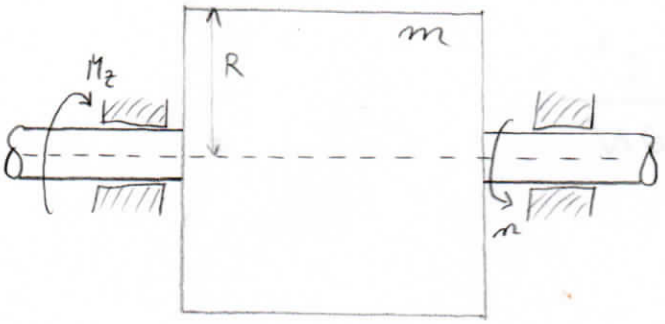
ZADATAK 1

Rotor elektromotora prilikom dizanja tereta mase m i polumjera R rotira kutnom brzinom n. Kolikim konstantnim momentom treba kočiti rotor da li se zaustavio nakon N punih okretaja



$\rightarrow z \qquad J_z = \frac{mR^2}{2}$

a) JEDNADŽBOM GIBANJA

* $-M_z = I_z \cdot \ddot{\varphi} = \frac{1}{2} m R^2 \cdot \ddot{\varphi}$

$\varepsilon = \ddot{\varphi} = -\frac{2M_z}{mR^2} \rightarrow \dot{\varphi} = -\frac{2M_z}{mR^2} t + C_1, \quad t=0 \quad \dot{\varphi} = \omega = \frac{n\pi}{30}$

$\rightarrow C_1 = \frac{n\pi}{30}$

$\omega = \frac{n\pi}{30} - \frac{2M_z}{mR^2} t \quad / \int dt$

$\varphi = \frac{n\pi}{30} t - \frac{M_z t^2}{mR^2} + C_2, \quad t=0 \quad \varphi=0 \Rightarrow C_2 = 0$

$\varphi = \frac{n\pi}{30} t - \frac{M_z t^2}{mR^2}$

$\varphi = 2\pi N, \quad \omega = 0$

$\downarrow$

$2\pi N = \frac{n\pi}{30} \cdot \frac{n\pi mR^2}{60 M_z} - \frac{M_z}{mR^2} \cdot \frac{n^2\pi^2 m^2 R^4}{60^2 M_z^2} \qquad t = \frac{n\pi mR^2}{60 M_z}$

$$M_z = \frac{n^2 \pi m R^2}{7200 N}$$

b) ZAKONOM KINETIČKE ENERGIJE

$$\frac{I_z \omega_2^2}{2} - \frac{I_z \omega_1^2}{2} = -\int_{\varphi_1=0}^{\varphi_2=2\pi N} M_z \, d\varphi$$

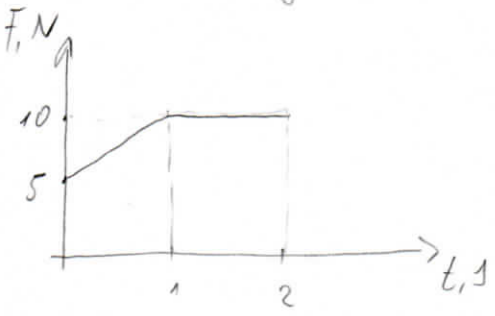
$\omega_2 = 0$

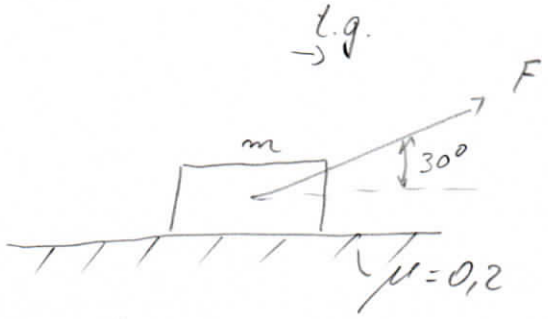
$$\frac{1}{2} m R^2 \cdot \frac{1}{2} \cdot \frac{n_1^2 \pi^2}{30^2} = 2\pi N M_z$$

$$\Rightarrow M_z = \frac{n^2 \pi m R^2}{7200 N}$$

ZADATAK 2

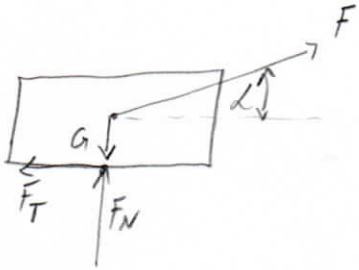
Blok mase $m = 2\,kg$ započinje gibanje bez početne brzine pod djelovanjem $F(t)$. Odredite brzinu bloka u $t = 2\,s$





