Ovaj PDF sadrži skenirane postupke svih 1.MI 2013.-2007.

Postupci su poredani od 2013. do 2007

Riješio i ustupio na skeniranje

fer0vac

skenirao

SipE

2013

$\boxed{1}$

1) $52 \searrow 5$

(a) bar jedam A (1 - miješam AS)

$$P(A) = 1 - \frac{\binom{48}{5}}{\binom{52}{5}}$$

(b) 5 karata različite jačine

$$P(B) = \frac{\binom{13}{5} 4^5}{\binom{52}{5}}$$

izbor boje

(c) 5 k iste boje

$$P(C) = \frac{\binom{13}{5}\binom{4}{1}}{\binom{52}{5}}$$

(d) 5 k ali nema svih boja 1 - sve boje

točno 1 boja + točno 2 boje + točno 3 boje

sve boje 1+1+1+2 → 2 karte

$$P(D) = 1 - \frac{\binom{4}{1}\binom{13}{2} 13^3}{\binom{52}{5}}$$

$\boxed{2}$ A i B nezavisni $\Rightarrow$ $\bar{A}$ i $\bar{B}$ nez.

$$P(AB)=P(A)P(B) \overset{?}{\Rightarrow} P(\bar{A})P(\bar{B})=P(\overline{AB})$$

krenimo: $P(\bar{A}\bar{B}) = P(\overline{A+B}) = 1 - P(A+B) =$

$= 1 - [P(A)+P(B)-P(AB)] = 1 - P(A) - P(B) + \boxed{P(AB)}$

$$\boxed{P(AB)} \;|||\; \boxed{P(A)P(B)}$$

$P(\bar{A})P(\bar{B}) = [1-P(A)][1-P(B)] = 1 - P(A) - P(B) + \boxed{P(A)P(B)}$

$\boxed{3}$ 4 kocke

I: 2 ispravne

L: 2 lažne (svi br. 6-ice)

(a) A = "pale su dvije šestice"

H_1 = "I I"

H_2 = "I L"

H_3 = "L L"

$$P(H_1) = \frac{\binom{2}{2}\binom{2}{0}}{\binom{4}{2}} = \frac{1}{\frac{4 \cdot 3}{2}} = \frac{1}{6}$$

$$P(H_2) = \frac{\binom{2}{1}\binom{2}{1}}{\binom{4}{2}} = \frac{4}{6} = \frac{4}{6}$$

$$P(H_3) = \frac{\binom{2}{0}\binom{2}{2}}{\binom{4}{2}} = \frac{1}{6}$$

$\Sigma = 1$ ✓

$P(A|H_1) = \left(\frac{1}{6}\right)^2 = \frac{1}{3…}$

$P(A|H_2) = \left(\frac{1}{6}\right) \cdot \left(\frac{6}{6}\right)…$

$P(A|H_3) = \left(\frac{6}{6}\right)^2 = …$

$$P(A) = \sum P(H_i) P(A|H_i) = \frac{1}{6} \cdot \frac{1}{36} + \frac{4}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot 1 = 0{,}2824$$

(b) $$P(H_3|A) = \frac{P(H_3)P(A|H_3)}{P(A)} = \frac{\frac{1}{6} \cdot 1}{0{,}2824} = 0{,}5901$$

MAN UTD

4 1B i 4C 1 po 1 dok ne B

X: broj izvlač.

(a) ne vraća

$$X \sim \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ \frac{1}{5} & \frac{4}{5}\frac{1}{4} & \frac{4}{5}\frac{3}{4}\cdot\frac{1}{3} & \frac{4}{5}\frac{3}{4}\frac{2}{3}\frac{1}{2} & \frac{4}{5}\frac{3}{4}\frac{2}{3}\frac{1}{2}\cdot 1 \end{pmatrix}$$

C B   C C B   C C C B

$\sum p_i = 1$ ✓ $E(X) = \sum_{i=1}^{\infty} x_i p_i = \frac{1}{5}(1+2+3+4+5) = \frac{15}{5} = 3$

(b) s vraćanjem

$$X \sim \begin{pmatrix} 1 & 2 & 3 & \dots & n & \dots \\ \frac{1}{5} & \left(\frac{4}{5}\right)\frac{1}{5} & \left(\frac{4}{5}\right)^2\frac{1}{5} & \dots & \left(\frac{4}{5}\right)^{n-1}\frac{1}{5} & \dots \end{pmatrix}$$

$$E(X) = \sum_{n=1}^{\infty} n\cdot\left(\frac{4}{5}\right)^{n-1}\frac{1}{5} = \frac{1}{5}\sum_{n=1}^{\infty} n\left(\frac{4}{5}\right)^{n-1} \overset{(1)}{=} \frac{1}{5}\left(\frac{1}{(1-\frac{4}{5})^2}\right) = 5$$

$$\sum_{n=0} x^n = 1 + x + x^2 + \dots + x^n = \frac{1}{1-x}$$

$$\sum_{n=1} n x^{n-1} = 0 + 1 + 2x + \dots + nx^{n-1} = -\frac{1}{(1-x)^2}(-1) = \frac{1}{(1-x)^2}$$

(2)

5 40 auto / h

1 pumpa

1 auto = 1 min

red ≡ 2 auta unutar 1 min

(a) λ = intenzitet pojavljanja (PROSJEK) = 40 auta / h = 40 auta / 60 min

$\lambda = \frac{2}{3}$ auta / min X = broj auta na benz. postaji unutar 1 min.

$P(X \geq 2) = ?$ $P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}$

$$P(X \geq 2) = 1 - P(X=1) - P(X=0)$$

$$P(X \geq 2) = 1 - \frac{\left(\frac{2}{3}\right)^1}{1!} e^{-\frac{2}{3}} - \frac{\left(\frac{2}{3}\right)^0}{0!} e^{-\frac{2}{3}} = 1 - \frac{2}{3}e^{-2/3} - e^{-2/3} = 1 - \frac{5}{3}e^{-2/3}$$

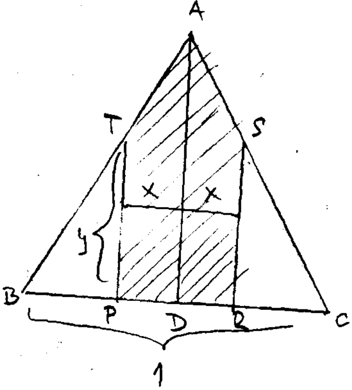
(b) benz postaja ima 2 pumpe ⇒ znači u minuti može primiti 2 auta

⇒ $P(X \geq 3) = ?$ (pojaviti će se red ako dođu 3 auta unutar 1 min)

$$P(X \geq 3) = \underline{1 - P(X=2) - P(X=1)} - P(X=0) = \left(1 - \frac{5}{3}e^{-2/3}\right) - \frac{\left(\frac{2}{3}\right)^2}{2!} e^{-2/3}$$

2/9

$$= 1 - e^{-2/3}\left(\frac{5}{3} + \frac{2}{9}\right) = 1 - \frac{17}{9}e^{-2/3}$$



ΔABC jednakostr.

IMI, 20..
(2)

$F(x)=P(X<x)=\dfrac{P_{\text{lik}}}{P_\Delta}=(*)$

$P_\Delta=\dfrac{a^2\sqrt3}{4}=[a=1]=\dfrac{\sqrt3}{4}$

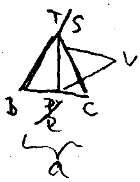
$P_{\text{lik}}=P_{\Delta BAC}-2P_{\Delta BTP}$

$|AD|=v=\dfrac{a\sqrt3}{2}=\dfrac{\sqrt3}{2}$ $\qquad P_{\Delta BTP}=?$

$|BP|=1-2x-|RC|$, a $|RC|=|BP|$

$2|BP|=1-2x$

$|BP|=\dfrac12-x$



$P=\dfrac{av}{2}$

$|PT|$ iz sličnosti:

$\dfrac{\frac12}{\frac12-x}=\dfrac{\frac{\sqrt3}{2}}{y}$ $\quad$ ($|BD|$, $|BP|$, $v=|AD|$)

$\dfrac{y}{2}=\dfrac{\sqrt3}{2}\cdot\left(\dfrac12-x\right)\Rightarrow y=2\left(\dfrac{\sqrt3}{4}-\dfrac{\sqrt3}{2}x\right)=\dfrac{\sqrt3}{2}-\sqrt3x$

