Zadaci za vježbu

1. Napon izvora izmjeren je 5 puta u istim uvjetima, digitalnim voltmetrom s prikazom 5 1/2 znamenke i granicama pogrješaka $\pm(5 \cdot 10^{-4}$ of reading $+ 4 \cdot 10^{-6}$ of range), na mjernom opsegu 1V. Aritmetička sredina svih rezultata bila je 0,2145 V, a standardno odstupanje (pojedine vrijednosti) 0,7 mV. Kolika je složena standardna nesigurnost u_c tako izmjerenog napona?

$n = 5$

5 1/2 znamenaka

m.o. = 1 V

$\bar{V} = 0{,}2145\,V$

$s = 0{,}7\,mV$

$u_c = ?$

$$a = \pm(5 \cdot 10^{-4} \cdot 0{,}2145\,V + 4 \cdot 10^{-6} \cdot 1\,V) = \pm 507{,}25 \cdot 10^{-6}\,V$$

$$u_1(U) = s_{\bar{x}}(U) = \frac{s}{\sqrt{n}} = \frac{0{,}7\,mV}{\sqrt{5}} = 313{,}05 \cdot 10^{-6}\,V$$

$$u_2(U) = \frac{a}{\sqrt{3}} = \frac{507{,}25 \cdot 10^{-6}\,V}{\sqrt{3}} = 292{,}86 \cdot 10^{-6}\,V$$

$$u_c(U) = \sqrt{u_1(U)^2 + u_2(U)^2} = \sqrt{(313{,}05 \cdot 10^{-6}\,V)^2 + (292{,}86 \cdot 10^{-6}\,V)^2}$$

$$= 428{,}68 \cdot 10^{-6}\,V = \underline{\underline{0{,}43\,mV}}$$

2. Otpor trošila određen je mjerenjem istosmjerne struje koja njime prolazi i pada napona na njemu. Kolika je relativna proširena nesigurnost tako izmjerenog otpora $U_{pr}(R)$ na razini pouzdanosti 95% ako su relativna mjerna nesigurnost i pripadni efektivni stupanj slobode izmjerenog napona i struje redom 0,34% i $\nu_{effU}=6$ te 0,33% i $\nu_{effI}=11$? Vrijednosti pripadnog obuhvatnog faktora $t_p(\nu_{eff})$ nalaze se u tablici

ν_{eff}	2	5	10	15	20	25	30	35	40	45	50
t_p	4,30	2,57	2,23	2,13	2,09	2,06	2,04	2,03	2,02	2,01	2,01

$p = 95\%$

$u_{cr}(U) = 0,34\%$

$\nu_{effU} = 6$

$u_{cr}(I) = 0,33\%$

$\nu_{effI} = 11$

$U_{pr}(R) = ?$

$$R = \frac{U}{I}$$

$$u_c(R) = \sqrt{\left(\frac{\partial R}{\partial U}\right)^2 \cdot u_c^2(U) + \left(\frac{\partial R}{\partial I}\right)^2 \cdot u_c^2(I)} = \sqrt{\frac{1}{I^2}\cdot u_c^2(U) + \left(-\frac{U}{I^2}\right)^2 u_c^2(I)}$$

$$u_{cr}(R) = \frac{100 \cdot u_c(R)}{R}$$

$$u_{cr}(R) = \frac{100}{R} \cdot \sqrt{\frac{1}{I^2} u_c^2(U) + \frac{U^2}{I^4} u_c^2(I)} =$$

$$= 100 \cdot \sqrt{\frac{I^2}{U^2} \cdot \frac{1}{I^2} \cdot u_c^2(U) + \frac{I^2}{U^2} \cdot \frac{U^2}{I^4} \cdot u_c^2(I)}$$

$$= 100 \cdot \sqrt{\frac{1}{U^2} \cdot u_c^2(U) + \frac{1}{I^2} \cdot u_c^2(I)}$$

$$= \sqrt{\left(100 \cdot \frac{u_c(U)}{U}\right)^2 + \left(100 \cdot \frac{u_c(I)}{I}\right)^2} = \sqrt{u_{cr}^2(U) + u_{cr}^2(I)}$$

$$u_{cr}(R) = \sqrt{(0,34\%)^2 + (0,33\%)^2} = 0,4738\%$$

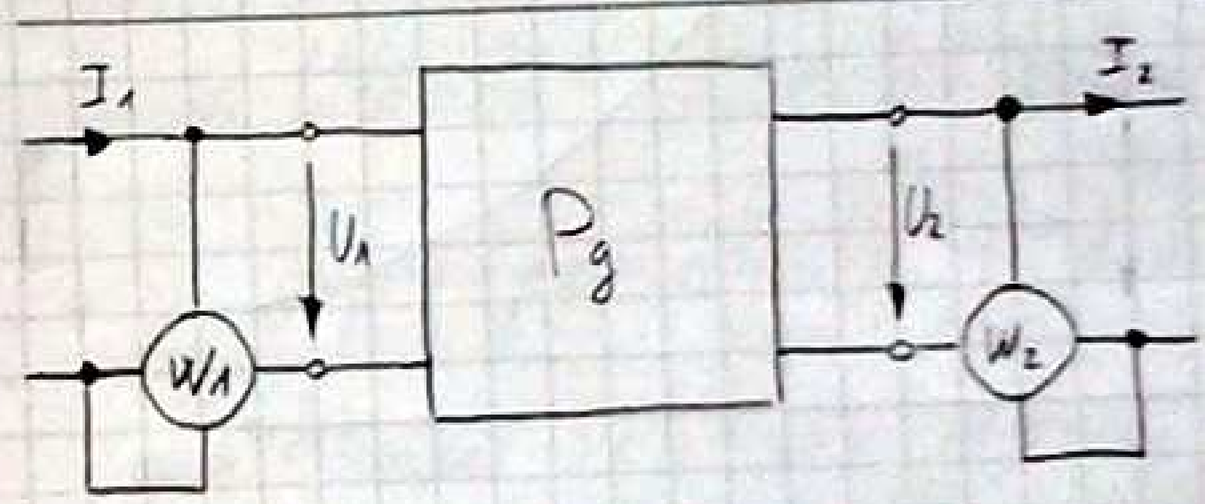
$$\nu_{effR} = \frac{u_{cr}^4(R)}{\frac{u_{cr}^4(U)}{\nu_{effU}} + \frac{u_{cr}^4(I)}{\nu_{effI}}} = \frac{(0,4738\%)^4}{\frac{(0,34\%)^4}{6} + \frac{(0,33\%)^4}{11}}$$

$$= 15,25 \approx 15$$

$t_p(v_{eff\,R}) = 2{,}13$

$U_{pr}(R) = t_p(v_{eff\,R}) \cdot U_{cR}(c) = 2{,}13 \cdot 0{,}4739\,\% = 1{,}0092\,\% \approx \underline{\underline{1{,}01\,\%}}$

3. Snagu gubitaka na četveropolu mjerimo kao razliku snaga na ulazu i izlazu. Ako je snaga od 50 W izmjerena vatmetrom W_1 na ulazu u četveropol uz relativnu nesigurnost 0,9 %, a snaga na izlazu 49 W izmjerena vatmetrom W_2 uz relativnu nesigurnost 1,2 %, kolika je mjerna nesigurnost snage gubitaka?



$$P_g = P_{UL} - P_{IZ}$$

$$P_{UL} = 50\,W \qquad u_{cR}(P_{UL}) = 0{,}9\,\%$$

$$P_{IZ} = 49\,W \qquad u_{cR}(P_{IZ}) = 1{,}2\,\%$$

$$u_c(P_g) = \sqrt{\left(\frac{\partial P_g}{\partial P_{UL}}\right)^2 u_c^2(P_{UL}) + \left(\frac{\partial P_g}{\partial P_{IZ}}\right)^2 \cdot u_c^2(P_{IZ})} =$$

$$= \sqrt{1^2 \cdot u_c^2(P_{UL}) + (-1)^2 \cdot u_c^2(P_{IZ})}$$

$$u_c(P_{UL}) = \frac{u_{cR}(P_{UL})}{100} \cdot P_{UL} = 0{,}45\,W$$

$$u_c(P_{IZ}) = \frac{u_{cR}(P_{IZ})}{100} \cdot P_{IZ} = 0{,}588\,W$$

$$u_c(P_g) = \sqrt{u_c^2(P_{UL}) + u_c^2(P_{IZ})} = \sqrt{(0{,}45\,W)^2 + (0{,}588\,W)^2} = \underline{\underline{0{,}74\,W}}$$

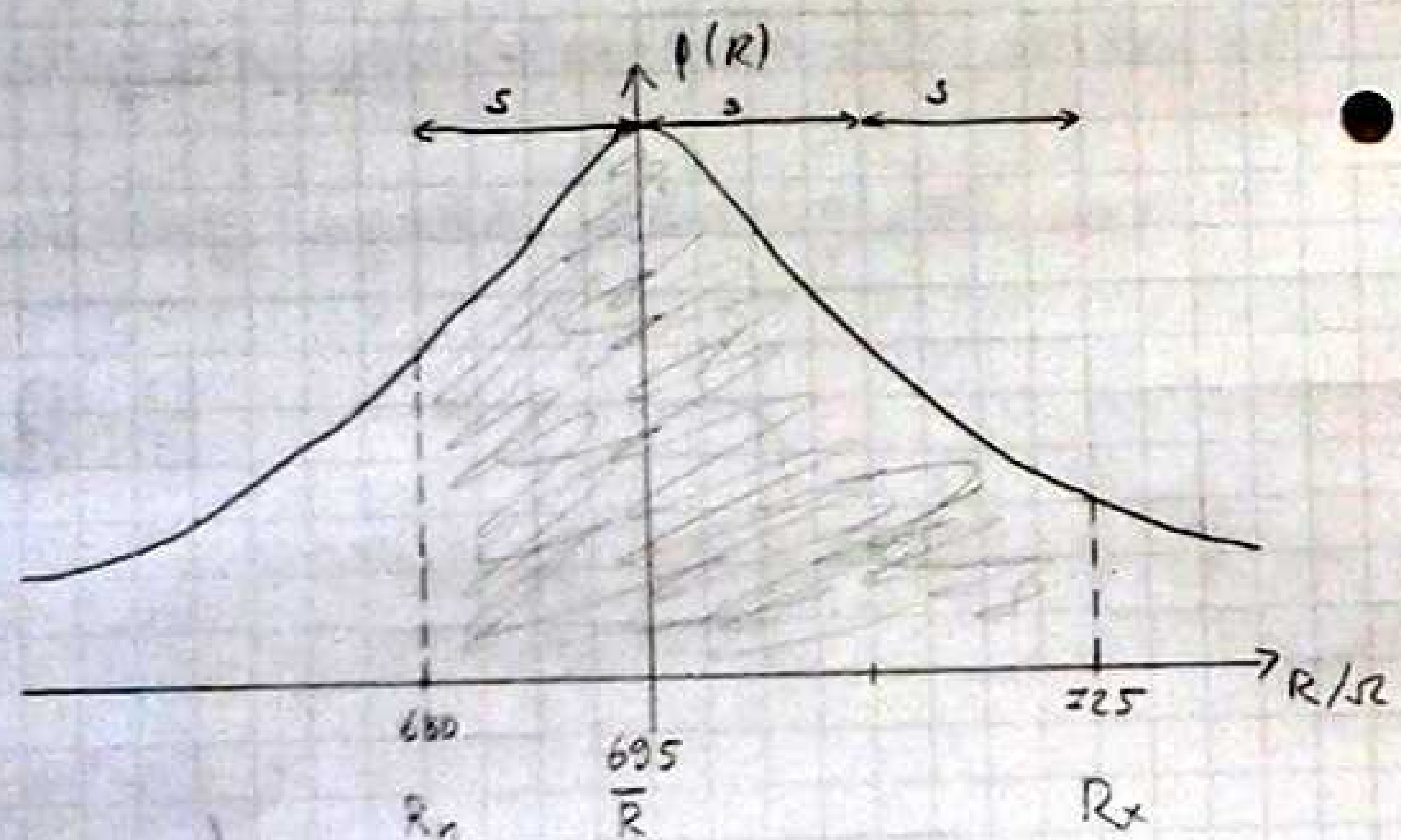
4. Izmjerili smo nekoliko desetaka otpornika nazivne vrijednosti 680 Ω te dobili aritmetičku sredinu 695 Ω i standardno odstupanje 15 Ω. Uzme li se nasumce jedan otpornik, vjerojatnost da je njegova vrijednost veća od nazivne vrijednosti, a manja od 725 Ω iznosi:

