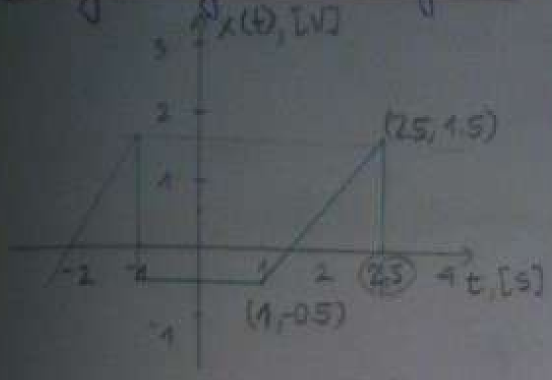
KOMUNIKACIJSKI KANALI U KONTINUIRANOM VREMENU

1) Srednja vrijednost signala



$$\boxed{P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} (x(t))\,dt}$$

$T_0 = 3,5$

$$x(t) = \begin{cases} -0,5 & t \in \langle -1, 1 \rangle \\ \frac{4}{3}t - \frac{11}{6} & t \in \langle 1, 2,5 \rangle \end{cases}$$

$x_1(1, -0.5)$
$x_2(2.5, 1.5)$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

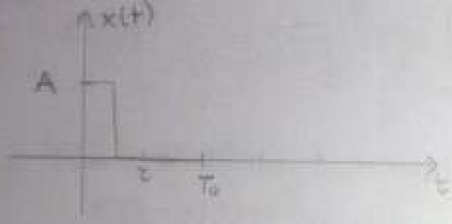
$$y + 0.5 = \frac{1.5 + 0.5}{2.5 - 1}(x - 1)$$

$$x(t) = \frac{4}{3}t - \frac{4}{3} - \frac{1}{2}$$

$$P = \frac{1}{3,5}\left(-\int_{-1}^{1} 0,5\, dt + \int_{1}^{2,5}\left(\frac{4}{3}t - \frac{11}{6}\right)dt\right) = \underline{-0,07\,V}$$

2)

$$x(t) = \begin{cases} A & 0 \le t \le \tau/2 \\ 0 & \tau/2 \le t \le T_0/2 \end{cases}$$



$$\boxed{\frac{\text{impuls}}{\text{pauza}} = \frac{\tau}{T_0 - \tau}}$$

kojim postotkom djeluje istosmjerna komponenta?

$$\frac{\tau}{T_0 - \tau} = \frac{1}{4}$$

$4\tau = T_0 - \tau$

$5\tau = T_0$

$$\frac{\tau}{T_0} = \frac{1}{5} = \underline{\underline{20\%}}$$

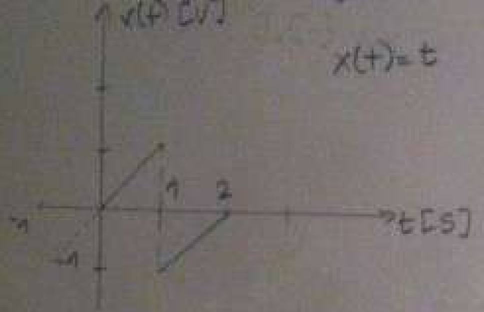
(4.4)

$$\int_{-\infty}^{\infty} \delta(t-10)\sin\left(\frac{\pi}{20}t\right)dt = ?$$

$$\boxed{\int_{-\infty}^{\infty} \delta(t-t_0) f(t) = f(t_0)}$$

$$\int_{-\infty}^{\infty} \delta(t-10)\sin\left(\frac{\pi}{20}t\right) = \sin\left(\frac{\pi}{20}\cdot 10\right) = \sin\left(\frac{\pi}{2}\right) = \underline{\underline{1}}$$

(4.5) $x(t) \Rightarrow$ signal snage ili energije?



$x(t) = t$

$$P = \frac{1}{T}\int_{-T_0/2}^{T_0/2} x(t)dt$$

$$P = \frac{1}{2}\int_{-1}^{1} t\,dt = \frac{1}{2}\frac{t^2}{2}\Big|_{-1}^{1}$$

$$P = \frac{1}{4}\left(1^2 - (-1)^2\right) = \underline{\underline{0}} \Rightarrow \text{signal energije}$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = 2\int_0^1 (t)^2 dt = 2\int_0^1 t^2 dt = 2\frac{t^3}{3}\Big|_0^1 = \underline{\underline{\frac{2}{3}\ Ws}}$$

(4.6) $y_1 = \sin(2\pi t)\,\text{rect}(t-0{,}5)$ V

$y_2 = \sin(2\pi t)\,\text{rect}(t-1)$ V

i) energija signala y_1

$$\text{rect}\left(\frac{t-x}{y}\right) = u\left(t-x+\frac{y}{2}\right) - u\left(t-x-\frac{y}{2}\right)$$

$$\text{rect}(t-0{,}5) = u(t-0{,}5+0{,}5) - u(t-0{,}5-0{,}5)$$

$$= u(t) - u(t-1)$$

$$y_1 = \sin(2\pi t)\left[u(t) - u(t-1)\right]$$

$$E = \int_0^1 \left(\sin^2(2\pi t)\right)dt = \frac{1}{2}\int_0^1 (1-\cos(4\pi t))dt = \frac{1}{2} + \frac{1}{2}\sin(4\pi t)\Big|_0^1 = \frac{1}{2}\ Ws$$

i) energiju signala y_2

$f_2(t) = \sin(2\pi t)[u(t-1+0{,}5) - u(t-1-0{,}5)]$

$= \sin(2\pi t)[u(t-0{,}5) - u(t-1{,}5)]$

$E = \int_{0,5}^{1,5} \sin^2(2\pi t)dt = \frac{1}{2}\int_{0,5}^{1,5}(1-\cos(4\pi t))dt = \frac{1}{2}(1{,}5-0{,}5) = \frac{1}{2}\,\underline{Ws}$

ii) energiju $y_1 + y_2$

$y_1 + y_2 = \sin(2\pi t)[u(t) - u(t-1) + u(t-0{,}5) - u(t-1{,}5)]$

$E = \int$

4.7 $x(t) = m(t)\cos(2\pi f_c t)$ [V] $\quad M(f) \neq 0$ na $[-B, B]$

$f_c \gg B$

$r(t) = x(t) \rightarrow$ primljeni = poslani

$E(r(t)) = ?$

$E(x(t)) = E_m$

Nema šuma

Množenje u vremenu je konvolucija u frekv. području

$R(f) = M(f) * \left[\frac{1}{2}(\delta(f-f_c) + \delta(f+f_c))\right] = \frac{1}{2}M(f-f_c) + \frac{1}{2}M(f+f_c)$

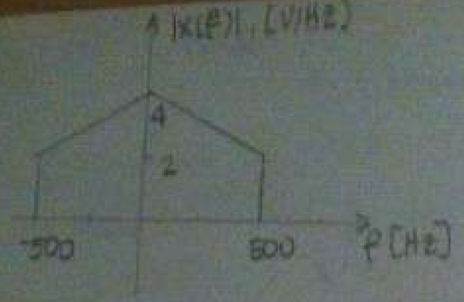
$E_{m1} = \int_{-B}^{B}|M(f)|^2 df = E_m \rightarrow$ energija poslanog signala

$E_{m2} = \int_{-f_c-B}^{-f_c+B}\left(\frac{M(f-f_c)}{2}\right)^2 df + \int_{f_c-B}^{f_c+B}\left(\frac{M(f+f_c)}{2}\right)^2 df$

$= \frac{1}{4}\int_{-f_c-B}^{-f_c+B}(M(f-f_c))^2 df + \frac{1}{4}\int_{f_c-B}^{f_c+B}(M(f+f_c))^2 df \rightarrow$ E_m samo na drugoj frekvenciji

$= \frac{1}{4}E_{m1} + \frac{1}{4}E_{m1} = \frac{1}{2}E_{m1}$

(4.8)



i) Odredi Fourierov transformat sljedećeg signala:

$$x_1(t) = -x(t) + x(t)\cos(2000\pi t) + 2x(t)\cos^2(3000\pi t)\ [V]$$

$$\boxed{X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt}$$

$$\cos^2(3000\pi t) = \frac{1}{2}\left(1 + \cos(6000\pi t)\right) = 0{,}5 + 0{,}5\cos(6000\pi t)$$

$$x_1(t) = \cancel{-x(t)} + x(t)\cos(2000\pi t) + \cancel{x(t)} + x(t)\cos(6000\pi t)$$

$$= x(t)\cos(2000\pi t) + x(t)\cos(6000\pi t)$$

$$\cos(2000\pi t) = \frac{1}{2}\left[e^{j2000\pi t} + e^{-j2000\pi t}\right]$$

$$\cos(6000\pi t) = \frac{1}{2}\left[e^{j6000\pi t} + e^{-j6000\pi t}\right]$$

$$x_1(t) = \underbrace{\frac{1}{2}x(t)\, e^{j2000\pi t}} + \frac{1}{2}x(t)\, e^{-j2000\pi t} + \frac{1}{2}x(t)\, e^{j6000\pi t} + \frac{1}{2}x(t)\, e^{-j6000\pi t}$$

$$X(f)_1 = \int_{-\infty}^{\infty} \frac{1}{2}x(t)\, e^{j2000\pi t} \cdot e^{-j2\pi f t} = \frac{1}{2}\int_{-\infty}^{\infty} x(t)\, e^{-j2\pi t(f-1000)}\, df = \frac{1}{2}X(f-1000)$$

