MI 2015./2016.

1.
$$Y(j\omega) = G(j\omega)U(j\omega) + V(j\omega)$$

$$u(t) = \mathcal{F}^{-1}\{U(j\omega)\} \qquad v(t) = \mathcal{F}^{-1}\{V(j\omega)\}$$

$$u(t) = u^*(t) + w(t)$$

$$v(t) = v^*(t) + w(t)$$

$$R_{uy} = \int_0^\infty R_{uu}(\tau-\sigma)g(\sigma)d\sigma \rightarrow \text{kad nema korelacije } u \text{ i } v$$

$$S_{uy}(j\omega) = S_{uu}(\omega)\cdot G(j\omega) \Rightarrow \hat{G}(j\omega) = \frac{S_{uy}(j\omega)}{S_{uu}(\omega)}$$

$$R_{uy}(\tau) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} u(t)\,y(t+\tau)\,dt$$

$$y = y_u + v$$

$$R_{uy}(\tau) = \underbrace{\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} u(t)\,y_u(t+\tau)dt}_{\text{nema šuma}} + \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} u(t)v(t+\tau)\,dt$$

$$R_{uy}(\tau) = \int_0^\infty R_{uu}(\tau-\sigma)g(\sigma)d\sigma + \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}(u^*(t)+w(t))(v^*(t+\tau)+w(t+\tau))\,dt$$

$v^*$ i $u^*$ ne koreliraju ni s čim (bijeli šum)

$$R_{uy}(\tau) = \underbrace{\int_0^\infty R_{uu}(\tau-\sigma)g(\sigma)d\sigma}_{R_{uu} * g} + \underbrace{\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} w(t)w(t+\tau)\,dt}_{R_{ww}(\tau)}$$

$$R_{uy}(\tau) = R_{uu} * g + R_{ww}(\tau)$$

$$S_{uy}(j\omega) = S_{uu}(\omega)\cdot G(j\omega) + S_{ww}(\omega) \quad /:S_{uu}$$

$$G(j\omega) = \frac{S_{uy}(j\omega)}{S_{uu}(\omega)} - \frac{S_{ww}(\omega)}{S_{uu}(\omega)}$$

$$\hat{G}(j\omega) - G(j\omega) = \frac{S_{ww}(\omega)}{S_{uu}(\omega)}$$

Razlika između stvarne i estimirane frekvencijske karakteristike je tim manja što je manji signal smetnje $w(t)$ i što je jača pobuda $u(t)$.

2.
$$\mathcal{L}(\mu,\sigma^2;x_i) = \prod_{i=1}^{n}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2}$$

$$\ln\mathcal{L}(\mu,\sigma^2;x_i) = \sum_{i=1}^{n}\left(-\ln(\sqrt{2\pi}\sigma) - \frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2\right)$$

$$= -\frac{n}{2}\ln(2\pi) - n\ln\sigma - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2 \quad \Big/\frac{d}{d\mu} \quad \Big/\frac{d}{d\sigma}$$

$$0 = -\frac{1}{2\sigma^2}\cdot 2\sum_{i=1}^{n}(x_i-\mu)(-1) + \frac{1}{\sigma^2}\left(\sum_{i=1}^{n}x_i - n\mu\right) = 0 \qquad \boxed{\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n}x_i}$$

$$0 = -\frac{n}{\sigma} + \frac{2}{2}\cdot\frac{1}{\sigma^3}\sum_{i=1}^{n}(x_i-\mu)^2 \quad /\cdot\sigma^3$$

$$n\sigma^2 = \sum_{i=1}^{n}(x_i-\mu)^2 \Rightarrow \qquad \boxed{\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\hat{\mu})^2}$$

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

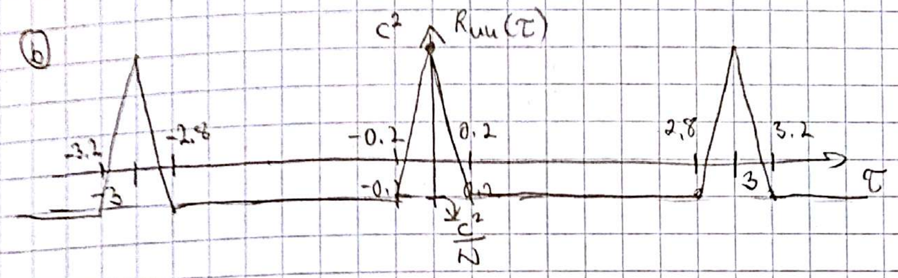
$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\hat{\mu})^2$$