$R_n = 680\ \Omega$

$\overline{R} = 695\ \Omega$

$s = 15\ \Omega$

$R_x = 725\ \Omega$



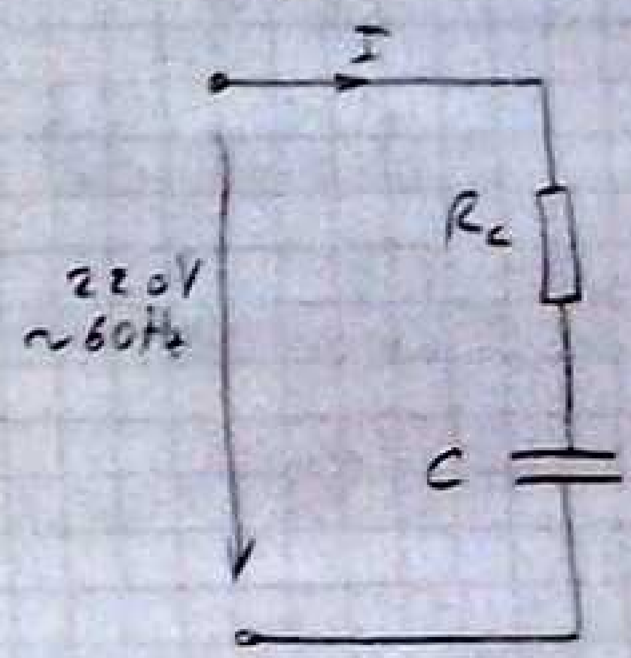
$p(R_n < R < R_x) = ?$

$$p(R_n < R < R_x) = p(-s < R < s) + \frac{1}{2}\left[P(-2s < R < 2s) - P(-s < R < s)\right] =$$

$$= 0{,}6827 + \frac{1}{2}\left[0{,}9545 - 0{,}6827\right] =$$

$$= 0{,}8186 = \underline{\underline{81{,}86\ \%}}$$

5. Koliki su gubitci kondenzatora kapaciteta $C = 10\,nF$ s $tg\,\delta = 0{,}012$ pri naponu frekvencije 60 Hz i efektivne vrijednost 220 V?



matematički model: $X_c = \frac{1}{\omega C} = \frac{1}{2\pi f \cdot C}$

$$X_c = \frac{1}{2 \cdot \pi \cdot 60\,Hz \cdot 10 \cdot 10^{-9}\,F} = 265\,258{,}2385\,\Omega$$

$$tg\,\delta = \frac{X_c}{R} \Rightarrow R = \frac{X_c}{tg\,\delta} = 22{,}10485321 \cdot 10^{6}\,\Omega$$

$$P_g = \frac{U^2}{R} = \frac{(220\,V)^2}{22{,}10485321 \cdot 10^{6}\,\Omega} = \underline{\underline{2{,}19\,mW}}$$

6. Otpornost žice od bakra kružnog presjeka određuje se mjerenjem otpora i dimenzija na pripremljenom uzorku. Ako su relativne mjerne nesigurnosti određivanja otpora, promjera i duljine žice redom 1,1%; 1,7%; 1,9%, kolika je relativna složena nesigurnost tako određene otpornosti materijala?

Matematički model:

$$R = \rho \cdot \frac{l}{S} = \rho \cdot \frac{l}{\frac{D^2 \pi}{4}} = \frac{\rho\, l \cdot 4}{D^2 \pi}$$

$$\rho = \frac{D^2 \cdot R}{l} \cdot \frac{\pi}{4}$$

ZADANO:

$u_{cr}(R) = 1{,}1\,\%$

$u_{cr}(D) = 1{,}7\,\%$

$u_{cr}(l) = 1{,}9\,\%$

$$u_c(\rho) = \sqrt{\left(\frac{\partial \rho}{\partial R}\right)^2 \cdot u_c^2(R) + \left(\frac{\partial \rho}{\partial D}\right)^2 \cdot u_c^2(D) + \left(\frac{\partial \rho}{\partial \ell}\right)^2 \cdot u_c^2(\ell)} =$$

$$= \sqrt{\left(\frac{D^2}{\ell}\cdot\frac{\pi}{4}\right)^2 u_c^2(R) + \left(\frac{2DR}{\ell}\cdot\frac{\pi}{4}\right)^2 u_c^2(D) + \left(-\frac{D^2 R}{\ell^2}\cdot\frac{\pi}{4}\right)^2 u_c^2(\ell)}$$

$$u_{cr}(\rho) = \frac{u_c(\rho)}{\rho} \cdot 100\,\%$$

$$u_{cr}(\rho) = \frac{1}{\frac{D^2 \cdot R \cdot \pi}{\ell \cdot 4}} \cdot \frac{D\pi}{4\ell} \cdot \sqrt{(D)^2 \cdot u_c^2(R) + (2R)^2 \cdot u_c(D) + \left(-\frac{DR}{\ell}\right)^2 \cdot u_c^2(\ell)} \cdot 100\%$$

$$= \frac{\ell \cdot 4}{D^2 \cdot R \cdot \pi} \cdot \frac{D\pi}{4\ell} \cdot \sqrt{D^2 u_c^2(R) + 4R^2 u_c^2(D) + \frac{D^2 R^2}{\ell^2} \cdot u_c^2(\ell)} \cdot 100\%$$

$$= \sqrt{\frac{D^2}{D^2 \cdot R^2} u_c^2(R) + \frac{4R^2}{D^2 R^2} \cdot u_c^2(D) + \frac{D^2 \cdot R^2}{\ell^2 \cdot D^2 \cdot R^2} \cdot u_c^2(\ell)} \cdot 100\%$$

$$= \sqrt{\left(\frac{u_c(R)}{R} \cdot 100\%\right)^2 + 4 \cdot \left(\frac{u_c(D)}{D} \cdot 100\%\right)^2 + \left(\frac{u_c(\ell)}{\ell} \cdot 100\%\right)^2}$$

$$= \sqrt{u_{cr}^2(R) + 4 \cdot u_{cr}^2(D) + u_{cr}^2(\ell)} = \sqrt{(1{,}1\%)^2 + 4 \cdot (1{,}7\%)^2 + (1{,}9\%)^2} =$$

$$= 4{,}047\% = \underline{\underline{4{,}05\%}}$$

7. Pri umjeravanju ampermetra na mjernom opsegu 1 A dobivene pogrješke pri njegovom pokazivanju prikazane su tablično. Kolika je mjerena struja ako on pokazuje 0,3 A?

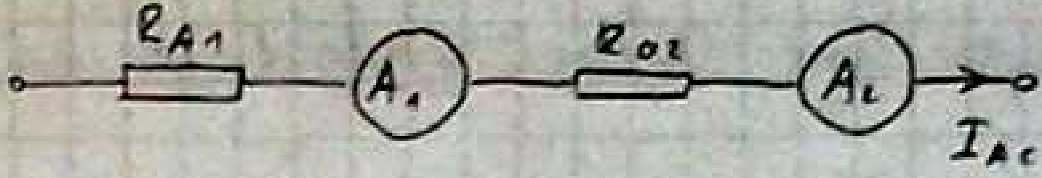
I/A	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
p/mA	3	2	-1	-2	4	1	3	2	2	-3

stvarna vrijednost = izmjerena vrijednost - pogrješka

$$I = I_m - p = 0{,}3\,A - (-1\,mA) = 0{,}3\,A + 1\,mA = \underline{\underline{0{,}301\,A}}$$

8. Dva ampermetra unutrašnjih otpora redom $R_{A1} = 0{,}01\,\Omega$, $R_{A2} = 1\,\Omega$, spojeni su serijski u strujni krug kojim prolazi izmjenična struja od 2A. Pokazivanje ampermetara,

uz zanemarivanje njihovih pogrješaka, je sljedeće:



$R_{A1} = 0{,}01\,\Omega$ $\qquad \underline{\underline{I_{A1} = I_{A2} = I_{Ac} = 2\,A}}$

$R_{A2} = 1\,\Omega$

$I_{Ac} = 2\,A$

9. Otpor jednog otpornika izmjeren je trima metodama: U-I metodom, digitalnim omometrom i usporedbom s poznatim otporom. Pritom su dobivene sljedeće aritmetičke sredine i pripadna standardna odstupanja sredine: 1,998 Ω (6 mΩ); 1,999 Ω (3 mΩ); 2,002 Ω (13 mΩ). Koja je najvjerojatnija vrijednost tog otpornika

$\overline{R}_1 = 1{,}998\,\Omega \qquad s_1 = 6\cdot 10^{-3}\,\Omega \qquad p_1 = \frac{1}{s_1^2} = \frac{1}{(6\cdot 10^{-3}\,\Omega)^2} = 27\,777{,}778\,\frac{1}{\Omega^2}$

