1.) b) gustoća > 1 , $f(x)>1$ DA

f) gustoća razdiobe prekidna DA

2.) a) $\frac{3}{4}+\frac{1}{2\pi}\operatorname{arctg} x$ NE, pomaknuta prema gore

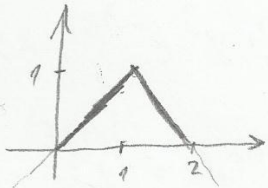
b) $\frac{1}{2}+\frac{1}{\pi}\operatorname{arctg} x$, DA $\frac{1}{2}-\frac{1}{2}=\emptyset$, $\frac{1}{2}+\frac{1}{2}=\underline{\underline{1}}$

c) $\frac{x}{1+x}$, $x>0$, DA , $\lim_{x\to\infty}\frac{1}{1-\frac{1}{x}}=\underline{1}$

d) $2^{-e^{-x}}$, DA , $2^{-e^{-\infty}}=2^0=1$, $2^{-e^{\infty}}=2^{-\infty}=\emptyset$

e) $1-e^{-x}$; $x>1$, DA , $1-e^{-\infty}=1-0=\underline{\underline{1}}$

3.) a) $1-|1-x|$, $0<x<2$



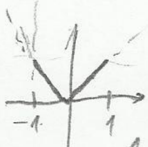
$1-|1-0|=0$ (pozit.) , $1-|1-2|=1-1=\emptyset$ (pozit.)

$$\int_0^2 1-|1-x|\,dx=\int_0^1 1-(1-x)dx+\int_1^2 1+(1-x)dx=$$

$$=\int_0^1 x\,dx+\int_1^2(2-x)dx=\frac{x^2}{2}\Big|_0^1+\left(2x-\frac{x^2}{2}\right)\Big|_1^2=$$

$$=\frac{1}{2}+\left(2\cdot2-\frac{4}{2}\right)-\left(2-\frac{1}{2}\right)=\frac{1}{2}+2-2+\frac{1}{2}=\underline{\underline{1}}\ \checkmark$$

b) $|x|$, $-1<x<1$



$$\int_{-1}^1|x|dx=\int_{-1}^0(-x)dx+\int_0^1 x\,dx=-\frac{x^2}{2}\Big|_{-1}^0+\frac{x^2}{2}\Big|_0^1=\frac{1}{2}+\frac{1}{2}=\underline{\underline{1}}\ \checkmark$$

c) $\frac{e^x}{(1+e^x)^2}$, $x\in R$

$$\int_{-\infty}^{\infty}\frac{e^x}{(1+e^x)^2}dx=\left|\begin{matrix}1+e^x=t\\ e^x dx=dt\end{matrix}\right|=\int\frac{dt}{t^2}=-\frac{1}{t}=-\frac{1}{1+e^x}\Big|_{-\infty}^{\infty}$$

$$=-\frac{1}{1+\infty}+\frac{1}{1+e^{-\infty}}=0+1=\underline{\underline{1}}\ \checkmark$$

d) $\frac{2}{\pi}\cdot\frac{e^x}{1+e^{2x}}$, $x\in R$

$$\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{e^x}{1+e^{2x}}=\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{dt}{1+t^2}=\frac{2}{\pi}\operatorname{arctg}(e^x)\Big|_{-\infty}^{\infty}=\frac{2}{\pi}(\operatorname{arctg}\infty-\operatorname{arctg}(\emptyset))$$

$$=\frac{2}{\pi}\cdot\frac{\pi}{2}=\underline{\underline{1}}$$

4.) a) $f(x)=C,\ x\in[a,b],\ C=?$

$$C\cdot\int_a^b dx = C\cdot x\Big|_a^b = C(b-a)=1 \Rightarrow \underline{C=\frac{1}{b-a}}$$

b) $f(x)=C|x-a|,\ x\in[c,d]$

$$\int_c^a C|x-a|\,dx-\int_a^d C(x-a)\,dx=\int_c^a (Cx-Ca)\,dx-\int_a^d (Cx-Ca)\,dx=$$

$$=C\int_c^a x\,dx-Ca\int_c^a dx-C\int_a^d x\,dx+Ca\int_a^d dx=C\left(\frac{x^2}{2}\Big|_a^c-ax\Big|_c^a-\frac{x^2}{2}\Big|_a^d+ax\Big|_a^d\right)$$

$$=\underline{C\left(\frac{a^2}{2}-\frac{c^2}{2}-a^2+ac-\frac{d^2}{2}+\frac{a}{2}+ad-ad\right)}$$

5.) $f(x)=C\cdot x^3\cdot e^{-\lambda x},\ x>0$

$$C\int_{-\infty}^{\infty} x^3 e^{-\lambda x}dx=\begin{vmatrix} u=x^3 & du=3x^2 \\ dv=e^{-\lambda x}dx & v=-\frac{1}{\lambda}e^{-\lambda x}\end{vmatrix}=C\cdot\left(-\frac{x^3}{\lambda}e^{-\lambda x}+\frac{3}{\lambda}\int_{-\infty}^{\infty}x^2e^{-\lambda x}\right)=\begin{vmatrix} u=x^2 & du=2x \\ dv=e^{-\lambda x} & v=-\frac{1}{\lambda}e^{-\lambda x}\end{vmatrix}$$

$$=C\cdot\left[-\frac{x^3}{\lambda}e^{-\lambda x}+\frac{3}{\lambda}\left(-\frac{x^2}{\lambda}e^{-\lambda x}+\frac{2}{\lambda}\int x\cdot e^{-\lambda x}\right)\right]=\begin{vmatrix} u=x & du=dx \\ dv=e^{-\lambda x} & v=-\frac{1}{\lambda}e^{-\lambda x}\end{vmatrix}$$

$$=C\cdot\left[-\frac{x^3}{\lambda}e^{-\lambda x}+\frac{3}{\lambda}\left(-\frac{x^2}{\lambda}e^{-\lambda x}+\frac{2}{\lambda}\left(-\frac{x}{\lambda}e^{-\lambda x}+\frac{1}{\lambda}\int_{-\infty}^{\infty}e^{-\lambda x}dx\right)\right)\right]$$

$$=C\cdot\left[-\frac{x^3}{\lambda}e^{-\lambda x}+\frac{3}{\lambda}\left(-\frac{x^2}{\lambda}e^{-\lambda x}+\frac{2}{\lambda}\left(-\frac{x}{\lambda}e^{-\lambda x}-\frac{1}{\lambda^2}e^{-\lambda x}\right)\right)\right]\Bigg|_0^{\infty}=$$

$$=C\cdot\left[0+\frac{3}{\lambda}\left(0+\frac{2}{\lambda}\left(0-\frac{1}{\lambda^2}+\frac{1}{\lambda^2}\right)\right)\right]\Rightarrow C\cdot\frac{6}{\lambda^2}=1\Rightarrow \underline{C=\frac{\lambda^2}{6}}$$

6.) $f(x)=\frac{C}{x^2},\ x>1,\ C=?,\ P(1<x<2)=?$

$$C\int_1^{\infty}\frac{1}{x^2}=C\cdot\frac{-1}{x}\Big|_1^{\infty}=C\cdot(0+1)=1\Rightarrow\boxed{C=1}$$

$$P(1<x<2)=F(2)-F(1)=1-\frac{1}{2}-\left(1-\frac{1}{1}\right)=\underline{\frac{1}{2}}$$

$$F(x)=\int_1^x\frac{1}{x^2}=\frac{-1}{x}\Big|_1^x=-\frac{1}{x}-\frac{-1}{1}=-\frac{1}{x}+1=\underline{1-\frac{1}{x}}$$

8.) a) $\sin x,\ 0<x<\frac{\pi}{2}$, $F(x)=?$

$$F(x)=\int_0^x \sin x = -\cos x\Big|_0^x = -\cos x+1 = \underline{1-\cos x}\ ✓$$

b) $x-\frac{1}{2},\ 1<x<2$

$$F(x)=\int_1^x\left(x-\frac{1}{2}\right)dx = \int_1^x x\,dx-\frac{1}{2}\int_1^x dx = \frac{x^2}{2}\Big|_1^x-\frac{1}{2}x\Big|_1^x = \frac{x^2}{2}-\frac{1}{2}-\frac{x}{2}+\frac{1}{2} = \underline{\frac{1}{2}(x^2-x)}$$

c) $3\sin 3x,\ \frac{\pi}{6}<x<\frac{\pi}{3}$

$$F(x)=3\int_{\pi/6}^x \sin 3x = -3\cdot\frac{1}{3}\cos 3x\Big|_{\pi/6}^x = \underbrace{\cos\frac{\pi}{2}}_{0}-\cos 3x = \underline{-\cos 3x}$$

