KOLIČINA INFORMACIJE

$$I(x_i) = -\log_2 p(x_i)\ \left[\frac{\text{bit}}{\text{simbol}}\right]$$

$$I(x_1x_2\dots x_k) = -\log_2[p(x_1)\cdot p(x_2)\cdot\ldots\cdot p(x_k)]\ \left[\frac{\text{bit}}{\text{poruka}}\right]$$

VJEROJATNOSTI U KOMUNIKACIJSKOM SUSTAVU

$$[p(y_j|x_i)] = \begin{bmatrix} p(y_1|x_1) & p(y_2|x_1) & \dots & p(y_m|x_1) \\ p(y_1|x_2) & p(y_2|x_2) & \dots & p(y_m|x_2) \\ \dots & \dots & \dots & \dots \\ p(y_1|x_n) & p(y_2|x_n) & \dots & p(y_m|x_n) \end{bmatrix} \begin{matrix} \}\,\Sigma = 1 \\ \}\,\Sigma = 1 \\ \dots \\ \}\,\Sigma = 1 \end{matrix}$$

$$[p(y_j)] = [p(x_i)][p(y_j|x_i)] \qquad [p(x_j)]^T = [p(x_i|y_j)][p(y_j)]^T$$

$$[p(x_i,y_j)] = [p(x_i)p(y_j|x_i)] = [p(x_i|y_j)p(y_j)]$$

$$p(x_i|y_j) = \frac{p(x_i,y_j)}{p(y_j)} = \frac{p(x_i,y_j)}{\sum_{i=1}^n p(x_i,y_j)} = \frac{p(x_i)p(y_j|x_i)}{\sum_{i=1}^n p(x_i)p(y_j|x_i)}$$

$$[p(x_i,y_j)] = \begin{bmatrix} p(x_1,y_1) & p(x_1,y_2) & \dots & p(x_1,y_m) \\ p(x_2,y_1) & p(x_2,y_2) & \dots & p(x_2,y_m) \\ \dots & \dots & \dots & \dots \\ \underbrace{p(x_n,y_1)}_{\Sigma = p(y_1)} & \underbrace{p(x_n,y_2)}_{\Sigma = p(y_2)} & \dots & \underbrace{p(x_n,y_m)}_{\Sigma = p(y_m)} \end{bmatrix} \begin{matrix} \}\,\Sigma = p(x_1) \\ \}\,\Sigma = p(x_2) \\ \dots \\ \}\,\Sigma = p(x_n) \end{matrix}$$

ENTROPIJA (Srednji sadržaj)

$$H(X) = -\sum_{i=1}^n p(x_i)\log_2 p(x_i)\ \left[\frac{\text{bit}}{\text{simbol}}\right]$$ (Vlastiti sadržaj uz neovisnost simbola)

$$H(Y) = -\sum_{j=1}^m p(y_j)\log_2 p(y_j)\ \left[\frac{\text{bit}}{\text{simbol}}\right] \quad \left(\begin{matrix} p(x_1) = p(x_2) = \dots = p(x_n) \\ \Rightarrow \max(H(X)) = \log_2(n) \end{matrix}\right)$$

$$H(X,Y) = -\sum_{i=1}^n\sum_{j=1}^m p(x_i,y_j)\log_2 p(x_i,y_j)\ \left[\frac{\text{bit}}{\text{simbol}}\right]$$

ENTROPIJA ŠUMA (IRELEVANTNOST)

$$H(Y|X) = -\sum_{i=1}^n\sum_{j=1}^m p(x_i,y_j)\log_2 p(y_j|x_i)\ \left[\frac{\text{bit}}{\text{simbol}}\right]$$

(Vlastiti sadržaj uz ovisnost simbola)

EKVIVOKACIJA (MNOGOZNAČNOST)

$$H(X|Y) = -\sum_{i=1}^n\sum_{j=1}^m p(x_i,y_j)\log_2 p(x_i|y_j)\ \left[\frac{\text{bit}}{\text{simbol}}\right]$$

SREDNJI SADRŽAJ INFORMACIJE (TRANSINFORMACIJA)

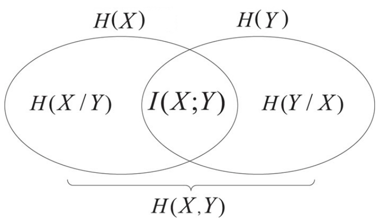
$$I(X;Y) = \sum_{i=1}^n\sum_{j=1}^m p(x_i,y_j)\log_2\frac{p(x_i,y_j)}{p(x_i)p(y_j)}\ \left[\frac{\text{bit}}{\text{simbol}}\right]$$

