Ovaj PDF sadrži skenirane postupke 3. KPZ-a 2012-2013.

Zadaci su poredani od po redom po zadacima iz svake grupe i godine.

Riješio i ustupio na skeniranje

fer0vac

skenirao

SipE

III – 12 – 1A, 12b

X\Y	−1	2	
−1	1/6	1/4	10/24
0	1/6	1/8	7/24
2	1/6	1/8	7/24
	$\frac{1}{2}$	$\frac{1}{2}$	1

X i Y su zavisne jer npr. $\frac{1}{6} \neq \frac{10}{24} \cdot \frac{1}{2}$

$Z = \frac{X}{X+Y}$ $\qquad W = X - Y \qquad P(Z > 0 \mid W \geq 0) = ?$

$\left.\begin{matrix} X=-1 \\ Y=-1 \end{matrix}\right\}$ $Z = \frac{1}{2}$ $\quad W = 0 \left(\frac{1}{6}\right)$

$\left.\begin{matrix} X=2 \\ Y=-1 \end{matrix}\right\}$ $Z = 2$ $\quad W = 3 \left(\frac{1}{6}\right)$

$\left.\begin{matrix} X=-1 \\ Y=2 \end{matrix}\right\}$ $Z = -1$ $\quad W = -3 \left(\frac{1}{4}\right)$

$\left.\begin{matrix} X=2 \\ Y=2 \end{matrix}\right\}$ $Z = \frac{1}{2}$ $\quad W = 0 \left(\frac{1}{8}\right)$

$\left.\begin{matrix} X=0 \\ Y=-1 \end{matrix}\right\}$ $Z = 0$ $\quad W = 1 \left(\frac{1}{6}\right)$

$\left.\begin{matrix} X=0 \\ Y=2 \end{matrix}\right\}$ $Z = 0$ $\quad W = -2 \left(\frac{1}{8}\right)$

Z\W	−3	−2	0	1	3	
−1	1/4	0	0	0	0	6/24
0	0	1/8	0	1/6	0	7/24
1/2	0	0	(1/6+1/8)	0	0	7/24
2	0	0	0	0	1/6	4/24
	$\frac{6}{24}$	$\frac{3}{24}$	$\frac{7}{24}$	$\frac{4}{24}$	$\frac{4}{24}$	1

$$P(Z>0 \mid W \geq 0) = \frac{P(Z>0, W\geq 0)}{P(W \geq 0)} = \frac{\left(\frac{1}{6}+\frac{1}{8}\right) + \frac{1}{6}}{\frac{1}{6} + \left(\frac{1}{6}+\frac{1}{8}\right) + \frac{1}{6}} = \frac{\frac{11}{24}}{\frac{15}{24}} = \frac{11}{15} = 0{,}73$$

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$\boxed{\text{III} - 12 - 13, 12h}$ X = ost. pri $/3$ od $\square$ - broj na kocki

$cov(X, Y) = ?$

$$Y = \begin{cases} 1, & \square / 3 = 0 \quad (3, 6) \\ -1, & \square / 3 \neq 0 \quad (1, 2, 4, 5) \end{cases}$$

$\square = 1 \Rightarrow X = 1$

$\square = 2 \Rightarrow X = 2$

$\square = 3 \Rightarrow X = 0$

$\square = 4 \Rightarrow X = 1$

$\square = 5 \Rightarrow X = 2$

$\square = 6 \Rightarrow X = 0$

$X \backslash Y$	-1	1	
0	0	$\frac{2}{6}$	$2/6$
1	$\frac{2}{6}$	0	$2/6$
2	$\frac{2}{6}$	0	$2/6$
	$\frac{4}{6}$	$\frac{2}{6}$	1

$X = 0$ za 3 i 6

$X = 1$ za 1 i 4

$X = 2$ za 2 i 5

X i Y su zavisne jer

npr. $\frac{2}{6} \neq \frac{2}{6} \cdot \frac{2}{6}$

$$E(X) = 0 \cdot \frac{2}{6} + 1 \cdot \frac{2}{6} + 2 \cdot \frac{2}{6} = 1 \qquad E(Y) = -1 \cdot \frac{4}{6} + 1 \cdot \frac{2}{6} = -\frac{1}{3}$$

$$E(XY) = 1 \cdot (-1) \frac{2}{6} + 2 \cdot (-1) \cdot \frac{2}{6} = -1$$

$$cov(X, Y) = E(XY) - E(X)E(Y) = -1 - \left(1 \cdot \left(-\frac{1}{3}\right)\right) = -\frac{2}{3}$$

...-12-...15 $X = \{0, 1, 2\}$ prvo bacanje broj 6-tica $Y = \{0, 1, 2\}$ oba bacanja broj 6-ica

$X \backslash Y$	0	1	2	
0	$\left(\frac{5}{6}\cdot\frac{5}{6}\right)\cdot\left(\frac{5}{6}\cdot\frac{5}{6}\right)$	$\left(\frac{5}{6}\right)^2\cdot\binom{2}{1}\frac{5}{6}\frac{1}{6}$	$\left(\frac{5}{6}\right)^2\cdot\binom{2}{2}\left(\frac{1}{6}\right)^2$	
1	0	$\binom{2}{1}\frac{5}{6}\frac{1}{6}\cdot\frac{5}{6}$	$\binom{2}{1}\frac{5}{6}\frac{1}{6}\cdot\frac{1}{6}$	
2	0	0	$\binom{2}{2}\frac{1}{6}\cdot\frac{1}{6}$	$\left(\frac{1}{6}\right)^2$
	$\left(\frac{5}{6}\right)^4$			

$\left.\begin{array}{l} x=1 \\ y=0 \end{array}\right\} = 0$

ne može u prvom bacanju biti jedna 6-ica, a u oba bacanja onda 0 6-ica.

$\left.\begin{array}{l} x=2 \\ y=0 \end{array}\right\} = 0$ slično

$\left.\begin{array}{l} x=2 \\ y=1 \end{array}\right\} = 0$ slično

X i Y su zavisne jer je npr. $0 \neq \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)^2$

$E(X+Y) = ?$

$Z = X + Y$

X	Y	X+Y
0	0	0
0	1	1
0	2	2
1	0	1
1	1	2
1	2	3
2	0	2
2	1	3
2	2	4

$$Z \sim \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ \left(\frac{5}{6}\right)^4 & \frac{250}{6^4} & \frac{325}{6^4} & \frac{10}{6^3} & \frac{1}{36} \end{pmatrix}$$

$Z = 1$ W W.A.

