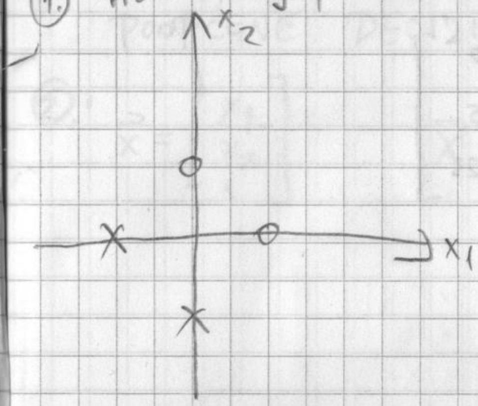
AUDITORNE 2. CIKLUS

1. Ho-Kashyap



$w_1 = \{ [1, 0]^T, [0, 1]^T \}$

$w_2 = \{ [-1, 0], [0, -1]^T \}$

$c = 1$

$\vec{b}(1) = [1\ 1\ 1\ 1]^T$

Proširivanje uzoraka

$$\vec{x}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad \vec{x}_2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad x_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad x_4 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$$[X] = \begin{bmatrix} \vec{x}_1^T \\ \vec{x}_2^T \\ \vec{x}_3^T \\ \vec{x}_4^T \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$[X]^T \cdot [X] = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$([X]^T \cdot [X])^{-1} = \begin{bmatrix} 0,5 & 0 & 0 \\ 0 & 0,5 & 0 \\ 0 & 0 & 0,25 \end{bmatrix}$$

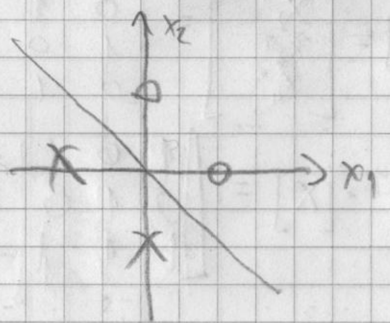
$$[X]^{\#} = ([X]^T \cdot [X])^{-1} \cdot [X]^T) = \begin{bmatrix} 0,5 & 0 & 0 \\ 0 & 0,5 & 0 \\ 0 & 0 & 0,25 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$$

$$[X]^{\#} = \begin{bmatrix} 0,5 & 0 & 0,5 & 0 \\ 0 & 0,5 & 0 & 0,5 \\ 0,25 & 0,25 & -0,25 & -0,25 \end{bmatrix}$$

$$\vec{w}(1) = [X]^{\#} \cdot \vec{b}(1) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\vec{e}(1) = [X] \cdot \vec{w}(1) - \vec{b}(1)$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



POOPĆENE DECIZIJSKE FUNKCIJE

2.1 $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$ $\vec{X}^{*} = \begin{bmatrix} x_1^2 \\ x_1 x_2 \\ x_2^2 \\ x_1 \\ x_2 \\ 1 \end{bmatrix}$

$$d(x) = w_1 x_1^2 + w_2 x_1 x_2 + w_3 x_2^2 + w_4 x_1 + w_5 x_2 + w_6$$

2.2

$$\vec{x}^{*} = \begin{bmatrix} x_1^3 \\ x_2^3 \\ x_1^2 x_2 \\ x_1 x_2^2 \\ x_1^2 \\ x_1 x_2 \\ x_2^2 \\ x_1 \\ x_2 \\ 1 \end{bmatrix}$$

2.3

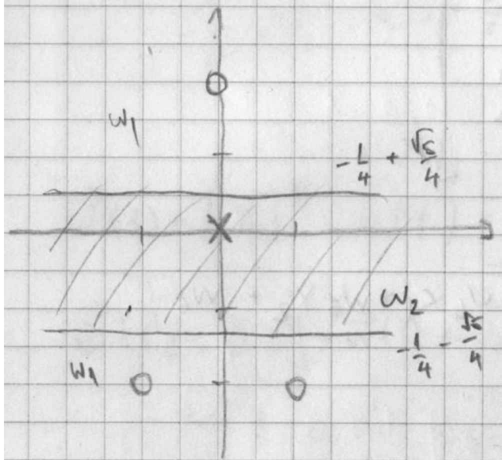
$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{x}^{*} = \begin{bmatrix} x_1^2 \\ x_2^2 \\ x_3^2 \\ x_1 x_2 \\ x_2 x_3 \\ x_1 x_3 \\ x_1 \\ x_2 \\ x_3 \\ 1 \end{bmatrix}$$

