$$p(\vec{x} \mid \vec{z}, \vec{\theta}) = \prod_{k=1} p(\vec{x} \mid \theta_k)^{z_k}$$

$$p(\vec{x}, \vec{z} \mid \vec{\theta}) = p(\vec{z}) \cdot p(\vec{x} \mid \vec{z}, \vec{\theta}) = \prod_{k=1}^{K} \pi_k^{z_k} \cdot \prod_{k=1}^{K} p(\vec{x} \mid \vec{\theta}_k)^{z_k} = \prod_{k=1}^{K} \pi_k^{z_k} p(\vec{x} \mid \vec{\theta}_k)^{z_k}$$

ko imamo više primjera (N), trebamo $\vec{z}^i_k$ (po 1 bin. vektor za svaki primjer)

potpuna log-izglednost

$$\ln \mathcal{L}(\theta \mid D, z) = \ln p(D, z \mid \theta) \overset{\text{iid}}{=} \ln \prod_{i=1}^{N} p(\vec{x}^i, \vec{z}^i \mid \vec{\theta})$$

$$= \ln \prod_{i=1}^{N} \prod_{k=1}^{K} \pi_k^{z_k^i} p(\vec{x}^i \mid \theta_k)^{z_k^i} = \underbrace{\sum_{i=1}^{N} \sum_{k=1}^{K} z^i_k (\ln \pi_k + \ln p(\vec{x}^i \mid \theta_k

$$P(z_k \mid \vec{x}^{\,i}, \vec{\theta}^{\,t}) = \frac{p(\vec{x}^{\,i} \mid z_k, \vec{\theta}^{\,t}) \cdot \pi_k^{t}}{\sum_{j=1}^{K} p(\vec{x}^{\,i} \mid z_j, \vec{\theta}^{\,t}) \cdot \pi_j^{t}} = h_k^i$$

π_k^t ~ $p(z_k)$

h_k^i ← ~~očekivanje~~ odgovornost

vjer. da primjer i komponenta k

$$Q(\theta \mid \theta^t) = \sum_{i=1}^{N} \sum_{k=1}^{K} h_k^i \left(\ln \pi_k^{\bullet} + \ln p(\vec{x}^{\,i} \mid \vec{\theta}_k^{\bullet}) \right)$$

$$= \sum_{i=1}^{N} \sum_{k=1}^{K} h_k^i \ln \pi_k + \underbrace{\sum_{i=1}^{N} \sum_{k=1}^{K} h_k^i \ln p(\vec{x}^{\,i} \mid \vec{\theta}_k)}_{A}$$

M-korak

- definicija ograničenja π_k, Lagrange

ML - procjena za parametre π_k, $\vec{\theta}_k$

$$\hat{\pi}_k = \frac{1}{N} \sum_{i=1}^{N} h_k^i \quad \text{- apriorna vj. klase } k$$

$$\nabla_{\vec{\theta}_k} A = 0$$

Gaussova mješavina (multi.)

$$p(\vec{x} \mid \vec{\theta}_k) = \frac{1}{2\pi^{n/2} |\Sigma|^{1/2}} \exp\left[(x-\mu)^T \Sigma^{-1} (x-\mu) \right]$$

$$\vec{\mu}_k = \frac{\sum h_k^i \vec{x}^{\,i}}{\sum h_k^i}$$

$$\Sigma_k = \frac{\sum_i h_k^i (x^i - \vec{\mu}_k)(x^i - \vec{\mu}_k)^T}{\sum h_k^i}$$