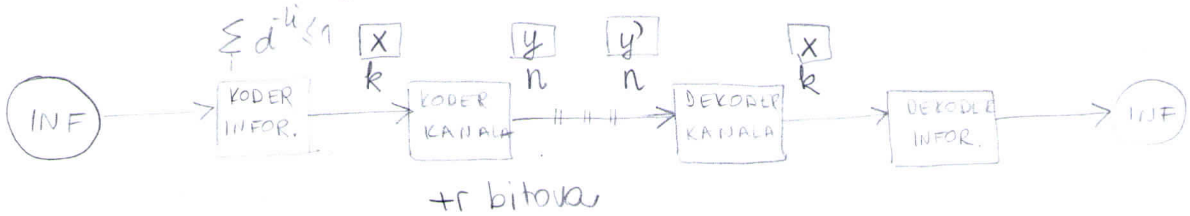
MASS

TINF - ZI

zaštitno kodiranje



$n = k + r$

blok kod — K

$M = 2^k$

n - broj bitova kodne riječi

k - broj informacijskih bitova

r - broj dodanih bitova (redundancija)

t - broj pogrešaka koje kod može ispraviti , $r \nearrow$ $t \nearrow$ kodna brzina $R \searrow$

s - broj pogrešaka koje kod može otkriti

d - min. Hamm. udaljenost

$K[n, M, d]$

$K[n, k, d]$

$K[n, k]$

Hamm. međa $\boxed{M \leq \dfrac{2^n}{\sum_{i=0}^{t} \binom{n}{i}}}$ uvjet postojanja blok koda

linearan blok kod K - sadrži kodnu riječ $\bar{0}$

$$K = \left\{ \begin{array}{l} 00\ldots\ldots \\ \\ \\ \end{array} \right\} 2^k$$

n

$t = \left\lfloor \frac{d-1}{2} \right\rfloor$

$s = d - 1$

$R(K) = \frac{k}{n}$

3.1.
$t=1$
$M=52$
$n=9$

$$52 \leqslant \frac{2^9}{\binom{9}{0}+\binom{9}{1}}$$

$\emptyset$ ne postoji

3.2.
$K[n,2]$
$k=2 \rightarrow M=2^k=4$
$d=5 \rightarrow t=\frac{d-1}{2}=2$
min n?

Krećemo od $n=d=5$ i provjeravamo nejednakost.

$$M \leqslant \frac{2^n}{\sum_{i}^{t}\binom{n}{i}}$$

3.4.

$$K=\begin{cases}00000\\10011\\11101\\01110\end{cases}$$

$M=4 \rightarrow k=\log_2 M=2$
$n=5$
$r=n-k=3$

i) $d=$ najmanja težina kodnih riječi $\rightarrow d=3 \rightarrow s=2$, $t=1$

ii) linearan? da jer sadrži kodnu riječ 0.

II uvjet: $G=k\times n$. generirajuća matrica

$$G=\underbrace{\begin{bmatrix}10011\\11101\end{bmatrix}}_{n=5}\Big\}k=2$$

a, b

linearnom kombinacijom vektora a i b je moguće dobiti sve kodne riječi

$0\cdot a \oplus 0\cdot b = 00000$
$0\cdot a \oplus 1\cdot b = 11101$
$1\cdot a \oplus 0\cdot b = 10011$
$1\cdot a \oplus 1\cdot b = 01110$

3.28 $M = 16$ S.F. — kod je blok kod

$k = 4$

$t = 4 \longrightarrow d = 2t + 1 = 9$

$R(K) = \frac{k}{n} = \frac{4}{n}$

max.n $n = \infty$ $R_1(K) = \frac{4}{\infty} = 0$

$r = \infty$

min. n. krećemo od $n = d = 9$ i provjeravamo nejednakost...

