UPRAVLJANJE ELEKTROMOTORNIM POGONIMA
1. međuispit 2009.

1. Zadani podaci su:
$P_n = 2200\ \mathrm{W}$
$U_{an} = 120\ \mathrm{V}$
$I_{an} = 22{,}5\ \mathrm{A}$
$n_n = 390\ \mathrm{r/min}$
$R_a = 0{,}7\ \Omega$
$J_M = 0{,}05\ \mathrm{kgm^2}$
$J_{mz} = 0{,}015\ \mathrm{kgm^2}$
$J_{b+vz} = J_b + J_{vz} = 1{,}3\ \mathrm{kgm^2}$
$F = 1300\ \mathrm{N}$
$i = 20$
$\eta_{zp} = 0{,}8$
$\eta_b = 0{,}95$
$r_b = 0{,}6\ \mathrm{m}$
$U_{DC} = 120\ \mathrm{V}$

(a)

$$J_{UK} = J_M + J_{mz} + J_{b+vz}\frac{1}{\eta_{zp}}\frac{1}{i^2} + \frac{F}{g}r_b^2\frac{1}{\eta_{zp}}\frac{1}{\eta_b}\frac{1}{i^2}$$

$$J_{UK} = 0{,}05 + 0{,}015 + 1{,}3\cdot\frac{1}{0{,}8}\cdot\frac{1}{20^2} + \frac{1300}{9{,}81}\cdot 0{,}6^2\cdot\frac{1}{0{,}8}\cdot\frac{1}{0{,}95}\cdot\frac{1}{20^2} = 0{,}226\ \mathrm{kgm^2}$$

$$P_t = \eta_{zp}\eta_b P_m = \eta_{zp}\eta_b M_m\omega_m$$

$$P_t = F v_t = F\omega_t r_b$$

$$\eta_{zp}\eta_b M_m\omega_m = F\omega_t r_b \rightarrow M_m = \frac{F\omega_t r_b}{\eta_{zp}\eta_b\omega_m} = \frac{F r_b}{\eta_{zp}\eta_b}\frac{\omega_t}{\omega_m} = \frac{F r_b}{\eta_{zp}\eta_b i}$$

$$M_m = \frac{F r_b}{\eta_{zp}\eta_b i} = \frac{1300\cdot 0{,}6}{0{,}8\cdot 0{,}95\cdot 20} = 51{,}316\ \mathrm{Nm}$$

(b) Radi se o bipolarnoj modulaciji pa je:

$$U_a = (2D-1)U_{DC} = (2\cdot 0{,}75 - 1)\cdot 120 = 60\ \mathrm{V}$$

Konstante motora su:

$$c_e = \frac{U_{an} - I_{an}R_a}{n_n} = \frac{120 - 22{,}5\cdot 0{,}7}{390} = 0{,}26731\ {}^{\mathrm{Vmin}}/_{\mathrm{r}}$$

$$c_m = \frac{30 c_e}{\pi} = 2{,}5526\ {}^{\mathrm{Nm}}/_{\mathrm{A}}$$

Nazivni moment motora je:

$$M_n = \frac{30P_n}{n_n\pi} = \frac{30 \cdot 220}{390 \cdot \pi} = 53{,}86783 \text{ Nm}$$

Nazivni elektromagnetski moment motora je:

$$M_{emn} = I_{an}c_m = 22{,}5 \cdot 2{,}5526 = 57{,}43351 \text{ Nm}$$

Moment trenja i ventilacije je:

$$M_{tr,v} = M_{emn} - M_n = 3{,}56478 \text{ Nm}$$

Za izračunati moment na osovini, $M_m = 51{,}316$ Nm, slijedi da je elektromagnetski moment jednak:

$$M_{em} = M_m + M_{tr,v} = 54{,}88078 \text{ Nm}$$

Struja armature tada iznosi:

$$I_a = \frac{M_{em}}{c_m} = \frac{54{,}88078}{2{,}5526} = 21{,}5 \text{ A}$$

Brzina motora iznosi:

$$n = \frac{U_a - I_aR_a}{c_e} = \frac{60 - 21{,}5 \cdot 0{,}7}{0{,}26731} = 168{,}157 \ {}^{\text{r}}/_{\text{min}}$$

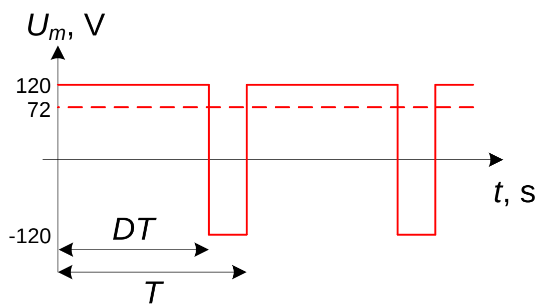
Brzina namatanja žice tada će biti:

$$\omega_m = \frac{n\pi}{30} = \frac{168{,}157 \cdot \pi}{30} = 17{,}61 \ {}^{\text{rad}}/_{\text{s}}$$

$$\omega_b = \frac{\omega_m}{i} = \frac{17{,}61}{20} = 0{,}8805 \ {}^{\text{rad}}/_{\text{s}}$$

$$v_b = \omega_b r_b = 0{,}8805 \cdot 0{,}6 = 0{,}528 \ {}^{\text{m}}/_{\text{s}}$$

