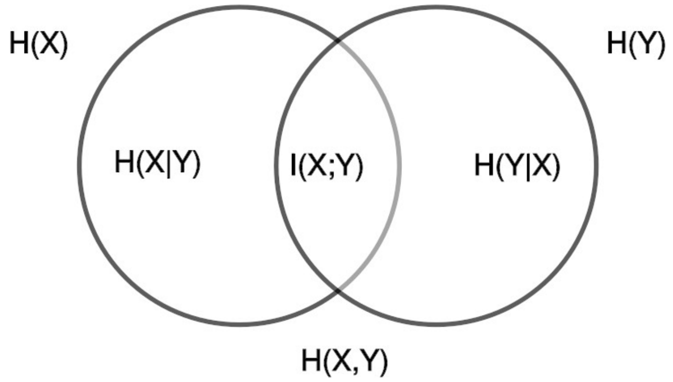
Diskretni kapaciteti

Kanal bez gubitka informacije	$p(X\mid Y)=1 \Vert 0$, *samo jedna* 1 *po stupcu* $C=\max\limits_{p(x_i)}(H(X)-\underbrace{H(X\mid Y)}_{0})=\max\limits_{p(x_i)}(H(X))=\log_2(n)$
Deterministički kanal	$p(Y\mid X)=1 \Vert 0$, *samo jedna* 1 *po retku* $C=\max\limits_{p(x_i)}(H(Y)-\underbrace{H(Y\mid X)}_{0})=\max\limits_{p(x_i)}(H(X))=\log_2(m)$ *ako je moguce postici distribuciju od* X
Diskretni bešumni	$m=n$ $p(X\mid Y)=p(Y\mid X)=I_n$ $H(X\mid Y)=H(Y\mid X)=0$ $C=\max\limits_{p(x_i)}(H(X))=\log_2(n)$
Kanal s neovisnim ulazom i izlazom	$p(Y\mid X)=1/n, \forall$ *broju matrici* $n\times m$ $H(Y\mid X)=\log_2(n)$ $H(Y)=\log_2(n)$ $C=\max\limits_{p(x_i)}(H(Y)-H(Y\mid X))=0$
WSC (generalizacija BSC)	*retci permutacije*, $\sum$ *ista* $\forall$ *stupac* $H(Y\mid x)=$ *entropija u jednom retku* $C=\max\limits_{p(x_i)}(H(Y)-H(Y\mid X))=\log_2(n)-H(Y\mid x)$
BEC	$p(X)=\begin{bmatrix}\alpha\\ 1-\alpha\end{bmatrix}$ $p(Y\mid X)=\begin{bmatrix}p & q & 0\\ 0 & q & p\end{bmatrix}$ $H(X\mid Y)=(1-p)H(X)$ $C=\max\limits_{p(x_i)}(H(X)-H(X\mid Y))=p$
k-BSC	$p(Y\mid X)=\begin{bmatrix}p & q\\ q & p\end{bmatrix}^k=SD^kS^{-1}$ $S=\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$ $S^{-1}=\begin{bmatrix}1/2 & 1/2\\ 1/2 & -1/2\end{bmatrix}$ $D^k=\begin{bmatrix}(p+q)^k & 0\\ 0 & (p-q)^k\end{bmatrix}$ $p(Y\mid X)=SD^kS^{-1}=\frac{1}{2}\begin{bmatrix}1+(p-q)^k & 1-(p-q)^k\\ 1-(p-q)^k & 1+(p-q)^k\end{bmatrix}$

Diskretne entropije

Entropija slučajne varijable	$H(X)=-\sum\limits_{i=1}^{n}p(x_i)\log_2(p(x_i))$
Združena entropija	$H(X,Y)=-\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{m}p(x_i,y_j)\log_2(p(x_i,y_j))$
Entropija šuma	$H(Y\mid X)=-\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{m}p(x_i,y_j)\log_2(p(y_j\mid x_i))$
Ekvivokacija	$H(X\mid Y)=-\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{m}p(x_i,y_j)\log_2(p(x_i\mid y_j))$
Transinformacija **(nema minusa)**	$I(X;Y)=\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{m}p(x_i,y_j)\log_2\left(\frac{p(x_i,y_j)}{p(x_i)p(y_j)}\right)$