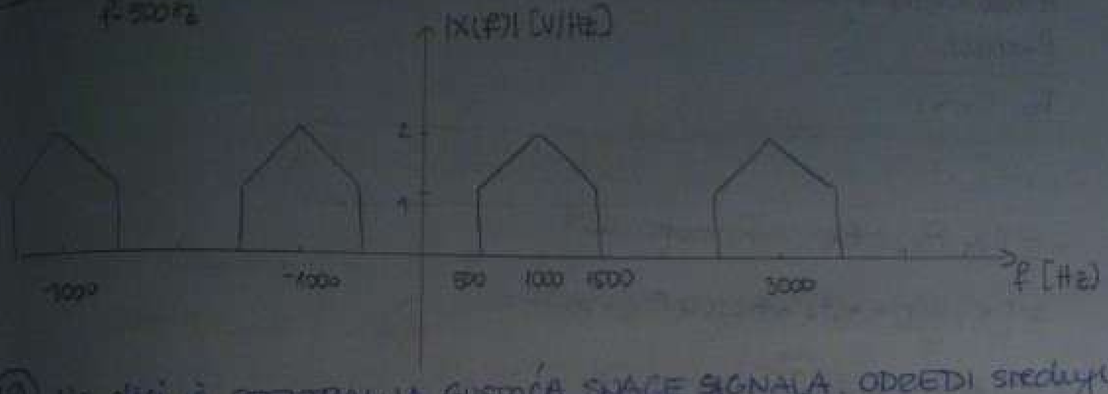
$$X(f)_2 = \frac{1}{2}X(f+1000)$$

$$X(f)_3 = \int \frac{1}{2}x(t)\, e^{j6000\pi t}\, e^{-j2\pi f t}\, df = \frac{1}{2}\int x(t)\, e^{-j2\pi t(f-3000)}\, df = \frac{1}{2}X(f-3000)$$

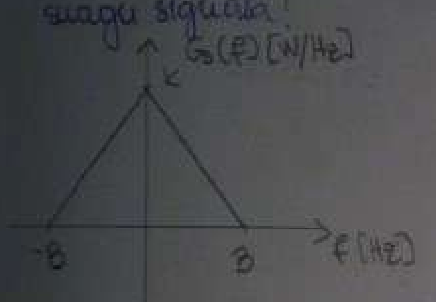
$$X(f)_4 = \frac{1}{2}X(f+3000)$$

$$\boxed{X(f)=\frac{1}{2}X(f-1000)+\frac{1}{2}X(f+1000)+\frac{1}{2}X(f-3000)+\frac{1}{2}X(f+3000)}$$

f=500Hz



(15) Na slici je SPEKTRALNA GUSTOĆA SNAGE SIGNALA. ODREDI srednju snagu signala!



$$\boxed{P=\int_{-\infty}^{\infty} G_S(f)\,df}$$

$$G_S(f)=\begin{cases}\frac{K}{B}f+K & \text{za } -B<f<0\\ -\frac{K}{B}f+K & \text{za } 0<f<B\end{cases}$$

$x_1(B,0)$
$x_2(0,K)$

$$y-0=\frac{K-0}{0-B}(x-B)$$

$$y=-\frac{K}{B}x+K$$

$$P=\int_{-B}^{0}\left(\frac{K}{B}f+K\right)df+\int_{0}^{B}\left(-\frac{K}{B}f+K\right)df$$

$$=\frac{K}{B}\frac{f^2}{2}\Big|_{-B}^{0}+Kf\Big|_{-B}^{0}-\frac{K}{B}\frac{f^2}{2}\Big|_{0}^{B}+Kf\Big|_{0}^{B}$$

$$=\frac{K}{2B}(0^2-B^2)+K(0+B)-\frac{K}{B2}(B^2-0)+K(B-0)$$

$$=-\frac{KB}{2}+KB-\frac{KB}{2}+KB=2KB-KB=\underline{\underline{KB\ [W]}}$$

(4.10) $x(t) = A\cos 2\pi f t$

A [0,1] → jednolika razdioba na [0,1]

f - konst.

$R_x, C_x = ?$

$$\boxed{R_x = E[x(t)\cdot x(t+\tau)]} \quad \boxed{C_x = R_x(t,\tau) - \mu_x(t)\mu_x(\tau)}$$

$$= E[A\cos 2\pi f t \cdot A\cos 2\pi f(t+\tau)]$$

$$= E[A^2 \cdot \cos(2\pi f t)\cos(2\pi f(t+\tau))]$$

$$= E[A^2]\underbrace{\cos(2\pi f t)\cdot\cos(2\pi f(t+\tau))}_{\text{konst.}}$$

$$f_A = \begin{cases} 1, & 0<t<1 \\ 0, & \text{inače} \end{cases}$$

$$E[A^2] = \int_0^1 A^2 dA = \frac{1}{3}$$

$$R_x = \frac{1}{3}\cos(2\pi f t)\cos(2\pi f(t+\tau))$$

$$\mu_x = E[x] = E(A\cos(2\pi f t))$$

$$= E[A]\cdot\underbrace{\cos(2\pi f t)}_{\text{konst}} \qquad E(A) = \int A\,dA = \frac{A^2}{2} = \frac{1}{2}$$

$$\mu_x(\tau+t) = E(x(t+\tau)) = E(A)\cdot\cos(2\pi f(t+\tau))$$

$$= \frac{1}{2}\cos(2\pi f(t+\tau))$$

$$C_x = R_x - \mu_x(t)\mu_x(\tau+t)$$

$$= \frac{1}{3}\cos(2\pi f t)\cos(2\pi f(t+\tau)) - \frac{1}{4}\cos(2\pi f t)\cos(2\pi f(t+\tau)$$

$$= \frac{4-3}{12}(\ldots) = \frac{1}{12}\cos(2\pi f t)\cos(2\pi f(t+\tau))$$

(2.11) $R_x(\tau) = e^{-\frac{|\tau|}{2}}$ $x(t)$ - stacionaran u širem smislu

$N(t)$ - Gaussov šum

$S_N(f) = \frac{N_0}{2}$ - spektralna gustoća snage

i) srednja snaga P slučajnog signala modeliranog stacionarnim stanjem $X(t)$

$$\boxed{P = \int_{-\infty}^{\infty} S_x(f)\,df = R_x(0)} = e^{0} = \underline{\underline{1}}$$

ii) $S_x(f)$ slučajnog signala modeliranog s $X(t)$

$$S_x(f) = \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f\tau} d\tau$$

$$= \int_{-\infty}^{0} e^{\frac{\tau}{2}} e^{-j2\pi f\tau} d\tau + \int_{0}^{\infty} e^{-\frac{\tau}{2}} \cdot e^{-j2\pi f\tau} d\tau$$

$$= \int_{-\infty}^{0} e^{\tau(0{,}5-2\pi fj)} d\tau + \int_{0}^{\infty} e^{-\tau(0{,}5+2\pi fj)} dt$$

$$= \frac{1}{0{,}5-2\pi fj} + \frac{1}{0{,}5+2\pi fj} = \frac{1}{0{,}25+4\pi^2 f^2} = \frac{4}{1+16\pi^2 f^2}$$

iii) na ulazu $N(t)$, na izlazu signal sa $S_x(f)$. Prijenosna karakteris?

S_N = spektr. gustoća ulaznog

S_x = -//- izlaznog

$H(f)$ = prijenosna f-ja

$$S_x(f) = S_N(f) \cdot |H(f)|^2$$

$$|H(f)|^2 = \frac{S_x(f)}{S_N(f)} = \frac{8}{N_0(1+16\pi^2 f^2)}$$

$$H(f) = \sqrt{\frac{8}{N_0(1+16\pi^2 f^2)}}$$

4.12 $r(t) = A_c \cos(2\pi f_c t) + n(t)$

$n(t)$ = Gaussov bijeli šum $\quad S_N(f) = \frac{N_0}{2}$

$r(t) \rightarrow \boxed{RC}$ $\quad B_{RC} = \frac{1}{4RC}$

filtar

omjer srednje snage korisnog prema srednjoj snazi šuma

$\frac{S}{N} = ?$

$N = N_0 \cdot B = N_0 \cdot \frac{1}{4RC}$

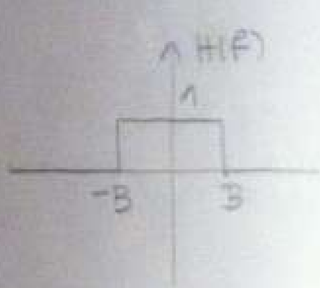
srednja snaga cos signala $\rightarrow S = \frac{A_c^2}{2}$

$$\boxed{\frac{S}{N} = \frac{\frac{A_c^2}{2}}{\frac{N_0}{4RC}} = \frac{2RC \cdot A_c^2}{N_0}}$$

4.13 NPF, $B\,[Hz]$, $|H(f)_{max}| = 1$

$R_x(\tau) = \delta(t)$ $\quad$ (Kanal LTI)

$R_y(\tau) = ?$



$$S_x(f) = \int_{-\infty}^{\infty} \delta(\tau) e^{-j2\pi f \tau} d\tau = 1 \ \frac{W}{Hz}$$

$$S_y(f) = \int_{-\infty}^{\infty} R_y(\tau) e^{-j2\pi f \tau} d\tau$$

$S_y(f) = S_x(f) \cdot |H(f)|^2$

$\boxed{S_y(f) = 1 \ \frac{W}{Hz}}$

$$R_y(\tau) = \int_{-\infty}^{\infty} S_y(f) \cdot e^{j2\pi f \tau} df = \int_{-B}^{B} e^{j2\pi f \tau} df = \frac{1}{2\pi\tau j} e^{j2\pi f \tau}$$

$$= \frac{1}{2\pi\tau j} \left(e^{j2\pi\tau B} - e^{-j2\pi\tau B} \right) = \boxed{\frac{\sin(2\pi\tau B)}{\pi\tau}}$$

štima

1.1) $y(t) = c \cdot t \cdot x(t) + d \cdot x(t-1)$

Je li linearan i vremenski nepromjenjiv?

$$\boxed{\begin{array}{l} y = a y_1(t) + b y_2(t) \\ y_1(t) = ct x_1(t) + d x_1(t-1) \\ y_2(t) = ct x_2(t) + d x_2(t-1) \\ x \to (a x_1 + b \cdot x_2) \end{array}}$$

$$y = ct\,(a \cdot x_1(t) + b \cdot x_2(t)) + d \cdot (a x_1(t-1) + b\, x_2(t-1))$$

$$= ct a x_1(t) + d \cdot a x_1(t-1) + ct b x_2(t) + d \cdot b \cdot x_2(t-1))$$

$$= a\,[\underbrace{ct x_1(t) + d x_1(t-1)}]\, + b\,[\underbrace{ct\,(x_2(t) + d x_2(t-1)}_{y_2}]$$