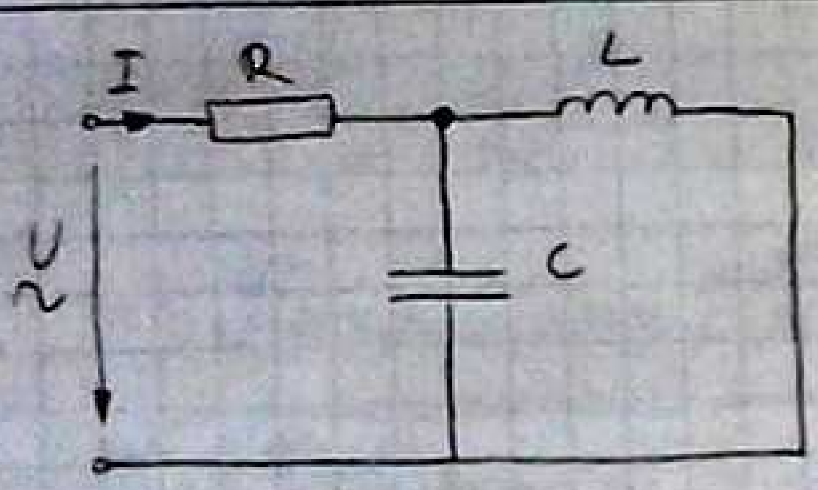
$\overline{R}_2 = 1{,}999\,\Omega \qquad s_2 = 3\cdot 10^{-3}\,\Omega \qquad p_2 = \frac{1}{s_2^2} = \frac{1}{(3\cdot 10^{-3}\,\Omega)^2} = 111\,111{,}111\,\frac{1}{\Omega^2}$

$\overline{R}_3 = 2{,}002\,\Omega \qquad s_3 = 13\cdot 10^{-3}\,\Omega \qquad p_3 = \frac{1}{s_3^2} = \frac{1}{(13\cdot 10^{-3}\,\Omega)^2} = 5917{,}16\,\frac{1}{\Omega^2}$

$$\overline{R} = \frac{\sum_{i=1}^{3} p_i \cdot \overline{R}_i}{\sum_{i=1}^{3} p_i} = \frac{27\,777{,}778\,\frac{1}{\Omega^2}\cdot 1{,}998\,\Omega + 111\,111{,}111\,\frac{1}{\Omega^2}\cdot 1{,}999\,\Omega + 5917{,}16\,\frac{1}{\Omega^2}\cdot 2{,}002\,\Omega}{27\,777{,}778\,\frac{1}{\Omega^2} + 111\,111{,}111\,\frac{1}{\Omega^2} + 5917{,}16\,\frac{1}{\Omega^2}} =$$

$$= 1{,}9989\,\Omega \approx \underline{\underline{1{,}999\,\Omega}}$$

10. Koliki je približno fazni pomak između napona na mjernom žičanom otporniku i struje koja njime prolazi pri 2,5 kHz, ako mu je otpor 100 Ω, vlastiti kapacitet 56 pF te vlastiti induktivitet 100 μH?



$$Z = R + \frac{\frac{1}{j\omega C}\cdot j\omega L}{\frac{1}{j\omega C} + j\omega L} = R + \frac{\frac{L}{C}}{\frac{1+j^2\cdot\omega^2 \cdot LC}{j\omega C}} =$$

$$= R + \frac{j\omega L}{1-\omega^2 LC} = \frac{R(1-\omega^2 LC) + j\omega L}{1-\omega^2 LC}$$

$$= R\cdot\frac{1-\omega^2 LC}{1-\omega^2 LC} + j\frac{\omega L}{1-\omega^2 LC} = \underline{R + j\frac{\omega L}{1-\omega^2 LC}}$$

$$\delta = \operatorname{atan}\left(\frac{Im(Z)}{Re(Z)}\right) = \operatorname{atan}\left(\frac{\frac{\omega L}{1-\omega^2 LC}}{\frac{R}{1}}\right) = \operatorname{atan}\left(\frac{\omega L}{R(1-\omega^2 LC}\right) =$$

$$= \operatorname{atan}\left(\frac{2\cdot\pi\cdot 2{,}5\cdot 10^3\,Hz\cdot 100\cdot 10^{-6}\,H}{100\,\Omega\cdot(1-(2\pi\cdot 2{,}5\cdot 10^3\,Hz)^2\cdot 100\cdot 10^{-6}\,H\cdot 56\cdot 10^{-12}\,F}\right) =$$

$$= 0{,}0157\,rad \approx \underline{\underline{0{,}016\,rad}}$$

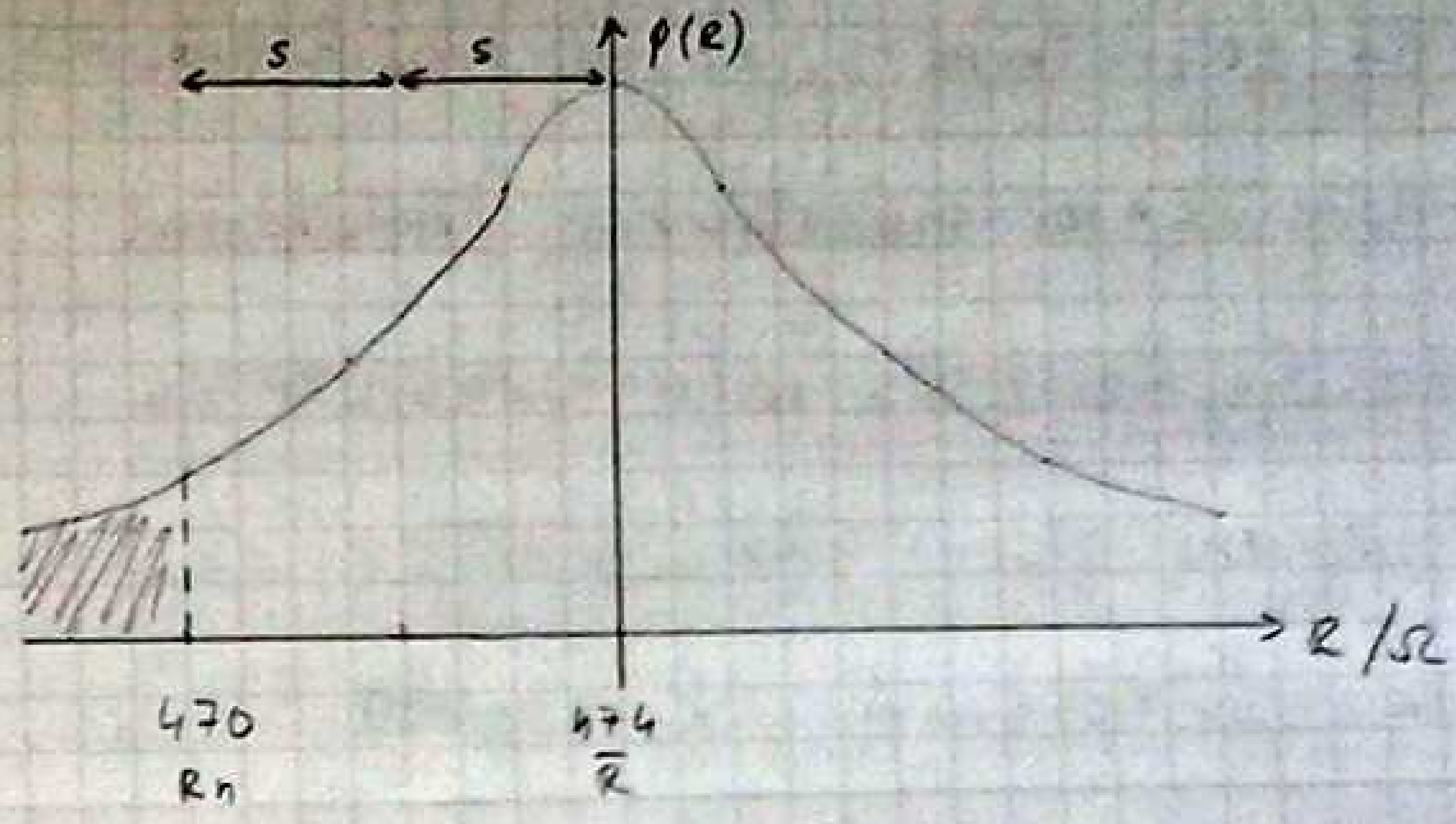
11. Izmjerili smo nekoliko desetaka otpornika nazivne vrijednosti 470 Ω te dobili aritmetičku sredinu 474 Ω i standardno odstupanje 2 Ω. Uzme li se nasumce jedan otpornik, vjerojatnost da je njegova vrijednost manja od nazivne vrijednosti iznosi:

$R_n = 470\,\Omega$

$\overline{R} = 474\,\Omega$

$s = 2\,\Omega$

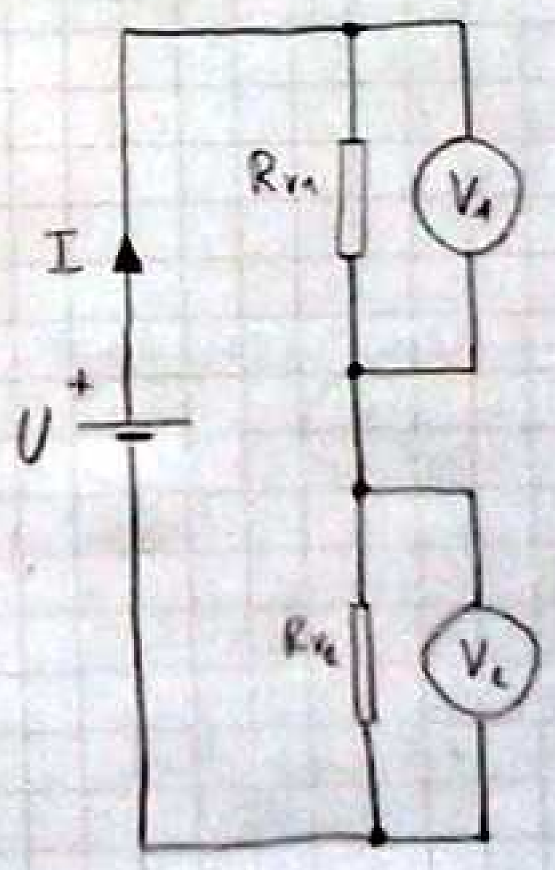
$p(R<R_n) = ?$



$$p(R<R_n) = p(R<-2s) = \frac{1}{2} - \frac{p(-2s<R<2s)}{2} =$$

$$= 0{,}5 - \frac{0{,}9545}{2} = 0{,}02275 = \underline{\underline{2{,}275\,\%}}$$

