Službeni izrazi za završni ispit iz Upravljanja elektromotornim pogonima

Istosmjerni stroj:

$$U = E + I_a R_a = c_e \cdot n + I_a R_a$$

$$M_{elm} = M_t + J\frac{d\omega}{dt} \qquad P_{tr,v} = 0 \Rightarrow M_{elm} = M_m = M_t + J\frac{d\omega}{dt}$$

$$M_{elm} = c_m \cdot I_a \qquad M_{elmn} = c_m \cdot I_n \qquad M_{tr,ven} = M_{elmn} - M_n$$

$$M_n = \frac{P_n}{\omega_n} \qquad M_m = \frac{P_2}{\omega} = \frac{P_2}{\omega_2} = \frac{P_{meh}}{\omega_{meh}}$$

$$P_1 = P_{el} = U \cdot I \qquad P_{meh} = \eta \cdot P_{el} \qquad P_g = P_{el} - P_{meh}$$

$$n = \frac{U}{c_e} - M_t \frac{R_a}{c_e c_m}$$

Faktor vođenja (opterećenja) istosmjernog pretvarača:

$$D = \frac{t_v}{T};\ t_v \rightarrow vrijeme\ vođenja\ jednog\ para\ sklopki;\ T \rightarrow ukupan\ period\ vođenja$$

Dizanje tereta:

$$P_t = F \cdot v = m \cdot g \cdot v$$

Ovisnost brzine kojom se giba predmet o brzini vrtnje navojnog vretena:

$$v_p = \frac{n_m}{60} \cdot h_v$$

Asinkroni stroj

$$I_2 = \frac{s \cdot E_{20}}{\sqrt{R_2^2 + (s \cdot X_{\sigma 2})^2}}$$

$$f_2 = s \cdot f_1 \qquad s = \frac{n_s - n_m}{n_s} \qquad n_s = \frac{60 \cdot f_1}{p}$$

$$M_m = M_{pr} \cdot \frac{2}{\frac{s_{pr}}{s} + \frac{s}{s_{pr}}} \qquad s_{pr} = \frac{R_2}{X_{\sigma 2}} = \frac{R_2}{\omega_1 \cdot L_{\sigma 2}}$$

$$P_1 = 3 \cdot U_1 \cdot I_1 \cdot \cos\varphi_1 \qquad P_{Cu1} = 3 \cdot I_1^2 \cdot R_1 \qquad P_{Fe1} = 3 \cdot \frac{E_1^2}{R_m}$$

$$P_{okr} = 3 \cdot \frac{R_2}{s} \cdot I_2^2 = M_{em} \cdot \omega_s \qquad P_{Cu2} = s \cdot P_{okr} = 3 \cdot I_2^2 \cdot R_2$$

$$P_{em} = (1-s) \cdot P_{okr} \qquad P_{em} = M_{em} \cdot \omega_m$$

$$P_2 = P_{em} - P_{tr,v} \qquad \eta = \frac{P_2}{P_1}$$

Dvomaseni elastični sustav

Prijenosne funkcije ulazno/izlaznog modela mehaničkog sustava određene su izrazima:

$$G_{11}(s) = \frac{\omega_{11}(s)}{m_1(s)} = \frac{\Omega_{02}^{-2}s^2 + 2\zeta_2\Omega_{02}^{-1}s + 1}{N(s)}, \qquad G_{21}(s) = \frac{\omega_{21}(s)}{m_1(s)} = \frac{2\zeta_2\Omega_{02}^{-1}s + 1}{N(s)},$$

$$G_{12}(s) = \frac{\omega_{12}(s)}{m_2(s)} = \frac{2\zeta_1\Omega_{01}^{-1}s + 1}{N(s)}, \qquad G_{22}(s) = \frac{\omega_{22}(s)}{m_2(s)} = \frac{\Omega_{01}^{-2}s^2 + 2\zeta_1\Omega_{01}^{-1}s + 1}{N(s)},$$

$$G_{t1}(s) = \frac{\omega_{21}(s)}{\omega_{11}(s)} = \frac{2\zeta_2\Omega_{02}^{-1}s + 1}{\Omega_{02}^{-2}s^2 + 2\zeta_2\Omega_{02}^{-1}s + 1}, \qquad G_{t2}(s) = \frac{\omega_{22}(s)}{\omega_{12}(s)} = \frac{\Omega_{01}^{-2}s^2 + 2\zeta_1\Omega_{01}^{-1}s + 1}{2\zeta_1\Omega_{01}^{-1}s + 1},$$

$$G_{\alpha\omega}(s) = \frac{\alpha_1(s)}{\omega_1(s)} = \frac{\alpha_2(s)}{\omega_2(s)} = \frac{1}{T_B s}.$$

