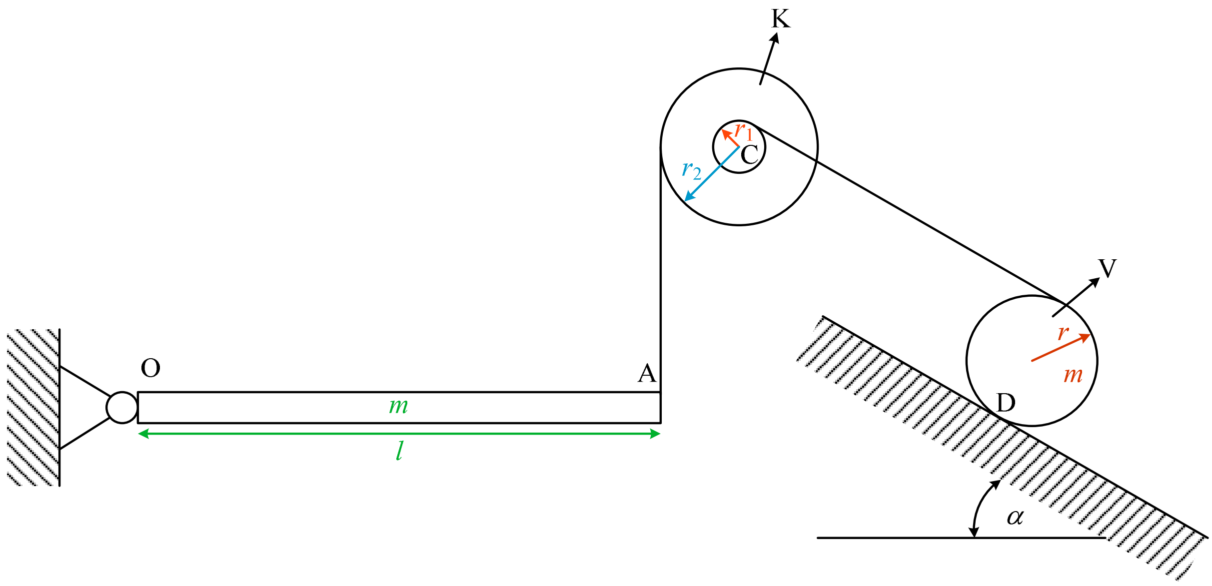
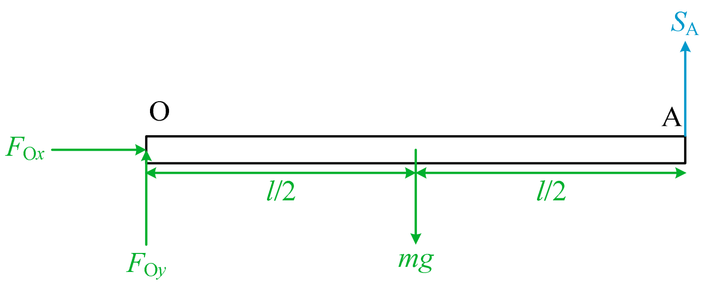
TEHNIČKA MEHANIKA
Završni ispit
4.2.2013.

1. Zadan je sustav prema slici. Štap OA ima masu m i duljinu l. Kolotura K ima zanemarivu masu. Valjak V ima polumjer r i masu m. Vrijedi: $l = 4r$, $r_2 = 3r_1$, $\alpha = 30°$. Potrebno je odrediti kutno ubrzanje ε_1 štapa OA, kutno ubrzanje valjka ε_2 i ubrzanje središta valjka a_2.

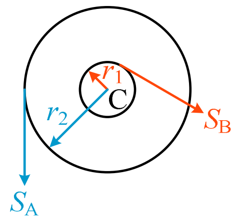


Rješenje:

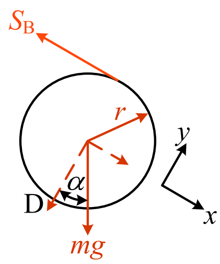
1. Statička analiza



$$\sum M_O = 0 \rightarrow -mg\frac{l}{2} + S_A l = 0 \rightarrow S_A = \frac{1}{2}mg \quad \mathbf{(1)}$$



$$\sum M_C = 0 \rightarrow S_A r_2 - S_B r_1 = 0 \rightarrow S_B = \frac{r_2}{r_1} S_A \quad \mathbf{(2)}$$



$$\sum M_D = 0 \rightarrow S_B \cdot 2r - mgr \sin\alpha \,?\, 0 \rightarrow 2S_B ? mg \sin\alpha \quad (\mathbf{3})$$