12. Dva voltmetra unutrašnjih otpora redom $R_{v1} = 25\,k\Omega$ i $R_{v2} = 50\,k\Omega$, spojeni su serijski izvoru napona od 5V. Pokazivanje voltmetra, uz zanemarenje njihovih pogrješaka, je sljedeće:



$$I = \frac{U}{R_{v1}+R_{v2}}$$

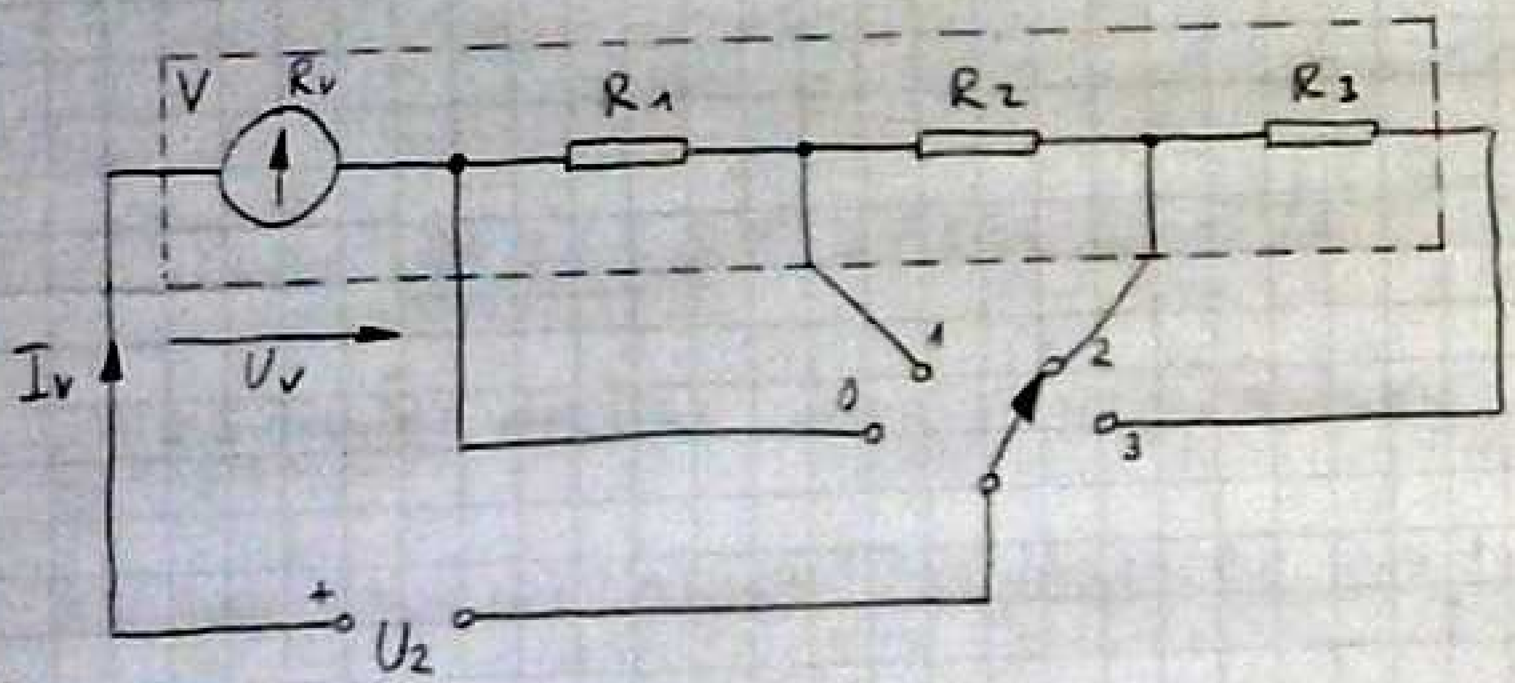
$$U_{v1} = I \cdot R_{v1} = U \cdot \frac{R_{v1}}{R_{v1}+R_{v2}} = 5V \cdot \frac{25\cdot 10^3\,\Omega}{25\cdot 10^3\,\Omega + 50\cdot 10^3\,\Omega} =$$

$$= \underline{\underline{1{,}67\,V}}$$

$$U_{v2} = I \cdot R_{v2} = U \cdot \frac{R_{v2}}{R_{v1}+R_{v2}} = 5V \cdot \frac{50\cdot 10^3\,\Omega}{25\cdot 10^3\,\Omega + 50\cdot 10^3\,\Omega} =$$

$$= \underline{\underline{3{,}33\,V}}$$

13. Instrumentu sa zakretnim svitkom proširujemo mjerni opseg dodavanjem višestrukog predotpora. Koje otpornike R_1, R_2 i R_3 valja odabrati da bi se dobili mjerni opsezi 1V, 2V i 5V, ako instrument ima $I_v = 0,5\,mA$ i $R_v = 500\,\Omega$?



I)

$$U_2' = I_v \cdot (R_v + R_1) = I_v \cdot R_v + I_v \cdot R_1$$

$$R_1 = \frac{U_2 - I_v \cdot R_v}{I_v} = \frac{U_2}{I_v} - R_v = \frac{1V}{0,5 \cdot 10^{-3} A} - 500\,\Omega = 1500\,\Omega = \underline{\underline{1,5\,k\Omega}}$$

II)

$$U_2'' = I_v \cdot (R_v + R_1 + R_2) = I_v \cdot R_v + I_v \cdot R_1 + I_v \cdot R_2$$

$$R_2 = \frac{U_2'' - I_v \cdot R_v - I_v \cdot R_1}{I_v} = \frac{U_2''}{I_v} - R_v - R_1 = \frac{2V}{0,5 \cdot 10^{-3} A} - 500\,\Omega - 1500\,\Omega =$$

$$= 2000\,\Omega = \underline{\underline{2\,k\Omega}}$$

III)

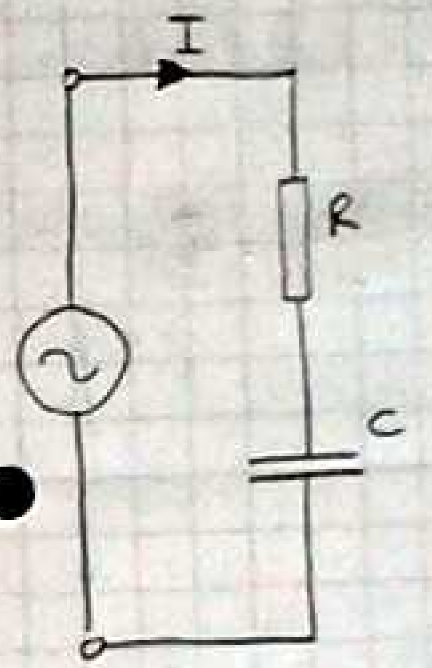
$$U_2''' = I_v (R_v + R_1 + R_2 + R_3) = I_v \cdot R_v + I_v \cdot R_1 + I_v \cdot R_2 + I_v \cdot R_3$$

$$R_3 = \frac{U_2''' - I_v \cdot R_v - I_v \cdot R_1 - I_v \cdot R_2}{I_v} = \frac{U_2'''}{I_v} - R_v - R_1 - R_2 = \frac{5V}{0,5 \cdot 10^{-3} A} - 500\,\Omega - 1500\,\Omega -$$

$$- 2000\,\Omega = 6000\,\Omega = \underline{\underline{6\,k\Omega}}$$

14. Koliki su gubitci kondenzatora kapaciteta $C = 1000\,nF$ sa $tg(\delta) = 0{,}008$ pri naponu $1\,kV$ frekvencije $50\,Hz$?

$$Z = R + \frac{1}{j\omega C} \qquad tg(\delta) = \left|\frac{Im(Z)}{Re(Z)}\right| = \frac{\frac{1}{\omega C}}{R} = \frac{1}{\omega C R}$$



$$R = \frac{1}{\omega C\, tg(\delta)} = \frac{1}{2\pi \cdot f \cdot C \cdot tg(\delta)} = \frac{1}{2\pi \cdot 50Hz \cdot 1000 \cdot 10^{-9} F \cdot 0{,}008} =$$

$$= 397\,887{,}358\,\Omega$$

$$P_g = \frac{U^2}{R} = \frac{(1 \cdot 10^3 V)^2}{397\,887{,}358\,\Omega} = 2{,}513\,W \approx \underline{2{,}5\,W}$$

15. Na izvor napona $u = [0{,}1 + 20\sin(\omega t)]\,V$ priključeni su paralelno univerzalni instrument za mjerenje izmjeničnog napona s odzivom na srednju vrijednost te instrument s odzivom na efektivnu vrijednost. Kolika je apsolutna razlika njihovih pokazivanja?

$$U_{ef_1} = \sqrt{0{,}1^2 + \frac{20^2}{2}} = 14{,}142\,V \qquad \xi = 1{,}11 = \text{TJEMENI FAKTOR}$$

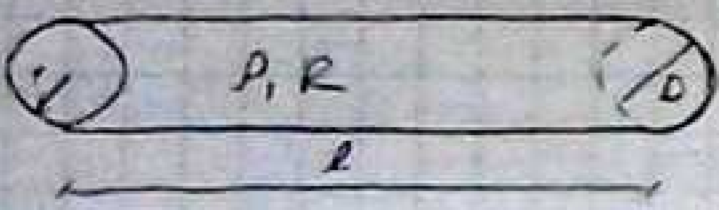
$$U_{sr} = \frac{2 \cdot 20V}{\pi} + 0{,}1 = 12{,}832\,V$$

$$U_{ef_2} = 1{,}11 \cdot U_{sr} = 14{,}244\,V$$

$$\Delta U = U_{ef_2} - U_{ef_1} = \underline{0{,}102\,V}$$

Zadatci za vježbu

1. Otpornost žice od bakra kružnog presjeka određuje se mjerenjem otpora i dimenzija na pripremljenom uzorku. Ako su relativne mjerne nesigurnosti određivanja otpora, promjera i duljine žice redom 0,2 %, 0,4 % i 0,8 %, kolika je relativna složena nesigurnost tako određene otpornosti materijala?



Matematički model:

$$R = \rho\cdot\frac{l}{S} = \rho\cdot\frac{l}{\frac{D^2\pi}{4}} = \frac{4\rho l}{D^2\pi}$$

