

Ovaj PDF sadrži skenirane postupke 2. KPZ-a 2012-2013.

Zadaci su poredani po po godinama, od 2013 prema 2012, od 12h do 13 sati, od A do B.

Bez prvog zadatka.

Riješio i ustupio na skeniranje

[fer0vac](#)

skenirao

[SipE](#)

$$\textcircled{2} P(X=2^k) = \frac{c}{5^k}, \quad k=5, 6, 7, \dots$$

II kolo
2013
A, 124

$$X \sim \left(\begin{array}{ccc} 2^5 & 2^6 & 2^7 \\ 32 & 64 & 128 \\ \hline \frac{c}{5^5} & \frac{c}{5^6} & \frac{c}{5^7} \end{array} \right) \quad \Sigma = 1$$

$$1 = \sum_{k=5}^{\infty} \frac{c}{5^k} \Rightarrow 1 = c \sum_{k=5}^{\infty} \frac{1}{5^{k+5}} = \frac{c}{3125} \sum_{k=0}^{\infty} \left(\frac{1}{5}\right)^k$$

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \Rightarrow 1 = \frac{c}{3125} \cdot \frac{1}{1-\frac{1}{5}}$$

$$1 = \frac{c}{2500} \Rightarrow c = 2500$$

$$c = 4 \cdot 5^4$$

$$E(X) = \sum_{k=5}^{\infty} 2^k \cdot \frac{(4 \cdot 5^4)}{5^k} = 2500 \sum_{k=5}^{\infty} \left(\frac{2}{5}\right)^k$$

povećaj / smanji

smanji / povećaj

$$= 2500 \sum_{k=0}^{\infty} \left(\frac{2}{5}\right)^{k+5} = 2500 \cdot \frac{32}{3125} \sum_{k=0}^{\infty} \left(\frac{2}{5}\right)^k =$$

$$= \frac{128}{5} \cdot \frac{1}{1-\frac{2}{5}} = \frac{128}{3} //$$

$$\sum_{k=5}^{\infty} \frac{(2^k)^2}{5^k} = c \sum_{k=5}^{\infty} \left(\frac{4}{5}\right)^k = 2500 \sum_{k=0}^{\infty} \left(\frac{4}{5}\right)^{k+5} =$$

$$= 2500 \cdot \frac{1024}{3125} \sum_{k=0}^{\infty} \left(\frac{4}{5}\right)^k = \frac{4096}{5} \cdot \frac{1}{1-\frac{4}{5}} = 4096$$

$$D(X) = 4096 - \left(\frac{128}{3}\right)^2 = \frac{20480}{9} //$$

MAN UTD
by ferovac

③ $\begin{matrix} 400 \text{ zł} & \text{sucho} & 0,5 \\ 150 \text{ zł} & \text{zima} & 0,5 \end{matrix}$ 3 sucho dan 0/5

71400 zł 5 dana \uparrow

$$\begin{aligned}
 & 3 \cdot 400 + 2 \cdot 150 \\
 & 4 \cdot 400 + 1 \cdot 150 \\
 & 5 \cdot 400 + 0 \cdot 150
 \end{aligned}$$

$$\Rightarrow \binom{5}{3} (0,5)^3 (0,5)^2 + \binom{5}{4} (0,5)^4 (0,5)^1 + \binom{5}{5} (0,5)^5 (0,5)^0$$

