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$$\begin{aligned} 1) \quad a) \quad (A+B)(B+C)(C+A) &= (AB+AC+BB+BC)(C+A) = \\ &= (ABC+ACC+BBC+BCA+AB+AA+AB+AB) = \\ &= ABC+AC+BC+AB = \\ &= ABC+ACC+AB+BC = \\ &= AC(B+C)+AB+BC = \\ &= \boxed{AC+AB+BC} \\ &\quad [AC \subseteq (B+C)] \end{aligned}$$

$$b) \quad \underbrace{(A+B)A}_{=A, A \subseteq A+B} + A(B+C) = A + A(B+C) = A(A+B+C) = \boxed{A}$$

$A \subseteq A+B+C$

$$\begin{aligned} 2) \quad P(A+B) &= 0,8 \\ P(AB) &= 0,2 \\ P(\bar{A}) &= 0,6 \end{aligned}$$

$$P(A) = 1 - P(\bar{A}) = 0,4$$

$$P(A+B) = P(A) + P(B) - P(AB) \Rightarrow \boxed{P(B) = P(A+B) + P(AB) - P(A) = 0,6}$$

$$P(\bar{B}) = 0,4$$

$$\boxed{P(A\bar{B}) = P(A) - P(AB) = 0,4 - 0,2 = 0,2}$$



3) Novčić 4x

a) tačno 1P

\Rightarrow

PGGG
GPGG
GGGP
GGPG

4 povoljna

$M=4$

$N=2^4=16$

$$\boxed{P(A) = \frac{M}{N} = \frac{4}{16}}$$

b) u drugom b. pismo \Rightarrow 2. kockica je nepoznata o ostalima, možemo promatrati kao 1 kockicu

$$\boxed{P(B) = \frac{1}{2}}$$

c) barem 1 pismo \Rightarrow računamo za niti 1 pismo = $\bar{E} = \bar{GGGG}$

$$\boxed{P(\bar{E}) = 1 - \frac{1}{16} = \frac{15}{16}}$$

d) barem 2P \Rightarrow računamo niti jedno, i jedno P

$$P(\bar{D}) = \frac{1}{16} + \frac{4}{16} = \frac{5}{16}$$

$$\boxed{P(D) = 1 - \frac{5}{16} = \frac{11}{16}}$$

4) 2 kocke

$N=36$

a) zbir = 8

$A = \{26, 35, 44, 53, 62\}$

$$M=5 \Rightarrow \boxed{P(A) = \frac{5}{36}}$$

b) bar 1 4orka

$B = \{x4, 4x, 44\}$
 $x \neq 4$

$$P(B) = \frac{11}{36}$$

c) zbir > 9

$C = \{46, 55, 64, 56, 65, 66\}$

$$P(C) = \frac{6}{36} = \frac{1}{6}$$

d) dijelovi 2 i 3

$D = \{11, 13, 23, 25, 31, 35, 41, 43, 53, 55, 61, 65\}$

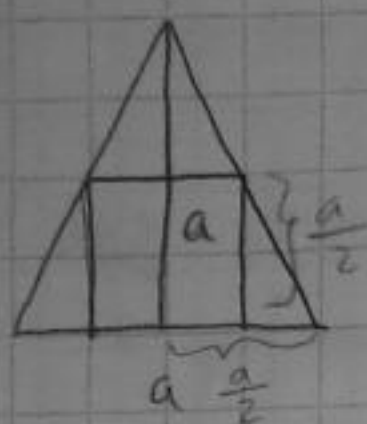
$$M=12 \Rightarrow 15, 11, 55, 51$$

(≥ 10 pismo)

$$P(D) = 1 - \frac{12}{36} = \frac{24}{36} = \frac{2}{3}$$

$$P(D) = 1 - \frac{4}{36} = \frac{32}{36}$$

5

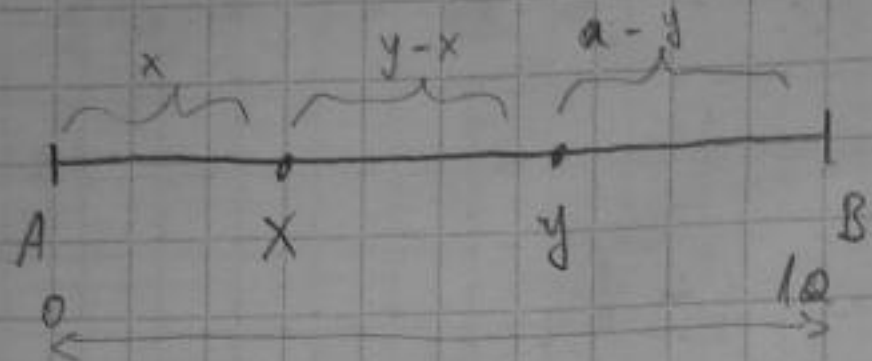


A = točka leži u kvadratu

$$P(A) = \frac{m(\square)}{m(\Delta)} = \frac{\frac{a}{2} \cdot \frac{a}{2}}{a \cdot a \cdot \frac{1}{2}} = \frac{1}{2}$$

