3.) n kuglica
1 bijela

$$X \sim \begin{pmatrix} 1 & 2 & \dots & k \\ \frac{1}{n} & \frac{n-1}{n}\cdot\frac{1}{n-1} & & \frac{n-k}{n}\cdot\frac{1}{n-k} \end{pmatrix}$$

$$P(X=k) = \frac{1}{n}, \text{ za } k=1,2,3,\dots n$$

$$E(X) = \frac{n+1}{2}$$

4.) prolaznost 40%, očekivanje je 2,5
DOKAŽI!

geom. r. $E(X) = \frac{1}{p} = \frac{1}{0,4} = 2,5$

7.) N kuglica, M bijelih, $E(M), D(M) = ?$

a) s vraćanjem

BINOMNA: $E(X) = n \cdot p = n \cdot \frac{M}{N}$

$$D(X) = npq = n\frac{M}{N}\left(1-\frac{M}{N}\right)$$

b) bez vraćanja $E(X) = np = n \cdot \frac{M}{N}$

8.) 3 KOCKE
U 5 POKUŠAJA, 2 PUTA POJAVE TOČNO 3 JEDINICE

BINOMNA: $P(A) = \binom{5}{2} \cdot \left(\frac{1}{6^3}\right)^2 \cdot \left(1-\frac{1}{6^3}\right)^3 = 0,000211$

$\binom{5}{2}$: 2 puta od 5 pokus.; $\left(\frac{1}{6^3}\right)^2$: točno 3 jedinica

9.) $E(X) = 12$
$D(X) = 4$
$B(n,p) = ?$

$E(X) = np$, $D(X) = npq$

$np = 12$, $np(1-p) = 4$

$n = \frac{12}{p}$ → $\frac{12}{p} \cdot p(1-p) = 4$

$12 - 12p = 4$

$12p = 8 \longrightarrow p = \frac{8}{12} = \frac{2}{3}$

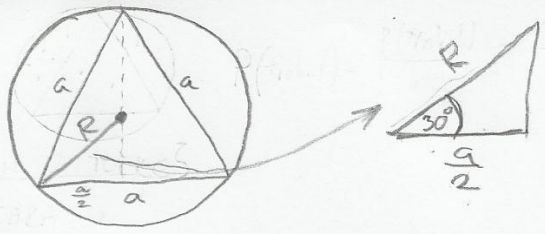
$np = 12$

$n = \frac{12}{\frac{2}{3}}$

$n = \frac{36}{2} = 18$

$B\left(18, \frac{2}{3}\right)$

10.)



$\cos(30°) = \frac{\frac{a}{2}}{R}$

$R = \frac{1}{2}a \cdot \frac{1}{\cos(\frac{\pi}{6})} = \frac{\sqrt{3}}{3}a$

A={barem 2 od 10 točaka u trokutu}

vj. da je točka u trokutu 

$P(\text{točka}) = \frac{P(\Delta)}{P(O)} = \frac{\frac{a^2\sqrt{3}}{4}}{r^2\pi} = \frac{\frac{a^2\sqrt{3}}{4}}{\frac{1}{3}a^2\pi} = \frac{3\sqrt{3}}{4\pi} = 0,4135$

BINOMNA:

$P(A) = 1 - \emptyset \text{ točaka} - 1 \text{ točka} = 1 - \binom{10}{0}(0.4135)^0(1-0,4135)^{10} - \binom{10}{1}(0,4135)^1(1-0,4135)^9$

$P(A) = 0,961$

11.) $X \sim B(n,p) \rightarrow p_k = \binom{n}{k}p^k q^{n-k}$

$Y = e^{2X+1}$

$E(Y), D(Y) = ?$

$\vartheta(t) = \sum p_k \cdot e^{itk} = \sum p_k e^{ite^{2k+1}}$

$E(X) = -i \cdot \vartheta(0)' \rightarrow \vartheta(t)' = \sum p_k \cdot e^{ite^{2k+1}} \cdot i \cdot e^{2k+1}$