ZAKON OČUVANJA KOLIČINE GIBANJA

$$m v_2 - m v_1 = \int_{t_1}^{t_2} F_{VK}\,dt$$



$v_1 = 0$

$$m v_2 = \int_{t_1}^{t_2} F_{VK} \cdot dt$$

$\Sigma F_x = 0 \quad F\cos\alpha - F_T = F_{VK}$

$\Sigma F_y = 0 \quad F\sin\alpha + F_N - mg = 0$ $\Big\}$ $F_{VK} = F\cos\alpha - \mu(mg - F\sin\alpha)$

$$m v_2 = \int_0^2 (F\cos\alpha - \mu(mg - F\sin\alpha))\,dt$$

$$= (\cos\alpha + \mu\sin\alpha)\int_0^2 F\,dt - \mu mg\,dt$$

$m v_2 = 9,057$

$v_2 = 4,529\ \frac{m}{s}$

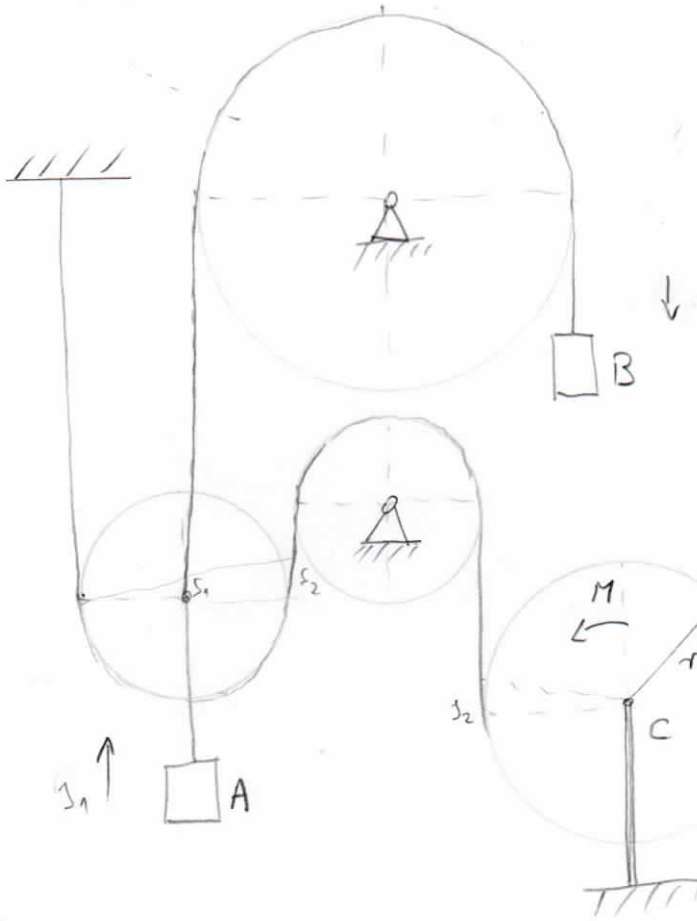
ZADATAK 3

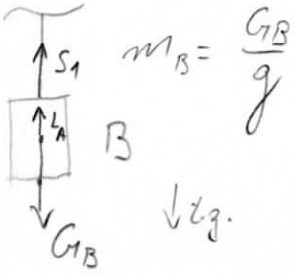
Uteg težine G_A podiže se pomoću elektromotora koji djeluje zakretnim momentom M na bubanj C polumjera r i pomoću protuutega B težine G_B. Mase kolotura, ležajeva i trenje se zanemaruju. Potrebno je odrediti iznos zakretnog momenta M elektromotora koji je potreban da se postigne ubrzanje utega A od $3\ m/s^2$ → D'ALAMBERT!

