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Closed Loop Steering of Unicycle-like Vehicles via Lyapunov Techniques

With a special choice for the system state equations, the use of the simplest quadratic form as candidate Lyapunov function directly leads to the definition of very simple, smooth and effective closed loop control laws for unicycle-like vehicles, suitable to be used for steering, path following, and navigation. The authors provide simulation examples to show the effectiveness and, in a sense, the "natural behavior" of the obtained closed loop motions (whenever compared with our everyday driving experience).

INTRODUCTION

The majority of the mobile robots made available for research or effectively employed within real applications are often characterized, for maneuverability reasons, by a unicycle-like structure. The problem of steering such vehicles has received a great deal of attention within the wide existing literature on mobile robots. Recently there has also been a great deal of interest in results obtained with the geometric-based approach to non-linear control theory, and the emerging field of the so-called non-holonomic motion planning and control. Most attention has been devoted to the determination of control laws that assure the attainment of a target position and orientation starting from any initial conditions (we can call it "parking problem"). There are two basic points of view: closed loop control and the planning. The first relates to the determination of steering laws assuring the asymptotic execution of the parking task and can be set as a stabilization problem. Regarding the determination of such laws, a well known work of Brockett [1] identifies a class of systems that cannot be stabilized via smooth state feedback. Cartesian state space representations of unicycles fall into this class. On this basis, for the control of vehicles represented in the Cartesian state space, a number of solutions have been found involving both discontinuous and/ or time varying control laws ([2], [3], Samson [4], [5]).

From the planning point of view, the natural framework is that of the non-linear control theory of non-holonomic systems. Murray and Sastry [6] provided a procedure to generate open-loop control actions for steering the vehicles. Obviously in this case, a finite time accomplishment of the task can be obtained.

Since for practical purposes an asymptotic convergence to the goal can be sufficient, we consider the problem from this point of view. As we have already noted, almost all previous works were based on the fact that, due to the limits imposed by the Brockett's result, a goal position and orientation are not reachable asymptotically by means of smooth and time invariant feedback control laws. If the vehicle is localized with a Cartesian set of variable, this is true. Nevertheless, if a different state-space representation is adopted, simpler approaches can be used, directly allowing smooth stabilization properties. For example, Badreddin and Mansour [7] use polar coordinates to localize the vehicle, but their analysis ends with the determination of a constant state feedback law that assures only local stabilizability around the final target position and orientation.

In this article, we explicitly show that, with a special choice of the system state variables, *global stability* properties can be guaranteed by *smooth* feedback control law. Such a result will be found using a very simple mathematical framework that can be considered a byproduct of the well known Lyapunov stability theory.

Moreover, an effective use of such a control law can also be devised for both the cases of path following and navigation among (possibly on-line) assigned via points, without requiring, for the latter case, of any sort of a-priori trajectory planning or re-planning. This will be possible by using the simple idea of making the goal move along a desired path while the vehicle follows it with the same control law used for the parking problem. The structure of the velocity of the goal will be an additional control variable that will be matter of discussion with reference to the desired degree of accuracy required for the tracking.

In section 2 the basic kinematic equations are reported, and in section 3 the Lyapunov analysis is carried out and the steering controls are deter-

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mined. In section 4 the behavior of the system is discussed with some simulation examples. In section 5 the use of the determined control laws for the path following problem and navigation among via-points are reported together with simulation experiments. We conclude with a short summary of the results and their use in future works.

KINEMATIC EQUATIONS

Consider a unicycle-like vehicle positioned at a non-zero distance with respect to a goal frame <g>, whose motion is governed by the combined action of both the angular velocity ω, and the linear velocity vector u always directed as one of the axis of its attached frame <a>, as depicted in Fig. 1.

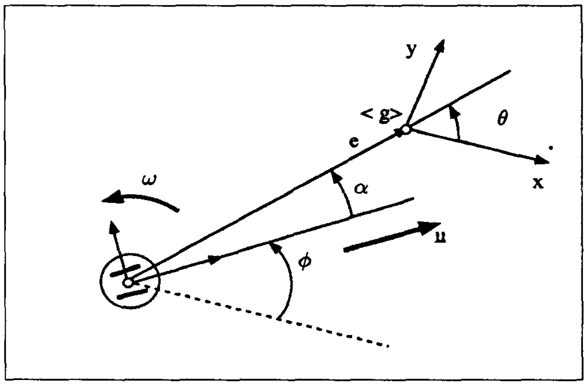

Figure 1. Vehicle's position and orientation with respect to the target frame.

Then, the usual set of kinematic equations, which involves the vehicle Cartesian position x, y, and its orientation angle ϕ, are:

$$\begin{cases} \dot{x} = u\cos\phi \\ \dot{y} = u\sin\phi \\ \dot{\phi} = \omega \end{cases} \tag{1}$$

where u is simply the component of u evaluated along its direction, and x, y, ϕ are all measured with respect to the target frame-point <g>.