9.) $X\sim\begin{pmatrix}-2 & -1 & 0 & 1 & 2\\ 0.1 & 0.2 & 0.2 & 0.3 & 0.2\end{pmatrix}$

$$F(x)=\begin{cases}0 & x\le -2\\ 0.1 & -1<x\le -1\\ 0.3 & -1<x\le 0\\ 0.5 & 0<x\le 1\\ 0.8 & 1<x\le 2\\ 1 & 2<x\end{cases}$$

$$P(|x|\le 1)=P(-1<x<1)=F(1)-F(-1) = 0{,}8-0{,}1=\underline{0{,}7}$$

10.) $f(x)=\frac{2}{\pi}\cos^2 x,\ x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, od 3 realizacije, da 2 padnu unutar $\left(0,\frac{\pi}{4}\right)$

$$F(x)=\frac{2}{\pi}\int_{-\pi/2}^x \cos^2 x\,dx = \left|\cos^2 x=\frac{1}{2}(1+\cos 2x)\right| = \ldots = \frac{2}{\pi}\left(\frac{x}{2}+\frac{1}{4}\sin 2x\right)\Big|_{-\frac{\pi}{2}}^x$$

$$=\frac{2}{\pi}\left(\frac{x}{2}+\frac{1}{4}\sin^2 x-\left(-\frac{\pi/2}{2}+\underbrace{\frac{1}{4}\sin 2\cdot\frac{\pi}{2}}_{0}\right)\right) = \underline{\frac{x}{\pi}+\frac{1}{2\pi}\sin 2x+\frac{1}{2}}$$

$$P\left(0\le x\le\frac{\pi}{4}\right)=F(\pi/4)-F(0)=\frac{\pi/4}{\pi}+\frac{1}{2\pi}\sin 2\cdot\frac{\pi}{4}+\frac{1}{2}-\frac{0}{\pi}-0-\frac{1}{2}=\frac{1}{4}+\frac{1}{2\pi}=\underline{\frac{\pi+2}{4\pi}}$$

$$P(2\text{ unutra}, 1\text{ vani})=\binom{3}{2}\cdot\left(\frac{\pi+2}{4\pi}\right)^2\cdot\left(1-\frac{\pi+2}{4\pi}\right)$$

11.) $P(X<1)=0{,}9$

$P(Y<2)=0{,}5$

$P(X+Y<3)\geq 0{,}4=?$

$P(X+Y<3)\geq P(X<1, Y<2)=P(X<1)+P(Y<2)-1=\underline{\underline{0{,}4}}$

12.) $P(0<X<1)=0{,}3$

$P(-1<Y<0)=0{,}9$

$P(-1<X+Y<1)\geq 0{,}2=?$

$$P(-1<X+Y<1)=P(-1<Y<0)+P(0<X<1)-\underbrace{P(0<X<1)\cup P(-1<Y<0)}_{\leq 1}$$

$$=0{,}3+0{,}9-1\geq\underline{\underline{0{,}2}}$$

13.) a) $f(x)=Cx,\quad 0<x<1$

$C\cdot\int_0^1 x=1 \Rightarrow C\cdot\frac{x^2}{2}\Big|_0^1=1 \Rightarrow C\left(\frac{1}{2}-0\right)=1 \rightarrow \underline{\underline{C=2}}$

$E(X)=2\cdot\int_0^1 x\cdot x\,dx=2\int_0^1 x^2dx=2\cdot\frac{x^3}{3}\Big|_0^1=\underline{\underline{\frac{2}{3}}}$

$D(X)=2\cdot\int_0^1 x^2x\,dx-\left(\frac{2}{3}\right)^2=2\cdot\int_0^1 x^3dx-\left(\frac{2}{3}\right)^2=2\cdot\frac{x^4}{4}\Big|_0^1-\left(\frac{2}{3}\right)^2=\frac{1}{2}-\frac{4}{9}=\underline{\underline{\frac{1}{18}}}$

b) $f(x)=Cx,\quad 0<x<2$

$C\cdot\int_0^2 x\,dx=C\cdot\frac{x^2}{2}\Big|_0^2 \Rightarrow 2C=1 \Rightarrow \underline{\underline{C=\frac{1}{2}}}$

$E(X)=\frac{1}{2}\int_0^2 x^2dx=\frac{1}{2}\cdot\frac{x^3}{3}\Big|_0^2=\underline{\underline{\frac{4}{3}}},$

$D(X)=\frac{1}{2}\int_0^2 x^3dx=\frac{1}{2}\frac{x^4}{4}\Big|_0^2=\frac{1}{2}\cdot\frac{2^4}{4}-\left(\frac{4}{3}\right)^2=\underline{\underline{\frac{2}{9}}}$

14.) a) $F(x)=\frac{1}{4}x$, $0<x<4$

$f(x)=\frac{dF(x)}{dx}=\frac{1}{4}$, $E(X)=\frac{1}{4}\cdot\int_0^4 x\,dx=\frac{1}{4}\frac{x^2}{2}\Big|_0^4=2$

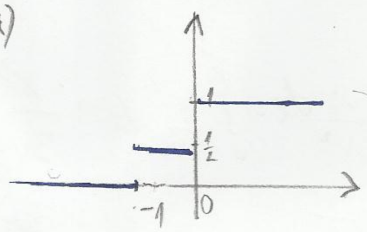
b) $F(x)=1-e^{-\lambda x}$, $x>0$

$f(x)=\lambda e^{-\lambda x}$

$$E(X)=\lambda\int_0^\infty x e^{-\lambda x}=\left|\begin{matrix} u=x & du=dx \\ v=e^{-\lambda x} & dv=-\frac{1}{\lambda}e^{-\lambda x}\end{matrix}\right|=\lambda\left(-\frac{x}{\lambda}e^{-\lambda x}\Big|_0^\infty+\frac{1}{\lambda}\int_0^\infty e^{-\lambda x}\right)$$

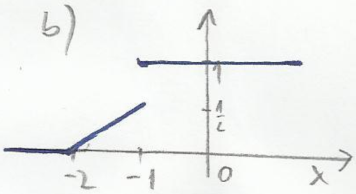
$$=\lambda\left(0+\frac{1}{\lambda}\cdot\left(-\frac{1}{\lambda}e^{-\lambda x}\Big|_0^\infty\right)\right)=\lambda\left(0+\frac{1}{\lambda}\left(0+\frac{1}{\lambda}\right)\right)=\lambda\cdot\frac{1}{\lambda^2}=\frac{1}{\lambda}$$

15.) a)



$E(X)=\int_{-\infty}^{\infty}x\cdot dF(x)=-1\cdot\frac{1}{2}+0\cdot\frac{1}{2}=-\frac{1}{2}$

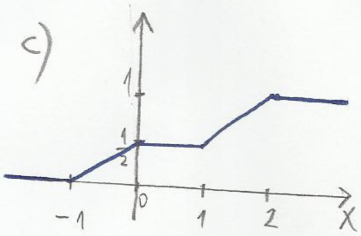
b)



$$E(X)=\frac{1}{2}\int_{-2}^{1}x+(-1)\cdot\frac{1}{2}=\frac{1}{2}\frac{x^2}{2}\Big|_{-2}^{-1}-\frac{1}{2}=\frac{1}{2}\left(\frac{1}{2}-\frac{4}{2}\right)-\frac{1}{2}$$

$$=-\frac{5}{4}$$

c)



$$E(X)=\frac{1}{2}\int_{-1}^{0}x\,dx+\frac{1}{2}\int_{1}^{2}x\,dx=\frac{1}{2}\frac{x^2}{2}\Big|_{-1}^{0}+\frac{1}{2}\frac{x^2}{2}\Big|_{1}^{2}$$

$$=0-\frac{1}{4}+1-\frac{1}{4}=\frac{1}{2}$$

16.) JEDNOLIKA R.

$E(X)=4$, $D(X)=3$

$E(X)=\frac{a+b}{2}$ $\qquad D(X)=\frac{(b-a)^2}{12}$

$\frac{a+b}{2}=4 \,/\cdot 2$ $\qquad \frac{(b-a)^2}{12}=3\,/\cdot 12$

$a+b=8$ $\qquad (b-a)^2=36$

$a=8-b$ $\qquad b^2-2ab+a^2=36$

$b^2-2(8-b)b+(8-b)^2=36$

$b^2-2(8b-b^2)+64-16b+b^2=36$

$b^2-16b+2b^2+64-16b+b^2=36$

$4b^2-32b+28=0$

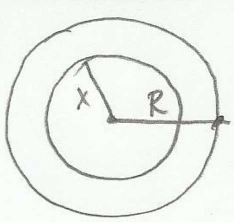
$b_{1,2}=\frac{32\pm\sqrt{32^2-4\cdot4\cdot28}}{2\cdot4}\to b_1=7,\ b_2=1$

$a_1=8-b_1=8-7=1 \to a_1=1,\ a_2=7$

$a_2=8-b_2=8-1=7$

$f(x)=\frac{1}{b-a}=\frac{1}{7-1}=\frac{1}{6}$, $1<x<7$

18.)



$F(x), E(x), D(x) = ?$

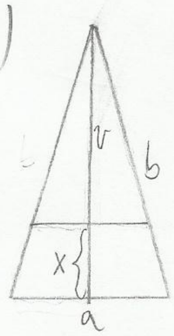
$$P(x) = F(x) = \frac{x^2\pi}{R^2\pi} = \left(\frac{x}{R}\right)^2$$

$$f(x) = \frac{dF(x)}{dx} = \frac{2}{R} \cdot x$$

$$E(x) = \frac{2}{R^2}\int_0^R x \cdot x\,dx = \frac{2}{R^2}\int_0^R x^2 dx = \frac{2}{R^2}\cdot\frac{x^3}{3}\Big|_0^R = \frac{2R^3}{3R^2} = \underline{\frac{2}{3}R}$$

$$D(x) = \frac{2}{R^2}\int_0^R x^3 dx - \left(\frac{2}{3}R\right)^2 = \frac{2}{R^2}\cdot\frac{R^4}{4} - \left(\frac{2}{3}R\right) = \frac{R^2}{2} - \frac{4}{9}R^2 = \underline{\frac{1}{18}R^2}$$

19.)