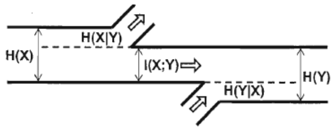
$$I(X;Y) = H(X) - H(X|Y) \quad I(X;Y) = H(Y) - H(Y|X) \quad I(X;X) = H(X)$$

$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$

$$H(X,Y) = H(X) + H(Y|X) \quad H(X,Y) = H(Y) + H(X|Y)$$

$$X\ i\ Y\ nezavisni \rightarrow H(X,Y) = H(X) + H(Y)$$





$$H(X|Y) \leq H(X)$$

$$\boldsymbol{I(X;Y) = I(Y;X) \geq 0}$$

$$\boldsymbol{I(X;X) = H(X)} \rightarrow \textit{vlastiti sadržaj}$$

$$\textit{Ergodičnost: } \left[\boldsymbol{p(x_i)}_{i,\dots,n}\right] = \left[\boldsymbol{p(x_1)\cdots p(x_n)}\right]\left[\boldsymbol{p(x_j\,|\,x_i)}\right]$$

RELATIVNA ENTROPIJA

$$D(p\|q) = \sum_{i=1}^n p(x_i)\log\frac{p(x_i)}{q(x_i)}$$

$$\boldsymbol{D(p\,\|\,q) \neq D(q\,\|\,p)}$$

KAPACITET DISKRETNOG KOMUNIKACIJSKOG KANALA

$$C = \max_{\{p(x_i)\}} I(X;Y) = \max_{\{p(x_i)\}}\left(H(Y) - H(Y|X)\right)\ \left[\frac{\text{bit}}{\text{simbol}}\right]$$

INFORMACIJSKA BRZINA IZVORIŠTA

$$\boldsymbol{R} = \frac{\boldsymbol{H(X)}}{\boldsymbol{T_s}}\ \left[\frac{bit}{s}\right] \qquad \boldsymbol{T_s} = \sum_{i=1}^n \boldsymbol{p_i\cdot t_i + t_s}\ \left[\frac{sekunda}{simbol}\right]$$ ($\boldsymbol{T_s}$ *- prosječno trajanje simbola)*

ENTROPIJSKO KODIRANJE

SREDNJA DULJINA KODNE RIJEČI

$$L(X) = \sum_{i=1}^n p(x_i)l_i\left[\frac{\text{bit}}{\text{simbol}}\right]$$

EFIKASNOST KODA

$$\boldsymbol{\varepsilon} = \frac{\boldsymbol{H(X)}}{\boldsymbol{L(X)}} \leq \boldsymbol{1}$$

KRAFTOVA NEJEDNAKOST (nužan i dovoljan uvjet za prefiksni kod)

$$\sum d^{-L_i} \leq 1$$ *($\boldsymbol{d_i}$ je baza $\rightarrow$ broj simbola u abecedi)*
(- $\boldsymbol{L_i}$ je duljina individualne kodne riječi)

OPTIMALNOST KODA (nužan i dovoljan optimalnosti koda)

$$H(X) \leq L(X) < H(X) + 1$$

$$\min\left[L(X) = \sum_{i=1}^n p(x_i)l_i\right]\ uz\ uvjet\ \sum_{i=1}^n d^{-L_i} \leq 1$$

SARDINAS-PATTERSONOV TEST

$C(y)$ *se dodaje u skup* S_{i+1} *ako i samo ako:*
$\exists\, C(x) \in S_0$ *tako da* $C(x)C(y) \in S_i$
ili
$\exists\, C(z) \in S_i$ *tako da* $C(z)C(y) \in S_0$
■ *Kod je JDK ako niti jedan* $S_i\ (i \geq 1)$ *ne sadrži kodne riječi iz* S_0 ■

HUFFMANNOVO KODIRANJE - ako baza ≠ 2

$$N - broj\ simbola;\quad B - baza\ kodiranja;\quad k = \left\lceil\frac{N-1}{B-1}\right\rceil$$

$$N' = (B-1)k + 1;\quad N' \neq N \Rightarrow dodaj\ N' - N\ simbola\ s\ \boldsymbol{p} = 0$$

ARITMETIČKO KODIRANJE

$$D' = D + (G - D)\cdot D_s$$

$$G' = D + (G - D)\cdot G_s$$

$$\left\lceil\log_2\left(\frac{1}{G' - D'}\right)\right\rceil + 1\ znamenki \rightarrow kod\ koji\ se\ može\ JDK$$

$$L_{a(d=10)} = \sum_{i=1}^N L_{a(d=2)}(i)\cdot 2^{-i}$$

SREDNJA DULJINA KODNE RIJEČI

$$L(X) = \frac{\sum_{i=1}^n p(x_i)\cdot l(x_i)}{\sum_{i=1}^n p(x_i)\cdot n(x_i)}\left[\frac{\text{bit}}{\text{simbol}}\right] \quad \begin{matrix} l(x_i) \rightarrow duljina\ kodne\ riječi \\ n(x_i) \rightarrow broj\ simbola \end{matrix}$$

