1. BAYES: ODREDITI GRANICU IZMEĐU DVA RAZREDA

UUKU A.1.1

$\omega_1 \dots \; p(\vec{x}|\omega_1) = \frac{1}{2\pi} e^{-\frac{1}{2}(\vec{x}-\vec{m_1})^T(\vec{x}-\vec{m_1})}$ $\qquad \vec{m_1} = [1\ 1]$

$\omega_2 \dots \; p(\vec{x}|\omega_2) = e^{-(x_1+x_2)}$ $\qquad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$p(\omega_2) = 2p(\omega_1)$

$p(\omega_1) = \frac{1}{3} \quad p(\omega_2) = \frac{2}{3}$

$$p(\omega_1|\vec{x}) = p(\vec{x}|\omega_1) \cdot p(\omega_1)$$

PRAVILO:

$$p(\vec{x}|\omega_1) \cdot p(\omega_1) > p(\vec{x}|\omega_2) \cdot p(\omega_2) \quad / \ln$$

$$\ln(p(\vec{x}|\omega_1)) + \ln(p(\omega_1)) > \ln(p(\vec{x}|\omega_2)) + \ln(p(\omega_2))$$

$$-\ln(2\pi) - \frac{1}{2}\underbrace{(\vec{x}-\vec{m_1})^T(\vec{x}-\vec{m_1})}_{\text{raspisano}} + \ln(p(\omega_1)) = +(x_1+x_2) + \ln 2 + \ln(p(\omega_1))$$

$$-\ln(2\pi) - \ln 2 - \frac{1}{2}\left[(x_1-1)^2+(x_2-1)^2\right] + x_1 + x_2 = 0$$

$$-2.531 - \frac{1}{2}(x_1^2 - 2x_1 + 1 + x_2^2 - 2x_2 + 1) + x_1 + x_2 = 0$$

$$-2,531 - \frac{1}{2}x_1^2 + x_1 - \frac{1}{2} - \frac{1}{2}x_2^2 + x_2 - \frac{1}{2} + x_1 + x_2 = 0$$

$$-\frac{1}{2}x_1^2 - \frac{1}{2}x_2^2 + 2x_1 + 2x_2 - 3,531 = 0 \implies \text{GRANICA IZMEĐU DVA RAZREDA}$$

SVE MOŽE BITI ZADANO GRAFIČKI !!!

UVRV A.2.2

2. ODREDITI LEVENSTEINOVU UDALJENOST IZMEĐU

S = acbcdb ; T = cabdb ⇒ DINAMIČKA

cost(S[i], -) = 1

cost(-, S[j]) = 1

cost(S[i], T[j]) = 1 za S[i] ≠ T[j] inače 0

umetanje T →

		0	1 c	2 a	3 b	4 d	5 b
0		0	1	2	3	4	5
1	a	1	1	1	2	3	4
2	c	2	1	2	2	3	4
3	b	3	2	2	2	3	3
4	c	4	3	3	3	3	4
5	d	5	4	4	4	3	4
6	b	6	5	5	4	4	3

brisanje S ↓

- horizontalno ⇒ dodavanje znakova iz T
- vertikalno ⇒ brisanje znakova iz S
- dijagonalno ⇒ zamjena znakova

→ najmanje zamjena

- na strelice bi trebalo pisati samo od kud se došlo, bilo bi preglednije

3. CYK ALGORITAM: pripada li babaa u ω_1 ili ω_2

ω_1	ω_2
S → AB	S → AB
A → AbB | a	A → AaB | b
B → bC | a	B → aC | b
C → a | b	C → a | b

1. pretvoriti u CNF!

