Ovaj PDF sadrži skenirane postupke svih 2.MI 2011.-2007.

Postupci su poredani od 2011. do 2007.

Riješio i ustupio na skeniranje

fer0vac

skenirao

SipE

(1)

$$f(x) = \begin{cases} x^2, & x \in [0,1\rangle \\ 2x - x^2, & x \in [1,2] \end{cases}$$

2MI, 2011 (1)

$F(x), E(X), P(0{,}5 < X < 1{,}5) = ?$

$$f(x) = F'(x) \Rightarrow F(x) = \int_{-\infty}^{x} f(t)\,dt$$

1° za $x \in [0,1\rangle$: $F(x) = \int_{-\infty}^{x} f(t)\,dt = \int_{0}^{x} t^2\,dt = \frac{x^3}{3}$

2° za $x \in [1,2]$:

$$F(x) = \int_{-\infty}^{x} f(t)\,dt = \underbrace{\int_{-\infty}^{0} 0\,dt}_{0} + \int_{0}^{1} t^2\,dt + \int_{1}^{x} (2t - t^2)\,dt =$$

$$= 0 + \left(\frac{1}{3} - \frac{0}{3}\right) + \left(\cancel{2}\frac{t^2}{\cancel{2}} - \frac{t^3}{3}\Big|_{1}^{x}\right) =$$

$$= \frac{1}{3} + \left(x^2 - \frac{x^3}{3}\right) - \left(1 - \frac{1}{3}\right) = \frac{1}{3} + x^2 - \frac{x^3}{3} - 1 + \frac{1}{3}$$

$$F(x) = -\frac{1}{3}x^3 + x^2 + \left(-\frac{1}{3}\right)$$

$$E(X) = \int_{-\infty}^{+\infty} x f(x)\,dx = \int_{0}^{1} x x^2\,dx + \int_{1}^{2} x(2x - x^2)\,dx =$$

$$= \frac{x^4}{4}\Big|_{0}^{1} + \left(2\frac{x^3}{3} - \frac{x^4}{4}\Big|_{1}^{2}\right) = \left(\frac{1}{4} - 0\right) + \left(\frac{2\cdot 8}{3} - \frac{16}{4} - \left(\frac{2}{3} - \frac{1}{4}\right)\right)$$

$$E(X) = \frac{1}{4} + \frac{16}{3} - 4 - \frac{2}{3} + \frac{1}{4} = \frac{7}{6} \qquad E(X) = \frac{7}{6}$$

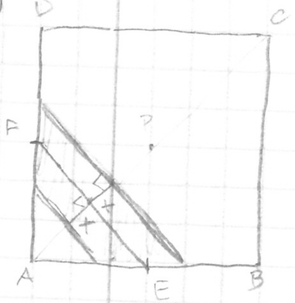
$$P(0{,}5 < X < 1{,}5) = \int_{1/2}^{1} x^2\,dx + \int_{1}^{3/2} 2x - x^2\,dx = \frac{x^3}{3}\Big|_{1/2}^{1} + \left(\cancel{2}\frac{x^2}{\cancel{2}} - \frac{x^3}{3}\right)\Big|_{1}^{3/2}$$

$$= \underbrace{\left[\frac{1}{3} - \frac{0{,}5^3}{3}\right]}_{\frac{7}{24}} + \underbrace{\left(\frac{9}{4} - \frac{\cancel{27/8}}{3}\right)}_{\frac{9}{8}} - \underbrace{\left(1 - \frac{1}{3}\right)}_{\frac{2}{3}}$$

$$= \frac{7}{24} + \frac{9}{8} - \frac{2}{3} = \frac{3}{4}$$

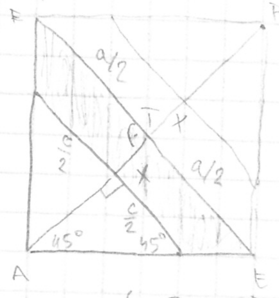
MANUTD
by ferdvac

②



$|AC| = 1\cdot\sqrt{2}$ (dijagonala)

$x \in \left[0, \frac{3}{4}\sqrt{2}\right]$ $\quad |AP| = \frac{2}{4}\sqrt{2}$



$|AT| = \frac{1}{4}\sqrt{2}$

1° $\quad x \in \left[0, \frac{1}{4}\sqrt{2}\right]$

$$P(X<x) = \frac{\frac{(a+c)}{2}\cdot v}{1^2}$$

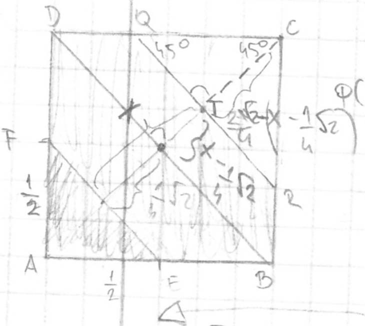
$$\frac{a}{2} = \left(\frac{1}{4}\sqrt{2} + x\right) \qquad \frac{c}{2} = \left(\frac{1}{4}\sqrt{2} - x\right)$$

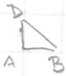
$$\boxed{a = 2\left(\frac{1}{4}\sqrt{2} + x\right)} \quad \boxed{v = 2x} \qquad \Downarrow \quad \boxed{c = 2\left(\frac{1}{4}\sqrt{2} - x\right)}$$

$$P(X<x) = \frac{\left(2\left(\frac{1}{4}\sqrt{2} + x\right) + 2\left(\frac{1}{4}\sqrt{2} - x\right)\right)\cancel{2}x}{\cancel{2}\cdot 1^2} =$$

$$= \left(\frac{2}{4}\sqrt{2} + \cancel{2x} + \frac{2}{4}\sqrt{2} - \cancel{2x}\right)\cdot x = x\sqrt{2}, \quad x \in \left[0, \frac{1}{4}\sqrt{2}\right]$$

