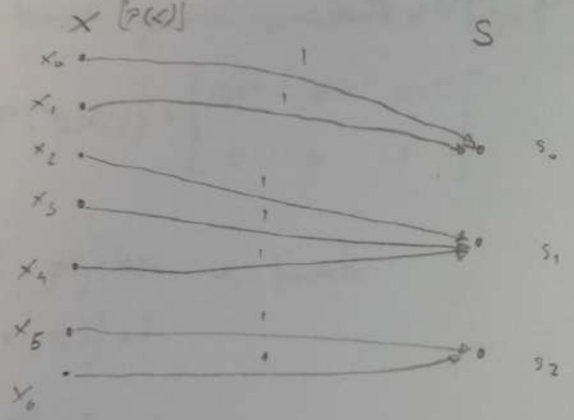
$[P(x)] = [0{,}04 \quad 0{,}25 \quad 0{,}10 \quad 0{,}21 \quad 0{,}20 \quad 0{,}15 \quad 0{,}05]$

$\{x_0, x_1\} \to s_0$

$\{x_2, x_3, x_4\} \to s_1$

$\{x_5, x_6\} \to s_2$

Odredite iznos izgubljene informacije $H(X|S) = ?$ u danom procesu kvantizacije.



$$[P(s_j|x_i)] = \begin{array}{c|ccc} & s_0 & s_1 & s_2 \\ \hline x_0 & 1 & 0 & 0 \\ x_1 & 1 & 0 & 0 \\ x_2 & 0 & 1 & 0 \\ x_3 & 0 & 1 & 0 \\ x_4 & 0 & 1 & 0 \\ x_5 & 0 & 0 & 1 \\ x_6 & 0 & 0 & 1 \end{array}$$

← popuni !

$H(X|S) = H(X) - I(X;S)$

$P(S|X) = \Big[\quad \Big]$

$I(X;S) = H(S) - \underbrace{H(S|X)}_{=0} = H(X) - H(X;S)$

$I(X;S) = H(S)$

$P(S) = [0.29 \quad 0{,}51 \quad 0{,}20]$

$H(X|S) = H(X) - H(S) = 2.53 - 1.47 = 1.17 \ \frac{bit}{simbol}$

$[P(Y|X)] = [$

simetričan ka

Primjer: (kaskadira

-dva binarna

$P(x_1) = P$

Odredite uk

$[P(Z)] = [P$

$[P(Y)] = [P$

$[P(Y|X)]$

X

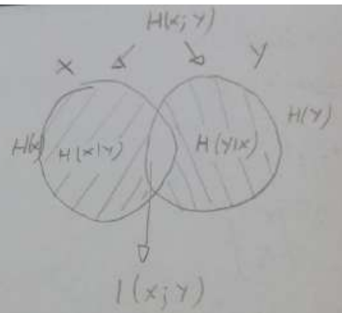
			Y	
p_1	x_1	i) $[p(Y\mid X)]$	y_1	$p(y_1)$
p_2	x_2	ii) $[p(X\mid Y)]$	y_2	$p(y_2)$
p_i	x_i	iii) $[p(X;Y)]$	y_j	$p(y_j)$
$\vdots$	$\vdots$		$\vdots$	$\vdots$
p_n	x_n	DKK	y_m	$p(y_m)$
$H(X)$				$H(Y)$

$H(X\mid Y)$

$H(Y\mid X)$

$H(X;Y)$

$I(X;Y) \rightarrow C$



X i Y nezavisne $\rightarrow I(X;Y)=0$

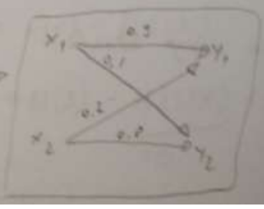
Primjer: Dis bezm. izv.

$X = \{x_1, x_2\}$

$[p(x)] = [0{,}25 \quad 0{,}75]$

DKK:

$\begin{bmatrix} x_1 & 0{,}25 \\ x_2 & 0{,}75 \end{bmatrix} \rightarrow$



$\rightarrow y_j$

Primjer: 4 poruke g

$X = \{x_1, \ldots, x_4\}$

$x_1 \rightarrow$ "00"

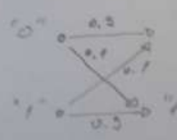
$x_2 \rightarrow$ "01"

$x_3 \rightarrow$ "10"