A → BC

A → a | b | c

ω_1		ω_2	
S → AB		S → AB	S → AB
A → EB | a	E → AD	A → ADB | b	A → EB | b
B → DC | a	D → b	B → DC | b	E → AD
C → a | b		D → a	D → a
		C → a | b	B → DC | b
			C → a | b

pripada li babaa u ω_1

5	∅				
4	∅	S			
3	∅	A,S	∅		
2	B	E	B	S	
1	C,D	A,B C	C,D	A,B C	A,B C

⇐ 1. korak

kolibo znakova: b 1, a 2, b 3, a 4, a 5 ⇒ od kud se počinje

2. korak

(b)(a)

(C,D) + (A,B,C) ⇒ B → CD

(a)(b)

(A,B,C) + (C,D) ⇒ E → AD

(b)(a) ⇒ B → CD

(C,D) + (A,B,C)

(a)(a)

(A,B,C) + (A,B,C) → S → AB

UURU
A.2.3

3. korak

bab

(b)(ab) ⇒ (C,D)+(E) ⇒ ∅ ⇒ ∅

(ba)(b) ⇒ (B)+(C,D) ⇒ ∅

---

aba

(ab)(a) ⇒ E + (A,B,C) ⇒ A ⇒ A,S

(a)(ba) ⇒ (A,B,C)+(B) ⇒ S

---

baa

(b)(aa) ⇒ (C,D) + S ⇒ ∅ ⇒ ∅

(ba)(a) ⇒ B + (A,B,C) ⇒ ∅

---

4. korak

baba

(b)(aba) ⇒ (A,B,C)+(A,S) ⇒ ∅

(ba)(ba) ⇒ B + B ⇒ ∅ ⇒ ∅

(bab)(a) ⇒ ∅ + (A,B,C) ⇒ ∅

---

abaa

(a)(baa) ⇒ (C,D)+(∅) ⇒ ∅

(ab)(aa) ⇒ E + S ⇒ ∅

(aba)(a) ⇒ (A,S)+(A,B,C) ⇒ S

---

5. korak

babaa

(b)(abaa) ⇒ (A,B,C)+(S) ⇒ ∅

(ba)(baa) ⇒ B + ∅ ⇒ ∅

(bab)(aa) ⇒ ∅ + S ⇒ ∅

(baba)(a) ⇒ ∅ + (A,B,C) ⇒ ∅

⇒ ∅

babaa ne pripada w1

pripada li babaa u w2

S				
E	∅			
S,A	∅	S		
E	B	E	B	
A,B,C	C,D	A,B,C	C,D	C,D
b	a	b	a	a

babaa pripada u w2

GRUPIRANJE

VURU A.2.4

4. MAX-MIN

$x_1 = [0, 0]^T$

$x_2 = [2\ 3]^T$

$x_3 = [5, 3]^T$

$x_4 = [3, 2]^T$

$x_5 = [1, 4]^T$

$x_6 = [4, 1]^T$

$x_7 = [1, 2]^T$

$x_8 = [3, 4]^T$

$x_9 = [4, 5]^T$

$x_{10} = [4, 2]^T$

$c = 0,5 \Rightarrow$ da bi algoritam stao maksimalna udaljenost najbližeg uzorka do središta svih grupa mora biti manja od $c \cdot d(\vec{z}_1, \vec{z}_2)$, općenito ovdje može biti bilo koji prag T

1) random uzorak (obično prvi)

$\vec{z}_1 = \vec{x}_1 = [0\ 0]^T$

2) udaljenost svih do početnog

	$D_{2,1}$	$D_{3,1}$	$D_{4,1}$	$D_{5,1}$	$D_{6,1}$	$D_{7,1}$	$D_{8,1}$	$D_{9,1}$	$D_{10,1}$
$\vec{z}_1$	$\sqrt{13}$	$\sqrt{34}$	$\sqrt{13}$	$\sqrt{17}$	$\sqrt{17}$	$\sqrt{5}$	$\sqrt{25}$	$\sqrt{41}$	$\sqrt{20}$

→ max od min

$\vec{z}_2 = \vec{x}_9 = [4\ 5]^T \quad d = \sqrt{41} = 6,403 = d(\vec{z}_1, \vec{z}_2)$