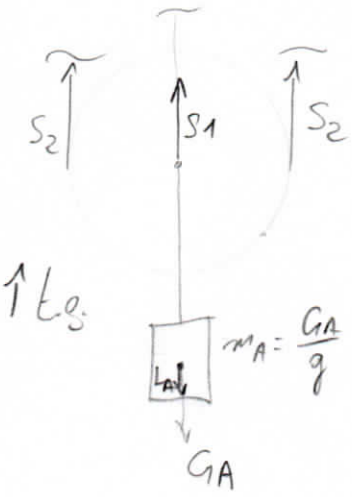
$G_A = 2\ kN$

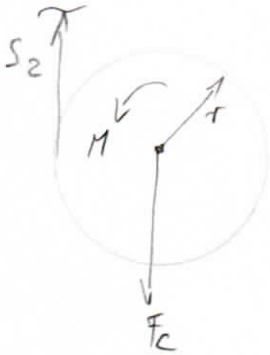
$G_B = 1{,}5\ kN$

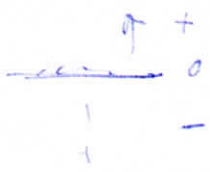
$r = 20\ cm$











$$\frac{G_B}{g} \cdot a_B + S_1 - G_B = 0$$

$$\sum M_c = 0 \qquad S_2 \cdot r = M \qquad a_A = a_B$$

$$S_1 + 2S_2 - \frac{G_A}{g} a_A - G_A = 0$$

$$-\frac{G_B}{g} \cdot a + G_B + 2S_2 - \frac{G_A}{g} \cdot a - G_A = 0$$

$$S_2 = \frac{1}{2}\left(G_A + \frac{G_A}{g} \cdot a - G_B + \frac{G_B}{g} \cdot a \right) = 785{,}17\ N$$

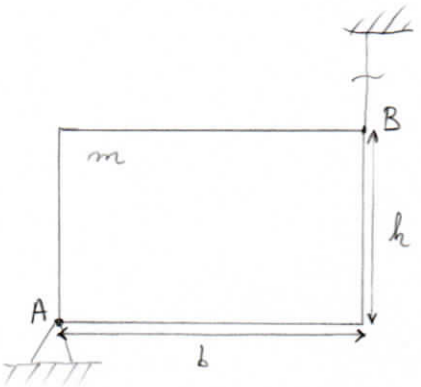
$$M = S_2 \cdot r = 157\ Nm$$

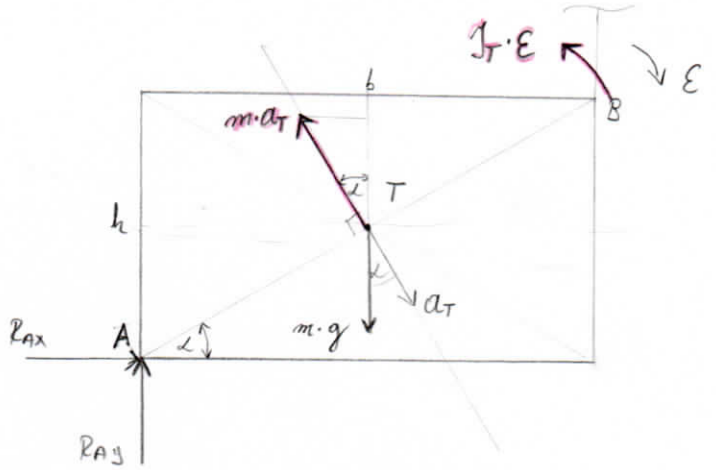
ZADATAK 4:

Homogena ploča mase m, širine b i visine h oslonjena je u zglobu A, a u B je vezana užetom. Odredite iznos reakcijske sile u osloncu A u trenutku prekida užeta u B.