$n = 15$ ✓

$R_2(K) = \frac{4}{15}$

3.6 $K = \begin{cases} 00000 \\ 10010 \\ 10100 \\ 00110 \end{cases}$

$G = [I_k | A]$ standardni oblik generirajuće matrice

$n = 5$

$k = 2, M = 4$

$d = 2$ (može se vidjeti u G)

$G_1 = \begin{bmatrix} 10010 \\ 10100 \end{bmatrix}$ $G_2 = \begin{bmatrix} 10100 \\ 00110 \end{bmatrix}$ $G_3 = \begin{bmatrix} 10010 \\ 00110 \end{bmatrix}$

00110 ✓

— matrica G se ne može napisati u stand. obliku za kod K

ekvivalentni kodovi $K_1 \equiv K_2$

$\boxed{t_1 = t_2}$

gener. mat. ekvival. kodova:

1. zamjena stupaca
2. zbrajanje redova
3. zamjena redova

$K' \rightarrow G_2' = \begin{bmatrix} 10100 \\ 01100 \end{bmatrix}$

$K' = \begin{cases} 00000 \\ 10100 \\ 01100 \\ 11000 \end{cases}$

3.8. $H = (n-k) \times n$ matrica provjere pariteta

$r \times n$

$$H^T = \underbrace{\left[\quad \right]}_{} \Big\} n$$

Retci H^T su sindromi.

$$H = \begin{bmatrix} 1&1&0&1&0&1 \\ 1&1&0&0&1&0 \\ 1&0&1&1&0&0 \end{bmatrix} \xrightarrow{H'} \overset{2,4 \text{ i } 6}{\begin{bmatrix} 1&1&0&1&0&1 \\ 1&1&0&0&1&0 \\ 1&0&1&0&0&1 \end{bmatrix}} \xrightarrow{H''} \overset{2,3 \text{ i } 6}{\begin{bmatrix} 1&1&1&1&0&0 \\ 1&1&0&0&1&0 \\ 1&0&1&0&0&1 \end{bmatrix}}$$

$A^T \quad I_r$

$n = 6$

$r = 3$

$k = 3 \rightarrow M = 2^k = 8$

$H = [A^T | I_r]$

Vraćamo se od G'' do G zamijenom stupaca (ne vraćamo promijene nad retcima)

$$G'' = \left[\begin{array}{ccc|ccc} 1&0&0&1&1&1 \\ 0&1&0&1&1&0 \\ 0&0&1&1&0&1 \end{array}\right]$$

$$G' = \left[\begin{array}{ccc|ccc} 1&0&1&1&1&0 \\ 0&1&0&1&1&0 \\ 0&0&1&1&0&1 \end{array}\right]$$

$$G = \begin{bmatrix} 1&0&1&0&1&1 \\ 0&1&0&0&1&1 \\ 0&0&1&1&0&1 \end{bmatrix} \rightarrow d = 3$$

$$t = \frac{d-1}{2} = 1$$

$$\boxed{G \cdot H^T = 0}$$

y = kodna riječ koju šaljem

y' = primljena kodna riječ

x = informacija

$y \cdot H^T = 0$

$y' \cdot H^T = s(y')$ sindrom

$y' = 110110$

$S(y')$ dimenzije r

$y' \cdot H^T = [110110] \cdot \begin{bmatrix} 1 & 1 & 1 \\ \boxed{1\ 1\ 0} \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = [110]$ – tražimo sindrom u H^T

pogreška se dogodila na 2. bitu

$y = 100110$

---

dualni kod $K^\perp$: $\boxed{X_K \cdot X_{K^\perp}^T = 0}$

$K \to G, H$

$K^\perp \to H, G$

↳ generirajuća matrica koda $K^\perp$

3.12.

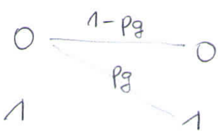
$K = \begin{cases} 0101 \\ 1010 \\ 1100 \end{cases}$

$K^\perp = \begin{cases} 0000 \\ 1111 \end{cases}$

kod nije linearan $M \neq 2^k$

! Svaka kodna riječ dualnog koda pomnožena sa svakom kodnom riječi zadanog koda daje nulu.

3.14.



$\begin{bmatrix} 1-p_g & p_g \\ p_g & 1-p_g \end{bmatrix}$ matrica prijelaza simetričnog kanala.

