DZ1. $X = \{x_1, x_2\}$

$p(x_1) = 0.2$

$p(x_2) = 0.8$

$$[p(y_j|x_i)] = \begin{bmatrix} 0.8 & 0.1 & 0 & 0.1 \\ 0.4 & 0.5 & 0.1 & 0 \end{bmatrix}$$

$$[p(z_k|y_j)] = \begin{bmatrix} 0 & 0.2 & 0.7 & 0 & 0.1 & 0 & 0 & 0 \\ 0.3 & 0 & 0.4 & 0.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0.4 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.8 & 0.1 & 0.1 \end{bmatrix}$$

a) $H(x) = ?$

$$H(x) = -\sum_{0}^{2} p(x_i) \log p(x_i)$$

$$= -0.2 \cdot \log_2 0.2 - 0.8 \cdot \log_2 0.8$$

$$\boxed{H(x) = 0.722 \ [\text{bit/simbol}]}$$

b) $$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.8 \end{bmatrix} \cdot \begin{bmatrix} 0.8 & 0.1 & 0 & 0.1 \\ 0.4 & 0.5 & 0.1 & 0 \end{bmatrix} = \begin{bmatrix} 0.16 & 0.02 & 0 & 0.02 \\ 0.32 & 0.4 & 0.08 & 0 \end{bmatrix}$$

$[p(x)] \cdot [p(y_j|x_i)] = p(x_i, y_j)$

$p(y_1)$ $p(y_2)$ $p(y_3)$ $p(y_4)$

$$\boxed{\begin{array}{ll} p(y_1) = 0.48 & p(y_3) = 0.08 \\ p(y_2) = 0.42 & p(y_4) = 0.02 \end{array}}$$

$$H(Y) = -\sum_{1}^{4} p(y_i) \log_2 p(y_i)$$

$$= -0.48 \cdot \log_2 0.48 - 0.42 \cdot \log_2 0.42 - 0.08 \log_2 0.08 - 0.02 \cdot \log_2 0.02$$

$$\boxed{H(Y) = 1.438 \ [\text{bit/simbol}]}$$

c) $H(Z) = ?$

$p(y_1) = 0.48$

$p(y_2) = 0.42$

$p(y_3) = 0.08$

$p(y_4) = 0.02$

$[p(y_j)]\,[p(z_k \mid y_i)] = [p(z_k, y_j)]$ $\quad p(z/y)$

$$\begin{bmatrix} 0.48 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 0 \\ 0 & 0 & 0.08 & 0 \\ 0 & 0 & 0 & 0.02 \end{bmatrix} \begin{bmatrix} 0 & 0.2 & 0.7 & 0 & 0.1 & 0 & 0 & 0 \\ 0.3 & 0 & 0.4 & 0.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0.4 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.8 & 0.1 & 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0.096 & 0.336 & 0 & 0.048 & 0 & 0 & 0 \\ 0.126 & 0 & 0.168 & 0.126 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.016 & 0.032 & 0.032 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.016 & 0.002 & 0.002 \end{bmatrix}$$

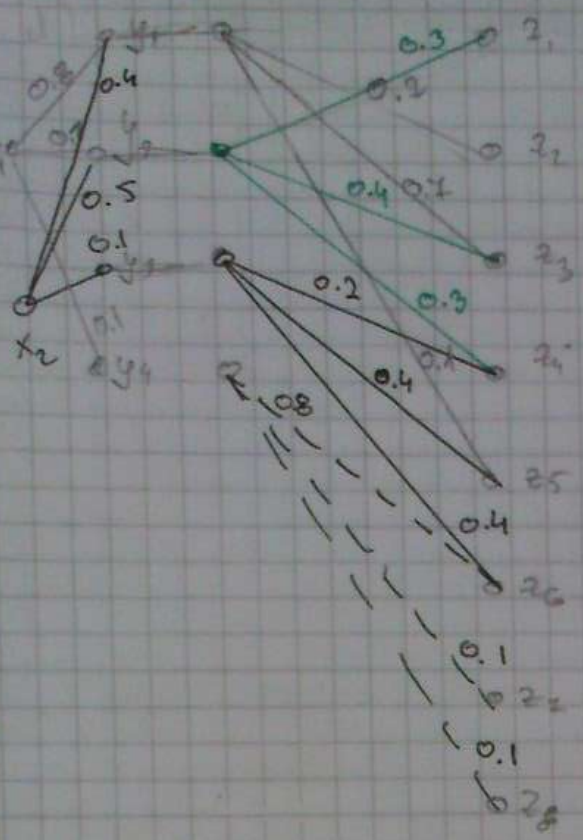
$p(z_1) = 0.126$ $\quad p(z_3) = 0.504$ $\quad p(z_5) = 0.08$ $\quad p(z_7) = 0.002$

$p(z_2) = 0.096$ $\quad p(z_4) = 0.142$ $\quad p(z_6) = 0.048$ $\quad p(z_8) = 0.002$

$$H(z) = -\sum_{1}^{8} p(z_k) \cdot \log p(z_k)$$

$$= -0.126 \cdot \log_2 0.126 - 0.096 \cdot \log_2 0.096 - 0.504 \log_2 0.504$$
$$- 0.142 \cdot \log_2 0.142 - 0.08 \cdot \log_2 0.08 - 0.048 \cdot \log_2 0.048$$
$$- 2 \cdot 0.002 \cdot \log_2 0.002$$

$$\boxed{H(z) = 2.137 \ \left[\text{bit} / \text{simbol}\right]}$$



e) transinformacija (x_2) $I(X;Z) = H(X) - H(X|Z)$

$= H(Z) + H(X) - H(X,Z) = 2.137 + 0.722 - 2.797 =$ **$I(X;Z) = 0.0624$ bit**

$p(z|x) = p(y|x) \cdot p(z|y)$

*(1)

$$p(z|x) = \begin{bmatrix} 0.03 & 0.16 & 0.6 & 0.03 & 0.08 & 0.08 & 0.01 & 0.01 \\ 0.15 & 0.08 & 0.48 & 0.14 & 0.08 & 0.04 & 0 & 0 \end{bmatrix}$$

$p(z,x) = p(x) \cdot p(z|x)$

$$= \begin{bmatrix} 0.006 & 0.032 & 0.12 & 0.006 & 0.016 & 0.016 & 0.002 & 0.002 \\ 0.12 & 0.064 & 0.384 & 0.136 & 0.064 & 0.032 & 0 & 0 \end{bmatrix}$$

$-0.043 \cdot 0.1589^{*2} - 0.3712 - 0.0443 - 0.0955 - 0.0955$

$-0.0179 \cdot 2 - 0.2538 \cdot 2 - 0.5302 - 0.3915$

$H(z,x) = 2.797$ bit / simbol