$Z = 1 \Rightarrow \left(\frac{5}{6}\right)^2\binom{2}{1}\frac{5}{6}\frac{1}{6} + 0 = \frac{250}{6^4}$

$Z = 2 \Rightarrow \left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^2 + 2\left(\frac{5}{6}\right)^2\frac{1}{6} + 0$

$(x,y) = (0,2)$ $\quad (x,y) = (1,1)$ $\quad (x,y) = (2,$

$Z = 3 \Rightarrow 2\cdot\frac{5}{6}\left(\frac{1}{6}\right)^2 + 0 = \frac{10}{6^3}$

$Z = 4 \Rightarrow \left(\frac{1}{6}\right)^2 = \frac{1}{36}$

$E(X+Y) = \frac{1333}{648}$

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III − 12 − 1A, 13h

dva broja iz skupa $\{0, 1, 2\}$ bez ponav.
I prvi II drugi

X = suma
Y = apsolutna vrijednost razlike

I	II	$X = I + II$	$Y = \lvert I - II \rvert$
0	1	1	1
0	2	2	2
1	0	1	1
1	2	3	1
2	0	2	2
2	1	3	1

3 mogućnosti
$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

$X \backslash Y$	1	2	
1	$\frac{1}{3}$	0	$\frac{1}{3}$
2	0	$\frac{1}{3}$	$\frac{1}{3}$
3	$\frac{1}{3}$	0	$\frac{1}{3}$
	$\frac{2}{3}$	$\frac{1}{3}$	1

X i Y su zavisne jer npr. $\frac{1}{3} \neq \frac{2}{3} \cdot \frac{1}{3}$

$E(X) = 1 \cdot \frac{1}{3} + 2 \cdot \frac{1}{3} + 3 \cdot \frac{1}{3} = \frac{1}{3}(1 + 2 + 3) = \frac{1}{3} \cdot 6 = 2$

$E(Y) = 1 \cdot \frac{2}{3} + 2 \cdot \frac{1}{3} = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$

$E(X \cdot Y) = 1 \cdot 1 \cdot \frac{1}{3} + 2 \cdot 2 \cdot \frac{1}{3} + 3 \cdot 1 \cdot \frac{1}{3} = \frac{1}{3} + \frac{4}{3} + 1 = \frac{5}{3} + \frac{3}{3} = \frac{8}{3}$

$\mathrm{cov}(X, Y) = E(XY) - E(X)E(Y)$

$= \frac{8}{3} - 2 \cdot \frac{4}{3} = 0$

III - 13 - 1A, cov(X, Y)

$E(X) = 3{,}9$

$X \backslash Y$	0	1	3	
2	0	0	0,3	0,3
4	p_1	0,1	0	
5	p_2	0,2	0,1	
		0,3	0,4	1

$2 \cdot 0{,}3 + 4 \cdot (p_1 + 0{,}1) + 5(p_2 + 0{,}2 + 0{,}1) = 3{,}9$

$0{,}6 + 4p_1 + 0{,}4 + 5p_2 + 1{,}5 = 3{,}9$

$4p_1 + 5p_2 = 3{,}9 - 1{,}5 - 1$

$(1)\ \boxed{4p_1 + 5p_2 = 1{,}4}$

$p_1 + p_2 = g_1 \qquad g_1 = 1 - 0{,}3 - 0{,}4$

$\boxed{p_1 + p_2 = 0{,}3 \ (2)} \qquad g_1 = 1 - 0{,}7 = 0{,}3$

X i Y su zavisni jer npr. $0 \neq 0{,}3 \cdot 0{,}3$

$4p_1 + 5p_2 = 1{,}4$

$p_1 + p_2 = 0{,}3 \Rightarrow p_1 = 0{,}3 - p_2$

$4(0{,}3 - p_2) + 5p_2 = 1{,}4$

$1{,}2 - 4p_2 + 5p_2 = 1{,}4$

$1p_2 = 0{,}2 \qquad \boxed{p_2 = 0{,}2}$

$p_1 = 0{,}3 - p_2 = 0{,}3 - 0{,}2$

$\boxed{p_1 = 0{,}1}$

$cov(X, Y) = E(XY) - E(X)E(Y)$

$E(X) = 3{,}9 \qquad E(Y) = 0 \cdot 0{,}3 + 1 \cdot 0{,}3 + 3 \cdot 0{,}4$

$\boxed{E(Y) = 1{,}5}$

$X \backslash Y$	0	1	3	
2	0	0	0,3	0,3
4	0,1	0,1	0	0,2
5	0,2	0,2	0,1	0,5
	0,3	0,3	0,4	1

$E(XY) = \cancel{2 \cdot 0 \cdot 0} + 2 \cdot 1 \cdot 0 + 2 \cdot 3 \cdot 0{,}3 + \cancel{4 \cdot 0 \cdot 0{,}1} +$

$+ 4 \cdot 1 \cdot 0{,}1 + 0 + 0 + 5 \cdot 1 \cdot 0{,}2 + 5 \cdot 3 \cdot 0{,}1 =$

$= 1{,}8 + 0{,}4 + 1 + 1{,}5 = \boxed{4{,}7 = E(XY)}$

$cov(X, Y) = E(XY) - E(X)E(Y) = 4{,}7 - 3{,}9 \cdot 1{,}5 = \boxed{-1{,}15 = cov(X, Y)}$

