Završni ispit [illegible] 2015/16

Andrea

1) $\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} dt$ = 1b [illegible] $+ \frac{1}{2}$ [illegible] $= \frac{1}{2} - \frac{1}{2}$ [illegible] 1b

b) [illegible] $\Phi(u_p)$ [illegible] $\int \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$ = [illegible] 1b

([illegible] funkciju normalne razdiobe)

c) $u_p = -0.484 \to p = \frac{1}{2} - \frac{1}{2}\Phi^*(0.484) = 0.314$ 1b

$u_p = 0.484 \to p = \frac{1}{2} - \frac{1}{2}\Phi^*(0.484) = 0.686$ 1b (ili $1 - 0.314$)

Snježana

[illegible] $2x\,dx$ [illegible] $\frac{x^2}{2}\Big|_1^3$ [illegible] $a = \frac{1}{4}$ 1b

$f_X(x) = \int_0^{\infty} \frac{1}{4} x e^{-y} dy = \frac{1}{4}x$, [illegible] 1b

$f_Y(y) = \int_1^3 \frac{1}{4} x e^{-y} dx = e^{-y}$, [illegible]

[illegible] X i Y su nezavisni 1b

$P(X<2 \mid Y \le$ [illegible] $) = P(X \le 2) = \frac{1}{4}$ [illegible] $= \frac{3}{8}$ 1b

(ili preko uvjetne)

Tomislav

3) [illegible] $X_i \in [0, 2]$ [illegible] $\frac{1}{2}$ [illegible]

[illegible] $P(\ldots < x)^n = (\frac{1}{2}x)^n = \frac{x^n}{2^n}$ 1b

[illegible]

$f_X(x) = F_X'(x) = \frac{n}{2^n} x^{n-1}$, $x \in [0, 2]$; $f_{Y|X=x}(y) = \frac{1}{2-x}$ 1b $y \in [x, 2]$

$E(Y \mid X=x) = \int_x^2 \frac{y}{2-x} dy = \frac{x+2}{2}$ 1b

$E(Y) = \int_0^2 \frac{x+2}{2} \cdot \frac{n}{2^n} x^{n-1}$ 1b $dx = \frac{n}{2^{n+1}} \left(\frac{x^{n+1}}{n+1} + 2\frac{x^n}{n} \right) \Big|_0^2 = \frac{n}{n+1} + 1 = \frac{2n+1}{n+1}$

1b Ivo

4) Koristeći Čebiševljevu nejednakost $P(|X-a| \ge \varepsilon) \le \frac{\sigma^2}{\varepsilon^2}$ 1b

slijedi $P(|X-a| < 3\sigma)$ 1b $\ge 1 - \frac{\sigma^2}{9\sigma^2} = \frac{8}{9}$ 1b

tj. $P(a - 3\sigma < X < a + 3\sigma) \ge \frac{8}{9}$

Ako je $X \sim N(a, \sigma^2)$:

$P(-3 < X^* < 3) =$ 1b $\Phi^*(3) = 0.9973$ 1b, tj. pravilo 3σ

1) Mario

a) $L(\lambda, x_1, \dots, x_n) = \frac{\lambda^{x_1}}{x_1!} e^{-\lambda} \cdots \frac{\lambda^{x_n}}{x_n!} e^{-\lambda} = \frac{\lambda^{x_1 + \dots + x_n}}{x_1! \cdots x_n!} e^{-n\lambda}$ 1b

$\ln L = (x_1 + \dots + x_n)\ln\lambda - n\lambda - \ln(x_1! \cdots x_n!)$

$\frac{\partial \ln L}{\partial \lambda} = \frac{x_1 + \dots + x_n}{\lambda} - n \overset{1b}{=} 0 \Rightarrow \lambda = \frac{x_1 + \dots + x_n}{n} = \bar{x}$ 1b

b) $E\left(\frac{X_1 + \dots + X_n}{n}\right) \underset{1b}{=} \frac{1}{n} \sum_{i=1}^{n} \underbrace{E(X_i)}_{\lambda} = \frac{1}{n} \cdot n \cdot \lambda = \lambda$ 1b procjena je nepristrana

Petar

$\hat{p} = \frac{314}{455} = 0.69$, $n = 455$, $p = 0.35 \to \alpha = 0.05$

$p_{1,2} = \hat{p} \mp \underbrace{u_{1-\frac{\alpha}{2}}}_{1.96} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \underset{1b}{=} 0.69 \mp 0.042 \Rightarrow P(0.648 \leq p \underset{1b}{\leq} 0.732) = 95\%$

$H_0 \dots p = 0.65$ 1b $\hat{u} = (\hat{p} - p_0)\sqrt{\frac{n}{p_0 q_0}} = 1.789 \overset{1b}{>} u_{1-\alpha} = 1.645$

$H_1 \dots p > 0.65$

Prihvaćamo H_1, 1b pivovara će povećati udio reklama...

2) Igor

$H_0 \dots \mu = \nu$ 1b $n = 6$, $\bar{x} = 305$, $s_x^2 = 38.8$ 1b $s_2^2 = 28.8$

$H_1 \dots \mu < \nu$ $m = 6$, $\bar{y} = 314$, $s_y^2 = 18.8$

$\hat{t} = \frac{\bar{x} - \bar{y}}{s_2}\sqrt{\frac{nm}{n+m}} = -2.905 \underset{1b}{<} -t_{10, 1-\alpha} = -2.764$ 1b ($\alpha = 0.01$)

Prihvaćamo H_1, 1b prosječna prodaja se povećala

3) Stjepan

	n_j	p_j	np_j	$(n_j - np_j)^2 / np_j$
O	24	$0.\dot{3}$	$26.\dot{6}$	$0.2\dot{6}$
A	39	0.4 1b	32	1.53
B	13 } 17	0.2	16 } 21.3 1b	0.88
AB	4	$0.0\dot{6}$	$5.\dot{3}$	
Σ	$n = 80$	$= 1$	$= 80$	$\chi_e^2 = 2.677$ 1b

5 : 6 : 3 : 1

Trebamo prihvatiti H_0 ($r = 3$, $r = 0$)

$\chi_e^2 = 2.677 < \chi^2_{2, 1-\alpha} = 2.773$ 1b

$1 - \alpha = 0.75 \Rightarrow \alpha = 25\%$ 1b