Mass ZI 23.06.2010.

PREDZAVRŠNI ISPIT ♡

1. $x_1, \ldots, x_n \in [3, \alpha]$ o procjenu duljine intervala

duljina $= \alpha - 3$

$$Z = \max\{x_1, \ldots, x_n\} - 3$$

varijabla po kojoj procjenjujemo

STATISTIKA JE NEPRISTRANA ako je očekivanje jednako pravoj vrijednosti:

$$E(Z) = \vartheta$$

$$\int x f(x)\,dx$$

$$X_i \sim U[3, \alpha]$$

$$f(x) = \frac{1}{\alpha - 3} \qquad F(x) = \frac{x-3}{\alpha-3}, \; x \in [3, \alpha]$$

$$X = \max\{x_1, \ldots, x_n\}$$

ako je max < x, onda je svaki manji od x

$$F_X(x) = P(X < x) = P(\max\{x_1, \ldots, x_n\} < x) =$$

$$= P(x_1 < x, \ldots, x_n < x) = \text{NEZAVISNOST!} =$$

$$= P(x_1 < x) \cdot \ldots \cdot P(x_n < x) =$$

$$= F_{x_1}(x) \cdot \ldots \cdot F_{x_n}(x) = \left(\frac{x-3}{\alpha-3}\right)^n$$

$$f_X(x) = F'(x) = n \cdot \left(\frac{x-3}{\alpha-3}\right)^{n-1} \cdot \frac{1}{\alpha-3} = \frac{n}{(\alpha-3)^n} \cdot (x-3)^{n-1}$$

$$E(Z) = E(X-3) = E(X) - 3 = \int_3^\alpha x \cdot \frac{n}{(\alpha-3)^n}(x-3)^{n-1}\,dx - 3 =$$

$$\left(\begin{matrix} x-3=t \\ dx = dt \end{matrix}\right) = \frac{n}{(\alpha-3)^n}\int_0^{\alpha-3}(t+3)t^{n-1}\,dt - 3 =$$

$$= \frac{n}{(\alpha-3)^n}\left[\frac{t^{n+1}}{n+1} + 3\cdot\frac{t^n}{n}\right]\Big|_0^{\alpha-3} = \frac{n}{n+1}(\alpha-3)$$

STATISTIKA NIJE NEPRISTRANA!

$$\frac{n}{n+1}(\alpha-3) \neq \alpha - 3$$

DA BUDE NEPRISTRANA:

$$Z = \frac{n+1}{n}(X-3)$$

- da bi bila valjana moramo računati disperziju

2

BINOMNA RAZDIOBA - događaj se dogodio k puta od n

$$X \sim B(n,p) = B(4,p)$$

(Poisson - intenzitet dolazaka... / u sat vremena x auti, dr sat y...)

METODOM NAJVEĆE IZGLEDNOSTI izračunavamo nepoznati parametar!

$$\mathcal{L}(\underbrace{p, x_1, x_2, x_3}_{\text{3 pokusa i parametar } p}) = P(X=x_1) \cdot P(X=x_2) \cdot P(X=x_3) =$$

$$\boxed{P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}}$$ ≈) BINOMNAAAA...

$$\mathcal{L}(p, x_1, x_2, x_3) = P(X=1) \cdot P(X=3) \cdot P(X=0) =$$

$$= \binom{4}{1} p (1-p)^3 \binom{4}{3} p^3 (1-p) \binom{4}{0} p^0 (1-p)^4 =$$

$$= 16 p^4 (1-p)^8$$

→ derivirati po parametru i izjednačiti s $\emptyset$

$$\ln L = \ln 16 + 4 \ln p + 8 \ln (1-p)$$

$$\frac{d \ln L}{dp} = \frac{4}{p} + \frac{8}{1-p}(-1) = \emptyset$$

$$4(1-p) = 8p$$

$$4 - 4p = 8p$$

$$4 = 12p$$

$$p = \frac{1}{3}$$

3. $f(x) = \lambda x^{\lambda-1}, \quad x \in (0,1)$

$$L(\lambda, x_1, \ldots, x_n) = f(x_1) \cdot \ldots \cdot f(x_n) =$$
$$= \lambda x_1^{\lambda-1} \cdot \ldots \cdot \lambda x_n^{\lambda-1} =$$
$$= \lambda^n (x_1 \cdot \ldots \cdot x_n)^{\lambda-1}$$

konstante!

$$\ln L = n \ln \lambda + (\lambda - 1) \ln (x_1 \cdot \ldots \cdot x_n)$$

$$\frac{d}{d\lambda} \ln L = \frac{n}{\lambda} + 1 \cdot \ln (x_1 \cdot \ldots \cdot x_n) = 0$$

$$\frac{n}{\lambda} = -\ln (x_1 \cdot \ldots \cdot x_n)$$

$$\lambda = \frac{-n}{\ln (x_1 \cdot \ldots \cdot x_n)} = \frac{-n}{\ln \prod_{i=1}^{n} x_i}$$

