Ponovljeni 2. MI – 2007

1.

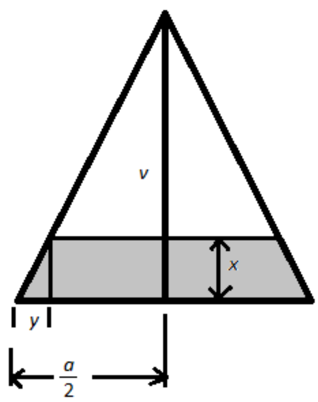
$$F(x) = \int f(x)dx = \int \frac{2}{\pi}\cos^2 x\, dx = \cdots = \frac{x}{\pi} + \frac{1}{2\pi}\sin(2x)$$

$$p\left(0 < X < \frac{\pi}{4}\right) = F\left(\frac{\pi}{4}\right) - F(0) = \frac{\frac{\pi}{4}}{\pi} + \frac{1}{2\pi}\sin(2\frac{\pi}{4}) - 0 = \frac{1}{4} + \frac{1}{2\pi}$$

p – vjerojatnost da padne u taj interval, (1 – *p*) – vjerojatnost da ne padne. Obzirom da terbamo dva od tri puta da padne, može pasti unutar intervala 1. i 2. put i 3. ne pasti, ili 1. i 3. pasti a 2. ne pasti, ili 1. ne pasti, pa pasti 2. i 3. put – dakle 3 načina.

$$P = 3\cdot p \cdot p \cdot (1-p) = 3p^2(1-p) = 3\left(\frac{1}{4} + \frac{1}{2\pi}\right)^2\left(1 - \frac{1}{4} + \frac{1}{2\pi}\right) = 3\left(\frac{\pi+2}{4\pi}\right)\left(\frac{3\pi-2}{4\pi}\right)$$

2.



$$\frac{a}{2} : v = y : x \rightarrow y = \frac{ax}{2v}$$

$$F(x) = \frac{m(A)}{m(B)} = \frac{2\left(\frac{a}{2} - y\right)x + \frac{2xy}{2}}{\frac{av}{2}} = \frac{2\left(\frac{a}{2} - \frac{ax}{2v}\right)x + x\frac{ax}{2v}}{\frac{av}{2}} = \frac{ax - \frac{ax^2}{v} + \frac{ax^2}{2v}}{\frac{av}{2}}$$
$$= \frac{\frac{2avx - 2ax^2 + ax^2}{2v}}{\frac{av}{2}} = \frac{2avx - ax^2}{av^2} = \frac{2vx - x^2}{v^2}$$

$$f(x) = \frac{dF(x)}{dx} = \frac{2}{v} - \frac{2x}{v^2}, \qquad x \in [0, v]$$

$$E(x) = \int_0^v xf(x)dx = \int_0^v x\left(\frac{2}{v} - \frac{2x}{v^2}\right)dx = \cdots = \frac{v}{3}$$

3.

$$X \sim \mathcal{E}(\lambda)$$

$$P(X < x + t | X > t) = \frac{P(t < X < x + t)}{P(X > t)} = \frac{F(x + t) - F(t)}{1 - P(X < t)} = \frac{1 - e^{-\lambda(x+t)} - \left(1 - e^{-\lambda t}\right)}{1 - (1 - e^{-\lambda t})} = \cdots =$$

$$= 1 - e^{-\lambda x} = F(x) = P(X < x)$$

4. knjiga, teorija

5.

