1.) 4.3, 4.5, 4.2, 4.6, 4.5, 4.4, 4.5, 4.4 ; $n=8$

$\bar{X}, \hat{S} = ?$

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i = \frac{1}{8}\sum_{n=1}^{8} X_i = \frac{1}{8}(4.3+4.5+\ldots+4.4) = \underline{4.425}$$

$$S^2 = \frac{1}{n-1}\sum_{k=1}^{n}(X_k-\bar{X})^2 = \frac{1}{8-1}\sum_{k=1}^{8}(X_k-4.425)^2 = \underline{0.01643} \Rightarrow \underline{S=0.128}$$

2.) $D^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i-a)^2 \qquad \sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^2$, dokaži: $D^2 = \sigma^2 + (\bar{x}-a)^2$!

$$D^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X}+\bar{x}-a) = \frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^2 + 2(X_i-\bar{x})(\bar{x}-a) + (\bar{x}-a)^2] =$$

$$= \frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^2 + \frac{2}{n}\sum_{i=1}^{n}(X_i-\bar{X})(\bar{X}-a) + \frac{1}{n}\sum_{i=1}^{n}(\bar{X}-a)^2 =$$

$$= \sigma^2 + \frac{2}{n}(\bar{X}-a)\cdot\sum_{n=1}^{n}(X_i-\bar{X}) + (\bar{X}-a)^2 = \sigma^2 + \frac{2}{n}(\bar{x}-a)\left(\sum_{i=1}^{n}X_i - n\bar{X}\right) + (\bar{x}-a)^2 =$$

$$= \sigma^2 + \frac{2}{n}(\bar{x}-a)\underbrace{(n\bar{x}-n\bar{X})}_{\emptyset} + (\bar{X}-a)^2$$

$$\underline{D^2 = \sigma^2 + (\bar{X}-a)^2}$$

3.) $h = 164.32\ m \Rightarrow E(X) = a = h = 164.32$

164.16, 164.33, 164.38, 164.44, 164.12, 164.30, 164.56, 164.47, 164.55, 164.22

$n=10$

$D=?$

$$D^2 = \frac{1}{n}\sum_{i=1}^{n}(X_k-a)^2 = \frac{1}{10}\sum_{k=1}^{10}(X_k-164.32)^2 = 0.02271$$

$$D = \sqrt{0.02271} = \underline{0,1507}$$

4.) očekivanje nije poznato $\Rightarrow$ podaci iz prošlog zadatka

$$S^2 = \frac{1}{n-1}\sum_{k=1}^{n}(X_k-\bar{X})^2 = \frac{1}{9}\cdot\sum_{k=1}^{8}(X_k-164.353)^2 = \underline{0,15499}$$

$$\bar{X} = \frac{\sum_{k=1}^{n} X_k}{8} = \underline{164,353}$$

5.)

interval	n_k
21.0 - 21.3	2
21.3 - 21.6	8
21.6 - 21.9	15
21.9 - 22.2	26
22.2 - 22.5	43
22.5 - 22.8	38
22.8 - 23.1	24
23.1 - 23.4	15
23.4 - 23.7	6
23.7 - 24.1	3

$\Rightarrow$

x_k	n_k
21.15	2
21.45	8
21.75	15
22.05	26
22.35	43
22.65	38
22.95	24
23.25	15
23.55	6
23.9	3

$\boxed{n = 180}$

$$\boxed{\bar{X} = \frac{\sum_{k=1}^{n} n_k \cdot x_k}{n}} \Rightarrow \underline{\bar{x} = 22.487}$$

$$\boxed{S^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} n_i x_i^2 - n\bar{x}^2\right)} \quad \underline{S^2 = .298}$$

9.) 5 puta pokus $\rightarrow$ A se dogodio 3 puta $\qquad$ 6 puta $\rightarrow$ A 4 puta

$$f(x,p) = \binom{5}{3} p^3 (1-p)^2 \qquad \binom{6}{4} p^4 (1-p)^2$$

$$L(X_1, \dots, X_n, p) = \binom{5}{3}\binom{6}{4} p^7 (1-p)^4 \quad / \ln$$

$$\ln L = \ln \binom{5}{3}\binom{6}{4} + 7\ln p + 4\ln(1-p) \quad /'$$

$$0 = 0 + \frac{7}{p} - \frac{4}{1-p} \Rightarrow \frac{7}{p} = \frac{4}{1-p}$$

$$7 - 7p = 4p$$

$$11p = 7 \Rightarrow \boxed{p = \frac{7}{11}}$$

10.) $f(x) = \lambda e^{-\lambda x}, \ \lambda > 0$

$$f(x_1) = \lambda e^{-\lambda x_1}$$

$$L(X_1, \lambda) = e^{-\lambda x_1} + \lambda(-x_1) e^{-\lambda x_1} = 0$$

$$\cancel{e^{-\lambda x_1}} = \lambda x_1 \cancel{e^{-\lambda x_1}} \Rightarrow 1 = \lambda x_1 \Rightarrow \boxed{x_1 = \frac{1}{\lambda}}$$

11.) 8, 12, 7, 10, 5 $\rightarrow \bar{x} = \frac{\sum_{i=1}^{5} x_k}{5} = \underline{8.4}$