$m, b, h, \quad J_T = \frac{m(b^2+h^2)}{12}$

• ROTACIJA + TRANSLACIJA (OKO T)





$$\frac{d}{2} = \frac{b}{2\cos\alpha}$$

$$a_T = \frac{d}{2} \cdot \varepsilon = \frac{b}{2\cos\alpha} \cdot \varepsilon$$

$\sum F_x = 0 \quad R_{Ax} - m a_T \sin\alpha = 0$

$\sum F_y = 0 \quad R_{Ay} + m a_T \cos\alpha - m g = 0$

$\sum M_A = 0 \quad J_T \cdot \varepsilon + m a_T \cdot \frac{b}{2\cos\alpha} - mg\frac{b}{2} = 0$

$$\frac{m d^2}{12} \cdot \varepsilon + m \cdot \frac{b}{2\cos\alpha} \cdot \frac{b}{2\cos\alpha} \cdot \varepsilon - mg\frac{b}{2} = 0$$

$$\frac{d^2}{12} \cdot \varepsilon + \frac{d^2}{4}\varepsilon - g \cdot \frac{b}{2} = 0$$

$$\frac{d^2}{3}\varepsilon = \frac{b}{2} \cdot g$$

$$\boxed{\varepsilon = \frac{3}{2} \cdot \frac{b}{b^2+h^2} \cdot g}$$

$R_{Ax} = m a_T \sin\alpha$

$R_{Ax} = m \cdot \frac{b}{2\cos\alpha} \cdot \sin\alpha \cdot \varepsilon$

$R_{Ax} = m \cdot \frac{b}{2} \cdot \varepsilon \cdot \operatorname{tg}\alpha$

$R_{Ax} = m \cdot \frac{b}{2} \cdot \frac{h}{b} \varepsilon$

$R_{Ax} = m \cdot \frac{h}{2} \cdot \varepsilon$

$$\boxed{R_{Ax} = \frac{3}{4} \cdot \frac{b\,h}{b^2+h^2} \cdot m \cdot g}$$

$R_{Ay} = m\left(g - \frac{b}{2\cos\alpha} \cdot \cos\alpha \cdot \varepsilon\right)$

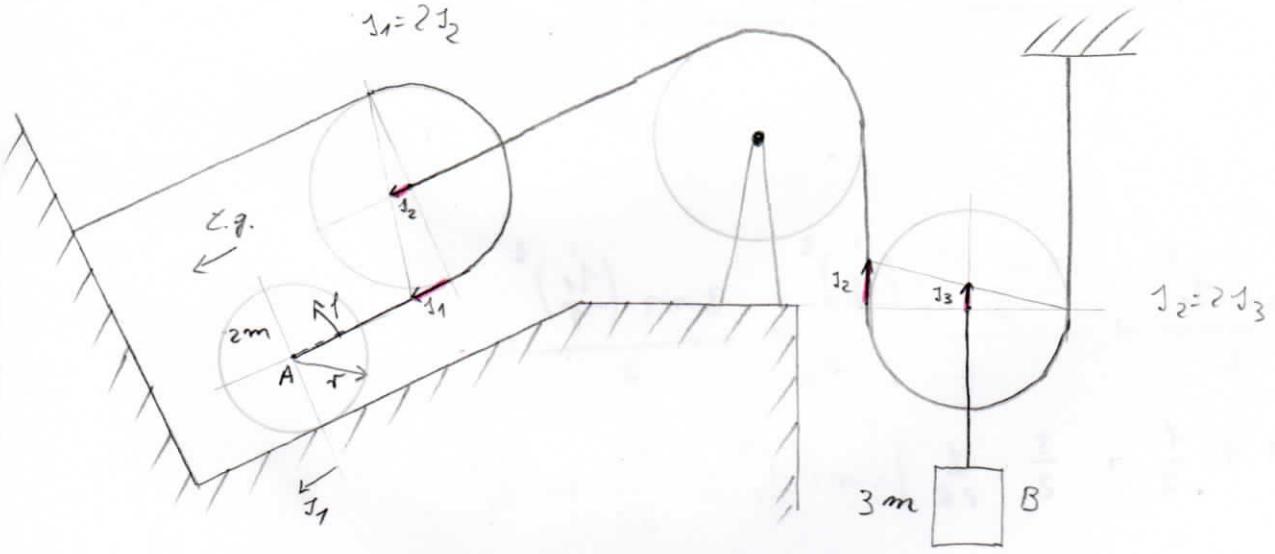
$R_{Ay} = m\left(g - \frac{b}{2} \cdot \frac{3}{2} \cdot \frac{b}{b^2+h^2} g\right)$

