Teorija električnih strojeva i transformatora

Ak. god. 2012./2013.

Službeni podsjetnik

$$U = 4{,}44 \cdot f \cdot N \cdot \Phi$$

$$P_{\mathrm{h}} = k_{\mathrm{h}} f B^2$$

$$P_{\mathrm{v}} = k_{\mathrm{v}} f^2 B^2$$

$$b = \frac{235 + \vartheta_2}{235 + \vartheta_1}$$

$$Z_{\mathrm{b}} = \frac{U_f}{I_f} = 3\frac{U_f^2}{S_n}$$

$$\eta = 1 - \frac{P_0 + \alpha^2 P_{\mathrm{tn}}}{\alpha S_{\mathrm{n}} \cos\varphi_2}$$

$$\alpha_{\max} = \sqrt{\frac{P_0}{P_{\mathrm{tn}}}} \text{ za } \eta = \eta_{\max}$$

$$S_{\mathrm{a}} = S_{\mathrm{n}} \frac{U_{1\mathrm{a}}}{U_{1\mathrm{a}} - U_{2\mathrm{a}}}$$

$$u_{\mathrm{ka}} = u_{\mathrm{k}} \frac{U_{1\mathrm{a}} - U_{2\mathrm{a}}}{U_{1\mathrm{a}}}$$

$$S_{\mathrm{T}} = S_{\mathrm{n}} \left(1 + \frac{a_{\%} + b_{\%}}{200} + \frac{S_3}{2S_{\mathrm{n}}}\right)$$

$$\frac{S_{\mathrm{v}}}{S_{\mathrm{nv}}} = \alpha_{\mathrm{v}} = \frac{S}{u_{\mathrm{kv}} \left(\sum_{i=1}^{n} \frac{S_{\mathrm{ni}}}{u_{\mathrm{ki}}}\right)}$$

$$\vartheta = \vartheta_0 + (\vartheta_{\mathrm{m}} - \vartheta_0) \cdot \left(1 - e^{-\frac{t}{T}}\right)$$

$$\Delta u = \alpha \Big[u_{\mathrm{r}} \cos\varphi_{\mathrm{T}} + u_{\sigma} \sin\varphi_{\mathrm{T}} + +0{,}005\alpha \left(u_{\sigma} \cos\varphi_{\mathrm{T}} - u_{\mathrm{r}} \sin\varphi_{\mathrm{T}}\right)^2\Big]$$

$$\Theta_{\mathrm{x,tA}} = \Theta_{1\mathrm{A}} \cos\omega t \, \sin\frac{\pi}{\tau_{\mathrm{p}}} x$$

$$\Theta_{\mathrm{x,tB}} = \Theta_{1\mathrm{B}} \cos(\omega t - \frac{2\pi}{3}) \, \sin\left(\frac{\pi}{\tau_{\mathrm{p}}} x - \frac{2\pi}{3}\right)$$

$$\Theta_{\mathrm{x,tC}} = \Theta_{1\mathrm{C}} \cos(\omega t - \frac{4\pi}{3}) \sin\left(\frac{\pi}{\tau_{\mathrm{p}}} x - \frac{4\pi}{3}\right)$$

$$f_{\mathrm{z}} = \frac{\sin\left(q\nu \frac{\alpha_{\mathrm{el}}}{2}\right)}{q \sin\left(\nu \frac{\alpha_{\mathrm{el}}}{2}\right)}$$

$$f_{\mathrm{t}} = \sin\left(\nu \frac{y}{\tau_p} \frac{\pi}{2}\right)$$

$$\Theta_{\mathrm{f1}} = \frac{4}{\pi} \frac{I\sqrt{2}}{2a} \frac{w}{p} f_{\mathrm{n}}$$

$$\sum \Theta_{\mathrm{x,tABC}} = \Theta_{\mathrm{d}} + \Theta_{\mathrm{i}} = \frac{3}{2} \Theta_1 \sin\left(\frac{\pi}{\tau_{\mathrm{p}}} x - \omega t\right) + 0$$

$$\Theta(\alpha) = \int_0^{\alpha} A(\alpha)\, d\alpha$$

$$B_{\delta} = \frac{\mu_0}{\delta} \Theta_{\delta}$$

$$\Phi = B_{\delta} \frac{Dl}{p}$$

$$E_{v1} = \frac{\pi}{\sqrt{2}} \Phi \frac{pn}{60}$$

$$E = 4{,}44 \cdot f \cdot f_n \cdot w \cdot \Phi$$

$$M_r = \frac{Dp\pi l}{2} B \Theta_{\mathrm{r}} \sin\delta_{\mathrm{r}} = \frac{\pi}{\tau_p} V B \Theta_{\mathrm{r}} \sin\delta_{\mathrm{r}} = V B A_{\mathrm{r}} \sin\delta_{\mathrm{r}}$$

