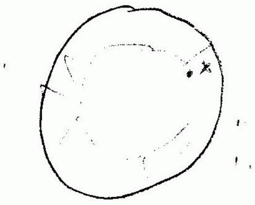
II. MI IZ VIS-a (8.5.2008.)

① $X \sim d(T, \square)$



$$F(x) = P(X < x) = \frac{m(\circledcirc)}{m(\bigcirc)} = \frac{\pi - (1-x)^2\pi}{\pi} = 2x - x^2$$

$0 \le x \le 1$

$$f(x) = F'(x) = 2 - 2x$$

$$E(X) = \int_0^1 x f(x)\, dx = \int_0^1 2x - 2x^2 dx = \dots = \frac{1}{3}$$

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② $Y = |X - 2|$

$X \sim f(x) = e^{-x}, \quad x > 0$

$G(y) = P(y < |X-2|)$

$\psi : \mathbb{R} \to \mathbb{R} \quad \psi(x) = |x-2|$

1) $x \in \langle 0, 2]$

$y(x) = -x + 2 \qquad x = -y + 2 \qquad x \in [0, 2]$

$$g_1(y) = \left|\frac{\partial x}{\partial y}\right| f(y) = f(y) = e^{y-2} \qquad y \in [0, 2]$$

2) $x \in [2, \infty)$

$y(x) = x - 2 \qquad y \in [0, +\infty)$

$$g_2(y) = \left|\frac{\partial x}{\partial y}\right| f(y) = e^{-2-y} \qquad y \in [0, +\infty)$$

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$$g(y) = g_1(y) + g_2(y) = \begin{cases} e^{-2}(e^{y} + e^{-y}) & y \in \langle 0, 2] \\ e^{-2-y} & y \in \langle 2, +\infty) \end{cases}$$

③ (a) $X \sim \mathcal{E}(\lambda)$

$f(x) = \lambda e^{-\lambda x} \quad x > 0$

$E(X) = \dfrac{1}{\lambda}$

$F(x) = 1 - e^{-\lambda x}$ u PARCIJALNOM

$$P(X < E(X)) = P\left(X < \frac{1}{\lambda}\right) = F\left(\frac{1}{\lambda}\right) = \underline{1 - e^{-1}}$$

(b) $X \sim \mathcal{E}(\lambda)$

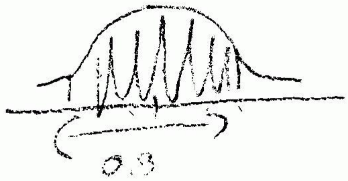
$E(X) = 3 \quad \Rightarrow \quad \lambda = \frac{1}{3}$

$$P(2 < X < 3 \mid X > 2) = \frac{P(2 < X < 3)}{P(X > 2)} =$$

$$= \frac{F(3) - F(2)}{1 - F(2)} = \frac{1 - e^{-1} - 1 + e^{-\frac{2}{3}}}{e^{-\frac{2}{3}}} = \underline{1 - e^{-\frac{1}{3}}}$$

④ $\mathcal{N}(a, \sigma)$

a) PRAVILO 3σ - KNJIŽICA STR 44



b) $X \sim \mathcal{N}(370, \sigma^2)$

$P(10 < X < 730) = 0{,}9973 \quad \approx 3\sigma$

$P(450 < X) =$

$3\sigma = 360 \rightarrow \sigma = 120$

$P(|X - a| < 3\sigma) = 0{,}9973$

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$$P(450 < X) = P\left(\frac{450 - 370}{120^2} < \frac{X - 370}{120^2}\right) =$$

$$\frac{1}{2}\left(1 - \Phi^*(\text{—})\right) = 0{,}252$$

(4c)

$X \sim N(0,1)$ $\quad f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$

$Y = X^2$

$Y > 0 \Rightarrow g(y) = 0 \quad y < 0$

$\varphi : \mathbb{R} \to \mathbb{R}$

$\varphi(x) = x^2$

$$\boxed{\begin{aligned} &F_Y(y) = 0 \quad x \le 0 \\ &F_Y(y) = P(X^2 < y) = P(-\sqrt{y} < X < \sqrt{y}) \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\sqrt{y}}^{\sqrt{y}} e^{-\frac{1}{2}x^2}\,dx = \frac{2}{\sqrt{2\pi}} \int_0^{\sqrt{y}} e^{-\frac{1}{2}x^2}\,dx \end{aligned}}$$

$$\boxed{\begin{aligned} &f_Y(y) = \frac{2}{\sqrt{2\pi}} e^{-\frac{1}{2}y} \frac{1}{2\sqrt{y}} \\ &= \end{aligned}}$$

str 103

1) $x \in (-\infty, 0)$ $\qquad$ 2) $x \in (0, +\infty)$

$y = x^2 \quad y \in (0, +\infty)$ $\qquad$ $x = \sqrt{y}$

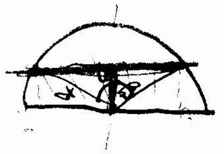
$x = -\sqrt{y}$

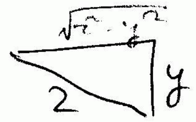
$g_1(y) = \left|\frac{\partial x}{\partial y}\right| f(y) =$ $\qquad$ $g_2(y) = \frac{1}{2\sqrt{y}} e^{-\frac{1}{2}y}$

