Popis formula iz kolegija *Teorija informacije*

Osnovne trigonometrijske jednakosti:

$$\sin\left(x \pm \frac{\pi}{2}\right) = \pm\cos x$$
$$\cos\left(x \pm \frac{\pi}{2}\right) = \mp\sin x$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$
$$\sin x - \sin y = 2\sin\frac{x-y}{2}\cos\frac{x+y}{2}$$
$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$
$$\cos x - \cos y = 2\sin\frac{x+y}{2}\sin\frac{y-x}{2}$$

$$\sin x \sin y = \frac{1}{2}(\cos(x-y) - \cos(x+y))$$
$$\cos x \cos y = \frac{1}{2}(\cos(x-y) + \cos(x+y))$$
$$\sin x \cos y = \frac{1}{2}(\sin(x-y) + \sin(x+y))$$

$$\sin(2x) = 2\sin x \cos x$$
$$\cos(2x) = cos^2 x - sin^2 x$$
$$2sin^2 x = 1 - \cos(2x)$$
$$2cos^2 x = 1 + \cos(2x)$$

Konačne sume:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$
$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$
$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$
$$\sum_{i=0}^{n} x^i = \frac{x^{n+1}-1}{x-1}$$
$$\sum_{i=0}^{n} \binom{n}{i} = \sum_{i=1}^{n} \frac{n!}{i!\,(n-i)!} = 2^n$$

Kompleksna eksponencijalna funkcija:

$$e^{\varphi + j\omega} = e^{\varphi}(\cos\omega + j\sin\omega)$$

$$\boldsymbol{sinwt} = \frac{\mathbf{1}}{\mathbf{2}\boldsymbol{j}}(\boldsymbol{e^{jwt}} - \boldsymbol{e^{-jwt}})$$
$$\boldsymbol{coswt} = \frac{\mathbf{1}}{\mathbf{2}}(\boldsymbol{e^{jwt}} + \boldsymbol{e^{-jwt}})$$

Entropija i njena svojstva:

$$H(X) = -\sum_{i=1}^{n} p(x_i)\log_2 p(x_i)\ \ [bit/simbol]$$
$$H \geq 0$$
$$H(X) \leq \log_2 n$$
$$H(XY) = H(X) + H(Y)$$

Prosječna duljina kodne riječi:

$$L = \sum_{i=1}^{n} p(x_i)\cdot l(x_i)\ \ [bit/simbol]$$

Vjerojatnosni opis diskretnog sustava:

$$D(p||q) = \sum_{i=1}^{n} p_i \log_2 \frac{p_i}{q_i}$$

$$p(x_i) = \sum_{j=1}^{m} p(x_i, y_j),\, p(y_j) = \sum_{i=1}^{n} p(x_i, y_j)$$
$$p(x_i, y_j) = p(x_i)p(y_j|x_i) = p(y_j)p(x_i|y_j)$$
$$p(x_i|y_j) = \frac{p(x_i, y_j)}{p(y_j)} = \frac{p(x_i, y_j)}{\sum_{i=1}^{n} p(x_i, y_j)} = \frac{p(x_i)p(y_j|x_i)}{\sum_{i=1}^{n} p(x_i)p(y_j|x_i)}$$

Informacijski opis diskretnog sustava:

$$H(X,Y) = -\sum_{i=1}^{n}\sum_{j=1}^{m} p(x_i, y_j)\log_2 p(x_i, y_j)$$
$$H(Y|X) = -\sum_{i=1}^{n}\sum_{j=1}^{m} p(x_i, y_j)\cdot\log_2 p(y_j|x_i)$$
$$I(X;Y) = \sum_{i=1}^{n}\sum_{j=1}^{m} p(x_i, y_j)\log_2\frac{p(x_i, y_j)}{p(x_i)p(y_j)}$$

$$I(X;Y) = H(X) - H(X|Y) = H(Y) - H(Y|X)$$
$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$
$$H(X,Y) = H(X) + H(Y|X) = H(Y) + H(X|Y)$$
$$I(X;Y) = I(Y;X)$$
$$I(X;X) = H(X)$$
$$I(X;Y) \geq 0$$
$$H(X|Y) \leq H(X)$$

