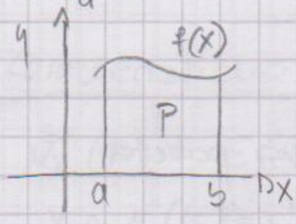
5, 6, 7 i 8 poglavlje
↳ ISPIT

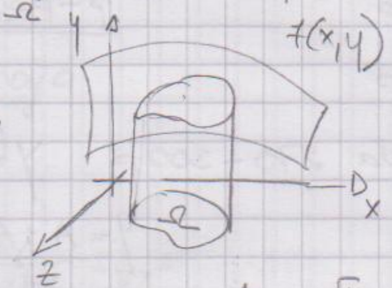
KONZULTACIJE – 20.04

DVOSTRUKI INTEGRALI – dvostruki užitak

$$\int_a^b f(x)\,dx \longmapsto \iint_\Omega f(x,y)\,dx\,dy$$



$\longmapsto$



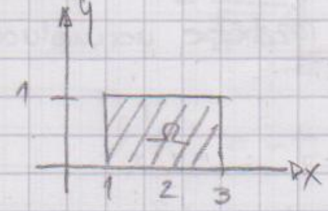
$$P = \iint_\Omega dx\,dy$$

$$\int_0^1 dx \int_x^{\operatorname{arctg} x} \sin(xy)\,dy \quad \text{ili} \quad \int_{-1}^1 dy \int_y^{\sqrt{y}} (x^2+y)\,dy$$

↳ integral u suprotnom poretku

GRANICE

Primjer 1.



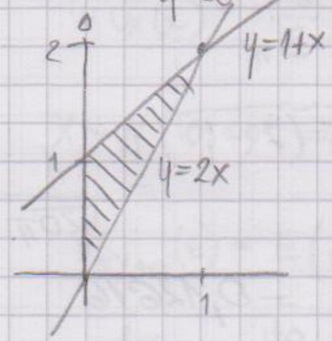
$$\int_1^3 dx \int_0^1 f(x,y)\,dy$$

suprotni poredak $\displaystyle\int_0^1 dy \int_1^3 f(x,y)\,dx$

Primjer 2. $y = 2x$

$y = 1+x$

$y - os$



$$\cancel{\int_0^1 dx \int_x^x f(x,y)\,dy}$$

$$\int_0^1 dx \int_{2x}^{1+x} f(x,y)\,dy$$

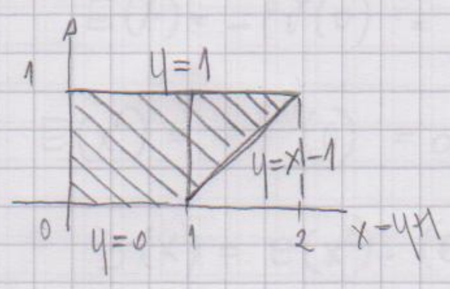
1. integral – konstante

općenito

– granice u 2. integralu su funkcije kojima je omeđeno

Primjer 3.

1. način $\displaystyle\int_0^1 dx \int_0^1 f(x,y)\,dy + \int_1^2 dx \int_{x-1}^1 f(x,y)\,dy$ – dijeli se



2. način $\displaystyle\int_0^1 dy \int_0^{y+1} f(x,y)\,dx$

→ potrebno podijeliti integral

RAČUNANJE

Primjer $\int_1^4 dx \int_0^{\sqrt{x}} \left(\frac{1}{x} + xy\right) dy = \int_1^4 \left(\frac{1}{x}\cdot y + x\frac{y^2}{2}\right)\Big|_0^{\sqrt{x}} dx =$

$$= \int_1^4 \left(\frac{\sqrt{x}}{x} + \frac{x^2}{2}\right) dx = \frac{x^{1/2}}{\frac{1}{2}} + \frac{1}{2}\frac{x^3}{3}\Big|_1^4 = \frac{13}{6}$$

– SLUČAJNI VEKTORI –

(X, Y) $\quad F(x,y) = P(X<x, Y<y)$

$$f(x,y) = \frac{\partial^2 F}{\partial x \partial y}$$

$$\boxed{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\, dx\, dy = 1}$$

$$P(a<X<b) = \int_a^b f(x)\, dx$$

$$\boxed{P((X,Y)\in D) = \iint_D f(x,y)\, dx\, dy}$$

$$\left.\begin{aligned} f_X(x) &= \int_{-\infty}^{\infty} f(x,y)\, dy \\ f_Y(y) &= \int_{-\infty}^{+\infty} f(x,y)\, dx \end{aligned}\right\} \text{marginalne gustoće}$$