2° $\quad x \in \left[\frac{1}{4}\sqrt{2}, \frac{3}{4}\sqrt{2}\right]$ $\qquad P(X<x) = \frac{\left(\frac{a+c}{2}\right)\cdot v}{1^2}$



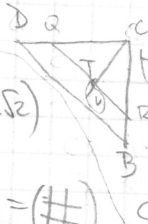
$P(X<x) =$  $+$  $= (*)$

$\frac{1\cdot 1}{2}$

$$P_{\triangle} = \left(\frac{a+c}{2}\right)\cdot v$$

$a = 1\cdot\sqrt{2}$ (dijagonala)

$V = x - \frac{1}{4}\sqrt{2}$



$$|TC| = \frac{2}{4}\sqrt{2} - \left(x - \frac{1}{4}\sqrt{2}\right) = \frac{3}{4}\sqrt{2} - x$$

$$|TC| = |TR| = \frac{c}{2}$$

$$c = 2\left(\frac{3}{4}\sqrt{2} - x\right) = \frac{3}{2}\sqrt{2} - 2x$$

$$P_{\triangle} = \left(\frac{\sqrt{2} + \frac{3}{2}\sqrt{2} - 2x}{2}\right)\cdot\left(x - \frac{1}{4}\sqrt{2}\right)$$

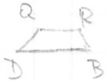
$$= \left(\frac{\frac{5\sqrt{2}}{2} - \frac{4x}{2}}{2}\right)\left(x - \frac{1}{4}\sqrt{2}\right) = (\#)$$

② (NASTAVAK) ⇒ $\boxed{1a}$ (samo nastavak DRUGOGA zad.)   ZM1, 2011   1a

$$(\#) = \left(\frac{5\sqrt{2}-4x}{4}\right)\left(x-\frac{1}{4}\sqrt{2}\right) = \left(\frac{5}{4}\sqrt{2}-x\right)\left(x-\frac{1}{4}\sqrt{2}\right)$$

$$= \frac{5}{4}\sqrt{2}\,x - \frac{5(\cancel{\sqrt{2}})^{\cancel{2}}}{16} - x^2 + \frac{1}{4}\sqrt{2}\,x = \frac{3}{2}\sqrt{2}\,x - x^2 - \frac{10}{16}$$

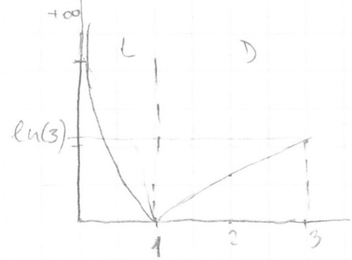
$$\frac{6}{4}\sqrt{2}x = \frac{3}{2}\sqrt{2}\,x$$

$F(x) = P(X < x) =$  $+$  $= \left[\frac{1}{2}\right] + \left[-x^2 + \frac{3}{2}\sqrt{2}\,x - \cancel{\frac{10}{16}}\,\frac{5}{8}\right]$

$$F(x) = -x^2 + \frac{3}{2}\sqrt{2}\,x + \left(-\frac{5}{8}+\frac{1}{2}\right) = -x^2 + \frac{3}{2}\sqrt{2}\,x - \frac{1}{8},\quad x \in \left[\frac{\sqrt{2}}{4}, \frac{3}{4}\sqrt{2}\right]$$

MAN UTD
by Rerlvac

③ $Y = |\ln|X||$ $\quad f(x) = \frac{2}{9}x, \ x \in \langle 0, 3 \rangle$



1° $x \in \langle 0, 1 \rangle$
L

$$-y < \ln(x) < y \quad / e$$

$$e^{-y} < x < e^{y}$$

L D

$$x = e^{-y} \quad \left|\frac{dx}{dy}\right| = \left|-1\, e^{-y}\right| = e^{-y}$$

$$g_1(y) = f(x)\left|\frac{dx}{dy}\right| = \frac{2}{9} x\, e^{-y} = \frac{2}{9} e^{-y} e^{-y} = \frac{2}{9} e^{-2y},$$

$$y \in (0, +\infty)$$

2° $x \in \langle 1, 3 \rangle$ $\quad x = e^{y} \quad \left|\frac{dx}{dy}\right| = \left|e^{y} \cdot 1\right| = e^{y}$
D

$$g_2(y) = f(x)\left|\frac{dx}{dy}\right| = \frac{2}{9} x\, e^{y} = \frac{2}{9} e^{y} e^{y} = \frac{2}{9} e^{2y},$$

$$y \in \langle 0, \ln(3) \rangle$$

$$g(y) = \begin{cases} \frac{2}{9} e^{-2y} + \frac{2}{9} e^{2y}, & y \in \langle 0, \ln(3) \rangle \\ \frac{2}{9} e^{-2y}, & y \in \langle \ln(3), +\infty \rangle \end{cases}$$

⑤ 30 pitanja, T i N
$P(X \geq 16) = ?$

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$X \sim B(n = 30, p = 0{,}5) \approx N(\mu = np = 15, \sigma^2 = npq = 7{,}5)$$

$$P(X \geq 16) = P\left(\frac{X - 15}{\sqrt{7{,}5}} \geq \frac{16 - 15}{\sqrt{7{,}5}}\right) = P(\hat{X} \geq 0{,}36514) =$$

$$= \frac{1}{2} - \frac{1}{2} \underbrace{\Phi^{*}(0{,}36514)}_{0{,}2845} = \frac{1}{2} - \frac{1}{2} \cdot 0{,}2845 = 0{,}3575$$