Karakteristični polinom *N(s)* i parametri ulazno/izlaznog modela povezani su s fizikalnim parametrima prema relacijama:

$$N(s) = T_{M\Sigma}s(\Omega_0^{-2}s^2 + 2\zeta\Omega_0^{-1}s + 1), \qquad T_{M\Sigma} = T_{M1} + T_{M2},$$

$$\Omega_0 = \sqrt{\frac{c}{T_B}\left(\frac{1}{T_{M1}} + \frac{1}{T_{M2}}\right)}, \qquad \zeta = \frac{d}{2c}T_B\Omega_0,$$

$$\Omega_{0i} = \sqrt{\frac{c}{T_B T_{Mi}}} < \Omega_0 \;, \quad i = 1,2 \;, \qquad \zeta_i = \frac{d}{2c}T_B\Omega_{0i} < \zeta.$$

Optimum dvostrukog odnosa

Opći oblik prijenosne funkcije razmatranog sustava je:

$$G(s) = \frac{y(s)}{y_R(s)} = \frac{1}{A(s)} = \frac{1}{a_n s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + 1}.$$

Odnosom vremenskih konstanti susjednih integralnih članova definirani su bezdimenzionalni *karakteristični odnosi*:

$$D_i \overset{!}{=} \frac{T_i}{T_{i-1}} = \frac{a_i a_{i-2}}{a_{i-1}^2} \quad , \quad i = 2,\ldots,n.$$

Izjednačenjem s najčešće korištenim oblikom oscilacijskog člana 2. reda dobije veza karakterističnog odnosa D_i i relativnog koeficijenta prigušenja ζ_i:

$$D_i = \frac{1}{4\zeta_i^2}.$$

Karakteristični polinom *A(s)* razmatranog sustava može se primjenom zapisati i u obliku

$$\begin{aligned} A(s) &= T_n T_{n-1} \cdots T_1 s^n + T_{n-1} T_{n-2} \cdots T_1 s^{n-1} + \ldots + T_2 T_1 s^2 + T_1 s + 1 = \\ &= D_n D_{n-1}^2 \cdots D_2^{n-1} T_e^n s^n + D_{n-1} D_{n-2}^2 \cdots D_2^{n-2} T_e^{n-1} s^{n-1} + \ldots + D_2 T_e^2 s^2 + T_e s + 1, \end{aligned}$$

pri čemu $T_e = T_1$ označava *nadomjesnu vremensku konstantu* ukupnog zatvorenog sustava. Veza nadomjesnih vremenskih konstanti pojedinih krugova kaskadne strukture sustava u općem i optimalnom obliku može se zapisati u obliku:

$$T_e = T_1 = \frac{1}{D_2} T_2 = \frac{1}{D_3 D_2} T_3 = \ldots = \frac{1}{D_i D_{i-1} \cdots D_2} T_i = \ldots = \frac{1}{D_n D_{n-1} \cdots D_2} T_n,$$

$$T_e = T_1 = 2T_2 = 4T_3 = \ldots = 2^{i-1} T_i = \ldots = 2^{n-1} T_n.$$

Prošireni optimum dvostrukog odnosa

Optimum dvostrukog odnosa može se proširiti na linearne sustave opisane prijenosnom funkcijom koji sadrže nule:

$$a_i^2 - 2a_{i-1}a_{i+1} = \frac{a_{i-1}^2}{b_{i-1}^2}(b_i^2 - 2b_{i-1}b_{i+1}).$$

Modulni optimum

Opći oblik prijenosne funkcije sustava kakav se razmatra je:

$$G(s) = \frac{y(s)}{y_R(s)} = \frac{1}{A(s)} = \frac{1}{a_n s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + 1}.$$

U ovom slučaju vrijede jednadžbe standardnog oblika *modulnog optimuma*:

$$\begin{aligned} &a_1^2 - 2a_0 a_2 = 0, \\ &a_2^2 - 2a_1 a_3 + 2a_0 a_4 = 0, \\ &a_3^2 - 2a_2 a_4 + 2a_1 a_5 - 2a_0 a_6 = 0, \\ &\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\ &a_{n-1}^2 - 2a_{n-2} a_n = 0. \end{aligned}$$

Prošireni modulni optimum

U slučaju da je proces opisan prijenosnom funkcijom koja sadrži nule, vrijede sljedeće jednadžbe *proširenog modulnog optimuma*:

$$
\begin{aligned}
&a_1^2 - 2a_0a_2 = b_1^2 - 2b_0b_2,\\
&a_2^2 - 2a_1a_3 + 2a_0a_4 = b_2^2 - 2b_1b_3 + 2b_0b_4,\\
&\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots\\
&a_i^2 + 2\sum_{j=1}^{i}(-1)^j a_{i-j}a_{i+j} = b_i^2 + 2\sum_{j=1}^{i}(-1)^j b_{i-j}b_{i+j},\\
&\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots\\
&a_{n-1}^2 - 2a_{n-2}a_n = b_{n-1}^2.
\end{aligned}
$$

