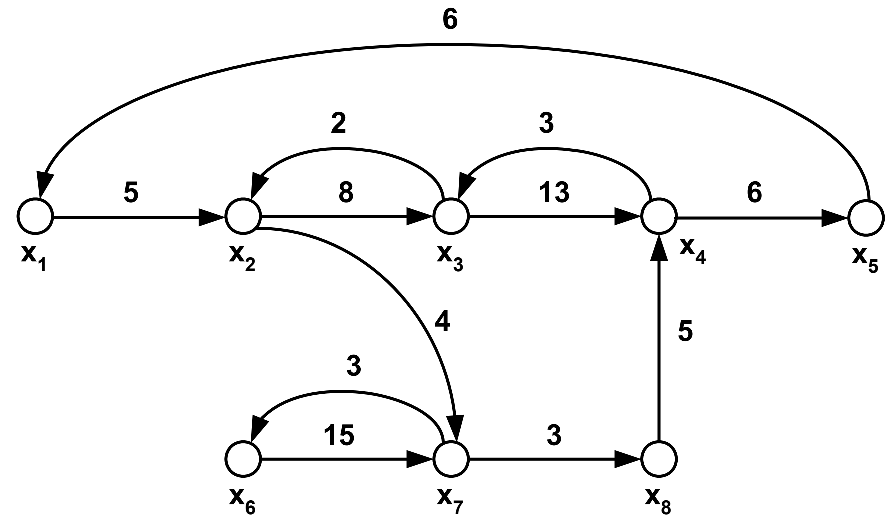
$$x_6(k) = 3x_7(k-1)$$

$$x_7(k) = 4x_2(k) \oplus 15x_6(k)$$

$$x_8(k) = 3x_7(k)$$

$$A_0 = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 5 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 8 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & 13 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 5 \\ \varepsilon & \varepsilon & \varepsilon & 6 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 4 & \varepsilon & \varepsilon & \varepsilon & 15 & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 3 & \varepsilon \end{bmatrix} \qquad A_1 = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & 6 & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & 2 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & 3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 3 & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \end{bmatrix}$$

Primjer: analiza FPS-a max-plus algebrom



$$A_0^* = \begin{bmatrix} e & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 5 & e & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 13 & 8 & e & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 26 & 13 & 13 & e & \varepsilon & 23 & 8 & 5 \\ 32 & 27 & 19 & 6 & e & 29 & 14 & 11 \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & e & \varepsilon & \varepsilon \\ 9 & 4 & \varepsilon & \varepsilon & \varepsilon & 15 & e & \varepsilon \\ 12 & 7 & \varepsilon & \varepsilon & \varepsilon & 18 & 3 & e \end{bmatrix}$$

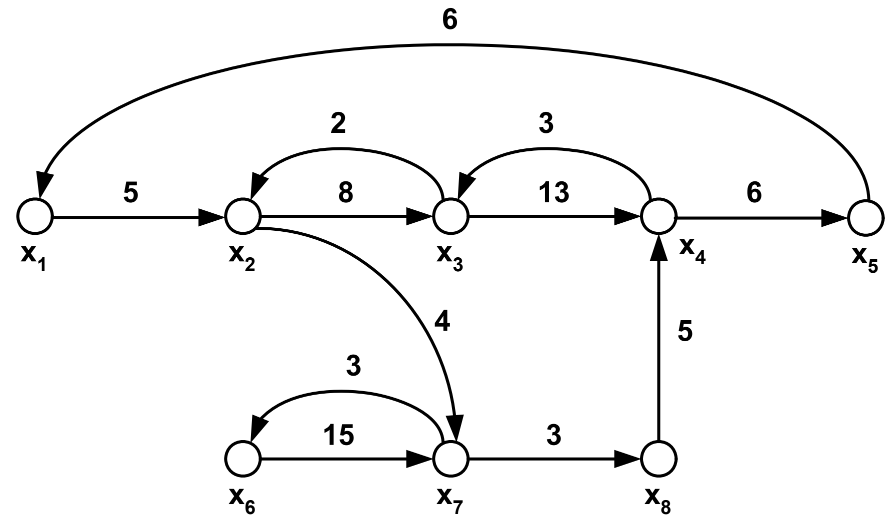
$$A = A_0^* \otimes A_1 = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & 6 & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & 2 & \varepsilon & 11 & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & 10 & 3 & 19 & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & 23 & 16 & 32 & \varepsilon & 26 & \varepsilon \\ \varepsilon & \varepsilon & 29 & 22 & 38 & \varepsilon & 32 & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 3 & \varepsilon \\ \varepsilon & \varepsilon & 6 & \varepsilon & 15 & \varepsilon & 18 & \varepsilon \\ \varepsilon & \varepsilon & 9 & \varepsilon & 18 & \varepsilon & 21 & \varepsilon \end{bmatrix}$$

Dobivena matrica razlikuje se od težinske matrice veza!