III - 13 - 1B, 12h

$X \backslash Y$	-1	2	4	
1	0,1	0,2	p_1	
3	0	0,2	0	0,2
4	p_2	0,3	0,1	
		0,7		

$E(Y) = 1{,}85$

$1{,}85 = -1(0{,}1 + p_2) + 2 \cdot 0{,}7 + 4(p_1 + 0{,}1)$

$1{,}85 = -0{,}1 - p_2 + 1{,}4 + 4p_1 + 0{,}4$

$\boxed{0{,}15 = 4p_1 - p_2} \ (2)$

$(0{,}1 + p_2) + 0{,}7 + (p_1 + 0{,}1) = 1$

$p_2 + p_1 = 1 - 0{,}7 - 0{,}1 - 0{,}1$

$(1)\ \boxed{p_1 + p_2 = 0{,}1} \Rightarrow p_1 = 0{,}1 - p_2$

(1) u (2) $\qquad 0{,}15 = 4(0{,}1 - p_2) - p_2$

$-0{,}25 = -5p_2$

$\boxed{p_2 = 0{,}05} \Rightarrow p_1 = 0{,}1 - 0{,}05 = \boxed{0{,}05 = p_1}$

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$X\backslash Y$	-1	2	4	
1	0,1	0,2	0,05	0,35
3	0	0,2	0	0,2
4	0,05	0,3	0,1	0,45
	0,15	0,7	0,15	1

$$E(X\cdot Y^2) = 1\cdot(-1)^2\cdot 0{,}1 + 1\cdot 2^2\cdot 0{,}2 + 1\cdot 4^2\cdot 0{,}05 +$$
$$+ 0 + 3\cdot 2^2\, 0{,}2 + 0$$
$$+ 4\,(-1)^2\, 0{,}05 + 4\cdot 2^2\, 0{,}3 + 4\cdot 4^2\cdot 0{,}1$$
$$= 1{,}7 + 2{,}4 + 11{,}4 = 15{,}5$$

X i Y su zavisne jer je npr. $0{,}1 \neq 0{,}35\cdot 0{,}15$

$\boxed{\text{III} - 13 - 1A, 13b}$ $X \sim \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ 0{,}2 & 0{,}1 & 0{,}4 & 0{,}1 & 0{,}2 \end{pmatrix}$

$X^2 \sim \begin{pmatrix} 4 & 1 & 0 & 1 & 4 \\ 0{,}2 & 0{,}1 & 0{,}4 & 0{,}1 & 0{,}2 \end{pmatrix} \sim \begin{pmatrix} 0 & 1 & 4 \\ 0{,}4 & 0{,}2 & 0{,}4 \end{pmatrix}$

$X^2+1 \sim \begin{pmatrix} 1 & 2 & 5 \\ 0{,}4 & 0{,}2 & 0{,}4 \end{pmatrix}$

X / X^2+1	1	2	5	
-2	0	0	0,2	0,2
-1	0	0,1	0	0,1
0	0,4	0	0	0,4
1	0	0,1	0	0,1
2	0	0	0,2	0,2
	0,4	0,2	0,4	1

$\underset{0{,}2}{X = -2}: (-2)^2+1 = 5 \Rightarrow 0{,}2$

$\underset{0{,}1}{X = -1}: (-1)^2+1 = 2 \Rightarrow 0{,}1$

$\vdots$

$\underset{0{,}2}{X = 2}: (2)^2+1 = 5 \Rightarrow 0{,}2$

$$\operatorname{cov}(X, X^2+1) = E(X\cdot(X^2+1)) - E(X)E(X^2+1)$$
$$E(X) = -2\cdot 0{,}2 + (-1)\cdot 0{,}1 + \dots = 0$$
$$E(X^2+1) = 1\cdot 0{,}4 + 2\cdot 0{,}2 + 5\cdot 0{,}4 = 2{,}8$$
$$E(X(X^2+1)) = -2\cdot 5\cdot 0{,}2 + (-1)\cdot 2\cdot 0{,}1 + 1\cdot 2\cdot 0{,}1 + 2\cdot 5\cdot 0{,}2$$
$$E(X(X^2+1)) = 0$$
$$\operatorname{cov}(X, X^2+1) = 0 - 0\cdot 2{,}8 = 0 - 0 = 0$$

X i X^2+1 su zavisne jer je npr. $0{,}2 \neq 0{,}2\cdot 0{,}4$

X je uniformno distribuirana na skupu {-1, 0, ...

$$X \sim \begin{pmatrix} -1 & 0 & 1 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \qquad X^2 \sim \begin{pmatrix} 1 & 0 & 1 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

$$X^2 \sim \begin{pmatrix} 0 & 1 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} \qquad X^2 - 1 \sim \begin{pmatrix} -1 & 0 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

X / X^2-1	-1	0	
-1	0	$\frac{1}{3}$	$\frac{1}{3}$
0	$\frac{1}{3}$	0	$\frac{1}{3}$
1	0	$\frac{1}{3}$	$\frac{1}{3}$
	$\frac{1}{3}$	$\frac{2}{3}$	1

$\underset{\text{redak}}{X} = \begin{pmatrix} -1 \\ \frac{1}{3} \end{pmatrix} \Rightarrow \underset{\text{stupac}}{X^2 - 1} = \begin{pmatrix} 0 \\ \frac{1}{3} \end{pmatrix}$

$(-1, 0) \Rightarrow \frac{1}{3}$

$X = \begin{pmatrix} 0 \\ \frac{1}{3} \end{pmatrix} \Rightarrow X^2 - 1 = \begin{pmatrix} -1 \\ \frac{1}{3} \end{pmatrix}$

$(0, -1) \Rightarrow \frac{1}{3}$

$X = 1 \quad X^2 - 1 = 0$

$(1, 0) \Rightarrow \frac{1}{3}$

$$\operatorname{cov}(X, X^2-1) = E(X(X^2-1)) - E(X)E(X^2-1)$$

$$E(X) = -1 \cdot \frac{1}{3} + 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} = 0$$

$$E(Y) = -1 \cdot \frac{1}{3} + 0 \cdot \frac{2}{3} = -\frac{1}{3}$$

$$E(X(X^2-1)) = (-1) \cdot (-1) \cdot 0 + (-1) \cdot 0 \cdot \frac{1}{3} + \dots = 0$$

$$\operatorname{cov}(X, X^2-1) = 0 - 0 \cdot \left(-\frac{1}{3}\right) = 0$$

X i $X^2 - 1$ su zavisni jer je npr. $\frac{1}{3} \neq \frac{1}{3} \cdot \frac{2}{3}$

2.A. 12. 2012

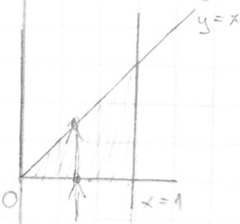
$c=?$

$f_Y(y)=?$

$g_Z(z)=?$

$f(x,y)=\dfrac{c}{x+y}$, $\quad 0\le y\le x\le 1$, $\quad Z=X\cdot Y$, $\quad z\in[0,1]$

