

Str 95.

$$1. X \sim N(\mu, \sigma^2 = 6)$$

$$n = 100$$

$$\bar{x} = 16,2$$

$$\alpha = 0,05$$

$$H_0: \mu = 15$$

$$H_1: \mu \neq 15$$

$$u_{1-\frac{\alpha}{2}} = u_{0,975} = 1,96$$

$$U = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{16,2 - 15}{\sqrt{\frac{6}{100}}} = \frac{1,2}{0,245} = 4,899$$

$$|U| > u_{0,975}$$

$$4,899 > 1,96 \Rightarrow H_0 \text{ se odbacuje}$$

$$2. X \sim N(\mu, \sigma^2)$$

$$H_0: \mu = 25$$

$$H_1: \mu < 25$$

$$n = 50$$

$$\bar{x} = 24,2$$

$$\sigma^2 = 4,8$$

$$\alpha = 0,05$$

$$T = \frac{\bar{x} - \mu}{\sqrt{\frac{\sigma^2}{n}}} = \frac{24,2 - 25}{\sqrt{\frac{4,8}{50}}} = -2,582$$

$$t_{0,95} =$$

$$|T| > t_{0,95}$$

$$t_{1-\frac{\alpha}{2}} = t_{0,95} = 1,678$$

H_0 se odbacuje

$$3. n = 10000$$

$$\bar{x} = 5120$$

$$\mu_0 = 5000$$

$$p_0 = 0,5$$

$$U = \left(\frac{m}{n} - p_0 \right) \sqrt{\frac{p_0 q_0}{n}}$$

$$= \left(\frac{5120}{10000} - 0,5 \right) \sqrt{\frac{10000}{0,25}}$$

$$= 0,012 \cdot 200 = 2,4$$

$$p = 1 - \frac{\alpha}{2} = 0,992$$

$$\alpha = 0,016$$

$$0,992 = 1 - \frac{\alpha}{2}$$

$$2,4 < u_{1-\frac{\alpha}{2}}$$

$$u_{0,992} = 2,409$$

$$p = 1 - \alpha$$

$$4. n = 500$$

$$m = 16$$

$$p_0 = 0,02$$

$$\alpha = 0,05$$

$$U = \left(\frac{m}{n} - p_0 \right) \sqrt{\frac{p_0 q_0}{n}} = \left(\frac{16}{500} - 0,02 \right) \sqrt{\frac{0,02 \cdot 0,98}{500}}$$

$$= 1,916$$

$$U > u_{0,95}$$

$$u_{1-\alpha} = u_{0,95} = 1,6445$$

Deklaracija je netrpeljiva odbacuje se.

$$5. n = 50$$

$$\bar{x} = 12,6$$

$$s = 0,53$$

$$\alpha = 0,05$$

$$H_0: \mu = 12$$

$$H_1: \mu \neq 12$$

$$U = \frac{\bar{x} - \mu}{\sqrt{\frac{s^2}{n}}} = \frac{12,6 - 12}{\sqrt{\frac{0,53^2}{50}}} = \frac{0,6}{0,075} = 8$$

$$u_{0,975} = 1,96$$

$$U > u_{0,975}$$

Hipoteza se odbacuje.

6.

G, a_i	a_{i+1}	x_i	n_i	$\frac{z_j}{n}$	$\Phi^*(z_j)$	p_i	$\frac{(n_i - np_i)^2}{np_i}$	$\bar{x} = \frac{15 \cdot 2,5 + 75 \cdot 7,5 + 100 \cdot 12,5 + 50 \cdot 17,5 + 10 \cdot 22,5}{250} = \frac{2500}{250} = 10$
0	5	2,5	15	-1,44	0,855	0,0725	0,5387	
5	10	7,5	75	-0,383	0,298	0,2785	0,4149	
10	15	12,5	100	0	0,503	0,4005	0,001	
15	20	17,5	50	1,74	0,918	0,2075	0,067	
20	∞	22,5	10	2	1	0,04	0,0067	
				250			1,0277	

$$p(a_i \leq X \leq a_{i+1}) = \frac{1}{2} [\Phi^*(a_{i+1}) - \Phi^*(a_i)]$$

$$= \frac{1}{2} [\Phi^*(\frac{a_{i+1} - \bar{x}}{\sigma}) - \Phi^*(\frac{a_i - \bar{x}}{\sigma})]$$

