

⑤

Godišnje godine od 540 studenata koji su slušali SIS i VIS, 480 je pohađalo VIS, a 360 SIS, a 330 oboje. Neka je slučajna X je 1 ako je pohađao SIS, a 0 ako nije, a Y je VIS. Odredi koef. korelacije,

$X \backslash Y$	0	1	
0	$\frac{60}{540}$	$\frac{30}{540}$	$\frac{90}{540}$
1	$\frac{120}{540}$	$\frac{330}{540}$	$\frac{450}{540}$
	$\frac{180}{540}$	$\frac{360}{540}$	1

$$E(X) = 0,83 \quad E(Y) = 0,67$$

$$D(X) = 0,141 \quad D(Y) = 0,221$$

$$\text{cov}(X, Y) = E(XY) - E(X)E(Y)$$

$$= 0,0559 \Rightarrow X \text{ i } Y \text{ su korelirani}$$

$$r(X, Y) = \frac{\text{cov}(X, Y)}{\sqrt{D(X)D(Y)}} = 0,305$$

$$E(XY) = \sum \sum x_i y_j p_{ij} = 0,61$$

21-08. 1a

bacanje kocke $\Rightarrow X = \{2 \cdot \text{broj na kocki}\}$ $Y = \begin{cases} 1 & \text{paran} \\ -1 & \text{neparan} \end{cases}$

a) i b)

$X \backslash Y$	-1	1	
2	$\frac{1}{6}$	0	$\frac{1}{6}$
4	0	$\frac{1}{6}$	$\frac{1}{6}$
6	$\frac{1}{6}$	0	$\frac{1}{6}$
8	0	$\frac{1}{6}$	$\frac{1}{6}$
10	$\frac{1}{6}$	0	$\frac{1}{6}$
12	0	$\frac{1}{6}$	$\frac{1}{6}$
	$\frac{2}{6}$	$\frac{2}{6}$	1

c) ispitaj nezavisnost

Nisu nezavisni jer $\frac{1}{6} \neq \frac{1}{6} \cdot \frac{1}{2}$

d) Odredi disperziju od $Z = X + Y$

$$Z \sim \begin{pmatrix} 1 & 3 & 5 & 7 & 9 & 11 & 13 \\ \frac{1}{6} & 0 & \frac{1}{6} & 0 & \frac{1}{6} & 0 & \frac{1}{6} \end{pmatrix}$$

\rightarrow to napisati bez sumnje

$$D(Z) = E(Z^2) - E(Z)^2 = \frac{44}{3}$$

$$E(Z) = 7$$

$X \backslash Y$	-1	1	
-1	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{10}{24}$
0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{7}{24}$
1	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{7}{24}$
	$\frac{13}{24}$	$\frac{11}{24}$	1

b) ispitaj nezavisnost

Zavisni jer $\frac{1}{4} \neq \frac{13}{24} \cdot \frac{10}{24}$

$$a) P(Y=1 | X \geq 0) = \frac{P(Y=1, X \geq 0)}{P(X \geq 0)} = \frac{\frac{1}{8} + \frac{1}{6}}{\frac{1}{6} + \frac{1}{3} + \frac{1}{6} + \frac{1}{3}} = \frac{1}{2}$$

b) $Z = |X - Y|$

$W = \frac{X}{Y}$

odredi razdiobu $(Z, W) = ?$

$Z \backslash W$	-1	0	1	
0	0	0	$\frac{10}{24}$	$\frac{10}{24}$
1	0	$\frac{7}{24}$	0	$\frac{7}{24}$
2	$\frac{7}{24}$	0	0	$\frac{7}{24}$
	$\frac{7}{24}$	$\frac{7}{24}$	$\frac{10}{24}$	1

$Z=0 : X=-1 Y=-1 \rightarrow W=1$
 $X=1 Y=1 \rightarrow W=1$

$Z=1 : X=0 Y=1 \rightarrow W=0$
 $X=0 Y=-1 \rightarrow W=0$

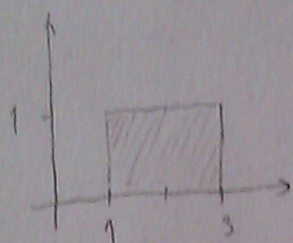
$Z=2 : X=-1 Y=1 \rightarrow W=-1$
 $X=1 Y=-1 \rightarrow W=-1$