$$= \frac{1}{2} //$$

ili

$$X \sim B(5, 0,5) \Rightarrow P(X \cdot 400 + (5-X) \cdot 150 > 1400) = ?$$

$$400x + 750 - 150x > 1400$$

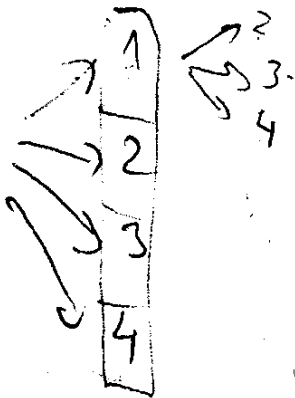
$$250x > 650$$

$$P(X > 2,6) = 1 - P(X=1) - P(X=0) - P(X=2)$$

3, 4, 5

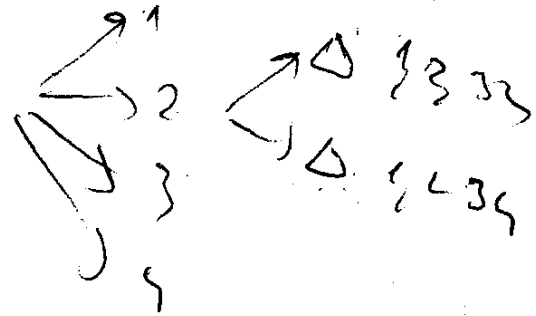
$$\begin{aligned}
 &= 1 - \binom{5}{1} (0,5)^1 (1-0,5)^{5-1} - \binom{5}{0} (0,5)^0 (1-0,5)^{5-0} \\
 &\quad - \binom{5}{2} (0,5)^2 (1-0,5)^{5-2} = \frac{1}{2}
 \end{aligned}$$

$$X \sim \begin{pmatrix} 2 & 3 & 4 & 5 \\ \frac{1}{4} & \frac{1}{3} & \frac{1}{3} & \frac{1}{12} \end{pmatrix}$$



$$\frac{1}{4} + \frac{1}{4} \cdot \frac{1}{3}$$

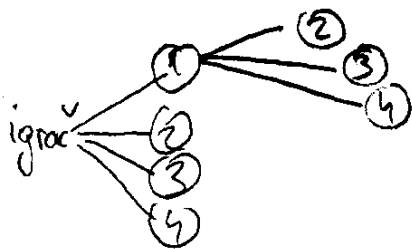
$$\frac{3}{12} + \frac{1}{12} = \frac{4}{12}$$



$$\frac{1}{4} + \frac{6}{64}$$

② ① ② ③ ④

$X \equiv$ zbroj rezultata oba izvlačenja



$$= 3 \text{ a}$$

$$= 4 \text{ a}$$

$$= 5 \text{ a}$$

$$= 2 \text{ b}$$

$$= 3 \text{ b}$$

$$= 4 \text{ b}$$

$$X \sim \begin{pmatrix} 2 & 3 & 4 & 5 \\ \frac{1}{4} & \frac{1}{3} & \frac{1}{3} & \frac{1}{12} \end{pmatrix}$$

$$X \sim \begin{pmatrix} 2 & 3 & 4 & 5 \\ \frac{1}{4} & \frac{4}{12} & \frac{4}{12} & \frac{1}{12} \end{pmatrix}$$

③ $X \sim \begin{pmatrix} 1 & 3 & 5 & 7 & 9 & \dots \\ \frac{1}{2} & (\frac{1}{2})^3 & (\frac{1}{2})^5 & (\frac{1}{2})^7 & (\frac{1}{2})^9 & \dots \end{pmatrix}$

$$P(X > 50) = ?$$

51, 52, ...

$$P(X > 50) = P(X = 51) + P(X = 52) + P(X = 53) + \dots$$

$$= \left(\frac{1}{2}\right)^{51} + \left(\frac{1}{2}\right)^{53} + \left(\frac{1}{2}\right)^{55} + \dots$$

$$= \left(\frac{1}{2}\right)^{50} \left[\frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \dots \right] = (*)$$

$$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{2n+1}$$

$$\frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{2n} = \frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = \frac{1}{2} \left(\frac{1}{1 - \frac{1}{4}} \right)$$

$$= (*) = \left(\frac{1}{2}\right)^{50} \cdot \frac{1}{2} \cdot \left(\frac{1}{1 - \frac{1}{4}} \right) = 5,92 \cdot 10^{-16} //$$