$E(X) = ?$

$$\frac{v}{a} = \frac{v-x}{b}$$

$$bv = a(v-x)$$

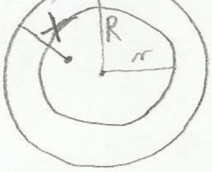
$$b = \frac{a(v-x)}{v}$$

$$P(X) = F(X) = \frac{\frac{av}{2}}{\frac{av}{2}} - \frac{\frac{a(v-x)}{v}}{\frac{a \cdot v}{2}} = 1 - \frac{2a(v-x)}{2av^2}$$

$$f(X) = \frac{x}{v^2}, \quad E(X) = \int_0^v x \cdot \frac{x}{v^2}\,dx = \int_0^v \frac{x^2}{v^2}\,dx = \frac{x^3}{3v^2}\Big|_0^v$$

$$E(X) = \frac{v^3}{3v^2} = \underline{\frac{v}{3}}$$

20.)

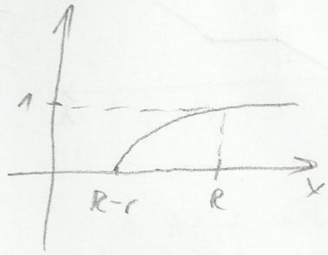


$X \sim$ udaljenost točke do veće kugle

$$V_{kugle} = \frac{4}{3}r^3\pi$$

$$P(X) = F(X) = \frac{\frac{4}{3}\pi R^3 - \frac{4}{3}\pi(R-x)^3}{\frac{4}{3}\pi r^3} = 1 - \frac{(R-x)^3}{r^3}$$

$$f(x) = -\frac{3}{r^3}(R-x)^2(-1) = \underline{\frac{3}{r^3}(R-x)^2}$$

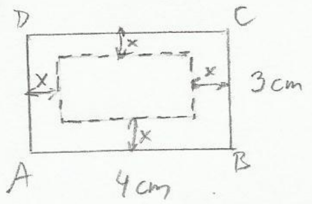


$$E(X) = \int_{R-r}^{R} \frac{3}{r^3}(R-x)^2 \cdot x = \frac{3}{r^3}\int_{R-r}^{R}(Rx - x^3)\,dx$$

$$= \frac{3}{r^3}\left(R\frac{x^2}{2} - \frac{x^4}{4}\right)\Big|_{R-r}^{R} = \frac{3}{r^3}R\left(\frac{R^2}{2} - \frac{(R-r)^2}{2}\right) - \frac{3}{r^3}R\left(\frac{R^4}{4} - \frac{(R-r)^4}{4}\right)$$

$$= \ldots = \underline{R - \frac{3}{4}r}$$

21.)



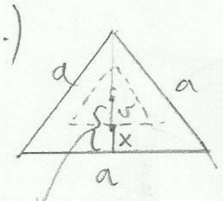
X ~ udaljenost točke do najbliže stanice

$$P(X) = F(x) = \frac{12-(4-x)(3-x)}{12} = \frac{12-(12-8x-6x+4x^2)}{12} = \frac{7}{6}x - \frac{1}{3}x^2$$

$$f(x) = \frac{dF(x)}{dx} = \frac{7}{6} - \frac{2}{3}x$$

$$E(X) = \int_0^{1,5}\left(\frac{7}{6}-\frac{2}{3}x\right)x\,dx = \int_0^{1,5}\left(\frac{7}{6}x-\frac{2}{3}x^2\right)dx = \frac{7}{6}\cdot\frac{x^2}{2}\Big|_0^{1,5} - \frac{2}{3}\cdot\frac{x^3}{3}\Big|_0^{1,5} = \frac{9}{16}$$

22.)

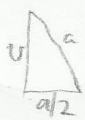


- jednakostraničan trokut

X ~ udaljenost točke T do najbliže stranice trokuta

$$\Omega = \frac{a\cdot v}{2} = \frac{a\cdot\frac{\sqrt{3}}{2}a}{2} = \frac{\sqrt{3}}{4}a^2$$

$$\frac{1}{3}\cdot\frac{\sqrt{3}}{2}a = \frac{\sqrt{3}}{6}a$$



$$v = \sqrt{a^2-\frac{a^2}{4}} = \frac{\sqrt{3}}{2}a$$

$$\frac{a}{\frac{1}{3}\cdot\frac{\sqrt{3}}{2}a} = \frac{a'}{\frac{1}{3}\cdot\frac{\sqrt{3}}{2}a - x}$$

$$\left.\begin{array}{l} x=0 \;\ldots\ldots\; a'=a \\ x=\frac{1}{3}\cdot\frac{\sqrt{3}}{2}a \;\ldots\ldots\; a'=0 \end{array}\right\} a' = a - \frac{6}{\sqrt{3}}x$$

$$a\left(\frac{\sqrt{3}}{6}a - x\right) = a'\,\frac{\sqrt{3}}{6}a$$

$$a' = \frac{\frac{\sqrt{3}}{6}a - x}{\frac{\sqrt{3}}{6}} = a - \frac{6}{\sqrt{3}}x$$

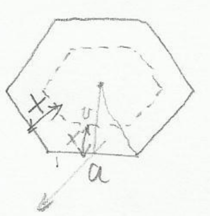
$$F(x) = 1 - \frac{\left(a-\frac{6}{\sqrt{3}}x\right)^2}{a^2}$$

$$P_\Delta = \frac{\sqrt{3}}{4}\cdot\left(a-\frac{6}{\sqrt{3}}x\right)^2$$

$$f(x) = \frac{4(\sqrt{3}a-6x)}{a^2}$$

$$E(X) = \int_0^{\frac{\sqrt{3}}{6}a} x\cdot\left(\frac{4(\sqrt{3}a-6x)}{a^2}\right)dx = \ldots = \frac{a\sqrt{3}}{18}$$

23.)

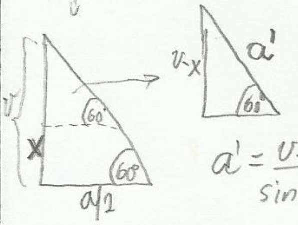


ŠESTEROKUT → 6 jednakostraničnih trokuta → $P_{\text{šest}} = 6 \cdot \frac{\sqrt{3}}{4} a^2$

$$F(x) = 1 - \frac{6\frac{\sqrt{3}}{4}\cdot\left(a - \frac{2}{\sqrt{3}}x\right)^2}{6\cdot\frac{\sqrt{3}}{4}a^2} = 1 - \frac{\left(a-\frac{2}{\sqrt{3}}x\right)^2}{a^2}$$

$v = \sqrt{a^2 - \left(\frac{a}{2}\right)^2} = \frac{\sqrt{3}}{2}a$

$$f(x) = \frac{dF(x)}{dx} = \frac{4(\sqrt{3}a - 2x)}{3a^2}$$

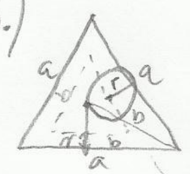


$a' = \frac{v-x}{\sin 60°} = \frac{\frac{\sqrt{3}}{2}a - x}{\frac{\sqrt{3}}{2}}$

$$E(x) = \int_0^{\frac{\sqrt{3}}{2}a} \frac{4(\sqrt{3}a - 2x)}{3a^2}\cdot x\, dx = \ldots = \frac{a\sqrt{3}}{6}$$

$a' = a - \frac{2}{\sqrt{3}}x$

26.)

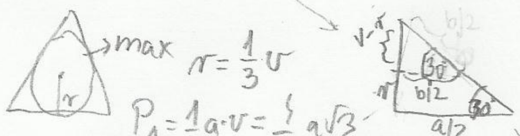


$X \sim$ površina max kruga upisanog u trokut, $x \in \left[0, \frac{a^2\pi}{12}\right]$

$X = r^2\pi$

$r = \sqrt{\frac{x}{\pi}}$

$$F(x) = P(X<x) = \frac{\frac{a^2\sqrt{3}}{4} - \frac{b^2\sqrt{3}}{4}}{\frac{a^2\sqrt{3}}{4}} = 1 - \frac{(a-2r\sqrt{3})^2}{a^2}$$



$r = \frac{1}{3}v$

$b = \frac{a}{2} - r\sqrt{3}$

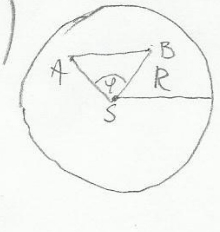
$$= \frac{4\sqrt{3}}{a}\sqrt{\frac{x}{\pi}} - \frac{12x}{a^2\pi}$$

$P_\Delta = \frac{1}{2}a\cdot v = \frac{1}{3}\cdot\frac{a\sqrt{3}}{2}$

$P_0 = r^2\pi = \left(\frac{1}{3}a\cdot\frac{\sqrt{3}}{2}\right)^2\cdot\pi = \frac{a^2\pi}{12}$

$\frac{b}{2}$ jer je b kateta vertanog trokuta na poč. slici, a tu u ovom primjeru imamo polovicu te katete!