$$M_r = \frac{Dp\pi l}{2} \left(B \cos\beta\right) \Theta_{\mathrm{s}} \sin\delta_{\mathrm{s}} + \frac{Dp\pi l}{2} B \left(\Theta_{\mathrm{s}} \cos\delta_{\mathrm{s}}\right) \sin\beta$$

Razvoj funkcije u Fourierov red:

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{n} a_k \cos k \frac{2\pi}{T} x + \sum_{k=1}^{n} b_k \sin k \frac{2\pi}{T} x$$

$$a_k = \frac{2}{T} \int_0^T f(x) \cos k \frac{2\pi}{T} x$$

$$b_k = \frac{2}{T} \int_0^T f(x) \sin k \frac{2\pi}{T} x$$

Funkcija namota, rotirajući vektori, asinkroni strojevi:

$$L_{aa} = \frac{\psi_{aa}}{i_a} = \frac{\mu_0 r_0 l}{\delta} \int_0^{2\pi} \left[N_a(\alpha) \right]^2 d\alpha$$

$$L_{ab} = \frac{\psi_{ab}}{i_b} = \frac{\mu_0 r_0 l}{\delta} \int_0^{2\pi} N_a(\alpha) N_b(\alpha) d\alpha$$

$$N_a(\alpha) = \frac{4}{\pi} \frac{w_s}{2p} \sum_{\nu=1,3,5,\ldots}^{\infty} \frac{f_{ns\nu}}{\nu} \cos\left[\nu p \left(\alpha + \alpha_{0a} \right) \right]$$

$$\overline{i}_{abcs} = \frac{2}{3}\left(i_{as} + a i_{bs} + a^2 i_{cs} \right) \;, \quad a = e^{j\frac{2\pi}{3}}$$

$$\overline{f}_{abc} = \frac{2}{3}\left(f_a + a f_b + a^2 f_c \right)$$

$$f_0 = \frac{1}{3}\left(f_a + f_b + f_c \right)$$

$$\Re e\left(\overline{f}_{abc} \right) = f_a - f_0 \;\Rightarrow\; f_a = \Re e\left(\overline{f}_{abc} \right) + f_0$$

$$\Re e\left(a^2 \overline{f}_{abc} \right) = f_b - f_0 \;\Rightarrow\; f_b = \Re e\left(a^2 \overline{f}_{abc} \right) + f_0$$

$$\Re e\left(a \overline{f}_{abc} \right) = f_c - f_0 \;\Rightarrow\; f_c = \Re e\left(a \overline{f}_{abc} \right) + f_0$$

$$\left.\begin{aligned} \overline{U}_{sp} &= \frac{1}{3}\left(\overline{U}_{as} + a\overline{U}_{bs} + a^2 \overline{U}_{cs} \right) \\ \overline{U}_{sn} &= \frac{1}{3}\left(\overline{U}_{as} + a^2\overline{U}_{bs} + a \overline{U}_{cs} \right) \end{aligned}\right\}$$

$$\overline{u}_{dqs}^{s} = \overline{U}_{sp} e^{j\omega_s t} + \overline{U}_{sn}^{*} e^{-j\omega_s t}$$

$$\overline{f}_{dqx}^{y} = f_{dx}^{y} + j f_{qx}^{y}$$

$$\overline{f}_{dqr}^{s} = \overline{f}_{dqr}^{r} e^{j\alpha_r}$$

$$\overline{f}_{dqr}^{r} = \overline{f}_{dqr}^{s} e^{-j\alpha_r}$$

$$\overline{f}_{dqs}^{s} = \overline{f}_{dqs} e^{j\alpha}$$

$$\overline{f}_{dqs} = \overline{f}_{dqs}^{s} e^{-j\alpha}$$

$$\overline{f}_{dqr}^{r} = \overline{f}_{dqr} e^{j(\alpha - \alpha_r)}$$

$$\overline{f}_{dqr} = \overline{f}_{dqr}^{r} e^{-j(\alpha - \alpha_r)}$$

$$P_{abcs} = \frac{3}{2} P_{dqs} + 3P_{0s}$$

$$P_{abcr} = \frac{3}{2} P_{dqr} + 3P_{0r}$$

$$P_{meh} = \frac{3}{2} \omega_r L_m \Im m\left(\overline{i}_{dqs}^{s} \overline{i}_{dqr}^{\prime s*} \right)$$

$$M_{em} = \frac{3}{2} p L_m \Im m\left(\overline{i}_{dqs}^{s} \overline{i}_{dqr}^{\prime s*} \right)$$

$$M_{em} = \frac{3}{2} p \left(\psi_{ds}^{s} i_{qs}^{s} - \psi_{qs}^{s} i_{ds}^{s} \right)$$