$\vartheta(0)' = \sum p_k \cdot i \cdot e^{2k+1}$

$E(Y) = -i \cdot \sum p_k \cdot i \cdot e^{2k+1}$

$E(Y) = \sum p_k e^{2k+1} = \sum \binom{n}{k} p^k \cdot (1-p)^{n-k} e^{2k+1} \rightarrow e \cdot e^{2k} = e \cdot (e^2)^k$

$E(Y) = e \cdot \sum \binom{n}{k} (pe^2)^k \cdot (1-p)^{n-k}$ (1−p = q) → slijedi po binomnom razvoju

$E(Y) = e \cdot (e^2 p + q)^n$

$D(Y) = -\vartheta(0)'' + (\vartheta(0)')^2$

$\vartheta(t)' = \sum p_k \cdot e^{ite^{2k+1}} \cdot i \cdot e^{2k+1}$

$\vartheta(t)'' = \sum p_k e^{ite^{2k+1}} \cdot i \cdot e^{2k+1} \cdot i \cdot e^{2k+1}$

$\vartheta(t)'' = \sum p_k \cdot e^{ite^{2k+1}} \cdot (-1) \cdot (e^4)^k \cdot e^2$

$\vartheta(0)'' = -1 \cdot e^2 \cdot \sum \binom{n}{k} p^k \cdot q^{n-k} \cdot (e^4)^k$

$\vartheta(0)'' = -e^2 \cdot (pe^4 + q)^n$

ZNAMO: $\vartheta(0)' = \sum p_k \cdot i \cdot e^{2k+1} = e \cdot i \cdot (pe^2 + q)^n$

$(\vartheta(0)')^2 = -e^2 (pe^2 + q)^{2n}$

$D(Y) = -\vartheta(0)'' + (\vartheta(0)')^2$

$= e^2 (pe^4 + q)^n - e^2 (pe^2 + q)^{2n}$

12.) POISSON → $P(X=k) = \frac{\lambda^k}{k!} e^{-\lambda}$

$P(X=1) = P(X=2)$

$\frac{\lambda^1}{1!} e^{-\lambda} = \frac{\lambda^2}{2!} e^{-\lambda}$

$\frac{\lambda}{1} = \frac{\lambda^2}{2}$

$2\lambda = \lambda^2$

$\boxed{\lambda = 2}$

$E(X) = \lambda$

$E(X) = 2$

$P(X \geq 4) = ?$

$P(X \geq 4) = 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3)$

$P(X \geq 4) = 1 - \frac{2^0}{0!} e^{-2} - \frac{2^1}{1!} e^{-2} - \frac{2^2}{2!} e^{-2} - \frac{2^3}{3!} e^{-2}$

$P(X \geq 4) = 0{,}1428$

13.) 220 grešaka u 200 stranica

$\lambda = \frac{220}{200} = 1{,}1$

a) niti jedna greška: $P(X=0) = \frac{\lambda^0}{0!} e^{-\lambda} = e^{-1{,}1} = 0{,}3328$

b) jedna greška: $P(X=1) = \frac{\lambda^1}{1!} e^{-\lambda} = 1{,}1 \cdot e^{-1{,}1} = 0{,}366$

c) barem 2 greške: $P(X \geq 2) = 1 - P(X=0) - P(X=1) = 0{,}3012$

14.) 200 LJUDI, 1 % LJEVAČI PROSJEČNO
BAREM 4 LJEVAKA?

