

$$1.) f(x) = 10e^{-10x}, x > 0$$

$$E(X), D(X) = ?$$

$$f(x) = \lambda e^{-\lambda x}$$

$$\lambda = 10$$

$$E(X) = \frac{1}{\lambda} = \frac{1}{10} = 0,1$$

$$D(X) = \frac{1}{\lambda^2} = \frac{1}{100} = 0,01$$

$$2.) P(X > 12) = 1 - P(X \leq 12) = 1 - (1 - e^{-12\lambda}) = e^{-12\lambda} = 0,4$$

$$P(X > 6) = 1 - P(X \leq 6) = 1 - (1 - e^{-6\lambda}) = e^{-6\lambda} = \sqrt{0,4}$$

Y (ispravni rad tijekom 6 mjeseci) $\sim B(50, \sqrt{0,4}) \rightarrow$ aproksimacija normalnom $\rightarrow n=50$

$$B(n, p) \sim N(np, npq) = N(31,623, 11,623)$$

$$P(Y \geq 40) = 0,5 (1 - \Phi(\frac{40 - 31,623}{\sqrt{11,623}})) = 0,5 (1 - \Phi(2,46)) = 0,007$$

$$p = \sqrt{0,4}$$

$$q = 1 - p$$

$$3.) f(x) = \frac{1}{3}e^{-\frac{x}{3}}, x > 0$$

$$\text{EKSPONENCIJALNA: } f(x) = \lambda e^{-\lambda x} \rightarrow \lambda = \frac{1}{3}$$

$$F(x) = 1 - e^{-\frac{x}{3}}$$

$$A = P(X > 3) = 1 - P(X \leq 3) = 1 - (1 - e^{-\frac{3}{3}}) = e^{-1} = \frac{1}{e}$$

$$B = P(X > 6 | X > 3) = \frac{P(X > 6, X > 3)}{P(X > 3)} = \frac{P(X > 6)}{P(X > 3)} = \frac{1 - P(X \leq 6)}{1 - P(X \leq 3)} = \frac{1 - F(6)}{1 - F(3)} = \frac{1 - e^{-\frac{6}{3}}}{1 - e^{-\frac{3}{3}}} = \frac{1}{e}$$

$$4.) E(X) = 6 \Rightarrow \lambda = \frac{1}{6}$$

$$Z = A + B \rightarrow F(z) = 1 - e^{-\lambda z} - \lambda z e^{-\lambda z}$$

$$P(Z > 12) = 1 - P(Z \leq 12) = 1 - (1 - e^{-\frac{12}{6}} - \frac{12}{6} e^{-\frac{12}{6}}) = e^{-2} + 2e^{-2} = 3e^{-2} = \frac{3}{e^2}$$

$$5.) \lambda = \frac{1}{200}$$

$$f(x) = C \cdot \lambda e^{-\lambda x} \rightarrow \int_0^{365} C \cdot \lambda e^{-\lambda x} dx$$

$$E(X) = \frac{1}{\lambda \cdot C} = 200 \cdot (1 - e^{-\frac{365}{200}})$$

$$= 167,75 \sim 168 \text{ dana}$$

$$\Rightarrow C = \frac{1}{1 - e^{-\frac{365}{200}}}$$

1. ⁶₃Li

$$a) P(0 < X < 1,42) = \Phi(1,42) - \Phi(0) = \frac{1}{2} (\phi^*(1,42) - \phi^*(0)) = \underline{\underline{0,42}}$$

$$b) P(-0,73 < x < 0) = \frac{1}{2}(\phi(0) + \phi^*(0,73)) = \frac{1}{2}(0 + 0,53461) = \underline{\underline{0,2673}}$$

$$c) P(-1,73 < X < 2,01) = \frac{1}{2} (\phi^*(2,01) + \phi^*(1,73)) = \frac{1}{2} (0,95557 + 0,91637) = \underline{0,9359}$$

$$d) P(0,65 \leq X \leq 1,26) = \frac{1}{2} (\Phi^*(1,26) - \Phi^*(0,65)) = \underline{0,1540}$$

$$c) P(-1,76 < X < -0,54) = \frac{1}{2} (-\phi^*(0,54) + \phi^*(1,76)) = \frac{1}{2} (-0,41020 + 0,92159) = 0,2557$$

$$d) P(X > 1,13) = 1 - \Phi(1,13) = \frac{1}{2} - \frac{1}{2} \Phi^*(1,13) = \underline{0,1292}$$

$$g) P(|X| \leq 0,5) = P(-0,5 \leq X \leq 0,5) = \frac{1}{2}(\phi'(0,5) + \phi'(-0,5)) = 0,383$$

12.) χ^2 jedinična normalna, $t = ?$

$$a) P(0 < X < t) = 0,4236 = \frac{1}{\sigma} (\Phi(t) - \Phi(0)) \Rightarrow \Phi(t) = 0,8472$$

$$b) P(X < t) = 0,7967 = \frac{1}{2} + \frac{1}{2} \Phi(t) \Rightarrow \frac{1}{2} \Phi(t) = 0,2967 \xRightarrow{t=1,43} \Phi(t) = 0,5934$$

c) $P(1 < X < 2) = 0,1 = \frac{1}{2}(\phi^*(2) - \phi^*(1)) \Rightarrow \phi^*(1) = 0,7545$ $t = 0,83$
 $t = 1,16$

13.) $X \sim \text{normalna}$ s oddivanjem 8 i odstopanjem 4.

$$a=8, G=4$$

$$a=8, b=4$$

$$a) P(5 < X < 10) = \frac{1}{2} \left(\Phi^* \left(\frac{10-8}{4} \right) - \Phi^* \left(\frac{5-8}{4} \right) \right) = \frac{1}{2} (\Phi^*(0.5) + \Phi^*(0.75)) = \underline{\underline{0.4649}}$$

$$b) P(10 < X < 15) = \frac{1}{2} (\phi^*(\frac{15-8}{4}) - \phi^*(\frac{10-8}{4})) = \frac{1}{2} (\phi^*(1,75) - \phi^*(0,5)) = \underline{0,26898}$$

$$c) P(X > 15) = 1 - \Phi\left(\frac{15-8}{4}\right) = \frac{1}{2} - \frac{1}{2} \Phi^*(1,75) = \underline{0,0401}$$

$$d) P(X \leq 5) = \frac{1}{2} + \frac{1}{2} \phi\left(\frac{5-8}{4}\right) = \frac{1}{2} + \frac{1}{2} \phi(-0.75) = \frac{1}{2} - \frac{1}{2} \phi(0.75) = \underline{\underline{0.226625}}$$