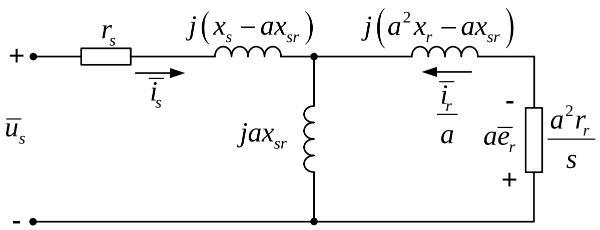
$$\boxed{M_{em(pu)} = \left| \overline{I}'_{rp(pu)} \right|^2 \frac{R'_{r(pu)}}{\omega_{s(pu)} s}}$$

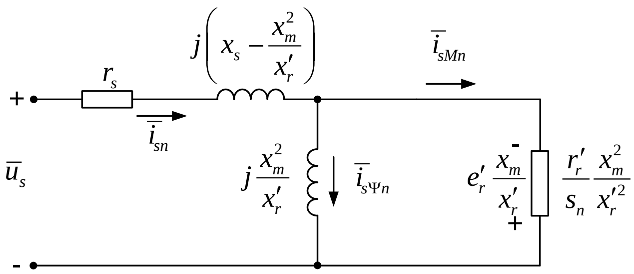
$$\boxed{M_{em} = \frac{3}{2} p \frac{L_{sr}}{L_r} \left| \overline{\Psi}_r \overline{I}_{sM} \right|}$$

$$\boxed{M_{em} = \frac{3}{2} p \frac{L_{sr}^2}{L_r} \left| \overline{I}_{s\psi} \right| \left| \overline{I}_{sM} \right|}$$

$$\boxed{s\omega_s = \frac{R_r}{L_r} \frac{\left| \overline{I}_{sM} \right|}{\left| \overline{I}_{s\psi} \right|}}$$

$$\boxed{\overline{\Psi}_r = L_{sr} \overline{I}_{s\psi}}$$





Sinkroni strojevi s trajnim magnetima:

$$u_{ds}^{r}=R_s i_{ds}^{r}+\frac{d\psi_{ds}^{r}}{dt}-\omega_r\psi_{qs}^{r}$$
$$u_{qs}^{r}=R_s i_{qs}^{r}+\frac{d\psi_{qs}^{r}}{dt}+\omega_r\psi_{ds}^{r}$$

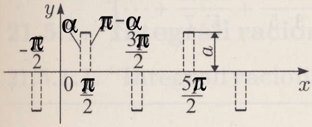
$$U_{ds}^{r}=R_s I_{ds}^{r}-X_q I_{qs}^{r}$$
$$U_{qs}^{r}=R_s I_{qs}^{r}+X_d I_{ds}^{r}+E_0$$
$$E_0=\omega_s\Psi_{md}$$

$$M_{em}=\frac{3}{2}\frac{p}{\omega_s}\left[E_0 I_{qs}^{r}+\left(X_d-X_q\right)I_{ds}^{r}I_{qs}^{r}\right]$$
$$M_{em}=\frac{3}{2}\frac{p}{\omega_s}\left[E_0 I_s\cos\gamma+\frac{1}{2}\left(X_d-X_q\right)I_s^2\sin\left(2\gamma\right)\right]$$
$$I_s=\sqrt{\left(I_{ds}^{r}\right)^2+\left(I_{qs}^{r}\right)^2}$$

$$\frac{\left(I_{ds}^{r}+\frac{\Psi_{md}}{L_d}\right)^2}{\left(\frac{U_{s\max}}{L_d\omega_s}\right)^2}+\frac{\left(I_{qs}^{r}\right)^2}{\left(\frac{U_{s\max}}{L_q\omega_s}\right)^2}=1$$

