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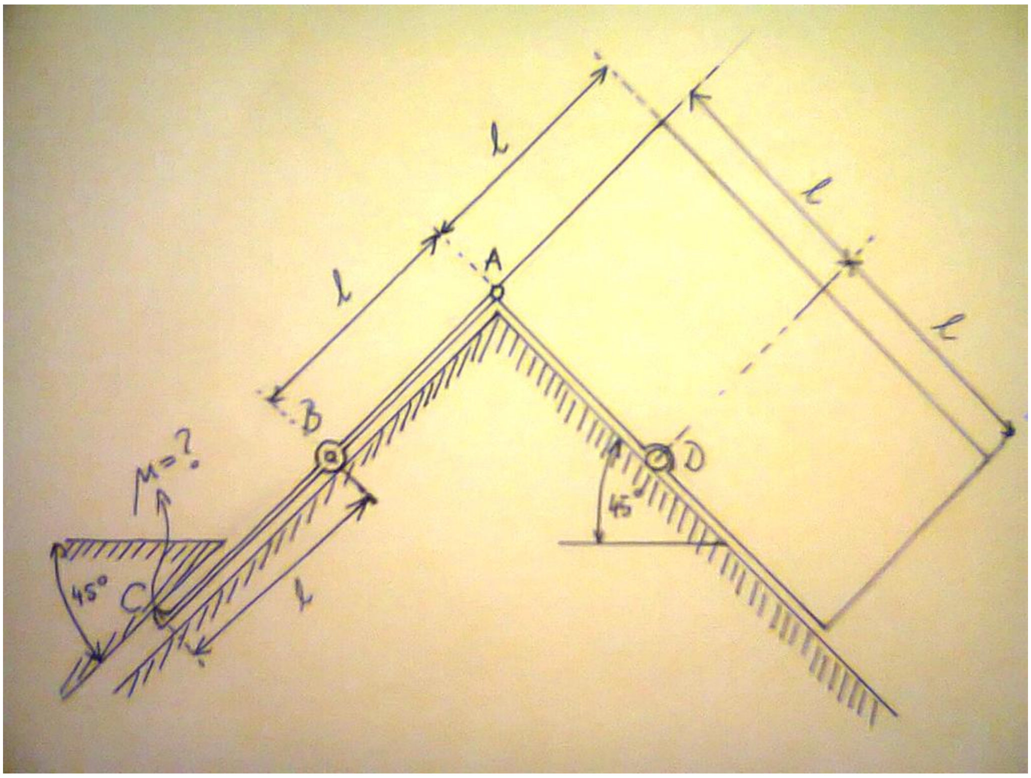
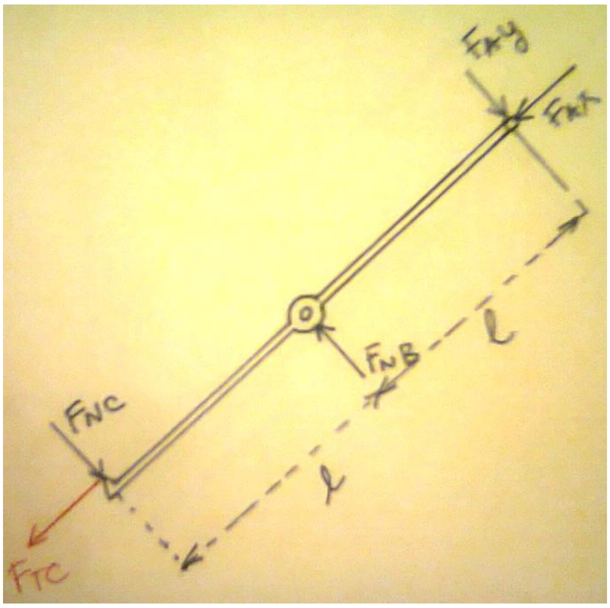
Međuispit
27.11.2012.

1. Štap ABC nema težine. Blok AD ima težinu G. Blok i štap zglobno su vezani u točci A. U točkama B i D nalaze se kotačići kod kojih nema trenja. Odredite minimalni iznos koeficijenta trenja μ tako da sustav ostane u ravnoteži.



Rješenje:

Štap ABC:



$\sum F_x = 0 \rightarrow -F_{Ax} - F_{TC} = 0 \rightarrow F_{Ax} = -F_{TC} = -\mu F_{NC}$ (1)