$x_4 \rightarrow$ "11"

prenose se bi

$I(X;Y) = ?$



$[p(x)] = [\frac{1}{4} \quad \frac{1}{4}$

$[p(Y\mid X)] = \begin{bmatrix} 0{,}9 \\ 0{,}1 \end{bmatrix}$

$H(X,Y)$

$I(X;Y) \rightarrow C$

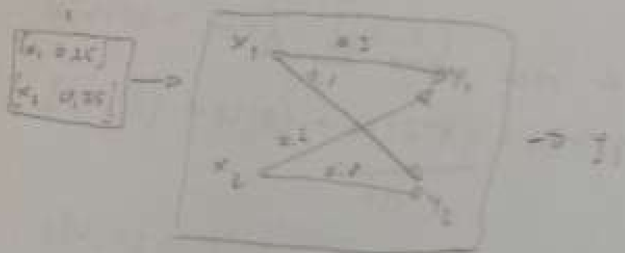
X, Y nezavisne

Primjer: Dis. bez. izv.

$X = \{x_1, x_2\}$

$[P(x)] = [0{,}25 \quad 0{,}75]$

Dkk:



1) vjerojatnost pojave pogrešnog simbola na izlazu

ii) matrica združenih vjerojatnosti $[P(x_i, y_j)]$

iii) iznos korisne inf. na izlazu kanala $I(X;Y)$

1) $[P(Y|X)] = \begin{bmatrix} 0{,}9 & 0{,}1 \\ 0{,}2 & 0{,}8 \end{bmatrix}$ $\quad [P(X,Y)]$

$[P(X)] = [0{,}25 \quad 0{,}75]$ $\quad P(x_i, y_j) = P(x_i) P(y_j|x_i)$

$x_1 \rightarrow y_2$

$x_2 \rightarrow y_1$

$P_g = P(x_1) \cdot P(y_2|x_1) + P(x_2) \cdot P(y_1|x_2) = \underline{0.175}$

ii) $[P(X,Y)] = \begin{bmatrix} 0{,}225 & 0{,}025 \\ 0{,}15 & 0{,}6 \end{bmatrix}$ $\downarrow \Sigma$

$[P(Y)] = [0.375 \quad 0.625]$

iii) $I(X;Y) = H(Y) - H(Y|X) = 0.9544 - 0.6586 = 0.2958$

Primjer: 4-pozicijsko generiranje od 4 jednako vjerojatna simbola

$X = \{x_1, \ldots, x_4\}$ kodirano binarnim kodom

$x_1 \to$ "00"

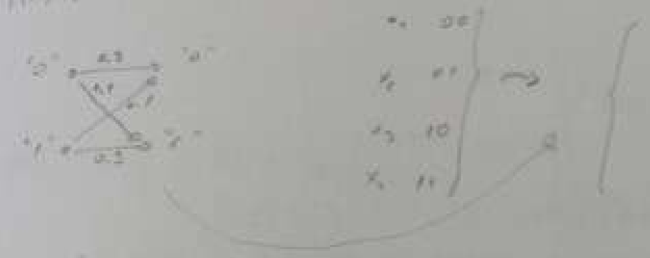
$x_2 \to$ "01"

$x_3 \to$ "10"

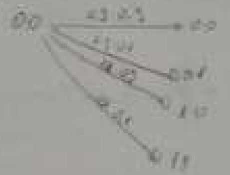
$x_4 \to$ "11"

prenose se binarnim simetričnim kan., gdje p greške prijenosa 0.1 ($p_g = 0.1$)

$I(X;Y) = ?$



$[p(X)] = \left[\frac{1}{4} \ \frac{1}{4} \ \frac{1}{4} \ \frac{1}{4}\right]$



$$[p(Y|X)] = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$$

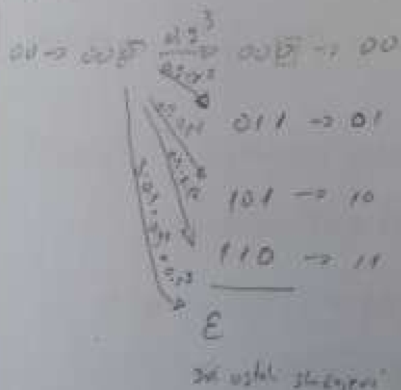
$$[p(y_j|x_i)] = \begin{bmatrix} 0{,}81 & 0{,}09 & 0{,}09 & 0{,}01 \\ & 0{,}81 & & \\ & & 0{,}81 & \\ & & & 0{,}81 \end{bmatrix}$$

