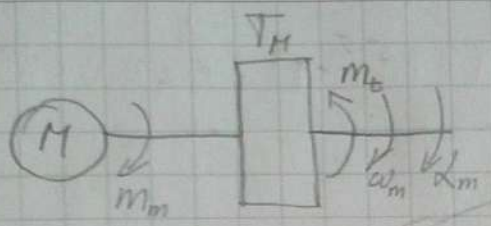
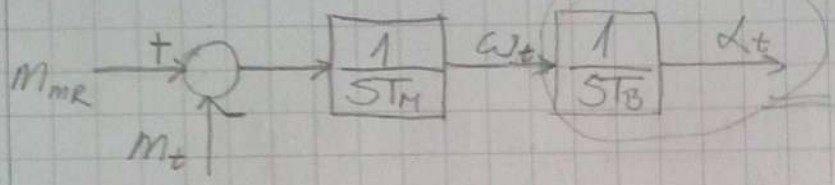
Z1 31.1.2012.

3.



a) $m_{mR}(t) - m_t(t) = \frac{d\omega_m(t)}{dt} \cdot T_H$

$\omega_m(t) = \frac{d\alpha_m(t)}{dt} \cdot T_B$



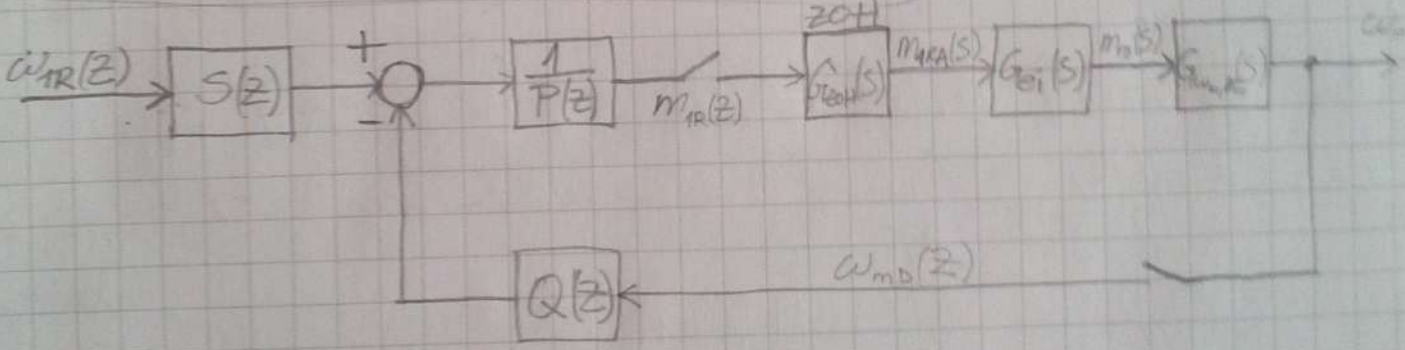
→ Izlaz je ω_m pa ovaj dio nije bitan.

$$\omega_m(s) = \frac{1}{sT_H}(M_m(s) - M_t(s))$$

$M_t = 0$

$$\frac{\omega_m(s)}{M_m(s)} = G_{\omega_t, M_m}(s) = \frac{1}{sT_H}$$

b)



$$G_{ei}(s) = \frac{1}{1+sT_{ei}}$$

$$G_P(s) = G_{ei}(s) \cdot G_{\omega_m, m_m}(s) = \frac{1}{1+sT_{ei}} \cdot \frac{1}{sT_H}$$

$$G_P(s) = \frac{1}{0{,}15 \cdot (1 + 2 \cdot 10^{-3} s)}$$

$$G_{ZOH}(s) = \frac{1 - e^{-sT}}{s}$$

DISKRETIZACIJA IT1 ČLANA

c) $T_e = 25\,ms$

$D_i = 0,5$

$T_\Sigma = 2 \cdot 10^{-3}\,s$; $\tau = \frac{T}{T_\Sigma} = \frac{10 \cdot 10^{-3}}{2 \cdot 10^{-3}} = \underline{5}$; $p_0 = e^{-\tau} = \underline{e^{-5}}$