$y = a y_1 + b y_2$ ✓✓ <u>SUSTAV JE LINEARAN</u>

b) <u>VREMENSKA NEPROMIJENJIVOST</u>

$x(t-t_0) = y(t-t_0)$

$y(t-t_0) = c(t-t_0) \cdot x(t-t_0) + d \cdot x(t-t_0-1)$

$x \to (t-t_0)$

$y(t) = c \cdot t \cdot x(t-t_0) + d \cdot x(t-t_0-1)$

$y(t) \neq y(t-t_0) \Rightarrow$ <u>sustav je vremenski promjenjiv</u>

4.15 $y(t)=\frac{1}{\sqrt{T}}\int\limits_{t-T}^{t} x(\tau)\,d\tau$ Dokaži da je linearan i vremenski nepromjenjiv

a) linearnost

$y_1(t)=\frac{1}{\sqrt{T}}\int\limits_{t-T}^{t} x_1(\tau)\,d\tau \qquad y_2(t)=\frac{1}{\sqrt{T}}\int\limits_{t-T}^{t} x_2(\tau)\,d\tau$

$x \to (ax_1+bx_2)$

$y(t)=\frac{1}{\sqrt{T}}\int\limits_{t-T}^{t}(a x_1(\tau)+b x_1(\tau))\,d\tau$

$=\frac{1}{\sqrt{T}}\int\limits_{t-T}^{t} a x_1(\tau)+\frac{1}{\sqrt{T}}\int\limits_{t-T}^{t} b x_2(\tau)\,d\tau = ay_1(t)+by_2(t)$ ✓

b) vremenska nepromjenjivost

$x(t)\to x(t-t_0) \qquad y=x(t)\to x(t-t_0)$

$y(t)=\frac{1}{\sqrt{T}}\int\limits_{t-t_0-T}^{t-t_0} x(\tau)\,d\tau$

= vrem. nepromjenjiv

$y(t-t_0)=\frac{1}{\sqrt{T}}\int\limits_{t-t_0-T}^{t-t_0} x(\tau)\,d\tau$

$h(t)=?$

$\int\limits_{-\infty}^{\infty} x(\tau)\cdot h(t-\tau)=\frac{1}{\sqrt{T}}\int\limits_{t-T}^{t} x(\tau)\,d\tau$

$\int\limits_{-\infty}^{\infty} x(\tau)\,d\tau\cdot h(t-\tau)=\frac{1}{\sqrt{T}}\int\limits_{t-T}^{t} x(\tau)\,d\tau$

(4.16) LTI SUSTAV

$X(f) \div R_x(\tau) = e^{-0{,}5|\tau|}$ $\quad E(x)=0$

$N(t) \div S_N(f) = \frac{N_0}{2}$ $\quad$ uaz

$\xrightarrow{} \boxed{NPF} \xrightarrow{}$ $S_x(f)$ $\qquad S_x(f) = \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f \tau} d\tau$

$|H(f)|^2 = ?$

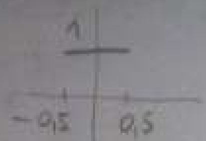
$$S_x(f) = \int_{-\infty}^{0} e^{\tau(0{,}5-2\pi f j)} d\tau + \int_{0}^{\infty} e^{-\tau(0{,}5+2\pi f j)} d\tau = \frac{1}{0{,}5^2+4\pi^2 f^2}$$

$$S_x(f) = \frac{4}{1+16\pi^2 f^2} \qquad \boxed{|H(f)|^2 = \frac{S_x(f)}{S_N(f)} = \frac{8}{N_0(1+16\pi^2 f^2)}}$$

(4.17) $h_1(t) = \mathrm{rect}(t-0{,}5) = u(t-0{,}5+0{,}5) - u(t-0{,}5-0{,}5)$

$h_1(t) = u(t) - u(t-1)$

$h_2(t) = \mathrm{rect}(t) = u(t+0+0{,}5) - u(t-0-0{,}5) = u(t+0{,}5) - u(t-0{,}5)$



$h(t) = ?$

$h(t) = h_1(t) * h_2(t) = \int h_1(\tau) h_2(t-\tau) d\tau$

$$h_1(t) * h_2(t) = \int_{-\infty}^{\infty} h_1(\tau) \cdot h_2(t-\tau) d\tau = \int_{-\infty}^{\infty} [u(\tau) - u(\tau-1)] \cdot [u(t-\tau+0{,}5) - u(t-\tau-0{,}5)]$$

$$= \int_{-\infty}^{\infty} u(\tau) u(t-\tau+0{,}5) d\tau - \int_{-\infty}^{\infty} u(\tau) u(t-\tau-0{,}5) d\tau - \int_{-\infty}^{\infty} u(\tau-1) u(t-\tau+0{,}5) d\tau$$

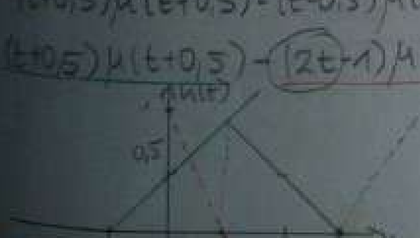
$$+ \int_{-\infty}^{\infty} u(t-1) u(t-\tau-0{,}5) d\tau$$

$y = -2t+1 \quad t=0 \quad y=1 \quad t=0{,}5 \quad y=0$

$$= \int_{0}^{t+0{,}5} u(t+0{,}5) dt - \int_{0}^{t-0{,}5} u(t-0{,}5) dt - \int_{+1}^{t+0{,}5} u(t-0{,}5) dt + \int_{+1}^{t-0{,}5} u(t-1{,}5) dt$$

$= (t+0{,}5)u(t+0{,}5) - (t-0{,}5)u(t-0{,}5) - (t-0{,}5)u(t-0{,}5) - (t-1{,}5)u(t-1{,}5)$

$= (t+0{,}5)u(t+0{,}5) - (2t-1)u(t-0{,}5) + (t-1{,}5)u(t-1{,}5)$



od -0,5 do 0,5 raste $t+0{,}5$

od 0,5 do 1,5 pada

4.18 $h(t) = \mathrm{rect}((t-3)/2)\ \mathrm{V}$

$x(t) = \mathrm{rect}((t-1)/3)\ \mathrm{V}$

$y(t) = ?$

$$h(t) = u(t-3+1) - u(t-3-1) = u(t-2) - u(t-4)$$

$$x(t) = u\left(t-1+\frac{3}{2}\right) - u\left(t-1-\frac{3}{2}\right) = u(t+0.5) - u(t-2.5)$$

$$\boxed{y(t) = h(t) * x(t)}$$

$$y = \int_{-\infty}^{\infty} [u(\tau-2) - u(\tau-4)] \cdot [u(t-\tau+0.5) - u(t-\tau-2,5)]$$

$$= \int_{-\infty}^{\infty} u(\tau-2)u(t-\tau+0.5)d\tau - \int_{-\infty}^{\infty} u(\tau-2)u(t-\tau-2,5) - \int_{-\infty}^{\infty} u(\tau-4)u(t-\tau+0,5)d\tau$$

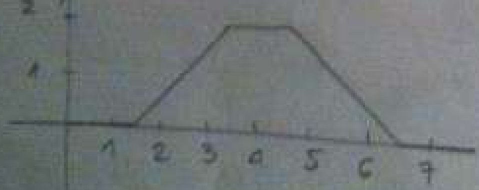
$$+ \int_{-\infty}^{\infty} u(\tau-4)u(t-\tau-2,5)d\tau$$

$$= \int_{2}^{t+0.5} u(t-1,5)d\tau - \int_{2}^{t-2.5} u(t-4,5)d\tau - \int_{4}^{t+0.5} u(t-3.5)d\tau + \int_{4}^{t-2.5} u(t-6,5)d\tau$$

$$= \underbrace{(t-1,5)u(t-1,5)}_{\text{rast od } 1,5 \text{ do } 3,5\ (2)} - (t-4,5)u(t-4,5) - \underbrace{(t-3,5)u(t-3,5)}_{\text{od } 3,5 \text{ do } 4,5 = 2} + (t-6,5)u(t-6,5)$$

do $1,5 = 0$

od 4,5 do 6,5 pad



13) $H(f) = 0{,}1\, e^{j\frac{\pi}{4}}, \quad \forall f \in \mathbb{R}$

$E = 0{,}1\ mWs = E(x)$

$$x(t-t_0) = \begin{cases} A, & za \quad 0 \le |t-t_0| < \tau/2 \\ 0, & za \quad |t-t_0| > \tau/2 \end{cases}$$

$E(x) \to \boxed{H(f)} \to E(y) = ?$

$E_{izlaz} = ? = E(y) = ?$

$X(f)$ = spektar ulaznog signala (pravokutnog)

$$X(f) = A\tau \frac{\sin\left(\frac{2\pi f \tau}{2}\right)}{\frac{2\pi f \tau}{2}} \qquad E(x) = A^2\tau = 0{,}1 \cdot 10^{-3}$$

$$Y(f) = X(f) \cdot H(f) = A\tau \frac{\sin\left(\frac{2\pi f \tau}{2}\right)}{\frac{2\pi f \tau}{2}} \cdot 0{,}1\, e^{j\frac{\pi}{4}}$$

$$E(y) = \int_{-\infty}^{\infty} |Y(f)|^2 df$$

$$E(y) = \int \left| A^2\tau^2 \cdot \frac{\sin^2\left(\frac{2\pi f \tau}{2}\right)}{\left(\frac{2\pi f \tau}{2}\right)^2} \cdot 0{,}01\, e^{j\frac{\pi}{2}} \right| df$$

$$|e^{j\frac{\pi}{2}}| = |\cos\tfrac{\pi}{2} + j\sin\tfrac{\pi}{2}| = \underline{\underline{1}} \qquad \boxed{|e^{ja}| = 1}$$

$$E(y) = A^2\tau^2 \cdot 0{,}01 \int_{-\infty}^{\infty} \underbrace{\left| \frac{\sin\left(\frac{2\pi f \tau}{2}\right)}{\frac{2\pi f \tau}{2}} \right|^2}_{} df$$

$$\boxed{\int_{-\infty}^{\infty} \left| \frac{\sin ax}{ax} \right|^2 dx = \frac{\pi}{a}} \qquad a = \frac{2\pi\tau}{2} = \underline{\underline{\pi\tau}}$$