14.) $\sqrt{(2,4)} \rightarrow a=2, G^2=4 \Rightarrow G=2$

$$P(-1 < X < 1 \mid 0 < X < 3) = \frac{P(0 < X < 1)}{P(0 < X < 3)} = \frac{\frac{1}{2}(\phi^*(\frac{1-2}{2}) - \phi^*(\frac{0-2}{2}))}{\frac{1}{2}(\phi^*(\frac{3-2}{2}) - \phi^*(\frac{0-2}{2}))} = \frac{(-\phi^*(0,5) + \phi^*(1))}{\phi^*(0,5) + \phi^*(1)} = \frac{-0,38292 + 0,68269}{0,38292 + 0,68269} = \frac{0,29977}{1,06561} = 0,28131$$

$$15.) a=0, \sigma=30 \text{ m}$$

$$P(|x| < 42) = P(-42 < X < 42) = \frac{1}{2} \left[\Phi\left(\frac{42-0}{30}\right) + \Phi\left(\frac{42-0}{30}\right) \right] = \frac{1}{2} [2 \cdot \Phi(1,4)]$$

$$= \underline{\underline{0,83849}}$$

$$16.) a=3, P(X < 5) = 0,6915$$

$$P(-1 < X < 6) = ?$$

$$0,6915 = \frac{1}{2} + \frac{1}{2} \Phi\left(\frac{5-3}{\sigma}\right)$$

$$0,1915 = \frac{1}{2} \Phi\left(\frac{2}{\sigma}\right)$$

$$\Phi\left(\frac{2}{\sigma}\right) = 0,383$$

$$\frac{2}{\sigma} = 0,5 \Rightarrow \underline{\underline{\sigma=4}}$$

$$P(-1 < X < 6) = \frac{1}{2} \left[\Phi\left(\frac{6-3}{4}\right) - \Phi\left(\frac{-1-3}{4}\right) \right]$$

$$= \frac{1}{2} [\Phi(0,75) + \Phi(1)] = \underline{\underline{0,61472}}$$

$$17.) N(4, \sigma^2) \Rightarrow a=4$$

$$P(2 < X < 6) = 0,8664$$

$$\sigma = ?$$

$$0,8664 = \frac{1}{2} \left[\Phi\left(\frac{6-4}{\sigma}\right) - \Phi\left(\frac{2-4}{\sigma}\right) \right] = \frac{1}{2} \left[\Phi\left(\frac{2}{\sigma}\right) + \Phi\left(\frac{2}{\sigma}\right) \right] = \frac{1}{2} \cdot 2 \cdot \Phi\left(\frac{2}{\sigma}\right)$$

$$0,8664 = \Phi\left(\frac{2}{\sigma}\right)$$

$$\frac{2}{\sigma} = 1,5 \Rightarrow \underline{\underline{\sigma=1,33}}$$

$$20.) Y = \sqrt{|X|}$$

$$X \sim (0,1)$$

$$X = \pm Y^2$$

$$f(y) = f(x) \cdot \left| \frac{dx}{dy} \right| = \frac{1}{\sigma \sqrt{2\pi}} \cdot e^{-\frac{(y^2-a)^2}{2\sigma^2}} \cdot |(y^2)'| + \frac{1}{\sigma \sqrt{2\pi}} \cdot e^{-\frac{(y^2-a)^2}{2\sigma^2}} \cdot |(-y^2)'|$$

$$f(y) = \frac{1}{\sigma \sqrt{2\pi}} \cdot e^{-\frac{(y^2-0)^2}{2\sigma^2}} \cdot 2y + \frac{1}{\sigma \sqrt{2\pi}} \cdot e^{-\frac{(y^2-0)^2}{2\sigma^2}} \cdot 2y$$

$$= \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}y^4} \cdot 2y + \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}y^4} \cdot 2y = \frac{2 \cdot 2y}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}y^4}$$

$$= \underline{\underline{\frac{4y}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}y^4}}}$$

$$21.) P(X > 52) = \frac{1}{2} - \frac{1}{2} \Phi^* \left(\frac{52-50}{\sigma} \right) = 0,1$$

$$-0,1 = -\frac{1}{2} \Phi^* \left(\frac{2}{\sigma} \right)$$

$$0,8 = \Phi^* \left(\frac{2}{\sigma} \right)$$

$$\left(\frac{2}{\sigma} \right) = 1,29$$

$$\sigma = 1,55038$$

$$\Phi^*(0,99) = 2,58$$

$$\Phi \left(\frac{|X-50|}{\sigma} \right) = 0,99$$

$$\frac{|X-50|}{\sigma} = 2,58$$

$$|X-50| = 2,58 \cdot \sigma$$

$$|X-50| = 4$$

$$50-4 < X < 50+4$$

$$46 < X < 54$$

$$22.) N(a, \sigma^2)$$

$$f(y) = ?, y = (x-a)^2$$

$$E(y) = ?$$

$$x = \pm \sqrt{y} + a, f(y) = f(x) \cdot \left| \frac{dx}{dy} \right| = \frac{1}{\sigma \sqrt{2\pi}} \cdot e^{-\frac{(\sqrt{y}+a-a)^2}{2\sigma^2}} \cdot \left| \frac{1}{2\sqrt{y}} \right|$$