$\rightarrow Y=\dfrac{Z}{X}$



$$\iint f(x,y)\,dx\,dy=1 \qquad \left|\frac{\partial y}{\partial z}\right|=\left|\frac{1}{x}\right|$$

=> prvo fiksiraj jednu var (npr. x) a onda vozi po y-u

$$\int_0^1 dx\int_0^x \frac{c}{x+y}\,dy = c\int_0^1 dx\int_0^x \frac{1}{x+y}\,dy = \left|\begin{matrix}x+y=u\\ dy=du\end{matrix}\right|$$

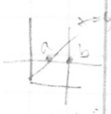
$$= c\int_0^1 dx\int_0 \frac{1}{u}\,du = c\int_0^1 dx\left(\ln|x+y|\Big|_0^x\right)$$

$$= c\int_0^1 dx\left(\ln|2x|-\ln|x|\right) = c\int_0^1 \ln|2x|-\ln|x|\,dx =$$

$$= c\int_0^1 \ln\left|\frac{2x}{x}\right|dx = c\ln(2)\int_0^1 dx = c\ln(2)(1-0) = c\ln(2)$$

$$1=c\ln(2) \Rightarrow c=\frac{1}{\ln(2)} \Rightarrow f(x,y)=\frac{1}{\ln(2)}\cdot\frac{1}{x+y}$$

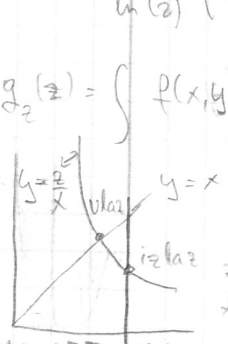
$$f_Y(y)=\int_a^b f(x,y)\,dx = \int_y^1 \frac{1}{\ln(2)}\,\frac{1}{\underbrace{x+y}_{u}}\,dx = \frac{1}{\ln 2}\int_y^1 \frac{1}{u}\,du =$$



$$= \frac{1}{\ln(2)}\left(\ln|x+y|\Big|_y^1\right) = \frac{1}{\ln(2)}\left(\ln|1+y|-\ln(2y)\right) =$$

$$= \frac{1}{\ln(2)}\left(\ln\left|\frac{1+y}{2y}\right|\right), \quad y\in[0,1], \quad x\in[y,1]$$

$$g_Z(z)=\int f(x,y)\left|\frac{\partial y}{\partial z}\right|dx = \int_{\sqrt z}^1 \frac{1}{\ln(2)}\,\frac{1}{x+\frac{z}{x}}\cdot\frac{1}{x}\,dx = \frac{1}{\ln 2}\int_{\sqrt z}^1 \frac{dx}{x^2+\underset{(\sqrt z)^2}{z}} =$$



$\dfrac{z}{x}=x$

$z=x^2$

$x=\sqrt z$

$$= \frac{1}{\ln 2}\cdot\left(\frac{1}{\sqrt z}\,\mathrm{arctg}\left(\frac{x}{\sqrt z}\right)\Big|_{\sqrt z}^1\right) = \frac{1}{\ln 2}\,\frac{1}{\sqrt z}\left(\mathrm{arctg}\left(\frac{1}{\sqrt z}\right)-\mathrm{arctg}(1)\right)$$

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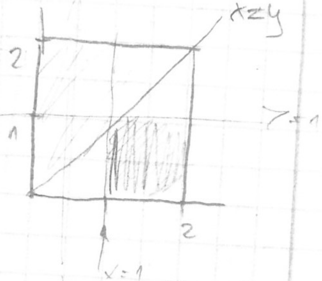
$$= \frac{1}{\ln 2}\cdot\frac{1}{\sqrt z}\left(\mathrm{arctg}\left(\frac{1}{\sqrt z}\right)-\frac{\pi}{4}\right), \quad z\in[0,1], \quad x\in[\sqrt z,1]$$

$$f(x,y)=\begin{cases}\frac{1}{3}, & x<y \\ Cxy, & x\geq y\end{cases}$$



$$1=\underbrace{\iint \frac{1}{3}\,dx\,dy}_{A}+\underbrace{\iint Cxy\,dx\,dy}_{B}$$

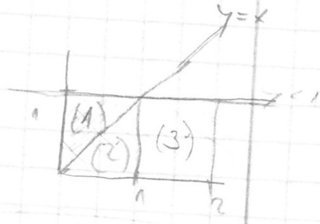
$$A=\frac{1}{3}\int_0^2 dx\int_x^2 dy=\frac{1}{3}\int_0^2 (2-x)\,dx=\frac{1}{3}\left[\int_0^2 2\,dx-\int_0^2 x\,dx\right]=$$

$$=\frac{1}{3}\left[2(2-0)-\frac{1}{2}(4-0)\right]=\frac{1}{3}\left[4-2\right]=\frac{2}{3}$$

$$B=C\int_0^2 x\,dx\int_0^x y\,dy=\dots=2C$$

$$1=\frac{2}{3}+2C \Rightarrow /3$$

$$3=2+6C$$
$$1=6C$$
$$C=\frac{1}{6}$$



$$P(X>1\,|\,Y<1)=\frac{P(X>1,\,Y<1)}{P(Y<1)}=\frac{\int_1^2 dx\int_0^1 \frac{1}{6}xy\,dy}{P(Y<1)} \quad \frac{3}{24}$$

$$P(Y<1)=\frac{1}{3}\int_0^1 dx\int_x^1 dy+\frac{1}{6}\int_0^1 x\,dx\int_0^x y\,dy+\frac{1}{6}\int_1^2 x\,dx\int_0^1 y\,dy$$