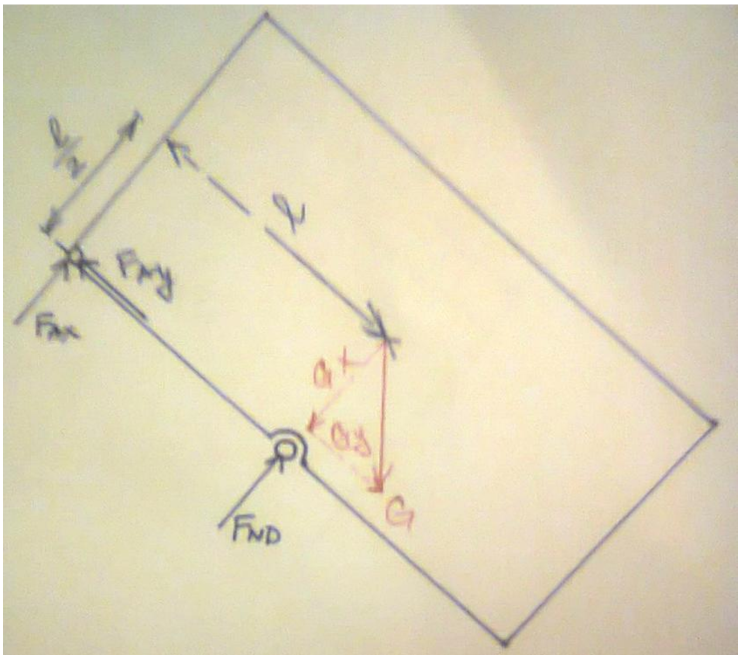
$\sum F_y = 0 \rightarrow -F_{Ay} + F_{NB} - F_{NC} = 0 \rightarrow F_{Ay} = F_{NB} - F_{NC}$ (2)

$\sum M_A = 0 \rightarrow -F_{NB} l + F_{NC} 2l = 0 \rightarrow F_{NB} = 2F_{NC}$ (3)

(3) → (2):

$F_{Ay} = F_{NB} - F_{NC} = 2F_{NC} - F_{NC} = F_{NC}$ (4)

Blok AD:



$\sum F_x = 0 \rightarrow F_{Ax} + F_{ND} - G_x = 0 \rightarrow F_{Ax} = G_x - F_{ND} = G\frac{\sqrt{2}}{2} - F_{ND}$ (5)

$\sum F_y = 0 \rightarrow F_{Ay} - G_y = 0 \rightarrow F_{Ay} = G_y = G\frac{\sqrt{2}}{2}$ (6)

$\sum M_A = 0 \rightarrow F_{ND} l - G_x l - G_y \frac{l}{2} = 0 \rightarrow F_{ND} = G_x + \frac{G_y}{2} = G\frac{\sqrt{2}}{2} + G\frac{\sqrt{2}}{4} = G\frac{3\sqrt{2}}{4}$ (7)

(7) → (5):

$F_{Ax} = G\frac{\sqrt{2}}{2} - F_{ND} = G\frac{\sqrt{2}}{2} - G\frac{3\sqrt{2}}{4} = -G\frac{\sqrt{2}}{4}$ (8)

Izjednače se (4) i (6):

$F_{NC} = G\frac{\sqrt{2}}{2}$ (9)

(9) se ubaci u (1):

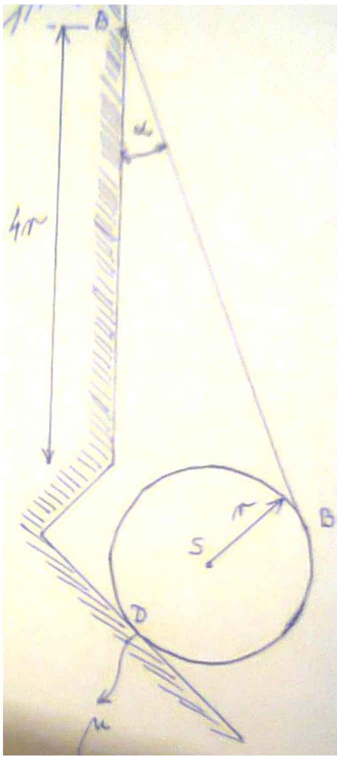
$F_{Ax} = -\mu F_{NC} = -\mu G \frac{\sqrt{2}}{2}$ (10)

Izjednače se (8) i (10):

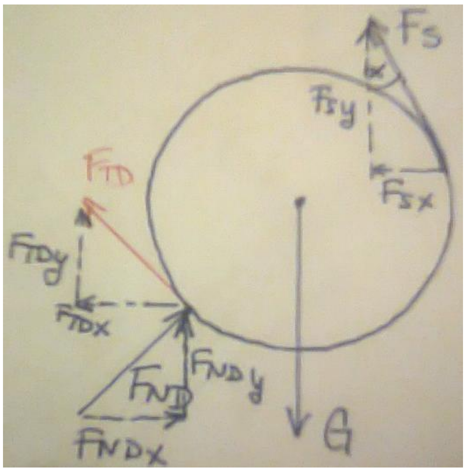
$$-G\frac{\sqrt{2}}{4} = -\mu G\frac{\sqrt{2}}{2}$$

$$\boldsymbol{\mu = 0,5}$$

2. Kugla ima težinu $G = 8\ kN$. Odrediti μ za koji je sustav u ravnoteži i odrediti silu u užetu AB.



Rješenje:



$\sum F_x = 0 \rightarrow F_{NDx} - F_{TDx} - F_{Sx} = 0 \rightarrow F_{ND}\frac{\sqrt{2}}{2} - \mu F_{ND}\frac{\sqrt{2}}{2} - F_S \sin\alpha = 0$

$F_S \sin\alpha = F_{ND}\frac{\sqrt{2}}{2}(1-\mu) \rightarrow F_{ND} = \frac{F_S \sin\alpha}{\frac{\sqrt{2}}{2}(1-\mu)}$ **(1)**

$\sum F_y = 0 \rightarrow F_{NDy} + F_{TDy} + F_{Sy} - G = 0 \rightarrow F_S \cos\alpha = -F_{ND}\frac{\sqrt{2}}{2} - \mu F_{ND}\frac{\sqrt{2}}{2} + G$

$F_S \cos\alpha = -F_{ND}\frac{\sqrt{2}}{2}(1+\mu) + G$ **(2)**

$\sum M_D = 0 \rightarrow -G\frac{r}{\sqrt{2}} + F_{Sx}\left(\frac{r}{\sqrt{2}} + r\sin\alpha\right) + F_{Sy}\left(\frac{r}{\sqrt{2}} + r\cos\alpha\right) = 0$

$G\frac{r}{\sqrt{2}} = F_S \sin\alpha\left(\frac{r}{\sqrt{2}} + r\sin\alpha\right) + F_S\cos\alpha\left(\frac{r}{\sqrt{2}} + r\cos\alpha\right) \rightarrow G = F_S\left(\sin\alpha + \cos\alpha + \sqrt{2}\right)$

$F_S = \frac{G}{\sin\alpha + \cos\alpha + \sqrt{2}}$ **(3)**

(1) → **(2)**:

$F_S \cos\alpha = -F_{ND}\frac{\sqrt{2}}{2}(1+\mu) + G = -\frac{F_S \sin\alpha}{\frac{\sqrt{2}}{2}(1-\mu)}\frac{\sqrt{2}}{2}(1+\mu) + G$

$F_S \cos\alpha = -\frac{F_S \sin\alpha}{(1-\mu)}(1+\mu) + G$

$(1-\mu)F_S\cos\alpha + (1+\mu)F_S\sin\alpha = G(1-\mu)$

$\mu = \frac{F_S\cos\alpha + F_S\sin\alpha - G}{F_S\cos\alpha - F_S\sin\alpha - G}$ **(4)**

$$\operatorname{tg}\alpha = \frac{r+c}{4r} = \frac{r + \frac{r}{\cos\alpha}}{4r} = \frac{\cos\alpha + 1}{4\cos\alpha}$$

$$\frac{\sin\alpha}{\cos\alpha} = \frac{\cos\alpha + 1}{4\cos\alpha} \rightarrow 4\sin\alpha - \cos\alpha = 1 \rightarrow \cos\alpha = 4\sin\alpha - 1$$

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\sin^2\alpha + (4\sin\alpha - 1)^2 = 1 \rightarrow \sin\alpha = \frac{8}{17}$$

$$\cos\alpha = \sqrt{1 - \left(\frac{8}{17}\right)^2} = \frac{15}{17}$$

Ovo se ubaci u **(3)** pa slijedi:

$$\boldsymbol{F_S} = \frac{G}{\sin\alpha + \cos\alpha + \sqrt{2}} = \frac{8\ kN}{\frac{8}{17} + \frac{15}{17} + \sqrt{2}} = \mathbf{2,89106}\ \boldsymbol{kN}$$

Ako se ti sinusi i kosinusi ubace u **(4)**, slijedi:

$$\boldsymbol{\mu} = \frac{F_S\cos\alpha + F_S\sin\alpha - G}{F_S\cos\alpha - F_S\sin\alpha - G} = \frac{\frac{G}{\sin\alpha + \cos\alpha + \sqrt{2}}(\cos\alpha + \sin\alpha) - G}{\frac{G}{\sin\alpha + \cos\alpha + \sqrt{2}}(\cos\alpha - \sin\alpha) - G} =$$

$$= \frac{\frac{\cos\alpha + \sin\alpha}{\sin\alpha + \cos\alpha + \sqrt{2}} - 1}{\frac{\cos\alpha - \sin\alpha}{\sin\alpha + \cos\alpha + \sqrt{2}} - 1} = \frac{\cos\alpha + \sin\alpha - \sin\alpha - \cos\alpha - \sqrt{2}}{\cos\alpha - \sin\alpha - \sin\alpha - \cos\alpha - \sqrt{2}} = \frac{-\sqrt{2}}{-2\sin\alpha - \sqrt{2}} = \frac{\sqrt{\mathbf{2}}}{\sqrt{\mathbf{2}} + \mathbf{2}\,\mathbf{sin}\,\boldsymbol{\alpha}}$$

$$\mu = \frac{\sqrt{2}}{\sqrt{2} + 2\frac{8}{17}} = \mathbf{0,600416}$$