$$f(y) = \frac{1}{\sigma \sqrt{2y\pi}} \cdot e^{-\frac{y}{2\sigma^2}}$$

$$\begin{aligned} E(y) &= E((x-a)^2) = E(x^2 - 2ax + a^2) = E(x^2) - 2aE(x) + a^2; E(x) = a \\ &= E(x^2) - 2a \cdot a + a^2 = E(x^2) - 2a^2 + a^2 = E(x^2) - a^2 = \\ &= \underbrace{E(x^2) - (E(x))^2}_{D(x)} \Rightarrow D(x) = \sigma^2 \end{aligned}$$

$$23.) a=1000$$

$$P(X > 1020p) = \frac{1}{2} - \frac{1}{2} \Phi^* \left(\frac{1020-1000}{\sigma} \right) = 0,1$$

$$0,1 = \frac{1}{2} - \frac{1}{2} \Phi^* \left(\frac{20p}{\sigma} \right)$$

$$0,8 = \Phi^* \left(\frac{20p}{\sigma} \right)$$

$$\frac{20p}{\sigma} = 1,29$$

$$\sigma = 15,50387$$

$$\Phi^* \left(\frac{|X-1000|}{\sigma} \right) = 0,99$$

$$\frac{|X-1000|}{15,50387} = 2,58$$

$$|X-1000| = 40p$$

$$960p < X < 1040p$$

$$24.) \mu = 0,5 \text{ €}$$

$$\sigma = 0,01 \text{ €}$$

5 elemenata $> 2,55 \text{ €}$

$$Y = X_1 + X_2 + X_3 + X_4 + X_5$$

$$X_i \sim N(0,5, 0,01^2)$$

$$P(X > 2,55) = \frac{1}{2} - \frac{1}{2} \left(\frac{2,55 - 2,5}{\sqrt{0,0005}} \right) = \frac{1}{2} - \frac{1}{2} \phi^*(2,23) = \underline{\underline{0,0128}}$$

$$25.) Z = X_1 + X_2 + X_3 + Y_1 + Y_2$$

$$X \sim N(\mu_x = 200 \text{ p, } \sigma_x = 10 \text{ p}), \quad Y \sim N(\mu_y = 100 \text{ p, } \sigma_y = 20 \text{ p})$$

$$P(Z > 750 \text{ p}) = ?$$

STABILNOST NORMALNE R.

$$Z \sim (200 + 200 + 200 + 100 + 100, 100 + 100 + 100 + 400 + 400)$$

$$Z \sim (800, 1100)$$

$$P(Z > 750 \text{ p}) = \frac{1}{2} - \frac{1}{2} \left(\frac{750 - 800}{\sqrt{1100}} \right) = \frac{1}{2} - \frac{1}{2} (-1,5) = \frac{1}{2} + \frac{1}{2} (1,5) = \underline{\underline{0,933}}$$

$$26.) X_1 \sim N(0,1)$$

$$X_2 \sim N(1,1)$$

$$X_3 \sim N(2,4)$$

$$P(X_1 < X_3 - X_2) = ?$$

$$0 < X_3 - X_2 - X_1$$

$$B \sim N(1 \cdot 2 - 1 \cdot 1 - 1 \cdot 0, 1^2 \cdot 4 + 1^2 \cdot 1 + 1^2 \cdot 1) \quad \leftarrow \text{aproksimacija binomne normalnom}$$

+ stabilnost normalne

$$B \sim N(1,6)$$

$$P(B < 0) = \frac{1}{2} + \frac{1}{2} \phi^*\left(\frac{1}{\sqrt{6}}\right) = \underline{\underline{0,6586}}$$

$$27.) X \sim N(1,1)$$

$$Y \sim N(4,4)$$

$$Z \sim N(9,9)$$

$$P(X \leq 3Y - 2Z - X) = ?$$

$$0 \leq 3Y - 2Z - X$$

$$B \sim N(3 \cdot 4 - 2 \cdot 9 - 1 \cdot 1, 3^2 \cdot 4 + 2^2 \cdot 9 + 1^2 \cdot 1)$$

$$B \sim N(-7, 73)$$

$$P(B < 0) = \frac{1}{2} + \frac{1}{2} \phi^*\left(\frac{-7}{\sqrt{73}}\right) = \underline{\underline{0,2063}}$$

$$28.) a_x = 2, a_y = 3, \sigma_x^2 = \sigma_y^2 = \sigma^2$$

$$P(X > Y) = 0,4$$

$$\sigma^2 = ?$$

$$0 > Y - X$$

$$B \sim N(3-2, \sigma^2 + \sigma^2)$$

$$B \sim N(1, 2\sigma^2)$$

$$P(B > 0) = 0,4$$

$$P(B > 0) = \frac{1}{2} + \frac{1}{2} \phi^*\left(\frac{1}{\sqrt{2}\sigma}\right) = 0,4$$

$$-0,1 = -\frac{1}{2} \phi^*\left(\frac{1}{\sqrt{2}\sigma}\right) \cdot (-1) \cdot 2$$

$$0,2 = \phi^*\left(\frac{1}{\sqrt{2}\sigma}\right) \Rightarrow \frac{1}{\sqrt{2}\sigma} = 0,26$$

$$\sigma = 2,71964 \Rightarrow \sigma^2 = 7,39$$

$$\sigma = 2,71964$$

$$29.) a_x = 1, a_y = 3, \sigma_x, \sigma_y = ?$$

$$P(0 < X < 1) = P(2 < Y < 4) = 0,4$$

$$P(2 < X+Y < 6) = ?$$

$$P(0 < X < 1) = \frac{1}{2} \left(\phi^*\left(\frac{1-1}{\sigma_x}\right) - \phi^*\left(\frac{0-1}{\sigma_x}\right) \right) = \frac{1}{2} \left(\phi^*(0) - \phi^*\left(-\frac{1}{\sigma_x}\right) \right) = 0,4$$

$$0,4 = \frac{1}{2} \phi^*\left(\frac{1}{\sigma_x}\right)$$

$$0,8 = \phi^*\left(\frac{1}{\sigma_x}\right)$$

$$\frac{1}{\sigma_x} = 1,28 \Rightarrow \sigma_x = 0,78125$$

$$P(2 < Y < 4) = \frac{1}{2} \left(\phi^*\left(\frac{4-3}{\sigma_y}\right) - \phi^*\left(\frac{2-3}{\sigma_y}\right) \right) = \frac{1}{2} \left(\phi^*\left(\frac{1}{\sigma_y}\right) + \phi^*\left(\frac{1}{\sigma_y}\right) \right) = 0,4$$

