TINF. DZ.1

1.

$$p(x_i) = [0.2 \quad 0.8]$$

$$p(y_j|x_i) = \begin{array}{c} \quad y_1 \quad\quad y_2 \quad\quad y_3 \quad\quad y_4 \\ \begin{bmatrix} 0.8 & 0.1 & 0 & 0.1 \\ 0.4 & 0.5 & 0.1 & 0 \end{bmatrix} \begin{array}{c} y_1 \\ y_2 \end{array} \end{array}$$

$$p(z_k|y_j) = \begin{array}{c} \quad z_1 \quad z_2 \quad\quad z_3 \quad z_4 \quad z_5 \quad z_6 \quad z_7 \quad z_8 \\ \begin{bmatrix} 0 & 0.2 & 0.7 & 0 & 0.1 & 0 & 0 & 0 \\ 0.3 & 0 & 0.4 & 0.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0.4 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.8 & 0.1 & 0.1 \end{bmatrix} \begin{array}{c} y_1 \\ y_2 \\ y_3 \\ y_4 \end{array} \end{array}$$

a) $H(x) = -(0.2 \log_2 0.2 + 0.8 \log_2 0.8)$

$\quad\quad = 0.72 \; bit/simbol$

$$p(y_j) = p(x_i) \cdot p(y_j|x_i) =$$

$$\begin{array}{c} 1 \times 2 \\ [0.2 \quad 0.8] \end{array} \begin{array}{c} 2 \times 4 \\ \begin{bmatrix} 0.8 & 0.1 & 0 & 0.1 \\ 0.4 & 0.5 & 0.1 & 0 \end{bmatrix} \end{array} =$$

$$[\,0.48 \quad 0.42 \quad 0.08 \quad 0.02\,] = p(y_j)$$

b)

$$p(z_g) = p(y_j) \cdot p(z_g | y_j) = \qquad 4 \times 8$$

$$[\,0.48 \quad 0.42 \quad 0.08 \quad 0.02\,]
\begin{bmatrix}
0. & 0.2 & 0.7 & 0 & 0.1 & 0 & 0 & 0 \\
0.3 & 0 & 0.4 & 0.3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.2 & 0.4 & 0.4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.8 & 0.1 & 0.1
\end{bmatrix} =$$

$$[\,0.126 \quad 0.096 \quad 0.504 \quad 0.142 \quad 0.08 \quad 0.048 \quad 0.002 \quad 0.002\,]$$

c) $H(z) = -\sum_{i=1}^{n} p(z_i) \cdot \log_2 p(z_i) = 2.1368$

d)

e) $I(X;Z) = H(X) + H(Z) - H(X,Z)$

$1 \times 2 \quad 2 \times 8$

$$p(z_k \mid x_i) = \begin{bmatrix} & z_1 & z_2 & z_3 & z_4 & z_5 & z_6 & z_7 & z_8 \\ 0.03 & 0.16 & 0.6 & 0.03 & 0.08 & 0.08 & 0.01 & 0.01 \\ 0.15 & 0.08 & 0.48 & 0.17 & 0.08 & 0.04 & 0 & 0 \end{bmatrix}$$
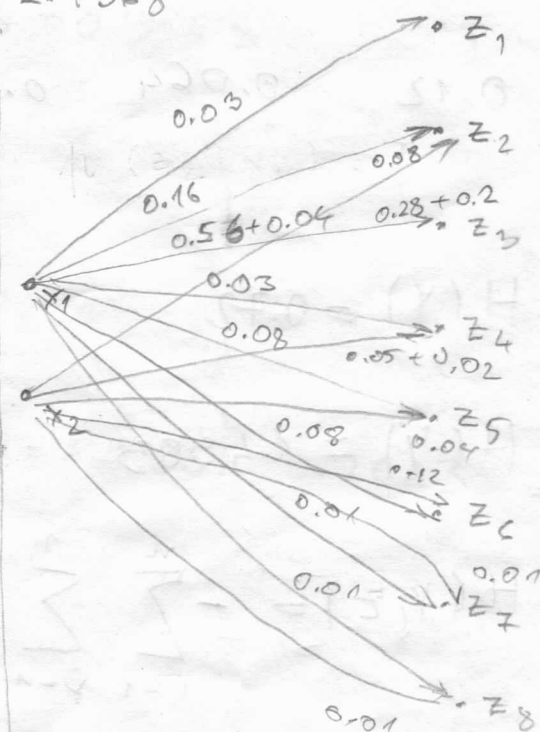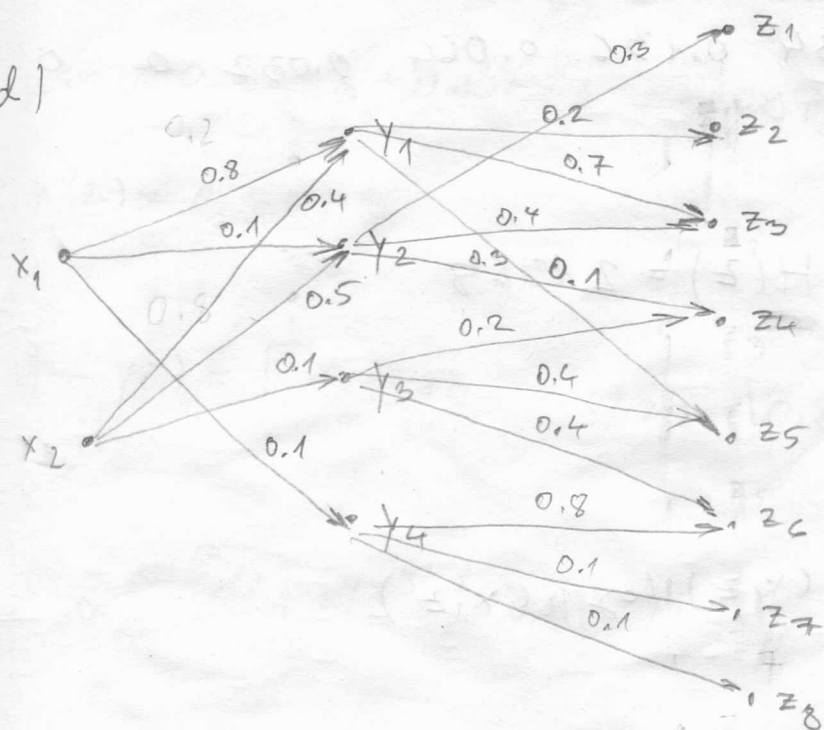
$p(x_i, z_k) = p(x_i) \cdot p(z_k \mid x_i) =$