3) udaljenosti do $\vec{z}_1, \vec{z}_2$

	$D_{2,1}$	$D_{3,1}$	$D_{4,1}$	$D_{5,1}$	$D_{6,1}$	$D_{7,1}$	$D_{8,1}$	$D_{10,1}$
$\vec{z}_1$	$\sqrt{13}$	$\sqrt{34}$	$\sqrt{13}$	$\sqrt{17}$	$\sqrt{17}$	$\sqrt{5}$	$\sqrt{25}$	$\sqrt{20}$
$\vec{z}_2$	$\sqrt{8}$	$\sqrt{5}$	$\sqrt{10}$	$\sqrt{10}$	$\sqrt{16}$	$\sqrt{18}$	$\sqrt{2}$	$\sqrt{9}$

○ → minimalni

$d = \sqrt{16} = 4 > c \cdot d(z_1, z_2) \Rightarrow$ nastavak

4) $\vec{z}_3 = \vec{x}_6 = [4\ 1]^T$

	$D_{2,1}$	$D_{3,1}$	$D_{4,1}$	$D_{5,1}$	$D_{7,1}$	$D_{8,1}$	$D_{10,1}$
$\vec{z}_1$	$\sqrt{13}$	$\sqrt{34}$	$\sqrt{13}$	$\sqrt{17}$	$\sqrt{5}$	$\sqrt{25}$	$\sqrt{20}$
$\vec{z}_2$	$\sqrt{8}$	$\sqrt{5}$	$\sqrt{10}$	$\sqrt{10}$	$\sqrt{18}$	$\sqrt{2}$	$\sqrt{9}$
$\vec{z}_3$	$\sqrt{8}$	$\sqrt{5}$	$\sqrt{2}$	$\sqrt{18}$	$\sqrt{10}$	$\sqrt{10}$	$\sqrt{1}$

biram bilo koji

$d = \sqrt{10} = 3,16 < c \cdot d(\vec{z}_1, \vec{z}_2) \Rightarrow$ kraj

GRUPE:

$S_1 = \{\vec{x}_1, \vec{x}_4\}$

$S_2 = \{\vec{x}_9, \vec{x}_2, \vec{x}_3, \vec{x}_8, \vec{x}_5\}$

$S_3 = \{\vec{x}_6, \vec{x}_7, \vec{x}_{10}\}$

5. K-MEANS ALGORITHM

UUU A.2.5

$\vec{x}_1 = [0,0]^T$
$\vec{x}_2 = [3,8]^T$
$\vec{x}_3 = [2\ 2]^T$
$\vec{x}_4 = [1\ 1]^T$
$\vec{x}_5 = [5\ 3]^T$
$\vec{x}_6 = [4\ 8]^T$
$\vec{x}_7 = [8\ 3]^T$
$\vec{x}_8 = [5\ 4]^T$
$\vec{x}_9 = [6\ 4]^T$
$\vec{x}_{10} = [7\ 5]^T$

$\boxed{K = 3}$ - postupak se ponavlja "sve dok traje „tranzicija"

0. odabir početnih središta (random)

$\vec{z}_1(1) = \vec{x}_4 = [0\ 0]^T$
$\vec{z}_2(1) = \vec{x}_2 = [3\ 8]^T$
$\vec{z}_3(1) = \vec{x}_3 = [2\ 2]^T$

1. proračun udaljenosti (korak 1)