$R_{Ay} = \frac{4b^2+4h^2-3b^2}{4(b^2+h^2)} \cdot m \cdot g$

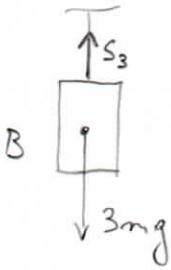
$$\boxed{R_{Ay} = \frac{b^2+4h^2}{4(b^2+h^2)} \cdot m \cdot g}$$

ZADATAK 5

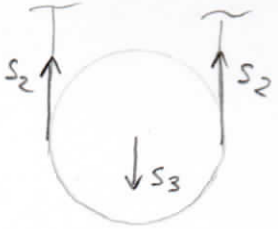
Odrediti ubrzanja valjka A mase $2m$ i utega B mase $3m$ u sustavu prema slici. Sva užad je neistezljiva, a čunjevi i mase kolotura se ne zanemaruju, uz pretpostavku da se valjak kotrlja po podlozi. $\alpha = 30°$, $r = 100\,mm$ $\quad J_C = \frac{m r^2}{2}$



(1.) STATIČKA ANALIZA DA SE ODREDI t.g.

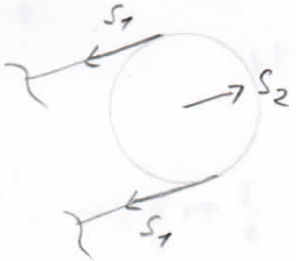


$S_3 = 3mg$



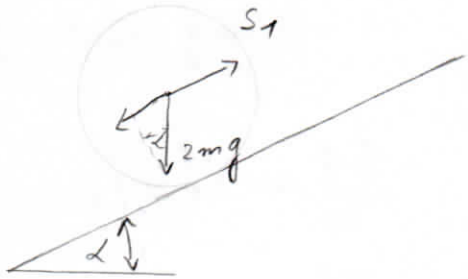
$2S_2 = S_3$

$S_2 = \frac{3}{2} mg$



$2S_1 = S_2$

$S_1 = \frac{3}{4} mg$



$2mg \sin\alpha \gtrless \frac{3}{4} mg$

$mg > \frac{3}{4} mg$

↳ t.g. PREMA DOLJE

(2.) ZAKON KINETIČKE ENERGIJE

$E_{K_2} - E_{K_1} = W_{12}$, $E_{K_1} = 0$ → KREĆE SE IZ NULE

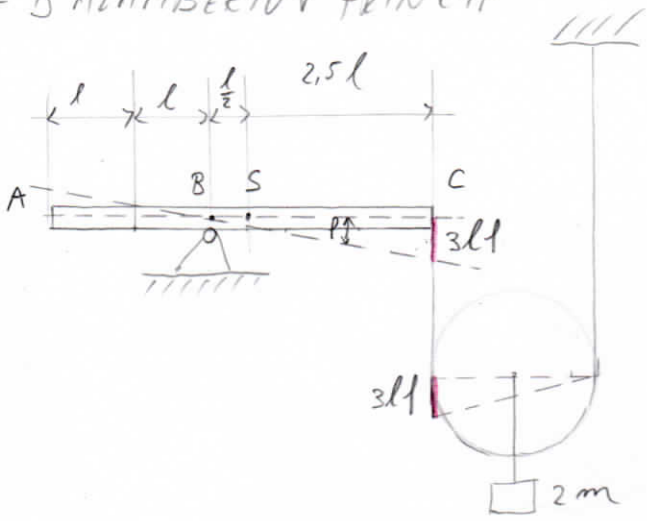
$$E_{K_2} = \frac{2m\,v_1^2}{2} + \frac{J_C\,\omega^2}{2} + \frac{3m\,v_3^2}{2}$$

