UPRAVLJANJE ELEKTROMOTORNIM POGONIMA Ponovljeni 1. međuispit 2009.

1. Zadani podaci su:

$$P_n = 33 \text{ kW}$$

$$U_{an} = 440 \text{ V}$$

$$I_{an} = 83 \text{ A}$$

 $n_n = 1040 \text{ r/min}$

$$R_a = 0.24 \Omega$$

$$M_{tr,red} = 15 \text{ Nm}$$

$$M_t = 150 \text{ Nm}$$

$$M_{tr,v} = \text{konst.}$$

- četverokvadrantni čoper; bipolarna modulacija

Konstante motora su:

$$c_e = \frac{U_{an} - I_{an}R_a}{n_n} = \frac{440 - 83 \cdot 0.24}{1040} = 0.4039 \text{ Vmin/r}$$

$$c_m = \frac{30c_e}{\pi} = 3.8572 \text{ Nm/A}$$

Nazivni moment motora je:

$$M_n = \frac{30P_n}{n_n \pi} = \frac{30 \cdot 33000}{1040 \cdot \pi} = 303,0065 \text{ Nm}$$

Nazivni elektromagnetski moment motora je:

$$M_{emn} = I_{an}c_m = 83 \cdot 3,8572 = 320,1460 \text{ Nm}$$

Moment trenja i ventilacije je:

$$M_{tr,v} = M_{emn} - M_n = 320,1460 - 303,0065 = 17,1395 \text{ Nm}$$

(a) Radi se o drugom kvadrantu. Izraz za momente je jednak:

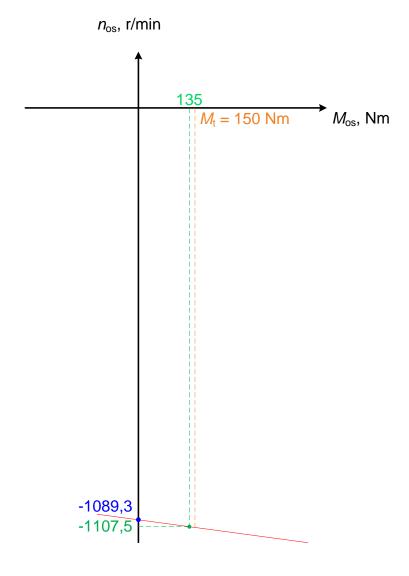
$$M_{em} = M_t - M_{tr,v} - M_{tr,red} = 150 - 17,1395 - 15 = 117,8605 \text{ Nm}$$

Struja armature tada iznosi:

$$I_a = \frac{M_{em}}{c_m} = \frac{117,8605}{3,8572} = 30,5561 A$$

Brzina motora iznosi:

$$n = \frac{U_a - I_a R_a}{c_e} = \frac{-440 - 30,5561 \cdot 0,24}{0,4039} = -1107,5 \text{ r/min}$$



$$n_0 = \frac{U_{an}}{c_e} = 1089,3 \text{ r/min}$$

$$M_{os} = M_t - M_{red} = 135 \text{ Nm}$$

(b) Radi se o dizanju tereta pa je jednadžba za momente jednaka:

$$M_{em} = M_t + M_{tr,v} + M_{tr,red} = 150 + 17,1395 + 15 = 182,1395 \text{ Nm}$$

Struja armature tada iznosi:

$$I_a = \frac{M_{em}}{c_m} = \frac{182,1395}{3,8572} = 47,2209 A$$

Vrijedi:

$$I_a(R_a + R_p) = 47,2209 \cdot (0,24 + 3) = 152,9957 \text{ V} < U_{an} = 440 \text{ V}$$

Radi se o motorskom načinu rada.