~~1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100.~~

$$\bar{x} = 11,8$$

$$m = \frac{1}{n} \sum n_i x_i^2 = \frac{1}{250} \cdot (15 \cdot 2,5^2 + 75 \cdot 7,5^2 + 100 \cdot 12,5^2 + 50 \cdot 17,5^2 + 10 \cdot 22,5^2)$$

$$= 161,25$$

$$s^2 = m - \bar{x}^2 = 161,25 - 139,24 = 22 \Rightarrow s = 4,69$$

$$s^2 = \frac{n}{n-1} \sigma^2 = 22,093 \Rightarrow s = 4,7$$

$$z_j = \frac{a_j - \bar{x}}{s} \quad z_n = -\infty$$

$$z_5 = \frac{5 - 11,8}{4,7} = -1,44$$

$$z_{10} = \frac{10 - 11,8}{4,7} = -0,38$$

$$z_{15} = \frac{15 - 11,8}{4,7} = 0,68$$

$$z_{20} = \frac{20 - 11,8}{4,7} = 1,74$$

Sad možemo popuniti tablicu:

$$P_0 + P_1 + P_2$$

j	n_j
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$$p_j = \frac{1}{2} \left[\Phi(z_{j+1}) - \Phi(z_j) \right]$$

$$p_0(-\infty, 5) = \frac{1}{2} (-0,855 + 1) = 0,0725$$

$$p_2(5, 10) = \frac{1}{2} (-0,298 + 0,855) = 0,2785$$

$$p_3(10, 15) = \frac{1}{2} (0,503 + 0,298) = 0,4005$$

$$p_4(15, 20) = \frac{1}{2} (0,918 - 0,503) = 0,2075$$

$$p_5(20, \infty) = \frac{1}{2} (1 - 0,918) = 0,041$$

$5 - r - 1$ stupnjevi slobode

$$r = 2 \quad (\bar{x}, s)$$

$$5 - 2 - 1 = 2 \text{ stupnja slobode}$$

$$\chi = 0,2 \Rightarrow p_1 0,8$$

$$\chi^2_{p(2, 0,8)} = 3,219$$

$$1,0277 < 3,219$$

ne odbacujemo, se

7. $n=8$

$\alpha_0 = 200$

$\bar{x}_0 = \frac{165 + 170 + 182 + 185 + 193 + 200 + 210 + 206}{8} = 188,5$

$\alpha = 0,05$

$t_{1-\alpha} = t_{0,9} = 2,306 = 1,895$

$\sigma = 15,8194$

$T = \frac{x - x_0}{\frac{\sigma}{\sqrt{n}}} = \frac{200 - 188,5}{\frac{15,8}{\sqrt{8}}} = 2,04$

$T < t_{1-\alpha}$ ne prihvaća se hipoteza

8. j 0 1 2 3 4 5 6 7 8 9

n_j 12 62 129 172 150 80 28 5 2

x_j	n_j	p_j	$p_j - \alpha p_j$	$\frac{(n_j - \alpha p_j)^2}{\alpha p_j}$
0	12	0,0163	+1,25	0,1453
1	62	0,0835	+4,669	0,378
2	129	0,209	-4,77	0,169
3	172	0,278	-6,362	0,227
4	150	0,232	-1,36	0,015
5	80	0,1238	0,727	0,007
6	28	0,041	1,57	0,013
7	5	0,0085	1,56	0,447
8	2	0,0017		
9	1	0,00085		
	640	1		$\Sigma 1,679$

$\alpha = 0,05$ $p = 0,1$

$8 - r - 1 = 7$

$\chi^2_p = 2,833$

$1,679 < 2,83$

prihvaća se

$p_i = \binom{8}{j} (0,4)^j (0,6)^{8-j}$

$p_0 = \binom{8}{0} (0,4)^0 (0,6)^8 =$

$p_1 = \binom{8}{1} (0,4)^1 (0,6)^7 =$

$p_2 = \binom{8}{2} (0,4)^2 (0,6)^6 =$

$p_3 = \frac{8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3} \cdot 0,4^3 \cdot 0,6^5$

$p_4 = \frac{8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4} \cdot 0,4^4 \cdot 0,6^4$

$p_5 = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} \cdot 0,4^5 \cdot 0,6^3$

$p_6 = \frac{8 \cdot 7}{1 \cdot 2} \cdot 0,4^6 \cdot 0,6^2$

$p_7 = 8 \cdot 0,4^7 \cdot 0,6 =$

9. $\alpha = 0,05$

j	n_j	p_j	$n_j - n p_j$	
0	132	0,599	8,89	0,64
1	48	0,306	-14,73	3,458
2	26	0,078	4,01	1
3,4	5	0,015	1,95	1,15
				6,26

