(4.) 2.9. (1)

$$P(|X - E(X)| < \varepsilon) > \underbrace{1 - \frac{\sigma^2}{\varepsilon^2}}_{P}$$

ČEBIŠJEVA NEJ.

$E(X) = 25$

$\sigma = 5,5$

A

$P(A) > 0,9$

$$0,9 = 1 - \frac{\sigma^2}{\varepsilon^2}$$

$$\varepsilon = 14,23$$

$$|X - 25| < 14,23$$

$$25 - 14,23 < X < 25 + 14,23$$

10.7/10.8. /20 s M.

PROGENA NEPRISTRANOST!

(8.)

$$c = \frac{a+b}{2}$$

$$\hat{c} = \frac{X_{(1)} + X_{(n)}}{2}$$

$$E(\hat{c}) = E\left(\frac{X_{(1)} + X_{(n)}}{2}\right)$$

$$= \frac{1}{2}\left[E(X_{(1)}) + E(X_{(n)})\right]$$

$$P\{\min(X_1, \dots, X_n) < x\}$$
$$= 1 - P\{\min > x\}$$
$$= 1 - P(X_1 > x, \dots, X_n > x)$$
$$= 1 - (1 - F_{X_1}(x))^n$$

$$E(X_{(n)}) = \int_{-\infty}^{\infty} x f_{(n)}(x)\,dx \; ; \quad f_{(n)}(x) = ?$$

$$F_{X_{(n)}}(x) = P\{X_{(n)} < x\} = P\{\max(X_1, \dots) < x\}$$

$$= P\{x_1 < x,\ x_2 < x, \cdots, x_n < x\}$$

$$= P\{x_1 < x\} \cdots P\{x_n < x\}$$

$$= F_{x_1}(x) \cdots F_{x_n}(x)$$

$$= [F_{x_1}(x)]^n$$

$$f_{x_1}(x) = \frac{1}{b-a}$$

$$F_{x_1}(x) = \frac{x-a}{b-a}$$

$$= \left(\frac{x-a}{b-a}\right)^n$$

$$f_{x_n} = n\left(\frac{x-a}{b-a}\right)^{n-1} \cdot \frac{1}{b-a}$$

$$E(x_n) = \frac{n}{(b-a)^n} \int_a^b x \left(\frac{x-a}{\quad}\right)^{n-1} dx$$

$$D(\bar{c}) = D\left(\frac{x_{n+1}+x_n}{2}\right)$$

PROGENA VARIJANSTI

$$= \frac{1}{4} D(x_n + x_{n+1})$$

$$= \frac{1}{4}\left[D(x_n) + D(x_{n+1}) - 2\,\mathrm{cov}(x_n x_{n+1})\right]$$

$\lim_{n\to\infty} D(x) = \sigma$ KONAČNA VARIJANSA!

$$f(x,y) = \frac{1}{\pi^2(x^2+y^2+x^2y^2+1)}, \quad x,y \in \mathbb{R}$$

$$F(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f(x,y)\,dx\,dy$$

$$= \frac{1}{\pi^2}\int_{-\infty}^{x} dx \int_{-\infty}^{y} \frac{1}{x^2(1+y^2)+y^2+1}$$

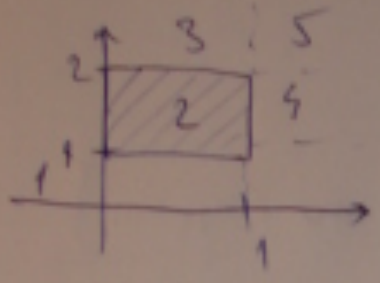
$$= \frac{1}{\pi^2}\int_{-\infty}^{x} dx \int_{-\infty}^{y} \frac{1}{(y^2+1)(1+x^2)}\,dy$$

$$= \frac{1}{\pi^2}\int_{-\infty}^{x} \frac{1}{1+x^2}\,dx \int_{-\infty}^{y} \frac{1}{1+y^2}\,dy$$

$$= \frac{1}{\pi^2}\,\mathrm{arctg}\,x\Big]_{-\infty}^{x}\,\mathrm{arctg}\,y\Big]_{-\infty}^{y}$$

$$= \frac{1}{\pi^2}\left(\mathrm{arctg}\,x + \frac{\pi}{2}\right)\left(\mathrm{arctg}\,y + \frac{\pi}{2}\right)$$