3. a) PARAMETRI PRBS SIGNALA:

$c = 2$

$\Delta t = 0.2\ s \qquad N = 2^m - 1 = 2^4 - 1 = 15$

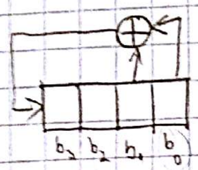
b)




$c^2 = 4$

$\frac{c^2}{N} = \frac{4}{15}$

c)




$b_0(t) = 0 \rightarrow u(t) = -c$

$b_0(t) = 1 \rightarrow u(t) = c$

početno stanje

1010

$b_0(0) = 0$

$b_0(1\cdot\Delta t) = b_1(0) = 1$

$b_0(2\cdot\Delta t) = b_1(\Delta t) = b_2(0) = 0$

$b_0(3\cdot\Delta t) = b_3(0) = 1$

$b_0(4\cdot\Delta t) = b_0(0) \oplus b_1(0)$

$\qquad = b_0((4-4)\Delta t) \oplus b_0((4-3)\Delta t)$

$b_0(k\cdot\Delta t) = b_0((k-4)\Delta t) \oplus b_0((k-3)\Delta t)$

$b_0(13\cdot\Delta t) = 0 \oplus 1 = 1 \qquad \rightarrow u(13\cdot\Delta t) = 2$

$b_0(14\cdot\Delta t) = 1 \oplus 0 = 1 \qquad \rightarrow u(14\cdot\Delta t) = 2$

4. $G_s(j\omega) = \frac{3}{2j\omega+1}, \quad S_{yy}(\omega) = \frac{9}{\omega^2+9}\cdot\frac{1}{4\omega^2+1}$

a) $S_{yy} = |G_s(j\omega)|^2 \cdot S_{uu} \qquad |G_s(j\omega)| = \frac{3}{\sqrt{4\omega^2+1}}$

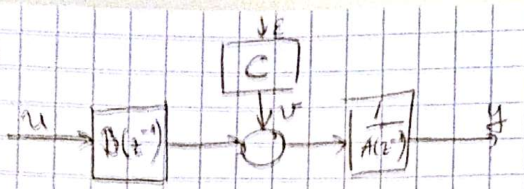
$\frac{\cancel{9}}{\omega^2+9}\cdot\frac{1}{\cancel{4\omega^2+1}} = \frac{\cancel{9}}{\cancel{4\omega^2+1}}\, S_{uu} \implies \boxed{S_{uu} = \frac{1}{\omega^2+9}}$

b) $\bar{P}_u = R_{uu}(0) = \frac{1}{\pi}\int_0^\infty S_{uu}(\omega)\,d\omega = \frac{1}{\pi}\int_0^\infty \frac{d\omega}{\omega^2+9} = \frac{1}{\pi}\int_0^\infty \frac{1}{9}\cdot\frac{d\omega}{\left(\frac{\omega}{3}\right)^2+1} = \left| \begin{array}{l} t = \frac{\omega}{3} \\ 3dt = d\omega \end{array} \right|$

$= \frac{1}{\pi}\int_0^\infty \frac{3dt}{9(t^2+1)} = \frac{1}{3\pi}\cdot \operatorname{arctg} t \Big|_0^\infty = \frac{1}{3\pi}\cdot\frac{\pi}{2} = \frac{1}{6}$