$\bar{x} = \lambda = \frac{48 + 40 + 8 + 3}{205}$

$= 0,512$

$\chi^2 = \frac{L - 1 - 1}{2 \cdot 0,05} = 5,999$

6,26 > 5,999

$p(x) = \frac{\lambda^x}{x!} e^{-\lambda}$

$f(0) = \frac{\lambda^0}{0!} e^{-\lambda} = e^{-\lambda} = e^{-0,51}$

$p(1) = \frac{\lambda^1}{1!} \cdot e^{-\lambda}$

$p(2) = \frac{\lambda^2}{2!} \cdot e^{-\lambda}$

$p(3) = \frac{\lambda^3}{3!} \cdot e^{-\lambda}$

10. MEDIANE

11. $a = 100$

✓ $n = 20$

$\bar{x} = 99,55$

$\sigma^2 = \frac{1}{19} \cdot \sum x_i^2 = 1,9131$

$t = 0,536$

$U = -\frac{0,45}{\frac{5}{19}} = 1,75$

~~$P(95,01 \leq a \leq 100,08) = 0,95$~~

$1,75 < 1,96$

Prima se hipoteza o ispravi

~~$t_{1-\frac{\alpha}{2}} = 2,093$~~

$U_c = t_{0,975} = 1,96$

$P_0 + P_1 + P_2$

12.

x_i	n_i	p_i	$n_i - n \cdot p_i$	$\frac{(n_i - n \cdot p_i)^2}{n \cdot p_i}$
0	505	0,4965	8,5	0,145
1	536	0,3476	-11,6	0,387
2	125	0,1216	3,4	0,095
3	24	0,0293	-4,3	0,653
4,5	10	0,0049	5,1	5,308
1000		1		$\sum 6,59$

$\bar{x} = \lambda = 0,7$

$P_0 = \frac{\lambda^0}{0!} e^{-\lambda} = e^{-\lambda}$

$P_1 = \lambda e^{-\lambda}$

$P_2 = \frac{\lambda^2}{2} e^{-\lambda} =$

$P_3 = \frac{\lambda^3}{6} e^{-\lambda} =$

$P_4 = \frac{\lambda^4}{24} e^{-\lambda} =$

$5 - 1 - 1 = 3$ stupnja slobode

$\alpha = 0,05$

$\chi^2_p = 7,815$

$6,59 < 7,815$

primala se hipoteza da je razlika pasova

13, 14, 15, 16. MEDIANE

~~$n = 1000$~~

j	n_j	p_j	$n_j - n \cdot p_j$	
0	130	0,602	+6,59	0,3519
1	52	0,305	-10,52	1,771
2	18	0,077	2,215	0,31
3,4	5	0,015	1,985	1,208
				$\sum 3,639$

$\bar{x} = \lambda = 0,5073$

$P_0 =$

$t_2 = \chi^2_{0,95}$

$\chi^2_{2,095} = 5,991$

hipoteza se prihvaća

17. $n=100$

$P=0,5$

x_i	n_i	p_i	$n_i - np_i$	$\frac{(n_i - np_i)^2}{np_i}$
0	8	0,03125	4,875	7,6
1	20	0,15625	4,375	1,225
2	42	0,3125	10,75	3,6
3	22	0,3125	-10,75	3,6
4	8	0,15625	-4,375	1,225
5	0	0,03125	-4,875	7,6
Σ	100	1	0	0,96875

~~$P_0 = \frac{100}{100} = 1$~~
 ~~$P_1 = \frac{20}{100} = 0,2$~~
 ~~$P_2 = \frac{42}{100} = 0,42$~~
 ~~$P_3 = \frac{22}{100} = 0,22$~~
 ~~$P_4 = \frac{8}{100} = 0,08$~~
 ~~$P_5 = \frac{0}{100} = 0$~~

$$P_0 = \binom{5}{0} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{5-0} = \frac{1}{2^5} = 0,03125$$

$$P_1 = 5 \cdot \frac{1}{2} \cdot \left(\frac{1}{2}\right)^4 = 0,15625$$

$$P_2 = 10 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \left(\frac{1}{2}\right)^3 = 0,3125$$

$$P_3 = \frac{5 \cdot 4}{1 \cdot 2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \left(\frac{1}{2}\right)^2 = 0,3125$$

$$P_4 = 5 \cdot \frac{1}{2} \cdot \left(\frac{1}{2}\right)^4 = 0,15625$$

$$\Sigma = 0,96875$$

5

1

54

12 20

543

123 10

5432

1234 5

0,9687

45

14

23

32

41

18.