$P(X > 5) = ?$

eksponencijalna: $f(x) = \lambda e^{-\lambda x}, \ x > 0 \qquad \lambda = \frac{1}{\bar{x}} = \frac{1}{8.4} = \underline{0.119}$

$$P(X>5) = 1 - F(5) = 1 - (1 - e^{-\lambda x}) = e^{-\lambda x} = e^{-\lambda \cdot 5} = e^{-0.119 \cdot 5} = \underline{0.55156}$$

12.) $X \sim P(\lambda)$ POISSON

$n=3$, $x_1=5$, $x_2=7$, $x_3=3$

$$f(x,\lambda) = P(X=x) = \frac{\lambda^x}{x!} e^{-\lambda}$$

$$L(\lambda, X_1,\dots,X_n) = \frac{\lambda^{x_1+\dots+x_n}}{x_1!x_2!\dots x_n!} e^{-n\lambda} = \frac{\lambda^{5+7+3}}{5!7!3!} e^{-3\lambda}$$

$$\frac{dL}{d\lambda} = -n + \frac{x_1+\dots+x_n}{\lambda} = 0 \qquad 3 = \frac{5+7+3}{\lambda} \Rightarrow 3\lambda = 15$$

$$\boxed{\lambda = 5}$$

13.) $f(x) = \lambda x^{\lambda-1}$, $0<x<1$

$\lambda = ?$

$$L(x_1,\dots,x_n,\lambda) = \lambda^n (x_1 \cdot x_2 \cdot \dots \cdot x_n)^{\lambda-1} \quad / \ln$$

$$\ln L = n \ln \lambda + (\lambda-1)(\ln x_1 + \ln x_2 + \dots + \ln x_n) \quad / \frac{dL}{d\lambda}$$

$$0 = n \cdot \frac{1}{\lambda} + \sum_{i=1}^{n} \ln x_i$$

$$\boxed{\lambda = -\frac{n}{\sum_{i=1}^{n} \ln x_i}}$$

SLUŽBENA RJEŠENJA:

§ 10. Matematička statistika

1. $\bar{x} = 4.425$, $\hat{s}_x = 0.128$.

3. $\bar{x} = 164.353$, $\hat{\sigma}^2 = 0.021624$, $\hat{d}^2 = \hat{\sigma}^2 + (\bar{x} - m)^2 = 0.02271$. Nepristrana korekcija je $\tilde{d} = k_{11}\hat{d} = 1.025 \cdot 0.1507 = 0.1545$.

4. $\bar{x} = 164.353$, $\hat{s} = 0.15499$. Nepristrana korekcija je $\tilde{s} = k_{10}\hat{s} = 0.15933$.

5. U računu koristi za x_k vrijednosti sredina intervala, i zatim pomakni podatke za $C = 22.5$. $\bar{x} = 22.487$, $\hat{s}^2 = .298$.

6. Broj pojavljivanja događaja ima razdiobu $m \sim \mathscr{B}(np, npq)$ Zato je disperzija frekvencija m/n jednaka pq/n . Maksimalna disperzija $1/4n$ dobiva se za $p = 0.5$.

7. Za disperziju vrijedi $D(\hat{x}) = \sum_{i=1}^{n} t_i^2 \sigma_i^2$. Treba minimizirati ovu funkciju, uz uvjet $t_1 + \ldots + t_n = 1$. Minimum se postiže ako je $t_i = \lambda / s_i^2$ za svaki i , a $\lambda = 1/(\sum 1/\sigma_j^2)$.

8. X ima jednoliku razdiobu na $[a, b]$ pa je njezina fnkcija razdiobe jednaka

$$F(x) = \frac{x-a}{b-a}.$$

Razdiobe varijabli x_m i x_M su:

$$F_{X_m}(x) = 1 - (1 - F(x))^n,$$
$$F_{X_M}(x) = F(x)^n$$

Odavde se izračunaju očekivanja i disperzije varijabli x_m i x_M . Dobivamo: $E(x_m) = a + \dfrac{b-a}{n+1}$ $E(x_M) = b - \dfrac{b-a}{n+1}$, pa je $E(\hat{c}) = \dfrac{a+b}{2} = c$ i procjena je nepristrana. Nadalje, $D(\hat{c}) = \frac{1}{4}D(x_m) + \frac{1}{4}D(x_M) + 2E(x_m x_M) = \dfrac{(b-a)^2}{2(n+1)(n+2)}$. Disperzija teži k nuli pa je procjena valjana.

9. Funkcija izglednosti je $f(p) = 10p^3(1-p)^2 \cdot 15p^4(1-p)^2$. Najizgledniji p je $p = \frac{7}{11}$.

11. Na temelju vrijednosti uzorka, izračuna se $\bar{x} = 8.4$. Vrijeme do sljedećeg poziva ima eksponencijalnu razdiobu. Procjena parametra je $\lambda = 1/\bar{x} = 0.119$. Tražena je vjerojatnost jednaka $1 - F(5) = e^{-\lambda 5} = 0.55$.

13. $\lambda = n \Big/ \sum_{i=1}^{n} \ln x_i$.

LITERATURA:

[1] Neven Elezović: Statistika i procesi, *Element 2010.godine*