$$x(k) = A \otimes x(k-1)\ ,\quad x(0) = x_0\ ,$$

$$y(k) = C \otimes x(k)\ ,$$

Primjer: analiza FPS-a max-plus algebrom



$$x(k) = A \otimes x(k-1)\ ,\ \ x(0) = x_0\ \ ,$$
$$y(k) = C \otimes x(k)\ ,$$

$\boldsymbol{x}(0) = [\varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ e\ \varepsilon\ e\ \varepsilon]^T$

$$x = \begin{bmatrix} x(1) & x(2) & x(3) & x(4) & \\ 6 & 44 & 82 & 120 & \\ 11 & 49 & 87 & 125 & \\ 19 & 57 & 95 & 133 & \\ 32 & 70 & 108 & 146 & \dots \\ 38 & 76 & 114 & 152 & \\ 3 & 21 & 56 & 94 & \\ 18 & 53 & 91 & 129 & \\ 21 & 56 & 94 & 132 & \end{bmatrix}$$

ciklus proizvodnje

$\boldsymbol{\lambda}_{21}$ = [38 38 38 38 38 18 35 35]^T

$\boldsymbol{\lambda}_{32}$ = [38 38 38 38 38 35 38 38]^T

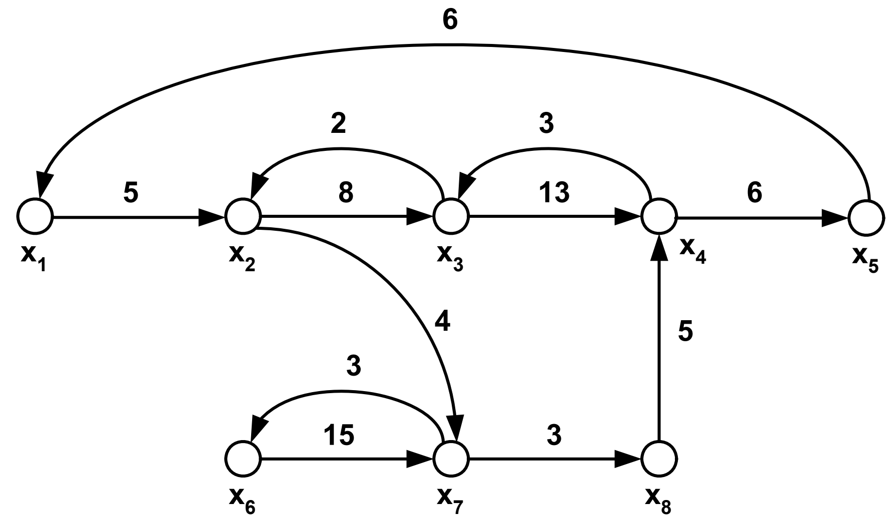
$\boldsymbol{\lambda}_{43}$ = [38 38 38 38 38 38 38 38]^T

$k_0 = 3$, $\lambda = 38$, $1/\lambda = 0.0263$

iskoristivost resursa

$$\eta_{M_1} = \frac{T_{M1}}{\lambda} = \frac{10}{38} = 0.263\ ,\ \eta_{M_2} = \frac{16}{38} = 0.421\ ,$$
$$\eta_{M_3} = \frac{18}{38} = 0.474\ ,\eta_{AGV} = \frac{29}{38} = 0.763\ .$$

Primjer: analiza FPS-a max-plus algebrom



Veza između fizikalnog sustava i max-plus modela?

$\sigma_A = (x_1, x_2, x_3, x_4, x_5, x_1)$, $\lambda = 38$

Podsjetnik - srednja težina staze

$$\overline{\sigma}_w = \frac{\sigma_w}{\sigma_\ell}$$

$$\lambda = \max_c(\overline{\sigma}_w)$$

5 veza staze σ_A => $\overline{\sigma}_{Aw} = \frac{\sigma_{Aw}}{\sigma_{A\ell}} = \frac{38}{5} = 7.6$

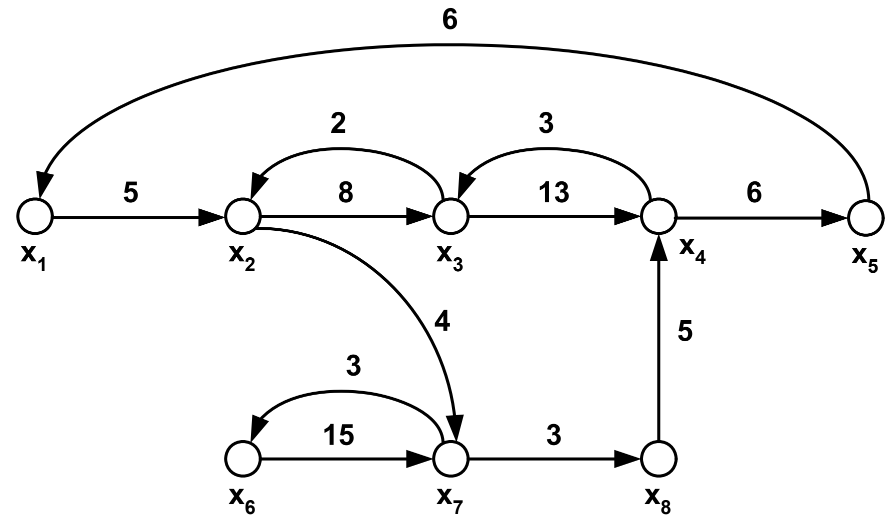
RAZLIKA! Zašto?

Da li svaka veza u grafu odgovara mjestu koje može primiti predmet?

Staza σ_A - 5 veza **ALI** samo 3 fizička mjesta – stroj M_1, stroj M_2 i AGV => 3 predmeta

Analiza s 3 predmeta => zaglavljenje sustava => zaključak – maksimalo dva predmeta na stazi $\sigma_{A,}$ => λ = 38/2 ; da li je to ciklus s najvećom težinom?

