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3. $n_1 = 5 \quad a = 3$

$n_2 = 4 \quad a = 2$

$n_3 = 3 \quad a = 2$

$f(p, x) = p^x (1-p)^{1-x}$

$f(p, x_1, x_2, x_3) = p^3 (1-p)^2 p^2 (1-p)^2 p^2 (1-p) = p^7 (1-p)^5$

$\ln f = 7 \ln p + 5 \ln (1-p) =$

$\frac{df}{dp} = \frac{7}{p} + \frac{5}{1-p} = 0 \quad / p(1-p)$

$7(1-p) - 5p = 0$

$7 - 7p - 5p = 0$

$-12p = -7 \quad \Rightarrow p = \frac{7}{12}$

4. $n = 400 \quad m = 300 \quad p = 0.9 \quad \alpha = 0.1$

$\hat{p} = \frac{m}{n} = 0.75 \quad u_{1-\frac{\alpha}{2}} = u_{1-0.05} = u_{0,95} = 1.64485$

$p_1 = 0.75 - 1.64485 \cdot \sqrt{\frac{0.75 \cdot 0.25}{400}} = 0.75 - 1.64485 \cdot 0.02165 = 0.75 - 0.0356 = 0.7144$

$p_2 = 0.75 + 0.0356 = 0.7856$

$P(0.7144 < p < 0.7856) = 0.9$

5. $n=100 \quad X \sim N(\mu, 25) \quad \bar{x}=27 \quad \alpha=0.05$

$H_0 \dots \mu = 26$

$H_1 \dots \mu \neq 26$

$u_{1-\frac{\alpha}{2}} = u_{0.975} = 1,96$

$U = \frac{27-26}{5} \cdot 10 = \frac{10}{5} = 2$

$|U| > |u_{1-\frac{\alpha}{2}}|$ Hipoteza H_0 se odbacuje

6. $\alpha = 0.02$ Poisson

j	n_j	p_j	$n_j - np_j$	$\frac{(n_j - np_j)^2}{np_j}$
0	95	0.25554	1,7279	0.032
1	140	0.34865	12,74275	1.27517
2	73	0,23785	−13.81525	2,19847
3	31	0,10817	−8,48205	1.82222
4	16	0.036896	2,53296	0.47641
5	7	0.012893	5.29406	5.95566
6	2			
7	0			
8	1			
	365			$\chi_1^2 = 11.76$

$p_j = \frac{\lambda^x}{x!} e^{-\lambda} \qquad \lambda = \bar{x} = \frac{140+146+93+64+35+12+8}{365} = 1.36438$

$p_j = \frac{1,36438^j \cdot 0,25554}{j!}$

$p_0 = 0.25554$

$p_1 = 1.36438 \cdot 0.25554 = 0.34865$

$p_2 = \frac{1,36438^2 \cdot 0.25554}{2} = 0.23785$

$p_3 = \frac{1.36438^3 \cdot 0,25554}{6} = 0.10817$

$p_4 = \frac{1,36438^3 \cdot 0,25554}{24} = 0.036896$

$p' = p_5 + p_6 + p_7 + p_8 = 0.012893$

$\alpha = 0.02 \qquad k = 6-1-1 = 4$

$\chi^2_{4,1-0.02} = \chi^2_{4,0.98} = 11.668$

$\chi_1^2 > \chi^2_{0.02}$ Hipoteza da je varijabla Poissonova se odbacuje

10.

x_j	115	120	125	130	135	140
n_j	3	4	7	6	3	2

$n=25$

a) $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i = 126.6$

$\hat{s}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X}) = 51.5$

b) $p=0.9$ $\alpha=0.1$ $n-1=24$

$t_{1-\frac{\alpha}{2}} = 1,711$

$t_{1-\frac{\alpha}{2}}\frac{\hat{s}}{\sqrt{n}} = 1,711 \cdot \frac{7.176}{\sqrt{25}} = 2,4557$

$P(126.6-2.4557 \le a \le 126.6+2,4557) = 0.9$

c) $p=0.9$ $\alpha=0.1$ $n-1=24$

$c_1 = \chi^2_{24,\,0.05} = 13.848$ $\quad c_2 = \chi^2_{24,\,0.95} = 36.415$

$s_1 = \frac{(n-1)\cdot\hat{s}^2}{c_2} = \frac{24\cdot 51.5}{36.415} = 33.94$

$s_2 = \frac{24\cdot 51.5}{13.848} = 89.25$

$P(33.94 \le \sigma^2 \le 89.25) = 0.9$

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3. $f(x) = \begin{cases} \frac{x}{a^2} e^{-\frac{x}{a}} & , x>0 \\ 0 & , x\le 0 \end{cases}$ $\quad a>0$

a) $f(a, x_1, x_2, \ldots, x_n) = \frac{x_1}{a^2} e^{-\frac{x_1}{a}} \cdot \frac{x_2}{a^2} e^{-\frac{x_2}{a}} \cdot \ldots \cdot \frac{x_n}{a^2} e^{-\frac{x_n}{a}} =$

$= \frac{(x_1 \cdot x_2 \cdot \ldots \cdot x_n)}{a^{2n}} e^{-\frac{\sum x_i}{a}} = (x_1 \cdot \ldots \cdot x_n)\cdot a^{-2n} \cdot e^{-\frac{\sum x_i}{a}}$ $\quad x>0$

$\ln f = \ln(x_1 \cdot \ldots \cdot x_n) - 2n \ln a - \frac{\sum x_i}{a}$

$$\frac{\partial \ln f}{\partial a} = -\frac{2n}{a} - \frac{(\sum x_i)' \cdot a - \sum x_i \cdot a'}{a^2} = -\frac{2n}{a} - \frac{0 - \sum x_i}{a^2} = -\frac{2n}{a} + \frac{\sum x_i}{a^2} = 0 \quad / \cdot a^2$$

$$-2na + \sum x_i = 0$$

$$2na = \sum x_i$$

$$a = \frac{\sum x_i}{2n} = \frac{\bar{x}}{2}$$

b)