$$\begin{bmatrix} 0.2 & 0.8 \end{bmatrix} \begin{bmatrix} 0.03 & 0.16 & 0.6 & 0.03 & 0.08 & 0.08 & 0.01 \\ 0.15 & 0.08 & 0.48 & 0.17 & 0.08 & 0.04 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0.006 & 0.032 & 0.12 & 0.006 & 0.016 & 0.016 & 0.002 & 0 \\ 0.12 & 0.064 & 0.384 & 0.136 & 0.064 & 0.032 & 0 & 0 \end{bmatrix}$$

$H(X) = 0.72$ $\qquad H(Z) = 2.1368$

$H(Y) = 1.4383$

$$H(X,Z) = -\sum_{i=1}^{n} \sum_{j=1}^{m} p(x,z) \log p(x,z)$$

$= 2.7966$

$$I(Y,Z) = H(X) + H(Z) - H(X,Z)$$

$$= 0.0602$$

2. $\quad X = \{4, 5, 6\}$

$$p(x_i, x_j) = \begin{array}{c} 4 \\ 5 \\ 6 \end{array} \begin{bmatrix} \overset{4}{0.1172} & \overset{5}{0.1172} & \overset{6}{0.1563} \\ 0.0713 & 0.2138 & 0.0713 \\ 0.2023 & 0.0253 & 0.0253 \end{bmatrix}$$

$$\begin{array}{ccc} 4-6 & \overset{4-5}{5-6} & \overset{5-4}{6-5} \\ & \overset{6-5}{} \end{array}$$

$$\begin{bmatrix} -2, & -1, & 0, & 1, & 2 \end{bmatrix} = Y$$

$$p(-2) = 0.1563$$

$$p(-1) = 0.1172 + 0.0713 = 0.1885$$

$$p(0) = 0.1172 + 0.2138 + 0.0253 = 0.3563$$

$$p(1) = 0.0713 + 0.0253 = 0.097$$

$$p(2) = 0.2023$$

$$p(y_i) = \begin{bmatrix} 0.1563 & 0.1885 & 0.3563 & 0.097 & 0.2023 \end{bmatrix}$$

$$H = -\sum p(y_i) \cdot \log_2 p(y_i)$$

$$= 2.1949$$

③ $p(x_1) : p(x_2) : p(x_3) : p(x_4) = 1:2:2:5$

$$p(x_i) = \begin{bmatrix} 0.1 & 0.2 & 0.2 & 0.5 \end{bmatrix}$$

$$p(y_j | x_i) = \begin{bmatrix} 0.1 & 0.2 & 0.2 & 0.5 \\ 0.1 & 0.2 & 0.5 & 0.2 \\ 0.1 & 0.5 & 0.2 & 0.2 \\ 0.5 & 0.1 & 0.2 & 0.2 \end{bmatrix}$$