$\lambda = 200 \cdot 0{,}01 = 2$

$P(X \geq 4) = 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3)$

$= 1 - \frac{2^0}{0!} e^{-2} - \frac{2^1}{1!} e^{-2} - \frac{2^2}{2!} e^{-2} - \frac{2^3}{3!} e^{-2} = 0{,}1428$

15.) $\lambda = 5000 \cdot 0{,}001 = 5$

$P(X \geq 2) = 1 - P(X=0) - P(X=1) = 1 - \frac{5^0}{0!} e^{-5} - \frac{5^1}{1!} e^{-5} = 0{,}96$

16.) 99,8 % ISPRAVNIH,
0,2 % NEISPRAVNIH, $\lambda = 500 \cdot 0{,}002 = 1$
500 PROIZVODA

$P(X \geq 3) = 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3)$

$P(X \geq 3) = 1 - \frac{1^0}{0!} e^{-1} - \frac{1^1}{1!} e^{-1} - \frac{1^2}{2!} e^{-1} - \frac{1^3}{3!} e^{-1}$

$P(X \geq 3) = 0{,}019$

17.) $\lambda = 10 \cdot 0{,}01 = \underline{0{,}1}$

a) nijedan pogrešan znak: $P(X=0) = \frac{0{,}1^0}{0!} e^{-0{,}1} = \underline{0{,}9048}$

b) barem 2 pogrešna znaka: $P(X \geq 2) = 1 - P(X=0) - P(X=1) = 1 - \frac{0{,}1^0}{0!} e^{-0{,}1} - \frac{0{,}1^1}{1!} e^{-0{,}1}$

$= \underline{0{,}00467}$

18.) POISSON

$E\left(\frac{1}{1+X}\right) = ?$

$\vartheta(t) = \sum_{k=0} p_k \cdot e^{it \cdot \frac{1}{1+k}}$

znamo da je: $p_k = \frac{\lambda^k}{k!} e^{-\lambda}$

$\vartheta(t)' = \sum_{k=0} \frac{\lambda^k}{k!} e^{-\lambda} \cdot e^{it \cdot \frac{1}{1+k}} \cdot \frac{i}{1+k}$

$\vartheta(0)' = e^{-\lambda} \cdot i \cdot \sum_{k=0} \frac{\lambda^k}{(k+1)!} \cdot 1$

$E\left(\frac{1}{1+X}\right) = -i \cdot \vartheta(0)'$, OPREZ: $(-i) \cdot i = 1$

$E\left(\frac{1}{1+X}\right) = e^{-\lambda} \sum_{k=0} \frac{\lambda^k}{(k+1)!} \quad / \cdot \frac{\lambda}{\lambda}$

$E\left(\frac{1}{1+X}\right) = e^{-\lambda} \cdot \frac{1}{\lambda} \cdot \sum_{k=0} \frac{\lambda^{k+1}}{(k+1)!}$

$\sum_{k=0} \frac{\lambda^{k+1}}{(k+1)!} = e^{\lambda} - 1$

$E\left(\frac{1}{1+X}\right) = e^{-\lambda} \cdot \frac{1}{\lambda} \cdot (e^{\lambda} - 1) = \underline{\frac{1 - e^{\lambda}}{\lambda}}$

19.) 2000 DIJELOVA, 0.0005

$P(X>3) = ?$

$\lambda = 2000 \cdot 0{,}0005 = \underline{1}$

$P(X>3) = 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3)$

$P(X>3) = \underline{0{,}019}$

20.) $\lambda = 3000 \cdot 0{,}001 = \underline{3}$

$P(X \geq 5) = 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3) - P(X=4)$

$P(X \geq 5) = 1 - \frac{3^0}{0!} e^{-3} - \frac{3^1}{1!} e^{-3} - \frac{3^2}{2!} e^{-3} - \frac{3^3}{3!} e^{-3} - \frac{3^4}{4!} e^{-4}$