VJEROJATNOST POJAVE POJEDINOG SIMBOLA

$$\boldsymbol{p}\,(pojave\ simbola) = \frac{\boldsymbol{p(x_i)\cdot n(x_i)}}{\boldsymbol{L(X)}}$$

$$\boldsymbol{n(x_i)} \rightarrow duljina\ simbola$$

VJEROJATNOST ISPRAVNOG/POGREŠNOG PRIJENOSA BINARNOG SIMETRIČNOG KANALA

$$\boldsymbol{P_{BSK}} = \frac{\boldsymbol{1}}{\boldsymbol{2}}(\boldsymbol{1 \pm (1 - 2\cdot p)^k})$$ *($\boldsymbol{p}$ $\rightarrow$ vjerojatnost pogreške)*
($\boldsymbol{k}$ $\rightarrow$ broj spojenih kanala)

NAJMANJA DULJINA KODNE RIJEČI (Aritmetički algoritam)

$$\boldsymbol{l(x)} = \left\lceil\boldsymbol{\log_2}\left(\frac{\boldsymbol{1}}{\boldsymbol{P(X)}}\right)\right\rceil + \boldsymbol{1}\ \ [bit]$$

$$\boldsymbol{P_i(x)} = \prod_{i=1}^n \boldsymbol{p(x_i)}$$

$$ako\ tražimo\ minimalnu\ duljinu \rightarrow tražimo\ \max(\boldsymbol{P_i(x)})$$

OSTALO

VJEROJATNOST PRIJELAZA

$$[p(y_j)] = [p(z_k)][p(y_j|z_k)]$$

$$[p(y_j)] = [p(x_i)][p(y_j|x_i)] \rightarrow [p(y_j|x_i)] = [p(z_k|x_i)][p(y_j|z_k)]$$

$$[p(z_k)] = [p(x_i)][p(z_k|x_i)]$$

HUFFMANOVO KODIRANJE (m simbola)

$$p(x_i) = \frac{1}{m}$$

$$2^n \leq m < 2^{n+1} \rightarrow k = m - 2^n$$

$$kodnih\ riječi\ duljine\ \mathbf{n}\ ima \rightarrow 2^n - k$$

$$kodnih\ riječi\ duljine\ \mathbf{n+1}\ ima \rightarrow 2k$$

SIMETRIČNI KANAL

$$\left[p(y_j|x_i)\right] = \begin{bmatrix} a & b & c \\ c & a & b \\ b & c & a \end{bmatrix}$$

Simetričan kanal $\rightarrow$ *stupci su permtuacije jednog drugog i redci također*

Slabo simetričan kanal $\rightarrow$ *redci su permutacije*

$\rightarrow$ *zbrojevi vjerojatnosti po stupcima međusobno su jednaki*

$$C = \log(card(Y)) - H(Y|x)$$

$$H(Y|x) = \sum_{i=1}^{m} p(y|x_i) \cdot \log_2(p(y|x_i))$$

(*računa se entropija za jedan redak matrice* $\left[p(y_j|x_i)\right]$)

BINARNI SIMETRIČNI KANAL – $p + q = 1$

$$\left[p(y_j|x_i)\right] = \begin{bmatrix} p & q \\ q & p \end{bmatrix}$$

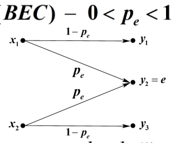
Binarni simetričan kanal $\rightarrow$ *dva ulaza i izlaza*

$\rightarrow$ *vjerojatnost pogreške ista za oba ulaza*

$$C = \log(card(Y)) - H(Y|x)$$

KANAL S BRISANJEM SIMBOLA (BEC) – $0 < p_e < 1$

$$\left[p(y_j|x_i)\right] = \begin{bmatrix} 1-p_e & p_e & 0 \\ 0 & p_e & 1-p_e \end{bmatrix}$$



$$C = 1 - p_e$$

Kanal s brisanjem simbola $\rightarrow$ *vjerojatnost* p_e *da obriše simbol*

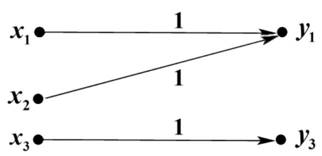
$\rightarrow$ *simbol se briše umjesto da se zamijeni*

$\rightarrow$ *kada je obrisan, izlaz dobije poseban e simbol*

KANAL BEZ ŠUMA – $H(Y|X) = 0$

$$\left[p(y_j|x_i)\right] = \begin{array}{c c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{array}{l} \Sigma = 1 \\ \Sigma = 1 \\ \Sigma = 1 \end{array} \end{array}$$

