Ovaj PDF sadrži skenirane postupke 1. KPZ-a 2012-2013.

Zadaci su poredani po potpunoj vjerojatnosti, Bayesova formula i na kraju ostali zadaci.

Riješio i ustupio na skeniranje

fer0vac

skenirao

SipE

POTPUNA VJ. I BAYESOVA FORM. (1)

2012-1-13hA

80 ljudi

50Ž — 100% sladoled voli sladoled

30M — 90% sladoled

A = "ispitana osoba voli sladoled"

H_1 = " -11- -11- je Ž"

H_2 = " -11- -11- je M"

$P(H_1) = \frac{50}{80} = 0,625$ $\quad P(H_2) = \frac{30}{80} = 0,375$

$\Sigma = 1$ ✓

$P(A|H_1) = 1$ (100%) $\quad P(A|H_2) = 0,9$

$P(A) = \sum P(H_i) P(A|H_i) = 0,625 \cdot 1 + 0,375 \cdot 0,9 = 0,9625$

$P(H_2|A) = \frac{P(H_2) P(A|H_2)}{P(A)} = \frac{0,375 \cdot 0,9}{0,9625} = 0,3506$

2012-1-13hB

3C2B → prebačen 1C3B → izvučena

A = "izvučena je B"

H_1 = "prebačena je C"

H_2 = " -11- je B"

$P(H_1) = \frac{3}{5}$

$P(H_2) = \frac{2}{5}$ } $\Sigma = 1$

$P(A|H_1)^{2C3B} = \frac{3}{5}$

$P(A|H_2)^{1C4B} = \frac{4}{5}$

$P(A) = \sum P(H_i) P(A|H_i) = \frac{17}{25}$

$P(H_1|A) = \frac{P(H_1) P(A|H_1)}{P(A)} = \frac{\frac{3}{5} \cdot \frac{3}{5}}{\frac{11}{25}} = \frac{9}{17}$

2013-1-12hB

A = "C pobjedio"

H_0 = "A nije sudje" $\quad H_1$ = "A je sudje"

$P(H_0) = 0,5$ $\quad P(H_1) = 0,5$

$P(A|H_0) = 0,25$ $\quad P(A|H_1) = \frac{1}{3}$

$P(A) = 0,5 \cdot 0,25 + 0,5 \cdot \frac{1}{3} = \frac{7}{24}$

$P(H_0|A) = \frac{0,5 \cdot 0,25}{\frac{7}{24}} = \frac{3}{7}$ ✓

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by fervac

2013-1-12zA

A = "izvučena B"

H_1 = "izgubljena C"

H_2 = " -11- B"

$P(H_1) = \frac{n+2}{n+n+2} = \frac{n+2}{2n+2}$

$P(H_2) = \frac{n}{2n+2}$

$\frac{n+2}{2n+2} + \frac{n}{2n+2} = \frac{2n+2}{2n+2} = 1$ ✓

$P(A|H_1) = \frac{n}{n+1+n} = \frac{n}{2n+1}$ ((n+1)C, (n)B)

$P(A|H_2) = \frac{n-1}{n+2+n-1} = \frac{n-1}{2n+1}$ ((n+2)C, (n-1)B)

$P(A) = \sum_1^2 P(H_i) P(A|H_i) =$

$= \frac{n+2}{2n+2} \cdot \frac{n}{2n+1} + \frac{n}{2n+2} \cdot \frac{n-1}{2n+1}$

$= \frac{n^2+2n+n^2-n}{(2n+2)(2n+1)} = \frac{2n^2+n}{(2n+2)(2n+1)}$

$= \frac{n(2n+1)}{(2n+2)(2n+1)} = \frac{n}{2n+2} = P(A)$

$P(H_1|A) = \frac{P(H_1) \cdot P(A|H_1)}{P(A)} = \frac{\frac{n+2}{2n+2} \cdot \frac{n}{2n+1}}{\frac{n}{2n+2}} =$

$P(H_1|A) = \frac{n+2}{2n+1}$

2013-1-13zB

6-6, 6-5, 6-4, 6-3, 6-2, 6-1 | 3-1, 3-2, 3-3, 3-4, 3-5, 3-6

Pobjeda I

A = " W I "