Kodiranja

Aritmetičko	*Dekodiranje*:
	p[ ] = granice vjerojatnosnih intervala //[0, p_1, p_2, ... 1]
	x = poruka
	for i in range(n):
	    a = index najveceg praga manjeg od x
	    x = $\frac{x-p[a]}{p[a+1]-p[a]}$
	    print(a)
	
	Kodiranje:
	p[ ] = granice vjerojatnostnih intervala //[0, p_1, p_2, ..., 1]
	x = poruka
	D, G = 0, 1
	for i in range(n):
	    L = G-D
	    a = x[i] // znak poruke; a je donja granica intervala, a+1 gornja
	    G += p[a]*L
	    D -= (1 - p[a+1]) * L
	$l(x)=\left\lceil\log_2\left(\frac{1}{P(x)}\right)\right\rceil+1$ *binarnih znamenki*
	$P(x)=$ *duljina intervala poruke* $=$ *vjerojatnost poruke*

Shannon-Fano	*Kodiranje*: *p[ ]=lista vjerojatnosti sortirana DESC* *rekurzivno podijeli ' popola ' na skupove s otprilike istom vj* *gore ide* 0, *dolje ide* 1 *Dekodiranje*: *citas redom znakove, nikad neces dobit viseznacnost*
Huffman	*Kodiranje*: *p[ ] lista vjerojatnosti sortirana DESC* *uzmi 2 najniža, zbroji, sortiraj* *veća vj – veća znamenka, manja vj – manja znamenka* *Dekodiranje*: *čitaj redom znakove, nećeš dobiti višeznačnost* $N=d+(d-1)K$ $N-$ *broj znakova* $d-$ *baza* $K-$ *visetratnik*

LZ77	*Kodiranje*:
	n = duljina posmični prozor PP
	m = duljina prozora za kodiranje PZK
	while len(poruka) > 0:
	    nadi pattern u PP koji je jednak
	    patternu u PZK (zadnji simbol mora biti u PZK)
	    if exists:
	        print(pomak_od_pocetka_PP, duljina, next)
	    else:
	        print(0, 0, next)
	    pp & pzk += duljina + 1
	
	Dekodiranje:
	t = trenutni index
	for x in kodirana_poruka:
	    pomak, duljina, next = x
	    for i in range(duljina):
	        print(dekodirano[t - duljina + i])
	        print(next)

LZW	*Kodiranje*:
	rr = sljedeci simbol
	d = dict() while len(poruka) > 0:
	    if rr + next in d:
	        rr += next
	    else:
	        print(d[rr])
	        dodaj rr+next u d rr = next
	
	Dekodiranje:
	citas kodove i pises kodne rijeci
	redom iz dekodirane poruke 'ponovo kodiras' tj popunjavas dict
	kad dode cudni brojcek dobro razmisli sto moze doci poslije

Kraftova nejednakost	$\sum\limits_{i=1}^{n}r^{-l_i}\leq 1$ $l_i=$ *duljina i-te kodne rijeci* $n=$ *broj kodnih rijeci* $r=$ *velicina abecede*
Srednja duljina, efikasnost, propustnost kanala	$\bar{L}=\sum\limits_{i=1}^{n}p(x_i)l_i$ $\bar{L}\geq H(X)\,\forall X$ $\epsilon=\frac{H(X)}{\bar{L}}$ $R=N\frac{H(X)}{t}$

Kvantizacijski šum

$S = snaga\,signala$

korak kvantizacije $\Delta = \frac{(b-a)}{L}$, L = broj kvantizacijskih razina = 2^r, r = broj bitova

$Q = uniformna\,slucajna\,varijabla\,na\,intervalu[\frac{-\Delta}{2}, \frac{\Delta}{2}]$

