TINF ZADNJE PREDAVANJE (prof. Ilić) 21. 1. 2014.

ZAD Koder kanala u nekom kom. sus. koristi lin. kod zadan matricom H Ham(r).

Odredi min r pa da kodna brzina ovako zadanog lin. blok koda bude veća od 0,904.

$HAM(r)$ $[n, k] = [2^r - 1,\ 2^r - r - 1]$

$R > 0,904$ $n = r + k$

$r = ?$

$$R = \frac{k}{n} \Rightarrow \frac{k}{r+k} > 0,904$$

$$\frac{2^r - r - 1}{\cancel{r} + 2^r - 1 - \cancel{r}} > 0,904$$

$$2^r - r - 1 > 0,904\,(2^r - 1)$$

...

$$r > 6$$

ZAD U nekom kanalu u kont. vremena omjer sr. snage signala prema sr. snazi šuma je 10^5

Koliko puta će se smanjiti prijenosna brzina u odnosu na kapacitet kanala uslijed korištenja neoptimalnog kodnog s.s koda koji unosi smanjenje omjera $\frac{S}{N}$ za 20 dB.

$\frac{S}{N} = 10^5$

$\Gamma = 20\ dB$

dB

$C = B \log_2\left(1 + \frac{S}{N}\right)$ ← u idealnom slučaju $\Gamma = 0\ dB$ ili $\Gamma = 1$

$R = B \log_2\left(1 + \frac{S}{\Gamma N}\right)$

$20\ dB = 10 \log(\Gamma)$

$\Rightarrow \Gamma = 100$

$$\frac{C}{R} = \frac{\log_2\left(1 + \frac{S}{N}\right)}{\log_2\left(1 + \frac{S}{\Gamma N}\right)}$$

$$\frac{C}{R} = 1,67$$

22V 4. poglavlje ([illegible])

(1.) i) $x_1(t) = -x(t) + x(t)\cos(2000\pi t) + \underline{2x(t)\cos^2(3000\pi t)}$ [V]

(4.2. zad. zbirka)

$$2x(t)\cos^2(3000\pi t) = \cancel{2}x(t)\left(1+\cos(6000\pi t)\right)\cdot\frac{1}{\cancel{2}} = x(t) + x(t)\cos(6000\pi t)$$

$$x_1(t) = x(t)\cos(2000\pi t) + x(t)\cos(6000\pi t)$$

$$X(f) = \int_{-\infty}^{\infty} x_1(t)e^{-j2\pi f t}\,dt = \int_{-\infty}^{\infty}\left[x(t)\cos(2000\pi t) + x(t)\cos(6000\pi t)\right]\cdot e^{-j2\pi f t}\,dt$$

$$= \left|\ \cos x = \frac{e^{jx}+e^{-jx}}{2}\ \right|$$

$$x_1(t) = x(t)\cos(2000\pi t) + x(t)\cos(6000\pi t)$$

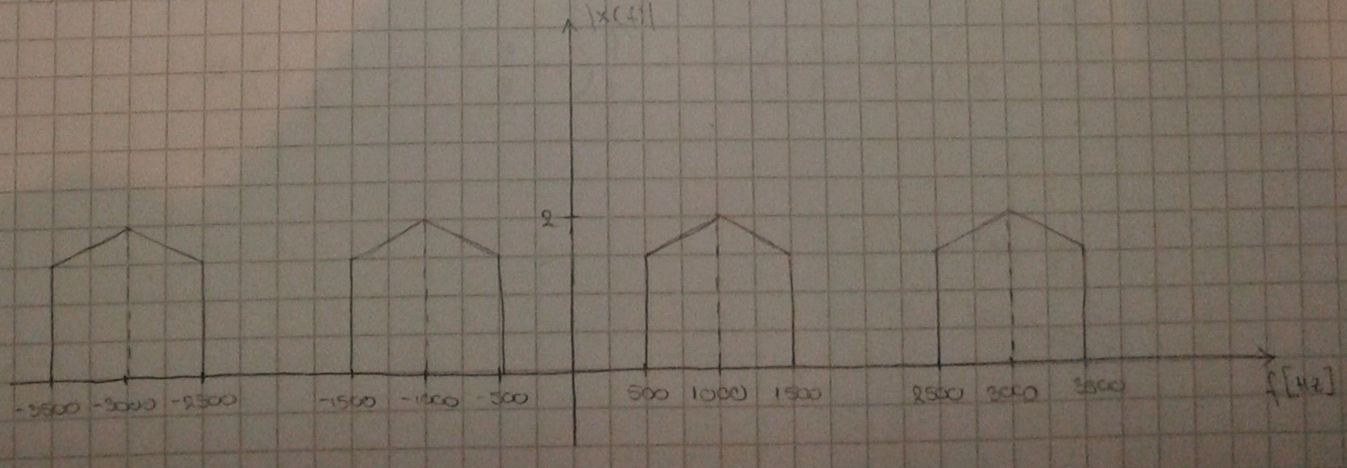
$$X(f) = \frac{1}{2}\int_{-\infty}^{\infty} x(t)\left[e^{j2000\pi t} + e^{-j2000\pi t}\right]\cdot e^{-j2\pi f t}\,dt + \frac{1}{2}\int_{-\infty}^{\infty} x(t)\left[e^{j6000\pi t} + e^{-j6000\pi t}\right]e^{-j2\pi f t}\,dt$$