Primjer: analiza FPS-a max-plus algebrom



Druga kružna staza

$\sigma_{AGV} = (x_1,\ x_2,\ x_7,\ x_8,\ x_4,\ x_5,\ x_1)$

σ_{AGVw} = 29 > 38/2 => novi ciklus!

σ_{AGV} – jedno vozilo = jedan predmet

Pretpostavke – a) predmet u M_2 spreman za obradu,
b) predmet na AGV spreman za transport u M_1
c) predmet u M_3 spreman za obradu,

$\boldsymbol{x}(0) = [e\ \varepsilon\ e\ \varepsilon\ \varepsilon\ e\ \varepsilon\ \varepsilon]^T$

$$
\begin{aligned}
x_1(k) &= 6x_5(k-1) \\
x_2(k) &= 5x_1(k) \oplus 2x_3(k-1) \\
x_3(k) &= 8x_2(k) \oplus 3x_4(k-1) \\
x_4(k) &= 13x_3(k) \oplus 5x_8(k) \\
x_5(k) &= 6x_4(k) \\
x_6(k) &= 3x_7(k-1) \\
x_7(k) &= 4x_2(k) \oplus 15x_6(k) \\
x_8(k) &= 3x_7(k)
\end{aligned}
\quad\Rightarrow\quad
\begin{aligned}
x_1(k) &= 6x_5(k) \\
x_2(k) &= 5x_1(k-1) \oplus 2x_3(k-1) \\
x_3(k) &= 8x_2(k) \oplus 3x_4(k) \\
x_4(k) &= 13x_3(k-1) \oplus 5x_8(k) \\
x_5(k) &= 6x_4(k) \\
x_6(k) &= 3x_7(k) \\
x_7(k) &= 4x_2(k) \oplus 15x_6(k-1) \\
x_8(k) &= 3x_7(k)
\end{aligned}
$$

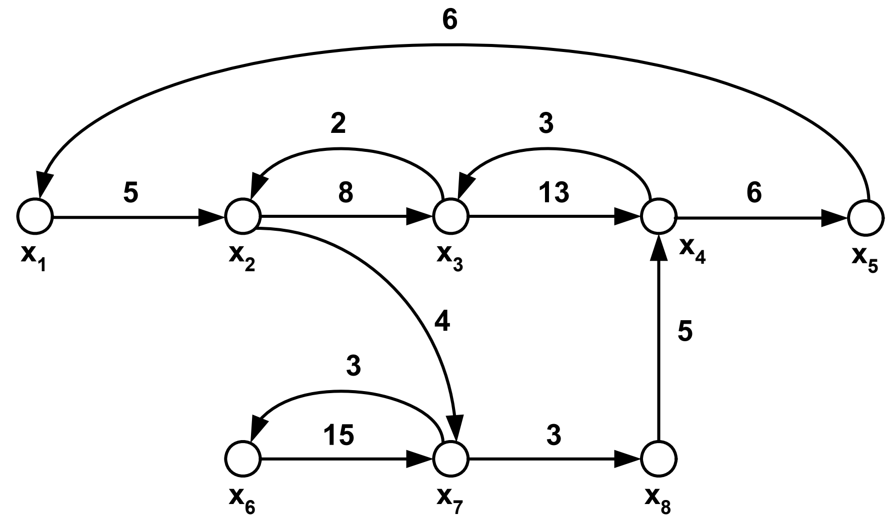
Opće pravilo

***n_1* predhodi *n_2*; veza (*n_1*, *n_2*) težine *a* predstavlja operaciju koja započinje s *n_1* i završava s *n_2* =>**

ako operacija “drži” predmet tada vrijedi

$$n_2(k) = a \otimes n_1(k-1)$$

Primjer: analiza FPS-a max-plus algebrom



$$A = \begin{bmatrix} 29 & \varepsilon & 26 & \varepsilon & \varepsilon & 35 & \varepsilon & \varepsilon \\ 5 & \varepsilon & 2 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 20 & \varepsilon & 17 & \varepsilon & \varepsilon & 26 & \varepsilon & \varepsilon \\ 17 & \varepsilon & 14 & \varepsilon & \varepsilon & 23 & \varepsilon & \varepsilon \\ 23 & \varepsilon & 20 & \varepsilon & \varepsilon & 29 & \varepsilon & \varepsilon \\ 12 & \varepsilon & 9 & \varepsilon & \varepsilon & 18 & \varepsilon & \varepsilon \\ 9 & \varepsilon & 6 & \varepsilon & \varepsilon & 15 & \varepsilon & \varepsilon \\ 12 & \varepsilon & 9 & \varepsilon & \varepsilon & 18 & \varepsilon & \varepsilon \end{bmatrix}$$

$\boldsymbol{x}(0) = [e\ \varepsilon\ e\ \varepsilon\ \varepsilon\ e\ \varepsilon\ \varepsilon]^T$

$$x = \begin{bmatrix} x(1) & x(2) & x(3) & x(4) & \\ 35 & 64 & 93 & 122 & \\ 5 & 40 & 69 & 98 & \\ 26 & 55 & 84 & 113 & \\ 23 & 52 & 81 & 110 & \dots \\ 29 & 58 & 87 & 116 & \\ 18 & 47 & 76 & 105 & \\ 15 & 44 & 73 & 102 & \\ 18 & 47 & 76 & 105 & \end{bmatrix}$$

