Zadatak 1. $x_1 \dots x_n$ $\quad f(x) = 2\lambda^2 x e^{-\lambda^2 x^2}$

$z = \sum_{i=1}^{n} x_i \qquad f(x) = 2\lambda^{2n} z e^{-\lambda^2 \cdot z^2}$

$L = 2\lambda^{2n} z e^{-\lambda^2 \cdot z^2} \qquad \frac{dL}{d\lambda} = n4\lambda^{2n-1} z e^{-\lambda^2 z^2} + (-z^2 2\lambda^{2n} z e^{-\lambda^2 z^2} \cdot 2\lambda)$

$4n\lambda^{2n-1} z e^{-\lambda^2 z^2} = 4 z^2 \lambda^{2n} \lambda e^{-\lambda^2 z^2}$

$n\frac{2n}{\lambda} = z^2 \lambda^{2n} \cdot \lambda \; / \qquad n = \lambda^2 z^2 \qquad \lambda = \sqrt{\frac{n}{z^2}} = \sqrt{\frac{n}{\sum_{i=1}^{n} x_i^2}}$

Zadatak 2.

x_i	112	114	116	118	120	122
n	2	3	7	5	2	3 $\Rightarrow n = 22$

Normalna $N(a, \sigma^2)$

a) $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i = 117 \qquad s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 = 8.667$

b) $p = 0.9 \qquad \alpha = 0.1$ 21-stupanj slobode

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$t_{1-\frac{\alpha}{2}} = t_{0.95} = 1.721 \quad t_{1-\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}} = 1.0201$

$P(115.92 \le a \le 118.08) = 0.9$

disperzija:

$c_1 = \chi^2_{n-1,\frac{\alpha}{2}} = \chi^2_{21;0.05} = 11.591$

$c_2 = \chi^2_{n-1,1-\frac{\alpha}{2}} = \chi^2_{21;0.95} = 32.671$

$s^2 = 8.667$

$p_1 = \frac{(n-1)s^2}{c_2} = \frac{21 \cdot 8.667}{32.671} = 5.57 \quad p_2 = \frac{(n-1)s^2}{c_1} = \frac{21 \cdot 8.667}{11.591} = 15.702$

$P(5.57 \le \sigma^2 \le 15.702) = 0.9$

Zadatak 3.

$n = 200 \quad m = 112 \quad p = 0.95 \quad \alpha = 0.05$

a)

$\hat{p} = \frac{m}{n} = 0.56$

$p_{1,2} = \hat{p} \pm u_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.56 \pm 0.0688 = 3$

$u_{1-\frac{\alpha}{2}} = u_{0.975} = 1.960$

$P(0.4912 \leq p \leq 0.6288) = 0.95$

b)

c)

$\alpha = 0.05$

$n = \frac{\hat{p}(1-\hat{p})}{\left(\frac{\Delta - b_5}{u_{1-\frac{\alpha}{2}}}\right)^2} \quad \Delta = \frac{0.56 \cdot 0.44}{\left(\frac{0.06}{1.960}\right)^2} = 862.936 \approx 863$

Zadatak 4.

$a_0 = 35 \text{ mm} \quad n = 20 \quad \alpha = 0.05 \quad H_1 (a_1 \neq 35)$

$N(a, \sigma^2)$

x_i	34.8	34.9	35	35.1	35.2	
n_i	2	3	4	6	5	$\sum n_i = 20$

$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i = 35.07 \quad S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$

$T = \frac{\bar{x} - a_0}{\frac{s}{\sqrt{n}}} = \frac{35.07 - 35}{\frac{0.1626}{\sqrt{20}}} = 1.9271$

$s^2 = 0.02644$

(1.960)

Zadatak 4.

$a_0 = 35\,mm$ $n = 20$ $\alpha = 0.05$ $H(a_0 \neq 35)$

$H(a \neq ...)$

x_i	34.8	34.9	35	35.1	35.2	
n	2	3	4	6	5	$= \sum n_i = 20$

$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i = 35.07$ $s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$

$T = \frac{\bar{x} - a_0}{\frac{s}{\sqrt{n}}} = \frac{35.07 - 35}{\frac{0.1626}{\sqrt{20}}} = 1.9251$

$s^2 = 0.02644$

$t_{1-\frac{\alpha}{2}} = t_{0.975} = 2.093$ $|T| < t_{1-\frac{\alpha}{2}}$ prihvaća se

Zadatak 5.

