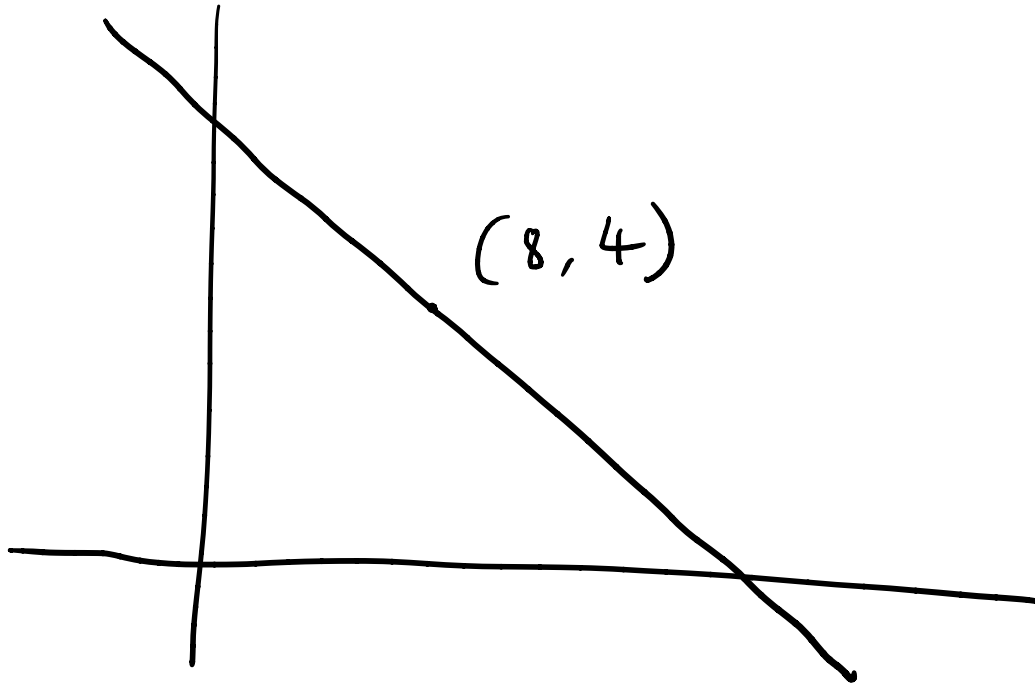


Consider triangles formed by lines passing through the point $(8, 4)$, the x -axis, and the y -axis. Find the dimensions that minimise area.



$$y - 4 = m(x - 8)$$

$$y = 0, \quad -4 = m(x - 8) \Rightarrow \begin{aligned} -4 &= mx - 8m \\ mx &= -4 + 8m \\ x &= -\frac{4}{m} + 8 \end{aligned}$$

$$x = 0, \quad y - 4 = -8m \Rightarrow y = -8m + 4$$

$$\begin{aligned} \text{Area} &= \frac{1}{2}xy \\ &= \frac{1}{2}x(m(x-8)+4) \\ &= \frac{1}{2}x(mx-8m+4) \end{aligned}$$

$$\begin{aligned} \text{Area} &= \frac{1}{2}x_0 y_0 \\ &= \frac{1}{2} \left(-\frac{4}{m} + 8 \right) (-8m + 4) \\ &= \frac{1}{2} \left(32 - 64m - \frac{16}{m} + 32 \right) \\ &= \frac{1}{2} \left(64 - 64m - \frac{16}{m} \right) \end{aligned}$$

$$= \frac{1}{2}mx^2 - 4mx + 2x$$

$$\frac{dA}{dx} = mx - 4m + 2$$

$$\frac{dA}{dx} = 0$$

$$mx - 4m + 2 = 0$$

$$\left(-\frac{1}{2}\right)x - 4\left(-\frac{1}{2}\right) + 2 = 0$$

$$-\frac{1}{2}x + 2 + 2 = 0$$

$$-\frac{1}{2}x + 4 = 0$$

$$-\frac{1}{2}x = -4$$

$$x = 8$$

(0, 8)

(8, 4)

(16, 0)

$$A = 8 \times 16 \times \frac{1}{2} = 64$$

$$= 32 - 32m - \frac{8}{m}$$

$$\frac{dA}{dm} = -32 + \frac{8}{m^2}$$

$$\frac{dA}{dm} = 0$$

$$-32 + \frac{8}{m^2} = 0$$

$$-32m^2 + 8 = 0$$

$$-32m^2 = -8$$

$$m^2 = \frac{1}{4}$$

$$m = \pm \frac{1}{2}$$

$m = -\frac{1}{2}$ because

line is sloping downwards

$$y - 4 = -\frac{1}{2}(x - 8)$$

when $x = 0$,

$$y = 8$$

when $y = 0$,

$$8 = x - 8$$

$$x = 16$$