H_1 = " um /3 " $P(H_1) = \frac{1}{3}$ ~~$\frac{5}{9}$~~

H_2 = " um. mije /3 " $P(H_2) = \frac{2}{3}$ ~~$\frac{4}{9}$~~

$P(A|H_1) = 1$

$P(A|H_2) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

I: PK, 5P, 66 → $= \frac{2}{4} = \frac{1}{2}$; PK, 5P, OO, 66 → $= \frac{1}{2}$

$P(A) = \frac{1}{3}$

$P(A) = \frac{5}{9} \cdot 1 + \frac{4}{9} \cdot \frac{1}{4} = \frac{2}{3}$

$P(\bar{A}) = \frac{2}{3}$

$P(H_1|A) = \frac{\frac{5}{9} \cdot 1}{2/3} = \frac{5}{6}$

10 uč. 6 odličnih 4 vrlo dobra
0,9 0,7

POTPUNA VJ
&
BAYESOVA FORM
2

A = " odabrali smo 2 uč. "
H_1 = " oba su odlična " $P(H_1) = \frac{\binom{6}{2}}{\binom{10}{2}} = \frac{1}{3} = \frac{5}{15}$ ili $\frac{6}{10}\cdot\frac{5}{9} = \frac{1}{3}$
H_2 = " -11- vrlo dobra "
H_3 = " 1 odl. i 1. vrlo " $P(H_2) = \frac{\binom{4}{2}}{\binom{10}{2}} = \frac{2}{15}$ $P(H_3) = \frac{\binom{6}{1}\binom{4}{1}}{\binom{10}{2}} = \frac{8}{15}$

$\Sigma = 1$ ✓

$P(A|H_1) = 0,9 \cdot 0,9$ $P(A|H_2) = 0,7 \cdot 0,7$ $P(A|H_3) = 0,9 \cdot 0,7$

$$P(A) = \sum_{i=1}^{3} P(H_i)\,P(A|H_i) = \frac{1}{3}(0,9)^2 + \frac{2}{15}(0,7)^2 + \frac{8}{15}(0,63)$$

$P(A) = 0,671\dot{3}$

2013-1-12hB

8 strijelaca
3 odl 5 dobrih
0,9 0,6

A = " meta je pogođena barem jednom (1+2) "
H_1 = " dva odlična su izabrana "
H_2 = " dva dobra -11- "
H_3 = " jedan odl. i jedan dobar -11- "

$P(H_1) = \frac{\binom{3}{2}}{\binom{8}{2}} = \frac{3}{28}$ $P(H_2) = \frac{\binom{5}{2}}{\binom{8}{2}} = \frac{5}{14}$ $P(H_3) = \frac{\binom{3}{1}\binom{5}{1}}{\binom{8}{2}} = \frac{15}{28}$

$\frac{3}{8}\cdot\frac{2}{7} = \frac{3}{28}$ $\frac{5}{8}\cdot\frac{4}{7} = \frac{5}{14}$ $\Sigma = 1$ ✓

$P(A|H_1) = \underset{\text{oba pog.}}{0,9 \cdot 0,9} + \underset{1 \text{ pog. } 2 \text{ pro.}}{0,9 \cdot 0,1} + \underset{1 \text{ pro } 2 \text{ pog.}}{0,1 \cdot 0,9} = 0,99$

$P(A|H_2) = \overset{1+\ 2+}{0,6 \cdot 0,6} + \overset{1+\ 2-}{0,6 \cdot 0,4} + \overset{1-\ 2+}{0,4 \cdot 0,6} = 0,84$

$P(A|H_3) = \overset{1+\ 2+}{0,9 \cdot 0,6} + \overset{1+\ 2-}{0,9 \cdot 0,4} + \overset{1-\ 2+}{0,1\ 0,6} = 0,96$

$$P(A) = \sum_{i=1}^{3} P(H_i)\cdot P(A|H_i) = \frac{3}{28}\cdot 0,99 + \frac{5}{14}\,0,84 + \frac{15}{28}\cdot 0,96 = 0,9204$$

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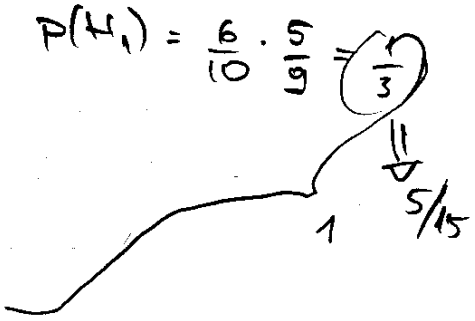
2013-1-13h A 10 kov 6×① i 4×②

A = "sve izvučene kov. istog iznosa"

H_1: "prvo izvlač. 2 kovanice po ①" $\Rightarrow P(H_1) = \frac{6}{10} \cdot \frac{5}{9} = \frac{1}{3}$