$$X(f) = \frac{1}{2}\int_{-\infty}^{\infty} x(t)e^{-j2\pi(f-1000)t}\,dt + \frac{1}{2}\int_{-\infty}^{\infty} x(t)e^{-j2\pi(f+1000)t}\,dt +$$

$$\frac{1}{2}\int_{-\infty}^{\infty} x(t)e^{-j2\pi(f-3000)t}\,dt + \frac{1}{2}\int_{-\infty}^{\infty} x(t)e^{-j2\pi(f+3000)t}\,dt$$

$$X(f) = \frac{1}{2}X(f-1000) + \frac{1}{2}X(f+1000) + \frac{1}{2}X(f-3000) + \frac{1}{2}X(f+3000)$$ //

ii) AMPLITUDNI SPEKTAR SIGNALA



(4.29)

(3)

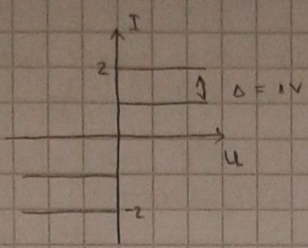
$$f_X(x) = \begin{cases} 0{,}5 - 0{,}25|x| & \text{za } -2V \leq x \leq 2V \\ 0 & \text{inače} \end{cases}$$

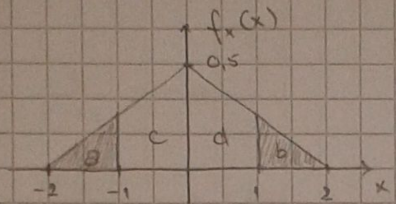
$L = 4$

Huff

$[-2,2] \Rightarrow \Delta = 1V$

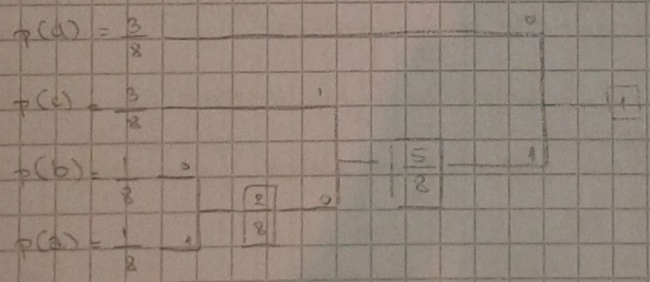
$u_{MAX} = 2V$

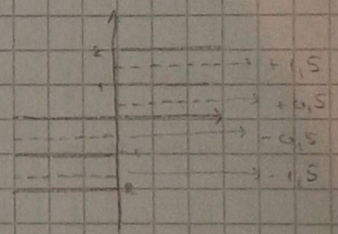




$$p(a) = p(b) = \int_{-2}^{-1} f_X(x)\,dx = \int_{-2}^{-1} (0{,}5 + 0{,}25)\,dx = \frac{1}{8}$$

$$p(c) = p(d) = \frac{3}{8}$$





$L(X) = 1{,}875$ bit / retna

$$\overline{N_q^2} = \int_{-2}^{-1} \left(x + \frac{3}{2}\right)^2 \left(\frac{1}{2} + \frac{1}{4}x\right) dx + \int_{-1}^{0} \left(x + \frac{1}{2}\right)^2 \left(\frac{1}{2} + \frac{1}{4}x\right) dx +$$

$$\int_{0}^{1} \left(x - \frac{1}{2}\right)^2 \left(\frac{1}{2} - \frac{1}{4}x\right) dx + \int_{1}^{2} \left(x - \frac{3}{2}\right)^2 \left(\frac{1}{2} - \frac{1}{4}x\right) dx$$