Kapacitet kanala:

$$C = \max_{\{p(x_i)\}} I(X;Y)\ [bit/simbol]$$

Komunikacijski kanali u kontinuiranom vremenu

Fourierova transformacija za **periodične signale**:

$$x(t) = \sum_{k=-\infty}^{\infty} c_k \cdot e^{jk\omega_0 t}$$

$$c_k = \frac{1}{T_0} \cdot \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t) \cdot e^{-jk\omega_0 t} dt$$

Spektar sinusnog signala:

$$X(f) = -j\frac{A}{2}[\delta(f - f_0) - \delta(f + f_0)]$$

Spektar kosinusnog signala:

$$X(f) = \frac{A}{2}[\delta(f - f_0) + \delta(f + f_0)]$$

Spektar periodičnog slijeda pravokutnih impulsa:

$$c_k = A\frac{\tau}{T_0}\frac{\sin(k\omega_0\tau/2)}{k\omega_0\tau/2}$$

Snaga i Parsevalova relacija za periodične signale:

$$P = \frac{1}{T_0} \cdot \int_0^{T_0} |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |c_k|^2$$

$$P = |c_0|^2 + 2 \cdot \sum_{k=1}^{\infty} |c_k|^2$$

Fourierova transformacija za **neperiodične signale**:

$$X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \cdot \int_{-\infty}^{\infty} X(\omega) \cdot e^{j\omega t} d\omega$$

Spektar pravokutnog impulsa:

$$X(f) = A\tau\frac{\sin(2\pi f\tau/2)}{2\pi f\tau/2}$$

Energija, snaga i Parsevalova relacija za neperiodične signale:

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \cdot \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \cdot \int_{-T}^{T} |x(t)|^2 dt$$

Slučajni signali:

$$\mu_X(t) = E[X(t)] = \int_{-\infty}^{+\infty} x f_X(x,t) dx$$

$$R_X(\tau) = E[X(t_1)X(t_2)] = \int_{-\infty}^{\infty} S_X(f) \cdot e^{j2\pi f\tau} df$$

$$C_X(t_1, t_2) = R_X(t_1, t_2) - E[X(t_1)]E[X(t_2)]$$

$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau) \cdot e^{-j2\pi f\tau}\, d\tau \ \left[\frac{W}{Hz}\right]$$

$$P = E[X^2] = R_X(0) = \int_{-\infty}^{\infty} S_X(f) df = 2BS_X(f)$$

$$var(X) = \sigma_X^2 = E[X^2] - (E[X])^2$$

Stacionarnost slučajnog procesa:

$$E[X(t)] = \mu_X, \forall t \in R$$

$$R_X(t_1, t_2) = K_X(|t_2 - t_1|) = K_X(\tau), \forall t_1, t_2 \in R$$

Impulsni odziv i prijenosna funkcija kanala:

$$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t - \tau) d\tau = x(t) * h(t)$$

$$H(f) = \int_{-\infty}^{\infty} h(t) \cdot e^{-j2\pi ft} dt = |H(f)| \cdot e^{-j\theta(f)}$$

$$Y(f) = H(f) \cdot X(f)$$

$$|H(-f)| = |H(f)|$$

$$\theta(-f) = -\theta(f)$$

$$\mu_Y = \mu_X \cdot H(0)$$

$$S_Y = S_X \cdot |H(f)|^2$$

$$P = \int_{-\infty}^{\infty} S_Y(f) df$$

$$|H(f)| = \left|\frac{U_2(f)}{U_1(f)}\right| = \frac{1}{\sqrt{1 + (2\pi fRC)^2}}$$

