1.)

x_j	20	21	22	23	24	25
n_j	2	1	3	2	1	1

$n = 2+1+3+2+1+1 = 10$

$\bar{X} = \frac{\sum_{j=1}^{6} n_j x_j}{n} = \frac{1}{10}\cdot(2\cdot 20 + 1\cdot 21 + \dots) = 22.2$

$p = 0,95 \rightarrow \alpha = 1 - p = 0,05$

→ nepoznata σ^2!

$$P\left(\bar{x} - t_{n-1,1-\frac{\alpha}{2}} \cdot \frac{\hat{s}}{\sqrt{n}} \leq a \leq \bar{x} + t_{n-1,1-\frac{\alpha}{2}} \frac{\hat{s}}{\sqrt{n}}\right) = 0,95$$

$t_{9,1-\frac{0,05}{2}} = 2,262$ ← (Kvantili Stud. razdiobe → $n=9$, $\alpha = 0,05$)

$\hat{s}^2 = \frac{1}{9}\left(\sum n_i x_i^2 - n\cdot\bar{x}^2\right) = 2,6222 \rightarrow \hat{s} = 1,61933$

$P(21,04 \leq a \leq 23,36) = 0,95$

$[21,04, 23,36]$

2.)

x_j	-2	-1	0	1	2	3	4	5
n_j	2	1	2	3	2	2	2	1

→ $n = 2+1+2+3+2+2+2+1 = 15$

$p = 0,95 \rightarrow \alpha = 0,05$

$\bar{X} = \frac{\sum_{k=1}^{8} x_j n_j}{15} = 1,4$

$t_{n-1,1-\frac{\alpha}{2}} = t_{14,1-\frac{0,05}{2}} = 2,145$

$s^2 = \frac{1}{14}\left(\sum n_i x_i^2 - n\bar{x}^2\right) = 4,6857 \Rightarrow s = 2,165$

$P(0,2 \leq a \leq 2,599)$

3.)

x_j	0	1	2	3	4
n_j	1	4	6	12	2

$n = 1+4+6+12+2 = 25$

$N(m, 4) \Rightarrow \sigma^2 = 4 \Rightarrow \boxed{\sigma = 2}$ POZNATI σ^2

$\bar{X} = \frac{1}{n}\sum_{j=1}^{n} n_j x_j = \frac{60}{25} = 2.4$

$\mu_{1-\frac{\alpha}{2}} = \mu_{0.95} = 1.64485$ $\quad P\left(\bar{X} - \mu_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} \leq a \leq \mu_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right) = P$

$P(1.742 \leq a \leq 3.058) = 0.9$

$\mu_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} = 0.658$

4.) 6.0, 9.2, 9.8, 9.9, 10.3, 10.3, 10.9, 11.6, 11.8, 12.5, 14.0

$n=11$

$p=0,95 \Rightarrow \alpha=0,05$

$$P\left(\bar{X}-t_{n-1,1-\frac{\alpha}{2}}\frac{S}{\sqrt{n}} \leq a \leq \bar{X}+t_{n-1,1-\frac{\alpha}{2}}\cdot\frac{S}{\sqrt{n}}\right)=p$$

$$\bar{X}=\frac{1}{n}\sum_{j=1}^{n}X_j=\frac{1}{11}\cdot 116,3=10,5\dot{7}\dot{2}$$

$$S^2=\frac{1}{n-1}\sum_{j=1}^{n}(X_j-\bar{X})^2=\frac{1}{10}\cdot 40,258=4,0258 \qquad S=2,00645$$

$$t_{10,1-\frac{0,05}{2}}=2.228$$

$$P\left(10,572-2.228\cdot\frac{2,00645}{\sqrt{10}}\leq a\leq 10.572+2.228\cdot\frac{2,00645}{\sqrt{10}}\right)$$

