UPRAVLJANJE ELEKTROMOTORNIM POGONIMA
Međuispit 2011.

1. Zadani podaci su:
M1: 33 kW; 440 V; 83 A; 1040 r/min; 0,24 Ω
M2: 34 kW; 440 V; 87 A; 1060 r/min; 0,3 Ω
$M_{tr,red} = 10$ Nm; $M_t = 400$ Nm

Konstante motora **M1**:

$$c_{e1} = \frac{U_{an1} - I_{an1}R_{a1}}{n_{n1}} = \frac{440 - 83 \cdot 0{,}24}{1040} = 0{,}4039 \ {}^{\text{Vmin}}/_{\text{r}}$$

$$c_{m1} = \frac{30c_{e1}}{\pi} = 3{,}8572 \ {}^{\text{Nm}}/_{\text{A}}$$

Nazivni moment:

$$M_{n1} = \frac{30P_{n1}}{n_{n1}\pi} = \frac{30 \cdot 33000}{1040 \cdot \pi} = 303{,}0065 \text{ Nm}$$

Nazivni elektromagnetski moment:

$$M_{emn1} = I_{an1}c_{m1} = 83 \cdot 3{,}8572 = 320{,}1460 \text{ Nm}$$

Moment trenja i ventilacije je:

$$M_{tr,v1} = M_{emn1} - M_{n1} = 17{,}1395 \text{ Nm}$$

Konstante motora **M2**:

$$c_{e2} = \frac{U_{an2} - I_{an2}R_{a2}}{n_{n2}} = \frac{440 - 87 \cdot 0{,}3}{1060} = 0{,}3905 \ {}^{\text{Vmin}}/_{\text{r}}$$

$$c_{m2} = \frac{30c_{e2}}{\pi} = 3{,}7287 \ {}^{\text{Nm}}/_{\text{A}}$$

Nazivni moment:

$$M_{n2} = \frac{30P_{n2}}{n_{n2}\pi} = \frac{30 \cdot 34000}{1060 \cdot \pi} = 306{,}2982 \text{ Nm}$$

Nazivni elektromagnetski moment:

$$M_{emn2} = I_{an2}c_{m2} = 87 \cdot 3{,}7287 = 324{,}3995 \text{ Nm}$$

Moment trenja i ventilacije je:

$$M_{tr,v2} = M_{emn2} - M_{n2} = 18{,}1013 \text{ Nm}$$

(a) Motori se vrte jednakim brzinama:

$$n_1 = n_2$$

$$\frac{U_{an1} - I_{a1}R_{a1}}{c_{e1}} = \frac{U_{an2} - I_{a2}R_{a2}}{c_{e2}}$$

$$\frac{440 - I_{a1} \cdot 0{,}24}{0{,}4039} = \frac{440 - I_{a2} \cdot 0{,}3}{0{,}3905} \rightarrow I_{a1} = 1{,}2931 I_{a2} - 63{,}1566$$

Izraz za momente je:

$$M_{em1} - M_{tr,v1} + M_{em2} - M_{tr,v2} = M_t + M_{tr,red}$$

$$I_{a1}c_{m1} - 17{,}1395 + I_{a2}c_{m2} - 18{,}1013 = 400 + 10$$

$$(1{,}2931 I_{a2} - 63{,}1566) \cdot 3{,}8572 + I_{a2} \cdot 3{,}7287 = 445{,}2408 \rightarrow I_{a2} = 79{,}0298 \text{ A}$$

$$I_{a1} = 39{,}0337 \text{ A}$$

$$M_{em1} = I_{a1}c_{m1} = 39{,}0337 \cdot 3{,}8572 = 150{,}5602 \text{ Nm}$$

$$M_{em2} = I_{a2}c_{m2} = 79{,}0298 \cdot 3{,}7287 = 294{,}6806 \text{ Nm}$$

$$n = n_1 = n_2 = \frac{440 - 39{,}0337 \cdot 0{,}24}{0{,}4039} = 1066{,}1235 \ {}^{\text{r}}/_{\text{min}}$$

$$M_{os1} = M_{em1} - M_{tr,v1} = 150{,}5602 - 17{,}1395 = 133{,}4207 \text{ Nm}$$

$$M_{os2} = M_{em2} - M_{tr,v2} = 294{,}6806 - 18{,}1013 = 276{,}5793 \text{ Nm}$$

