) 15 kuglica → označene brojevima $\{1, 2, \dots, 15\}$ | MI 2016 |

⇒ izvlačimo 5 kuglica

a) A – {izvukli točno 2 parna broja}

$$P(A) = \frac{\binom{7}{2}\binom{8}{3}}{\binom{15}{5}} = 0.39$$

b) B – {izvukli bar 2 parna}

$$P(B) = 1 - P(\overline{B}) = 1 - \frac{\binom{7}{1}\binom{8}{4} + \binom{8}{5}}{\binom{15}{5}} = 0.82$$

c) C – {zbroj svih brojeva je paran}

$$\left.\begin{array}{l} N+N=P \\ P+P=P \\ N+P=N \end{array}\right\} \to \begin{array}{l} P+P+P+P+P=P \\ P+P+P+\underbrace{N+N}_{P}=P \\ P+\underbrace{N+N}_{P}+\underbrace{N+N}_{P}=P \end{array}$$

$$P(C) = \frac{\binom{7}{5} + \binom{7}{3}\binom{8}{2} + \binom{7}{1}\binom{8}{4}}{\binom{15}{5}} = 0.4965$$

d) D – {zbroj 2 najveća broja >26}

$$\left.\begin{array}{l} 15+12 \\ 15+13 \\ 15+14 \\ 14+13 \end{array}\right\} > 26 \qquad P(D) = \frac{\binom{13}{3} + 2\binom{12}{3} + \binom{11}{3}}{\binom{15}{3}} = 0.2967$$

2. $P(\text{pogotka}) = 0.4$ → za sva 4 topa

$P(A|H_1) = 0.3$
$P(A|H_2) = 0.6$
$P(A|H_3) = 0.8$
$P(A|H_4) = 0.9$
} vjerojatnost uništenja cilja ovisno o broju topova koji ga pogode

A – {cilj je uništen}
H_i – {pogodio ga je i top}

$P(H_1|A) = ?$

$$P(A) = P(H_1) + P(H_2) + P(H_3) + P(H_4)$$
$$= 4\cdot 0.3 \cdot 0.4 \cdot 0.6^3 + 6 \cdot 0.6 \cdot 0.4^2 \cdot 0.6^2 + 4 \cdot 0.8 \cdot 0.6 \cdot 0.4^3 + 0.9 \cdot 0.4^4$$
$$= 0.45696 = 45.696\%$$

$$P(H_1|A) = \frac{P(H_1A)}{P(A)} = \frac{4\cdot 0.3\cdot 0.4\cdot 0.6^3}{0.45696} = 0.2269 = 22.69\% \,//$$

3. $P(X = 2^k) = 6c^{-k}$; $k = 2, 3, \dots$

$c = ?$
$E(X) = ?$
$P(X \geq 5) = ?$

$$X \sim \begin{pmatrix} 4 & 8 & \dots & 2^n \\ \frac{6}{c^2} & \frac{6}{c^3} & \dots & \frac{6}{c^n} \end{pmatrix}$$

$$6\left(\frac{1}{c^2} + \frac{1}{c^3} + \dots + \frac{1}{c^n}\right) = 1$$

$$6\sum_{n=2}^{\infty} \frac{1}{c^n} = 1 \longrightarrow \sum_{n=1}^{\infty} \frac{1}{x^n} = \frac{1}{x-1}$$

$$\sum_{2}^{\infty} \frac{1}{c^n} = \frac{1}{6} \qquad \sum_{n=2}^{\infty} \frac{1}{x^{n+1}} = \sum_{n=2}^{\infty} \frac{1}{x^n \cdot x}$$

$$\frac{1}{c(c-1)} = \frac{1}{6} \qquad x \sum_{2}^{\infty} \frac{1}{x^n} = \frac{1}{x-1}\cdot\frac{1}{x}$$

$$1 = \frac{1}{6}(c^2 - c) \qquad \sum_{2}^{\infty} \frac{1}{x^n} = \frac{1}{x(x-1)}$$

$$\frac{1}{6}c^2 - \frac{1}{6}c - 1 = 0 \qquad |x| > 1+1$$

$$\boxed{c = 3} \,// \qquad \cancel{c = -2} \qquad |x| > 2$$

⑥ Može li ...

a) Funkcija razdiobe

b) Funkcija gustoće

... biti strogo padajuća na nekom intervalu?

