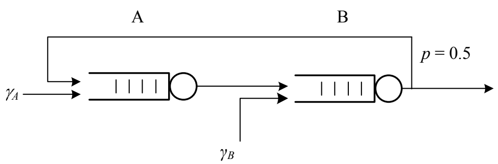
# Primjeri 2

16. Zadano je $M$ identičnih poslužiteljskih sustava spojenih u zatvorenu petlju (prsten). Odredite vjerojatnost da red prvog poslužiteljskog sustava nije prazan (tj. u njegovom redu je barem jedan korisnik) ako u takvom sustavu cirkulira $N$ korisnika.
***Rezultat*:** $N/(N + M - 1)$.

17. Komutacijski čvor ima tri izlazna (odlazna) linka A, B i C. Poruke koje dođu u čvor mogu se odaslati jednim od izlaznih linkova uz jednake vjerojatnosti. Svaki izlazni link ima različitu brzinu pa su i trajanja prijenosa na linkovima A, B i C redom 1, 2 i 3 ms. Zbog razlika u rutama kojima prolaze pojedini linkovi vjerojatnosti pogrešaka na linkovima A, B i C su redom 0.2, 0.3 i 0.1. Kolika je vjerojatnost da će poruka biti ispravno prenesena u 2 ms?
***Rezultat*:** 7/30.

18. Zadan je sustav posluživanja sastavljen od dva poslužiteljska sustava povezana na način prikazan na slici. Korisnici dolaze u sustave A i B brzinama $\gamma_A$ i $\gamma_B$. Ako je $p$ vjerojatnost grananja na izlazu sustava B odredite efektivne brzine dolazaka $\lambda_A$ i $\lambda_B$ za sustave A i B.
***Rezultat*:** $\lambda_A = 2\gamma_A + \gamma_B$, $\lambda_B = 2(\gamma_A + \gamma_B)$.




19. Tramvaji dolaze na stajalište sukladno Poissonovom procesu sa srednjim međudolaznim vremenom od 10 minuta. (a) Ako putnik dođe na stanicu i netko mu kaže da je zadnji tramvaj došao prije 9 minuta, koliko će prosječno trebati čekati slijedeći tramvaj? (b) Kolika je vjerojatnost da će ući u tramvaj u sljedećih 5 minuta? (c) Kolika je vjerojatnost da će čekati tramvaj između 5 i 9 minuta?
***Rezultat*:** (a) 10 min (Markovljevo svojstvo); (b) 0.393; (c) 0.2.

20. Pacijenti dolaze u ambulantu sukladno Poissonovom procesu sa srednjim međudolaznim vremenom od 18 minuta. U čekaonici se formira red, u ambulanti radi jedan liječnik, a prosječno trajanje jednog pregleda (ili konzultacije) iznosi 7 minuta i ravna se po eksponencijalnoj razdiobi. (a) Kolika je vjerojatnost da će pacijent čekati da bi „vidio" liječnika? (b) Koliko je prosječno pacijenata u čekaonici? (c) Kolika je vjerojatnost da je u ambulanti 5 ili više pacijenata uključujući onog na pregledu? (d) Kolika je vjerojatnost da će pacijent čekati više od 10 minuta na pregled? (e) Ambulanta će zaposliti dodatnog liječnika ako je trajanje čekanja na pregled veće od 7 minuta. Kolika bi u tom slučaju mogla biti brzina dolazaka u ambulantu pa da se opravda zapošljavanje novog liječnika?
***Rezultat*:** (a) 7/18; (b) 49/198; (c) 0.0089; (d) 0.162; (e) 1/14 pacijent/min.



21. Paketi varijabilne duljine dolaze u komunikacijski čvor sukladno Poissonov procesu srednjom brzinom 10 paket/s. Jedan izlazni komunikacijski link radi na brzini 64 kbit/s. Za duljinu paketa može se pretpostaviti da se ravna po eksponencijalnoj razdiobi srednje duljine 480 znakova u 8 bitnom ASCII formatu. Izračunajte glavne parametre performanci opisanog komunikacijskog linka pretpostavljajući da on ima vrlo veliki ulazni spremnik (buffer). Kolika je vjerojatnost da 6 ili više paketa čeka na slanje?
***Rezultat*:** $N$ = 1.5 paket, $N_Q$ = 0.9 paket, $T$ = 0.15 s, $W$ = 0.09 s; 0.028.

22. U paketskoj mreži dvije vrste paketa, govorni i podatkovni, dolaze na jednokanalni prijenosni sutav. Govorni paketi uvijek ulaze u spremnik za prijenos, dok se podatkovnim paketima dopušta ulaz u spremnik samo kad je ukupni broj paketa u sustavu manji od $N$. Odredite stacionarne vjerojatnosti da je $k$ paketa u sustavu ako se oba toka paketa ravnaju sukladno Poissonovom procesu s brzinama $\lambda_1$ i $\lambda_2$. Pretpostavite da svi paketi imaju eksponencijalnu razdiobu duljine te da se odašilju prosječnom brzinom $\mu$.

23. Fakultetski računski centar ima jedno računalo rezervirano samo za posebna testiranje studentskih programa. Kad je gužva na računalu studenti odustaju od testiranja programa pa se zato dolazni proces može modelirati kao Poissonov brzine dolazaka $\lambda_k = \lambda/(k + 1)$ za $k$ = 0,1,2,..., gdje je $k$ broj programa na testiranju. (Studenti mogu programe slati mrežom ili ih donjeti na nekom memorijskom mediju; red čekanja čine svi učitani programi koji se obrađuju jedan po jedan u redosljedu učitavanja; u svakom slučaju, trajanje prijenosa programa ili njegovog učitavanja može se zanemariti). Trajanje testiranja pojedinog programa ravna se po eksponencijalnoj razdiobi sa srednjom vrijednosti $1/\mu$ i to bez obzira na broj programa u sustavu. (a) Nacrtajte Markovljev lanac te napišite jednadžbe lokalne i globalne ravnoteže. (b) Odredite vjerojatnos da je $k$ programa u računalu i srednji broj programa. (c) Odredite srednje vrijeme boravka (od predaje do završetka testiranja) programa u računalu.
***Rezultat*:** (b) $p_k = (\rho^k/k!)e^{-\rho}$, $N = \rho$. (c) $T = \rho/\mu(1 - e^{-\rho})$.

24. Paketi varijabilne duljine dolaze u komutacijski čvor srednjom brzinom 125 paket/s. Duljine paketa se ravnaju po eksponencijalnoj razdiobi sa srednjom vrijednosti 88 bit a jedan izlazni link komutacije ima brzinu 19.2 kbit/s. (a) Kolika je vjerojatnost *preliva spremnika* (*buffer overflow*) ako je njegova veličina dovoljna za spremanje samo 11 paketa? (b) U prosjeku, koliko je paketa u spremniku? (c) Koliko bi veliki trebao biti spremnik (izražen brojem paketa) pa da gubitak paketa bude manji od $10^{-6}$ (na milijun paketa jedan je izgubljen)?
***Rezultat*:** (a) $5.35\times10^{-4}$. (b) 0.75 (c) 24.