	$\vec{x}_1$	$\vec{x}_2$	$\vec{x}_3$	$\vec{x}_4$	$\vec{x}_5$	$\vec{x}_6$	$\vec{x}_7$	$\vec{x}_8$	$\vec{x}_9$	$\vec{x}_{10}$
$\vec{z}_1(1)$	(0)	$\sqrt{73}$	$\sqrt{8}$	($\sqrt{2}$)	$\sqrt{34}$	$\sqrt{80}$	$\sqrt{45}$	$\sqrt{41}$	$\sqrt{52}$	$\sqrt{74}$
$\vec{z}_2(1)$	$\sqrt{73}$	(0)	$\sqrt{37}$	$\sqrt{53}$	$\sqrt{29}$	($\sqrt{1}$)	$\sqrt{34}$	$\sqrt{20}$	$\sqrt{25}$	($\sqrt{25}$)
$\vec{z}_3(1)$	$\sqrt{8}$	$\sqrt{36}$	(0)	$\sqrt{2}$	($\sqrt{10}$)	$\sqrt{40}$	($\sqrt{117}$)	($\sqrt{13}$)	($\sqrt{20}$)	$\sqrt{34}$

$S_1(1) = \{\vec{x}_1, \vec{x}_4\}$
$S_2(1) = \{\vec{x}_2, \vec{x}_6, \vec{x}_{10}\}$
$S_3(1) = \{\vec{x}_3, \vec{x}_5, \vec{x}_7, \vec{x}_8, \vec{x}_9\}$

$\vec{z}_1(2) = \frac{1}{2}\left(\begin{bmatrix}0\\0\end{bmatrix} + \begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}1/2\\1/2\end{bmatrix}$

$\vec{z}_2(2) = \frac{1}{3}\left(\begin{bmatrix}3\\8\end{bmatrix} + \begin{bmatrix}4\\8\end{bmatrix} + \begin{bmatrix}7\\5\end{bmatrix}\right) = \begin{bmatrix}14/3\\7\end{bmatrix}$

$\vec{z}_3(2) = \frac{1}{5}\left(\begin{bmatrix}2\\2\end{bmatrix} + \begin{bmatrix}5\\3\end{bmatrix} + \begin{bmatrix}8\\3\end{bmatrix} + \begin{bmatrix}5\\4\end{bmatrix} + \begin{bmatrix}6\\4\end{bmatrix}\right) = \begin{bmatrix}4.2\\4.8\end{bmatrix}$

2. korak

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
$\vec{z}_1$	(0,7)	11,9	(2,1)	(0,7)	5,15	8,3	~8	~5,5	~6,5	~8
$\vec{z}_2$	8,4	(1,9)	5,65	7,0	4,35	(~1)	~5	~3	~3	3,1
$\vec{z}_3$	6,4	3,4	3,6	5,0	(2,0)	~2	(~4)	(~1)	(~2)	(2,8)

- nigdje ne treba točna udaljenost, može se samo na grafu uzeti približne vrijednosti, tek ako su vrijednosti jako slične onda je potrebno finije određivanje

