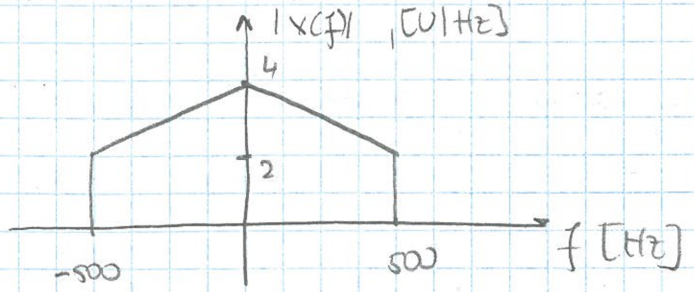
DOMAĆE ZADAĆE

Zi-6



[V]

signal: $x_1(t) = -x(t) + x(t)\cos(2000\pi t) + 2x(t)\cos^2(3000\pi t)$

$2\cos^2(x) = \cos(2x) + 1$

$x_1(t) = \cancel{-x(t)} + x(t)\cos(\underbrace{2000\pi t}_{\omega_0}) \cancel{+x(t)} + x(t)\cos(6000\pi t)$

$x(t) = A\cos(\omega_0 t) \qquad \omega_0 = 2\pi f$

$X(t) = \frac{A}{2}\left[\delta(f - f_0) + \delta(f + f_0)\right]$

konvolucija

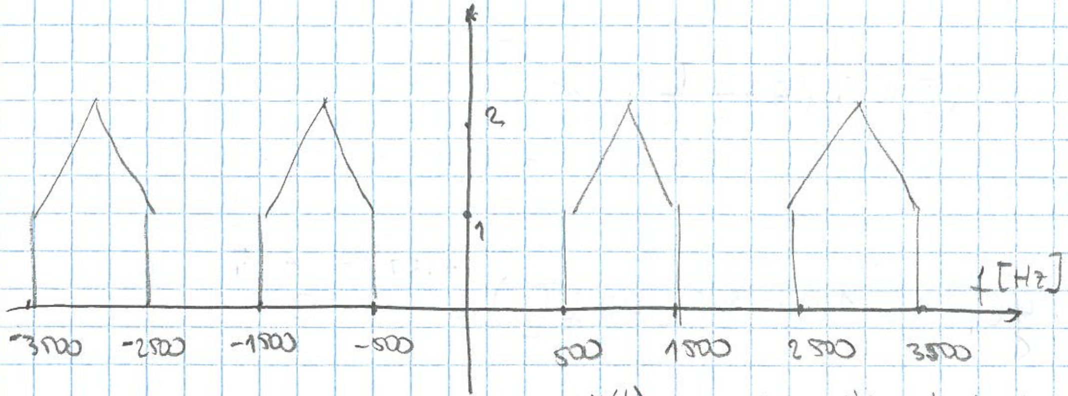
$X_1(f) = X(f) * \left[\frac{1}{2}\delta(f-1000) + \frac{1}{2}\delta(f+1000)\right]$

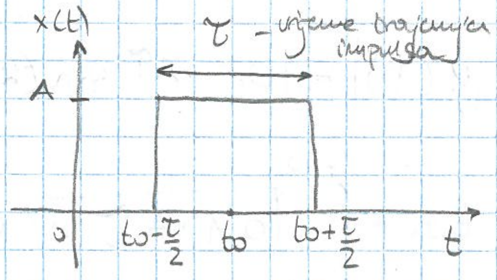
$+ X(f) * \left[\frac{1}{2}\delta(f-3000) + \frac{1}{2}\delta(f+3000)\right]$

$\rightarrow X_1(f) = \int_{-\infty}^{+\infty} \left[X(f-\tau)\frac{1}{2}\delta(\tau-1000) + X(f-\tau)\frac{1}{2}\delta(\tau+1000)\right] d\tau$

$+ \int_{-\infty}^{+\infty} \left[X(f-\tau)\frac{1}{2}\delta(\tau-3000) + X(f-\tau)\frac{1}{2}\delta(\tau+3000)\right] d\tau$

$\Rightarrow X_1(f) = \frac{1}{2}\left[X(f-1000) + X(f+1000) + X(f-3000) + X(f+3000)\right]$ [V/Hz]





Zi-10

$H(f) = 0.1 e^{j\frac{\pi}{4}}, \quad \forall f \in R$

~~Ato~~ = 0.1 mWs = Eul

$x(t - t_0) = \begin{cases} A & \text{za } 0 \le |t - t_0| < \frac{\tau}{2} \\ 0 & \text{za } |t - t_0| > \frac{\tau}{2} \end{cases}, \quad t \in R$

pravokutni impuls

$E_{izl} = ?$

$E_{izl} = \int_{-\infty}^{+\infty} y^2(t)\,dt = \int_{-\infty}^{+\infty} |Y(f)|^2 df$