Brzina vrtnje je jednaka:

$$n = \frac{U_a - I_a R_a}{c_e} = \frac{440 - 47,2209 \cdot 3,24}{0,4039} = 710,542 \text{ r/min}$$

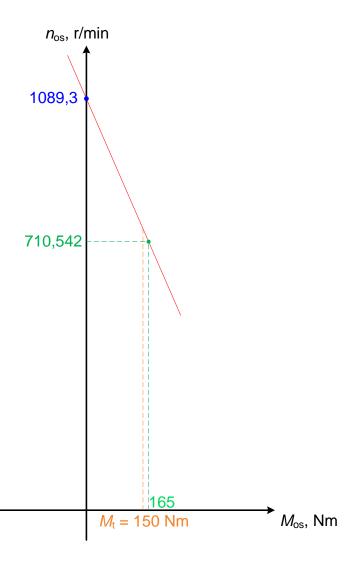
Za korisnost je potrebno naći snagu uzetu iz mreže i snagu na osovini:

$$P_m = U_{an}I_a = 440 \cdot 47,2209 = 20777 \text{ W}$$

$$M_{os} = M_t + M_{red} = 150 + 15 = 165 \text{ Nm}$$

$$P_{os} = M_{os} \frac{n\pi}{30} = 165 \cdot \frac{710,542 \cdot \pi}{30} = 12277 \text{ W}$$

$$\eta = \frac{P_{os}}{P_m} \cdot 100\% = \frac{12277}{20777} \cdot 100\% = 59,09\%$$



(c)
$$U_a = nc_e + I_a R_a = 710 \cdot 0,4039 + 47,2209 \cdot 0,24 = 298,1184 \text{ V}$$

Radi se o bipolarnoj modulaciji pa je:

$$U_a = (2D - 1)U_{DC} \to D = \frac{1}{2} \left(\frac{U_a}{U_{DC}} + 1 \right)$$

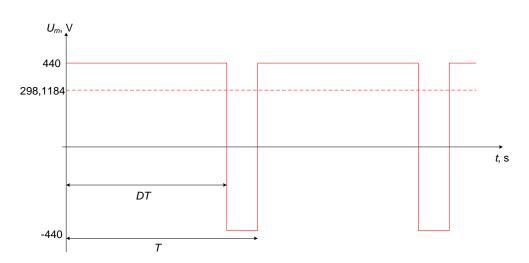
$$D = \frac{1}{2} \left(\frac{298,1184}{440} + 1 \right) = 0,8388$$

Za korisnost je potrebno naći snagu uzetu iz mreže i snagu na osovini:

$$P_m = U_a I_a = 298,1184 \cdot 47,2209 = 14077 \text{ W}$$

$$P_{os} = 12277 \text{ W}$$

$$\eta = \frac{P_{os}}{P_m} \cdot 100\% = \frac{12277}{14077} \cdot 100\% = 87,21\%$$



2. Zadani podaci su:

$$U = 550 \text{ V}$$

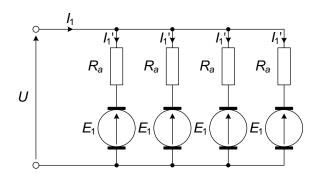
$$I_1 = 100 \text{ A}$$

$$R_a=0.3~\Omega$$

$$v_1 = 45 \text{ km/h}$$

$$M_2 = 4M_1$$

Shema za prvi slučaj je:

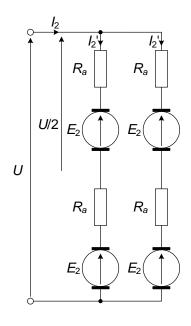


$$U - I_1' R_a = E_1$$

$$4I_{1}^{'} = I_{1} \rightarrow I_{1}^{'} = \frac{1}{4}I_{1} = 25 \text{ A}$$

$$E_1 = 550 - 25 \cdot 0.3 = 542.5 \text{ V}$$

Za drugi slučaj shema je:



$$\frac{U}{2} - I_2' R_a = E_2$$

$$2I_{2}^{'}=I_{2}\rightarrow I_{2}^{'}=\frac{1}{2}I_{2}$$

$$M_{1} = 4M_{1}^{'} = 4I_{1}^{'}c_{m} = 100c_{m}$$

$$M_{2} = 2M_{2}^{''} = 2 \cdot 2M_{2}^{'} = 2 \cdot 2I_{2}^{'}c_{m} = 4I_{2}^{'}c_{m}$$

