MI 2018

① $P(A) = \frac{1}{4}$

$P(A \cup B) = \frac{1}{3}$

$P(B) = ?$

a) $A \to B$

$P(A \cup B) = P(B) = \frac{1}{3}$ //

b) A, B disjunkti

$P(B) = P(A \cup B) - P(A) = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$ //

c) A, B nezavisni

$P(A \cup B) = P(A) P(B)$

$= P(A) - P(B) - P(AB)$

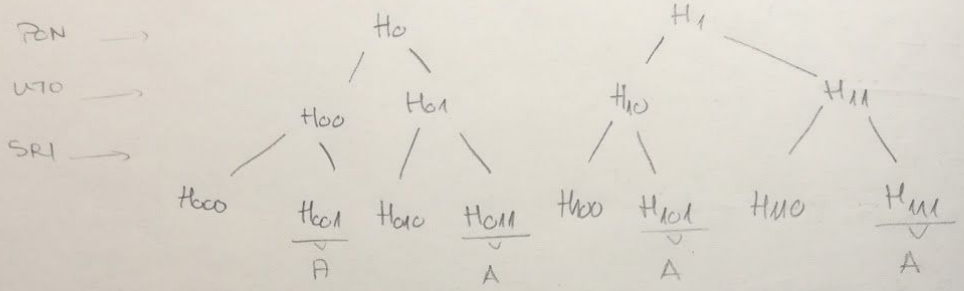
$= P(A) + P(B) - P(A) P(B)$

$= P(A) + P(B) \cdot (1 - P(A))$

$\Downarrow$

$P(B) = \frac{P(A \cup B) - P(A)}{1 - P(A)} = \frac{\frac{1}{3} - \frac{1}{4}}{1 - \frac{1}{4}} = \frac{1}{9}$ //

② a) A - {u odjedu izvučena čista čarapa}



$$P(A) = P(H_{001}) + P(H_{011}) + P(H_{101}) + P(H_{111})$$

$$= \left(\frac{4}{10}\right)^2 \cdot \frac{6}{10} + \frac{4}{10} \cdot \frac{6}{10} \cdot \frac{5}{10} + \frac{6}{10} \cdot \left(\frac{5}{10}\right)^2 + \frac{6}{10} \cdot \frac{5}{10} \cdot \frac{4}{10}$$

$$= 0.486 = 48.6\%$$

b) B - {čiste čarape izvučene sva 3 dana} $\rightarrow H_{111}$

$$P(B|A) = \frac{P(B)}{P(A)} = \frac{P(H_{111})}{P(A)} = \frac{\frac{6}{10} \cdot \frac{5}{10} \cdot \frac{4}{10}}{0.486} = 0.247 = 24.7\%$$

③ a) $\overline{P} = 0.064$ (3 pokušaja)

$P = ?$

$\overline{P} = (1-P)(1-P)(1-P)$

$0.064 = (1-P)^3$

$1 - P = \frac{2}{5}$

$P = 0.6$

b) $P = 0.6$ → ne ovisi o prijašnjim bacanjima

c) $y \sim \begin{pmatrix} 2 & 3 & 4 & \cdots & n \\ p^2 & p^2(1-p)\cdot 2 & p^2(1+p)^2 \cdot 3 & \cdots & p^2(1-p)^{n-2}(n-1) \end{pmatrix}$

$\sum x^n = \frac{1}{1-x} \quad / \frac{d}{dx}$

$\sum n x^{n-1} = \frac{1}{(1-x)^2} \quad / \frac{d}{dx}$

$\sum n(n-1)x^{n-2} = \frac{2(1-x)}{(1-x)^4}$

$= \frac{2-2x}{(1-x)^4}$

$E(y) = np^2(1-p)^{n-2}(n-1)$

$= p^2 \cdot \frac{2-2(1-p)}{(1-1+p)^{p^2}} = \frac{2p}{p^2} = \frac{2}{p} = \frac{2}{0.6} = 3.33$

4. $X \sim \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ c & 2c & 2c & 3c & c^2 & 2c^2 & 7c^2+c \end{pmatrix}$

a) $c = ?$

$c + 2c + 2c + 3c + c^2 + 2c^2 + 7c^2 + c = 1$

/* kalkulator */ $\rightarrow \boxed{c = \frac{1}{10}}$

b) $P(x \in k) \geq \frac{2}{5}$, $k = ?$

$P(x < 0) = 0$

$P(x \leq 1) = \frac{1}{10} < \frac{2}{5}$

$P(x < 2) = \frac{1}{10} + \frac{1}{5} = \frac{3}{10} < \frac{2}{5}$

$P(x \leq 3) = \frac{1}{10} + \frac{1}{5} + \frac{1}{5} = \frac{1}{2} > \frac{2}{5} \rightarrow \boxed{k = 3}$

c) $E(y) = ?$, $D(y) = ?$ $\rightarrow y = \frac{1}{x}$

$\frac{1}{x} = y \sim \begin{pmatrix} 1 & \frac{1}{2} & \cdots & \frac{1}{7} \\ c & 2c & \cdots & 7c^2 + c \end{pmatrix}$