27.)



$P_\Delta = \frac{1}{2}\cdot R^2 \cdot \sin\varphi$

$\varphi \in [0, 2\pi] \to$ jednolika $f(\varphi) = \frac{1}{2\pi}$

$X = P_\Delta = \frac{1}{2}R^2\sin\varphi$

$\varphi = \arcsin\left(\frac{2x}{R^2}\right)$

$$\frac{d\varphi}{dx} = \frac{1}{\sqrt{1-\frac{4x^2}{R^4}}}\cdot\frac{2}{R^2} = \frac{2}{\sqrt{R^4-4x^2}}$$

$$f(x) = f(\varphi)\cdot\frac{d\varphi}{dx} = \frac{1}{2\pi}\cdot\frac{2}{\sqrt{R^4-4x^2}} = \frac{1}{\pi\cdot\sqrt{R^4-4x^2}}$$

podijelimo na intervale jer $\sin\varphi$ nije injekcija:

$\left[0, \frac{\pi}{2}\right] \Rightarrow f(x_1) = \frac{1}{\pi\sqrt{R^4-4x^2}}$

$\left[\frac{\pi}{2}, \pi\right] \Rightarrow f(x_2) =$ –||–

$\left[\pi, \frac{3\pi}{2}\right] \Rightarrow f(x_3) =$ –||–

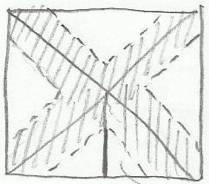
$\left[\frac{3\pi}{2}, 2\pi\right] \Rightarrow f(x_4) =$ –||–

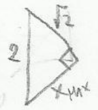
$$f(x) = f(x_1) + f(x_2) + f(x_3) + f(x_4) = \frac{4}{\pi\sqrt{R^4-4x^2}}$$

$$E(x) = \int_0^{R^2} \frac{4}{\pi\sqrt{R^4-4x^2}}\cdot x\, dx = \ldots = \frac{R^2}{\pi}$$

31.) $a = 4$ cm $X \sim$ udaljenost točke do bliže dijagonale kvadrata

$x \in [0, \sqrt{2}]$

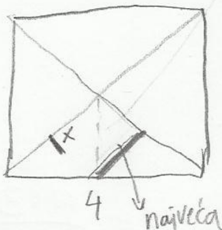


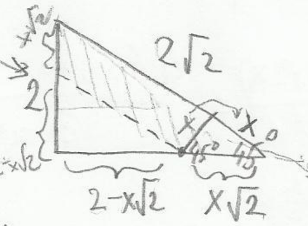


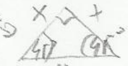
$x_{MAX} = \sqrt{4-2} = \sqrt{2}$

$$F(x) = P(X<x) = \frac{16 - 4\cdot\frac{1}{2}(2-x\sqrt{2})\cdot 2\cdot(2-x\sqrt{2})}{16}$$

$$F(x) = x\sqrt{2} - \frac{1}{2}x^2$$





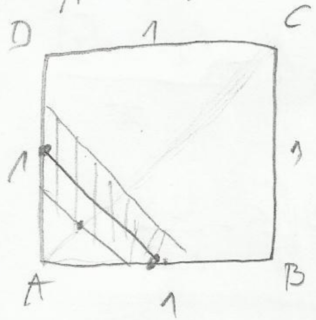


y - ? $\sin\alpha = \frac{x}{y} \to y = \frac{x}{\sin\alpha} = \frac{x}{\frac{\sqrt{2}}{2}} = \frac{2x}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{2x\sqrt{2}}{2} = x\sqrt{2}$

$$f(x) = \frac{dF(x)}{dx} = \sqrt{2} - x, \quad x \in [0, \sqrt{2}], \quad E(X) = \int_0^{\sqrt{2}} x\cdot(\sqrt{2}-x)\,dx = \frac{\sqrt{2}}{3}$$

32.) $X \sim$ udaljenost točke do pravca koji prolazi polovištima E, F, tj. stranica $\overline{AB}$ i $\overline{AD}$.

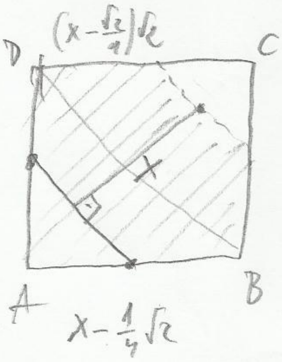
$x \in [0, \frac{3}{4}\sqrt{2}]$ → od pravca do vrha C, $\frac{3}{4}$ dijagonale



1. slučaj za $x \in [0, \frac{1}{4}\sqrt{2}]$:

$$F(x) = P(X<x) = \frac{\frac{1}{2}(x\sqrt{2}+\frac{1}{2})^2 - \frac{1}{2}(\frac{1}{2}-x\sqrt{2})^2}{1} = x\sqrt{2}$$

ili $\frac{P_{trapeza}}{1} = \frac{\text{srednjica}\cdot\text{visina}}{1} = \frac{\frac{\sqrt{2}}{2}\cdot 2x}{1} = x\sqrt{2}$



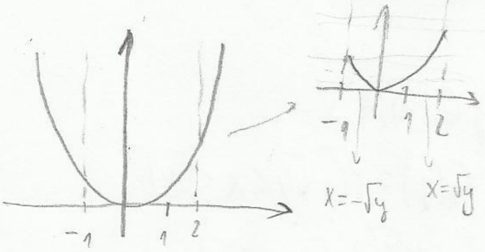
2. slučaj $x \in [\frac{\sqrt{2}}{4}, \frac{3}{4}\sqrt{2}]$

$$F(x) = P(X<x) = \frac{1 - \frac{1}{2}b^2}{1} = 1 - \frac{1}{2}\left(1 - \sqrt{2}\left(x - \frac{\sqrt{2}}{4}\right)\right)$$

$$= -x^2 + \frac{3\sqrt{2}}{2}x - \frac{1}{8}$$

64.) $[-1,2]$ $\quad f(x)=\frac{1}{b-a}=\frac{1}{2-(-1)}=\underline{\underline{\frac{1}{3}}}$

a) $Y=X^2$



$1^\circ \; y\in[0,1]$

$G(y)=P(Y<y)$

$=P($

$=\int_{-\sqrt{y}}^{2}\frac{1}{3}\,dx=\frac{1}{3}x\Big|_{-\sqrt{y}}^{2}=\underline{\underline{\frac{2+\sqrt{y}}{3}}}$

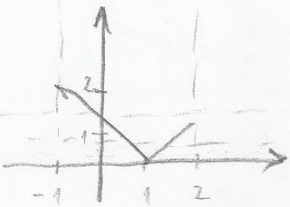
$2^\circ \; y\in[1,4]$

$G(y)=P(Y<y)$

$=P(-1<X<\sqrt{y})$

$=\int_{-1}^{\sqrt{y}}\frac{1}{3}\,dx=\underline{\underline{\frac{\sqrt{y}+1}{3}}}$

b) $Y=|X-1|$



$1^\circ \; y\in[0,1]$

$G(y)=P(|X-1|<y)=$

$=P(-y<X-1<y)$

$=P(1-y<X<y+1)$

$\int_{1-y}^{1+y}\frac{1}{3}\,dx=\frac{1}{3}(1+y-1+y)=\underline{\underline{\frac{2}{3}y}}$

$2^\circ \; y\in(1,2]$

$G(y)=P(Y<y)=P(1-X<y)$

$=P(X>1-y)=1-P(X<1-y)$

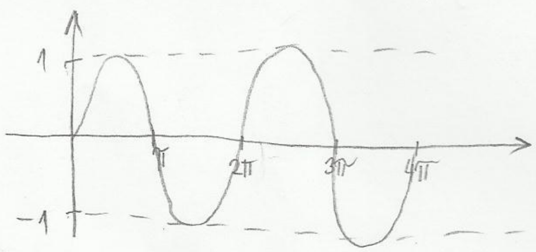
$=\underline{\underline{\frac{1+y}{3}}}$

65.) $X\sim$ jed. raz. na intervalu $[0,4\pi]$

$Y=\sin X$

$\rightarrow$ lakše je rješavati na 1. način (preko funkcije razdiobe) po definiciji : $G(y)=P(Y<y)=P(\sin X<y$

$y\in[-1,1]$



$f(x)=\frac{1}{b-a}=\frac{1}{4\pi}$

$G(y)=P(\sin X<y)=P(0<x<\arcsin y)+(\pi-\arcsin y<x<2\pi+\arcsin y)+(3\pi-\arcsin y<x<4\pi)$

$=P(0<x<\arcsin y)+P(\pi-\arcsin y<x<2\pi+\arcsin y)+P(3\pi-\arcsin y<x<4\pi)$

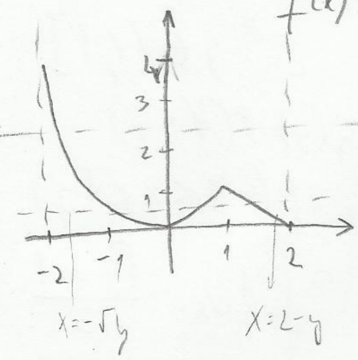
$G(y)=\frac{1}{4\pi}\left[\arcsin y-0+2\pi+\arcsin y-(\pi-\arcsin y)+4\pi-(3\pi-\arcsin y)\right]$