$$0,4 = \frac{1}{2} \cdot 2 \cdot \phi^*\left(\frac{1}{\sigma_y}\right)$$

$$\frac{1}{\sigma_y} = 0,525 \Rightarrow \sigma_y = 1,90476$$

STABILNOST: $X+Y$

$$N(a_x + a_y, \sigma_x^2 + \sigma_y^2)$$

$$N(4, 2,0587^2)$$

$$P(2 < X+Y < 6) = \frac{1}{2} \left(\phi^*\left(\frac{6-4}{2,0587}\right) - \phi^*\left(\frac{2-4}{2,0587}\right) \right) = \phi^*(0,971) = 0,668$$

$$30.) X_1 \sim N(5, 4) \\ X_2 \sim N(4, 9)$$

$$Z = 2X_1 + X_2$$

$$Z \sim N(2 \cdot 5 + 1 \cdot 4, 2^2 \cdot 4 + 1^2 \cdot 9)$$

$$Z \sim N(14, 25)$$

$$W = X_1 + 2X_2$$

$$W \sim N(1 \cdot 5 + 2 \cdot 4, 1^2 \cdot 4 + 2^2 \cdot 9)$$

$$W \sim N(13, 40)$$

$$P(Z > W) = ?$$

$$0 > W - Z$$

$$B \sim N(13 - 14, 40 - 25)$$

$$B \sim N(-1, 15)$$

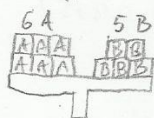
$$P(B > 0) = \frac{1}{2} - \frac{1}{2} \Phi^*\left(\frac{-1}{\sqrt{15}}\right)$$

$$= \frac{1}{2} + \frac{1}{2} \Phi^*(0, 258)$$

$$= \frac{1}{2} + \frac{1}{2} \cdot 0, 20359 = \underline{\underline{0, 601}}$$

$$31.) A \rightarrow a_A = 3, \sigma_A = 0, 7$$

$$B \rightarrow a_B = 4, \sigma_B = 0, 2$$



$$P(B > A) = ?$$

$$P(B > 0) = \frac{1}{2} - \frac{1}{2} \Phi^*\left(\frac{-2}{\sqrt{3, 14}}\right)$$

$$0 > A - B > A + A + A + A + A - (B + B + B + B) = \frac{1}{2} + \frac{1}{2} \Phi^*(4, 12)$$

$$B \sim N(3 + 3 + 3 + 3 + 3 - (4 + 4 + 4 + 4), 6 \cdot 0, 7^2 + 5 \cdot 0, 2^2) = \frac{1}{2} + \frac{1}{2} \cdot 0, 73729 = \underline{\underline{0, 868}}$$

$$B \sim N(-2, 3, 14)$$

$$32.) X \sim \text{broj pojavljenih devetki}$$

$$\text{vjerovatnost pojavljivanja 9-ke: } P(9) = \frac{1}{10} = 0, 1$$

$$n = 10000 \text{ izabranih znamenki}$$

$$q = 1 - p = 0, 9$$

APROKSIMACIJA BINOMNE NORMALNOM ZA VELIKI n :

$$X \sim B(n, p) \sim B(10000, 0, 1) \sim N(np, npq) \sim N(1000, 900)$$

$$P(940 < X < 1060) = \frac{1}{2} \left(\Phi^*\left(\frac{1000 - 940}{\sqrt{900}}\right) - \Phi^*\left(\frac{1000 - 1060}{\sqrt{900}}\right) \right) = \frac{1}{2} \cdot 2 \cdot \Phi^*\left(\frac{60}{\sqrt{900}}\right)$$

$$= \Phi^*(2) = \underline{\underline{0, 9545}}$$

33.) $X \sim$ broj pojavljivanja 1-ice na kocki

$$p = P(X) = \frac{1}{6}$$

$$n = 1200$$

$$q = \frac{5}{6}$$

$$B(n, p) \sim N(np, npq) \sim N(200, 166,66)$$

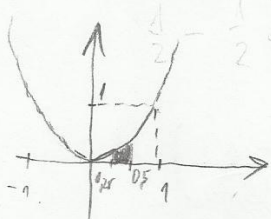
$$P(X \leq 140) = \frac{1}{2} + \frac{1}{2} \Phi^* \left(\frac{140 - 200}{\sqrt{166,66}} \right) = \frac{1}{2} - \frac{1}{2} \Phi^*(4,64) = \frac{1}{2} - \frac{1}{2} \cdot 1 = 0$$

NE PRIHVATAMO HIPOTEZU!

34.) $F(x) = x^2, 0 < x < 1$

$$n = 50$$

$$P(X \geq 10), 0,25 < x < 0,5 = ?$$



$$P(0,25 < x^2 < 0,5) = F(0,5) - F(0,25) = 0,5^2 - 0,25^2 = 0,1875$$

$$B(n, p) \sim N(np, npq) = N(50 \cdot 0,1875, 50 \cdot 0,1875 \cdot (1 - 0,1875))$$

$$N(9,375, 7,617)$$

$$P(X \geq 10) = \frac{1}{2} - \frac{1}{2} \Phi^* \left(\frac{9,5 - 9,375}{\sqrt{7,617}} \right)$$

$$= \frac{1}{2} - \frac{1}{2} \Phi^*(0,045) = 0,5 - \frac{1}{2} \cdot 0,03589 = 0,482$$

! KOREKCIJA \rightarrow za n -ove između 10 i 50 uzimamo pomaknute vrijednosti: $P(X \geq 10) = P(X \geq 9,5)$, da je bilo $P(X \leq 10) = P(X \leq 10,5)$

