

$$P(\vec{x} | \vec{z}, \vec{\theta}) = \prod_{k=1}^K P(\vec{x} | \theta_k)^{z_k}$$

$$P(\vec{x}, \vec{z} | \vec{\theta}) = P(\vec{z}) \cdot P(\vec{x} | \vec{z}, \vec{\theta}) = \prod_{k=1}^K \pi_k^{z_k} \cdot \prod_{k=1}^K P(\vec{x} | \theta_k)^{z_k} = \prod_{k=1}^K \pi_k^{z_k} P(\vec{x} | \theta_k)^{z_k}$$

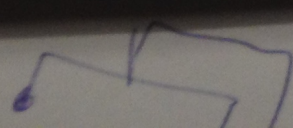
bo musíme viete pripraviť ( $K$ ), teda  $\vec{\theta}_k$  (po 1 bin. vektor zo svoj. priemer)

potom log-likelihood

$$\begin{aligned} \ln L(\theta | D, z) &= \ln P(D, z | \theta) = \ln \prod_{i=1}^N P(\vec{x}^i, \vec{z}^i | \vec{\theta}) \\ &= \ln \prod_{i=1}^N \prod_{k=1}^K \pi_k^{z_k^i} P(\vec{x}^i | \theta_k)^{z_k^i} = \sum_{i=1}^N \sum_{k=1}^K z_k^i (\ln \pi_k + \ln P(\vec{x}^i | \theta_k)) \end{aligned}$$

↑  
(1) potom log. likelihood za

12.10.2023





$$P(z_k | \vec{x}^i, \vec{\theta}) = \frac{p(\vec{x}^i | z_k, \vec{\theta}) \cdot \pi_k^p(z_k)}{\sum_{j=1}^K p(\vec{x}^i | z_j, \vec{\theta}) \cdot \pi_j^p} = h_k^i$$

odgovornost  
- odgovor  
vjerodajnosni  
komponenta

$$Q(\theta | \theta^*) = \sum_{i=1}^N \sum_{k=1}^K h_k^i (\ln \pi_k^* + \ln p(\vec{x}^i | \vec{\theta}_k^*))$$

$$= \sum_{i=1}^N \sum_{k=1}^K h_k^i \ln \pi_k^* + \underbrace{\sum_{i=1}^N \sum_{k=1}^K h_k^i \ln p(\vec{x}^i | \vec{\theta}_k^*)}_{(A)}$$

M-bud

- derivacije ograniči  $\pi_k$ , Lagrange

ML- program za parametre  $\pi_k, \vec{\theta}_k$

$$\hat{\pi}_k = \frac{1}{N} \sum_{i=1}^N h_k^i \quad - \text{odgovor na pitanje } k$$

$$\nabla \vec{\theta}_k (A) = 0$$

Gaussova mešavina (mult.)

$$p(\vec{x}^i | \vec{\theta}_k) = \frac{1}{2\pi^{d/2} |\Sigma|^{1/2}} \exp \left[ -\frac{1}{2} (\vec{x} - \vec{\mu})^T \Sigma^{-1} (\vec{x} - \vec{\mu}) \right]$$

$$\vec{\mu}_k = \frac{\sum h_k^i \vec{x}^i}{\sum h_k^i}$$

$$\Sigma_k = \frac{\sum h_k^i (\vec{x}^i - \vec{\mu}_k)(\vec{x}^i - \vec{\mu}_k)^T}{\sum h_k^i}$$