$F(x)=\dfrac{\frac{\sqrt3}{4}-2\cdot\left[\frac{\left(\frac12-x\right)\left(\frac{\sqrt3}{2}-\sqrt3x\right)}{2}\right]}{\frac{\sqrt3}{4}}=\dfrac{\frac{\sqrt3}{4}-\cancel{2}\frac{1}{\cancel{2}}\left(\frac12-x\right)\sqrt3\left(\frac12-x\right)}{\frac{\sqrt3}{4}}=$

$=\dfrac{\frac{\sqrt3}{4}-\left(\frac12-x\right)^2\sqrt3}{\frac{\sqrt3}{4}}=\dfrac{\frac{\sqrt3}{4}-\sqrt3\left[\frac14-\cancel{2}\frac{1}{\cancel{2}}x+x^2\right]}{\frac{\sqrt3}{4}}=$

$=\dfrac{\cancel{\frac{\sqrt3}{4}}-\cancel{\frac{\sqrt3}{4}}+\frac{\sqrt3x-\sqrt3x^2}{1}}{\frac{\sqrt3}{4}}=4x-4x^2,\quad x\in[0,1/2]$

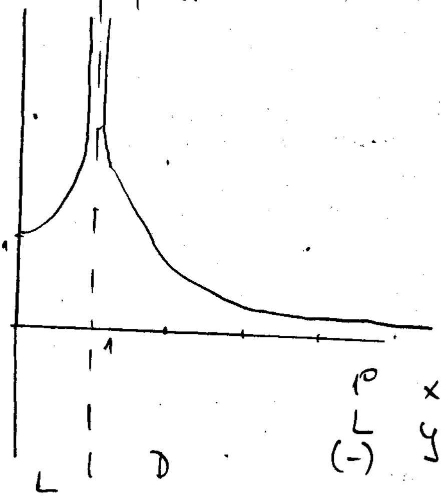
$f(x)=4-8x,\quad x\in[0,1/2]$

$E(x)=\displaystyle\int_0^{1/2}x(4-8x)\,dx=\frac16$

7

7. $Y = \frac{1}{(X-1)^2}$, $X \Rightarrow f(x) = e^{-x}$, $x > 0$

nije def.



inverz:

$y = \frac{1}{(x-1)^2} \Big/ ^{-1}$

$\frac{1}{y} = (x-1)^2 \Big/ \sqrt{\ }$

$\pm\sqrt{\frac{1}{y}} = (x-1)$

$1 \pm \sqrt{\frac{1}{y}} = x$

$\left(\frac{1}{t}\right)' = t^{-1} = 1t^{0}$

1° $x \in \langle 0, 1\rangle$ L (−) $y \in \langle 1, +\infty\rangle$

$x = 1 - \sqrt{\frac{1}{y}} = 1 - \frac{1}{y}^{\frac{1}{2}}$

$\left|\frac{dx}{dy}\right| = \left|0 - \frac{1}{2}\left(\frac{1}{y}\right)^{3/2}\right| = \left|-\frac{1}{2}\cdot\frac{1}{y^{3/2}}\right| = \frac{1}{2y\sqrt{y}}$

$g_1(y) = f(x)\left|\frac{dx}{dy}\right| = e^{-\left(1-\sqrt{\frac{1}{y}}\right)}\frac{1}{2y\sqrt{y}}$

2° $x \in \langle 1, +\infty\rangle$ D (+) $y \in \langle 0, +\infty\rangle$

$x = 1 + \sqrt{\frac{1}{y}}$

$g_2(y) = f(x)\cdot\left|\frac{dx}{dy}\right| = e^{-\left(1+\sqrt{\frac{1}{y}}\right)}\frac{1}{2y\sqrt{y}}$

$$g(y) = \begin{cases} e^{-\left(1-\sqrt{\frac{1}{y}}\right)}\frac{1}{2y\sqrt{y}} + e^{-\left(1+\sqrt{\frac{1}{y}}\right)}\frac{1}{2y\sqrt{y}}, & y \in \langle 1, +\infty\rangle \\ e^{-\left(1+\sqrt{\frac{1}{y}}\right)}\frac{1}{2y\sqrt{y}}, & y \in \langle 0, 1\rangle \end{cases}$$

8. $\mathcal{N}(a, \sigma^2)$ $a = 174$ cm

$P(165 < X < 183) = 0{,}68269$

$P(X > 170) = ?$

$\Rightarrow P\left(\frac{165-174}{\sigma} < \tilde{X} < \frac{183-174}{\sigma}\right) = P\left(-\frac{9}{\sigma} < \tilde{X} < \frac{9}{\sigma}\right) = \frac{1}{2}\Phi^*\left(\frac{9}{\sigma}\right) - \frac{1}{2}\Phi^*\left(-\frac{9}{\sigma}\right)$

$= \frac{1}{2}\Phi^*\left(\frac{9}{\sigma}\right) + \frac{1}{2}\Phi^*\left(\frac{9}{\sigma}\right) = 1\Phi^*\left(\frac{9}{\sigma}\right) = 0{,}68269 \Rightarrow \frac{9}{\sigma} = 1{,}00 \Rightarrow \sigma = 9$

$P(X > 170) = P\left(\tilde{X} > \frac{170-174}{9}\right) = P\left(\tilde{X} > -\frac{4}{9}\right) = \frac{1}{2} + \frac{1}{2}\Phi^*\left(\frac{4}{9}\right) = \frac{1}{2} + \frac{1}{2}\,0{,}343 = 0{,}6715$

1 (b) $P(A + \bar{A}) = P(\Omega) = \boxed{1}$

$P(A + \bar{A}) = P(A) + P(\bar{A})$

$1 = P(A) + P(\bar{A}) \Rightarrow P(\bar{A}) = 1 - P(A)$

2 7 bračnih parova → 5 osoba

(a) svih 5 osoba istog spola

$$P(A) = \frac{\binom{2}{1}\binom{7}{5}}{\binom{14}{5}}$$

(b) među tih 5 niti jedan bračni par

od 7 parova biramo 5

$$P(B) = \frac{\binom{7}{5} \, 2^5}{\binom{14}{5}}$$

2^5 → tih 5 da li je M ili Ž

3 ispitanik ima bolest test će biti POZITIVAN 0,90
— " — nema — " — — " — POZITIVAN 0,05

0,02 POPULACIJE IMA BOLEST
0,98 — " — NEMA — " —

A = "TEST POZITIVAN"
H_1 = "ISPITANIK IMA BOLEST" $P(H_1) = 0{,}02$
H_2 = " — " — NEMA — " — " $P(H_2) = 0{,}98$ } $\Sigma = 1$ ✓

$P(A|H_1) = 0{,}9$ $P(A|H_2) = 0{,}05$

$$P(A) = \sum P(H_i)P(A|H_i) = 0{,}02 \cdot 0{,}9 + 0{,}98 \cdot 0{,}05 = 0{,}067$$

$$P(H_1|A) = \frac{P(H_1)P(A|H_1)}{P(A)} = \frac{0{,}02 \cdot 0{,}9}{0{,}067} = 0{,}268$$

4 1, 2, ..., 10 i izvlačimo 4 kuglice (bez vraćanja
X = drugi najveći izvučeni broj

_ 3 _

$$X \sim \begin{pmatrix} 3 & 4 & 5 & \dots & 9 \\ \frac{1 \cdot 1 \cdot 1 \binom{7}{1}}{\binom{10}{4}} & \frac{18}{\binom{10}{4}} & & & \frac{28}{210} \end{pmatrix}$$

$E(X) = 6.6$

X=3
1 2 3 4
1 2 3 5
1 2 3 6
7
8
9
1 2 3 10

X=4
1 2 4 5
1 3 4 5 } 5
2 3 4 5
1 2 4 6
1 3 4 6 } 6
2 3 4 6
7
8
9
10

6 · 3 = 12

$$P(X=3) = \frac{\binom{2}{2}\binom{7}{1}}{\binom{10}{4}}$$

$$P(X=4) = \frac{\binom{3}{2}\cdot\binom{6}{1}}{\binom{10}{4}}$$

HAN UTD
by ferOvac

$\boxed{5}$ 0,99 I   od 300 proizvoda barem 4 neispravna

0,01 N

X = broj neispravnih

$$X \sim B(n, p) = B(300, 0{,}01) \approx P(\lambda = np) = P(300 \cdot 0{,}01) = P(\lambda = 3)$$

$$P(X \geq 4) = 1 - P(X=3) - P(X=2) - P(X=1) - P(X=0)$$

$$P(X=k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

$$P(X \geq 4) = 1 - \frac{3^3}{3!}e^{-3} - \frac{3^2}{2!}e^{-3} - \frac{3^1}{1!}e^{-3} - \frac{3^0}{0!}e^{-3}$$

$$= 1 - e^{-3}\left(\frac{27}{6} + \frac{9}{2} + 3 + 1\right) = 1 - 13e^{-3}$$

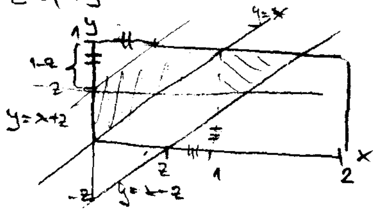
$\boxed{6}$ dva broja $\begin{cases} X\ [0,2] \\ Y\ [0,1] \end{cases}$ Z = apsolutna vrijednost razlike ta dva broj