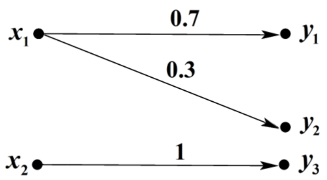
$$\left[p(x_i|y_j)\right] = \begin{array}{c c} & \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{array}{c} y_1 \\ y_2 \end{array} & \begin{bmatrix} p(x_1) & p(x_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{l} \Sigma = 1 \\ \Sigma = 1 \end{array} \end{array}$$



KANAL BEZ MNOGOZNAČNOSTI – $H(X|Y) = 0$

$$\left[p(y_j|x_i)\right] = \begin{array}{c c} & \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.7 & 0.3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{l} \Sigma = 1 \\ \Sigma = 1 \end{array} \end{array}$$

$$\left[p(x_i|y_j)\right] = \begin{array}{c c} & \begin{array}{cc} x_1 & x_2 \end{array} \\ \begin{array}{c} y_1 \\ y_2 \\ y_3 \end{array} & \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{array}{l} \Sigma = 1 \\ \Sigma = 1 \\ \Sigma = 1 \end{array} \end{array}$$



Kodiranja

Aritmetičko	*Dekodiranje*:
	p[ ] = granice vjerojatnosnih intervala //[0, p_1, p_2, ... 1]
	x = poruka
	for i in range(n):
	    a = index najveceg praga manjeg od x
	    x = $\frac{x - p[a]}{p[a+1] - p[a]}$
	    print(a)
	
	Kodiranje:
	p[ ] = granice vjerojatnostnih intervala //[0, p_1, p_2, ..., 1]
	x = poruka
	D, G = 0, 1
	for i in range(n):
	    L = G-D
	    a = x[i] // znak poruke; a je donja granica intervala, a+1 gornja
	    G += p[a]*L
	    D -= (1 - p[a+1]) * L
	$l(x) = \lceil \log_2\left(\frac{1}{P(x)}\right) \rceil + 1\ binarnih\ znamenki$
	$P(x) = duljina\ intervala\ poruke = vjerojatnost\ poruke$

Shannon-Fano	*Kodiranje*: *p[ ]=lista vjerojatnosti sortirana DESC* *rekurzivno podijeli 'popola' na skupove s otprilike istom vj* *gore ide 0, dolje ide 1* *Dekodiranje*: *citas redom znakove, nikad neces dobit viseznacnost*
Huffman	*Kodiranje*: *p[ ]listavjerojatnosti sortirana DESC* *uzmi 2 najniža, zbroji, sortiraj* *veća vj – veća znamenka, manja vj – manja znamenka* *Dekodiranje*: *čitaj redom znakove, nećeš dobiti višeznačnost* $N = d + (d-1)K$ *N – broj znakova* *d – baza* *K – visetratnik*

LZ77	*Kodiranje*:
	n = duljina posmični prozor PP
	m = duljina prozora za kodiranje PZK
	while len(poruka) > 0:
	    nadi pattern u PP koji je jednak
	    patternu u PZK (zadnji simbol mora biti u PZK)
	    if exists:
	        print(pomak_od_pocetka_PP, duljina, next)
	    else:
	        print(0, 0, next)
	    pp & pzk += duljina + 1
	
	Dekodiranje:
	t = trenutni index
	for x in kodirana_poruka:
	    pomak, duljina, next = x
	    for i in range(duljina):
	        print(dekodirano[t - duljina + i])
	        print(next)

LZW	*Kodiranje*:
	rr = sljedeci simbol
	d = dict() while len(poruka) > 0:
	    if rr + next in d:
	        rr += next
	    else:
	        print(d[rr])
	        dodaj rr+next u d rr = next
	
	Dekodiranje:
	citas kodove i pises kodne rijeci
	redom iz dekodirane poruke 'ponovo kodiras' tj popunjavas dict
	kad dode cudni brojcek dobro razmisli sto moze doci poslije

LINEARNO BINARNI BLOK KODOVI

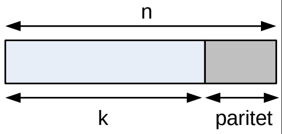
$$(n, M, d) \leftrightarrow [n, k, d] \leftrightarrow [n, k]$$

$$M = 2^k$$

$$R(K) = \frac{k}{n} \leq 1$$

$$d(K) = \min_{x,y \in K} (d(x, y) | x \neq y)$$

$$d(\boldsymbol{x}, \boldsymbol{y}) = w(\boldsymbol{x} - \boldsymbol{y})$$



Uvjeti lineranosti binarnog blok koda

1) $\boldsymbol{x} + \boldsymbol{y} \in \quad , \quad \boldsymbol{x}, \boldsymbol{y} \in$
2) $a \cdot \boldsymbol{x} \in \quad , \quad a \in \{0,1\}$
3) $000 \ldots 0 \in$

k – broj informacijskih bitova u kodnoj riječi
n - duljina kodne riječi
M - broj kodnih riječi u kodu
d – distanca (udaljenost) koda
R – kodna brzina
w - težina kodne riječi

