PZI 08 $\boxed{3}$

(7) bubanj, 4B, 1C

a) bez vraćanja

$X=1 \quad \frac{1}{5}$

$X=2 \quad \frac{4}{5}\cdot\frac{1}{4}=\frac{1}{5}$

$X=3 \quad \frac{4}{5}\cdot\frac{3}{4}\cdot\frac{1}{3}=\frac{1}{5}$

$X=4 \quad \frac{4}{5}\cdot\frac{3}{4}\cdot\frac{2}{3}\cdot\frac{1}{2}=\frac{1}{5}$

$X=5 \quad \frac{4}{5}\cdot\frac{3}{4}\cdot\frac{2}{3}\cdot\frac{1}{2}\cdot 1=\frac{1}{5}$

$p_i=\frac{1}{5}, \quad i=1\dots 5$

$E(X)=\frac{1}{5}(1+2+3+4+5)=\underline{3}$

b) s vraćanjem

$X=1 \quad \frac{1}{5}$

$X=2 \quad \frac{4}{5}\cdot\frac{1}{5}$

$X=3 \quad \frac{4}{5}\cdot\frac{4}{5}\cdot\frac{1}{5}$

$X=4 \quad \left(\frac{4}{5}\right)^3\frac{1}{5}$

$X=5 \quad \left(\frac{4}{5}\right)^4\cdot\frac{1}{5}$

...

$p_i=\left(\frac{4}{5}\right)^{i-1}\cdot\frac{1}{5} \quad i=1,2,\dots$

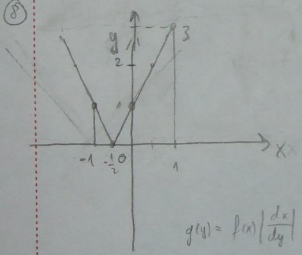
$E(X)=\sum_{i=1}^{\infty} i\cdot\left(\frac{4}{5}\right)^{i-1}\cdot\frac{1}{5}=\frac{1}{5}\sum_{i=1}^{\infty} i\left(\frac{4}{5}\right)^{i-1}=$

$=\frac{1}{5}\sum_{i=0}^{\infty} i\left(\frac{4}{5}\right)^{i}=\frac{1}{5}\,\frac{1}{\left(1-\frac{4}{5}\right)^2}=\boxed{5}$

$\left(\sum_{i=0}^{\infty}x^i=\frac{1}{1-x} \quad /'\right.$

$\left.\sum i x^{i-1}=\frac{1}{(1-x)^2}\right)$

(8)



$y=|2x+1| \qquad y\in\langle 0,3)$

$f(x)=\frac{1}{2}x+\frac{1}{2}, \ x\in(-1,1)$

$g(y)=f(x)\left|\frac{dx}{dy}\right|$

$x\in(-1,-\frac{1}{2})\to y=-2x-1 \qquad x=-\frac{y+1}{2} \qquad \left|\frac{dx}{dy}\right|=\frac{1}{2}$

$g(y)=\left(\frac{1}{2}\left(-\frac{y+1}{2}\right)+\frac{1}{2}\right)\cdot\frac{1}{2}= \qquad y\in(0,1)$

$=\underline{\frac{-y+1}{8}}$

$x\in(-\frac{1}{2},1)\to y=2x+1 \qquad x=\frac{y-1}{2} \qquad \left|\frac{dx}{dy}\right|=\frac{1}{2}$

$g(y)=\left(\frac{1}{2}\,\frac{y-1}{2}+\frac{1}{2}\right)\frac{1}{2}= \qquad y\in(0,3)$

$=\underline{\frac{y+1}{8}}$

$\begin{cases} y\in(0,1)\to g(y)=\frac{1}{4} \\ y\in(1,3)\to g(y)=\frac{y+1}{8}\end{cases}$

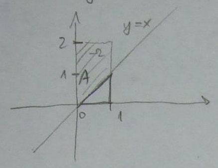
(9) $f(x,y)=cx \qquad [x,y]\in[0,1]\times[0,2]$

$1=c\int_0^1 x\,dx\int_0^2 dy=c\int_0^1 2x=cx^2\Big|_0^1 \qquad \boxed{c=1}$

b) $f_X(x)=\int_{-\infty}^{\infty}f(x,y)\,dy=\int_0^2 x\,dy=\underline{2x} \qquad \underline{x\in(0,1)}$

$f_Y(y)=\int_{-\infty}^{\infty}f(x,y)\,dx=\int_0^1 x\,dx=\frac{x^2}{2}\Big|_0^1=\frac{1}{2} \qquad \underline{y\in(0,2)}$

c) $P(X<y)$



$P(X<Y)=\int_0^1 x\,dx\int_x^2 dy=\int_0^1 x(2-x)\,dx=\left(x^2-\frac{1}{3}x^3\right)\Big|_0^1=\boxed{\frac{2}{3}}$

(10) $[\alpha,1]$ $\quad n$ brojeva $\quad x_1\dots x_n$

$Z=1-\min\{x_1,x_2\dots x_n\} \qquad \Rightarrow$ prava vrijednost $E(z)=1-\alpha$

a) $X_i\sim U[\alpha,1] \qquad F_{X_i}(x)=\frac{x-a}{b-a}=\frac{x-\alpha}{1-\alpha}$

