2. DZ TINF

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1. PERFEKTAN KÔD

Binarni $(n, M, 2t+1)$ kôd koji zadovoljava izraz:

$$M = \frac{2^n}{\binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{t}}$$

Perfektan kôd je onaj kôd udaljenosti $2t+1$ za kojeg vrijedi da su sve moguće kodne riječi u jednoj od kugli radijusa t

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

G ima 4 riječi

K ima 2^4 riječi

$0 \cdot [11110100] + 0[01101011] + 1[00100111] + 1[11100001] =$
$= 00100111 + 11100001 = 11000110$

$0 \cdot [11110100] + 1[01101011] + 0[00100111] + 1[11100001] =$
$= 01101011 + 11100001 = 10001010$

$1[11110100] + 0[01101011] + 0[00100111] + 1[11100001] =$
$= 11110100 + 11100001 = 00010101$

$0[11110100] + 1[01101011] + 1[00100111] + 0[11100001]$
$= 01101011 + 00100111 = 01001100$

$1[11110100] + 0[01101011] + 1[00100111] + 0[11100001] = 11110100 + 00100111$
$= 11010011$

$$K = \left\{ \begin{array}{l} 00000000 \\ 11100001 \\ 00100111 \\ 11000110 \\ 01101011 \\ 10001010 \\ 01001100 \\ 10101101 \\ 11110100 \\ 00010101 \\ 11010011 \\ 00110010 \\ 10011111 \\ 01111110 \\ 10111000 \\ 01011001 \end{array} \right.$$

$$\text{kôd} \begin{array}{c} n \quad M \quad d \\ (8, 16, 3) \end{array}$$

B) $d(x,y) = 3$ između $\begin{matrix} 11000110 \\ 10001010 \end{matrix}$

Kod može otkriti $d-1 = 2$ greške, a ispraviti može $\left\lfloor \frac{d-1}{2} \right\rfloor = 1$.

C) Da li je kod perfektan? $M = 16$

kôd $(8, 16, 3)$

$3 = 2t+1$
$2 = 2t$
$t = 1$

$$M = \frac{2^n}{\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\dots+\binom{n}{t}}$$

$$M = \frac{2^8}{\binom{8}{0}+\binom{8}{1}}$$

$$M = \frac{2^8}{1+8} = 28.44$$

$28.44 \neq 16$ kod nije perfektan!

D)

$$G = \begin{bmatrix} 1&1&1&1&0&1&0&0 \\ 0&1&1&0&1&0&1&1 \\ 0&0&1&0&0&1&1&1 \\ 1&1&1&0&0&0&0&1 \end{bmatrix} = \left(\begin{bmatrix} 1&0&1&1&1&1&0&0 \\ 0&1&1&0&1&0&1&1 \\ 0&0&1&0&0&1&1&1 \\ 0&0&1&1&1&0&0&1 \end{bmatrix}\right)^{*}$$

$$= \begin{bmatrix} 1&0&0&1&1&0&1&1 \\ 0&1&1&0&1&0&1&1 \\ 0&0&1&0&0&1&1&1 \\ 0&0&1&1&1&0&0&1 \end{bmatrix} = \left(\begin{bmatrix} 1&0&0&1&1&0&1&1 \\ 0&1&0&0&1&1&1&1 \\ 0&0&1&0&0&1&1&1 \\ 0&0&0&1&1&1&0&1 \end{bmatrix}\right)^{*} = \begin{bmatrix} 1&0&0&0&0&1&1&0 \\ 0&1&0&0&1&1&1&1 \\ 0&0&1&0&0&1&1&1 \\ 0&0&0&1&1&1&0&1 \end{bmatrix} = G'''$$

$$H^{*} = [A^T \mid I_4] = \left[\begin{array}{cccc|cccc} 0&1&0&1&1&0&0&0 \\ 1&1&1&1&0&1&0&0 \\ 1&1&1&0&0&0&1&0 \\ 0&1&1&1&0&0&0&1 \end{array}\right] = \begin{bmatrix} 0&1&0&1&1&0&0&0 \\ 1&1&1&1&0&1&0&0 \\ 1&1&0&0&0&1&1&0 \\ 0&1&0&1&0&1&0&1 \end{bmatrix}$$

$$H = \begin{bmatrix} 1&1&0&0&1&0&0&0 \\ 1&0&1&1&1&1&0&0 \\ 0&0&0&1&1&1&1&0 \\ 1&0&0&0&1&1&0&1 \end{bmatrix}$$

