BURIĆEV PREDMEĐUISPIT

1. a) $f: \{1,2,3\dots,100\} \to \{1,2,\dots 100\}$

- svaki od ovih brojeva možemo preslikat na 100 načina

$$|\Omega| = 100^{100} \rightarrow \text{ukupan broj funkcija}$$

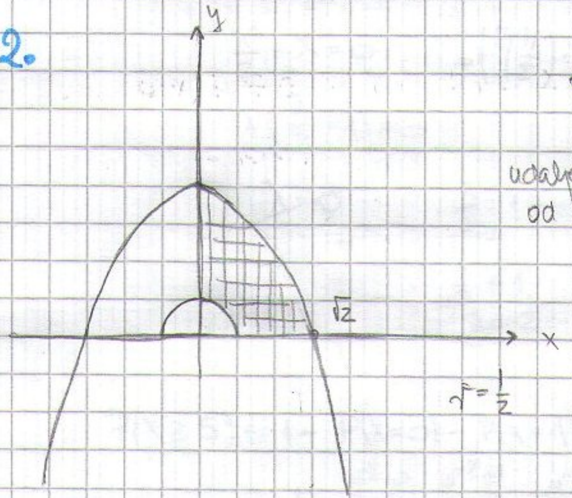
- svaki parni/neparni možemo preslikati na 50 načina

$$P(A) = \frac{50^{50}\cdot 50^{50}}{100^{100}} = \frac{1}{2^{100}}$$

b) injekcija → svaki broj preslikati u "svoje"

$$P(B) = \frac{50!\cdot 50!}{100\cdot 99\cdot 98\cdot\ldots\cdot 1} = \frac{50!\,50!}{100!}$$

2.



prvi kvadrant

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{P_{AB}}{P_B} =$$

udaljenost veća od $\frac{1}{2}$

$$= 1 - \frac{P_D}{P_B} = 1 - \frac{1}{4}\cdot\frac{r^2\pi}{\int_0^{\sqrt{2}}(2-x^2)\,dx} =$$

$$= 1 - \frac{1}{4}\cdot\frac{1}{4}\pi\cdot\frac{1}{2\sqrt{2}-\frac{2\sqrt{2}}{3}} =$$

$$= 1 - \frac{\pi}{16}\cdot\frac{3}{4\sqrt{2}}$$

$$= 1 - \frac{\pi}{16}\cdot\frac{3\sqrt{2}}{8} = \frac{128 - 3\sqrt{2}\pi}{128}$$

Bayesova formula

3. → prvi 0.3
→ ostali 0.2
(4 lovca)

- potrebna barrem 2
- jedan od lovaca izvršio gađanje tri puta
- P(pogodio_prvi_lovac) = ?

$P(H_1) = \frac{1}{4}$

$P(H_2) = P(H_3) = P(H_4) = \frac{1}{4}$

$P(A \mid H_1) = \binom{3}{2} \underbrace{0.3 \cdot 0.3}_{2\times \text{pogodio}} \cdot \underset{\text{fulao}}{0.7} + \binom{3}{3} \cdot \underbrace{0.3 \cdot 0.3 \cdot 0.3}_{3\times \text{pogodio}}$

vepar je uginuo

$P(A \mid H_2) = P(A \mid H_3) = P(A \mid H_4) = \binom{3}{2} 0.2^2 \cdot 0.8 + 0.2^3$

$P(H_1 \mid A) = \frac{P(H_1) \cdot P(A \mid H_1)}{P(H_1) \cdot P(A \mid H_1) + 3P(H_2) \cdot P(A \mid H_2)} =$

ako je vepar uginuo, kolika je vjerojatnost da je bio prvi

4.

$$X \sim \begin{pmatrix} 1 & 2 & \cdots & n & \cdots \\ 0.686 \cdot p & [0.314 + 0.686(1-p)] \cdot (0.686 \cdot p) & & \underbrace{[0.314 + 0.686(1-p)]}_{1-0.686p}{}^{n-1} (0.686p) & \end{pmatrix}$$

p je vjerojatnost da smo prošli na usmenom dijelu

$E(X) = \sum_{n=1}^{\infty} n \cdot (1 - 0.686p)^{n-1} \cdot 0.686p =$

$= 0.686p \cdot \frac{1}{[1 - 1 + 0.686p]^2} = \frac{1}{0.686p} = 2$

$p = 0.73$

5.

X\Y	0	1	2	3	4	5	x
0	$\frac{6}{36}$	0	0	0	0	0	$\frac{6}{36}$
1	0	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{10}{36}$
2	0	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	0	$\frac{8}{36}$
3	0	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	0	0	$\frac{6}{36}$
4	0	$\frac{2}{36}$	$\frac{2}{36}$	0	0	0	$\frac{4}{36}$
5	0	$\frac{2}{36}$	0	0	0	0	$\frac{2}{36}$
y	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$	1

→ zavisne

$$r(X,Y) = \frac{\operatorname{cov}(X,Y)}{\sigma_X \sigma_Y}$$

$$\operatorname{cov}(X,Y) = E(XY) - E(X)\cdot E(Y)$$

$$E(X) = \frac{10}{36} + 2\cdot\frac{8}{36} + 3\cdot\frac{6}{36} + 4\cdot\frac{4}{36} + 5\,\frac{2}{36} = \frac{70}{36} = \frac{35}{18} = E(Y)$$

$$E(X^2) = E(Y^2) = \frac{10}{36} + 4\cdot\frac{8}{36} + 9\cdot\frac{6}{36} + 16\cdot\frac{4}{36} + 25\cdot\frac{2}{36} = \frac{35}{6}$$

$$D(X) = D(Y) = E(X^2) - (E(X))^2 = \frac{665}{324}$$

$$E(XY) = 15\cdot\frac{2}{36} + 20\,\frac{2}{36} + 18\cdot\frac{2}{36} + 12\cdot\frac{2}{36} + 5\cdot\frac{2}{36} = \frac{35}{9}$$

$$r(X,Y) = \frac{\frac{35}{9} - \frac{35^2}{18}}{\frac{665}{324}} = 0.052$$

6 o podatke koji su dani prebacit u jedinicu u koju trebamo :D

$$X \sim P(\lambda)$$

$$\lambda = \frac{960}{60} \cdot \frac{1}{2} = 8 \text{ automobila/pola minute}$$

$$P(X>4) = 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3) - P(X=4)$$
$$= 1 - e^{-8}\left(1 + 8 + 64\frac{1}{2} + 8^3 \cdot \frac{1}{6} + 8^4 \cdot \frac{1}{24}\right) = 0.9$$

7. Čim imamo n događaja, a nama treba k → BINOMNA RAZDIOBA

$n = 7$
$p = 0.05$ $\qquad B(n, p) = B(7, 0.05)$

a) $P(x \geq 2) = 1 - P(x=0) - P(x=1) =$
$$= 1 - \binom{7}{0} \cdot 1 \cdot (1-0.05)^7 - \binom{7}{1} \cdot 0.05 \cdot (1-0.05)^6 =$$
$$= 0.0444$$

b) $X \sim B(700, 0{,}05) \approx P(np) = P(35)$

Binomna razdioba se za velike n aproksimira Poissonom.

$$P(x \geq 3) = 1 - P(x=0) - P(x=1) - P(x=2) =$$
$$= 1 - e^{-35} - 35e^{-35} - \frac{35^2}{2!}e^{-35} = 1$$