$\underline{\ldots,12,2012}$ X = radijus $\sim \mathcal{U}[2,4]$

$$f_X(x) = \frac{1}{4-2} = \frac{1}{2}$$

$$P\left(Y < \frac{1}{2} \,\middle|\, X = x\right) = \frac{x^2\cancel{\pi} - \left(x - \frac{1}{2}\right)^2\cancel{\pi}}{x^2\cancel{\pi}} = (*)$$

y - da je udaljenost

BLA uz uvjet da smo fixirali X na $[2,4]$ npr. $x = 3$

$$= (*) = \frac{x^2 - \left(x^2 - x + \frac{1}{4}\right)}{x^2} = \frac{\cancel{x^2} - \cancel{x^2} + x - \frac{1}{4}}{x^2} = \frac{x - \frac{1}{4}}{x^2}$$

$$P(A) = \int_{-\infty}^{+\infty} P(A \mid X = x) \cdot f_X(x)\, dx$$

$$\int x^{-2}\, dx = \frac{x^{-1}}{-1}$$

$$P\left(Y < \frac{1}{2}\right) = \int_2^4 \left(\frac{1}{x} - \frac{1}{4x^2}\right) \cdot \frac{1}{2}\, dx = \frac{1}{2}\left[\int_2^4 \frac{1}{x}\, dx - \frac{1}{4}\int_2^4 \frac{1}{x^2}\, dx\right] =$$

$$= \frac{1}{2}\left[\ln|x|\Big|_2^4 + \frac{1}{4}\left(\frac{1}{x}\right)\Big|_2^4\right] = \frac{1}{2}\left[\ln|4| - \ln|2| + \frac{1}{4}\left(\frac{1}{4} - \frac{1}{2}\right)\right.$$

$$= \frac{1}{2}\left[\ln\left|\frac{\overset{2}{\cancel{4}}}{\cancel{2}}\right| + \frac{1}{4}\left(-\frac{1}{4}\right)\right] = \frac{1}{2}\ln(2) - \frac{1}{32}$$ ✓

$\boxed{28,13,2012}$ $f(x) = \frac{2}{3}x$, $x \in [1,2]$ radijus polukruga

$$P\left(Y > \frac{1}{3} \,\middle|\, X = x\right) = \frac{\cancel{\frac{x^2}{2}}\cancel{\pi} - \left(\frac{1}{3}\right)^2\frac{\pi}{2}}{x^2\frac{\pi}{2}} = \frac{x^2 - \frac{1}{9}}{x^2} = 1 - \frac{1}{9x^2}$$

$$P\left(Y > \frac{1}{3}\right) = \int_1^2 \left(1 - \frac{1}{9x^2}\right)\frac{2}{3}x\, dx$$

$$= \int_1^2 \frac{2}{3}x\, dx - \frac{1}{9}\cdot\frac{2}{3}\int_1^2 \frac{1}{x}\, dx =$$

$$= \frac{2}{3}\cdot\frac{1}{2}\left(x^2\Big|_1^2\right) - \frac{2}{27}\left(\ln|x|\Big|_1^2\right) = \underbrace{\frac{\cancel{2}}{\cancel{3}}\cdot\frac{1}{\cancel{2}}(\cancel{3})}_{1} - \frac{2}{27}\underbrace{\left(\ln|2| - \ln|1|\right)}_{\ln\left[\frac{2}{1}\right]\ \ln|2|}$$

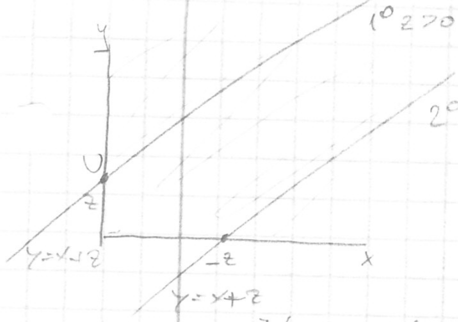
$$= 1 - \frac{2}{27}\ln|2|$$

MAN UTD
by Ferdinac

24.12.2013 $f(x,y) = 9e^{-3x-3y}$, $x>0$, $y>0$

$z = y - x$ $\quad z \in \langle -\infty, +\infty \rangle$, tj. $z \in \mathbb{R}$

$y = z + x$



$$1^\circ \underset{z>0}{g(z)} = \int_U^{I\ (\text{nema izlaza} = 0 + \infty)} f(x,y)\left|\frac{\partial y}{\partial z}\right| dx \qquad y = x+z$$

$$\left|\frac{\partial y}{\partial z}\right| = |1| = 1 \qquad U \Rightarrow y = z \ \text{ i } \ y = z + x$$

$$\cancel{z} = \cancel{z} + x$$
$$0 = x$$

$$= \int_0^{+\infty} 9e^{-3x-3(x+z)}|1|\,dx = 9\int_0^{\infty} e^{-3x-3x-3z} = 9\int_0^{\infty} e^{-6x}\cdot e^{-3z}\,dx$$

$$= 9e^{-3z}\left(\frac{e^{-6x}}{-6}\Big|_0^{\infty}\right) = \cancel{9}^{3} e^{-3z}\left(-\frac{1}{\cancel{6}_2}\right)\left(\frac{1}{e^{\infty}} - \frac{1}{e^0}\right) = -\frac{3}{2}e^{-3z}(-1)$$

$$= \frac{3}{2}e^{-3z}, \quad z \in [0, +\infty\rangle$$

$$G(z) = \int_0^z \frac{3}{2}e^{-3z}\,dz = \frac{3}{2}\left(-\frac{1}{3}\right)\left(e^{-3z} - e^{0}\right) = \frac{1}{2} - \frac{1}{2}e^{-3z}, \ z>0$$

2° $z<0$ $\quad y = x + z$, ali $z<0$ pa je $-z$

$U \Rightarrow y = 0$ i $y = x + z$

$0 = x + z \Rightarrow x = -z$

$$= \int_{-z}^{\infty} 9e^{-3x-3(x+z)}\,dx$$

$$= 9e^{-3z}\int_{-z}^{\infty} e^{-6x}\,dx = \cancel{9}^{3}e^{-3z}\left(-\frac{1}{\cancel{6}_2}\left(e^{-6x}\Big|_{-z}^{\infty}\right)\right) =$$

$$= -\frac{3}{2}e^{-3z}\left(\frac{1}{e^{\infty}}^{\to 0} - e^{-6(-z)}\right) = -\frac{3}{2}e^{-3z}\left(-e^{6z}\right) = \frac{3}{2}e^{-3z}e^{6z} =$$

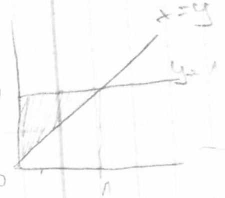
$$= \frac{3}{2}e^{3z}, \quad z \in \langle -\infty, 0\rangle \qquad G(z) = \frac{3}{2}\cdot\frac{1}{3}e^{3z} = \frac{1}{2}e^{3z}, \ z<0$$

$$P(-1 < Z < 1) = G(1) - G(-1) = \left(\frac{1}{2} - \frac{1}{2}e^{-3(1)}\right) - \frac{1}{2}e^{3(-1)}$$

