Pois. Bin, geom, eks, norm

(2)

2. Binomna – n puta ponavljam pokus, k puta se nešto stvarno

$X \sim B(n, p)$ $\qquad P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$

pokusaji ~~godine~~

$n = 4 \quad p = ?$

$L(p, x_1, x_2, x_3) = P(x_1 = 1) \cdot P(x_2 = 3) \cdot P(x_3 = 0)$

1 godina, 2. god, 3 god

$= \binom{4}{1} p (1-p)^3 \cdot \binom{4}{3} p^3 (1-p) \cdot \binom{4}{0} p^0 (1-p)^4$

$= 16 p^4 (1-p)^8$

$\ln L = \ln 16 + 4 \ln p + 8 \ln(1-p) \;/\; \frac{d}{dp}$

$= 0 + \frac{4}{p} + 8 \frac{1}{1-p} \cdot (-1) = 0 \;/ \cdot (p - p^2)$

$4 - 4p - 8p = 0$

$4 = 12p$

$p = \frac{1}{3}$ //

3.

x_j	115	120	125	130	135	140
n_j	3	4	7	6	3	2

$n = 25$

a) $\bar{X} = \frac{\sum n_j \cdot x_j}{n} = \frac{3 \cdot 115 + \ldots + 2 \cdot 140}{25} = 126.6$

$s^2 = \frac{1}{n-1} \sum n_j (x_j - \bar{x})^2 = 51.5$

pouzdanost – nivo pouzdanosti – p, nivo značajnosti – α, $\alpha + p = 1$

b) $p = ?$

$P\left(\overbrace{\bar{x} - t_{n-1, 1-\frac{\alpha}{2}} \cdot \frac{\hat{s}}{\sqrt{n}}}^{d} \le a \le \overbrace{\bar{x} + t_{n-1, 1-\frac{\alpha}{2}} \cdot \frac{\hat{s}}{\sqrt{n}}}^{c}\right) = p$

$\bar{x} = 126{,}6 \qquad \hat{s} = \sqrt{51{,}5} \qquad \sqrt{n} = \sqrt{25}$

$c - d < 4{,}912$

$t_{n-1, 1-\frac{\alpha}{2}}$

$2 t_{n-1, 1-\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}} < 4{,}912$

$t_{n-1,1-\frac{\alpha}{2}} \leq 1,711$

$t_{24,1-\frac{\alpha}{2}} \leq 1,711$ $\alpha = 0,1$

$p = 0,9$

c)
$$P\left(\frac{(n-1)\hat{s}^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}} \leq \sigma^2 \leq \frac{(n-1)\hat{s}^2}{\chi^2_{n-1,\frac{\alpha}{2}}}\right) = p$$

$$P\left(\frac{24 \cdot 51,5}{36,415} \leq \sigma^2 \leq \frac{24 \cdot 51,5}{13,848}\right) = p$$

$\chi^2_{24,\,0.95} = 36,415$ $\chi^2_{24,\,0.05} = 13,848$

$$P\left(33,94 \leq \sigma^2 \leq 89,25\right) = p = 0.9.$$

4. $n = 215$

$m = 5$

$p = 0,95$

$\alpha = 0,05$

$u_{1-\frac{\alpha}{2}} = u_{0,975} = 1,96$

$$p_{1,2} = \hat{p} \mp u_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} , \quad \hat{p} = \frac{m}{n}$$

$n = 215$

postotak zadan u zadatku $\hat{p} = \frac{5}{215}$

$$p_{1,2} = \frac{5}{215} \mp 1,96 \cdot \sqrt{\frac{\frac{5}{215} \cdot \frac{210}{215}}{215}}$$

$$= 0,0233 \pm 0.02$$

$$P(0.0033 < p < 0,0433) = 95\%$$

b) $p = 0.95$

$\alpha = 0.05$

$n = ?$

$$\hat{p} + u_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq 0,04$$

$$\frac{5}{215} + 1,96 \sqrt{\frac{\frac{5}{215} \cdot \frac{210}{215}}{n}} \leq 0,04$$