MAN UTD
by FerOvac

(1) c, $P\left(\frac{1}{2} < x < \frac{3}{2}\right)$, $D(X) = ?$

X, $f(x) = 2 - cx$, $x \in [0, 1]$

$$\int_{-\infty}^{+\infty} f(x)\,dx = 1 \Rightarrow 1 = \int_0^1 2 - cx = \left(2x - c\frac{x^2}{2}\Big|_0^1\right) =$$

$$= \left(2 - c\frac{1}{2}\right) - (0 - 0) = 2 - \frac{c}{2}$$

$$1 = 2 - \frac{c}{2} \Rightarrow -1 = -\frac{c}{2} \quad / \cdot (-2) \Rightarrow c = 2$$

$$P\left(\frac{1}{2} < x < \frac{3}{2}\right) = \int_{1/2}^{3/2} f(x)\,dx \overset{\text{PAZI} \Rightarrow x \in [0,1]}{\underset{\text{PAZI} \Rightarrow x \in [0,1]}{=}} = \int_{1/2}^{1} 2 - 2x\,dx =$$

$$= \left(2x - 2\frac{x^2}{2}\right)\Big|_{1/2}^{1} = \left(2 \cdot 1 - \cancel{2} \cdot \frac{1}{\cancel{2}}\right) - \left(\cancel{2} \cdot \frac{1}{\cancel{2}} - \cancel{2} \cdot \frac{\frac{1}{4}}{\cancel{2}}\right) =$$

$$= (2 - 1) - \underbrace{\left(1 - \frac{1}{4}\right)}_{3/4} = 1 - \frac{3}{4} = \frac{1}{4}$$

$$D(X) = \int_{-\infty}^{+\infty} x^2 f(x)\,dx - \underbrace{\left(\int_{-\infty}^{+\infty} x f(x)\,dx\right)^2}_{(E(X))^2} =$$

$$= \int_0^1 \underbrace{x^2(2 - 2x)}_{2x^2 - 2x^3}\,dx - \left(\int_0^1 \underbrace{x(2 - 2x)}_{2x - 2x^2}\,dx\right)^2 =$$

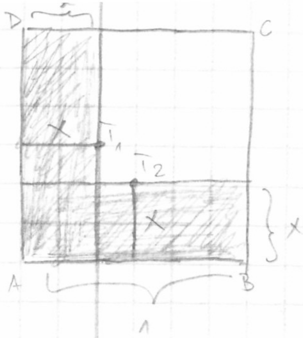
$$= \left(2\frac{x^3}{3} - 2\frac{x^4}{4}\right)\Big|_0^1 - \left(\left(2\frac{x^2}{2} - 2\frac{x^3}{3}\right)\Big|_0^1\right)^2 =$$

$$= \left(2 \cdot \frac{1}{3} - \cancel{2} \cdot \frac{1}{\cancel{4}_2}\right) - \left(\left(\cancel{2} \cdot \frac{1}{\cancel{2}} - 2 \cdot \frac{1}{3}\right)\right)^2 =$$

$$= \frac{1}{6} - \left(\left(1 - \frac{2}{3}\right)\right)^2 = \frac{1}{6} - \left(\frac{1}{3}\right)^2 = \frac{1}{6} - \frac{1}{9} = \frac{1}{18}$$

MAN UTD
by ferChac

(2)



$x \in [0, 1]$

$F(x) = P(X < x) = \dfrac{1^2 - (1-x)(1-x)}{1^2} =$

$= \dfrac{1 - (1-x)^2}{1} = \dfrac{1 - (1 - 2x + x^2)}{1} =$

$= 1 - 1 + 2x - x^2 = 2x - x^2$

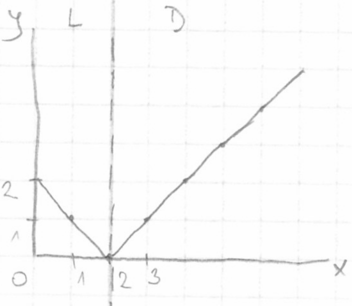
$F(x) = 2x - x^2 \quad , \quad x \in [0, 1]$

$f(x) = F'(x) = 2 - 2x \quad , \quad x \in [0, 1]$

$E(X) = \int_{-\infty}^{+\infty} x f(x)\, dx = \int_0^1 2x - 2x^2\, dx = \left(2\dfrac{x^2}{2} - \dfrac{2x^3}{3}\right)\Big|_0^1 =$

$E(X) = \left(1 - \dfrac{2}{3}\right) - (0 - 0) = \dfrac{1}{3}$

(3)(b) $Y = |X - 2| \qquad f(x) = e^{-x} \, , \; x > 0$



inverz

$-Y < X - 2 < Y \quad / +2$

$2 - Y < X < Y + 2$

ili

$-(x-2) < Y < x - 2$

$-x + 2 < Y < x - 2$

$-x + 2 = Y \qquad x - 2 = Y$

$2 - Y = x \qquad x = Y + 2$

1° L $\quad x \in \langle 0, 2] \quad y \in \langle 0, 2] \qquad x = 2 - y \; (L) \qquad \left|\dfrac{dx}{dy}\right| = |0 - 1| = 1$