2° slučaj zbog granica 1° slučaj

MAN UTD
by Ferčvac

13.12.2013 $f(x,y) = C(x+y)$ $0 \le x \le y \le 1$



$$1 = \iint f(x,y)\,dx\,dy = C\int_0^1 dy \int_0^y (x+y)\,dx$$

$$= C\int_0^1 dy \left[\int_0^y x\,dx + y\int_0^y dy\right] = C\int_0^1 dy \left(\frac{1}{2}y^2 + \underbrace{y(y)}_{y^2}\right)$$

$$= C\int_0^1 \frac{3}{2}y^2\,dy = C\frac{3}{2}\int_0^1 y^2\,dy = C\frac{3}{2}\left(\frac{y^3}{3}\Big|_0^1\right) = \frac{C}{2}\cdot(1-0) = 1$$

$$\frac{C}{2} = 1 \Rightarrow \boxed{C=2}$$

$$f_X(x) = \int f(x,y)\,dy = \int_x^1 2(x+y)\,dy = 2\left[x\int_x^1 dy + \int_x^1 y\,dy\right]$$

$$= 2\left[x(1-x) + \frac{1}{2}(1-x^2)\right] = 2\left[x - x^2 + \frac{1}{2} - \frac{x^2}{2}\right] =$$

$$= 2\left[\frac{1}{2} + x - \frac{3}{2}x^2\right] = 1 + 2x - 3x^2 = f_X(x)$$

$Z = XY$ hiperbola $Y = \frac{Z}{X}$ $\left|\frac{\partial y}{\partial z}\right| = \left|\frac{1}{x}\right| = \frac{1}{x}$ $z \in [0,1]$

x z y

$\frac{1}{4}$ $\frac{3}{4}$

$$P(Z<z) = P(XY<z) = P\left(Y < \frac{z}{X}\right)$$

$$g_Z(z) = \int_U^I f(x,y)\left|\frac{\partial y}{\partial z}\right|dx$$

$U \Rightarrow y = \frac{z}{x}$ $y = 1$

$\frac{z}{x} = 1 \Rightarrow \underline{z = x}$

$I \Rightarrow y = x$ $y = \frac{z}{x}$

$\frac{z}{x} = x$

$z = x^2 \Rightarrow \underline{x = \sqrt{z}}$

$$g_Z(z) = \int_z^{\sqrt{z}} 2\left(x + \left(\frac{z}{x}\right)\right)\cdot\frac{1}{x}\,dx = (*)$$

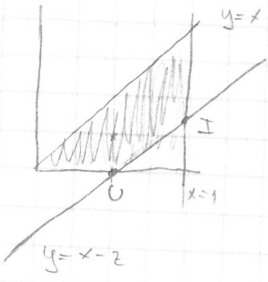
$$(*) = 2\int_z^{\sqrt{z}} \left(1 + \frac{z}{x^2}\right)dx = 2\left(x - \frac{z}{x}\right)\Bigg|_z^{\sqrt{z}} = 2\left(\sqrt{z} - \frac{z}{\sqrt{z}} - z + \frac{\cancel{z}}{\cancel{z}}\right) = 2(1-z),$$

$z \in [0,1]$

$z \in [0,1]$

$$E(Z) = \int_0^1 z\,g(z)\,dz = \int_0^1 (2z - 2z^2)\,dz = \left(\cancel{2}\frac{1}{\cancel{2}}z^2 - 2\frac{z^3}{3}\right)\Bigg|_0^1 = 1 - \frac{2}{3} = \frac{1}{3}$$

$0 \le y \le x \le 1$ $\qquad z = x - y$



$P\left(\frac{1}{2} < z < \frac{3}{4}\right)$ $\qquad f(x) = \frac{1}{P_p \text{(površina)}} = \frac{1}{\frac{1}{2}} = 2$

$z = x - y \qquad z \in [0, 1]$

$$P(Z < z) = P(x - y < z) = P(x - z < y) = \iint_D f(x,y)\,dx\,dy$$

$$= 2\int_0^z dx \int_z^1 dy + 2\int_z^1 dx \int_{x-z}^x dy = 2z - z^2, \quad z \in [0, 1]$$

$U \Rightarrow \quad y = 0 \quad i \quad y = x - z \qquad$ I: uvijek 1

$0 = x - z \Rightarrow x = z$

$$g(z) = \int f(x,y) \left|\frac{\partial y}{\partial z}\right| dx = \int_z^1 2 \cdot |1|\, dx = 2(1 - z) = 2 - 2z, \quad z \in [0,1]$$

$$G(z) = \int_{-\infty}^z g(z)\,dz = \int_0^z (2 - 2z)\,dz = 2z - z^2, \quad z \in [0,1]$$

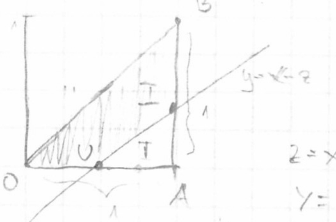
DONJA GR. $[0,1] \in z$

$$P\left(\frac{1}{2} < z < \frac{3}{4}\right) = G\left(\frac{3}{4}\right) - G\left(\frac{1}{2}\right) = \cancel{2}\,\frac{3}{\cancel{4}_2} - \left(\frac{3}{4}\right)^2 - \left(\cancel{2}\,\frac{1}{\cancel{2}} - \left(\frac{1}{2}\right)^2\right)$$

$$P\left(\frac{1}{2} < z < \frac{3}{4}\right) = \frac{3}{16}$$

MAN UTD
by FerOvac

2A, 13, 2013



jednolika $f_x(x) = \frac{1}{P_p} = \frac{1}{\frac{1}{2}} = 2$

površina $\leftarrow P_p$
područja

$Z = X - Y$ $\quad \left|\frac{\partial y}{\partial z}\right| = 1$ $\quad P(Z<z) = (X - z < Y)$

$Y = X - Z$ $\quad z \in [0,1]$

Izlaz

$$g(z) = \int_{\text{Ulaz}} f(x,y) \left|\frac{\partial y}{\partial z}\right| dx = \int_z^1 2 \cdot 1\, dy = 2(1-z), \quad z \in [0,1]$$