a)

$$F(x, y) = \int_{-\infty}^{x} dx \int_{-\infty}^{y} \frac{dy}{\pi^2(x^2 + y^2 + x^2y^2 + 1)} = \int_{-\infty}^{x} dx \int_{-\infty}^{y} \frac{dy}{\pi^2(x^2 + 1)(y^2 + 1)} =$$

$$= \frac{1}{\pi^2} \int_{-\infty}^{x} \frac{dx}{x^2 + 1} \int_{-\infty}^{y} \frac{dy}{y^2 + 1} = \cdots = \left(\frac{1}{\pi} arctgx + \frac{1}{2}\right)\left(\frac{1}{\pi} arctgy + \frac{1}{2}\right)$$

b) kriterij nezavisnoti:

$$f_X(x) f_Y(y) = f(x, y)$$

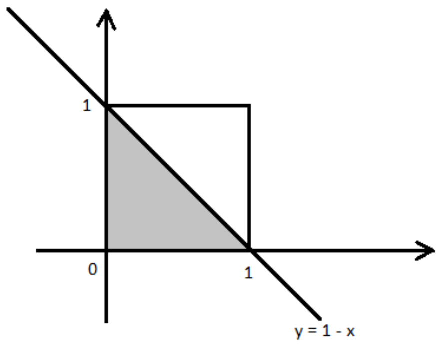
$$f_X(x) = \int_{-\infty}^{\infty} \frac{dy}{\pi^2(x^2 + y^2 + x^2y^2 + 1)} = \cdots = \frac{1}{\pi(x^2 + 1)}$$

$$f_Y(y) = \int_{-\infty}^{\infty} \frac{dx}{\pi^2(x^2 + y^2 + x^2y^2 + 1)} = \cdots = \frac{1}{\pi(y^2 + 1)}$$

Uvrsti se u $f_X(x) f_Y(y) = f(x, y)$ i vidi se da vrijedi → nezavisne su.

6.

a)



$$\int_0^1 dx \int_0^{1-x} Cxydy = 1$$

$$C\int_0^1 \left(\frac{xy^2}{2}\Big|_{y=0}^{y=1-x}\right)dx = 1$$

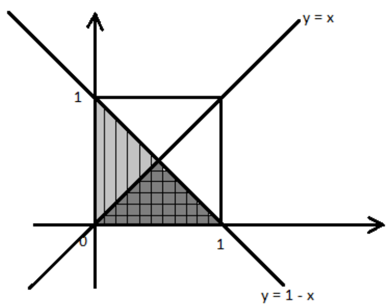
$$C\int_0^1 \frac{x(1-x)^2}{2}dx = 1$$

$$C = nakon\ računanja = 24$$

b)

$$f_X(x) = \int_0^{1-x} 24xydy = \cdots = 12x(1-x)^2, \qquad x \in [0,1]$$

c) P(X > Y) = ?



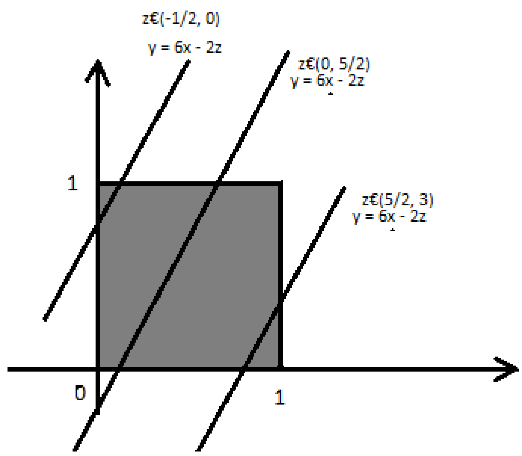
$$P(X > Y) = \int_0^{\frac{1}{2}} dy \int_y^{1-y} 24xydx = \cdots = \frac{1}{2}$$

7.

a) Z = 3X – Y/2 → y = 6x – 2z

$$g_Z(z) = \int_{-\infty}^{\infty} f(x,y)\left|\frac{dy}{dz}\right| dx = \int_{-\infty}^{\infty} 2f(x, 6x - 2z)dx$$

b)



$$\int_0^1 dx \int_0^1 (x^2 + Cy)dy = 1$$

Nakon računanja dobije se *C* = 4/3.

c) Kao što rješenje kaže, integrirati izraz pod a) za ove različite *z*-ove.