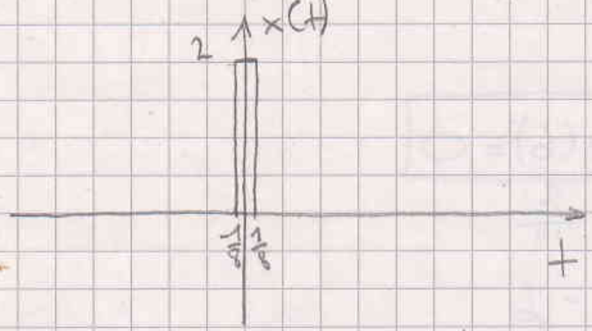
3. DOMAĆA ZADAĆA

(1.) $x(t) = \begin{cases} A, & t \in [-\frac{\tau}{2}, \frac{\tau}{2}] \\ 0, & \text{inače} \end{cases}$ $A=2,\ \tau = 1/4, 1/2, 1$

a) $\tau = 1/4$



$$X(f) = A \cdot \tau \frac{\sin\left(2\pi f \frac{\tau}{2}\right)}{2\pi f \cdot \frac{\tau}{2}} =$$

$$= \frac{1}{2} \frac{\sin\left(f \cdot \frac{\pi}{4}\right)}{f \cdot \frac{\pi}{4}}$$

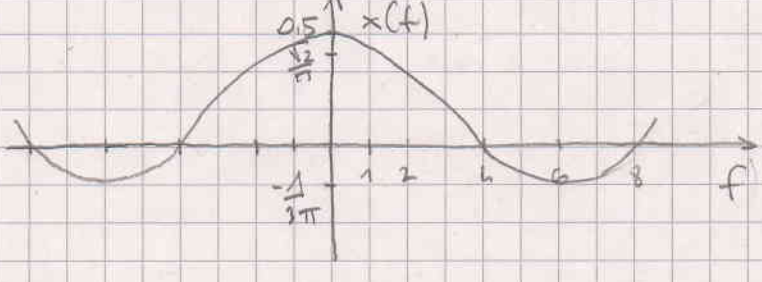
$X(0) = 1/2$

$\boxed{X(4) = X(8) = X(12) = 0}$

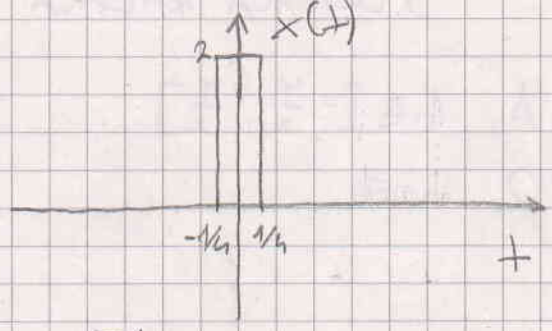
$X(1) = X(-1) = \frac{\sqrt{2}}{\pi}$

$X(2) = X(-2) = \frac{1}{\pi}$

$X(6) = X(-6) = -\frac{1}{3\pi}$



b) $\tau = \frac{1}{2}$



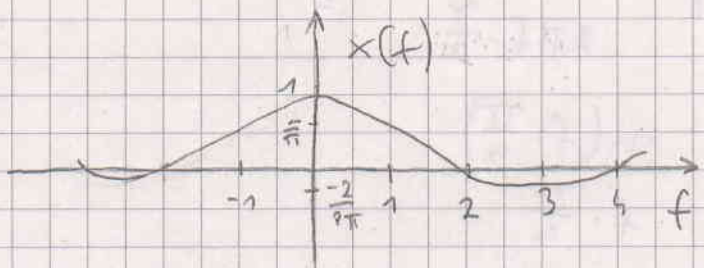
$$x(f) = \frac{\sin\left(f \frac{\pi}{2}\right)}{f \frac{\pi}{2}}$$