x_i	n_i	p_i
0	6	0,0156
1	32	0,09375
2	71	0,2343
3	69	0,3125
4	35	0,2343
5	7	0,09375

$$\Sigma = 0,984375$$

$$X \sim B\left(6, \frac{1}{2}\right)$$

$$P_0 = \binom{6}{0} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{6-0} = \left(\frac{1}{2}\right)^6 = 0,015625 \quad P_4 = \binom{6}{4} \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{2} = 0,15625$$

$$P_1 = \binom{6}{1} \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{5-1} = 6 \cdot \frac{1}{2} \cdot \left(\frac{1}{2}\right)^4 = 0,09375 \quad P_5 = \binom{6}{5} \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^1 = 0,09375$$

$$P_2 = \binom{6}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-2} = \frac{6 \cdot 5}{1 \cdot 2} \cdot \frac{1}{2} \cdot \frac{1}{2} = 0,2343$$

$$\frac{6 \cdot 5 \cdot 3 \cdot 2}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} = 0,2343$$

$$P_3 = \binom{6}{3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{3-3} = \frac{6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3} = 20 = 0,3125$$

$$\Sigma = 0,984375 \checkmark$$

21.

$$n = 560$$

j	n	p	$n_i - np$	$\frac{(n_i - np)^2}{np}$
0	317	$\frac{9}{16}$	2	$\frac{4}{560 \cdot \frac{9}{16}}$ 0,127
1	109	$\frac{3}{16}$	4	0,1523
2	102	$\frac{2}{16}$	-3	0,0857
3	32	$\frac{1}{16}$	-3	0,257

$$\sum 0,507$$

$$\chi^2_{3, \alpha=0,05} = 7,815$$

$$p = 0,95$$

$$0,5 < 7,8$$

priljavica se!

22.

j	n	p	$n_i - np$	$\frac{(n_i - np)^2}{np}$
1	34	$\frac{1}{6}$	4	0,533
2	28	$\frac{1}{6}$	-2	0,133
3	26	$\frac{1}{6}$	-4	0,533
4	32	$\frac{1}{6}$	2	0,133
5	27	$\frac{1}{6}$	-3	0,3
6	33	$\frac{1}{6}$	3	0,3

$$\sum 1,933$$

$$\alpha = 0,9 \Rightarrow p = 0,1$$

$$\chi^2_{5, 0,1} = 1,61$$

Ne priljavica se!

5 stopnjeva svobode

$$p = 0,1$$

$$\chi^2_{5, 0,1} = 1,61$$

$$34 - 180 \cdot \frac{1}{6} = 4$$

$$\frac{16}{180 \cdot \frac{1}{6}} = \frac{16}{30} = 0,533$$

24.

x_i	n_i
0	1

x_i	n_i	p_i	$n_i - np_i$	$(n_i - np_i)^2$
0	20	0,131	-4,36	0,782
1	55	0,2663	0,6036	
2	48	0,2706	-2,33	0,108
3	55	0,1833	0,9067	0,024
4	18	0,1131	0,6839	0,027
5	10	0,1131	2,96	1,2539
	186	0,0378		

$$\sum 2,799$$

$$P = \frac{\sum x_i^2}{\sum x_i} \cdot c = \lambda$$

$$\chi^2_{6-1-1, 1-0,05} = \chi^2_{4,0,95} = 9,488$$

$$2,799 < 9,488$$

prilivati se H_0/H_1

25. $n=180$ Ist kao i 23, samo $p=0,1$

x_i	n_i	p_i	$n_i - np_i$
1	28	1	
2	26	1	
3	52	6	
4	27		
5			
6			

$$\chi^2_2 = 1,93$$

$$\chi^2_{5,0,95} = 9,236$$

prilivati se!

26. $n=800$

i	n_i	p_i	$n_i - np_i$	$(n_i - np_i)^2$
0	74	1	-6	0,45
1	92	10	12	1,8
2	83	1	3	0,1125
3	79	10	-1	0,0125
4	80		0	0
5	73	1	-7	0,0125
6	77	10	-3	0,1125
7	75	1	-5	0,3125
8	76	10	-4	0,2
9	91		11	1,5125

$$\sum 800 \quad 1$$

$$\chi^2_2 = 5,125$$

$$\alpha = 0,1 \Rightarrow P = 0,9$$

$$10 - r - 1 = 10 - 0 - 1 = 9 \text{ stupnjeva slobode}$$

$$\chi^2_{9,0,9} = 14,684$$

$$\chi^2_E < \chi^2_\alpha$$

$$5,125 < 14,684 \Rightarrow \text{Hipoteza se prihvata}$$