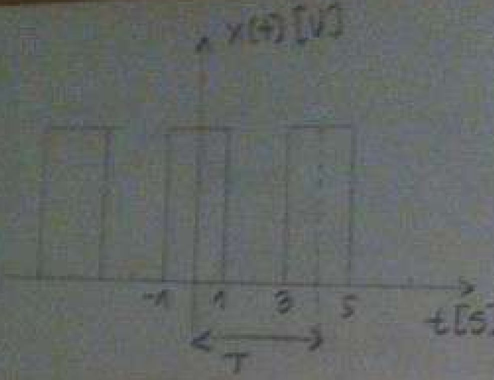
$$E(y) = A^2 \cdot \tau^{\cancel{2}} \cdot 0{,}01 \cdot \frac{\cancel{\pi}}{\cancel{\pi\tau}} = A^2\tau \cdot 0{,}01$$

$E(y) = 0{,}01\, E(x)$

$E(y) = 10^{-6}\ Ws$

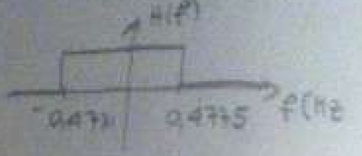
$\boxed{E(y) = 1 0 \mu Ws}$

4.20



T = 4s

$$H(f) = \begin{cases} 1 & |f| \le 0{,}4775\,\text{Hz} \\ 0, & \text{inače} \end{cases}$$



$x(t) \rightarrow$ NPF $\rightarrow y(t)$ $\boxed{y(t) = ?}$

$$X(f) = A\tau \cdot \frac{\sin\left(\frac{2\pi f \tau}{2}\right)}{\frac{2\pi f \tau}{2}} \qquad Y(f) = X(f) \cdot H(f)$$

$$X(f) = \frac{A}{2} \frac{\sin\left(\frac{\pi f}{2}\right)}{\frac{\pi f}{2}} \quad A = 1 \quad \boxed{X(f) = \frac{1}{2} \frac{\sin\left(\frac{\pi f}{2}\right)}{\frac{\pi f}{2}}}$$

za $f = 2\,\text{Hz}$

$X(f) = 0$ ali filtar to ne propušta

$X(f) = Y(f)$

$$y(t) = \frac{1}{2} \underbrace{\int_{-\infty}^{\infty} \frac{\sin\left(\frac{\pi f}{2}\right)}{\frac{\pi f}{2}} e^{j2\pi f \tau} \, df}_{2}$$

4.21

4.22

$N = 4{,}5$

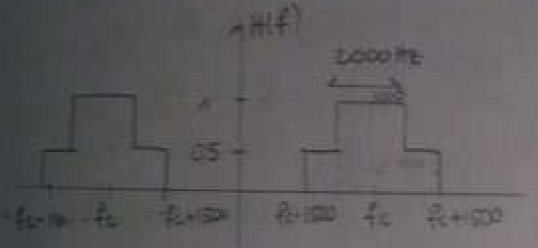
7.21 $x(t) = \underbrace{2\sqrt{E}\cos(2\pi\, 0{,}25 t)}_{\text{željeni signal}} + w(t)$

$S_w(f) = 10^{-3}\, W/Hz = \sigma^2 = \frac{N_0}{2}$

$y(t) = \int_{t-2}^{t} x(t)\,dt$

$\left(\frac{S}{N}\right)_{dB} = ?\quad 10\log\left(\frac{S}{N}\right)$

7.22



$\frac{N_0}{2} = S_w = 10^{-12}\,\frac{W}{Hz}$

$H(f)$

snaga šuma = ?

$$N = \int_{-\infty}^{\infty} S_w(f)\cdot |H(f)|^2\, df$$

$$N = 2\cdot\left[\int_{f_c-1500}^{f_c-1000} (0{,}5)^2\cdot 10^{-12}\, df + \int_{f_c-1000}^{f_c+1000} 1^2\cdot 10^{-12}\, df + \int_{f_c+1000}^{f_c+1500} 0{,}5^2\cdot 10^{-12}\, df\right]$$

$$N = \left[2{,}5\cdot 10^{-13}(f_c - 1000 - f_c + 1500) + 10^{-12}\, 2000 + 2{,}5\cdot 10^{-13}\cdot 500\right]\cdot 2$$

$$= \left[2{,}5\cdot 10^{-13}\cdot 500\cdot 2 + 10^{-12}\, 2000\right] 2 = 2{,}25\cdot 10^{-9}\cdot 2$$

$$\boxed{N = 4{,}5\cdot 10^{-9}\, W}$$

4.23 $x(t) = \begin{cases} e^{-at} & t \geq 0 \\ 0 & t < 0 \end{cases}, \quad a \in \mathbb{R}$

$X(f) = ?$ $f = 10\,Hz$; $a = 30$

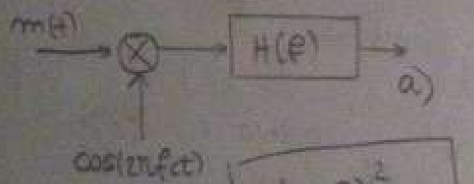
$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t}\,dt = \int_0^{\infty} e^{-at} e^{-j2\pi f t}\,dt = \int_0^{\infty} e^{-t(a+2\pi f j)}\,dt$$

$$= \frac{-1}{a+2\pi f j} e^{-t(a+2\pi f j)}\Big|_0^{\infty} = \frac{1}{a+2\pi f j}$$

$$|X(f=10)| = \frac{1}{\sqrt{a^2+(2\pi f)^2}} = \underline{0{,}0144}$$

4.24 $m(t) = 4\cos(2\pi f_1 t) + 4\cos(2\pi f_2 t)$ [V]

$f_1 = \frac{f_2}{2}$



$$H(f) = \begin{cases} 1, & |f| = f_c - f_2 \\ \frac{1}{4}, & |f| = f_c - f_1 \\ \frac{1}{2}, & |f| = f_c \\ \frac{3}{4} & |f| = f_c + f_1 \\ 1 & |f| = f_c + f_2 \end{cases}$$

$\boxed{S \cdot |H(f)|^2}$

Odredi snagu na izlazu iz sklopa a)

$x(t) = m(t)\cos(2\pi f_c t) = 4\cos(2\pi f_1 t)\cos(2\pi f_c t) + 4\cos(2\pi f_2 t)\cos(2\pi f_c t)$

$x(t) = 2[\cos 2\pi t(f_1 - f_c) + \cos 2\pi t(f_1 + f_c)] + 2[\cos(2\pi t(f_2 - f_c)) + \cos 2\pi t(f_2 + f_c)]$

$$= \frac{2^2}{2}\left(\left|\frac{1}{4}\right|^2 + \left(\frac{3}{4}\right)^2\right) + \frac{2^2}{2}\left(|1|^2 + |1|^2\right)$$

4.25. $f_g = 10 MHz$

$S_N = 10^{-10} W/Hz = \frac{N_0}{2}$

$|H(f)| = 0.2$

$N = ?$

$N = N_0 \cdot B = 2 \cdot 10^{-10} \cdot 10 \cdot 10^{6} = 0{,}002 \frac{W}{Hz}$ → srednja snaga šuma na ulazu

$S_x = 0{,}02 \frac{W}{Hz}$

$S_y = S_x \cdot |H(f)|^2$ → srednja snaga šuma na izlazu

$S_y = 0{,}002 \cdot 0{,}04 = 8 \cdot 10^{-5} \frac{W}{Hz}$

4.26. $x_a(t)$ → rekonstruira iz $x_a(nT_u)$ $T_u = 1 ms$.

f_{max} $X_a(f) = ?$

$f_u \geq 2 f_{max}$ $\quad f_u = \frac{1}{10^{-3}} = 10^{3} Hz$

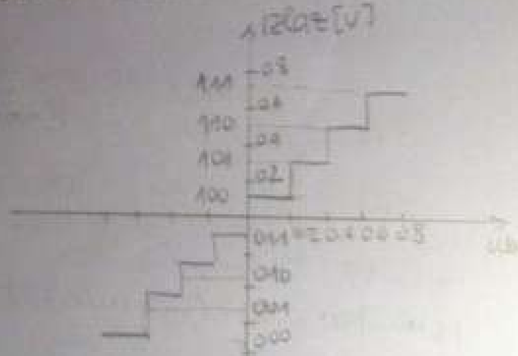
$f_{max} = 500 Hz$

4.27. $u_m(t) = 0{,}8 \sin(2\pi \cdot 4000t + \frac{\pi}{4})$ [V] → prigušen za 5 dB

$|u(t)| \leq 0{,}8$ [V] jednoliko kvantiziranje

L = 8

Odredi kodnu kombinaciju na izlazu kodera u t = 0 s.