? je potrebno odrediti. Prvu jednadžbu ubacimo u drugu, a zatim drugu u treću:

$$S_B = \frac{r_2}{r_1} S_A = \frac{1}{2}\frac{r_2}{r_1} mg$$

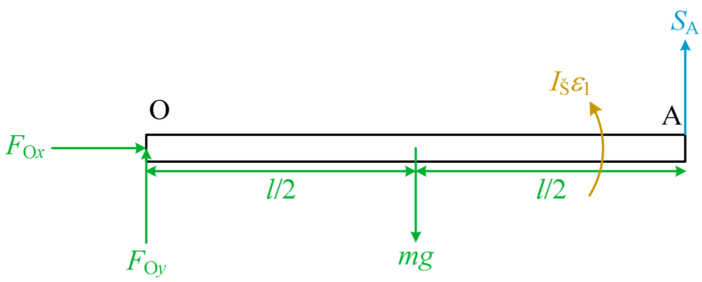
$$2\frac{1}{2}\frac{r_2}{r_1} mg ? mg \sin\alpha$$

$$3 ? \sin 30°$$

$$3 ? \frac{1}{2}$$

Slijedi da je ? zapravo >. T**endencija gibanja je uz kosinu**.

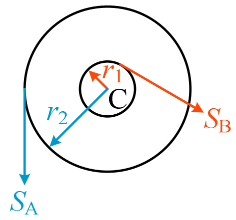
<u>2. Jednadžbe ravnoteže</u>



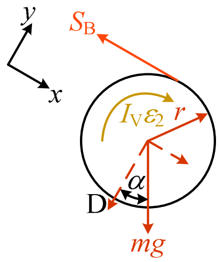
$$\sum M_O = 0 \rightarrow -mg\frac{l}{2} + S_A l + I_{š}\varepsilon_1 = 0$$

$$I_{š} = I_{CMš} + md^2 = \frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2 = \frac{ml^2}{12} + \frac{ml^2}{4} = \frac{1}{3}ml^2$$

$$-mg\frac{l}{2} + S_A l + \frac{1}{3}ml^2\varepsilon_1 = 0 \rightarrow -3mg + 6S_A + 2ml\varepsilon_1 = 0 \quad (\mathbf{4})$$



$$\sum M_C = 0 \rightarrow S_A r_2 - S_B r_1 = 0 \rightarrow S_B = \frac{r_2}{r_1} S_A \quad \mathbf{(5)}$$



$$\sum M_D = 0 \rightarrow S_B \cdot 2r - mgr \sin\alpha - I_V \varepsilon_2 = 0$$

$$I_V = I_{CMV} + md^2 = \frac{mr^2}{2} + mr^2 = \frac{3}{2} mr^2$$

$$a_2 = \varepsilon_2 r$$

$$S_B \cdot 2r - mgr \sin\alpha - \frac{3}{2} mr^2 \varepsilon_2 = 0 \rightarrow 4S_B = 2mg \sin\alpha + 3mr\varepsilon_2 \quad \mathbf{(6)}$$

(5) u **(6)**:

$$4\frac{r_2}{r_1} S_A = 3mg \sin\alpha + 3mr\varepsilon_2 \rightarrow S_A = \frac{1}{4}\frac{r_1}{r_2}(2mg \sin\alpha + 3mr\varepsilon_2) \quad \mathbf{(7)}$$

(7) u **(4)**:

$$-3mg + 6\frac{1}{4}\frac{r_1}{r_2}(2mg \sin\alpha + 3mr\varepsilon_2) + 2ml\varepsilon_1 = 0$$

$$-3g + 6\frac{1}{4}\frac{1}{3}(2g \sin\alpha + 3r\varepsilon_2) + 2l\varepsilon_1 = 0 \rightarrow -3g + \frac{1}{2}(2g \sin\alpha + 3r\varepsilon_2) + 2l\varepsilon_1 = 0$$