$x(0) = 1$

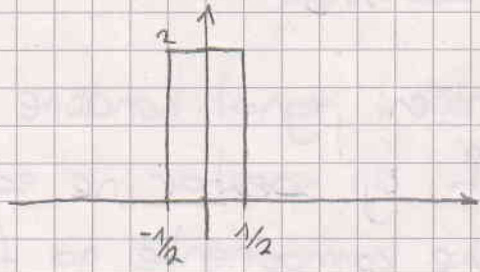
$\boxed{x(2) = x(4) = x(6) = 0}$

$x(1) = x(-1) = \frac{2}{\pi}$

$x(3) = x(-3) = -\frac{2}{3\pi}$



c) $T=1$



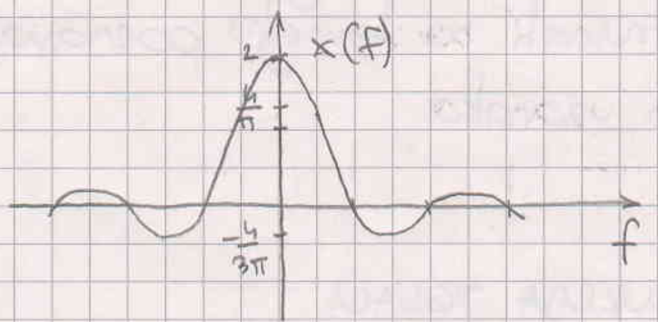
$$x(f) = 2 \cdot \frac{\sin(f\pi)}{f\pi}$$

$x(0) = 2$

$\boxed{x(1) = x(2) = x(3) = 0}$

$x(1/2) = \frac{4}{\pi}$

$x(3/2) = -\frac{4}{3\pi}$



Što je signal širi u vremenskoj domeni, to je uži u frekvencijskoj (obrnuto su proporcionalni).

Heisenbergovo načelo neodređenosti govori da što je koncentriranija funkcija $f(x)$, to je njezin spektar raspršeniji.

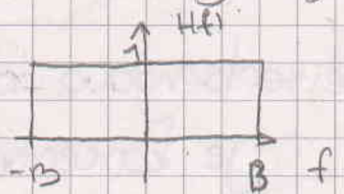
2. TEOREM O UZORKOVANJU:

Pojasno ograničeni signal konačne energije $x(t)$, $t \in \mathbb{R}$, čiji spektar ne sadrži frekvencijske komponente na frekvencijama iznad B Hz ($X(f) = 0$ za $|f| > B$), u potpunosti je i na jednoznačan način opisan pomoću vrijednosti tog signala uzetih u diskretnim vremenskim trenutcima $T_n = n/2B$, gdje je B gornja granična frekvencija signala a n cijeli broj. Također, takav signal moguće je u potpunosti i na jednoznačan način rekonstruirati na temelju poznavanja njegovih uzoraka.

a) REKONSTRUKCIJA SIGNALA

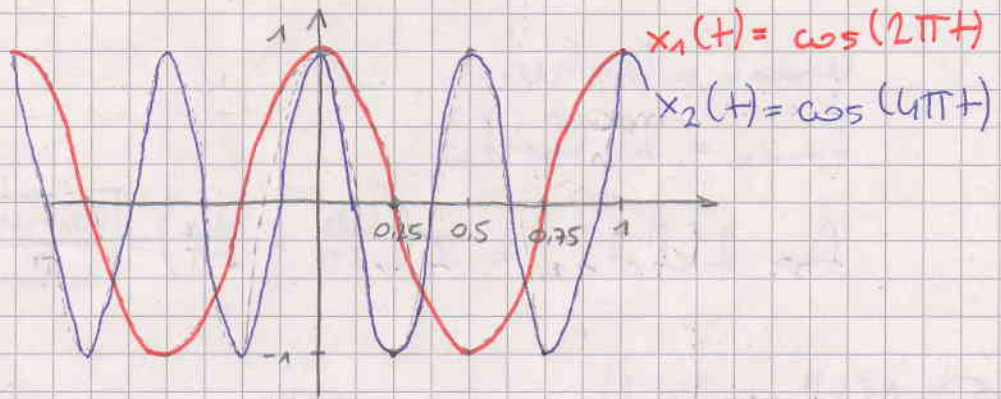
Ako signal uzorkujemo sukladno teoremu rekostrukciju provodimo njegovim propuštanjem kroz niskopropusni filtar za koji vrijedi

$$H(f) = \begin{cases} 1, & |f| < B \\ 0, & \text{inače} \end{cases}$$



Kao rezultat dobit ćemo signal $x(t)$, koji je pojasno ograničen frekvencijom B, a $A=1$ nam omogućava da se frekv. spektar signala u tom području ne mijenja. Na taj način opet dobijemo signal $x(t)$. Obrat teorema ne vrijedi.