$P_1 = \frac{A^2}{2} = \frac{0{,}8^2}{2} = 0{,}32 W$

$\frac{P_1}{P_2} = 10\log\left(\frac{P_1}{P_2}\right) = 5 dB$

$10^{0,5} = \frac{P_1}{P_2} \quad P_2 = \frac{0{,}32}{10^{0,5}} = 0{,}101 W$

$P_2 = \frac{A_2^2}{2} \quad A_2 = \sqrt{2P_2} = 0{,}449 \approx 0{,}45$

$u_{m_2}(t) = 0{,}45 \sin(2\pi \cdot 4000t + \frac{\pi}{4})$ [V] → amplituda koja ulazi u kvantizator

$t = 0$

$u_{m_2}(t) = 0{,}45 \sin\left(\frac{\pi}{4}\right) = 0{,}318$ [V] → 101

4.28 $v_m(t) = \sin\left(2\pi 1000t + \frac{\pi}{4}\right)$ [V] $f_u = 4\,kHz$

$L = 32$ $[-3, 3]$ $A_{max} = 3$ $L = 2^{5}$ $5 = r$

$$\frac{S}{N} = ? \quad \frac{S}{N} = \frac{3S}{A_{max}^2} 2^{2r} = \frac{3\frac{A^2}{2}}{3^2} 2^{10} = \frac{\frac{1}{2}}{3} 2^{10}$$

$$\frac{S}{N} = 10\log\left(\frac{1}{6}\cdot 2^{10}\right) = \underline{22{,}32\,dB}$$

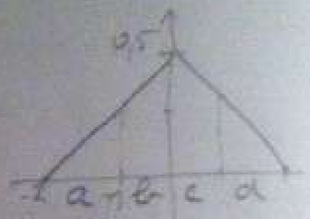
4.29

$$f_x(x) = \begin{cases} 0{,}5 - 0{,}25|x| & ; -2V \le x \le 2V \\ 0 & ; \text{inače} \end{cases}$$

vjerojatnosti razina signala

$L = 4$

Huffman kod



a) srednja duljina kodne riječi   a, b, c i d → razine

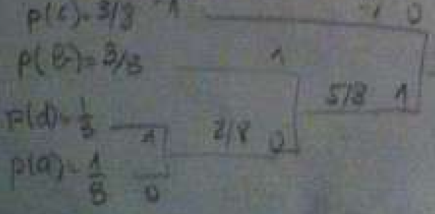
$p(a) = p(d)$

$p(b) = p(c)$

$$p(a) = \int_{-2}^{-1} f_x(x)\,dx = \int_{-2}^{-1}(0{,}5 + 0{,}25x) = 0{,}5\cdot 1 + 0{,}25\frac{x^2}{2}\Big|_{-2}^{-1}$$

$$= 0{,}5 + 0{,}125(1^2 - 4) = \underline{\underline{\frac{1}{8}}} = p(d)$$

$$p(b) = \int_{-1}^{0} f_x(x)\,dx = \int_{-1}^{0}(0{,}5 + 0{,}25x)\,dx = 0{,}5 + \frac{0{,}25x^2}{2}\Big|_{-1}^{0} = \underline{\underline{\frac{3}{8}}} = p(c)$$



$p(a) = 100$

$p(d) = 101$

$p(c) = 0$

$p(b) = 11$

$\bar{L} = \sum$ bita $p(x) = \underline{1{,}875\text{ bit/razina}}$

3) srednja kvadratna pogreška kvantiziranja

$$\overline{N_q^2} = \sum_{x_{qi}} \int_{x_{qi}-\frac{\Delta}{2}}^{x_{qi}+\frac{\Delta}{2}} (x-x_{qi})^2 (f_X(x))\,dx$$

$$= \int_{-1,5-0,5}^{-1,5+0,5} (x+1,5)^2(0,5+0,25x)\,dx$$

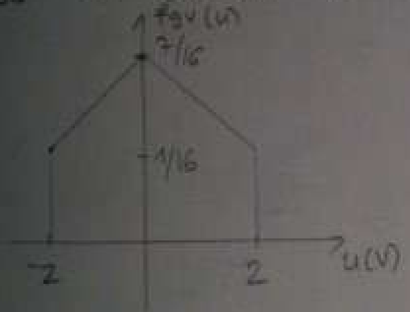
$$+ \int_{-1}^{0} (x+0,5)^2(0,5+0,25x)\,dx$$

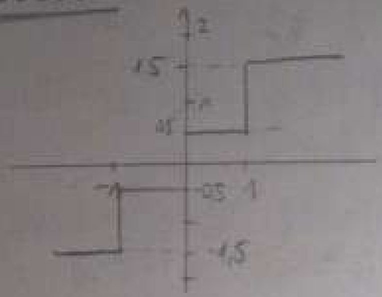
$$\Delta = \frac{2m_{max}}{L} = \frac{4}{4} = 1$$

$$+ \int_{0}^{1} (x-0,5)^2(x-0,25x)\,dx + \int_{1}^{2} (x-1,5)^2(0,5-0,25x)\,dx$$

$$= \frac{1}{96} + \frac{1}{32} + \frac{1}{32} + \frac{1}{96} = \frac{1}{12} = \underline{0,08333\,V^2}$$

4.30





L=4

$f_{uz} = 2f_{max} = 8\,kHz$

f) srednja snaga signala g(t)

$$f_{gv} = \begin{cases} -\frac{3}{16}|u| + \frac{7}{16} & -2 < u < 2 \\ \frac{1}{16}, & |u| = 2 \\ 0, & \text{inače} \end{cases}$$

1 (2, 1/16)
2 (0, 7/16)

$$y - \frac{1}{16} = \frac{7/16 - 1/16}{0-2}(x-2)$$

$$y - 1/16 = -\frac{3}{16}(x-2)$$

$$y = -\frac{3}{16}x + \frac{7}{16}$$

c) $f(u) = \left(-\frac{3}{16}|u| + \frac{7}{16}\right) \quad |u|<2$

$\boxed{P_S = \int_{-\infty}^{\infty} u^2 f(u)\,du}$ → srednja snaga signala

$$P_S = \int_{-2}^{0} u^2\left(\frac{3}{16}u+\frac{7}{16}\right)du + \int_{0}^{2}\left(-\frac{3}{16}u+\frac{7}{16}\right)u^2\,du$$

$$= \frac{3}{16}\int_{-2}^{0} u^3\,du + \frac{7}{16}\int_{-2}^{0} u^2\,du - \frac{3}{16}\int_{0}^{2} u^3\,du + \frac{7}{16}\int_{0}^{2} u^2\,du$$

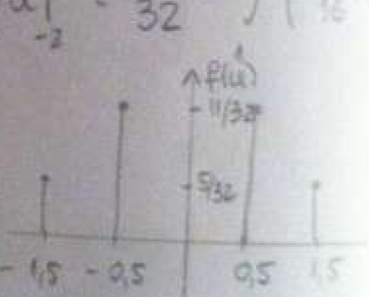
$$= \frac{3}{16}\frac{u^4}{4}\Big|_{-2}^{0} + \frac{7}{16}\frac{u^3}{3}\Big|_{-2}^{0} - \frac{3}{16}\frac{u^4}{4}\Big|_{0}^{2} + \frac{7}{16}\frac{u^3}{3}\Big|_{0}^{2}$$

$$= \frac{3}{16}\frac{1}{4}(0-16) + \frac{7}{16\cdot 3}(8) - \frac{3}{16\cdot 4}16 + \frac{7}{16\cdot 3}\cdot 8 = \frac{5}{6} = \underline{0{,}8333\,W}$$

ii) skiciraj graf f-je gustoće vjerojatnosti razine signala na izlazu iz kvantizatora

$$\int_{-2}^{-1} f_x(u)\,du = \int_{-2}^{-1}\left(\frac{3}{16}u+\frac{7}{16}\right)du = \frac{3}{16}\frac{u^2}{2}\Big|_{-2}^{-1} + \frac{7}{16}u\Big|_{-2}^{-1} = \frac{5}{32} = \int_{1}^{2}\left(-\frac{3}{16}u+\frac{7}{16}\right)du$$

$$\int_{-1}^{0} f_x(u)\,du = \int_{-1}^{0}\left(\frac{3}{16}u+\frac{7}{16}\right)du = \frac{11}{32}$$



iii) omjer srednje snage signala i srednje snage kvantizacijskog šuma

$P = \frac{5}{6} \qquad L = 4 = 2^{2} \quad r=2$

$A_{max} = 2$

$$\frac{S}{N} = \frac{3S}{A_{max}^2}\cdot 2^{2r} = \frac{3\cdot\frac{5}{6}}{4}\cdot 2^{4} = \underline{\underline{10}}$$

(4.31) $B = 10\,kHz$

$f_{uz} = 20\,kHz$

$L = 256$ razina

trajanje 1 bita na izlazu iz kodera?

$L = 2^8 \quad r = 8$

$R_b = f_{uz} \cdot r = 160\,\frac{kbit}{s} \quad T_b = \frac{1}{R_b} = \underline{6{,}25\,\mu s}$

(4.32) $x(t) = 10\cos(600\pi t)\cos^2(1600\pi t)$ uzorkujemo $f_u = 4\,kHz$

i) srednja snaga na jediničnom otporu

$x(t) = 10\cos(600\pi t)\,\frac{1}{2}\left(1 + \cos(3200\pi t)\right)$

$x(t) = 5\cos(600\pi t) + 5\cos(600\pi t)\cdot\cos(3200\pi t)$

$\quad = 5\cos(600\pi t) + 5\cdot\frac{1}{2}\left[\cos(2600\pi t) + \cos(3800\pi t)\right]$

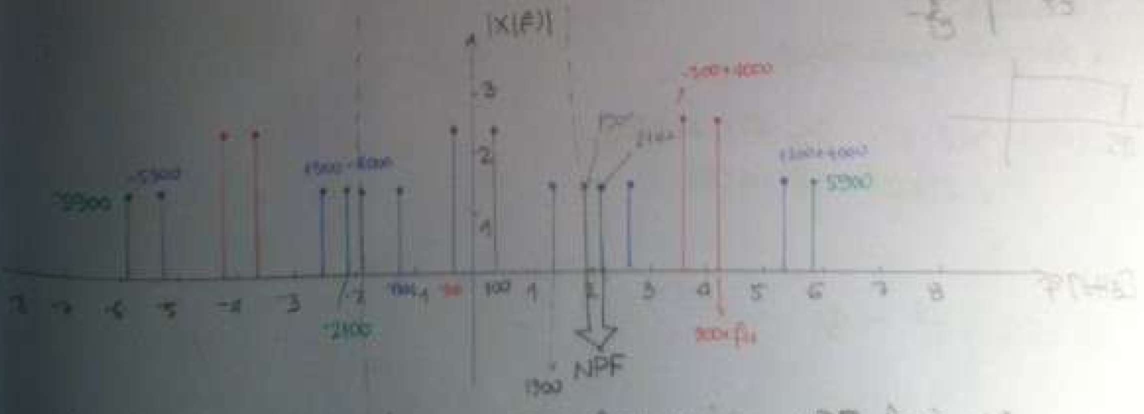
$$\boxed{x(t) = 5\cos(600\pi t) + 2.5\cos(2600\pi t) + 2.5\cos(3800\pi t)}$$

