Compute the definite integrals

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan x \, dx \quad \text{and} \quad \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \tan x \, dx$$

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7/8/25

$$F'(x) = f(x) = \tan x$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan x \; dx$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x}{\cos x} \; dx$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} -\frac{1}{u} \; du$$

$$= -\ln|\cos x| \Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$= -\ln\left|\cos \frac{\pi}{3}\right| + \ln\left|\cos \frac{\pi}{6}\right|$$

$$= -\ln\left|\frac{1}{2}\right| + \ln\left|\frac{\sqrt{3}}{2}\right|$$

$$= -\ln\left|\frac{1}{\sqrt{3}}\right|$$

$$= 0 + \ln(\sqrt{3})$$

$$= \frac{\ln 3}{2}$$

Let $u = \sin x \Rightarrow du = \cos x \; dx$

$u = \sec x \Rightarrow du = \tan x \sec x \; dx$

$u = \cos x \Rightarrow du = -\sin x \; dx$

$$\Rightarrow \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \tan x \; dx$$

$$= \ln|\cos x| \Big|_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$$

$$= \ln\left|\cos \frac{\pi}{3}\right| - \ln\left|\cos\left(-\frac{\pi}{3}\right)\right|$$

$$= \ln\left|\frac{1}{2}\right| - \ln\left|\frac{1}{2}\right|$$

$$= 0$$