2.) Kreiranje Ham(2)

$$H^T = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Valjanost matrice H:

$$G \cdot H^T = 0$$

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$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} //$$

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2) Koristimo Ham(2)

A) H matrica dimenzija $2\times(2^2-1) = 2\times 3$, 2 retka i 3 stupca

$$H = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$H = \left[\begin{array}{c|cc} 1 & 1 & 0 \\ 1 & 0 & 1 \end{array}\right]$$

$$A^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

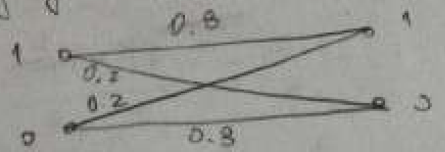
$$G = [I_1 \mid A]$$

$$G = [1 \mid 1\ 1]$$

$$K = \begin{cases} 0\,0\,0 \\ 1\,1\,1 \end{cases} \qquad (3, 2, 3)$$

može ispraviti $\left\lfloor \frac{d-1}{2} \right\rfloor = 1$ grešku

B) Vjerojatnost ispravnog dekodiranja



$$p(K) = \binom{3}{0}0.2^0\cdot 0.8^3 + \binom{3}{1}0.2^1\cdot 0.8^2 = 0.896$$

C) Posebni oblik retransmisije

Točno ćemo dekodirati ako: odmah na izlazu dobijemo ono što smo poslali ili imamo tri greške (retransmisija se neće dogoditi) ili ako se dogodi jedna greška, a retransmisijom dobijemo dobro, ili ako se dogode 2 greške, a dobijemo OK retransm.

$$p(K) = 0.8^3 + 3\cdot 0.2\cdot 0.8^2\left(0.8^3 + 3\cdot 0.2\cdot 0.8^2\right) +$$
$$+\ 3\cdot 0.2^2\cdot 0.8\left(0.8^3 + 3\cdot 0.2\cdot 0.8^2\right) = 0.94208$$

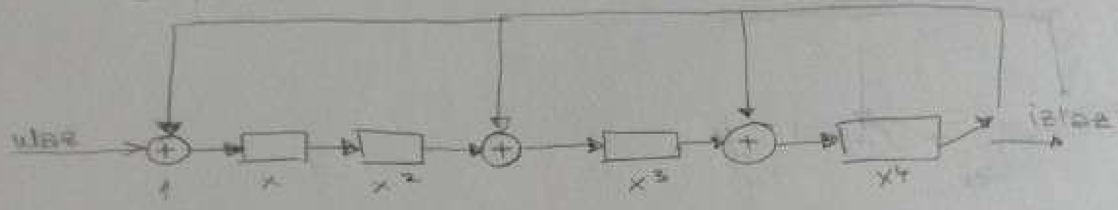
Vjerojatnost raste

3) Ciklički kod [10,k]

$r=4$

$k = 10-4 = 6$

[10,6]



A) $g(x) = 1 + x^2 + x^3 + x^4$

[11101]

B) Kodirati poruku [100101]

$$r(x) = \text{ost}\ \frac{x^4 \cdot d(x)}{g(x)}$$

$$x^4(1+x^2+x^5) : x^4+x^3+x^2+1 =$$

$$\cancel{x^9}+x^6+x^4 : x^4+x^3+x^2+1 = x^5+x^4+x+1$$

$$\cancel{x^9}+x^8+x^7+x^5$$

$$\cancel{x^8}+\cancel{x^7}+\cancel{x^6}+x^5+x^4$$

$$\cancel{x^8}+\cancel{x^7}+\cancel{x^6}+x^4$$

$$x^5$$

$$x^5+\cancel{x^4}+x^3+x$$

$$\cancel{x^4}+\cancel{x^3}+x$$

$$\cancel{x^4}+\cancel{x^3}+x^2+1$$

$$x^2+x+1$$

$x^2+x+1 \rightarrow [0111]$

$c = [1001010111]$