$$p(y_j) = \begin{bmatrix} 0.1 & 0.2 & 0.2 & 0.5 \end{bmatrix} \begin{bmatrix} 0.1 & 0.2 & 0.2 & 0.5 \\ 0.1 & 0.2 & 0.5 & 0.2 \\ 0.1 & 0.5 & 0.2 & 0.2 \\ 0.5 & 0.1 & 0.2 & 0.2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.3 & 0.21 & 0.26 & 0.23 \end{bmatrix}$$

a) $H(x) = - \sum_i p(x_i) \cdot \log p(x_i) = 1.761$

$H(y) = - \sum_j p(y_j) \cdot \log p(y_j) = 1.9869$

b)

$$p(x_i, y_j) = p(x_i) \cdot p(y_j | x_i) = \begin{bmatrix} 0.1 (0.1 & 0.2 & 0.2 & 0.5) \\ 0.2 (0.1 & 0.2 & 0.5 & 0.2) \\ 0.2 (0.1 & 0.5 & 0.2 & 0.2) \\ 0.5 (0.5 & 0.1 & 0.2 & 0.2) \end{bmatrix}$$

$$= \begin{bmatrix} 0.01 & 0.02 & 0.02 & 0.05 \\ 0.02 & 0.04 & 0.1 & 0.04 \\ 0.02 & 0.1 & 0.04 & 0.04 \\ 0.25 & 0.05 & 0.1 & 0.1 \end{bmatrix}$$

$$H(X,Y) = -\sum_i \sum_j p(x_i, y_j) \cdot \log p(x_i, y_j)$$

$$= \underline{3.5219 \text{ bits}}$$

$$I(X,Y) = H(X) + H(Y) - H(X,Y) = 1.761 + 1.9869 - 3.5219$$

$$= 0.226$$

$$H(Y|X) = H(X,Y) - H(X) = 3.5219 - 1.761$$

$$= 1.761 \text{ bits}$$

4.

$$p(x_i, y_j) = \begin{bmatrix} \frac{1}{8} & \frac{1}{16} & \frac{1}{32} & \frac{1}{32} \\ \frac{1}{16} & \frac{1}{8} & \frac{1}{32} & \frac{1}{32} \\ \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} \\ \frac{1}{4} & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \frac{1}{4} \\ \frac{1}{4}\ \vec{p}(x_i) \\ \frac{1}{4} \\ \frac{1}{4} \end{matrix}$$

$$\begin{matrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \end{matrix}$$

$$\vec{p}(y_j)$$

$$p(x_i) = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

$$p(y_j) = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \end{bmatrix}$$

a) $H(X) = - \sum_i p(x_i) \cdot \log_2 p(x_i) = 2$ bit/ symbol

$H(Y) = - \sum_j p(y_j) \log_2 p(y_j) = 1.75$ bit / symbol

b) $H(X,Y) = - \sum_i \sum_j p(x_i, y_j) \cdot \log_2 p(x_i, y_j) = 3.375$

$I(X,Y) = H(X) + H(Y) - H(X,Y) = 0.375$

$H(Y|X) = - \sum_{i=1}^{n} \sum_{j=1}^{m} p(x_i, y_j) \cdot \log_2 p(y_j | x_i)$

$$H(Y|X) = H(X,Y) - H(X) = 1.375 \text{ bit / symbol}$$

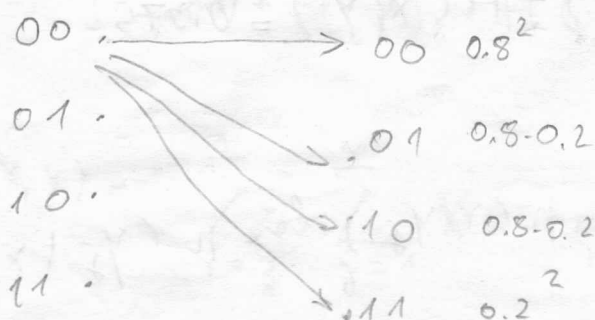c) $H(C) = H(Y) - H(Y|X)$

$$C = 1.75 - 1.375 = 0.375 \quad ??$$

5.

x_i	1	2	3	4	5	6	7	8	9
t_i	2500	500	1000	5000	4000	5000	3500	1500	2000
$p(x_i)$	0.0926	0.0185	0.037	0.1852	0.1481	0.1852	0.1296	0.0556	0.0741

$$\Sigma = 27\,000$$

a) $H(X) = 3.0798$ bit / symbol

b) $\dfrac{27000 \cdot 3.0798}{56\,000} = 1.4849 \text{ D}$

6.