2.4 $w_1 = \{[0,2]^T, [-1,-2]^T, [1,-2]^T\}$

$w_1 = \{[0,0]^T\}$

$c = 1$

$\vec{J}(1) = \vec{0}$



$$X^* = \begin{bmatrix} x_1^2 \\ x_2^2 \\ x_1 x_2 \\ x_1 \\ x_2 \\ 1 \end{bmatrix}$$

$$x_1^* = [0 \quad 4 \quad 0 \quad 0 \quad 2 \quad 1]^T$$

$$x_2^* = [1 \quad 4 \quad -2 \quad -1 \quad -2 \quad 1]^T$$

$$x_3^* = [1 \quad 4 \quad -2 \quad 1 \quad -2 \quad 1]^T$$

$$x_4^* = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad -1]^T$$

perceptron!

$$\vec{w}_1^T \cdot \vec{x}_1^* = \vec{0}$$

$$\vec{w}_2 = \vec{w}_1 + c \cdot x_1^* = [0, 4, 0, 0, 2, 1]^T$$

$$\vec{w}_2^T \cdot \vec{x}_2^* = [0, 4, 0, 0, 2, 1]^T \cdot [1, 4, -2, -1, -2, 1] = 13$$

$$\vec{w}_3^T \cdot \vec{x}_3^* = [0, 4, 0, 0, 2, 1]^T \cdot [0, 0, 0, 0, 0, -1] = -1$$

$$\vec{w}_5 = \vec{w}_4 + c\vec{x}_4 = [0, 4, 0, 0, 2, 0]^T$$

12. korak: $\vec{w} = [0, 4, 0, 0, 2, -1]^T$

$$d(\vec{x}) = 4x_2^2 + 2x_2 - 1$$

$$4x_2^2 + 2x_2 - 1 = 0$$

$$4x_2^2 + 2x_2 + \frac{1}{4} - \frac{5}{4} = 0$$

$$\left(2x_2 + \frac{1}{2}\right)^2 = \frac{5}{4}$$

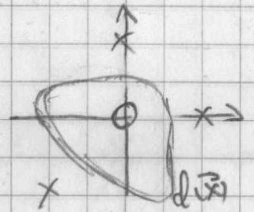
$$2x_2 + \frac{1}{2} = \pm \frac{\sqrt{5}}{2}$$

$$x_2 = -\frac{1}{4} \pm \frac{\sqrt{5}}{4}$$

3. POTENCIJALNE FUNKCIJE

(2) $w_1 = \{[0,0]^T\}$

$w_2 = \{[0,1]^T, [1,0]^T, [-1,-1]^T\}$



$K(\vec{x}, \vec{x}_k) = \frac{1}{1+\|\vec{x}-\vec{x}_k\|^2}$

$d_0(\vec{x})=0$

1. $d_0(\vec{x}_1)=0$

$d_1(\vec{x}) = d_0(\vec{x}) + K(\vec{x},\vec{x}_1) = \frac{1}{1+\|\vec{x}-\vec{x}_1\|^2} = \frac{1}{1+x_1^2+x_2^2}$

2. $d_1(\vec{x}_2) = \frac{1}{1+0^2+1^2} = \frac{1}{2} \geq 0$

$d_2(\vec{x}) = d_1(\vec{x}) - K(\vec{x},\vec{x}_2)$

$d(\vec{x}) = d_1(\vec{x}) - \frac{1}{1+x_1^2+(x_2-1)^2}$

3. $d_2(\vec{x}_3) = \frac{1}{1+1^2+0^2} - \frac{1}{1+1^2+1^2} = \frac{1}{6}$

$d_3(\vec{x}) = d_2(\vec{x}) - K(\vec{x},\vec{x}_3)$

$d_3(\vec{x}) = d_2(\vec{x}) - \frac{1}{1+(x_1-1)^2+x_2^2}$

4. $d_3(\vec{x}_4) = \frac{1}{3} - \frac{1}{6} - \frac{1}{6} = 0$

$d_4(\vec{x}) = d_3(\vec{x}) - K(\vec{x},\vec{x}_4)$

5. $d_4(\vec{x}_5) = 1 - \frac{1}{2} - \frac{1}{2} - \frac{1}{3} = -\frac{1}{3}$

$d_5(\vec{x}) + K(\vec{x},\vec{x}_5)$

$$\boxed{d(\vec{x}) = d_5(\vec{x}) = \frac{2}{1+x_1^2+x_2^2} - \frac{1}{1+x_1^2+(x_2-1)^2} - \frac{1}{1+(x_1-1)^2+x_2} - \frac{1}{1+(x_1+1)^2+(x_2+1)^2}}$$

