1. IZ GRAFA OČITAMO TABLICU UVJETNIH VJEROJATNOSTI:

$$[P(Y/X)] = \begin{array}{c} \\ X_1 \\ X_2 \end{array}\begin{bmatrix} Y_1 & Y_2 & Y_3 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad P(X) = [\ P_1 \quad P_2\ ]$$

$$[P(X,Y)] = \begin{bmatrix} 0.5P_1 & 0.5P_1 & 0 \\ 0 & 0 & P_2 \end{bmatrix}$$

$$\downarrow$$

$$[P(Y)] = [\ 0.5P_1 \quad 0.5P_1 \quad P_2\ ]$$

$$C = \max_{P(x_i)} I(X;Y)$$

(1) $I(X;Y) = H(X) + H(Y) - H(X;Y)$

$$H(X,Y) = - \sum_{i=1}^{m} \sum_{j=1}^{m} P(x_i, y_j) \log_2 P(x_i, y_j)$$

$$H(Y) = - \sum_{i=1}^{m} P(x_i) \log_2 P(x_i)$$

KADA SE OVO IZRAČUNA, DOBIJE SE DA JE $H(Y) = H(X,Y)$
ŠTO SE MOŽE VIDJETI I POZNATOM OSOBITIVOM METODOM.
BUDUĆI DA JE $H(Y) = H(X,Y)$ IZ (1) SLIJEDI DA JE

$$I(X;Y) = H(X)$$

$$C = \max I(X;Y) = \max H(X) = \log_2 m = \log_2 2 = 1 \ bit/s$$

→ ENTROPIJA JE NAJVEĆA KADA SU VJEROJATNOSTI
POJAVLJIVANJA SVIH SIMBOLA JEDNAKE. TADA JE
ENTROPIJA $H(X) = \log_2 n$ GDJE JE n BROJ ULAZNIH SIMBOLA

(2.)

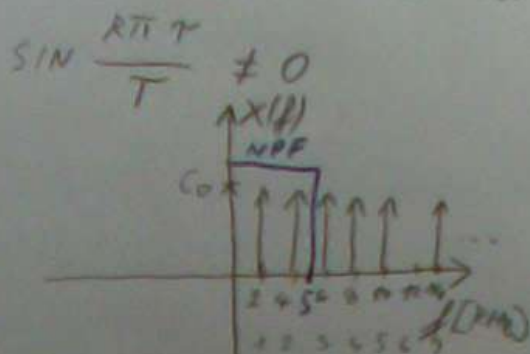IZ FREKVENCIJA POJAVLJIVANJA POJEDINOG SIMBOLA TREBA IZRAČUNATI VJEROJATNOSTI POJAVLJIVANJA SVAKOG.

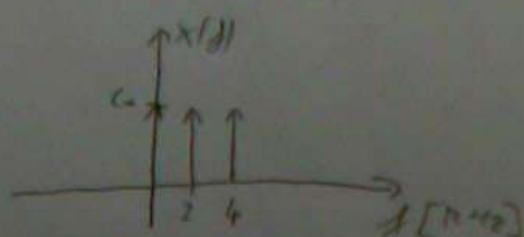$$H(x) = -\sum_{i=1}^{m} P(x_i) \log_2 P(x_i) = \boxed{2.4578 \text{ bit/simbol}}$$

(3.) $f = 2 \text{kHz}$

$$\frac{\tau}{T} = \frac{1}{6}$$

1. NACRTATI SPEKTAR (JEDNOSTRANI) PRAVOKUTNOG IMPULSA.

PRAVOKUTNI IMPULS IMA KOMPONENTE KADA VRIJEDI

$$SIN \frac{k\pi \tau}{T} \neq 0$$

2. NAKON PROLASKA KROZ NPF GRANIČNE FREKVENCIJE $f_g = 5 \text{kHz}$ OSTAJE:

3. OVO UZORKUJEMO S $f_s = 10 \text{kHz}$:

4. PROPUŠTAMO OVO KROZ PPF $f_c = 6 \text{kHz}$, TJ. $B \in [3, 9] \text{kHz}$ GUBIMO ONO ŠTO JE IZMEĐU 3 I 9 kHz.

ŽELENO NA 3

OSTALE SU KOMPONENTE NA 4, 6 i 8 kHz.

