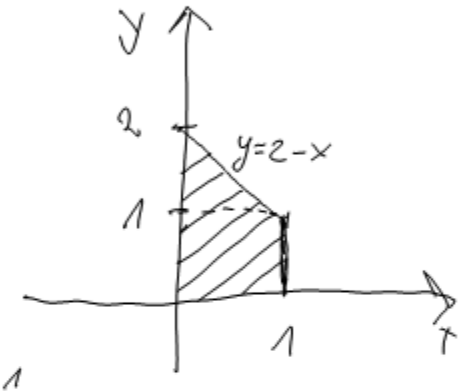
6. svibanj 2009
11:56

Burić / 8. konz

(1.) $f(x,y) = Cxy$

$$D = \{(x,y) : 0 \le x \le 1,\ 0 \le y \le 2,\ x+y \le 2\}$$

a) $C = ?$



$$\iint_D f(x,y)\,dx\,dy = 1$$

$$\int_0^1 dx \int_0^{2-x} Cxy\,dy = \frac{C}{2}\int_0^1 x(2-x)^2\,dx = \frac{1}{24}C = 1$$

$$\Rightarrow C = \frac{24}{11}$$

b) $$f_X(x) = \int_0^{2-x} \frac{24}{11} xy\,dy = \frac{12}{11} x(2-x)^2, \quad x \in (0,1)$$

$$f_Y(y) = \int \frac{24}{11} xy\,dx$$

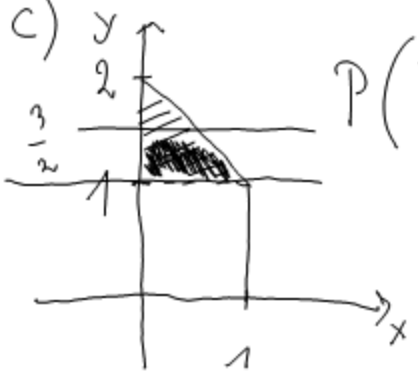
$y \in (0,1)$ $f_Y(y) = \int_0^{2-y} \frac{24}{11} x y \, dx = \frac{12}{11} y$

$y \in (1,2)$ $f_Y(y) = \int_0^{2} \frac{24}{11} x y \, dx = \frac{12}{11} y (2-y)^2$

NEodvisnost

$f(x,y) \neq f_X(x) \cdot f_Y(y)$ – zbog 2 intervala ne možemo množit.

c)



$$P\left(Y < \frac{3}{2} \mid Y > 1\right) = \frac{P\left(Y < \frac{3}{2}, Y > 1\right)}{P(Y > 1)} = \frac{\int_1^{\frac{3}{2}} dy \int_0^{2-y} \frac{24}{11} x y \, dx}{\int_0^1 dx \int_1^{2-x} \frac{24}{11} x y \, dy} =$$

$$= \frac{\frac{67}{176}}{\frac{5}{11}} = \frac{67}{80}$$

$$f_{X|Y=y}(x) = \frac{f(x,y)}{f_Y(y)}$$

2. MI 2007 / 7. vaja

$Y \in (0,2)$ $\quad f_X(x) = ?$

$X \in (y,2)$ $\quad E(X) = ?$

$x > y$

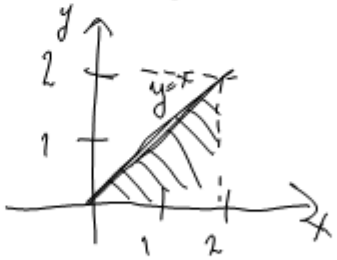
$$f_Y(y) = \frac{1}{2}$$

$$f_{X|Y=y}(x) = \frac{1}{2-y}$$ } enakomerna razdelba

$x \in (0,2)$

$$f(x,y) = f_{X|Y=y}(x) \cdot f_Y(y) = \frac{1}{2(2-y)}$$

$$f_X(x) = \int_0^x \frac{1}{2(2-y)}\,dy =$$



$$= \frac{1}{2}\ln 2 - \frac{1}{2}\ln|2-x|$$

$x \in (0,2)$

$$E(X) = \int x \cdot f(x)\,dx = \int x\left(\frac{1}{2}\ln 2 - \frac{1}{2}\ln|2-x|\right)dx$$

ima : lažji način

$$E(X) = \int_{-\infty}^{\infty} E(X|Y=y) f_Y(y)\,dy$$

$$E(X|Y=y) = \int x \cdot f_{X|Y=y}(x)\,dx = \int_{y}^{2} x \cdot \frac{1}{2-y}\,dx$$

$$= \frac{2+y}{2}$$

$$E(X) = \int_{0}^{2} \frac{1}{2} \cdot (2+y) \cdot \frac{1}{2}\,dy = \frac{3}{2}$$