$K = \begin{cases} 00000 \\ 11010 \\ 01111 \\ 10101 \end{cases}$

$G = 2 \times 5$

$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$

$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

$H = 3 \times 5$

$M = 4$

$k = 2$

$n = 5$

$r = 3$

$t = 1$

$d = 3$

$s = 2$

Standardni niz

kod može ispraviti sve jednostruke pogreške

vektor pogreške $\bar{e}$				sindrom
00000	11010	01111	10101	000
00001				001
00010				010
00100	...	...	...	100
01000	...	...	...	111
10000	01010	11111	00101	101

$S(y') = y' \cdot H^T$

$$H^T = \begin{bmatrix} 101 \\ 111 \\ 100 \\ 010 \\ 001 \end{bmatrix} \begin{array}{l} \to \text{pogr. na 1. bitu} \\ \text{2. bitu} \\ \text{3. bitu} \\ \text{4. bitu} \\ \text{5. bitu} \end{array}$$

$$\boxed{p = \sum_{i=0}^{t} \binom{n}{i} p_g^i (1-p_g)^{i-t}}$$

$$p_{isp} = \boxed{p(0 \text{ pogrešaka})} + \boxed{p(1 \text{ pogreška})} + \boxed{p(2 \text{ pogreške sa sindromima koji nisu u } H^T)}$$

$$= \binom{5}{0} p_g^0 (1-p_g)^5 + \binom{5}{1} p_g^1 (1-p_g)^4 + 2 \cdot p_g^2 (1-p_g)^2$$

vj. ispravnog dekodiranja (ako koristimo sindromsko dekodiranje)

011
110 — ne nalaze se u H^T

— mogu ispraviti dvije dvostruke pogreške

$$\begin{array}{rl} 100 & \to \text{3. bit gr.} \\ +\,010 & \to \text{4.} \\ \hline 011 & \checkmark \end{array} \qquad \begin{array}{rl} 111 & \to \text{2. bit} \\ +\,001 & \to \text{5. bit} \\ \hline 110 & \checkmark \end{array}$$

$\bar{e} = 00110$ $\qquad$ $\bar{e} = 01001$

→ Rješenje ne mora biti jednoznačno

$$\begin{array}{rl} 100 & \to \text{3. bit} \\ 010 & \to \text{4. bit} \\ \hline 110 & \end{array}$$

3.21

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$H^T = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

+ 011
010

svi mogući sindromi

(000 — 00000)
100 — 10000
110 — 01000
111 — 00100
011 001 — 00010
101 — 00001

011 — 10100 ili 01001
010 — 00101 ili 00101

$y' = 11010$

$y' \cdot H^T = 011$

$y = 01100$ ili 10011 (rj. nije jednoznačno)

3.24 Hammingov koder → $d=3$
Ham(r)

$t=1$
$s=2$
zaštitni bitovi se nalaze na 2^k pozicijama

r_1 r_2 k_1 r_3 k_2 k_3

Ham [7,4]

$r=3$
$M=16$

1010101... → $\underset{1.}{\_}\ \underset{2.}{\_}\ 1\ \underset{4.}{\_}\ 0\ 1\ 0$

k bitova ulazi u koder!

$\xrightarrow{k}$ [KOD.] $\xrightarrow{n}$

$H = r \times n$
$\quad 3 \times 7$

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

1 0 1 1 0 1 0

$P_g = 0{,}004$

paritetno kodiranje $\quad n = k+1 \quad , \quad \begin{matrix} t=0 \\ S=1 \end{matrix} \quad \rightarrow \quad n = 4+1 = 5\,!$

parno

neparno

Vj. ispravnog dekodiranja ?

$P_{Ham} = \binom{7}{0} P_g^0 (1-P_g)^7 + \binom{7}{1} P_g^1 (1-P_g)^6$ $\quad$ $n=7$ za Ham

$\rightarrow$ isto bi bilo i za sindromsko dekodiranje jer nemo neiskorištenih sindroma u H^T.

$P_{par} = \binom{5}{0} P_g^0 (1-P_g)^5$ $\quad$ $n=5$! za par

$\Delta p = ?$

3.27 $\quad M = 128 \rightarrow k = 7$

$\underbrace{1111011000100\ldots}_{y'}$ na ulazu dekodera $\quad \xrightarrow{n} \boxed{\text{DEKOD.}} \xrightarrow{k}$

$$\boxed{\begin{matrix} 2^r - 1 = \\ 2^r - 1 - r = n \end{matrix}}$$

$>$ _ _ k_1 _ k_2 k_3 k_4 _ k_5 k_6 k_7

$n = 11$

$r = 4$

$y' \cdot H^T = s(y') \rightarrow$ tražimo ga u H^T $\rightarrow$ ispravimo grešku $\rightarrow$ nađemo poslanu riječ $k_1 k_2 \ldots k_7$

$$H = \begin{array}{c} \begin{matrix} 1. & 2. & 3. & \ldots & 11. \end{matrix} \\ \begin{bmatrix} 1 & 0 & 1 & \ldots & 1 \\ 0 & 1 & 1 & \ldots & 1 \\ 0 & 0 & 0 & & 0 \\ 0 & 0 & 0 & & 1 \end{bmatrix} \end{array}$$