$x \in [0,2]$

$y = [0,1]$

$P(Z < z) = ?$

$Z = |X - Y|$



(I) slika

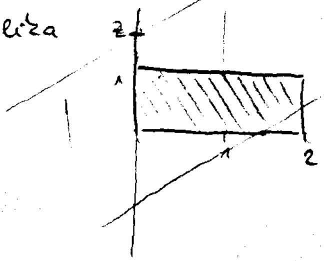
$$P(|X-Y| < z) = \frac{m(G_z)}{m(\Omega)} = \begin{cases} \dfrac{1^2 - \frac{(1-z)^2}{2} - \frac{(1-z)^2}{2}}{1 \cdot 1}, & z \in [0,1] \\ \end{cases}$$

$|x - y| < z$

$-z < y - x < z \quad / + x$

$x - z < y < x + z$

(II) slika



$|x-y| < z$

$x - z < y \quad y < x + z$

$\boxed{7}$ $Y = X^2$

X = jednoliku razdiobu $[-2, 1]$

$\to$ 2MI, 2009

$\boxed{8}$ $X \sim \mathcal{E}(\lambda)$

$E(X) = 3 = \frac{1}{\lambda} \Rightarrow \lambda = \frac{1}{3}$

$P(2 < X < 3 \mid X > 2) = ?$

$P(2 < X < 3) = F(3) - F(2) = (*)$

$F(x) = 1 - e^{-\lambda x} = 1 - e^{-\frac{1}{3}x}$

$F(2) = 1 - e^{-\frac{2}{3}}$

$F(3) = 1 - e^{-3/3} = 1 - e^{-1}$

$(*) = \cancel{1} - e^{-1} - (\cancel{1} - e^{-2/3}) = -e^{-1} + e^{-2/3}$

$P(X > 2) = 1 - F(2) = \cancel{1} - (\cancel{1} - e^{-2/3}) = e^{-2/3}$

$$P(2 < X < 3 \mid X > 2) = \frac{P(2 < X < 3)}{P(X > 2)} = \frac{-e^{-1} + e^{-2/3}}{e^{-2/3}} = 0{,}28$$

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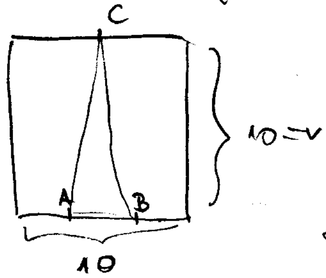
by ferOvac

$\boxed{1}$ (a)

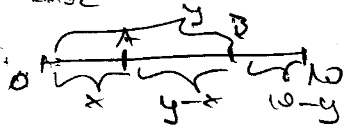
$$P(A) = \frac{\binom{15}{15}\binom{75}{n-15} n!}{\binom{90}{n} n!}$$ za $n=90$ to je 100%

(b) $$P(b) = \frac{15\binom{75}{n-15}(n-1)!}{\binom{90}{n} n!}$$ za $n=90$ to je $\frac{1}{6}$

$\boxed{2}$



$P_{\Delta ABC} < 25$



$$P_{\Delta ABC} = \frac{|y-x| \cdot v}{2} < 25$$

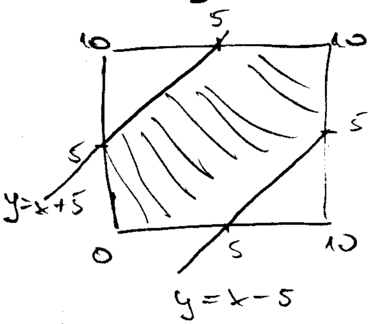
$|y-x| v < 50 / : v$ $v = 10$ cm

$|y-x| < 5 \longleftarrow |y-x| < \frac{50}{10}$

$-5 < y-x < 5 \; / +x$

$x-5 < y < x+5$

$y > x-5$ $\quad$ $y < x+5$



$$P(A) = \frac{10^2 - \frac{5^2}{2} - \frac{5^2}{2}}{10^2} = 0{,}75$$

MAN UTD
by ferdvao

$\boxed{3}$ 1000 ljudi

950 ISTINA 0,95 LAŽE 50 0,05

DETEKTOR GRIJEŠI u 0,01

A = " DETEKTOR LAŽI kaže da ISPITANIK LAŽE"

H_1 = " ISPITANIK GOVORI ISTINU"

H_2 = " —ll— —ll— LAŽ"

$P(H_1) = 0.95$ $\quad p(H_2) = 0{,}05$

$p(A|H_1) = 0{,}01$ $\quad p(A|H_2) = 0{,}99$

(a) $P(A) = \sum P(H_i)\, P(A|H_i) = 0{,}059$

(b) $P(H_2|A) = \dfrac{P(H_2)\, P(A|H_2)}{P(A)} = \dfrac{0{,}05 \cdot 0{,}99}{0{,}059} = 0{,}839$

$\boxed{217}$ kocka je bačena 7 puta

(a) šestica dobivena tek u 7. bac.

$$P(A) = \frac{5^6 \cdot \overset{\text{šestica}}{1}}{6^7}$$

(b) šestica barem jednom (tj jednom, dva puta ....)

$P(B) = 1 - P(\text{nijednom šestica})$

$$= 1 - \frac{5^7}{6^7}$$

(c) šestica točno jednom

$$P(C) = \frac{5^6 \cdot \overset{\text{šestica}}{1} \cdot \boxed{\binom{7}{1}}}{6^7}$$

u svim bacanju je dobivena šestica

4 (b) X, Y nez.

$D(X)=2$

$D(Y)=4$

$D(2X-Y)=?$

$$D(2X-Y)=D(2X)+D(-Y)=2^2D(X)+(-1)^2D(Y)=$$

$$=4D(X)+1D(Y)=4\cdot 2+1\cdot 4=8+4=12$$

6 (b) 52 karte 1 po 1, s vraćanjem

sve dok A ili tref boju

$P(X>E(X))=?$ X = broj izvlačenja

52 - 16 = 36

$As=\{$pik, herc, karo, tref$\}$ ⇒ As tref

tref boja $=\{$ 13 kareta $\}$ ⇒ As tref

$4+13-1=16$

$$X\sim\begin{pmatrix} 1 & 2 & 3 & 4 & \dots & n & \dots \\ \frac{16}{\binom{52}{1}} & \frac{36}{52}\cdot\frac{16}{52} & \left(\frac{36}{52}\right)^2\frac{16}{52} & \left(\frac{36}{52}\right)^3\frac{16}{52} & & \left(\frac{36}{52}\right)^{n-1}\frac{16}{52} & \end{pmatrix}$$

$$X\sim\begin{pmatrix} 1 & 2 & 3 & \dots & n & \dots \\ \frac{4}{13} & \frac{9}{13}\,\frac{4}{13} & \left(\frac{9}{13}\right)^2\frac{4}{13} & \dots & \left(\frac{9}{13}\right)^{n-1}\frac{4}{13} & \dots \end{pmatrix}$$

$$E(X)=\sum_{n=1}^{\infty} n\left(\frac{9}{13}\right)^{n-1}\cdot\frac{4}{13}=\frac{4}{13}\sum_{n=1}^{\infty} n\left(\frac{9}{13}\right)^{n-1}$$

$$\sum_{n=0} x^n=1+x+x^2+\dots+x^n=\frac{1}{1-x}\quad /\,'$$

$$\sum_{n=1} nx^{n-1}=0+1+2x+\dots+nx^{n-1}=\frac{1}{(1-x)^2}$$

$$E(X)=\frac{4}{13}\cdot\left(\frac{1}{(1-9/13)^2}\right)=\frac{13}{4}$$

ili prepoznati da se radi o geom. razd. $E(X)=\frac{1}{p}=\frac{1}{\frac{4}{13}}=\frac{13}{4}$