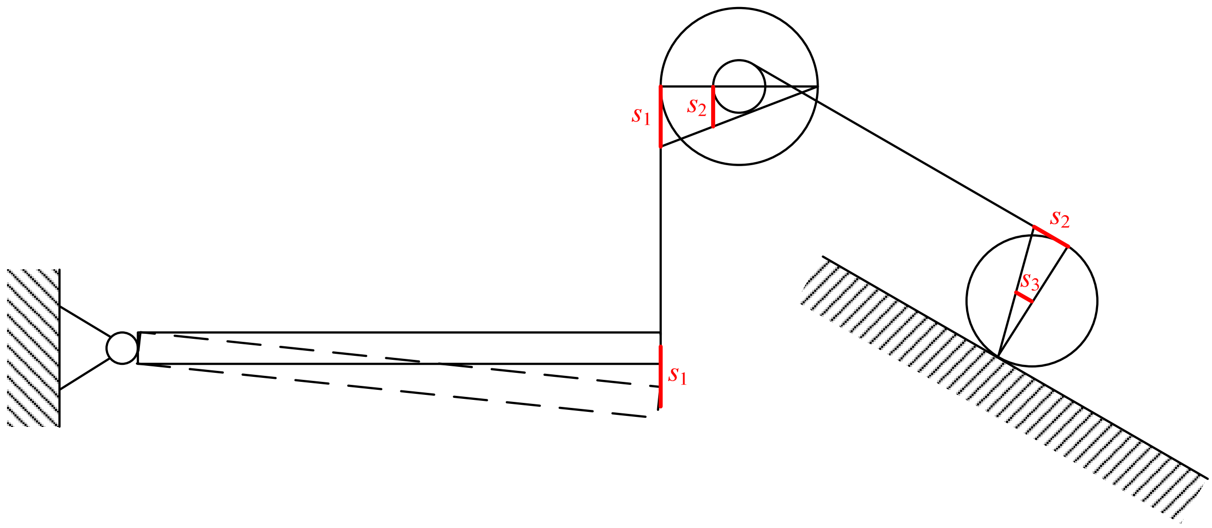
$$-6g + g + 3r\varepsilon_2 + 16r\varepsilon_1 = 0 \rightarrow 16r\varepsilon_1 + 3r\varepsilon_2 = 5g \quad \mathbf{(8)}$$

3. Kinematička veza

(Slika na sljedećoj stranici.)

$$s_1 : r_2 = s_2 : r_1 \rightarrow s_1 r_1 = s_2 r_2 \rightarrow s_2 = \frac{r_1}{r_2} s_1 \quad \mathbf{(9)}$$

$$s_2 : 2r = s_3 : r \rightarrow s_2 r = 2 s_3 r \rightarrow s_3 = \frac{1}{2} s_2 \quad \mathbf{(10)}$$



(9) u **(10)**:

$$s_3 = \frac{1}{2}\frac{r_1}{r_2}s_1 = \frac{1}{6}s_1 \rightarrow r\varphi_2 = \frac{1}{6}l\varphi_1 \rightarrow r\varphi_2 = \frac{1}{6}4r\varphi_1 \rightarrow \varphi_2 = \frac{2}{3}\varphi_1 \rightarrow \ddot{\varphi}_2 = \frac{2}{3}\ddot{\varphi}_1 \rightarrow \varepsilon_2 = \frac{2}{3}\varepsilon_1 \quad (\mathbf{11})$$

(11) u **(8)**:

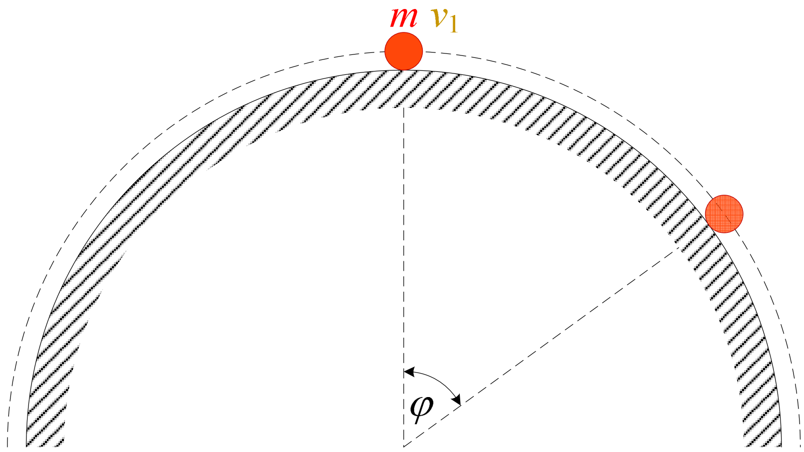
$$16r\varepsilon_1 + 3r\frac{2}{3}\varepsilon_1 = 5g \rightarrow 18r\varepsilon_1 = 5g \rightarrow \boldsymbol{\varepsilon_1 = \frac{5}{18}\frac{g}{r}} \quad (\mathbf{12})$$

(12) u **(11)**:

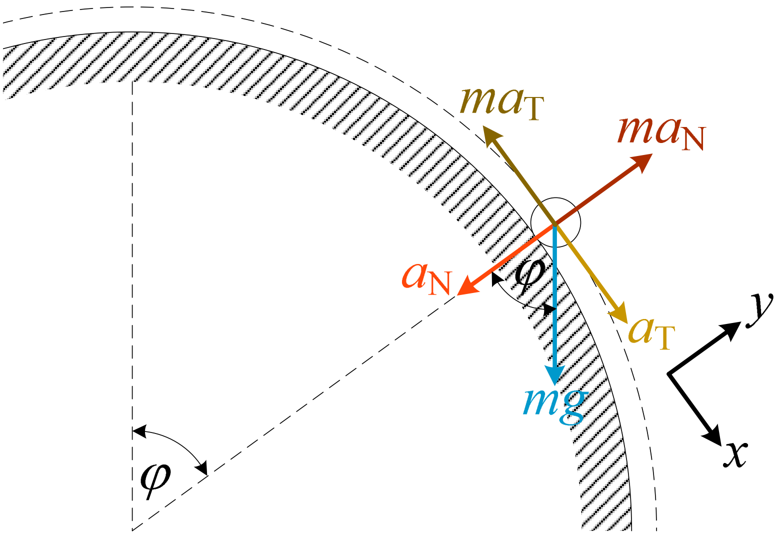
$$\boldsymbol{\varepsilon_2} = \frac{2}{3}\varepsilon_1 = \frac{2}{3}\frac{5}{18}\frac{g}{r} = \boldsymbol{\frac{5}{27}\frac{g}{r}} \quad (\mathbf{13})$$

$$\boldsymbol{a_2} = \varepsilon_2 r = \boldsymbol{\frac{5}{27}g} \quad (\mathbf{14})$$

2. Malo tijelo mase *m* nalazi se na vrhu glatkog cilindra i počinje klizati po cilindru. Nađi kut φ u odnosu na vertikalu pod kojim se tijelo odvaja od cilindra. Početna brzina tijela je jednaka v_1 = 2 m/s, a r = 1 m.



Rješenje:



Granični slučaj:

$$\sum F_x = 0 \rightarrow mg\sin\varphi - ma_T = 0 \rightarrow g\sin\varphi = a_T \rightarrow g\sin\varphi = r\varepsilon \rightarrow \varepsilon = \frac{g}{r}\sin\varphi \quad (\mathbf{1})$$

$$\sum F_y = 0 \rightarrow mg\cos\varphi - ma_N = 0 \rightarrow g\cos\varphi = a_N \rightarrow g\cos\varphi = r\omega_2^2 = \frac{v_2^2}{r} \rightarrow v_2^2 = gr\cos\varphi \quad (\mathbf{2})$$

$$\varepsilon = \frac{d\omega}{dt} \quad (\mathbf{3})$$

$$\omega = \frac{d\varphi}{dt} \rightarrow dt = \frac{d\varphi}{\omega} \quad (\mathbf{4})$$

$$v = \omega r \rightarrow \omega = \frac{v}{r} \rightarrow d\omega = \frac{1}{r}dv \quad (\mathbf{5})$$

(4) u **(3)**:

$$\varepsilon = \frac{d\omega}{dt} = \frac{d\omega}{\frac{d\varphi}{\omega}} \rightarrow \varepsilon d\varphi = \omega d\omega \quad (\mathbf{6})$$

(5) u **(6)**:

$$\varepsilon d\varphi = \omega d\omega = \frac{v}{r}\frac{1}{r}dv = \frac{v}{r^2}dv \quad (\mathbf{7})$$

(1) u **(7)**:

$$\varepsilon d\varphi = \frac{v}{r^2}dv$$

$$\frac{g}{r}\sin\varphi\, d\varphi = \frac{v}{r^2}dv \quad (\mathbf{8})$$

Integriranje obiju strana:

$$\int_0^{\varphi} \frac{g}{r}\sin\varphi\, d\varphi = \int_{v_1}^{v_2} \frac{v}{r^2}dv$$

$$-\frac{g}{r}\cos\varphi\Bigg|_0^{\varphi} = \frac{1}{2r^2}v^2\Bigg|_{v_1}^{v_2}$$

$$-\frac{g}{r}(\cos\varphi - 1) = \frac{1}{2r^2}(v_2^2 - v_1^2) \quad (\mathbf{9})$$

U **(9)** se ubace $v_1 = 2$ m/s, $r = 1$ m i izraz **(2)**:

$$-\frac{g}{r}(\cos\varphi - 1) = \frac{1}{2r^2}(v_2^2 - v_1^2)$$

$$-\frac{g}{r}(\cos\varphi - 1) = \frac{1}{2r^2}(gr\cos\varphi - v_1^2)$$

$$-g(\cos\varphi - 1) = \frac{1}{2}(g\cos\varphi - 4)$$

$$g + 2 = \frac{3}{2}g\cos\varphi$$

$$\cos\varphi = \frac{g+2}{\frac{3}{2}g} = \frac{9{,}81 + 2}{\frac{3}{2}\cdot 9{,}81} \rightarrow \boldsymbol{\varphi} = \arccos 0{,}8026 = \mathbf{36{,}623^\circ} \quad (\mathbf{10})$$