(c)



2. Zadani podaci su:
$P_n = 28$ kW
$U_{an} = 400$ V
$I_{an} = 80$ A
$n_n = 1000$ r/min
$R_a = 0{,}27\ \Omega$

Konstante motora su:

$$c_e = \frac{U_{an} - I_{an}R_a}{n_n} = \frac{400 - 80 \cdot 0{,}27}{1000} = 0{,}3784\ {}^{\text{Vmin}}/_{\text{r}}$$

$$c_m = \frac{30c_e}{\pi} = 3{,}6135\ {}^{\text{Nm}}/_{\text{A}}$$

Nazivni moment motora je:

$$M_n = \frac{30P_n}{n_n\pi} = \frac{30 \cdot 28000}{1000 \cdot \pi} = 267{,}38\ \text{Nm}$$

Nazivni elektromagnetski moment motora je:

$$M_{emn} = I_{an}c_m = 80 \cdot 3{,}6135 = 289{,}08\ \text{Nm}$$

Nazivni moment trenja i ventilacije je:

$$M_{tr,vn} = M_{emn} - M_n = 21{,}7\ \text{Nm}$$

S obzirom na to da se radi o viskoznom trenju, slijedi:

$$M_{tr,vn} = b\omega_n = b\frac{n_n\pi}{30} \rightarrow b = \frac{30M_{tr,vn}}{n_n\pi} = \frac{30 \cdot 21{,}7}{1000 \cdot \pi} = 0{,}20722\ {}^{\text{Nms}}/_{\text{rad}}$$

(a) Radi se o elektrodinamičkom kočenju pa je $U_a = 0$, dok je smjer struje suprotan (predznak minus):

$$n = \frac{U_a - I_a(R_a + R_p)}{c_e} = \frac{0 - (-0{,}7I_{an})(0{,}27 + 2)}{0{,}3784} = \frac{0{,}7 \cdot 80 \cdot 2{,}27}{0{,}3784} = 335{,}941\ {}^{\text{r}}/_{\text{min}}$$

(b) Vrijedi:

$$I_a(R_a + R_p) = 60 \cdot (0{,}27 + 13) = 796{,}2\ \text{V} > U_{an} = 400\ \text{V}$$

Radi se o protustrujnom kočenju. Brzina vrtnje je jednaka:

$$n = \frac{U_{an} - I_a(R_a + R_p)}{c_e} = \frac{400 - 796{,}2}{0{,}3784} = -1047{,}04\ {}^{\text{r}}/_{\text{min}}$$

Iznos elektromagnetsko momenta je:

$$M_{em} = I_a c_m = 60 \cdot 3{,}6135 = 216{,}81 \text{ Nm}$$

Moment trenja i ventilacije je:

$$M_{tr,v} = b\omega = b\frac{n\pi}{30} = 0{,}20722\frac{1047{,}04 \cdot \pi}{30} = 22{,}72 \text{ Nm}$$

Radna točka je u četvrtom kvadrantu. Vrijedi:

$$M_{os} = M_{em} + M_{tr,v} = 216{,}81 + 22{,}72 = 239{,}53 \text{ Nm}$$

Snaga uzeta iz mreže je:

$$P_{mr} = U_{an} I_a = 400 \cdot 60 = 24000 \text{ W}$$

Snaga utrošena na otporima je:

$$P_{otp} = I_a^2 (R_a + R_p) = 60^2 \cdot 13{,}27 = 47772 \text{ W}$$

(c) Iznos struje je:

$$I_a = \frac{U_{an} - c_e n}{R_a} = \frac{400 - 0{,}3784 \cdot 1100}{0{,}27} = -60{,}15 \text{ Nm}$$

***n* − *M* dijagram**

Brzina praznog hoda za **(b)** i **(c)** dio zadatka je:

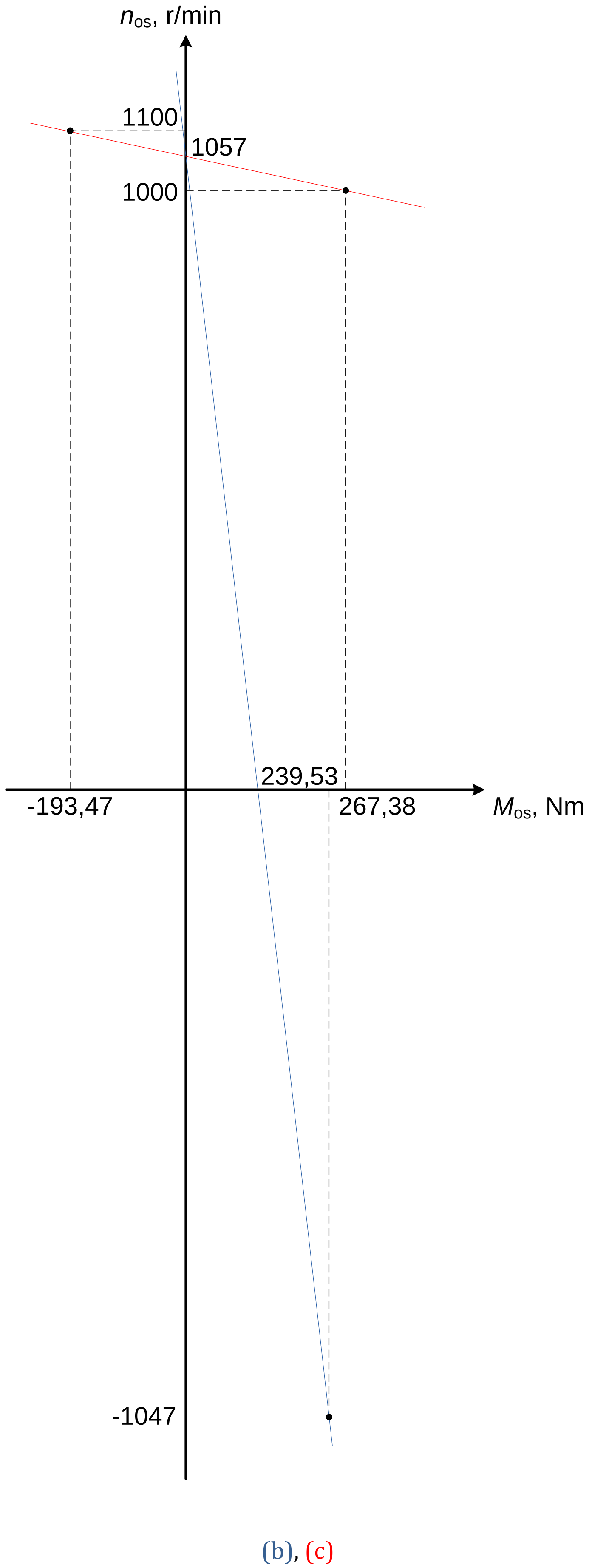
$$n = \frac{U_{an}}{c_e} = \frac{400}{0{,}3784} = 1057{,}082 \ {}^{\text{r}}/_{\text{min}}$$

Moment na osovini za **(c)** dio zadatka:

$$M_{tr,v} = b\omega = b\frac{n\pi}{30} = 0{,}20722\frac{1100 \cdot \pi}{30} = 23{,}87 \text{ Nm}$$

$$M_{em} = I_a c_m = -60{,}15 \cdot 3{,}6135 = -217{,}34 \text{ Nm}$$

$$M_{os} = M_{em} + M_{tr,v} = -193{,}47 \text{ Nm}$$

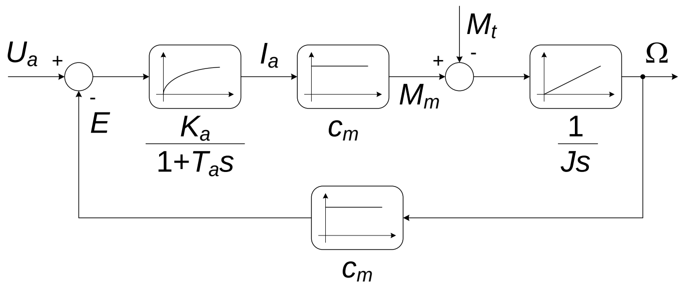


(b), (c)

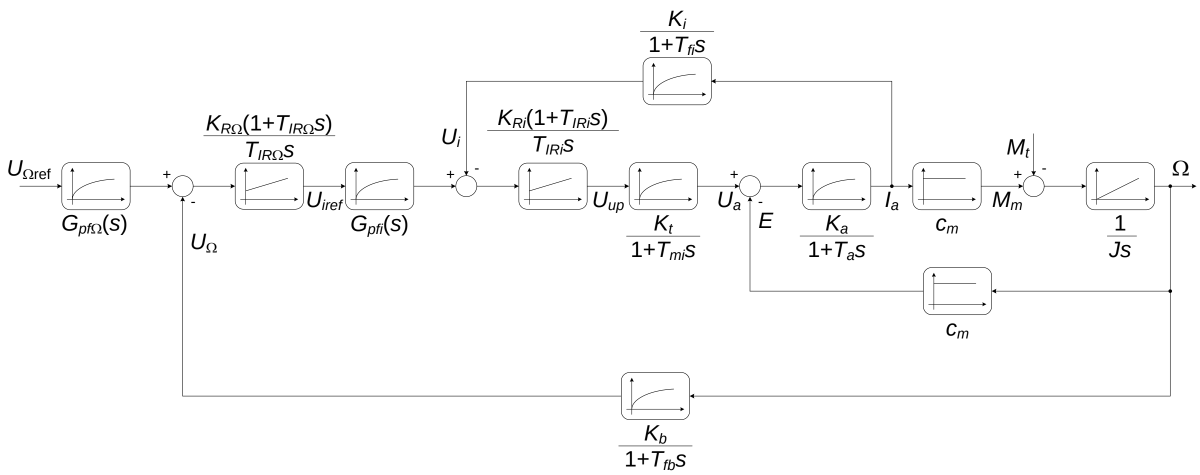
3. Zadani podaci su:

$P_n = 13{,}5\ \mathrm{kW}$
$U_{an} = 420\ \mathrm{V}$
$I_{an} = 40{,}6\ \mathrm{A}$
$U_{un} = 242\ \mathrm{V}$
$I_{un} = 2{,}3\ \mathrm{A}$
$n_n = 1480\ \mathrm{r/min}$
$R_a = 1{,}83\ \Omega$
$L_a = 10{,}71\ \mathrm{mH}$
$c_e = c_m = 2{,}147\ \mathrm{Vs/rad}$
$J = 0{,}151\ \mathrm{kgm^2}$
$L_a = 10{,}71\ \mathrm{mH}$
$K_t = 0{,}25$
$T_{mi} = 1\ \mathrm{ms}$
$K_i = 0{,}5\ \mathrm{V/A}$
$T_{fi} = 2\ \mathrm{ms}$
$K_b = 1\ \mathrm{Vs}$
$T_{fb} = 15\ \mathrm{ms}$

(a)



(b) Strukturni blokovski prikaz regulacijskog sustava kaskadne regulacije brzine vrtnje istosmjernog motora s nezavisnom i konstantnom uzbudom dan je na slici ispod. Pri tome su $G_{pfi}(s)$ i $G_{pf\Omega}(s)$ prefiltri za struju armature, odnosno brzinu vrtnje motora, koji se dodaju po potrebi.



Sinteza regulatora struje armature

Vrijede sljedeći izrazi (u ovoj fazi izvoda se ne koristi prefiltar $G_{pfi}(s)$):

$$I_a(s) = [U_a(s) - E(s)]\frac{K_a}{1+T_a s}$$

$$U_i(s) = \frac{K_i}{1+T_{fi}s} I_a(s)$$

$$U_a(s) = [U_{iref}(s) - U_i(s)]K_{Ri}\frac{1+T_{IRi}s}{T_{IRi}s}\frac{K_t}{1+T_{mi}s}$$

Kada se drugi izraz ubaci u treći izraz, a potom to sve u prvi izraz, dobije se:

$$I_a(s) = \left\{\left[U_{iref}(s) - \frac{K_i}{1+T_{fi}s}I_a(s)\right]K_{Ri}\frac{1+T_{IRi}s}{T_{IRi}s}\frac{K_t}{1+T_{mi}s} - E(s)\right\}\frac{K_a}{1+T_a s}$$

Za dovoljno veliki odnos $\frac{T_m}{T_a}$ može se pri sintezi regulatora struje armature uzeti da je $E = 0$. Vrijedi $\frac{T_m}{T_a} = \frac{\frac{JR_a}{c_e c_m}}{\frac{L_a}{R_a}} = \frac{JR_a^2}{c_e c_m L_a} = \frac{0{,}151 \cdot 1{,}83^2}{2{,}147^2 \cdot 10{,}71 \cdot 10^{-3}} = 10{,}243$ pa će se uzeti $E = 0$:

$$I_a(s) = \left\{\left[U_{iref}(s) - \frac{K_i}{1+T_{fi}s}I_a(s)\right]K_{Ri}\frac{1+T_{IRi}s}{T_{IRi}s}\frac{K_t}{1+T_{mi}s}\right\}\frac{K_a}{1+T_a s}$$

$$I_a(s) = U_{iref}(s)K_{Ri}\frac{1+T_{IRi}s}{T_{IRi}s}\frac{K_t}{1+T_{mi}s}\frac{K_a}{1+T_a s} - \frac{K_i}{1+T_{fi}s}I_a(s)K_{Ri}\frac{1+T_{IRi}s}{T_{IRi}s}\frac{K_t}{1+T_{mi}s}\frac{K_a}{1+T_a s}$$

$$I_a(s)\left[1 + \frac{K_{Ri}K_iK_tK_a}{T_{IRi}s}\frac{1+T_{IRi}s}{(1+T_{fi}s)(1+T_{mi}s)(1+T_a s)}\right] = U_{iref}(s)\frac{K_{Ri}K_tK_a}{T_{IRi}s}\frac{1+T_{IRi}s}{(1+T_{mi}s)(1+T_a s)}$$

$$\frac{I_a(s)}{U_{iref}(s)} = \frac{\frac{K_{Ri}K_tK_a}{T_{IRi}s}\frac{1+T_{IRi}s}{(1+T_{mi}s)(1+T_a s)}}{1 + \frac{K_{Ri}K_iK_tK_a}{T_{IRi}s}\frac{1+T_{IRi}s}{(1+T_{fi}s)(1+T_{mi}s)(1+T_a s)}}$$