$F_{X_m}(x)=p(X_m<x)=p(\min\{x_1\dots x_n\}<x)=1-P(\min\{X_1\dots X_n\}>x)=$

$=1-P(X_1>x)P(X_2>x)\dots P(X_n>x)=$

$=1-(1-F_{X_i}(x))^n=1-\left(1-\frac{x-\alpha}{1-\alpha}\right)^n=1-\left(\frac{1-x}{1-\alpha}\right)^n$

$f_{X_m}(x)=+n\left(\frac{1-x}{1-\alpha}\right)^{n-1}\cdot\frac{1}{1-\alpha}=\frac{n(1-x)^{n-1}}{(1-\alpha)^n}$

$E(z)=1-E(X_m)=1-\int_{\alpha}^{1}x\,\frac{n(1-x)^{n-1}}{(1-\alpha)^n}\,dx=\left|\begin{matrix}1-x=t\\ -dx=dt\end{matrix}\right|=$

$=1+\frac{n}{(1-\alpha)^n}\int_{1-\alpha}^{0}(1-t)t^{n-1}\,dt=1+\frac{n}{(1-\alpha)^n}\int_{1-\alpha}^{0}\left(t^{n-1}-t^n\right)dt=$

$=1+\frac{n}{(1-\alpha)^n}\left[\frac{1}{n}t^n-\frac{1}{n+1}t^{n+1}\right]\Bigg|_{1-\alpha}^{0}=1+\frac{n}{(1-\alpha)^n}\left[\frac{1}{n+1}(1-\alpha)^{n+1}-\frac{1}{n}(1-\alpha)^n\right]=$

$=\cancel{1}+\frac{n}{n+1}(1-\alpha)\cancel{-1}=\underline{\frac{n}{n+1}(1-\alpha)} \qquad \Rightarrow$ nije nepristrana

$\Rightarrow$ pomnožiti sa $\underline{\frac{n+1}{n}}$

Z1 07

(3) $p = ?$

$n_1 = 5 \quad m_1 = 3$

$n_2 = 4 \quad m_2 = 2$

$n_3 = 3 \quad m_3 = 2$

$$L = \binom{5}{3} p^3 (1-p)^2 \binom{4}{2} p^2 (1-p)^2 \binom{3}{2} p^2 (1-p) = 180\, p^7 (1-p)^5$$

$$L' = \cancel{180}\left[7p^6(1-p)^5 - 5p^7(1-p)^4\right] = 0 \quad \Big| : p^6(1-p)^4 \cdot 180$$

$$7(1-p) = 5p$$

$$7 - 7p = 5p$$

$$\boxed{p = \frac{7}{12}}$$

(4) $n = 400$, $m = 300$, $p = 0{,}9$

$$\hat{p} = \frac{m}{n} = \frac{3}{4}$$

$$u_{1-\alpha/2} = u_{0,95} = 1{,}64485$$

$$p_{1,2} = \hat{p} \pm u_{1-\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$p_1 = 0{,}7144$

$p_2 = 0{,}7856$

$$\boxed{P(0{,}7144 \le p \le 0{,}7856) = 0{,}9}$$

(5) $n = 100$

$X \sim N(\mu, 25)$ $\quad \bar{X} = 27$ $\quad \sigma^2 = 25$ $\quad \alpha = 0{,}05$

$H_0 \quad \mu = 26$

$H_1 \quad \mu \ne 26$

$$u_{1-\alpha/2} = u_{0,975} = 1{,}96$$

$$\hat{u} = \frac{\bar{X} - \mu_0}{\sigma}\sqrt{n} = \frac{27-26}{5} \cdot 10 = 2$$

$\boxed{|\hat{u}| > u_{1-\alpha/2}} \Rightarrow$ odbacuje se H_0

(6) $\alpha = 0{,}02$ $\quad \mathcal{P}(\lambda)$?

$\lambda = \bar{X} = 1{,}3644$

$$p_j = \frac{\lambda^j}{j!} e^{-\lambda} = \frac{1{,}3644^j}{j!} e^{-1{,}3644} \quad \text{teorijske}$$

$n = 365$

$\hat{s} = 1{,}2823$

$$\boxed{k = 6 - 1 - 1 = 4}$$

x_j	n_j	p_j	$n_j - np_j$	$\frac{(n_j-np_j)^2}{np_j}$
0	95	0,255534	1,7301	0,03209
1	140	0,34865	12,74275	1,276
2	73	0,23785	-13,81525	2,1985
3	31	0,10817	-8,48205	1,8222
4	16	0,036898	2,53223	0,47611
5	7	0,01007	3,32445	
6	2	0,00229	1,16415	5,9698
7	0	0,000446	-0,16279	
8	1	0,000076	0,97226	
	10	0,012882	5,29807	$\chi^2_g = 11{,}975$