$n_1 = 30$ $\bar{x}_1 = 74$ $\sigma_1 = 8$ $\alpha = 0.05$ postoji li bitna razlika

$n_2 = 40$ $\bar{x}_2 = 77$ $\sigma_2 = 7$

$\sigma_z^2 = \frac{\sigma_x^2}{n_1} + \frac{\sigma_y^2}{n_2} = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = \frac{64}{30} + \frac{49}{40} = 3.358\dot{3}$

$\sigma_z = 1.83257$

$\hat{u} = \frac{\bar{x}_1 - \bar{x}_2}{\sigma_z} = \frac{74 - 77}{1.83257} = -1.637$ $u_{1-\frac{\alpha}{2}} = 1.960$

$|\hat{u}| < u_{1-\frac{\alpha}{2}}$ ne postoji bitna razlika

hipoteza o jednakosti ne može se odbaciti

Zadatak 6.

$n = 130$ $B(n, p)$ $\alpha = ?$

x_i	0	1	2	3	4	5
n	75	77	20	6	1	1 ⇒ n = 130

$p_i = \binom{n}{i} p^i (1-p)^{n-i}$

$p_i = \binom{5}{i} \frac{1}{6}^i \left(\frac{5}{6}\right)^{5-i}$

j	n_j	p_j	$\frac{(n_j - np_j)^2}{np_j}$
0	75	0.40187	0.0241
1	77	0.40187	$5.4209 \cdot 10^{-3}$
2	30	0.16075	$9.642649 \cdot 10^{-3}$
3	6	0.03215	0.234
4	1	3.215^{-3}	
5	1	$1.286 \cdot 10^{-4}$	
		0.9999 ≈ 1	$\chi^2_q = 0.273108$

$r = 0$ $m = 4$

$f = m - r - 1 = 4 - 0 - 1 = 3$

$\chi^2_{3,1-\alpha} = \chi^2_{3,1-\alpha}$

$\chi^2_q < \chi^2_{kr}$ podrugova za $\alpha = ?$

$0.216 < \alpha < 0.352$

$1 - \alpha = 0.05$

$\alpha = 1 - 0.05$

$= \underline{0.95}$

Zadatak 7.

0.4, 0.6, 0.7 0.8

Zadatak 1.

0.4, 0.6, 0.7, 0.8

a) $P(\bar{A}) = 0.6 \cdot 0.4 \cdot 0.3 \cdot 0.2 = 0.0144$ $\quad P(A) = 1 - P(\bar{A}) = 0.9856$

b) $P(B) = 0.4 \cdot 0.6 \cdot 0.7 \cdot 0.2 + 0.4 \cdot 0.6 \cdot 0.3 \cdot 0.2 + 0.4 \cdot 0.4 \cdot 0.7 \cdot 0.8 + 0.6 \cdot 0.6 \cdot 0.7 \cdot 0.8 = 0.3824$

$P(B|S_4) = 0.4 \cdot 0.6 \cdot 0.7 \cdot 0.2 = 0.0336$

$P(S_4|B) = \frac{P(B|S_4)}{P(B)} = \frac{0.0336}{0.3824} = 0.0878766$

Zadatak 2.