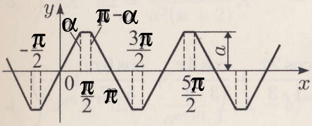
$$M_{em}=\Psi_{md}I_{qs}+\left(L_d-L_q\right)I_{ds}I_{qs},\ \text{pu}$$

6. $y = 0$ za $0 \le x < \alpha$ i za $\pi - \alpha < x \le \pi + \alpha$ i $2\pi - \alpha < x \le 2\pi$
$y = a$ za $\alpha < x < \pi - \alpha$; $y = -a$ za $\pi + \alpha < x \le 2\pi - \alpha$



$$y = \frac{4a}{\pi}\left(\cos\alpha\sin x + \frac{1}{3}\cos 3\alpha\sin 3x + \frac{1}{5}\cos 5\alpha\sin 5x + \cdots\right)$$

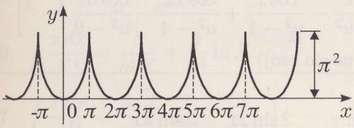
7. $y = \dfrac{ax}{\alpha}$ za $-a \le x \le a$
$y = a$ za $\alpha \le x \le \pi - \alpha$,
$y = \dfrac{a(\pi - x)}{\alpha}$ za $\pi - \alpha \le x \le \pi + \alpha$,
$y = -a$ za $\pi + \alpha \le x \le 2\pi - \alpha$



$$y = \frac{4}{\pi}\frac{a}{\alpha}\left(\sin\alpha\sin x + \frac{1}{3^2}\sin 3\alpha\sin 3x + \frac{1}{5^2}\sin 5\alpha\sin 5x + \cdots\right)$$

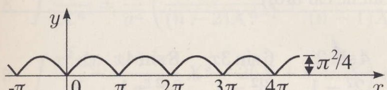
Posebno vrijedi za $\alpha = \dfrac{\pi}{3}$:
$$y = \frac{6\sqrt{3}a}{\pi^2}\left(\sin x - \frac{1}{5^2}\sin 5x + \frac{1}{7^2}\sin 7x - \frac{1}{11^2}\sin 11x + \cdots\right)$$

8. $y = x^2$ za $-\pi \le x \le \pi$



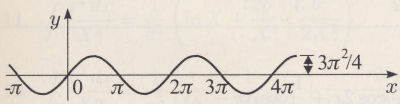
$$y = \frac{\pi^2}{3} - 4\left(\frac{\cos x}{1} - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} - \cdots\right)$$

9. $y = x(\pi - x)$ za $0 \le x \le \pi$



$$y = \frac{\pi^2}{6} - \left(\frac{\cos 2x}{1^2} + \frac{\cos 4x}{2^2} + \frac{\cos 6x}{3^2} + \cdots\right)$$

10. $y = x(\pi - x)$ za $0 \le x \le \pi$
$y = (\pi - x)(2\pi - x)$ za $\pi \le x \le 2\pi$



$$y = \frac{8}{\pi}\left(\sin x + \frac{1}{3^3}\sin 3x + \frac{1}{5^3}\sin 5x + \cdots\right)$$

Sustav jediničnih vrijednosti primijenjen na asinkroni stroj

Prikaz varijabli asinkronog stroja u jediničnim (per-unit) vrijednostima olakšava usporedbu strojeva različitih snaga te daje bolji uvid u relativne veličine njihovih parametara. Najčešće korišteni sustav jediničnih vrijednosti je baziran na nazivnoj snazi na osovini stroja. Osnovne bazne vrijednosti su onda:

Bazni napon: $U_B = \sqrt{2} U_{fn}$ – vršna vrijednost nazivnog faznog napona

Bazna snaga: $P_B = P_n$ – nazivna mehanička snaga na osovini motora

Bazna električna kutna brzina: $\omega_B = \omega_s = 2\pi f_n$ – nazivna električna kutna brzina.

Iz osnovnih vrijednosti slijede izvedene bazne vrijednosti:

Bazna struja: $I_B = \frac{2}{3}\frac{P_B}{U_B} = \frac{\sqrt{2}}{3}\frac{P_n}{U_{fn}}$

Bazni induktivitet: $L_B = \frac{Z_B}{\omega_B}$

Bazna impedancija: $Z_B = \frac{U_B}{I_B} = \frac{3U_{fn}^2}{P_n}$

Bazna mehanička kutna brzina: $\omega_{mB} = \frac{\omega_B}{p}$ (p – broj pari polova)

Bazni moment: $M_B = \frac{P_B}{\omega_{mB}} = p\frac{P_n}{\omega_s}$