35.) $P(\text{dječaka}) = 0,515$

$$n = 100$$

$$P(50 < X < 55) = ?$$

$$X \sim B(n, p) \rightarrow \text{APROKSIMACIJA BINOMNE NORMALNOM} \sim N(np, npq)$$

$$N(100 \cdot 0,515, 100 \cdot 0,515 \cdot 0,485) \sim N(51,5, 24,9775)$$

! KOREKCIJA ZA 0,5 JER n NIJE "VELIK"

$$P(49,5 < X < 55,5) = \frac{1}{2} \left(\Phi^* \left(\frac{55,5 - 51,5}{\sqrt{24,9775}} \right) - \Phi^* \left(\frac{49,5 - 51,5}{\sqrt{24,9775}} \right) \right)$$

$$= \frac{1}{2} \left(\Phi^*(0,8) - \Phi^*(0,4) \right) = 0,443$$

37.) $F(x) = ax^2, 0 \leq x \leq 3$

$$n = 100$$

$$P(X \geq 30) = ?, 1 < x < 2$$

$$1 = \int_0^3 ax^2 dx = a \int_0^3 x^2 dx = a \cdot \frac{x^3}{3} \Big|_0^3 = a \cdot 9$$

$$1 = 9a \Rightarrow \boxed{a = \frac{1}{9}} \rightarrow \underline{\underline{F(x) = \frac{1}{9} X^2}}$$

$$P(1 < X < 2) = F(2) - F(1) = \frac{4}{9} - \frac{1}{9} = \frac{1}{3}$$

$$B(n, p) \sim N(np, npq) \sim N(100 \cdot \frac{1}{3}, 100 \cdot \frac{1}{3} \cdot \frac{2}{3}) \sim N(33,33, 22,22)$$

$$P(X \geq 30) = P(X \geq 29,5) = \frac{1}{2} - \frac{1}{2} \Phi^* \left(\frac{29,5 - 33,33}{\sqrt{22,22}} \right)$$

$$= \frac{1}{2} + \frac{1}{2} \Phi^*(0,813) = \frac{1}{2} + \frac{1}{2} \cdot 0,58378 = 0,7918$$

$$38.) X \sim B(n, p) \sim N(np, npq)$$

$$N(75 \cdot 0.6, 75 \cdot 0.6 \cdot 0.4)$$

$$N(45, 18)$$

$$P(X \geq 40) = P(X \geq 40) = \frac{1}{2} - \frac{1}{2} \phi^* \left(\frac{40-45}{\sqrt{18}} \right) = \frac{1}{2} + \frac{1}{2} \phi^*(1,17) = \frac{1}{2} + \frac{1}{2} \cdot 0.758$$

$$= \underline{\underline{0,879}}$$

$$39.) P(A) = 0.1$$

$$n = ?, P(A \geq 5) = 0.8$$

$$X \sim B(n, 0.1) \sim N(0.1n, 0.09n)$$

$$P(X > 5) = 0.8 \Rightarrow 0.8 = \frac{1}{2} - \frac{1}{2} \phi^* \left(\frac{5-0.1n}{\sqrt{0.09n}} \right) \Rightarrow 0.6 = \phi^* \left(\frac{0.1n-5}{0.3\sqrt{n}} \right)$$

$$0.2526\sqrt{n} = 0.1n - 5$$

$$\begin{cases} n=t^2 \\ \sqrt{n}=t \end{cases} \rightarrow t^2 - 2.526t - 50 = 0$$

$$t = 8.446$$

$$n=t^2 \rightarrow \underline{\underline{n=71.33}} \rightarrow \text{POKUS TREBA IZVESTI 72 PUTA!}$$

$$40.) P(X) = 0.4$$

$$n = ?, P(X \geq 30) = 0.7$$

$$X \sim B(n, 0.4) \sim N(0.4n, 0.24n)$$

$$P(X \geq 30) = 0.7 \Rightarrow 0.7 = \frac{1}{2} - \frac{1}{2} \phi^* \left(\frac{30-0.4n}{\sqrt{0.24n}} \right) \Rightarrow 0.4 = \phi^* \left(\frac{0.4n-30}{\sqrt{0.24n}} \right)$$

$$0.525 = \frac{0.4n-30}{\sqrt{0.24n}} \Rightarrow 0.2572\sqrt{n} = 0.4n - 30 \rightarrow \begin{cases} t^2=n \\ t=\sqrt{n} \end{cases}$$

$$4t^2 - 2.572t - 300 = 0$$

$$t_{1,2} = \frac{2.572 \pm \sqrt{6.615 + 4800}}{8} = \frac{2.572 \pm 69.33}{8}$$

$$t = 8.987 \rightarrow \underline{\underline{n=80.78}} \rightarrow \text{TREBALI BI PROIZVESTI 81 PROIZVOD!}$$

41.) $A = \{\text{zbroj brojeva na obje kocke barem 10}\}$

$P(A) = ?$

$$P(A) = \frac{6}{36} = \frac{1}{6}$$

11	21	31	41	51	61
12	22	32	42	52	62
13	23	33	43	53	63
14	24	34	44	54	64
15	25	35	45	55	65
16	26	36	46	56	66

$$P(A > 1) = 0.8, n = ?$$

zbog dovoljno "malog" n -a koristimo binomnu razdiobu

$$A \sim B(n, \frac{1}{6})$$

$$P(A > 1) = 1 - P(A = 0) = 0.8 \Rightarrow 0.8 = 1 - \binom{n}{0} \left(\frac{1}{6}\right)^0 \left(1 - \frac{1}{6}\right)^{n-0}$$

$$0.8 = 1 - \left(\frac{5}{6}\right)^n \Rightarrow \left(\frac{5}{6}\right)^n = 0.2 \quad / \ln \Rightarrow \ln\left(\frac{5}{6}\right)^n = \ln 0.2$$

$$n \cdot \ln\left(\frac{5}{6}\right) = \ln 0.2 \Rightarrow n = \frac{\ln 0.2}{\ln \frac{5}{6}} = 8.827 \sim \underline{\underline{9}}$$