$$\chi^2_{4;\,0,98} = 11{,}668$$

$$\boxed{\chi^2_g > \chi^2_{kr}}$$

odbacuje se

(7) špil – 52 karte, vadimo 5 karata

a) 4 asa $\Rightarrow \dfrac{\binom{4}{4}\binom{48}{1}}{\binom{52}{5}}$

b) 3 asa, 2 kralja $\Rightarrow \dfrac{\binom{4}{3}\binom{4}{2}}{\binom{52}{5}}$

c) barem 1 as $\Rightarrow 1 - \dfrac{\binom{48}{5}}{\binom{52}{5}}$

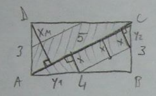
(8) a) $X \sim \mathcal{P}(\lambda = 2)$

$$P(X>2) = 1 - P(X=0) - P(X=1) - P(X=2)$$
$$= 1 - \frac{2^0}{0!}e^{-2} - \frac{2^1}{1!}e^{-2} - \frac{2^2}{2!}e^{-2} = \boxed{1 - 5e^{-2}}$$

b) $X \sim \mathcal{E}(\lambda = 2)$

$$P(X>2) = 1 - P(X<2) = 1 - F(2) = 1 - (1 + e^{-\lambda \cdot 2}) = \boxed{e^{-4}}$$

(9)



$$\frac{3}{5} = \frac{x}{y_1} \qquad y_1 = \frac{5}{3}x$$

$$\frac{4}{5} = \frac{x}{y_2} \qquad y_2 = \frac{5}{4}x$$

$$F(x) = P(X<x) = \frac{12 - (4 - \frac{5}{3}x)(3 - \frac{5}{4}x)}{3 \cdot 4} = \frac{12 - (12 - 10x + \frac{25}{12}x^2)}{12} =$$

$$= \frac{10}{12}x - \frac{25}{144}x^2$$

$$\frac{x_M}{3} = \frac{4}{5} \qquad \boxed{x_M = \frac{12}{5}}$$

$$f(x) = F'(x) = \frac{10}{12} - \frac{50}{144}x \qquad \boxed{0 \le x \le \frac{12}{5}}$$

$$E(X) = \int_0^{12/5} x\left(\frac{10}{12} - \frac{50}{144}x\right)dx = \frac{10}{24}x^2\Big|_0^{12/5} - \frac{50}{3 \cdot 144}x^3\Big|_0^{12/5} = \boxed{0{,}8}$$

(10) $X \sim N(a, \sigma^2)$

a) $\bar{X} = 126{,}6 \quad n = 25$

$\hat{s} = 7{,}17635$

b) $p = 0{,}9 \quad \alpha = 0{,}1$

$$t_{1-\alpha/2} \cdot \frac{\hat{s}}{\sqrt{n}} = 1{,}711 \cdot \frac{7{,}17635}{5} = 2{,}4557$$

$$\boxed{P(124{,}14 \le a \le 129{,}056) = 0{,}9}$$

c) $c_1 = \chi^2_{24;\,0,05} = 13{,}848$ $\qquad \beta_1 = \dfrac{24 \cdot 7{,}17635^2}{36{,}415} = 33{,}942$

$c_2 = \chi^2_{24;\,0,95} = 36{,}415$ $\qquad \beta_2 = \dfrac{24 \cdot 7{,}17635^2}{13{,}848} = 89{,}255$

$$\boxed{P(33{,}942 \le \sigma^2 \le 89{,}255) = 0{,}9}$$

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08

1. $f(x) = 2\pi^2 x e^{-\pi^2 x^2}, \quad x > 0 \qquad x_1 \ldots x_n$

$$L = 2^n \pi^{2n} (x_1 x_2 \ldots x_n) e^{-\pi^2 \sum x_i^2}$$

$$\ln L = \ln 2^n + \ln \pi^{2n} + \ln(x_1 \ldots x_n) + \ln e^{-\pi^2 \sum x_i^2}$$

$$= n \ln 2 + 2n \ln \pi + \ln(x_1 \ldots x_n) - \pi^2 \sum x_i^2$$

$$\frac{\partial \ln L}{\partial \pi} = \frac{\cancel{2}n}{\pi} - \cancel{2}\pi \sum x_i^2 = 0$$

$$\boxed{\hat{\pi} = \sqrt{\frac{n}{\sum x_i^2}}}$$

2. a) $\bar{x} = 117$

$\hat{s} = 2{,}94392$

$n = 22$

b) $p = 90\% \qquad \alpha = 0{,}1 \qquad t_{1-\alpha/2} = 1{,}721$

$t_{1-\alpha/2} \cdot \frac{\hat{s}}{\sqrt{n}} = 1{,}0802$

$$\boxed{P(115{,}92 \le a \le 119{,}362) = 0{,}9}$$

$c_1 = \chi^2_{n-1, \alpha/2} = 11{,}591$

$c_2 = \chi^2_{n-1, 1-\alpha/2} = 32{,}671$

$\beta_2 = \frac{(n-1)\hat{s}^2}{c_1} = 15{,}7018$

$\beta_1 = \frac{(n-1)\hat{s}^2}{c_2} = 5{,}5706$

$$\boxed{P(5{,}5706 \le \sigma^2 \le 15{,}7018) = 0{,}9}$$

3. $n = 200 \qquad m = 112$

a) $p = 0{,}95 \qquad \alpha = 0{,}05$

$\hat{p} = \frac{112}{200} = 0{,}56$

$p_{1,2} = \hat{p} \pm u_{1-\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} =$