$$P(\bar{A}) = 1 - P(A) = \frac{1}{2}$$

6



1) $y > x$

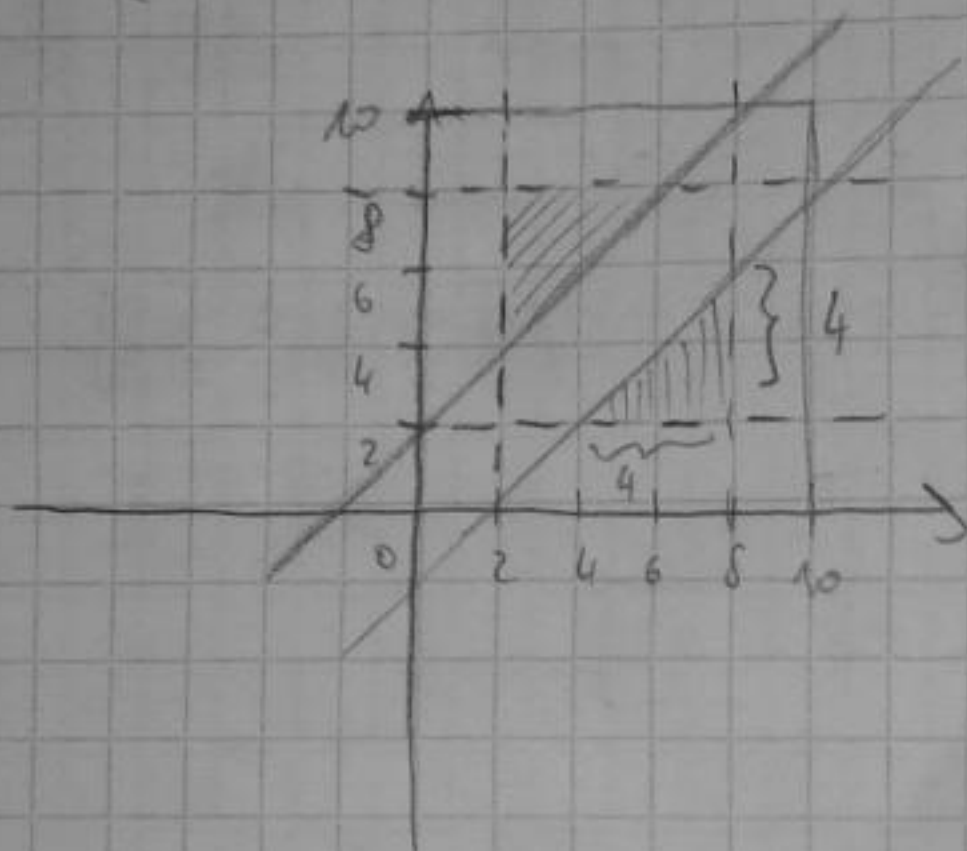
$$\begin{aligned} x &> 2 \\ y - x &> 2 \\ 10 - y &> 2 \end{aligned}$$

$$\begin{aligned} x &> 2 \\ y &> x + 2 \\ y &< 8 \end{aligned}$$

2) $x > y$

$$\begin{aligned} y &> 2 \\ x - y &> 2 \\ 10 - x &> 2 \end{aligned}$$

$$\begin{aligned} y &> 2 \\ y &< x - 2 \\ x &< 8 \end{aligned}$$



$$\begin{aligned} \Omega &= [0, 10] \times [0, 10] \\ m(\Omega) &= 100 \end{aligned}$$

$$m(A) = \frac{2 \cdot 4 \cdot 4}{2} = 16$$

$$P(A) = \frac{m(A)}{m(\Omega)} = \frac{16}{100} = 0,16$$

7) 5C, 4B

6 knjiga - 3C, 3B

$$P(A) = \frac{\binom{5}{3} \binom{4}{3}}{\binom{9}{6}} = \frac{2 \cdot \frac{5 \cdot 4 \cdot 3}{3 \cdot 2} \cdot \frac{4 \cdot 3 \cdot 2}{3 \cdot 2}}{\frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} = \frac{60}{84} = \frac{5}{7}$$

8

6C, 4P

bitno 3, P(pov 1 plana 2)

$$P(A) = 1 - P(\text{nijedna planeta}) = 1 - \frac{\binom{6}{3}}{\binom{10}{3}} = \frac{5 \cdot 4 \cdot 3}{10 \cdot 9 \cdot 8} = \frac{5}{6}$$