$$P_1 = M_{os1}\omega = M_{os1}\frac{n\pi}{30} = 133{,}4207 \cdot \frac{1066{,}1235 \cdot \pi}{30} = 14895{,}6443 \text{ W}$$

$$P_{1\%} = \frac{P_1}{P_{1n}} \cdot 100\% = \frac{14895{,}6443}{33000} \cdot 100\% = 45{,}8190\%$$

$$P_2 = M_{os2}\omega = M_{os2}\frac{n\pi}{30} = 276{,}5793 \cdot \frac{1066{,}1235 \cdot \pi}{30} = 30878{,}4764 \text{ W}$$

$$P_{2\%} = \frac{P_2}{P_{2n}} \cdot 100\% = \frac{30878{,}4764}{34000} \cdot 100\% = 90{,}8190\%$$

(b) Svaki motor na osovini će preuzeti:

$$M_{os1} = M_{os2} = \frac{410}{2} = 205 \text{ Nm}$$

Vrijedi:

$$M_{em1} = M_{os1} + M_{tr,v1} = 205 + 17{,}1395 = 222{,}1395 \text{ Nm}$$

$$I_{a1} = \frac{M_{em1}}{c_{m1}} = \frac{222{,}1395}{3{,}8572} = 57{,}5912 \text{ A}$$

$$n = n_1 = n_2 = \frac{U_{an1} - I_{a1}R_{a1}}{c_{e1}} = \frac{440 - 57{,}5912 \cdot 0{,}24}{0{,}4039} = 1055{,}0972 \ {}^{\text{r}}/_{\text{min}}$$

$$M_{em2} = M_{os2} + M_{tr,v2} = 205 + 18{,}1013 = 223{,}1013 \text{ Nm}$$

$$I_{a2} = \frac{M_{em2}}{c_{m2}} = \frac{223{,}1013}{3{,}7287} = 59{,}8331 \text{ A}$$

$$U_{a2} = nc_{e2} + I_{a2}R_{a2} = 1055{,}0972 \cdot 0{,}3905 + 59{,}8331 \cdot 0{,}3 = 429{,}9355 \text{ V}$$

(c) Općenito vrijedi $c_e = k_e\Phi$ i $c_m = k_m\Phi$. Ako se iznos toka motora **M2** promijeni na $\Phi_2' = 0{,}95\Phi_2$, slijedi i da se konstante motora promijene:

$$c_{e2}' = 0{,}95c_{e2} = 0{,}95 \cdot 0{,}3905 = 0{,}3709 \ {}^{\text{Vmin}}/_{\text{r}}$$

$$c_{m2}' = 0{,}95c_{m2} = 0{,}95 \cdot 3{,}7287 = 3{,}5423 \ {}^{\text{Nm}}/_{\text{A}}$$

Motori se vrte jednakim brzinama:

$$n_1 = n_2$$

$$\frac{U_{an1} - I_{a1}R_{a1}}{c_{e1}} = \frac{U_{an2} - I_{a2}R_{a2}}{c_{e2}}$$

$$\frac{440 - I_{a1} \cdot 0{,}24}{0{,}4039} = \frac{440 - I_{a2} \cdot 0{,}3}{0{,}3709} \rightarrow I_{a1} = 1{,}3611I_{a2} - 162{,}9718$$

Izraz za momente je:

$$M_{em1} - M_{tr,v1} + M_{em2} - M_{tr,v2} = M_t + M_{tr,red}$$

$$I_{a1}c_{m1} - 17{,}1395 + I_{a2}c_{m2} - 18{,}1013 = 400 + 10$$

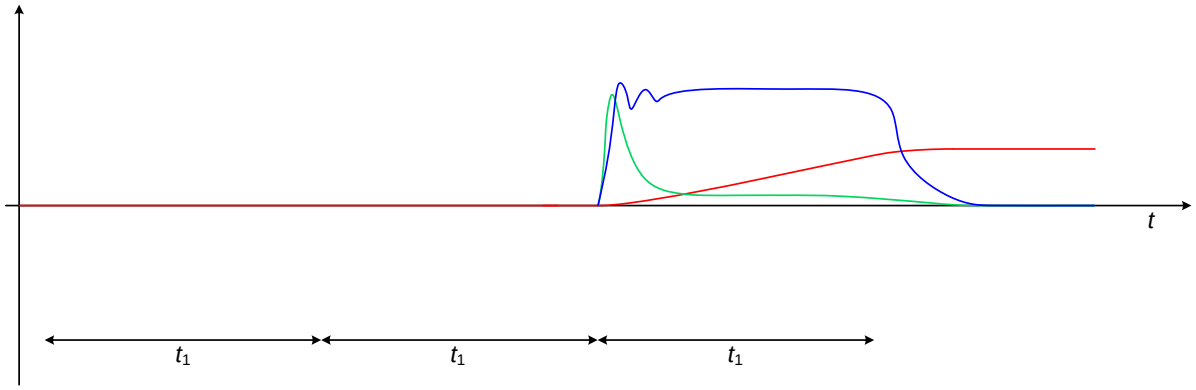
$$(1{,}3611I_{a2} - 162{,}9718) \cdot 3{,}8572 + I_{a2} \cdot 3{,}5423 = 445{,}2408 \rightarrow I_{a2} = 122{,}1346 \text{ A}$$