By instead representing the position of the vehicle in terms of its polar coordinates, involving the error distance $e > 0$ and its orientation θ with respect to <g>, the following others are easily obtained:

$$\begin{cases} \dot{e} = -u\cos(\theta - \phi) \\ \dot{\theta} = u\dfrac{\sin\alpha}{e} \\ \dot{\phi} = \omega \end{cases} \tag{2}$$

Now, letting $\alpha = \theta - \phi$ be the angle measured between the vehicle principal axis and the distance vector e, we finally have

$$\begin{cases} \dot{e} = -u\cos\alpha \\ \dot{\alpha} = -\omega + u\dfrac{\sin\alpha}{e} \\ \dot{\theta} = u\dfrac{\sin\alpha}{e} \end{cases} \tag{3}$$

Notwithstanding the fact that an infinite number of other basic kinematic equations might obviously be devised, within this work particular attention will be devoted to the last obtained equations (3), since, as it will be shown later, their form will be suitable for easily designing appropriate closed loop control laws for the vehicle manoeuvering. As a matter of fact, equations (3) allow for a set of state variables which closely resemble the same ones regularly used within our car-driving experience.

Note however that, since they are based on the use of polar coordinates, kinematic equations (3) (as well as (2) and contrary to (1)) are actually valid only for non zero values assumed by the distance errors e, since both angles α and θ are undefined when $e = 0$; thus implying that the generally existing one-to-one correspondence with (1) is actually lost in correspondence of such singular points.

CLOSED LOOP STEERING

Now we consider the basic problem of finding suitable strategies which allow the unicycle vehicle to reach the goal. In particular, we want to determine steering strategies that *asymptotically* drive the unicycle toward a desired position and orientation. First, note that every set of kinematic equations mentioned before falls in the class of systems characterized by the general structure

$$\dot{z} = \sum_{i=1}^{m} f_i(z)\,u_i$$

with $z \in \Re^n$ and $u_i \in \Re$. In other words, we have dynamic systems without drift with n state variables and m control variables.

For such systems, a well known result of Brockett [1] states that a point z^* cannot be asymptotically stabilized using smooth and time invariant feedback laws if in a compact neighborhood of z^*, the vectors $f_i(z)$ are independent, continuously differentiable and $m<n$. It is easy to see that the Cartesian representation (1) allows the application of the Brockett's result for any (x^*, y^*, ϕ^*). This is the reason why most of the previous approaches provided discontinuous and/or time varying control laws.

On the other hand, we can see that, whenever equations (3) are used to localize the vehicle with respect to its goal, the basic parking manoeuver can be accomplished reaching asymptotically the limiting point (0,0,0). For system (3) the regularity assumptions needed to apply the Brockett's result do not hold. Then the asymptotic stabilizability of such a limiting point by means of smooth and time invariant feedback laws is not prevented.

Given these considerations, we can now better specify the aforementioned closed loop steering problem in the following general terms:

Parking Problem: Let the unicycle-like vehicle be initially positioned at any non zero distance from the target frame <g> and assume state variables $[e, \alpha, \theta]'$ to be directly measurable in correspondence with any $e > 0$. Find a suitable state dependent control law $[u, \omega]' = g(e, \alpha, \theta)$ which guarantees the state to be asymptotically driven to the null *limiting point* $[0, 0, 0]'$, without attaining the condition $e=0$ in finite time.

Clearly, while the first specification expresses the requirement of reaching the target frame with the appropriate orientation, the second one technically serves only to avoid the complexities that could arise in correspondence with any, finite time, loss of validity of the considered model (3). In this respect, also note how it was necessary to use the more general concept of "limiting point" to correctly specify the need of obtaining an asymptotic convergence of the state toward a point, namely the point $[0,0,0]'$, which is actually located on the frontier of the open set of validity of model equations (3) (i.e., the subset of R^3 where $e > 0$).

One of the most commonly used methods to study the asymptotic behavior of a dynamic system is based on the Lyapunov stability theory. In our case, terms like "Lyapunov analysis" and "candidate Lyapunov function" can be considered an abuse of terminology. Nevertheless, the concept of limiting point and the possibility of projecting the motion of (3) on a suitable scalar function survive. In the following, a Lyapunov-like framework will be used, even if we continue to use standard terms just to avoid cumbersome terminology.

With the above aims in mind, let us now start by considering the *simplest* choice for the structure of a candidate Lyapunov function related to the considered control problem; i.e., the positive definite quadratic form

$$V = V_1 + V_2 = \frac{1}{2}\lambda e^2 + \frac{1}{2}(\alpha^2 + h\theta^2)\ ; \quad \lambda, h > 0 \tag{4}$$

whose terms $\dot{V}_1, \dot{V}_2$ represent (one half of) the squared weighted norms of both the "error distance vector" e and the so-called "alignment error vector" $[\alpha, \sqrt{h}\theta]'$ exhibited by the vehicle with respect to the target frame <g>.

Then, consider its time derivative $\dot{V}$ along (3), given by

$$\begin{aligned} \dot{V} &= \dot{V}_1 + \dot{V}_2 = \lambda e\dot{e} + \left(\alpha\dot{\alpha} + h\theta\dot{\theta}\right) \\ &= \lambda e u\cos\alpha + \alpha\left[-\omega + u\frac{\sin\alpha}{\alpha}\frac{(\alpha + h\theta)}{e}\right] \end{aligned} \tag{5}$$

From the latter expression we see that the first term, corresponding to $\dot{V}_1$, can be made non-positive by letting the linear velocity u have the smooth form (independent from both the parameters λ, h)

$$u = (\gamma\cos\alpha)\,e;\ \gamma > 0 \tag{6}$$

In this way, the term $\dot{V}_1$ becomes

$$\dot{V}_1 = -(\lambda\sin^2\alpha)\,e^2 \le 0\ . \tag{7}$$

This means that the first term $\dot{V}_1$ of (4) is always non increasing in time and consequently, since it is lower bounded by zero, it is asymptotically converging toward a non negative finite limit.