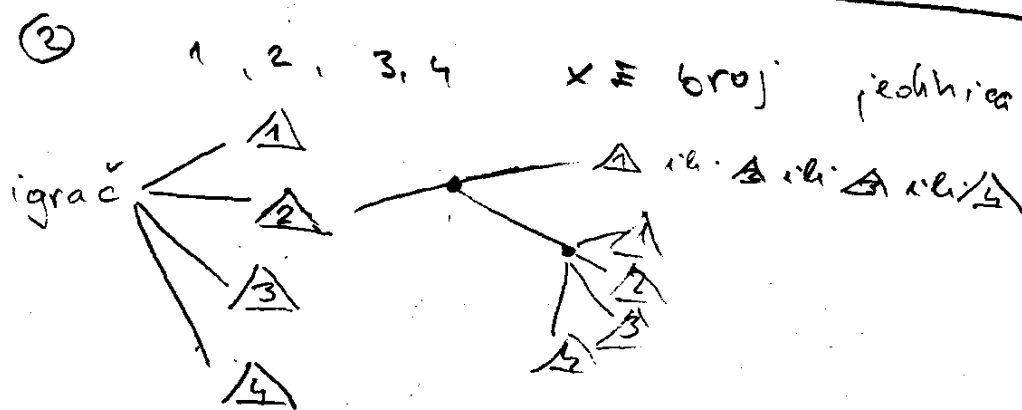
$X \sim \begin{pmatrix} 0 & 1 & 2 \\ \frac{41}{64} & \frac{22}{64} & \frac{1}{64} \end{pmatrix}$

$x=0 \Rightarrow \frac{2}{4} + \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{41}{64}$

$x=1 \Rightarrow \frac{1}{4} + \frac{1}{4} \cdot \left(\frac{3}{4} \cdot \frac{1}{4} \right) \cdot 2$

$x=2 \Rightarrow \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}$

$x=1 \Rightarrow \frac{1}{4} + \frac{1}{4} \cdot \left(\frac{3}{4} \cdot \frac{1}{4} \right) \cdot 2$



$X \sim \begin{pmatrix} 0 & 1 & 2 \\ \frac{41}{64} & \frac{22}{64} & \frac{1}{64} \end{pmatrix}$

na zapreku tetraedru je pola jedinica

$X \sim \begin{pmatrix} 0 & 1 & 2 \\ \frac{41}{64} & \frac{22}{64} & \frac{1}{64} \end{pmatrix}$

- ③ 300 putnika
 $p = 0,01$ kašnjenje
 \Rightarrow 302 karte prodano

$$X \sim B(302, 0,99)$$

$$P(X \leq 300) = ?$$

$$P(X \leq 300) = 1 - P(X = 301) - P(X = 302)$$

$$= 1 - \binom{302}{1} 0,99^{301} 0,01 - 0,99^{302} \binom{302}{0} = 0,8053$$

↓
 koja osoba je zakašnula

ili

Poisson

osoba je zakašnula

$$X \sim P(\lambda) = P(\mu \cdot p) = P(302 \cdot 0,01) = P(\lambda = 3,02)$$

$$P(X = 2) = \frac{\lambda^k}{k!} e^{-\lambda}$$

$$P(X \geq 2) = 1 - P(X = 1) - P(X = 0)$$

$$= 1 - \frac{3,02^1}{1!} e^{-3,02} - \left(\frac{3,02^0}{0!} \right) e^{-3,02}$$

$$= 0,803219 //$$

② $P(X=2^k) = \frac{c}{k!}, k=0,1,2,\dots$

$$X \sim \begin{pmatrix} 1 & 2 & 4 & \dots \\ \frac{c}{0!} & \frac{c}{1!} & \frac{c}{2!} & \dots \end{pmatrix}$$

$$1 = \sum_{k=0}^{\infty} \frac{c}{k!} = c \sum_{k=0}^{\infty} \frac{1}{k!} = c \cdot e \Rightarrow \boxed{c = \frac{1}{e}}$$

$$E(X) = \sum_{n=0}^{\infty} 2^n \left(\frac{1}{e}\right) \cdot \frac{1}{n!}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$= \frac{1}{e} \sum_{n=0}^{\infty} \frac{2^n}{n!} = \frac{1}{e} e^2 = e //$$

$$D(X) = \sum_{n=0}^{\infty} (2^n)^2 \frac{c}{n!} - (E(X))^2$$

$$= \frac{1}{e} \sum_{n=0}^{\infty} \frac{4^n}{n!} - e^2 = e^3 - e^2 = e^2(e-1) > 0 //$$

