7. c) A função de logverossimilhança é dada por

$$l(\mu, \Sigma) = \text{const} + \frac{n}{2} \ln|V| - \frac{1}{2}\left[\sum_{i=1}^{n}(x_i-\mu)'\overset{V}{\,}(x_i-\mu)\right]$$

$$= \text{const} + \frac{n}{2} \ln|V| - \frac{1}{2} tr\left[V \sum_{i=1}^{n}(x_i-\mu)(x_i-\mu)'\right],$$

onde $V = \Sigma^{-1}$. Definindo $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_c)'$ como o vetor com os multiplicadores de lagrange, temos que a logverossimilhança com os multiplicadores de lagrange ~~são~~ é

$$l(\mu, \Sigma, \lambda) = \text{const} + \frac{n}{2} \ln|V| - \frac{1}{2} tr\left[V \sum_{i=1}^{n}(x_i-\mu)(x_i-\mu)'\right] - \lambda'(R\mu - b)$$

- lembrando que $\frac{\partial \ln|V|}{\partial V} = V^{-1} = \Sigma$, $\frac{\partial\, tr(VA)}{\partial V} = A'$,

$\frac{\partial A\mu}{\partial \mu} = A$, ($A'$, se A for um vetor linha), $\frac{\partial f(g)}{\partial \mu} = \frac{\partial f}{\partial g}\frac{\partial g}{\mu}$,

$\frac{\partial \mu' B \mu}{\partial \mu} = \mu'(B+B^t)$, $[(B+B')\mu ?]$ $\hookrightarrow p\times 1$, $\frac{\partial \lambda' A}{\partial \lambda} = A'$, ($A$, se A for um vetor coluna)

- Estimadores de máxima verossimilhança:

- $S(\mu) = \frac{\partial l(\mu, \Sigma, \lambda)}{\partial \mu} = -\frac{1}{2}\sum_{i=1}^{n}\left[2\Sigma^{-1}(x_i-\mu)\right] - R'\lambda$

$$= n\Sigma^{-1}\mu - n\Sigma^{-1}\bar{x} - R'\lambda$$

- $S(V) = \frac{\partial l(\mu, \Sigma, \lambda)}{\partial V} = \frac{n\Sigma}{2} - \frac{1}{2}\sum_{i=1}^{n}(x_i-\mu)(x_i-\mu)'$

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$$S(\lambda) = \frac{\partial \ell(\mu, \Sigma, \lambda)}{\partial \lambda} = -(R\mu + b)$$

• Igualando as derivadas à matriz de 0 compatível, temos

$$S(\mu) = n\Sigma^{-1}\mu - n\Sigma^{-1}\bar{x} - R'\lambda = 0 \qquad (1)$$

$$S(V) = \frac{n\Sigma}{2} - \frac{n}{2}S^2 = 0 \qquad (2)$$

$$S(\lambda) = -(R\mu - b) = 0 \qquad (3)$$

• De (1), temos que

$$\Sigma^{-1}\mu = \Sigma^{-1}\bar{x} + \frac{R'\lambda}{n} \Rightarrow \mu = \bar{x} + \frac{\Sigma R'\lambda}{n} \qquad (4)$$

• De (3), temos que

$$R(\mu = \underbrace{R(\bar{x} + \frac{1}{n}\Sigma R'\lambda) = b}$$

$$\Rightarrow \quad R\Sigma R'\lambda = nb - nR\bar{x}$$

$$\Rightarrow \quad \lambda = n(R\Sigma R')^{-1}(b - R\bar{x}) \qquad (5)$$

Colocando (5) em (4), temos

$$\hat{\mu} = \bar{x} + \frac{1}{n}\Sigma R'[n(R\Sigma R')^{-1}(b - R\bar{x})]$$

$$= \bar{x} + \Sigma R'(R\Sigma R')^{-1}(b - R\bar{x}) \qquad (6)$$

$$= \bar{x} + \underbrace{(R'R)^{-1}R'R}_{I}\underbrace{\Sigma R'(R\Sigma R')^{-1}}_{I}(b - R\bar{x})$$

$$= \bar{x} + (R'R)^{-1}R'(b - R\bar{x}) = \bar{x} + (RR)^{-1}R'b - \bar{x}$$

$$= (RR)^{-1}R'b$$

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• De (2), temos que

$$\Sigma = S^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)(x_i-\mu)'$$

Logo, os EMVs de μ e Σ são

$$\hat{\mu} = (R R)^{-1} R' b$$

$$\hat{\Sigma} = \frac{1}{n}\sum_{i=1}^{n}(X_i-\hat{\mu})(X_i-\hat{\mu})'$$