$y(t) = x(t) * h(t)$

$Y(f) = X(f) \cdot H(f)$

$$X(f) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-j2\pi ft}\, dt = \int_{b_0-\frac{\tau}{2}}^{b_0+\frac{\tau}{2}} x(t) \cdot e^{-j2\pi ft}\, dt =$$

$$= A\tau \sin\left(2\pi f \frac{\tau}{2}\right) \cdot \underbrace{e^{-j2\pi ft}}_{\text{fazni pomak}}$$

$$y(f) = 0.1\, A\tau \sin\left(2\pi f \frac{\tau}{2}\right) \underbrace{e^{-j2\pi ft}}_{①}\, e^{-j\frac{\pi}{4}}$$

$$|y(f)| = |0.1 \cdot A\tau \sin\left(2\pi f \tfrac{\tau}{2}\right)| \cdot 1 \cdot 1$$

$$E_{IZL} = \int_{-\infty}^{+\infty} |0.1 \cdot A\tau \sin\left(2\pi f \tfrac{\tau}{2}\right)|^2 df =$$

$$= 0.1^2 \underbrace{\int_{-\infty}^{+\infty} |A\tau \sin\left(2\pi f \tfrac{\tau}{2}\right)|^2 df}_{E_{ul}}$$

$$E_{IZL} = 0.1^2 \cdot E_{ul} = 0.1^2 \cdot 0.1 \cdot 10^{-3} = 1\,\text{mWs}$$

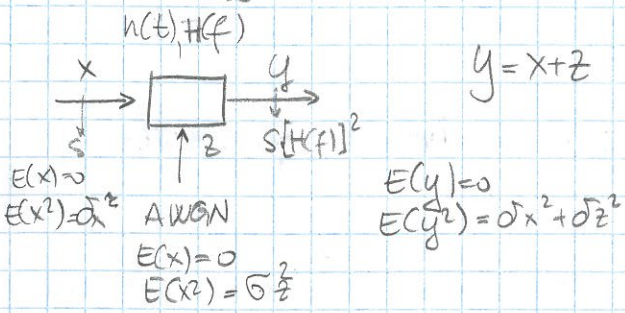
22.1.2013 god. Pr.

PRIPREMA ZA ZAVRŠNI ISPIT

$$C = f(B, SNR)$$

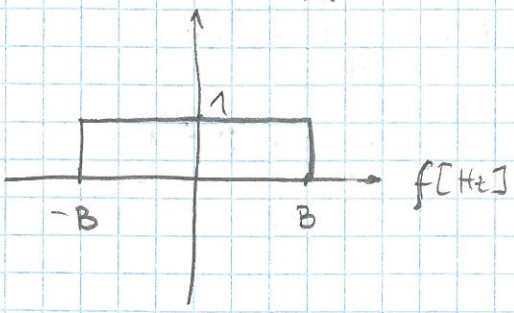
bandwith — signal to noise ratio

$$C = B \log_2 \left(1 + \frac{S}{N}\right) \quad \left[\frac{bit}{s}\right]$$



$y = x + z$

$$C = B \log_2 \left(1 + \frac{S}{N} \cdot [H(f)]^2\right)$$



$f_c = 4\,kHz = B$

govorni signal → A/D →

- Sklop za uzorkovanje $f_0 = 2f_c$
- Sklop za kvantizaciju $L = 256$
- Koder $r = 8\,bit/uzorak$

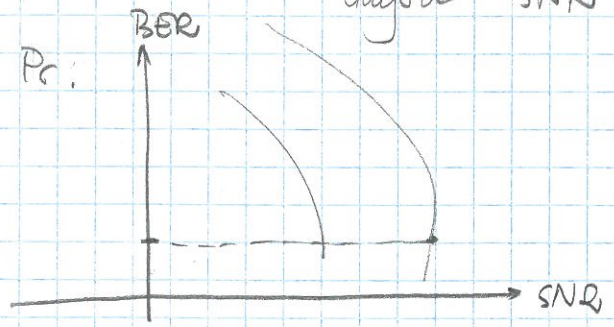
$f_0 = 8\,kHz = 8000$ uzoraka/s

$R = f_0 \cdot r = 2B \cdot D$

$r \equiv D$ DINAMIKA (broj bitova po uzorku)

$$D = \frac{1}{2} \log \left(1 + \frac{S}{N}\right)$$

$\Gamma \rightarrow C$ (SNR gap) - parametar realizacije



$$\Rightarrow C = B \log_2 \left(1 + \frac{S}{\Gamma \cdot N} \cdot [H(f)]^2\right).$$