$$P(9.158 \leq a \leq 11.985)$$

5.) $\sigma=0.5$

$n=8$

16, 16, 16, 16.2, 16.2, 16.2, 16.5, 16.5

$p=90 \Rightarrow \alpha=0,1$

$$\bar{X}=\frac{1}{8}\sum_{j=1}^{8}X_j=16,2$$

$$\mu_{1-\frac{\alpha}{2}}=\mu_{0.95}=1,64485$$

$$P\left(\bar{X}-\mu_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} \leq a \leq \bar{X}+\mu_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right)=p$$

$$\boxed{P(15,91 \leq a \leq 16,49)=0,9}$$

6.) $a=\emptyset$

$\sigma \rightarrow$ NEPOZNATO, $p=0,95 \Rightarrow \alpha=0,05$

$$S^2=\frac{1}{21}\left(\sum_{i=1}^{n}n_i X_i^2-n\bar{X}^2\right)=1,31597 \rightarrow S=1,147$$

X_j	64	65	66	67	68
n_j	3	4	8	5	2

$$t_{21,1-\frac{0,05}{2}}=2,08$$

$n=22$

$$\bar{X}=\frac{1}{n}\sum_{j=0}^{n}n_j X_j=65,955$$

$$P(65,446 \leq a \leq 66.464)=0,95$$

7.)

x_j	110	115	120	125	130	135
n_j	2	3	6	5	2	2

$\rightarrow n = 20$

$p = 0,9 \Rightarrow \alpha = 0,1$

$\bar{X} = \frac{1}{20} \sum_{j=1}^{n} n_j x_j = \underline{122}$

$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (n_i x_i^2 - n\bar{x}^2) = \frac{1}{19} \Sigma (n_i x_i^2 - 20 \cdot 122^2)$

$s^2 = 51,053 \Rightarrow \underline{s = 7,14511}$

$D^2 = \frac{1}{20} \sum_{i=1}^{n} (x_k - \bar{x}) = \underline{21.95}$

$t_{19,\, 1-\frac{0,1}{2}} = \underline{1,729}$

$\boxed{P(119,24 \leq a \leq 124,76) = 0,9}$

8.) $N(a,4) \rightarrow \sigma^2 = 4 \Rightarrow \boxed{\sigma = 2}$

x_j	0	1	2	3	4	5
n_j	1	4	6	10	5	2

$\rightarrow n = 28$

$p = 0,9 \Rightarrow \alpha = 0,1$

$\bar{X} = \frac{1}{28} \sum_{j=1}^{6} n_j x_j = \underline{2,714}$

$P\left(\bar{X} - \mu_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \leq a \leq \bar{X} + \mu_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}\right) = 0,9$

$\mu_{1-\frac{\alpha}{2}} = \mu_{0,95} = 1,64485$

$\boxed{P(2,092 \leq a \leq 3,336) = 0,9}$

9.) $N(a,3) \Rightarrow \sigma^2 = 3 \Rightarrow \underline{\sigma = \sqrt{3}}$

x_j	0	1	2	3	4
n_j	1	5	8	10	3

$\rightarrow n = 27$

$p = 0.9 \Rightarrow \underline{\alpha = 0,1}$

$\bar{X} = \frac{1}{27} \sum_{i=1}^{5} n_i x_i = \underline{2,333}$

$\mu_{1-\frac{0,1}{2}} = \mu_{0,95} = 1,64485$

interval: $\underline{[1,785,\ 2.881]}$

10.) $n = 200$, $m = 110$ $\Big\} \hat{p} = \frac{m}{n} = 0,55$

b) $p = 0,99 \Rightarrow \alpha = 0,01$

$\mu_{1-\frac{\alpha}{2}} = \mu_{0.995} = 2.57583$

$p_{1,2} = \hat{p} \pm \mu_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0,55 \pm 0.09$

