1.) n kuglica
1 bijela kuglica

kuglica koja nije bijela
kuglica koja je bijela

$$X \sim \begin{pmatrix} 1 & 2 & & n \\ \frac{1}{n} & \frac{n-1}{n}\cdot\frac{1}{n-1} & \cdots\cdots & \frac{1}{n} \end{pmatrix} \leftarrow \text{pokušaji}$$

2.) $1,2,\dots,n$ → izvlačimo 3 broja
X → prima vrijednost najvećeg od ta 3 broja

$$X \sim \begin{pmatrix} 3 & 4 & & n \\ \frac{1}{\binom{n}{3}} & \frac{3}{\binom{n}{3}} & \cdots & \frac{\binom{n-1}{2}}{\binom{n}{3}} \end{pmatrix}$$

samo 123   124, 234, 134 → nije bitan poredak

4.)
$$X \sim \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ 0{,}2 & 0{,}1 & 0{,}4 & 0{,}1 & 0{,}2 \end{pmatrix}$$

$E(X) = ?$
$D(X) = ?$

$$E(X) = \sum_k x_k p_k = -2\cdot 0{,}2 + (-1)\cdot 0{,}1 + 0\cdot 0{,}4 + 1\cdot 0{,}1 + 2\cdot 0{,}2$$

$$\underline{\underline{E(X) = \emptyset}}$$

$$D(X) = \sum_k x_k^2 p_k - E(X)^2 = 4\cdot 0{,}2 + 0{,}1 + 0{,}1 + 4\cdot 0{,}2 = \underline{\underline{1{,}8}}$$

5.)
$X \sim x = (n-1)^2, n \in N$
$P(X = (n-1)^2) = \frac{2^{-n}}{n\ln 2}$

$$E(X) = (1-1)^2\frac{2^{-1}}{1\ln 2} + (2-1)^2\frac{2^{-2}}{2\ln 2} + \dots = (n-1)^2\cdot\frac{2^{-n}}{n\ln 2}$$

$E(X) = ?$

$$E(X) = \sum_{n=1}^{n} (n-1)^2 \frac{2^{-n}}{n\ln 2} = \sum_{n=1}^{n} (n^2 - 2n + 1)\cdot\frac{1}{n}\cdot\frac{2^{-n}}{\ln 2} =$$

$$E(X) = \frac{1}{\ln 2}\left[\sum_{n=1}^{n} \frac{n^2}{n}\left(\frac{1}{2}\right)^n - 2\sum_{n=1}^{n} \frac{n}{n}\left(\frac{1}{2}\right)^n + \sum_{n=1}^{n} \frac{1}{n}\cdot\left(\frac{1}{2}\right)^n\right]$$
① ② ③

① $\sum_{n=1}^{n} x^n = \frac{1}{1-x} \Big/ '$
$= n\cdot x^{n-1} = \frac{1}{(1-x)^2} \Big/ \cdot x$
$= n\cdot x = \frac{x}{(1-x)^2}$

② $\sum_{n=1}^{n} x^n = \frac{1}{1-x}$

③ $\sum_{n=0}^{n} x^n = \frac{1}{1-x} \Big/ \int = \sum_{n=0}^{n} \frac{x^{n+1}}{n+1} = -\ln(1-x) = \sum_{\boxed{n=1}}^{n} \frac{x^n}{n} = -\ln(1-x)$

$$\Rightarrow E(X) = \frac{1}{\ln 2}\cdot\left[\frac{\frac{1}{2}}{(1-\frac{1}{2})^2} - 2\cdot\frac{\frac{1}{2}}{1-\frac{1}{2}} - \ln\left(1-\frac{1}{2}\right)\right]$$

$+\ln(2)^{-1}$

$$E(X) = \frac{1}{\ln 2}\left[2 - 2 + \ln 2\right] = \underline{\underline{1}} \quad *$$

(u rješenjima je $1 + \frac{2}{\ln 2}$)

