**LINEARNO BINARNI BLOK KODOVI**

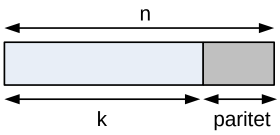
$K:(n,M,d) \leftrightarrow [n,k,d] \leftrightarrow [n,k]$

$M = 2^k$

$R(K) = \frac{k}{n} \leq 1$

$d(K) = \min_{x,y \in K}(d(x,y) | x \neq y)$

$d(\boldsymbol{x},\boldsymbol{y}) = w(\boldsymbol{x} - \boldsymbol{y})$




$k$ – broj informacijskih bitova u kodnoj riječi

$n$ - duljina kodne riječi

$M$ - broj kodnih riječi u kodu

$d$ – distanca (udaljenost) koda

$R$ – kodna brzina

$w$ - težina kodne riječi

Uvjeti lineranosti binarnog blok koda

1) $\boldsymbol{x} + \boldsymbol{y} \in K, \ \boldsymbol{x}, \boldsymbol{y} \in K$
2) $a \cdot \boldsymbol{x} \in K, \ a \in \{0,1\}$
3) $000 \ldots 0 \ \in K$

HAMMINGOVA MEĐA

$$M \leq \frac{2^n}{\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{t}}$$

PERFEKTAN KÔD

$$M = \frac{2^n}{\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{t}}$$

VJEROJATNOST ISPRAVNOG DEKODIRANJA

$$p(K) = \sum_{i=0}^{t} \binom{n}{i} \, p_g^i \left(1 - p_g\right)^{n-i}$$

<u>DEKODIRANJE LINEARNOG BINARNOG KODA:</u>

1) Metoda najbližeg susjeda

$d(K) \geq s + 1$

$s = d(K) - 1$

$t = \left\lfloor \frac{d(K)-1}{2} \right\rfloor$

$d(K) \geq 2t + 1$

$$2^{n-k} \geq \sum_{i=0}^{t} \binom{n}{i}$$

$s$ - najveći broj pogrešaka koje kôd može otkriti

$t$ - najveći broj pogrešaka koje kôd *K* može ispraviti

2) Sindromsko dekodiranje

$\boldsymbol{e} = \boldsymbol{y} - \boldsymbol{x}$

$\boldsymbol{e}$ - vektor pogreške

$\boldsymbol{x}$ - poslana kodna riječ

$\boldsymbol{y}$ - primljena kodna riječ

$\boldsymbol{G} = [\boldsymbol{I_k} \mid \boldsymbol{A}]$

$\boldsymbol{H} = [\boldsymbol{A}^T \mid \boldsymbol{I}_{n-k}]$

$\boldsymbol{S}(\boldsymbol{y}) = \boldsymbol{y} \cdot \boldsymbol{H^T}$

$\boldsymbol{G}$ – generirajuća matrica koda dimenzija $k \times n$

$\boldsymbol{H}$ - matrica provjere pariteta

$\boldsymbol{S}$ - sindrom

HAMMINGOV KÔD

$\boldsymbol{H}$ - matrica provjere pariteta dimenzija $r \times (2^r - 1)$

$r = n - k$

CIKLIČKI KÔD

Uvjeti:

1. $\forall \ a(x), b(x) \ \in \ K,$ *vrijedi* $a(x) \ + \ b(x) \ \in K$
2. $\forall \ a(x) \ \in K \ i \ \forall \ r(x) \ \in \ Rn,$ *vrijedi* $r(x) \cdot a(x) mod(xn - 1) \in K.$

$x^n \ - 1 = \ g(x) \cdot h(x)$

$r$ - stupanj generirajućeg polinoma

$h(x)$ - polinom za provjeru pariteta cikličnog koda K.

$d(x) \cdot x^r = g(x)q(x) + r(x) = c(x)$

$r(x) = d(x) \cdot x^r \ mod[g(x)]$

$S\big(c'(x)\big) = \frac{x^{n-k} c'(x)}{g(x)}$

$\boldsymbol{c} = [d|r]$

$g(x)$ - generirajući polinom

$q(x)$ - kvocijent

*d*(*x*) – polinom kodirane poruke

$r(x)$ – ostatak nakon dijeljenja s $g(x)$

$c(x)$ – kodna riječ

$S\big(c'(x)\big)$ - sindrom primljene kodne riječi

FAKTORIZACIJE NEKIH POLINOMA OBLIKA $x^n - 1$

| n | aritmetika | Faktorizacija u aritmetici modulo 2 |
|---|---|---|
| 1 | $x - 1$ | $x + 1$ |
| 2 | $x^2 - 1$ | $(x + 1)^2$ |
| 3 | $x^3 - 1$ | $(x + 1)(x^2 + x + 1)$ |
| 5 | $x^5 - 1$ | $(x + 1)(x^4 + x^3 + x^2 + x + 1)$ |
| 7 | $7 - 1$ | $(x + 1)(x^3 + x + 1)(x^3 + x^2 + 1)$ |
| 9 | $x^9 - 1$ | $(x + 1)(x^2 + x + 1)(x^6 + x^3 + 1)$ |
| 11 | $x^{11} - 1$ | $(x + 1)(x^{10} + x^9 + \cdots + x + 1)$ |
| 13 | $x^{13} - 1$ | $(x + 1)(x^{12} + x^{11} + \cdots + x + 1)$ |
| 15 | $x^{15} - 1$ | $(x + 1)(x^2 + x + 1)(x^4 + x + 1)(x^4 + x^3 + 1)(x^4 + x^3 + x^2 + 1)$ |
| 17 | $x^{17} - 1$ | $(x + 1)(x^8 + x^5 + x^4 + x^3 + 1)(x^8 + x^7 + x^6 + x^4 + x^2 + x + 1)$ |
| 19 | $x^{19} - 1$ | $(x + 1)(x^{18} + x^{17} + \cdots + x + 1)$ |

HAM [7,4]

$$\boldsymbol{G} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{H} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

HAM [15,11]

$$\boldsymbol{G} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{H} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$