H_2: "①②" $P(H_2) = 2 \cdot \frac{6}{10} \cdot \frac{4}{9} = \frac{8}{15}$ (①② i ②①)

H_3: "②②" $P(H_3) = \frac{4}{10} \cdot \frac{3}{9} = \frac{2}{15}$



ostale su 4 po ① i 4 po ②, ali moraju biti istog iznosa

$P(A|H_1) = \frac{4}{8} \cdot \frac{3}{7} = \frac{3}{14}$

$P(A|H_2) = 0$

$P(A|H_3) = \frac{2}{8} \cdot \frac{1}{7} = \frac{1}{28}$ → ostale su 6 po ① i 2 po ②

$P(A) = \frac{1}{3} \cdot \frac{3}{14} + 0 + \frac{2}{15} \cdot \frac{1}{28} = \frac{8}{105}$

$P(H_1|A) = \frac{\frac{1}{3} \cdot \frac{3}{14}}{\frac{8}{105}} = \frac{15}{16}$

2 32 ↘ 5

$P(A) = \{\text{sve karte iste boje}\} = \dfrac{\binom{8}{5}\binom{4}{1}}{\binom{32}{5}} = 0011$ (izbor karte, izbor boja)

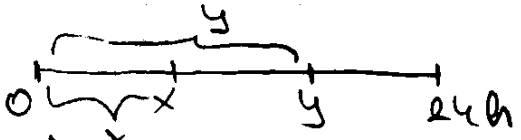
$P(\text{sve boje}) = \dfrac{\binom{4}{1}\binom{8}{2}\cdot\binom{8}{1}\binom{8}{1}}{\binom{32}{5}} = 0{,}2847$ (izbor boje gdje će biti 2 karte)

1,1,1,2

3 $x, y \in [0,24]$ $\quad \Omega \in [0,24]^2$

$x \equiv$ 1 sat $\quad y \equiv$ 2 sata

(a) $x < y$



npr. x u 9h $\Rightarrow$ 10h

y mora doći između 9 i 10

$y - x < 1 \Rightarrow y < 1 + x$

$x < y \Rightarrow y > x$ iznad $x = y$

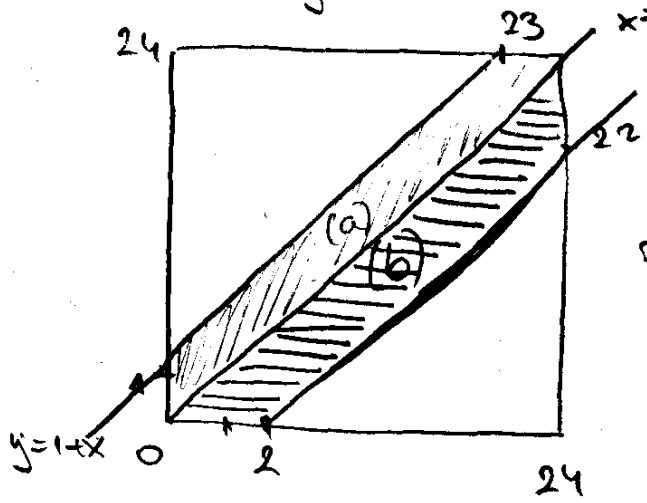
(b) $y < x$

npr y u 13h $\Rightarrow$ 15h

x mora doći između 13 i 15

$x - y < 2 \Rightarrow x - 2 < y$

$y < x$



$$P(A) = \frac{24^2 - \frac{23^2}{2} - \frac{22^2}{2}}{24^2} = 0{,}1206$$

2) 7 puta kocka

(a) 3-puta 6-ica $P(A) = \binom{7}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^4$ → 6-ica, u svojim bacanjima; ostala 4 bacanja

$P(A) = 0{,}078$

(b) pojave svi brojevi — broj koji se pojavio 2 puta; ostali 5

$P(B) = \left(\frac{1}{6}\right)^6\left(\frac{1}{6}\right)^1 \cdot \binom{7}{2} \cdot 6! = 0{,}054$

3) 200m

$v = 1200\,m/min$

22:00 ⇔ 22:30

30 min = 1800 s

$x, y \in [0, 1800]$

$v = \frac{s}{t} \Rightarrow t = \frac{s}{v} = \frac{200}{1200} = \frac{1}{6}$ min

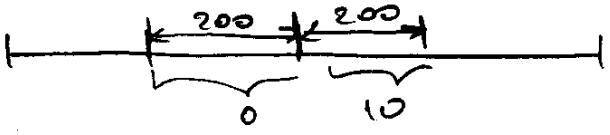
$t = \frac{1}{6}$ min $\Rightarrow t = 10$ s

Vlaku je potrebno da prođe svoju dužinu 10 s.