Iskoristivost pojasa prijenosa

$$\frac{C}{B} = \log_2\left(1 + \frac{S}{N}[H(f)]^2\right) \quad [bit/s/Hz]$$

Zadatak 4 $\quad (n,k) = [2^r - 1,\ 2^r - r - 1]$

Hammingov kod $\quad r = n - k$

$\varepsilon > 0{,}904 \quad n = r + k$

$r = ?$ - najmanji br. zaštitnih bitova

$$\varepsilon = \frac{k}{n} \Rightarrow \frac{k}{n} > 0{,}904$$

$$\frac{k}{r+k} > 0{,}904$$

$$\frac{2^r - r - 1}{r + 2^r - r - 1} > 0{,}904$$

$$\frac{2^r - r - 1}{2^r - 1} > 0{,}904$$

$$2^r - r - 1 > 0{,}904 \cdot 2^r - 0{,}904$$

$$2^r(1 - 0{,}904) > r + 1 - 0{,}904$$

$$2^r(0{,}096) > r + 0{,}096 \Rightarrow r = 4\ bit$$

Zadatak 5 $\quad \frac{S}{N}\Big|_{dB} = 10\log\frac{S}{N} = 50\ dB$

$$\frac{S}{N} = 10^5$$

smanjenje $\Rightarrow$ $T_2 = 30\ dB$

$$C = B\log_2\left(1 + \frac{S}{N}\right)$$

$$R = B \cdot \log_2\left(1 + \frac{S}{\Gamma N}\right)$$

$$\Big\} \quad \frac{C}{R} = 1{,}67$$

Signal

$$u_n(t) = 0.8\sin\left(2\pi\, 400t + \frac{\pi}{4}\right),\ [V]$$

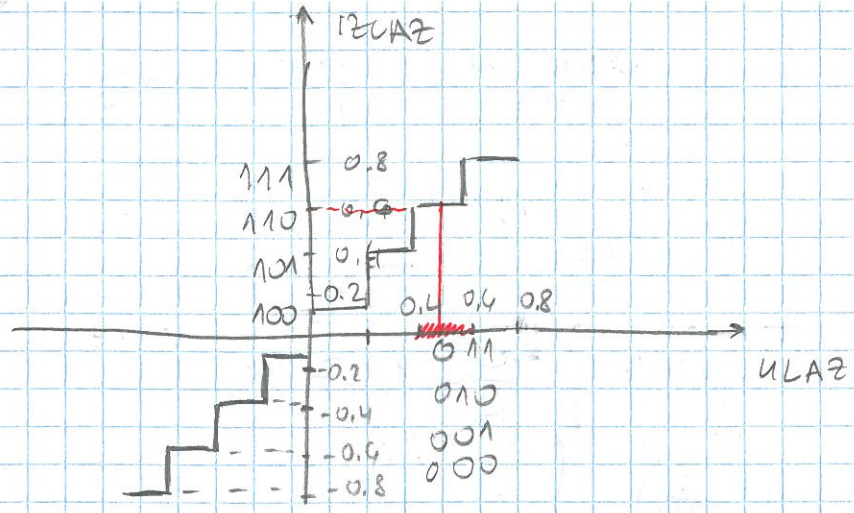
PRIGUŠENJE ZA 30 dB

Obr. kodna sekvenca iz A/D pretvornika za uzorak uzet za $t = 0s$

Amplitude uzoraka u intervalu $|n(t)| \le 0.8,\ [V]$

i kvantizirane u kvantizatoru sa $L = 8$

$$L = 2^r \Rightarrow r = 3$$



Svi pozitivni br. počinju s 1; neg. br. sa 0.

$U_m(0) = 0.8 \sin\frac{\pi}{4} = 0.8 \cdot \frac{\sqrt{2}}{2} = 0.5657\ V$

$20\log(0.5657) - 20\log(x) = 3$ * prigušenje za 3dB

$\Rightarrow x = 0{,}4005\ V$ ulaz kvantizatora

$\Rightarrow$ IZLAZ **110**

Zadatak 1. rok, veljača 2012. polaz: 0%

* digitalizacija signala

$x(f) \neq 0$ za $0 \leq f \leq 4\,kHz$

$x(t)$ → uzima uzorak → kvantizira → kodira → izlaz

$f_0 = 1.25 f_u = 1.25 \cdot 2B = 10\,kHz$

$L = 256$

a) $H(x)_{max} = ?$

$H(x)_{max} = \log_2 L = \log_2 25 = 8\ \frac{bit}{uzorak}$

b) $I(x_i) = ?$

$I(x_i) = \log_2 \frac{1}{P_{xi}}$

$P(x_i) = \frac{1}{2^8} = \frac{1}{256}$ (vjerojatnost u svakoj razini)