$g_1(y) = f(x) \left|\dfrac{dx}{dy}\right| = e^{-(2-y)} \cdot |1| = e^{y-2} \quad , \; y \in \langle 0, 2]$

2° D $\quad x \in \langle 2, +\infty\rangle \quad y \in \langle 0, +\infty\rangle \qquad x = 2 + y \; (D) \qquad \left|\dfrac{dx}{dy}\right| = |0 + 1| = 1$

$g_2(y) = f(x) \left|\dfrac{dx}{dy}\right| = e^{-(2+y)} |1| = e^{-y-2} \quad , \; y \in \langle 0, +\infty\rangle$

$$g(y) = \begin{cases} e^{y-2} + e^{-y-2} & , \; y \in \langle 0, 2] \\ e^{-y-2} & , \; y \in \langle 2, +\infty\rangle \end{cases}$$

(4) (b)

DOMAĆE $\mu_1 = 180$, $\sigma_1 = 20$ $\mathcal{N}_1(180, 400)$

GMOL $\mu_2 = 220$, $\sigma_2 = 5$ $\mathcal{N}_2(220, 25)$

paketi po 4 jabuke = 2 domaće i 2 GMOL (X)

$P(820 < X < 1000) = ?$

$X = X_1 + X_1 + X_2 + X_2$ $\quad \cancel{X = 2X_1 + 2X_2}$ KRIVO

$\sim \mathcal{N}_{1122}(\mu_1 + \mu_1 + \mu_2 + \mu_2, \sigma_1^2 + \sigma_1^2 + \sigma_2^2 + \sigma_2^2)$

$\mathcal{N}_{1122}(180 + 180 + 220 + 220, 20^2 + 20^2 + 5^2 + 5^2)$

$X \sim \mathcal{N}_{1122}(800, 850)$

$\quad\; \mathcal{N} \quad\; \mu \quad\; \sigma^2$

$$P(820 < X < 1000) = P\left(\frac{820 - 800}{\sqrt{850}} < \frac{X - 800}{\sqrt{850}} < \frac{1000 - 800}{\sqrt{850}}\right)$$

$$= P\left(\frac{20}{5\sqrt{34}} < \tilde{X} < \frac{200}{5\sqrt{34}}\right) = \frac{1}{2}\underbrace{\Phi^*(6{,}85994)}_{1} - \frac{1}{2}\Phi^*(0{,}68599)$$

$$= \frac{1}{2} \cdot 1 - \frac{1}{2}\, 0{,}50729 = 0{,}246 \Rightarrow 24{,}6\ \%$$

MAN UTD
by ferovac

1. $X \quad f(x) = \frac{c}{x^3}, \quad x > 1$

$P(2 < X < 3), \ E(X) = ?$

$$1 = \int_{-\infty}^{\infty} f(x)\,dx = \int_{1}^{+\infty} \frac{c}{x^3}\,dx = c\int_{1}^{+\infty} x^{-3}\,dx = c\left(\frac{x^{-2}}{2}\Big|_{1}^{+\infty}\right) =$$

$$1 = c\left(\underbrace{\frac{1}{\infty(-2)}}_{\lim \to 0} - \frac{1}{1(-2)}\right) = c\left(\frac{-1}{-2}\right)^{\frac{1}{2}} = 1 \Rightarrow \boxed{c = 2}$$

$$f(x) = \frac{2}{x^3}, \quad x > 1$$

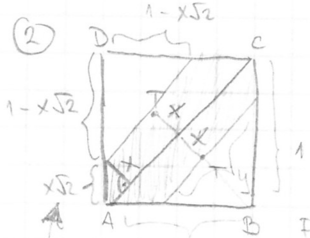
$$E(X) = \int_{-\infty}^{+\infty} x\, f(x)\,dx = \int_{1}^{+\infty} \cancel{x} \cdot \frac{2}{x^{\cancel{3}\,2}}\,dx = 2\int_{1}^{+\infty} x^{-2}\,dx =$$

$$= 2\left[\frac{x^{-1}}{-1}\Big|_{1}^{+\infty}\right] = -2\left(\underbrace{\frac{1}{\infty}}_{\lim \to 0} - \frac{1}{1}\right) = 2$$

$$P(2 < X < 3) = \int_{2}^{3} f(x)\,dx = \int_{2}^{3} 2x^{-3}\,dx = 2\left(\frac{x^{-2}}{-2}\Big|_{2}^{3}\right) =$$

$$= \frac{2}{-2}\left(\frac{1}{3^2} - \frac{1}{2^2}\right) = -1\left(\frac{1}{9} - \frac{1}{4}\right) = -\frac{1}{9} + \frac{1}{4} = \frac{5}{36}$$

2.

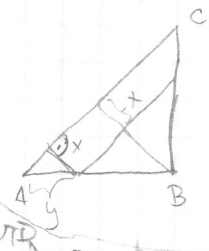


$$x \in \left[0, \frac{\sqrt{2}}{2}\right]$$

$$F(x) = P(X < x) = \frac{m(G_x)}{m(\Omega)} = \frac{1^2 - \cancel{2}\cdot\frac{1}{\cancel{2}}(1 - x\sqrt{2})^2}{1^2}$$

$$F(x) = 2x\sqrt{2} - 2x^2, \quad x \in \left[0, \frac{\sqrt{2}}{2}\right]$$

$$f(x) = 2\sqrt{2} - 4x, \quad x \in \left[0, \frac{\sqrt{2}}{2}\right]$$



$$\frac{x}{\frac{\sqrt{2}}{2}} = \frac{y}{1} \Rightarrow \frac{\sqrt{2}}{2}\, y = x \ \Big/ \cdot \frac{2}{\sqrt{2}}$$