$= 0{,}56 \pm 1{,}96 \sqrt{\frac{0{,}56 \cdot 0{,}44}{200}}$

$u_{1-\alpha/2} = u_{0{,}975} = 1{,}96$

$$\boxed{P(0{,}4912 \le p \le 0{,}6288) = 0{,}95}$$

b) $p_h \le \hat{p} + u_{1-\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$0{,}5 < 0{,}56 - u_{1-\alpha/2} \sqrt{\frac{0{,}56 \cdot 0{,}44}{200}} \qquad u_{1-\alpha/2} < +0{,}06 \cdot \sqrt{\frac{200}{0{,}56 \cdot 0{,}44}} = 1{,}7094$

$1 - \alpha/2 = 0{,}955$

$\boxed{\alpha = 0{,}09} \Rightarrow \boxed{p = 0{,}91}$

c) Izabran $\alpha = 0{,}05$

$0{,}5 = 0{,}56 - u_{0{,}975} \sqrt{\frac{0{,}56 \cdot 0{,}44}{n}}$

$\sqrt{\frac{0{,}56 \cdot 0{,}44}{n}} = \frac{0{,}06}{1{,}96} \quad /^2$

$n = \frac{0{,}2464}{0{,}0009371} = 262{,}93 \Rightarrow \boxed{n = 263}$

4. $H_0 \quad \mu = 35 \qquad \alpha = 0{,}05$

$H_1 \quad \mu \ne 35 \qquad n = 20 \qquad \bar{x} = 35{,}07 \qquad \hat{s} = 0{,}16575$

$\hat{t} = \frac{35{,}07 - 35}{0{,}16575} \sqrt{20} = \underline{-1{,}8887}$

$t_{1-\alpha/2} = 2{,}093 \qquad |\hat{t}| < t_{1-\alpha/2} \Rightarrow$ <u>H_0 se prihvaća</u>

$n - 1 = 19$

5. $n_1 = 30 \qquad a_1 = 74 \qquad \sigma_1 = 8$

$n_2 = 40 \qquad a_2 = 77 \qquad \sigma_2 = 7$

$\alpha = 0{,}05$

$\sigma_z^2 = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = \underline{3{,}3583}$

$\hat{u} = \frac{74 - 77}{1{,}8326} = \underline{-1{,}637}$

$u_{1-\alpha/2} = u_{0{,}975} = 1{,}96$

$|\hat{u}| < u_{1-\alpha/2} \Rightarrow$ prihvaća se (razlika ne postoji)

6. $n = 190$

5 kocaka $\qquad X$ – broj šestica $\qquad X \sim (5, \frac{1}{6})$

x_j	n_j	p_j	$\frac{(n_j - np_j)^2}{np_j}$
0	75	0,40188	0,02412
1	77	0,40188	0,00541
2	30	0,16075	0,00964
3	6	0,03215	0,23397
4	1	0,003215	
5	1	0,000129	

$h = 4 - 0 - 1 = 3$

$p_j = \binom{5}{j} \left(\frac{1}{6}\right)^j \left(\frac{5}{6}\right)^{5-j}$

$$\boxed{\chi^2_q = 0{,}27044}$$

$\chi^2_q < \chi^2_{krit}$

razreda $\boxed{3} \qquad 0{,}27044 < \chi^2_{p=0{,}05} \Rightarrow \boxed{\alpha = 0{,}95}$

7. 4 strijelca ista meta $\qquad p_1 = 0{,}4$

$p_2 = 0{,}8$

a) meta pogođena – A $\qquad p_3 = 0{,}7$

$P_A = 1 - p(\text{svi promaše}) \qquad p_4 = 0{,}8$

$$\boxed{P_A = 1 - 0{,}6 \cdot 0{,}4 \cdot 0{,}3 \cdot 0{,}2 = 0{,}9856}$$

b) 3 metka, p(falio 4-ti strijelac) $\Rightarrow p = 0{,}4 \cdot 0{,}6 \cdot 0{,}7 \cdot 0{,}2 = \underline{0{,}0336}$ ✗

C – meta s 3 metka

H_4 – falio 4-ti

$P(H_4|C) = \frac{P(H_4, C)}{P(C)} = \frac{0{,}0336}{0{,}6 \cdot 0{,}6 \cdot 0{,}7 \cdot 0{,}8 + 0{,}4 \cdot 0{,}4 \cdot 0{,}7 \cdot 0{,}8 + 0{,}4 \cdot 0{,}6 \cdot 0{,}3 \cdot 0{,}8 + 0{,}4 \cdot 0{,}6 \cdot 0{,}7 \cdot 0{,}2}$

$$\boxed{P(H_4|C) = 0{,}0879}$$

27 08

2

(8) $f(x) = c(3-x) \qquad x \in (0,3)$

a) $c = ?$

$\int_0^3 c(3-x)\,dx = 1 \qquad c\int_0^3 (3-x)\,dx = c\left(3x - \frac{x^2}{2}\right)\Big|_0^3 = c\left(9 - \frac{9}{2}\right) \Rightarrow \boxed{c = \frac{2}{9}}$

b) $P(X<1) = F(x) = \frac{2}{9}\int_0^1 (3-x)\,dx = \frac{2}{9}\left(3x - \frac{x^2}{2}\right)\Big|_0^1 = \frac{2}{9}\left(3 - \frac{1}{2}\right) = \boxed{\frac{5}{9}}$

c) $E(x) = \int_0^3 x f(x)\,dx = \frac{2}{9}\int_0^3 x(3-x)\,dx = \frac{2}{9}\left[\frac{3}{2}x^2 - \frac{x^3}{3}\right]\Big|_0^3 = \frac{2}{9}\left(\frac{27}{2} - \frac{27}{3}\right) = 1$