$$I_{a1} = 3{,}2677 \text{ A}$$

Nije dozvoljen trajni rad pogona u ovoj radnoj točci jer je struja armature motora **M2** veća od nazivne struje tog motora. Motori će teret dizati brzinom:

$$n = n_1 = n_2 = \frac{440 - 3{,}2677 \cdot 0{,}24}{0{,}4039} = 1087{,}3747 \ {}^{\text{r}}/_{\text{min}}$$

2. Iz slike je vidljivo da se radi o trofaznom punoupravljivom usmjerivaču koji omogućuje dvokvadrantni rad sa samo jednim smjerom struje. Kvalitativni vremenski odzivi struje armature $i_a(t)$, brzine vrtnje $\omega(t)$ i kuta zakreta $\varphi(t)$ neopterećenog motora dani su na slici ispod.



$i_a(t)$, $\omega(t)$, $\varphi(t)$

3. (a) Vrijednosti faznih struja statora su:

$$i_{sa}(t = 0{,}014\ \text{s}) = 20\sqrt{2}\sin(2\pi \cdot 50 \cdot 0{,}014) = -26{,}8999\ \text{A}$$

$$i_{sb}(t = 0{,}014\ \text{s}) = 20\sqrt{2}\sin\left(2\pi \cdot 50 \cdot 0{,}014 - \frac{2\pi}{3}\right) = 21{,}0193\ \text{A}$$

$$i_{sc}(t = 0{,}014\ \text{s}) = 20\sqrt{2}\sin\left(2\pi \cdot 50 \cdot 0{,}014 + \frac{2\pi}{3}\right) = 5{,}8806\ \text{A}$$

Vrijednosti α i β komponenata struja statora su:

$$i_{s\alpha}(t = 0{,}014\ \text{s}) = i_{sa}(t = 0{,}014\ \text{s}) = -26{,}8999\ \text{A}$$

$$i_{s\beta}(t = 0{,}014\ \text{s}) = \frac{i_{sb} - i_{sc}}{\sqrt{3}} = \frac{21{,}0193 - 5{,}8806}{\sqrt{3}} = 8{,}7403\ \text{A}$$

Vrijednosti d i q komponenata struja statora su:

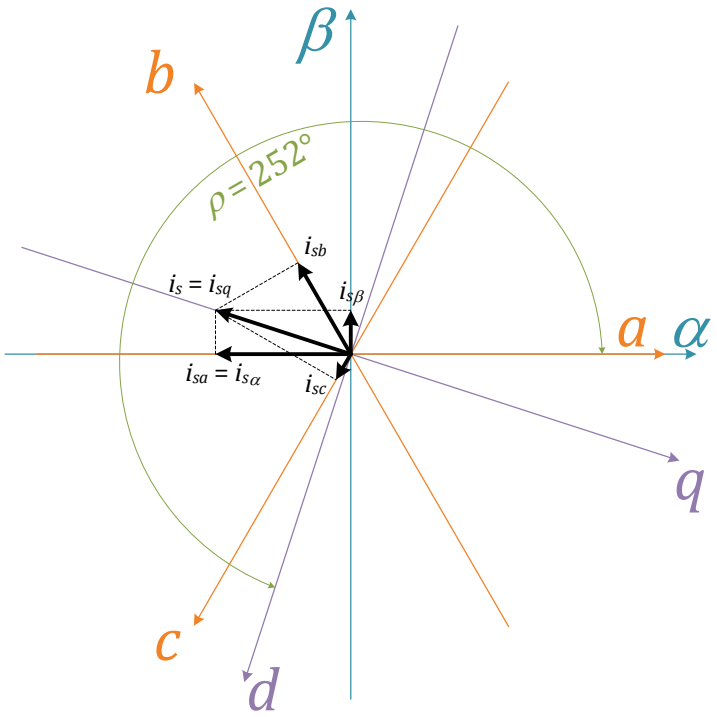
$$i_{sd}(t = 0{,}014\ \text{s}) = i_{s\alpha}(t = 0{,}014\ \text{s})\cos\rho + i_{s\beta}(t = 0{,}014\ \text{s})\sin\rho$$