b) $x_1 = \cos(2\pi t)$
$x_2 = \cos(4\pi t)$



$\omega_1 = 2\pi$

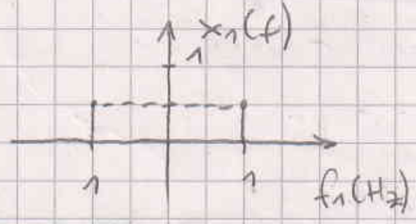
$\cancel{2\pi} f_1 = \cancel{2\pi}$

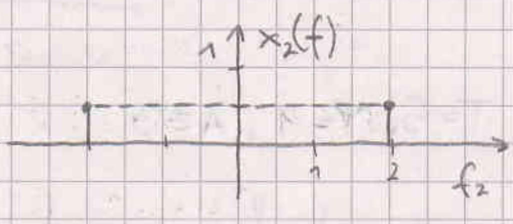
$f_1 = 1$

$\omega_2 = 4\pi$

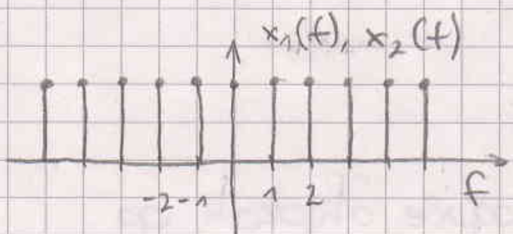
$\cancel{2\pi} f_2 = \cancel{4\pi}$

$f_2 = 2$





- spektri nakon otipkavanja



→ dolazi do preklapanja

c) 1. $\cos(\omega t)$

$$\omega = 2\pi f \qquad f_s = 2f = \boxed{\frac{\omega}{\pi}}$$

$$f = \frac{\omega}{2\pi}$$

2. $\cos(\omega_1 t)\cos(\omega_2 t) = \quad \omega_1 << \omega_2$

$$= \frac{1}{2}\left(\cos(\omega_1 - \omega_2)t + \cos(\omega_1 + \omega_2)t\right)$$

$$\omega_{max} = \omega_1 + \omega_2$$

$$f_{max} = 2\pi(f_1 + f_2)$$

$$f_s = 2(f_1 + f_2) = 2\left(\frac{\omega_1}{2\pi} + \frac{\omega_2}{2\pi}\right) = \boxed{\frac{\omega_1 + \omega_2}{\pi} \sim \frac{\omega_2}{\pi}}$$

3. $A(t)\cos(\omega t) \qquad \omega_A << \omega$

$$f_{max} = \frac{\omega + \omega_A}{2\pi}$$

$$f_s = 2f_{max} = \boxed{\frac{\omega + \omega_A}{\pi} \sim \frac{\omega}{\pi}}$$

4. $T = 5,\ \tau = 1,\ A = 3$

- pravokutni impulsi nisu pojasno ograničeni, nije ih moguće otipkati

5. $\tau = 1,\ A = 3$

- isto kao 4., nije moguće otipkati ga

d) $\Omega = [0,2]$

$f_s = 1$

$\cos(\Omega t) + \cos(\Omega^2 t)$

$f_s > 2 f_{max} - \frac{\Omega^2}{2\pi}$

$$p(f_s > 2f_{max}) = p\left(1 > \frac{\Omega^2}{\pi}\right) = p(\pi > \Omega^2) =$$

$$= p(\Omega < \sqrt{\pi}) = \int_0^{\sqrt{\pi}} \frac{1}{2}\, dt = \frac{1}{2}\sqrt{\pi} =$$

