2.MI 2013.

1. $P\left(\frac{1}{2} < X < \frac{3}{2}\right)$, $D(X) = ?$

$f(x) = 2 - cx$, $x \in [0,1]$

$\int_0^1 (2-cx)\,dx = 2 - c \cdot \frac{1}{2} \Rightarrow \boxed{c = 2}$

$F(x) = \int_0^x 2 - 2t\,dt = 2x - x^2$, $x \in (0,1)$

$F(x) = 0$, $x < 0$

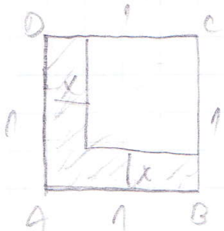
$F(x) = 1$, $x > 1$

$P\left(\frac{1}{2} < X < \frac{3}{2}\right) = F(1) - F\left(\frac{1}{2}\right) = 2 - 1 - 1 + \frac{1}{4} = \boxed{\frac{1}{4}}$

$E(X) = \int_0^1 2(x - x^2)\,dx = 2 \cdot \left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{3}$

$D(X) = \int_0^1 2(x^2 - x^3)\,dx - \left(\frac{1}{3}\right)^2 = 2\left(\frac{1}{3} - \frac{1}{4}\right) - \frac{1}{9} = \frac{1}{6} - \frac{1}{9} = \boxed{\frac{1}{18}}$

2.



$x \in (0,1)$ $X = \min(x,y)$

$F(x) = P(X < x) = \frac{1 - (1-x)^2}{1} = \frac{\cancel{1} - \cancel{1} + 2x - x^2}{1} = \boxed{2x - x^2}$ $x \in (0,1)$

$f(x) = F'(x) = 2 - 2x$ $x \in (0,1)$

$E(X) = \int_0^1 x(2-2x)\,dx = \boxed{\frac{1}{3}}$

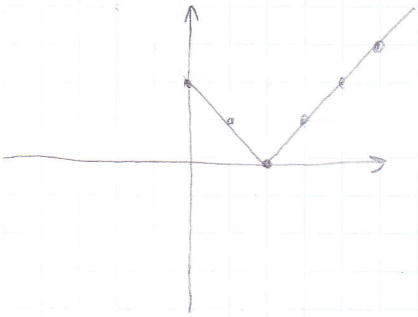
3. a) $Y = f(X)$

$G(y) = F(x)$

$$g(y) = \frac{d}{dy} G(y) = \frac{d}{dx} F(x) \cdot \frac{dx}{dy} = f(x) \frac{dx}{dy}$$

b) $Y = |X - 2|$

$f(x) = e^{-x}, \ x > 0$



$x \in (0,2)$ $x \in (2,\infty)$

$y \in (0,2)$ $y \in (0,\infty)$

$y = 2 - x \Rightarrow x = 2 - y$ $y = x - 2 \Rightarrow x = y + 2$

$\left|\frac{\partial y}{\partial x}\right| = 1$ $\left|\frac{\partial y}{\partial x}\right| = 1$

$g_1(y) = e^{y-2}$ $g_2(y) = e^{-y-2}$

$$g(y) = \begin{cases} g_1(y) + g_2(y), & y \in (0,2) \\ g_2(y), & y \in (2,\infty) \end{cases}$$

4. a) ako su X_1 i X_2 nezavisne onda vrijedi:

$$X_1 \sim N(a_1, \sigma_1^2), \quad X_2 \sim N(a_2, \sigma_2^2)$$

$$s_1 X_1 + s_2 X_2 \sim N(s_1 a_1 + s_2 a_2, s_1^2 \sigma_1^2 + s_2^2 \sigma_2^2) \qquad s_1, s_2 \in \mathbb{R}$$