$P(B) = \frac{6 \cdot \frac{7!}{2!}}{6^7}$ — znamenka koja je 2 puta; sve permutacije riječi

A A B C D E F

⇓ $\frac{7!}{2!}$ (slova)

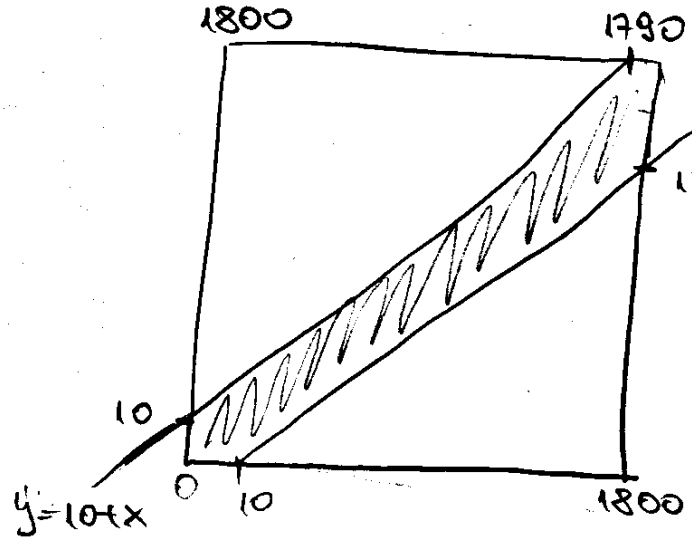


$|y - x| < 10$ ako je razlika manja od 10 s dogoditi će se sudar

$-10 < y - x < 10 \quad /+x$

$x - 10 < y < 10 + x$

$y = x - 10$ $\quad y = 10 + x$



$P(A) = \frac{1800^2 - \frac{1790^2}{2} - \frac{1790^2}{2}}{1800^2}$

$P(A) = 0{,}01108$

2 5 žatova 7 osoba

a) prvi žat točno 3 osobe

$$P(A) = \frac{\binom{7}{3}\binom{4}{4}}{5^7}$$

$\binom{7}{3}$ → izbor 3 osobe za prvi žat; $\binom{4}{4}$ → 4 osobe → izbor žatova II, III, IV, V, ~~VI~~

b) svaki žat barem jedna osoba

1113 ili 1122

$$P(B) = \frac{\binom{7}{3}\binom{5}{1}\cdot 4!}{5^7} + \frac{\binom{7}{2}\binom{5}{2}\binom{3}{2}\cdot 3!}{5^7}$$

$\binom{7}{3}$ → izbor 3 osobe za prvi žat; $\binom{5}{1}$ → 1 osoba za drugi žat; $4!$ → preostala žata

$\binom{7}{2}$ → 2 osobe za prvi žat; $\binom{5}{2}$ → 2 osobe za drugi žat; $\binom{3}{2}$ → 2 osobe za treći žat; $3!$ → preostala žata

prva osoba 4 preostala žata,
drug -||- 3 -||- -||-
⋮

$$+ \frac{\binom{7}{2}\binom{5}{1}\binom{5}{2}\binom{4}{1}3!}{5^7}$$

KRIVO jer su žatovi jednake ravnopravnosti, tj. međusobno su ravnopravni pa je $\binom{5}{2}$

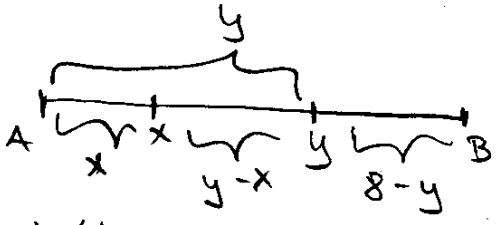
DA je pisalo npr. jedan žat 3 osobe a drugi 2 osobe a ostali BLA BLA

