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$$\frac{x-11}{(x^2+9)(x+2)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+9}$$

$x = -2$:

$$\frac{-2-11}{13} = \frac{-2B+C}{13}$$

$$-13 = -2B+C$$

$$C = 2B-13$$

$x-11 = Ax^2 + 9A + Bx^2 + 2Bx + Cx + 2C$

$x-11 = (A+B)x^2 + (2B+C)x + 9A + 2C$

$\Rightarrow A+B = 0$

$2B+C = 1$

$9A + 2C = -11$

$\Rightarrow A = -B$

$\Rightarrow 2B+C = 1$

$-9B + 2C = -11$

$\Rightarrow 13B = 13$

$B = 1$

$A = -1$

$C = 1-2B = -1$

$$\int \frac{x-11}{(x^2+9)(x+2)} \, dx$$

$$= \int -\frac{1}{x+2} \, dx + \int \frac{x-1}{x^2+9} \, dx$$

$$= -\ln|x+2| + \ln \left| \frac{3}{\sqrt{x^2+9}} \right|$$

$$-\frac{\arctan \frac{x}{3}}{3} + C$$

$$\int \frac{x-1}{x^2+9} \, dx \qquad \begin{array}{l} x = 3\tan u \\ dx = 3\sec^2 u \, du \end{array}$$

$$= 3\int \frac{(3\tan u - 1)\sec^2 u}{9\tan^2 u + 9} \, du$$

$$= \frac{1}{3} \int \frac{(3\tan u - 1)\sec^2 u}{\sec^2 u} \, du$$

$$= \frac{1}{3} \int 3\tan u - 1 \, du$$

$$= \ln|\cos u| - \frac{u}{3} + C$$

$$= \ln \left| \frac{3}{\sqrt{x^2+9}} \right| - \frac{\arctan \frac{x}{3}}{3} + C$$

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