$f(x,y)$ innn jedn. rozdiosu na $S = \{0 < x < 1, 1 < y < 2\}$



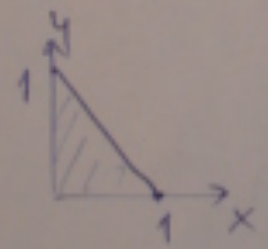
$$f(x,y) = \frac{1}{P_a} = 1$$

$$F_1(x,y) = 0$$

$$F_5(x,y) = 1$$

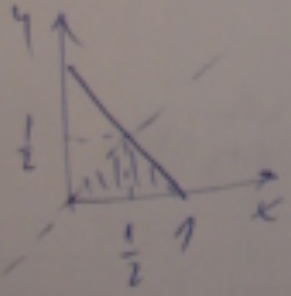
$$F_2: \int_{0}^{x}\int_{1}^{y} 1\,dx\,dy \quad | \quad F_3: \int_{0}^{x}\int_{1}^{2} 1\,dx\,dy \quad | \quad F_4: \int_{0}^{1}\int_{1}^{y} 1\,dx\,dy$$

(2. HI) $f(x,y) = 24xy$



$P(x > y) = ?$

$$= \iint_D f(x,y)\,dx\,dy$$



$$= \int_0^{1/2} dy \int_y^{1-y} 24xy\,dx = \frac{1}{2}$$

$f(x,y) = \dots$

$z = y + x$

$$g(z) = \int_{-\infty}^{\infty} f(x,y) \cdot \left| \frac{\partial y}{\partial z} \right| dx$$

---

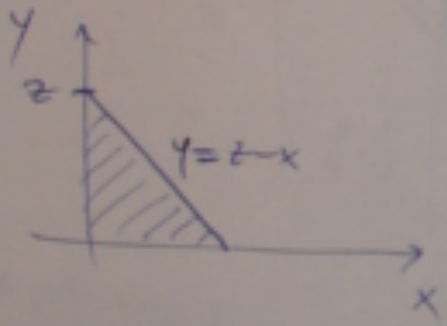
pr. x i y $\Rightarrow$ $\lambda = 1$ (exp. rozd.)

$z = x + y$; $f(z) = ?$

$X \sim E(1)$ $\quad f_x(x) = e^{-x}$, $x > 0$

$Y \sim E(1)$ $\quad f_y(y) = e^{-y}$, $y > 0$

$$f(x,y) = f(x) \cdot f(y) = e^{-x-y}$$



I. podst.

$$G(z) = P\{Z < z\} = P\{X + Y < z\}$$

$$= \iint_D f(x,y)\,dx\,dy = \int_0^z dx \int_0^{z-x} e^{-x-y}\,dy = 1 - e^{-z}(1+z)$$

$$g(z) = G'(z) = z e^{-z}$$

II. $g(z) = \int_{-\infty}^{\infty} f(x, y) \left| \frac{\partial y}{\partial z} \right| dx$

$y = z - x \quad , \quad \frac{\partial y}{\partial z} = 1$

$$g(z) = \int_{0}^{z} e^{-x-(z-x)} dx = \underline{\underline{z e^{-z}}}$$

$x > 0$

$z - x > 0$

$x < z$

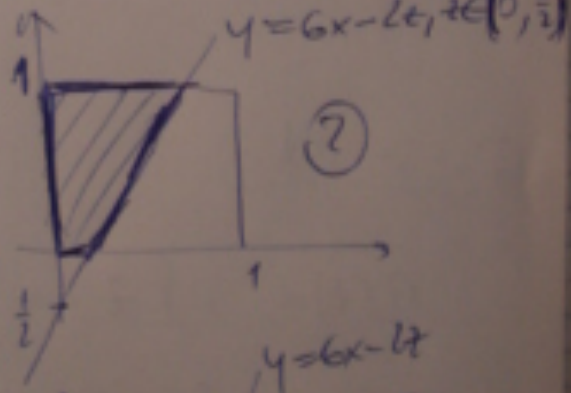
$f(x,y) = x^2 + Cy \qquad (x,y) \in [0,1]^2$