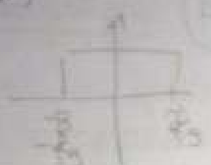
$P = \frac{5^2}{2} + 2\cdot\frac{2.5^2}{\cancel{2}} = \underline{18{,}75\,W}$

ii) amplitudni spektar uzorkovanog signala u području $[-9, 9]\,kHz$

$f_u = 4\,kHz$

$x(t) = 5\cos(\underbrace{300}_{f_1}\cdot 2\pi t) + 2{,}5\cos(\underbrace{1300}_{f_2}\cdot 2\pi t) + 2{,}5\cos(\underbrace{1900}_{f_3}\,2\pi t)$





iii) interval za gornju granicu frekvencije NPF koji se koristi za rekonstrukciju $x(t)$

$1900 \le f_g \le 2100\,Hz$

4.33

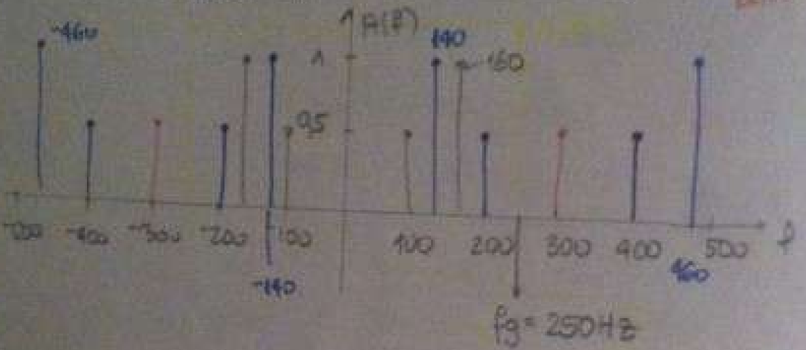
$s(t) = \cos(200\pi t) + 2\cos(320\pi t)$ [V]

$f_u = 300 Hz$

$f_g = 250 Hz$

Na kojim frekv. se nalaze komp signala koje su propuštene kroz filtar?

$s(t) = \cos(100 \cdot 2\pi t) + 2\cos(160 \cdot 2\pi t)$



* propuštaju se komponente na frekvencijama od 100, 140, 160 i 200 Hz!

4.34 Uređaj za digitalizaciju signala

- sklop za uzimanje uzoraka
- kvantizator
- sklop za kodiranje

$x(t) \rightarrow X(f) \neq 0 \quad 0 \leq |f| \leq 4 kHz$

$X(f) = 0 \quad |f| > 4 kHz$

$f_{uz} = 2 \cdot f_N$

$L = 256 \rightarrow 2^8 \quad r = 8$

(4.35) A/D pretvorba

u_1 [-8, 8] V $\qquad \frac{\Delta}{2} = 20\mu V$ $\qquad$ Amax = amplitude ulaznog signala

u_2 [-20, +20] μV

r = ?

Δ = korak kvantizacije $= \frac{2A_{max}}{L} = \frac{2A_{max}}{2^r}$

$$2^r = \frac{2A_{max}}{\Delta} \quad / \log_2$$

$$r = \log_2\left(\frac{2A_{max}}{\Delta}\right) = \log_2\left(\frac{2\cdot 8}{40\cdot 10^{-6}}\right) = 18{,}61$$

$$\boxed{r = 19}$$

(4.36) B = 4kHz

f_{u} = min

L = 8 razina = 2^{3} r = 3

p(a) = p(b) = 0.25

p(c) = p(d) = 0.125

p(e) = p(f) = p(g) = p(h) = 0,0625

minimalni protok na izlazu iz kodera

f_{uz} = 8kHz

$R_b = f_{uz} \cdot r$ $\qquad -H(x) = 2\cdot 0{,}25 \log_2 0{,}25 + 2\cdot 0{,}125 \log_2 0{,}125$

$R_b = f_{uz} \cdot H(x)$ $\qquad + 4\cdot 0{,}0625 \log_2 (0{,}0625)$

$H(x) = 2{,}75$ bit/simbol

$R_b = 8000 \cdot 2{,}75 = 22000$ bit/s

$$\boxed{R_b = 22 \text{ kbit/s}}$$

437) $s(t) = 4\sin(2\pi \cdot 35000t)$ [V]

A/D pretvorba $\quad f = 35000$ Hz

$\frac{S}{N} = 65$dB $\quad \underline{f_{uz} = 70000\text{Hz}}$

$R_b = ?$ $\quad R_b = f_{uz} \cdot r$

$S = \frac{4^2}{2} = 8W \quad 65 = 10\log\left(\frac{S}{N}\right)$

$\frac{S}{N} = 10^{6.5} = \frac{3S}{A^2_{max}} \cdot 2^{2r} \quad 3S = 3 \cdot \frac{A^2}{2} = 3 \cdot 8 = 24$

$A^2_{max} = 16$

$10^{6.5} = \frac{24}{16} \cdot 2^{2r} \; / \log_2$

$2r = \log_2(2108185)$

$r \approx 10{,}5 \quad \underline{r = 11}$

$R_b = r \cdot f_u = 77000$ b/s

$\boxed{R_b = 770 \text{ kbit/s}}$

438) $g(t) = \text{sinc}(100t) = \frac{\sin(100t)}{100t}$

4.39 $f_u = 10\,kHz$

$r = 10$ bit/uzorak

$\sigma^2 = 4 - [E(x)]^2 \qquad \sigma = 2$

$E(x) = 0$

i) prijenosna brzina

$R_b = f_u \cdot r = 100\,kbit/s$

ii) vrijednost kvantizacijske razine $[\pm\sigma 4V]$

$A_{max} = 8V$

$$\Delta = \frac{2A_{max}}{L} = \frac{2 \cdot 8}{2^r} = \frac{2 \cdot 8}{2^{10}} = \underline{0{,}0156V}$$

iii) srednji snagu kvantizacijskog šuma

$$\boxed{N_q = \frac{\Delta^2}{12} = \frac{1}{3}A^2_{max}2^{-2r}} = \underline{2{,}035 \cdot 10^{-5}\ W}$$

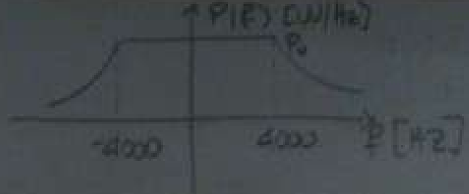
iv) vjerojatnost da je uzorak izvan granica ii)

$$f_Q = \begin{cases} \frac{1}{\Delta} & za \quad -\frac{\Delta}{2} < q < \frac{\Delta}{2} \\ 0 & inače \end{cases}$$

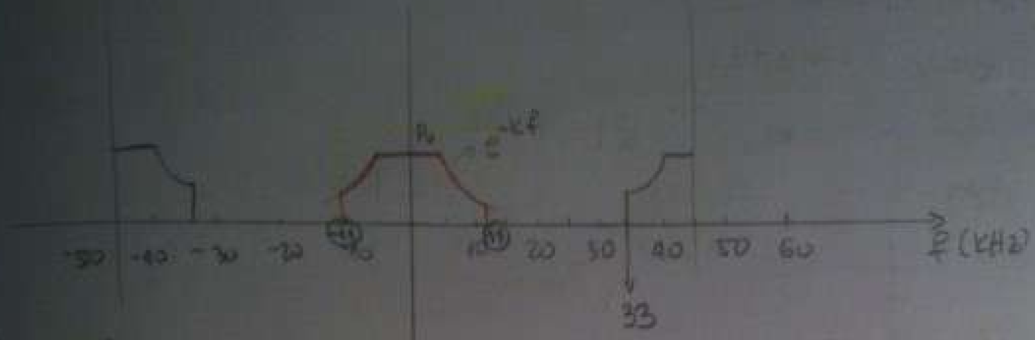
4.10

$f_g = 11\,kHz$

$f_u = 44\,kHz \Rightarrow r = 16$



i) spektar uzorkovanog signala na (-44, 44)



ii) omjer snage na ulazu i izlazu NPF-a u dB

$P_0 = 3 \cdot 10^{-3}\,W/Hz$

$|f| \geq 4\,kHz \quad S_x = e^{kf}$

$$P_{prije} = 2 \cdot \int_0^4 3 \cdot 10^{-3}\,df + 2\int_4^{\infty} e^{-kf}\,df = 0$$

$$P_{poslije} = 2\int_0^4 3 \cdot 10^{-3}\,df + 2\int_4^{11} e^{-kf}\,df$$

$$\frac{0{,}024 - 2e^{-4}}{0{,}024 - 2e^{-15}}$$

STEREO $\underline{\times 2}$

4.41 $f\ [-6, 6]$ kHz

i) $f_{uz} = 2 f_{max} = 12$ kHz

ii) zaštitni pojas od 2 kHz, fmin uzorkovanja

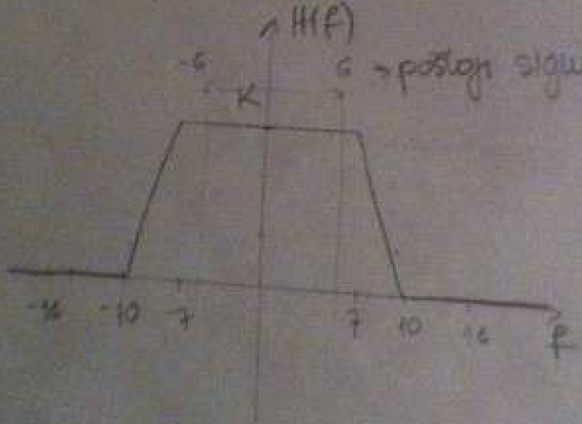
$f\ [-7, 7]$ kHz

$f_{uz} = 2 \cdot 7 = 14$ kHz

iii)

$$H(f) = \begin{cases} K, & |f| < 7\text{ kHz} \\ K - K\dfrac{|f| - 7000}{3000}, & 7 \leq |f| < 10\text{ kHz} \\ 0, & \text{inače} \end{cases}$$