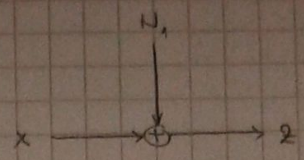
$$= \frac{1}{12}\,V^2$$

(10.) (4.51 zbirka)

$E[N_1^2] = E[N_2^2] = \sigma^2$

$E[N_1] = E[N_2] = 0$

$Y = \dfrac{Y_1 + Y_2}{2}$



$\underline{var(Y) - var(Z) = ?}$

$var(X) = E(X^2) - [E(X)]^2$

varijanca slučajne varijable

$E[Z] = E[X] + E[N_1] = E[X]$

$$\begin{aligned} var(Z) &= E(Z^2) - [E(Z)]^2 \\ &= E[(X+N_1)^2] - [E(X+N_1)]^2 \\ &= E[X^2 + 2XN_1 + N_1^2] - [E(X)]^2 \\ &= E[X^2] + \cancel{2E[X]E[N_1]} - [E(X)]^2 + \sigma^2 \\ &= var(X) + \sigma^2 \end{aligned}$$

SLIKA A:

$Y = \dfrac{Y_1 + Y_2}{2} = \dfrac{X + N_1 + X + N_2}{2} = X + \dfrac{N_1}{2} + \dfrac{N_2}{2}$

$$\begin{aligned} var(Y) &= var\left(\frac{Y_1+Y_2}{2}\right) \\ &= E\left[\left(X + \frac{N_1}{2} + \frac{N_2}{2}\right)^2\right] - \left[E\left(X + \frac{N_1}{2} + \frac{N_2}{2}\right)\right]^2 \\ &= E[X^2] - [E(X)]^2 + \frac{1}{4}E[N_1^2] + \frac{1}{4}E[N_2^2] \\ &= var(X) + \frac{1}{4}\sigma^2 + \frac{1}{4}\sigma^2 = var(X) + \frac{1}{2}\sigma^2 \end{aligned}$$

$$\underline{\underline{var(Y) - var(Z) = \frac{1}{2}\sigma^2}}$$

7.) $y(t) = \cos(2t) \cdot x(t+1)$

$$y(t) = a \cdot x_1(t) - b \cdot x_2(t)$$

$$x_1(t) \longrightarrow \boxed{LTI} \longrightarrow y_1(t)$$

$$x_1(t) \longrightarrow \boxed{LTI} \longrightarrow y_2(t)$$

$$\longrightarrow \boxed{LTI} \longrightarrow y(t) = a\,y_1(t) + b\,y_2(t)$$

$$x(t) = ax_1(t) + bx_2(t)$$

<u>LINEARNOST:</u>

$$y_1(t) = \cos(2t) \cdot x_1(t+1)$$

$$y_2(t) = \cos(2t)\, x_2(t+1)$$

$$y = y_1(t) + y_2(t) = \cos(2t) \cdot \left[x_1(t+1) + x_2(t+1)\right]$$

$$y(t) = \cos(2t) \cdot x(t+1) = \cos(2t) \cdot \left(ax_1(t+1) + bx_2(t+1)\right)$$

$$= \underbrace{\cos(2t)\, ax_1(t+1)} + \underbrace{\cos(2t) \cdot b \cdot x_2(t+1)}$$

$$= a\,y_1(t) + b\,y_2(t)$$

$\Rightarrow$ SUSTAV LINEARAN! ($y = ay_1 + by_2$ dokazano)

<u>VREMENSKA NEPROMJENJIVOST:</u>

$$y(t) = \cos(2t) \cdot x(t+1)$$

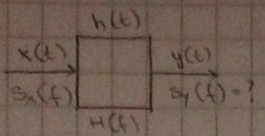
$$y(t-t_0) = \cos(2[t-t_0]) \cdot x[(t-t_0)+1]$$

$$x(t) \rightarrow x(t-t_0)$$

$$y'(t) = \cos(2t) \cdot x[(t-t_0)+1]$$

$\rightarrow$ VREMENSKI PROMJENJIV SUSTAV!