6.)
$$X \sim \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 7 & 8 & 9 & 10 & 11 & 12 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6^2} & \frac{1}{6^2} & \frac{1}{6^2} & \frac{1}{6^2} & \frac{1}{6^2} & \frac{1}{6^2} \end{pmatrix}$$

$$E(X) = \sum_i x_i p_i = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + \ldots + 12 \cdot \frac{1}{6^2} = \underline{4{,}08}$$

$$D(X) = E(X^2) - [E(X)]^2 = \underline{8{,}04}$$

$$\sigma_X = \sqrt{D(X)} = \sqrt{8{,}04} = \underline{2{,}83}$$

7.)
$$X \sim \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ p_0 & p_1 & p_2 & p_3 & p_4 & p_5 \end{pmatrix} \leftarrow \text{broj jedinica}$$

$$p_0 = \frac{5^5}{6^5} \cdot \frac{5^5}{6^5}, \quad p_1 = \frac{\binom{5}{1} \cdot 5^4}{6^5} \cdot \frac{5^4}{6^4} + \frac{5^5}{6^5} \cdot \frac{\binom{5}{1} \cdot 5^4}{6^5}, \quad p_2 = \frac{\binom{5}{2} \cdot 5^3}{6^5} \cdot \frac{5^3}{6^3} + \frac{\binom{5}{1} 5^4}{6^5} \cdot \frac{\binom{4}{1} 5^3}{6^5} + \frac{5^5}{6^5} \frac{\binom{5}{2} \cdot 5^3}{6^5}$$

$$p_3 = \frac{\binom{5}{3} 5^2}{6^5} \cdot \frac{5^{-2}}{6^2} + \frac{\binom{5}{2} 5^3}{6^5} \cdot \frac{\binom{3}{1} 5^2}{6^3} + \frac{\binom{5}{1} \cdot 5^4}{6^5} \cdot \frac{\binom{4}{2} 5^2}{6^4} + \frac{5^5}{6^5} \cdot \frac{\binom{5}{3} 5^{-2}}{6^5}$$

$$p_4 = \frac{\binom{5}{4} 5}{6^5} \cdot \frac{5}{6} + \frac{\binom{5}{3} 5^2}{6^5} \cdot \frac{\binom{2}{1} 5}{6^2} + \frac{\binom{5}{2} 5^3}{6^5} \cdot \frac{\binom{3}{2} 5}{6^3} + \frac{\binom{5}{1} \cdot 5^4}{6^5} \cdot \frac{\binom{4}{3} 5}{6^4} + \frac{5^5}{6^5} \cdot \frac{\binom{5}{4} \cdot 5}{6^5}$$

$$p_5 = \frac{\binom{5}{5}}{6^5} + \frac{\binom{5}{4} 5}{6^5} \cdot \frac{1}{6} + \frac{\binom{5}{3} 5^2}{6^5} \cdot \frac{1}{6^2} + \frac{\binom{5}{2} 5^3}{6^5} \frac{1}{6^3} + \frac{\binom{5}{1} 5^4}{6^5} \cdot \frac{1}{6^4} + \frac{5^5}{6^5} \cdot \frac{1}{6^5}$$

$$E(X) = \sum_{i=0}^{5} x_i p_i = \underline{1{,}5277}$$

8.)
$$X \sim \begin{pmatrix} 1 & 2 & \ldots & n \\ \frac{1}{n^2} & & & \frac{(2n+1)}{n^2} \end{pmatrix}$$

$$E(X) = \sum_{k=0}^{n} \frac{k(2k+1)}{n^2} = \frac{1}{n^2} \cdot \left(\sum_{k=0}^{n} k + 2 \sum_{k=0}^{n} k^2 \right)$$

$$② \sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$① \sum_{k=0}^{n} k = 1 + 2 + \ldots + n = \frac{(n+1)n}{2}$$

$$E(X) = \frac{1}{n^2} \cdot \left(\frac{(n+1)n}{2} + \frac{\cancel{2} \cdot n(n+1)(2n+1)}{\cancel{6}\,3} \right) = \frac{1}{n^{\cancel{2}}} \cdot \frac{3\cancel{n}(n+1) + 2\cancel{n}(n+1)(2n+1)}{6}$$

$$= \frac{3(n+1) + (2n+2)(2n+1)}{6n} = \frac{3n + 3 + 4n^2 + 2n + 4n + 2}{6n} = \frac{4n^2 + 9n + 5}{6n} = \underline{\frac{(n+1)(4n+5)}{6n}}$$