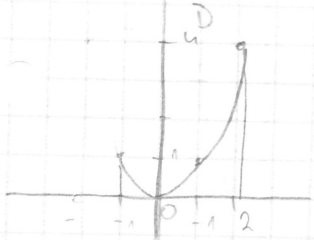
$$y = x\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}\,x = x\sqrt{2}$$

MAN UTD
by FerOvac

③ $Y = X^2$

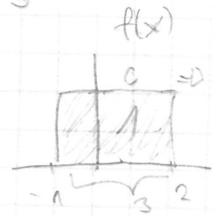
X ima jednoliku $[-1, 2]$

$$f(x) = \frac{1}{b-a} = \frac{1}{2-(-1)} = \frac{1}{3}$$



inverz

$$Y = x^2 \quad / \sqrt{\ }$$
$$\Rightarrow \sqrt{y} = x$$



$\Rightarrow c = \frac{1}{3}$

1° $x \in [0, 2]$ $\quad x = +\sqrt{y} = y^{1/2} = \sqrt{y}$

$\Rightarrow$ $y \in [0, 4]$

$$\frac{dx}{dy} = \frac{1}{2} y^{-1/2} = \frac{1}{2} \frac{1}{\sqrt{y}}$$

$$g_1(y) = f(x) \left| \frac{dx}{dy} \right| = \frac{1}{3} \cdot \left| \frac{1}{2\sqrt{y}} \right| = \frac{1}{6\sqrt{y}}, \; y \in [0, 4]$$

2° $x \in [-1, 0]$

$y \in [0, 1]$

$$x = -\sqrt{y} \qquad \frac{dx}{dy} = -\frac{1}{2} \frac{1}{\sqrt{y}}$$

$$g_2(y) = f(x) \left| \frac{dx}{dy} \right| = \frac{1}{3} \left| -\frac{1}{2\sqrt{y}} \right| = \frac{1}{6\sqrt{y}}, \; y \in [0, 1]$$

$$g(y) = \begin{cases} \frac{1}{6\sqrt{y}} + \frac{1}{6\sqrt{y}}, & y \in [0, 1] \\ \frac{1}{6\sqrt{y}}, & y \in \langle 1, 4] \end{cases}$$

⑤ (b)

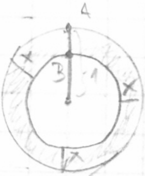
$X \sim N(4, \sigma^2)$; $P(2 < X < 6) = 0{,}86639$

$$0{,}86639 = P\left(\frac{2-4}{\sigma} < \frac{X-4}{\sigma} < \frac{6-4}{\sigma} \right) = P\left(-\frac{2}{\sigma} < \tilde{X} < \frac{2}{\sigma} \right) =$$

$$= \frac{1}{2} \left(\Phi^*\left(\frac{2}{\sigma}\right) - \Phi^*\left(-\frac{2}{\sigma}\right) \right) = \cancel{\frac{1}{2}} \cdot \cancel{2}\, \Phi^*\left(\frac{2}{\sigma}\right) = 0{,}86639$$

$$\frac{2}{\sigma} = 1{,}50 \Rightarrow \sigma = \frac{2}{1{,}5} = \frac{4}{3}$$

(1) 2MI 2008 (1)



$F(x)$, $E(x) = ?$

$x \in [0,1]$

$|AB| = 1 - x$

$$F(x) = P(X < x) = \frac{1^2\pi - (1-x)^2\pi}{1^2\pi} =$$

$$F(x) = 1 - (1-x)^2 = 1 - (1 - 2x + x^2) = \cancel{1} - \cancel{1} + 2x - x^2$$

$$F(x) = 2x - x^2 \quad , \quad x \in [0,1]$$

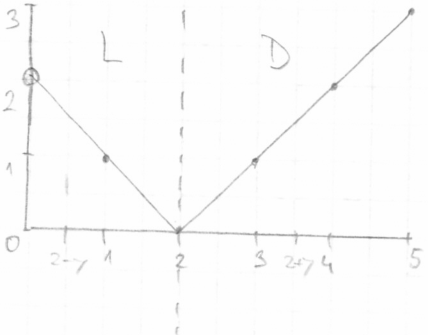
$$f(x) = F'(x) = 2 - 2x \quad , \quad x \in [0,1]$$

$$E(X) = \int_{-\infty}^{+\infty} x f(x)\, dx = \int_0^1 x(2-2x)\, dx = \int_0^1 (2x - 2x^2)\, dx =$$

$$E(X) = 2\left(\frac{x^2}{2}\Big|_0^1\right) - 2\left(\frac{x^3}{3}\Big|_0^1\right) = 2\,\frac{1}{2} - 2\,\frac{1}{3} = 1 - \frac{2}{3} = \frac{1}{3}$$

(2)

$Y = |X - 2|$, X $f(x) = e^{-x}$, $x > 0$



inverz:

$$-Y < X - 2 < +Y \quad / +2$$

$$2 - Y < X < 2 + Y$$

ili:

$$-(x-2) < Y < x - 2$$

$$-x + 2 < Y < x - 2$$

$Y = -x + 2$ $\qquad$ $Y = x - 2$

$x = 2 - Y$ $\qquad$ $x = Y + 2$

L 1° $x \in [0,2]$ $\quad \Rightarrow x = 2 - y$

$y \in [0,2]$

$$g_1(y) = f(x)\left|\frac{dx}{dy}\right| = e^{-x}\,|0 - 1| = e^{-(2-y)} \cdot 1$$

