1. MASS BY BURIĆ {2. CIKLUS
{V I S}

NEPREKINUTE SLUČAJNE VARIJABLE

$F(x)$ – f-ja razdiobe vjerojatnosti

$\boxed{F(x) = P(X < x)}$ – vjerojatnost da sl. var. X poprimi vrijednosti manje od x

– poprima vrijednosti između 0 i 1

– rastuća f-ja

– $\lim\limits_{x \to -\infty} F(x) = 0 \qquad \lim\limits_{x \to \infty} F(x) = 1$

$$P(a < X < b) = F(b) - F(a) = \int_a^b f(x)\,dx$$

$f(x)$ – f-ja gustoće vjerojatnosti

$\boxed{f(x) = F'(x)} \iff F(x) = \int_{-\infty}^{x} f(t)\,dt$

– može biti veća od 1, ali ne smije biti negativna

– površina ispod f-je gustoće u intervalu $\langle -\infty, \infty \rangle$ mora biti 1

1. $f(x) = C - |x-2| \quad , x \in (1,3)$

a) odredi konstantu C

$$f(x) = \begin{cases} C - (-(x-2)) & , x \in (1,2) \\ C - (x-2) & , x \in (2,3) \end{cases} = \begin{cases} C + x - 2 & , x \in (1,2) \\ C - x + 2 & , x \in (2,3) \end{cases}$$

$$\int_1^2 C\,dx + \int_1^2 x\,dx - 2\int_1^2 dx + \int_2^3 C\,dx - \int_2^3 x\,dx + 2\int_2^3 dx = 2C - 1$$

$2C - 1 = 1$

$\boxed{C = 1}$

b) Izračunaj $E(X)$ i $D(X)$

$$\boxed{E(X)=\int_{-\infty}^{\infty} x f(x)\,dx}$$

$$E(X)=\int_1^2 x(x-1)\,dx+\int_2^3 x(3-x)\,dx=\int_1^2 x^2dx-\int_1^2 x\,dx+3\int_2^3 x\,dx-\int_2^3 x^2dx=\boxed{2}$$

$$\boxed{D(X)=\int_{-\infty}^{\infty} x^2 f(x)\,dx-(E(X))^2}$$

$$D(X)=\int_1^2 x^2(x-1)\,dx+\int_2^3 x^2(3-x)\,dx-2^2=\boxed{\frac{1}{6}}$$

c) odredi $F(x)$

$$F(x)=\int_{-\infty}^{x} f(t)\,dt=\int_1^x (t-1)\,dt=\frac{1}{2}t^2-t\Big|_1^x=\boxed{\frac{1}{2}x^2-x+\frac{1}{2}}$$ za $x\in(1,2)$

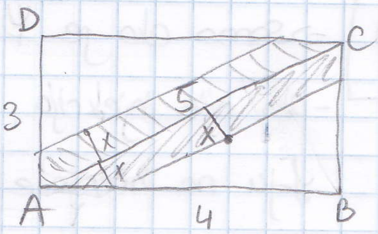
za $x<1 \Rightarrow F(x)=0$, za $x>3 \Rightarrow F(x)=1$

$$F(x)=\int_1^2 (t-1)\,dt+\int_2^x (3-t)\,dt=\boxed{-\frac{1}{2}x+3x-\frac{7}{2}}$$

$$\boxed{F(x)=\begin{cases}0 & ; \quad x<1\\ \frac{1}{2}x^2-x+\frac{1}{2} & ; \quad x\in(1,2)\\ -\frac{1}{2}x^2+3x-\frac{7}{2} & ; \quad x\in(2,3)\\ 1 & ; \quad x>3\end{cases}}$$

d) $P\left(\frac{3}{2}<X<\frac{5}{2}\right)=F\left(\frac{5}{2}\right)-F\left(\frac{3}{2}\right)=\boxed{\frac{3}{4}}$

(2.) U pravokutniku sa stranicama duljina 3 i 4 biramo točku. X je udaljenost do dijagonale AC. Izračunaj očekivanje od X.