$$i_{sd}(t = 0{,}014\ \text{s}) = -26{,}8999\cos(252°) + 8{,}7403\sin 252° = 0\ \text{A}$$

$$i_{sq}(t = 0{,}014\ \text{s}) = -i_{s\alpha}(t = 0{,}014\ \text{s})\sin\rho + i_{s\beta}(t = 0{,}014\ \text{s})\cos\rho$$

$$i_{sq}(t = 0{,}014\ \text{s}) = 26{,}8999\sin(252°) + 8{,}7403\cos 252° = -28{,}2843\ \text{A}$$

(b)



4. Zadani podaci su:
$P_n = 5$ kW
$U_n = 400$ V
$n_n = 1430$ r/min
$f_n = 50$ Hz
$M_{pr}/M_n = 3$
$M_t = k/n$ Nm
- spoj Y; skalarno upravljanje U/f metodom u otvorenoj petlji
$f_n \rightarrow 0{,}5M_n$

(a)

$$p = \frac{60f_s}{n_s} = \frac{60 \cdot 50}{1500} = 2$$

$$M_n = \frac{30P_n}{n_n\pi} = \frac{30 \cdot 5000}{1430 \cdot \pi} = 33{,}3891 \text{ Nm}$$

Na linearnom dijelu momentne karakteristike vrijedi (karakteristike vidjeti u **(b)** dijelu zadatka):

$$\frac{M_n}{M_t} = \frac{s_n}{s_t} \rightarrow \frac{M_n}{0{,}5M_n} = \frac{s_n}{s_t} \rightarrow s_t = 0{,}5s_n$$

Nazivno klizanje iznosi:

$$s_n = \frac{n_s - n_n}{n_s} = \frac{1500 - 1430}{1500} = 0{,}0467$$

Brzina vrtnje pri nazivnoj frekvenciji iznosi:

$$n_t = n_s(1 - s_t) = 1500(1 - 0{,}5 \cdot 0{,}0467) = 1465 \ {}^{\text{r}}/_{\text{min}}$$

Sada se može odrediti konstanta k za momentnu karakteristiku tereta:

$$M_t = \frac{k}{n_t} \rightarrow k = M_t n_t = 0{,}5 \cdot 33{,}3891 \cdot 1465 = 2{,}4458 \cdot 10^4 \ {}^{\text{Nmmin}}/_{\text{r}}$$

Iz Klossove jednadžbe slijedi:

$$\frac{M_n}{M_{pr}} = \frac{2}{\frac{s_n}{s_{pr}} + \frac{s_{pr}}{s_n}} = \frac{2}{x + \frac{1}{x}} \rightarrow x + \frac{1}{x} = 6 \rightarrow x_{1,2} = 3 \pm \sqrt{8}$$

Odabire se rješenje $x = 3 - \sqrt{8} = 0{,}1716$ iz čega se dobije prekretno klizanje:

$$s_{pr} = \frac{s_n}{0{,}1716} = \frac{0{,}0467}{0{,}1716} = 0{,}2720$$

Slijedi da je iznos brzine na prekretnom klizanju jednak:

$$n_{pr} = n_s\left(1 - s_{pr}\right) = 1500(1 - 0{,}2720) = 1092{,}0101 \ {}^{\text{r}}/_{\text{min}}$$