(9) a) svojstvo odsustva pamćenja expo

$P(X < x+t \mid X > t) = P(X < x)$

b) dokaz

$Q(x) = P(X > x) = 1 - F(x)$

$Q(0) = P(X>0) = 1 \qquad Q'(x) = -f'(x) < 0 \quad , x > 0$

$Q'(0) = -\lambda$

$\cancel{1-} P(X > x+t \mid X>t) = \cancel{1-} P(X>x)$

$\frac{P(X > x+t, X>t)}{P(X>t)} = P(X>x)$

$P(X > x+t) = P(X>x)P(X>t)$

$Q(x+t) = Q(t)Q(x) \qquad / \frac{d}{dt}$

$Q'(x+t) = Q'(t)Q(x)$

$t=0 \qquad Q'(x) = Q'(0)Q(x) = -\lambda Q(x) \qquad Q(x) = Ce^{-\lambda x}$

$Q(0) = 1 \qquad C = 1$

$\boxed{F(x) = 1 - Q(x) = 1 - e^{-\lambda x}}$

c) $f(x) = \lambda e^{-\lambda x}$

$E(x) = \int_0^\infty x\lambda e^{-\lambda x}\,dx = \left| \begin{matrix} x = u & e^{-\lambda x}dx = dv \\ dx = du & -\frac{1}{\lambda}e^{-\lambda x} = v \end{matrix} \right| = \cancel{\lambda}\left[\cancel{-\frac{1}{\lambda}xe^{-\lambda x}\Big|_0^\infty} + \frac{1}{\cancel{\lambda}}\int_0^\infty e^{-\lambda x}dx\right] =$

$= -\frac{1}{\lambda}e^{-\lambda x}\Big|_0^\infty = \boxed{\frac{1}{\lambda}}$

(10) kocka $\qquad X$ – 2x vrijednost

Y – 1 za nepar, 3 za par

X | Y	1	3
2	$\frac{1}{6}$	0
4	0	$\frac{1}{6}$
6	$\frac{1}{6}$	0
8	0	$\frac{1}{6}$
10	$\frac{1}{6}$	0
12	0	$\frac{1}{6}$
		1

$Z = X + Y$

$Z \sim \begin{pmatrix} 3 & 7 & 11 & 15 \\ \frac{1}{6} & \frac{2}{6} & \frac{2}{6} & \frac{1}{6} \end{pmatrix} \quad \begin{matrix} x_i \\ p_i \end{matrix}$

$E(z) = 9$

$D(z) = E(z^2) - E^2(z) = 14{,}66667 = \boxed{\frac{44}{3}}$ ✓

PZI 07

1

(3) $X \; - \; f(x) = \begin{cases} \frac{x}{a^2}e^{-\frac{x}{a}} , & x > 0 \\ 0 , & x \le 0 \end{cases} \qquad a > 0$

$L = \frac{x_1}{a^2}e^{-\frac{x_1}{a}} \cdot \frac{x_2}{a^2}e^{-\frac{x_2}{a}} \cdots \frac{x_n}{a^2}e^{-\frac{x_n}{a}} = \frac{x_1x_2\cdots x_n}{a^{2n}}e^{\frac{-(x_1+x_2+\dots x_n)}{a}}$

$\ln L = \ln(x_1x_2x_n) - \ln a^{2n} - \frac{(x_1+x_2+\dots x_n)}{a}$

$\ln' L = 0 - \frac{2n}{a} + \frac{(x_1+x_2 \dots x_n)}{a^2} = 0$

$\boxed{a = \frac{\bar{X}}{2}}$

$E(X) = \int_0^\infty \frac{x^2}{a^2}e^{-\frac{x}{a}}dx = \frac{1}{a^2}\int_0^\infty x^2e^{-\frac{x}{a}}dx = \left| \begin{matrix} x^2 = u & e^{-\frac{x}{a}}dx = dv \\ 2x\,dx = du & -ae^{-\frac{x}{a}} = v \end{matrix} \right| =$

$= \frac{1}{a^2}\left(\cancel{-x^2ae^{-\frac{x}{a}}}\Big|_0^\infty + 2a\int_0^\infty xe^{-\frac{x}{a}}dx\right) = \left| \begin{matrix} x = u & e^{-\frac{x}{a}}dx = dv \\ dx = du & -ae^{-\frac{x}{a}} = v \end{matrix} \right| =$

$= \frac{2}{\cancel{a}}\left[\cancel{-xae^{-\frac{x}{a}}}\Big|_0^\infty + \cancel{a}\int_0^\infty e^{-\frac{x}{a}}dx\right] = 2(-a)e^{-\frac{x}{a}}\Big|_0^\infty = \boxed{2a}$