$$M_{2} = 4M_{1} \rightarrow 4I_{2}^{'}c_{m} = 400c_{m} \rightarrow I_{2}^{'} = 100 \text{ A}$$

$$E_{2} = \frac{550}{2} - 100 \cdot 0.3 = 245 \text{ V}$$

Vrijedi:

$$\frac{E_2}{E_1} = \frac{c_e n_2}{c_e n_1} = \frac{n_2}{n_1} = \frac{v_2}{v_1} \to v_2 = v_1 \frac{E_2}{E_1}$$

$$v_2 = 45 \cdot \frac{245}{542,5} = 20,3226 \text{ km/h}$$

3. Zadani podaci su:

3. Zadani podaci su:
$$P_n = 13.5 \text{ kW}$$
 $U_{an} = 420 \text{ V}$ $I_{an} = 40.6 \text{ A}$ $U_{un} = 242 \text{ V}$ $I_{un} = 2.3 \text{ A}$ $n_n = 1480 \text{ r/min}$ $P_g = 11 \text{ kW}$ $U_g = 220 \text{ V}$ $I_g = 45 \text{ A}$ $R_a = 0.2 \Omega$ $L_a = 4 \text{ mH}$ $K = c_e = c_m = 2.66$

 $K = c_e = c_m = 2,66 \text{ Vs/rad}$

$$J = 0,478 \text{ kgm}^2$$

$$K_t = 44$$

$$T_{mi} = 1,67 \text{ ms}$$

$$K_i = 0.1 \text{ V/A}$$

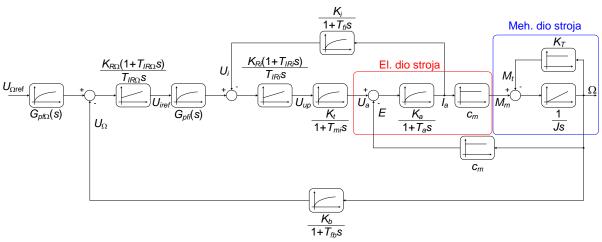
$$T_{fi} = 2 \text{ ms}$$

$$K_b = 0.031 \, \text{Vs}$$

$$T_{fb} = 15 \text{ ms}$$

$$M_t = K_T \omega$$

(a) Strukturni blokovski prikaz regulacijskog sustava kaskadne regulacije brzine vrtnje istosmjernog motora s nezavisnom i konstantnom uzbudom dan je na slici ispod. Pri tome su $G_{pfi}(s)$ i $G_{pf\Omega}(s)$ prefiltri za struju armature, odnosno brzinu vrtnje motora, koji se dodaju po potrebi.



(b)
$$P_g = M_t \omega_n = K_T \omega_n^2 \to K_T = \frac{P_g}{\omega_n^2}$$

$$K_T = \frac{11000}{154.98^2} = 0.4579 \text{ Nms/rad}$$

Sinteza regulatora struje armature

Vrijede sljedeći izrazi (u ovoj fazi izvoda se ne koristi prefiltar $G_{pfi}(s)$):

$$I_a(s) = [U_a(s) - E(s)] \frac{K_a}{1 + T_a s}$$

$$U_i(s) = \frac{K_i}{1 + T_{fi}s} I_a(s)$$

$$U_a(s) = \left[U_{iref}(s) - U_i(s) \right] K_{Ri} \frac{1 + T_{IRi}s}{T_{IRi}s} \frac{K_t}{1 + T_{mi}s}$$

Kada se drugi izraz ubaci u treći izraz, a potom to sve u prvi izraz, dobije se:

$$I_{a}(s) = \left\{ \left[U_{iref}(s) - \frac{K_{i}}{1 + T_{fi}s} I_{a}(s) \right] K_{Ri} \frac{1 + T_{IRi}s}{T_{IRi}s} \frac{K_{t}}{1 + T_{mi}s} - E(s) \right\} \frac{K_{a}}{1 + T_{a}s}$$