tada $\binom{7}{3}\binom{5}{1}\binom{4}{2}\binom{2}{1}2!$ ostalo

3 osobe | 2 osobe | ŽATOVI

3 $|\overline{AB}| = 8$

(a) $x < y$

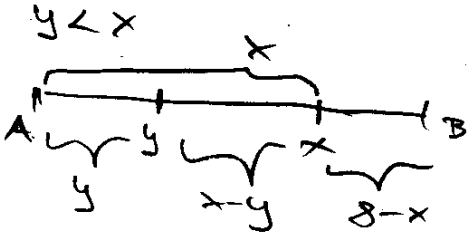


$x < 4$

$y - x < 4 \Rightarrow y < 4 + x$

$8 - y < 4 \Rightarrow y > 4$

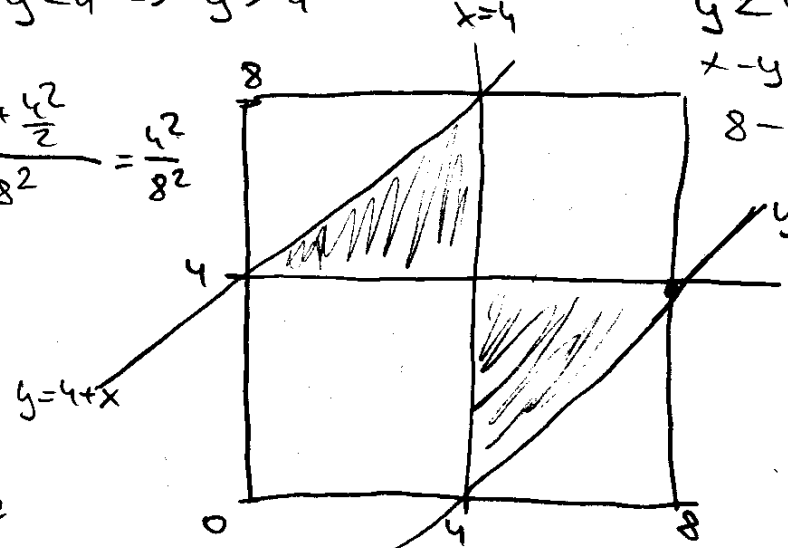
(b) $y < x$



$y < 4$

$x - y < 4 \Rightarrow y > x - 4$

$8 - x < 4 \Rightarrow 4 < x$



$$P(A) = \frac{\frac{4^2}{2} + \frac{4^2}{2}}{8^2} = \frac{4^2}{8^2}$$

$$P(A) = \frac{1}{4}$$

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by ferdvac

1KP2
2012
1343

a) u prvom vagonu točno 3 osobe

$$P(A) = \frac{\binom{6}{3}\,3^3}{4^6} = 0,1318$$

b) u svakom vagonu barem jedna osoba

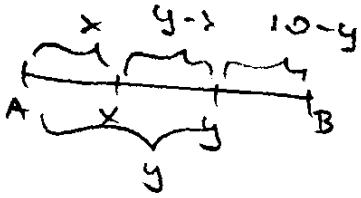
1113 + 1122

$$P(B) = \frac{\binom{6}{3}\binom{4}{1}\,3!}{4^6} + \frac{\binom{6}{2}\binom{4}{2}\binom{4}{2}\,2!}{4^6} = 0,3808$$

($\binom{4}{1}$: izbor vagona u kojem će biti 3 osobe; ostali.
$\binom{6}{2}$: osobe; $\binom{4}{2}\binom{4}{2}2!$: ravnopravni vagoni)

