1.) $E(X)=1$

$\sigma(X)=0.2$

$A=\{0,5<X<1,5\}$

$P(A)=?$

ČEBIŠEVA NEJEDNAKOST:

$$P(|X-E(X)|\geq\varepsilon)\leq\frac{D(X)}{\varepsilon^2}=P(|X-E(X)|<\varepsilon)\geq 1-\frac{D(X)}{\varepsilon^2}$$

$$P(|X-E(X)|<\varepsilon)\geq 1-\frac{\sigma^2(X)}{\varepsilon^2}=1-\frac{0.2^2}{\varepsilon}$$

$$P(-\varepsilon<X-E(X)<\varepsilon)\geq 1-\frac{0.2^2}{\varepsilon}$$

$$P(\underbrace{-\varepsilon+E(X)}_{-\varepsilon+1=0.5,\ \varepsilon=0.5}<X<\underbrace{\varepsilon+E(X)}_{\varepsilon+1=1.5,\ \varepsilon=0.5})\geq 1-\frac{0.04}{\varepsilon}=1-\frac{0.04}{0.5}=0.84$$

2.) $E(X)=75$

$P(X\leq 200)=\frac{5}{8}$

$$P(X\leq 200)\geq\underbrace{1-\frac{75}{\varepsilon}}$$

$$1-\frac{75}{200}=0,625=\frac{5}{8}\Rightarrow P(X\leq 200)=\frac{5}{8}$$

3.) $E(X)=1$

$\sigma(X)=0.4$

$P(0<X<3)>0.9=?$

$$P(|X-E(X)|\geq\varepsilon)\leq\frac{\sigma^2}{\varepsilon^2}\Rightarrow P(|X-E(X)|<\varepsilon)>1-\frac{\sigma^2}{\varepsilon^2}$$

$$P(\underbrace{-\varepsilon+E(X)}_{-\varepsilon+1=\emptyset,\ \varepsilon=1}<X<\underbrace{\varepsilon+E(X)}_{\varepsilon+1=3,\ \boxed{\varepsilon=2}\ \text{(uzimamo veći)}})>1-\frac{0.4^2}{\varepsilon^2}$$

$$P(0<X<3)>1-\frac{0.4^2}{2}=0,96$$

DOGAĐAJ $A\{0<x<3\}$ IMA VJEROJATNOST VEĆU OD 90%

4.) $E(X) = 25$

$\sigma(X) = 4.5$

$P \geq 0.9$

$P(|X - E(X)| < \varepsilon) > 1 - \underbrace{\frac{D(X)}{\varepsilon^2}}_{0,9} \rightarrow 1 - \frac{4.5^2}{\varepsilon^2} = 0.9$

$\varepsilon = 14,23$

$-14,23 < X - E(X) < 14,23$

$\boxed{10,77 < X < 39,23}$

5.) $E(X) = 1$

$\sigma(X) = 0,4$

$A = \{X < 3\} > 0.95$

$P(|X - E(X)| < \varepsilon) > 1 - \frac{\sigma^2(X)}{\varepsilon^2} = 1 - \frac{0.4^2}{4} = 0,96$ ✓

$X < \varepsilon + E(X)$

$X < 3$

$\varepsilon + E(X) = 3$

$\varepsilon = 3 - 1$

$\boxed{\varepsilon = 2}$

6.) $P(\underbrace{|X - E(X)|}_{a} < \underbrace{3\sigma}_{\varepsilon}) > 1 - \frac{\sigma^2}{\varepsilon^2} = 1 - \frac{\sigma^2}{(3\sigma)^2} = 1 - \frac{\sigma^2}{9\sigma^2} = 1 - \frac{1}{9} = 0,889$

$E(X) = 0$

$\sigma(X) = 1$

} normalna jedinična razdioba $\sim N(0,1)$

$P(-3\sigma < a < 3\sigma) = P\left(\frac{-3\sigma - 0}{1} < a^* < \frac{3\sigma - 0}{1}\right) = P(-3 < a^* < 3)$

$= \frac{1}{2}[\phi^*(3) + \phi^*(3)] = \phi^*(3) = 0,997$

SLUŽBENA RJEŠENJA:

§9. Konvergencija nizova slučajnih varijabli

1. $\mathrm{P}(A) > 0.84$.

2. $\mathrm{P}\{X > a\} < \dfrac{\boldsymbol{E}(X)}{a}$, nejednakost Markova.

3. $\mathrm{P}(A) = \mathrm{P}\{|X - \boldsymbol{E}(X)| < 2\} \geqslant 0.96$.

4. $10.8 - 39.2$ km/h.

5. Primjeni Čebiševljevu nejednakost, $\mathrm{P}(A) \geqslant 0.96$.

6. 0.889, 0.997.

12. $\dfrac{2(1-\cos t)}{t^2}$.

13. $\vartheta(t) = \begin{cases} 0, & |t| > 1 \\ 1 - |t|, & |t| \leqslant 1 \end{cases}$.

14. $\vartheta(t) = \dfrac{e^{it\pi} + 1}{2(1-t^2)}$, $\boldsymbol{E}(X) = \dfrac{\pi}{2}$,
$\boldsymbol{D}(X) = \dfrac{\pi^2 - 8}{4}$.

15. $\vartheta_X(t) = \dfrac{1}{1-it}$, $\boldsymbol{E}(X^n) = n!$.

16. 0, za neparni n; $1 \cdot 3 \cdots (n-1)$ za parni n.

17. $\vartheta_X(t) = \dfrac{e^{it}}{1+t^2}$, $\boldsymbol{E}(X) = 1$.

18. $\dfrac{b}{\pi[b^2 + (x-a)^2]}$.

20. $\boldsymbol{P}\{X = \pm k\} = \frac{1}{2}a_k$, $\boldsymbol{P}\{X = 0\} = a_0$.

21. $\dfrac{9}{32}$.

22. $\theta_Y(t) = \frac{1}{2} + \frac{1}{4}e^{2it} + \frac{1}{4}e^{6it}$

24. Iskoristi Bochnerov teorem

25. Razvij u red po ϑ i primjeni prethodni zadatak.

26. Iskoristi Bochnerov teorem.

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