D 2° $x \in [2, +\infty]$

$y \in [0, +\infty]$ $\quad \Rightarrow x = 2 + y$

$$g_2(y) = f(x)\left|\frac{dx}{dy}\right| = e^{-x}\,|0 + 1| = e^{-(2+y)} \cdot 1 = e^{-y-2}$$

$$g(y) = \begin{cases} e^{-2+y} + e^{-y-2} & , \ y \in [0,2] \\ e^{-y-2} & , \ y \in [2, +\infty) \end{cases}$$

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③ (a) $X \sim \mathcal{E}(\lambda)$

$P(X < E(X))$ ne ovisi o λ

$$F(x) = P\left(X < \frac{1}{\lambda}\right) = \int_{-\infty}^{x} \underbrace{f(t)}\,dt = \int_0^{1/\lambda} \underbrace{\lambda e^{-\lambda x}}\,dx = \cancel{\lambda}\left(\frac{1}{-\cancel{\lambda}}\right)\left(e^{-\lambda x}\Big|_0^{1/\lambda}\right)$$

$$= -1\left(e^{-\lambda \frac{1}{\lambda}} - e^{0}\right) = -1\left(e^{-1} - 1\right) = 1 - \frac{1}{e}$$

(b) $X \sim \mathcal{E}(\lambda)$ $\quad E(X) = 3$

$$P(2 < X < 3 \mid X > 2) = ?$$

$$E(X) = \frac{1}{\lambda} = 3 \Rightarrow \lambda = \frac{1}{3}$$

$$P(a < X < b) = F(b) - F(a)$$

$$F(x) = 1 - e^{-\lambda x}$$

$$F(x) = 1 - e^{-\frac{x}{3}}$$

$$P(2 < X < 3) = F(3) - F(2) = 1 - e^{-\frac{3}{3}} - \left(1 - e^{-\frac{2}{3}}\right) =$$

$$= \cancel{1} - e^{-1} - \cancel{1} + e^{-2/3}$$

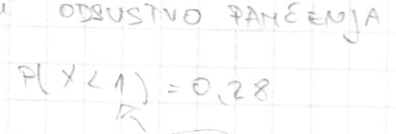
$$P(X < a) = F(a) \Rightarrow P(X < 2) = F(2) = 1 - e^{-2/3}$$

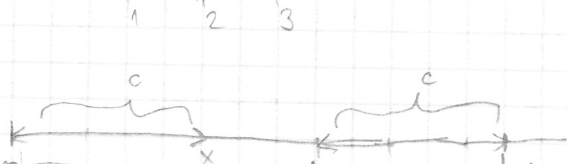
$$P(X > a) = 1 - F(a) \Rightarrow 1 - \left(1 - e^{-2/3}\right) = e^{-2/3}$$

$$P(2 < X < 3 \mid X > 2) = \frac{P(2 < X < 3)}{P(X > 2)} = \frac{e^{-2/3} - e^{-1}}{e^{-2/3}} = 0,28$$

ili ODSUSTVO PAMĆENJA

$$P(X < 1) = 0,28$$





ako nema kvarova od 0 do trenutka t

(4) (b)

2M1, 2008 (2)

$X \sim N(\underset{E(X)}{\mu}, \underset{D(X)}{\sigma^2})$, $E(X) = 370\ \ell/m^2$

$\mu = 370$

$P(10 < X < 730) = 99{,}73\% = 0{,}9973$

$P(X > 450) = ?$

$$P\left(\frac{10-\overset{370}{\mu}}{\sigma} < \frac{X-\mu}{\sigma} < \frac{730-''}{\sigma}\right) = 0{,}9973$$

$$P\left(\frac{-360}{\sigma} < \tilde{X} < \frac{360}{\sigma}\right) = 0{,}9973$$

$$\frac{1}{2}\left(\Phi^*\left(\frac{360}{\sigma}\right) - \Phi^*\left(-\frac{360}{\sigma^2}\right)\right) = 0{,}9973$$

$$\frac{1}{\cancel{2}}\left(\cancel{2}\,\underbrace{\Phi^*\left(\frac{360}{\sigma}\right)}_{3{,}00}\right) = 0{,}9973$$

$$\frac{360}{\sigma} = 3{,}00 \quad /^{-1}$$

$$\frac{\sigma}{360} \quad \frac{1}{3} \Rightarrow \boxed{\sigma = 120}$$

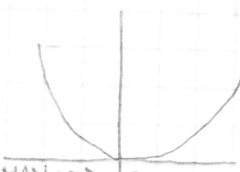
$$P(X > 450) = P\left(\frac{X-370}{120} > \frac{450-370}{120}\right) =$$

$$P\left(\tilde{X} > \frac{2}{3}\right) = \frac{1}{2} - \frac{1}{2}\Phi^*\left(\frac{2}{3}\right) = \frac{1}{2} - \frac{1}{2}\cdot 0{,}49459 =$$

$= 0{,}252705$

(c) $X \sim N(0,1)$ $\quad Y = X^2$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad x \in \mathbb{R}$$



1° $x \in [0, +\infty)$

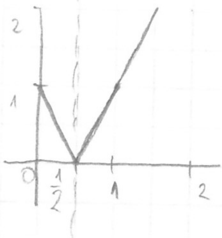
$y \in [0, +\infty)$

$$y \cdot \sigma\sqrt{2\pi} = e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