$P(p_1 \leq p \leq p_2) = 0.99$

$\boxed{P(0,46 \leq p \leq 0.64) = 0,99}$

d) $p = 0,95 \Rightarrow \alpha = 0,05$

$\mu_{1-\frac{\alpha}{2}} = \mu_{0.975} = 1.95996$

$n = \frac{\hat{p}(1-\hat{p})}{\left(\frac{p-0.5}{\mu_{1-\frac{\alpha}{2}}}\right)^2} = \frac{0.55 \cdot 0.45}{\left(\frac{0.05}{1.95996}\right)^2} = 380$

SLUŽBENA RJEŠENJA:

§11. Intervalne procjene

1. $21{,}04 \leqslant a \leqslant 23{,}36$
2. $0.2 \leqslant a \leqslant 2.6$
3. $1.742 < a < 3.058$
4. $\chi^2 = 7.54\,,\ \chi^2_\alpha = 11.07\,,$ ne može se odbaciti
5. $\overline{x} = 16.2\,,\ 15.91 < a < 16.49$
6. $\overline{x} = 65.955\,,\ 65.434 < a < 66.475$
7. $\overline{x} = 122\,;\ 118.7 < a < 125.3$
8. $\overline{x} = 2.714\,,\ [2.092\,, 3.336]$
9. $1.79 \leqslant a \leqslant 2.88$

KVANTILI STUDENTOVE RAZDIOBE:

Tablica 3. Kvantili Studentove razdiobe $t_{1-\alpha/2}$

	nivo značajnosti α						
n	0.90	0.8	0.70	0.60	0.50	0.40	0.30
1	0.158	0.325	0.510	0.727	1.000	1.376	1.963
2	0.142	0.289	0.445	0.617	0.816	1.061	1.386
3	0.137	0.277	0.424	0.584	0.765	0.978	1.250
4	0.134	0.271	0.414	0.569	0.741	0.941	1.190
5	0.132	0.267	0.408	0.559	0.727	0.920	1.156
6	0.131	0.265	0.404	0.553	0.718	0.906	1.134
7	0.130	0.263	0.402	0.549	0.711	0.896	1.119
8	0.130	0.262	0.399	0.546	0.706	0.889	1.108
9	0.129	0.261	0.398	0.543	0.703	0.883	1.100
10	0.129	0.260	0.397	0.542	0.700	0.879	1.093
11	0.129	0.260	0.396	0.540	0.697	0.876	1.088
12	0.128	0.259	0.395	0.539	0.695	0.873	1.083
13	0.128	0.259	0.394	0.538	0.694	0.870	1.079
14	0.128	0.258	0.393	0.537	0.692	0.868	1.076
15	0.128	0.258	0.393	0.536	0.691	0.866	1.074
16	0.128	0.258	0.392	0.535	0.690	0.865	1.071
17	0.128	0.257	0.392	0.534	0.689	0.863	1.069
18	0.127	0.257	0.392	0.534	0.688	0.862	1.067
19	0.127	0.257	0.391	0.533	0.688	0.861	1.066
20	0.127	0.257	0.391	0.533	0.687	0.860	1.064
21	0.127	0.257	0.391	0.532	0.686	0.859	1.063
22	0.127	0.256	0.390	0.532	0.686	0.858	1.061
23	0.127	0.256	0.390	0.532	0.685	0.858	1.060
24	0.127	0.256	0.390	0.531	0.685	0.857	1.059
25	0.127	0.256	0.390	0.531	0.684	0.856	1.058
26	0.127	0.256	0.390	0.531	0.684	0.855	1.058
27	0.127	0.256	0.389	0.531	0.684	0.855	1.057
28	0.127	0.256	0.389	0.530	0.683	0.855	1.056
29	0.127	0.256	0.389	0.530	0.683	0.854	1.055
30	0.127	0.256	0.389	0.530	0.683	0.854	1.055
40	0.126	0.255	0.388	0.529	0.681	0.851	1.050
60	0.126	0.254	0.387	0.527	0.679	0.848	1.046
120	0.126	0.254	0.386	0.526	0.677	0.845	1.041
$u_{1-\alpha/2}$	0.126	0.253	0.385	0.524	0.674	0.842	1.036