00 . $\longrightarrow$ 00 $\quad 0.8^2$

01 . $\quad$ .01 $\quad 0.8 \cdot 0.2$

10 . $\quad$ .10 $\quad 0.8 \cdot 0.2$

11 . $\quad$ .11 $\quad 0.2^2$

a)

$$p(y_i \mid x_j) = \begin{array}{c} \\ 00 \\ 01 \\ 10 \\ 11 \end{array} \begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} \left[\begin{array}{cccc} 0.64 & 0.16 & 0.16 & 0.04 \\ 0.16 & 0.64 & 0.04 & 0.16 \\ 0.16 & 0.04 & 0.64 & 0.16 \\ 0.04 & 0.16 & 0.16 & 0.64 \end{array} \right]$$

$$p(y_i, y_j) = \frac{1}{4} \left[\begin{array}{cccc} 0.64 & 0.16 & 0.16 & 0.04 \\ 0.16 & 0.64 & 0.04 & 0.16 \\ 0.16 & 0.04 & 0.64 & 0.16 \\ 0.04 & 0.16 & 0.16 & 0.64 \end{array} \right] = \left[\begin{array}{cccc} 0.16 & 0.04 & 0.04 & 0.01 \\ 0.04 & 0.16 & 0.01 & 0.04 \\ 0.04 & 0.01 & 0.16 & 0.04 \\ 0.01 & 0.04 & 0.04 & 0.16 \end{array} \right]$$

$$H(X,Y) = H(Y) + H(X \mid Y)$$

$$H(X \mid Y) = H(X,Y) - H(Y) = \underline{1.4439 \text{ bit}}$$

$$p(y_j) = p(x_i)\, p(y_j \mid x_i) = \begin{array}{c} 1 \times 4 \end{array} \left[\begin{array}{cccc} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{array} \right] \begin{array}{c} 4 \times 4 \end{array} \left[\begin{array}{cccc} 0.64 & 0.16 & 0.16 & 0.04 \\ 0.16 & 0.64 & 0.04 & 0.16 \\ 0.16 & 0.04 & 0.64 & 0.16 \\ 0.04 & 0.16 & 0.16 & 0.64 \end{array} \right]$$

$$= \left[\begin{array}{cccc} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{array} \right]$$

$$H(X,Y) = - \sum_i \sum_j p(x_i, y_j) \cdot \log p(x_i, y_j) = 3.4439$$

$$H(Y) = 2$$

b) paritet.

1 kriv → 0.128

2 kriva → 0.032

$$p(y_j \mid x_i) = \begin{array}{c} \\ 000 \\ 011 \\ \\ 101 \\ 110 \end{array} \begin{array}{cccccccc} 000 & 001 & 010 & 011 & 100 & 101 & 110 & 111 \end{array} \left[\begin{array}{cccccccc} 0.512 & 0.128 & 0.128 & 0.032 & 0.128 & 0.032 & 0.032 & 0.008 \\ 0.032 & 0.128 & 0.128 & 0.512 & 0.008 & 0.032 & 0.032 & 0.128 \\ 0.032 & 0.128 & 0.008 & 0.032 & 0.128 & 0.512 & 0.032 & 0.128 \\ 0.032 & 0.008 & 0.128 & 0.032 & 0.128 & 0.032 & 0.512 & 0.128 \end{array} \right]$$

$$p(x_i, y_j) = \frac{1}{4} p(y_j \mid x_i)$$

$$4 \times 8$$

$$= \begin{bmatrix} 0.128 & 0.032 & 0.032 & 0.008 & 0.032 & 0.008 & 0.008 & 0.0 \\ 0.008 & 0.032 & 0.032 & 0.128 & 0.02 & 0.008 & 0.068 & 0.0 \\ 0.008 & 0.032 & 0.002 & 0.08 & 0.032 & 0.128 & 0.008 & 0.0 \\ 0.008 & 0.002 & 0.032 & 0.08 & 0.032 & 0.008 & 0.128 & 0.0 \end{bmatrix}$$

$$\overset{1 \times 4}{p(y_j)} = \overset{}{p(x_i)} \cdot \overset{4 \times 8}{p(y_j \mid x_i)} =$$

$$\begin{bmatrix} 0.152 & 0.098 & 0.098 & 0.152 & 0.098 & 0.152 & 0.152 & 0.0 \end{bmatrix}$$

$$H_2(Y) = 2.9661 \ \text{bit/o}$$

$$H_2(X, Y) = 4.1658$$

$$H_2(X \mid Y) = H_2(Y, X) - H_2(Y) = 1.1997 \ \text{bit/o}$$

razlika ekvivalencije

$$\Delta H(X \mid Y) = H_2(X \mid Y) - H_1(X \mid Y)$$

$$= 1.4439 - 1.1997$$

$$= 0.2442 \ \text{bit/simbol}$$

7.

$$p(y_j \mid x_i) = \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C = \max I(X;Y) = \max \{ H(Y) - H(Y|X) \}$$

$$H(Y|X) = 0 \quad \text{bit} / \text{simbol}$$

$$\underset{\text{nos } Y}{\downarrow}$$

$$C = \max H(Y) = \log_2 4 = 2 \quad \text{bit} / \text{simbol}$$