9.) 6 CRVENIH } 10 KUGLICA
4 PLAVE

→ na sreću 3 kuglice

$$X \sim \begin{pmatrix} 0 & 1 & 2 & 3 \\ \frac{\binom{6}{3}}{\binom{10}{3}} & \frac{3\binom{6}{2}\binom{4}{1}}{\binom{10}{3}} & \frac{\binom{3}{2}\binom{6}{1}\binom{4}{2}}{\binom{10}{3}} & \frac{\binom{4}{3}}{\binom{10}{3}} \end{pmatrix} \leftarrow \text{plave kug.}$$

$$E(X) = 0 + 1{,}5 + 0{,}9 + 0{,}0\dot{3} = \underline{2{,}4\dot{3}}$$ *(u rješenjima je 1,2)

10.) 5 KLJUČEVA

$$X \sim \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ \frac{1}{5} & \frac{4}{5}\cdot\frac{1}{\cancel{4}} & \frac{4}{5}\cdot\frac{3}{4}\cdot\frac{1}{3} & \frac{4}{5}\cdot\frac{3}{4}\cdot\frac{2}{3}\cdot\frac{1}{2} & \frac{4}{5}\cdot\frac{3}{4}\cdot\frac{2}{3}\cdot\frac{1}{2}\cdot\frac{1}{1} \end{pmatrix} \leftarrow \text{broj pokušaja}$$

$$E(X) = \frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5} + 1 = \underline{\underline{3}}$$

očekivani broj pokušaja je 3 puta

11.) 6 ŽARULJA
2 ISPRAVNE, 4 NEISPRAVNE

$$X \sim \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ p_1 & p_2 & p_3 & p_4 & p_5 \end{pmatrix}$$

$p_1 = \frac{2}{6}$ ← odmah ispravna, $p_2 = \frac{4}{6}\cdot\frac{2}{5} = \frac{8}{30}$, $p_3 = \frac{4}{6}\cdot\frac{3}{5}\cdot\frac{2}{4} = \frac{24}{120}$,

$p_4 = \frac{4}{6}\cdot\frac{3}{5}\cdot\frac{2}{4}\cdot\frac{2}{3} = \frac{16}{120}$, $p_5 = \frac{4}{6}\cdot\frac{3}{5}\cdot\frac{2}{4}\cdot\frac{1}{3}\cdot\frac{2}{2} = \frac{8}{120}$

$$E(X) = \sum_k x_k \cdot p_k = \frac{7}{3} = \underline{2{,}\dot{3}}$$

12.) b) $E(X) = a$

$P(X=k) = ?$

$$E(X) = \sum_{k=0}^{\infty} k \cdot p(1-p)^k = p\sum_{k=0}^{\infty} k(1-p)^k$$

$$\boxed{\begin{aligned} &\sum x^n = \frac{1}{1-x} \quad /' \\ &\sum n x^{n-1} = \frac{1}{(1-x)^2} \quad / \cdot x \\ &\sum n \cdot x^n = \frac{x}{(1-x)^2} \end{aligned}}$$

$$\Rightarrow \cancel{p}\cdot\frac{1-p}{p^{\cancel{2}}} = a$$

$$\frac{1-p}{p} = a$$

$$ap = 1-p$$

$$1 = ap + p$$

$$p = \frac{1}{1+a}$$

$$1 - p = 1 - \frac{1}{1+a} = \frac{a}{1+a}$$

$$P(X=k) = p\cdot(1-p)^k = \frac{1}{1+a}\cdot\left(\frac{a}{1+a}\right)^k$$

13.) $X \sim \begin{pmatrix} 1 & 2 & 3 & 4 & \dots & n \\ \frac{1}{2} & \frac{1}{2}\cdot\frac{1}{2} & \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} & \left(\frac{1}{2}\right)^4 & & \left(\frac{1}{2}\right)^n \end{pmatrix}$