ZADANO:

$$\rho = \frac{D^2\pi R}{4l}$$

$$\mu_{cR}(R) = 0{,}2\,\% = \frac{\mu_c(R)}{R}\cdot 100\,\%$$

$$\mu_{cR}(D) = 0{,}4\,\% = \frac{\mu_c(D)}{D}\cdot 100\,\%$$

$$\mu_{cR}(l) = 0{,}8\,\% = \frac{\mu_c(l)}{l}\cdot 100\,\%$$

$$\mu_c(\rho) = \sqrt{\left(\frac{\partial\rho}{\partial R}\right)^2\cdot\mu_c^2(R) + \left(\frac{\partial\rho}{\partial D}\right)^2\cdot\mu_c^2(D) + \left(\frac{\partial\rho}{\partial l}\right)^2\cdot\mu_c^2(l)} =$$

$$= \sqrt{\left(\frac{D^2\pi}{4l}\right)^2\cdot\mu_c^2(R) + \left(\frac{2D\pi R}{4l}\right)^2\cdot\mu_c^2(D) + \left(-\frac{D^2\pi R}{4l^2}\right)^2\cdot\mu_c^2(l)}$$

$$\mu_{cR}(\rho) = \frac{\mu_c(\rho)}{\rho}\cdot 100\,\%$$

$$\mu_{cR}(\rho) = \frac{1}{\frac{D^2\pi R}{4l}}\cdot 100\,\%\cdot\frac{D\pi}{4l}\sqrt{(D)^2\cdot\mu_c^2(R) + (2R)^2\cdot\mu_c^2(D) + \left(-\frac{D\cdot R}{l}\right)^2\cdot\mu_c^2(l)} =$$

$$= \sqrt{\frac{D^2}{D^2R^2}\cdot\mu_c^2(R)\cdot(100\,\%)^2 + \frac{(2R)^2}{D^2R^2}\cdot\mu_c^2(D)\,(100\,\%)^2 + \frac{D^2R^2}{l^2}\cdot\frac{1}{D^2R^2}\cdot\mu_c(l)\,(100\,\%)^2}$$

$$= \sqrt{\left(\frac{\mu_c(R)}{R}\cdot 100\,\%\right)^2 + 4\left(\frac{\mu_c(D)}{D}\cdot 100\,\%\right)^2 + \left(\frac{\mu_c(l)}{l}\cdot 100\,\%\right)^2}$$

$$u_{cR}(\rho) = \sqrt{u_{cR}^2(R) + 4u_{cR}^2(D) + u_{cR}^2(l)} = \sqrt{(0{,}2\%)^2 + 4\cdot(0{,}4\%)^2 + (0{,}8\%)^2} =$$

$$= 1{,}149\,\% \approx \underline{\underline{1{,}15\,\%}}$$

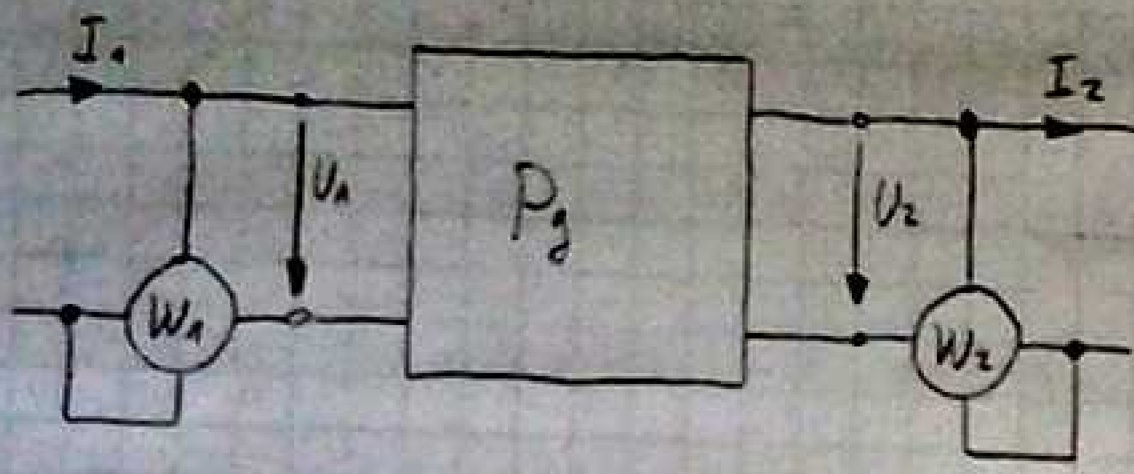
2. Pri umjeravanju ampermetra na mjernom opsegu 1 A dobivene pogrješke pri njegovom pokazivanju prikazane su tablično. Kolika je mjerena struja ako on pokazuje 0,9 A?

I/A	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
p/mA	3	2	-1	-2	4	1	3	2	-2	-3

$$p = U_{iz} - U_{pr}$$

$$U_{pr} = U_{iz} - p = 0{,}9\,A - (-2\cdot10^{-3}\,A) = \underline{\underline{0{,}902\,A}}$$

3. Snagu gubitaka na četveropolu mjerimo kao razliku snaga na ulazu i izlazu. Ako je snaga od 680 W izmjerena vatmetrom W_1 na ulazu u četveropol uz relativnu nesigurnost 0,5 %, a snaga na izlazu 656 W izmjerena vatmetrom W_2 uz relativnu nesigurnost 0,5 %, kolika je mjerna nesigurnost snage gubitaka?



Matematički model

$P_g = P_{UL} - P_{IZ}$

$P_{UL} = 690\,W \quad u_{cR}(P_{UL}) = 0{,}8\,\%$

$P_{IZ} = 656\,W \quad u_{cR}(P_{IZ}) = 0{,}5\,\%$

$$u_c(P_g) = \sqrt{\left(\frac{\partial P_g}{\partial P_{UL}}\right)^2 \cdot u_c^2(P_{UL}) + \left(\frac{\partial P_g}{P_{IZ}}\right)^2 \cdot u_c^2(P_{IZ})} =$$

$$= \sqrt{(1)^2 \cdot u_c^2(P_{UL}) + (-1)^2 \cdot u_c^2(P_{IZ})}$$

$$u_c(P_{UL}) = \frac{u_{cR}(P_{UL})}{100\,\%} \cdot P_{UL} \quad , \quad u_c(P_{IZ}) = \frac{u_{cR}(P_{IZ})}{100\,\%} \cdot P_{IZ}$$

$$u_c(P_g) = \sqrt{\left(\frac{u_{cR}(P_{UL})}{100\,\%} \cdot P_{UL}\right)^2 + \left(\frac{u_{cR}(P_{IZ})}{100\,\%} \cdot P_{IZ}\right)^2}$$

$$= \frac{1}{100\,\%} \sqrt{u_{cR}^2(P_{UL}) \cdot P_{UL}^2 + u_{cR}^2(P_{IZ}) \cdot P_{IZ}^2}$$

$$= \frac{1}{100\,\%} \cdot \sqrt{(0{,}8\,\%)^2 \cdot (680\,W)^2 + (0{,}5\,\%)^2 \cdot (656\,W)^2} =$$

$$= \underline{\underline{6{,}35\,W}}$$

4. Otpor trošila određen je mjerenjem istosmjerne struje koja njime prolazi i pada napona na njemu. Kolika je relativna proširena nesigurnost tako izmjerenog otpora $U_{pr}(R)$ na razini pouzdanosti 95 % ako su relativna mjerna nesigurnost i pripadni efektivni stupanj slobode izmjerenog napona i struje redom 0,11 % i $\nu_{effU} = 22$ te 0,17 % i $\nu_{effI} = 11$? Vrijednosti pripadnog obuhvatnog faktora $t_p(\nu_{eff})$ nalaze se u tablici.

ν_{eff}	2	5	10	15	20	25	30	35	40	45	50
t_p	4,30	2,57	2,23	2,13	2,09	2,06	2,04	2,03	2,02	2,01	2,01

Matematički model: $R = \frac{U}{I}$

$$u_c(R) = \sqrt{\left(\frac{\partial R}{\partial U}\right)^2 u_c^2(U) + \left(\frac{\partial R}{\partial I}\right)^2 \cdot u_c^2(I)} = \sqrt{\left(\frac{1}{I}\right)^2 u_c^2(U) + \left(-\frac{U}{I^2}\right)^2 u_c^2(I)}$$

$$u_{cR}(R) = \frac{u_c(R)}{R} \cdot 100\%$$

$$u_{cR}(R) = \frac{I}{U} \cdot 100\% \cdot \sqrt{\frac{1}{I^2} \cdot u_c^2(U) + \frac{U^2}{I^4} u_c^2(I)} =$$

$$= \sqrt{\frac{I^2}{U^2} \cdot \frac{1}{I^2} \cdot u_c^2(U) \cdot (100\%)^2 + \frac{I^2}{U^2} \cdot \frac{U^2}{I^4} \cdot u_c^2(I) \cdot (100\%)^2} =$$

$$= \sqrt{\left(\frac{u_c(U)}{U} \cdot 100\%\right)^2 + \left(\frac{u_c(I)}{I} \cdot 100\%\right)^2} = \sqrt{u_{cR}^2(U) + u_{cR}^2(I)}$$

$$= \sqrt{(0,11\%)^2 + (0,17\%)^2} = 0,202\%$$

$$\nu_{eff}(R) = \frac{u_{cR}^4(R)}{\frac{u_{cR}^4(U)}{\nu_{effU}} + \frac{u_{cR}^4(I)}{\nu_{effI}}} = \frac{(0,202\%)^4}{\frac{(0,11\%)^4}{22} + \frac{(0,17\%)^4}{11}} = 20,36 \approx 20$$

$$t_p(\nu_{eff}(R)) = 2,09$$

$$U_{pr} = t_p \cdot u_{cR}(R) = 2,09 \cdot 0,202\% = \underline{\underline{0,422\%}}$$

5. Otpor jednog otpornika izmjeren je trima metodama: U-I metodom, digitalnim omometrom i usporedbom s poznatim otporom. Pritom su dobivene sljedeće aritmetičke sredine i pripadna standardna odstupanja sredine: 1,821 Ω (11 mΩ); 1,802 Ω (6 mΩ) i 1,765 Ω (15 mΩ). Koja je najvjerojatnija vrijednost tog otpornika?

$\bar{R}_1 = 1{,}821\,\Omega \qquad s_1 = 11\cdot 10^{-3}\,\Omega \qquad p_1 = \frac{1}{s_1^2} = \frac{1}{(11\cdot 10^{-3}\,\Omega)^2} = 8264{,}46\ 1/\Omega^2$

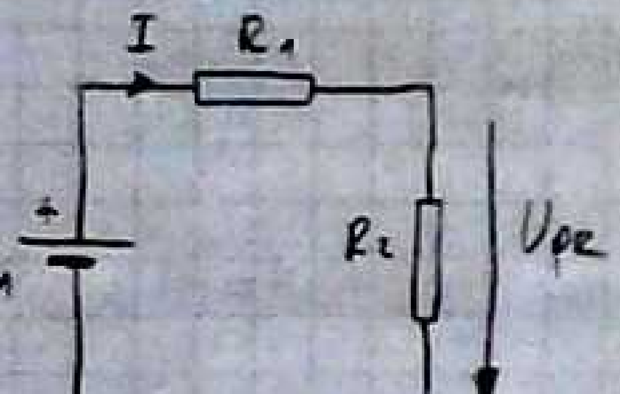
$\bar{R}_2 = 1{,}802\,\Omega \qquad s_2 = 6\cdot 10^{-3}\,\Omega \qquad p_2 = \frac{1}{s_2^2} = \frac{1}{(6\cdot 10^{-3}\,\Omega)^2} = 27\,777{,}78\ 1/\Omega^2$

$\bar{R}_3 = 1{,}795\,\Omega \qquad s_3 = 15\cdot 10^{-3}\,\Omega \qquad p_3 = \frac{1}{s_3^2} = \frac{1}{(15\cdot 10^{-3}\,\Omega)^2} = 4444{,}44\ 1/\Omega^2$

$$R = \frac{\sum_{i=1}^{3} p_i \cdot \bar{R}_i}{\sum_{i=1}^{3} p_i} = \frac{8264{,}46\ 1/\Omega^2 \cdot 1{,}821\,\Omega + 27\,777{,}78\ 1/\Omega^2 \cdot 1{,}802\,\Omega + 4444{,}44\ 1/\Omega^2 \cdot 1{,}795\,\Omega}{8264{,}46\ 1/\Omega^2 + 27\,777{,}78\ 1/\Omega^2 + 4444{,}44\ 1/\Omega^2}$$