④

$B = 4 \, kHz$

$S_N = 10^{-12} \, \frac{W}{Hz} \implies \frac{N_0}{2} = 10^{-12} \frac{W}{Hz} \implies N_0 = 2 \cdot 10^{-12} \frac{W}{Hz}$

$N = B \, N_0 = 8 \cdot 10^{-9} \, W = \boxed{8 \cdot 10^{-6} \, mW}$

$S = 0.1 \, mW$

$C = B \cdot \log_2 \left(1 + \frac{S}{N}\right) = 4000 \cdot \log_2 \left(1 + \frac{0.1}{8 \cdot 10^{-6}}\right) = \boxed{54.44 \, kHz}$
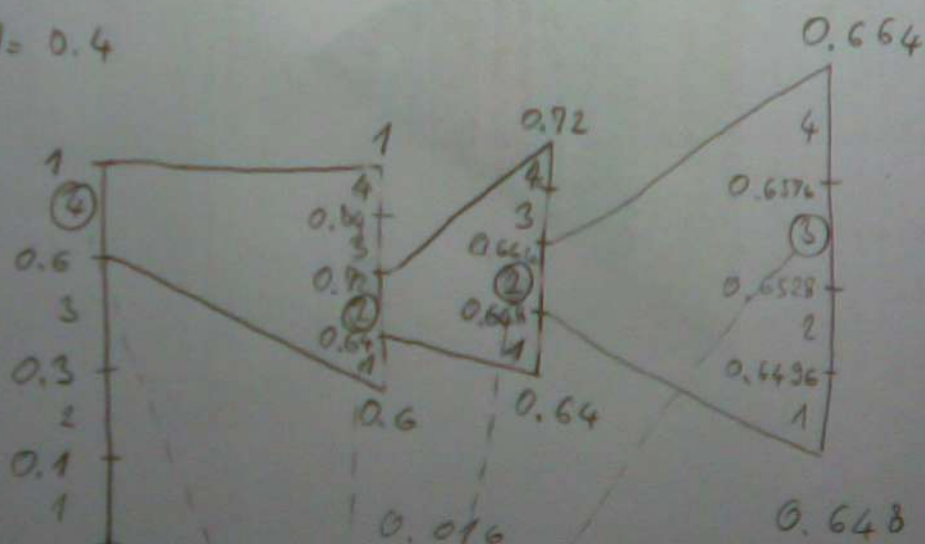
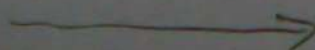**5.**

$X = \{1, 2, 3, 4\}$

$P(1) = 0.1$

$P(2) = 0.2$

$P(3) = 0.3$

$P(4) = 0.4$

$(0.101010)_2 = (0.65625)_{10}$

Očitano: 4   2   2   3

(6.)

Tree diagram with values:
530
116 1
114 ...... 0 211
104 ...... 1 .... 252 ...1 .. 680 1
78 .... 0 .... 136 0
54 .... 0 58 1
2 ... 0 4 1
2 ... 0

530   530   530
136   211   252
116   136   211
114   116
104

|≥1000| 530 - 1   (0)
         116 - 3   (101)
         114 - 3   (110)
         104 - 3   (111)
         78   - 4   (1000)
         54   - 5   (10010)
         2   - 6   (100110)
         2   - 6   (100111)

$$Z = \sum_{i=1}^{n} p_i \cdot l_i$$

$$\overline{Z} = 2.138 \text{ bit}$$

$$\overline{Z} = 1 \cdot 0{,}530 + 3 \cdot 0{,}116 + \ldots + 6 \cdot 0{,}002$$

(17.)   [15,11]

$g(x) = x^4 + x^3 + 1$

$4 = n - k$

$4 = 15 - k \Rightarrow k = 11$

HOD [15,11]

PRVO RIJEČ NA ULASKU U KODER JE: $\underset{4\,3\,2\,1}{1}\,\underset{11}{0}\,\underset{10}{1}\,\underset{9}{0}\,\underset{8}{1}\,\underset{7}{0}\,\underset{6}{1}\,\underset{5}{0}\,\underset{4}{1}\,\underset{3}{0}\,\underset{2}{1}\,\underset{1}{0}\,\underset{0}{1}$

$d(x) = x^{10} + x^8 + x^6 + x^4 + x^2 + 1$

$v(x) = \dfrac{x^{n-k} \cdot d(x)}{g(x)} = \dfrac{x^4 \cdot (x^{10} + x^8 + x^6 + x^4 + x^2 + 1)}{x^4 + x^3 + 1}$

$v(x) = \dfrac{x^{14} + x^{12} + x^{10} + x^8 + x^6 + x^4}{x^4 + x^3 + 1}$

$x^{14} + x^{12} + x^{10} + x^8 + x^6 + x^4 : x^4 + x^3 + 1 = x^{10} + x^9 + x^5 + x^2 + 1$

$\underline{x^{14} + x^{13} + x^{10}}$

$\quad \underline{x^{13} + x^{12} + x^8}$
$\quad \phantom{x^{13}} x^{13} + x^{12} + x^9$