4. $\hat{D}'^2 = \hat{\sigma}^2 + (\bar{x} - a)^2 \qquad \hat{D}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - a)^2 \qquad \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

$$\hat{D}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - a)^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i^2 - 2x_i a + a^2) = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\sum_{i=1}^{n} 2x_i a + \frac{1}{n}\sum_{i=1}^{n} a^2 =$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - 2a\bar{x} + a^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \bar{x}^2 + \bar{x}^2 - 2a\bar{x} + a^2 =$$

$$= \frac{1}{n}\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right) + (\bar{x} - a)^2 = \frac{1}{n}\left(\sum_{i=1}^{n} x_i^2 - 2n\bar{x}^2 + n\bar{x}^2\right) + (\bar{x} - a)^2 =$$

$$= \frac{1}{n}\left(\sum_{i=1}^{n} x_i^2 - 2\frac{1}{n}\sum_{i=1}^{n} x_i\bar{x} + \sum_{i=1}^{n}\bar{x}^2\right) + (\bar{x} - a)^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 + (\bar{x} - a)^2$$

5. $n = 17 \quad X \sim N(a, \sigma^2) \quad \sum_{i=1}^{17} x_i = 680 \quad \sum_{i=1}^{17} x_i^2 = 34000 \quad p = 0.9 \quad \alpha = 0.1$

Odredi dvostrani interval za σ^2

$$\bar{x} = \frac{680}{17} = 40$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i^2 - 2\bar{x}x_i + \bar{x}^2) = \frac{1}{16}\left(34000 - 2\sum_{i=1}^{17}\bar{x}x_i + 17 \cdot 1600\right) =$$

$$= \frac{1}{16}(34000 + 27200 - 2 \cdot 40 \cdot 680) = \frac{1}{16} \cdot 6800 = 425$$

$n-1=16$

$c_1 = \chi_{16, 0.05} = 7.962 \qquad c_2 = \chi_{16, 0.95} = 26.296$

$\beta_1 = \frac{16 \cdot 425}{26.296} = 258.59 \qquad \beta_2 = \frac{16 \cdot 425}{7.962} = 854.06$

$P(258.59 \leq \sigma^2 \leq 854.06) = 0.9$

6. $n = 288$ Poisson $\alpha = 0.05$

j	n_j	p_j	$n_j - np_j$	$(n_j-np_j)^2/np_j$
0	154	0.520597	4.06806	0.11038
1	93	0.33983	-4.87104	0.24243
2	31	0.110917	-0.9441	0.0279
3	7	0.028655	1.74448	0.36863
4	3			
5	0			
	288			$\chi_g = 0.7493$

$p_j = \frac{\lambda^j}{j!} e^{-\lambda} \qquad \lambda = \bar{x} = \frac{93+62+21+12}{288} = 0.652\dot{7}$

$p_j = \frac{0.65278^j \cdot 0.520597}{j!}$

$p_0 = 0.520597$

$p_1 = 0.65278 \cdot 0.520597 = 0.33983$

$p_2 = \frac{0.65278^2 \cdot 0.520597}{2} = 0.110917$

$p' = p_3 + p_4 + p_5 = 1 - (p_0 + p_1 + p_2) = 0.028655$

$k = 4 - 1 - 1 = 2 \qquad \alpha = 0.05 \qquad \chi_{2, 1-0.05} = \chi_{2, 0.95} = 5.991$

$\chi_g < \chi_{2, 0.95}$ Hipotezu ne možemo odbaciti

10.

	1	2	3	4	5
A	7	16	28	14	11
B	10	13	30	16	9

$\alpha = ?$

$n_1 = 76 \qquad n_2 = 78$

$$\overline{X}_1 = \frac{7+32+84+56+55}{76} = \frac{234}{76} = 3.07894$$

$$\hat{s}_1^2 = \frac{7\cdot 2.079^2 + 16\cdot 1.079^2 + 28\cdot 0.079^2 + 14\cdot 0.921^2 + 11\cdot 1.921^2}{75} =$$

$$= \frac{30.26 + 18.63 + 0.17 + 11.87 + 40.59}{75} = \frac{101.52}{75} = 1.3536$$

$$\overline{X}_2 = \frac{10+26+90+64+45}{78} = \frac{235}{78} = 3.01282$$

$$\hat{s}_2^2 = \frac{10\cdot 2.013^2 + 13\cdot 1.013^2 + 30\cdot 0.013^2 + 16\cdot 0.987^2 + 9\cdot 1.987^2}{77} =$$

$$= \frac{40.52 + 13.34 + 0.01 + 15.59 + 35.53}{77} = \frac{104.99}{77} = 1.3636$$

$$\hat{s}_{ul}^2 = \frac{\hat{s}_1^2}{n_1} + \frac{\hat{s}_2^2}{n_2} = \frac{1.3536}{76} + \frac{1.3636}{78} = 0.03529 \qquad \hat{s}_{ul} = 0.18786$$

$$\hat{u} = \frac{3.079 - 3.013}{0.18786} = \frac{0.066}{0.18786} = 0.3513$$

$$u_{1-\frac{\alpha}{2}} \geq 0.3513$$

$$u_{1-\frac{\alpha}{2}} \geq u_{0.65}$$

$$1 - \frac{\alpha}{2} = 0.65$$

$$\frac{\alpha}{2} = 0.35$$

$$\alpha = 0.7$$