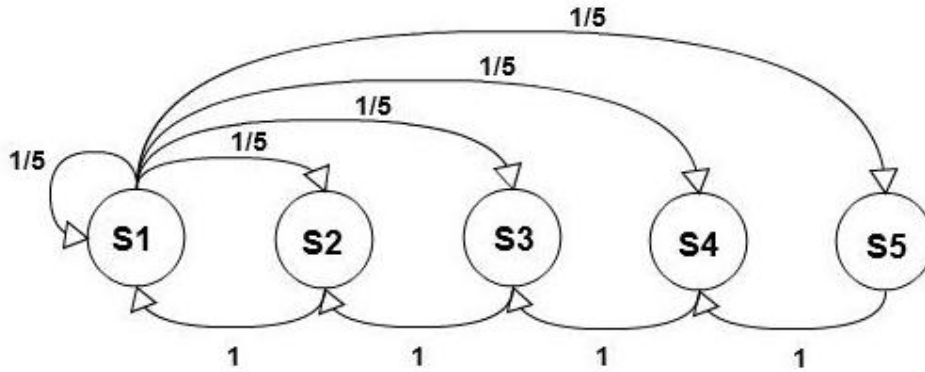


1. ZADATAK 1 Informacijski izvor u pravilnim vremenskim intervalima generira jedan od 5 simbola: s1, s2, s3, s4, s5. Ako je u generiran simbol s1, u sljedećem koraku bit će generiran bilo koji od 5 simbola s jednakom vjerojatnošću (uključujući s1). Ako je generiran simbol si, i > 1, sljedeći simbol koji će se generirati je si1. Pretpostavite da se ovaj izvor može modelirati Markovljevim lancem s diskretnim parametrom.

a) Dijagram stanja izvora



b) Matrica prijelaznih vjerojatnosti izvora

$$P = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

- c) Postavljena matrična jednadžbu oblika $A \cdot \hat{p} = \hat{b}$ čijim rješavanjem dolazimo do vektora stacionarnih vjerojatnosti.

$$P^T \cdot \hat{p} = \hat{p} \Rightarrow (P^T - I) \cdot \hat{p} = 0 \quad \sum \hat{p}_i = 1$$

$$A \cdot \hat{p} = \hat{b}$$

$$(P^T - I) = \begin{bmatrix} -\frac{4}{5} & 1 & 0 & 0 & 0 \\ \frac{1}{5} & -1 & 1 & 0 & 0 \\ \frac{1}{5} & 0 & -1 & 1 & 0 \\ \frac{1}{5} & 0 & 0 & -1 & 1 \\ \frac{1}{5} & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$A = \begin{bmatrix} -\frac{4}{5} & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ \frac{1}{5} & 0 & -1 & 1 & 0 \\ \frac{1}{5} & 0 & 0 & -1 & 1 \\ \frac{1}{5} & 0 & 0 & 0 & -1 \end{bmatrix} \quad \hat{p} = \begin{bmatrix} \hat{p}_1 \\ \hat{p}_2 \\ \hat{p}_3 \\ \hat{p}_4 \\ \hat{p}_5 \end{bmatrix} \quad \hat{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{4}{5} & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ \frac{1}{5} & 0 & -1 & 1 & 0 \\ \frac{1}{5} & 0 & 0 & -1 & 1 \\ \frac{1}{5} & 0 & 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} \hat{p}_1 \\ \hat{p}_2 \\ \hat{p}_3 \\ \hat{p}_4 \\ \hat{p}_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

- d) Postavljen nehomogen sustav jednadžbi čijim rješavanjem bismo dobili stacionarne vjerojatnosti stanja lanca.

$$P^{\top} \cdot \hat{p} = \hat{p}$$

$$\begin{bmatrix} \hat{p}_1 \\ \hat{p}_2 \\ \hat{p}_3 \\ \hat{p}_4 \\ \hat{p}_5 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & 1 & 0 & 0 & 0 \\ \frac{1}{5} & 0 & 1 & 0 & 0 \\ \frac{1}{5} & 0 & 0 & 1 & 0 \\ \frac{1}{5} & 0 & 0 & 0 & 1 \\ \frac{1}{5} & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \hat{p}_1 \\ \hat{p}_2 \\ \hat{p}_3 \\ \hat{p}_4 \\ \hat{p}_5 \end{bmatrix}$$

$$\hat{p}_1 = \frac{1}{5}\hat{p}_1 + \hat{p}_2 \quad \hat{p}_1 + \hat{p}_2 + \hat{p}_3 + \hat{p}_4 + \hat{p}_5 = 1$$