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Novčići tisuć dala se 2x zaredom na popari istih znak. P(parno kockanje)

$$\begin{aligned} w_1 &= \frac{GG}{PP} \rightarrow \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \cdot 2 \\ w_2 &= \frac{PG}{GP} \rightarrow \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \cdot 2 \\ w_3 &= \frac{GPGG}{PGPP} \rightarrow \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \left(\frac{1}{2}\right)^2 \cdot \frac{1}{4} \cdot 2 \\ w_4 &= \frac{PGPG}{GPGG} \rightarrow \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \left(\frac{1}{2}\right)^2 \cdot \frac{1}{4} \cdot 2 \\ w_5 &= \frac{GPGPGG}{PGPGPP} \rightarrow \left(\frac{1}{2}\right)^4 \cdot \frac{1}{4} \cdot 2 \end{aligned}$$

$$\begin{aligned} w_{2k-1} &= \frac{GP \dots GPGG}{PG \dots PGPP} \\ w_{2k} &= \frac{PG \dots GPGG}{GP \dots PGPP} \end{aligned} \left. \vphantom{\begin{aligned} w_{2k-1} \\ w_{2k} \end{aligned}} \right\} \left(\frac{1}{2}\right)^{n-1} \cdot \frac{1}{4}$$

(repariran index - parni broj kockanja)

$$\begin{aligned} P(A) &= \{w_1, w_3, w_5, \dots\} = 2 \left(\frac{1}{4} + \left(\frac{1}{2}\right)^2 \cdot \frac{1}{4} + \left(\frac{1}{2}\right)^4 \cdot \frac{1}{4} + \dots \right) \\ &= 2 \cdot \frac{1}{4} \left(1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \dots \right) = \frac{2}{4} \cdot \frac{1}{1 - \frac{1}{4}} \\ &= \frac{2}{4} \cdot \frac{4}{3} = \frac{2}{3} \end{aligned}$$

10) 52h, 3 baro

$$a) \text{ bar 1 AS} \Rightarrow P(\bar{A}) = \frac{\binom{48}{3}}{\binom{52}{3}} = \frac{48 \cdot 47 \cdot 46}{52 \cdot 51 \cdot 50} = 0,982624$$

$$P(A) = 1 - P(\bar{A}) = 0,21738$$

$$b) 3 karte iste boje \Rightarrow P(A) = \frac{4 \cdot \binom{13}{3}}{\binom{52}{3}} = 0,0518$$

$$c) 3 karte razl. boje \Rightarrow P(A) = \frac{4 \cdot \binom{13}{1} \cdot 3 \cdot \binom{12}{1} \cdot 2 \cdot \binom{11}{1} \cdot \frac{1}{3!}}{\binom{52}{3}}$$

nakon razdelit odabra boje

11) A i B nezavisni, $A \subset B$ Dokazi $P(A) = 0$ ili $P(B) = 1$

$$\hookrightarrow P(AB) = P(A) \cdot P(B)$$

$$A \subset B \rightarrow AB = A \Rightarrow P(AB) = P(A) = P(A) P(B) \quad \boxed{P(B)=1} \quad \text{XII}$$

12) $P(ABC) = P(A)P(B)P(C)$

$$\stackrel{2}{\Rightarrow} P(A(B+C)) = P(A)P(B+C) \text{ dokazi}$$

$$P(AB) = P(A)P(B)$$

$$P(AC) = P(A)P(C)$$

$$P(BC) = P(B)P(C)$$

$$P(B+C) = P(B) + P(C) - P(BC) = P(B) + P(C) - P(B)P(C)$$

$$P(A(B+C)) = P(AB+AC) = P(AB) + P(AC) - P(ABC) = P(AB) + P(AC) - P(A)P(B)P(C) = P(A)P(B) + P(A)P(C) - P(A)P(B)P(C) \quad *$$

\checkmark

$$P(A)P(B+C) = P(A)[P(B) + P(C) - P(BC)] = P(A)P(B) + P(A)P(C) - P(A)P(B)P(C) \quad *$$

13) 10 x coin toss

$$P(10x \text{ pravo, ali } \bar{x} \text{ 9x pravo}) = P(\text{pravo}) = \frac{1}{2} \quad [\text{nezavisni}]$$

14) 10 štigloca, 4 celina, 6 sh \Rightarrow biramo [1]

$$P(\text{razdelah}) = \frac{\binom{4}{1}}{\binom{10}{1}} \cdot 0,9 + \frac{\binom{6}{1}}{\binom{10}{1}} \cdot 0,7 = 0,98$$