ukupni br komb. koji se može prezentirati sa 8 uzoraka

$\Rightarrow I(x_i) = -\log_2(P_{xi}) = -\log_2\left(\frac{1}{256}\right) = 8\ bit/uz$

c) $R = ?$ $\quad f_0 = 1.25 \cdot f_{uz} = 10\,kHz$

$R = f_{uz} \cdot r = f_0 \cdot r$

$L = 2^r \Rightarrow r = 8\ bit/uzorak$

$\Rightarrow R = 10 \cdot 10^3 \cdot 8 = 80k\ bit/simbol$

d) AWGN kanal
* max prijenosna brzina (R) $\frac{S}{N} = ?$

$H(f) \neq 0 \quad za \quad f \in [-10, 10]$ kHz

$C = B \log_2 (1 + \frac{S}{N})$

$C \geq R = 8 \cdot 10^4$

$\Rightarrow 10^4 \cdot \log_2 (1 + \frac{S}{N}) \geq 8 \cdot 10^4$

$1 + \frac{S}{N} \geq 256$

$\frac{S}{N} \geq 255$

$\Rightarrow \frac{S}{N}\Big|_{dB} \geq 24{,}0824\,dB$

e) $E = \frac{R}{B} = \frac{80\,000 \text{ bit/simbol}}{10000 \text{ Hz}} = 8 \text{ bit/s/Hz}$

Zadatak 2

$x(t) = -1 + 2\cos^2(5 \cdot 10^3 t)$ [V]

$$H(f) = \begin{cases} 0.5 \cdot e^{-j2\pi f} & 0 \leq |f| \leq 10^4 \text{ Hz} \\ 0 & |f| > 10^4 \text{ Hz} \end{cases}$$

a) $x(f) = ?$

$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$

$x(t) = 1 + 2\frac{1}{2}(1 + \cos(2 \cdot 5 \cdot 10^3 t)) - \cos(10^4 t)$

$x(f) = \frac{1}{2}\left[\delta\left(f - \frac{10^4}{2\pi}\right) + \delta\left(f + \frac{10^4}{2\pi}\right)\right]$ $\left[\frac{W}{Hz}\right]$

b) $y(t) = ?$

$Y(f) = H(f) \cdot X(f)$

$y(t) = \int_{-\infty}^{+\infty} Y(f)\, e^{j2\pi f t}\, df = \int_{-\infty}^{+\infty} H(f) \cdot X(f) e^{j2\pi f t}\, df$

$\Rightarrow y(t) = \frac{1}{4}\int \left[\delta\left(f - \frac{10^4}{2\pi}\right) + \delta\left(f + \frac{10^4}{2\pi}\right)\right] e^{j2\pi f} \cdot e^{-j2\pi f}\, df =$

$= \cdots = \frac{1}{2}\cos[10^4(t-1)]$ [V]

c) impulsni odziv $h(t<0)=0$

$$h(t)=\int_{-\infty}^{+\infty} H(f)\, e^{j2\pi ft}\, df = \int_{-\infty}^{+\infty} \frac{1}{2}\, e^{-j2\pi f}\cdot e^{j2\pi ft}\, df =$$

$$= \dots = 2\cdot 10^4 \frac{\sin[2\pi\cdot 10^4(t-1)]}{2\pi\cdot 10^4(t-1)}$$

$\Rightarrow$ dvoji f-ja bilo 0, $2\pi\cdot 10^4(t-1)=0+k\pi$ argument fje množi bili 0.

$$\Rightarrow t=\frac{k}{2\cdot 10^4}+1\ [s],\quad k\in\mathbb{Z},\ k\neq 0$$

d) $\mu_y=\mu_x\cdot H(0)$

srednje vrij.

$$f_x(x)=\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{(x-0.1)^2}{2}}$$ (μ_x = 0.1)

$$\mu_y=\mu_x\cdot H(0)=0{,}1\cdot 0{,}5=0{,}05$$

e) $S_y(f)=S_x(f)\,|H(f)|^2$? bijeli Gaussov šum

$$S_x(f)=\int_{-\infty}^{+\infty} R_x(\tau)\cdot e^{-j2\pi f\tau}\, d\tau$$

$$R_x(\tau)=\sigma_x^2\,\delta(\tau)=\delta(\tau)$$

$$S_x(f)=\int_{-\infty}^{+\infty} R_x(\tau)\, e^{-j2\pi f\tau}\, d\tau =$$

$$=\int_{-\infty}^{+\infty}\delta(\tau)\, e^{-j2\pi f\tau}\, d\tau = 1\ \frac{W}{Hz}$$

$$S_y(f)=S_x(f)\cdot|H(f)|^2=\begin{cases}0.25\ \frac{W}{Hz}, & |f|\le 10^4\ Hz\\ 0, & |f|>10^4\ Hz\end{cases}$$