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Predmet: Teorija informacije (34315)
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Zadatak
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Zadatak /zi03/:

Komunikacijskim kanalom prenose se četiri poruke generirane iz skupa od četiri simbola **X** = $\{x_1,...,x_4\}$. Vjerojatnosti pojavljivanja simbola su sljedeće: $\mathbf{p}_X = [p/2,\ p/2,\ (1-p)/2,\ (1-p)/2]$, slijedno gledano ($p \in (0, 1)$). Matrica uvjetnih vjerojatnosti prijelaza u kanalu je:

$$\left[p(y_j|x_i)\right] = \begin{bmatrix} 1-f & f & 0 & 0 \\ f & 1-f & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ uz } 0 \le f \le 1.$$

Odredite općeniti izraz za varijablu p koji osigurava maksimalnu količinu informacije po simbolu koja se u prosjeku može prenijeti danim kanalom. (**Napomena:** H(f) = f*log(1/f) + (1-f)*log(1/(1-f)))

C= max I (X;Y)

I(X;Y)= H(Y) – H(Y|X)

$$\left[p(y_j, x_i)\right] = \left[p(x_i)\right] * \left[p(y_j \mid x_i)\right]$$

$$\left[p(y_j, x_i)\right] = \begin{bmatrix} p/2 \\ p/2 \\ (1-p)/2 \\ (1-p)/2 \end{bmatrix} \begin{bmatrix} 1-f & f & 0 & 0 \\ f & 1-f & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} (1-f)p/2 & fp/2 & 0 & 0 \\ fp/2 & (1-f)p/2 & 0 & 0 \\ 0 & 0 & (1-p)/2 & 0 \\ 0 & 0 & 0 & (1-p)/2 \end{bmatrix}$$

$$\left[p(y_j)\right] = \left[p/2 \quad p/2 \quad (1-p)/2 \quad (1-p)/2\right]$$

$$H(Y) = \sum_{j=1}^{4} p(y_j) * \log_2 \frac{1}{p(y_j)}$$

H(Y) = p/2*log(2/p) + p/2*log(2/p) + (1-p)/2*log(2/(1-p)) + (1-p)/2*log(2/(1-p)) =

= p*log(2/p) + (1-p)*log(2/(1-p))

$$H(Y \mid X) = \sum_{j=1}^{4}\sum_{i=1}^{4} p(y_j, x_i) * \log_2 \frac{1}{p(y_j \mid x_i)}$$

H(Y|X) = (1-f)*p/2*log(1/(1-f)) + f*p/2*log(1/f) + f*p/2*log(1/f) + (1-f)*p/2*log(1/(1-f)) +

+ (1-p)/2*log 1 + (1-p)/2*log 1 =

= (1-f)*p*log(1/(1-f)) + f*p*log(1/f)

I(X;Y) = H(Y) – H(Y|X) =

$= p*\log(2/p) + (1-p)*\log(2/(1-p)) - (1-f)*p*\log(1/(1-f)) - f*p*\log(1/f) =$

$= H(X) -(1-f)*p*\log(1/(1-f)) -f*p*\log(1/f)$?? ← [ovaj zadnji red bi zanemario]

C tj količina informacija je maksimalna kad je derivacija I(X;Y) po p jednaka 0.

$dI(X;Y)/dp = (p*\log(2/p))' + ((1-p)*\log(2/(1-p)))' –((1-f)*p*\log(1/(1-f)) -f*p*\log(1/f))' =$

$=(\log(2/p) – 1/\ln 2) + (\log(2/(1-p)) + 1/\ln 2) - (1-f)*\log(1/(1-f)) - f*\log(1/f) =$

$= \log (2/p) + \log (2/(1-p)) - (1-f)*\log(1/(1-f)) - f*\log(1/f) =$

$= \log ((1-p)/p) - (1-f)*\log(1/(1-f)) - f*\log(1/f) = 0$

$\log ((1-p)/p) = (1-f)*\log(1/(1-f)) + f*\log(1/f)$

$(1-p)/p = 2^{(1-f)*\log(1/(1-f)) + f*\log(1/f)}$

$p = 1/(1 + 2^{(1-f)*\log(1/(1-f)) + f*\log(1/f)}) =1/(1+2\text{^}H(f))$