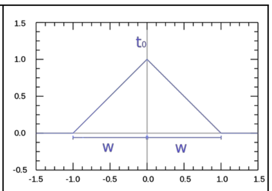
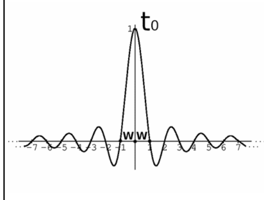
$\sigma_Q^2 = \frac{\Delta^2}{12}$

$(S/N) = \frac{S}{\sigma_Q^2}$

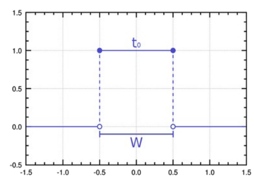
za sin(x) [$-m_{max}$, $+m_{max}$] vrijedi:

$(S/N) = \left(\frac{3\,S}{m_{max}^2}\right) 2^{2r} = \frac{3}{2} 2^{2r}$

$(S/N)_{dB} = 10\log_{10}(S/N) = 1.76 + 6.02\,r$

$\Delta\left(\frac{t-t_0}{W}\right) = max\left(1 - \left\lvert\frac{t-t_0}{W}\right\rvert, 0\right), a.k.a.\,\text{šator}$ $\Delta(t) = rect(t) * rect(t)$	
$sinc(x) = \frac{\sin(x\pi)}{x\pi}$ ¿ $x = \frac{t-t_0}{W}$	

Fourierove transformacije

$f(t)$	$F(2\pi f)$
$e^{-at}\mu(t), a>0$	$\frac{1}{a + j2\pi f}$
$e^{at}\mu(-t), a>0$	$\frac{1}{a - j2\pi f}$
$e^{-a\lvert t\rvert}, a>0$	$\frac{2a}{a^2 + (2\pi f)^2}$
$e^{-at}\mu(t), a>0$	$\frac{1}{(a + j2\pi f)^2}$
$t^n e^{-at}\mu(t), a>0$	$\frac{n!}{(a + j2\pi f)^{n+1}}$
$rect\left(\frac{t}{\tau}\right)$	$\tau sinc\left(\frac{2\pi f\,\tau}{2}\right)$
$sinc(Wt)$	$\frac{\pi}{W} rect\left(\frac{2\pi f}{2W}\right)$
$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2} sinc^2\left(\frac{2\pi f\,\tau}{4}\right)$
$sinc^2\left(\frac{Wt}{2}\right)$	$\frac{2\pi}{W}\Delta\left(\frac{2\pi f}{2W}\right)$
$e^{-at}\sin(\omega_0 t)\mu(t), a>0$	$\frac{\omega_0}{(a + j2\pi f)^2 + \omega_0^2}$
$e^{-at}\cos(\omega_0 t)\mu(t), a>0$	$\frac{a + j2\pi f}{(a + j2\pi f)^2 + \omega_0^2}$
$e^{-\frac{t^2}{2\sigma^2}}$	$\sigma\sqrt{2\pi}\,e^{-\frac{(2\sigma\pi f)^2}{2}}$
$\delta(t)$	1
1	$\delta(f)$
$\delta(t - t_0)$	$e^{-j2\pi f t_0}$
$e^{j\omega_0 t}$	$\delta\left(f - \frac{\omega_0}{2\pi}\right)$
$\sin(2\pi f_0 t)$	$\frac{1}{2j}(\delta(f - f_0) - \delta(f + f_0))$
$\cos(2\pi f_0 t)$	$\frac{1}{2}(\delta(f - f_0) + \delta(f + f_0))$
$\mu(t)$	$\frac{1}{2}\delta(f)$
$rect\left(\frac{t-t_0}{W}\right) = \mu((t-t_0) - W/2) - \mu((t-t_0) + W/2)$	