$f_{uz} = 2 f_{max} = ?$



(4.12) $H(x) = 0{,}75 \cdot 10^6$ bit
AWGN
$S_N = 4 \cdot 10^{-18}$ W/kHz $\qquad S_N = \frac{N_0}{2} \quad N_0 = 8 \cdot 10^{-18}$ W/kHz
$B = 1$ kHz
$T = 375$ s $\qquad C = B \log_2 \left(1 + \frac{S}{N_0 B}\right)$

$S = ?$

$R_b = \frac{H(x)}{T} = \frac{0{,}75 \cdot 10^6 \text{ bit}}{375 \text{ s}} = 2000 \text{ bit/s} = 2 \text{ kbit/s}$ + prijenosna brzina

$R_b \leq C \quad \boxed{R_b = C}$

$2 = 1 \log_2 \left(1 + \frac{S}{N_0 B}\right) \Big/ 2^x$

$2^2 = \left(1 + \frac{S}{N_0 B}\right) \qquad 3 = \frac{S}{N_0 B} \qquad S = 3 \cdot N_0 \cdot B$

$S = 3 \cdot 8 \cdot 10^{-18} \frac{W}{\cancel{kHz}} \cdot 1\cancel{kHz} = \underline{\underline{24 \cdot 10^{-18} W}}$

(4.13) $S \Rightarrow$ se poveća x puta
$S_{novo} = x \cdot S$ [W]
$\frac{S}{N} \gg 1$
N[W] = srednja snaga šuma
B

za koliko se promijeni kapacitet signala?

$C_1 = B \log_2 \left(\frac{S}{N}\right) \qquad C_2 - C_1 = B \log_2 \left(\frac{xS}{N}\right) - B \log_2 \left(\frac{S}{N}\right)$

$C_2 = B \log_2 \left(\frac{xS}{N}\right) \qquad = B \log_2 \left(\frac{\frac{xS}{\cancel{N}}}{\frac{S}{\cancel{N}}}\right) = \boxed{B \log_2 x}$

poveća se za $\underline{B \log_2 x}$

4.44 $S = 1{,}9W$ AWGN

$S_N = 7{,}5 \cdot 10^{-9} W/Hz = \frac{N_0}{2}$

$N_0 = 15 \cdot 10^{-9} W/Hz$

$C_{max} = ?$

$C = \log_2\left(1 + \frac{S}{N_0}\right) = \infty$

$B \to \infty$

$\lim_{B \to \infty} C = \frac{S}{N_0} \log_2 e$ $\boxed{C = 182{,}74 \frac{bit}{s}}$

4.45 NPF, B, $H(f) = 0{,}8$

$$S_s(f) = \begin{cases} a\frac{|f|}{B} & |f| \le B \\ 0, & \text{inače} \end{cases}$$

$\frac{N_0}{2} = a\,10^{-10} W/Hz$ $\Gamma = 0dB = 1$

$c = ?$

$N_0 = 2a \cdot 10^{-10} W$

$N = N_0 B = 2aB \cdot 10^{-10}$

$S_y(f) = S_s(f) \cdot |H(f)|^2$

$S =$ srednja snaga $= \int_{-B}^{B} S_y(f)\,df = \int_{-B}^{B} a \cdot \frac{|f|}{B} \cdot 0{,}8^2$

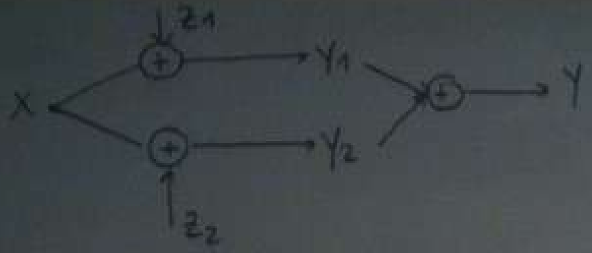
$S = 0{,}8^2 \cdot 2 \cdot \int_0^B a\,\frac{f}{B}\,df = \frac{0{,}8^2 \cdot \cancel{2} a}{B} \frac{f^2}{\cancel{2}}\Big|_0^B$

$S = \frac{0{,}8^2 a B^2}{B} = \boxed{0{,}64aB}$

$C = B\log_2\left(1 + \frac{S}{N}\right) = B\log_2\left(1 + \frac{0{,}64\cancel{aB}}{2\cancel{aB} \cdot 10^{-10}}\right)$

$\boxed{C = B \cdot 31{,}58\ bit/s}$

1.47



$E(z_1)=E(z_2)=0$

$C_z=\begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix}$ → kovarijantna matrica

$x+z_1=y_1$
$x+z_2=y_2$ $>$ $y=2x+z_1+z_2$

$$E(y^2)=E\left(2x+(z_1+z_2)\right)^2$$

$$=4E(x^2)+\cancel{4E(x)E(z_1+z_2)}^{\,0}+E(z_1^2)+2E(z_1)E(z_2)+E(z_2^2)$$

$$=4P+\sigma^2+2\rho\sigma^2+\sigma^2$$

$$=\underbrace{4P}_{\text{ulaz}}+\underbrace{2\sigma^2(1+\rho)}_{\text{šum}}$$

$$\frac{S}{N}=\frac{4P}{2\sigma^2(1+\rho)}$$

$$C=\frac{1}{2}\log_2\left(1+\frac{S}{N}\right)$$

$$C=\frac{1}{2}\log_2\left(1+\frac{2P}{\sigma^2(1+\rho)}\right)\frac{bit}{s}$$

4.46 kom. kanal → podijeljen na 2 dijela

$B_1 \neq B_2$

$P_1 = P_2$

$\alpha_1 = 2\alpha_2$ → ampl. odzivi

$S_{N1} = S_{N2} = \frac{N_0}{2} \left[\frac{W}{Hz}\right]$ $N_0 = 2S_{N1} = 2S_{N2}$

$\frac{B_2}{B_1} = ?$ tako da $R_{B1max} = R_{B2max} \Rightarrow C_1 = C_2$

$$\boxed{\frac{E_{b1}}{N_0} = \frac{\alpha_1^2 P_1}{C_1 N_0} = 100}$$

$$\frac{E_{b2}}{N_0} = \frac{\left(\frac{\alpha_1}{2}\right)^2 P_2}{C_1 \cdot N_0} = \frac{\alpha_1^2 \cdot P_1}{C_1 N_0 \, 4} \quad = 100$$

$$\boxed{\frac{E_{b2}}{N_0} = 25} \qquad E_b = \frac{S}{R_b}$$

$$\frac{E_b}{N_0} = \frac{2^{C/B} - 1}{C/B} \qquad C_1 = C_2$$

$$\frac{E_{b1}}{N_0} = \frac{2^{C/B_1} - 1}{C/B_1} \qquad \frac{C}{B_1} = \log_2\left(1 + \frac{E_{b1} C_1}{N_0 \cdot B_1}\right)$$

$$\frac{E_{b2}}{N_0} = \frac{2^{C/B_2} - 1}{C/B_2} \qquad \frac{C}{B_1} = \log_2\left(1 + 100\frac{C}{B_1}\right)$$

$$2^{\frac{C}{B_1}} = 1 + 100\frac{C}{B_1} \qquad \frac{C}{B_1} = x$$

$$\boxed{2^x = 1 + 100x}$$

(4.48) $E[z_1^2]=0{,}5 \quad E(z_1)=E(z_2)=0$

$E(z_2^2)=0{,}7$

$E(x_1)=E(x_2)=0$ $\quad x_1 \to$ prvi kanal (z_1)

$\underline{E[x_1^2]+E[x_2^2]=0{,}4}$ $\quad x_2 \to$ drugi kanal (z_2)

maksimalna dinamika sustava

Dinamika u sustavu paralelnih kanala jednaka je zbroju dinamika pojedinih kanala

$$D=D_1+D_2=\frac{1}{2}\log_2\left(1+\frac{\sigma_{x_1}^2}{\sigma_{z_1}^2}\right)+\frac{1}{2}\log_2\left(1+\frac{\sigma_{x_2}^2}{\sigma_{z_2}^2}\right)$$

$$=\frac{1}{2}\log_2\left(1+\frac{\sigma_{x_1}^2}{0{,}5}\right)+\frac{1}{2}\log_2\left(1+\frac{0{,}4-\sigma_{x_1}^2}{0{,}7}\right)$$

$\boxed{\rightarrow \text{MAKSIMUM DINAMIKE } \frac{\partial D}{\partial \sigma_{x_1}}=0}$ $\quad \boxed{(\log_a x)'=\frac{1}{x\ln a}}$

$$\frac{\partial D}{\partial \sigma_{x_1}}=0{,}5\frac{1}{\left(1+\frac{\sigma_{x_1}^2}{0{,}5}\right)\ln 2}\cdot\frac{2\sigma_{x_1}}{0{,}5}+\frac{1}{2}\frac{1}{\left(1+\frac{0{,}4-\sigma_{x_1}^2}{0{,}7}\right)\ln 2}\cdot\left(-\frac{2\sigma_{x_1}}{0{,}7}\right)$$

$$\cancel{0{,}5}\frac{1}{\left(1+\frac{\sigma_{x_1}^2}{0{,}5}\right)\cancel{\ln 2}}\cdot\frac{2\sigma_{x_1}}{0{,}5}=\cancel{0{,}5}\cdot\frac{\cancel{2\sigma_{x_1}}}{\left(1+\frac{0{,}4-\sigma_{x_1}^2}{0{,}7}\right)\ln 2}\cdot\frac{1}{0{,}7}$$

$$\left(1+\frac{\sigma_{x_1}^2}{0{,}5}\right)\cdot 0{,}5=\left(1+\frac{0{,}4-\sigma_{x_1}^2}{0{,}7}\right)\cdot 0{,}7$$

$$0{,}5+\sigma_{x_1}^2=0{,}7+0{,}4-\sigma_{x_1}^2$$

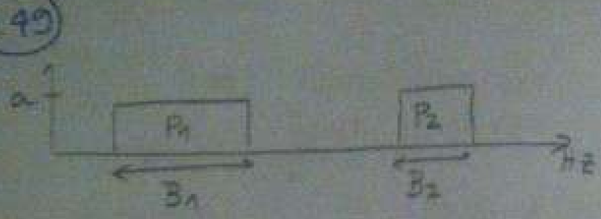
$$2\sigma_{x_1}^2=0{,}6$$

$\boxed{\sigma_{x_1}^2=0{,}3}=E(x_1^2)$ $\quad \boxed{E(x_2^2)=\sigma_{x_2}^2=0{,}1}$

$$D(max)=\frac{1}{2}\log_2\left(1+\frac{0{,}3}{0{,}5}\right)+\frac{1}{2}\log_2\left(1+\frac{0{,}1}{0{,}7}\right)$$

