① $f: \{1, \dots, 100\} \to \{1, \dots, 100\}$

$p \to p \qquad np \to np$

$|\Omega| = 100 \cdot 100 \cdot \ldots \cdot 100 = 100^{100}$

a) $|A| = 50 \cdot 50 \cdot 50 \cdots 50 \cdot 50 \cdot 50 \cdot \ldots \cdot 50$

$= 50^{50} \cdot 50^{50} \Longrightarrow \quad P(A) = \frac{50^{100}}{100^{100}} = \left(\frac{1}{2}\right)^{100}$

b) Izračunaj vjerojatnost da f-ja preslika $p \to p$, $np \to np$

$|B| = 50 \cdot 49 \cdot 48 \cdot \ldots \cdot 1 \cdot 50 \cdot 49 \cdot 48 \cdot \ldots \cdot 1 = 50!\,50!$

$|\Omega| = 100 \cdot 99 \cdot \ldots \cdot 1 = 100!$

$P(B) = \frac{50!\,50!}{100!}$

② Izračunaj vjerojatnost da [illegible] od 12 osoba dvije po dvije budu rođene u istom mjesecu.

a) $|\Omega| = 12 \cdot 12 \cdot \ldots \cdot 12 = 12^{12}$

$|A| = \binom{12}{6}\binom{12}{2}\binom{10}{2}\binom{8}{2}\binom{6}{2}\binom{4}{2}\binom{2}{2} = \binom{12}{6}\frac{12!}{2!\,2!\,2!\,2!\,2!\,2!}$

b) $|B| = \binom{12}{6}\binom{30}{3}\binom{27}{3}\binom{24}{3}\binom{21}{3}\binom{18}{3}\binom{15}{3}\binom{12}{2}\binom{10}{2}\cdots\binom{2}{2} = \binom{12}{6}\frac{30!}{(3!)^6(2!)^6}$

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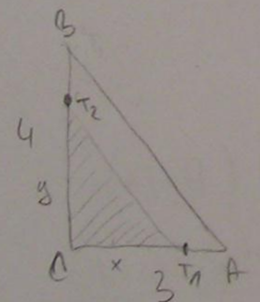
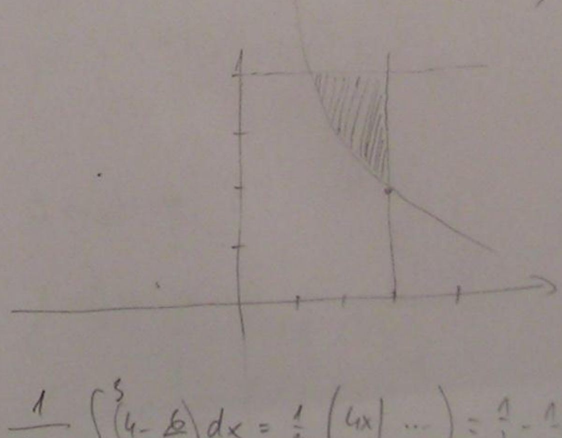
③ U pravokutnom trokutu ABC. Točka T_1 bira se na kateti $\overline{AC}$, a T_2 na $\overline{BC}$. Kolika je vjerojatnost da je $P(P_{CT_1T_2} > \frac{1}{2}P_{ABC}) = ?$



$\frac{1}{2} \cdot xy > \frac{1}{2} \cdot \frac{1}{2} \cdot 4 \cdot 3$

$\frac{1}{2} xy > 3$

$xy > 6 \qquad y = \frac{6}{x}$



$x \in \langle 0, 3 \rangle$

$y \in \langle 0, 4 \rangle$

Ω

$P = \frac{P_A}{P_\Omega} = \frac{1}{12}\int_{\frac{3}{2}}^{3}\left(4 - \frac{6}{x}\right)dx = \frac{1}{12}\left(4x \,\Big|\, \ldots\right) = \frac{1}{2} - \frac{1}{2}\ln 2$

$= 0{,}15$

(4.) $H_1 = \{Au\}$ $\quad P(H_1) = \frac{1}{4} \quad \longrightarrow \quad P(A|H_1) = 0{,}6$

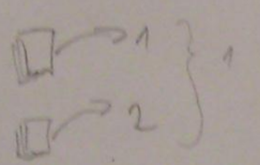
$H_2 = \{cc\}$ $\quad P(H_2) = \frac{1}{4} \quad\quad P(A|H_2) = 0{,}8$

$H_3 = \{Am\}$ $\quad P(H_3) = \frac{2}{4} = \frac{1}{2} \quad\quad P(A|H_3) = 0{,}9$

$A = \{$potrošit će limit$\}$

$$P(H_2|A) = \frac{P(H_2) \cdot P(A|H_2)}{\sum P(H_i) P(A|H_i)} = \frac{1}{4} = 0{,}25$$

(8.) Dva mopa karata po 52 karte. Iz 1. izvučemo 1 kartu, iz 2. 2 karte. Ako je otkriven as, kolika je vjerojatnost da su sve 3 bili asevi