$K_p = 1$

$T_i = 0,1\,s$

$K_z = K_p T_\Sigma (\tau + e^{-\tau} - 1) = 2 \cdot 10^{-3}(5 + e^{-5} - 1)$; $K_z = 8,013476 \cdot 10^{-3}$

$$z_0 = \frac{1 - \tau e^{-\tau} - e^{-\tau}}{1 - \tau - e^{-\tau}} = \frac{1 - e^{-\tau}(\tau + 1)}{1 - \tau - e^{-\tau}} = \frac{1 - e^{-5} \cdot 6}{1 - 5 - e^{-5}}$$

$z_0 = -0,239490$

$$G_{IT1}(z) = \frac{K_z}{T_i} \frac{z - z_0}{(z-1)(z - p_0)} = \frac{8,013476 \cdot 10^{-3}}{0,1} \frac{z + 0,239490}{(z-1)(z - e^{-5})}$$

$$G_{IT1}(z) = \frac{0,080135\,z + 0,019191}{z^2 - 1,006738\,z + e^{-5}} = \frac{B(z)}{A(z)}$$

Proces se diskretizira preko ZOH-a

Regulator — II — — II — Eulera i Tustina

Treba pronaći modelsku prijenosnu fiju

$$G_m(z) = \frac{B_m(z)}{A_m(z)}$$; $A_m(z) \Rightarrow \deg A(s) = \deg A(z)$

$$B_m(z) = B(z)\frac{A_m(1)}{B(1)}$$

$A(s) = 0$

$A(s) = 1 + T_e s + D_2 T_e^2 s^2$

$A(s) = 1 + 25 \cdot 10^{-3} s + 0,5 (25 \cdot 10^{-3})^2 s^2$

$A(s) = 3,125 \cdot 10^{-4} s^2 + 0,025 s + 1$

$$A_m(z) = \prod_{i=1}^{n} (z - z_i)$$

$$z_i = e^{s_{pi} T}$$

$3,125 \cdot 10^{-4} s^2 + 0,025 s + 1 = 0$

$s_{p1} = -40 + j40$

$s_{p2} = -40 - j40$

$z_1 = e^{(-40 + j40) \cdot 10^{-2}}$

$z_1 = e^{(-0,4 + j0,4)}$

$z_1 = e^{-0,4} \cdot e^{j0,4}$

$z_1 = 0,670320\, e^{j0,4}$

$z_2 = e^{(-40 - j40) \cdot 10^{-2}}$

$z_2 = e^{(-0,4 - j0,4)}$

$z_2 = e^{-0,4} \cdot e^{-j0,4}$

$z_2 = 0,670320\, e^{-j0,4}$

$A_m(z) = (z - z_1)(z - z_2) = (z - A e^{j0,4})(z - A e^{-j0,4})$

$A_m(z) = z^2 - A(e^{j0,4} + e^{-j0,4})z + A^2 \cdot e^{j0,4} \cdot e^{-j0,4}$

$A_m(z) = z^2 - 2A\cos(0,4)z + A^2$

$A_m(z) = z^2 - 1,234811\,z + 0,449329$

$B_m(z) = 2,153937\,(0,080135\,z + 0,019191)$

$$G_m(z) = \frac{(0,080135\,z + 0,019191) \cdot 2,153937}{z^2 - 1,234811\,z + 0,449329}$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

$$\sin x = \frac{e^{jx} - e^{-jx}}{2j}$$

$B_m(z) = (0,080135z + 0,019191) \cdot \frac{1 - 1,234811 + 0,449329}{0,080135 + 0,019191}$

Datum/Date: | Seite/Page:

d) Ostvarivanje observera

$A_o(z) \Rightarrow n = \deg A_o(z) = \deg P(z) = \deg Q(z) = \deg S(z)$
$= \deg A_m(z) - 1 + i$

Stac. točnost s obzirom na brzinu $i = 0$

$= 2 - 1 + 0 = 1$

$n = 1$

Traži se komp. poremećaja u najkraćem mogućem vremenu.

$A_o(z) = z^n$ (Deadbeat)

$\boxed{A_o(z) = z} \Rightarrow$ Observerski polinom

$A(z) \cdot P(z) + B(z) Q(z) = A_m(z) A_o(z)$

$B(z) S(z) = B_m(z) A_o(z)$

$P(z) = Az + B$; $Q(z) = Cz + D$; $S(z) = Ez + F$

$(z^2 - 1{,}006738z + e^{-5})(Az + B) + (0{,}080135z + 0{,}019191)(Cz + D) = (z^2 - 1{,}234811z + 0{,}449329)z$