$E(y) = 1c + \frac{1}{2} \cdot 2c + \cdots \frac{1}{7}(7c^2 + c) = \underline{0.371}$

$D(y) = E(y^2) - (E(y))^2$

$= 0.195 - 0.371^2$

$= \underline{0.0578}$

⑤ 3 novčića

X - {broj palih glava} $\rightarrow$ 0, 1, 2 ili 3

Y - {parnost palih pisama} $\rightarrow$ 2 - parna, 1 - inače

a) Razdioba? (0 - pismo, 1 - glava)

1	2	3	X	Y
0	0	0	0	1
0	0	1	1	2
0	1	0	1	2
0	1	1	2	1
1	0	0	1	2
1	0	1	2	1
1	1	0	2	1
1	1	1	3	2

8 slučajeva

X \ Y	1	2	
0	$\frac{1}{8}$		$\frac{1}{8}$
1		$\frac{3}{8}$	$\frac{3}{8}$
2	$\frac{3}{8}$		$\frac{3}{8}$
3		$\frac{1}{8}$	$\frac{1}{8}$
	$\frac{4}{8}$	$\frac{4}{8}$	$\Sigma = 1$

b) $\tau(X,Y) = ?$

$\tau(X,Y) = \frac{E(XY) - E(X)E(Y)}{\sqrt{D(X)D(Y)}}$

$= \frac{\frac{9}{4} - \frac{3}{2} \cdot \frac{3}{2}}{\sqrt{\frac{3}{4} \cdot \frac{1}{4}}}$

$= \frac{0}{\sqrt{\frac{3}{4} \cdot \frac{1}{4}}} = 0$

$X \sim \begin{pmatrix} 0 & 1 & 2 & 3 \\ \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \end{pmatrix}$

$E(X) = \frac{3}{2}$

$D(X) = \frac{3}{4}$

$Y \sim \begin{pmatrix} 1 & 2 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

$E(Y) = \frac{3}{2}$

$D(Y) = \frac{1}{4}$

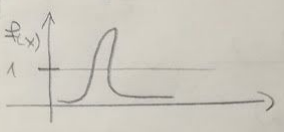
$XY \sim \begin{pmatrix} 0 & 2 & 6 \\ \frac{1}{8} & \frac{6}{8} & \frac{1}{8} \end{pmatrix}$

$E(XY) = \frac{9}{4}$

c) $P(y>x \mid y=1) = \frac{P(y>x,\ y=1)}{P(y=1)} = \frac{\frac{1}{8}}{\frac{4}{8}} = \frac{1}{4}$

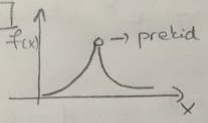
⑥ a) Fja gustoće sl. varijable ne može biti >1

NETOČNO



b) Fja gustoće mora biti neprekinuta

NETOČNO



c) Za fju gustoće vrijedi $\lim_{x\to\pm\infty} f(x) = 0$

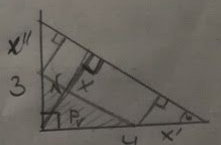
TOČNO

$\Rightarrow$ mora vrijediti $\int_{-\infty}^{+\infty} f(x)\,dx = 1$,

a ako je $\lim_{x\to\pm\infty} f(x) = a$; $a \neq 0$, integral divergira.

⑦ X - {udaljenost od $T(x,y)$ do hipotenuze}

$F(x) = ?$ $\quad a = 3$ $\quad b = 4$



$P = \frac{ab}{2} = 6$

$x \in [0, r]$

/* trigonometrija */

$r = 2.4$

$x \in [0, 2.4]$

$\frac{x'}{x} = \frac{5}{3}$

$x' = \frac{5x}{3}$

$x'' = \frac{5x}{4}$

$P_V = \frac{(3-x'')(4-x')}{2}$

$= \frac{\left(3-\frac{5x}{4}\right)\left(4-\frac{5x}{3}\right)}{2}$

$= \frac{12 - 5x - 5x + \frac{25x^2}{12}}{2} = 6 - 5x + \frac{25x^2}{24}$

$P_X = P - P_V = 6 - 6 + 5x + \frac{25x^2}{24} = 5x + \frac{25^2}{24}$

$F(x) = \frac{P_V}{P} = \frac{5x - \frac{25x^2}{24}}{6} = \frac{5x}{6} - \frac{25x^2}{144}$

(8) Eksponencijalna razdioba

$P(X>1) = \frac{1}{e^X}$ $\longrightarrow$ $F(1) = 1 - e^{-\lambda \cdot 1} = \frac{1}{e^2}$

$e^{-\lambda} = 1 - \frac{1}{e^2} \quad / \cdot e^{\lambda}$

$\lambda = 2$

$P(|X-1|<2) = ?$

$P(|X-1|<2) = P(-2<X-1<2)$

$= P(-1<X<3)$

$= P(0<X<3)$ $\longrightarrow$ e^X ide od 0 do $+\infty$

$= F(3) - F(0)$

$= 1 - e^{-3 \cdot 2} - \underbrace{1 - e^{-0 \cdot 1}}_{=0} = 1 - e^{-6}$

$= 0.9975$