Z1 2008.

1. $f(x) = 2\lambda^2 x e^{-\lambda^2 x^2} \qquad x > 0$

$f(x_1, \dots, x_n) = 2\lambda^2 x_1 e^{-\lambda^2 x_1^2} \cdot 2\lambda^2 x_2 e^{-\lambda^2 x_2^2} \cdot \dots =$

$= 2^n \lambda^{2n} (x_1 \cdot x_2 \cdot \dots \cdot x_n) \cdot e^{-\lambda^2 \sum x_i}$

$\ln f = n \ln 2 + 2n \ln \lambda + \ln(x_1 \cdot \dots \cdot x_n) - \lambda^2 \sum x_i \; / '$

$\frac{d}{d\lambda} f = \frac{2n}{\lambda} - 2\lambda \sum x_i = 0$

$n - \lambda^2 \sum x_i = 0$

$\lambda^2 \sum x_i = n \Rightarrow \lambda = \sqrt{\frac{n}{\sum x_i}}$

3,

2. normalna razdioba

x_i	112	114	116	118	120	122
n_i	2	3	7	5	2	3

$\Rightarrow n = 22$

a) $\bar{X} = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i = 117$

$\hat{s}^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 = 8,6$

b) $p = 0.9 \qquad \alpha = 0.1$

I $t_{n-\frac{\alpha}{2}} = t_{0.95} = \{ \text{uz } n-1=21 \} = 1,721$

$t_{1-\frac{\alpha}{2}} \cdot \frac{\hat{s}}{\sqrt{n}} = 1,721 \cdot \frac{2,944}{\sqrt{22}} = 1,08$

$P(117 - 1.08 \le a \le 117 + 1.08) = 0,9$

II $c_1 = \chi^2_{n-1, \alpha/2} = \chi^2_{21, 0,05} = 11,591$

$c_2 = \chi^2_{n-1, 1-\frac{\alpha}{2}} = \chi^2_{21, 0,95} = 32,671$

$\beta_1 = \frac{21 \cdot 8.6}{32.671} = 5.57 \qquad \beta_2 = \frac{21 \cdot 8.6}{11.591} = 15.7$

$P(5.57 \leq \sigma^2 \leq 15.7) = 0.9$

3. $n = 200 \quad m = 112$ 1/r 38.

a) $p = 0.95 \quad \alpha = 0.05$

$\hat{p} = \frac{112}{200} = 0.56 \qquad u_{1-\frac{\alpha}{2}} = u_{1-0.025} = u_{0.975}$

$p_1 = 0.56 - u_{0.975}\sqrt{\frac{0.56 \cdot 0.44}{200}} = 0.56 - 1.95996 \cdot 0.035 =$

$= 0.56 - 0.0688 = 0.491$

$p_2 = 0.56 + 0.0687 = 0.6288$

$P(0.491 < p < 0.6288) = 0.95$

b)

$0.5 = 0.56 - u\sqrt{\frac{0.56 \cdot 0.44}{200}}$

$u \cdot 0.35 = 0.06$

$u = \frac{0.06}{0.035} = 1.714$

$u_{1-\frac{\alpha}{2}} = 1.714$

$1 - \frac{\alpha}{2} = 0.96 \qquad \frac{\alpha}{2} = 0.04 \qquad \alpha = 0.08 \qquad P = 1.92 = 92\%$

c) $p = 0.95 \quad \alpha = 0.05 \qquad u_{1-\frac{\alpha}{2}} = u_{1-0.025} = 0.975$

$0.5 = 0.56 - 1.96 \cdot \sqrt{\frac{0.56 \cdot 0.44}{N}}$

$1.96\sqrt{\frac{0.2464}{N}} = 0.06 \;/ : 1.96$

$\sqrt{\frac{0.2464}{N}} = 0.0306 \;/^2$

$\frac{0.2464}{N} = 0.000937 \qquad N = \frac{0.2464}{0.000937} = 262.96 \approx 293$