25. Usporedite performanse statističkog (SM) i vremenskog multipleksiranja (TDM). Svaki multipleksor ima 4 ulaza, brzina dolazaka paketa je 5 paket/s, a multipleksirani izlaz ima brzinu 64 kbit/s. SM ima spremnik veličine 3 paketa za kombinirani tok ulaznih paketa, a TDM ima spremnik za 3 paketa u svakom ulaznom kanalu. Srednja duljina paketa je 2000 bit.

26. Trgovačka kuća treba instalirati novu kućnu centralu (PBX) s 300 lokalnih priključaka. Nova PBX ima grupu dvosmjernih vanjskih linija (vodova). Vanjski promet (dolazni i odlazni) dijeli se na jednake dijelove po grupi dvosmjernih linija. Ustanovljeno je da je svaki interni telefon tijekom uobičajenog radnog dana zauzet 20 minuta bilo da zove ili je pozvan. Također je ustanovljeno da se u glavnom prometnom satu obavi 14% dnevnog prometa. Kod dimenzioniranja vanjskih linija

treba dobiveni promet uvećati 10% zbog (neuspješnih) ponovljenih i nekompletnih poziva tijekom faze biranja odredišnog korisnika Koliko vanjskih linija (vodova) treba osigurati da bi vjerojatnost blokiranja bila bolja od 0.02?
***Rezultat*:** 23.

27. Na „vruću" telefonsku liniju u turističkoj agenciji pozivi dolaze sukladno Poissonovom procesu. Tu liniju koriste tri turistička koordinatora koji sa svakim potencijalnim korisnikom razgovaraju u prosjeku 6 minuta. Ustanovljeno je da se u jednom satu primi 9 poziva, te da se trajanje poziva ravna po eksponencijalnoj razdiobi. Koliko dugo će korisnik čekati na razgovor s koordinatorom pretpostavljajući da će korisnik čekati (poziv na čekanju) u slučaju zauzetosti svih koordinatora? U prosjeku, koliko korisnika čeka razgovor s koordinatorom?
***Rezultat*:** 0.2 min, 0.03 korisnik

28. Multinacionalna naftna kompanija iznajmljuje određeni satelitski kapacitet za svoju mobilnu telefonsku mrežu. U toj mreži raspoloživi satelitski kapacitet se dijeli na $N_v$ govornih kanala brzine 1200 bit/s i $N_d$ podatkovnih kanala brzine 2400 bit/s. Mobilni telefoni kolektivno generiraju Poissonov govorni promet s prosječno 200 govornih poziva u sekundi, te Poissonov podatkovni promet s prosječno 40 poruka u sekundi. Trajanje govornih poziva i podatkovnih poruka može se aproksimirati eksponencijalnim razdiobama s prosječnim duljinama 54 bit, odnosno 240 bit. Govorni pozivi se prijenose odmah nakon generiranja ili se blokiraju ako nema slobodvih kanala, dok se podatkovne poruke zadržavaju u velikom spremniku ako nema slobodnog kanala. (a) Odredite $N_v$ uz uvjet da vjerojatnost blokiranja poziva bude manja od 0.02. (b) Odredite $N_d$ uz uvjet da srednje kašnjenje poruka bude manje od 0.115 s, a vjerojatnost da uopće dođe do kašnjenja poruka bude 0.3, a vjerojatnost da uopće dođe do kašnjenja poruka bude 0.3.
***Rezultat*:** (a) $N_v \geq 15$. (b) $N_d \geq 6$.

29. U jednom našem malom gradiću postoje dvije radio-taksi službe. Svaka od tih tvrtki ima dva taksija a njihovi udjeli na tržištu su približno jednaki. Svaka tvrtka ima vlastiti dispečerski centar. Ustanovljeno je da u svaki dispečerski centar pozivi dolaze brzinom 10 poziva na sat i to sukladno Poissonovoj razdiobi. Prosječno trajanje jedne vožnje je u obje tvrtke 11.5 minuta. I sad se događa već poznato: obje su tvrtke zapale u poslovne teškoće pa ih je kupio jedan gradski biznismen. Njegova prva akcija nakon preuzimanja tih dviju tvrtki je da od dvije stvori jednu tvrtku, te da konsolidira dva dispečerska centra u jedan u cilju poboljšanja usluge. Naravno, bez dodatnih ulaganja u novu tvrtku! Izračunajte: (a) Iskoristivost (tj. omjer primljenih poziva i brzine posluživanja) svake tvrtke prije integracije u jednu; (b) Iskoristivost nakon integracije; (c) Očekivano vrijeme čekanja pojedinog korisnika (od trenutka poziva do dolaska taksija) kad postoje dvije tvrtke; (d) Isto kao i u (c) ali nakon integracije u jednu tvrtku; (e) Odredite postotak zauzetosti svih taksija u svakoj od dviju tvrtki; (f) Isto kao i pod (e) samo za jednu konsolidiranu tvrtku; (g) Komentirajte riječima (dakako, kratko, ali uz svu moguću maštu!) dobivene rezultate. Što bi Vi učinili da ste na mjestu ovog tajkuna?
***Rezultat*:** (a) 95.8%. (b) 95.8%. (c) 2.16 h. (d) 1.05 h. (e) 93.8%. (f) 90.93%.

30. U konsolidiranoj taksi tvrtki iz Zadatka 29 vlasnik je uočio da je vrijeme čekanja taksija predugo. Međutim, banke mu zbog recesije ne žele dati kredit za kupnju dodatnih vozila čime bi smanjio čekanje. Da bi ipak smanjio predugo čekanje vlasnik je uveo novu strategiju poslovanja: kad lista čekanja korisnika dosegne 16, osoblje dispečerskog centra treba se svakom novom potencijalnom



korisniku ljubezno ispričati! Odredite: (a) očekivano vrijeme čekanja; (b) očekivan broj korisnika u redu čekanja; (c) očekivani broj praznih vozila i (d) gubitke, tj. vjerojatnost da se dispečer ispričao potencijalnom korisniku jer nema slobodno vozilo.
***Rezultat*:** (a) 0.303 h. (b) 5.85. (c) 0.2986. (d) 3.4% (0.03433).

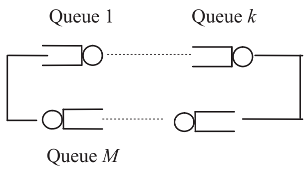



**Figure 1.1** A closed loop of $M$ queues

There are some fundamental results that can be deduced from this axiomatic definition of probability and we summarize them without proofs in the following propositions.