3.29. $\quad M = 128 \rightarrow k = 7$

$r = 4$

$n = 11$

poruka = 11 bita

$\frac{C}{r} = 3624 \frac{\text{poruka}}{s}$

$B = 4$ kHz

$\left(\frac{S}{N}\right)_{dB} = 30$ dB

$10 \log(x) = 30$

$x = 1000$

uvijek aps. vrijednost

$$\boxed{C = B \cdot \log_2\left(1 + \frac{S}{N}\right)}$$

$C = \ldots = 39860 \frac{bit}{s}$ = max. brzina u kanalu

CIKLIČNI KOD.

$$G = \begin{bmatrix} \vec{1}010 \\ 0101 \end{bmatrix} \qquad G = \begin{bmatrix} g_r & g_{r-1} & g_{r-2} & \cdots & g_0 & 0 & \cdots & 0 \\ \vdots & & & & & & & \vdots \\ 0 & \cdots & 0 & g_r & g_{r-1} & \cdots & \cdots & g_0 \end{bmatrix}$$

$$H = \begin{bmatrix} h_0 & h_1 & \cdots & h_k & 0 & \cdots & 0 \\ 0 & \cdots & 0 & h_0 & h_1 & \cdots & h_k \end{bmatrix}$$

$c = [d|r]$

$\underset{3.\,2.\,1.\,0.}{1010} \to x^3 + x$

$d(x)$ informacija
$r(x)$ zaštitni bitovi
$c(x)$ kodna riječ
$g(x)$ generirajući polinom
$h(x)$

$$(x^n + 1) = g(x) \cdot h(x)$$

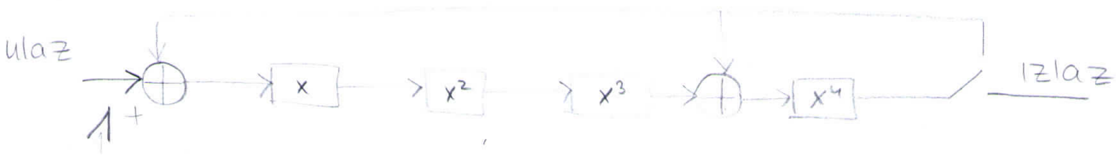
$$r(x) = d(x) \cdot x^r \bmod [g(x)] = CRC$$

$$s(x) = c(x) \cdot x^r \bmod [g(x)]$$

$$g(x) = x^r + \dots + 1$$

$$h(x) = x^k + \dots + 1$$

3.30



$g(x) = x^4 + x^3 + 1$

$r = 4$

$K[15, k]$

$k = n - r = 11$

na ulazu kodera: $\underbrace{10101010101}_{k=11},01\dots$

$$d(x) = x^{10} + x^8 + x^6 + x^4 + x^2 + 1$$

$$r(x) = d(x) \cdot x^r \bmod [g(x)]$$

dijeljenje polinoma...

Ostatak = $r(x)$ $= x^3+x^2+1$

$r=4$ 1 1 0 1

$c(x) = d(x)\cdot x^r + r(x)$

$c(x) = \underbrace{x^{14}+x^{12}+x^{10}+x^{8}+x^{6}+x^{4}}+x^3+x^2+1$

3.31. $g(x) = x^3+x^2+1$

$r=3$

$K[7,k]$

$k=7-3=4$

na ulazu dekodera: $\underbrace{1001110}_{n=7}0\ldots$

$c(x) = x^6+x^3+x^2+x$

$s(x) = c(x)\cdot x^r \bmod [g(x)]$

3.32. $g(x) = x^8+x^7+x^6+x^4+1$; $K[\ ,\ ]$

$\boxed{(x^n+1):g(x)}$ ako nema ostatka, $g(x)$ je generirajući polinom koda K