Vrijedi $T_a = \frac{L_a}{R_a} = \frac{10{,}71 \cdot 10^{-3}}{1{,}83} = 5{,}852$ ms, $T_{mi} = 1$ ms i $T_{fi} = 2$ ms. Iz toga slijedi da je T_a dominantna vremenska konstanta. **Regulacijski krug struje armature** pogodan je za projektiranje regulatora po **tehničkom optimumu.** Kompenzira se dominantna vremenska konstanta pa je:

$$T_{IRi} = T_a = 5{,}852 \text{ ms}$$

Vrijedi $T_\Sigma = T_{mi} + T_{fi} = 3$ ms. Strukturnim pojednostavljenjem, uz uvrštavanje $T_{IRi} = T_a$, se dobiva:

$$\frac{I_a(s)}{U_{iref}(s)} = \frac{\dfrac{K_{Ri}K_tK_a}{T_as(1+T_{mi}s)}}{1+\dfrac{K_{Ri}K_iK_tK_a}{T_as\left(1+T_{fi}s\right)(1+T_{mi}s)}} = \frac{\dfrac{K_{Ri}K_tK_a}{T_as(1+T_{mi}s)}}{\dfrac{T_as\left(1+T_{fi}s\right)(1+T_{mi}s)+K_{Ri}K_iK_tK_a}{T_as\left(1+T_{fi}s\right)(1+T_{mi}s)}}$$

$$\frac{I_a(s)}{U_{iref}(s)} = \frac{K_{Ri}K_tK_a\left(1+T_{fi}s\right)}{T_as\left(1+T_{fi}s\right)(1+T_{mi}s)+K_{Ri}K_iK_tK_a} = \frac{K_{Ri}K_tK_a\left(1+T_{fi}s\right)}{T_as(1+T_\Sigma s)+K_{Ri}K_iK_tK_a}$$

$$\frac{I_a(s)}{U_{iref}(s)} = \left(1+T_{fi}s\right)\frac{K_{Ri}K_tK_a}{T_aT_\Sigma s^2+T_as+K_{Ri}K_iK_tK_a}$$

$$\frac{I_a(s)}{U_{iref}(s)} = \left(1+T_{fi}s\right)\frac{1}{K_{Ri}K_iK_tK_a}\frac{K_{Ri}K_tK_a}{\dfrac{T_aT_\Sigma}{K_{Ri}K_iK_tK_a}s^2+\dfrac{T_a}{K_{Ri}K_iK_tK_a}s+1}$$

$$\frac{I_a(s)}{U_{iref}(s)} = \frac{1+T_{fi}s}{K_i}\frac{1}{\dfrac{T_aT_\Sigma}{K_{Ri}K_iK_tK_a}s^2+\dfrac{T_a}{K_{Ri}K_iK_tK_a}s+1}$$

Promatramo dio $\frac{1}{\frac{T_aT_\Sigma}{K_{Ri}K_iK_tK_a}s^2+\frac{T_a}{K_{Ri}K_iK_tK_a}s+1}$ koji je oblika $\frac{1}{1+\frac{2\zeta}{\omega_n}s+\frac{s^2}{\omega_n^2}}$, pa slijedi:

$$\frac{1}{\omega_n^2} = \frac{T_aT_\Sigma}{K_{Ri}K_iK_tK_a}$$

$$\frac{2\zeta}{\omega_n} = \frac{T_a}{K_{Ri}K_iK_tK_a}$$

Izbor $\zeta = \frac{\sqrt{2}}{2}$ predstavlja tehnički najprihvatljiviji izbor za većinu primjena:

$$\frac{2\zeta}{\omega_n} = \frac{T_a}{K_{Ri}K_iK_tK_a} \rightarrow \frac{1}{\omega_n} = \frac{1}{2\zeta}\frac{T_a}{K_{Ri}K_iK_tK_a} = \frac{1}{\sqrt{2}}\frac{T_a}{K_{Ri}K_iK_tK_a}$$

Kada se gornji izraz kvadrira, slijedi:

$$\frac{1}{\omega_n^2} = \frac{1}{2}\frac{T_a^2}{(K_{Ri}K_iK_tK_a)^2}$$

Izjednačavanjem izraza $\frac{1}{\omega_n^2} = \frac{T_aT_\Sigma}{K_{Ri}K_iK_tK_a}$ i $\frac{1}{\omega_n^2} = \frac{1}{2}\frac{T_a^2}{(K_{Ri}K_iK_tK_a)^2}$ slijedi:

$$\frac{T_aT_\Sigma}{K_{Ri}K_iK_tK_a} = \frac{1}{2}\frac{T_a^2}{(K_{Ri}K_iK_tK_a)^2}$$

$$T_\Sigma = \frac{1}{2}\frac{T_a}{K_{Ri}K_iK_tK_a} \rightarrow K_{Ri} = \frac{T_a}{2T_\Sigma K_iK_tK_a} = \frac{\frac{L_a}{R_a}}{2(T_{mi}+T_{fi})K_iK_t\frac{1}{R_a}} = \frac{L_a}{2(T_{mi}+T_{fi})K_iK_t}$$

$$K_{Ri} = \frac{10{,}71}{2\cdot 3\cdot 0{,}5\cdot 0{,}25} = 14{,}28$$

Prijenosna funkcija $\frac{I_a(s)}{U_{iref}(s)}$ sada postaje (uvrštavanjem $K_{Ri} = \frac{T_a}{2T_\Sigma K_iK_tK_a}$):

$$\frac{I_a(s)}{U_{iref}(s)} = \frac{1+T_{fi}s}{K_i}\frac{1}{\frac{T_aT_\Sigma}{\frac{T_a}{2T_\Sigma K_iK_tK_a}K_iK_tK_a}s^2 + \frac{T_a}{\frac{T_a}{2T_\Sigma K_iK_tK_a}K_iK_tK_a}s + 1}$$

$$\frac{I_a(s)}{U_{iref}(s)} = \frac{1+T_{fi}s}{K_i}\frac{1}{1+2T_\Sigma s+2T_\Sigma^2 s^2}$$