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$$P\left(X > \frac{13}{4}\right) = P(\overset{4,5,6,7,\ldots}{X > 3{,}25}) = 1 - P(X=3) - P(X=2) - P(X=1) - \underset{\text{nema ga}}{P(X=0)}$$

$$= 1 - \left(\frac{9}{13}\right)^2 \frac{4}{13} - \left(\frac{9}{13}\right)^1 \frac{4}{13} - \left(\frac{9}{13}\right)^0 \frac{4}{13} = \left(\frac{9}{13}\right)^3$$

$$P(Y=1 \mid X \geq 0) = \frac{P(Y=1, X \geq 0)}{P(X \geq 0)} = \frac{P(Y=1, X=0) + P(Y=1, X=1)}{P(X=0) + P(X=1)}$$

1) 52 karte

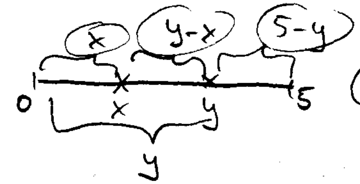
(a) FLUSH
BOJA JAČINA
$$P(A) = \frac{\binom{4}{1}\binom{13}{5}}{\binom{52}{5}}$$

(b) ROYAL FLUSH
$$P(B) = \frac{\binom{4}{1}}{\binom{52}{5}}$$

(c) FULLHOUSE
$$P(C) = \frac{\binom{4}{3}\binom{13}{1}\binom{4}{2}\binom{12}{1}}{\binom{52}{5}}$$

(d) DVA PARA
$$P(D) = \frac{\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{1}\binom{4}{1}}{\binom{52}{5}}$$

2)



(a) $x<y \Rightarrow x>1$

$y-x>1 \Rightarrow y>1+x$

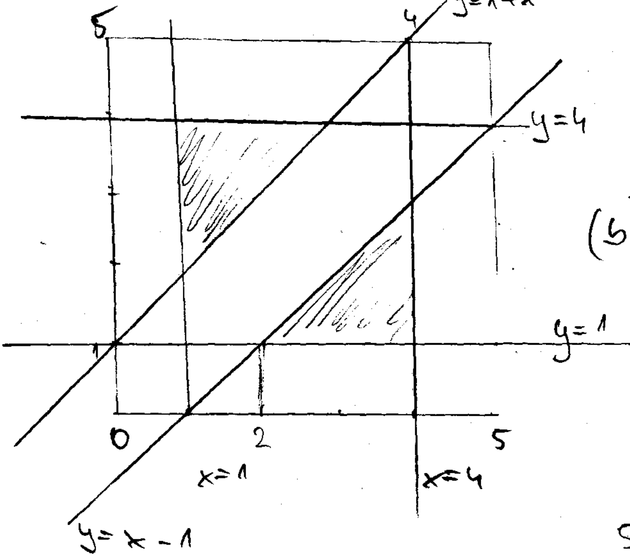
$5-y>1 \quad /-4$

$\Rightarrow -y > 1-5$

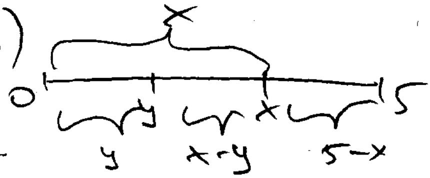
$y<4$

$y=1+x$

$5=1+x \Rightarrow x=4$



(b)



$y>1$

$x-y>1 \Rightarrow -y>1-x$

$y<x-1$

$5-x>1 \Rightarrow -x>-4$

$x<4$

$$P(A) = \frac{\frac{2^2}{2}+\frac{2^2}{2}}{5^2} = \frac{\frac{4}{2}+\frac{4}{2}}{25} = \frac{4}{25} = \boxed{0{,}16}$$

$y=x-1$

$0=x-1 \Rightarrow x=1$

$y=5-1 \Rightarrow y=4$

MAN UTD
by ferOvac

(3) 34 % O → O
37 % A → A, O
21 % B → B, O
8 % AB → O, A, B, AB

(a) $P(A) = ?$

$H_0 = "O"$ $\quad p(H_0) = 0{,}34$
$H_A = "A"$ $\quad p(H_A) = 0{,}37$
$H_B = "B"$ $\quad p(H_B) = 0{,}21$
$H_{AB} = "AB"$ $\quad p(H_{AB}) = 0{,}08$

$\Sigma = 1$ ✓

A = "uspješna transfuzija"

$p(A|H_0) = 0{,}34 \quad p(A|H_A) = 0{,}34 + 0{,}37 = 0{,}71$
$p(A|H_B) = 0{,}21 + 0{,}34 = 0{,}55 \quad p(A|H_{AB}) = 1$

$$p(A) = \sum_{i=1}^{4} p(H_i)\,p(A|H_i) = 0{,}34 \cdot 0{,}34 + 0{,}37 \cdot 0{,}71 + 0{,}21 \cdot 0{,}55 + 0{,}08 \cdot 1 = 0{,}5738 = P(A)$$

(b) $$P(H_{AB}|A) = \frac{P(H_{AB})\,P(A|H_{AB})}{P(A)} = \frac{0{,}08 \cdot 1}{0{,}5738} = 0{,}1394$$

$\boxed{Z17}$ 4 bacanja barem jedna 1-ca

$$P(A) = 1 - P(\text{0 jedinica}) = 1 - \frac{5^4}{6^4} = 1 - \left(\frac{5}{6}\right)^4 = 0{,}518$$

sve osim jedinica

P(B) 24 bacanja dvije kocke suma 2 tj dvije 1-ce

sve mogućnosti osim 1 1

$$P(B) = 1 - \left(\frac{35}{6^2}\right)^{24} = 0{,}491$$

[4] 1B i 4C 1 pol s vraćanjem

AM1 2010 (2)

(a) X = broj izvlač.

$$X \sim \begin{pmatrix} 1 & 2 & 3 & 4 & \dots & n & \dots \\ \frac{1}{5} & \underset{C\ B}{\frac{4}{5}\cdot\frac{1}{5}} & \frac{4}{5}\cdot\frac{4}{5}\cdot\frac{1}{5} & \left(\frac{4}{5}\right)^3\frac{1}{5} & \dots & \left(\frac{4}{5}\right)^{n-1}\frac{1}{5} \end{pmatrix}$$

X=2 CB

X=3 CCB $\frac{4}{5}\cdot\frac{4}{5}\cdot\frac{1}{5}$

$P(X \le 15 \mid X > 10) = ?$ $\qquad P(AB) = P(B)\,P(A|B)$

$$P(\underset{A}{X \le 15} \mid \underset{B}{X > 10}) = \frac{P(\overbrace{10 < X \le 15}^{x=11,12,13,14,15})}{P(\underbrace{X > 10}_{x=11,12,\dots})} = \frac{\left(\frac{4}{5}\right)^{11-1}\left(\frac{1}{5}\right)^1 + \dots + \left(\frac{4}{5}\right)^{15-1}\left(\frac{1}{5}\right)}{\left(\frac{4}{5}\right)^{11-1}\left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)^{12-1}\left(\frac{1}{5}\right) + \dots}$$

$$\frac{1}{5}\left(\sum_{k=10}^{\infty}\left(\frac{4}{5}\right)^k\right) = \frac{1}{5}\sum_{i=0}^{\infty}\underbrace{\left(\frac{4}{5}\right)^{i+10}}_{\left(\frac{4}{5}\right)^i\cdot\left(\frac{4}{5}\right)^{10}} = \frac{1}{5}\cdot\left(\frac{4}{5}\right)^{10}\sum_{i=0}^{\infty}\left(\frac{4}{5}\right)^i$$

ODSUSTVO pamćenja samo GEOM.

(b) 1B 4C 1 pol s vraćanjem, sve dok drugi put B

$$Y \sim \begin{pmatrix} 2 & 3 & 4 & \dots & n & \dots \\ \left(\frac{1}{5}\right)^2 & 2\cdot\frac{4}{5}\left(\frac{1}{5}\right)^2 & 3\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right)^2 & \dots & (n-1)\left(\frac{4}{5}\right)^{n-2}\left(\frac{1}{5}\right)^2 \end{pmatrix}$$

Y=2 BB

Y=3 BCB $\frac{1}{5}\frac{4}{5}\frac{1}{5}$

CBB $\frac{4}{5}\frac{1}{5}\frac{1}{5}$

Y=4 BCCB

CCBB

CBCB

$$E(Y) = \sum_{n=2}^{\infty} n\cdot(n-1)\left(\frac{4}{5}\right)^{n-2}\left(\frac{1}{5}\right)^2 = \frac{1}{25}\sum_{n=2}^{\infty} n(n-1)\left(\frac{4}{5}\right)^{n-2}$$