Proposition 1.1

(i) $P(\varnothing) = 0$
(ii) $P(\bar{E}) + P(E) = 1$ for any event $E$ in $\Omega$, where $\bar{E}$ is the compliment of $E$.
(iii) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$, for any events $E$ and $F$.
(iv) $P(E) \le P(F)$, if $E \subseteq F$.
(v) $P\left(\bigcup_i E_i\right) = \sum_i P(E_i)$, for $E_i \cap E_j = \varnothing$, when $i \neq j$.

Example 1.1

By considering the situation where we have a closed loop of $M$ identical queues, as shown in Figure 1.1, then calculate the probability that Queue 1 is non-empty (it has at least one customer) if there are $N$ customers circulating among these queues.

Solution

To calculate the required probability, we need to find the total number of ways of distributing those $N$ customers among $M$ queues. Let $X_i(>0)$ be the number of customers in Queue $i$, then we have

$$X_1 + X_2 + \ldots + X_M = N$$

The problem can now be formulated by having these $N$ customers lined up together with $M$ imaginary zeros, and then dividing them into $M$ groups. These $M$ zeros are introduced so that we may have empty queues. They also ensure that one of the queues will contain all the customers, even in the case where



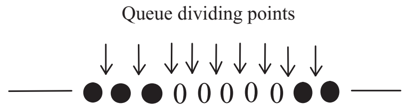



**Figure 1.2** $N$ customers and $M$ zeros, $(N + M - 1)$ spaces

all zeros are consecutive because there are only $(M - 1)$ spaces among them, as shown in Figure 1.2.

We can select $M - 1$ of the $(N + M - 1)$ spaces between customers as our separating points and hence the number of combinations is given by

$$\binom{N+M-1}{M-1} = \binom{N+M-1}{N}.$$

When Queue 1 is empty, the total number of ways of distributing $N$ customers among $(M - 1)$ queues is given by

$$\binom{N+M-2}{N}.$$

Therefore, the probability that Queue 1 is non-empty:

$$\begin{aligned} &= 1 - \binom{N+M-2}{N} \Big/ \binom{N+M-1}{N} \\ &= 1 - \frac{M-1}{N+M-1} = \frac{N}{N+M-1} \end{aligned}$$

Example 1.2

Let us suppose a tourist guide likes to gamble with his passengers as he guides them around the city on a bus. On every trip, there are about 50 random passengers. Each time he challenges his passengers by betting that if there is at least two people on the bus that have the same birthday, then all of them would have to pay him \$1 each. However, if there were none for that group on that day, he would repay each of them \$1. What is the likelihood (or probability) of the event that he wins his bet?

Solution

Let us assume that each passenger is equally likely to have their birthday on any day of the year (we will neglect leap years). In order to solve this problem we need to find the probability that nobody on that bus has the same birthday. Imagine that we line up these 50 passengers, and the first passenger has 365 days to choose as his/her birthday. The next passenger has the remainder of 364 days to choose from in order for him/her not to have the same birthday as the first person (i.e. he has a probability of 364/365). This number of choices reduces until the last passenger. Therefore:

$$P(\text{None of the 50 passengers has the same birthday}) = \underbrace{\left(\frac{364}{365}\right)\left(\frac{363}{365}\right)\left(\frac{362}{365}\right)\cdots\left(\frac{365-49}{365}\right)}_{49 terms}$$

Therefore, the probability that the tourist guide wins his bet can be obtained by Proposition 1.1 (ii):

$$P(\text{At least 2 passengers out of 50 has the same birthday}) = 1-\left(\frac{\prod_{j=1}^{49}(365-j)}{365^{49}}\right) = 0.9704.$$

The odds are very much to the favour of tourist guide, although we should remember this probability has a limiting value of (1.1) only.

### 1.1.2 Conditional Probability and Independence

In many practical situations, we often do not have information about the outcome of an event but rather information about related events. Is it possible to infer the probability of an event using the knowledge that we have about these other events? This leads us to the idea of *conditional probability* that allows us to do just that!

Conditional probability that an event $E$ occurs, given that another event $F$ has already occurred, denoted by $P(E|F)$, is defined as

$$P(E|F) = \frac{P(E\cap F)}{P(F)} \quad \text{where} \quad P(F) \neq 0 \tag{1.2}$$

Conditional probability satisfies the axioms of probability and is a probability measure in the sense of those axioms. Therefore, we can apply any results obtained for a normal probability to a conditional probability. A very useful expression, frequently used in conjunction with the conditional probability, is the so-called *Law of Total Probability*. It says that if $\{A_i \in \Omega, i = 1, 2, \ldots, n\}$ are events such that



(i) $A_i \cap A_j = \varnothing$ *if* $i \neq j$

(ii) $P(A_i) > 0$

(iii) $\bigcup_{i=1}^{n} A_i = \Omega$

then for any event $E$ in the same sample space:

$$P(E) = \sum_{i=1}^{n} P(E\cap A_i) = \sum_{i=1}^{n} P(E|A_i)P(A_i) \tag{1.3}$$

This particular law is very useful for determining the probability of a complex event $E$ by first conditioning it on a set of simpler events $\{Ai\}$ and then by summing up all the conditional probabilities. By substituting the expression (1.3) in the previous expression of conditional probability (1.2), we have the well-known *Bayes' formula*:

$$P(E|F) = \frac{P(E\cap F)}{\sum_i P(F|A_i)P(A_i)} = \frac{P(F|E)P(E)}{\sum_i P(F|A_i)P(A_i)} \tag{1.4}$$

Two events are said to be statistically independent if and only if

$$P(E \cap F) = P(E)P(F).$$

From the definition of conditional probability, this also implies that

$$P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{P(E)P(F)}{P(F)} = P(E) \tag{1.5}$$

Students should note that the statistical independence of two events $E$ and $F$ does not imply that they are mutually exclusive. If two events are mutually exclusive then their intersection is a null event and we have

$$P(E|F) = \frac{P(E\cap F)}{P(F)} = 0 \quad \text{where} \quad P(F) \neq 0 \tag{1.6}$$

Example 1.3

Consider a switching node with three outgoing links A, B and C. Messages arriving at the node can be transmitted over one of them with equal probability. The three outgoing links are operating at different speeds and hence message transmission times are 1, 2 and 3 ms, respectively for A, B and C. Owing to an average call duration of 2.5 minutes, what is the offered load presented to this PBX?