vj. da je 1. igrač pobjedio? → može pobjediti u 1, 4, 7, 11... bacanju:
(od 3 igrača)

$$P = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^{3k+1} \leftarrow 1,4,7,11\dots$$

$$P = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^{3k+1} = \sum_{k=0}^{\infty} \left(\frac{1}{8}\right)^k \cdot \frac{1}{2} = \frac{1}{2} \sum_{k=0}^{\infty} \left(\frac{1}{8}\right)^k = \frac{1}{2} \cdot \frac{8}{7} = \frac{4}{7} = 0{,}5714$$

$$E(X) = \sum_{k=0}^{\infty} k \cdot p_k = \frac{10}{7}$$

22.) $X = 3 \times \text{broj}$

$Y = \begin{cases} 3, & \text{broj} > 2 \\ 0, & \text{broj} \le 2 \end{cases}$

X\Y	0	3	
3	1/6	0	1/6
6	1/6	0	1/6
9	0	1/6	1/6
12	0	1/6	1/6
15	0	1/6	1/6
18	0	1/6	1/6
	2/6	4/6	1

$Z = X + Y$

$Z \sim \begin{pmatrix} 3 & 6 & 9 & 12 & 15 & 18 & 21 \\ \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$D(Z) = E(X^2) - E(X)^2 = 196{,}5 - 156{,}25 = 40{,}25$

23.) $X = a^2$

$Y = \begin{cases} -1, & \text{broj} \le 2 \\ +1, & \text{broj} > 2 \end{cases}$

X\Y	-1	1	
1	1/6	0	1/6
4	1/6	0	1/6
9	0	1/6	1/6
16	0	1/6	1/6
25	0	1/6	1/6
36	0	1/6	1/6
	2/6	4/6	1

$Z = X + Y$

$Z \sim \begin{pmatrix} 0 & 2 & 3 & 5 & 8 & 10 & 15 & 17 & 24 & 26 & 35 & 37 \\ \frac{1}{6} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{6} & 0 & \frac{1}{6} & 0 & \frac{1}{6} \end{pmatrix}$

$D(Z) = 407{,}167 - 240{,}25 = 166{,}92$

24.)

$X \backslash Y$	0	1	2	
0	1/8	0	0	1/8
1	1/8	1/8	0	1/4
2	0	1/8	0	1/8
3	0	1/4	1/4	1/2
	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	1

$\boxed{r=?}$

$$r = \frac{cov(X,Y)}{\sigma_X \sigma_Y} = \frac{cov(X,Y)}{\sqrt{D_X D_Y}} = \frac{E(XY)-E(X)E(Y)}{\sqrt{D_X} \cdot \sqrt{D_Y}}$$

$$E(X) = 0 \cdot \frac{1}{8} + 1 \cdot \frac{1}{4} + 2 \cdot \frac{1}{8} + 3 \cdot \frac{1}{2} = \underline{\underline{2}}$$

$$E(Y) = 0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} = \underline{\underline{1}}$$

$$Z = X \cdot Y \sim \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 6 \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix} \quad \underline{\underline{E(XY) = 2,625}}$$

$$D(X) = E(X^2) - 4 = \underline{\underline{1,25}}$$

$$D(Y) = E(Y^2) - 1 = \underline{\underline{0,5}}$$

$$\Rightarrow r = \frac{2,625-2}{\sqrt{\frac{5}{8}}} = \underline{\underline{0,79}}$$