Traži se prijenosna funkcija $\frac{I_a(s)}{U_{iref}(s)}$ pa će se uzeti E=0:

$$I_{a}(s) = \left\{ \left[U_{iref}(s) - \frac{K_{i}}{1 + T_{fi}s} I_{a}(s) \right] K_{Ri} \frac{1 + T_{IRi}s}{T_{IRi}s} \frac{K_{t}}{1 + T_{mi}s} \right\} \frac{K_{a}}{1 + T_{a}s}$$

$$I_{a}(s) = U_{iref}(s)K_{Ri}\frac{1 + T_{IRi}s}{T_{IRi}s}\frac{K_{t}}{1 + T_{mi}s}\frac{K_{a}}{1 + T_{a}s} - \frac{K_{i}}{1 + T_{fi}s}I_{a}(s)K_{Ri}\frac{1 + T_{IRi}s}{T_{IRi}s}\frac{K_{t}}{1 + T_{mi}s}\frac{K_{a}}{1 + T_{a}s}$$

$$I_{a}(s)\left[1+\frac{K_{Ri}K_{i}K_{t}K_{a}}{T_{IRi}s}\frac{1+T_{IRi}s}{\left(1+T_{fi}s\right)(1+T_{mi}s)(1+T_{a}s)}\right]=U_{iref}(s)\frac{K_{Ri}K_{t}K_{a}}{T_{IRi}s}\frac{1+T_{IRi}s}{(1+T_{mi}s)(1+T_{a}s)}$$

$$\frac{I_a(s)}{U_{iref}(s)} = \frac{\frac{K_{Ri}K_tK_a}{T_{IRi}s}\frac{1 + T_{IRi}s}{(1 + T_{mi}s)(1 + T_as)}}{1 + \frac{K_{Ri}K_iK_tK_a}{T_{IRi}s}\frac{1 + T_{IRi}s}{(1 + T_{fi}s)(1 + T_{mi}s)(1 + T_as)}}$$

Vrijedi $T_a = \frac{L_a}{R_a} = \frac{4 \cdot 10^{-3}}{0.2} = 20$ ms, $T_{mi} = 1,67$ ms i $T_{fi} = 2$ ms. Iz toga slijedi da je T_a dominantna vremenska konstanta. **Regulacijski krug struje armature** pogodan je za projektiranje regulatora po **tehničkom optimumu**. Kompenzira se dominantna vremenska konstanta pa je:

$$T_{IRi} = T_a = 5.852 \text{ ms}$$

Vrijedi $T_{\Sigma} = T_{mi} + T_{fi} = 3,67$ ms. Strukturnim pojednostavljenjem, uz uvrštavanje $T_{IRi} = T_a$, se dobije:

$$\frac{I_a(s)}{U_{iref}(s)} = \frac{\frac{K_{Ri}K_tK_a}{T_as(1+T_{mi}s)}}{1+\frac{K_{Ri}K_iK_tK_a}{T_as(1+T_{fi}s)(1+T_{mi}s)}} = \frac{\frac{K_{Ri}K_tK_a}{T_as(1+T_{fi}s)(1+T_{mi}s)}}{\frac{T_as(1+T_{fi}s)(1+T_{mi}s)+K_{Ri}K_iK_tK_a}{T_as(1+T_{fi}s)(1+T_{mi}s)}}$$