HAN UTD

boy fervac

(3) $Y = |2X - 1|$ X, $f(x) = 2e^{-2x}$, $x > 0$



nije injekcija

INVERZNA!

$$Y = |2X - 1|$$

$$-Y < 2X - 1 < Y \quad / +1$$

$$1 - Y < 2X < 1 + Y \quad / :2$$

$$\frac{1-Y}{2} < X < \frac{1+Y}{2}$$

1°

$x \in (0, \frac{1}{2})$

$y \in (0, 1)$

$$x = \frac{1-y}{2} = \frac{1}{2} - \frac{y}{2} \quad \left|\frac{dx}{dy}\right| = \left|\frac{1-y}{2}'\right| =$$

$$= \left|0 - \frac{1}{2}\right| = \left|\frac{1}{2}\right|$$

$$g_1(y) = f(x)\left|\frac{dx}{dy}\right| = \cancel{2}e^{-2x}\left|\frac{1}{\cancel{2}}\right| = e^{-2\left(\frac{1-y}{2}\right)} =$$

$$g_1(y) = e^{-\frac{2}{2} + \frac{2y}{2}} = e^{y-1}, \quad y \in (0, 1)$$

2° $x \in (\frac{1}{2}, +\infty)$

$y \in (0, +\infty)$

$$x = \frac{1+y}{2} = \frac{1}{2} + \frac{y}{2}$$

$$\left|\frac{dx}{dy}\right| = \left|\left(\frac{1}{2} + \frac{y}{2}\right)' \frac{d}{dy}\right| = \left|0 + \frac{1}{2} \cdot 1\right| = \left|\frac{1}{2}\right|$$

$$g_2(y) = f(x)\left|\frac{dx}{dy}\right| = \cancel{2}e^{-2x}\left|\frac{1}{\cancel{2}}\right| = e^{-2\left(\frac{1}{2} + \frac{y}{2}\right)} = e^{-y-1}, \quad y \in (0, +\infty)$$

$$g(y) = \begin{cases} g_1(y) + g_2(y) \\ g_2(y) \end{cases} = \begin{cases} e^{y-1} + e^{-y-1}, & y \in (0, 1) \\ e^{-y-1}, & y \in (1, +\infty) \end{cases}$$

MAN UTD
by ferOvac

① $f(x) = \begin{cases} cx^2, & 0 \le x \le 2 \\ 0, & \text{inače} \end{cases}$, $F(x), P(0 < x < 1) = ?$

$$1 = \int_{-\infty}^{+\infty} f(x)\,dx = \int_0^2 cx^2\,dx = c\left(\frac{x^3}{3}\Big|_0^2\right) = c\left(\frac{8}{3}\right) = 1 \Rightarrow \boxed{c = \frac{3}{8}}$$

$$F(x) = \int_{-\infty}^{x} f(t)\,dt = \int_0^x \underbrace{\frac{3}{8}t^2}_{f(t)}\,dt = \frac{3}{8}\left(\frac{t^3}{3}\Big|_0^x\right) = \frac{\cancel{3}}{8}\frac{x^3}{\cancel{3}} = \frac{x^3}{8}$$

$$F(x) = \frac{x^3}{8}$$

$$P(0 < x < 1) = F(1) - F(0) = \frac{1}{8} - \frac{0}{8} = \frac{1}{8}$$

$$P(0 < x < 1) = \int_0^1 \frac{3}{8}x^2\,dx = \frac{3}{8}\left(\frac{x^3}{3}\Big|_0^1\right) = \frac{3}{8}\cdot\frac{1}{3} = \frac{1}{8}$$

⑤ b) $X \sim N(2, 8) \Rightarrow \sigma = \sqrt{8}$

$Y = aX + b$ ima jediničnu normal. razd.

$$Y = aX + b = \frac{X-2}{\sqrt{8}} = \frac{X}{\sqrt{8}} - \frac{2}{\sqrt{8}} = \frac{X}{2\sqrt{2}} - \frac{\cancel{2}}{\cancel{2}\sqrt{2}}$$

$$Y = aX + b = \frac{X}{2\sqrt{2}} - \frac{1}{\sqrt{2}} = \underbrace{\left(\frac{1}{2\sqrt{2}}\right)}_{a}X + \underbrace{\left(-\frac{1}{\sqrt{2}}\right)}_{b}$$

$$a = \frac{1}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4} \qquad b = -\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

d)

$X \sim B(n = 15000, p = \frac{1}{2})$, $P(\underbrace{X > 7600}_{\text{broj prsima}}) = ?$

$$\approx N(\underbrace{np}_{\mu}, \underbrace{npq}_{\sigma^2})$$

$$N\left(15000\cdot\frac{1}{2},\ 15000\cdot\frac{1}{2}\left(1-\frac{1}{2}\right)\right) = N(\overset{\mu}{7500}, \overset{\sigma^2}{3750})$$

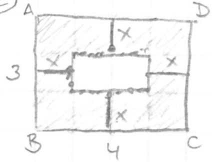
$$P(X > 7600) = P\left(\frac{X-7500}{\sqrt{3750}} > \frac{7600-7500}{\sqrt{3750}}\right) = P\left(X^* > \frac{100}{25\sqrt{6}}\right)$$

$$= \frac{1}{2} - \frac{1}{2}\underbrace{\Phi^*(1,63299)}_{\substack{\text{Tablica} \\ 0,89690}} = \frac{1}{2} - \frac{1}{2}(0,8969) = 0,05155$$