42.) $P(A) = 0.8$

$n = ?$, $P(A > 75) = 0.9$

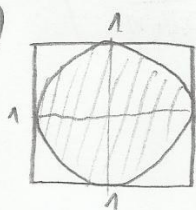
$$X \sim B(n, 0.8) \sim N(0.8n, 0.16n)$$

$$P(A > 75) = 0.9 \Rightarrow 0.9 = \frac{1}{2} - \frac{1}{2} \phi\left(\frac{75 - 0.8n}{0.4\sqrt{n}}\right) \Rightarrow 0.8 = \phi\left(\frac{0.8n - 75}{0.4\sqrt{n}}\right)$$

$$1.29 = \frac{0.8n - 75}{0.4\sqrt{n}} \Rightarrow 0.516\sqrt{n} = 0.8n - 75 \quad \left| \begin{array}{l} t^2 = n \\ t = \sqrt{n} \end{array} \right| \Rightarrow 8t^2 - 5.16t - 750 = 0$$

$$t_{1,2} = \frac{5.16 \pm \sqrt{26.6256 + 24000}}{16} = \frac{5.16 \pm 155}{16} \Rightarrow t = 10.01 \Rightarrow \underline{\underline{n = t^2 = 100.2 \sim 101 \text{ pokus}}}$$

43.)



$n = 50$

$P(X \geq 40) = ?$

$A = \{\text{točke unutar kruga}\}$

$$P(A) = \frac{P_0}{P_{\square}} = \frac{\pi^2 \pi}{1} = \frac{0.5^2 \cdot \pi}{1} = \underline{\underline{0.7854}}$$

$$A \sim B(n, p) \sim N(np, npq) \sim N(50 \cdot 0.7854, 50 \cdot 0.7854 \cdot (1 - 0.7854))$$

$$A \sim N(39.27, 8.4274)$$

$$P(X \geq 40) = \frac{1}{2} - \frac{1}{2} \phi\left(\frac{40 - 39.27}{\sqrt{0.7854}}\right) = \frac{1}{2} - \frac{1}{2} \cdot 0.1381 = \underline{\underline{0.4}}$$

§ 6. Primjeri neprekinutih razdioba

1. 0.1; 0.01.

2. $Y \sim B(50, \sqrt{0.4})$; $P(Y \geq 40) = 0.00705$

3. $\frac{1}{e}$ u sva tri slučaja.

4. $F_Z(z) = 1 - e^{-\lambda z} - \lambda z e^{-\lambda z}$, $\lambda = \frac{1}{6}$;
 $P(Z > 12) = \frac{3}{e^2}$.

5. 168 dana

6. $\frac{k!}{\lambda^k}$

9. $F(\lambda) = 1 - P(X > \lambda) = 1 - \frac{1}{n!} \int_{\lambda}^{\infty} x^n e^{-x} dx$.

Parcijalnom integracijom dobivamo

$$F(\lambda) = 1 - e^{-\lambda} \sum_{i=0}^n \frac{\lambda^i}{i!} = 1 - G(n).$$

10. $\frac{1}{e^4 \sqrt{\pi}}$, 2; $\frac{1}{2}$.

11. A. 0.422; B. 0.2673; C. 0.9359; D. 0.1540;
E. 0.2579; F. 0.1292; G. 0.3830.

12. A. 1.43; B. 0.83; C. 1.16.

13. A. 0.4649; B. 0.2684; C. 0.0401; D. 0.2266.

14. 0.281.

15. 0.839.

16. 0.6147.

17. $\frac{4}{3}$.

18. $\sqrt{\frac{b^2 - a^2}{2(\ln b - \ln a)}}$.

19. $\sigma \sqrt{2/\pi}$.

20. $\frac{4y}{\sqrt{2\pi}} \exp(-\frac{1}{2}y^4)$, $y > 0$.

21. (45.825, 54.175)

22. $f_Y(y) = \frac{1}{\sigma \sqrt{2\pi y}} \exp(-\frac{y}{2\sigma^2})$, $y > 0$;
 $E(Y) = \sigma^2$.

23. (960 p, 1040 p)

24. 0.0128.

25. 0.934.

26. 0.658.

27. $P(A) = \frac{1}{2}[1 - \Phi^*(7/\sqrt{73})] = 0.2063$.

28. 7.75.

29. 0.6681.

30. 0.60924.

31. $\frac{1}{2} + \frac{1}{2}\Phi^*\left(\frac{2}{\sqrt{3.14}}\right) = 0.87$.

32. 0.95

33. Ne, vjerojatnost da se broj 1 pojavi ≤ 140 puta
je $1 - \Phi^*(4.65) = 0$.

34. 0.483.

35. 0.443.

36. $n \geq 632$.

37. $a = \frac{1}{9}$; 0.792.

38. $0.5 + 0.5\Phi^*(5/\sqrt{18}) = 0.8806$.

39. 71.

40. 81.

41. $P(A) = \frac{1}{6}$, $n \geq \frac{\log 5}{\log 1.2} \implies n = 9$,
 $E(Y) = 6$.

42. 101 pokus.

43. 0.398.

44. $E(X) = \alpha$, $D(X) = 2\alpha^2$.

45. $E(X) = e^{a+\sigma^2/2}$, $E(X^2) = e^{2a+2\sigma^2}$,
 $D(X) = e^{2a}(e^{\sigma^2} - 1)e^{\sigma^2}$.

46. $E(X) = \frac{\sqrt{\pi}}{2h}$, $D(X) = \frac{4-\pi}{4h^2}$.

47. $E(X) = \frac{2}{h\sqrt{\pi}}$, $D(X) = \frac{1}{h^2}\left(\frac{3}{2} - \frac{4}{\pi}\right)$.

48. a) α/β ; b) α/β^2 ;
c) $\alpha(\alpha+1) \cdots (\alpha+n-1)/\beta^n$.

49. $x_m = \frac{\gamma}{m}$, $E(X) = \frac{\gamma}{m-2}$,

$D(X) = \frac{\gamma^2}{(m-2)^2(m-3)}$.