3) $|AB| = 10$

$x < y$



$x > 2$

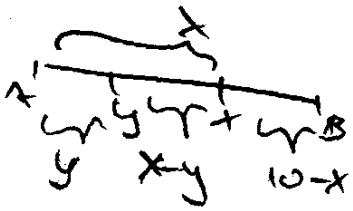
$y - x > 2 \Rightarrow y > 2 + x$

$10 - y > 2 \Rightarrow -y > 2 - 10$

$-y > -8$

$y < 8$

$y < x$



$y > 2$

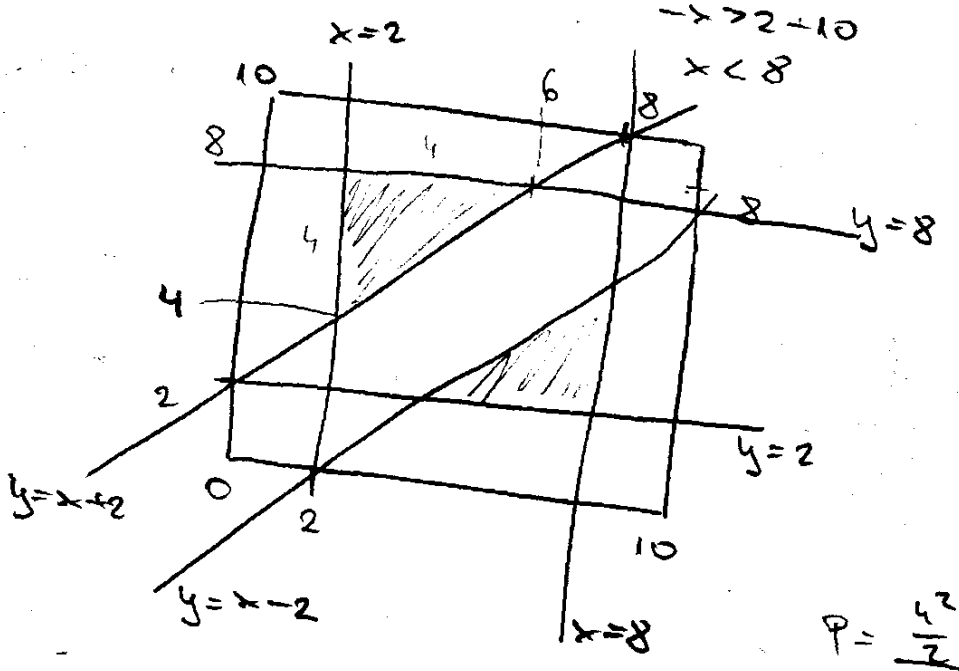
$x - y > 2 \Rightarrow -y > 2 - x$

$y < x - 2$

$10 - x > 2$

$-x > 2 - 10$

$x < 8$



$$P = \frac{\frac{4^2}{2} + \frac{4^2}{2}}{10^2} = \frac{4}{25} = 0,16$$

$y = x + 2$ i $x = 2$

$y = 2 + 2 = 4$

$y = x + 2$ i $y = 8$

$8 = x + 2 \Rightarrow x = 6$

[2] 52 ↘ 3

(a) barem 1. AS (1 − P(0 aceva))

$$P(A) = 1 - \frac{\binom{4}{0}\binom{48}{3}}{\binom{52}{3}}$$

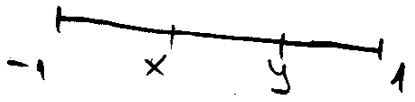
(b) jedna 2-jka, 3-jka, 4-ka

$$P(B) = \frac{\binom{4}{1}\binom{4}{1}\binom{4}{1}}{\binom{52}{3}}$$

(c) sve karte različite boje (odabir boja)

$$P(C) = \frac{\binom{4}{3}\binom{13}{1}\binom{13}{1}\binom{13}{1}}{\binom{52}{3}}$$

[3] $\subset -1, 1 \supset$



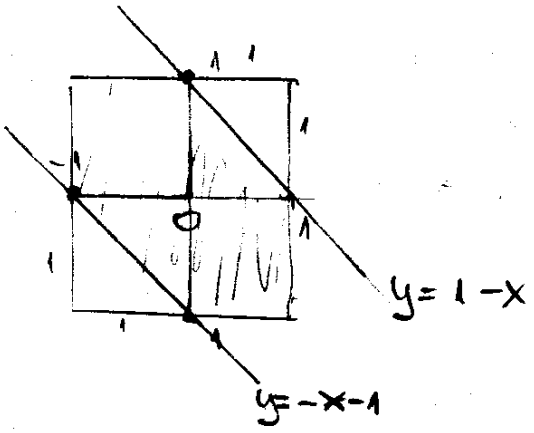
$|x+y| < 1$

$-1 < x+y < 1 \quad / -x$

$\rightarrow -1 < y < 1-x$

$y = -x-1 \qquad y = 1-x$

$$P(A) = \frac{2^2 - \frac{1^2}{2}\frac{1^1}{2}}{2^2} = \frac{4-1}{4} = \frac{3}{4}$$



2) 52 karte ∋ 4

(a) točno jedan $\clubsuit$ $\quad P(A) = \dfrac{\binom{\clubsuit}{1}\binom{\text{ostale}}{3}}{\binom{52}{4}}$

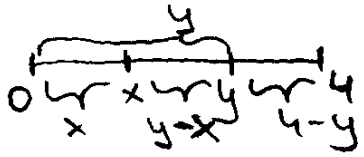
(b) barem dva asa (2AS + 3AS + 4AS tj. 1 − 0AS − 1AS)

$$P(B) = 1 - \frac{\binom{4}{0}\binom{48}{4}}{\binom{52}{4}} - \frac{\binom{4}{1}\binom{48}{3}}{\binom{52}{4}}$$

(c) sve karte iste boje $\quad P(C) = \dfrac{\binom{\text{izbor karata}}{13}\binom{\text{izbor boje}}{1}}{\binom{52}{4}}$