Regulacija brzine vrtnje i položaja elektromotornog pogona s elastičnim prijenosnim mehanizmom korištenjem regulatora po varijablama stanja

Prijenosna funkcija zatvorenog regulacijskog kruga

$$G_{cl}(s) = \frac{\omega_2(s)}{\omega_R(s)} = \frac{B(s)}{A(s)} = \frac{2\zeta_2\Omega_{02}^{-1}s + 1}{a_5s^5 + a_4s^4 + a_3s^3 + a_2s^2 + a_1s + 1}.$$

Integralna konstanta

$$T_I = T_e - 2\zeta_2\Omega_{02}^{-1} \ .$$

Nadomjesna vremenska konstanta otvorenog kruga

$$T_\Sigma = T_{ei} + T$$

<u>*Slučaj PI regulatora*</u>

Vrijednosti parametara karakterističnog polinoma:

$$
\begin{aligned}
a_1 &= T_I + 2\zeta_2\Omega_{02}^{-1},\\
a_2 &= K_{\omega 1}^{-1}T_IT_{M\Sigma} + 2\zeta_2T_I\Omega_{02}^{-1} + \Omega_{02}^{-2},\\
a_3 &= K_{\omega 1}^{-1}T_IT_{M\Sigma}(T_\Sigma + 2\zeta\Omega_0^{-1}) + T_I\Omega_{02}^{-2},\\
a_4 &= K_{\omega 1}^{-1}T_IT_{M\Sigma}(2\zeta\Omega_0^{-1}T_\Sigma + \Omega_0^{-2}),\\
a_5 &= K_{\omega 1}^{-1}T_IT_{M\Sigma}T_\Sigma\Omega_0^{-2}.
\end{aligned}
$$

Nadomjesna vremenska konstanta zatvorenog kruga (približni izraz)

$$T_e \approx \frac{3}{2}T_\Sigma + \sqrt{\frac{21}{4}T_\Sigma^2 + 8\Omega_{02}^{-2}}$$

Pojačanje PI regulatora

$$K_{\omega 1} = \frac{T_IT_{M\Sigma}\Omega_{02}^2}{D_2T_e^2\Omega_{02}^2 - 2\zeta_2(T_e\Omega_{02} - 2\zeta_2) - 1}$$

Slučaj PIm regulatora

Vrijednosti parametara karakterističnog polinoma:

$$
\begin{aligned}
a_1 &= T_I + 2\zeta_2\Omega_{02}^{-1},\\
a_2 &= K_{\omega1}^{-1}T_I T_{M\Sigma} + K_{\omega1}^{-1}K_m T_I T_{M\Sigma} + 2\zeta_2 T_I\Omega_{02}^{-1} + \Omega_{02}^{-2},\\
a_3 &= K_{\omega1}^{-1}T_I T_{M\Sigma}(T_\Sigma + 2\zeta\Omega_0^{-1}) + 2\zeta_2 K_m T_I T_{M2} K_{\omega1}^{-1}\Omega_{02}^{-1} + T_I\Omega_{02}^{-2},\\
a_4 &= K_{\omega1}^{-1}T_I T_{M\Sigma}(2\zeta\Omega_0^{-1}T_\Sigma + \Omega_0^{-2}),\\
a_5 &= K_{\omega1}^{-1}T_I T_{M\Sigma}T_\Sigma\Omega_0^{-2}.
\end{aligned}
$$

Nadomjesna vremenska konstanta zatvorenog kruga

$$
T_e = \frac{8}{3D_4T_\Sigma\Omega_0^2}\cos\left(\frac{\arccos\left(\frac{27D_4^2T_\Sigma^2\Omega_0^4}{4\Omega_{02}^2}-1\right)+\pi}{3}\right)+\frac{4}{3D_4T_\Sigma\Omega_0^2}
$$

Pojačanja regulatora

$$
\begin{aligned}
K_{\omega1} &= \frac{T_I T_{M\Sigma}(1+2\zeta T_\Sigma\Omega_0)}{D_4D_3^2D_2^3T_e^4\Omega_0^2}\\
K_m &= \frac{D_2T_e^2\Omega_{02}^2 - K_{\omega1}^{-1}T_{M\Sigma}T_I\Omega_{02}^2 - 2\zeta_2(T_e\Omega_{02}-2\zeta_2)-1}{K_{\omega1}^{-1}T_{M2}T_I\Omega_{02}^2}
\end{aligned}
$$