$$n \geq 313,47$$

n: treba ih proizvesti 314

5. $n = 15$ 3 2 4 5 4 1 3 5 1 5 2 3 4 5 3

$\bar{x} = 3.5$

H_0

$\alpha = 0,1$

a) 1 prosj. ocjena

$H_0 \ldots \quad a = 3.5$

$H_1 : \quad a < 3.5$

$$T = \frac{\bar{x} - a_0}{\frac{s}{\sqrt{n}}} = -0,468$$

$\bar{x} = 3.33$

$s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2 = 1,976$

$n = 15$

$\hat{t} < -t_{n-1, 1-\alpha}$

$-0,468 < -t_{14, 0.1}$

$-0,468 < -1,345$

ako vrijedi odbacij
ako ne vrijedi ne odbacuj

H_0 se ne odbacuje

! $\boxed{\text{jednostrana}}$ hipoteza $\alpha \to$ dva puta veći

prof. govori istinu.

b) 2 post će manje od 10%

$H_0 \ldots \quad a = \overset{p_0}{0.1} \qquad q_0 = 1 - p_0$

$H_1 \ldots \quad a > 0.1$

$$u = \left(\frac{m}{n} - p_0\right)\sqrt{\frac{n}{p_0 q_0}}$$

$$u = \left(\frac{2}{15} - 0.1\right)\sqrt{\frac{15}{0.1 \cdot 0.9}} = 0,43$$

$\hat{u} > u_{1-\alpha}$

$0,43 > u_{0.9}$

$0,43 > 1,281$ Ne, H_0 ne odbacujemo, prof. govori istinu.

6. $n=60$

$\overline{x}=11{,}68 \qquad s_x^2=22{,}9 \Rightarrow n=60$

$\overline{y}=13{,}55 \qquad s_y^2=23{,}38 \Rightarrow m=60$

$\alpha=0{,}05$

$H_0 \ldots \quad \overline{x}=\overline{y}$

$H_1 \ldots \quad \overline{x}\neq\overline{y}$

$$S_z^2=\frac{1}{n+m-2}\left[(n-1)s_x^2+(m-1)s_y^2\right]$$

$$=\frac{1}{118}\left[59\cdot 22{,}9+59\cdot 23.38\right] = 23{,}14 = e^{\pi}$$

$$T=\frac{\overline{x}-\overline{y}}{\sqrt{S_z^2}}\sqrt{\frac{m\cdot n}{m+n}} = \frac{11{,}68-13{,}55}{\sqrt{23{,}14}}\cdot\sqrt{\frac{\cancel{60\cdot 60}}{\frac{120}{2}}}^{\sqrt{30}} = -2{,}186$$

$$|\hat{t}| > t_{n+m-2,\,1-\frac{\alpha}{2}} = t_{118,\,1-\frac{\alpha}{2}} \quad \alpha=0{,}05$$

$2{,}186 > 1{,}98$ ✓ H_0 možemo odbaciti

7. $n=800$ $\qquad \alpha=0{,}1$

x_j	n_j	p_j	$n_j p_j$	$\frac{(n_j-np_j)^2}{np_j}$
0	74	$\frac{1}{10}$	80	$\frac{(74-80)^2}{2}=0{,}45$
1	92	.	.	$\frac{(92-80)^2}{80}=1{,}8$
2	83	.	.	$\frac{(83-80)^2}{80}=0{,}1125$
3	79	.	.	
4	80	.	.	
5	73	.	.	
6	77	.	.	
7	75	.	.	
8	76	.	.	0,2
9	91	1/10		1,515
			$=800$	$\Sigma = 5{,}152 = \chi^2_\alpha$

korisška upi.
koliko se puta
treba pojaviti
koliko puta
– ovisi o razdiobi u zadatku
ovdje je normalna

Ako parametar izračunam $r=1,2,\ldots$ / ako je zadan $r=0$.