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \dots + x^n = \frac{1}{1-x} \quad /'$$

$$\sum_{n=1}^{\infty} n x^{n-1} = 0 + 1 + 2x + \dots + n x^{n-1} = \frac{1}{(1-x)^2} \quad /'$$

$$\boxed{\sum_{n=2}^{\infty} n(n-1) x^{n-2}} = 0 + 0 + 2 + \dots + n(n-1)x^{n-2} = \boxed{\frac{2}{(1-x)^3}}$$

$$\frac{1}{(1-x)^2}\Big/' \Rightarrow (1-x)^{-2}\Big/' = -2(1-x)^{-3}\cdot(-1) = \frac{2}{(1-x)^3}$$

$$E(Y) = \frac{1}{25}\cdot\sum_{n=2}^{\infty} n(n-1)\left(\frac{4}{5}\right)^{n-2} = \frac{1}{25}\left[\frac{2}{(1-4/5)^3}\right] = 10$$

MARK UPD by ferhac

$\boxed{6}$. $X_1 \sim P(\lambda_1)$
$X_2 \sim P(\lambda_2)$

IMI 2009 (1)

1 6B i 4C $\xrightarrow{\text{barem}}$ 5B

(a) odjednom 7kug

$$P(A) = \frac{\binom{6}{5}\binom{4}{2}}{\binom{10}{7}} + \frac{\binom{6}{6}\binom{4}{1}}{\binom{10}{7}} = 0{,}3\dot{3}$$

(b) 7puta s vraćanjem

PONAVLJANJE POKUSA   u i-tom izvlačenju je bila C

$$P(B) = \underbrace{\left(\frac{6}{10}\right)^7}_{7B} + \underbrace{\left(\frac{6}{10}\right)^6\left(\frac{4}{10}\right)\binom{7}{1}}_{6B} + \underbrace{\left(\frac{5}{10}\right)^5\left(\frac{4}{10}\right)^2\binom{7}{2}}_{5B}$$

2 A, B, C da bi bili nezavisni

(a)

$$P(ABC) = P(A)P(B)P(C)$$
$$P(AB) = P(A)P(B)$$
$$P(AC) = P(A)P(C)$$
$$P(BC) = P(B)P(C)$$

(b) $P(A(B+C)) = P(AB + AC) = P(AB) + P(AC) - P(AABC) =$ $\check{z}\cdot\check{z} = 1$

$$= P(A)P(B) + P(A)P(C) - P(A)P(B)P(C)$$

$$= P(A)[\underbrace{P(B) + P(C) - P(B)P(C)}_{P(B+C)}] \Longrightarrow P(BC)$$

$$= P(A)\cdot P(B+C)$$

3 Provalnik 2:50 3:00 $x \in [0, 10]$ treba 2

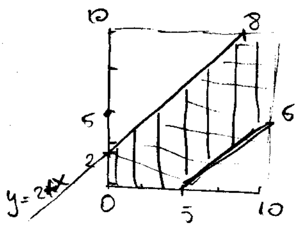
Policajac -||- $y \in [0, 10]$ -||- 5

(a) $(x < y)$ — x, y, $y-x$, 10 $(y < x)$ — y, x, $x-y$

$y - x < 2$ $x - y < 5$

$y < 2 + x$ $-y < 5 - x$

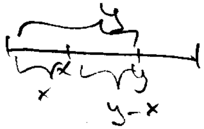
$y > x - 5$



$$P(A) = \frac{10^2 - \frac{8^2}{2} - \frac{5^2}{2}}{10^2} = 0{,}555$$

MAN UTD by Perčvac

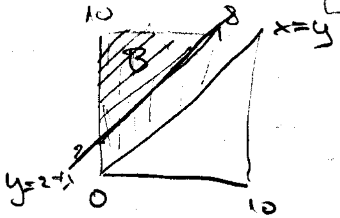
(b) $\overset{!}{x} < \overset{?}{y}$



prije dolaska $x < y$

$y - x > 2$

$y > 2 + x$



$$P(B) = \frac{\frac{8^2}{2}}{10^2} = 0{,}32$$

$\boxed{4}$ ZA 41% $\longrightarrow$ 62% V.O.

PROTIV 37% $\longrightarrow$ 40% V.O.

SUZDRŽANO 22% $\longrightarrow$ 15% V.O.

A = "ispitanik je V.O."

H_1 = "ZA" $\qquad H_2$ = "PROTIV" $\qquad H_3$ "ispitanik je SUZDRŽAN"

$$\underbrace{P(H_1) = 0{,}41 \qquad P(H_2) = 0{,}37 \qquad P(H_3) = 0{,}22}_{0{,}41 + 0{,}37 + 0{,}22 = 1}$$

$P(A|H_1) = 0{,}62$ — ako je ispitanik "za" koliko vj. da je ispitanik V.O. $\qquad P(A|H_2) = 0{,}40 \qquad P(A|H_3) = 0{,}15$

$$P(A) = \sum_{1}^{3} P(H_i) P(A|H_i) = 0{,}41 \cdot 0{,}62 + 0{,}4 \cdot 0{,}37 + 0{,}15 \cdot 0{,}22$$

$$P(A) = 0{,}429$$

$$P(H_2|A) = \frac{P(H_2) P(A|H_2)}{P(A)} = \frac{0{,}37 \cdot 0{,}4}{0{,}429} = 0{,}345$$

$\boxed{5}$ X = maks od 2 broja $\{1, \dots, 7\}$

$$X \sim \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \left(\frac{1}{7}\right)^2 & \frac{3}{49} & \frac{5}{49} & \frac{7}{49} & \frac{9}{49} & \frac{11}{49} & \frac{13}{49} \end{pmatrix}$$

$X=2 \Rightarrow$ 12, 21, 22

$E(X) = 1 \cdot \frac{1}{49} + 2 \cdot \frac{3}{49} + \dots + 7 \cdot \frac{13}{49} = \frac{36}{7} = 5.14$

$X=3 \Rightarrow$ 13, 23, 33, 31, 32

$\boxed{6}$ $X \sim \begin{pmatrix} -1 & 1 \\ \frac{3}{6} & \frac{3}{6} \end{pmatrix}$ $\qquad Y \sim \begin{pmatrix} -1 & 1 \\ \frac{3}{6} & \frac{3}{6} \end{pmatrix}$

$Y \backslash X$	-1	1
-1	$P(X=-1, Y=-1)$	$P(X=1, Y=-1)$
1	$P(X=-1, Y=1)$	$P(X=1, Y=1)$

$-1 - (1)$

$1 - (-1)$

$1 - 1$

$-1 - (-1)$

$X-Y$ poprima $-2, 0, 2$

$P(X-Y=0) = P(X=1, Y=-1) + P(X=-1, Y=1)$

MAN UTD
by ferOvac

1MI 2008

1

2) 12 kopčica $\to$ 7

8 dobrih, 4 defektno

(a) točno 1. def

$$P(A) = \frac{\binom{4}{1}\binom{8}{6}}{\binom{12}{7}}$$

točno 1 def; od 8 dobrih 6 biramo

(b) najviše 1. def

0 def + 1 def

$$P(B) = \frac{\binom{4}{0}\binom{8}{7}}{\binom{12}{7}} + \frac{\binom{4}{1}\binom{8}{6}}{\binom{12}{7}}$$

(c) barem 1. def   1 def + 2 def + 3 def + 4 def

tj. 1 − 0 def

$$P(C) = 1 - \frac{\binom{4}{0}\binom{8}{7}}{\binom{12}{7}}$$

3

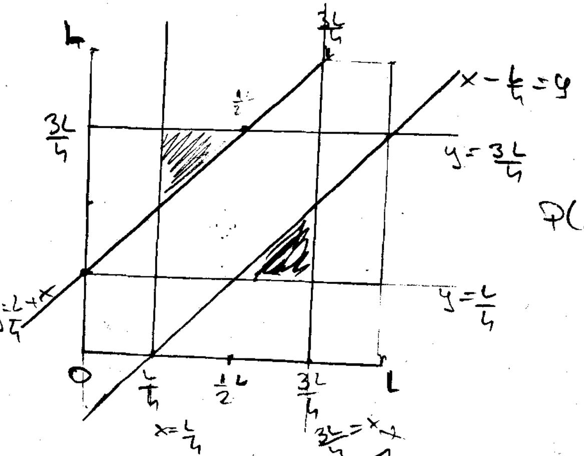
I) $x < y$

0 — x — y — L; $\underbrace{x}$, $\underbrace{y-x}$, $\underbrace{L-y}$, $\overbrace{y}$