6. $d_5(\vec{x}_6) = 1-1-\frac{1}{3}-\frac{1}{6} = -\frac{1}{2}$ ✓

7. $d_6(\vec{x}_7) = -\frac{1}{2}$ ✓

8. $d_7(\vec{x}_8) = \frac{2}{3} - \frac{1}{6} - \frac{1}{6} - 1 = \frac{-2}{3}$ ✓

9. $d_8(\vec{x}_9) = 2 - \frac{1}{2} - \frac{1}{2} - \frac{1}{3} = \frac{2}{3}$ ✓

SVI UZORCI ISPRAVNO RAZVRSTANI

$d(\vec{x}) = d_5(\vec{x})$

4. BAYESOV KLASIFIKATOR

$$P(A|B)\cdot P(B) = P(B|A)\cdot P(A)$$

$$\boxed{P(A|B)=\frac{P(B|A)\cdot P(A)}{P(B)}}$$

$$P(\omega_1|\vec{x}) = \frac{P(\vec{x}|\omega_1)\cdot P(\omega_1)}{P(\vec{x})}$$

$\vec{x} \in \omega_i$ ako $P(\omega_i|\vec{x}) > P(\omega_j|\vec{x}) \quad \forall j\neq i$

odnosno:

ako $P(\vec{x}|\omega_i)\cdot P(\omega_i) > P(\vec{x}|\omega_j)\cdot P(\omega_j) \quad \forall j\neq i$

$$P(x)=\frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$$

NORMALNA RAZDIOBA

$$\boxed{P(\vec{x})=\frac{1}{(2\pi)^{\frac{1}{2}}|C|^{\frac{1}{2}}} e^{-\frac{1}{2}(\vec{x}-\vec{m})^T C^{-1}(\vec{x}-\vec{m})}}$$

VIŠEDIMENZIONALNA NORMALNA RAZDIOBA

$$P(\vec{x}|\omega_i)=\frac{1}{(2\pi)^{\frac{1}{2}}|C_i|^{\frac{1}{2}}} e^{-\frac{1}{2}(\vec{x}-\vec{m}_i)^T C_i^{-1}(\vec{x}-\vec{m}_i)}$$

$$C_i = \underset{\vec{x}\in\omega_i}{E}\{(\vec{x}-\vec{m}_i)(\vec{x}-\vec{m}_i)^T\}$$

$$\approx \frac{1}{N_i}\sum(\vec{x}-\vec{m}_i)(\vec{x}-\vec{m}_i)^T$$

Logaritamsko Bayesovo pravilo

$\vec{x}\in\omega_i$ ako $\ln[P(\vec{x}|\omega_i)\cdot P(\omega_i)] > \ln[P(\vec{x}|\omega_j)\cdot P(\omega_j)]$

$\forall j\neq i$

$$d_i = \ln[P(\vec{x}|\omega_i)P(\omega_i)] = \ln(\omega_i) + \ln P(\vec{x}|\omega_i)$$

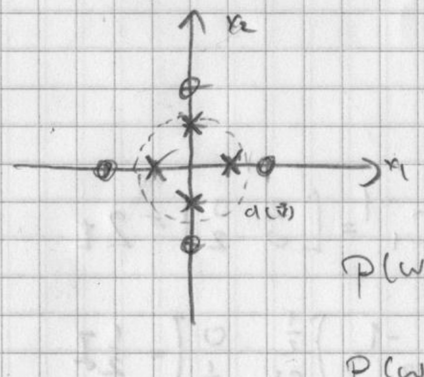
$$= \ln P(\omega_i) - \underbrace{\cancel{\frac{n}{2}\ln(2\pi)}}_{\text{konst}} - \frac{1}{2}\ln|C_i| - \frac{1}{2}[(\vec{x}-\vec{m}_i)^T C_i (\vec{x}-\vec{m}_i)]$$

$$\boxed{d_i(\vec{x}) = \ln P(\omega_i) - \frac{1}{2}\ln|C_i| - \frac{1}{2}[(\vec{x}-\vec{m}_i)^T C_i (\vec{x}-\vec{m}_i)]}$$