Tablica 3. Kvantili Studentove razdiobe $t_{1-\alpha/2}$

	nivo značajnosti α						
n	0.20	0.10	0.05	0.02	0.01	0.005	0.001
1	3.078	6.314	12.706	31.821	63.657	127.32	639.619
2	1.886	2.920	4.303	6.965	9.925	14.09	31.598
3	1.638	2.353	3.182	4.541	5.841	7.45	12.941
4	1.533	2.132	2.776	3.747	4.604	5.60	8.610
5	1.476	2.015	2.571	3.365	4.032	4.77	6.859
6	1.440	1.943	2.447	3.143	3.707	4.32	5.959
7	1.415	1.895	2.365	2.998	3.499	4.03	5.405
8	1.397	1.860	2.306	2.896	3.355	3.83	5.041
9	1.383	1.833	2.262	2.821	3.250	3.69	4.781
10	1.372	1.812	2.228	2.764	3.169	3.58	4.587
11	1.363	1.796	2.201	2.718	3.106	3.50	4.437
12	1.356	1.782	2.179	2.681	3.055	3.43	4.318
13	1.350	1.771	2.160	2.650	3.012	3.37	4.221
14	1.345	1.761	2.145	2.624	2.977	3.33	4.140
15	1.341	1.753	2.131	2.602	2.947	3.29	4.073
16	1.337	1.746	2.120	2.583	2.921	3.25	4.015
17	1.333	1.740	2.110	2.567	2.898	3.22	3.965
18	1.330	1.734	2.101	2.552	2.878	3.20	3.922
19	1.328	1.729	2.093	2.539	2.861	3.17	3.883
20	1.325	1.725	2.086	2.528	2.845	3.15	3.850
21	1.323	1.721	2.080	2.518	2.831	3.14	3.819
22	1.321	1.717	2.074	2.508	2.819	3.12	3.792
23	1.319	1.714	2.069	2.500	2.807	3.10	3.767
24	1.318	1.711	2.064	2.492	2.797	3.09	3.745
25	1.316	1.708	2.060	2.485	2.787	3.08	3.725
26	1.315	1.706	2.056	2.479	2.779	3.07	3.707
27	1.314	1.703	2.052	2.473	2.771	3.06	3.690
28	1.313	1.701	2.048	2.467	2.763	3.05	3.674
29	1.311	1.699	2.045	2.462	2.756	3.04	3.659
30	1.310	1.697	2.042	2.457	2.750	3.03	3.646
40	1.303	1.684	2.021	2.423	2.704	2.97	3.551
60	1.296	1.671	2.000	2.390	2.660	2.91	3.460
120	1.289	1.658	1.980	2.358	2.617	2.86	3.373
$u_{1-\alpha/2}$	1.282	1.645	1.960	2.326	2.576	2.81	3.291

KVANTILI STANDARDNE NORMALNE RAZDIOBE:

Tablica 4. Kvantili standardne normalne razdiobe u_p

p	u_p	p	u_p	p	u_p
0.55	0.12566	0.92	1.40507	0.994	2.51214
0.60	0.25335	0.93	1.47579	0.995	2.57583
0.65	0.38532	0.94	1.55477	0.996	2.65207
0.70	0.52440	0.95	1.64485	0.997	2.74778
0.75	0.67449	0.96	1.75069	0.9975	2.80703
0.80	0.84162	0.97	1.88079	0.998	2.87816
0.82	0.91537	0.975	1.95996	0.999	3.09023
0.84	0.99446	0.98	2.05375	0.9995	3.29053
0.86	1.08032	0.99	2.32635	0.9999	3.71901
0.88	1.17499	0.991	2.36562	0.99995	3.89059
0.90	1.28155	0.992	2.40892	0.99999	4.26489
0.91	0.34076	0.993	2.45726	0.999999	4.75342

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