$E\left(\frac{\bar{X}}{2}\right) = \frac{1}{2}E\left(\sum_{i=1}^n X_i\right) = \frac{1}{2n}\cdot n\,E(X_i) = \frac{1}{2}\cdot 2a = a$ $\Rightarrow$ nepristrana ✓

(4) $\hat{s}^2 = \frac{1}{n}\sum(x_i - a)^2 \Rightarrow n\hat{s}^2 = \sum(x_i^2 - 2x_ia + a^2) = \sum x_i^2 - 2a\bar{x}n + na^2$

$\hat{\sigma}^2 = \frac{1}{n}\sum(x_i - \bar{x})^2 \Rightarrow n\hat{\sigma}^2 = \sum(x_i^2 - 2x_i\bar{x} + \bar{x}^2) = \sum x_i^2 - 2\bar{x}\bar{x}n + n\bar{x}^2$

$-2a\bar{x}\cancel{n} + 2\bar{x}^2\cancel{n} + \cancel{n}(a^2 - \bar{x}^2) = \cancel{n}(\hat{s}^2 - \hat{\sigma}^2)$

$\boxed{\hat{s}^2 = \hat{\sigma}^2 + (\bar{x} - a)^2}$

(5) $n = 17$

$X \sim N(a, \sigma^2) \qquad \boxed{\bar{x} = \frac{680}{17} = 40}$

$\boxed{\hat{s}^2} = \frac{1}{n-1}\sum(x_i - \bar{x})^2 = \frac{1}{n-1}\sum(x_i^2 - 2x_i\bar{x} + \bar{x}^2) = \frac{1}{16}\sum x_i^2 - \frac{1}{8}\sum x_i\bar{x} + \frac{1}{16}\sum\bar{x}^2 =$

$= \frac{1}{16}\cdot 34000 - \frac{1}{8}\bar{x}\cdot\underbrace{\sum x_i}_{n\bar{x}} + \frac{1}{16}\cdot n\bar{x}^2 = 2125 - 3400 + 1700 = \boxed{425}$

$p = 0{,}9 \qquad \alpha = 0{,}1$

$c_1 = \chi^2_{16, 0,05} = 7{,}962 \qquad \beta_2 = \frac{16\cdot 425}{7{,}962} = 854{,}057$

$c_2 = \chi^2_{16, 0,95} = 26{,}296 \qquad \beta_1 = \frac{16\cdot 425}{26{,}296} = 258{,}594$

$\boxed{P(258{,}594 \le \sigma^2 \le 864{,}057) = 0{,}9}$

(6) $n = 288$

$\alpha = 0{,}05$

x_j	n_j	p_j	$n_j - np_j$	$\frac{(n_j - np_j)^2}{np_j}$
0	154	0,52091	3,97792	0,10548
1	93	0,34004	−4,93152	0,24834
2	31	0,11092	−0,94496	0,02795
3	7	0,02414		
4	3	0,003939	1,76608	0,3788
5	0	0,000514		
	$\Downarrow$ 10	$\Downarrow$ 0,02859		

$\boxed{\chi^2_g = 0{,}76057}$

$\lambda = \bar{x} = 0{,}65278$

$\hat{s} = 0{,}84577$

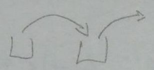
$p_j = \frac{\lambda^j}{j!} e^{-\lambda} = \frac{0{,}65278^j}{j!} e^{-0{,}652}$

$h = 4 - 1 - 1 = 2$ $\qquad \chi^2_{krit} = \chi^2_{2,\,0,95} = \underline{5{,}991}$ $\qquad \boxed{\chi^2_g < \chi^2_{krit}}$

prihvaća se

(7) 1. vrč 2B 1C

2. vrč 1B 5C



B $\qquad p(\text{iz } 1. \text{ u } 2.\ c)$

H_1 – prebacili crnu $\Rightarrow p(H_1) = \frac{1}{3}$

H_2 – prebacili bijelu $\Rightarrow p(H_2) = \frac{2}{3}$

$p(A|H_1) = \frac{1}{7}$ $\qquad p(A|H_2) = \frac{2}{7}$ $\qquad \Rightarrow p(A) = \sum p(H_i)\,p(A|H_i) = \frac{1}{3}\cdot\frac{1}{7} + \frac{2}{3}\cdot\frac{2}{7} = \boxed{\frac{5}{21}}$

$p(H_1|A) = \frac{p(A|H_1)\cdot p(H_1)}{p(A)} = \frac{\frac{1}{7}\cdot\frac{1}{3}}{\frac{5}{21}} = \frac{1}{5}$

(8) $X \sim N(16, 6)$ $\qquad \mu = 16$ $\qquad p = 0{,}95$

$P(16 - a < X < 16 + a) = 0{,}95$

$P\left(-\frac{a}{\sqrt{6}} < \frac{X - 16}{\sqrt{6}} < \frac{a}{\sqrt{6}}\right) = 0{,}95$ $\qquad 0{,}95 = \Phi^*\left(\frac{a}{\sqrt{6}}\right)$