$$\iint f(x,y)\, dx\, dy = 1$$

$$\int_0^1 dx \int_0^1 (x^2 + Cy)\, dy = 1$$

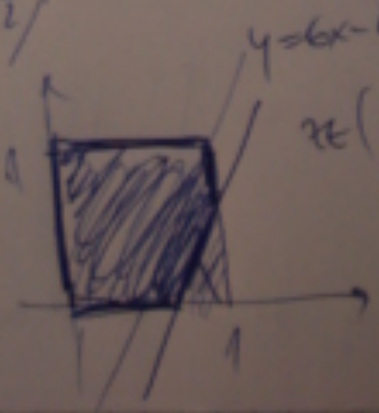
$$\underline{\underline{C = \frac{4}{3}}}$$

$z = 3x - \frac{1}{2}y \quad , \quad z \in \left[-\frac{1}{2}, 3\right]$



I. $F(z) = P\{Z < z\} = P\{3x - \frac{1}{2}y < z\}$

$y > 6x - 2z$



(7)

(1) $G(z) = \int_{\varnothing}^{\frac{1}{6}+\frac{z}{3}} dx \int_{6x-2z}^{1} \left(x^2 + \frac{2}{3}y\right) dy$

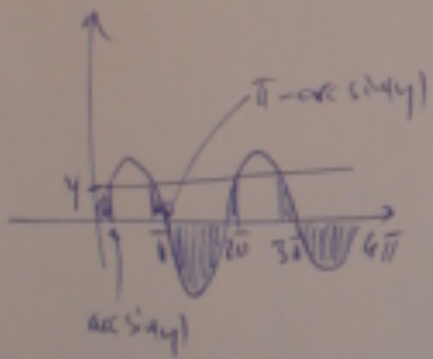
(2) $G(z) = \int_{\varnothing}^{1} dy \int_{\varnothing}^{\frac{y}{6}+\frac{z}{3}} f(x,y)\, dx$

(3) $G(z) = 1 - \iint_{\Delta} f(x,y)\, dx\, dy$

S. d. 10/11

$x \in (0, 4\pi)$, $g(x) = \frac{1}{4\pi}$

$Y = \sin X$, $g(y) = ?$



$g(y) = f(x) \left| \frac{dx}{dy} \right|$

$G(y) = P(Y < y) = P(\sin X < y)$

$= P\{(0 < x < \arcsin y) \cup (\pi - \arcsin y < x < 2\pi + \arcsin y) \cup (3\pi - \arcsin y < x < 4\pi)\}$

$= \int_{a}^{S} f(x)\, dx$

1. CIKLUS

SYLVESTEROVA FORMULA!

BAYESOVA FORMULA!

BESKONAČNA SUMA!

LOŽ. STV. 1. CIKLUS

X \ Y	0	1	2	3	4	5	
0	$\frac{0}{36}$	0	0	0	0	0	0
1	0	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	10
2	0	⋮	⋮	⋮	⋮	0	8
3	0	⋮	$\frac{2}{36}$	⋮	0	0	6
4	0	⋮	⋮	0	0	0	4
5	0		0	0	0	0	2
	$\frac{6}{36}$	10	8	6	4	2	1

$r(X,Y) = ?$

$$\rho(X,Y) = \frac{\mathrm{cov}(X,Y)}{\sqrt{\sigma_x \sigma_y}}$$

$\sqrt{\sigma_x \sigma_y} = D(X)\,D(Y)$

$$\mathrm{cov}(X,Y) = E(XY) - E(X)\,E(Y)$$