③ 10 zł ili 5 zł \Rightarrow 4 gasta < 30 zł

$$\frac{1}{12} P_v = \frac{2}{12} P_m = \frac{3}{12} \Rightarrow 100:3 = 33\%$$

$$\Rightarrow P_m = 0.66 = \frac{2}{3}, P_v = \frac{1}{3}$$

$$0 \cdot 10 + 4 \cdot 5 = 20 \Rightarrow$$

$$1 \cdot 10 + 3 \cdot 5 = 25 \Rightarrow$$

$$\left(\frac{1}{3}\right)^0 \cdot \left(\frac{2}{3}\right)^4 \binom{4}{4}$$

$$\left(\frac{1}{3}\right)^1 \cdot \left(\frac{2}{3}\right)^3 \binom{4}{3}$$

$$\left. \begin{matrix} \left(\frac{1}{3}\right)^0 \cdot \left(\frac{2}{3}\right)^4 \binom{4}{4} \\ \left(\frac{1}{3}\right)^1 \cdot \left(\frac{2}{3}\right)^3 \binom{4}{3} \end{matrix} \right\} + = \frac{16}{27}$$

ili \nearrow y. vece

$$X \sim B\left(4, \frac{1}{3}\right)$$

$$P(X=2) = \binom{4}{2} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{4-2}$$

$$10x + 5(4-x) < 30$$

$$20 + 5x < 30$$

$$5x < 10$$

$$P(X < 2) = P(X=0) + P(X=1) = \frac{16}{27}$$

mean value = $\binom{4}{0} \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^4 + \binom{4}{1} \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^3 = \frac{16}{27}$
by Perchac

$$X \sim \left(\begin{array}{c} 1 \\ \frac{2}{6} \end{array}, \begin{array}{c} 2 \\ \frac{4}{6} \cdot \frac{2}{5} \end{array}, \begin{array}{c} 3 \\ \frac{4}{6} \cdot \frac{2}{5} \cdot \frac{2}{4} \end{array}, \begin{array}{c} 4 \\ \frac{4}{6} \cdot \frac{2}{5} \cdot \frac{2}{4} \cdot \frac{2}{3} \end{array}, \begin{array}{c} 5 \\ \frac{4}{6} \cdot \frac{2}{5} \cdot \frac{2}{4} \cdot \frac{2}{3} \cdot \frac{2}{2} \end{array} \right)$$

dobar salter = 2

Envi = 4

$$E(X) = 1 \cdot \frac{2}{6} + 2 \left(\frac{4}{6} \cdot \frac{2}{5} \right) + \dots + 5 \left(\frac{4}{6} \cdot \frac{2}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} \cdot \frac{2}{2} \right) =$$

$$= \frac{1}{3} + \frac{8}{15} + \frac{3}{5} + \frac{8}{15} + \frac{1}{3} = \frac{7}{3} = 2,3$$

$$D(X) = \sum x^2 \cdot p - (E(X))^2$$

$$= 1^2 \cdot \left(\frac{2}{6} \right) + 2^2 \left(\frac{4}{6} \cdot \frac{2}{5} \right) + \dots - (2,3)^2$$

$$= \frac{1}{3} + \frac{16}{15} + \frac{3}{5} + \frac{32}{15} + \frac{5}{3} - \frac{49}{9} = \frac{14}{9} = 1,5$$

③ 200 ljudi, barem u lipovda
ato lipovda ima prosječno 1%
prosjечно 1%

$$X \sim B(200, 0,01)$$

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k=0,1,2,\dots$$

$$P(X \geq 4) = ?$$

$$P(X \geq 4) = 1 - P(X=3) - P(X=2) - P(X=1) - P(X=0)$$

$$= 1 - \binom{200}{3} 0,01^3 (0,99)^{197} - \binom{200}{2} 0,01^2 (0,99)^{198} -$$

$$\binom{200}{1} 0,01^1 (0,99)^{199} - \binom{200}{0} 0,01^0 (0,99)^{200} =$$

$$= 1 - 0,18136 - 0,27203 - 0,27067 - 0,13398 = 0,14196$$

Il uaciu Poisson

$$X \sim P(\lambda) \quad X \sim B(200, 0,01) \approx P(\lambda = n \cdot p = 200 \cdot 0,01 = 2)$$

$$P(X \geq 4) = 1 - \frac{2^3}{3!} e^{-2} - \frac{2^2}{2!} e^{-2} - \frac{2^1}{1!} e^{-2} - \frac{2^0}{0!} e^{-2}$$

