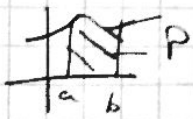
Uvod u dvostruke integrale

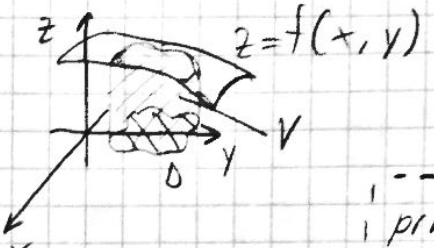
Podsjetimo se: $\int_a^b f(x)\,dx = P$



$$\iint_{(D)} f(x,y)\,dx\,dy = V$$

↘ područje

Volumen → ispod plohe



⇒ bilo koji poredak

$$\int_a^b dx \int_{y_1(x)}^{y_2(x)} f(x,y)\,dy = \int_c^d dy \int_{x_1(y)}^{x_2(y)} f(x,y)\,dx$$

prvo unutarnji, onda vanjski

granice unutarnjeg su funkcije

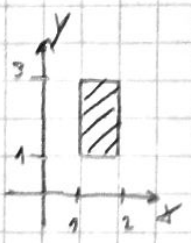
⇒ rješenje je uvijek broj, nema neodređenih

Pr.
$$\int_0^1 dx \int_x^2 (2x + x^2 y)\,dy = \int_0^1 \left(2xy + \frac{1}{2}x^2y^2\right)\Big|_x^2 dx$$

$$= \int_0^1 \left(4x - \cancel{2x^2} + \cancel{2x^2} - \frac{x^4}{2}\right) dx = \left(2x^2 - \frac{x^5}{10}\right)\Big|_0^1 = \ldots = \frac{19}{10}$$

Postavljanje granica

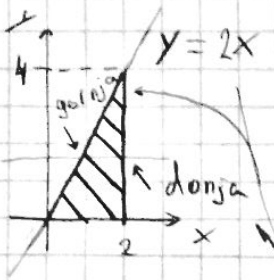
① $D = \{(x,y) : [1,2] \times [1,3]\}$ → pravokutnik



$$\int_1^2 dx \int_1^3 f(x,y)\,dy = \int_1^3 dy \int_1^2 f(x,y)\,dx$$

↘ gornja i donja krivulja $\left.\begin{matrix} y=3 \\ y=1 \end{matrix}\right\}$ u oba slučaja

② $y = 2x$, $y = 0$; $x = 2$

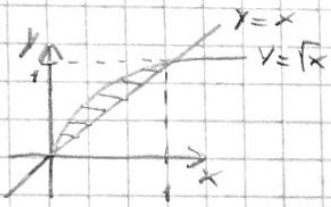


$$\int_0^2 dx \int_0^{2x} f(x,y)\,dy = \int_0^4 dy \int_{\frac{y}{2}}^2 f(x,y)\,dx$$

za drugi poredak

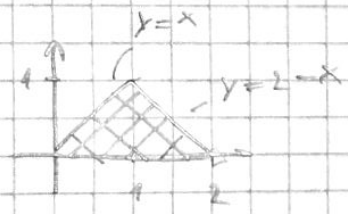
donja granica

③ $y = \sqrt{x}$, $y = x$



$$\int_0^1 dx \int_x^{\sqrt{x}} f(x,y)\,dy = \int_0^1 dy \int_{y^2}^{y} f(x,y)\,dx$$

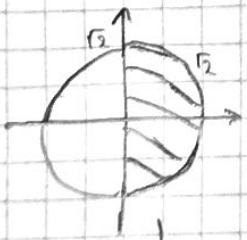
④



$$\int_0^1 dx \int_0^x f(x,y)\,dy + \int_1^2 dx \int_0^{2-x} f(x,y)\,dy$$

$$= \int_0^1 dy \int_y^{2-y} f(x,y)\,dx \quad \leftarrow \text{drugi poredak je povoljniji}$$

⑤ $x^2 + y^2 = 2$, $x > 0$



$$\int_0^{\sqrt{2}} dx \int_{-\sqrt{2-x^2}}^{+\sqrt{2-x^2}} f(x,y)\,dy = \int_{-\sqrt{2}}^{\sqrt{2}} dy \int_0^{\sqrt{2-y^2}} f(x,y)\,dx$$

$y = \pm\sqrt{2-x^2}$

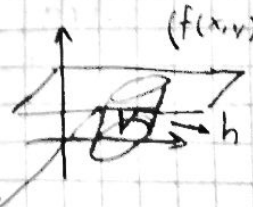
$x = \pm\sqrt{1-y^2}$, ali gleda se samo + grana

Nije površina, nema simetrija

→ ne može se npr. 2x jedan kvadrant

Osim $\iint_{(P)} dx\,dy = P$

$(f(x,y) = 1)$



$V = B \cdot 1 = B$