$\boldsymbol{\lambda}_{21} = [29\ 35\ 29\ 29\ 29\ 29\ 29\ 29]^T$
$\boldsymbol{\lambda}_{32} = [29\ 29\ 29\ 29\ 29\ 29\ 29\ 29]^T$

$k_0 = 2,\ \lambda = 29$ $\quad \sigma_{AGV} = (x_1, x_2, x_7, x_8, x_4, x_5, x_1)$

$\sigma_{AGVw} = 29 = \lambda$

$$\eta_{M_1} = \frac{T_{M1}}{\lambda} = \frac{10}{29} = 0.345\ ,\ \eta_{M_2} = \frac{16}{29} = 0.552\ ,$$

$$\eta_{M_3} = \frac{18}{29} = 0.621\ ,\eta_{AGV} = \frac{29}{29} = 1\ .$$

Maksimalna iskoristivost AGV-a.

Primjena max-plus algebre

- ograničena na “linearne” sustave s diskretim događajima (označeni graf, graf događaja – *marked graph, event graph*)
- planiranje
- komunikacije
- proizvodni procesi
- promet
- programiranje
- ...

Automati

$$A = \left\{ E, X, f, x_0, X_{\mathrm{m}} \right\}$$

E = {e_1, e_2, ..., e_m} - skup svih događaja (*events*) u automatu,

X = {x_1, x_2, ..., x_m} - skup svih stanja (*states*) automata,

$f : X \times E \rightarrow X$ – prijelazna funkcija (*transition function*) automata,

x_0 – početno stanje (*initial state*) automata,

X_m - skup markiranih stanje (*marked states*) automata => stanja s osobitim svojstvima.

Automati

f (*x*,*e*) = *y* => događaj *e* je uzrok prelaska automata iz stanje *x* u stanje *y*

=> funkcija *f* definirana samo na dijelu domene => ako događaj *e* ne utječe na stanje *x* tada *f* (*x*,*e*) nije definirana => *Γ*(*x*) ={*e* : *postoji f* (*x*,*e*)}

=> ako *f* (*x*,*e*) može poprimiti više vrijednosti => nedeterministički automat

$$f(y,b) = f(f(x,a),b) = f(x,ab) = z$$

niz događaj (*event sequence*)

*E** - skup nizova događaja => jezici

Automati

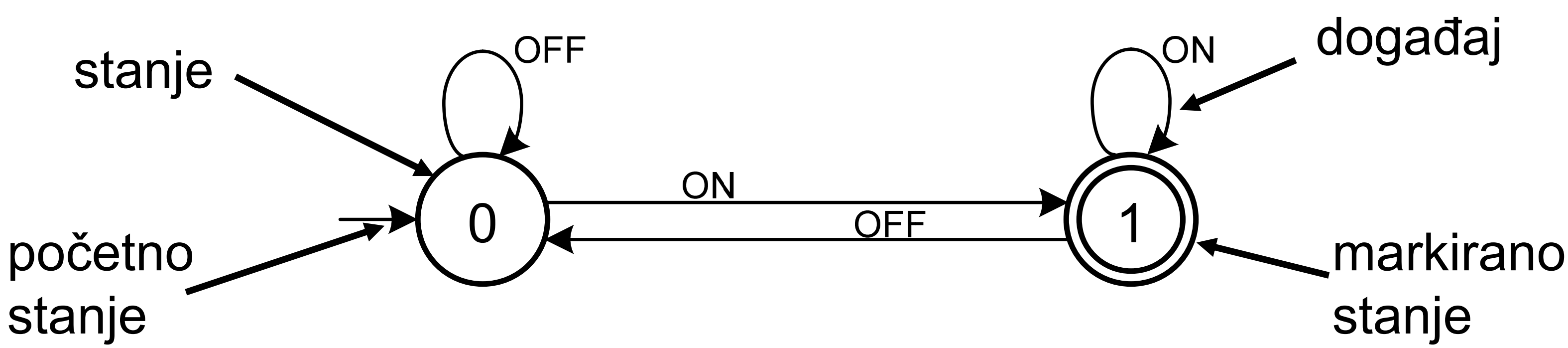
Primjer: automat koji opisuje ventilator u regulaciji zagađenja tunela

$$A_{\mathrm{F}} = \left\{ E_{\mathrm{F}}, X_{\mathrm{F}}, f_{\mathrm{F}}, x_{\mathrm{F0}}, X_{\mathrm{Fm}} \right\}$$