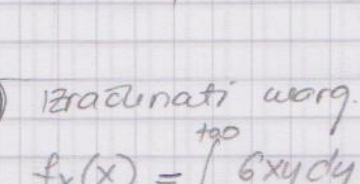
→ ispitivanje nezavisnosti:

$$f(x)\cdot f(y) = f(x,y)$$

↳ umnožak = gustoći na cijelom području

1 $f(x,y) = Cx\cdot y$ na $\Omega = \{(x,y): 0\le x\le 1,\ x^2 \le y \le 1\}$

a) odredi konstantu C $\qquad y = x^2 \quad y = 1$



$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} Cxy\, dx\, dy = \int_0^1 dx \int_{x^2}^1 Cxy\, dy = \int_0^1 Cx\frac{y^2}{2}\Big|_{x^2}^1 dx =$$

$$= \frac{C}{6} = 1 \qquad C = 6$$

b) Izračunati marg. gustoće i ispitaj nezavisnost

$$f_X(x) = \int_{-\infty}^{+\infty} 6xy\, dy = \int_{x^2}^1 6xy\, dy = \frac{6}{2}xy^2\Big|_{x^2}^1 = 3x - 3x^5, \quad x\in[0,1]$$

↳ potrebno je definirati za koje x vrijedi!!

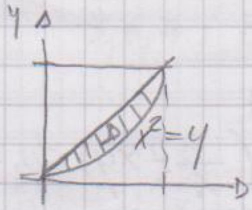
$$f_Y(y) = \int_{-\infty}^{\infty} 6xy\, dx = \cancel{\int_0^1 6xy\, dx} = \int_0^{\sqrt{y}} 6xy\, dx = 3x^2y\Big|_0^{\sqrt{y}} = 3y^2, \quad y\in[0,1]$$

→ X i Y su zavisne jer $f_X(x)\cdot f_Y(y) \neq f(x,y)$

c) $P(Y<X) = \iint_D f(x,y)\,dx\,dy$

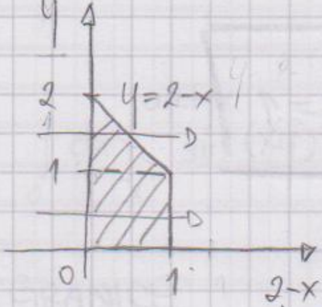
$= \iint 6xy\,dx\,dy = \int_0^1 dx \int_{x^2}^{x} 6xy\,dy$

$= \int_0^1 \frac{6}{2}xy^2 \Big|_{x^2}^{x} dx = \frac{1}{4}$ → nije isto što i površina !



$\boxed{2.}$ (X,Y) jednoliko razdioba na $\{(x,y): 0 \le x \le 1, 0 \le y < 2, x+y < 2\}$

Odredi marginalne gustoće i očekivanje $E(X,Y)$ $\qquad y = 2-x$



$f(x,y) = \frac{1}{P_\Omega} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$ $\qquad f = \frac{1}{m(\Omega)}$

$f_X(x) = \int_0^{2-x} \frac{2}{3}\,dy = \frac{2}{3}y \Big|_0^{2-x} = \frac{2}{3}(2-x), \quad x \in [0,1)$

$\cancel{f_Y(y) = \int \frac{2}{3}dx + \int \frac{2}{3}\,dx}$ → NE!

$f_Y(y) = \int_0^1 \frac{2}{3}\,dx = \frac{2}{3}, \quad y \in (0,1)$

$f_Y(y) = \int_0^{2-y} \frac{2}{3}\,dx = \frac{2}{3}(2-y), \quad y \in (1,2)$

$E(X) = \int_0^1 x \cdot f_X(x,y)\,dx = \int_0^1 x \cdot \frac{2}{3}\,dx = \frac{1}{3}x^2 \Big|_0^1 = \frac{1}{3}$ → granice su brojevi

$E(Y) = \int_0^1 y \cdot f_Y(x,y)\,dy + \int_1^2 y \cdot f_Y(x,y)\,dy = \int_0^1 \frac{2}{3}y\,dy + \int_1^2 \frac{2}{3}(2-y)\,dy =$

$= \frac{1}{3}y^2 \Big|_0^1 + \frac{2}{3}\left(2y - \frac{y^2}{2}\right)\Big|_1^2 = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$