$U \Rightarrow \quad y = 0 \quad i \quad y = x - z$

$0 = x - z$

$x = z$

$I \Rightarrow 1$ (uvijek će izaći u 1)

$$G(z) = \int_{-\infty}^{z} g(z)\, dz \Rightarrow G(z) = \int_0^z (2-2z)\, dz = 2(z-0) - 2\cdot\frac{1}{2}(z^2 - 0)$$

$-\infty \to$ donja granica od $z \in [0,1]$

$$G(z) = 2z - z^2, \quad z \in [0,1]$$

$$f_x(x) = \int_{-\infty}^{+\infty} f(x,y)\, dy = \int_0^x 2\, dy = 2(x-0) = 2x, \quad x \in [0,1]$$

važić

$$E(X) = \int_0^1 x\, f_x(x)\, dx = \int_0^1 2x^2\, dx = 2\left(\frac{x^3}{3}\Big|_0^1\right) = \frac{2}{3}$$

PO DEFINICIJI:

$$G(z) = P(Z<z) = P(X - Y < z) = P(X - z < Y) = \iint_D f(x,y)\, dx\, dy =$$

važić važić

$$2\int_0^z dx \int_0^x dy + 2\int_z^1 dx \int_{x-z}^{x} dy = \ldots = 2z - z^2, \quad z \in [0,1]$$

$X \Rightarrow$ aritm. sred. 100 slučajnih $\quad E(x_i)=4 \quad D(x_i)=4$

CGT

CGT $\quad \dfrac{\sum_{i=1}^{n} x_i - n \cdot m}{\sigma \sqrt{n}}$ $\quad m = E(X)$, $\sigma = \sqrt{D(X)}$

aritm. sred.

$$\frac{\sum_{i=1}^{100} x_i - 100 \cdot 4 \quad / :100}{2 \cdot \sqrt{100} \quad / :100} = \frac{\frac{\sum_{i=1}^{100} x_i}{100} - \frac{\cancel{100} \cdot 4}{\cancel{100}}}{\frac{2 \cancel{\sqrt{100}}^{1}}{\cancel{100}_{10}}}$$

$$= \frac{\frac{\sum_{i=1}^{100} x_i}{100} - 4}{\frac{1}{5}} \xrightarrow{D} N(0,1)$$

$\Rightarrow$

$$P(3{,}9 < X < 4{,}1) = P\left(\frac{3{,}9-4}{\frac{1}{5}} < \frac{X-4}{\frac{1}{5}} < \frac{4{,}1-4}{\frac{1}{5}}\right)$$

$$= P(-0{,}5 < X^* < 0{,}5) = \Phi^*(0{,}5) = 0{,}38292$$

28.12.2012 $\quad X_i \sim G(p) = G\left(1 - \frac{1}{4}\right) = G\left(\frac{3}{4}\right)$

neispravni

$S_n = X_1 + \dots + X_n$

$E(X_i) = \frac{1}{p} = \frac{4}{3} \quad D(X) = \frac{1-p}{p^2} = \frac{4}{9}$

$P(S_{200} > t) = 0{,}97$

$$P\left(\frac{S_{200} - \frac{4}{3} \cdot 200}{\frac{2}{3} \sqrt{200}} > \frac{t - \frac{4}{3} 200}{\frac{2}{3} \sqrt{200}}\right) = 0{,}97 \Rightarrow$$

$$\Rightarrow = \frac{1}{2} - \frac{1}{2} \Phi^*\left(\frac{t - \frac{800}{3}}{\frac{20\sqrt{2}}{3}}\right) = 0{,}97 \Rightarrow -\frac{1}{2} \Phi^*\left(\frac{t - \frac{800}{3}}{\frac{20\sqrt{2}}{3}}\right) = 0{,}47$$

$$\Phi^*\left(\frac{\frac{800}{3} - t}{\frac{20\sqrt{2}}{3}}\right) = 0{,}94 \qquad 0{,}65278 = \frac{\frac{800}{3} - t}{\frac{20\sqrt{2}}{3}} \Rightarrow -260{,}5 = -t$$

$t = 260{,}512$

MAN UTD
by ferUrac

3A, 13, 2012 $n=100$ $m = 110$ cm $\sigma = 3$ cm CGT

X = prosječna visina

$P(105 < X < 115) = ?$ $X = \dfrac{\sum_{k=1}^{100} x_k}{100}$

CGT $\dfrac{\sum_{k=1}^{100} x_k - nm}{\sigma\sqrt{n}} = \dfrac{\sum_{k=1}^{100} x_k - 100 \cdot 110 \;\; / :100}{3\sqrt{100} \;\; / :100} =$

$= \dfrac{\overbrace{\dfrac{\sum_{k=1}^{100} x_k}{100}}^{=X} - \dfrac{\cancel{100} \cdot 110}{\cancel{100}}}{\dfrac{3 \cdot 1\cancel{0}}{10\cancel{0}}} = \dfrac{X - 110}{\frac{3}{10}}$

$P(105 < X < 115) = P\left(\dfrac{105-110}{\frac{3}{10}} < \dfrac{X-110}{\frac{3}{10}} < \dfrac{115-110}{\frac{3}{10}}\right)$

$P\left(-\dfrac{50}{3} < X^* < \dfrac{50}{3}\right) = \Phi^*\left(\dfrac{50}{3} = 16{,}666\right) = 1$

$\boxed{3B, 13, 2012}$ 2 žetona $\frac{1}{6}$ 1 žeton $\frac{4}{6}$ 0 žetona $\frac{1}{6}$

$X_k \sim \begin{pmatrix} 2 & 1 & 0 \\ \frac{1}{6} & \frac{4}{6} & \frac{1}{6} \end{pmatrix} \begin{cases} E(X_i) = 2 \cdot \frac{1}{6} + 1 \cdot \frac{4}{6} + 0 \cdot \frac{1}{6} = 1 \\ D(X_i) = 4 \cdot \frac{1}{6} + 1 \cdot \frac{4}{6} - 1 = \frac{1}{3} \end{cases}$