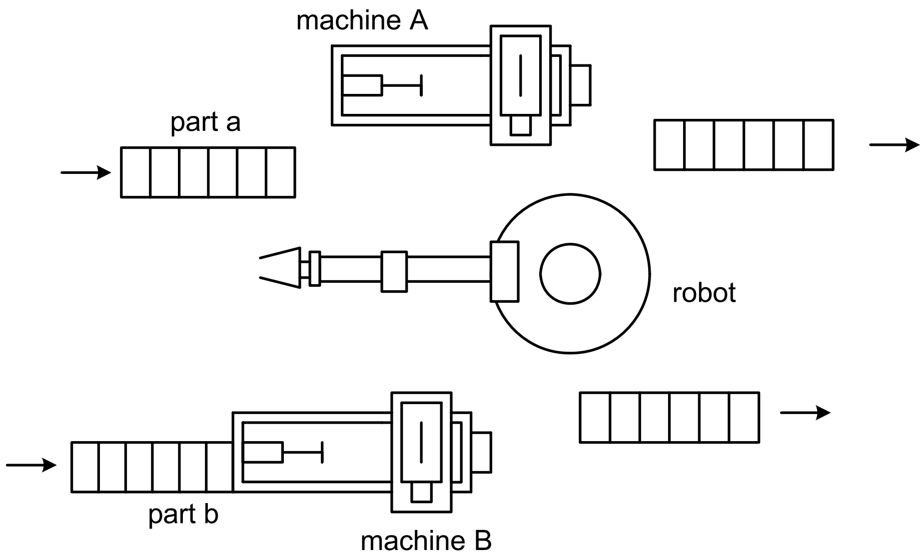
$$E_{\mathrm{F}} = \left\{ \mathrm{ON}, \mathrm{OFF} \right\},\ X_{\mathrm{F}} = \left\{ 0,1 \right\},\ X_{\mathrm{Fm}} = \left\{ 1 \right\}$$

$$f_{\mathrm{F}}(0, \mathrm{ON}) = 1,\ f_{\mathrm{F}}(0, \mathrm{OFF}) = 0,\ f_{\mathrm{F}}(1, \mathrm{OFF}) = 0,\ f_{\mathrm{F}}(1, \mathrm{ON}) = 1,\ x_{\mathrm{F0}} = 0$$

dijagram prelaska stanja => graf



Primjer: automat proizvodnog sustava



State	Description
I	machine A (B) idle
W	machine A (B) – work in progress
A	robot availabe
M	moving part *a* in machine A
2	removing part *b* from machine B
1	removing part *a* from machine A

Event	Description
α	arrival of part *a*
β	arrival of part *b* in machine B (processing started)
m	processing of part *a* in machine A started
f	replacement of part *b* from machine B started
r	replacement of part *b* from machine B completed
	replacement of part *a* from machine A completed
c	replacement of part *a* from machine A started

stanje automata opisano s tri znaka:

Prvi znak – stanje robota (A,M,1,2)

Drugi znak – stanje stroja A (I,W)

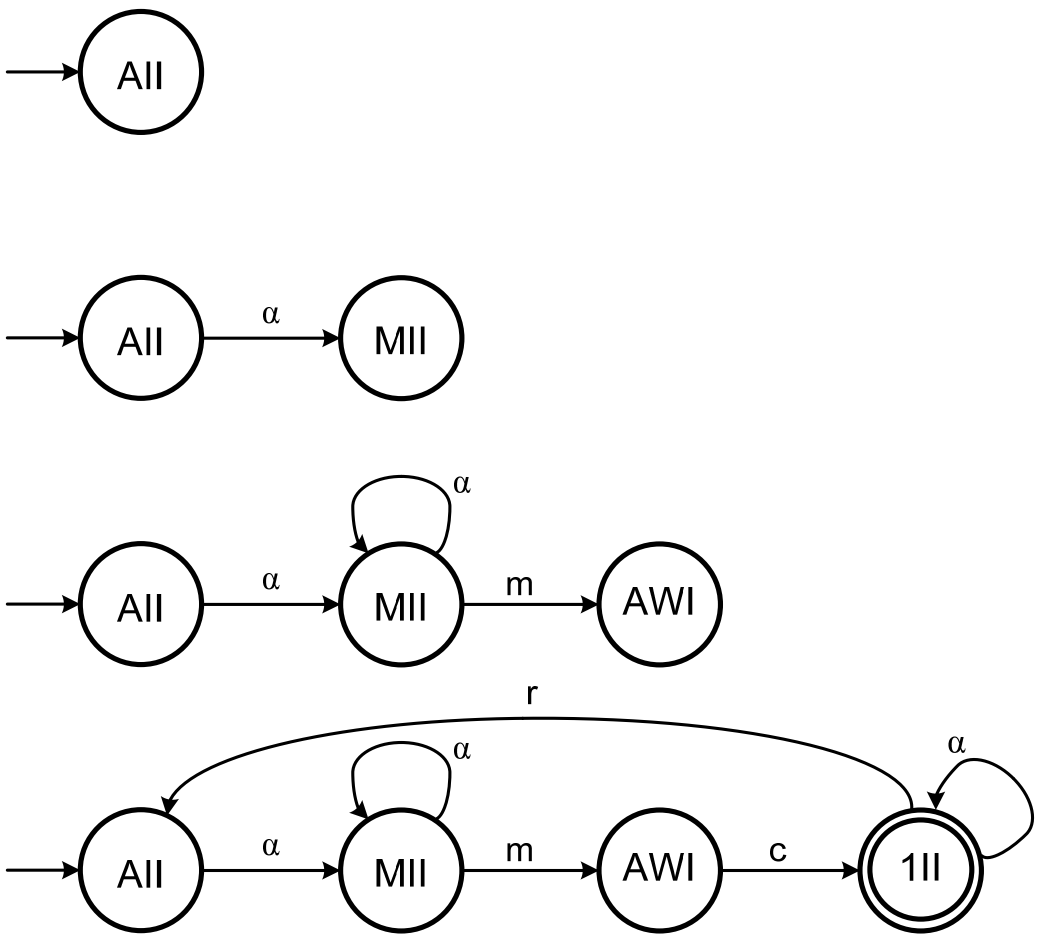
Treći znak – stanje stroja B (I,W)