$P(X \geq 5) = \underline{0{,}185}$

21.) $\lambda = 4000 \cdot 0{,}005 = \underline{20}$

$P(10 \leq X \leq 40) = ?$

$$P(X) = \sum_{k=10}^{40} \frac{20^k}{k!} e^{-20} = \underline{0{,}99498}$$

22.) 90 poziva/h

$$\lambda = \frac{90}{60} \cdot 2 = 1{,}5 \cdot 2 = \underline{3}$$

$$P(X \geq 5) = 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3) - P(X=4)$$

$$P(X \geq 5) = 1 - \frac{3^0}{0!}e^{-3} - \frac{3^1}{1!}e^{-3} - \frac{3^2}{2!}e^{-3} - \frac{3^3}{3!}e^{-3} - \frac{3^4}{4!}e^{-3}$$

$$P(X \geq 5) = \underline{0{,}1847}$$

23.) 300 STR., 1100 GREŠAKA

$$\lambda = \frac{1100}{300} = \underline{3{,}\dot{6}}$$

$$P(X > 3) = 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3)$$

$$= 1 - \frac{3{,}\dot{6}^0}{0!}e^{-3{,}\dot{6}} - \frac{3{,}\dot{6}^1}{1!}e^{-3{,}\dot{6}} - \frac{3{,}\dot{6}^2}{2!}e^{-3{,}\dot{6}} - \frac{3{,}\dot{6}^3}{3!}e^{-3{,}\dot{6}} =$$

$$= \underline{0{,}81}$$

$$E(X) = \lambda = 3{,}6 = \underline{3 \text{ GREŠKE}}$$

24.) $\lambda = \frac{3{,}87}{7{,}5} = 0{,}516$

$$P(X \geq 1) = 1 - P(X=0) = 1 - e^{-0{,}516} = \underline{0{,}403}$$

$$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) = \underline{0{,}9843}$$

25.) STABILNOST POISSON:

DJEČACI λ_1, DJEVOJČICE λ_2

$$P(X_1 = k \mid X_1 + X_2 = n) = \frac{P(X_1 = k, X_2 = n-k)}{P(X_1 + X_2 = n)} = \frac{\frac{\lambda_1^k}{k!}e^{-\lambda_1} \cdot \frac{\lambda_2^{n-k}}{(n-k)!}e^{-\lambda_2}}{\frac{(\lambda_1+\lambda_2)^n}{n!}e^{-\lambda_1-\lambda_2}} =$$

$$= \underline{\binom{n}{k}\left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^k\left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^{n-k}}$$

SLUŽBENA RJEŠENJA:

§ 4. Primjeri dskretnih razdioba

2. $E(X) = r\frac{q}{p}$, $D(X) = r\frac{q}{p^2}$.

3. $P(X = k) = \frac{1}{n}$, $k = 1, \ldots, n$;
$E(X) = \frac{n+1}{2}$.

4. $E(X) = \frac{1}{p} = \frac{1}{0.4} = 2.5$

5. $E(X) = a$; $D(X) = a(a+1)$.

6. $p_k = P(X = k) = \frac{\binom{M}{k}\binom{N-M}{n-k}}{\binom{N}{n}}$.

7. a) $E(X) = \frac{nM}{N}$, $D(X) = \frac{nM}{N} \cdot \frac{N-M}{N}$.
b) $E(X) = \frac{nM}{N}$, $D(X) = \frac{nM}{N} \cdot \frac{N-M}{N} \cdot \frac{N-M}{N-1}$.

8. $2.11 \cdot 10^{-4}$.

9. 18, $\frac{2}{3}$.

10. 0.961.

11. $E(Y) = e(e^2p + q)^n$,
$D(Y) = e^2(e^4p + q)^n - e^2(e^2p + q)^{2n}$.

12. 2; 0.143

13. 0.333; 0.366; 0.301

14. 0.143

15. 0.960

16. 0.019

17. 0.9044; 0.0043

18. $\frac{1 - e^{-\lambda}}{\lambda}$

19. 0.019.

20. 0.185.

21. 0.55

22. 0.185.

23. 0.4989, 3 greške.

24. 0.4031, 0.9844.

25. $\binom{n}{k}\left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^k\left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^{n-k}$

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