ViS thingies

$X \sim \begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ p(x_1) & p(x_1) & \cdots & p(x_n) \end{bmatrix}$ $n \in \mathbb{N} \vee n = +\infty$ $\sum_{i=1}^{n} p(x_i) = 1$	
$\mu = E[X] = \sum_{i=1}^{n} p(x_i)x_i$, *ovaj* μ *nije* $\mu(t)$*! nego je oznaka za očekivanje*	
$\sigma^2 = D[X] = E[(X-\mu)^2] = E[X^2] - (E[X])^2$ $D[X] = \sum_{i=1}^{n}(p(x_i)x_i^2) - \mu^2$	
Uniformna	$f_X(x) = \frac{1}{b-a}, \forall x \in [a,b], else\ 0$ $E[X] = \frac{a+b}{2}$ $D[X] = \frac{(b-a)^2}{12}$ $H(X) = \ln(b-a)$ ***nat***
Eksponencijalna	$f_X(x) = \lambda e^{-\lambda x}, \forall x > 0, else\ 0$ $E[X] = \frac{1}{\lambda}$ $D[X] = \frac{1}{\lambda^2}$ $H(X) = 1 - \ln(\lambda)$ ***nat***
Gaussova (normalna)	$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \forall x \in \mathbb{R}$ $E[X] = \mu$ $D[X] = \sigma^2$ $H(X) = \frac{1}{2}\ln(2\pi e\,\sigma^2)$ ***nat***
"Šator" (simetrični)	$f_X(x) = \frac{1}{W} - \frac{1}{W}\left\lvert\frac{x}{W}\right\rvert,\ W = \frac{b-a}{2}$ $E[X] = \frac{a+b}{2}$ $D[X] = \frac{(b-a)^2}{24} = \frac{W^2}{6}$ $H(X) = \frac{1}{2} + \ln\left(\frac{b-a}{2}\right)$ ***nat***
Geometrijska	$X \sim G(p) \rightarrow p(x_i) = p(1-p)^{k-1}, k \in 1..+\infty$ $E[X] = 1/p$ $D[X] = \frac{1-p}{p^2}$ $H[X] = \frac{-(1-p)\log_2(1-p) - p\,\log_2(p)}{p}$ ***bit***
Binomna	$X \sim B(n,p) \rightarrow p(x_i) = \binom{n}{k} p^k (1-p)^{n-k}, k \in 1..n$ $E[X] = np$ $D[X] = np(1-p)$ $H[X] = \frac{1}{2}\log_2(2\pi e\,n\,p(1-p))$ ***bit***
Poisson	$X \sim P(\lambda) \rightarrow p(x_i) = \frac{\lambda^i e^{-\lambda}}{i!}, i \in 1..+\infty$ $E[X] = \lambda$ $D[X] = \lambda$ $H[X] =$ nes uzasno komplicirano, nvm

SiS thingies

$$\mathcal{F}(x(t))=\int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft}\,dt=X(f),\ x(t)\ \textit{neperiodična}$$

$$\mathcal{F}(x(t))=\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)e^{-jk\frac{2\pi}{T_0}t}\,dt=X(k),\ x(t)\ \textit{periodična}$$

$$\mathcal{F}^{-1}(X(f))=\int_{-\infty}^{+\infty} X(f)e^{j2\pi ft}\,df=x(t),\ X(f)\ \textit{kontinuirana}$$

$$\mathcal{F}^{-1}(X(k))=\sum_{k=-\infty}^{+\infty} X(k)e^{jk\frac{2\pi}{T_0}t},\ X(k)\ \textit{diskretna}$$

$$E=\int_{-\infty}^{+\infty}|x(t)|^2\,dt,\ \textit{modul je tu zbog kompleksnih brojeva, može se ignorirati}$$

$$P=\lim_{T\to+\infty}\frac{1}{2T}\int_{-T}^{T}|x(t)|^2\,dt,\ \textit{ista stvar za modul vrijedi i tu}$$

$$x(t)*y(t)=\int_{-\infty}^{+\infty}x(\tau)y(t-\tau)\,d\tau$$

$$x(t)*y(t)=X(f)Y(f)$$

$$Y(f)=X(f)*H(f)$$

izlaz je konvolucija ulaznog signala i impulsnog odziva

$A\ rect\left(\frac{t-t_0}{W}\right)$	*neperiodički*: $E=A^2W$, $P=0$ *periodički*: $E=+\infty$, $P=A^2\frac{W}{T_0}$
$A\ \Delta\left(\frac{t-t_0}{W}\right)$	*neperiodički*: $E=\frac{2}{3}A^2W$, $P=0$ *periodički*: $E=+\infty$, $P=\frac{2}{3}A^2\frac{W}{T_0}$
$A\sin(2\pi f t)$ $A\cos(2\pi f t)$	$E=+\infty$ $P=\frac{A^2}{2}$