$\boxed{\bar{P}_u = \frac{1}{6}}$

5. a)




$$Y = \frac{B}{A} U + \frac{C}{A} E \quad , \quad V = CE$$

$$AY = BU + \underbrace{CE}_{V} \rightarrow \quad V = AY - BU$$

$$y(k) = -0.615\, y(k-1) - 0.0851\, y(k-2) + 0.2u(k-1) + v(k)$$

$$v(k) = \varepsilon(k) + 0.4\, \varepsilon(k-1)$$

b) $E[y(3)] = -0.615\, y(2) - 0.0851\, y(1) + 0.2u(2) + 0.4 \cdot \varepsilon(2) + \overset{=0}{E[\varepsilon(3)]}$ bijeli šum

$$\varepsilon(k) = y(k) + 0.615\, y(k-1) + 0.0851\, y(k-2) - 0.2u(k-1) + 0.4\, \varepsilon(k-1)$$

$$\varepsilon(0) = y(0) = 0.1$$

$$\varepsilon(1) = y(1) + 0.615\, y(0) - 0.2u(0) - 0.4\, \overset{=y(0)}{\varepsilon(0)}$$

$$\varepsilon(2) = y(2) + 0.615\, y(1) + 0.0851\, y(0) - 0.2u(1) - 0.4\, \varepsilon(1)$$

$$\varepsilon(1) = 0.2 + 0.215 \cdot 0.1 - 0.2 \cdot 2 =$$

$$\varepsilon(2) =$$

c) Deterministički dio $\frac{B}{A}$ $\Rightarrow$ B ima jednu nulu

A nema tu nulu

Dakle, red modela ne može se smanjiti

6. $Y = \frac{B}{A} U + \frac{V}{A} \qquad V = \frac{1}{D} E$

$$v(k) - d_1 v(k-1) = \varepsilon(k)$$

$$v(k) = y(k) + a_1 y(k-1) - b_1 u(k-1)$$

$$\varepsilon(k) = y(k) + a_1 y(k-1) - b_1 u(k-1) + d_1 \left(y(k-1) + a_1 y(k-2) - b_1 u(k-2)\right)$$

$k = 1, \dots, 5$

$$\varepsilon(N) = [\varepsilon(3) \quad \varepsilon(4) \quad \varepsilon(5)]^T$$

$$Y(N) = \Phi(N) \cdot \underline{\theta} + \varepsilon(N) \quad , \quad \underline{\theta} = d_1$$

$$\varphi(k) = -y(k-1) - a_1 y(k-2) + b_1 u(k-2) \qquad \Phi(N) = \begin{bmatrix} \varphi(3) \\ \varphi(4) \\ \varphi(5) \end{bmatrix}$$

$$Y(N) = \begin{bmatrix} y(3) + a_1 y(2) - b_1 u(2) \\ y(4) + a_1 y(3) - b_1 u(3) \\ y(5) + a_1 y(4) - b_1 u(4) \end{bmatrix}$$

(7.) $G(z) = \dfrac{(1+a_1) z^{-1}}{1+a_1 z^{-1}}$

$$UG + V = Y$$

$$(1+a_1) z^{-1} U + (1+a_1 z^{-1}) V = (1+a_1 z^{-1}) Y$$

$$y(k) = -a_1 y(k-1) + (1+a_1) u(k-1) \overbrace{+ v(k) + a_1 v(k-1)}^{v'(k)}$$

Po metodi IV ne zanimaju nas statistička svojstva smetnje $v'(k)$

$$y(k) - u(k-1) = a_1(-y(k-1) + u(k-1)) + \underbrace{v'(k)}_{\text{ne zanima me}}$$

$$v' = Y - \phi \vartheta$$

$$v' = \begin{bmatrix} v'(2) \\ v'(3) \\ v'(4) \\ v'(5) \end{bmatrix} \quad Y = \begin{bmatrix} y(2) - u(1) \\ y(3) - u(2) \\ y(4) - u(3) \\ y(5) - u(4) \end{bmatrix} \quad \phi = \begin{bmatrix} -y(1) + u(1) \\ -y(2) + u(2) \\ -y(3) + u(3) \\ -y(4) + u(4) \end{bmatrix}$$

$$W = \begin{bmatrix} -y_H(1) + u(1) \\ -y_H(2) + u(2) \\ -y_H(3) + u(3) \\ -y_H(4) + u(4) \end{bmatrix}$$

$y_H(1) = 0$ , $y_H(2) = 0.4 u(1)$ , $y_H(3) = 0.24 u(1) + 0.4 u(2)$

$$y_H(k) = -\hat{a}_1 y_H(k-1) + (1+\hat{a}_1) u(k-1)$$

$$= 0.6 y_H(k-1) + 0.4 u(k-1)$$

$$y_H(4) = 0.144 u(1) + 0.24 u(2) + 0.4 u(3)$$