$=\frac{1}{2}+\frac{1}{\pi}\arcsin y$

$g(y)=\frac{dG(y)}{dy}=\frac{1}{\pi}\cdot\frac{1}{\sqrt{1-y^2}}\,,\quad y\in\langle-1,1\rangle$

66.) $X \sim U[-2,2]$, odredi f-ju razdiobe $Y=\min\{x^2, -x+2\}$

$\hookrightarrow f(x) = \frac{1}{2-(-2)} = \frac{1}{4}$



$1^\circ \; y \in [1,4]$

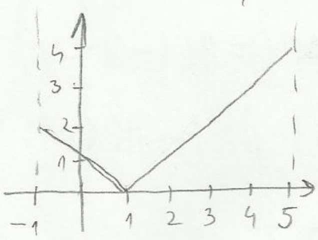
$G(y) = P(Y<y)$
$= P(-\sqrt{y} < X < 2)$
$= \int_{-\sqrt{y}}^{2} \frac{1}{4}\,dx = \frac{1}{2} + \frac{1}{4}\sqrt{y}$

$2^\circ \; y \in [0,1]$

$G(y) = P(-\sqrt{y} < X < \sqrt{y})$
$+ P(2-y < X < 2)$
$= \int_{-\sqrt{y}}^{\sqrt{y}} \frac{1}{4}\,dx + \int_{2-y}^{2} \frac{1}{4}\,dx$

$= \frac{1}{2}\sqrt{y} + \frac{1}{2} - \frac{1}{4}(2-y)$

67.) $X \sim U[-1,5]$, $Y = |X-1|$, $g(y) = ?$



$f(x) = \frac{1}{5-(-1)} = \frac{1}{6}$

$1^\circ \; y \in [0,2]$

$G(y) = P(|X-1|<y)$
$= P(-y < X-1 < y) = P(1-y < X < 1+y)$
$= \int_{1-y}^{1+y} \frac{1}{6}\,dx = \frac{1}{6}\cdot 2y = \frac{1}{3}y$

$g_1(y) = \frac{dG_1(y)}{dy} = \frac{1}{3}$

$2^\circ \; y \in [2,4]$

$G(y) = P(Y<y) = P(1-X<y)$
$= P(X>1-y) = 1 - P(X<1-y)$
$= \frac{1+y}{6}$

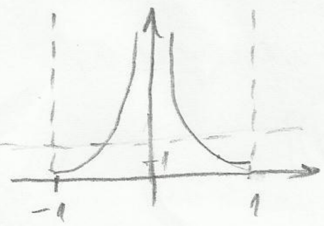
$g_2(y) = \frac{dG_2(y)}{dy} = \frac{1}{6}$

$g(y) = \begin{cases} \frac{1}{3}, & y \in [0,2] \\ \frac{1}{6}, & y \in [2,4] \end{cases}$

$E(Y) = \int_0^2 \frac{1}{3}x\,dx + \int_2^4 \frac{1}{6}x\,dx =$
$= \frac{1}{3}\frac{x^2}{2}\Big|_0^2 + \frac{1}{6}\frac{x^2}{2}\Big|_2^4 = \frac{2}{3} + \frac{4}{3} - \frac{1}{3} = \frac{5}{3}$

68.) $X \sim U[-1,1]$, $Y = \frac{1}{x^2}$, $G(y) = ?$

$f(x) = \frac{1}{2}$



$y \in [1, +\infty]$

$G(Y<y) = P\left(\frac{1}{x^2} < y\right)$
$= P\left(X > \frac{1}{\sqrt{y}}\right) = 1 - P\left(X < \frac{1}{\sqrt{y}}\right)$
$= 1 - \frac{1}{\sqrt{y}}$

69.) $f(x)=?$

a) $Y=X^3$

$X=\sqrt[3]{Y}$ $\quad g(y)=f(\sqrt[3]{y})\cdot\frac{1}{3\sqrt[3]{y^2}}\ ,\ y\in\langle 0,+\infty\rangle$

$\frac{dx}{dy}=\frac{1}{3\sqrt[3]{y^2}}$

b) $Y=\frac{1}{X}\ ,\ X=\frac{1}{y}\ ,\ \frac{dx}{dy}=\left|-\frac{1}{y^2}\right|=\frac{1}{y^2}$ $\quad g(y)=f(\frac{1}{y})\cdot\frac{1}{y^2}\ ,\ y\in\langle 0,+\infty\rangle$

c) $Y=\sqrt{X}\ ,\ X=Y^2\ ,\ \frac{dx}{dy}=2y \rightarrow g(y)=f(y^2)\cdot 2y\ ,\ y\in\langle 0,+\infty\rangle$

d) $Y=e^X\ ,\ X=\ln Y\ ,\ \frac{dx}{dy}=\frac{1}{y} \rightarrow g(y)=f(\ln y)\cdot\frac{1}{y}\ ,\ y\in\langle 1,+\infty\rangle$

e) $y=e^{-x}\rightarrow x=-\ln y\ ,\ \frac{dx}{dy}=\left|-\frac{1}{y}\right|=\frac{1}{y}\rightarrow g(y)=f(-\ln y)\cdot\frac{1}{y}\ ,\ y\in\langle 1,+\infty\rangle$

f) $y=\ln x\ ,\ x=e^y\ ,\ \frac{dx}{dy}=e^y \rightarrow g(y)=f(e^y)\cdot e^y\ ,\ y\in\langle -\infty,+\infty\rangle$

70.) a) $Y=\mathrm{arctg}\,X \rightarrow X=\mathrm{tg}\,Y \rightarrow \frac{dx}{dy}=\frac{1}{\cos^2 y} \rightarrow g(y)=f(\mathrm{tg}\,y)\cdot\frac{1}{\cos^2 y}\ ,\ y\in\langle -\frac{\pi}{2},\frac{\pi}{2}\rangle$

b) $Y=\mathrm{tg}\,X \rightarrow X=\mathrm{arctg}\,Y \rightarrow \frac{dx}{dy}=\frac{1}{y^2+1} \rightarrow g(y)=f(\mathrm{arctg}\,y)\cdot\frac{1}{y^2+1}\ ,\ y\in\langle -\frac{\pi}{2},\frac{\pi}{2}\rangle$

c) $Y=|X| \rightarrow X=\pm y\ ,\ \frac{dx}{dy}=1 \rightarrow g(y)=f(-y)\cdot 1+f(y)\cdot 1$?

d) $Y=X^2 \rightarrow X=\sqrt{y}\ ,\ \frac{dx}{dy}=\frac{1}{2\sqrt{y}} \rightarrow g(y)=f(\sqrt{y})\cdot\frac{1}{2\sqrt{y}}$ $\quad x\in\langle 0,+\infty\rangle$

e) $Y=\frac{1}{X^2}\rightarrow X=\frac{\pm 1}{\sqrt{y}}\ ,\ \frac{dx}{dy}=\left|-\frac{1}{2\sqrt{y^3}}\right|=\frac{1}{2\sqrt{y^3}} \rightarrow g(y)=f(\frac{1}{\sqrt{y}})\cdot\frac{1}{2\sqrt{y^3}}+f(-\frac{1}{\sqrt{y}})\cdot\frac{1}{2\sqrt{y^3}}\ ,\ y>0$

f) $Y=\frac{1}{1+X^2}\rightarrow X=\pm\sqrt{\frac{1-y}{y}}\ ,\ \frac{dx}{dy}=\frac{1}{2y^2\sqrt{\frac{1-y}{y}}} \rightarrow g(y)=f\left(\sqrt{\frac{1-y}{y}}\right)\cdot\frac{1}{2y^2\sqrt{\frac{1-y}{y}}}+f\left(-\sqrt{\frac{1-y}{y}}\right)\cdot\frac{1}{2y^2\sqrt{\frac{1-y}{y}}}$