4. $a_0 = 35$ $n = 20$ $\alpha = 0,05$ $H_0 \dots \mu = 0,35$

x_i	34,8	34,9	35	35,1	35,3
n_i	2	3	4	6	5

$H_1 \dots \mu \neq 0,35$

$X \dots N(\mu, \sigma^2)$

$$\bar{x} = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i = 35,07$$

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 = 0,0275 \qquad s = 0,1658$$

$$T = \frac{\bar{x} - a_0}{s/\sqrt{n}} = \frac{(\bar{x} - a_0)\sqrt{n}}{s} = \frac{0,07 \cdot \sqrt{20}}{0,1658} = 1,888$$

$n - 1 = 19$ $t_{1-\frac{\alpha}{2}} = 2,093$

$T < t_{1-\frac{\alpha}{2}}$ Hipotezu H_0 ne može se odbaciti

5. $n_1 = 30$ $a_1 = 74$ $\sigma_1 = 8$

$n_2 = 40$ $a_2 = 77$ $\sigma_2 = 7$ $\alpha = 0,05$

$$\sigma_{a_1 - a_2}^2 = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = \frac{64}{30} + \frac{49}{40} = 3,358$$

$$\sigma_{a_1 - a_2} = 1,833$$

$$u = \frac{a_1 - a_2}{\sigma_{a_1 - a_2}} = -\frac{3}{1,833} = -1,636$$

$$u_{1-\frac{\alpha}{2}} = u_{1-\frac{0,05}{2}} = u_{1-0,025} = u_{0,975} = 1,96$$

$|u_{1-\frac{\alpha}{2}}| > |u|$ Ne može se odbaciti pretpostavka da ne postoji bitna razlika u brzinama

6. $n = 190$ 5 kocaka X - št. pojavljanja šestice

x_j	0	1	2	3	4	5
n_j	75	77	30	6	1	1

$\sim B(5, \frac{1}{6})$

x_j	n_j	p_j	$n_j - np_j$	$(n_j - np_j)^2 / np_j$
0	75	0,40188	-1,3572	0,02412
1	77	0.40188	0,6428	0.00541
2	30	0,16075	-0.5425	0,00964
3	6	0.0355	1.255	0,23351
4	1			
5	1			
	190			$\chi^2_g = 0.27268$

$$p_j = \binom{5}{j} \left(\frac{1}{6}\right)^j \left(1 - \frac{1}{6}\right)^{n-j}$$

$$p_0 = \binom{5}{0} \cdot \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^5 = \left(\frac{5}{6}\right)^5 = \frac{3125}{7776} = 0,40188$$

$$p_1 = \binom{5}{1} \cdot \left(\frac{1}{6}\right)^1 \cdot \left(\frac{5}{6}\right)^4 = 5 \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^4 = 0,40188$$

$$p_2 = \binom{5}{2} \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^3 = 10 \cdot \frac{1}{36} \cdot \frac{5 \cdot 5 \cdot 5}{6 \cdot 6 \cdot 6} = \frac{1250}{7776} = 0,16075$$

$$p_3' = \binom{5}{3} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^2 = 10 \cdot \frac{1}{6 \cdot 6 \cdot 6} \cdot \frac{25}{6 \cdot 6} = \frac{250}{7776} = 0.0321$$

$$p_4' = \binom{5}{4} \left(\frac{1}{6}\right)^4 \cdot \left(\frac{5}{6}\right)^1 = 5 \cdot \frac{1}{6^4} \cdot \frac{5}{6} = \frac{25}{7776} = 0.00321$$

$$p_5' = \binom{5}{5} \left(\frac{1}{6}\right)^5 \cdot \left(\frac{5}{6}\right)^0 = \frac{1}{6^5} = 0,00013$$

$$p' = p_3' + p_4' + p_5' = 0,0355$$

$$(n' - np') = -0,091 + 0,3901 + 0,9753 = 1,2664$$

← br. parametara zadani

$k = 4 - 0 - 1 = 3$

br. razreda pročitao u tablici

$\chi^2_{3,\,0{,}05} = 0.352 > 0.27268$

Nivo značajnosti $\alpha = 0.95$