Definirajmo:

$$\Delta n = n_s - n_{pr} = 1500 - 1092{,}0101 = 407{,}9899\ {}^{\mathrm{r}}/_{\mathrm{min}} = n_s' - n_{pr}'$$

gdje su n_s' i n_{pr}' nova sinkrona brzina i brzina na novom prekretnom klizanju pri novoj frekvenciji f'. Prekretni moment kod nove i stare karakteristike su jednaki:

$$M_{pr}' = M_{pr} = 3M_n = 100{,}1674\ \mathrm{Nm}$$

Iz izraza $\Delta n = n_s' - n_{pr}'$ slijedi $\Delta n = n_s' - n_{pr}' = n_s' - n_s'(1 - s_{pr}') = n_s'(1 - 1 + s_{pr}') = n_s' s_{pr}'$. Kada se pronađe n_s', bit će nađena i tražena frekvencija. s_{pr}' se može dobiti iz Klossove jednadžbe:

$$\frac{M_t'}{M_{pr}'} = \frac{2}{\frac{s_t'}{s_{pr}'} + \frac{s_{pr}'}{s_t'}} \rightarrow \frac{\frac{k}{n_t'}}{M_{pr}'} = \frac{2}{x + \frac{1}{x}} \rightarrow \frac{2{,}4458 \cdot 10^4}{100{,}1674 \cdot 900} = \frac{2}{x + \frac{1}{x}} \rightarrow x^2 - 7{,}372x + 1 = 0$$

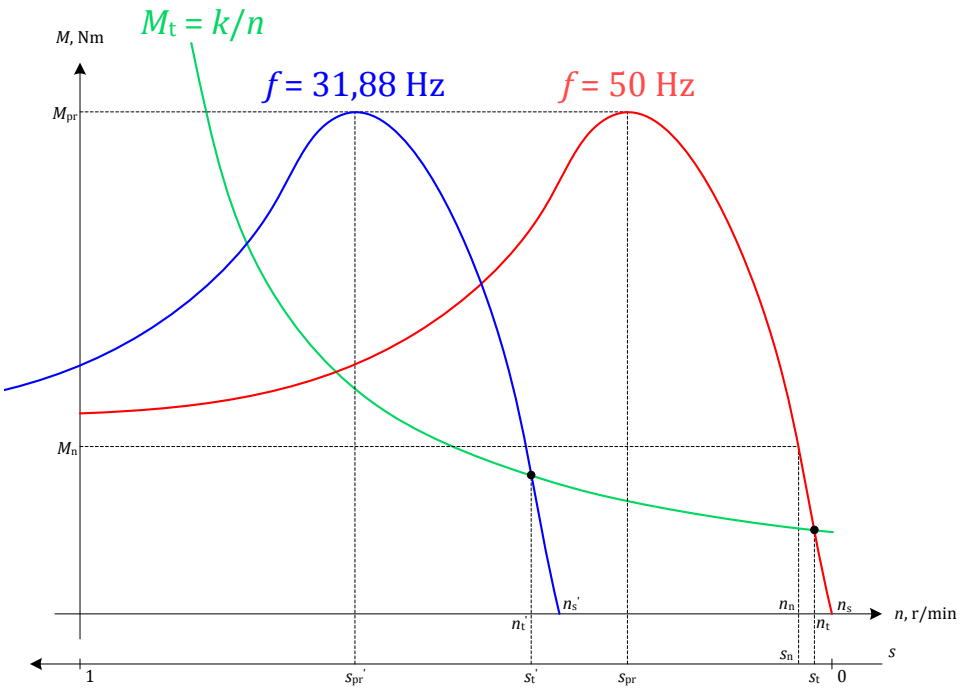
Odabire se rješenje $x = 0{,}1382$. Slijedi:

$$\frac{s_t'}{s_{pr}'} = 0{,}1382 \rightarrow \frac{\frac{n_s' - n_t'}{n_s'}}{\frac{\Delta n}{n_s'}} = 0{,}1382 \rightarrow \frac{n_s' - n_t'}{\Delta n} = 0{,}1382$$

$$n_s' = 0{,}1382\Delta n + n_t' = 0{,}1382 \cdot 407{,}9899 + 900 = 956{,}4007\ {}^{\mathrm{r}}/_{\mathrm{min}}$$

$$f' = \frac{n_s' p}{60} = \frac{956{,}4007 \cdot 2}{60} = 31{,}88\ \mathrm{Hz}$$

(b)



5. Zadani podaci su:
$u_\alpha = 200\ \text{V}$
$u_\beta = -145\ \text{V}$
$T_s = 2\ \mu\text{s}$
$U_{DC} = 600\ \text{V}$