The fact that $\dot{V}_1$ is simply proportional to the square of the positive scalar variable e also implies that e is also monotonically non increasing in time and that the null value cannot ever be approached in finite time, thereby assuring the validity of (3) for all the parking process.

Furthermore, in correspondence with the choice (6), the second term $\dot{V}_2$ of (5) turns out to be

$$\dot{V}_2 = \alpha\left[-\omega + \gamma\frac{\cos\alpha\sin\alpha}{\alpha}(\alpha + h\theta)\right]. \tag{8}$$

This can also be made non-positive, by letting the angular velocity ω take on the smooth form (it is also independent from the parameter λ, but not from h)

$$\omega = k\alpha + \gamma\frac{\cos\alpha\sin\alpha}{\alpha}(\alpha + h\theta);\ (k > 0) \tag{9}$$

Hence,

$$\dot{V}_2 = k\alpha^2 \le 0 \tag{10}$$

finally leading to the following expression for the time derivative of the original global Lyapunov function V

$$\dot{V} = \dot{V}_1 + \dot{V}_2 = -\lambda(\gamma\cos^2\alpha)e^2 - k\alpha \le 0 \tag{11}$$

which results in a negative semi-definite form.

From this it follows that Lyapunov function V is always non increasing in time and consequently, since lower bounded by zero, asymptotically converging toward a finite non-negative limit. This fact, joined with the radial unboundedness of V itself, in turn guarantees the boundedness of the state trajectory corresponding to any bounded initial condition. Moreover, due to the boundedness of the state trajectory, the uniform continuity in time of $\dot{V}$ directly follows.

Finally, as a consequence of the uniform continuity in time of $\dot{V}$, and of the existence of a convergence limit for $\dot{V}_1$ by applying Barbalat's Lemma it follows that $\dot{V}$ necessarily converges to zero for increasing time; thus in turn implying the convergence of the state trajectory toward some subset of the line $[e, \alpha, \theta]' = [0, 0, \theta]'$ (i.e., toward a part of the subspace where function $\dot{V}$ can attain the null value; see (11)).

At this point, to show that the only possible convergence subset within the line $[e, \alpha, \theta]' = [0, 0, \theta]'$ is actually constituted by the sole origin point $[0, 0, 0]'$, and that such convergence occurs with null time derivatives for the state trajectory, let us consider the state equations (3) in the presence of the established feedback laws (6), (9); that is the closed loop equations, having the form

$$\begin{cases} \dot{e}=-(\gamma\cos^2\alpha)e \\ \dot{\alpha}=\left(-k\alpha-\gamma h\dfrac{\cos\alpha\sin\alpha}{\alpha}\right) \\ \dot{\theta}=\gamma\cos\alpha\sin\alpha \end{cases} \qquad e(0)>0 \qquad (12)$$

Then, due to the convergence to zero of both e and α, it immediately follows from the first and third of (12) that both the time derivatives $\dot{e}$ and $\dot{\theta}$ converge to zero. Moreover, due to the boundedness of the state trajectory, the convergence to zero of $\dot{\theta}$ also implies that θ tends to some finite limit $\bar{\theta}$ for increasing time. Then, due to this, from the second of (12) it follows that also $\dot{\alpha}$ must necessarily tend to the finite limit $-\gamma h\bar{\theta}$.

At this point, by still keeping into account the convergence to zero of α, and also noting that $\dot{\alpha}$ is actually a uniformly continuous time function, as implied by the boundedness of the state trajectory, by Barbalat's Lemma it follows that $\dot{\alpha}$ actually also converges to zero; thus in turn implying that the finite limit $\bar{\theta}$, for θ, it must also be necessarily zero. This is sufficient for completely proving the fulfillment of the control requirements by the established state feedback law (6), (9).

Remark 1. We note once again that feedback control law (6), (9) actually results in a very simple form, which also can be easily implemented by simple on-board computing devices (and relatively simple sensor systems for evaluating distances, as well as angular orientations). Moreover, as it can be possibly recognized by the set of simulation examples reported below, it also exhibits a closed loop behavior that could be considered very "natural," whenever compared with those that we could expect as the result of our everyday car-driving experience. It is our opinion that such a behavior also arises as a direct consequence of the use, within the feedback loop, of a set of state variables that, as it has been already mentioned, apparently coincides with the ones which are presumably used by any human operator involved in driving tasks.

Remark 2. Note that the convergence toward *any one* of the limiting points $[0, \pm n\pi, \pm n\pi]' n \in N$, could also have been considered as appropriate, whenever seen from the point of view of the task accomplishment. Notwithstanding this last fact, we shall not deal with such additional possibilities that can be taken into consideration when using "multiple Lyapunov functions" as has been suggested in [8].