$\boxed{D=0{,}435\ \frac{\text{bit}}{\text{simbol}}}$

4.49



$B_1 > B_2$

$P = P_1 + P_2$

$C = C_1 + C_2$

$P_1, P_2 = ?$

$P = \int_B S_N \, df \qquad S_{N izlaz} = S_{N ulaz} \cdot |H(f)|^2$

$\boxed{P_1 = S_N \cdot B_1 \cdot a^2} \quad \boxed{P_2 = S_N \cdot B_2 \cdot a^2} \Rightarrow S_N a^2 = \frac{P_2}{B_2}$

$P_1 = \frac{P_2}{B_2} \cdot B_1 \qquad P_2 = P - P_1$

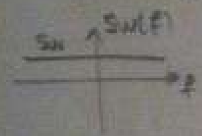
$P_1 = \frac{P - P_1}{B_2} B_1 \qquad P_1 B_2 = P B_1 - P_1 B_1$

$P_1 (B_1 + B_2) = P B_1$

$\boxed{P_1 = \frac{P \cdot B_1}{B_1 + B_2}}$

$P_2 = P - P_1 = \frac{P(B_1 + B_2) - P B_1}{B_1 + B_2}$

$\boxed{P_2 = \frac{P \cdot B_2}{B_1 + B_2}}$



⇒ mala šuma prijenosa, tj kapacitet zato ulaz i izlaz imaju Gaussovu razdiobu ⇒ S_N - konst.

4.50 $B = 4\,kHz$

$|H(f)| = 1$ za $|f| < 4\,kHz$

$P_1 = -10\,dBm$

$\frac{P_1}{P_2} = 10\,dB$

$N_0 = -80\,dBm/Hz$

$-10 = 10 \log \left(\frac{P_1 (mW)}{1\,mW} \right)$

$10^{-1} = P_1 (mW) = 0{,}1\,mW$

$\boxed{P_1 = 10^{-4}\,W}$

$-80 = 10 \log \left(\frac{N_0 (mW/Hz)}{1\,mW} \right)$

$N_0 = 10^{-8}\,mW/Hz = 10^{-11}\,W/Hz$

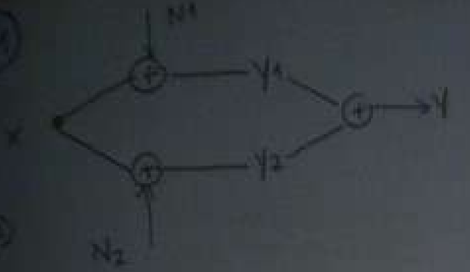
$N_0 B = 4 \cdot 10^{-8}\,W$

$10 = 10 \log \left(\frac{P_1}{P_2} \right)$

$10 = \frac{P_1}{P_2} \quad \boxed{P_2 = 10^{-5}\,W} = S$

$$\eta = \frac{C}{B} = \log_2 \left(1 + \frac{E_b \cdot C}{N_0 B} \right) = \log_2 \left(1 + \frac{\frac{S}{C} \cdot C}{N_0 B} \right)$$

$$\boxed{\eta = \log_2 \left(1 + \frac{S}{N_0 B} \right) = 7.97 \, \frac{bit/s}{Hz}}$$



a)

b)

$E(N_1) = E(N_2) = 0$

$E[N_1^2] = E[N_2^2] = \sigma^2$

$Y = \dfrac{Y_1 + Y_2}{2}$

$var(Y) - var(Z) = ?$

$var(Z) = E(Z^2) - [E(Z)]^2 \qquad Z = X + N_1$

$= E((X+N_1)^2) - [E(X+N_1)]^2$

$= E(X^2) + \cancel{2E(X)\cdot E(N_1)} + E(N_1^2) - [E(X)]^2$

$\boxed{var(Z)} = E(X^2) + E(N_1^2) - [E(X)]^2 = \boxed{var(X) + \sigma^2}$

$var(Y) = E(Y^2) - [E(Y)]^2 = E\left(\dfrac{(Y_1+Y_2)^2}{4}\right) - \left[E\left(\dfrac{Y_1+Y_2}{2}\right)\right]^2$

$= E\left(X + \dfrac{N_1}{2} + \dfrac{N_2}{2}\right)^2 - \left[E(X) + \cancel{E\left(\dfrac{N_1}{2}\right)} + \cancel{E\left(\dfrac{N_2}{2}\right)}\right]^2$

$= E(X^2) + \cancel{E(X)E(N_1)E(N_2)} + \dfrac{E}{4}(N_1^2 + \cancel{2N_1N_2} + N_2^2) - [E(X)]^2$

$= E(X^2) + \dfrac{\sigma^2}{4} + \dfrac{\sigma^2}{4} - [E(X)]^2$

$= var(X) + \dfrac{\sigma^2}{2}$

$var(Y) - var(Z) = -\dfrac{\sigma^2}{2}$ //

(4.52) $P_{iz} = 250 kW$

a) $P_{iz}(dB) = 10\log(250\cdot10^3)$

$\underline{P_{iz} = 53{,}979 dB}$

$P_{iz}(dBm) = 10\log\left(\frac{250\cdot10^6 mW}{1 mW}\right) = \underline{83{,}98 dBm}$

b) $L_A(d) = 30\log\left(30\frac{d}{km}\right)$ [dB]

$d = 250 km$

$\underline{L_A(d) = 116{,}25 dB}$

c) snaga signala u dBm na prijamnoj strani

$L_A = \frac{P_{iz}}{P_{pz}}$ $\quad L_A(dB) = 10\log\left(\frac{P_{iz}}{P_{pz}}\right)$

$116{,}25 = 10\log\left(\frac{P_{iz}}{P_{pz}}\right)$

$P_{pz} = \frac{P_{iz}}{10^{11{,}625}} = 5{,}926\cdot10^{-7} W$

$P_{pz}(dBm) = 10\log(5{,}926\cdot10^{-4}) = \underline{-32{,}3 dBm}$

(4.53) $s_1(t) = \cos(2\pi f_1 t)$ [V]
$s_2(t) = \cos(2\pi f_2 t)$ [V] AM modulacija

$f_2 = 2f_1 = 20 kHz$

$S_{AM} = [A + s_1(t) + s_2(t)]\cos(2\pi f_c t)$ [V] $\quad f_c = 1000 kHz$

skiciraj amplitudni spektar AM signala!

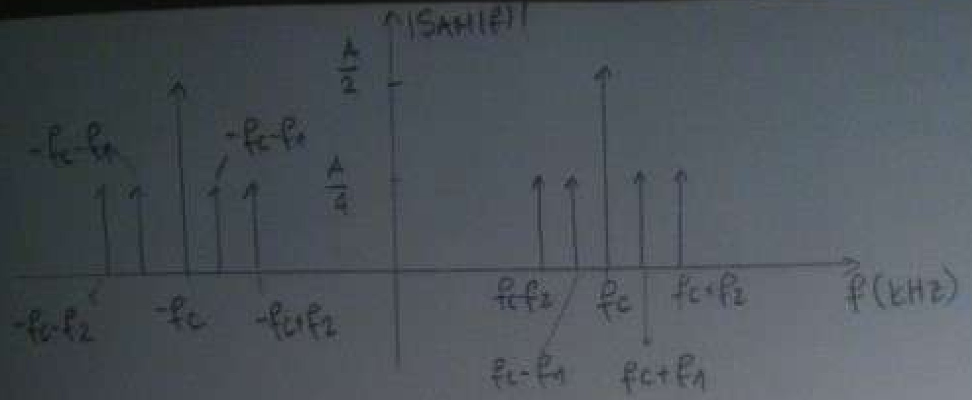
$s_1(t) = \cos(20\pi t)$

$s_2(t) = \cos(40\pi t)$

$S_{AM} = A\cos(2\pi f_c t) + \cos(20\pi t)\cos(2\pi f_c t) + \cos(2\pi f_2 t)\cos(2\pi f_c t)$

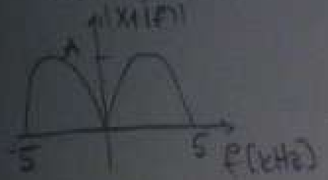
$= A\cos(2\pi f_c t) + \frac{1}{2}\left[\cos(2\pi t(f_c - 10)) + \cos(2\pi t(f_c + 10))\right]$

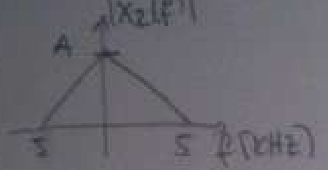
$+ \frac{1}{2}\left[\cos(2\pi t(f_c - 20)) + \cos(2\pi t(f_c + 20))\right]$

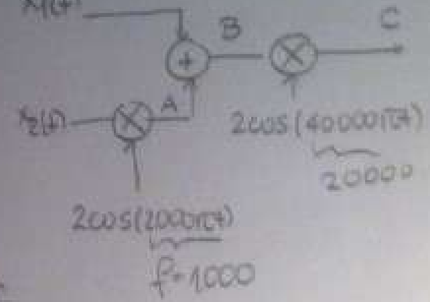


4.54

$x_1(t)$, $x_2(t)$







1) amplitudni spektar u točkama A, B, C