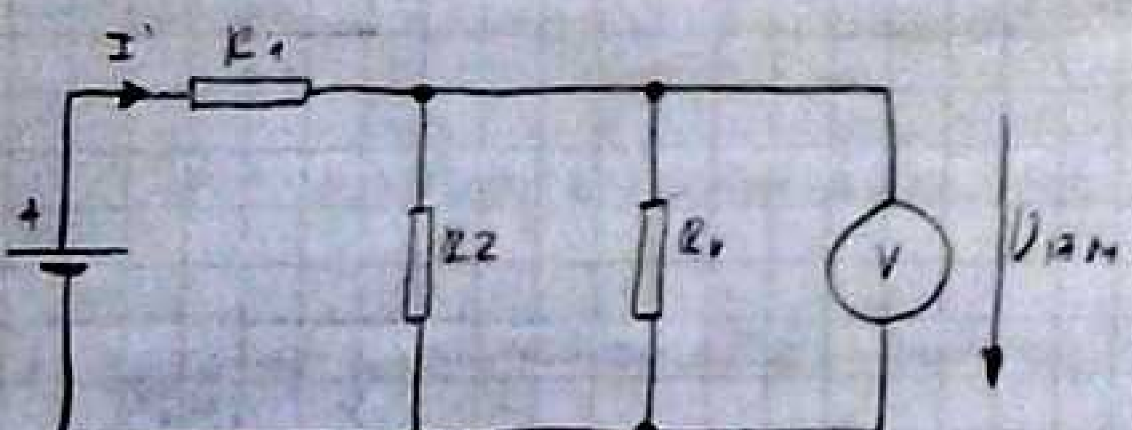
$$= \underline{\underline{1{,}804\,\Omega}}$$

3. Za mjerenje istosmjernog napona $U_1 = 1000\,V$ koristimo otporničko djelilo sastavljeno od $R_1 = 100\,k\Omega$ i $R_2 = 1\,k\Omega$. Kolika relativna pogrješka nastaje pri mjerenju tog napona na donjoj grani djelila voltmetrom unutrašnjeg otpora $150\,k\Omega$?



$$U_{pr} = \frac{R_2}{R_1 + R_2}\cdot U_1 = \frac{1000\,\Omega}{100\cdot 10^3\,\Omega + 1000\,\Omega}\cdot 1000\,V =$$

$$= \underline{9{,}901\,V}$$



$$U_{izm} = \frac{\frac{R_2\cdot R_v}{R_2 + R_v}}{R_1 + \frac{R_2 R_v}{R_2 + R_v}}\cdot U = \frac{\frac{R_2\cdot R_v}{R_2 + R_v}}{\frac{R_1(R_2+R_v) + R_2\cdot R_v}{R_2 + R_v}}\cdot U =$$

$$= \frac{R_2\cdot R_v}{R_1\cdot(R_2 + R_v) + R_2\cdot R_v}\cdot U$$

$$U_{izm} = \frac{10^3\,\Omega\cdot 150\cdot 10^3\,\Omega}{100\cdot 10^3\,\Omega\cdot(10^3\,\Omega + 150\cdot 10^3\,\Omega) + 10^3\,\Omega\cdot 150\cdot 10^3\,\Omega}\cdot 1000\,V = 9{,}836\,V$$

$$p = U_{izm} - U_{pr} = 9{,}836\,V - 9{,}901\,V = -0{,}065\,V$$

$$p\% = \frac{p}{U_{pr}}\cdot 100\,\% = \frac{-0{,}065\,V}{9{,}901\,V}\cdot 100\,\% = -0{,}656\,\% \approx \underline{\underline{-0{,}66\,\%}}$$

7. Napon dijagonale neuravnoteženog mosta, kojeg spajamo na ulaz pojačala nazivnog pojačanja 60 dB, mjerimo na izlazu pojačala digitalnim voltmetrom na mjernom opsegu 20 V. Poznati su nam sljedeći podaci: točnost pojačanja pojačala ± 2,6 te točnost instrumenta $\pm(4 \cdot 10^{-4}$ of reading $+ 4 \cdot 10^{-4}$ of range). Ako smo voltmetrom izmjerili napon 4,568 V, izračunajte složenu standardnu nesigurnost $u_c(y)$ napona dijagonale!

$A = 60\,dB$ $\quad a_r(A) = 2,5\,mV$ $\quad u_{cr}(A) = \frac{a(A)}{\sqrt{3}} = 1,5 \cdot 10^{-3}\,mV$

$U_{iz} = 4,568\,V$ $\quad a(U_{iz}) = 4 \cdot 10^{-4} \cdot 4,568\,V + 4 \cdot 10^{-4} \cdot 20\,V = 9,8272\,mV$

$A = 20 \log\left(\frac{U_{iz}}{U_{ul}}\right)$ $\quad u_c(U_{iz}) = \frac{a(U_{iz})}{\sqrt{3}} = 5,6737\,mV$

$\frac{U_{iz}}{U_{ul}} = 10^{\frac{60}{20}} = 10^{3}$ $\quad u_{cr}(U_{iz}) = \frac{u_c(U_{iz})}{U_{iz}} = 1,242\,mV$

Matematički model: $A = \frac{U_{iz}}{U_{ul}} \Rightarrow U_{ul} = \frac{U_{iz}}{A}$

$$u_{cr}(U_{ul}) = \sqrt{u_{cr}^2(A) + u_{cr}^2(U_{iz})} = \sqrt{(1,5 \cdot 10^{-3}\,V)^2 + (1,242 \cdot 10^{-3}\,V)^2} =$$

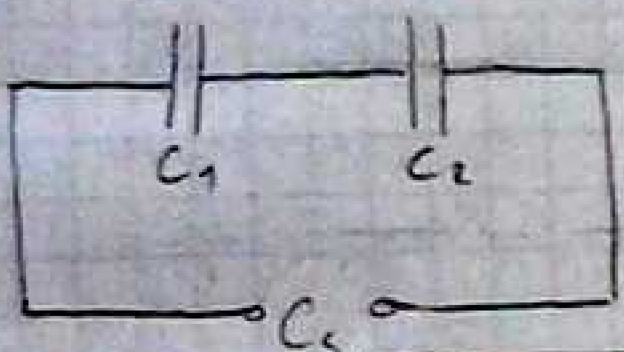
$$= 1,947 \cdot 10^{-3}\,V$$

$$u_c(U_{ul}) = u_{cr}(U_{ul}) \cdot U_{ul} = u_{cr}(U_{ul}) \cdot \frac{U_{iz}}{10^3} = 1,947 \cdot 10^{-3}\,V \cdot \frac{4,568\,V}{10^3} =$$

$$= \underline{\underline{8,894\,\mu V}}$$

8. Kondenzator kapaciteta $1\,\mu F$ ima relativnu mjernu nesigurnost 1%, a kondenzator kapaciteta $0{,}2\,\mu F$ ima relativnu mjernu nesigurnost 5%. Kolika je relativna mjerna nesigurnost kapaciteta njihove serijske kombinacije, odnosno njihove paralelne kombinacije?

I) SERIJSKI SPOJ



Matematički model

$$C_s = \frac{C_1 \cdot C_2}{C_1 + C_2}$$

$$u_c(C_s) = \sqrt{\left(\frac{\partial C_s}{\partial C_1}\right)^2 \cdot u_c^2(C_1) + \left(\frac{\partial C_s}{\partial C_2}\right) \cdot u_c^2(C_2)} =$$

$$u_c(C_s) = \sqrt{\left(\frac{C_2 \cdot (C_1 + C_2) - 1 \cdot C_1 C_2}{(C_1 + C_2)^2}\right)^2 \cdot u_c^2(C_1) + \left(\frac{C_1 (C_1 + C_2) - 1 \cdot C_1 C_2}{(C_1 + C_2)^2}\right)^2 \cdot u_c^2(C_2)}$$

$$u_{cR}(C_s) = \frac{u_c(C_s)}{\frac{C_1 \cdot C_2}{C_1 + C_2}} \cdot 100\,\%$$

$$u_{cR}(C_s) = \frac{C_1 + C_2}{C_1 \cdot C_2} \cdot 100\,\% \cdot \sqrt{\left(\frac{C_2}{C_1 + C_2} - \frac{C_1 C_2}{(C_1 + C_2)^2}\right)^2 \cdot u_c^2(C_1) + \left(\frac{C_1}{C_1 + C_2} - \frac{C_1 C_2}{(C_1 + C_2)^2}\right)^2 \cdot u_c^2(C_2)} =$$

$$= \sqrt{\left(\frac{C_1 + C_2}{C_1 \cdot C_2} \cdot \left(\frac{C_2}{C_1 + C_2} - \frac{C_1 C_2}{(C_1 + C_2)^2}\right)\right)^2 \cdot u_c^2(C_1)(100\,\%)^2 + \left(\frac{C_1 + C_2}{C_1 \cdot C_2} \cdot \left(\frac{C_1}{C_1 + C_2} - \frac{C_1 C_2}{(C_1 + C_2)^2}\right)\right)^2 \cdot u_c^2(C_2) \cdot (100\,\%)^2} =$$

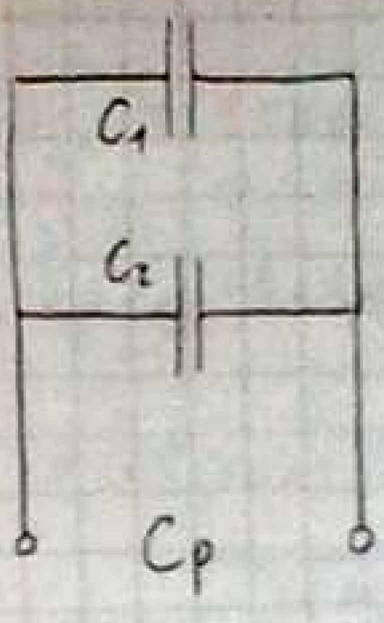
$$= \sqrt{\left(\frac{u_c(C_1)}{C_1} \cdot 100\,\%\right)^2 \cdot \left(1 - \frac{C_1}{C_1 + C_2}\right)^2 + \left(\frac{u_c(C_2)}{C_2} \cdot 100\,\%\right)^2 \cdot \left(1 - \frac{C_2}{C_1 + C_2}\right)^2}$$

$$= \sqrt{u_{cR}^2(C_1) \cdot \left(1 - \frac{C_1}{C_1 + C_2}\right)^2 + u_{cR}^2(C_2) \cdot \left(1 - \frac{C_2}{C_1 + C_2}\right)^2} =$$

$$= \sqrt{(1\,\%)^2 \cdot \left(1 - \frac{10^{-6}\,F}{10^{-6}\,F + 0{,}2 \cdot 10^{-6}\,F}\right)^2 + (5\,\%)^2 \cdot \left(1 - \frac{0{,}2 \cdot 10^{-6}\,F}{10^{-6}\,F + 0{,}2 \cdot 10^{-6}\,F}\right)^2} =$$

$$= 4{,}1699\,\% \approx \underline{\underline{4{,}17\,\%}}$$

II PARALELNI SPOJ



Matematički model:

$$C_p = C_1 + C_2$$

$$u_c(C_p) = \sqrt{\left(\frac{\partial C_p}{\partial C_1}\right)^2 \cdot u_c^2(C_1) + \left(\frac{\partial C_p}{\partial C_2}\right)^2 \cdot u_c^2(C_2)}$$

$$= \sqrt{1^2 \cdot u_c^2(C_1) + 1^2 \cdot u_c^2(C_2)}$$

$$u_{cR}(C_p) = \frac{u_c(C_p)}{C_1 + C_2} \cdot 100\%$$

$$= \sqrt{\frac{1}{(C_1+C_2)^2} \cdot u_c^2(C_1) \cdot (100\%)^2 + \frac{1}{(C_1+C_2)^2} \cdot u_c^2(C_2) \cdot (100\%)^2} =$$

$$= \sqrt{\frac{C_1^2}{C_1^2 \cdot (C_1+C_2)^2} \cdot u_c^2(C_1) \cdot (100\%)^2 + \frac{C_2^2}{C_2^2 \cdot (C_1+C_2)^2} \cdot u_c^2(C_2) \cdot (100\%)^2} =$$

