$$\left[\begin{array}{l} P(X \geq \varepsilon) \leq \frac{E(x)}{\varepsilon} \\ \\ P(|X-E(x)| \geq \varepsilon) \leq \frac{D(x)}{\varepsilon^2} \end{array}\right] \Leftrightarrow P(|X-E(x)|<\varepsilon) \geq 1-\frac{D(x)}{\varepsilon^2}$$

// Čebišev

3.

$E(x)=1$

$\sigma(x)=0,4$

$P(0<X<3)=?$ // nemamo $F(x)$, $X>0$

$0<x<3$

$P(|X-E(x)| \geq \varepsilon) \Rightarrow \quad -2 \leq x-1 \leq 2$

$-1 \leq x \leq 3$

$0 \leq x \leq 3$

$$\Rightarrow P(|X-1| \leq 2) \geq 1-\frac{D(x)}{\varepsilon^2}$$

$$\geq 1-\frac{D(x)}{2^2}$$

$$\geq 1-\frac{\sigma^2}{4}$$

$$\geq 1-\frac{0,16}{4}$$

$$\geq 0,96$$

4.

$E(X) = 25$ km/h

$\sigma(X) = 4{,}5$ km/h

$P \geq 0{,}9$

$P(|X - E(X)| < \varepsilon) \geq 0{,}9 = 1 - \dfrac{D(X)}{\varepsilon^2}$

$\varepsilon = 14{,}2$

$P(|X - 25| < 14{,}2) = ?$

$|X - 25| < 14{,}2$

$-14{,}2 < X - 25 < 14{,}2$

$-10{,}8 < X < 39{,}2$

2. 2. iz zadaće

$E(X) = 1$

$\sigma(X) = 0{,}4$

$P(X < 3) = ?$

— isti zadatak...

- centralni granični teorem:

$$\frac{\sum_{i=1}^{n} (X_i - m)}{\sigma\sqrt{n}} \xrightarrow{D} \mathcal{N}(0,1)$$

3. rač. iz zadatka

$$X = \frac{1}{n}\sum_{k=1}^{n} X_k$$

$$m = 20$$

$$\sigma^2 = 4$$

$$P(19{,}95 < X < 20{,}08) = ?$$

$$\frac{\sum X_k - n\cdot m \ /:n}{\sigma\sqrt{n} \ \ /:n} = \frac{\frac{1}{n}\sum X_k - m}{\frac{\sigma}{\sqrt{n}}} = \frac{X-m}{\frac{\sigma}{\sqrt{n}}} \to \mathcal{N}(0,1)$$

$$X \sim \mathcal{N}\left(m, \frac{\sigma^2}{n}\right)$$

$$P\left(\frac{19{,}95-20}{\frac{\sigma}{\sqrt{n}}} < \frac{X-20}{\frac{\sigma}{\sqrt{n}}} < \frac{20{,}08-20}{\frac{\sigma}{\sqrt{n}}}\right) = \text{izračunati..}$$

$$\vartheta(t) = \int_{-\infty}^{\infty} e^{itx} f(x)\,dx$$

7.

- uvrstiti $f(x)$ i riješiti parcijalnom integracijom

$$f(x) = \frac{1}{2}e^{-|x-1|}, \quad x \in \mathbb{R}, \quad E(x) = ?, \quad \vartheta(t) = ?$$

$$\vartheta(t) = \int_{-\infty}^{1} e^{itx}\cdot\frac{1}{2}e^{x-1}\,dx + \int_{1}^{\infty} e^{itx}\cdot\frac{1}{2}e^{1-x}\,dx$$

$$= \frac{1}{2}\frac{1}{e}\int_{-\infty}^{1} e^{x(1+it)}\,dx + \frac{1}{2}e\int_{1}^{\infty} e^{x(it-1)}\,dx \quad // \text{integrirati}$$

$$= \ldots = \frac{e^{it}}{1+t^2} \Rightarrow \quad \vartheta'(t) = \frac{ie^{it}(1+t^2) - 2te^{t}}{(1+t^2)^2}$$

$$E(x) = \frac{\vartheta'(0)}{i} = \frac{1}{i}\,\frac{i(1+0)-2+1}{1^2} = 1$$

18. statistika

$$\Theta = f(x_1, x_2, \dots, x_n)$$

$$E(\Theta) = \vartheta$$

parametar kojeg procjenjujemo

- procjena očekivanja

X ima očekivanje a

$$\bar{x} = \frac{\sum_{k=1}^{n} x_k}{n} \qquad E(\bar{x}) = a$$ // ako vrijedi, statistika je nepristrana

$$D(\bar{x}) = \frac{a^2}{n} \to 0$$ // ako $D(\bar{x})$ teži nuli, statistika je valjana

- procjena disperzije:

- ako je poznato a

$$D^2 = \frac{1}{n}\sum_{i=1}^{n} (x_i - a)^2$$

- ako je a nepoznato

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n} (x_i - \bar{x})^2$$

8. zad. iz zadaće

$a = 164{,}32\ m$ // očekivanje = prava visina tornja

– procjena odstupanja tj. standardne devijacije se traži

$$\tilde{\sigma} = k_{n+1} \cdot D = k_{n+1} \cdot \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - a)^2}$$

$$\Gamma(n) = (n-1)!\ ,\ n \in \mathbb{N}$$

$$\Gamma\left(\frac{n}{2}+1\right) = \frac{n!!}{2^{\frac{n+1}{2}}}$$

$$\tilde{\sigma} = k_n \cdot \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$k_n = \sqrt{\frac{n-1}{2}} \cdot \frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}$$

11. $\tilde{\sigma} = 0{,}1545$

12. $\tilde{\sigma} = 1{,}0280 \cdot 0{,}15559 = 0{,}15933$

$$\left[\begin{array}{l} L(\vartheta, x_1, x_2, \ldots, x_n) = f(\vartheta, x_1) \cdot f(\vartheta, x_2) \cdot \ldots \cdot f(\vartheta, x_n) \\ \dfrac{\partial \ln L}{\partial \vartheta} = 0 \end{array}\right]$$

17. $P(A) = p = ?$

$$f(p) = f_1(p) \cdot f_2(p)$$

$$f(p) = \binom{5}{3} p^2 (1-p)^2 \cdot \binom{6}{4} p^4 (1-p)^2$$ // f'a izglednosti

$$\ln f = \ln p^3 + \ln(1-p)^2 + \ln p^4 + \ln(1-p)^2$$

$$\ln f = 7 \ln p + 4 \ln(1-p)$$

$$\frac{\partial \ln f}{\partial p} = \frac{7}{p} + \frac{-4}{1-p} = 0 \implies p = \frac{7}{11}$$