$[p(Y)] = [p(X)]$

$I(X;Y) = 1{,}062 \ \frac{\text{bit}}{\text{simbol}}$

Primjer: $I(X;Y) = ?$ ako se u prijenosu uvede jedan paritetni bit (parni paritet)

$$\begin{matrix} 00 & \boxed{0} \\ 01 & 1 \\ 10 & 1 \\ 11 & \boxed{0} \end{matrix} \to \text{dekod.}$$



$$[p(y_j|x_i)] = \begin{array}{c|ccccc} & y_1 & y_2 & y_3 & y_4 & E \\ \hline x_1 & & & & & 0{,}244 \\ x_2 & & & & & 0{,}244 \\ x_3 & & & & & 0{,}244 \\ x_4 & & & & & 0{,}244 \end{array}$$

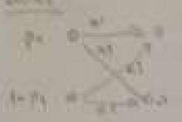
$I(X;Y) = 1{,}302 \ \frac{\text{bit}}{\text{simbol}}$

Kapacitet DKK

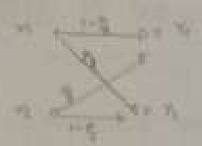
$C = \max I(X;Y)$

$I(X;Y) = H(Y) - H(Y|X) = H(X) - H(X|Y)$

BSKK:



Primjer kapacitet DBSKK



$$P(Y|X) = \begin{bmatrix} 1-p_g & p_g \\ p_g & 1-p_g \end{bmatrix}$$

$$[p(X)] = \begin{bmatrix} p & 1-p \end{bmatrix}$$

$p(x_1)$ $p(x_2)$

$$[p(X,Y)] = \begin{bmatrix} p(1-p_g) & p \cdot p_g \\ (1-p) \cdot p_g & (1-p)(1-p_g) \end{bmatrix}$$

$p(x_1, y_1) = p(x_1) \cdot p(y_1|x_1) = p \cdot (1-p_g)$

$p(x_1, y_2) = p(x_1) \, p(y_2|x_1) = p \, p_g$

$$[p(Y)] = \begin{bmatrix} p(1-p_g) + (1-p)p_g & p \, p_g + (1-p)(1-p_g) \end{bmatrix}$$

$I(X;Y) = H(Y) - H(Y|X)$

$$H(Y) = -\left\{ (p(1-p_g) + (1-p)p_g) \log_2 (p(1-p_g) + (1-p)p_g) + (p p_g + (1-p)(1-p_g)) \log_2 (p p_g + (1-p)(1-p_g)) \right\}$$

$$H(Y|X) = -\sum_i \sum_j p(x_i, y_j) \log p(y_j|x_i) = -\left\{ p(1-p_g)\log_2(1-p_g) + p p_g \log_2 p_g + (1-p) p_g \log_2 p_g + (1-p)(1-p_g) \log_2 (1-p_g) \right\}$$

$I(X;Y) = 1 \frac{\text{bit}}{\text{simbol}} \Rightarrow p_1 = p_2 = \frac{1}{2}$

potpuno sim kanal

$P(Y|X) =$ 

$C = \log_2 n - H(r)$

$p(x_1) = p(x_2) = \ldots$

Primjer: Specifični kanal = ?

$$\begin{tikzcd}[row sep=small] x_1 \arrow[r, "1/2"] \arrow[dr, "1/2"'] & y_1 \\ & y_2 \\ x_2 \arrow[r] & y_3 \end{tikzcd}$$

$$C = \max_{\{p_i\}} I(X;Y)$$

$$p(x_1) = p_1$$

$$p(x_2) = 1 - p_1 = p_2$$

$$[P(y_j|x_i)] = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightsquigarrow [P(x_i, y_j)] = \begin{bmatrix} \frac{1}{2}p_1 & \frac{1}{2}p_1 & 0 \\ 0 & 0 & p_2 \end{bmatrix}$$

$$I(X;Y) = H(Y) - H(Y|X) \qquad [P(y_j)] = \begin{bmatrix} \frac{1}{2}p_1 & \frac{1}{2}p_1 & p_2 \end{bmatrix}$$

$$H(Y) = -\sum_j p(y_j) \log_2 p(y_j) = -\left\{ 2 \cdot \frac{1}{2} p_1 \log_2 \frac{p_1}{2} + p_2 \log_2 p_2 \right\}$$