4. Normalni zakon s nepoznatim parametrima: ...

x_i	115	120	125	130	135	140
n_i	3	4	7	6	3	2

$n = 25$

a) točkaste procjene za očekivanje i

$$\overline{X} = \frac{\sum n_i x_i}{n} = 126.6$$

disperziju:

MORAMO PISATI FORMULE !!
(a ne samo uvrstiti :))

$$s^2 = \frac{1}{n-1} \sum n_i (x_i - \overline{X})^2 = 51.5$$

b) 90%-tni interval povjerenja za očekivanje (formulicee)

$p = 0.9$

korjenovati (s^2 !) $= \sqrt{51.5}$

$$\rightarrow P\Big(\underbrace{\overline{X}}_{126.6} - t_{n-1, 1-\frac{\alpha}{2}} \frac{s}{\underbrace{\sqrt{n}}_{\sqrt{25}}} \le a \le \overline{X} + t_{n-1, 1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}}\Big) = p = 0.9$$

$$t_{n-1, 1-\frac{\alpha}{2}} = (n-1 = 24, \alpha = 0.1) = 1.711$$

$$\rightarrow P(124.15 \le a \le 129.06) = 0.9$$

c) 90%-tni interval (DVOSTRUKI) za disperziju

(Ako ne naglase da je jednostruki, radimo DVOSTRUKI!)

$$P\left(\frac{(n-1)\hat{s}^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}} \le \sigma^2 \le \frac{(n-1)\hat{s}^2}{\chi^2_{n-1,\frac{\alpha}{2}}}\right) = p$$

hi-kvadrat razdioba :)

$$\chi^2_{n-1,1-\frac{\alpha}{2}} = (n-1 = 24,\ 1-\tfrac{\alpha}{2} = 0.95) = 36.415$$

$$\chi^2_{n-1,\frac{\alpha}{2}} = (n-1 = 24,\ \tfrac{\alpha}{2} = 0.05) = 13.848$$

$$P(33.94 \le \sigma^2 \le 89.25) = 0.9$$

• za disperziju su dosta veliki intervali :)

↳ 90% sam siguran da će se disperzija nalaziti između ovih vrijednosti

5. $n = 100$ žarulja 3 loše

a) 95%-tni interval povjerenja za postotak loših žarulja

$$p_{1,2} = \hat{p} \pm u_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$\hat{p} = \frac{3}{100} = 0.03$ • poznati postotak dan u zadatku

$p = 0.95$ $\alpha = 1-p = 0.05$

$u_{0.975} = 1.95996$

$P(0.0102 < p < 0.846)$ ~ mislim da je ovo duža formula

b) koliki mora biti n da bi sa 0.95 mogli tvrditi da u cijeloj pošiljci nema više od 0.05 loših žarulja

$$\hat{p} + u_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{\underset{?}{\underset{\shortparallel}{n}}}} \leq 0.05$$

$\hat{p}$ sa početka zadatka,
nije bitno što i $\hat{p}$ ovisi o n

$n \geq 279.48$
$n = 280$

(da je bilo $n \leq 279.48$
onda $n = 279$)

6. TESTIRANJE HIPOTEZA

prosječna ocjena barem 3.5   barem = lijevi

$n = 10$

2, 4, 4, 3, 5, 1, 5, 2, 3, 5 $\alpha = 0.1$

$H_0 \ldots \quad a = 3.5 \rightarrow a_0$

$H_1 \ldots \quad a < 3.5$ LIJEVI TEST

$$\bar{x} = \frac{\sum x_i}{n} = 3.4$$

$$\hat{t} = \frac{\bar{x} - a_0^{=3.5}}{\frac{\hat{s}}{\sqrt{n}}} \qquad \hat{s}^2 = \frac{1}{n-1} \sum n_i (x_i - \bar{x})^2 = 2.04$$

$$\hat{t} = -0.221$$

GLE!!

KRITERIJ ZA ODBACIVANJE H_0

U TABLICI GLEDAMO za 2α zbog $(1-\alpha)$

$$\hat{t} < -t_{n-1,1-\alpha} = -1.383$$

$-0.221 < -1.383$ X   nije istina

H_0 se prihvaća, profesor nije prekršio obećanje :)

↓ MORA SE NAPISATI ZAKLJUČAK!

7. 2% - škarta → tvornica tvrdi

Na uzorku od 500 bilo je 16 škartnih.

$\alpha = 0.05$

Deklaracija neće valjati ako ima više od 2% škarta.

$H_0 \quad .. \quad p = 0.02$ HIPOTEZA O PROPORCIJI...