Parni paritet - suma bitova $= \mathbf{0} \rightarrow$ *Paritet: 0*
Neparni paritet - suma bitova $= \mathbf{0} \rightarrow$ *Paritet: 1*

***HAMMINGOVA MEĐA** - kôd perfektan ako vrijedi samo '='*

$$M \leq \frac{\mathbf{2}^n}{\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{t}}$$

VJEROJATNOST NEOTKRIVENE POGREŠKE U PRIJENOSU

$$p_{np} = \binom{n}{2} p^2 (1-p)^{n-2} + \binom{n}{4} p^4 (1-p)^{n-4} + \cdots + \binom{n}{n} p^n \quad \text{n-parno}$$

$$p_{np} = \binom{n}{2} p^2 (1-p)^{n-2} + \binom{n}{4} p^4 (1-p)^{n-4} + \cdots + \binom{n}{n-1} p^{n-1} (1-p) \quad \text{n-neparno}$$

MINIMALNI r ZA ZADANU KODNU BRZINU

$$\frac{\mathbf{2}^r - \mathbf{1} - r}{\mathbf{2}^r - \mathbf{1}} > R_b \qquad (r = n - k)$$

VJEROJATNOST ISPRAVNOG DEKODIRANJA - *neispravno kodiranje* $\rightarrow \sum_{i=t+1}^{n}$

$$p(K) = \sum_{i=0}^{t} \binom{n}{i} p_g^i (1 - p_g)^{n-i}$$

DEKODIRANJE LINEARNOG BINARNOG KODA:

1) Metoda najbližeg susjeda

$$d(K) \geq s + 1$$
$$s = d(K) - 1$$

$$t = \left\lfloor \frac{d(K) - 1}{2} \right\rfloor$$
$$d(K) \geq 2t + 1$$
$$2^{n-k} \geq \sum_{i=0}^{t} \binom{n}{i}$$

s - najveći broj pogrešaka koje kôd može otkriti
t - najveći broj pogrešaka koje kôd K može ispraviti

2) Sindromsko dekodiranje

$\boldsymbol{e} = \boldsymbol{y} - \boldsymbol{x} = \boldsymbol{y} \oplus \boldsymbol{x}$
$\boldsymbol{e}$ *- vektor pogreške - označava poziciju pogreške*
$\boldsymbol{x}$ *- poslana kodna riječ*
$\boldsymbol{y}$ *- primljena kodna riječ*

$$\boldsymbol{G} = [\overbrace{\boldsymbol{I}_k}^{k} | \overbrace{\boldsymbol{A}}^{n-k}] \} \, \boldsymbol{k}$$

$$\boldsymbol{H} = [\overbrace{-\boldsymbol{A}^T | \boldsymbol{I}_{n-k}}^{n}] \} \, \boldsymbol{n-k}$$

$$\boldsymbol{S}(\boldsymbol{y}) = \boldsymbol{y} \cdot \boldsymbol{H}^T$$

$\boldsymbol{G}$ *– generirajuća matrica kôda (dimenzija* $\boldsymbol{k} \times \boldsymbol{n}$*)*
$\rightarrow$ *generira kôd K*
$\boldsymbol{H}$ *– matrica provjere pariteta (dimenzija* $(\boldsymbol{n} - \boldsymbol{k}) \times \boldsymbol{n}$*)*
$\rightarrow$ *generira dualni kôd* $K^{\perp}$
$\boldsymbol{S}$ *– sindrom*
$\rightarrow$ *ako različit od* $\mathbf{0}$*, onda postoji greška*
$\boldsymbol{A}$ *– kvadratna matrica*

$\boldsymbol{G} \cdot \boldsymbol{H}^T = \mathbf{0}$ *(rezultantni vektor dimenzija* $\boldsymbol{k} \times (\boldsymbol{n} - \boldsymbol{k})$*)*
$\boldsymbol{x} \cdot \boldsymbol{H}^T = \mathbf{0}$ *(ako je primljena kodna riječ ispravna)*
$\boldsymbol{c} = \boldsymbol{d} \cdot \boldsymbol{G}$ *(***c*** - kodna riječ,* ***d*** *- poruka s k bitova)*