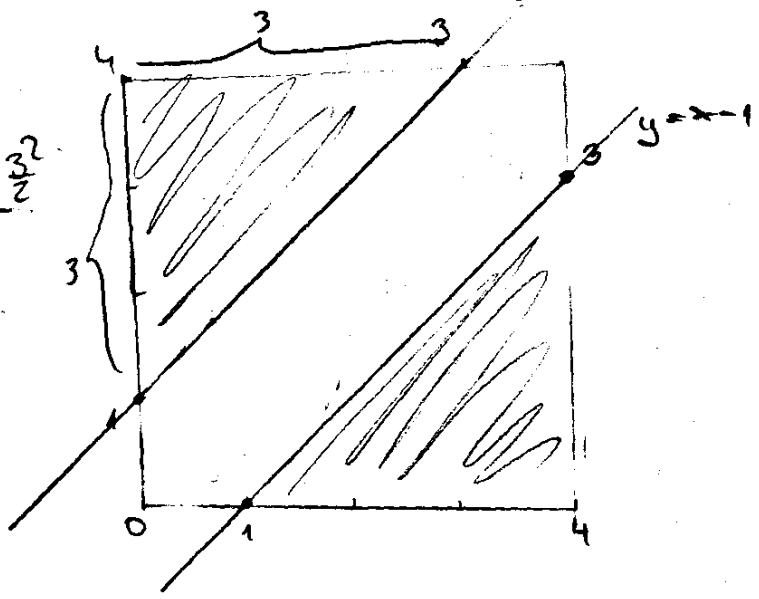
3) $[0,4]$ biramo 2 realna broja $\quad$ tj. $|y-x|>1$?

(a) $x<y$



$$|y-x|>1$$
$$-1 > y-x > 1 \quad /+x$$
$$x-1 > y > 1+x$$

$y = x-1$ $\quad$ $y = 1+x$ $\quad$ $y = 1+x$



$$P(A) = \frac{m(A)}{m(\Omega)} = \frac{\frac{3^2}{2} + \frac{3^2}{2}}{4^2}$$

$$P(A) = \frac{9}{16}$$

2: 5C, 6B, 7Z ↘ 4

(a) nema crnih $P(A) = \frac{\binom{5}{0}\binom{13}{4}}{\binom{18}{4}}$

(b) nisu zastupljene sve boje

I 1 1 2
II 1 2 1
III 2 1 1

1 − P(zastupljene su sve boje)

$$P(B) = 1 - \underbrace{\frac{\binom{5}{1}\binom{6}{1}\binom{7}{2}}{\binom{18}{4}}}_{I} - \underbrace{\frac{\binom{5}{1}\binom{6}{2}\binom{7}{1}}{\binom{18}{4}}}_{II} - \underbrace{\frac{\binom{5}{2}\binom{6}{1}\binom{7}{1}}{\binom{18}{4}}}_{III}$$