4. Normalna distr.

$\omega_1 = \{[-1,0]^T, [0,-1]^T, [1,0]^T, [0,1]^T\}$

$\omega_2 = \{[-2,0]^T, [0,-2]^T, [2,0]^T, [0,2]^T\}$



$N_1 = 4$ – broj uzoraka u skupu za učenje

$N_2 = 4$

$P(\omega_i) = \frac{N_1}{N} = \frac{1}{2}$

$P(\omega_2) = \frac{1}{2}$

$$\vec{m}_1 = \frac{1}{N_1}\sum_{x\in\omega_1}\vec{x} = \frac{1}{4}\left(\begin{bmatrix}-1\\0\end{bmatrix}+\begin{bmatrix}0\\-1\end{bmatrix}+\begin{bmatrix}1\\0\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}\right)$$

$$= \vec{0}$$

$$\vec{m}_2 = \frac{1}{N_2}\sum_{x\in\omega_2}\vec{x} = \vec{0}$$

$$C_1 = \frac{1}{N_1}\sum_{x_i\in\omega_1}(\vec{x}_i-\vec{m}_1)(\vec{x}_i-\vec{m}_1)^T = \frac{1}{4}\left(\begin{bmatrix}-1\\0\end{bmatrix}[-1\ 0]+\begin{bmatrix}0\\-1\end{bmatrix}[0\ -1]+\begin{bmatrix}1\\0\end{bmatrix}[1\ 0]+\begin{bmatrix}0\\1\end{bmatrix}[0\ 1]\right)$$

$$= \frac{1}{4}\left(\begin{bmatrix}1&0\\0&0\end{bmatrix}+\begin{bmatrix}0&0\\0&1\end{bmatrix}+\begin{bmatrix}1&0\\0&0\end{bmatrix}+\begin{bmatrix}0&0\\0&1\end{bmatrix}\right)$$

$$= \frac{1}{2}I$$

$$C_2 = \frac{1}{N_2}\sum_{x_i\in\omega_2}(\vec{x}_i-\vec{m}_2)(\vec{x}_i-\vec{m}_2)^T$$

$$= 2I$$

$$|C_1| = \begin{vmatrix}\frac{1}{2}&0\\0&\frac{1}{2}\end{vmatrix} = \frac{1}{4} \qquad C_1^{-1} = \begin{bmatrix}2&0\\0&2\end{bmatrix} = 2I$$

$$|C_2| = \begin{vmatrix}2&0\\0&2\end{vmatrix} = 4 \qquad C_2^{-1} = \begin{bmatrix}\frac{1}{2}&0\\0&\frac{1}{2}\end{bmatrix} = \frac{1}{2}I$$

$$d_1 - d_2 = 0$$

$$-\frac{1}{2}\ln|C_1| - \frac{1}{2}[(\vec{x}-\vec{m}_1)^T C_1^{-1}(\vec{x}-\vec{m}_1)] + \frac{1}{2}\ln|C_2| + \frac{1}{2}[(\vec{x}-\vec{m}_2)^T C_2^{-1}(\vec{x}-\vec{m}_2)] = 0$$

$$-\frac{1}{2}\ln\frac{1}{4} - \frac{1}{2}\vec{x}^T\cdot 2I\cdot\vec{x} + \frac{1}{2}\ln 4 + \frac{1}{2}\vec{x}^T\frac{1}{2}I\cdot\vec{x} = 0$$

$$0{,}693 - \vec{x}^T\vec{x} + 0{,}693 + \frac{1}{4}\vec{x}^T\vec{x} = 0$$

$$1{,}386 - 3/4\ \vec{x}^T\vec{x} = 0$$

$$1{,}386 - \frac{3}{4}(x_1^2 + x_2^2) = 0 \quad \cdot\frac{4}{3}$$

$$x_1^2 + x_2^2 = 1{,}848$$

kružnica

$r^2 = 1{,}848$

$r = 1{,}36$