25.) $X = 2a$

$$Y = \begin{cases} 1 & \text{neparan} \\ 3 & \text{paran} \end{cases}$$

$X \backslash Y$	1	3	
2	1/6	0	$\frac{1}{6}$
4	0	1/6	$\frac{1}{6}$
6	1/6	0	$\frac{1}{6}$
8	0	1/6	$\frac{1}{6}$
10	1/6	0	$\frac{1}{6}$
12	0	1/6	$\frac{1}{6}$
	$\frac{1}{2}$	$\frac{1}{2}$	1

$$Z = X + Y$$

$$Z \sim \begin{pmatrix} 3 & 5 & 7 & 9 & 11 & 13 & 15 \\ 1/6 & 0 & 2/6 & 0 & 2/6 & 0 & 1/6 \end{pmatrix}$$

$$E(Z) = 3 \cdot \frac{1}{6} + 7 \cdot \frac{2}{6} + 11 \cdot \frac{2}{6} + 15 \cdot \frac{1}{6} = \underline{\underline{9}}$$

$$E(Z^2) = 9 \cdot \frac{1}{6} + 49 \cdot \frac{2}{6} + \frac{121 \cdot 2}{6} + 225 \cdot \frac{1}{6} = \underline{95,\dot{6}}$$

$$D(Z) = E(Z^2) - E(Z)^2 = 95,6 - 81 = \underline{\underline{14,\dot{6}}}$$

26.) $S = \{1,2,3\}$, dva sluč. broja i,j

$X = \max\{i,j\}$

$Y = \min\{i,j\}$

$X \backslash Y$	1	2	3	
1	$\frac{1}{9}$	0	0	1/9
2	$\frac{2}{9}$	$\frac{1}{9}$	0	3/9
3	2/9	2/9	1/9	5/9
	5/9	3/9	1/9	1

$$E(XY) = 4$$

$$E(X) = \frac{22}{9}$$

$$E(Y) = \frac{14}{9}$$

$$D(X) = \frac{38}{81}$$

$$D(Y) = \frac{38}{81}$$

$$r = \frac{E(XY) - E(X)E(Y)}{\sqrt{D_X D_Y}} = \frac{4 - \frac{22 \cdot 14}{81}}{\frac{38}{81}}$$

$$r = \frac{16}{38} = \underline{\underline{0,42}}$$

27.) $X = \begin{cases} 1 & \text{paran} \\ -1 & \text{neparan} \end{cases}$ $\quad Y = \begin{cases} 1 & \leq 3 \\ -1 & > 3 \end{cases}$

$Z = X - Y, \; D(Z) = ?$

X\Y	1	-1	
1	1/6	2/6	1/2
-1	2/6	1/6	1/2
	$\frac{1}{2}$	$\frac{1}{2}$	1

$$Z \sim \begin{pmatrix} -2 & 0 & 2 \\ 2/6 & 2/6 & 2/6 \end{pmatrix} \qquad E(Z) = 0 \qquad E(Z^2) = 2,\dot{6}$$

$$D(Z) = E(Z^2) - E(Z)^2 = 2,\dot{6} - 0 = \underline{\underline{2,\dot{6}}}$$

28.) $X = \max(a,b)$
$Y = \min(a,b)$

X\Y	1	2	3	4	5	6	
1	1/36	0	0	0	0	0	1/36
2	2/36	2/36	0	0	0	0	3/36
3	2/36	2/36	1/36	0	0	0	5/36
4	2/36	2/36	2/36	1/36	0	0	7/36
5	2/36	2/36	2/36	2/36	1/36	0	9/36
6	2/36	2/36	2/36	2/36	2/36	1/36	11/36
	11/36	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$	1

$$E(XY) = \frac{441}{36} \qquad D(X) = 1,97$$

$$E(X) = \frac{161}{36} \qquad D(Y) = 1,97$$

$$E(Y) = \frac{91}{36}$$

$$r(X,Y) = \frac{E(XY) - E(X)E(Y)}{\sigma_X \sigma_Y} = \frac{E(XY) - E(X)E(Y)}{\sqrt{D_X D_Y}}$$

$$= \underline{\underline{0,48}}$$

29.)