TABLICA NORMALNE RAZDIOBE

TABLICA NORMALNE RAZDIOBE: FUNKCIJA Φ^*

	0	1	2	3	4	5	6	7	8	9
0.00	00000	00080	00160	00239	00319	00399	00479	00559	00638	00718
0.01	00798	00878	00957	01037	01117	01197	01277	01356	01436	01516
0.02	01596	01675	01755	01835	01915	01995	02074	02154	02234	02314
0.03	02393	02473	02553	02633	02712	02792	02872	02951	03031	03111
0.04	03191	03270	03350	03430	03510	03589	03669	03749	03828	03908
0.05	03988	04067	04147	04227	04306	04386	04466	04545	04625	04705
0.06	04784	04864	04944	05023	05103	05183	05262	05342	05421	05501
0.07	05581	05660	05740	05819	05899	05979	06058	06138	06217	06279
0.08	06376	06456	06535	06615	06694	06774	06853	06933	07012	07092
0.09	07171	07251	07330	07410	07489	07569	07648	07727	07807	07886
0.10	07966	08045	08124	08204	08283	08362	08442	08521	08600	08680
0.11	08759	08838	08918	08977	09076	09155	09235	09314	09393	09472
0.12	09552	09631	09710	09789	09868	09948	10027	10106	10185	10264
0.13	10343	10422	10502	10581	10660	10739	10818	10897	10976	11055
0.14	11134	11213	11292	11371	11450	11529	11608	11687	11766	11845
0.15	11924	12002	12081	12160	12239	12318	12397	12476	12554	12633
0.16	12712	12791	12869	12948	13027	13106	13184	13263	13342	13420
0.17	13499	13578	13656	13735	13813	13892	13971	14049	14128	14206
0.18	14285	14363	14442	14520	14599	14677	14756	14834	14912	14991
0.19	15069	15147	15226	15304	15382	15461	15539	15617	15695	15774
0.20	15852	15930	16008	16086	16165	16243	16321	16399	16477	16555
0.21	16633	16711	16789	16867	16945	17023	17101	17179	17257	17335
0.22	17413	17491	17569	17646	17724	17802	17880	17958	18035	18113
0.23	18191	18269	18346	18424	18502	18579	18657	18734	18812	18889
0.24	18967	19044	19122	19199	19277	19354	19432	19509	19587	19664
0.25	19741	19819	19896	19973	20050	20128	20205	20282	20359	20436
0.26	20514	20591	20668	20745	20822	20899	20976	21053	21130	21207
0.27	21284	21361	21438	21515	21592	21668	21745	21822	21899	21976
0.28	22052	22129	22206	22282	22359	22436	22512	22589	22665	22742
0.29	22818	22895	22971	23048	23124	23201	23277	23353	23430	23506
0.30	23582	23659	23735	23811	23887	23963	24040	24116	24192	24268
0.31	24344	24420	24496	24572	24648	24724	24800	24876	24952	25027
0.32	25103	25179	25255	25330	25406	25482	25558	25633	25709	25784
0.33	25860	25936	26011	26087	26162	26237	26313	26388	26464	26539
0.34	26614	26690	26765	26840	26915	26991	27066	27141	27216	27291
0.35	27366	27441	27516	27591	27666	27741	27816	27891	27966	28040
0.36	28115	28190	28265	28340	28414	28489	28563	28638	28713	28787
0.37	28862	28936	29011	29085	29160	29234	29308	29383	29457	29531
0.38	29605	29680	29754	29828	29902	29976	30050	30124	30198	30272
0.39	30346	30420	30494	30568	30642	30716	30789	30863	30937	31011
0.40	31084	31158	31232	31305	31379	31452	31526	31599	31673	31746
0.41	31819	31893	31966	32039	32113	32186	32259	32332	32405	32478
0.42	32551	32624	32697	32770	32843	32916	32989	33062	33135	33208
0.43	33280	33353	33426	33499	33571	33644	33716	33789	33861	33934
0.44	34006	34079	34151	34223	34296	34368	34440	34512	34585	34657
0.45	34729	34801	34873	34945	35017	35089	35161	35233	35305	35377
0.46	35448	35520	35592	35664	35735	35807	35878	35950	36022	36093
0.47	36164	36236	36307	36379	36450	36521	36593	36664	36735	36806
0.48	36877	36948	37019	37090	37161	37232	37303	37374	37445	37516
0.49	37587	37657	37728	37799	37869	37940	38011	38081	38152	38222
0.50	38292	38363	38433	38504	38574	38644	38714	38785	38855	38925
0.51	38995	39065	39135	39205	39275	39345	39415	39484	39554	39624
0.52	39694	39763	39833	39903	39972	40042	40111	40181	40250	40319
0.53	40389	40458	40527	40587	40666	40735	40804	40873	40942	41011
0.54	41080	41149	41218	41287	41356	41425	41493	41562	41631	41699
0.55	41768	41837	41905	41974	42042	42111	42179	42247	42316	42384
0.56	42452	42520	42588	42657	42725	42793	42861	42929	42997	43064
0.57	43132	43200	43268	43336	43403	43471	43538	43606	43674	43741
0.58	43809	43876	43943	44011	44078	44145	44212	44280	44347	44414
0.59	44481	44548	44615	44682	44749	44816	44882	44949	45016	45083
0.60	45149	45216	45283	45349	45416	45482	45549	45615	45681	45748
0.61	45814	45880	45946	46012	46078	46145	46211	46277	46342	46408
0.62	46474	46540	46606	46672	46737	46803	46869	46934	47000	47065
0.63	47131	47196	47261	47327	47392	47457	47522	47588	47653	47718
0.64	47783	47848	47913	47978	48042	48107	48172	48237	48302	48366