Širina prijenosnog pojasa kanala:

$$A(f) = \frac{1}{|H(f)|}$$

$$Y(f) = |Y(f)| \cdot e^{j\vartheta(f)}$$

$$|Y(f)| = |X(f)| \cdot |H(f)|$$

$$\vartheta(f) = \varphi(f) - \theta(f)$$

Uzorkovanje:

$$x(t)_\delta = \sum_{n=-\infty}^{\infty} x(nT_u) \cdot \delta(t - nT_u)$$

$$f_u = \frac{1}{T_u} = 2B$$

Kvantizacija:

$$m_k < m(nT_u) \le m_{k+1}, k = 1,2, \dots, L$$

$$m_k < v_k \le m_{k+1}, k = 1,2, \dots, L$$

$$v_k = \frac{m_k + m_{k+1}}{2}$$

$$\Delta = \frac{2m_{max}}{L}$$

$$var(Q) = \sigma_Q^2 = \frac{\Delta^2}{12} = \frac{1}{3} m_{max}^2 \cdot 2^{-2r}$$

$$\frac{S}{N} = \frac{S}{\sigma_Q^2} = \left(\frac{3S}{m_{max}^2}\right) \cdot 2^{2r}$$

$$10\log_{10}\left(\frac{S}{N}\right) = 1.76 + 6.02 \cdot r$$

Informacijske mjere:

$$f_1 = \int_{-\infty}^{\infty} f(x, y) dy$$

$$f_2 = \int_{-\infty}^{\infty} f(x, y) dx$$

$$H(X) = -\int_{-\infty}^{\infty} f_1(x) \cdot \log_2 f_1(x) dx$$

$$H(Y) = -\int_{-\infty}^{\infty} f_2(y) \cdot \log_2 f_2(y)\, dy$$

$$H(X|Y) = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\cdot\log_2\frac{f(x,y)}{f_2(y)}dxdy$$

$$H(Y|X) = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\cdot\log_2\frac{f(x,y)}{f_1(x)}dxdy$$

$$H(X,Y) = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\cdot\log_2 f(x,y)dxdy$$

Kapacitet kanala u prisutnosti Gaussovog aditivnog šuma:

$$C = \frac{1}{2}\log_2\left(1+\frac{S}{N}\right)\left[\frac{bit}{simbol}\right]$$

Informacijski kapacitet AWGN kanala:

$$I(X;Y) = H(Y) - \sum_{k=1}^{n}\log_2\left(\sigma_{z_k}\cdot\sqrt{2\pi e}\right)$$

$$I_{max}(X;Y) = \frac{n}{2}\log_2\left(1+\frac{S}{N}\right)\left[\frac{bit}{simbol}\right]$$

$$C = B\cdot\log_2\left(1+\frac{S}{N}\right) = 2BD\ [bit/s]$$

$$D = \frac{1}{2}\log_2\left(1+\frac{S}{N}\right)\left[\frac{bit}{uzorak}\right]$$

pri čemu je S srednja snaga signala na izlazu predajnika, a N srednja snaga signala. Ako je zadana spektralna gustoća snage $S_N(f) = \frac{N_0}{2}$, vrijedi:

$$C = B\cdot\log_2\left(1+\frac{S}{N_0 B}\right)\ [bit/s]$$

$$N = \frac{N_0}{2}\ 2B$$

Učinkovitost prijenosnog pojasa:

$$E = \frac{R_b}{B}$$

Shannonovo ograničenje AWGN kanala (uz nepostojanje zalihosti):

$$\lim_{B\to\infty} C = \frac{S}{N_0}\log_2 e\ [bit/s]$$

Odnos prijenosne brzine i kapaciteta kanala:

$$\Gamma = \frac{2^{2C}-1}{2^{2R}-1} = \frac{\left(\frac{S}{N}\right)}{2^{2R}-1}$$

$$R = \frac{1}{2}\log_2\left(1+\frac{S}{\Gamma N}\right)\left[\frac{bit}{simbol}\right]$$

$$R_b = B\log_2\left(1+\frac{S}{\Gamma N}\right)\left[\frac{bit}{s}\right]$$

pri čemu je Γ funkcija dozvoljene vjerojatnosti pogreške i kodnog sustava.