MAN UTO
by forbac = 0,14287

② 7 cărți pe 2 + 5 =

$$X \sim \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \frac{1}{21} & \frac{2}{21} & \frac{3}{21} & \frac{4}{21} & \frac{5}{21} & \frac{6}{21} \end{pmatrix}$$

$$\frac{6+5+4+3+2+1}{21} = \frac{21}{21} = 1 \checkmark$$

$$E(X) = \frac{6}{21} + 2 \cdot \frac{5}{21} + 3 \cdot \frac{4}{21} + 4 \cdot \frac{3}{21} + 5 \cdot \frac{2}{21} + 6 \cdot \frac{1}{21}$$

$$= \frac{1}{21} (6 + 10 + 12 + 12 + 10 + 6) = \frac{8}{3} = 2,6 \checkmark$$

$$D(X) = \sum x^2 \cdot p - (E(X))^2$$

$$= 1 \cdot \frac{6}{21} + 4 \cdot \frac{5}{21} + 9 \cdot \frac{4}{21} + \dots - \left(\frac{8}{3}\right)^2$$

$$= \frac{1}{21} (6 + 20 + 36 + 16 \cdot 3 + 25 \cdot 2 + 36) - \left(\frac{8}{3}\right)^2$$

$$= \frac{28}{3} - \left(\frac{8}{3}\right)^2 = \frac{20}{9} = 2,2 \checkmark$$

③ 3 cărți = 1 poezie

$$X \sim B\left(5, \frac{1}{6}\right)$$

$$\rightarrow 3 \text{ poezii} = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216}$$

$$P(X=2) = ?$$

$$P(X=2) = \binom{5}{2} \left(\frac{1}{216}\right)^2 \left(1 - \frac{1}{216}\right)^3 = 0,00021137 \checkmark$$

$$X \sim \begin{pmatrix} 1 & 2 & 3 & \dots & n & \dots \\ p & p^2 & p^3 & \dots & p^n & \dots \end{pmatrix}, p < 1$$

$$p = ? , E(X) = ? , D(X) = ?$$

$$p + p^2 + p^3 + \dots + p^n = 1$$

$$\sum_{n=1}^{\infty} p^n = 1$$

$$\frac{1}{1-p} = \sum_{n=1}^{\infty} p^n = \left(\frac{1}{1-p} - 1 \right) \quad \text{with } p=0 \quad \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \dots + x^n = \frac{1}{1-x}$$

$$1 = \frac{1}{1-p} - 1$$

$$2 = \frac{1}{1-p} \quad \Rightarrow \quad \frac{1}{2} = 1-p \Rightarrow \boxed{p = \frac{1}{2}}$$

$$E(X) = \sum_{n=1}^{\infty} n \cdot p^n$$

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \dots + x^n = \frac{1}{1-x}$$

$$\sum_{n=1}^{\infty} n x^{n-1} = 0 + 1 + 2x + \dots + n x^{n-1} = \frac{1}{(1-x)^2} \cdot x$$

$$\sum_{n=1}^{\infty} n x^n = 0 + x + 2x^2 + \dots + n x^n = \frac{x}{(1-x)^2}$$

$$E(X) \left(p = \frac{1}{2} \right) = \frac{\frac{1}{2}}{\left(1 - \frac{1}{2} \right)^2} = \left(\frac{\frac{1}{2}}{\frac{1}{4}} \right) = 2$$

$$D(X) = \sum n^2 p - (E(X))^2$$

$$\sum_{n=1}^{\infty} n x^n = 0 + x + 2x^2 + \dots + n x^n = \frac{x}{(1-x)^2}$$

$$\sum_{n=1}^{\infty} n^2 x^{n-1} = 0 + 1 + 4x + \dots + n^2 x^{n-1} = (*)$$

$$(*) = \frac{1 \cdot (1-x)^2 + x(2(1-x)(1))}{(1-x)^4} = \frac{(1-x)^2 - 2x - 2x^2}{(1-x)^4}$$