$= \frac{\partial 1}{2\sqrt{y}\sqrt{2\pi}} e^{-\frac{1}{2}y}$ $\qquad$ $g(y) = \frac{1}{2\sqrt{y}\sqrt{2\pi}} e^{-\frac{1}{2}y}$

$$g(y) = g_1(y) + g_2(y) = \frac{1}{\sqrt{2\pi y}} e^{-\frac{1}{2}y} \qquad y \in (0, +\infty)$$

(5) $f(x,y) = C \qquad x^2 + y^2 \le 4 \quad y > 0$





$$1 = \iint_{(1)} c\,dx\,dy = C\frac{2^2\pi}{2} \quad \therefore \quad C = \frac{1}{2\pi}$$

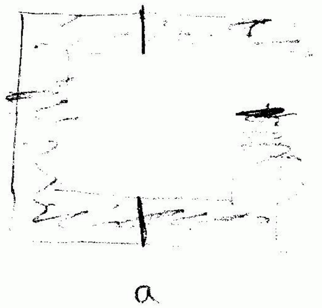
$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \frac{1}{2\pi}\,dx = \frac{1}{\pi}\sqrt{4-y^2} \quad \boxed{0 \le y \le 2}$$

$$E(Y) = \int_0^2 y f_Y(y)\,dy = \frac{1}{\pi}\int_0^2 y\sqrt{4-y^2}\,dy$$

$$= \begin{bmatrix} t = 4 - y^2 & y = 0 \Rightarrow t = 4 \\ dt = -2y\,dy & y = 2 \Rightarrow t = 0 \end{bmatrix} = -\frac{1}{2\pi}\int_4^0 \sqrt{t}\,dt = \frac{t^{3/2}}{-2\pi\frac{3}{2}}\Big|_4^0 = \frac{8}{3\pi}$$

⑥ a) $f(x|y) = \dfrac{f(x,y)}{\int_{-\infty}^{\infty} f(x,y)\,dx}$

b)



$X \quad f_X(x) = 1, \text{ na } x \in [1,2]$

$$P(A \mid X=a) = \frac{m(▨)}{m(□)} = \frac{a^2-(a-1)^2}{a^2} = \frac{2a-1}{a^2} = $$

$$= \frac{2}{a} - \frac{1}{a^2}$$

$\boxed{1 \le a \le 2}$

$$P(A) = \int_{-\infty}^{\infty} \left(\frac{2}{x} - \frac{1}{x^2}\right) f_X(x)\,dx = \int_1^2 \frac{2}{x} - \frac{1}{x^2}\,dx =$$

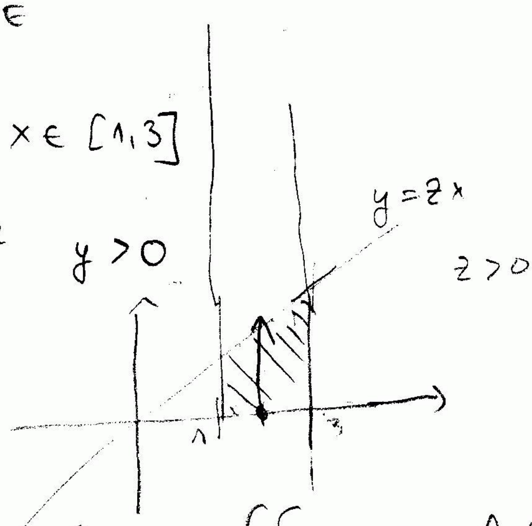
$$= 2\ln x + \frac{1}{x}\Big|_1^2 = 2\ln 2 - \frac{1}{2}$$

⑦ X, Y NEZAVISNE

$X \quad f_X(x) = \frac{1}{2} \quad x \in [1,3]$

$Y \quad f_Y(y) = e^{-y} \quad y > 0$

$Z = \dfrac{Y}{X} \quad zx = y$



$G_Z(z) = P(Z<z)$

$$G_Z(z) = \iint_{G_z} f(x,y)\,dx\,dy = \iint_{G_z} f_X(x) \cdot f_Y(y)\,dx\,dy =$$

$$= \frac{1}{2}\int_1^3 \int_0^{zx} e^{-y}\,dy\,dx =$$

$$= \frac{1}{2}\int_1^3 -e^{-zx} + 1\,dx = 1 + \frac{e^{-zx}}{2z}\Big|_1^3 = 1 + \frac{1}{2z}\left(e^{-3z} - e^{-z}\right)$$