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Exercício da lista: 18 i

$$\int_1^4 \left(\frac{x}{\sqrt{2+4x}} \right) dx$$

$$F(x) = \int \left(\frac{x}{\sqrt{2+4x}} \right) dx$$

$$u = 2 + 4x \rightsquigarrow du = 4dx$$

$$\int \left(\frac{x}{\sqrt{u}} \frac{du}{4} \right) = \frac{1}{4} \int \frac{x}{\sqrt{u}} du$$

Multiplicando por $2/2$ e somando $2/\sqrt{u} - 2/\sqrt{u}$

$$\frac{1}{16} \int \frac{2+4x}{\sqrt{u}} - \frac{2}{\sqrt{u}} du = \frac{1}{16} \int \frac{u}{\sqrt{u}} du - 2 \int \frac{du}{\sqrt{u}}$$

Como
$$\int u^{\frac{1}{2}} du = \frac{2}{3} u^{\frac{3}{2}}$$
e
$$\int u^{\frac{-1}{2}} = 2u^{\frac{1}{2}}$$
então
$$\frac{1}{16} \left(\int u^{\frac{1}{2}} du - 2 \int u^{\frac{-1}{2}} du \right) = \frac{1}{16} \left(\frac{2u^{\frac{3}{2}}}{3} - 2u^{\frac{1}{2}} \right)$$

Temos então que:
$$F(x) = \frac{\sqrt{(2+4x)^3}}{24} - \frac{\sqrt{2+4x}}{4}$$

$$F(1) = \frac{\sqrt{(2+4 \cdot 1)^3}}{24} - \frac{\sqrt{2+4 \cdot 1}}{4}$$

$$F(4) = \frac{(\sqrt{2+4 \cdot 4})^3}{24} - \frac{\sqrt{2+4 \cdot 4}}{4}$$

O valor da integral é, portanto

$$\int_1^4 \left(\frac{x}{\sqrt{2+4x}} \right) dx = F(4) - F(1) \approx 2.1213$$