Slučaj $PI_{\Delta\omega}$ regulatora

Vrijednosti parametara karakterističnog polinoma:

$$
\begin{aligned}
a_1 &= T_I + 2\zeta_2\Omega_{02}^{-1},\\
a_2 &= K_{\omega1}^{-1}T_I T_{M\Sigma} + 2\zeta_2 T_I\Omega_{02}^{-1} + \Omega_{02}^{-2},\\
a_3 &= K_{\omega1}^{-1}T_I T_{M\Sigma}(T_\Sigma + 2\zeta\Omega_0^{-1}) + K_{\Delta\omega}T_I K_{\omega1}^{-1}\Omega_{02}^{-2} + T_I\Omega_{02}^{-2},\\
a_4 &= K_{\omega1}^{-1}T_I T_{M\Sigma}(2\zeta\Omega_0^{-1}T_\Sigma + \Omega_0^{-2}),\\
a_5 &= K_{\omega1}^{-1}T_I T_{M\Sigma}T_\Sigma\Omega_0^{-2}.
\end{aligned}
$$

Nadomjesna vremenska konstanta zatvorenog kruga

$$
\begin{aligned}
r_M &= \frac{T_{M2}}{T_{M1}} \le 3 \Rightarrow \frac{5,5}{\Omega_0} \ge T_e \ge \frac{4}{\Omega_0}\\
r_M &= \frac{T_{M2}}{T_{M1}} > 3 \Rightarrow T_e = \frac{2}{\Omega_0}\sqrt{1+r_M}
\end{aligned}
$$

Pojačanja regulatora

$$
K_{\omega1} = \frac{T_I T_{M\Sigma}\Omega_{02}^2}{D_2T_e^2\Omega_{02}^2 - 2\zeta_2(T_e\Omega_{02}-2\zeta_2)-1}
$$

$$K_{\Delta\omega} = \frac{D_3 D_2^2 T_e^3 \Omega_0 \Omega_{02}^3 - K_{\omega 1}^{-1} T_{M\Sigma} T_I \Omega_{02}^3 (T_\Sigma \Omega_0 + 2\zeta) - \Omega_0 (T_e \Omega_{02} - 2\zeta_2)}{K_{\omega 1}^{-1} T_I \Omega_0 \Omega_{02}}$$

Regulator stanja punog reda

Vrijednosti parametara karakterističnog polinoma:

$$\begin{aligned}
a_1 &= T_I + 2\zeta_2 \Omega_{02}^{-1},\\
a_2 &= (K_{\omega 1} + K_{\omega 2})^{-1} T_I T_{M\Sigma} + 2\zeta_2 T_I \Omega_{02}^{-1} + (K_{\omega 1} + K_{\omega 2})^{-1} K_{\Delta\alpha} T_I T_B^{-1} \Omega_{02}^{-2},\\
a_3 &= (K_{\omega 1} + K_{\omega 2})^{-1} T_I T_{M\Sigma} (T_\Sigma + 2\zeta \Omega_0^{-1}) + (K_{\omega 1} + K_{\omega 2})^{-1} K_{\omega 1} T_I \Omega_{02}^{-2},\\
a_4 &= (K_{\omega 1} + K_{\omega 2})^{-1} T_I T_{M\Sigma} (2\zeta \Omega_0^{-1} T_\Sigma + \Omega_0^{-2}),\\
a_5 &= (K_{\omega 1} + K_{\omega 2})^{-1} T_I T_{M\Sigma} T_\Sigma \Omega_0^{-2}.
\end{aligned}$$

Nadomjesna vremenska konstanta zatvorenog kruga

$$T_e = \frac{T_\Sigma}{D_5 D_4 D_3 D_2 (1 + 2\zeta T_\Sigma \Omega_0)}$$

Pojačanja regulatora

$$\begin{aligned}
K_{\omega 1} &= \frac{T_{M\Sigma} \Omega_{02}^2}{\Omega_0} \left(\frac{1 + 2\zeta T_\Sigma \Omega_0}{D_4 D_3 D_2 T_e \Omega_0} - T_\Sigma \Omega_0 - 2\zeta \right)\\
K_{\omega 2} &= \frac{T_I T_{M\Sigma} (1 + 2\zeta T_\Sigma \Omega_0)}{D_4 D_3^2 D_2^3 T_e^4 \Omega_0^2} - K_{\omega 1}\\
K_{\Delta\alpha} &= \frac{D_2 T_e^2 \Omega_{02}^2 - (K_{\omega 1} + K_{\omega 2})^{-1} T_{M\Sigma} T_I \Omega_{02}^2 - 2\zeta_2 (T_e \Omega_{02} - 2\zeta_2)}{(K_{\omega 1} + K_{\omega 2})^{-1} T_B^{-1} T_I}
\end{aligned}$$