$Az^3 - 1{,}006738Az^2 + e^{-5}Az + Bz^2 - 1{,}006738Bz + Be^{-5} + 0{,}080135Cz^2 + 0{,}080135Dz + 0{,}019191Cz + 0{,}019191D$
$= z^3 - 1{,}234811z^2 + 0{,}449329z$

$Az^3 + (-1{,}006738A + B + 0{,}080135C)z^2 + (Ae^{-5} - 1{,}006738B + 0{,}080135D + 0{,}019191C)z + Be^{-5} + 0{,}019191D$
$= z^3 - 1{,}234811z^2 + 0{,}449329z$

$\boxed{A = 1}$

(I.) $-1{,}006738 + B + 0{,}080135C = -1{,}234811$

(II.) $e^{-5} - 1{,}006738B + 0{,}080135D + 0{,}019191C = 0{,}449329$

(III.) $Be^{-5} + 0{,}019191D = 0 \Rightarrow Be^{-5} = -0{,}019191D \Rightarrow B = -2{,}848197D \Rightarrow$ u (I.)

$-2{,}848197D + 0{,}080135C = -1{,}234811 + 1{,}006738 \Rightarrow 0{,}080135C = 2{,}848197D - 0{,}228073$

$C = 35{,}542485D - 2{,}846140$

III. i IV. u II.

$-1{,}006738\cdot(-2{,}848197D)+0{,}080135D+0{,}019191\cdot(35{,}542485D-2{,}846110)=0{,}449329$

$2{,}867388D+0{,}080135D+0{,}682096D=0{,}503949$

$$D=0{,}138843$$

$B=-2{,}848197D$ $\qquad$ $C=35{,}542485D-2{,}846110$

$$B=-0{,}395453 \qquad C=2{,}088730$$

$B(z)S(z)=B_m(z)A_0(z)$

$(0{,}080135z+0{,}019191)\cdot(Ez+F)=2{,}159737(0{,}080135z+0{,}019191)z$

$F=0$

$E=2{,}159737$

$$P(z)=z-0{,}395453$$

$$Q(z)=2{,}088730z+0{,}138843$$

$$S(z)=2{,}159737z$$

e)

$\deg A_0=\deg P=\deg Q=\deg S=n$ (zbog dod. int. (n=2))

$P(z)=(z-1)(Az+B)$

$Q(z)=Cz^2+Dz+E$

$S(z)=Fz^2+Gz+H$

$n=\deg A_m-1+1=2$

$A_0(z)=z^{n-1}(z-a)$

$A_0(z)=z(z-a)$

$$A_0(z)=z^2-0{,}6z$$

$B(z)\cdot S(z)=B_m(z)A_0(z)$

$(0{,}080135z+0{,}019191)\cdot(Gz^2+Hz+I)=$

$2{,}159737(0{,}080135z+0{,}019191)\cdot(z^2-0{,}6z)$

$Gz^2+Hz+I=2{,}159737z^2-1{,}295842z$

$$S(z)=2{,}1597z^2-1{,}2958$$

$a\to r=\frac{T_{M2}}{T_{M1}}<<1\Rightarrow T_0\approx(\frac{1}{3}\div1)T_e$

$\to r\geq1\Rightarrow T_0\simeq(0\div\frac{1}{3})T_e$

$a=e^{-\frac{T}{T_0}}$

Naš slučaj

$r=0<<1$

$T_0\approx0{,}5T_e$

$T_0\simeq0{,}5\cdot25\cdot10^{-3}$

$T_0=0{,}0125$

$a=e^{-\frac{10}{12{,}5}}$

$a=0{,}4493$

zbog preglednosti rj. uzima se $a=\frac{6}{10}=\frac{3}{5}$ da bi se dobio rez. kao u rj.

$(z^2-1,0067z+10^{-5})(z-1)(Az+B)+(0,0801z+0,0192)(Cz^2+Dz+E)=(z^2-1,2548z+0,4493)(z^2-0,62)$

$(z^3-z^2-1,0067z^2+1,0067z+10^{-5}z-10^{-5})(Az+B)+(0,0801z+0,0192)(Cz^2+Dz+E)=z^4-0,62z^3-1,2548z^3+0,7709z^2+0,4493z^2-0,2696z$

$(z^3-2,0067z^2+1,0134z-10^{-5})(Az+B)+(0,0801z+0,0192)(Cz^2+Dz+E)=z^4-1,8348z^3+1,1902z^2-0,2696z$