S obzirom na to da se u prijenosnoj funkciji $\frac{I_a(s)}{U_{iref}(s)}$ pojavljuje nula zbog mjernog člana struje armature, koristi se prefiltar:

$$G_{pfi}(s) = \frac{1}{1+T_{fi}s}$$

pa se dobije:

$$\frac{I_a(s)}{U_{iref}(s)} = \frac{1}{K_i}\frac{1}{1+2T_\Sigma s+2T_\Sigma^2 s^2}$$

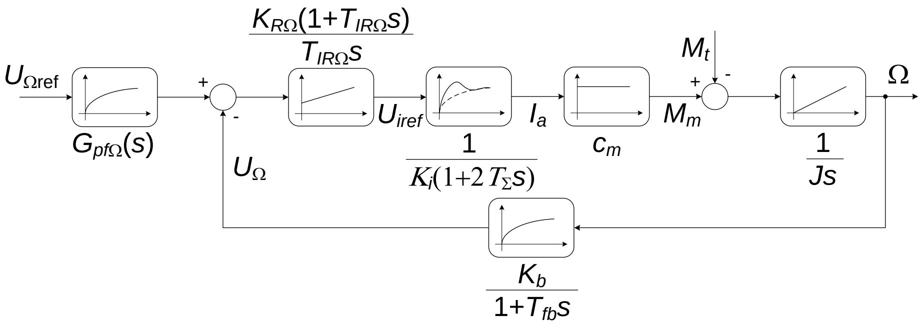
S obzirom na to da je regulacijski krug struje armature podređen regulacijskom krugu brzine vrtnje, vrlo je praktično, sa stajališta sinteze regulatora brzine vrtnje, strukturno pojednostaviti prijenosnu funkciju, tj. nadomjestiti je prijenosnom funkcijom:

$$\frac{I_a(s)}{U_{iref}(s)} \approx \frac{1}{K_i}\frac{1}{1+T_s s}$$

gdje je $T_s = 2T_\Sigma = 6$ ms nadomjesna vremenska konstanta.

Sinteza regulatora brzine vrtnje

Uz prethodno uzeta pojednostavljena, strukturna shema postaje:



Proces kojim upravlja regulator brzine vrtnje ima prijenosnu funkciju:

$$G_{p\Omega}(s)=\frac{1}{K_i}\frac{1}{1+2T_\Sigma s}c_m\frac{1}{Js}\frac{K_b}{1+T_{fb}s}$$

S obzirom na to da je $T_m=\frac{JR_a}{c_e c_m}$, slijedi $J=\frac{T_m c_e c_m}{R_a}=T_m K_a c_m^2$. To se uvrsti u gornju jednadžbu pa slijedi:

$$G_{p\Omega}(s)=\frac{1}{K_i}\frac{1}{1+2T_\Sigma s}\frac{1}{T_m K_a c_m s}\frac{K_b}{1+T_{fb}s}=\frac{K_x}{(1+2T_\Sigma s)\left(1+T_{fb}s\right)}\frac{1}{T_m s}$$

pri čemu je $K_x=\frac{K_b}{K_i K_a c_m}$.

Vrijedi $T_m=\frac{JR_a}{c_e c_m}=\frac{0{,}151\cdot 1{,}83}{2{,}147^2}=59{,}947$ ms, $2T_s=4T_\Sigma=12$ ms i $T_{fb}=15$ ms. Iz toga slijedi da je T_m dominantna vremenska konstanta. Izraz za proces može se pojednostaviti na sljedeći način:

$$G_{p\Omega}(s)=\frac{K_x}{1+T_\Sigma^* s}\frac{1}{T_m s}$$

pri čemu je $T_\Sigma^*=2T_\Sigma+T_{fb}=6+15=21$ ms.

Regulacijski krug brzine vrtnje pogodan je za projektiranje regulatora po **simetričnom optimumu.** Vrijedi:

$$G_0(s)=G_{R\Omega}(s)G_{p\Omega}(s)=K_{R\Omega}\frac{1+T_{IR\Omega}s}{T_{IR\Omega}s}\frac{K_x}{1+T_\Sigma^* s}\frac{1}{T_m s}$$

$$G_0(j\omega)=-\frac{K_{R\Omega}K_x}{T_{IR\Omega}T_m\omega^2}\frac{1+j\omega T_{IR\Omega}}{1+j\omega T_\Sigma^*}$$

Slijedi da je izraz za fazno-frekvencijsku karaktersitiku jednak:

$$\varphi_0(j\omega)=-180°+\mathrm{arctg}(\omega T_{IR\Omega})-\mathrm{arctg}(\omega T_\Sigma^*)$$