2M1, 2007

$\boxed{2}$

2



$x \in \left[0, \frac{3}{2}\right]$

$$F(x) = P(X<x) = \frac{m(G_x)}{m(\Omega)} = \frac{3\cdot 4 - (3-2x)(4-2x)}{3\cdot 4}$$

$$= \frac{12 - (12 - 6x - 8x + 4x^2)}{12} = \frac{\cancel{12} - \cancel{12} + 14x - 4x^2}{12} = \frac{\cancel{14}\,7}{\cancel{12}\,6}x - \frac{\cancel{4}\,1}{\cancel{12}\,3}x^2$$

$$F(x) = \frac{7}{6}x - \frac{1}{3}x^2, \quad x \in \left[0, \frac{3}{2}\right]$$

$$f(x) = F'(x) = \frac{7}{6}\cdot 1 - \frac{1}{3}\cdot 2x = \frac{7}{6} - \frac{2}{3}x$$

$$E(X) = \int_{-\infty}^{+\infty} x f(x)\,dx = \int_0^{3/2} x\left(\frac{7}{6} - \frac{2}{3}x\right)dx = \int_0^{3/2}\left(\frac{7}{6}x - \frac{2}{3}x^2\right)dx =$$

$$= \int_0^{3/2} \frac{7}{6}x\,dx + \int_0^{3/2}\left(-\frac{2}{3}\right)x^2\,dx = \frac{7}{6}\left(\frac{x^2}{2}\Big|_0^{3/2}\right) + \left(-\frac{2}{3}\right)\left(\frac{x^3}{3}\Big|_0^{3/2}\right) = \frac{9}{16}$$

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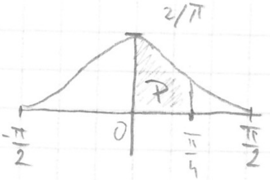
by ferdvac

① $f(x)=\frac{2}{\pi}\cos^2 x \quad , \quad x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$



$$P=\frac{2}{\pi}\int_0^{\pi/4}\cos^2 x\,dx=\frac{\cancel{2}}{\pi}\int_0^{\pi/4}\frac{1+\cos(2x)}{\cancel{2}}\,dx=$$

$$=\frac{1}{\pi}\int_0^{\pi/4}1+\cos(2x)\,dx=\frac{1}{\pi}\left[\left(x+\frac{1}{2}\sin(2x)\right)\Big|_0^{\pi/4}\right]=$$

$$=\frac{1}{\pi}\left[\left(\frac{\pi}{4}+\frac{1}{2}\underbrace{\sin\left(\frac{2\pi}{4_2}\right)}_{1}\right)-\left(0+\frac{1}{2}\overbrace{\sin 0}^{0}\right)\right]=\frac{1}{\pi}\left[\frac{\pi}{4}+\frac{1}{2}\cdot 1\right]$$

$$=\frac{1}{\pi}\left(\frac{\pi}{4}+\frac{1\cdot 2}{2\cdot 2}\right)=\frac{1}{\pi}\left(\frac{\pi+2}{4}\right)=\frac{\pi+2}{4\pi}$$

$$p=P=\frac{\pi+2}{4\pi}$$

da od 3 točno 2 padnu unutar int $\left[0,\frac{\pi}{4}\right]$

$$X\sim B\left(n=3,\ p=\frac{\pi+2}{4\pi}\right)$$

$$P(X=2)=\binom{3}{2}\left(\frac{\pi+2}{4\pi}\right)^2\left(\overset{\frac{4\pi}{4\pi}}{1}-\frac{\pi+2}{4\pi}\right)^{3-2}$$

$$=\binom{3}{2}\left(\frac{\pi+2}{4\pi}\right)^2\left(\frac{3\pi+2}{4\pi}\right)^1$$

$$P(X=2)=3\left(\frac{\pi+2}{4\pi}\right)^2\left(\frac{3\pi+2}{4\pi}\right)$$

↓

točno 2

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②

$$P(X<x) = 1 - \frac{\frac{1}{2}(v-x)\left[(v-x)\frac{a}{v}\right]}{\frac{av}{2}} = (*)$$

sličnost: $\frac{v-x}{v} = \frac{y}{a} \Rightarrow vy = (v-x)a \;/:v$

$$y = \frac{(v-x)a}{v}$$

$$(*) = 1 - \frac{\frac{1}{2}(v-x)^2\frac{a}{v}}{\frac{av}{2}} = 1 - \frac{(v-x)^2}{v^2}$$

$$F(x) = 1 - \frac{(v-x)^2}{v^2},\; x\in[0,v]$$

$$f(x) = F'(x) = 0 - \frac{1}{v^2}\,2(v-x)(-1) = \frac{2}{v^2}(v-x),\; x\in[0,v]$$

$$E(x) = \int_{-\infty}^{+\infty} x f(x)\,dx = \int_0^v x\frac{2}{v^2}(v-x)\,dx = \frac{2}{v^2}\int_0^v x(v-x)\,dx$$

$$= \frac{2}{v^2}\left[\int_0^v xv\,dx - \int_0^v x^2\,dx\right] = \frac{2}{v^2}\left[v\cdot\left(\frac{x^2}{2}\Big|_0^v\right) - \left(\frac{x^3}{3}\Big|_0^v\right)\right] =$$

$$= \frac{2}{v^2}\left[\frac{v\cdot v^2}{2} - \frac{v^3}{3}\right] = \frac{\cancel{2}}{v^2}\left[\frac{3v^3-2v^3}{\cancel{6}\,3}\right] = \frac{1}{v^2}\frac{v^3}{3} = \frac{v}{3}$$

$$E(x) = \frac{v}{3}$$ //