MIW

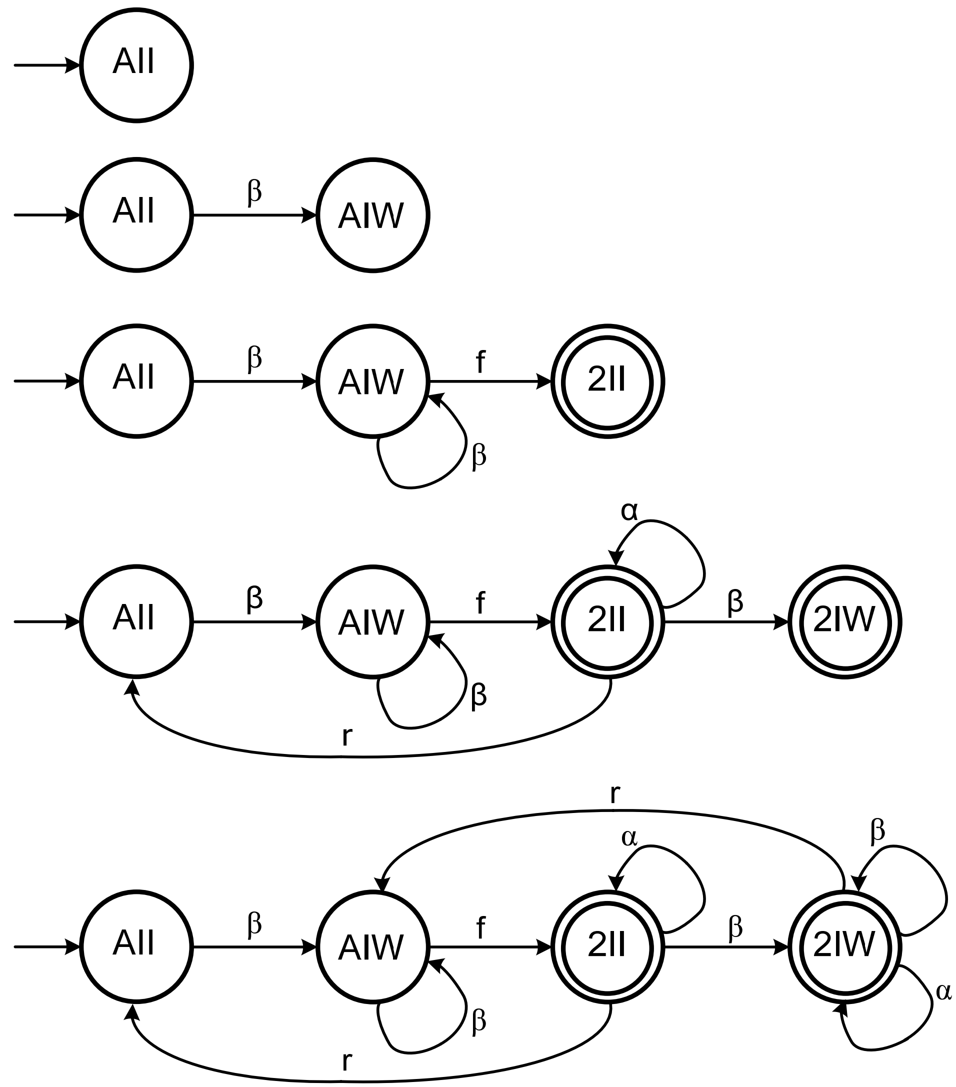
Primjer: automat proizvodnog sustava

- neformalni pristup određivanja automata

obrada predmeta *a*



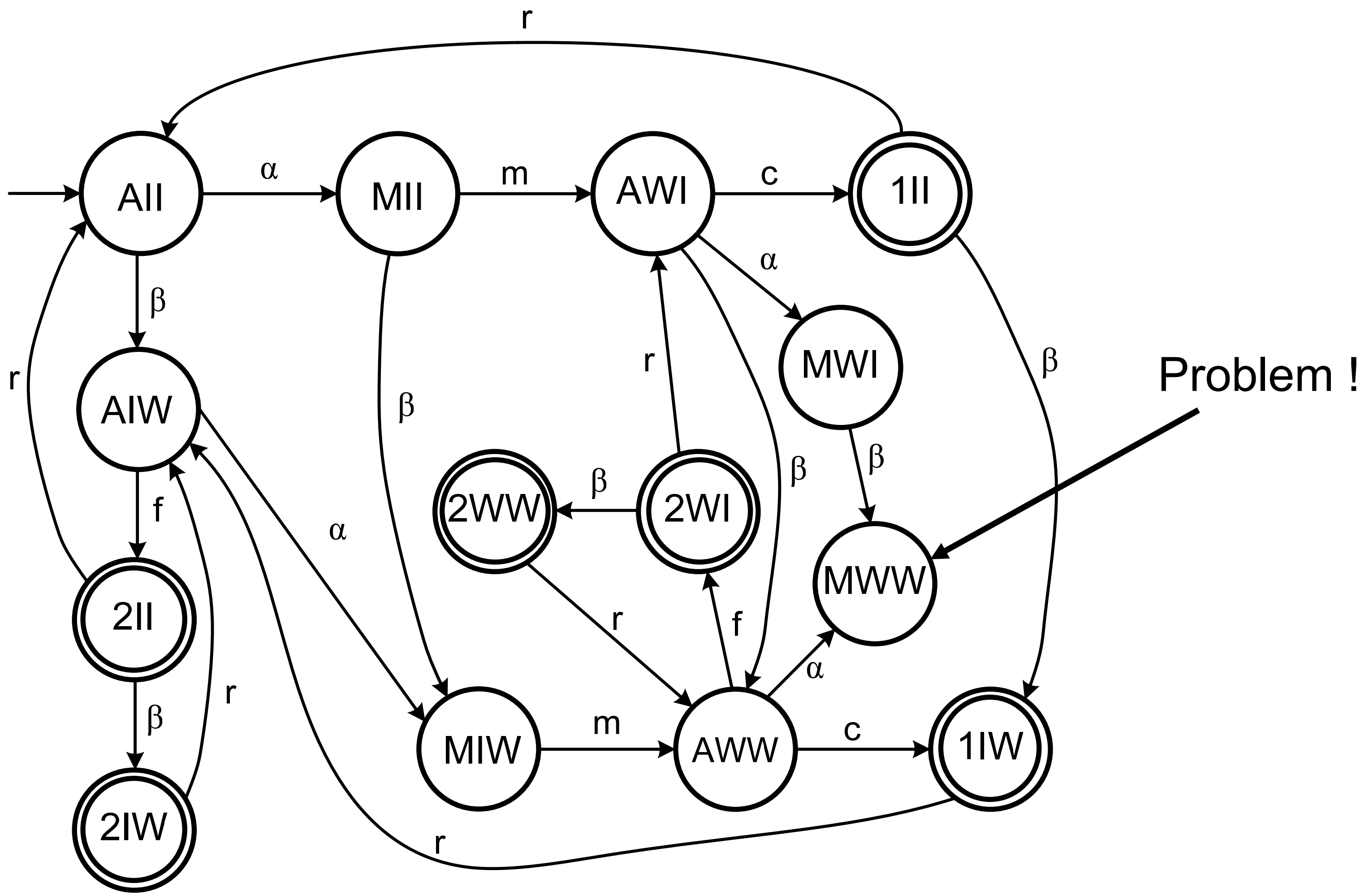
obrada predmeta *b*



Primjer: automat proizvodnog sustava

- neformalni pristup određivanja automata

konačni oblik automata (nisu prikazani događaji koji ne mijenjaju stanje)



Formalni pristup određivanja automata

– paralelna kompozicija automata

$$A_1 \parallel A_2 = Ac\left(X_1 \times X_2, E_1 \cup E_2, f((x_1 x_2), e), x_{01} x_{02}, X_{m1} \times X_{m2}\right)$$