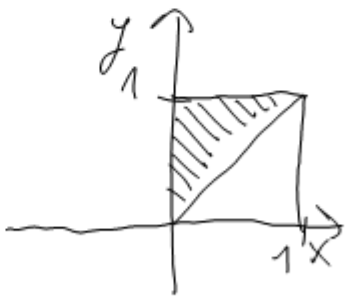
$$Z = f(X, Y)$$

$$g(z) = \int_{-\infty}^{\infty} f(x, y) \cdot \left| \frac{\partial y}{\partial z} \right| dx$$

$$y = \gamma(x, z)$$

5 DZ / 19. $f(x,y) = C(x+y)$

$0 \le x \le y \le 1$

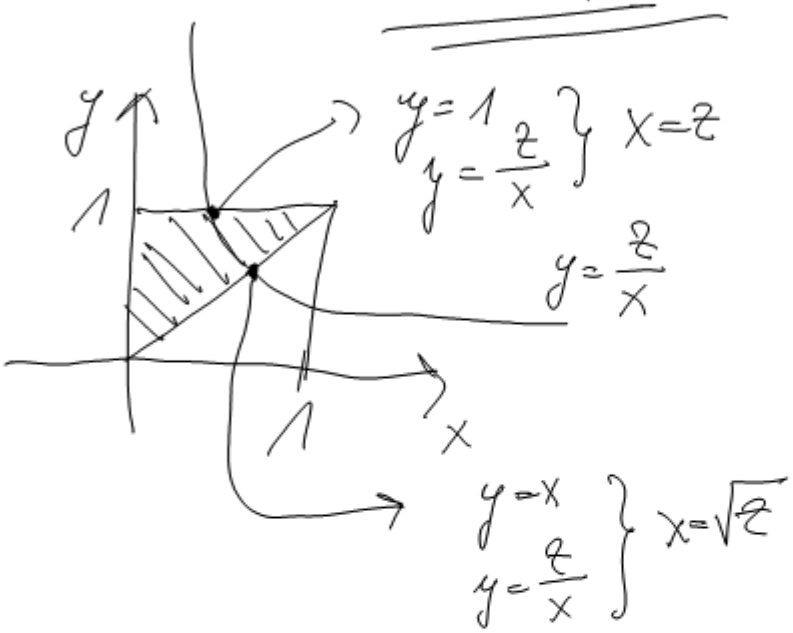


$$\int_0^1 dx \int_x^1 C(x+y)\,dy = 1 \Rightarrow C = 2$$

$$y = \frac{z}{x}, \quad \frac{\partial y}{\partial z} = \frac{1}{x}$$

$$g(z) = \int_{z}^{\sqrt{z}} 2\left(x + \frac{z}{x}\right) \cdot \frac{1}{x}\, dx$$

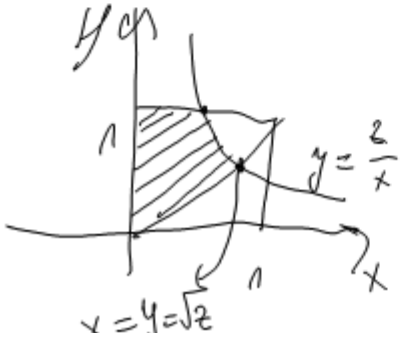
Burićev princip :)



$$g(z) = 2 - 2z$$

$$G(z) = P(Z < z) = P(X \cdot Y < z) =$$

$$y < \frac{z}{x}$$

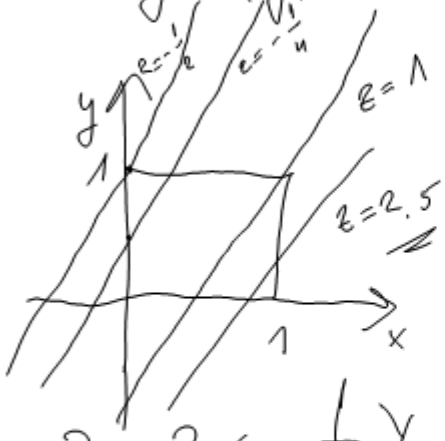


$$= \int_0^{\sqrt{2}} dy \int_0^{y} 2(x+y)\,dx + \int_{\sqrt{2}}^{1} dy \int_0^{\frac{2}{y}} 2(x+y)\,dx =$$

$$= 2z - z^2$$

2. MI 2007.

(7.) $f(x,y) = x^2 + Cy$, $x \in (0,1)$, $y \in (0,1)$



$$\int_0^1 dx \int_0^1 (x^2 + Cy)\,dy = 1$$

$$C = \frac{4}{3}$$

$$z = 3x - \frac{1}{2}y, \quad z \in \left(-\frac{1}{2}, 3\right)$$

$$\Big(y = 6x - 2z$$

$$\frac{\partial y}{\partial z} = |-2| = 2$$

$$g(z) = \int_{-\infty}^{\infty} \left(x^2 + \frac{4}{3}(6x - 2z)\right) \cdot 2 \, dx =$$

= 3 razl. podr. integracije

1° $z \in \left(-\frac{1}{2}, 0\right)$ $\quad \frac{1+2z}{6}$

$$g(z) = 2\int_{0}^{\frac{1+2z}{6}} \left(x^2 + 8x - \frac{8}{3}z\right) dx =$$

$$= \frac{73}{324} + \frac{z}{54} - \frac{23z^2}{27} + \frac{2z^3}{81}$$

2° $z \in \left(0, \frac{5}{2}\right)$ $\quad \frac{1+2z}{6}$

$$g(z) = 2\int_{\frac{1}{3}z}^{\frac{1+2z}{6}} \left(x^2 + 8x - \frac{8}{3}z\right) dx = \ldots$$

3° $z \in \left(\frac{5}{2}, 3\right)$

$$g(z) = 2\int_{\frac{1}{3}z}^{1} \left(x^2 + 8x - \frac{8}{3}z\right) dx = \ldots$$