To conclude this section, we note that, due to the particular structure assigned to the Lyapunov function V, the corresponding control law results in a form which is actually independent from the positive value assigned to the λ parameter appearing in the first term V_1 of the global function V. In a later section we will show how such parameters instead play a very important role within the problems of closed loop path following and navigation among via points. Relationships among the positive parameters h, γ, k, which determine the limiting behavior of the curvature of the resulting path can be found in [8].

SIMULATION EXAMPLES FOR THE PARKING PROBLEM

Let us start with the simple example of a vehicle located in $(x,y,\phi) = (-1,1,3\pi/4)$ with respect to the reference frame. The target is in the origin of the reference frame, i.e., $(x,y,\phi)=(0,0,0)$. The corresponding initial conditions for (e,α,θ) are $e(0)=\sqrt{2}$, $\theta(0) = -\pi/4$, and $\alpha(0) = \theta(0) - \phi(0) = -\pi$. The resulting manoeuver, corresponding to $\gamma=3$; $h=1$, and $k=6$, is shown in Figures 2 and 3.

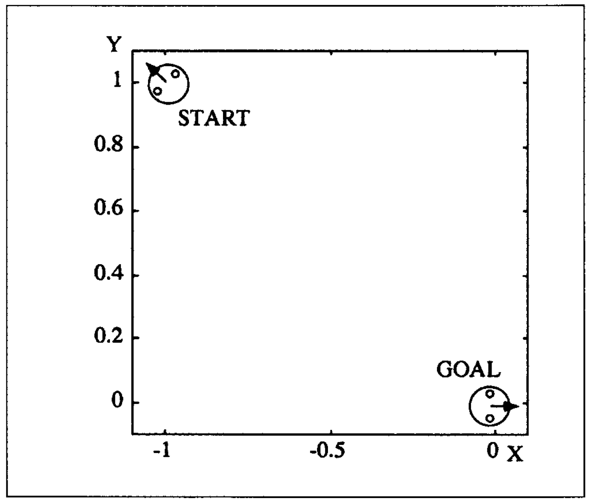

Figure 2. Starting position and goal.

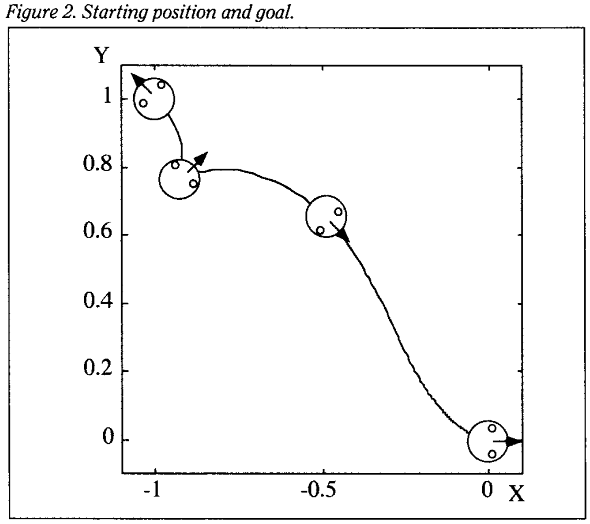

Figure 3. Resulting parking maneuver.

Note now that even if in the (x,y) plane the trajectory has an angular point the relevant controls over the time horizon result in continuous and smooth signals as reported in the previous section. If we had assumed $\phi(0) = -5\pi/4$ (which gives the same absolute orientation), we would have obtained a different manoeuver, since in this case $\alpha(0)$ would have been equal to $+\pi$. Nevertheless, the convergence to the final goal and the smoothness of the control signals would have been always assured.

We now present some simulation examples which correspond to a couple of cases where the unicycle vehicle is required to reach $(x,y,\phi)=(0,0,0)$, starting from many different initial configurations, all characterized by a unitary initial value for $e(0)$. In particular, in Figs. 4 and 5 the initial orientations $\phi(0)$ of the vehicle with respect to the target have been assumed to be always zero and $\pi/2$ respectively. As we can see, the vehicle eventually approaches the target always with a positive linear velocity. This is due to the fact that the steering strategy has been designed on the basis of the Lyapunov function (4) without taking into account the considerations made in Remark 2. In fact, since the steering laws continuously drive α to zero, it is easily understood that such a situation can be obtained only by using, in the last part of the manoeuver, a positive velocity. Even if it might seem a limitation as to the parking problem, this possibility positively contributes to solving the path following problem, as will be apparent in the next section.