Za maksimalnu vrijednost fazno-frekvencijske karakteristike dobije se:

$$\frac{d\varphi_0(j\omega)}{d\omega}=\frac{T_{IR\Omega}}{1+(\omega T_{IR\Omega})^2}-\frac{T_\Sigma^*}{1+(\omega T_\Sigma^*)^2}=0\rightarrow T_{IR\Omega}+T_{IR\Omega}(\omega_m T_\Sigma^*)^2-T_\Sigma^*-T_\Sigma^*(\omega_m T_{IR\Omega})^2=0$$

$$T_{IR\Omega}+T_{IR\Omega}\omega_m^2 T_\Sigma^{*2}-T_\Sigma^*-T_\Sigma^*\omega_m^2 T_{IR\Omega}^2=0\rightarrow\omega_m^2 T_{IR\Omega}T_\Sigma^*(T_\Sigma^*-T_{IR\Omega})=T_\Sigma^*-T_{IR\Omega}$$

$$\omega_m=\frac{1}{\sqrt{T_{IR\Omega}T_\Sigma^*}}$$

Ako se odabere da je presječna frekvencija $\omega_c=\omega_m$, dobiju se simetrična amplitudno-frekvencijska i simetrična fazno-frekvencijska karakteristika. Neka je integracijska vremenska konstanta regulatora $T_{IR\Omega}=a^2T_\Sigma^*$. Uz odabir $a=2$, slijedi:

$$T_{IR\Omega}=4T_\Sigma^*=4\cdot 21=84\text{ ms}$$

Presječna frekvencija jednaka je $\omega_c = \frac{1}{\sqrt{4T_\Sigma^* T_\Sigma^*}} = \frac{1}{2T_\Sigma^*} = \frac{1}{42} = 0{,}0238\ \mathrm{s}^{-1}$.

Na presječnoj frekvenciji vrijedi:

$$|G_0(j\omega_c)| = 1$$

$$\left|\frac{K_{R\Omega}K_x}{T_{IR\Omega}T_m\omega_c^2}\frac{1+j\omega_c T_{IR\Omega}}{1+j\omega_c T_\Sigma^*}\right| = 1 \rightarrow \frac{K_{R\Omega}K_x}{T_{IR\Omega}T_m\omega_c^2}\sqrt{\frac{1+\omega_c^2T_{IR\Omega}^2}{1+\omega_c^2T_\Sigma^{*2}}} = 1$$

$$K_{R\Omega} = \frac{T_{IR\Omega}T_m\omega_c^2}{K_x}\sqrt{\frac{1+\omega_c^2T_\Sigma^{*2}}{1+\omega_c^2T_{IR\Omega}^2}} = \frac{4T_\Sigma^*T_m\frac{1}{4T_\Sigma^{*2}}}{K_x}\sqrt{\frac{1+\frac{1}{4T_\Sigma^{*2}}T_\Sigma^{*2}}{1+\frac{1}{4T_\Sigma^{*2}}16T_\Sigma^{*2}}} = \frac{\frac{T_m}{T_\Sigma^*}}{K_x}\sqrt{\frac{\frac{5}{4}}{5}} = \frac{1}{2K_x}\frac{T_m}{T_\Sigma^*}$$

$$K_{R\Omega} = \frac{1}{2\frac{K_b}{K_iK_ac_m}}\frac{T_m}{T_\Sigma^*} = \frac{K_iK_ac_m}{2K_b}\frac{T_m}{T_\Sigma^*} = \frac{K_ic_m}{2K_bR_a}\frac{\frac{JR_a}{c_ec_m}}{T_\Sigma^*} = \frac{K_i}{2K_bc_m}\frac{J}{T_\Sigma^*}$$

$$K_{R\Omega} = \frac{0{,}5}{2\cdot 1\cdot 2{,}147}\frac{0{,}151}{21\cdot 10^{-3}} = 0{,}8373$$

Vrijede sljedeći izrazi (u ovoj fazi izvoda se ne koristi prefiltar $G_{pf\Omega}(s)$):

$$\Omega(s) = [M_m(s) - M_t(s)]\frac{1}{Js}$$

$$U_\Omega(s) = \frac{K_b}{1+T_{fb}s}\Omega(s)$$

$$M_m(s) = [U_{\Omega ref}(s) - U_\Omega(s)]K_{R\Omega}\frac{1+T_{IR\Omega}s}{T_{IR\Omega}s}\frac{1}{K_i}\frac{1}{1+2T_\Sigma s}c_m$$

Kada se drugi izraz ubaci u treći izraz, a potom to sve u prvi izraz, dobije se:

$$\Omega(s) = \left\{\left[U_{\Omega ref}(s) - \frac{K_b}{1+T_{fb}s}\Omega(s)\right]K_{R\Omega}\frac{1+T_{IR\Omega}s}{T_{IR\Omega}s}\frac{1}{K_i}\frac{1}{1+2T_\Sigma s}c_m - M_t(s)\right\}\frac{1}{Js}$$