$H_1 \quad p > 0.02$

$$\hat{u} = (\hat{p} - p_0)\sqrt{\frac{n}{\underbrace{p_0(1-p_0)}_{q_0}}} \qquad \hat{p} = \frac{m}{n} = \frac{16}{500}$$

$\hat{u} = 1.917$

$\hat{u} > u_{1-\alpha}$ → KRITERIJ ZA ODBACIVANJE H_0

$u_{1-\alpha} = u_{0.95} = 1.64485$

$1.917 > 1.64485$ ✓ Odbacujemo H_0. Tvornica laže.

8. $\alpha = ?$ za prihvaćanje hipoteza o jednakosti srednjih ocjena

	1	2	3	4	5
E1	7	16	28	14	11
R1	10	13	30	16	9

$n = 76$ $\bar{x} = 3.078$ $s_x^2 = 1.354$

$m = 78$ $\bar{y} = 3.013$ $s_y^2 = 1.363$

$H_0 \ldots \quad a_1 = a_2$

$H_1 \ldots \quad a_1 \neq a_2$

$$S_z^2 = \frac{1}{n+m-2}\left[(n-1)s_x^2 + (m-1)s_y^2\right] = 1.358$$

$$\hat{t} = \frac{\bar{x}-\bar{y}}{\sqrt{S_z^2}}\sqrt{\frac{nm}{n+m}} \sim t(m+n-2) = 0.3518$$

KRITERIJ ZA ODBACIVANJE H_0

$$|\hat{t}| > t_{n+m-2,\,1-\frac{\alpha}{2}}$$

o nemamo α nego pita kad možemo prihvatiti hipotezu

H_0 prihvaćamo: $|\hat{t}| < t_{n+m-2,\,1-\frac{\alpha}{2}} = (n+m-2 = 152,\ 1-\frac{\alpha}{2} = ?)$

$$0.3518 < \underbrace{t_{n+m-2,\,1-\frac{\alpha}{2}}}_{\text{prvi veći od ovog}}$$

$\rightarrow$ najbliži za $\alpha = 0.7$

npr. 15, 4, 9

9. χ^2–test ; Poisson , $\alpha = 0.05$

x_i	n_i	p_j	np_j	$\frac{(n_j-np_j)^2}{np_j}$
0	130	0.61	125.05	0.196
1	52	0.3	61.5	1.47
2	19	0.076	15.58	
3	3	0.013	2.665	1.06
4	1	0.016	0.32	
Σ	205	1	≈ 205	$\chi^2 = 2.726$

RAZREDI SE GRUPIRAJU AKO IMA MANJE od 5 ELEMENATA

Poisson: $p = \frac{\lambda^k}{k!} \cdot e^{-\lambda}$ $\quad \lambda = \bar{x} = E(x) = \frac{\sum n_i x_i}{n} = 0.5$

$f = \underset{3}{m} - \underset{1}{r} - 1 = 1$

r - broj parametara koje smo sami računali

$\chi^2_{1,0.95} = 3.841$

$\chi^2 = 2.726 < \chi^2_{1,0.95} = 3.841$

Ho prihvaćamo, podaci se ravnaju po Poissonovoj razdiobi

10. (zadnji zzv)

imamo 10 znamenki => jednako su vjerojatne

x_i	n_j	p_j	np_j	$\frac{(n_j-np_j)^2}{np_j}$
0	74	1/10	80	
1	92	1/10	80	
⋮	⋮	⋮	⋮	
9	81	1/10	80	
	800			

- ima na materijalima negdje riješeno

$\chi^2 = 5{,}152$

svaka bi se trebala pojaviti 80 puta

$\alpha = 0.1$

$f = 10 - 0 - 1 = 9$

$\chi^2_{f,1-\frac{\alpha}{2}} = \chi^2_{9,0.9} = 14.684$

$5.152 < 14.684$ Prihvaćamo H_0 !)

KRITERIJ ZA ODBACIVANJE $\chi^2 > \chi^2_{f,1-\frac{\alpha}{2}}$

h. Najzanimljiviji zadatak → H. "mogao bi se pojaviti"

$\alpha = 0.95$

Moram provjeriti ravnaju li se podaci po nekoj razdiobi → radim χ^2 (hi-kvadrat)!

- nije zadana razdioba po kojoj se ravna
- kako odrediti teoriju vjerojatnosti (e pa po principu iz prvog semestra, svaki slučaj promatramo)

x_i	n_j	p_j	np_j	$\frac{(n-np_j)^2}{np_j}$
0	4	$\left(\frac{1}{2}\right)^4$	6	0
1	26	$\binom{4}{1}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^3$	24	
2	35	$\binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2$	36	1/36
3	24	$\binom{4}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)$	24	0
4	7	$\left(\frac{1}{2}\right)^4$	6	1/6
	96			7/36

GLE! 

↓

skrivena je BINOMNA RAZDIOBA

$f = 4 - 0 - 1 = 3$

$\alpha = 0.95$

$\chi^2_{3,0.05} = 0.352$ Prihvaćamo /ne odbacujemo?)