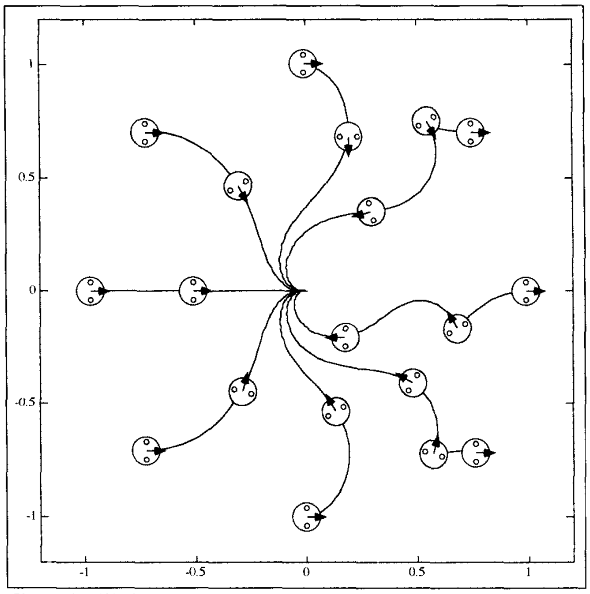

Figure 4. $\phi(0)=0$

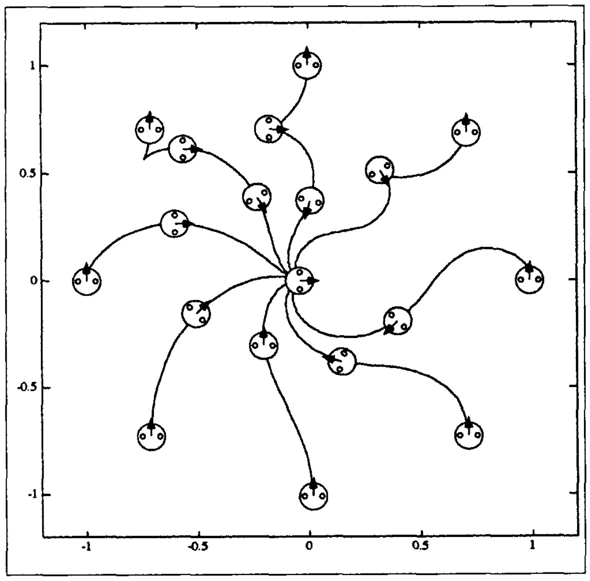

Figure 5. $\phi(0)=\pi/2$

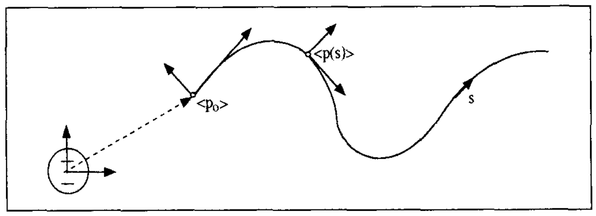

Figure 6.

PATH FOLLOWING AND NAVIGATION AMONG VIA-POINTS

The first control problem that will be addressed in this section corresponds to the requirement, for the vehicle, of approximately following an assigned *smooth* directed path, by approaching it from an initial frame $<p_0>$ tangently located on the path itself, as indicated in Fig 6.

By assuming the oriented path parametrized by the curvilinear abscissa s, and denoting with $<p(s)>$ the corresponding frame tangently located on the oriented path, let us now consider the possibility of continuously moving $<p(s)>$ from its initial position $<p_0>$ with a non-negative velocity $\dot{s}$ to be appropriately assigned.

Then, by considering $<p(s)>$ as the current target for the vehicle, we can easily verify that in this case, due to the superimposed target motion, the original kinematic equations (3) modify as follows:

$$\begin{cases} \dot{e} = -u\cos\alpha + \dot{s}\cos\theta \\ \dot{\alpha} = -\omega + u\dfrac{\sin\alpha}{e} - \dot{s}\dfrac{\sin\theta}{e} \qquad e(0)>0 \\ \dot{\theta} = u\dfrac{\sin\alpha}{e} - \dot{s}\dfrac{\sin\theta}{e} - \dfrac{\dot{s}}{R(s)} \end{cases} \tag{13}$$

where $R(s)$ is the current "signed" curvature radius of the path (positive or negative depending from its location on the right or left side of the path itself) and where all the additional terms depending on $\dot{s}$ just represent the "perturbations" which are introduced by the assumed motion of the target.

By assuming now that the control laws (6), (9) are applied also in the present case, we immediately have that in such closed loop conditions the corresponding equations (13) become

$$\begin{cases} \dot{e} = -(\gamma\cos^2\alpha)e + \dot{s}\cos\theta \\ \dot{\alpha} = -k\alpha + \gamma h\dfrac{\cos\alpha\sin\alpha}{\alpha}\theta - \dot{s}\dfrac{\sin\theta}{e} \qquad e(0)>0 \\ \dot{\theta} = \gamma\sin\alpha\cos\alpha - \dot{s}\dfrac{\sin\theta}{e} - \dfrac{\dot{s}}{R} \end{cases} \tag{14}$$

where the rate $\dot{s}$ now represents the sole control action which is made available for the accomplishment of the required path following task.

To this aim, assuming for simplicity h>1, the following closed loop policy is then proposed for the on-line specification of the target motion rate $\dot{s} \in [0, v]$; $v>0$:

$$\dot{s} = \begin{cases} 0 & V=\lambda e^2 + (\alpha^2 + h\theta^2) > \varepsilon \\ f(e,\alpha,\theta) & V=\lambda e^2 + (\alpha^2 + h\theta^2) \le \varepsilon \end{cases} \qquad 0 < \varepsilon < \frac{\pi^2}{4} \tag{15}$$

where $f(e,\alpha,\theta)$ is any continuous radial function centered on the ellipsoidal domain $\lambda e^2 + (\alpha^2 + h\theta^2) = \varepsilon$, which attains its maximum value $v_{\max}$ (equal to the largest admissible one for $\dot{s}$ in correspondence with the origin, and a null minimum value in correspondence on the border of the ellipsoid itself.

The rationale underlying the proposed policy structure for $\dot{s}$ can be now explained on the basis of the following considerations.