(12) $h(t) = \delta(t) - \delta(t-T)$, T = konst.



$$S_y(f) = |H(f)|^2 S_x(f)$$

$$H(f) = \int_{-\infty}^{\infty} h(t) e^{-j2\pi f t} dt = \int_{-\infty}^{\infty} \delta(t) e^{-j2\pi f t} dt - \int_{-\infty}^{\infty} \delta(t-T) e^{-j2\pi f t} dt$$

$$= 1 - e^{-j2\pi f T} = (1 - \cos(2\pi f T)) + j\sin(2\pi f T)$$

$$|H(f)|^2 = \left(\sqrt{(1-\cos(2\pi f T))^2 + (\sin(2\pi f T))^2}\right)^2$$

$$= (1-\cos(2\pi f T))^2 + (\sin(2\pi f T))^2$$

$$= 1 - 2\cos(2\pi f T) + \underbrace{\cos^2(2\pi f T) + \sin^2(2\pi f T)}_{1}$$

$$= 2(1-\cos(2\pi f T))$$

$$= 2(2\sin^2(\pi f T))$$

$$= 4\sin^2(\pi f T)$$

$$S_y(f) = 4\sin^2(\pi f T)\, S_x(f) \quad [W/Hz]$$

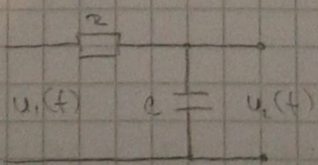
ZAD. Neki kom. kanal u kontinuiranom režimu ima karakteristiku RC kruga pri čemu je $R = 100\ \Omega$ i $C = 50$ nF. Prijenosna f-ja $|H(f)| = \frac{U_2(f)}{U_1(f)}$. Odredi graničnu frekvenciju f_g tog kanala ako je amplitudni odziv RC kruga 100 puta manji od $|H(0)|$.

$R = 100\ \Omega$

$C = 50$ nF

$|H(f)| = \frac{U_2(f)}{U_1(f)}$

$f_g = ?$



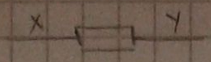
$|H(0)| = 1$

$|H(f_g)| = \frac{1}{100}$

$$|H(f)| = \frac{1}{\sqrt{1+(2\pi f R C)^2}}$$

$$\sqrt{1+(2\pi f R C)^2} = 100 \Rightarrow f_g = 3{,}18 \cdot 10^6\ Hz$$

2. $H(f) = 0{,}1 \cdot e^{j\frac{\pi}{4}}$

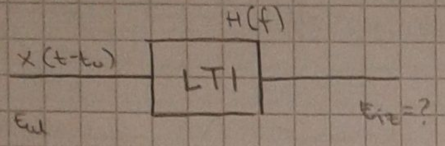


(4.19 iz knjige)

$E(x) = 0{,}1 \text{ mWs}$

$$x(t-t_0) = \begin{cases} A, & \text{za } 0 \leq |t-t_0| < \frac{\tau}{2} \\ 0, & \text{za } |t-t_0| > \frac{\tau}{2} \end{cases}, \quad t \in \mathbb{R}$$

$E_{ul} = A^2 \tau = 0{,}1 \text{m}$



$Y(f) = X(f) \cdot H(f)$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-2\pi f t} dt = \int_{-\frac{\tau}{2}+t_0}^{\frac{\tau}{2}+t_0} A e^{-j2\pi f t} dt = A e^{-j2\pi f t_0} \frac{\tau \cdot \sin\left(2\pi f \cdot \frac{\tau}{2}\right)}{2\pi f \frac{\tau}{2}}$$

$Y(f) = X(f) \cdot H(f) = A e^{-j2\pi} \ldots$ (nedovršen zadatak!)

3.34 (zbirka → NOVA 3.35)

$g(x) = x^4 + x^2 + 1$

$[n, k]$

011010...

U ispit ne dolaze konvolucijsko kodiranje i 3.3.3 iz knjige iz signala