MI 2017

1) a) Ako je $P(A|B) = P(A)$, onda je $P(B|A) = P(B)$

TOČNO

⇒ uvjetna vjerojatnost:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(A|B)\,P(B) = P(B|A)\,P(A)$$

$$P(A|B)\,P(B) = P(B|A)\,P(A) \quad / : P(A)$$

$$\underbrace{\frac{P(A|B)}{P(A)}}_{=1}\,P(B) = P(B|A)$$

$$P(B) = P(B|A)$$

$$P(A|B) = P(A)$$

b) A i B disjunktni ⟶ A i B nezavisni

NETOČNO

Nezavisni ⟶ $P(A \cap B) = P(A)\,P(B)$
Disjunktni ⟶ $P(A \cap B) = 0$

$P(A) \cdot P(B) = 0 \longleftrightarrow P(A)$ i/ili $P(B) =$

c) Vrijedi $P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$ ⟶ A, B i C su nezavisni

NETOČNO

Da bi $P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$ vrijedilo, onda A, B i C moraju biti nezavisni i podskupovi AB, AC i BC.

② 25 pitanja na kvizu

⇒ 1. igrač → 9 (2 točna)

⇒ 2. igrač → 5 (4 točna)

⇒ 3. igrač → 6 (3 točna)

⇒ 4. igrač → 5 (sva točna)

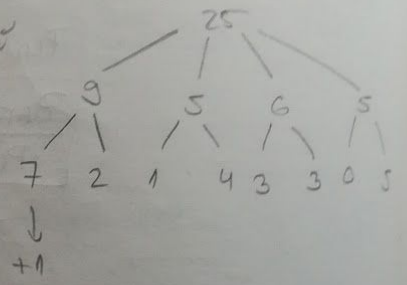
⇒ postavljeno samo pitanje i **krivo** je odgovoreno

$P(B|\bar{A}) = ?$

$$P(B|\bar{A}) = 1 - \frac{P(B)}{P(A)}$$

$P(B)$ → odgovorio 1. igrač

$P(A)$ → krivo odgovorio



$$P(A) = \frac{7}{25} + \frac{1}{25} + \frac{3}{25} + \frac{0}{25} = \frac{11}{25}$$

$$P(B) = \frac{\cancel{9}}{25} \cdot \frac{7}{\cancel{9}} = \frac{7}{25}$$

$$P(B|\bar{A}) = 1 - \frac{\frac{7}{25}}{\frac{11}{25}} = 1 - \frac{7}{11} = \frac{4}{11}$$

④ GEOMETRIJSKA RAZDIOBA

$$P(X=m) = p(1-p)^{m-1} \quad |k = 1,2,3\ldots$$

=> očekivanje:

$$E(X) = \sum_{m=1}^{\infty} m(1-p)^{m-1}p$$

$$= p \sum_{m=1}^{\infty} m(1-p)^{m-1}$$

$$= p \cdot \frac{1}{p^2}$$

$$= \frac{1}{p} \quad //$$

=> disperzija:

$$D(X) = E(X^2) - (E(X))^2$$

$$= \sum_{k\geq 1} k^2 p q^{k-1} - \frac{1}{p^2} \qquad \Rightarrow q = p-1$$

$$= \sum_{k\geq 1} kpq^{k-1} - \frac{1}{p^2}$$

$$= \sum k(k+1)pq^{k-1} - \frac{1}{p} - \frac{1}{p^2}$$

$$= p\frac{2}{(1-q)^3} - \frac{1}{p} - \frac{1}{p^2}$$

$$= \frac{2}{p^2} - \frac{1}{p} - \frac{1}{p^2}$$

$$= \frac{1}{p^2} - \frac{1}{p}$$

$$= \frac{1-p}{p^2} \quad //$$

(3)

X|Y	0	1
-1	p_1	0.13
0	0.21	p_2
1	0.15	0.28

$p_1 + p_2 = 0.23$

$p_1 = 0.23 - p_2$

a) $cov(X,Y) = 0.0828$

$p_1 \cdot p_2 = ?$

$E(XY) = -1 \cdot 0.13 + 0.28 = 0.15$

$EX = -p_1 - 0.13 + 0.43 = 0.3 - p_1 = p_2 + 0.07$

$EY = p_2 + 0.41$

$0.15 - (p_2 + 0.07)(p_2 + 0.41) = 0.0828$

$0.15 - (p_2^2 + 0.41 p_2 + 0.07 p_2 + 0.0287) = 0.0828$

$p_2^2 + 0.48 p_2 - 0.0385 = 0$

$p_2 = 0.07$ // $p_1 = 0.16$ //

b)

X|Y	0	1	
-1	0.16	0.13	0.29
0	0.21	0.07	0.28
1	0.15	0.28	0.43
	0.52	0.48	

$(U,V) \sim ?$

$U = X + Y$

$V = X^2 + Y^2$

$U = X + Y \sim \begin{pmatrix} -1 & 0 & 1 & 2 \\ 0.16 & 0.13 + 0.21 & 0.07 + 0.15 & 0.28 \end{pmatrix}$

$U \sim \begin{pmatrix} -1 & 0 & 1 & 2 \\ 0.16 & 0.34 & 0.22 & 0.28 \end{pmatrix}$

$V = X^2 + Y^2 \rightarrow V \sim \begin{pmatrix} 0 & 1 & 2 \\ 0.21 & 0.38 & 0.41 \end{pmatrix}$

U|V	0	1	2	
-1		0.16		0.16
0	0.21		0.13	0.34
1		0.22		0.22
2			0.28	0.28
	0.21	0.38	0.41	

⑥ $F(x) = a\frac{e^x - 1}{e^x + 2} + b \quad ; x \in \mathbb{R}$

a, b = ?