First, when the vehicle state is outside the ellipsoidal domain (as it could be, for instance, during the beginning of the motion, when the vehicle has to approach the starting frame $<p_o>$) the target is maintained in a fixed absolute position as specified by the first of (15), thus implying that in this case the vehicle results in being driven by control laws (6), (9) acting within their usual "fixed target" operating conditions. These force the state trajectory to evolve continuously in such a way to asymptotically converge toward the origin, while maintaining $e > 0$ as explained earlier. Due to such behavior, it then necessarily follows that the vehicle state will reach the ellipsoid surface in a *finite time* (say t*) and within the relevant subset of points corresponding to $e > 0$. Then, by noting that the rate $\dot{s}$ is also zero on the whole ellipsoid surface as prescribed by $f(e,\alpha,\theta)$ evaluated on it, and due to the fact that in correspondence with the reached point we actually have $\dot{V} < 0$ due to $e > 0$, we can also immediately conclude that, successive to t^* but connected with it, a time interval exists during which the state trajectory evolves continuously *strictly inside* the ellipsoid, while still maintaining $e > 0$.

Once the state is inside the ellipsoid with $e > 0$, a non-zero positive value is consequently assigned to the rate $\dot{s}$ by function $f(e,\alpha,\theta)$ as prescribed by (15). Then, since within the ellipsoid both $|\theta|$ and $|\alpha|$ are less than $\pi/2$ (due to the assumption h>1), it immediately follows that a positive contribution is actually added to $\dot{e}$, as established by the first of (14). Hence, during the motion inside of the ellipsoid, the state trajectory cannot consequently reach the condition $e = 0$ in a whatever finite time; thus guaranteeing the continuity of the state trajectory evolution also within the ellipsoidal domain itself.

Moreover, inside the ellipsoid the Lyapunov function V is certainly no more guaranteed to have a negative semi-definite time derivative (as instead it occurs outside and on the surface of it). This induces the obvious possibility that, from the inside of such domain and within a certain time interval, the state norm might also increase continuously and in a way that can allow the state trajectory itself to eventually reach again the ellipsoid surface, by now proceeding from its internal side.

But, if this occurs in correspondence with a time instant, we can again see, on the basis of the previous consideration, that it must be $e > 0$. Consequently, the application of the same previous reasoning line allows, once more, to show that an immediate future time interval certainly exists, where the state trajectory is again strictly located *inside* the ellipsoid.

As a net result, we are led by induction to conclude that, once the vehicle state trajectory has been "captured" for the first time within the inside of the ellipsoid, it will remain continuously confined within such domain throughout all future time instants, by always maintaining $e > 0$. Moreover, the possibility of lying on its border surface is represented by the occasional occurrence of "events" which are characterized by a null duration time.

Due to this latter fact it also follows that, once the state vehicle is confined within the ellipsoid, the rate $\dot{s}$ might actually attain the zero value only in correspondence of a subset of the time axis characterized by a null measure. The obvious consequence is that since $\dot{s}$ is a non negative quantity, the curvilinear abscissa s will be certainly a monotonically increasing function of time; thus finally implying that the goal frame-point $<p(s)>$ will proceed monotonically along the assigned path, till its complete covering. Moreover, during its travel along the path, $<p(s)>$ will maintain the vehicle behind of it, with an error distance and misalignment whose (squared) norms are, at each time instant, measured by the actual value assumed by the Lyapunov function V, which is however maintained within the a-priori assigned threshold $\varepsilon \in (0, \pi^2/4)$.

Also note how in the present path following case (and on the contrary of the parking one considered earlier) an important role is now played by the parameter λ appearing in the Lyapunov function V: by assigning to it a specific positive value, a general upper bound ε/λ is consequently established to the (squared) value for the admissible error distance occurring during the motion of $<p(s)>$. It is worth pointing out that such an upper bound is not to be confused with the actual maximum value attainable by e^2, this is just the value beyond which $\dot{s}$ is set to zero by (15) even in presence of null angular errors.

A different matter is the determination of the actual value that e can take on. As a particular case, consider the asymptotic behavior of e when a straight line has to be tracked. Once the angular quantities have reached values that can be neglected, the first of (14) becomes

$$\dot{e} = -\gamma e + \dot{s} \tag{16}$$

Hence, in case of constant and perfect alignment with the path, the eventual value for e can be found solving the equation

$$-\gamma e + \dot{s} = 0; \text{ ie. } f(e,0,0) = \gamma e \tag{17}$$

It is easy to see that, due to the structure of f(·)set in equation 15, equation (17) has always a positive solution fulfilling the condition $e < \sqrt{\varepsilon/\lambda}$. Finally, it should also be noted how the general vehicle behavior induced by the proposed closed loop control law for path following actually turns out to be qualitatively similar to the behavior apparently adopted within our everyday car-driving experience, where, in presence of good alignment with the road path, we are naturally led to progressively point our sight toward farther goals on the road itself, provided they remain sufficiently aligned with our car and within a certain maximum distance range; thus inducing a progressive increment in our car speed. On the contrary, when the misalignments begin to increase, as happens when we approach a curve of the road, we reduce our speed, and in the same time point our sight toward progressively closer goals in front of us; thus reducing the error distance, while maintaining the misalignments within an acceptable range. Furthermore, when the misalignments start again to reduce, as it happens at the outlet of a curve, we can in turn restart to increase our car speed. Figure 7 represents a path following experiment corresponding to the following choices:

$x(0)$=-2; $y(0)$=0; $\phi(0)$=0;
λ=0.001; ε=0.03;

Path equation y = Atan(x^4), x>0
$\dot{s}$ = max(0,1-V/ε); v_{max}=1

Figure 7 deserves some discussion. First note that it shows good accuracy in path tracking. Second, we have plotted the velocity u in order to obtain a one-to-one correspondence between the actual position of the vehicle and its velocity (both denoted with dashed lines). To better understand the behavior, note that the current value of ε is such that, once the state has been captured by the ellipsoid, the maximum value of $|\alpha|$ and $|\theta|$ is such that both cos α and cos θ can be considered almost everywhere equal to 1. Then the evolution of the vehicle velocity u can be considered the same as that of the error distance e (recall that γ=1). On this basis, given a certain position of the vehicle in Fig. 7, the corresponding position of the target frame can be found at any time by adding to the position of the vehicle a segment of length u with the other edge on the reference path. If we want to explain why at x=1 the vehicle begins to slow its speed, we can simply note that for that position, the error e takes on a value close to 0.5 (see $u(1)$). Hence, we can see that when the vehicle is in x=1 the x of the target frame is close to 1.5, that is at the beginning of the second curve. Due to this, the angular errors will increase and $\dot{s}$ will be reduced. As a consequence e will decrease and then u also. Finally, note that with the chosen values of the parameters, the eventual value of e can be easily computed by solving equation (17), that yields e=0.9687. Obviously, as ε increases, we get less accuracy in the tracking.

We now wish to consider the case of navigation among via points (possibly assigned by an external off-line or on-line acting roadmap planner) as depicted in Fig. 8. Let us first explicitly state that we will completely neglect (since it is too cum-

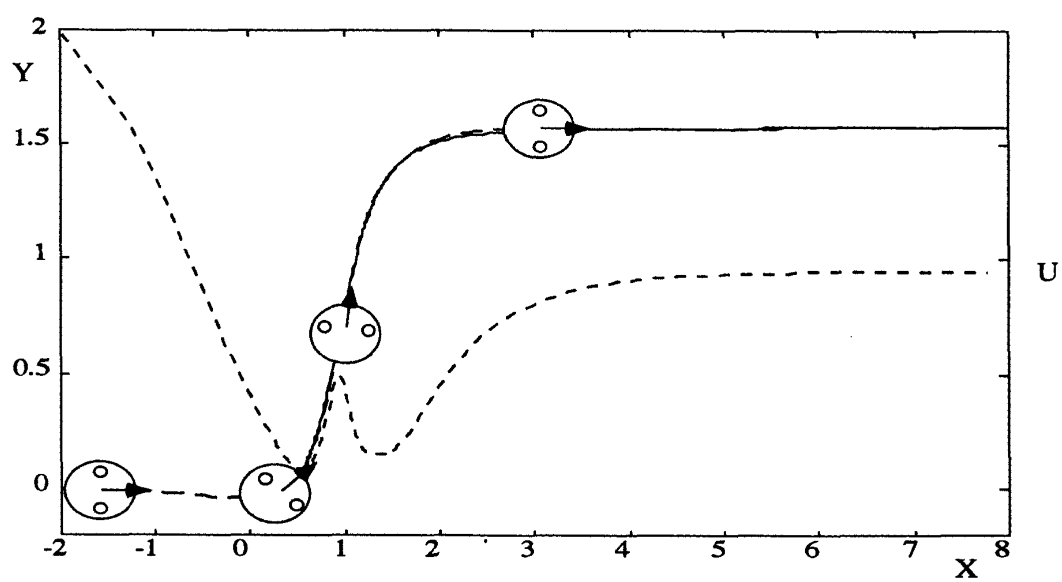

Figure 7. Reference trajectory = ———; Trajectory travelled = — — —; Velocity = - - - - - - -

bersome or even infeasible, especially within an on-line context) the obvious possibility of transforming the assigned problem into a path following one, consequently with the preliminary evaluation of some sort of smooth interpolating curve among the assigned via points. On the contrary, what will be considered here simply corresponds to the trivial idea of applying the above developed (smooth) path following technique also to the present case of navigation among via points, once the same are considered as connected each others by straight line segments (the most trivial and computationally zero-expensive form of interpolation) as it has been actually indicated also in Fig. 8.

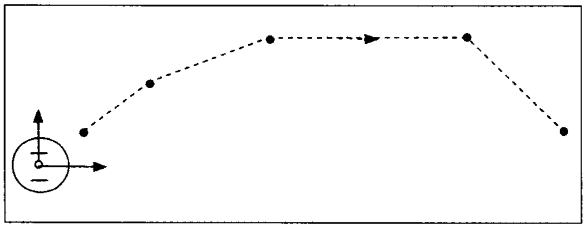

Figure 8.