(mod 2) $-1 = 1 \rightarrow A = \text{-}A$
(mod 3) $-1 = 2 \rightarrow -2 = 1$
Matrica k redova i n stupaca:
$(\boldsymbol{k} \times \boldsymbol{q}) \cdot (\boldsymbol{q} \times \boldsymbol{n}) = (\boldsymbol{k} \times \boldsymbol{n})$

$$\boldsymbol{d} = [011], \ \boldsymbol{c} = \{\boldsymbol{d}, \boldsymbol{d} \cdot \boldsymbol{A}\}$$

$$\boldsymbol{G} = \begin{array}{c} \\ \\ \\ \end{array}\begin{array}{cccccc|l} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & \\ 1 & 0 & 0 & 0 & 1 & 1 & d_1 \\ 0 & 1 & 0 & 1 & 0 & 1 & d_2 \\ 0 & 0 & 1 & 0 & 1 & 1 & d_3 \end{array}$$

$$\boldsymbol{c} = [011] \cdot \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} = [011 | 110]$$

$$[011] \cdot \overbrace{\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}}^{A} = \overbrace{[1 \ \ 1 \ \ 0]}^{\text{Zaštitni bitovi}} \qquad [011] \cdot \overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{I_3} = \overbrace{[0 \ \ 1 \ \ 1]}^{\text{Poruka}}$$

$$\boldsymbol{c}_4 = \boldsymbol{d}_2$$
$$\boldsymbol{c}_5 = \boldsymbol{d}_1 \oplus \boldsymbol{d}_3$$
$$\boldsymbol{c}_6 = \boldsymbol{d}_1 \oplus \boldsymbol{d}_2 \oplus \boldsymbol{d}_3$$

HAMMINGOV KÔD

$\boldsymbol{H}$ - matrica provjere pariteta dimenzija $r \times (2^r - 1)$

$r = n - k \qquad \boldsymbol{HAM}[\boldsymbol{n}, \boldsymbol{k}] \Leftrightarrow \boldsymbol{HAM}[\boldsymbol{r}] \qquad$ ***r*** *– broj zaštitnih bitova*

Generirajuću matricu G je iz matrice H moguće dobiti sljedećim postupkom:

1. U matrici H izbrisati sve stupce koji se nalaze na pozicijama s indeksom jednakim potenciji broja 2 (pozicije 1,2,4, 8, 16, itd).
2. Dobivenu matricu transponirati.
3. Stupce dobivene matrice smjestiti na pozicije generirajuće matrice G čiji indeksi odgovaraju potencijama broja 2.
4. Ostale stupce popuniti redom stupcima jedinične matrice.

HAM [7,4]

$$\boldsymbol{G} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{cc} \boldsymbol{H} = & \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \\ & \begin{array}{ccccccc} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} & \mathbf{7} \end{array} \end{array}$$

$$\boldsymbol{S}(\boldsymbol{y}) = \begin{bmatrix} \overset{1}{\mathbf{0}} & \overset{2}{\mathbf{1}} & \overset{4}{\mathbf{1}} \end{bmatrix} \rightarrow \mathbf{110}_{(2)} = \mathbf{6}_{(10)}$$

Ispravi bit na 6. poziciji s lijeva

$$r \geq 2 \begin{cases} \boldsymbol{d}(\boldsymbol{K}) = \mathbf{3} \\ \boldsymbol{t} = \mathbf{1} \\ \boldsymbol{s} = \mathbf{1,2} \\ \textit{perfektan kod} \\ \textit{linearan blok kod} \end{cases}$$

CIKLIČKI KÔD

Uvjeti:

1. $a(x), b(x) \in K$*, vrijedi* $a(x) + b(x) \in K$
2. $a(x) \in K \ i \ \forall \ r(x) \in Rn$*, vrijedi* $r(x) \cdot a(x) mod(xn - 1) \in K.$

$$x^n - 1 = g(x) \cdot h(x)$$

r - stupanj generirajućeg polinoma
$h(x)$ - polinom za provjeru pariteta cikličnog koda K.

$$d(x) \cdot x^r = g(x)q(x) + r(x) = c(x)$$

$$r(x) = d(x) \cdot x^r \ mod[g(x)] = \frac{\boldsymbol{x}^{n-k} \cdot \boldsymbol{c}'(\boldsymbol{x})}{\boldsymbol{g}(\boldsymbol{x})}$$ $\rightarrow$ *metoda cikličke redundantne zaštita* $\rightarrow$ *gleda se ostatak*

$$S(c'(x)) = \frac{x^{n-k} c'(x)}{g(x)}$$

$\boldsymbol{c} = [d | r] \rightarrow$ *kodna riječ dobivena metodom cikličke redundantne zaštite*

$g(x)$ - generirajući polinom
$q(x)$ - kvocijent
$d(x)$ – polinom kodirane poruke
$r(x)$ – ostatak nakon dijeljenja s $g(x)$
$c(x)$ – kodna riječ
$S(c'(x))$ - sindrom primljene kodne riječi