P(x>1) = ?

$$\lim_{x\to+\infty}\left(a\frac{e^x-1}{e^x+2}+b\right) = b + a\lim_{x\to+\infty}\frac{e^x-1}{e^x+2}$$

$$\overset{L'H}{=} b + a\lim_{x\to+\infty}\frac{\frac{d}{dx}(e^x-1)}{\frac{d}{dx}(e^x+2)} = b + a\lim_{x\to+\infty}\frac{e^x}{e^x} = 1$$

$$= b + a$$

$$\lim_{x\to-\infty}\left(a\frac{e^x-1}{e^x+2}+b\right) = b + a\lim_{x\to-\infty}\frac{e^x-1}{e^x+2} \overset{\text{Sub. } x=-x}{=} [x=-x]$$

$$= b + a\left(\lim_{x\to-\infty}\frac{-1+e^{-x}}{2+e^{-x}}\right) = \left(\lim_{x\to-\infty}(-1+e^{-x})\cdot\lim_{x\to-\infty}\frac{1}{2+e^{-x}}\right)a + b$$

$$= b + a\left((-1+\lim_{x\to-\infty}e^{-x})\cdot\frac{\lim_{x\to+\infty}1}{\lim_{x\to-\infty}(2+e^{-x})}\right) = \left((-1+\lim_{x\to-\infty}e^{-x})\cdot\frac{1}{(2+\lim_{x\to+\infty}e^{-x})}\right)a + b$$

$$= b + a\left((-1+e^{-\lim_{x\to-\infty}x})\cdot\frac{1}{2+e^{-\lim x}}\right) = \left((-1+e^{-\infty})\cdot\frac{1}{2+e^{-\infty}}\right)a + b$$

$$= \left(-1\cdot\frac{1}{2}\right)a + b = -\frac{1}{2}a + b$$

$$\left.\begin{aligned} F(+\infty) &= 1 \\ F(-\infty) &= 0 \end{aligned}\right\} \longrightarrow \left.\begin{aligned} b + a &= 1 \\ b - \frac{1}{2}a &= 0 \end{aligned}\right\}$$

$\rightarrow b = \frac{1}{2}a$

$1 = \frac{1}{2}a + a$

$1 = \frac{3}{2}a \quad | :\frac{3}{2}$

$\boxed{a = \frac{2}{3}}$

$b = \frac{1}{2}\cdot\frac{2}{3}$

$\boxed{b = \frac{1}{3}}$

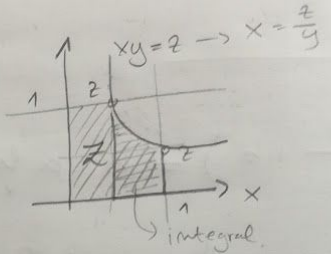
$$P(x>1) = 1 - P(x<1)$$

$$= 1 - F(1) = 1 - \frac{2}{3}\cdot\frac{e^1-1}{e^1+2} + \frac{1}{3} = 0.424$$

⑦ 2 broja iz $(0,1)$ intervala na sreću

$Z = X \cdot Y$

$F(z)$ $\quad y = ?$
$E(z)$



$$F(z) = 1 \cdot z + \int_z^1 \frac{z}{y}\,dy$$

$$= z + z\,(\ln y|_z^1)$$

$$= z - z\ln z$$ //

$$E(z) = E(xy) \overset{\text{nezavisne}}{=} E(x)E(y)$$

$$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$ //

jednolika razdioba

⑧ Eksponencijalna razdioba

a) $P(x>b \mid x>a) = P(x>b-a) \quad \forall b,a>0$

=> Dokaz: (odsustvo pamćenja)

$$P(x>b \mid x>a) = \frac{P(x>b \cap x>a)}{P(x>a)}$$

$$= \frac{P(x>b)}{P(x>a)} = \frac{e^{-\lambda b}}{e^{-\lambda a}} = e^{-\lambda(b-a)}$$

$$= P(x>b-a)$$ //

b) $E(x) = 4\lambda$

$P(x>4) = ?$

$$E(x) = \frac{1}{\lambda}$$

$$\lambda = \frac{1}{4}$$

$$P(x>4) = 1 - F(4) = 1 - \left(1 - e^{-\frac{1}{4}\cdot 4}\right) = 1 - 1 + \frac{1}{e}$$

$$= \frac{1}{e} = 0.3678$$

$$= 36.78\%$$ //