$$\int_{-\infty}^{+\infty}\frac{\sin(a\pi x)}{a\pi x}=\frac{1}{a}$$

$$\int_{-\infty}^{+\infty}\frac{\sin(a\pi x)^2}{(a\pi x)^2}=\frac{1}{a}$$

Kontinuirane slučajne varijable

Diferencijalna entropija	$H(X)=-\int_{-\infty}^{+\infty} f_X(x)\log_2(f_X(x))\,dx\ bit$
Združena entropija	$H(X,Y)=-\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\log_2(f(x,y))\,dxdy$
Ekvivokacija	$H(X\vert Y)=-\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\log_2\left(\frac{f(x,y)}{f_Y(y)}\right)dxdy$
Entropija šuma	$H(Y\vert X)=-\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\log_2\left(\frac{f(x,y)}{f_X(x)}\right)dxdy$
Transinformacija	$I(X;Y)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\log_2\left(\frac{f(x,y)}{f_X(x)f_Y(y)}\right)dxdy$

Kapacitet kontinuiranih kanala

Pojasno neograničeni	$C=\frac{1}{2}\ln\left(1+\frac{S}{N}\right)$
Pojasno ograničeni	$C=B\ln\left(1+\frac{S}{N_0B}\right)=2BD$
Srednja energija po bitu	$E_b=\frac{S}{R_b},\ idealno\ R_b=C$
Učinkovitost prijenosog pojasa	$\frac{C}{B}=\log_2\left(1+\frac{E_bC}{N_0B}\right)$

Slučajni signali

Šum na izlazu iz kanala	$N=\int_{-\infty}^{+\infty}S_N(f)\vert H(f)^2\vert df$
Autokorelacijska funkcija	$R_X(t_1,t_2)=E[X(t_1)X(t_2)]$
Autokovarijanca	$C_X(t_1,t_2)=R_X(t_1,t_2)-E[X(t_1)]E[X(t_2)]$
Stacionarni procesi	$R_X(t_1,t_2)=f(\tau),\ \tau=\vert t_2-t_1\vert$ $S_X(f)=\mathcal{F}\{R_X(\tau)\}\Leftrightarrow R_X(\tau)=\mathcal{F}^{-1}\{S_X(f)\}$ $P=R_X(0)=\int_{-\infty}^{+\infty}S_X(f)df$ $Gaussov\ šum: S_N=\frac{N_0}{2}$

Matematičke thingies

$$\sum_{i=1}^{n}x^i=\frac{1-x^n}{1-x},|x|<1$$

$$\sum_{i=0}^{+\infty}x^i=\frac{1}{1-x},|x|<1$$

$$\sum_{i=0}^{+\infty}ix^{i-1}=\frac{d}{dx}=\frac{1}{(1-x)^2},|x|<1$$

$$\sum_{i=0}^{+\infty}i(i-1)x^{i-2}=\frac{d^2}{dx^2}=\frac{2}{(1-x)^3},|x|<1$$

$$\cos(a)\cos(b)=\frac{1}{2}(\cos(a-b)+\cos(a+b))$$

$$\sin(a)\sin(b)=\frac{1}{2}(\cos(a-b)-\cos(a+b))$$

$$\sin(a)\cos(b)=\frac{1}{2}(\sin(a-b)+\sin(a+b))$$

$$\cos(a)+\cos(b)=2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\cos(a)-\cos(b)=-2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$\sin(a)+\sin(b)=2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\sin(a)-\sin(b)=2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$\frac{d}{dx}\log_a(x)=\frac{1}{\ln(a)x}$$

$$\log_b(x)=\frac{\log_a(x)}{\log_a(b)}$$

(v2.0)