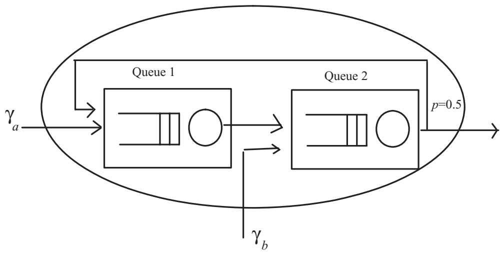
Solution

(a) The average transmission time
$= (0.2 \times 48 + 0.8 \times 480)/9600$
$= 0.041$ s
$\rho = 15 \times 0.041 = 61.5\%$

(b) *Offered load* = *Arrival rate* × *Service time*
= 300 (*users*) × 1.5 (*calls per user per hour*)
× 2.5 (minutes *percall*) ÷ 60 (minutes *perhour*)
= 18.75 *erlangs*

Example 2.3

Consider the queueing system shown below where we have customers arriving to Queue 1 and Queue 2 with rates $\gamma_a$ and $\gamma_b$, respectively. If the branching probability $p$ at Queue 2 is 0.5, calculate the effective arrival rates to both queues.




Solution

Denote the effective arrival rates to Queue 1 and Queue 2 as $\lambda_1$ and $\lambda_2$, respectively, then we have the following expressions under the principal of flow conservation:

$$\lambda_1 = \gamma_a + 0.5\lambda_2$$

$$\lambda_2 = \lambda_1 + \gamma_b$$



Hence, we have

$$\lambda_1 = 2\gamma_a + \gamma_b$$

$$\lambda_2 = 2(\gamma_a + \gamma_b)$$

## 2.7 POISSON PROCESS

Poisson process is central to physical process modelling and plays a pivotal role in classical queuing theory. In most elementary queueing systems, the inter-arrival times and service times are assumed to be exponentially distributed or, equivalently, that the arrival and service processes are Poisson, as we shall see below. The reason for its ubiquitous use lies in the fact that it possesses a number of marvellous probabilistic properties that give rise to many elegant queueing results. Secondly, it also closely resembles the behaviour of numerous physical phenomenon and is considered to be a good model for an arriving process that involves a large number of similar and independent users.

Owing to the important role of Poisson process in our subsequent modelling of arrival processes to a queueing system, we will take a closer look at it and examine here some of its marvellous properties.

Put simply, a Poisson process is a counting process for the number of randomly occurring point events observed in a given time interval $(0, t)$. It can also be deemed as the limiting case of placing at random $k$ points in the time interval of $(0, t)$. If the random variable $X(t)$ that counts the number of point events in that time interval is distributed according to the well-known Poisson distribution given below, then that process is a Poisson process:

$$P[X(t) = k] = \frac{(\lambda t)^k}{k!} e^{-\lambda t} \tag{2.21}$$

Here, $\lambda$ is the rate of occurrence of these point events and $\lambda t$ is the mean of a Poisson random variable and physically it represents the average number of occurrences of the event in a time-interval $t$. Poisson distribution is named after the French mathematician, Simeon Denis Poisson.

### 2.7.1 The Poisson Process – A Limiting Case

The Poisson process can be considered as a limiting case of the Binomial distribution of a Bernoulli trial. Assuming that the time interval $(0, t)$ is divided into time slots and each time slot contains only one point, if we place points

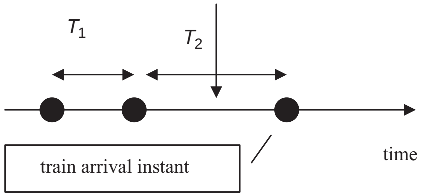



**Figure 2.13** Sample train arrival instants

arrival. In other words, the process starts afresh at the time of observation and has no memory of the past. Before we deal with the formal definition, let us look at an example to illustrate this concept.

Example 2.4

Consider the situation where trains arrive at a station according to a Poisson process with a mean inter-arrival time of 10 minutes. If a passenger arrives at the station and is told by someone that the last train arrived 9 minutes ago, so on the average, how long does this passenger need to wait for the next train?

Solution

Intuitively, we may think that 1 minute is the answer, but the correct answer is 10 minutes. The reason being that Poisson process, and hence the exponential inter-arrival time distribution, is memoryless. What have happened before were sure events but they do not have any influence on future events.

This apparent 'paradox' lies in the renewal theory and can be explained qualitatively as such. Though the average inter-arrival time is 10 minutes, if we look at two intervals of inter-train arrival instants, as shown in Figure 2.13, a passenger is more likely to arrive within a longer interval $T_2$ rather than a short interval of $T_1$. The average length of the interval in which a customer is likely to arrive is twice the length of the average inter-arrival time. We will re-visit this problem quantitatively in Chapter 5.

Mathematically, the 'memoryless' property states that the distribution of remaining time until the next arrival, given that $t_0$ units of time have elapsed since the last arrival, is identically equal to the unconditional distribution of inter-arrival times (Figure 2.14).

Assume that we start observing the process immediately after an arrival at time 0. From Equation (2.21) we know that the probability of no arrivals in $(0, t_0)$ is given by

$$P[\textit{no arrival in } (0, t_0)] = e^{-\lambda t_0}$$



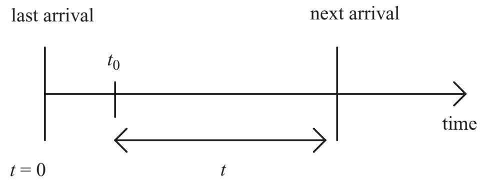



**Figure 2.14** Conditional inter-arrival times

Let us now find the conditional probability that the first arrival occurs in $[t_0, t_0 + t]$, given that $t_0$ has elapsed; that is

$$P[\textit{arrival in } (t_0, t_0+t) | \textit{no arrival in} (0, t_0)] = \frac{\int_{t_0}^{t_0+t} \lambda e^{-\lambda t}}{e^{-\lambda t_0}} = 1 - e^{-\lambda t}$$

But the probability of an arrival in $(0, t)$ is also $\int_0^t \lambda e^{-\lambda t} dt = 1 - e^{-\lambda t}$

Therefore, we see that the conditional distribution of inter-arrival times, given that certain time has elapsed, is the same as the unconditional distribution. It is this memoryless property that makes the exponential distribution ubiquitous in stochastic models. Exponential distribution is the only continuous function that has this property; its discrete counterpart is the geometry distribution.

## 2.8.5 Poisson Arrivals During a Random Time Interval

Consider the number of arrivals ($N$) in a random time interval $I$. Assuming that $I$ is distributed with a probability density function $A(t)$ and $I$ is independent of the Poisson process, then

$$P(N = k) = \int_0^\infty P(N = k | I = t) A(t) dt$$

But
$$P(N = k | I = t) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}$$

Hence
$$P(N = k) = \int_0^\infty \frac{(\lambda t)^k}{k!} e^{-\lambda t} A(t) dt$$

Taking the z-transform, we obtain

$$N(z) = \sum_{k=0}^{\infty}\left(\int_0^{\infty}\frac{(\lambda t)^k}{k!}e^{-\lambda t}A(t)dt\right)z^k$$
$$= \int_0^{\infty} e^{-\lambda t}\sum_{k=0}^{\infty}\frac{(\lambda tz)^k}{k!}A(t)dt$$
$$= \int_0^{\infty} e^{-(\lambda-\lambda z)t}A(t)dt$$
$$= A^*(\lambda - \lambda z)$$

where $A^*(\lambda - \lambda z)$ is the Laplace transform of the arrival distribution evaluated at the point $(\lambda - \lambda z)$.