$$\begin{split} \frac{I_{a}(s)}{U_{iref}(s)} &= \frac{K_{Ri}K_{t}K_{a}\left(1 + T_{fi}s\right)}{T_{a}s\left(1 + T_{fi}s\right)\left(1 + T_{mi}s\right) + K_{Ri}K_{i}K_{t}K_{a}} = \frac{K_{Ri}K_{t}K_{a}\left(1 + T_{fi}s\right)}{T_{a}s\left(1 + T_{\Sigma}s\right) + K_{Ri}K_{i}K_{t}K_{a}} \\ &\frac{I_{a}(s)}{U_{iref}(s)} = \left(1 + T_{fi}s\right)\frac{K_{Ri}K_{t}K_{a}}{T_{a}T_{\Sigma}s^{2} + T_{a}s + K_{Ri}K_{i}K_{t}K_{a}} \\ &\frac{I_{a}(s)}{U_{iref}(s)} = \left(1 + T_{fi}s\right)\frac{1}{K_{Ri}K_{i}K_{t}K_{a}}\frac{K_{Ri}K_{t}K_{a}}{\frac{T_{a}T_{\Sigma}}{K_{Ri}K_{i}K_{t}K_{a}}s^{2} + \frac{T_{a}}{K_{Ri}K_{i}K_{t}K_{a}}s + 1} \\ &\frac{I_{a}(s)}{U_{iref}(s)} = \frac{1 + T_{fi}s}{K_{i}}\frac{1}{\frac{T_{a}T_{\Sigma}}{K_{Ri}K_{i}K_{t}K_{a}}s^{2} + \frac{T_{a}}{K_{Ri}K_{i}K_{t}K_{a}}s + 1} \end{split}$$

Promatramo dio $\frac{1}{\frac{T_aT_{\Sigma}}{K_{Ri}K_iK_tK_a}s^2 + \frac{T_a}{K_{Ri}K_iK_tK_a}s + 1}$ koji je oblika $\frac{1}{1 + \frac{2\zeta}{\omega_n}s + \frac{s^2}{\omega_n^2}}$, pa slijedi:

$$\frac{1}{\omega_n^2} = \frac{T_a T_{\Sigma}}{K_{Ri} K_i K_t K_a}$$
$$\frac{2\zeta}{\omega_n} = \frac{T_a}{K_{Ri} K_i K_t K_a}$$

Izbor $\zeta = \frac{\sqrt{2}}{2}$ predstavlja tehnički najprihvatljiviji izbor za većinu primjena:

$$\frac{2\zeta}{\omega_n} = \frac{T_a}{K_{Ri}K_iK_tK_a} \to \frac{1}{\omega_n} = \frac{1}{2\zeta} \frac{T_a}{K_{Ri}K_iK_tK_a} = \frac{1}{\sqrt{2}} \frac{T_a}{K_{Ri}K_iK_tK_a}$$

Kada se gornji izraz kvadrira, slijedi:

$$\frac{1}{\omega_n^2} = \frac{1}{2} \frac{T_a^2}{(K_{Ri} K_i K_t K_a)^2}$$

Izjednačavanjem izraza $\frac{1}{\omega_n^2} = \frac{T_a T_{\Sigma}}{K_{Ri} K_i K_t K_a}$ i $\frac{1}{\omega_n^2} = \frac{1}{2} \frac{T_a^2}{(K_{Ri} K_i K_t K_a)^2}$ slijedi:

$$\frac{T_a T_\Sigma}{K_{Ri} K_i K_t K_a} = \frac{1}{2} \frac{T_a^2}{(K_{Ri} K_i K_t K_a)^2}$$

$$T_{\Sigma} = \frac{1}{2} \frac{T_a}{K_{Ri} K_i K_t K_a} \to K_{Ri} = \frac{T_a}{2 T_{\Sigma} K_i K_t K_a} = \frac{\frac{L_a}{R_a}}{2 (T_{mi} + T_{fi}) K_i K_t \frac{1}{R_a}} = \frac{L_a}{2 (T_{mi} + T_{fi}) K_i K_t}$$

$$K_{Ri} = \frac{4}{2 \cdot 3.67 \cdot 0.1 \cdot 44} = 0.124$$

Prijenosna funkcija $\frac{I_a(s)}{U_{iref}(s)}$ sada postaje (uvrštavanjem $K_{Ri} = \frac{T_a}{2T_\Sigma K_i K_t K_a}$):

$$\frac{I_a(s)}{U_{iref}(s)} = \frac{1 + T_{fi}s}{K_i} \frac{1}{\frac{T_a T_{\Sigma}}{2T_{\Sigma} K_i K_t K_a} K_i K_t K_a} s^2 + \frac{T_a}{\frac{T_a}{2T_{\Sigma} K_i K_t K_a} K_i K_t K_a} s + 1}$$

$$\frac{I_a(s)}{U_{iref}(s)} = \frac{1 + T_{fi}s}{K_i} \frac{1}{1 + 2T_{\Sigma}s + 2T_{\Sigma}^2 s^2}$$