X\Y	0	1
-1	1/4	1/6
0	1/6	1/8
1	1/8	1/6
		1

$Z = 2X + Y \qquad Z \sim -2 \;\; -1 \;\; 0 \;\; 1 \;\; 2 \;\; 3$

$T = 2X - Y \qquad T \sim -2 \;\; -3 \;\; 0 \;\; -1 \;\; 1 \;\; 2$

Z/T	-3	-2	-1	0	1	2	
-2	0	$\frac{1}{4}$	0	0	0	0	1/4
-1	$\frac{1}{6}$	0	0	0	0	0	1/6
0	0	0	0	$\frac{1}{6}$	0	0	1/6
1	0	0	$\frac{1}{8}$	0	0	0	1/8
2	0	0	0	0	0	$\frac{1}{8}$	1/8
3	0	0	0	0	$\frac{1}{6}$	0	1/6
	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{8}$	1

30.)

$X = 2a$

$Y = \begin{cases} 0 & \text{paran} \\ 1 & \text{neparan} \end{cases}$

X\Y	0	1	
2	0	1/6	1/6
4	1/6	0	1/6
6	0	1/6	1/6
8	1/6	0	1/6
10	0	1/6	1/6
12	1/6	0	1/6
	1/2	1/2	1

$Z = XY$

$$XY \sim \begin{pmatrix} 0 & 2 & 4 & 6 & 8 & 10 & 12 \\ 1/2 & 1/6 & 0 & 1/6 & 0 & 1/6 & 0 \end{pmatrix}$$

$$E(XY) = 2 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6} + 10 \cdot \frac{1}{6} = \underline{\underline{3}}$$

$E(X) = 7$ $\quad D(X) = \frac{35}{3}$ $\quad \sigma_X = \sqrt{D(X)} = 3{,}4156$

$E(Y) = \frac{1}{2}$ $\quad D(Y) = \frac{1}{4}$ $\quad \sigma_Y = \sqrt{D(Y)} = 0{,}5$

$$r(X,Y) = \frac{E(XY) - E(X)E(Y)}{\sigma_X \sigma_Y} = \frac{3 - 3{,}5}{1{,}7078} = \underline{\underline{-0{,}293}}$$

31.)

$$X \sim \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{pmatrix}$$

$$\boxed{\cos t = \frac{e^{it} + e^{-it}}{2} \qquad \sin t = \frac{e^{it} - e^{-it}}{2}}$$

$$\vartheta(t) = \frac{1}{5} e^{-2it} + \frac{1}{5} e^{-it} + \frac{1}{5} e^{-\cancel{0}it} + \frac{1}{1} e^{it} + \frac{1}{5} e^{2it}$$

$$\vartheta(t) = \frac{1}{5} e^{-2it} + \frac{1}{5} e^{2it} + \frac{1}{5} e^{-it} + \frac{1}{5} e^{it} + \frac{1}{5}$$

$$\vartheta(t) = \frac{1}{5} \left(e^{-2it} + e^{2it} + e^{-it} + e^{it} + 1 \right) \Big/ \cdot \frac{2}{2}$$

$$\vartheta(t) = \frac{2}{5} \left(\frac{e^{-2it} + e^{2it}}{2} + \frac{e^{it} + e^{-it}}{2} + \frac{1}{2} \right) \Rightarrow \vartheta(t) = \frac{2}{5} \left(\cos 2t + \cos t + \frac{1}{2} \right)$$

35.)

$$\vartheta(t) = \frac{1}{3} \cos t + \frac{1}{6} \cos 2t + \frac{1}{2} \cos 3t$$

$$\vartheta(t) = \frac{1}{3} \left(\frac{e^{it} + e^{-it}}{2} \right) + \frac{1}{6} \left(\frac{e^{2it} + e^{-2it}}{2} \right) + \frac{1}{2} \left(\frac{e^{3it} + e^{-3it}}{2} \right)$$

$$\vartheta(t) = \frac{1}{6} e^{it} + \frac{1}{6} e^{-it} + \frac{1}{12} e^{2it} + \frac{1}{12} e^{-2it} + \frac{1}{4} e^{3it} + \frac{1}{4} e^{-3it}$$

$$X \sim \begin{pmatrix} -3 & -2 & -1 & 1 & 2 & 3 \\ \frac{1}{4} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{4} \end{pmatrix}$$

SLUŽBENA RJEŠENJA:

§ 3. Diskretne slučajne varijable i vektori

1. $P(X=k)=\dfrac{1}{n}$, $k=1,\dots,n$.