$y\in\langle 0,1\rangle$

71.) $Y=e^X$

$X=\ln Y$

$G(y)=P(Y<y)=P(e^X<y)=F(\ln y)$

72.) $Y=|1+X|\ ,\ y>0$

$g_1(y)=f(x)\left|\frac{dx}{dy}\right|=f(y-1)\cdot 1\ ,\ y>\cancel{0}$

$X_1=Y-1$

$g_2(y)=f(x)\left|\frac{dx}{dy}\right|=f(-y-1)\cdot 1\ ,\ y>0$

$X_2=-Y-1$

$g(y)=g_1(y)+g_2(y)=f(y-1)+f(-y-1)\ ,\ y\geq\cancel{0}$

75.) $F(x) = 1-e^{-2x}, \ x>0, \ G(x) = !$

$Y = x^2$

$X = \pm\sqrt{y}$

$G(y) = P(Y<y) = P(-\sqrt{y} < X < \sqrt{y}) = F(\sqrt{y}) - F(-\sqrt{y})$

zbog $x>0$ uzima se samo $F(\sqrt{y})$, odbacujemo $F(-\sqrt{y})$

$G(x) = F(\sqrt{y}) = \underline{1-e^{-2\sqrt{y}}}, \ y>0$

76.) $f(x) = C|\sin x|, \quad |x| \leq \pi$

$Y = x^2$

$X = \pm\sqrt{Y}$

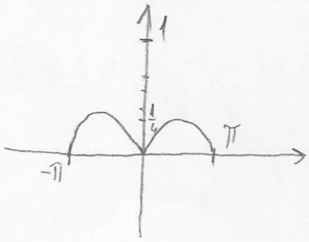
$-\pi \leq x \leq \pi \rightarrow 0 \leq y \leq \pi^2$

$$F(x) = \int_{-\pi}^{\pi} f(x)dx = -\int_{-\pi}^{0} C\sin x\,dx + \int_{0}^{\pi} C\sin x\,dx$$

$$= C + C - (-C-C) = 4C$$

$$4C = 1 \rightarrow \boxed{C = \frac{1}{4}}$$

$f(x) = \frac{1}{4}\sin x$



1° $x \in [-\pi, 0]$

$y \in [0, \frac{1}{4}]$

$x = -\sqrt{y}$

$\left|\frac{dx}{dy}\right| = \frac{1}{2\sqrt{y}}$

$g_1 = \frac{1}{4}\sin(\sqrt{y}) \cdot \frac{1}{2\sqrt{y}}$

2° $x \in [0, \pi]$

$y \in [0, \frac{1}{4}]$

$x = \sqrt{y}$

$\left|\frac{dx}{dy}\right| = \frac{1}{2\sqrt{y}}$

$g_2 = \frac{1}{4}\sin(\sqrt{y}) \cdot \frac{1}{2\sqrt{y}}$

$$g(y) = g_1(y) + g_2(y) = \frac{1}{4\sqrt{y}}\sin(\sqrt{y}), \quad 0 < y \leq \pi^2$$

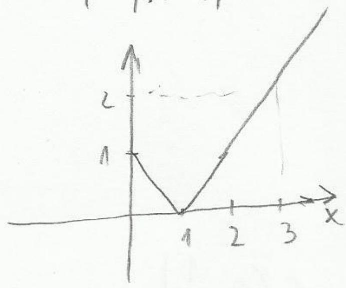
$$E(y) = \int_0^{\pi} \frac{1}{4\sqrt{y}}\sin(\sqrt{y}) \cdot y = \left| \begin{array}{l} t = \sqrt{y} \\ y = t^2 \\ dy = 2t\,dt \end{array} \right| = \frac{1}{4}\int_0^{\pi} t \cdot \sin(t) \cdot 2t\,dt = \frac{1}{2}\int_0^{\pi} t^2 \sin t\,dt$$

$$= \left| \begin{array}{ll} u = t^2 & du = 2t \\ dv = \sin t\,dt & v = -\cos t \end{array} \right| = \frac{1}{2}\left(-t^2\cos t\Big|_0^{\pi} + 2\int t\cos t\right) = \left| \begin{array}{ll} u = t & du = dt \\ dv = \cos t\,dt & v = \sin t \end{array} \right|$$

$$= \frac{\pi^2}{2} + t\sin t\Big|_0^{\pi} - \int_0^{\pi} \sin t\,dt = \frac{\pi^2}{2} + \cos t\Big|_0^{\pi} = \frac{\pi^2}{2} - 1 - 1 = \underline{\frac{\pi^2}{2} - 2}$$

77.) $f(x)=e^{-x},\ x>0$

$Y=|X-1|$



$1^\circ\ x\in(0,1)$

$y\in(0,1)$

$y=|x-1|$

$-y=x-1$

$x=1-y$

$\left|\frac{dx}{dy}\right|=1$

$g_1=e^{+y-1}\cdot 1$

$2^\circ\ x\in(1,+\infty)$

$y\in(0,+\infty)$

$y=x-1$

$x=y+1$

$\left|\frac{dx}{dy}\right|=1$

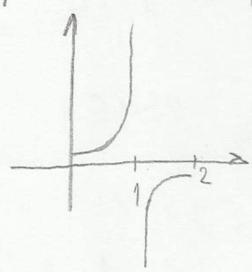
$g_2=e^{-y-1}\cdot 1$

$$g(y)=\begin{cases} e^{+y-1}+e^{-y-1}=\frac{1}{e}e^{y}+e^{-y}\cdot\frac{2}{2}=\frac{2}{e}\operatorname{ch}y, & y\in(0,1)\\ e^{-y-1}, & y\in(1,+\infty)\end{cases}$$

78.) $f(x)=Ce^{-2x},\ x\geq 1$

$Y=\frac{1}{1-x}$

$$1=\int_1^\infty Ce^{-2x}dx=C\int_1^\infty e^{-2x}dx=C\left(-\frac{1}{2}e^{-2x}\Big|_1^\infty\right)=\frac{1}{2}e^{-2}\cdot C\ \rightarrow\ C=2e^2$$



$f(x)=2e^{2-2x}$

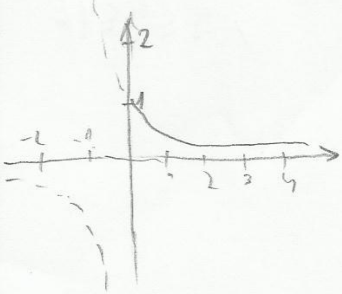
$X=1-\frac{1}{y}$

$g(y)=f(x)\left|\frac{dx}{dy}\right|=f\left(1-\frac{1}{y}\right)\frac{1}{y^2},\ y>0$

$$g(y)=2e^{2-2\left(1-\frac{1}{y}\right)}\cdot\frac{1}{y^2}=2e^{\frac{2}{y}}\cdot\frac{1}{y^2},\ y<0$$

79.) $f(x)=e^{-x},\ x>0$

$Y=\frac{1}{1+x}$



$x\in[0,+\infty)$

$y\in[0,1]$

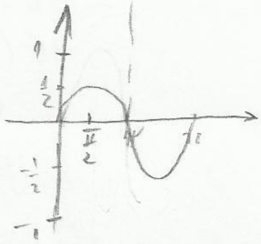
$X=\frac{1}{y}-1$

$\left|\frac{dx}{dy}\right|=\left|-\frac{1}{y^2}\right|=\frac{1}{y^2}$

$$g(y)=e^{-\frac{1}{y}+1}\cdot\frac{1}{y^2}=\frac{e}{y^2}\cdot e^{-\frac{1}{y}},\ y\in[0,1]$$

82.) $f(x)=\frac{1}{2}\sin x\,,\quad 0\le x\le \pi$

$Y=\frac{1}{2}\sin X$



$X=\arcsin 2y$

$\frac{dx}{dy}=\frac{1}{\sqrt{1-4y^2}}$

$1^\circ\ x\in(0,\frac{\pi}{2})$ $\qquad$ $2^\circ\ x\in(\frac{\pi}{2},\pi)$

$g_1(y)=\frac{2y}{\sqrt{1-4y^2}}$ $\qquad$ $g_2(y)=\frac{2y}{\sqrt{1-4y^2}}$ $\Big\}\ g(y)=\frac{4y}{\sqrt{1-4y^2}}\,,\quad y\in(0,\frac{1}{2})$

$E(y)=\int_0^{1/2}\frac{4y}{\sqrt{1-4y^2}}\cdot y=\int_0^{1/2}\frac{4y^2}{\sqrt{1-4y^2}}dy=\left|\begin{matrix}u=y & du=dy\\ dv=\frac{4y}{\sqrt{1-4y^2}} & v=-\sqrt{1-4y^2}\end{matrix}\right|$

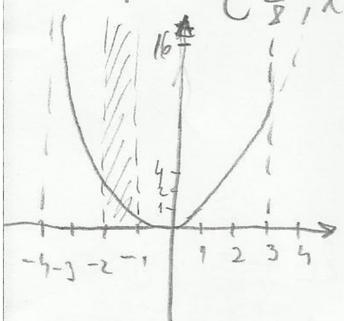
$\int\frac{4y}{\sqrt{1-4y^2}}=\left|\begin{matrix}t=1-4y^2\\ dt=-8y\,dy\\ 4y\,dy=-\frac{1}{2}dt\end{matrix}\right|=\int\frac{-\frac{1}{2}dt}{\sqrt{t}}=-\frac{1}{2}\cdot\frac{t^{\frac{1}{2}}}{\frac{1}{2}}=-\sqrt{t}=-\sqrt{1-4y^2}$

$=\underbrace{-y\cdot\sqrt{1-4y^2}\Big|_0^{1/2}}_{0}+\int\sqrt{1-4y^2}\,dy=\left|\begin{matrix}y=\frac{1}{2}\sin x\\ dy=\frac{1}{2}\cos x\,dx\end{matrix}\right|=\int_0^{1/2}\sqrt{1-\sin^2x}\cdot\frac{1}{2}\cos x\,dx$

$=\frac{1}{2}\int_0^{1/2}\sqrt{\cos^2x}\cdot\cos x\,dx=\frac{1}{2}\int_0^{1/2}\cos x\cdot\cos x\,dx=\frac{1}{2}\int_0^{1/2}\cos^2x\,dx=\int_0^{1/2}\frac{1+\cos 2x}{2}\,dx$