=> a) NE

=> Vrijedi monotonost vjerojatnosti:

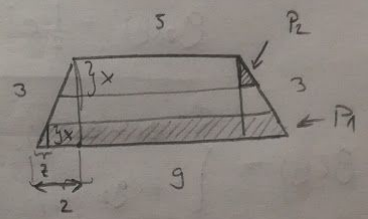
$A, B \in F$ tako da $A \subseteq B \longrightarrow P(A) \leq P(B)$

=> b) DA

=> npr. fja. eksponencijalne razdiobe

⑦ JEDNAKOKRAČNI TRAPEZ

$a = 9$
$b = 3$
$c = 5$



$X \sim \{$udaljenost (x, y) do bliže osnovice$\}$

$X \sim ?$

$P(X \geq 1) = ?$

$F(x) = \dfrac{P_1 + P_2}{P(\square)}$

$P(\square) = \left(\dfrac{9+5}{2}\right) \cdot 3 = 21$

$\dfrac{3}{2} = \dfrac{x}{z}$

$3z = 2x$

$z = \dfrac{2}{3}x$

$P_1 = (9 - 2z)x + 2 \cdot \dfrac{x \cdot z}{2}$

$= \left(9 - \dfrac{4}{3}x\right)x + \dfrac{2}{3}x^2 = 9x - \dfrac{4}{3}x^2 + \dfrac{2}{3}x^2$

$= 9x - \dfrac{2}{3}x^2$

$P_2 = 5x + 2 \cdot \dfrac{x \cdot z}{2}$

$P_2 = 5x + \dfrac{2}{3}x^2$

$F(x) = \dfrac{14x}{21}$

$P_1 - P_2 = 9x - \cancel{\dfrac{2}{3}x^2} + 5x + \cancel{\dfrac{2}{3}x^2} = 14x$

$F(x) = \dfrac{2}{3}x$ //

$P(X \geq 1) = 1 - P(X < 1)$

$= 1 - F(1) = 1 - \dfrac{2}{3}$

$= \dfrac{1}{3}$ //

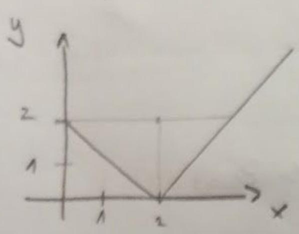
⑧ $E(x)=2$ → eksp. razdioba

$y=|2-x|$

$E(y)=?$

$$E(x)=\frac{1}{\lambda}$$

$$2=\frac{1}{\lambda x} \Rightarrow \lambda x=\frac{1}{2}$$



$y=|2-x|$

$\langle 0,2\rangle \Rightarrow x=2-y$

$$g(x)=f(x)\cdot\left|\frac{dv}{dy}\right| \qquad \left|\frac{dx}{dy}\right|=1$$

$$g_1(y)=\frac{1}{2}e^{-\frac{1}{2}(2-y)}$$

$[2,+\infty)$: $x=y+2$

$$\left|\frac{dv}{dy}\right|=1$$

$$g_2(y)=\frac{1}{2}e^{-\frac{1}{2}(2+y)}$$

$$g(y)=\begin{cases} g_1+g_2 & ; \ 0\le y\le 2 \\ g_2 & ; \ y>2 \end{cases}$$

$$E(y)=\int_0^{+\infty} y\,g(y)\,dy$$

$$=\int_0^2 y\,g_1(y)\,dy+\int_0^2 y\,g_2(y)\,dy+\int_2^{+\infty} g_2(y)\,dy$$

$$=\int_0^2 y\frac{1}{2}e^{-\frac{1}{2}(2-y)}\,dy+\int_0^2 y\frac{1}{2}e^{-\frac{1}{2}(2+y)}\,dy$$

$$=2e^{-1}+2e^{-1}$$

$$=\frac{4}{e}$$