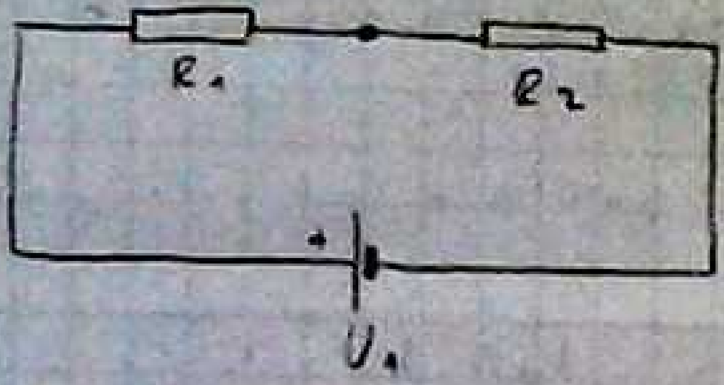
$$= \sqrt{\left(\frac{u_c(C_1)}{C_1} \cdot 100\%\right)^2 \cdot \frac{C_1^2}{(C_1+C_2)^2} + \left(\frac{u_c(C_2)}{C_2} \cdot 100\%\right)^2 \cdot \frac{C_2^2}{(C_1+C_2)^2}} =$$

$$= \sqrt{u_{cR}^2(C_1) \cdot \frac{C_1^2}{(C_1+C_2)^2} + u_{cR}^2(C_2) \cdot \frac{C_2^2}{(C_1+C_2)^2}} =$$

$$= \sqrt{(1\%)^2 \cdot \left(\frac{10^{-6}F}{10^{-6}F + 0{,}2 \cdot 10^{-6}F}\right)^2 + (5\%)^2 \cdot \left(\frac{0{,}2 \cdot 10^{-6}F}{10^{-6}F + 0{,}2 \cdot 10^{-6}F}\right)^2} =$$

$$= 1{,}1785\% \approx \underline{\underline{1{,}18\%}}.$$

9. Za mjerenje istosmjernog napona U_1 koristimo otporničko djelilo omjera 1:10 sastavljeno od $R_1 = 900\,\Omega$ i $R_2 = 100\,\Omega$. Ako želimo da relativna mjerna nesigurnost omjera dijeljenja bude 0,1%, a relativna mjerna nesigurnost otpora R_2 iznosi 0,02%, kolika je dozvoljena mjerna nesigurnost otpora R_1?



Matematički model

$$K = \frac{R_2}{R_1 + R_2}$$

$$M_c(K) = \sqrt{\left(\frac{\partial k}{\partial R_1}\right)^2 \cdot M_c^2(R_1) + \left(\frac{\partial k}{\partial R_2}\right)^2 \cdot M_c^2(R_2)}$$

$$= \sqrt{\left(\frac{0 - 1 \cdot R_2}{(R_1+R_2)^2}\right)^2 \cdot M_c^2(R_1) + \left(\frac{1 \cdot (R_1+R_2) - 1 \cdot R_2}{(R_1+R_2)^2}\right)^2 \cdot M_c^2(R_2)}$$

$$M_{cR}(k) = \frac{M_c(k)}{\frac{R_2}{R_1+R_2}} \cdot 100\,\%$$

$$M_{cR}(k) = \frac{R_1+R_2}{R_2} \cdot 100\,\% \cdot \sqrt{\left(\frac{R_2}{(R_1+R_2)^2}\right)^2 \cdot M_c^2(R_1) + \left(\frac{R_1+R_2-R_2}{(R_1+R_2)^2}\right)^2 \cdot M_c^2(R_2)} =$$

$$= \sqrt{\left(\frac{R_1+R_2}{R_2}\right)^2 \cdot \left(\frac{R_2}{(R_1+R_2)^2}\right)^2 \cdot M_c^2(R_1) \cdot (100\%)^2 + \left(\frac{R_1+R_2}{R_2}\right)^2 \cdot \left(\frac{R_1}{(R_1+R_2)^2}\right)^2 M_c^2(R_2) \cdot (100\%)^2} =$$

$$= \sqrt{\frac{R_1^2}{R_1^2 \cdot (R_1+R_2)^2} \cdot M_c^2(R_1)(100\%)^2 + \frac{R_1^2}{R_2^2 (R_1+R_2)^2} \cdot M_c^2(R_2) \cdot (100\%)^2} =$$

$$= \sqrt{\left(\frac{M_c(R_1)}{R_1} \cdot 100\%\right)^2 \cdot \left(\frac{R_1}{R_1+R_2}\right)^2 + \left(\frac{M_c(R_2)}{R_2} \cdot 100\%\right)^2 \cdot \left(\frac{R_1}{R_1+R_2}\right)^2} =$$

$$M_{cR}(k) = \sqrt{M_{cR}^2(R_1) \cdot \frac{R_1^2}{(R_1+R_2)^2} + M_{cR}^2(R_2) \cdot \frac{R_1^2}{(R_1+R_2)^2}} \quad /^2$$

$$M_{cR}^2(k) = \frac{(R_1)^2}{(R_1+R_2)^2} \cdot \left(M_{cR}^2(R_1) + M_{cR}^2(R_2)\right)$$

$$\left(\frac{R_1+R_2}{R_1}\right)^2 \cdot M_{cR}^2(k) = M_{cR}^2(R_1) + M_{cR}^2(R_2)$$

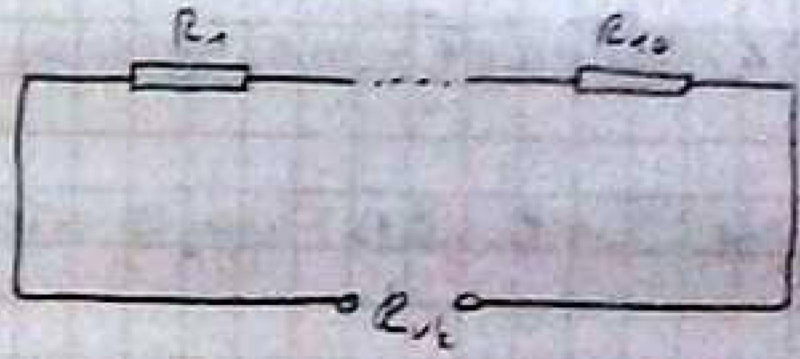
$$M_{cR}(R_1) = \sqrt{\left(1 + \frac{R_2}{R_1}\right)^2 \cdot M_{cR}^2(k) - M_{cR}^2(R_2)} =$$

$$= \sqrt{\left(1 + \frac{100\,\Omega}{900\,\Omega}\right)^2 \cdot (0{,}1\%)^2 - (0{,}02\%)^2} =$$

$$= 0{,}1093\,\%$$

$$M_c(R_1) = \frac{M_{cR}(R_1)}{100\%} \cdot R_1 = \frac{0{,}1093\,\%}{100\,\%} \cdot 900\,\Omega = 0{,}9837\,\Omega \approx \underline{\underline{0{,}984\,\Omega}}$$

10. Deset jednakih otpornika, nazivne vrijednosti $1\,k\Omega$ i mjerne nesigurnosti $100\,m\Omega$, spajamo u seriju kako bismo dobili otpor od $10\,k\Omega$. Kolika je relativna mjerna nesigurnost tako ostvarenog otpora?



$u_c(R_i) = 100\,m\Omega$, $R_j = 1\,k\Omega$

$$u_{cR}(R_i) = \frac{u_c(R_i)}{R_i}\cdot 100\% =$$

$$= \frac{100\cdot 10^{-3}\,\Omega}{10^{3}\,\Omega}\cdot 100\% = 0{,}01\%$$

Matematički model: $R_{uk} = \sum_{j=1}^{10} R_j = 10R_i$

$$u_c(R_{uk}) = \sqrt{\sum_{j=1}^{10}\left(\frac{\partial R_{uk}}{\partial R_j}\right)^2\cdot u_c^2(R_j)} = \sqrt{\sum_{j=1}^{10}(1)^2\cdot u_c^2(R_j)} =$$

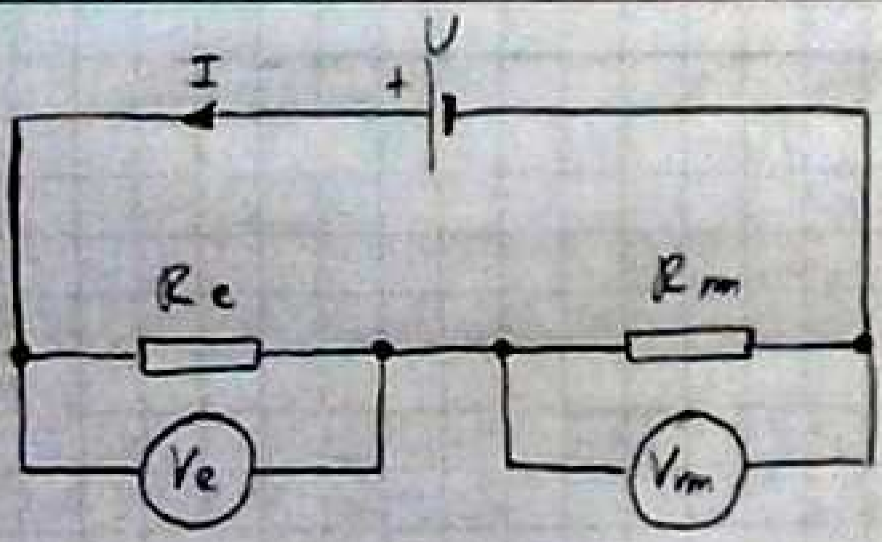
$$= \sqrt{10\cdot u_c^2(R_i)} = u_c(R_i)\cdot\sqrt{10}$$

$$u_{cR}(R_{uk}) = \frac{u_c(R_{uk})}{R_{uk}}\cdot 100\% = \frac{u_c(R_i)\cdot\sqrt{10}}{10R_i}\cdot 100\% =$$

$$= \frac{100\cdot 10^{-3}\,\Omega\cdot\sqrt{10}}{10\cdot 10^{3}\,\Omega}\cdot 100\% = 10^{-3}\cdot\sqrt{10}\,\% = \underline{\underline{0{,}00316\%}}$$

