

1. a) Neka su  $A$  i  $B$  nezavisni,  $A \subset B$ , pokaži  
 $P(A) = 0$  ili  $P(B) = 1$

b)  $P(A \cap B) \neq 0$  pokaži da vrijedi

$$P(A|B) = P(B|A) \Leftrightarrow P(A) = P(B)$$

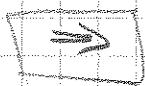
R:

$$a) \quad P(A) \stackrel{A \subset B}{=} P(A \cap B) = P(A) \cdot P(B)$$

$$P(A)(1 - P(B)) = 0$$

$$\Rightarrow P(A) = 0 \quad \text{ili} \quad 1 - P(B) = 0 \\ P(B) = 1$$

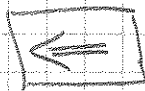
$$b) \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$



$$P(A|B) = P(B|A)$$

$$\frac{P(A \cap B)}{P(B)} = \frac{P(B \cap A)}{P(A)}$$

$$\Rightarrow P(B) = P(A)$$



$$P(A) = P(B) \Rightarrow \frac{P(A \cap B)}{P(A)} = \frac{P(A \cap B)}{P(B)}$$

$$P(B|A) = P(A|B)$$

2. Od 10 učenika u učionici 6 je odličnih, a 4 vrlo dobra.  
 Odlični učenici rješ. zad. s vjer. 0,9, a vrlo dobri 0,7.  
 Prof. na sreću odabire 2 učenika da rješ. isti zad.

Kolika je vjer. da su oba učenika rješila zad.  
 $A = \{\text{oba učenika rješila zad.}\}$

$$H_1 = \{\text{oba odlična}\} \quad P(H_1) = \frac{\binom{6}{2}}{\binom{10}{2}} = \frac{1}{3}$$

$$H_2 = \{\text{oba vrlo dobra}\} \quad P(H_2) = \frac{\binom{4}{2}}{\binom{10}{2}} = \frac{2}{15}$$

$$H_3 = \{\text{1 odličan, 1 vrlo dobar}\} \quad P(H_3) = \frac{\binom{6}{1} \binom{4}{1}}{\binom{10}{2}} = \frac{8}{15}$$

Formula potpune vjer.  

$$P(A) = P(A|H_1) \cdot P(H_1) + P(A|H_2) \cdot P(H_2) + P(A|H_3) \cdot P(H_3)$$

$$P(A|H_1) = 0.9 \cdot 0.9 = 0.81$$

$$P(A|H_2) = 0.7 \cdot 0.7 = 0.49$$

$$P(A|H_3) = 0.9 \cdot 0.7 = 0.63$$

$$P(A) = P(A|H_1) \cdot P(H_1) + P(A|H_2) \cdot P(H_2) + P(A|H_3) \cdot P(H_3) \\ = 0.81 \cdot \frac{1}{3} + 0.49 \cdot \frac{2}{15} + 0.63 \cdot \frac{8}{15} =$$

3. U poslovnici A nalazi se 100 srećki od kojih je 25 dobitnih, a u poslovnici B, 55 srećki od kojih je 5 dobitnih. Marko baca simetričnu kocku, ako na kocki padne 1 kupuje 2 srećke u poslovnici A, a ako padne 2 kupuje 2 u poslovnici B, inače u svakoj kupuje po 1. Kolika je vjer. da je tačno 1 kupljena srećka dobitna.

Rj:  $A = \{ \text{tačno jedna kupljena srećka dobitna} \}$

$$H_1 = \{ \text{pao br. 1} \} \quad P(H_1) = \frac{1}{6} \quad P(A|H_1) = \frac{\binom{25}{1} \binom{75}{1}}{\binom{100}{2}} = \frac{25}{66}$$

$$H_2 = \{ \text{pao br. 2} \} \quad P(H_2) = \frac{1}{6} \quad P(A|H_2) = \frac{\binom{5}{1} \binom{50}{1}}{\binom{55}{2}} = \frac{50}{297}$$

$$H_3 = \{ \text{pao je 3, 4, 5 ili 6} \} \quad P(H_3) = \frac{2}{3} \quad P(A|H_3) = \frac{\binom{25}{1} \binom{50}{1} + \binom{5}{1} \binom{75}{1}}{\binom{100}{1} \binom{55}{1}} = \frac{13}{44}$$

$$P(A) = P(A|H_1) \cdot P(H_1) + P(A|H_2) \cdot P(H_2) + P(A|H_3) \cdot P(H_3) \\ = \frac{25}{66} \cdot \frac{1}{6} + \frac{50}{297} \cdot \frac{1}{6} + \frac{13}{44} \cdot \frac{2}{3} = 0.2882$$

4. Bacamo 3 novčića one novčiće na kojima je pala glava stavimo sa strane, a ostale bacimo još jednom. Ako su pale 3 glave, odredite vjer. da su bar 2 pale u prvom bacanju. 2 ili 3

$$A = \{ \text{pale su 3 glave} \}$$

$$H_i = \{ \text{u prvom bacanju je pala } i \text{ glava} \}, \quad i = 0, 1, 2, 3$$

$$P(H_0) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

$$P(A|H_0) = \frac{1}{8}$$

$$P(H_1) = \binom{3}{1} \cdot \frac{1}{2} \left( \frac{1}{2} \right)^2 = \frac{3}{8}$$

$$P(A|H_1) = \frac{1}{2^2} = \frac{1}{4}$$

$$P(H_2) = \binom{3}{2} \left( \frac{1}{2} \right)^2 \frac{1}{2} = \frac{3}{8}$$

$$P(A|H_2) = \frac{1}{2}$$

$$P(H_3) = \left( \frac{1}{2} \right)^3 = \frac{1}{8} \quad P(A|H_3) = 1$$

$$P(H_2/A) + P(H_3/A) = \frac{P(H_2) \cdot P(A/H_2)}{P(A)} + \frac{P(H_3) \cdot P(A/H_3)}{P(A)}$$

$$P(A) = \frac{1}{8} \cdot \frac{1}{8} + \frac{3}{8} \cdot \frac{1}{9} + \frac{3}{8} \cdot \frac{1}{2} + \frac{1}{8} = \frac{27}{64}$$

$$\begin{aligned} P(H_2 \cup H_3/A) &= P(H_2/A) + P(H_3/A) \\ &= \frac{\frac{3}{8} \cdot \frac{1}{2} + \frac{1}{8} \cdot 1}{\frac{27}{64}} = \frac{20}{27} = 0,741 \end{aligned}$$

5. Iz kutiji u kojoj je bilo  $n$  bijelih i  $n+2$  crnih kuglica je izgubljena kuglica nepoznate boje. Ako je nakon toga izvučena bijela kuglica, kolika je vjer. da je izgubljena kuglica crne.

$A = \{ \text{izvučena je bijela} \}$

$H_0 = \{ \text{izgubljena je crna} \}$

$H_1 = \{ \text{izgubljena bijela} \}$

$$P(H_0) = \frac{n+2}{2n+2}$$

$$P(A/H_0) = \frac{n}{2n+1}$$

$$P(H_1) = \frac{n}{2n+2}$$

$$P(A/H_1) = \frac{n-1}{2n+1}$$

$$P(H_0/A) = \frac{P(H_0) P(A/H_0)}{P(A)}$$

$$= \frac{\frac{n+2}{2n+2} \cdot \frac{n}{2n+1}}{\frac{n}{2n+1} \cdot \frac{n+2}{2n+2} + \frac{n-1}{2n+1} \cdot \frac{n}{2n+2}}$$

$$= \frac{n+2}{2n+1}$$