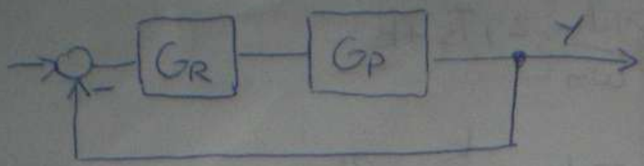


$k_s = 0.1$

$T = 10s$

$\xi = \frac{1}{\sqrt{2}}$

$\omega_n = 1$ rad/s

$$G_P(s) = \frac{k_s}{(1+Ts)\left(1+\frac{2\xi}{\omega_n}s + \frac{1}{\omega_n^2}s^2\right)}$$

$$G_R(s) = k_R \cdot \frac{1+T_I s}{T_I s}$$

$$G_0 = G_R \cdot G_P$$

$$k = k_s \cdot k_R$$

$$G(s) = \frac{G_0(s)}{1+G_0(s)} = \frac{k(1+T_I s)}{\underbrace{k}_{a_0} \cancel{+1} + \underbrace{\frac{T_I T}{\omega_n^2}}_{a_4} s^4 + \underbrace{T_I\left(\frac{1}{\omega_n^2}+\frac{2\xi T}{\omega_n}\right)}_{a_3} s^3 + \underbrace{T_I\left(T+\frac{2\xi}{\omega_n}\right)}_{a_2} s^2 + \underbrace{T_I(k+1)}_{a_1} s}$$

$$a_i^2 - 2a_{i-1}a_{i+1} = \frac{a_{i-1}^2}{b_{i-1}^2}\left(b_i^2 - 2b_{i-1}b_{i+1}\right)$$

$$T_I^{\cancel{2}}(k+1)^2 - 2k\cancel{T_I}\left(T+\frac{2\xi}{\omega_n}\right) = k^2 T_I^{\cancel{2}}$$ $i=1$ // mogli smo izvući i relaciju $i=3$ $\Rightarrow$ PD

$$\cancel{T_I^2}\left(T+\frac{2\xi}{\omega_n}\right)^2 - 2\cancel{T_I^2}(k+1)\left(\frac{1}{\omega_n^2}+2\xi\frac{T}{\omega_n}\right) = 0$$ $i=2$

$$k = \frac{\left(T+\frac{2\xi}{\omega_n}\right)^2}{\frac{2}{\omega_n^2}+4\frac{\xi}{\omega_n}} - 1 = 3.3$$

$$T_I = \frac{2k\left(T+\frac{2\xi}{\omega_n}\right)}{2k+1} = 9.9s$$

$$k_R = \frac{k}{k_s} = 33$$

u slučaju da smo odmah uzeli $T_1 = T$ imali bi

$$G(s) = \frac{G_0(s)}{1+G_0(s)} = \frac{k}{\frac{T}{\omega_n^2}s^3 + 2\xi\frac{T}{\omega_n}s^2 + Ts + k}$$

$$\Rightarrow k = \frac{T\omega_n}{4\xi} = 35.4 \Rightarrow k_R = \frac{k}{k_S} = 35.4$$

sada tražimo PID regulator:

$$G_R(s) = K_R \cdot \frac{1 + T_I s + T_I T_D s^2}{T_I s}$$

$$= k_R\left[1 + \frac{1}{T_I s} + T_D s\right]$$

$$G(s) = \frac{b_2 s^2 + b_1 s + b_0}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

$b_0 = k$ $\quad a_0 = k$ $\quad a_3 = T_I(1 + 10\sqrt{2})$

$b_1 = k \cdot T_I$ $\quad a_1 = T_I(k+1)$ $\quad a_4 = 10T_I$

$b_2 = kT_I T_D$ $\quad a_2 = T_I(kT_D + 10 + \sqrt{2})$

$$a_i^2 - 2a_{i-1}a_{i+1} = \frac{a_{i-1}^2}{b_{i-1}^2}(b_i - 2b_{i-1}b_{i+1})$$

$$T_I^2(k+1)^2 - 2k(kT_D + 10 + \sqrt{2}) = k^2 T_I^2 - 2k^2 T_I T_D \quad , i = 1$$