$$\hat{p}_2 = \frac{1}{5}\hat{p}_1 + \hat{p}_3 \quad \hat{p}_2 = \frac{4}{5}\hat{p}_1$$

$$\hat{p}_3 = \frac{1}{5}\hat{p}_1 + \hat{p}_4 \quad \hat{p}_3 = \frac{3}{5}\hat{p}_1$$

$$\hat{p}_4 = \frac{1}{5}\hat{p}_1 + \hat{p}_5 \quad \hat{p}_4 = \frac{2}{5}\hat{p}_1$$

$$\hat{p}_5 = \frac{1}{5}\hat{p}_1 \quad \hat{p}_5 = \frac{1}{5}\hat{p}_1$$

e) Stacionarne vjerojatnosti stanja lanca

$$\left(1 + \frac{4}{5} + \frac{3}{5} + \frac{2}{5} + \frac{1}{5}\right) = \frac{1}{\hat{p}}$$

$$\hat{p}_1 = \frac{1}{3} \quad \hat{p}_2 = \frac{4}{15} \quad \hat{p}_3 = \frac{1}{5}$$

$$\hat{p}_4 = \frac{2}{15} \quad \hat{p}_5 = \frac{1}{15}$$

f) Funkcija vjerojatnosti varijable koja mjeri broj intervala koje lanac provodi u istom stanju bez prekida za svako od stanja.

$$P(A_i) = q^{i-1} \cdot p$$

$$q = \frac{1}{5} \quad p = \frac{4}{5}$$

$$P(A_i) = \frac{4}{3125} \quad za \quad i = 1$$

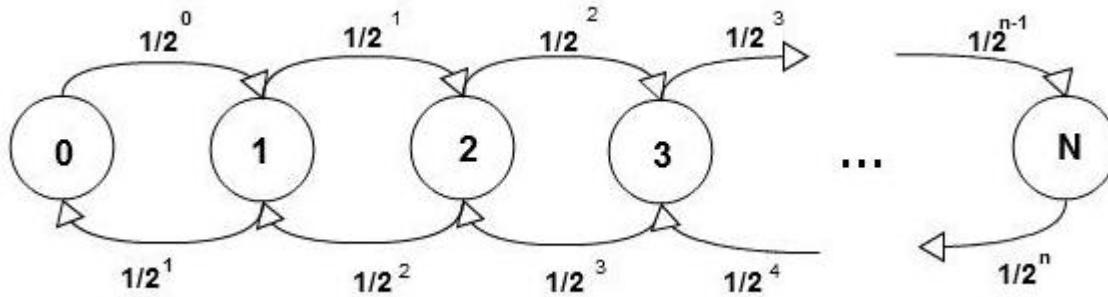
$$P(A_i) = 0 \quad za \quad i > 1$$

g) Prosječan broj koraka koje lanac bez prekida boravi u istom stanju

$$E[N] = \frac{p}{(1-q)^2} = \frac{1}{p} = \frac{5}{4} \quad za \quad i = 1$$

$$E[N] = 1 \quad za \quad i > 1$$

2. ZADATAK 2 Promatrajte Markovljev lanac diskretan po parametru sa stanjima 0, 1, 2,..., N. Vjerojatnost prijelaza iz stanja i u stanje i + 1 kada se lanac nalazi u stanju i je $(1/2)^i$, $i < N$. Vjerojatnost prijelaza iz stanja i u stanje i+1 je također $(1/2)^i$, $i > 0$. Ostali prijelazi nisu mogući. Izračunajte stacionarne vjerojatnosti ovog lanca.



$$p_0 = 2p_1$$

$$p_1 = 2p_0$$

$$\left(\frac{1}{2} + \frac{1}{2}\right)p_1 = \frac{1}{4}p_2 + p_0$$

$$p_2 = 4p_0$$

$$\left(\frac{1}{4} + \frac{1}{4}\right)p_2 = \frac{1}{2}p_1 + \frac{1}{8}p_0$$

$$p_3 = 8p_0$$

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$$\left(\frac{1}{2}\right)^n p_n = \left(\frac{1}{2}\right)^{n-1} p_{n-1}$$

$$p_n = 2p_{n-1} = 2^n p_0$$

$$p_0 + p_1 + p_2 + p_3 + \dots + p_n = 1$$

$$p_0 + 2p_0 + 4p_0 + 8p_0 + \dots + 2^n p_0 = 1$$

$$p_0 \cdot \sum_0^n 2^n = 1$$

$$p_0 = \frac{1}{\sum_0^n 2^n} = \frac{1}{2^{n+1}-1}$$

$$p_i = \frac{2^i}{2^{n+1}-1}$$