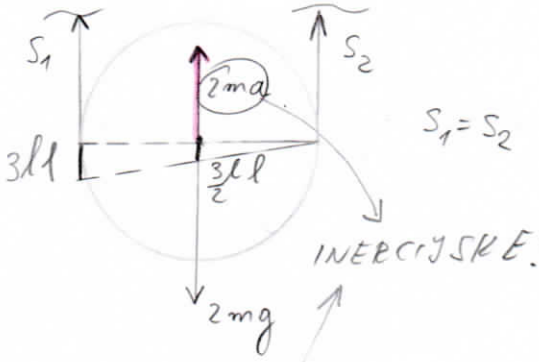
↓ ZBOG A TRANSLACIJE ; ↓ ZBOG ROTACIJE A ; ↓ ZBOG B

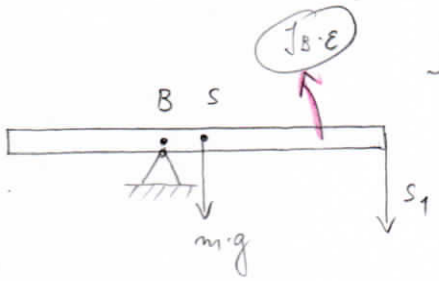
ZADATAK 6

Odredite kutno ubrzanje štapa AC mase m, vezanog kolotursom za masu 2m prema slici. Mase kolotura i užeta se zanemaruju

- D'ALAMBERTOV PRINCIP







ROTACIJA OKO B

STEINEROV POUČAK

$$J = J_{cm} + m \cdot d^2$$

$$J_S = \frac{m l^2}{12}$$

$$J_B = J_S + m\left(\frac{l}{2}\right)^2 = \frac{m \cdot (5l)^2}{12} + m\,\frac{l^2}{4} = ml^2\left(\frac{25}{12} + \frac{1}{4}\right) = \frac{7}{3} m l^2$$

→ za kolaturu:

$$\Sigma F_y = 0 \quad 2S_1 + 2ma - 2mg = 0$$

$$S_1 = mg - ma = mg - \frac{3}{2} m l \varepsilon$$

KINEMATIČKA VEZA

$$y = \frac{3}{2} l \varphi$$

$$\dot{y} = \frac{3}{2} l \dot{\varphi}$$

$$\ddot{y} = \frac{3}{2} l \ddot{\varphi} \qquad a = \frac{3}{2} l \varepsilon$$

→ za štap:

$$\Sigma M_B = 0 \quad J_B \cdot \varepsilon - mg\frac{l}{2} - S_1 \cdot 3l = 0$$

$$\frac{7}{3} m l^2 \varepsilon - mg\frac{l}{2} - 3mgl + \frac{9}{2} m l^2 \varepsilon = 0$$

$$\frac{41}{6} \cdot l \varepsilon = \frac{7}{2} g$$

$$\varepsilon = \frac{21}{41} \frac{g}{l}$$

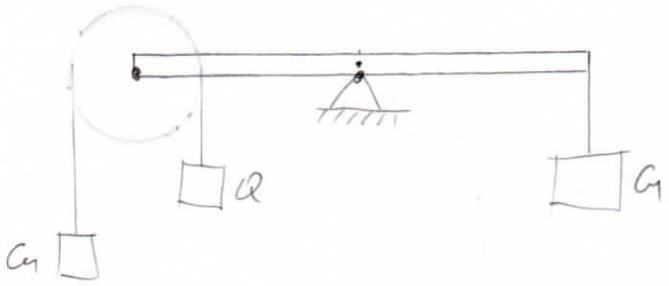
$$a = \frac{3}{2} l \cdot \frac{21}{41} \frac{g}{l}$$

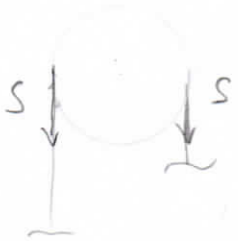
$$a = \frac{63}{82} g$$

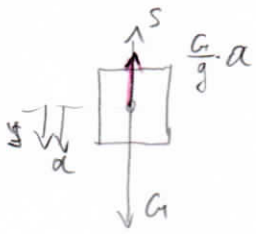
ZADATAK 7

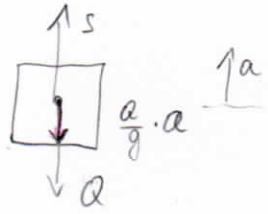
Tri užeta spojena su polugom i kolaturom prema slici. Masa poluge i kolature je zanemariva. Odrediti težinu užeta Q tako da poluga AB u prikazanom položaju miruje.