$=\frac{1}{4}x+\frac{1}{4}\cdot\frac{1}{2}\sin 2x=\left|x=\arcsin 2y\right|=\frac{1}{4}\arcsin 2y+\frac{1}{8}\sin(2\cdot\arcsin 2y)\Big|_0^{1/2}=\underline{\underline{\frac{\pi}{8}}}$

83.) $f(x)=\begin{cases}\frac{1}{4}\,, & x\in[-4,-2]\\ \frac{1}{8}\,, & x\in[-1,3]\end{cases}\,,\quad Y=X^2$



$1^\circ\ x\in[-4,-2]$

$y\in[4,16]$

$y=x^2,\ x=-\sqrt{y}$

$\left|\frac{dx}{dy}\right|=\frac{1}{2\sqrt{y}}\,,\ g_1(y)=\frac{1}{8}\cdot\frac{1}{\sqrt{y}}$

$2^\circ\ x\in[-1,0]$

$y\in[0,1]$

$x=-\sqrt{y}$

$\left|\frac{dx}{dy}\right|=\frac{1}{2\sqrt{y}}$

$g_2(y)=\frac{1}{16}\cdot\frac{1}{\sqrt{y}}$

$3^\circ\ x\in[0,3]$

$y\in[1,9]$

$x=\sqrt{y}$

$\left|\frac{dx}{dy}\right|=\frac{1}{2\sqrt{y}}\,,\ g_3(y)=\frac{1}{16\sqrt{y}}$

$g(y)=\begin{cases}\frac{1}{8}\cdot\frac{1}{\sqrt{y}}\,, & 0\le y\le 1\\ 1/16\cdot\frac{1}{\sqrt{y}}\,, & 1\le y\le 4\\ \frac{3}{16}\cdot\frac{1}{\sqrt{y}}\,, & 4\le y\le 9\\ \frac{1}{8}\cdot\frac{1}{\sqrt{y}}\,, & 9\le y\le 16\end{cases}$

$F(y_1)=\frac{1}{8}\int_0^y\frac{1}{\sqrt{y}}dy=\frac{1}{4}\sqrt{y}\quad 0\le y\le 1$

$F(y_2)=\frac{1}{8}+\frac{1}{8}\sqrt{y}\,,\quad 1\le y\le 4$

$F(y_3)=\frac{1}{4}+\frac{1}{8}\cdot[2-1]+\frac{3}{8}(\sqrt{y}-2)=\frac{1}{4}+\frac{1}{8}-\frac{5}{4}+\frac{3}{8}\sqrt{y}$

$=\frac{3}{8}\sqrt{y}-\frac{3}{8}\,,\quad 4\le y\le 9$

$F(y_4)=\frac{1}{4}+\frac{1}{8}+\frac{3}{8}[3-2]+\frac{1}{4}\cdot[\sqrt{y}-3]=\frac{1}{4}\sqrt{y}\,,\quad 9\le y\le 16$

SLUŽBENA RJEŠENJA:

§ 5. Neprekinute slučajne varijable

1. Ne, da, ne, ne, da , da.

2. Ne, da, da, da, da.

3. Funkcije su pozitivne i integral po $\mathbf{R}$ jednak im je jedinici.

5. $\dfrac{\lambda^4}{6}$, $\dfrac{1}{2\sqrt{-\alpha\pi}}$.

6. 1 , 0.5 .

7. $\dfrac{a^3}{2}$, $1-\dfrac{5}{2e}$.

8. Funkcije su definirane formulama na istim intervalima, i iznose: a) $1-\cos x$, b) $\frac{1}{2}(x^2-x)$, c) $-\cos 3x$.

9. $$F(x)=\begin{cases} 0, & x\leqslant -2,\\ 0.1, & -1<x\leqslant -1,\\ 0.3, & -1<x\leqslant 0,\\ 0.5, & 0<x\leqslant 1,\\ 0.8, & 1<x\leqslant 2,\\ 1, & 2<x. \end{cases}$$
$P\{|X|\leqslant 1\}=0.7$.

10. $3\left(\dfrac{\pi+2}{4\pi}\right)^2\dfrac{3\pi-2}{4\pi}$.

11. Uputa: $\{X+Y<3\}\supset\{X<1,\ Y<2\}$.

12. Uputa: Ako je $AB\subset C$, tada vrijedi $P(C)\geqslant P(A)+P(B)-P(A+B)\geqslant P(A)+P(B)-1$

13. a) 2 , $\dfrac{2}{3}$, $\dfrac{1}{18}$; b) $\dfrac{1}{2}$, $\dfrac{4}{3}$, $\dfrac{2}{9}$;
c) $\dfrac{3}{4}$, $\dfrac{11}{16}$, 0.052 ; d) $\dfrac{1}{2}$, 0 , 2 .

14. 2 , $1/\lambda$.

15. $-\dfrac{1}{2}$, $-\dfrac{5}{4}$, $\dfrac{1}{2}$.

16. $f(x)=\frac{1}{6}, \qquad 1\leqslant x\leqslant 7$,
$F(x)=\frac{1}{6}x-\frac{1}{6}, \qquad 1\leqslant x\leqslant 7$.

17. $n+1$, $n+1$.

18. $(x/R)^2$, $0<x<R$, $\frac{2}{3}R$, $\frac{1}{18}R^2$.

19. $v/3$

20. $F_X(x)=1-\dfrac{(R-x)^3}{r^3}$, $R-r\leqslant x\leqslant R$;
$E(X)=R-\frac{3}{4}r$.

21. $\dfrac{9}{16}$.

22. $\dfrac{a\sqrt{3}}{18}$.

23. $f_X(x)=\dfrac{4}{a\sqrt{3}}\left(1-\dfrac{2x}{a\sqrt{3}}\right)$, $x\in[0,a\sqrt{3}/2]$;
$E(X)=\dfrac{a\sqrt{3}}{6}$.

24. $1-(1-2\sqrt{x/\pi})^2$, $0\leqslant x\leqslant\frac{\pi}{4}$; 0.25 .

25. $F(x)=1-(1-2\sqrt[3]{3x/4\pi})^3$, $0\leqslant x\leqslant\pi/6$.

26. $f_X(x)=\sqrt{\dfrac{12}{a^2\pi}\cdot\dfrac{1}{\sqrt{x}}-\dfrac{12}{a^2\pi}}$, $x\in[0,\frac{a^2\pi}{12}]$.

27. $f_X(x)=\dfrac{4}{\pi\sqrt{R^4-4x^2}}$, $0\leqslant x\leqslant\dfrac{R^2}{2}$;
$E(X)=\dfrac{R^2}{\pi}$; $P\{X>\dfrac{R^2}{4}\}=\dfrac{2}{3}$.

28. $F_X(x)=\frac{20\sqrt{3}}{9}x-\frac{32}{9}x^2$, $\quad x\in[0,\sqrt{3}/4]$.

30. $F(x)=\frac{5}{144}(24x-5x^2)$, $x\in[0,\frac{12}{5}]$

31. $\dfrac{a\sqrt{2}}{12}$.

32. $$f(x)=\begin{cases} \dfrac{\sqrt{2}}{a}, & x\in[0,\frac{\sqrt{2}}{4}a]\\ \dfrac{3\sqrt{2}}{2a}-\dfrac{2}{a^2}x, & x\in[\frac{\sqrt{2}}{4}a,\frac{3\sqrt{2}}{4}a] \end{cases}$$

33. $$f_X(x)=\begin{cases} \dfrac{\pi}{2a^2}x, & x\in[0,a],\\ \dfrac{\pi}{2a^2}x-\dfrac{2x}{a^2}\arccos\dfrac{a}{x}, & x\in[a,a\sqrt{2}]. \end{cases}$$
$E(X)=\dfrac{a}{3}[\sqrt{2}+\ln\operatorname{tg}(\dfrac{3\pi}{8})]$

34. $f_X(x)=\frac{2}{3}-\frac{2}{9}x$, $x\in[0,3]$; $E(X)=1$.

35. $F(x)=2x-x^2$, $x\in[0,1]$; $E(X)=\frac{1}{3}$;
$E(X^n)=\dfrac{2}{(n+1)(n+2)}$; $D(X)=\frac{1}{18}$.

36. $f_X(x) = \dfrac{6(x-a)(b-x)}{(b-a)^3}$, $x \in [a, b]$

37. $f_k(x) = \binom{n}{1}\binom{n-1}{k-1} x^{k-1}(1-x)^{n-k}$.

38. $f_X(x) = 3x^2$, $E(X) = \dfrac{3}{4}$.

39. $F(x) = 2x - 1$, $\frac{1}{2} < x < 1$; $E(X) = 0.75$.

40. $1 - \dfrac{(T-x)^n}{T^n}$, $0 \leqslant x \leqslant T$.

41. $f_X(x) = \dfrac{1}{l}$, $x \in [0, l]$; $E(X) = \frac{1}{2}l$;
$D(X) = \frac{1}{12}l^2$

42. $f_X(x) = \begin{cases} \frac{\pi}{2}x, & x \in [0,1], \\ \frac{\pi}{2}x - 2x \arccos \dfrac{1}{x}, & x \in [1, \sqrt{2}]. \end{cases}$

43. $f_X(x) = \begin{cases} \frac{\pi}{4}x, & x \in [0, \sqrt{2}], \\ \frac{\pi}{4}x - x \arccos \dfrac{\sqrt{2}}{x}, & x \in [\sqrt{2}, 2]. \end{cases}$

44. $\dfrac{4a}{\pi} \ln \operatorname{tg}\left(\dfrac{3\pi}{8}\right)$.