Z, W zavisne jer $\frac{10}{24} \cdot \frac{10}{24} \neq \frac{10}{24}$

Dvostruki integral

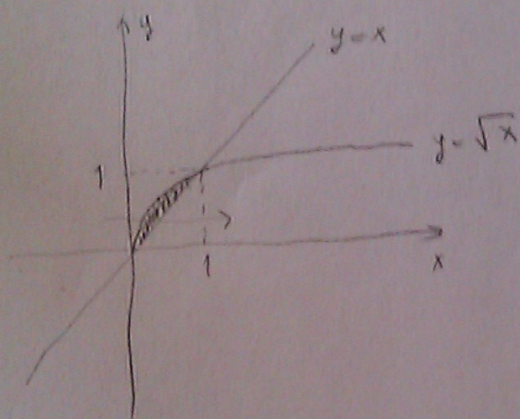
$$\iint_{\Omega} f(x,y) dx dy \quad \rightarrow \text{ dvostruki integral funkcije } f(x,y)$$

Pr.) $\Omega = [1, 3] \times [0, 1]$



$$\int_1^3 dx \int_0^1 f(x,y) dy = \int_0^1 dy \int_1^3 f(x,y) dx$$

Pr.) $\Omega = \{ (x,y) : x \leq y \leq \sqrt{x} \}$



ne najprijetliji granice su uvijek konstante
ne najprijetliji uvijek fije

$$\int_0^1 dx \int_x^{\sqrt{x}} f(x,y) dy$$

koja kruzja medu y=x i y=sqrt(x)

$$= \int_0^1 dy \int_{y^2}^y f(x,y) dx$$

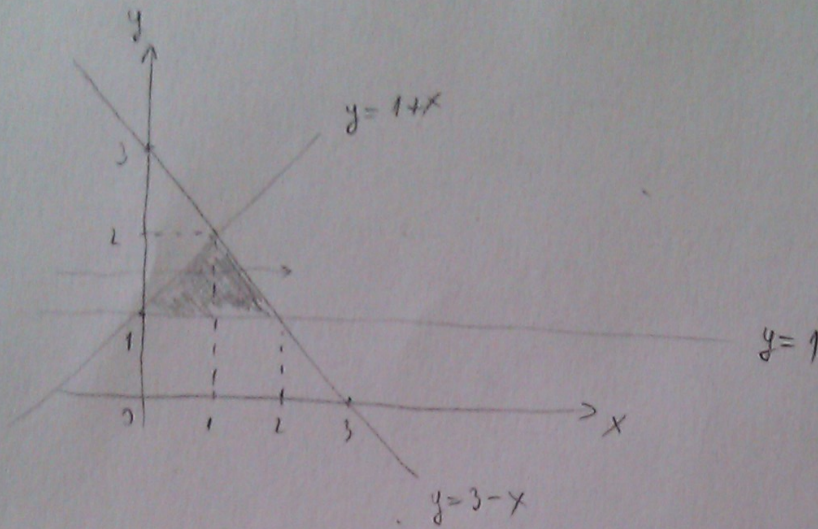
Pr.)

 Ω omeđeno pravougaono

$$y = 1 + x$$

$$y = 3 - x$$

$$y = 1$$



$$\int_0^1 dx \int_1^{1+x} f(x, y) dy + \int_1^2 dx \int_1^{3-x} f(x, y) dy = \int_1^2 dy \int_{y-1}^{3-y} f(x, y) dx$$

Računanje

$$\begin{aligned} \int_0^1 dx \int_x^2 (2x + x^2 y) dy &= \int_0^1 dx \left[2xy \Big|_x^2 + x^2 \frac{y^2}{2} \Big|_x^2 \right] = \int_0^1 \left[4x - 2x^2 + 2x^2 - \frac{1}{2} x^4 \right] dx \\ &= 4 \frac{x^2}{2} \Big|_0^1 - 2 \frac{x^3}{3} \Big|_0^1 + 2 \frac{x^3}{3} \Big|_0^1 - \frac{1}{2} \cdot \frac{x^5}{5} \Big|_0^1 = \\ &= 2 - \frac{2}{3} + \frac{2}{3} - \frac{1}{10} = \frac{19}{10} \end{aligned}$$