Example 2.5

Let us consider again the problem presented in Example 2.4. When this passenger arrives at the station:

a) What is the probability that he will board a train in the next 5 minutes?
b) What is the probability that he will board a train in 5 to 9 minutes?

Solution

a) From Example 2.4, we have $\lambda = 1/10 = 0.1\ \text{min}^{-1}$, hence for a time period of 5 minutes we have

$$\lambda t = 5 \times 0.1 = 0.5 \quad \text{and}$$
$$P[0\ \textit{train in}\ 5\,\text{min}] = \frac{e^{-\lambda t}(\lambda t)^k}{k!} = \frac{e^{-0.5}(0.5)^0}{0!} = 0.607$$

He will board a train if at least one train arrives in 5 minutes; hence

$$P[\textit{at least}\ 1\ \textit{train in}\ 5\ \text{min}] = 1 - P[0\ \textit{train in}\ 5\ \text{min}] = 0.393$$

b) He will need to wait from 5 to 9 minutes if no train arrives in the first 5 minutes and board a train if at least one train arrives in the time interval 5 to 9 minutes. From (a) we have

$$P[0\ \textit{train in}\ 5\ \text{min}] = 0.607$$



and $P[\textit{at least}\ 1\ \textit{train in next}\ 4\ \text{min}] = 1 - P[0\ \textit{train in next}\ 4\ \text{min}]$

$$= 1 - \frac{e^{-0.4}(0.4)^0}{0!} = 0.33$$

Hence, $P[0\ \textit{train in}\ 5\ \text{min}\ \&\ \textit{at least}\ 1\ \textit{train in next}\ 4\ \text{min}]$
$= P[0\ \textit{train in}\ 5\ \text{min}] \times P[\textit{at least}\ 1\ \textit{train in next}\ 4\ \text{min}]$
$= 0.607 \times 0.33 = 0.2$

Example 2.6

Pure Aloha is a packet radio network, originated at the University of Hawaii, that provides communication between a central computer and various remote data terminals (nodes). When a node has a packet to send, it will transmit it immediately. If the transmitted packet collides with other packets, the node concerned will re-transmit it after a random delay $\tau$. Calculate the throughput of this pure Aloha system.

Solution

For simplicity, let us make the following assumptions:

(i) The packet transmission time is one (one unit of measure).
(ii) The number of nodes is large, hence the total arrival of packets from all nodes is Poisson with rate $\lambda$.
(iii) The random delay $\tau$ is exponentially distributed with density function $\beta e^{-\beta\tau}$, where $\beta$ is the node's retransmission attempt rate.

Given these assumptions, if there are $n$ node waiting for the channel to re-transmit their packets, then the total packet arrival presented to the channel can be assumed to be Poisson with rate $(\lambda + n\beta)$ and the throughput $S$ is then given by

$$S = (\lambda + n\beta)P[\textit{a successful transmission}]$$
$$= (\lambda + n\beta)P_{succ}$$

From Figure 2.15, we see that there will be no packet collision if there is only one packet arrival within two units of time. Since the total arrival of packets is assumed to be Poisson, we have

$$P_{succ} = e^{-2(\lambda+n\beta)}$$

and hence

$$S = (\lambda + n\beta)e^{-2(\lambda+n\beta)}$$

Taking Laplace transform, we have

$$\frac{\mu-\lambda}{s+(\mu-\lambda)} = L[f_w(s)]\frac{\mu}{s+\mu}$$

$$L(f_w(s)] = \frac{(s+\mu)(1-\rho)}{s+(\mu-\lambda)}$$

$$= (1-\rho) + \frac{\lambda(1-\rho)}{s+(\mu-\lambda)} \qquad (4.29)$$

Inverting

$$f_w(t) = (1-\rho)\delta(t) + \lambda(1-\rho)e^{-(\mu-\lambda)t} \qquad (4.30)$$

where $\delta(t)$ is the impulse function or Dirac delta function.

Integrating we have the waiting time cumulative distribution function $F_w(t)$:

$$F_w(t) = 1 - \rho e^{-(\mu-\lambda)t} \qquad (4.31)$$

Example 4.1

At a neighhourhood polyclinic, patients arrive according to a Poisson process with an average inter-arrival time of 18 minutes. These patients are given a queue number upon arrival and will be seen, by the only doctor manning the clinic, according to their queue number. The length of a typical consultation session is found from historical data to be exponentially distributed with a mean of 7 minutes:

(i) What is the probability that a patient has to wait to see the doctor?
(ii) What is the average number of patients waiting to see the doctor?
(iii) What is the probability that there are more than 5 patients in the clinic, including the one in consultation with the doctor?
(iv) What is the probability that a patient would have to wait more than 10 minutes for this consultation?
(v) The polyclinic will employ an additional doctor if the average waiting time of a patient is at least 7 minutes before seeing the doctor. By how much must the arrival rate increase in order to justify the additional doctor?

Solution

Assuming the clinic is sufficiently large to accommodate a large number of patients, the situation can be modelled as an M/M/1 queue. Given the following parameters:



$$\lambda = 1/18 \text{ person/min} \qquad \text{and} \qquad \mu = 1/7 \text{ person/min}$$

We have $\rho = 7/18$:

(i) $P$[patient has to wait] $= \rho = 1 - P_0 = 7/18$
(ii) The waiting-queue length $N_q = \rho^2/(1-\rho) = 49/198$
(iii) $P[N \geq 5] = \rho^5 = 0.0089$
(iv) Since $P[waitingtime \leq 10] = 1 - \rho e^{-\mu(1-\rho)t}$,

$$P[\text{queueing time} > 10] = \frac{7}{18}e^{-1/7(1-7/18)\times 10} = 0.162$$

(v) Let the new arrival rate be $\lambda'$

then $\dfrac{7\lambda'}{\frac{1}{7}-\lambda'} \geq 7 \Rightarrow \lambda' = \dfrac{1}{14}$ person/min person/min.

Example 4.2

Variable-length data packets arrive at a communication node according to a Poisson process with an average rate of 10 packets per second. The single outgoing communications link is operating at a transmission rate of 64 kbits per second. As a first-cut approximation, the packet length can be assumed to be exponentially distributed with an average length of 480 characters in 8-bit ASCII format. Calculate the principal performance measures of this communication link assuming that it has a very large input buffer. What is the probability that 6 or more packets are waiting to be transmitted?

