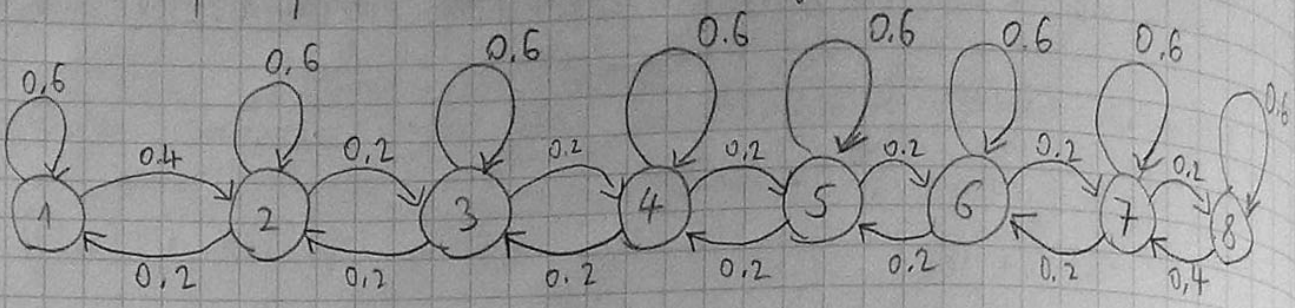
2. DZ

8. Markovljev proces 1. reda s 8 stanja

a)



b) stacionarne vjerojatnosti pojave stoja u nekom položaju

$$[p(x_j|x_i)] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 0.6 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0.6 & 0.2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0.6 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0.6 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0.6 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.2 & 0.6 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.2 & 0.6 & 0.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.4 & 0.6 \end{bmatrix} \end{array}$$

-> matrica vjetnih vjerojatnosti prijelaza (izvora)

$$[p_1\ p_2 \dots p_8] = [p_1\ p_2 \dots p_8] \cdot [p(x_j|x_i)]$$

$p_1 = 0.6p_1 + 0.2p_2$ $\Rightarrow$ $p_2 = 2p_1$

$p_2 = 0.4p_1 + 0.6p_2 + 0.2p_3$ $p_3 = 2p_1$

$p_3 = 0,2p_2 + 0.6p_3 + 0.2p_4$ $p_4 = 2p_1$

$p_4 = 0.2p_3 + 0.6p_4 + 0,2p_5$ $p_5 = 2p_1$

$p_5 = 0,2p_4 + 0.6p_5 + 0,2p_6$ $p_6 = 2p_1$

$p_6 = 0.2p_5 + 0.6p_6 + 0.2p_7$ $p_7 = 2p_1$

$p_7 = 0.2p_6 + 0.6p_7 + 0.4p_8$ $p_8 = p_1$

$p_8 = 0.2p_7 + 0.6p_8$

$\sum_{i=1}^{8} p_i = 1$

$\Downarrow$

$14p_1 = 1$

$p_1 = \frac{1}{14}$

$p_1 = 1/14$

$p_2, \dots, p_7 = \frac{2}{14} = \frac{1}{7}$

$p_8 = 1/14$

c) $$H'(X) = -\sum_{i=1}^{n}\sum_{j=1}^{m} p(x_i,x_j)\log_2 p(x_j|x_i) \quad [bit/simbol]$$

$$[p(x_i,x_j)] = [p(x_i)\cdot p(x_j|x_i)] \qquad p(x_i) = \left[\frac{1}{14}\ \frac{1}{7}\ \frac{1}{7}\ \frac{1}{7}\ \frac{1}{7}\ \frac{1}{7}\ \frac{1}{7}\ \frac{1}{14}\right]$$

$$[p(x_i,x_j)] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 0.0428 & 0.0285 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0285 & 0.0857 & 0.0285 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0285 & 0.0857 & 0.0285 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0285 & 0.0857 & 0.0285 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0285 & 0.0857 & 0.0285 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0285 & 0.0857 & 0.0285 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0285 & 0.0857 & 0.0285 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0285 & 0.0428 \end{bmatrix} \end{array}$$

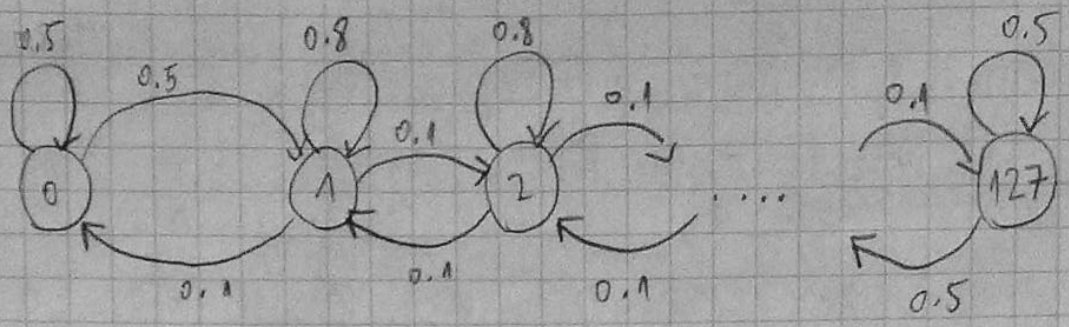
$$H(x) = -\sum_{i=1}^{8} p(x_i)\log_2 p(x_i) \quad [bit/simbol]$$

$$H(x) = \underline{2.9502} \ [bit/simbol]$$

$$H'(x) = 1,3141 \ [bit/simbol]$$

9. piksel → 128 vrijednosti

a)



b)

$$[p(x_j|x_i)] = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ \vdots \\ 126 \\ 127 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & \cdots & 126 \quad 127 \end{array} \\ \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & & 0 \quad 0 \\ 0.1 & 0.8 & 0.1 & 0 & & 0 \quad 0 \\ 0 & 0.1 & 0.8 & 0.1 & & 0 \quad 0 \\ 0 & 0 & 0.1 & 0.8 & & 0 \quad 0 \\ & & & & & \\ 0 & 0 & 0 & 0 & & 0.8 \quad 0.1 \\ 0 & 0 & 0 & 0 & & 0.5 \quad 0.5 \end{bmatrix} \end{array}$$

$$[P_0 \; P_1 \; \dots \; P_{127}] = [P_0 \; P_1 \; \dots \; P_{127}] \cdot [p(x_j|x_i)]$$