S.S. $m-r-1$
$10-0-1=9$

tu nismo računali nikakav parametar

$\chi^2_{9,\,0.9}=14{,}684$

8.

x_j°	0	1	2	3	4
n_j'	130	52	19	3	1

$P \sim \frac{\lambda^k}{k!} e^{-\lambda}$

$\boxed{\bar{x} = E(x) = \lambda}$

$\alpha = 0{,}05$

RAZREDI < 5

	x_j	n_j	p_j°	$n_j \cdot p_j$	$\frac{(n_j - np_j)^2}{np_j^\circ}$
$k=$	0	130	$\frac{\lambda^0}{0!} e^{-0.5} = 0.61$	$0{,}61 \cdot 205 = 125{,}05$	0,196
$k=$	1	52	$\frac{\lambda^1}{1!} e^{-0.5} = 0.3$	61,5	1,47
$k=$	2	19	$\frac{\lambda^2}{3!} e^{-0.5} = 0.076$	15,58	$\frac{(23-18{,}565)^2}{18{,}565} = 1.06$
$k=$	3	3 } 23	$= 0{,}013$	2,67 } 18,565	
$k=$	4	1	$= 0{,}0016$	0,32	
		$n=205$	≈ 1	≈ 205	$\chi_q^2 \quad \Sigma = 2{,}726$

$\bar{x} = \lambda = 0{,}5$

$\chi^2_{m-r-1,\,1-\alpha} = \chi^2_{3-1-1,\,0.95}$

RAZRED NAKON SPAJANJA

$= \chi^2_{1,\,0.95} = 3{,}841$

$2{,}726 < 3{,}841$ ✓

H0 prihv.

9.

x_j	$[a, b]$	n_j	p_j°
5	0–10	9	$e^{0\lambda}\ e^{-10\lambda}$
15	10–20	27	
25	20–30	33	
35	30–40	16	
		$n=85$	

srednja vrijednost svakog intervala

ista duljina između svakog

$\lambda = \frac{1}{\bar{x}} = \frac{1}{21{,}588}$

$\bar{x} = \frac{\Sigma n_j \cdot x_j}{n} = \frac{5 \cdot 9 + 27 \cdot 15 + 33 \cdot 25 + 35 \cdot 16}{85}$

$= 21{,}588$

PREDZAVRŠNI

① 2007 21 | PZ1
g. pog.
konvergencije sve
nejednakosti sve
C.G.T
prva 4 zd. iz 6 zb.

1. $x_1, \dots, x_n$

$$f(x) = \frac{x}{\lambda^2} e^{-x/\lambda} \qquad , x > 0$$

a) $\lambda = ?$

fja. najveće izglednosti $\quad L(\lambda, x_1, \dots, x_n) = f(x_1) \cdot \dots \cdot f(x_n)$

↓

traženje maximuma (derivira) po parametru koji računaš $= 0$)

$$= \frac{x_1}{\lambda^2} \cdot e^{-\frac{x_1}{\lambda}} \cdot \dots \cdot \frac{x_n}{\lambda^2} \cdot e^{-\frac{x_n}{\lambda}}$$

$$= \frac{x_1 \cdot \dots \cdot x_n}{\lambda^{2n}} \cdot e^{-\frac{1}{\lambda}(x_1 \cdot \dots \cdot x_n)}$$

$$\ln L = \ln(x_1 \cdot \dots \cdot x_n) - \ln(\lambda^{2n}) + \ln e^{-\frac{1}{\lambda}(x_1 + \dots + x_n)}$$

$$\ln L = \underbrace{\ln(x_1 \cdots x_n)}_{\text{konstanta}} - 2n \ln \lambda - \frac{1}{\lambda}(x_1 + \dots + x_n) \cdot \overset{1}{\cancel{\ln e}} \quad \Big/ \frac{d}{d\lambda}$$

$$= 0 - 2n \cdot \frac{1}{\lambda} + \frac{1}{\lambda^2}(x_1 + \dots + x_n) = 0 \quad / \cdot \lambda^2$$

$$\lambda = \frac{x_1 + \dots x_n}{2n} = \frac{1}{2} \cdot \bar{x} \quad //$$

b) $E(\Theta) = \vartheta \quad$ očekivanje od statistike, mora biti jednako pravoj vrijednosti

$$E\left(\frac{\bar{x}}{2}\right) = \lambda$$

$$\frac{1}{2} E(\bar{x}) = \frac{1}{2} E(x) = \frac{1}{2} \int_{-\infty}^{+\infty} x \cdot f(x)\, dx = \frac{1}{2} \int_0^{\infty} \frac{x^2}{\lambda^2} e^{-\frac{x}{\lambda}} dx = \text{2 put parc.}$$

$$= \frac{1}{2} \cdot 2\lambda = \lambda \quad // \checkmark$$