By acting in this way, we can however note a fundamental difference with the previous smooth path following case: finite "jumps" always occur in the angular component θ of the state, and in correspondence of the "corner points" of the path, coinciding with the via points themselves. As it can be trivially verified, the effect introduced by the presence of such jumps is simply makes the state trajectory generally discontinuous (in the θ variable only) in correspondence with the transitions of the curvilinear abscissa s from one segment to the next one. When the angle measured between two successive segments is very wide, the corresponding jump of the θ variable can also be so high that the state is instantaneously transferred outside of the ellipsoidal domain, where, once the rate $\dot{s}$ has been consequently and instantaneously zeroed, the control law will bring back the state itself on the ellipsoidal surface, and further on the inside of it, where it will remain successively confined until the possible occurrence of another "sufficiently high jump" of the θ variable.

Obviously, provided the error distance e is sufficiently close to its largest upper bound $\sqrt{\varepsilon/\lambda}$, at the time of occurrence of a large jump for θ, sensibly wide deviations from the segmented path can actually occur.

This phenomenon is clearly evidenced in the motion example reported in Fig. 9a, where a large jump of θ occurs just when the vehicle, being previously well aligned with the antecedent straight line path segment, had already allowed the error distance to almost attain its admissible maximum value.

Such a phenomenon can however be reduced to any desired extent, by acting, for example, in the following way: at the instant when a large jump for θ is detected (in correspondence of a corner point), the value of the λ parameter is simultaneously and suitably increased (i.e., the dimension of the ellipsoid in the direction of *e* is suddenly reduced) while the same target frame preceding the jump is still maintained as the current target. This will have the instantaneous effect of letting the current state at the outside of the new ellipsoid, within a direction sufficiently close to that of the *e* axis. As a consequence of this, the rate $\dot{s}$ will be instantaneously zeroed, thus allowing the state to re-enter inside the new ellipsoid with a reduced value for e, and without a substantial change for the alignment relevant to the maintained old target frame. Once the state is inside the new ellipsoid, the abrupt change for θ is inserted (i.e., the current target frame is instantaneously changed with the new one corresponding to the starting point of the new path segment) and λ is re-established in its original value. Even in this new condition, the state will be outside of the ellipsoid. Then, the null value of $\dot{s}$ will be kept, allowing the alignment with respect to the new current frame-point to re-enter within the admissible range while continuing to de-

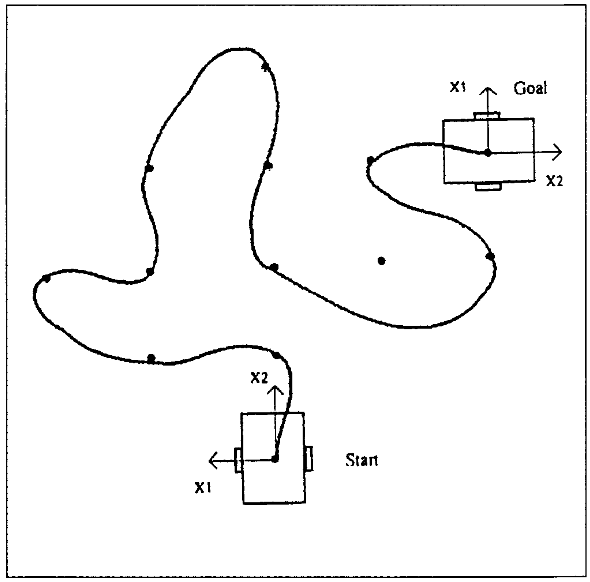

Figure 9a.

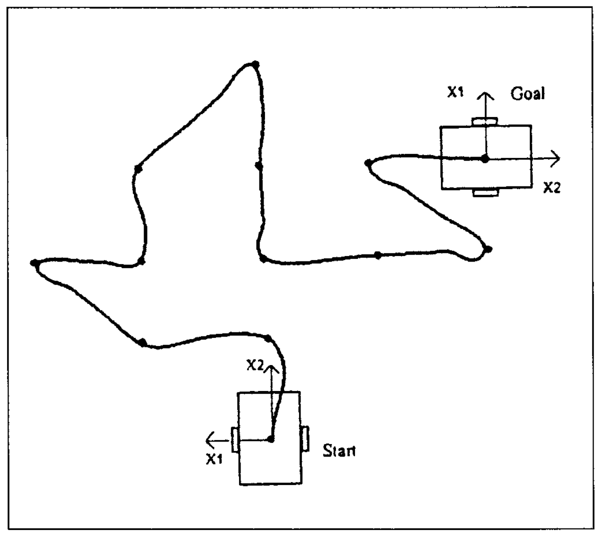

Figure 9b.

crease the error distance, till the ellipsoid is not reached. Once this happens, everything will proceed in the usual way till the eventual occurrence of a new large jump for θ. The results corresponding to the application of such additional technique are reported in Fig. 9b. As the above two examples show, the first one could be considered as acceptable only when a large space for manoeuvering actually exists around the vehicle; the second one is more indicated for maneuvering within constrained environments, for instance in the case of navigation along narrow corridors.

CONCLUSIONS

This paper has presented evidence that a special choice for the system state equations allows the almost straightforward use of Lyapunov theory, leading to the definition of very simple and effective closed loop control laws for unicycle-like vehicles, suitable for both steering, path following, and navigation among assigned via points; without requiring, in the latter case, of any sort of trajectory planning or re-planning, as it has been instead very often proposed.

The simplicity of the approach, whenever compared with the more sophisticated ones based on advanced non linear systems concepts and differential geometric techniques, also seems to suggest the possibility of an extension toward the more complex case of car-like or articulated vehicles.

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