Solution

Again the situation can be modelled as an M/M/1 with the communication link being the server.

The average service time:

$$\bar{x} = \frac{480\times 8}{64000} = 0.06\,\text{s}$$

and the arrival rate

$$\lambda = 10 \text{ packets/second}$$

thus

$$\rho = \lambda\bar{x} = 10 \times 0.06 = 0.6$$

The parameters of interest can be calculated easily as:

$$N = \frac{\rho}{1-\rho} = \frac{3}{2} \text{ packets} \quad \text{and} \quad N_q = \frac{\rho^2}{1-\rho} = \frac{9}{10} \text{ packets}$$

$$T = \frac{1}{\mu-\lambda} = \frac{1}{2} \text{ seconds} \quad \text{and} \quad W = \frac{\bar{x}}{\rho^{-1}-1} = \frac{3}{10} \text{ seconds}$$

$$\begin{aligned} &P[\text{number of packets in the system} \geq 7] \\ &= \rho^7 = (0.6)^7 \\ &= 0.028 \end{aligned}$$

Example 4.3

In an ATM network, two types of packets, namely voice packets and data packets, arrive at a single channel transmission link. The voice packets are always accepted into the buffer for transmission but the data packets are only accepted when the total number of packets in the system is less than $N$.

Find the steady-state probability mass function of having k packets in the system if both packet streams are Poisson with rates $\lambda_1$ and $\lambda_2$, respectively. You may assume that all packets have exponentially distributed lengths and are transmitted at an average rate $\mu$.

Solution

Using local balance concept (Figure 4.5), we obtain the following equations:

$$P_k = \begin{cases} \dfrac{\lambda_1+\lambda_2}{\mu} P_{k-1} & k \leq N \\ \dfrac{\lambda_1}{\mu} P_{k-1} & k > N \end{cases}$$

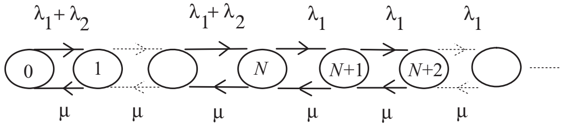



**Figure 4.5** Transition diagram for Example 4.3



Using back substitution, we obtain

$$\begin{aligned} P_k &= \left(\frac{\lambda_1+\lambda_2}{\mu}\right)P_{k-1} = \left(\frac{\lambda_1+\lambda_2}{\mu}\right)^2 P_{k-1} \quad \text{where} \quad \rho = \frac{\lambda_1+\lambda_2}{\mu} \\ &= \rho^k P_0 \end{aligned}$$

$$\begin{aligned} P_k &= \left(\frac{\lambda_1}{\mu}\right)P_{k-1} = \left(\frac{\lambda_1}{\mu}\right)^2 P_{k-2} \qquad \text{where} \quad \rho_1 = \frac{\lambda_1}{\mu} \\ &= \rho_1^{k-N} P_N = \rho_1^{k-N}\rho^N P_0 \end{aligned}$$

To find $P_0$, we sum all the probabilities to 1:

$$P_0\sum_{k=0}^{N}\rho^k + P_0\sum_{k=N+1}^{\infty}\rho_1^{k-N}\rho^N = 1$$

$$P_0\left\{\sum_{k=0}^{N}\rho^k + \left(\frac{\rho}{\rho_1}\right)^N\left[\sum_{k=0}^{\infty}\rho_1^k - \sum_{k=0}^{N}\rho_1^k\right]\right\} = 1$$

$$P_0 = \left[\frac{1-\rho^{N+1}}{1-\rho} + \frac{\rho^N\rho_1}{1-\rho_1}\right]^{-1}$$

Therefore, we have

$$P_k = \begin{cases} \rho^k\left[\dfrac{1-\rho^{N+1}}{1-\rho} + \dfrac{\rho^N\rho_1}{1-\rho_1}\right]^{-1} \\ \rho_1^{k-N}\rho^N\left[\dfrac{1-\rho^{N+1}}{1-\rho} + \dfrac{\rho^N\rho_1}{1-\rho_1}\right]^{-1} \end{cases}$$

Example 4.4

A computing facility has a small computer that is solely dedicated to batch-jobs processing. Job submissions get discouraged when the computer is heavily used and can be modelled as a Poisson process with an arrival rate $\lambda_k = \lambda/(k+1)$ for $k = 0,1,2\ldots$ when there are $k$ jobs with the computer. The time taken to process each job is exponentially distributed with mean $1/\mu$, regardless of the number of jobs in the system.

(i) Draw the state transitional-rates diagram of this system and write down the global as well as the local balance equation.

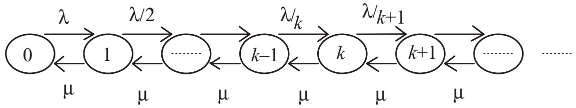



**Figure 4.6** Transition diagram for Example 4.4

(ii) Find the steady-state probability $P_k$ that there are $k$ jobs with the computer and then find the average number of jobs.
(iii) Find the average time taken by a job from submission to completion.

Solution

(i) With reference to node $k$ (Figure 4.6), we have

$$\text{Global:} \qquad P_k\left(\mu + \frac{\lambda}{k+1}\right) = P_{k-1}\frac{\lambda}{k} + P_{k+1}\mu$$

With reference to the boundary between nodes $k-1$ and $k$, we have

$$\text{Local:} \qquad \frac{\lambda}{k}P_k = \mu P_{k-1}$$

(ii) From the local balance equation, we obtain

$$P_k = \frac{\rho^2}{k(k-1)}P_{k-2} = \frac{\rho^k}{k!}P_0 \quad \text{where} \quad \rho = \lambda/\mu$$

Summing all the probabilities to 1, we have

$$\sum_{k=0}^{\infty}\frac{\rho^k}{k!}P_0 = 1 \quad \Rightarrow \quad P_0 = e^{-\rho}$$

Therefore

$$P_k = \frac{\rho^k}{k!}e^{-\rho}$$

Since $P_k$ is Poisson distributed, the average number of jobs $N = \rho$.



(iii) To use Little's theorem, we need to find the average arrival rate:

$$\bar{\lambda} = \sum_{k=0}^{\infty}\left(\frac{\lambda}{k+1}\right)\frac{\rho^k}{k!}e^{-\rho} = \mu e^{-\rho}\sum_{k=0}^{\infty}\frac{\rho^{k+1}}{(k+1)!}$$
$$= \mu e^{-\rho}(e^{\rho}-1) = \mu(1-e^{-\rho})$$

Therefore

$$T = \frac{N}{\bar{\lambda}} = \frac{\rho}{\mu(1-e^{-\rho})}$$

## 4.4 M/M/1/S QUEUEING SYSTEMS

The M/M/1 model discussed earlier is simple and useful if we just want to have a first-cut estimation of a system's performance. However, it becomes a bit unrealistic when it is applied to real-life problems, as most of them do have physical capacity constraints. Often we have a finite waiting queue instead of one that can accommodate an infinite number of customers. The M/M/1/S that we shall discuss is a more accurate model for this type of problem.