45. $F_X(x) = \frac{16}{9} + \frac{2}{3}\sqrt{x^2-16} - \frac{1}{9}x^2$, $x \in [4, 5]$.

46. $F_X(x) = \frac{1}{7}x,\ x \in [0, 3]$;
$\frac{1}{14}(x + 3 + \sqrt{x^2-9})$, $x \in [3, 4]$;
$\frac{1}{14}(7 + \sqrt{x^2-9} + \sqrt{x^2-16})$, $x \in [4, 5]$.

47. $F(x) = \sqrt{x-3}$, $x \in [3, 4]$; $E(X) = \frac{10}{3}$.

48. $F_X(x) = \dfrac{x}{2a}$, $x \in [0, a/2]$;
$\dfrac{1}{4} + \dfrac{1}{4a}\sqrt{4x^2-a^2}$, $x \in [a/2, a]$;
$\dfrac{1}{4} + \dfrac{1}{4a}\sqrt{4x^2-a^2} + \dfrac{1}{2a}\sqrt{x^2-a^2}$, $x \in [a, a\frac{\sqrt{5}}{2}]$.

49. $f_X(x) = \begin{cases} \dfrac{\pi x}{2}, & x \in [0,1], \\ \dfrac{\pi x}{2} - 2x \arccos \dfrac{1}{x}, & x \in [1, \sqrt{2}]. \end{cases}$

50. $F_X(x) = \dfrac{\pi}{300\sqrt{3}}x^2$, $0 \leqslant x \leqslant 10$; $\dfrac{\pi x^2}{300\sqrt{3}} + \dfrac{\sqrt{x^2-100}}{10\sqrt{3}} - \dfrac{x^2 \arccos 10/x}{100\sqrt{3}}$; $10 \leqslant x \leqslant 20$.

51. $F_{X_2}(x_2) = \dfrac{4x_2^2}{a^2}$, $x_2 \in [0, \frac{a}{2}]$; $E(X_2) = \frac{1}{3}a$

52. $E(X) = \dfrac{5\sqrt{3}a}{18}$.

53. $f_X(x) = \dfrac{2x}{\pi\sqrt{4R^2d^2 - (d^2 + R^2 - x^2)^2}}$.

54. $F_X(x) = \dfrac{4}{\pi^2} \arcsin^2\left(\dfrac{x}{2R}\right)$, $0 \leqslant x \leqslant 2R$.

55. $f_X(x) = \dfrac{4}{\pi\sqrt{R^2-x^2}}$, $x \in [0, \dfrac{R}{\sqrt{2}}]$;
$E(X) = \dfrac{2}{\pi}(2 - \sqrt{2})R$.

56. $f_X(x) = \begin{cases} \dfrac{8}{5\sqrt{3}d}, & x \in [0, \frac{d\sqrt{3}}{2}], \\ \dfrac{12}{5\sqrt{3}d} - \dfrac{16x}{15d^2}, & x \in [\frac{d\sqrt{3}}{2}, \frac{3d\sqrt{3}}{4}]. \end{cases}$

59. $E(X) = E(Y) = 0$.

64. a) $\dfrac{2y}{3}$, $0 \leqslant y \leqslant 1$; $\dfrac{\sqrt{y}+1}{3}$, $1 \leqslant y < 4$;
b) $\dfrac{2y}{3}$, $0 \leqslant y < 1$; $\dfrac{1}{3} + \dfrac{y}{3}$, $1 \leqslant y \leqslant 2$.

65. $F_Y(x) = \frac{1}{2} + \frac{1}{\pi} \arcsin x$, $-1 < x < 1$;
$f_Y(x) = \dfrac{1}{\pi\sqrt{1-x^2}}$, $-1 < x < 1$.

66. $F_Y(y) = \begin{cases} \frac{1}{4}y + \frac{1}{2}\sqrt{y}, & 0 \leqslant y \leqslant 1, \\ \frac{1}{2} + \frac{1}{4}\sqrt{y}, & 1 \leqslant y \leqslant 4. \end{cases}$

67. $g_Y(y) = \begin{cases} \frac{1}{3}, & y \in [0, 2], \\ \frac{1}{6}, & y \in [2, 4]. \end{cases}$ $E(Y) = \frac{5}{3}$.

68. $F_Y(y) = 1 - \dfrac{1}{\sqrt{y}}$, $y \geqslant 1$.

69. a) $\dfrac{1}{3\sqrt[3]{y^2}} f(\sqrt[3]{y})$, $0 < y < \infty$;
b) $\dfrac{1}{y^2} f(\dfrac{1}{y})$, $0 < y < \infty$;
c) $2yf(y^2)$, $0 < y < \infty$;
d) $\dfrac{1}{y} f(\ln y)$, $1 < y < \infty$;
e) $\dfrac{1}{y} f(\ln \dfrac{1}{y})$, $0 < y < 1$;
f) $e^y f(e^y)$, $-\infty < y < \infty$.

71. $F(\ln x)$, $x > 0$.

72. $g(y) = f(y-1) + f(-y-1)$, $y \geqslant 0$

73. $f(y)$, $y \notin (0, 1)$; $f(\sqrt{y})\dfrac{1}{2\sqrt{y}}$, $y \in (0, 1)$.

74. a) $F\left(\dfrac{x-b}{a}\right)$, $a > 0$;
$1 - F\left(\dfrac{x-b}{a} + 0\right)$, $a < 0$;
b) $F(\sqrt{x}) - F(-\sqrt{x} + 0)$, $x \geqslant 0$;
c) $F(x) - F(-x + 0)$, $x \geqslant 0$;
e) $F(g^{-1}(x))$.

75. $1 - e^{-2\sqrt{y}}$, $y > 0$.

76. $g(y) = \dfrac{\sin\sqrt{y}}{4\sqrt{y}}$, $0 \leqslant y \leqslant \pi^2$; $E(Y) = 1$.

77. $g_Y(y) = \begin{cases} \dfrac{2}{e}\operatorname{ch} y, & y \in [0,1], \\ e^{-y-1}, & y \in [1, \infty). \end{cases}$

$E(Y) = \dfrac{2}{e}$.

78. $C = 2e^2$, $g_Y(y) = \dfrac{2}{y^2}\exp(\dfrac{2}{y})$, $y < 0$.

79. $g_Y(y) = \dfrac{e}{y^2}\exp(-\dfrac{1}{y})$, $0 < y < 1$.

80. $G_Y(y) = \begin{cases} -\dfrac{1}{\pi}\operatorname{arc\,tg}\dfrac{1}{y}, & y < 0, \\ \dfrac{1}{2} + \dfrac{2}{\pi}\operatorname{arc\,tg} y, & y > 0. \end{cases}$

81. $g_Y(y) = \dfrac{1}{\sqrt{2\pi}}$

82. $g_Y(y) = \dfrac{4y}{\sqrt{1-4y^2}}$, $0 \leqslant y \leqslant \frac{1}{2}$;

$E(Y) = \dfrac{\pi}{8}$.

83. $F_Y(y) = \begin{cases} \frac{1}{4}\sqrt{y}, & 0 \leqslant y \leqslant 1, \\ \frac{1}{8}\sqrt{y} + \frac{1}{8}, & 1 \leqslant y \leqslant 4, \\ \frac{3}{8}\sqrt{y} - \frac{3}{8}, & 4 \leqslant y \leqslant 9, \\ \frac{1}{4}\sqrt{y}, & 9 \leqslant y \leqslant 16. \end{cases}$

84. $g_Y(y) = \dfrac{1}{4\sqrt{\pi(y-1)}}\Big[\exp\Big(-\dfrac{(\sqrt{y-1}+1)^2}{4}\Big)$

$+\exp\Big(-\dfrac{(\sqrt{y-1}-1)^2}{4}\Big)$, $y \geqslant 1$; 0.1193.

85. $P\{Y = k\} = \dfrac{e^\lambda - 1}{e^\lambda}e^{-k\lambda}$, $E(Y) = \dfrac{1}{e^\lambda - 1}$.

86. $Y \sim N(0,1)$.

87. Uputa: Ako X ima Cauchyjevu razdiobu, tada je $X = \operatorname{tg}\alpha$ i α ima jednoliku razdiobu.

88. $\dfrac{\pi}{4} \cdot \dfrac{a^2 + ab + b^2}{3}$,

$\dfrac{\pi^2}{720}(b-a)^2(4b^2 + 7ab + 4a^2)$.

89. $C = \dfrac{1}{\pi}$; $F(y) = \dfrac{3}{\pi}\arcsin\dfrac{y}{100}$, $0 \leqslant y \leqslant 50\sqrt{2}$; $\dfrac{1}{2} + \dfrac{1}{\pi}\arcsin\dfrac{y}{100}$, $50\sqrt{2} \leqslant y \leqslant 100$; 0.5.

90. $F_X(x) = 1 - \sqrt{1-2x}$, $0 < x < 0.5$.

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http://en.wikipedia.org/wiki/Uniform_distribution_(continuous)