$P_0 = 0.5P_0 + 0.1P_1$ $\Rightarrow 0.5P_0 = 0.1P_1 \rightarrow P_1 = 5 \cdot P_0$

$P_1 = 0.5P_0 + 0.8P_1 + 0.1P_2$ $P_0 = P_{127}$

$P_2 = 0.1P_1 + 0.8P_2 + 0.1P_3$ $P_1 = P_2 = \dots = P_{126}$

$P_{126} = 0.1P_{125} + 0.8P_{126} + 0.5P_{127}$

$P_{127} = 0.1 \cdot P_{126} + 0.5 \cdot P_{127}$

$P_1 + P_2 + \dots + P_{127} = 1$

$2P_0 + 126 \cdot 5P_0 = 1$

$P_0 = P_{127} = \frac{1}{632} = 0.001582$

$P_1 = P_2 = \dots P_{126} = \frac{5}{632} = 0.00791$

c) $H(X) = ?$ $H'(X) = ?$

$$H(X) = -\sum_{i=1}^{127} p(x_i) \log_2 p(x_i) = 6.989 \text{ bit/simb.}$$

$$H'(X) = -\sum_i \sum_j p(x_i, x_j) \log_2 p(x_j|x_i)$$

$$[p(x_i x_j)] = \begin{bmatrix} \frac{0.5}{632} & \frac{0.5}{632} & \cdots & 0 & 0 \\ \frac{0.5}{632} & \frac{4}{632} & \frac{0.5}{632} & \cdots & 0 & 0 \\ \vdots & & & & & \\ & & & & \frac{4}{632} & \frac{0.5}{632} \\ & & & & \frac{0.5}{632} & \frac{0.5}{632} \end{bmatrix} \begin{matrix} \\ \\ \\ 126 \\ 127 \end{matrix}$$

$H'(X) = 0.9221$ bit/simb.

2) 640 × 480

$v = 25$ slika/s

minimalni kapacitet kanala = ?

NE ZNAM JE LI TOČAN OVAJ ZADATAK

Ukupno piksela = $640 \times 480 \times 25 = 7{,}68 \cdot 10^6$ piksela

$C = 7{,}68 \cdot 10^6 \cdot 0.9221 = 7.0817$ Mbit/simb.

10.

a) $L = \{5, 3, 4, 2, 1, 4\}$

$\sum d^{-l_i} \leq 1$ da bi kod bio prefiksni

$\sum d^{-l_i} = 2^{-5} + 2^{-3} + 2^{-4} + 2^{-2} + 2^{-1} + 2^{-4} = 1.03125 > 1$

Kod ne postoji /

b) $L = \{2, 4, 2, 3, 4, 2\}$

$\sum d^{-l_i} = 2^{-2} + 2^{-4} + 2^{-2} + 2^{-3} + 2^{-4} + 2^{-2} = 1 \leq 1$

Kod postoji /

L =	2	0 0	p_1
	2	0 1	p_2
	2	1 0	p_3
	3	1 1 0	p_4
	4	1 1 1 1	p_5
	4	1 1 1 0	p_6

c) $X = \{x_1, \dots, x_6\}$ $p_i(x_i) = ?$ $\Rightarrow L = H$

$L = H(X)$

$H(X) = ?$

$L = \sum_{i=1}^{6} p(x_i) \cdot l_i = p_1 \cdot l_1 + p_2 \cdot l_2 + p_3 \cdot l_3 + p_4 \cdot l_4 + p_5 \cdot l_5 + p_6 \cdot l_6$

$= p_1 \cdot 2 + p_2 \cdot 2 + p_3 \cdot 2 + p_4 \cdot 3 + p_5 \cdot 4 + p_6 \cdot 4$

$p_1 + p_2 + p_3 + p_4 + p_5 + p_6 = 1$

$H(X) = -\sum_{i=1}^{6} p(x_i) \cdot \log_2 p(x_i)$

$H(X) = -(p_1 \cdot \log_2 p_1 + p_2 \cdot \log_2 p_2 + p_3 \cdot \log_2 p_3 + p_4 \cdot \log_2 p_4 + p_5 \cdot \log_2 p_5 + p_6 (\log_2 p_6)$

$2 = -\log_2 p_1$

$2 = \log_2 \frac{1}{p_1}$

$\log_2 4 = \log_2 \frac{1}{p_1}$

$\therefore 4 = \frac{1}{p_1}$

$p_1 = \frac{1}{4}$ ✓

$2 = -\log_2 p_2$

$p_2 = \frac{1}{4}$ ✓

$p_3 = \frac{1}{4}$ ✓

$3 = -\log_2 p_4$

$p_4 = \frac{1}{8}$

$p_5 = \frac{1}{16}$

$p_6 = \frac{1}{16}$

$$[p(x_i)] = \left[\frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{16} \quad \frac{1}{16} \right]$$

$$H(x) = -\sum_{i=1}^{6} p(x_i) \log_2 p(x_i)$$

$$= 2.375 \quad \text{bit/simbol}$$