$Az^4+Bz^3-2,0067Az^3-2,0067Bz^2+1,0134Az^2+1,0134Bz-10^{-5}Az-10^{-5}B+0,0801Cz^3+0,0801Dz^2+0,0801Ez+0,0192Cz^2+0,0192Dz+0,0192E=z^4-1,8348z^3+1,1902z^2-0,2696z$

$Az^4+(B-2,0067A+0,0801C)z^3+(-2,0067B+1,0134A+0,0801D+0,0192C)z^2+(1,0134B-10^{-5}A+0,0801E+0,0192D)z-10^{-5}B+0,0192E=z^4-1,8348z^3+1,1902z^2-0,2696z$

① $A=1$

② $B-2,0067A+0,0801C=-1,8348$ ⟹ $B+0,0801C=0,1719 \Rightarrow C=2,1461-12,4844B$

③ $-2,0067B+1,0134A+0,0801D+0,0192C=1,1902$ ⟹ $2,0067B+0,0192C+0,0801D=0,1768$ ← ⟹ $-2,0067B+0,0192(2,1461-12,4844B)+0,0801D=0,1768$

④ $1,0134B-10^{-5}A+0,0801E+0,0192D=-0,2696$ ⟹ $1,0134B+0,0192D+0,0801E=-0,2629$ ⟹ $1,0134B+0,0192D+0,0801\cdot0,3509B=-0,2629$

⑤ $0,0192E-10^{-5}B=0 \Rightarrow B=2,885E$ ⟹ $E=0,3509B$ →

$-2,0067B+0,0412-0,2397B+0,0801D=0,1768$ ⟹ $0,0801D-2,2464B=0,1356$ ⟹ $0,0801(54,2148B-13,6927)-2,2464B=0,1356$

$0,0192D+1,0415B=-0,2629$ ⟹ $D=54,2148B-13,6927$ →

$-4,3503B-2,2464B=0,1356+1,0968 \Rightarrow B=-0,1870$

$E=0,3509\cdot(-0,1870)$

$E=-0,0656$

$C=2,1461-12,4844\cdot(-0,1870)$

$C=4,4807$

$D=-54,2148\cdot(-0,1870)-13,6927$

$D=-3,5789$

$P(z)=(z-1)(z-0,1870)$

$Q(z)=4,4807z^2-3,5789z-0,0656$

$S(z)=2,1597z^2-1,2958$

$$G_{cl}(s) = \frac{G_o(s)}{1+G_o(s)} = \frac{\frac{K_R c(1+sT_I)}{sT_I(T_H s^2+ds+c)}}{1+\frac{K_R c(1+sT_I)}{sT_I(T_H s^2+ds+c)}}$$

$$G_{cl}(s) = \frac{K_R c(1+sT_I)}{sT_I(T_H s^2+ds+c)+K_R c T_I s+K_R c}$$

$$G_{cl}(s) = \frac{K_R c(1+sT_I)}{K_R c\left(\frac{T_I}{K_R c} s(T_H s^2+ds+c)+T_I s+1\right)}$$

$$G_{cl}(s) = \frac{1+sT_I}{\frac{T_I}{K_R}\cdot\frac{T_H}{c}s^3+\frac{T_I}{K_R}\frac{d}{c}s^2+\left(\frac{T_I}{K_R}+T_I\right)s+1}$$

$$A(s) = 1+T_e s+D_2 T_e^2 s^2+D_3 D_2^2 T_e^3 s^3$$

$$T_e = \frac{T_I}{K_R}+T_I \Rightarrow T_I = T_e-\frac{T_I}{K_R} = 1{,}6-51{,}2 = -49{,}6\ s$$

II. $D_2 T_e^2 = \frac{T_I}{K_R}\frac{d}{c}$

III. $D_3 D_2^2 T_e^3 = \frac{T_I}{K_R}\cdot\frac{T_H}{c}$

$$\frac{III}{II}: \quad D_2^2 T_e = \frac{T_H}{d}$$

$$T_e = \frac{T_H}{d D_2^2} = \frac{0{,}1}{0{,}5^2\cdot 0{,}25} = 1{,}6$$

II: $\frac{T_I}{K_R} = D_2 T_e^2\cdot\frac{c}{d} = 0{,}5\cdot 1{,}6^2\cdot\frac{10}{0{,}5}$

$$\frac{T_I}{K_R} = 51{,}2$$

$$K_R = -\frac{51}{32}$$