dobiveni automat sadrži kombinacije svih stanja automata iz kojih je nastao

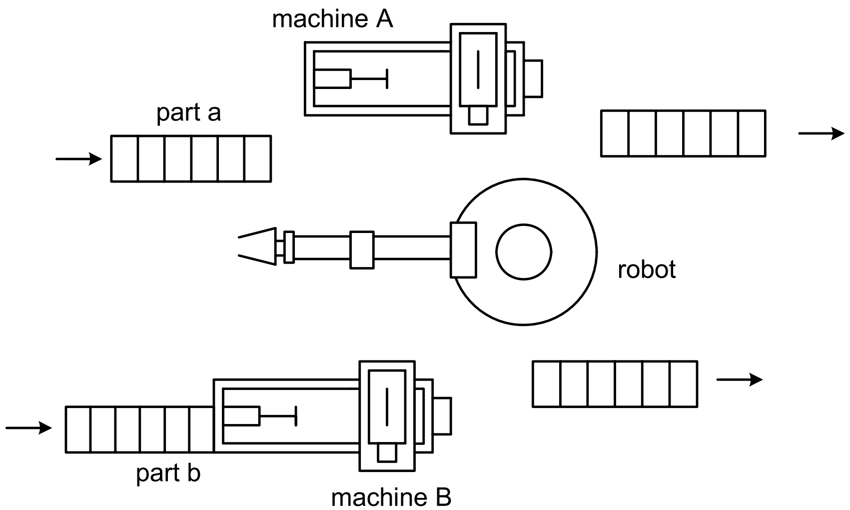
dobiveni automat sadrži sve događaje automata iz kojih je nastao

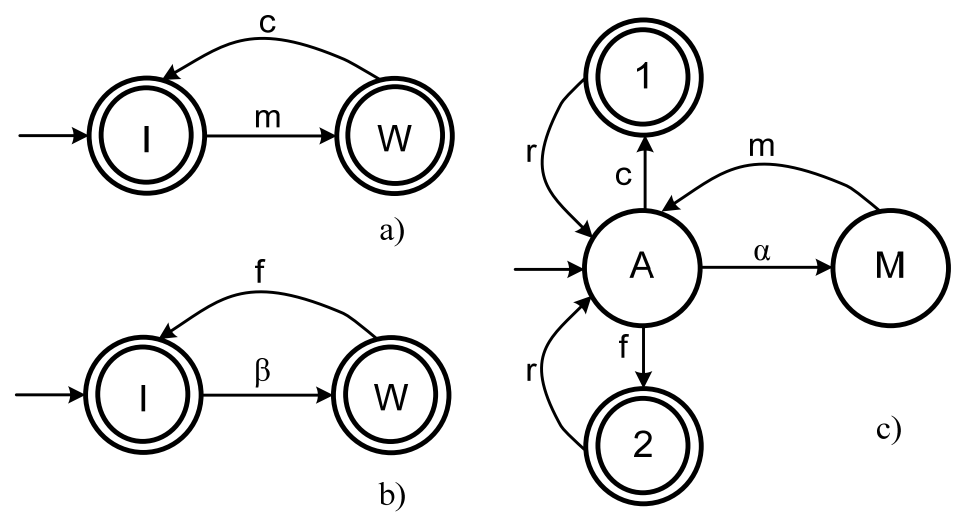
$$f((x_1 x_2), e) = \begin{cases} \left(f_1(x_1, e) f_2(x_2, e)\right) & \text{if } e \in \Gamma_1(x_1) \cap \Gamma_2(x_2) \\ \left(f_1(x_1, e) x_2\right) & \text{if } e \in \Gamma_1(x_1) \setminus E_2 \\ \left(x_1 f_2(x_2, e)\right) & \text{if } e \in \Gamma_2(x_2) \setminus E_1 \end{cases}$$