$$= -p_1 \log \frac{p_1}{2} - p_2 \log p_2$$

$$= -p_1 \log p_1 + \underbrace{p_1 \log 2}_{p_1} - p_2 \log p_2$$

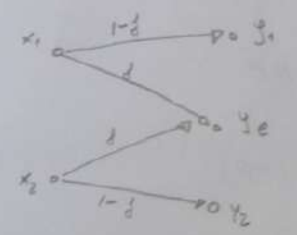
$$= p_1 - p_1 \log p_1 - p_2 \log p_2 = p_1 + H(X)$$

$$H(Y|X) = -\sum_i \sum_j p(x_i, y_j) \log_2 p(y_j|x_i) = -\left\{ \frac{p_1}{2} \log \frac{1}{2} + \frac{p_1}{2} \log \frac{1}{2} + \cancel{p_2 \log 1} \right\} = p_1$$

$$C = \max_{\{p_i\}} (p_1 + H(X) - p_1) = \max_{\{p_i\}} H(X) \rightarrow p_1 = p_2 = \frac{1}{2} \rightarrow I(X,Y) = 1 \ \frac{bit}{simbol}$$

Primjer:

$$p(x_1) = p$$



$P(x_1) = p$

$P(x_2) = 1-p$

$$[P(Y|X)] = \begin{bmatrix} 1-\delta & \delta & 0 \\ 0 & \delta & 1-\delta \end{bmatrix}$$

$$[P(X;Y)] = \begin{bmatrix} p(1-\delta) & p\delta & 0 \\ 0 & \delta(1-p) & (1-p)(1-\delta) \end{bmatrix} \downarrow \Sigma$$

$$[P(Y)] = [p(1-\delta) \quad \delta \quad (1-p)(1-\delta)]$$

$I(X;Y) = H(Y) - H(Y|X)$

$$H(Y) = -\sum_j P(y_j)\log_2 P(y_j) = -\{p(1-\delta)\log(p(1-\delta)) + \delta\log(\delta) + (1-p)(1-\delta)\log((1-p)(1-\delta))\}$$

$$H(Y|X) = -\sum_i\sum_j P(x_i;y_j)\log_2 P(y_j|x_i) = -\{p(1-\delta)\log(1-\delta) + p\delta\log(\delta) + \delta(1-p)\log(\delta) + (1-p)(1-\delta)\log(1-\delta)\}$$

$$I(X;Y) = -p(1-\delta)\log(p) - \cancel{p(1-\delta)\log(1-\delta)} - \delta\log(\delta) - (1-p)(1-\delta)\log((1-p)(1-\delta)) + \cancel{p(1-\delta)\log(1-\delta)} + p\delta\log(\delta)$$
$$+ \delta(1-p)\log(\delta) + (1-p)(1-\delta)\log(1-\delta)$$
$$= -p(1-\delta)\log(p) - \delta\log(\delta) - (1-p)(1-\delta)\log(1-p) - \cancel{(1-p)(1-\delta)\log(1-\delta)} + p\delta\log(\delta) + \delta(1-p)\log(\delta)$$
$$+ \cancel{(1-p)(1-\delta)\log(1-\delta)}$$
$$= \underline{-p(1-\delta)\log(p)} - \delta\log(\delta) - \underline{(1-p)(1-\delta)\log(1-p)} + p\delta\log(\delta) + \delta(1-p)\log(\delta)$$
$$= -(1-\delta)(p\log(p) + (1-p)\log(1-p))$$
$$= (1-\delta)H(X)$$

$$C = (1-\delta)\max_{\{p_i\}} H(X) = 1-\delta \ \frac{\text{b.t}}{\text{simbol}}$$

Primjer: DBKK

$X = \{x_1, x_2\}$

$$P(Y|X) = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

a) $p(x_1) = ?$ $p(x_2) = ?$

$$p(x) = [p \quad 1-p]$$

$I(X;Y) = I_{max}(X;Y)$

b) $C = ?$

a) $I(X;Y) = H(Y) - H(Y|X)$

$$[P(X,Y)] = \begin{bmatrix} p & 0 \\ \frac{1}{2}(1-p) & \frac{1}{2}(1-p) \end{bmatrix}$$

$$[P(Y)] = \begin{bmatrix} \frac{1}{2}(1+p) & \frac{1}{2}(1-p) \end{bmatrix} \rightarrow \Sigma = 1 \checkmark$$

