11. ožujak 2009
12:08

Geometrijska vjerojatnost

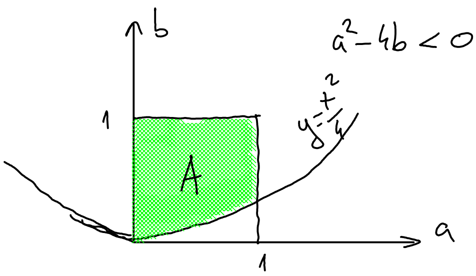
cca 1 zad na M1

$$P(A) = \frac{m(A)}{m(\Omega)}$$

Primjer 1. Biramo na sreću 2 broja iz $[0,1]$. Kolika je vjerojatnost da kvadratna jednadžba bude stogo veća od nule.

$$x^2+ax+b>0$$

$$D<0$$



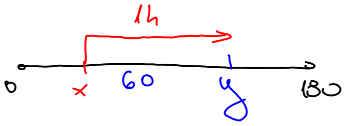
$$a^2-4b<0 \qquad b>\frac{a^2}{4} \Rightarrow y>\frac{x^2}{4}$$

$$P=\frac{m(A)}{m(\Omega)}=\frac{1-\int_0^1 \frac{1}{4}x^2dx}{1}=\frac{11}{12}$$

Pr 2: Simona i Ante izlaze van, neovisno jedno o drugom, u terminu od 00:00, do 03:00, i zadrže se 1h. Kolika je vjerojatnost susreta.

y - Ante
x - Simona

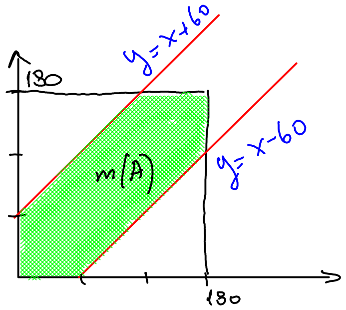
$$x,y \in [0,180]$$



$$|y-x|<60$$

$$y-x<60 \qquad y-x>-60$$

$$y<x+60 \qquad y>x-60$$



$$P(A)=\frac{m(A)}{m(\Omega)}=\frac{180^2-2\cdot\frac{1}{2}\cdot 120^2}{180^2}=\frac{5}{9}=0.55$$

11. ožujak 2009
12:39

Uvjetna vjerojatnost

cca 1 zad na M1

$$P(A|B) = \frac{P(AB)}{P(B)}$$

znamo da se dogodilo

≈ 15 c) Bacamo 4 kocke, ako znamo da je zbroj 8, kolika je vjerojatnost da su manji od 3

$B = \{\text{zbroj} < 8\}$, $A = \{\text{svi brojevi manji od } 3\}$

$P(A|B) = ?$ $|\Omega| = 6^4$

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{\frac{15}{\cancel{64}}}{\frac{35}{\cancel{64}}} = \frac{15}{35} = \frac{3}{7}$$

1, 1, 1, 1 → 1 slučaj
2, 1, 1, 1 → 4 slučaja
2, 2, 1, 1 → permutacije $\frac{4!}{2!\,2!} = 6$
2, 2, 2, 1 → 4
3, 1, 1, 1 → 4
3, 2, 1, 1 → $\frac{4!}{2!} = 12$
4, 1, 1, 1 → 4

$\sum = 35$

$|\Omega| = 64$

$P(B) = \frac{35}{64}$

svi manji od 3

$\sum = 15$

$P(AB) = \frac{15}{64}$

11. ožujak 2009
12:53

Potpuna vjerojatnost

$$P(A) = \sum_i P(H_i)\, P(A|H_i)$$

Primjer: $\approx$ 17Dz

Iz prve prebacis u 2 kutiju 2 cokolade, te izvucis iz druge kutije jednu cokoladu. Vjerojatnost da izvuces Milku

$$\boxed{\begin{matrix} 2\,M \\ 3\,D \end{matrix}} \xrightarrow{2} \boxed{\begin{matrix} 1M \\ 4D \end{matrix}} \xrightarrow{1} \quad P(M) = ?$$

$$H_1 = \{2M\} \qquad P(H_1) = \frac{\binom{2}{2}}{\binom{5}{2}}$$

$$H_2 = \{1M \quad 1D\} \quad P(H_2) = \frac{\binom{2}{1}\binom{3}{1}}{\binom{5}{2}} = \frac{6}{10}$$

$$H_3 = \{2D\} \qquad P(H_3) = \frac{\binom{3}{2}}{\binom{5}{2}} = \frac{3}{10}$$

Dobra kontrola

$\sum_i H_i = 1$

mora biti $\underline{\underline{1}}$

$$P(M|H_1) = \frac{\binom{3}{1}}{\binom{7}{1}} = \frac{3}{7} \qquad P(M|H_2) = \frac{\binom{2}{1}}{\binom{7}{1}} = \frac{2}{7}$$

$$P(M|H_3) = \frac{\binom{1}{1}}{\binom{7}{1}} = \frac{1}{7}$$

$$P(M) = P(H_1)P(M|H_1) + P(H_2)P(A|H_2) + P(H_3)P(A|H_3)$$

$$= 0.257 \checkmark$$

11. ožujak 2009
13:06

Bayesova formula

$$P(H_i|A) = \frac{P(H_i)P(A|H_i)}{\sum_i P(H_i)P(A|H_i)}$$

Kolika je vjerojatnost hipoteze, ako se dogodio događaj

Primjer 4 zad | MI 2008
4 boda

Bacamo kocku, te u ovisnosti o rezultatu, bacamo još n puta. Ako su ukupno pale točno 2 petice. Kolika je vjerojatnost da je jedna od njih pala u prvom bacanju

$H_i = \{$ pao je i-ti broj $\}$

(1) $P(H_i) = \frac{1}{6}$ $\quad i = 1,2,3,4,5,6$

$A = \{$ pale su točno dvije petice $\}$

$P(H_5|A) = ?$

(2) $P(A|H_1) = 0$

$P(A|H_2) = \left(\frac{1}{6}\right)^2$

$P(A|H_3) = \binom{3}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^1$

$P(A|H_4) = \binom{4}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2$

$P(A|H_5) = \binom{5}{1}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^4$

jer je u prvom bacanju bila petica trebamo još samo jednu

$P(A|H_6) = \binom{6}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^4$

$$P(H_5|A) = \frac{P(H_5)P(A|H_5)}{\sum_{i=1}^{6} P(H_i)P(A|H_i)} = 0.49263$$