$x \in (0, \frac{12}{5})$

$\frac{3 \cdot 4}{2} = \frac{5 \cdot v}{2} \Rightarrow v = \frac{12}{5}$

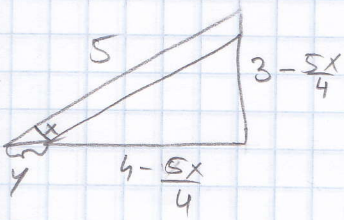
$F(x) = 0 \quad , x < 0$

$F(x) = 1 \quad , x > \frac{12}{5}$

$\frac{x}{y} = \frac{3}{5}$

$y = \frac{5x}{3}$

$F(x) = P(X < x) = \frac{12 - P_\Delta \cdot 2}{12}$



$F(x) = \frac{12 - 2 \cdot \frac{1}{2}(3 - \frac{5x}{4})(4 - \frac{5x}{3})}{12}$

$\boxed{F(x) = \frac{5}{6}x - \frac{25}{144}x^2 \quad , x \in (0, \frac{12}{5})}$

$\boxed{f(x) = F'(x) = \frac{5}{6} - \frac{25}{72}x}$ $\quad x \in (0, \frac{12}{5})$

$\boxed{E(X) = \int_0^{\frac{12}{5}} x f(x) dx = \boxed{\frac{4}{5}}}$

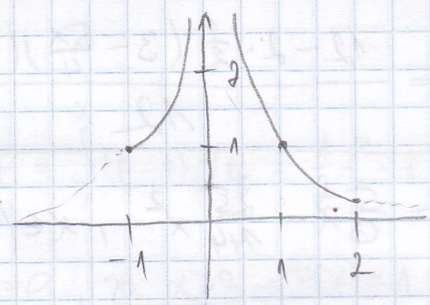
- ako imamo $Y = \Psi(X)$, a znamo $f(x)$ i $F(X)$, kolika je f-ja gustoće $g(y) = ?$

$$\boxed{g(y) = f(x) \cdot \left|\frac{dx}{dy}\right|, \quad x = \Psi^{-1}(y)}$$

-> samo ako je Ψ injekcija!
- ako nije injekcija, rastavimo f-ju na intervale gdje je f-ja injektivna

(3.) Sl. var. X zadana je f-jom gustoće $f(x) = \frac{x}{6} + \frac{1}{4}$, $x \in (-1, 2)$.
Odredi gustoću od sl. var. $Y = \frac{1}{X^2}$.

$g(y) = ?$



$x^2 = \frac{1}{y}$

$x = \pm\sqrt{\frac{1}{y}}$

$x \in (-1, 0)$
$y \in (1, \infty)$

$x = -\frac{1}{\sqrt{y}}$

$\left|\frac{dx}{dy}\right| = \frac{1}{2} y^{-\frac{3}{2}} = \frac{1}{2y\sqrt{y}}$

$g_1(y) = \left(-\frac{1}{6\sqrt{y}} + \frac{1}{4}\right) \cdot \frac{1}{2y\sqrt{y}}$

$$\boxed{g_1(y) = -\frac{1}{12y^2} + \frac{1}{8y\sqrt{y}}}$$

$x \in (0, 2)$
$y \in (\frac{1}{4}, \infty)$

$x = \frac{1}{\sqrt{y}}$

$\left|\frac{dx}{dy}\right| = \frac{1}{2y\sqrt{y}}$

$g_2(y) = \left(\frac{1}{6\sqrt{y}} + \frac{1}{4}\right) \cdot \frac{1}{2\sqrt{y} \cdot y}$

$$\boxed{g_2(y) = \frac{1}{12y^2} + \frac{1}{8y\sqrt{y}}}$$

$$g(y) = \begin{cases} g_2, & y \in (\frac{1}{4}, 1) \\ g_1 + g_2, & y \in (1, \infty) \end{cases}$$

$$\boxed{g(y) = \begin{cases} \frac{1}{12y^2} + \frac{1}{8y\sqrt{y}}, & y \in (\frac{1}{4}, 1) \\ \frac{1}{4y\sqrt{y}}, & y \in (1, \infty) \end{cases}}$$