$$T_I^2(kT_D + 10 + \sqrt{2})^2 - 2T_I(k+1)(1 + 10\sqrt{2}) = \frac{(k+1)}{\cancel{k^2}}\cancel{k^2} T_I^2 T_D^2 \quad , i = 2$$

$$T_I^2(1 + 10\sqrt{2})^2 - 20T_I^2(kT_D + 10\sqrt{2}) = 0 \quad i = 3$$

$\downarrow$

$kT_D = 0.05$ (3) $\Rightarrow$ (2)

$$T_D^3 + 0.1T_D^2 - 101{,}14T_D + 1.51 = 0$$

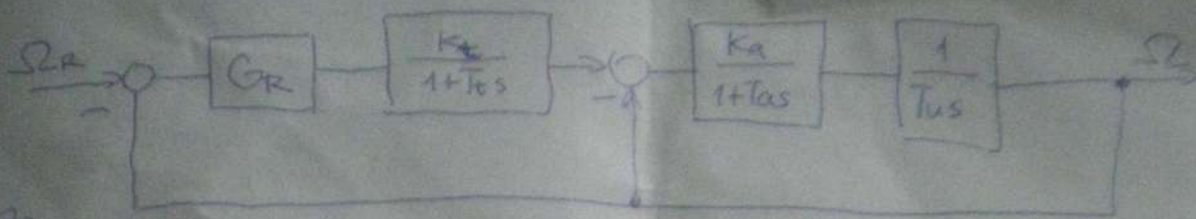
$\Rightarrow T_D = \cancel{10}$, $T_D = 10s$, $T_D = 15ms$

$k = 0.005$, $T_I = 0.11$

pojačanje premalo, a derivacijska konst. prevelika, ova treća vrij. konst. najbolje zadovoljava

$$T_I = \frac{2k}{2k+1}(10 + \sqrt{2}) \Rightarrow \begin{array}{l} k = 3.33 \\ k_R = 33.3 \\ T_I = 9.9s \end{array}$$

2) modelni optimum



$$G_R(s) = k \cdot \left(1 + \frac{1}{T_I s}\right)\left(1 + T_D s\right) = k_R \frac{(T_I s + 1)(T_D s + 1)}{T_I s}$$

$$G_{RM}(s) = \frac{k_a}{T_u T_a s^2 + T_u s + k_a} \qquad k = k_a \cdot k_T \cdot k_R$$

$$G_o(s) = G_R \cdot \frac{k_t}{1 + T_t s} \cdot G_{RM} \qquad G_{z}(s) = \frac{G_o(s)}{1 + G_o(s)}$$

$b_0 = k$ $\qquad a_0 = k$

$b_1 = k(T_I + T_D)$ $\qquad a_1 = k(T_I + T_D) + T_I k_a$

$b_2 = k T_I T_D$ $\qquad a_2 = T_I(k T_D + k_a T_t + T_u)$

$a_3 = T_I T_u (T_a + T_t)$

$a_4 = T_I T_t T_u T_a$

$$a_i^2 + 2 \sum_{j=1}^{i} (-1)^j a_{i-j} a_{i+j} = b_i^2 + 2 \sum_{j=1}^{i} (-1)^j b_{i-j} b_{i+j}$$

$$\boxed{\begin{aligned} a_1^2 - 2a_0 a_2 &= b_1^2 - 2b_0 b_1 \\ a_2^2 - 2a_1 a_3 + 2a_0 a_4 &= b_2^2 \\ a_3^2 - 2a_2 a_4 &= 0 \end{aligned}}$$

123) $k \cdot T_D = \dfrac{T_u (T_a + T_t)^2}{2 T_a T_t} - k_a T_t - T_u$

$k T_D = 0.98763$

$k T_I = 0.13765 + 0.004265\, T_I^2$

~~$T_I = 18000$~~

$T_I = 5$[illegible] ms $\quad T_D = 3.6$ ms $\quad k = 2.76$

$T_I = 14$ ms $\quad T_D = 0.1 \quad k = 9.9$