događaj *e* є $E_1 \cap E_2$ može biti ostvaren samo ako novi automat dođe u stanje sastavljeno od stanja koja u polaznim automatima iniciraju događaj *e*

Ac – operacija dohvatljivosti – briše sva stanja koja nisu dohvatljiva iz početnog stanja

Primjer: automat proizvodnog sustava – formalni pristup





Automati elemenata proizvodnog sustava:
a) stroj A, b) stroj B i c) robot

paralelna kompozicija automata a) i c)

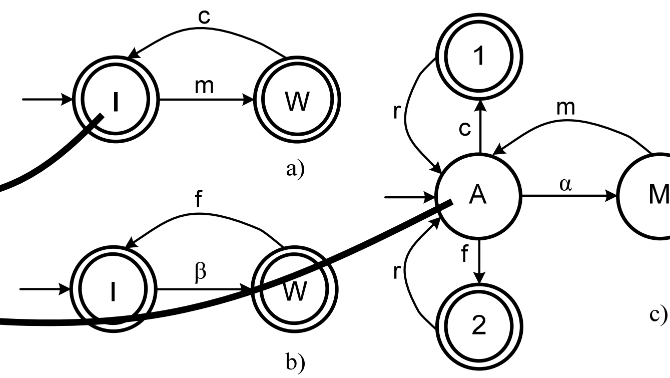
- 8 stanja (4 robot x 2 stroj A): AI, MI, 1I, 2I, AW, MW, 1W i 2W
- zajednički događaji => $E_A \cap E_R = \{c, m\}$

Primjer: automat proizvodnog sustava – formalni pristup

novo stanje AI => $\Gamma(A) = \{c, f, \alpha\}$ i $\Gamma(I) = \{m\}$

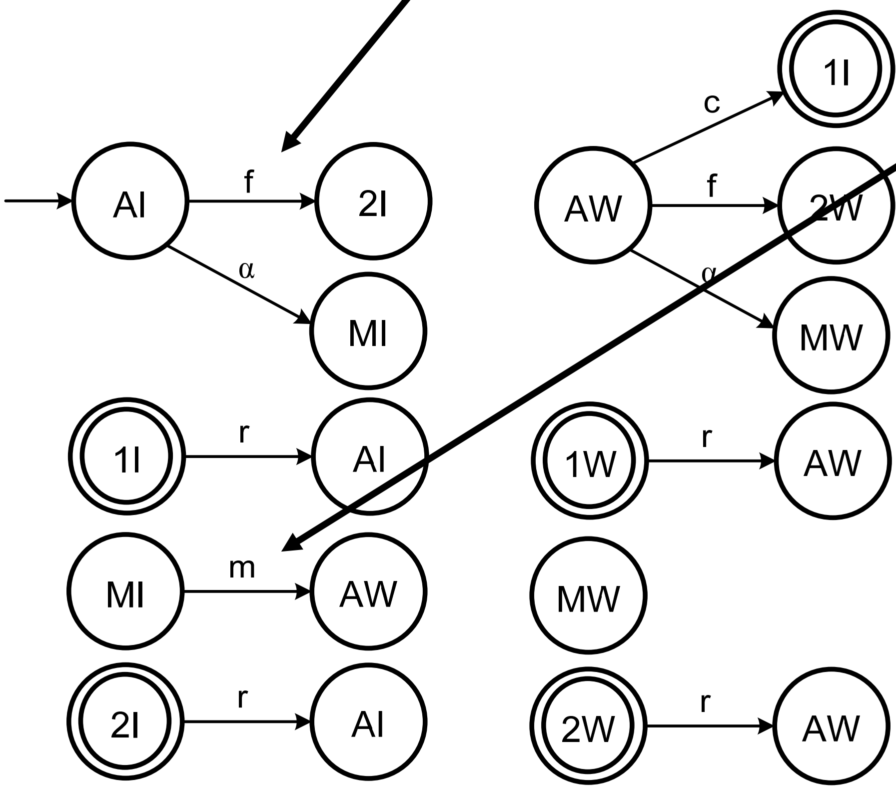
$E_A \cap E_R = \{c, m\}$

budući da ne vrijedi $c \in \Gamma(A) \cap \Gamma(I)$, c ne može biti iniciran; isto vrijedi i za *m*



novo stanje MI => $\Gamma(M) = \{m\}$ i $\Gamma(I) = \{m\}$

budući da je $m \in \Gamma(M) \cap \Gamma(I)$, *m* može biti iniciran



konačni automat