$\frac{a}{\sqrt{6}} = 1{,}96$ $\qquad \boxed{a = 4{,}8}$

$\boxed{P(11{,}2 < X < 20{,}8) = 0{,}95}$

(9) $X, Y \sim \mathcal{E}(\lambda = 1)$ $\qquad f(x) = e^{-x}$ $\qquad f(y) = e^{-y}$ $\qquad f(x,y) = e^{-(x+y)}$

$z = x + y$ $\qquad y = z - x$

$g_z(z) = \int_{-\infty}^{\infty} e^{-(x+z-x)} \left|\frac{\partial y}{\partial z}\right| dx = \int_0^z e^{-z}\,dx = x e^{-z}\Big|_0^z = \underline{z e^{-z}}$ $\quad \underline{z > 0}$

PZI 07 $\qquad$ 2

(10) $\bar{x}_1 = 3{,}07895$ $\qquad n_1 = 76$ $\qquad \hat{s}_1 = 1{,}16348$

$\bar{x}_2 = 3{,}01282$ $\qquad n_2 = 78$ $\qquad \hat{s}_2 = 1{,}16768$

$\hat{s}_z^2 = \frac{1}{76 + 78 - 2}\left(75 \cdot 1{,}16348^2 + 77 \cdot 1{,}16768^2\right) = \underline{1{,}358646}$

$\hat{t} = \frac{\bar{x}_1 - \bar{x}_2}{\hat{s}_z}\sqrt{\frac{76 \cdot 78}{76 + 78}} = \underline{0{,}351997}$

$t_{1-\alpha/2} < \hat{t}.$ $\qquad t_{1-\alpha/2} < 0{,}351997$ $\qquad h = n + m - 2 \Rightarrow \underline{152}$

$\Downarrow$ (tab)

$f = 0{,}7 \quad = 70\,\%$

PZI 08

(1) $f(x) = \lambda x^{\lambda - 1}$

$L = \lambda x_1^{\lambda-1} \cdot \lambda x_2^{\lambda-1} \cdot \ldots \cdot \lambda x_n^{\lambda-1} = \lambda^n (x_1 x_2 \cdots x_n)^{\lambda-1}$

$\ln L = \ln \lambda^n + \ln (x_1 \cdots x_n)^{\lambda-1} = n \ln \lambda + (\lambda - 1)\ln(x_1 \cdots x_n)$

$(\ln L)' = \frac{n}{\lambda} + \ln(x_1 x_2 \cdots x_n) = 0$ $\qquad \boxed{\lambda = \frac{-n}{\sum_{i=1}^{n} \ln x_i}}$

(2) $\sigma^2 = 0{,}0025$ $\qquad \sigma = 0{,}05$

$p = 0{,}95$ $\qquad (a - 0{,}01 < a < a + 0{,}01) = 0{,}95$

$\alpha = 0{,}05$

$u_{1-\alpha/2}\frac{\sigma}{\sqrt{n}} = 0{,}01$

$\sqrt{n} = \frac{\sigma \cdot u_{0,975}}{0{,}01}$

$n = \frac{0{,}0025 \cdot 1{,}96^2}{0{,}0001} = 96{,}04 \Rightarrow \boxed{n \geq 97}$

(3) $\boxed{\bar{x} = 25}$

a) $n = 22$

$\boxed{\hat{s} = 2{,}94392}$ $\qquad \boxed{\hat{s}^2 = 8{,}6667}$

$p = 0{,}8$ $\qquad \boxed{\alpha = 0{,}2}$

b) $t_{1-\alpha/2} = t_{0,9}$ $\qquad n - 1 = 21$ $\qquad t_{1-\alpha/2}\cdot\frac{\hat{s}}{\sqrt{n}} = 1{,}323 \cdot \frac{2{,}94392}{\sqrt{22}} = 0{,}83038$

$\boxed{P(24{,}1696 \leq a \leq 25{,}8304) = 0{,}8}$

$c = \chi^2_{0,2,\,21} = 15{,}445$ $\qquad \beta = \frac{21 \cdot 8{,}6667}{15{,}445} = 11{,}7838$

$\boxed{P(0 \leq \sigma^2 \leq 11{,}7838) = 0{,}8}$

PZI 07

2

(10) $\bar{X}_1 = 3{,}07895$ $n_1 = 76$ $\hat{s}_1 = 1{,}16348$

$\bar{X}_2 = 3{,}01282$ $n_2 = 78$ $\hat{s}_2 = 1{,}16768$

$$\boxed{\hat{S}_Z^2} = \frac{1}{76+78-2}\left(75 \cdot 1{,}16348^2 + 77 \cdot 1{,}16768^2\right) = \underline{1{,}358646}$$

$$\hat{t} = \frac{\bar{X}_1 - \bar{X}_2}{\hat{S}_Z}\sqrt{\frac{76 \cdot 78}{76+78}} = \underline{0{,}351997}$$

$t_{1-\alpha/2} < \hat{t}$. $t_{1-\alpha/2} < 0{,}351997$ $k = n+m-2 \Rightarrow \underline{152}$

$\Downarrow$ (tab)