S obzirom na to da se u prijenosnoj funkciji $\frac{I_a(s)}{U_{iref}(s)}$ pojavljuje nula zbog mjernog člana struje armature, koristi se prefiltar:

$$G_{pfi}(s) = \frac{1}{1 + T_{fi}s}$$

pa se dobije:

$$\frac{I_a(s)}{U_{iref}(s)} = \frac{1}{K_i} \frac{1}{1 + 2T_\Sigma s + 2T_\Sigma^2 s^2}$$

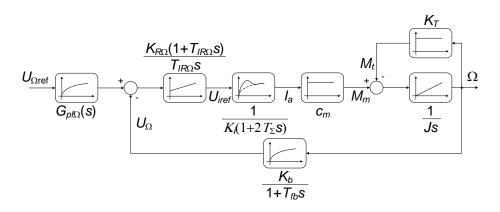
S obzirom na to da je regulacijski krug struje armature podređen regulacijskom krugu brzine vrtnje, vrlo je praktično, sa stajališta sinteze regulatora brzine vrtnje, strukturno pojednostaviti prijenosnu funkciju, tj. nadomjestiti je prijenosnom funkcijom:

$$\frac{I_a(s)}{U_{iref}(s)} \approx \frac{1}{K_i} \frac{1}{1 + T_s s}$$

gdje je $T_s = 2T_{\Sigma} = 7,34$ ms nadomjesna vremenska konstanta.

Sinteza regulatora brzine vrtnje

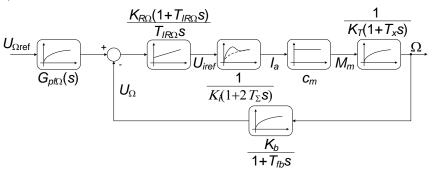
Uz prethodno uzeta pojednostavljena, strukturna shema postaje:



Prijenosna funkcija $\frac{\Omega(s)}{M_m(s)}$ glasi:

$$\frac{\Omega(s)}{M_m(s)} = \frac{1}{Js + K_T}$$

Shema se mijenja u:



pri čemu je $T_{\chi} = \frac{J}{K_T}$. U zadatku se traži i da se sinteza **regulatora brzine vrtnje** obavi prema **tehničkom optimumu**. Vrijede sljedeći izrazi (u ovoj fazi izvoda se ne koristi prefiltar $G_{pf\Omega}(s)$):

$$\Omega(s) = \frac{1}{K_T(1 + T_x s)} M_m(s)$$

$$U_{\Omega}(s) = \frac{K_b}{1 + T_{fb}s} \Omega(s)$$

$$M_m(s) = \left[U_{\Omega ref}(s) - U_{\Omega}(s) \right] K_{R\Omega} \frac{1 + T_{IR\Omega}s}{T_{IR\Omega}s} \frac{1}{K_i} \frac{1}{1 + 2T_{\Sigma}s} c_m$$

Kada se drugi izraz ubaci u treći izraz, a potom to sve u prvi izraz, dobije se:

$$\Omega(s) = \frac{1}{K_T(1+T_x s)} \left[U_{\Omega \text{re} f}(s) - \frac{K_b}{1+T_{fb} s} \Omega(s) \right] K_{R\Omega} \frac{1+T_{IR\Omega} s}{T_{IR\Omega} s} \frac{1}{K_i} \frac{1}{1+2T_\Sigma s} c_m$$