$x > \frac{L}{4}$ $y - x > \frac{L}{4}$ $L - y > \frac{L}{4}$

$y > \frac{L}{4} + x$ $\frac{4L}{4} - \frac{L}{4} > y$

$\frac{3L}{4} > y$



$$P(A) = \frac{m(A)}{\Omega(A)} = \frac{2 \cdot \frac{\left(\frac{1}{4}L\right)^2}{2}}{L^2} = \frac{2\,\frac{\frac{1}{16}L^2}{2}}{L^2} = 2\,\frac{\frac{L^2}{32}}{L^2} = \frac{2}{32}$$

$$\boxed{P(A) = \frac{1}{16}}$$

II) $y < x$ 0 — y — x — L; y, $x - y$, $L - x$

$y < \frac{L}{4}$

MANUUD $x - y < \frac{L}{4} \Rightarrow x - \frac{L}{4} \le y$

by fenOkee $x < \frac{3L}{4} \Rightarrow \frac{3L}{4} > x$

$y = \frac{3L}{4}$ $y = \frac{L}{4} + x$

$\frac{3L}{4} = \frac{L}{4} + x$

$3L - L = 4x$ $2L = 4x \Rightarrow x = \frac{1}{2}L$

$\boxed{4}$. A = " pale su dvije 5-ice "

H_i = " u prvom bacanju pao je broj i "

$$p(H_i) = \frac{1}{6} \Rightarrow \underbrace{\frac{1}{6} + \frac{1}{6} + \dots + \frac{1}{6}}_{6} = \frac{6}{6} = 1$$

$p(A|H_1) = 0 \Rightarrow$ ako je u prvom bacanju pao br. 1 ; kolika je vjeroj. da su pale 2-ije petice

$p(A|H_2) = \frac{\binom{2}{2}}{6^2}$ → na dvije će pasti dvije 5-ice

$p(A|H_3) = \frac{\binom{3}{2} \cdot 5}{6^3}$ → od 3 kocke dvije će biti 5-ice ; → bilo što osim 5 na ovoj kocki koja nije 5

$$p(A|H_4) = \frac{\binom{4}{2} 5^2}{6^4} \qquad p(A|H_6) = \frac{\binom{6}{2} 5^4}{6^6}$$

5 bacanja

$$p(A|H_5) = \frac{\binom{5}{1} 5^4}{6^5}$$

$$P(A) = \sum_{i=1}^{6} P(H_i) P(A|H_i) = 0,136$$

$$P(H_5|A) = \frac{P(H_5) P(A|H_5)}{P(A)} = \frac{\frac{1}{6} \cdot \frac{\binom{5}{1} 5^4}{6^5}}{} = 0,492$$

$\boxed{Z17}$

I	0,4
II	0,6
III	0,7
IV	0,8

H_1 = " I, ~~II~~, ~~III~~, ~~IV~~ "

H_2 = " ~~I~~ II ~~III~~ ~~IV~~ "

⋮

(a) meta će biti pogođena 1 − P(nije pogođena)

$$P(A) = 1 - 0,6 \cdot 0,4 \cdot 0,3 \cdot 0,2 = 1 - \frac{9}{625} = 0,9856$$

(b) B = " meta je pogođena s točno 3 metka "

$H_1 = "ABC\bar{D}"$ $\quad P(H_1) = 0,4 \cdot 0,6 \cdot 0,7 \cdot 0,2 = 0,0336 \quad P(B|H_1) = 1$

$H_2 = "AB\bar{C}D"$ $\quad P(H_2) = 0,4 \cdot 0,6 \cdot 0,3 \cdot 0,8 = 0,0576 \quad P(B|H_2) = 1$

$H_3 = "A\bar{B}CD"$ $\quad P(H_3) = 0,4 \cdot 0,4 \cdot 0,7 \cdot 0,8 = 0,0896 \quad P(B|H_3) = 1$

$H_4 = "\bar{A}BCD"$ $\quad P(H_4) = 0,6 \cdot 0,6 \cdot 0,7 \cdot 0,8 = 0,2016 \quad P(B|H_4) = 1$

$H_5 \dots H_{16}$ NEBITNO ! $\Rightarrow P(B|H_5) = 0$

$P(B|H_6) = 0$

$P(B|H_{16}) = 0$

$$P(H_1|B) = \frac{0,0336}{0,0336 + 0,0576 + 0,0896 + 0,2016}$$

$P(H_1|B) = 0,0879$

6 $X_1 \sim G(p) \qquad X_2 \sim G(p)$

(a) $Y = X_1 + X_2$

$$X_{2)} X_1 \sim \begin{pmatrix} 1 & 2 & 3 & \dots n \dots \\ p & (1-p)p & (1-p)^2 p & \dots (1-p)^{n-1} p \dots \end{pmatrix}$$

$$Y = X_1 + X_2 \sim \Bigg($$

7 (c) 1h 180 poruka

$\lambda =$ 180 mailova / 1h = 180 / 60 = 3 maila / min

$$P(X \geq 3) = 1 - P(X=0) - P(X=1) - P(X=2)$$

$$= 1 - \frac{3^0}{0!} e^{-3} - \frac{3^1}{1!} e^{-3} - \frac{3^2}{2!} e^{-3}$$

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by ferOvac

1. 2Z, 3C, 4P

(a) 2 raznobojne kuglice

$$P(A) = \frac{\binom{2}{1}\binom{3}{1}}{\binom{9}{2}} + \frac{\binom{2}{1}\binom{4}{1}}{\binom{9}{2}} + \frac{\binom{3}{1}\binom{4}{1}}{\binom{9}{2}} = \frac{6+8+12}{36} = \frac{26}{36} = \frac{13}{18}$$

(b) 10 puta po 2 kuglice

barem 2 puta bile izvučene kuglice iste boje

RAČUNAMO suprotno: nijedanput i jedanput se izvuku istobojne

$$P(B) = 1 - \underbrace{\left(\frac{13}{18}\right)^{10}}_{\text{10 puta različite boje}} - \underbrace{\left(\frac{5}{18}\right)^{1}}_{\text{1. put istobojne}} \boxed{\left(\frac{13}{18}\right)^{9}} \binom{10}{1}$$

10 puta različite boje

9. puta raznobojne

1. put istobojne $= 1 - \frac{13}{18} = \frac{5}{18}$

$\binom{10}{1}$ – odabir pokusa u kojem su pale iste kuglice (1 od 10 pokusa)

2. A i M $[0,60]$, 20 min na trgu
x y

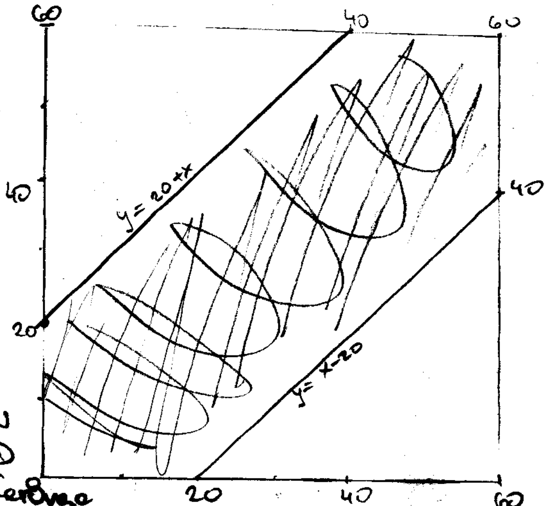
$x, y \in [0, 60]$

$|x-y| < 20$ tada će se sresti

$-20 < x - y < 20 \quad / -x$

$-20 - x < -y < 20 - x \quad / (-1)$

$20 + x > y > x - 20$



$$P(A) = \frac{m(A)}{\Omega(A)} = \frac{60^2 - \frac{40^2}{2} - \frac{40^2}{2}}{60^2}$$

$P(A) = \frac{5}{9} = 0{,}5\dot{5}$

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$\boxed{3}$ (a) Dva događaja A i B su nezavisni ako vrijedi bilo koja od jednakosti:

(1) $P(A|B) = P(A)$ ili (2) $P(B|A) = P(B)$

Nužan i dovoljan uvjet za nezavisnost je

$$P(AB) = P(A)\,P(B)$$

(b)

$$P(AB) = P(A)P(B)$$

$$P(\overline{A}\overline{B}) = P(\overline{A+B}) = 1 - P(A+B) = 1 - \left(P(A) + P(B) - P(AB)\right) =$$

$$= 1 - P(A) - P(B) + P(AB) = \left(1 - P(A)\right)\left(1 - P(B)\right) = P(\overline{A})\,P(\overline{B})$$