$$H(Y) = -\left\{ \frac{1+p}{2} \log\left(\frac{1+p}{2}\right) + \frac{1-p}{2} \log\left(\frac{1-p}{2}\right) \right\} = -\left\{ \frac{1+p}{2}\left(\log(1+p) - \log 2\right) + \frac{1-p}{2}\left(\log(1-p) - \log 2\right) \right\}$$

$$= -\left\{ \frac{1+p}{2} \log(1+p) - \frac{1+p}{2} + \frac{1-p}{2} \log(1-p) + \frac{1-p}{2} \right\}$$

$$= -\left\{ -1 + \frac{1+p}{2} \log(1+p) + \frac{1-p}{2} \log(1-p) \right\}$$

$$H(Y|X) = -\sum_i \sum_j p(x_i, y_j) \log P(y_j|x_i) = -\left\{ \frac{1}{2}(1-p) \log\left(\frac{1}{2}\right) + \frac{1}{2}(1-p) \log\left(\frac{1}{2}\right) \right\} = 1-p$$

$$I(X;Y) = 1 - \frac{1+p}{2} \log(1+p) - \frac{1-p}{2} \log(1-p) - 1 + p = p - \frac{1+p}{2} \log(1+p) - \frac{1-p}{2} \log(1-p)$$

$$\frac{d\,I(X;Y)}{dp} = 1 - \frac{1}{\ln 2}\left(\frac{1}{2} \cdot \frac{1}{1+p} + \frac{1}{2} \log(1+p) + \frac{p}{2} \cdot \frac{1}{1+p} \right) - \frac{1}{\ln 2}\left(\frac{-1}{2} \cdot \frac{1}{1-p} + \frac{1}{2} \log(1-p) - \frac{p}{2} \cdot \frac{1}{1-p} \right)$$

$$= 0 \Rightarrow p = \frac{3}{5} \quad 1-p = \frac{2}{5}$$

Primjer

a)

$$[P(Z|X)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow H(Z|X) =$$

$$[P(Z,X)] = \begin{bmatrix} p_1 & 0 & 0 \\ 0 & p_2 & 0 \\ 0 & 0 & p_3 \end{bmatrix}$$

$$[P(Z)] = [p_1 \quad p_2 \quad p_3]$$

$$C = \max_{\{p(x_i)\}} I(X;Z) = \max_{\{p(x_i)\}} H(Z) =$$

b) $C_{(X,Y)} = ?$

$$[P(Y|Z)] = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[P(Y|X)] = [P(Z|X)][P(Y|Z)] =$$

$$I(X;Y) = H(Y) - H(Y|X) =$$

$$C_{(X,Y)} = \max_{\{p(x_i)\}} H(Y) = 1 \frac{\text{bit}}{\text{sim}}$$

Primjer: SKK sa smetnjama

Primjer:

a) $C_{XZ} = ?$

$I(X;Z) = H(Z) - H(Z|X) = H(Z)$

$$[p(Z|X)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow H(Z|X) = 0$$

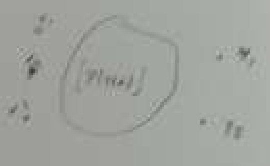
$$[p(Z;X)] = \begin{bmatrix} p_1 & 0 & 0 \\ 0 & p_2 & 0 \\ 0 & 0 & p_3 \end{bmatrix}$$

$$[p(Z)] = [p_1 \ p_2 \ p_3]$$

$$C = \max_{[p(x)]} I(X;Z) = \max_{[p(x)]} H(Z) = \log_2 3 \ \frac{bit}{simbol}$$

b) $C_{(X,Y)} = ?$

$$[p(Y|Z)] = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$$[p(Y|X)] = [p(Z|X)][p(Y|Z)] = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow H(Y|X) = 0$$

$$I(X;Y) = H(Y) - H(Y|X) = H(Y)$$

$$C_{(X,Y)} = \max_{[p(x)]} H(Y) = 1 \ \frac{bit}{simbol}$$

Primjer: BSK sa smetnjama

$X = \{x_1, x_2, x_3\}$

$Y = \{y_1, y_2, y_3\}$

$$[p(x_i, y_j)] = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}$$

odredite p-ove za koje se postiže max $H(X,Y)$

$p(x_1) = \frac{1}{2}$ $\quad p(y_1) = \frac{1}{3}$

$p(x_2) = \frac{1}{5}$ $\quad p(y_2) = \frac{1}{6}$

$$H(Y,X) = H(X) - \underbrace{I(X;Y)}_{=0} = H(X)$$

X i Y nezavisne