$\qquad \underline{x^9 + x^8 + x^6}$
$\qquad x^9 + x^8 + x^5$

$\qquad\quad \underline{x^6 + x^3 + x^4}$
$\qquad\quad x^6 + x^5 + x^2$

$\qquad\qquad x^4 + x^2$

$\qquad\qquad \underline{x^4 + x^3 + 1}$

$\qquad\qquad\quad x^3 + x^2 + 1 \rightarrow \boxed{[1\ 1\ 0\ 1]}$

(8.)

$$X = \{1, 2, \ldots, 2048\}$$

JEDNAKE VJEROJATNOSTI POJAVLJIVANJA SVAKOG X_i

↳ BUDUĆI DA SU VJEROJATNOSTI POJAVLJIVANJA JEDNAKE ZA SVAKI SIMBOL NA ULAZU, SVE KODNE RIJEČI BIT ĆE JEDNAKE DULJINE, TJ. JEDNAKE DULJINE KAO DA SMO IH KODIRALI JEDNOLIKIM KODOM.

DA BISMO KODIRALI 2048 SIMBOLA, TREBA NAM [11] BITA JER JE $2^{11} = 2048$.

ULAZNU RIJEČ OD 11 BITA KODIRAMO CIKLIČKIM KODOM S $g(x) = x^4 + x + 1$. ZA CIKLIČKE KODOVE VRIJEDI DA JE NAJVIŠA POTENCIJA $g(x)$ JEDNAKA $n-k$ GDJE JE n DULJINA KODIRANE RIJEČI, k DULJINA „KORISNOG" DIJELA PORUKE. U NAŠEM SLUČAJU:

$$\rightarrow 4 = n - 11 \Rightarrow \boxed{n = 15}$$

↳ RIJEČ KODIRANA CIKLIČKIM KODOM KOJU ŠALJEMO U KOMUNIKACIJSKI KANAL DUGA JE 15 BITA

U 1 SEKUNDI ŠALJEMO 4000 KODNIH RIJEČI, TJ.

$4000 \cdot 15$ BITA $= 60000$ BITA $\Rightarrow C = 60000$ BIT/s

$$C = B \cdot \log_2\left(\frac{S}{N} + 1\right)$$

$$B = \frac{C}{\log_2\left(1 + \frac{S}{N}\right)}$$

$$B = \frac{60000}{11.63} = 5160.3 \text{ Hz}$$

$$= \boxed{5.16 \text{ kHz}}$$

| OMJER S/N TREBA OSLOBODITI OD dB!

$$\frac{S}{N}\bigg|_{dB} = 35 dB$$

$$35 dB = 10 \log_{10} \frac{S}{N}$$

$$\log_{10} \frac{S}{N} = 3.5$$

$$\frac{S}{N} = 10^{3.5} = \boxed{3162,28}$$

9.

AAAA BBB CCD⨯   (0, 0, A)

Λ ÀÀÀ BCB CCD⨯   (1, 3, B)

AAAA BBB CCD⨯   (1, 2, C)

AA AA BBB CC D⨯   (1, 1, 0)

AAAA BBB CC D⨯   (0, 0, ⨯)

RJEŠENJE: (0,0,A), (1,3,B), (1,2,C), (1,1,0), (0,0,⨯)

10.

$D[0] = A$

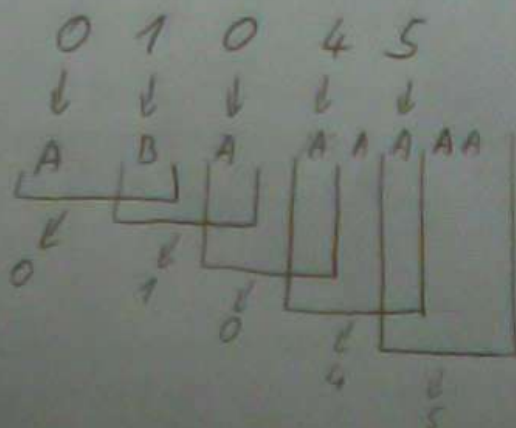$D[1] = B$

$D[2] = AB$

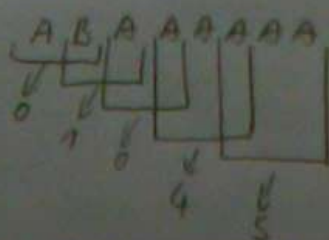$D[3] = BA$

$D[4] = AB$

DEKODIRANJE:

0 1 0 4 5

A B A AAA AA

$D[2] = AB$
$D[3] = BA$
$D[4] = AB$
$D[5] = ABA$

KODIRANJE:

A B A AA AAA

RJEŠENJE: AB AAAAAA