$\boxed{4}$

$1 \to 0{,}75$
$0 \to 0{,}25$
na izlazu 0,05 pogreške

A = "primljena 1"
H_0 = "poslana je 0" H_1 = "poslana je 1"

$$\underbrace{p(H_0) = 0{,}25 \qquad p(H_1) = 0{,}75}_{0{,}25 + 0{,}75 = 1}$$

$p(A|H_0) = 0{,}05$ $p(A|H_1) = 0{,}95$

$$p(A) = p(H_0)p(A|H_0) + p(H_1)p(A|H_1)$$

$$= 0{,}25 \cdot 0{,}05 + 0{,}75 \cdot 0{,}95 = 0{,}725$$ 0,725

$$p(H_1|A) = \frac{p(H_1)p(A|H_1)}{P(A)} = \frac{0{,}75 \cdot 0{,}95}{0{,}725} = 0{,}983$$

$\boxed{ZI\ 7}$ (a) 4A 52 ↘ 5

$$P(A) = \frac{1 \cdot \binom{4}{4} \cdot \binom{48}{1}}{\binom{52}{5}}$$

(b) $$P(B) = \frac{\overset{A}{\binom{4}{3}}\overset{K}{\binom{4}{2}}}{\binom{52}{5}}$$ 3A i 2K

(c) barem jedan AS

$$P(C) = 1 - \frac{\binom{48}{5}}{\binom{52}{5}}$$

5

X/Y	0	1	Σ
-1	1/12	1/12	1/6
0	1/12	1/4	1/3
1	1/6	1/3	1/2
Σ	1/3	2/3	1

1MI
2007
2

(a) X i Y su nezavisne

$$X \sim \begin{pmatrix} -1 & 0 & 1 \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \end{pmatrix} \qquad Y \sim \begin{pmatrix} 0 & 1 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

(b) $U = X^2 \sim \begin{pmatrix} 1 & 0 & 1 \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \end{pmatrix} \sim \begin{pmatrix} 0 & 1 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} \sim x^2$

$E(U) = 0 \cdot \frac{1}{3} + 1 \cdot \frac{2}{3} = \frac{2}{3}$

(c) $V = x^2 + y^2$; $W = X \cdot Y$

$$Y^2 \sim \begin{pmatrix} 0 & 1 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} \qquad V \sim \begin{pmatrix} \overset{0+0}{0} \\ \frac{1}{3} \cdot \frac{1}{3} \end{pmatrix}$$

$x^2 + y^2$?

$P(x^2+y^2=k)$, $k = 0, 1, 2$

$P(x^2+y^2=0) = P(x^2=0, y^2=0) = P(X=0, Y=0) = \frac{1}{12}$

$P(x^2+y^2=1) = P(x^2=0, y^2=1) + P(x^2=1, y^2=0)$

$= \cancel{P(X=0, Y=-1)} + P(X=0, Y=1) + P(X=-1, Y=0)$

$+ P(X=1, Y=0) =$ iščitati

$P(x^2+y^2=2) = P(x^2=1, y^2=1) = P(X=-1, Y=1) + P(X=1, Y=1)$

$\cancel{P(X=-1) \cdot P(Y=1)}$

MAN
UTD
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$P(X=-1, Y=0) \neq P(X=-1) \cdot P(Y=0)$

$\boxed{6}$

$$X \sim \begin{pmatrix} 2 & 3 & 4 & \dots & n & \dots \\ 2\cdot\left(\frac{1}{2}\right)^2 & 2\left(\frac{1}{2}\right)^3 & 2\cdot\left(\frac{1}{2}\right)^4 & \dots & 2\cdot\left(\frac{1}{2}\right)^n & \dots \end{pmatrix}$$

2 puta $\Rightarrow$ PP ili GG

3 puta $\Rightarrow$ PGG ili GPP

$$E(X) = \sum_{n=2}^{\infty} n \cdot 2\left(\frac{1}{2}\right)^n = 2\sum_{n=2}^{\infty} n\left(\frac{1}{2}\right)^n$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad /'$$

$1 + x + x^2 + \dots + x^n$

$$\left(\frac{1}{t}\right)' = (t^{-1})' = -1t^{-2}\cdot t'$$

$$\sum_{n=1}^{\infty} n x^{n-1} = 0 + 1 + 2x + \dots + n x^{n-1} = -\frac{1}{(1-x)^2}(-1) \quad / \cdot x$$

$$\sum_{n=1}^{\infty} n x^n = \underset{n=0}{0} + \underset{n=1}{x} + \underset{n=2}{2x^2} + \dots + n x^n = \frac{x}{(1-x)^2}$$

$$E(X) = 2\sum_{n=2}^{\infty} n\left(\frac{1}{2}\right)^n = 2\left(\sum_{n=1}^{\infty} n\left(\frac{1}{2}\right)^n - \Big(\overbrace{1\cdot\left(\tfrac{1}{2}\right)^1}^{n=1}\Big)\right)$$

$$E(X) = 2\left(\frac{\frac{1}{2}}{\left(1-\frac{1}{2}\right)^2} - \frac{1}{2}\right)$$

$$E(X) = 2\left(2 - \frac{1}{2}\right) = 2\left(\frac{3}{2}\right) = 3$$

$\boxed{8}$ b)

$E(X) = 3 \Rightarrow \lambda_1 = E(X) = 3$

$E(Y) = 4 \Rightarrow \lambda_2 = E(Y) = 4$

$P(X+Y=10) = ?$

$\lambda = \lambda_1 + \lambda_2 = 7$ (jer su nez.)

$$P(X+Y=10) = \frac{7^{10}}{10!}e^{-7} = 0{,}071$$

(1) $P(A\cap B)=0{,}72 \Rightarrow P(AB)=0{,}72$

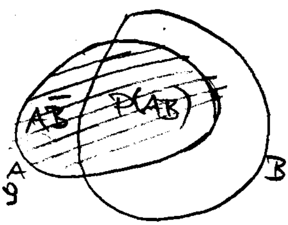
(a) $P(A\cap\bar{B})=0{,}18 \Rightarrow P(A\bar{B})=0{,}18$

$P(A)=?$

$P(A)=0{,}72+0{,}18=0{,}9$

$$\boxed{P(A\bar{B})=P(A)-P(AB)}$$

$P(A)=P(A\bar{B})+P(AB)=0{,}72+0{,}18=0{,}9$



(b)

$P(\bar{B}|\bar{A})=P(B|A)$ tada su A i B nezavisni

$$P(B|\bar{A})=\frac{P(\bar{A}B)}{P(\bar{A})} \qquad P(B|A)=\frac{P(AB)}{P(A)}$$

$$\frac{P(\bar{A}B)}{P(\bar{A})}=\frac{P(AB)}{P(A)} \Rightarrow P(A)\,\underbrace{P(\bar{A}B)}_{P(B)-P(AB)}=\underbrace{P(\bar{A})}_{1-P(A)}\,P(AB)$$

$$\Rightarrow P(A)\left[P(B)-P(AB)\right]=\left[1-P(A)\right]P(AB)$$

$$\Rightarrow P(A)P(B)-\cancel{P(A)P(AB)}=P(AB)-\cancel{P(A)P(AB)}$$

$$\Rightarrow \boxed{P(A)P(B)=P(AB)} \text{ def } \checkmark$$

(2) 5B i 10C $\to 5$

barem dvije B $\Rightarrow$ tj 2B + 3B + 4B + 5B ili $1-0B-1B$

$$P(A)=1-\frac{\binom{5}{0}^{B}\binom{10}{5}^{C}}{\binom{15}{5}}-\frac{\binom{5}{1}\binom{10}{4}}{\binom{15}{5}}=\frac{81}{143}\approx 0{,}566$$

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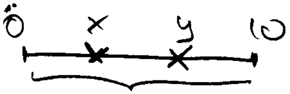
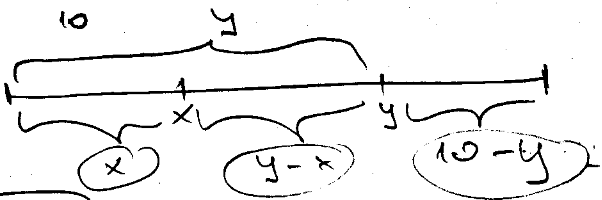
3 str 56, $\boxed{1.18}$



