Vamos considerar a parábola genérica $y = ax^2 + bx + c$.

Here is a sketch with the picture on the interval $[x_{i-1}, x_{i+1}]$ (Assume that the points x_i are evenly distributed in the interval $[a, b]$):

![enter image description here][1]

$$p_i(x) = f(x_{i-1})\frac{(x-x_i)(x-x_{i+1})}{(x_{i-1}-x_i)(x_{i-1}-x_{i+1})} + f(x_i)\frac{(x-x_{i-1})(x-x_{i+1})}{(x_i-x_{i-1})(x_i-x_{i+1})} + f(x_{i+1})\frac{(x-x_{i-1})(x-x_i)}{(x_{i+1}-x_{i-1})(x_{i+1}-x_i)}$$

Integrating $p_i(x)$ gives

$$\int_{x_{i-1}}^{x_{i+1}} dx = \frac{\Delta x}{3}\,[f(x_{i-1}) + 4f(x_i) + f(x_{i+1})]$$

Now if the points are from x_0 to x_{2n}, then you add the n integrals together:

$$\int_{x_0}^{x_2} + ... + \int_{x_{2n-2}}^{x_{2n}} = \frac{\Delta x}{3}\,[f(x_0) + 4f(x_1) + f(x_2) + f(x_2) + 4f(x_3) + f(x_4) + f(x_4) + 4f(x_5) + f(x_6) + ... + f(x$$

I have a different form because I used $0, ..., 2n$.

Edit: There is another way to derive the two-subinterval integral without using the $p_i(x)$.

Let the interval be $[-h, h]$. We represent the curve using a polynomial $ax^2 + bx + c$. Then the integral is

$$\int_{-h}^{h} ax^2 + bx + c\,dx = 2\int_0^h ax^2 + c\,dx = \frac{h}{3}(2ah^2 + 6c)$$

You can see that

$$f(-h) = ah^2 - bh + cf(0) = cf(h) = ah^2 + bh + c$$

Some algebraic manipulating gives you

$$f(-h) + 4f(0) + f(h) = \frac{h}{3}(2ah^2 + 6c)$$

This can of course be shifted to get similar formulas on other subintervals. Then you can add them together to get the composite formula as above.

[1]: http://i.stack.imgur.com/Y8twg.png