$H_i = \{$izvučeno sam i-asa$\}$
$i = 0,1,2,3$

$A = \{$izvučen je as$\}$

$$P(H_0) = \frac{48}{52} \cdot \frac{\binom{48}{2}}{\binom{52}{2}}, \quad P(H_1) = \frac{4}{52} \cdot \frac{\binom{48}{2}}{\binom{52}{2}} + \frac{48}{52} \frac{\binom{4}{1}\binom{48}{1}}{\binom{52}{2}}$$

$$P(H_2) = \frac{4}{52} \frac{\binom{4}{1}\binom{48}{1}}{\binom{52}{2}} + \frac{48}{52} \cdot \frac{\binom{4}{2}}{\binom{52}{2}}$$

$$P(H_3) = \frac{4}{52} \cdot \frac{\binom{4}{2}}{\binom{52}{2}}$$

$P(A|H_0) = 0$ $\quad\quad P(A|H_2) = \frac{2}{3}$

$P(A|H_1) = \frac{1}{3}$ $\quad\quad P(A|H_3) = 1$

$$P(H_3|A) = \frac{P(H_3) P(A|H_3)}{\sum P(H_i) P(A|H_i)}$$

6.

$$X \sim \begin{pmatrix} 1 & 2 & 3 & \dots n \dots \\ 0{,}686 \cdot x & \underbrace{[0{,}314 + 0{,}686(1-x)]}_{\text{pao}} \cdot \underbrace{0{,}686x}_{\text{prošao}} & \underbrace{(1-0{,}686x)^2}_{\text{pao}} \cdot \underbrace{0{,}686x}_{\text{prošao}} & (1-0{,}686x)^{n-1} \, 0{,}686x \end{pmatrix}$$

$\Rightarrow X \sim G(p)$

$p = 0{,}686x$

$E(x) = 3$

$\frac{1}{p} = 3$

$\frac{1}{0{,}686x} = 3$

$x = 0{,}486$

7. Bacamo 2 kocke, X je broj parnih brojeva, Y je ostatak zbroja dijeljenog sa 4

X\Y	0	1	2	3	
0	$\frac{4}{36}$	0	$\frac{5}{36}$	0	$\frac{9}{36}$
1	0	$\frac{8}{36}$	0	$\frac{10}{36}$	$\frac{18}{36}$
2	$\frac{5}{36}$	0	$\frac{4}{36}$	0	$\frac{9}{36}$
	$\frac{9}{36}$	$\frac{8}{36}$	$\frac{9}{36}$	$\frac{10}{36}$	1

zbroj = 4, 8, 12

4: (4,2), (3,1), (1,3)

8: (4,4), (2,6), (6,2), (5,3), (3,5)

12: (6,6)

zbroj = 5, 9

5: 23, 32, 41, 14

9: 63, 36, 54, 45, 4

zbroj = 6, 10, 2

6: 2,4; 4,2; 5,1; 1,5; 3,3; 4

10: 6,4; 4,6; 5,5

2: 1,1

zbroj = 7, 11, 3

7: 16, 61, 52, 25, 43, 34

11: 65, 56

3: 12, 21

b) $U = \max\{X, Y\}$, $V = X \cdot Y$ $\Big\}$ $(U, V) = ?$

$U \backslash V$	0	1	2	3	4	6	
0	$\frac{1}{36}$	0	0	0	0	0	
1	0	$\frac{8}{36}$	0	0	0	0	1
2	$\frac{5}{36}+\frac{5}{36}$	0	0	0	$\frac{4}{36}$	0	
3	0	0	0	$\frac{10}{36}$	0	0	
							1

$U = 0$: $X = 0, Y = 0$
$V = 0$

$U = 1$, $X = 0$, $Y = 1 \to V = 0$
$X = 1$, $Y = 0 \to V = 0$
$X = 1$, $Y = 1 \to V = 1$

$U = \max$
$V = X \cdot Y$

8. 960 a/h

0,5 min

$X \sim P(\lambda)$

~~$2 \cdot 60 \cdot 12 = 1440$~~

$\lambda \to$ intenzitet u jedinici vremena

$\lambda = \frac{960}{60 \cdot 2} = 8$ u pola minute

$P(X \geq 5) = 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3) - P(X=4)$

$P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}$

$= 1 - e^{-8} - \frac{8}{1} e^{-8} - \frac{8^2}{2!} e^{-8} - \frac{8^3}{3!} e^{-8} - \frac{8^4}{4!} e^{-8} = 0,9$

9. $p = 0,5\ \%$

$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$

a) $X \sim B(n, p)$

$X \sim B(8, 0{,}005)$

$P(X \geq 2) = 1 - P(X=0) - P(X=1)$

$= 1 - 0{,}995^8 - \binom{8}{1} 0{,}005 \cdot 0{,}995^7 = 0{,}07\ \%$