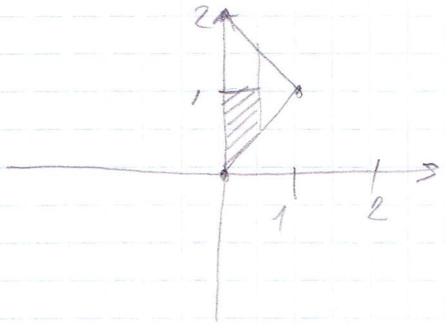
b) $X_0 \sim N(180, 20^2)$, $X_i \sim N(220, 5^2)$, $Y \sim N(800, 850)$

$$P(820 < Y < 1000) = \left(\frac{820-800}{\sqrt{850}} < Y < \frac{1000-800}{\sqrt{850}}\right) = \frac{1}{2}\left(\phi^*(6{,}86) - \phi^*(0{,}686)\right)$$

$$= \boxed{0{,}24635}$$

5. $(X,Y) \sim U$

$O(0,0)$, $A(1,1)$, $B(0,2)$



$f(x,y) = \frac{1}{P} = \frac{1}{\frac{1}{2}+\frac{1}{2}} = 1$

$P(X<\frac{1}{2} \mid Y<1) = \frac{P(X<\frac{1}{2}, Y<1)}{P(Y<1)}$

$= \frac{\int_0^{\frac{1}{2}} dx \int_x^1 dy}{\int_0^1 y\,dy} = \frac{\int_0^{\frac{1}{2}} (1-x)\,dx}{\frac{1}{2}} = \frac{\frac{1}{2} - \frac{\frac{1}{4}}{2}}{\frac{1}{2}} = \frac{\frac{3}{8}}{\frac{1}{2}} = \frac{6}{8} = \boxed{\frac{3}{4}}$

$f_X(x) = \int_x^{2-x} dy = 2-x-x = \boxed{2-2x}$, $x \in (0,1)$

$f_Y(y) = \int_0^y dx = \boxed{y}$, $y\,(0,1)$

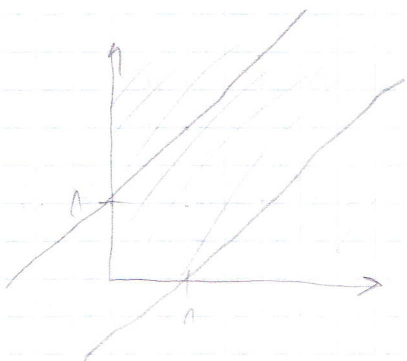
nisu nezavisne

$f_Y(y) = \int_0^{2-y} dy = \boxed{2-y}$, $y\,(1,2)$

6. $X,Y \sim \mathcal{E}(\frac{1}{2})$, $Z = Y - X \to Y = Z + X$

$f(x) = \frac{1}{2} e^{-\frac{1}{2}x}$ $\quad f(y) = \frac{1}{2} e^{-\frac{1}{2}y}$

$f(x,y) = f(x) \cdot f(y) = \frac{1}{4} e^{-\frac{1}{2}(x+y)}$, $x>0, y>0$



$F_Z(z) = P(Z<z) = P(Y-X<z) = P(Y<Z+X)$

$F_Z(z) = \int_0^\infty dx \int_0^{x+z} \frac{1}{4} e^{-\frac{1}{2}(x+y)} dy = \int_0^\infty \frac{1}{2} e^{-\frac{1}{2}x} \cdot (e^{-\frac{1}{2}(x+z)} - 1)\,dx$

$= \int_0^\infty \frac{1}{2} (e^{-x-\frac{1}{2}z} - e^{-\frac{1}{2}x})\,dx$

$= \frac{1}{2}(e^{-\frac{1}{2}z} \cdot (-1) + 2(-1)) = -\frac{1}{2} e^{-\frac{1}{2}z} - 1$, $z>0$

$F_Z(z) = \int_{-z}^\infty dx \int_0^{x+z} \frac{1}{4} e^{-\frac{1}{2}(x+y)} dy = \frac{1}{2}(e^{-\frac{1}{2}z} \cdot (-e^{z}) + 2 \cdot (-e^{-\frac{1}{2}z})) = \frac{1}{2} e^{\frac{z}{2}}$, $z<0$