$f(x) = C(3-x), \quad x \in (0,3)$

a) $1 = \int_0^3 C(3-x)\,dx = C\left[\int_0^3 3\,dx - \int_0^3 x\,dx\right] = C\left(3x\Big|_0^3 - \frac{x^2}{2}\Big|_0^3\right) = C\left(9 - \frac{9}{2}\right)$

$1 = C \cdot \frac{9}{2} \qquad 9C = 2 \qquad C = \frac{2}{9}$

b) $P(X<1) = ? \qquad f(x) = \frac{2}{9}(3-x) \qquad F(x) = \int_0^x \frac{2}{9}(3-x)\,dx$

$= \frac{2}{9}x\Big|_0^x - \frac{2}{9}\frac{x^2}{2}\Big|_0^x$

$= \frac{2}{3}x - \frac{x^2}{9}$

$F(1) = \frac{2}{3} - \frac{1}{9} = \frac{6-1}{9} = \frac{5}{9}$

c) $E(x) = ? \qquad E(x) = \int_0^3 x\,f(x)\,dx = \int_0^3 \frac{2}{9}(3x - x^2)\,dx$

$= \frac{2}{9}\left[\frac{3x^2}{2}\Big|_0^3 - \frac{x^3}{3}\Big|_0^3\right] = \frac{2}{9}\left[\frac{27}{2} - \frac{27}{3}\right] = 3 - 2 = 1$

[zadatak 9.]

a) $P(X<x+t \mid x>t) = P(X<x)$

b)

c) $f(x) = \lambda e^{-\lambda x} \quad x>0 \qquad E(x) = \frac{1}{\lambda}$

$$E(x) = \int_0^\infty x \cdot \lambda e^{\lambda x} dx = \lambda \int_0^\infty x e^{-\lambda x} dx = \left| \begin{matrix} u = x & dv = e^{-\lambda x} dx \\ du = dx & v = -\frac{1}{\lambda} e^{-\lambda x} \end{matrix} \right|$$

$$= \lambda \cdot v\Big|_0^\infty - \int_0^\infty v\,du = \lambda \left[x \cdot \left(-\frac{1}{\lambda}\right) e^{-\lambda x}\Big|_0^\infty + \frac{1}{\lambda} \int_0^\infty e^{-\lambda x} dx \right]$$

$$= \lambda \left[0 + \frac{1}{\lambda}\left(-\frac{1}{\lambda}\right) e^{-\lambda x}\Big|_0^\infty \right] = \lambda \left[-\frac{1}{\lambda^2} e^{-\lambda x}\Big|_0^\infty \right] = \lambda \left[-\frac{1}{\lambda^2} \cdot 0 + \frac{1}{\lambda^2} e^0 \right]$$

$$= \lambda \left(0 + \frac{1}{\lambda^2} \right) = \frac{1}{\lambda} \checkmark$$

[zadatak 10.]

$p = \frac{1}{6}$

Y \ X	2	4	6	8	10	12
1	$\frac{1}{6}$	0	$\frac{1}{6}$	0	$\frac{1}{6}$	0

$Z = X + Y \sim \begin{pmatrix} 2+1 & 4+1 & 6+1 & 8+1 & 10+1 & 12+1 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$$= \lambda \left[0 + \frac{1}{\lambda}\left(-\frac{1}{\lambda}\right) e^{-\lambda x} \Big|_0^{\infty} \right] = \lambda \left[-\frac{1}{\lambda^2} e^{-\lambda x} \Big|_0^{\infty} \right] = \lambda \left[-\frac{1}{\lambda^2} \cdot 0 + \frac{1}{\lambda^2} e^{0} \right]$$

$$= \lambda \left(0 + \frac{1}{\lambda^2} \right) = \frac{1}{\lambda}$$ W

Zadatak 10.

$P \; \frac{1}{6}$

$y \backslash x$	2	4	6	8	10	12
1	$\frac{1}{6}$	0	$\frac{1}{6}$	0	$\frac{1}{6}$	0
3	0	$\frac{1}{6}$	0	$\frac{1}{6}$	0	$\frac{1}{6}$

$$z = x + y \sim \begin{pmatrix} 2+1 & 4+3 & 6+1 & 8+3 & 10+1 & 12+3 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$$z \sim \begin{pmatrix} 3 & 7 & 7 & 11 & 11 & 15 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$$z \sim \begin{pmatrix} 3 & 7 & 11 & 15 \\ \frac{1}{6} & \frac{1}{6}+\frac{1}{6} & \frac{1}{6}+\frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$$z \sim \begin{pmatrix} 3 & 7 & 11 & 15 \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$$