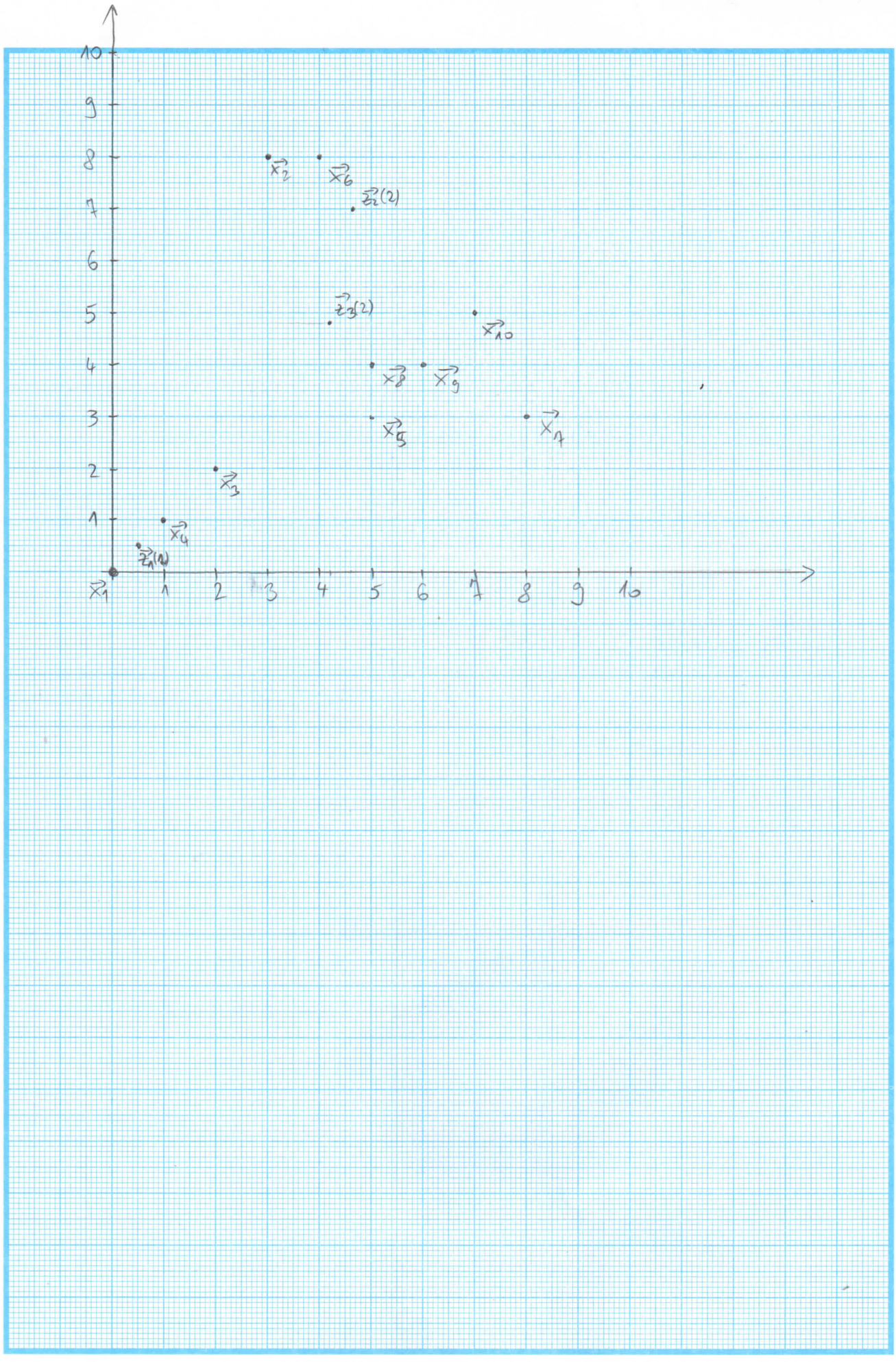
$S_1(2) = \{\vec{x}_1, \vec{x}_3, \vec{x}_4\}$
$S_2(2) = \{\vec{x}_2, \vec{x}_6\}$
$S_3(2) = \{\vec{x}_5, \vec{x}_7, \vec{x}_8, \vec{x}_9, \vec{x}_{10}\}$

$\vec{z}_1(3) = \frac{1}{3}\left(\begin{bmatrix}0\\0\end{bmatrix} + \begin{bmatrix}2\\2\end{bmatrix} + \begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\1\end{bmatrix}$

$\vec{z}_2(3) = \frac{1}{2}\left(\begin{bmatrix}3\\8\end{bmatrix} + \begin{bmatrix}4\\8\end{bmatrix}\right) = \begin{bmatrix}3,5\\8\end{bmatrix}$

$\vec{z}_3(3) = \frac{1}{5}\left(\begin{bmatrix}5\\3\end{bmatrix} + \begin{bmatrix}8\\3\end{bmatrix} + \begin{bmatrix}5\\4\end{bmatrix} + \begin{bmatrix}6\\4\end{bmatrix} + \begin{bmatrix}7\\5\end{bmatrix}\right) = \begin{bmatrix}6,2\\3,8\end{bmatrix}$

itd. :)


10
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