2. $P(X=k)=\dfrac{C_{k-1}^2}{C_n^3}$, $k=3,\dots,n$.

3. $P(X=k)=\dfrac{k^n-(k-1)^n}{N^n}$, $k=1,\dots,N$.

4. 0; 1.80.

5. $\dfrac{2}{\ln 2}+1$.

6. 4.08; 2.837.

7. 1.53

8. $\dfrac{(n+1)(4n-1)}{6n}$.

9. $\frac{6}{5}$.

10. 3.

11. $\frac{7}{3}$.

12. a) $D(X)=E(X)+E(X)^2$;
b) $P(X=k)=\dfrac{a^k}{(a+1)^k}$.

13. $\frac{4}{7}$, $\frac{10}{7}$.

14. $\displaystyle\sum_{k=0}^{n}\binom{n}{k}^2\frac{1}{2^{2n}}=\frac{1}{2^{2n}}\binom{2n}{n}\approx\frac{1}{\pi n}$

15. $P(Z=k)=\begin{cases}\dfrac{k+1}{(n+1)^2}, & k=0,1,\dots,n\\[2mm] \dfrac{2n+1-k}{(n+1)^2}, & k=n+1,\dots,2n\end{cases}$

17. Uputa. Dokaži da vrijedi

$$n^{[k]}=k\sum_{m=1}^{n-1}m^{[k-1]},\quad \forall n$$

i zatim transformiraj desnu stranu u jednakosti

$$E(X^{[k]})=\sum_{n=1}^{\infty}n^{[k]}P(X=n).$$

18. $\dfrac{n-1}{2n}$, $n+1$.

19. Poissonova s parametrom $\lambda=3$, $E(Z)=3$, $D(Z)=3$.

20. $P(Y=k)=\dbinom{n}{k}\left(\dfrac{5}{6}\right)^k\left(\dfrac{1}{6}\right)^{n-k}$;
$E(Y)=\dfrac{5n}{6}$;
$D(Y)=\dfrac{5n}{36}$; $\dfrac{1+5n+12.5n(n-1)}{6^n}$.

21. $E(X)=0$, $E(X^2)=n/2$, $E(X^3)=0$

22. $D(Z)=\frac{161}{4}$.

23. 166.92.

24. $r(X,Y)=\frac{1}{4}\sqrt{10}$.

25. $D(Z)=\frac{44}{3}$.

26. $r(X,Y)=0.5$.

27. $D(Z)=\frac{8}{3}$.

28. $r(X,Y)=\frac{35}{73}$.

29. $f(-2,-2)=\frac{1}{4}$, $f(-1,-3)=\frac{1}{6}$, $f(0,0)=\frac{1}{6}$, $f(1,-1)=\frac{1}{8}$, $f(2,2)=\frac{1}{8}$, $f(3,1)=\frac{1}{6}$.

30. -0.293.

31. $\frac{2}{5}(\cos 2t+\cos t+\frac{1}{2})$

32. $\vartheta_X(t)=\dfrac{e^{it}}{n+1}\cdot\dfrac{e^{2it(n+1)}-1}{e^{2it}-1}$.

33. $\vartheta_X(t)=\dfrac{1}{4e^{2it}-1}+\dfrac{2e^{it}}{4-e^{2it}}$; $E(X)=\dfrac{2}{9}$.

34. $\vartheta_X(t)=\dfrac{p}{1-(1-p)e^{it}}$; $E(X)=\dfrac{1}{p}-1$.

35. $X\sim\begin{pmatrix}-3 & -1 & 0 & 1 & 3\\ \frac{1}{16} & \frac{3}{16} & \frac{8}{16} & \frac{3}{16} & \frac{1}{16}\end{pmatrix}$.

36. $X\sim\begin{pmatrix}-3 & -2 & -1 & 1 & 2 & 3\\ \frac{1}{4} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{4}\end{pmatrix}$.

LITERATURA:

[1] Neven Elezović: Diskretna vjerojatnost, *Element 2010.godine*