Uzima se $M_t(s) = 0$:

$$\Omega(s) = \left\{\left[U_{\Omega ref}(s) - \frac{K_b}{1+T_{fb}s}\Omega(s)\right]K_{R\Omega}\frac{1+T_{IR\Omega}s}{T_{IR\Omega}s}\frac{1}{K_i}\frac{1}{1+2T_\Sigma s}c_m\right\}\frac{1}{Js}$$

$$\Omega(s) = U_{\Omega ref}(s)K_{R\Omega}\frac{1+T_{IR\Omega}s}{T_{IR\Omega}s}\frac{1}{K_i}\frac{1}{1+2T_\Sigma s}c_m\frac{1}{Js} - \frac{K_b}{1+T_{fb}s}\Omega(s)K_{R\Omega}\frac{1+T_{IR\Omega}s}{T_{IR\Omega}s}\frac{1}{K_i}\frac{1}{1+2T_\Sigma s}c_m\frac{1}{Js}$$

$$\Omega(s)\left[1+\frac{K_{R\Omega}K_b c_m(1+T_{IR\Omega}s)}{K_i T_{IR\Omega}s\left(1+T_{fb}s\right)(1+2T_\Sigma s)Js}\right]=U_{\Omega ref}(s)\frac{K_{R\Omega}c_m(1+T_{IR\Omega}s)}{K_i T_{IR\Omega}s(1+2T_\Sigma s)Js}$$

$$\frac{\Omega(s)}{U_{\Omega ref}(s)}=\frac{\dfrac{K_{R\Omega}c_m(1+T_{IR\Omega}s)}{K_i T_{IR\Omega}s(1+2T_\Sigma s)Js}}{1+\dfrac{K_{R\Omega}K_b c_m(1+T_{IR\Omega}s)}{K_i T_{IR\Omega}s\left(1+T_{fb}s\right)(1+2T_\Sigma s)Js}}$$

$$\frac{\Omega(s)}{U_{\Omega ref}(s)}=\frac{K_{R\Omega}c_m(1+T_{IR\Omega}s)\left(1+T_{fb}s\right)}{K_i T_{IR\Omega}s\left(1+T_{fb}s\right)(1+2T_\Sigma s)Js+K_{R\Omega}K_b c_m(1+T_{IR\Omega}s)}$$

$$\frac{\Omega(s)}{U_{\Omega ref}(s)}=\frac{K_{R\Omega}c_m(1+T_{IR\Omega}s)\left(1+T_{fb}s\right)}{K_i T_{IR\Omega}s(1+T_\Sigma^* s)Js+K_{R\Omega}K_b c_m(1+T_{IR\Omega}s)}$$

$$\frac{\Omega(s)}{U_{\Omega ref}(s)}=\left(1+T_{fb}s\right)\frac{K_{R\Omega}c_m+K_{R\Omega}c_m T_{IR\Omega}s}{K_i T_{IR\Omega}Js^2+K_i T_{IR\Omega}JT_\Sigma^* s^3+K_{R\Omega}K_b c_m+K_{R\Omega}K_b c_m T_{IR\Omega}s}$$

$$\frac{\Omega(s)}{U_{\Omega ref}(s)}=\left(1+T_{fb}s\right)\frac{K_{R\Omega}c_m(1+T_{IR\Omega}s)}{K_{R\Omega}K_b c_m\left(1+T_{IR\Omega}s+\dfrac{K_i T_{IR\Omega}J}{K_{R\Omega}K_b c_m}s^2+\dfrac{K_i T_{IR\Omega}JT_\Sigma^*}{K_{R\Omega}K_b c_m}s^3\right)}$$

$$\frac{\Omega(s)}{U_{\Omega ref}(s)}=\frac{1+T_{fb}s}{K_b}\frac{1+T_{IR\Omega}s}{1+T_{IR\Omega}s+\dfrac{K_i T_{IR\Omega}J}{K_{R\Omega}K_b c_m}s^2+\dfrac{K_i T_{IR\Omega}JT_\Sigma^*}{K_{R\Omega}K_b c_m}s^3}$$

$$\frac{\Omega(s)}{U_{\Omega ref}(s)}=\frac{1+T_{fb}s}{K_b}\frac{1+4T_\Sigma^* s}{1+4T_\Sigma^* s+\dfrac{4T_\Sigma^* K_i T_m c_m^2 K_a}{\dfrac{K_i K_a c_m}{2K_b}\dfrac{T_m}{T_\Sigma^*}K_b c_m}s^2+\dfrac{4{T_\Sigma^*}^2 K_i T_m c_m^2 K_a}{\dfrac{K_i K_a c_m}{2K_b}\dfrac{T_m}{T_\Sigma^*}K_b c_m}s^3}$$

$$\frac{\Omega(s)}{U_{\Omega ref}(s)}=\frac{1+T_{fb}s}{K_b}\frac{1+4T_\Sigma^* s}{1+4T_\Sigma^* s+8{T_\Sigma^*}^2 s^2+8{T_\Sigma^*}^3 s^3}$$

U prijenosnoj funkciji $\frac{\Omega(s)}{U_{\Omega ref}(s)}$ pojavljuju se dvije nule koje kvare vladanje sustava. Jedna nula javlja se zbog utjecaja mjernog člana brzine vrtnje, dok se druga nula javlja se zbog utjecaja PI regulatora. Iz tog razloga koristi se prefiltar:

$$G_{pf\Omega}(s)=\frac{1}{\left(1+T_{fb}s\right)(1+4T_\Sigma^* s)}$$