MAN UTD by farvac

$$\frac{(1-x)^2 + 2x(1-x)}{(1-x)^4} = \frac{(1-x)(1-x+2x)}{(1-x)^4} = \frac{1+x}{(1-x)^3}$$

$$\sum_{n=1}^{\infty} n^2 x^{n-1} = \frac{1+x}{(1-x)^3} \cdot x$$

$$\sum_{n=1}^{\infty} n^2 x^n = \frac{x(1+x)}{(1-x)^3} \quad \left(p = \frac{1}{2}\right) = 6$$

$$D(x) = 6 - 2^2 = 2 //$$

③ 20 dijelova $n = 20$
 tj. kvota 0,3 barem jednog

$$P(X \geq 1) = 0,3 \quad \text{Poisson}$$

$$P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

$$0,3 = P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{\lambda^0}{0!} e^{-\lambda}$$

$$0,3 = 1 - e^{-\lambda}$$

$$0,7 = e^{-\lambda} / \ln$$

$$\ln 0,7 = -\lambda$$

$$\lambda = -\ln 0,7 = 0,35667$$

$$\lambda = n \cdot p \Rightarrow p = \frac{\lambda}{n} = \frac{0,35667}{20} = 0,017833 //$$

II način

$$X \sim B(20, p)$$

$$P(X = 0) = \binom{20}{0} p^0 (1-p)^{20} = (1-p)^{20}$$

$g \equiv$ tj. da je
 pojedini uređaj
 porvaren

$$P(X = 0) = 1 - \underbrace{P(X \geq 1)}_{0,3} = 0,7$$

$$g^{20} = 0,7 / g$$

$$g = \sqrt[20]{0,7} = 0,98232$$

$$1 - p = 0,98232$$

$$p = 1 - 0,98232 = 0,01767 //$$

$\left. \begin{matrix} 17 \\ 12 \\ 20 \end{matrix} \right\} 5$

odabir prve c

$$X \sim \left(\begin{matrix} 2 & 3 & 4 & 5 \\ \frac{2}{5} \cdot \frac{1}{2} & \frac{2}{5} \cdot \frac{2}{4} & \frac{2}{5} \cdot \frac{2}{3} \cdot \frac{1}{2} & \frac{2}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} \cdot \frac{2}{2} \\ C & C & C & C \end{matrix} \right)$$

$$\frac{1}{10} \quad \frac{1}{5} = \frac{2}{10} \quad \frac{3}{10} \quad \frac{2}{5} = \frac{4}{10}$$

$$\frac{10}{10} = 1$$

$$E(X) = \frac{1}{10} (2 \cdot 1 + 3 \cdot 2 + 4 \cdot 3 + 5 \cdot 4) = 4 //$$

$$D(X) = \frac{1}{10} (4 \cdot 1 + 9 \cdot 2 + 16 \cdot 3 + 25 \cdot 4) - 4^2 = 17 - 16 = 1 //$$

③ $p = 0,10$
 $P(X \geq 1) = 0,469$

$X \sim B(n, 0,10)$
 $P(X = \sum) = \binom{n}{\sum} p^{\sum} (1-p)^{n-\sum}, \sum = 0, 1, 2, \dots$

$P(X \geq 1) = 1 - P(X = 0)$
 $1, 2, 3, \dots$

$$0,469 = 1 - \binom{n}{0} p^0 (1-p)^{n-0}$$

$$0,531 = (0,9)^n / \log_{0,9}$$

$$\log_{0,9} 0,531 = \log_{0,9} 0,9^n = n$$

$$n = \log_{0,9} 0,531 = 6,0078 //$$

Treba ispitati 6 ljudi //

MAN UTD
by Ferhac

II uaci
=>

Poisson

$$0.469 = 1 - e^{-\lambda}$$

$$0.531 = e^{-\lambda} / e$$

$$\lambda = 0.63299$$

$$p = 0.10$$

$$\lambda = n \cdot p$$

$$n = \frac{\lambda}{0.10} = \frac{0.63299}{0.10} = 6.3299$$

Treba ispitati 6 ljudi //