15) 4 loche $N = 6^4$

$$A = \{2 \text{ jedinice, 1 dvojica}\} = \{112x \mid x \in \{3,4,5,6\}\} \Rightarrow \text{perm. s pom. 4} = \frac{4!}{2!} \cdot 4 = 48 = M$$

$$P(A) = \frac{48}{6^4} = 0,9375$$

$$B = \{x_i < 4\} \quad M = 3^4 \quad P(B) = \frac{3^4}{6^4} = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

$$C = \{\text{zaj. na sum } > 6\} \quad \bar{C} = \{1111, 1211, 1311, 1221\} \quad M = 15 \quad P(C) = 1 - \frac{15}{6^4} \quad P(C) = \frac{1281}{6^4}$$

$$P(C|B) = \frac{P(CB)}{P(B)}$$

$$CB = \{X_i < 4, \text{ suma} > 6\} = \{1123, 1133, 1222, 1223, 1233, 1333, 2333, 3333, 2283 \rightarrow (4), 2223 \rightarrow 4, 2222 \rightarrow 1\}$$

$$P(C|B) = \frac{\frac{N_{CB}}{N}}{\frac{N_B}{N}} = \frac{N_{CB}}{N_B} = \frac{66}{34} = 0,8148$$

$$N_{CB} = 3 \cdot \frac{4^3}{2^3} + 2 \cdot \binom{4}{2} + 4 \cdot 4 \cdot 2 = 66$$

17) $\underbrace{2B, 3C}_{1 \text{ hit}} \quad \underbrace{1B, 4C}_{2 \text{ hit}} \rightarrow B$

$$P(A) = \left\{ \begin{array}{l} P(2B) \sim \frac{2}{5} \cdot \frac{1}{4} \cdot \frac{3}{7} \\ P(B,C) \sim \frac{2}{5} \cdot \frac{3}{4} \cdot \frac{2}{7} \cdot 2 \\ P(C,C) \sim \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{1}{7} \end{array} \right\} \quad \frac{6}{140} + \frac{24}{140} + \frac{6}{140} = \frac{36}{140} = \frac{9}{35}$$

$$P(B,C) = \frac{\binom{3}{1} \binom{2}{1}}{\binom{5}{2}} = \frac{6}{5 \cdot 4 / 2} = \frac{3}{5}$$

18) 2 losche

$$P(A) = ? \quad \text{Suma} = 8$$

$$P(B) = \{20, 35, 44, 53, 62\}$$

$$P(A|B) = \frac{2}{5}$$

19) 4 shpilea 0,4, 0,6, 0,7, 0,8

$$A - \text{meta pozitivna}$$

$$P(A) = 0,6 \cdot 0,4 \cdot 0,3 \cdot 0,2 = 0,0144 \Rightarrow P(A) = 0,3856$$

$$B - 3 \text{ nella hit} \Rightarrow$$

$$P(\bar{S}_4 | B) = \frac{P(B \bar{S}_4)}{P(B)}$$

$$P(B) = 0,4 \cdot 0,6 \cdot 0,7 \cdot 0,2 + 0,4 \cdot 0,6 \cdot 0,3 \cdot 0,2 + 0,4 \cdot 0,4 \cdot 0,7 \cdot 0,2 + 0,6 \cdot 0,6 \cdot 0,7 \cdot 0,8 = 0,3824$$

$$P(B \bar{S}_4) = 0,4 \cdot 0,6 \cdot 0,7 \cdot 0,2 = 0,0336$$

$$P(\bar{S}_4 | B) = \frac{0,0336}{0,3824} = 0,0879$$

20) 2 od 3 flazala

$$p(1,2 | 2 \text{ od } 3) \quad 0,2, 0,4, 0,7 \text{ - flazna}$$

$$p(2 \text{ od } 3) = 0,2 \cdot 0,4 \cdot 0,7 + 0,2 \cdot 0,6 \cdot 0,3 + 0,8 \cdot 0,4 \cdot 0,3 = 0,188$$

$$p(1,2 | 2 \text{ od } 3) = \frac{p(1,2 \cdot 2 \text{ od } 3)}{p(2 \text{ od } 3)} = \frac{0,2 \cdot 0,4 \cdot 0,7}{0,188} = 0,298$$

16 11

pala

1

:

$$\frac{1}{6} \cdot \frac{1}{6}$$

2

:

$$\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6} \cdot 2 \cdot \frac{1}{3}$$

3

:

$$\frac{1}{6} \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^2 \cdot \binom{2}{1} \cdot \frac{1}{2}$$

4

:

$$\frac{1}{6} \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^3 \cdot \binom{3}{1} \cdot \frac{2}{3}$$

5

:

$$\frac{1}{6} \cdot \left(\frac{5}{6}\right)^4$$

6

:

$$\frac{1}{6} \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^5 \cdot \binom{5}{1}$$

1.500

5.120

$$P(A) = 0,8302$$