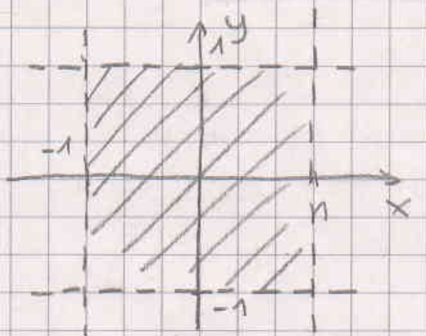
$$= \boxed{0{,}886227}$$

3. a) $F(u,v) = \iint\limits_{-\infty}^{\infty} f(x,y)\, e^{-j2\pi(ux+vy)}\, dx\, dy$

$f(x,y) = \iint\limits_{-\infty}^{\infty} F(u,v)\, e^{j2\pi(ux+vy)}\, du\, dv$

- u i v su prostorne funkcije u x i y smjeru
- F(u,v) označava 2D spektar funkcije f(x,y)

b) $I(x,y) = \begin{cases} 1, & (x,y) \in [-1,1] \times [-1,1] \\ 0, & \text{inače} \end{cases}$



$F(u,v) = \int_{-1}^{1}\int_{-1}^{1} e^{-j2\pi(ux+vy)}\, dx\, dy =$

$= \int_{-1}^{1} e^{-j2\pi ux}\, dx \int_{-1}^{1} e^{-j2\pi vy}\, dy =$

$= -\frac{e^{-j2\pi ux}}{j2\pi u}\Big|_{-1}^{1} \cdot \frac{-e^{-j2\pi vy}}{j2\pi v}\Big|_{-1}^{1} =$

$= \frac{\cancel{4\pi^2}}{\cancel{\pi^2}} \frac{\sin(2\pi u)}{\cancel{2}\pi u} \cdot \frac{\sin(2\pi v)}{\cancel{2}\pi v} =$

$= 4 \cdot \frac{\sin(2\pi u)}{2\pi u} \cdot \frac{\sin(2\pi v)}{2\pi v}$

c) POTPUNA REKONSTRUKCIJA

Ako je signal $I_{m,n}(mx_s, ny_s)$ pojasno ograničen, onda je moguće rekonstruirati početni signal $I(x,y)$.

Neka su mx_s i ny_s granične frekvencije. Moguće je rekonstruirati signal I samo ako za udaljenosti među uzorcima w i h (w - smjer x-osi, h - smjer y-osi) vrijedi:

$$w \leq \frac{1}{2mx_s}, \quad h \leq \frac{1}{2ny_s}$$

(4.) $x(t) = A\cos(2\pi f t)$

$f_s = 4000$

$A = 2$

$f = 1000$

$L = 32 = 2^r \Rightarrow r = 5$

$10\log_{10}(S/N) = 1{,}76 + 6{,}02 \cdot r = 31{,}86 \text{ dB}$

$$\boxed{(S/N) = 1534{,}61}$$

(5.) a) $H(X) = 10^8$ bitova

$S_N(f) = 10^{-18}$ W/Hz

$B = 1000$ Hz

$t = 1000$ s

$S = ?$

$S_N(f) = N_0/2$

$N_0 = 2 \cdot 10^{-18}$

$N = N_0 \cdot B = 2 \cdot 10^{-15}$ W

$$C = \frac{H}{t} = 10^5 = B \log_2 (1 + S/N)$$

$$1 + S/N = 2^{\frac{C}{B}}$$

$$S = N\left(2^{C/B} + 1\right) = \boxed{2{,}535 \cdot 10^{15}\ W}$$

b) $S = 1{,}9$ W

$S_N(f) = 7{,}5 \cdot 10^{-9}$ W/Hz

$N_0 = 2 \cdot S_N(f) = 1{,}5 \cdot 10^{-8}$ W

$B \to \infty$

$$\lim_{B \to \infty} C = \frac{S}{N_0} \cdot \log_2 e = \boxed{1{,}827\ \text{Mbit/s}}$$