In *M/M/1/S*, the system can accommodate only $S$ customers, including the one being served. Customers who arrive when the waiting queue is full are not allowed to enter and have to leave without being served. The state transition diagram is the same as the classical M/M/1 queue except that it is truncated at state $S$, as shown in Figure 4.7. This truncation of state transition diagram will affect the queueing results through $P_0$.

From last section, we have

$$P_k = \rho^k P_0 \qquad \text{where} \qquad \rho = \lambda/\mu$$

Using the normalization equation but sums to S state, we have

$$P_0\sum_{k=0}^{S}\rho^k = 1 \quad \Rightarrow \quad P_0 = \frac{1-\rho}{1-\rho^{S+1}} \tag{4.32}$$

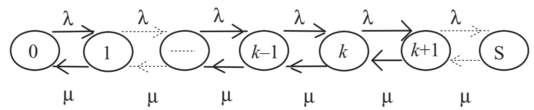



**Figure 4.7** M/M/1/S transition diagram

$$W = \frac{N_q}{\lambda'} = \frac{\rho}{\mu - \lambda} - \frac{S\rho^{S+1}}{\lambda - \mu\rho^{S+1}} \tag{4.43}$$

Example 4.5

Variable-length packets arrive at a network switching node at an average rate of 125 packets per second. If the packet lengths are exponentially distributed with a mean of 88 bits and the single outgoing link is operating at 19.2 kb/s, what is the probability of buffer overflow if the buffer is only big enough to hold 11 packets? On average, how many packets are there in the buffer? How big should the buffer be in terms of number of packets to keep packet loss below $10^{-6}$?

Solution

Given $\lambda = 125$ pkts/s $\quad \mu^{-1} = 88/19200 = 4.6 \times 10^{-3}$ s
$\rho = \lambda\mu^{-1} = 0.573$

$$P_S = \frac{(1-\rho)\rho^S}{1-\rho^{S+1}} = \frac{(1-0.573)(0.573)^{12}}{1-(0.573)^{13}}$$
$$= 5.35 \times 10^{-4}$$

$$N_q = \frac{\rho^2}{1-\rho} - \frac{S+\rho}{1-\rho}P_d = 0.75$$

$$P_d = \frac{(1-0.573)(0.573)^S}{1-(0.573)^{S+1}} \le 10^{-6}$$

The above equation is best solved by trial and error. By trying various numbers, we have

$$S = 23 \ Pd = 1.2 \times 10^{-6}$$

$$S = 24 \ Pd = 6.7 \times 10^{-7}$$

Therefore, a total of 24 buffers is required to keep packet loss below 1 packet per million.

Example 4.6

Let us compare the performance of a statistical multiplexer with that of a frequency-division multiplexer. Assume that each multiplexer has 4 inputs of 5



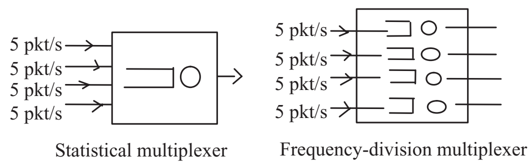



**Figure 4.9** System model for a multiplexer

packets/second and a multiplexed output at 64 kbps. The statistical multiplexer has a buffer of 3 packets for the combined stream of input packets, whereas the frequency-division multiplexer has a buffer of 3 packets for each of the channels.

Solution

From the multiplexing principles, we see that the statistical multiplexer can be modelled as an M/M/1/4 queue and the equivalent frequency-division multiplexer as an 4 M/M/1/4 queue with the service rate of each server equal to a quarter of the statistical multiplexer, as shown in Figure 4.9.

(a) Statistical multiplexer

We have $\lambda = \sum_{i=1}^{4} \lambda_i = 20$ pkts/s and $\mu = \frac{64000}{2000} = 32$ pkts/s

hence

$$\rho = 20/32 = 0.625$$

$$P_b = \frac{(1-\rho)\rho^4}{1-\rho^5} = 0.06325$$

$$N = \frac{\rho}{1-\rho} - \frac{\rho}{1-\rho}(S+1)P_b = 1.14 \text{ packets}$$

$$N_q = \frac{\rho^2}{1-\rho} - \frac{(S+\rho)\rho}{1-\rho}P_b = 0.555 \text{ packet}$$

Since there are 4 inputs, on average there are (1.14/4) = 0.285 packet per input in the system and (0.555/4) = 0.1388 packet per input in the buffer:

$$T = \frac{N}{\lambda(1-P_b)} = 0.06$$

$$W = \frac{N_q}{\lambda(1-P_b)} = 0.03$$

(b) Frequency-division multiplexer
We have $\lambda = 5$ pkts/s and $\mu = 32/4 = 8$ pkts/s, hence $\rho = 0.625$

$$P_b = \frac{(1-\rho)\rho^4}{1-\rho^5} = 0.06325$$

$$N = \frac{\rho}{1-\rho} - \frac{\rho}{1-\rho}(S+1)P_b = 1.14$$

$$N_q = \frac{\rho^2}{1-\rho} - \frac{(s+\rho)\rho}{1-\rho}P_b = 0.555$$

$$T = \frac{N}{\lambda(1-P_b)} = 0.24$$

$$W = \frac{N_q}{\lambda(1-P_b)} = 0.118$$

We see that the number of packets per input in the system as well as in the waiting queue increases four-fold. The delays have also increased by four times.

## 4.5 MULTI-SERVER SYSTEMS – M/M/m

Having examined the classical single-server queueing system, it is natural for us now to look at its logical extension; the multi-server queueing system in which the service facility consists of $m$ identical parallel servers, as shown in Figure 4.10. Here, identical parallel servers mean that they all perform the same functions and a customer at the head of the waiting queue can go to any of the servers for service.

If we again focus our attention on the system state $N(t)$, then $\{N(t), t \geq 0\}$ is a birth-death process with state-dependent service rates. When there is one customer, one server is engaged in providing service and the service rate is $\mu$. If there are two customers, then two servers are engaged and the total service rate is $2\mu$, so the service rate increases until $m\mu$ and stays constant thereafter.

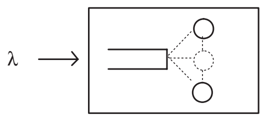



**Figure 4.10** A multi-server system model



process of some other real-life problems, but so long as the discrepancy is small, it can be treated as the first-cut approximation.