Zaštitno kodiranje

Hammingova međa i perfektan kod:

$$K(n,M,2t+1)$$

$$M \le \frac{2^n}{\binom{n}{0}+\binom{n}{1}+\cdots+\binom{n}{t}}$$

Paritetni bitovi:

$$R = x_1 + x_2 + \cdots + x_k$$

$$R = x_1 + x_2 + \cdots + x_k + 1$$

Zbroj vektora:

$$\vec{x}+\vec{y} = [x_1,x_2,\dots,x_n] + [y_1,y_2,\dots,y_n] = [x_1+y_1,x_2+y_2,\dots,x_n+y_n]$$

Umnožak vektora skalarom:

$$a\cdot\vec{x} = a\cdot[x_1,x_2,\dots,x_n] = [a\cdot x_1, a\cdot x_2,\dots,a\cdot x_n]$$

Skalarni produkt vektora:

$$\vec{x}\cdot\vec{y} = \sum_{i=1}^{n} x_i\cdot y_i$$

Vjerojatnost ispravnog dekodiranja:

$$P(K) = \sum_{i=0}^{n} N_i\cdot p_g^i\cdot(1-p_g)^{n-i}$$

Standardni oblik generirajuće matrice i matrice provjere pariteta:

$$G = [I_k|A]$$

$$G\cdot H^T = \mathbf{0}$$

$$H = [A^T|I_{n-k}]$$

$$H^T = \begin{bmatrix} A \\ I_{n-k}\end{bmatrix}$$

$$H = [B|I_{n-k}]$$

Kodiranje linearnim kodovima:

$$\boldsymbol{x} = \boldsymbol{m}\cdot\boldsymbol{G}\ \ O(nk)$$

$$\boldsymbol{x} = \boldsymbol{m}\cdot[\boldsymbol{I_k}|\boldsymbol{A}] = [\boldsymbol{m},\boldsymbol{m}\cdot\boldsymbol{A}]\ \ O(nk-k^2)$$

Sindrom:

$$S(\boldsymbol{y}) = \boldsymbol{y}\cdot\boldsymbol{H^T}$$

Vjerojatnost ispravnog dekodiranja putem **standardnog niza**:

$$P(K) = \sum_{i=0}^{n}\binom{n}{i}p_g^i\left(1-p_g\right)^{n-i}$$

$$P_e(K) = 1 - P(K)$$

Kodna brzina zaštitnog koda:

$$R(K) = \frac{k}{n} \le 1$$

Shannonov teorem o kodiranju za binarni simetrični kanal:

$$C(p_g) = 1 + p_g\log_2 p_g + \left(1-p_g\right)\log_2\left(1-p_g\right)$$

$$R \le \frac{C(p_g)}{1+P_e\log_2 P_e + (1-P_e)\log_2(1-P_e)}$$

$$P_e(K) \ge 1 - \frac{1}{R(K)}\cdot\left(\frac{1}{n}+C\right)$$

Hammingov kod:

$$Ham(r), r\ge 2, [2^r-1, 2^r-1-r]$$

$$k + r \le 2^r - 1$$

Ciklični kodovi:

$$x^n - 1 = g(x)h(x)$$

Faktorizacija u aritmetici modulo 2:

$x^1 - 1$	$x + 1$
$x^2 - 1$	$(x + 1)^2$
$x^3 - 1$	$(x + 1)(x^2 + x + 1)$
$x^4 - 1$	$(x - 1)(x + 1)(x^2 + 1)$
$x^5 - 1$	$(x + 1)(x^4 + x^3 + x^2 + 1)$
$x^6 - 1$	$(x - 1)(x + 1)(x^2 - x + 1)(x^2 + x + 1)$
$x^7 - 1$	$(x + 1)(x^3 + x + 1)(x^3 + x^2 + 1)$
$x^8 - 1$	$(x - 1)(x + 1)(x^2 + 1)(x^4 + 1)$
$x^9 - 1$	$(x + 1)(x^2 + x + 1)(x^6 + x^3 + 1)$
$x^{10} - 1$	$(x - 1)(x + 1)(x^4 - x^3 + x^2 - x + 1)(x^4 + x^3 + x^2 + x + 1)$
$x^{11} - 1$	$(x + 1)(x^{10} + x^9 + x^8 + x^7 + x^6 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$
$x^{12} - 1$	$(x - 1)(x + 1)(x^2 + 1)(x^2 - x + 1)(x^2 + x + 1)(x^4 - x^2 + 1)$
$x^{13} - 1$	$(x + 1)(x^{12} + x^{11} + x^{10} + x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$
$x^{14} - 1$	$(x - 1)(x + 1)(x^6 - x^5 + x^4 - x^3 + x^2 - x + 1)(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$
$x^{15} - 1$	$(x + 1)(x^2 + x + 1)(x^4 + x + 1)(x^4 + x^3 + 1)(x^4 + x^3 + x^2 + x + 1)$
$x^{16} - 1$	$(x - 1)(x + 1)(x^2 + 1)(x^4 + 1)(x^8 + 1)$
$x^{17} - 1$	$(x + 1)(x^8 + x^5 + x^4 + x^3 + 1)(x^8 + x^7 + x^6 + x^4 + x^2 + x + 1)$
$x^{18} - 1$	$(x - 1)(x + 1)(x^2 - x + 1)(x^2 + x + 1)(x^6 - x^3 + 1)(x^6 + x^3 + 1)$
$x^{19} - 1$	$(x + 1)(x^{18} + x^{17} + \cdots + x^3 + x^2 + x + 1)$

Entropijsko kodiranje:

Nesingularni kodovi: Svaki simbol poruke x_i ima jedinstveno dodijeljenu kodnu riječ $C(x_i)$.

Jednoznačno dekodabilni kodovi: Kod je *jednoznačno dekodabilan* ako je preslikavanje $C^+ : X^+ \rightarrow C(X)^+$ takvo da je $\forall \boldsymbol{x}$i $\forall \boldsymbol{x}'$ iz X^+ i $\boldsymbol{x} \neq \boldsymbol{x}'$, $C(\boldsymbol{x})^+ \neq C(\boldsymbol{x}')^+$.
Sardinas-Pattersonov test za provjeru jednoznačne dekodabilnosti koda: Neka je dan kôd K, tj. neka je S_0 polazni skup kodnih riječi. Potrebno je stvoriti skup S_{i+1} iz S_i (počevši od $i = 0$ pa nadalje) na sljedeći način: kodna riječ $C(y)$ dodaje se u skup S_{i+1} ako i samo ako postoji kodna riječ $C(x)$ iz S_0 takva da je $C(x)C(y)$ iz S_i ili ako postoji kodna riječ $C(z)$ iz S_i takva da je $C(z)C(y)$ iz S_0. Nadalje, kôd je *jednoznačno dekodabilan* ako niti jedan $S_i (i \geq 1)$ ne sadrži kodne riječi iz S_0. Također, ako je $S_{i+1} = \{\emptyset\}$ ili je svaki sljedeći S_{i+1} jednak prethodnom S_i jasno je da u konačnom broju iteracije skup S_{i+1} neće sadržavati element iz skupa S_0, što znači da je kôd *jednoznačno dekodabilan*.

Prefiksni kodovi: Kôd nazivamo *prefiksnim* (ili *trenutnim*), ukoliko niti jedna kodna riječ nije prefiks bilo koje druge kodne riječi. Prefiksni kodovi su podgrupa jednoznačno dekodabilnih kodova.

Kraft-McMillanova nejednakost: Nužan i dovoljan uvjet postojanja prefiksnog koda. Jednoznačno dekodabilni kodovi zadovoljavaju ovu nejednakost.

$$\sum_{i=1}^{n} d^{-l_i} \leq 1$$

Nužan i dovoljan uvjet optimalnosti koda:

$$H(X) \leq L(X) < H(X) + 1$$

$$min\left[L(X) = \sum_{i=1}^{n} p(x_i) \cdot l_i\right] uz\ uvjet \sum_{i=1}^{n} d^{-l_i} \leq 1$$

Efikasnost koda:

$$\varepsilon_{(d)} = \frac{H_{(d)}(X)}{L_{(d)}(X)} \leq 1$$

$$k' = \left\lceil \frac{N-1}{d-1} \right\rceil, N' = k'(d-1) + 1$$