$$\Omega(s) \left[1 + \frac{K_{R\Omega} K_b c_m (1 + T_{IR\Omega} s)}{K_T K_i T_{IR\Omega} s (1 + T_x s) (1 + 2T_\Sigma s) (1 + T_{fb} s)} \right] = U_{\Omega ref}(s) \frac{K_{R\Omega} c_m (1 + T_{IR\Omega} s)}{K_T K_i T_{IR\Omega} s (1 + T_x s) (1 + 2T_\Sigma s)}$$

$$\frac{\Omega(s)}{U_{\Omega ref}(s)} = \frac{\frac{K_{R\Omega}c_{m}(1 + T_{IR\Omega}s)}{K_{T}K_{i}T_{IR\Omega}s(1 + T_{x}s)(1 + 2T_{\Sigma}s)}}{1 + \frac{K_{R\Omega}K_{b}c_{m}(1 + T_{IR\Omega}s)}{K_{T}K_{i}T_{IR\Omega}s(1 + T_{x}s)(1 + 2T_{\Sigma}s)(1 + T_{fb}s)}}$$

Vrijedi: $T_S=2T_\Sigma=0.00734$ ms, $T_{fb}=0.015$ ms i $T_x=\frac{J}{K_T}=\frac{0.478}{0.4579}=1.0438$ ms. Kompenzira se dominantna vremenska konstanta pa je:

$$T_{IR\Omega} = T_x = 1,0438 \text{ ms}$$

Strukturnim pojednostavljenjem, uz uvrštavanje $T_{IR\Omega} = T_x$ i $T_{\Sigma}^* = T_s + T_{fb}$, se dobije:

$$\frac{\Omega(s)}{U_{\Omega ref}(s)} = \frac{\frac{K_{R\Omega}c_m}{K_T K_i T_{IR\Omega} s (1 + 2T_{\Sigma}s)}}{1 + \frac{K_{R\Omega}K_b c_m}{K_T K_i T_{IR\Omega} s (1 + 2T_{\Sigma}s) (1 + T_{fb}s)}}$$

$$\frac{\Omega(s)}{U_{\Omega ref}(s)} = \frac{\frac{K_{R\Omega}c_m}{K_TK_iT_{IR\Omega}s(1+2T_{\Sigma}s)}}{\frac{K_TK_iT_{IR\Omega}s(1+2T_{\Sigma}s)\left(1+T_{fb}s\right)+K_{R\Omega}K_bc_m}{K_TK_iT_{IR\Omega}s(1+2T_{\Sigma}s)\left(1+T_{fb}s\right)}}$$

$$\frac{\Omega(s)}{U_{\Omega ref}(s)} = \frac{K_{R\Omega}c_m \left(1 + T_{fb}s\right)}{K_T K_i T_{IR\Omega}s(1 + 2T_{\Sigma}s)\left(1 + T_{fb}s\right) + K_{R\Omega}K_b c_m} = \frac{K_{R\Omega}c_m \left(1 + T_{fb}s\right)}{K_T K_i T_{IR\Omega}s(1 + T_{\Sigma}^*s) + K_{R\Omega}K_b c_m}$$

$$\frac{\Omega(s)}{U_{\Omega \text{re}f}(s)} = \left(1 + T_{fb}s\right) \frac{1}{K_{R\Omega}K_bc_m} \frac{K_{R\Omega}t_m}{\frac{K_TK_iT_{IR\Omega}T_{\Sigma}^*}{K_{R\Omega}K_bc_m}} s^2 + \frac{K_TK_iT_{IR\Omega}}{K_{R\Omega}K_bc_m} s + 1$$

$$\frac{\Omega(s)}{U_{\Omega \text{re}f}(s)} = \left(1 + T_{fb}s\right) \frac{1}{K_b} \frac{1}{\frac{K_T K_i T_{IR\Omega} T_{\Sigma}^*}{K_{R\Omega} K_b c_m}} s^2 + \frac{K_T K_i T_{IR\Omega}}{K_{R\Omega} K_b c_m} s + 1$$