FAKTORIZACIJE NEKIH POLINOMA OBLIKA $x^n - 1$

n	aritmetika	Faktorizacija u aritmetici modulo 2
1	$x - 1$	$x + 1$
2	$x^2 - 1$	$(x + 1)^2$
3	$x^3 - 1$	$(x + 1)(x^2 + x + 1)$
5	$x^5 - 1$	$(x + 1)(x^4 + x^3 + x^2 + x + 1)$
7	$7 - 1$	$(x + 1)(x^3 + x + 1)(x^3 + x^2 + 1)$
9	$x^9 - 1$	$(x + 1)(x^2 + x + 1)(x^6 + x^3 + 1)$
11	$x^{11} - 1$	$(x + 1)(x^{10} + x^9 + \cdots + x + 1)$
13	$x^{13} - 1$	$(x + 1)(x^{12} + x^{11} + \cdots + x + 1)$
15	$x^{15} - 1$	$(x + 1)(x^2 + x + 1)(x^4 + x + 1)(x^4 + x^3 + 1)(x^4 + x^3 + x^2 + 1)$
17	$x^{17} - 1$	$(x + 1)(x^8 + x^5 + x^4 + x^3 + 1)(x^8 + x^7 + x^6 + x^4 + x^2 + x + 1)$
19	$x^{19} - 1$	$(x + 1)(x^{18} + x^{17} + \cdots + x + 1)$

ALGORITAM ZA DOBIVANJE GENERIRAJUĆE MATRICE $\boldsymbol{G} = [\boldsymbol{I}_k | \boldsymbol{A}]$ ***IZ*** $\boldsymbol{g}(\boldsymbol{x})$

1. upiši g(x) u binarnom obliku u k-ti (zadnji) redak tako da zadnji bit bude na zadnjem stupcu
2.a) redak k-1 (jedan iznad) se dobije posmakom k-tog retka za 1 u lijevo
2.b) Ako k-ti stupac u k-1 retku ima '1' onda je potrebno dodati k-ti (donji) red na k-1 (trenutni) redak
3. ponavljaj dok G nije pun

Srednja snaga i energija

(Ako nije drugačije zadano, R = 1Ω!)

$$E = \int_{-\infty}^{\infty} Ri^2(t)dt = \int_{-\infty}^{\infty} \frac{u^2(t)}{R}dt$$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} Ri^2(t)dt$$

Periodični signali

$$\cos x = \frac{e^{jx}+e^{-jx}}{2} \qquad \sin x = \frac{e^{jx}-e^{-jx}}{2j}$$

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

$$c_k = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)e^{-jk\omega_0 t}dt$$

Granice mogu ići i od 0 do T_0, ili po bilo kojem intervalu duljine T_0

$$c_k = |c_k|e^{-j\theta_k}$$

$$P = \sum_{k=-\infty}^{\infty}|c_k^2| = |c_0|^2 + 2\sum_{k=1}^{\infty}|c_k^2| = \frac{1}{2}(A_1^2 + \cdots + A_n^2)$$

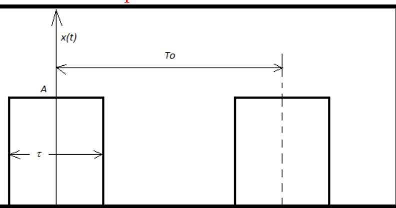
Snaga istosmjerne komponente: $|c_0|^2$.

$\omega_0 = \frac{2\pi}{T_0} \to T_0$ - osnovni period

Periodičan slijed pravokutnih impulsa

$$x(t) = \begin{cases} A, & 0 \le |t| \le \frac{\tau}{2} \\ 0, & \frac{\tau}{2} < |t| \le \frac{T_0}{2} \end{cases}$$

τ - trajanje signala
T_0 - osnovni period
A - amplituda



$$P = A^2\frac{\tau}{T_0}$$

Omjer impuls/pauza:

$$\frac{\tau}{T_0 - \tau}$$

$$c_k = A\frac{\tau}{T_0}\frac{\sin\left(\frac{k\omega_0\tau}{2}\right)}{\frac{k\omega_0\tau}{2}} \leftrightarrow |c_k| = A\frac{\tau}{T_0}\left|\frac{\sin\left(\frac{k\omega_0\tau}{2}\right)}{\frac{k\omega_0\tau}{2}}\right|$$

Kroz 0 prolazi u $\frac{k}{\tau}, k \in \mathbb{Z}$.