$\alpha = 0{,}7 = 70\%$

PZI 08

(1) $f(x) = \mathcal{R} x^{\mathcal{R}-1}$

$L = \mathcal{R} x_1^{\mathcal{R}-1} \cdot \mathcal{R} x_2^{\mathcal{R}-1} \cdot \ldots \cdot \mathcal{R} x_n^{\mathcal{R}-1} = \mathcal{R}^n (x_1 x_2 \ldots x_n)^{\mathcal{R}-1}$

$\ln L = \ln \mathcal{R}^n + \ln (x_1 \ldots x_n)^{\mathcal{R}-1} = n \ln \mathcal{R} + (\mathcal{R}-1) \ln (x_1 \ldots x_n)$

$(\ln L)' = \frac{n}{\mathcal{R}} + \ln(x_1 x_2 \ldots x_n) = 0$ $\boxed{\mathcal{R} = \frac{-n}{\sum_{i=1}^{n} \ln x_i}}$

(2) $\sigma^2 = 0{,}0025$ $\sigma = 0{,}05$

$p = 0{,}95$ $(a - 0{,}01 < a < a + 0{,}01) = 0{,}95$

$\alpha = 0{,}05$

$$u_{1-\alpha/2} \frac{\sigma}{\sqrt{n}} = 0{,}01$$

$$\sqrt{n} = \frac{\sigma \cdot u_{0{,}975}}{0{,}01}$$

$$n = \frac{0{,}0025 \cdot 1{,}96^2}{0{,}0001} = 96{,}04 \Rightarrow \boxed{n \geq 97}$$

(3) $\boxed{\bar{X} = 25}$

a) $n = 22$

$\boxed{\hat{s} = 2{,}94392}$ $\boxed{\hat{s}^2 = 8{,}6667}$

$p = 0{,}8$ $\boxed{\alpha = 0{,}2}$

b) $t_{1-\alpha/2} = t_{0{,}9}$ $n-1 = 21$ $t_{1-\alpha/2} \cdot \frac{\hat{s}}{\sqrt{n}} = 1{,}323 \cdot \frac{2{,}94392}{\sqrt{22}} = 0{,}83038$

$\boxed{P(24{,}1696 \leq a \leq 25{,}8304) = 0{,}8}$

$c = \chi^2_{0{,}2,\,21} = 15{,}445$ $\beta = \frac{21 \cdot 8{,}6667}{15{,}445} = 11{,}7838$

$\boxed{P(0 \leq \sigma^2 \leq 11{,}7838) = 0{,}8}$

(4) $n = 100$, 3 loše $\hat{p} = 0{,}03$ $c = 1{,}96$

$\alpha = 0{,}05$

$$p_{1,2} = \frac{\hat{p} + \frac{c^2}{2n} \mp c\sqrt{\frac{\hat{p}(1-\hat{p})}{n} + \frac{c^2}{4n^2}}}{1 + \frac{c^2}{n}}$$

$$p_{1,2} = \frac{0{,}03 + \frac{1{,}96^2}{200} \mp 1{,}96\sqrt{\frac{0{,}03 \cdot 0{,}97}{100} + \frac{1{,}96^2}{4 \cdot 10000}}}{1 + \frac{1{,}96^2}{100}}$$

$p_1 = 0{,}010254$ $p_2 = 0{,}08452$

$\boxed{P(0{,}010254 \leq p \leq 0{,}08452) = 0{,}95}$

$p_2 = 0{,}05$ $0{,}05 > \hat{p} + \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \cdot u_{1-\alpha/2}$

$0{,}05 - 0{,}03 > \sqrt{\frac{0{,}03 \cdot 0{,}97}{n}} \cdot 1{,}96$

$0{,}0004 > \frac{0{,}0291 \cdot 1{,}96^2}{n}$ $n > 279{,}476$ $\boxed{n \geq 280}$

(5) $n = 60$ $\bar{X} = 34$ $\hat{s} = 4{,}8$

$\mu = 32$ $\alpha = 0{,}01$ $H_0 \ldots \mu = 32$ $X \sim N(\mu, \sigma^2)$ σ^2 nepoznat

$H_1 \ldots \mu \neq 32$

$$\hat{t} = \frac{\bar{X} - \mu_0}{\hat{s}}\sqrt{n} = \frac{34-32}{4{,}8}\sqrt{60} = \underline{3{,}2275}$$

$t_{59,\,1-\alpha/2} = \underline{2{,}660}$ $\boxed{|\hat{t}| > t_{1-\alpha/2}} \Rightarrow H_0$ se odbacuje

(6) $n = 120$

x_j	m_j	p_j	$m_j - np_j$	$\frac{(m_j - np_j)^2}{np_j}$
1	20	$\frac{1}{6}$	0	0
2	14	$\frac{1}{6}$	−6	1,8
3	23	$\frac{1}{6}$	3	0,45
4	12	$\frac{1}{6}$	−8	3,2
5	26	$\frac{1}{6}$	6	1,8
6	25	$\frac{1}{6}$	5	1,25

$k = 6 - 1 = \underline{5}$

$\boxed{\chi^2_g = 8{,}5}$ $\chi^2_g < \chi^2_{krit}$

$\chi^2_{krit} = \underline{9{,}236}$

$1 - \alpha = 0{,}9$

$\boxed{\alpha = 0{,}1}$