Promatramo dio $\frac{1}{\frac{K_TK_iT_{IR\Omega}T_{\Sigma}^*}{K_{R\Omega}K_bc_m}S^2 + \frac{K_TK_iT_{IR\Omega}}{K_{R\Omega}K_bc_m}S + 1}$ koji je oblika $\frac{1}{1 + \frac{2\zeta}{\omega_n}S + \frac{S^2}{\omega_n^2}}$, pa slijedi:

$$\frac{1}{\omega_n^2} = \frac{K_T K_i T_{IR\Omega} T_{\Sigma}^*}{K_{R\Omega} K_h c_m}$$

$$\frac{2\zeta}{\omega_n} = \frac{K_T K_i T_{IR\Omega}}{K_{R\Omega} K_h c_m}$$

Izbor $\zeta=\frac{\sqrt{2}}{2}$ predstavlja tehnički najprihvatljiviji izbor za većinu primjena:

$$\frac{2\zeta}{\omega_n} = \frac{K_T K_i T_{IR\Omega}}{K_{R\Omega} K_b c_m} \rightarrow \frac{1}{\omega_n} = \frac{1}{2\zeta} \frac{K_T K_i T_{IR\Omega}}{K_{R\Omega} K_b c_m} = \frac{1}{\sqrt{2}} \frac{K_T K_i T_{IR\Omega}}{K_{R\Omega} K_b c_m}$$

Kada se gornji izraz kvadrira, slijedi:

$$\frac{1}{\omega_n^2} = \frac{1}{2} \frac{K_T^2 K_i^2 T_{IR\Omega}^2}{K_{R\Omega}^2 K_h^2 c_m^2}$$

Izjednačavanjem izraza $\frac{1}{\omega_n^2} = \frac{K_T K_i T_{IR\Omega} T_\Sigma^*}{K_{R\Omega} K_b c_m}$ i $\frac{1}{\omega_n^2} = \frac{1}{2} \frac{K_T^2 K_i^2 T_{IR\Omega}^2}{K_{R\Omega}^2 K_b^2 c_m^2}$ slijedi:

$$\frac{K_T K_i T_{IR\Omega} T_{\Sigma}^*}{K_{R\Omega} K_b c_m} = \frac{1}{2} \frac{K_T^2 K_i^2 T_{IR\Omega}^2}{K_{R\Omega}^2 K_b^2 c_m^2}$$

$$T_{\Sigma}^* = \frac{1}{2} \frac{K_T K_i T_{IR\Omega}}{K_{R\Omega} K_b c_m} \rightarrow \frac{K_{R\Omega}}{2T_{\Sigma}^* K_b c_m} = \frac{0.4579 \cdot 0.1 \cdot 1.0438}{2 \cdot 0.02234 \cdot 0.031 \cdot 2.66} = \frac{12.9778}{2 \cdot 0.02234 \cdot 0.031 \cdot 2.66}$$

Prijenosna funkcija $\frac{I_a(s)}{U_{iref}(s)}$ sada postaje (uvrštavanjem $K_{R\Omega} = \frac{K_T K_i T_{IR\Omega}}{2T_\Sigma^* K_b c_m}$):

$$\begin{split} \frac{\Omega(s)}{U_{\Omega ref}(s)} &= \left(1 + T_{fb} s\right) \frac{1}{K_b} \frac{1}{\frac{K_T K_i T_{IR\Omega} T_{\Sigma}^*}{K_T K_i T_{IR\Omega}} s^2 + \frac{K_T K_i T_{IR\Omega}}{\frac{K_T K_i T_{IR\Omega}}{2 T_{\Sigma}^* K_b c_m} K_b c_m} s + 1} \\ &\frac{\Omega(s)}{U_{\Omega ref}(s)} = \frac{1 + T_{fb} s}{K_b} \frac{1}{1 + 2 T_{\Sigma}^* s + 2 T_{\Sigma}^{*2} s^2} \end{split}$$

S obzirom na to da se u prijenosnoj funkciji $\frac{\Omega(s)}{U_{\Omega ref}(s)}$ pojavljuje nula zbog mjernog člana brzine vrtnje, koristi se prefiltar:

$$G_{pf\Omega}(s) = \frac{1}{1 + T_{fb}s}$$