$$c_0 = A\frac{\tau}{T_0}$$

Snaga istosmjerne komponente: $P_0 = A^2\left(\frac{\tau}{T_0}\right)^2$

Neperiodični signali

$$E = \int_{-\infty}^{\infty}[x(t)]^2dt = \lim_{T\to\infty}\int_{-T}^{T}[x(t)]^2dt$$

$$P = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}[x(t)]^2dt$$

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

Pravokutni impuls:

$$X(f) = A\tau\frac{\sin\left(\frac{2\pi f\tau}{2}\right)}{\frac{2\pi f\tau}{2}}$$

Signal energije: $E < \infty \to P = 0$

Signal snage: $P > 0 \to E \to \infty$

Ni jedno ni drugo: $E \to \infty,\ P \to \infty$

Slučajni signali

$$\mu_X(t) = \int_{-\infty}^{\infty} xf(x,t)dx$$

$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau)e^{-j2\pi f\tau}d\tau$$

$$R_X(\tau) = \int_{-\infty}^{\infty} S_X(f)e^{j2\pi f\tau}df$$

$$P = R_X(0) = \int_{-\infty}^{\infty} S_X(f)df$$

$$S_Y(f) = S_X(f)|H(f)|^2$$

$$S_N(f) = \frac{N_0}{2}$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$H(f) = \int_{-\infty}^{\infty} h(t)e^{-j2\pi ft}dt$$

$$h(t) = \int_{-\infty}^{\infty} H(f)e^{j2\pi ft}df$$

NPK: $B = f_g$
PPK: $B = f_g - f_d$

μ_X – srednja vrijednost slučajnog procesa
R_X – autokorelacijska funkcija
$S_{X,Y,N}$ – spektralna gustoća snage
X – ulaz; Y – izlaz; može biti i obrnuto!!
S ili P_S – srednja snaga signala
P ili P_N – srednja snaga šuma
h(t) – impulsni odziv
H(f) – prijenosna funkcija
H(X) – entropija
N_0 – spektralna gustoća AWGN
N_q – srednja snaga kvantizacijskog šuma
u – uzorkovanje
m – multipleksiranje
n – broj kanala koji se multipleksiraju
r – broj bitova
L – broj razina
B – širina pojasa
A – pojačanje
E_b – energija bita
C – kapacitet kanala
D – dinamika
m_{max} – maksimalna amplituda ulaznog signala
Δ - korak kvantizacije
σ_q^2 - srednja kvadratna greška (brže i lakše)
$\overline{N_q}^2$ - srednja kvadratna greška (sporije i teže)
f(u) ili p(u) – funkcija gustoće vjerojatnosti razine signala
f(q) – funkcija gustoće vjerojatnosti razine kvantizacijskog šuma
(1) – ako imamo odstupanje od polazne vrijednosti
(2) – kod odstupanja od vrha do vrha;
U_p = |zadano u postocima|
Frekvencijsko miješalo poduplava frekvencije!

Uzorkovanje

$$f_u = 2B$$

$$R = \frac{H(X)}{T}$$

$$R_m = nR$$

$$L = 2^r$$

$$N = N_0 B$$

$$H(X) = -\sum_{i=1}^{n} p(x_i)\log_2 p(x_i)$$

$$A = \frac{S_2}{S_1}$$

$$C = B\log_2\left(1 + \frac{S}{N}\right)$$

$$C \ge R$$

$$C = \log_2 e\frac{S}{N_0}, \text{kad } B \to \infty$$

$$\frac{E_b}{N_0} = \frac{2^{\frac{C}{B}} - 1}{\frac{C}{B}}$$

$$D = 0.5\log\left(1 + \frac{S}{N}\right) = \frac{C}{2B}$$

Kvantizacija

$$\frac{S}{N} = \frac{3}{2}2^{2r} = \left(\frac{3S}{m_{max}^2}\right)2^{2r}$$

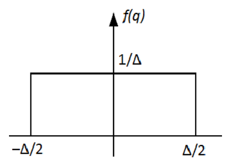
$$\left(\frac{S}{N_q}\right)_{dB} = 1.76 + 6.02r$$

$$\Delta = \frac{2m_{max}}{L}$$

$$-\frac{\Delta}{2} \le q \le \frac{\Delta}{2}$$

$$P_S = \int_{-\infty}^{\infty} u^2 f(u)du$$

$$P_N = \int_{-\infty}^{\infty} q^2 f(q)dq$$



(1) $\frac{\Delta}{2} = \frac{m_{max}}{L}$

(2) $\frac{\Delta}{2} = U_p 2m_{max}$

$$\overline{N_q}^2 = \int_{u_{qi}-\frac{\Delta}{2}}^{u_{qi}+\frac{\Delta}{2}} (u - u_{qi})^2 p(u)du$$

$$\sigma_q^2 = \frac{1}{3}m_{max}^2 2^{-2r}$$