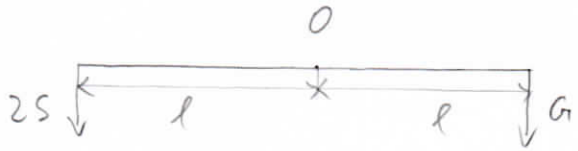
$G, l, G > Q$











$\sum F_y = 0 \qquad S + \frac{G}{g}\cdot a - G = 0$

$\sum F_y = 0 \qquad S - \frac{Q}{g}\cdot a - Q = 0$

$$\frac{G}{2} - G + G\frac{a}{g} = 0$$

$$\frac{a}{g} = \frac{1}{2} \Rightarrow a = \frac{1}{2}g$$

$$\frac{G}{2} - Q\cdot\frac{1}{2} - Q = 0$$

$$\frac{G}{2} = \frac{3}{2}Q$$

$$\boxed{Q = \frac{1}{3}G}$$

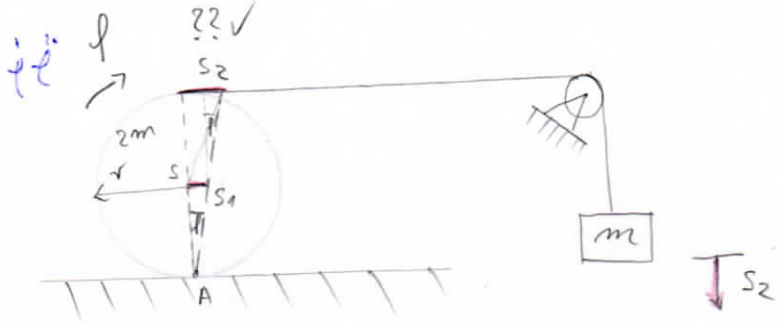
$\sum M_O = 0$

$2Sl = G\cdot l$

$2S = G$

ZADATAK 8

Za sistem zadan prema slici potrebno je odrediti ubrzanja utega i valjka ako je zadano m i r. Treba se zanemariti i valjak se kotrlja po podlozi



KINEMATIČKA VEZA

$2 s_1 = s_2$

$s_1 = r \cdot \varphi$

$\dot{s}_1 = r \cdot \dot{\varphi}$

$\ddot{s}_1 = r \cdot \ddot{\varphi}$

ZOE: $E_k + E_p = konst.$

$E_k = \frac{m \cdot \dot{s}_2^2}{2} + \frac{2m \dot{s}_1^2}{2} + \frac{J_S \cdot \dot{\varphi}^2}{2}$

$\frac{J_A \cdot \dot{\varphi}^2}{2}$ ← STEINER?!

$J_A = J_S + m \cdot r^2$

$E_k = \frac{m \cdot (2\dot{s}_1)^2}{2} + \frac{2m \dot{s}_1^2}{2} + \frac{\frac{2mr^2}{2} \cdot \dot{\varphi}^2}{2}$

$= \frac{m \cdot 4 \dot{s}_1^2}{2} + \frac{2m \dot{s}_1^2}{2} + \frac{2mr^2 \cdot (\frac{\dot{s}_1}{r})^2}{2} = \frac{7}{2} m \cdot \dot{s}_1^2$