TABLICA NORMALNE RAZDIOBE: FUNKCIJA Φ^*

	0	1	2	3	4	5	6	7	8	9
0.65	48431	48495	48560	48624	48689	48753	48818	48882	48946	49010
0.66	49075	49139	49203	49267	49331	49395	49459	49523	49587	49650
0.67	49714	49778	49842	49905	49969	50032	50096	50159	50223	50286
0.68	50350	50413	50476	50539	50602	50666	50729	50792	50855	50918
0.69	50981	51043	51106	51169	51232	51294	51357	51420	51482	51545
0.70	51607	51670	51732	51794	51857	51919	51981	52043	52105	52168
0.71	52230	52292	52354	52415	52477	52539	52601	52663	52724	52786
0.72	52848	52909	52971	53032	53093	53155	53216	53277	53339	53400
0.73	53461	53522	53583	53644	53705	53766	53827	53888	53947	54009
0.74	54070	54131	54191	54252	54312	54373	54433	54494	54554	54614
0.75	54675	54735	54795	54855	54915	54975	55035	55095	55155	55215
0.76	55275	55334	55394	55454	55513	55573	55632	55692	55751	55811
0.77	55870	55929	55989	56048	56107	56166	56225	56284	56343	56402
0.78	56461	56520	56579	56637	56696	56755	56813	56872	56930	56989
0.79	57047	57106	57164	57222	57280	57339	57397	57455	57513	57571
0.80	57629	57687	57745	57803	57860	57918	57976	58033	58091	58148
0.81	58206	58263	58321	58378	58436	58493	58550	58607	58664	58721
0.82	58778	58835	58892	58949	59006	59063	59120	59176	59233	59290
0.83	59346	59403	59459	59516	59572	59628	59685	59741	59797	59853
0.84	59909	59965	60021	60077	60133	60189	60245	60300	60356	60412
0.85	60467	60523	60579	60634	60690	60745	60800	60856	60911	60966
0.86	61021	61076	61131	61186	61241	61296	61351	61406	61461	61515
0.87	61570	61625	61679	61734	61788	61843	61893	61951	62006	62060
0.88	62114	62168	62222	62276	62330	62384	62438	62492	62546	62600
0.89	62653	62707	62761	62814	62868	62921	62975	63028	63081	63135
0.90	63188	63241	63294	63347	63400	63453	63506	63559	63612	63665
0.91	63718	63770	63823	63876	63928	63981	64033	64086	64138	64190
0.92	64243	64295	64347	64399	64451	64503	64555	64607	64659	64711
0.93	64763	64815	64866	64918	64970	65021	65073	65124	65176	65227
0.94	65278	65330	65381	65432	65483	65534	65585	65636	65687	65738
0.95	65789	65840	65890	65941	65992	66042	66093	66143	66194	66244
0.96	66294	66345	66395	66445	66495	66546	66596	66646	66696	66745
0.97	66795	66845	66895	66945	66994	67044	67094	67143	67193	67242
0.98	67291	67341	67390	67439	67488	67538	67587	67636	67685	67734
0.99	67783	67831	67880	67929	67978	68026	68075	68124	68172	68221
1.0	68269	68750	69227	69699	70166	70628	71086	71538	71986	72429
1.1	72867	73300	73729	74152	74571	74986	75395	75800	76200	76595
1.2	76986	77372	77754	78130	78502	78870	79233	79592	79945	80295
1.3	80640	80980	81316	81648	81975	82298	82617	82931	83241	83547
1.4	83849	84146	84439	84728	85013	85294	85571	85844	86113	86370
1.5	86639	86896	87149	87398	87644	87886	88124	88358	88589	88817
1.6	89040	89260	89477	89690	89899	90106	90309	90508	90704	90897
1.7	91087	91273	91457	91637	91814	91988	92159	92327	92492	92655
1.8	92814	92970	93124	93275	93423	93596	93711	93852	93989	94124
1.9	94257	94387	94514	94639	94762	94882	95000	95116	95230	95341
2.0	95450	95557	95662	95764	95865	95964	96060	96155	96247	96338
2.1	96427	96514	96599	96683	96765	96844	96923	96999	97074	97148
2.2	97217	97289	97358	97425	97491	97555	97618	97679	97739	97798
2.3	97855	97911	97966	98019	98072	98123	98173	98221	98269	98315
2.4	98360	98405	98448	98490	98531	98571	98611	98649	98686	98723
2.5	98758	98793	98826	98859	98891	98923	98953	98983	99012	99040
2.6	D0678	D0946	D1207	D1462	D1709	D1951	D2186	D2415	D2638	D2855
2.7	D3066	D3272	D3472	D3667	D3856	D4040	D4220	D4394	D4564	D4729
2.8	D4890	D5046	D5198	D5345	D5489	D5628	D5764	D5895	D6023	D6148
2.9	D6268	D6386	D6500	D6610	D6718	D6822	D6924	D7022	D7118	D7210
3.0	D7300	D7388	D7472	D7554	D7634	D7712	D7787	D7859	D7930	D7998
3.1	D8065	D8129	D8191	D8252	D8311	D8367	D8422	D8476	D8527	D8577
3.2	D8626	D8673	D8718	D8762	D8805	D8846	D8886	D8925	D8962	D8998
3.3	T0332	T0670	T0998	T1315	T1622	T1919	T2206	T2483	T2751	T3011
3.4	T3261	T3504	T3738	T3964	T4183	T4394	T4598	T4795	T4986	T5170
3.5	T5347	T5519	T5685	T5844	T5999	T6148	T6291	T6430	T6564	T6693
3.6	T6818	T6938	T7054	T7166	T7274	T7378	T7478	T7574	T7668	T7757
3.7	T7844	T7927	T8008	T8085	T8160	T8232	T8301	T8368	T8432	T8494
3.8	T8553	T8610	T8665	T8719	T8770	T8819	T8866	T8912	T8955	T8998
3.9	C0381	C0770	C1145	C1505	C1852	C2185	C2505	C2813	C3108	C3393
4.0	C3666	C3928	C4180	C4422	C4655	C4878	C5093	C5299	C5496	C5686

Napomena**

U tablicama za normalnu razdiobu slova D, T i Č označavaju koliko znamenka broja 9 stoji ispred broja. D označava "dva" ili dvije devetke 99xxx, T označava "tri" ili tri devetke 999xxx....., Č označava "četiri" devetke ili 9999xxx..... Primjerice: pod 3.00 imamo vrijednost D7300 iliti 0.997300

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