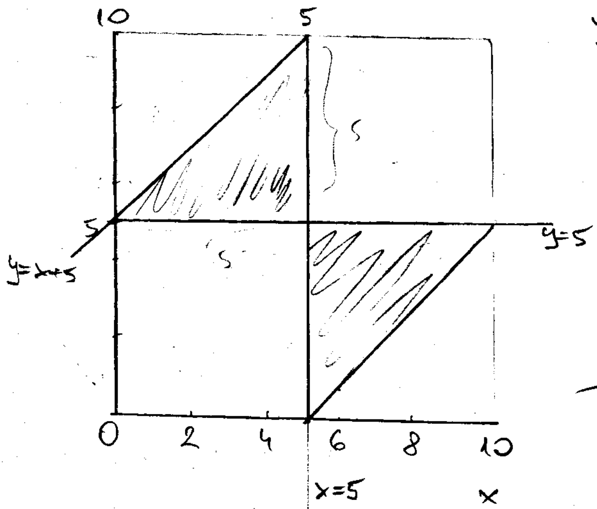


I) $\boxed{x<y}$

$x < \cancel{y} - x + 10 - \cancel{y} \Rightarrow 2x < 10 \Rightarrow x < 5$

$y - x < x + 10 - y \Rightarrow 2y < 2x + 10$
$y < x + 5$

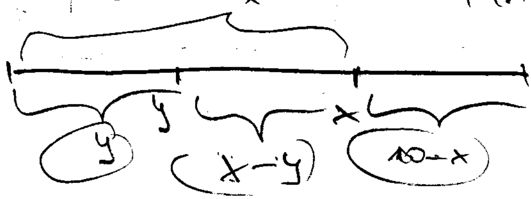
$10 - y < \cancel{x} + y - \cancel{x} \Rightarrow 2y > 10$
$y > 5$



$$P(A) = \frac{m(A)}{m(\Omega)} = \frac{\frac{5^2}{2} + \frac{5^2}{2}}{10^2} = \frac{25}{100}$$

$P(A) = \frac{1}{4}$

II $y < x$



$y < \cancel{x} - y + 10 - \cancel{x} \Rightarrow y < 5$

$\cancel{x} - y < y + 10 - x \Rightarrow x < y + 5$

$10 - x < \cancel{y} + x - \cancel{y} \Rightarrow x > 5$

$\boxed{4}$ 2B i 3C

[illegible]MI
2009
$\boxed{2}$

$\boxed{2B \text{ i } 3C} \xrightarrow{\circ\circ\circ} \boxed{\text{PRAZNA}} \xrightarrow{\circ}$

A = " IZVUČENA JE C "

H_1 = " 2B1C "

H_2 = " 1B2C "

H_3 = " 3C "

$P(H_3 | A) = ?$

$$P(H_1) = \frac{\overset{B}{\binom{2}{2}}\overset{C}{\binom{3}{1}}}{\binom{5}{3}} = \frac{3}{\binom{5}{3}} = \frac{3}{10}$$

$$P(H_2) = \frac{\overset{B}{\binom{2}{1}}\overset{C}{\binom{3}{2}}}{\binom{5}{3}} = \frac{2 \cdot \frac{3 \cdot \cancel{2}}{\cancel{2} \cdot 1}}{\frac{5 \cdot 4 \cdot \cancel{3}}{\cancel{3} \cdot 2 \cdot 1}} = \frac{6}{10}$$

$$P(H_3) = \frac{\overset{B}{\binom{2}{0}}\overset{C}{\binom{3}{3}}}{\binom{5}{3}} = \frac{1}{10}$$

$\sum P(H_i) = 1$ ✓

$$P(A | \overset{2B1C}{H_1}) = \frac{\overset{C}{\binom{1}{1}}}{\binom{3}{1}} = \frac{1}{3}$$

$$P(A | \overset{1B2C}{H_2}) = \frac{\overset{C}{\binom{2}{1}}}{\binom{3}{1}} = \frac{2}{3}$$

$$P(A | H_3) = \frac{\binom{3}{1}}{\binom{3}{1}} = 1$$

$$P(A) = \sum_{1}^{3} P(H_i) P(A | H_i)$$

$$P(A) = \frac{\cancel{3}}{10} \cdot \frac{1}{\cancel{3}} + \frac{\cancel{6}^{2}}{10_{5}} \cdot \frac{2}{\cancel{3}_{1}} + \frac{1}{10} \cdot 1 = \frac{3}{5}$$

$$P(H_3 | A) = \frac{P(H_3) P(A | H_3)}{P(A)} = \frac{\frac{1}{10} \cdot 1}{\frac{3}{5}} = \frac{1}{6}$$

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PZ17

2B 1C $\xrightarrow{O}$ 1B 5C $\xrightarrow{}$ O

A = " izvučena je na kraju B kuglica"

H_1 = " prva kuglica je B" 1B 5C

H_2 = " — " — " je C"

$$P(H_1) = \frac{\binom{2}{1}}{\binom{3}{1}} = \frac{2}{3} \qquad P(H_2) = \frac{\binom{1}{1}}{\binom{3}{1}} = \frac{1}{3}$$

$$\frac{2}{3} + \frac{1}{3} = 1$$

$$P(A|H_1) = \frac{\binom{2}{1}}{\binom{7}{1}} = \frac{2}{7} \qquad P(A|H_2) = \frac{\binom{1}{1}}{\binom{7}{1}} = \frac{1}{7}$$

$$P(A) = \frac{2}{3} \cdot \frac{2}{7} + \frac{1}{3} \cdot \frac{1}{7} = \frac{5}{21}$$

$$P(H_2|A) = \frac{P(H_2)P(A|H_2)}{P(A)} = \frac{\frac{1}{\cancel{3}} \cdot \frac{1}{\cancel{7}}}{\frac{5}{\cancel{21}}} = \frac{1}{5}$$

PIMI
2007
(2)

[5] 6 i 3 su $/3 = \frac{2}{6}$ vj.

X = broj bacanja

$$X \sim \begin{pmatrix} 1 & 2 & 3 & \dots & n & \dots \\ \frac{2}{6} & \frac{4}{6}\cdot\frac{2}{6} & \frac{4}{6}\cdot\frac{4}{6}\cdot\frac{2}{6} & & \left(\frac{4}{6}\right)^{n-1}\frac{2}{6} \end{pmatrix}$$

$$E(X) = \sum_{n=1}^{\infty} n \cdot \left(\frac{4}{6}\right)^{n-1} \frac{2}{6} = \frac{1}{3}\sum_{n=1}^{\infty} n\left(\frac{2}{3}\right)^{n-1}$$

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \dots + x^n = \frac{1}{1-x} \quad /'$$

$$\sum_{n=1}^{\infty} n x^{n-1} = 0 + 1 + 2x + \dots + n x^{n-1} = -\frac{1}{(1-x)^2}(-1)$$

$$\sum_{n=1}^{\infty} n x^{n-1} = \frac{1}{(1-x)^2}$$

$$\text{za } x = \frac{2}{3} \Rightarrow \frac{1}{\left(\frac{1}{3}\right)^2} = 9$$

$$E(X) = \frac{1}{3}\sum_{n=1}^{\infty} n\left(\frac{2}{3}\right)^{n-1} = \frac{1}{3}\cdot 9 = 3$$

[6] X = kvadrat broja na kocki

$Y = -1 \ \{1,2\}$ i $1 \ \{3,4,5,6\}$

(a)
$$X \sim \begin{pmatrix} 1 & 4 & 9 & 16 & 25 & 36 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \qquad Y \sim \begin{pmatrix} -1 & 1 \\ \frac{2}{6} & \frac{4}{6} \end{pmatrix}$$

$$E(X) = 1\cdot\frac{1}{6} + 4\cdot\frac{1}{6} + 9\cdot\frac{1}{6} + \dots + 36\cdot\frac{1}{6} = \frac{1}{6}(1+4+9+16+25+36) = \frac{91}{6}$$

$$E(Y) = -1\cdot\frac{2}{6} + 1\cdot\frac{4}{6} = \frac{1}{3}$$

$$D(X) = E(X^2) - E(X)^2 = \frac{1}{6}(1 + 4^2 + 9^2 + \dots + 36^2) - \left(\frac{91}{6}\right)^2 = 149{,}14 > 0$$

$$D(Y) = E(Y^2) - E(Y)^2 = \left((-1)^2\cdot\frac{2}{6} + 1^2\frac{4}{6}\right) - \left(\frac{1}{3}\right)^2 = \frac{8}{9} > 0$$

(b)
$$r(X,Y) = \frac{cov(X,Y)}{\sigma_X \sigma_Y} = \frac{E(XY) - E(X)E(Y)}{\sqrt{D(X)}\sqrt{D(Y)}} =$$

$$\frac{91}{6}\cdot\frac{1}{3}$$

$$= \frac{\frac{91}{18} - }{\sqrt{149{,}14}\cdot\sqrt{8/9}}$$

Ako je $3=0$ koef kol $=0$ nije kol NEZAV.

$$X\cdot Y \sim \begin{pmatrix} -1 & -4 & -9 & -16 & -25 & -36 & 1 & 4 & 9 & 16 & 25 & 36 \\ \frac{2}{6}\cdot\frac{1}{6} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{6}\cdot\frac{4}{6} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{pmatrix}$$

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$$E(XY) = \frac{91}{18}$$