Vrijedi:

$$u_{sa} = u_{s\alpha} = 200\ \text{V}$$

$$u_{s\beta} = \frac{u_{sb} - u_{sc}}{\sqrt{3}}$$

$$u_{sa} + u_{sb} + u_{sc} = 0 \rightarrow u_{sb} = -u_{sa} - u_{sc}$$

$$u_{s\beta} = \frac{u_{sb} - u_{sc}}{\sqrt{3}} = \frac{-u_{sa} - 2u_{sc}}{\sqrt{3}} \rightarrow u_{sc} = -\frac{u_{sa} + \sqrt{3}u_{s\beta}}{2}$$

$$u_{sc} = -\frac{200 - \sqrt{3} \cdot 145}{2} = 25{,}5737\ \text{V}$$

$$u_{sb} = -200 - 25{,}5737 = -225{,}5737\ \text{V}$$

Također je:

$$\vartheta = \text{arctg}\left(\frac{u_{s\beta}}{u_{s\alpha}}\right) = \text{arctg}\left(\frac{-145}{200}\right) = -54{,}0579°$$

Slijedi da se referentni vektor napona u_{ref} nalazi u II. sektoru. Za svaki kratki period T_s srednja vrijednost na izlazu iz izmjenjivača treba biti jednaka srednjoj vrijednosti referentnog vektora napona u_{ref}:

$$\frac{1}{T_s}\int_0^{T_s} u_{ref}\,dt = \frac{1}{T_s}\int_0^{T_1} u_1\,dt + \frac{1}{T_s}\int_{T_1}^{T_1+T_6} u_6\,dt$$

$$\frac{1}{T_s} \cdot u_{ref} \cdot (T_s - 0) = \frac{1}{T_s} \cdot u_1 \cdot (T_1 - 0) + \frac{1}{T_s} \cdot u_6 \cdot (T_1 + T_6 - T_1)$$

$$u_{ref} = u_1 \frac{T_1}{T_s} + u_6 \frac{T_6}{T_s}$$

$$u_{s\alpha} + ju_{s\beta} = \frac{2U_{DC}}{3}\frac{T_1}{T_s} + \left(\frac{U_{DC}}{3} - j\frac{U_{DC}}{\sqrt{3}}\right)\frac{T_6}{T_s} = \frac{2U_{DC}}{3}\frac{T_1}{T_s} + \frac{U_{DC}}{3}\frac{T_6}{T_s} + j\left(-\frac{U_{DC}}{\sqrt{3}}\frac{T_6}{T_s}\right)$$

$$u_{s\alpha} = \frac{2U_{DC}}{3}\frac{T_1}{T_s} + \frac{U_{DC}}{3}\frac{T_6}{T_s}$$

$$u_{s\beta} = -\frac{U_{DC}}{\sqrt{3}}\frac{T_6}{T_s}$$

Iz druge jednakosti slijedi:

$$T_6 = -\frac{\sqrt{3}u_{s\beta}}{U_{DC}}T_s = \frac{\sqrt{3}\cdot 145}{600}\cdot 2 = 0{,}8372\ \mu s$$

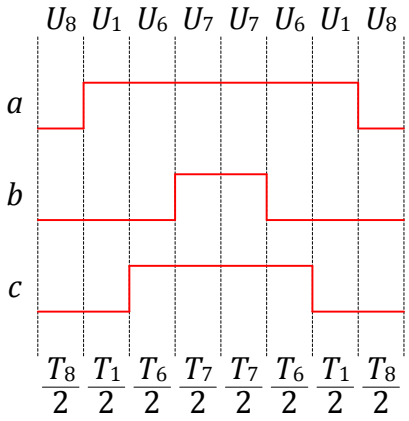
Iz prve jednakosti slijedi:

$$T_1 = \frac{3u_{s\alpha}T_s - U_{DC}T_6}{2U_{DC}} = \frac{3\cdot 200\cdot 2 - 600\cdot 0{,}8732}{2\cdot 600} = 0{,}5814\ \mu s$$

Zadovoljen je uvjet $T_1 + T_6 = 1{,}4186\ \mu s \leq T_s = 2\ \mu s$.

$$T_7 = T_8 = \frac{T_s - T_1 - T_6}{2} = \frac{2 - 1{,}4186}{2} = 0{,}2907\ \mu s$$

Valni oblici upravljačkih signala sklopki izmjenjivača prikazani su na slici ispod.



$$t_a = T_s - T_8 = 2 - 0{,}2907 = 1{,}7039\ \mu s$$

$$t_b = T_7 = 0{,}2907\ \mu s$$

$$t_c = T_s - T_1 - T_8 = 2 - 0{,}5184 - 0{,}2907 = 1{,}1279\ \mu s$$

Izmjenjivač sa sklopkama prikazan je na slici ispod.