3 $ax^2+bx+1=0$ $a,b \in [0,1]$ nema realna rj

$ax^2+bx+c=0$

$a=a \quad b=b \quad c=1$

$b^2-4ac<0$

$b^2-4a\cdot 1<0$

$b^2-4a<0$

$y^2-4x<0$

$y^2<4x$

$y^2=4x$

x	y
0	0
1	2
1	−2

ovo što je ma kvadrat stavi na x-os

$D = b^2-4ac<0$

$y^2<4x$

npr. $T(1,1)$

$1^2<4$ ✓ vrijedi

$T(0,1)$

$1<0$ NE

$T(1,0)$

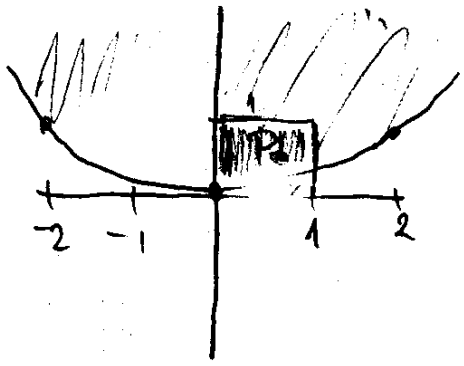
$0<4$ ✓

b^2 ×2 $-4a<0$

$-4y<0$

$-4y<-x^2$

$y>\frac{1}{4}x^2$

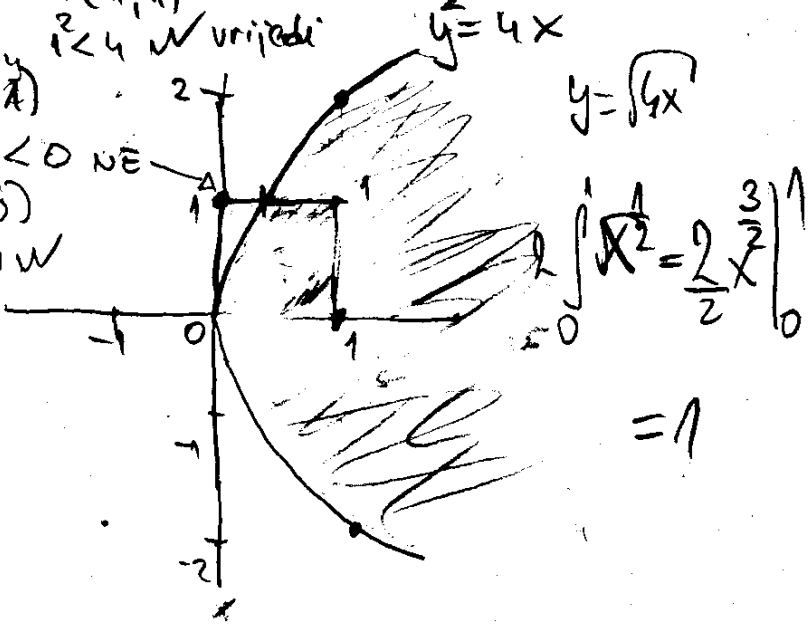


$y=\frac{1}{4}x^2$

$P_\bullet = P_\square - P_\curvearrowright = 1^2 - \int_0^1 \frac{1}{4}x^2\,dx$

$P_\bullet = 1 - \frac{1}{4}\frac{x^3}{3}\Big|_0^1 = 1 - \frac{1}{4}\frac{1}{3} = 1 - \frac{1}{12}$

$P_\bullet = \frac{11}{12}$



$2\int_0^1 x^{\frac{1}{2}} = 2\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\Big|_0^1$

$=1$

(2) 3C, 4B, 5Z ↓4

(a) najviše dvije crne ⇒ 0, 1 ili 2 C

$$P(A) = \frac{\binom{3}{0}\binom{9}{4}}{\binom{12}{4}} + \frac{\binom{3}{1}\binom{9}{3}}{\binom{12}{4}} + \frac{\binom{3}{2}\binom{9}{2}}{\binom{12}{4}} = \frac{54}{55}$$

ili

$$P(A) = 1 - \frac{\binom{3}{3}\binom{9}{1}}{\binom{12}{4}}$$ (1 − P(izvučene su 3C i jedna bilo koja))

(b) zastupljene su sve boje (CBZ 112, CBB 121, CBZ 211)

$$P(B) = \frac{\binom{3}{1}\binom{4}{1}\binom{5}{2}}{\binom{12}{4}} + \frac{\binom{3}{1}\binom{4}{2}\binom{5}{1}}{\binom{12}{4}} + \frac{\binom{3}{2}\binom{4}{1}\binom{5}{1}}{\binom{12}{4}}$$

(3) $x^2 + ax + b = 0$ $a, b \in [0, 1]$ ima realna rj.

$Ax^2 + Bx + C = 0$ $D = B^2 - 4AC \geq 0$

$B^2 - 4AC \geq 0$

$A = 1$ $B = a$ $C = b$

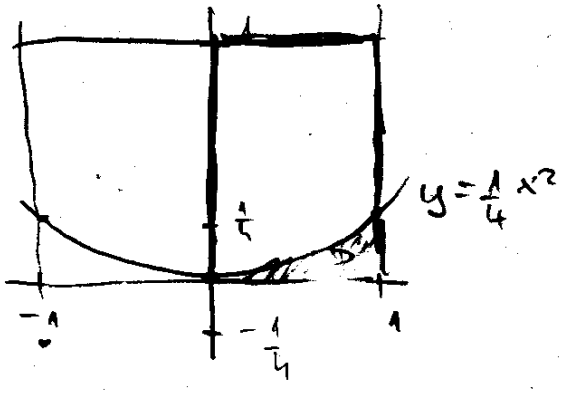
$a^2 - 4 \cdot 1 \cdot b \geq 0$

$-4b \geq -a^2 \ /(-1)$

$4b \leq a^2$

$b \leq \frac{1}{4}a^2$

$y = \frac{1}{4}x^2$



$$P = \int_0^1 \frac{1}{4}x^2 = \frac{1}{4} \cdot \frac{x^3}{3}\Big|_0^1 = \frac{1}{12}$$

MAN UTD
by farkvac