$Y = \sum_{k=1}^{100} x_k$ $E(Y) = 100 \cdot 1$

$D(Y) = 100 \cdot \frac{1}{3} = \frac{100}{3}$

$P(Y \leq 90) = ?$

CGT $n = 100$ partija

$\dfrac{\sum_{k=1}^{100} x_k - n \cdot m}{\sigma\sqrt{n}} = \dfrac{Y - 100 \cdot 1}{\sqrt{\frac{1}{3}} \cdot \sqrt{100}} = \dfrac{Y - 100}{\frac{10\sqrt{3}}{3}} \sim N(0,1)$

$P(Y < 90) = P\left(\dfrac{Y-100}{\frac{10\sqrt{3}}{3}} < \underbrace{\dfrac{90-100}{\frac{10\sqrt{3}}{3}}}_{= -\sqrt{3}}\right)$

$= \dfrac{1}{2} + \dfrac{1}{2}\Phi^*(-\sqrt{3}) = \dfrac{1}{2} - \dfrac{1}{2}\Phi(1{,}732) = \dfrac{1}{2} - \dfrac{1}{2} \cdot 0{,}91637 = 0{,}042$

3A, 13, 2013 $\square\,\square$ $x+y \geq 7$ gubi 1 ž, $x+y < 7$ dobije 1 ž

300 igara

11	21	31	41	51
12	22	32	42	
13	23	33		
14	24			
15				

$2\cdot(5+4+3+2+1)$

$30 \,/\!/\, 72$

$$X_i \sim \begin{pmatrix} -1 & 1 \\ \frac{42}{72} & \frac{30}{72} \end{pmatrix} \qquad E(X_i) = -1\cdot\frac{42}{72} + 1\cdot\frac{30}{72} = -\frac{1}{6}$$

$$D(X_i) = (-1)^2\,\frac{42}{72} + (1)\cdot\frac{30}{72} - \left(-\frac{1}{6}\right)^2 = \frac{35}{36}$$

CGT
$$\frac{\sum_{i=1}^{n} x_i - n\cdot\mu}{\sigma\sqrt{n}} = \frac{\sum_{i=1}^{300} x_i - 300\cdot\left(-\frac{1}{6}\right)}{\sqrt{\frac{35}{36}}\;\sqrt{300}} = \frac{\sum_{i=1}^{300} x_i + 50}{\frac{5\sqrt{105}}{3}}$$

$$P\left(10 + \sum x_i > 1\right) = P\left(\sum x_i > -9\right) = P\left(\frac{\sum_{i=1}^{300} x_i + 50}{\frac{5\sqrt{105}}{3}} > \underbrace{\frac{-9+50}{\frac{5\sqrt{105}}{3}}}_{=2{,}400}\right)$$

$$= P(X > 2{,}400) = \frac{1}{2} - \frac{1}{2}\,\Phi^*(2{,}400) = \frac{1}{2} - \frac{1}{2}\cdot 0{,}98360 = 0{,}0082$$

3B, 13, 2013 $\square$ $x/2$ 0 ž, x nije/2 x ž

$n = 100$ igara

$P\left(150 < \sum_{1}^{100} x_i < 170\right) = ?$

$$X_i \sim \begin{pmatrix} 0 & 1 & 3 & 5 \\ \frac{3}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \qquad E(X_i) = \frac{1}{6}\cdot 1 + \frac{1}{6}\cdot 3 + \frac{1}{6}\cdot 5 = \frac{3}{2}$$

2, 4, 6

$$D(X_i) = 1^2\cdot\frac{1}{6} + 3^2\cdot\frac{1}{6} + 5^2\,\frac{1}{6} - \left(\frac{3}{2}\right)^2 = \frac{43}{12}$$

CGT
$$\frac{\sum_{i=1}^{n} x_i - n\cdot m}{\sigma\sqrt{n}} = \frac{\sum_{i=1}^{100} x_i - 100\cdot\frac{3}{2}}{\sqrt{\frac{43}{12}}\;\sqrt{100}} = \boxed{\frac{\sum_{i=1}^{100} x_i - 150}{\frac{5\sqrt{129}}{3}}} \sim N(0,1)$$

$$P\left(150 < \sum_{i=1}^{100} x_i < 170\right) = P\left(\underbrace{\frac{150-150}{\frac{5\sqrt{129}}{3}}}_{=0} < X^* < \frac{170-150}{\frac{5\sqrt{129}}{3}}\right)$$

$$P(0 < X^* < 1{,}0565) = \frac{1}{2}\,\Phi^*(0) + \frac{1}{2}\,\Phi^*(1{,}056)$$

$$= \left(\frac{1}{2}\cdot 0 + \frac{1}{2}\cdot 0{,}71086\right) = 0 + 0{,}35543 = 0{,}35543$$

MAN UTD
by [illegible]

CGT

3A, 12, 2012

max 1000 kg

$n = 11$ kutija $m_i = 80$ $\sigma_i = 15$

CGT $\dfrac{\sum_{i=1}^{11} x_i - n \cdot m}{\sigma \sqrt{n}} = \dfrac{\overbrace{\sum_{i=1}^{11} x_i}^{= X} - 11 \cdot 80}{15 \cdot \sqrt{11}} \sim N(0,1)$

$P(X < 1000) = ?$ $P\left(\dfrac{X - 880}{15\sqrt{11}} < \overbrace{\dfrac{1000 - 880}{15\sqrt{11}}}^{= 2{,}412} \right)$

$= \dfrac{1}{2} + \dfrac{1}{2} \Phi^*(2{,}412) = \dfrac{1}{2} + \dfrac{1}{2} \, 0{,}98405 = 0{,}992$

3B, 12, 2012

$m = E(x_i) = 2{,}4$ $\sigma_{x_i} = 2$

$n = 100$ ljudi

$P(X < 250)$

CGT $\dfrac{\sum_{i=1}^{100} x_i - nm}{\sigma \sqrt{n}} = \dfrac{\sum_{i=1}^{100} x_i - 100 \cdot 2{,}4}{2 \cdot \sqrt{100}} = \dfrac{\overbrace{\sum_{1}^{100} x_i}^{X} - 240}{2 \cdot 10} \sim N(0,1)$

$P(X < 250) = P\left(\dfrac{X - 240}{20} < \overbrace{\dfrac{250 - 240}{20}}^{= 0{,}5} \right) = \dfrac{1}{2} + \dfrac{1}{2}\Phi(0{,}5)$

$= \dfrac{1}{2} + \dfrac{1}{2} \cdot 0{,}38292 = 0{,}69146$