11. Otpornik nazivne vrijednosti $R_N = 100\,\Omega$ uspoređuje se s etalonskim otpornikom iste nazivne vrijednosti tako da se, uz njihov serijski spoj, dvama voltmetrima istodobno mjere padovi napona na oba otpornika. Poznati su nam podaci o etalonu: otpor $R = R_N(1-4{,}5\cdot 10^{-6})$, proširena nesigurnost

$U(R) = 2,5\ m\Omega$, obuhvatni faktor $k = 2$ i pripadni efektivni stupanj slobode $\nu_{eff} = 13$. Nakon ponovljenih mjerenja dobili smo aritmetičke sredine napona, izmjerenih na mjerenom i etalonskom otporniku, redom 0,999853 V i 0,999736 V, čije su složene standardne nesigurnosti redom 76 μV i 55 μV, a $\nu_{eff} = 11$ za obje vrijednosti. Efektivni stupanj slobode ν_{eff} mjerne nesigurnosti mjerenog otpornika iznosi:



$R_e = R_N \cdot (1 - 4,5 \cdot 10^{-6})$ $\quad U(R_e) = 2,5\ m\Omega$

$R_N = 100\ \Omega$, $k_e = 2$, $\nu_{eff}(R_e) = 13$

$\bar{U}_m = 0,999853\ V$ $\quad u_c(U_m) = 76\ \mu V$

$\bar{U}_e = 0,999736\ V$ $\quad u_c(U_e) = 55\ \mu V$

$\nu_{effU} = 11$

Matematički model:

$$U = U_e + U_m$$

$$u_c(U) = \sqrt{\left(\frac{\partial U}{\partial U_e}\right)^2 \cdot u_c^2(U_e) + \left(\frac{\partial U}{\partial U_m}\right)^2 \cdot u_c^2(U_m)} = \sqrt{1^2 \cdot u_c^2(U_e) + 1^2 \cdot u_c^2(U_m)} =$$

$$= \sqrt{(55 \cdot 10^{-6}\ V)^2 + (76 \cdot 10^{-6}\ V)^2} = \underline{93,814\ \mu V}$$

$$\nu_{eff} = \frac{u_c^4(U)}{\frac{u_c^4(U_e)}{\nu_{effU}} + \frac{u_c^4(U_m)}{\nu_{effU}}} = \frac{(93,814 \cdot 10^{-6}\ V)^4}{\frac{(55 \cdot 10^{-6}\ V)^4}{11} + \frac{(76 \cdot 10^{-6}\ V)^4}{11}} = \underline{\underline{20,04}}$$

12. Istosmjernu struju trošila određujemo mjerenjem pada napona na poznatom otporu. Ako su aritmetička sredina, pripadna nesigurnost i efektivni stupanj slobode za napon redom (0,3198 V; 0,3 mV, 11), a za otpor redom (0,1122 Ω, 0,18 mΩ, 15), koliko iznosi složena standardna nesigurnost mjerene struje?

Matematički model: $I = \frac{U}{R}$

$$u_c(I) = \sqrt{\left(\frac{\partial I}{\partial U}\right)^2 \cdot u_c^2(U) + \left(\frac{\partial I}{\partial R}\right)^2 \cdot u_c(R)} = \sqrt{\frac{1}{R^2} \cdot u_c^2(U) + \left(-\frac{U}{R^2}\right)^2 u_c(R)} =$$

$$= \sqrt{\frac{1}{(0{,}1122\,\Omega)^2} \cdot (0{,}3 \cdot 10^{-3}\,V)^2 + \frac{(0{,}3198\,V)^2}{(0{,}1122\,\Omega)^4} \cdot (0{,}18 \cdot 10^{-3}\,\Omega)^2} =$$

$$= 5{,}297\,mA \approx \underline{\underline{5{,}3\,mA}}$$

13. Napon izvora izmjeren je 12 puta u istim uvjetima, digitalnim voltmetrom s prikazom 5 ½ znamenke i granicama pogrješaka $\pm(4 \cdot 10^{-4}$ of reading $+ 6 \cdot 10^{-4}$ of range$)$, na mjernom opsegu 1 V. Aritmetička sredina svih rezultata bila je 0,78614 V, a standardno odstupanje (pojedine vrijednosti) 0,6 mV. Kolika je složena standardna nesigurnost u_c tako izmjerenog napona?

$N = 12$

$a = \pm 4 \cdot 10^{-4} \cdot 0{,}78614\,V + 6 \cdot 10^{-4} \cdot 1\,V = 914{,}456\,\mu V$

$u_2 = \frac{a}{\sqrt{3}} = 527{,}961\,\mu V$

$u_1 = \frac{s}{\sqrt{N}} = \frac{0{,}6 \cdot 10^{-3}\,V}{\sqrt{12}} = 173{,}205\,\mu V$

$u_c(U) = \sqrt{u_1^2 + u_2^2} = \sqrt{(527{,}961 \cdot 10^{-6}\,V)^2 + (173{,}205 \cdot 10^{-6}\,V)^2} =$

$= 555{,}646\,\mu V \simeq \underline{\underline{0{,}56\,mV}}$

14. Od ukupno 3000 otpornika nazivne vrijednosti $1000\,\Omega$ i dozvoljenog odstupanja od nazivne vrijednosti do $\pm 0{,}5\%$ uzeli smo uzorak od 60 otpornika. Koliko će otpornika, od ukupne količine, odstupati više od dozvoljenog, ako je aritmetička sredina uzorka $999\,\Omega$, a standardno odstupanje uzorka $2{,}1\,\Omega$?

$R_N = 1000\,\Omega \pm 0{,}5\%$ $\quad$ $N = 60$

$N = 3000$ $\quad$ $\bar{R} = 999\,\Omega$, $s = 2{,}1\,\Omega$

$R_N = 1000\,\Omega \pm 5\,\Omega$

$\bar{R} + k_{p1} \cdot s < 1005\,\Omega$, $\bar{R} - k_{p2} \cdot s > 995\,\Omega$

$\Rightarrow k_{p1} = 2{,}576$ $\quad$ $\Rightarrow k_{p2} = 1{,}96$

$999\,\Omega + 2{,}576 \cdot 2{,}1\,\Omega = 1004{,}4\,\Omega$ ✓

$999\,\Omega - 1{,}96 \cdot 2{,}1\,\Omega = 994{,}89\,\Omega \approx 995\,\Omega$ ✓

$$P(x) = 1 - \left(p(-t_{p_2}s < \bar{R} < t_{p_2}s) + \frac{1}{2}\left(p(-t_{p_1}s < \bar{R} < t_{p_1}s) - p(-t_{p_2}s < \bar{R} < t_{p_2}s) \right) \right)$$

$$= 1 - \left(\frac{p(-t_{p_2}s < \bar{R} < t_{p_2}s)}{2} + \frac{p(-t_{p_1}s < \bar{R} < t_{p_1}s)}{2} \right)$$

$$= 1 - \frac{0{,}95 + 0{,}99}{2} = 0{,}03 = 3\%$$

$$N_x = p(x) \cdot N = 90$$

$$N_x' = P(x) \cdot N' = 1{,}8 \approx 2$$

$$N_{x_{uk}} = N_x + N_x' = 90 + 2 = \underline{\underline{92}}.$$

Otpor jednog otpornika mjeren je U-I metodom pomoću ampermetra razreda točnosti 0,2 na mjernom opsegu 1,5 A te minivoltmetrom razreda točnosti 0,2 na mjernom opsegu 150 mV. Mjerenje je provedeno više puta kod struja od (0,3; 0,6; 0,9; 1,2; 1,5) A. Pri tom su dobivene sljedeće srednje vrijednosti pada napona na mjerenom otporniku: (29,7; 59,45; 89,2; 119,05; 149,9) mV. Uz pretpostavku da vrijednost otpora nije strujno ovisna na području primjenjenih struja, koja je najvjerojatnija vrijednost mjerenog otpornika?

	I/A	U/mV
1	0,3	29,7
2	0,6	59,45
3	0,9	89,2
4	1,2	119,05
5	1,5	148,9

$$R_1 = \frac{U_1}{I_1} = \frac{29,7\,mV}{0,3\,A} = 99,0\ m\Omega$$

$$R_2 = \frac{U_2}{I_2} = \frac{59,45\,mV}{0,6\,A} = 99,083\ m\Omega$$

$$R_3 = \frac{U_3}{I_3} = \frac{89,2\,mV}{0,9\,A} = 99,111\ m\Omega$$

$$R_4 = \frac{U_4}{I_4} = \frac{119,05\,mV}{1,2\,A} = 99,2083\ m\Omega$$

$$R_5 = \frac{U_5}{I_5} = \frac{148,9\,mV}{1,5\,A} = 99,2667\ m\Omega$$

$$\bar{R} = \frac{1}{n}\sum_{i=1}^{n} R_i = \frac{1}{5}\sum_{i=1}^{5} R_i = 99,1339\ m\Omega$$

$$s_1 = R_1 - \bar{R} = -0,1339\ m\Omega \qquad p_1 = \frac{k}{s_1^2} = \frac{10^{-6}}{(-0,1339\cdot 10^{-3}\,\Omega)^2} = 55,7749\ 1/\Omega^2$$

$$s_2 = R_2 - \bar{R} = -0,0506\ m\Omega \qquad p_2 = \frac{k}{s_2^2} = \frac{10^{-6}}{(-0,0506\cdot 10^{-3}\,\Omega)^2} = 390,5201\ 1/\Omega^2$$

$$s_3 = R_3 - \bar{R} = -0,0228\ m\Omega \qquad p_3 = \frac{k}{s_3^2} = \frac{10^{-6}}{(-0,0228\cdot 10^{-3}\,\Omega)^2} = 1923,6639\ 1/\Omega^2$$

$$s_4 = R_4 - \bar{R} = 0,0744\ m\Omega \qquad p_4 = \frac{k}{s_4^2} = \frac{10^{-6}}{(0,0744\cdot 10^{-3}\,\Omega)^2} = 180,6567\ 1/\Omega^2$$

$$s_5 = R_5 - \bar{R} = 0,1328\ m\Omega \qquad p_5 = \frac{k}{s_5^2} = \frac{10^{-6}}{(0,1328\cdot 10^{-3}\,\Omega)^2} = 56,7027\ 1/\Omega^2$$

k = proizvoljan broj, npr $k = 10^{-6}$

$$R = \frac{\sum_{i=1}^{n} p_i \cdot R_i}{\sum_{i=1}^{n} p_i} = \underline{\underline{99,1146\ m\Omega}}$$