The exponential service time assumption appears to be less ideal than the Poisson assumption but still offers fairly good results as far as voice networks are concerned. However, it may be inadequate in modelling packets or messages in data networks as the length of a packet is usually constrained by the physical implementation. The length could even be a constant, as in the case of ATM (Asynchronous Transfer Mode) networks. Nevertheless, the exponential distribution can be deemed as the worst-case scenario and will give us the first-cut estimation.

The M/M/1 queue and its variants can be used to study the performance measure of a switch with input buffer in a network. In fact, its multi-server counterparts have traditionally been employed in capacity planning of telephone networks. Since A K Erlang developed them in 1917, the M/M/m and M/M/m/m models have been extensively used to analyse the 'lost-calls-cleared' (or simply blocked-calls) and 'lost-calls-delayed' (queued-calls) telephone systems, respectively. A 'queued-calls' telephone system is one which puts any arriving call requests on hold when all the telephone trunks are engaged, whereas the 'blocked-calls' system rejects those arriving calls.

In a 'blocked-calls' voice network, the main performance criterion is to determine the probability of blocking given an offered load or the number of trunks (circuits) needed to provide certain level of blocking. The performance of a 'queued-calls' voice network is characterized by the Erlang C formula and its associated expressions.

Example 4.6

A trading company is installing a new 300-line PBX to replace its old existing over-crowded one. The new PBX will have a group of two-way external circuits and the outgoing and incoming calls will be split equally among them.

It has been observed from past experience that each internal telephone usually generated (call or receive) 20 minutes of voice traffic during a typical busy day. How many external circuits are required to ensure a blocking probability of 0.02?

Solution

In order for the new PBX to handle the peak load during a typical busy hour, we assume that the busy hour traffic level constitutes about 14% of a busy day's traffic.

Hence the total traffic presented to the PBX:

$$= 300 \times 20 \times 14\% \div 60$$
$$= 14 erlangs$$

The calculated traffic load does not account for the fact that trunks are tied up during call setups and uncompleted calls. Let us assume that these amount to an overhead factor of 10%.

Then the adjusted traffic = $14 \times (1 + 10\%) = 15.4 erlangs$

Using the Erlang B formula:

$$P_m = \frac{(15.4)^m / m!}{\sum_{k=0}^{m} (15.4)^k / k!} \leq 0.02$$

Again, we solve it by trying various numbers for $m$ and we have

$$m = 22 \quad P_m = 0.0254$$
$$m = 23 \quad P_m = 0.0164$$

Therefore, a total of 23 lines is needed to have a blocking probability of less than or equal to 0.02.

Example 4.7

At the telephone hot line of a travel agency, information enquiries arrive according to a Poisson process and are served by 3 tour coordinators. These tour coordinators take an average of about 6 minutes to answer an enquiry from each potential customer.

From past experience, 9 calls are likely to be received in 1 hour in a typical day. The duration of these enquiries is approximately exponential. How long will a customer be expected to wait before talking to a tour coordinator, assuming that customers will hold on to their calls when all coordinator are busy? On the average, how many customers have to wait for these coordinators?

Solution

The situation can be modelled as an M/M/3 queue with

$$\rho = \frac{\lambda}{m\mu} = \frac{9/60}{3 \times (1/6)} = 0.3$$



$$P = \left[\sum_{k=0}^{m-1} \frac{(m\rho)^k}{k!} + \frac{(m\rho)^m}{m!}\left(\frac{1}{1-\rho}\right)\right]^{-1}$$
$$= \left[\sum_{k=0}^{2} \frac{(0.9)^k}{k!} + \frac{(0.9)^3}{3!}\left(\frac{1}{1-0.3}\right)\right]^{-1}$$
$$= 0.4035$$

$$P_d = P_0 \frac{(m\rho)^m}{m!}\left(\frac{1}{1-\rho}\right)$$
$$= 0.4035 \times \frac{(0.9)^3}{3!} \times \frac{1}{1-0.3}$$
$$= 0.07$$

$$N_q = \frac{\rho}{1-\rho} P_d = 0.03$$

$$W = \frac{N_q}{\lambda} = \frac{0.03}{9/60} = 0.2 \text{ minute}$$

Example 4.8

A multi-national petroleum company leases a certain satellite bandwidth to implement its mobile phone network with the company. Under this implementation, the available satellite bandwidth is divided into $N_v$ voice channels operating at 1200 bps and $N_d$ data channels operating at 2400 bps. It was forecast that the mobile stations would collectively generate a Poisson voice stream with mean 200 voice calls per second, and a Poisson data stream with mean 40 messages per second. These voice calls and data messages are approximately exponentially distributed with mean lengths of 54 bits and 240 bits, respectively. Voice calls are transmitted instantaneously when generated and blocked when channels are not available, whereas data messages are held in a large buffer when channels are not available:

(i) Find $N_v$, such that the blocking probability is less than 0.02.
(ii) Find $N_d$, such that the mean message delay is less than 0.115 second.

Solution

(i) $\lambda_v = 200$ and $\mu_v^{-1} = 54/1200 = 9/200$
$(\lambda_v/\mu_v) = 9$

$$P_m = \frac{(\lambda_v/\mu_v)^m/m!}{\sum_{k=0}^{m}(\lambda_v/\mu_v)^k/k!}$$

$$\frac{(9)^{N_v}/N_v!}{\sum_{k=0}^{N_v}(9)^k/k!} \le 0.02 \text{ therefore } N_v \ge 15$$

(ii) $\lambda_d = 40$ and $\mu_d^{-1} = 240/2400 = 0.1$
$(\lambda_d/\mu_d) = 4$

$$T = \frac{1}{\mu_d} + \frac{P_d}{m\mu_d - \lambda_d}$$

$$\frac{1}{10} + \frac{P_d}{10N_d - \lambda_d} \le 0.115 \text{ and } N_d \ge 6$$

Example 4.9

A group of 10 video display units (VDUs) for transactions processing gain access to 3 computers ports via a data switch (port selector), as shown in Figure 4.14. The data switch merely performs connections between those VDUs and the computer ports.

If the transaction generated by each VDU can be deemed as a Poisson stream with rates of 6 transactions per hour, the length of each transaction is approximately exponentially distributed with a mean of 5 minutes. Calculate:

(i) The probability that all three computer ports are engaged when a VDU initiates a connection;
(ii) The average number of computer ports engaged;

It is assumed that a transaction initiated on a VDU is lost and will try again only after an exponential time of 10 minutes if it can secure a connection initially.

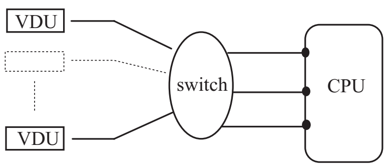



**Figure 4.14** A VDU-computer set up