$E_p = -mg s_2 = -mg 2 s_1$

$\frac{7}{2} m \cdot \dot{s}_1^2 - 2mg s_1 = konst. \; / \cdot \frac{d}{dt}$

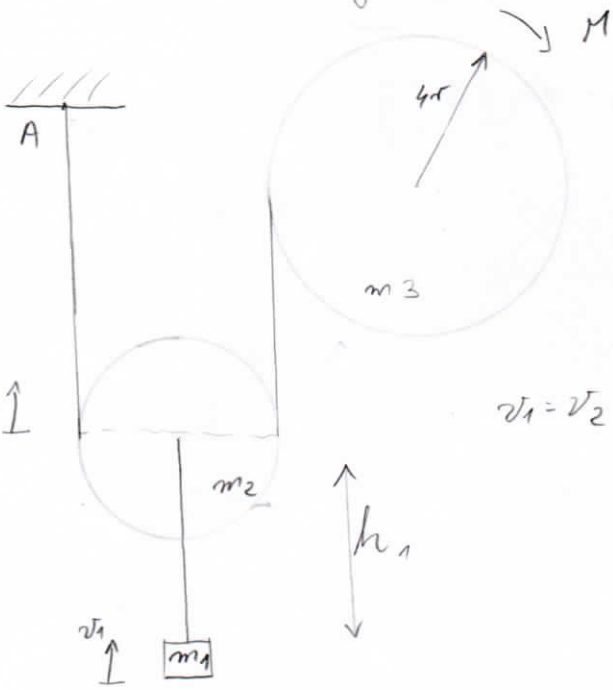
$7m \dot{s}_1 \cdot \ddot{s}_1 - 2mg \dot{s}_1 = 0$

$\ddot{s}_1 = \frac{2}{7} g$

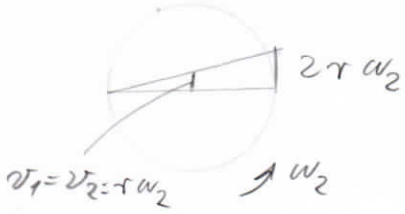
$\ddot{s}_2 = \frac{4}{7} g$

ZADATAK 9.

Teret mase m_1 obješen je na kolotur mase m_2 koji visi na užetu čiji je jedan kraj pričvršćen u A, a drugi je namotan na obod bubnja mase m_3. Ako na bubanj djeluje konstantni moment $M = 10\,mgr$, odredite vrijeme t_1 potrebno da se teret m_1 podigne na visinu $h = 10r$ iz stanja mirovanja. Kolotur i bubanj smatrati homogenim valjcima ($J = \frac{mr^2}{2}$). m, $m_1 = 4m$, $m_2 = 0{,}5m$, $m_3 = m$



$v_1 = v_2$



$4r\omega_3 = 2r\omega_2$

$\omega_3 = \frac{1}{2}\omega_2$

$v_1 = v_2 = r\omega_2$

$\omega_2 = \frac{v_2}{r} = \frac{v_1}{r}$

$\varphi_3 = \frac{h_1}{2r}$

$\varepsilon_2 = \frac{a_1}{r} = \ddot{\varphi}$

$\varepsilon_3 = \frac{a_1}{2r}$

ZOE:

$E_{K_1} = 0$ → kreće iz stanja mirovanja

$E_{K_2} = \underbrace{\frac{m_1 v_1^2}{2}}_{T} + \underbrace{\frac{m_2 v_2^2}{2}}_{T} + \underbrace{\frac{\frac{m_2 r^2}{2}\cdot\omega_2^2}{2}}_{R} + \underbrace{\frac{\frac{m_3\cdot(4r)^2}{2}\cdot\omega_3^2}{2}}_{R}$

$= \frac{27}{8} m v_1^2$

$E_{K_2} - E_{K_1} = W = W_G + W_{MOM}$

$W_G = -m_1 g\cdot h_1 - m_2\cdot g\cdot h_2 = -4mgh_1 - \frac{1}{2}mgh_1 = -\frac{9}{2}mgh_1$

$W_{MOM} = M\cdot\varphi_3 = 10\,mgr\cdot\frac{h_1}{2r} = 5mgh_1$

$$E_{K2} - E_{K1} = W_{a_1} + W_{MOM}$$

$$\frac{27}{8} m v_1^2 = -\frac{9}{2} mgh_1 + 5gh_1$$

$$\frac{27}{8} m v_1^2 = \frac{1}{2} mgh_1 \quad / \frac{d}{dt}$$

$$\frac{27}{8} m \cdot 2 v_1 a_1 = \frac{1}{2} m g v_1$$

$$a_1 = \frac{2}{27} g \qquad h_1 = a_1 \frac{t_1^2}{2} \Rightarrow t_1 = \sqrt{\frac{2h_1}{a_1}}$$

$$t_1 = 16{,}53 \sqrt{\frac{r}{g}}$$

$$v = \int_0^t a \, dt$$

$$v = a \cdot t$$

$$s = a \frac{t^2}{2}$$