Comparing Linear Approximations to Calculator Computations

In lecture, we explored linear approximations to common functions at the point $x = 0$. In this worked example, we use the approximations to calculate values of the sine function near $x = 0$ and compare the answers to those on a scientific calculator.

Find the linear approximation to $\sin(x)$ at the point $x = 0$ and use your answer to approximate the values of $\sin(.01), \sin(.1)$ and $\sin(1)$. Check your answer on a calculator.

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$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$ when $x \approx x_0$

$f(x) = \sin x, \; x_0 = 0$

$\Rightarrow f(x) \approx \sin 0 + \cos 0 \; (x - 0)$

$= 0 + 1 \; (x)$

$= x$

$\therefore \sin x \approx x$ when $x \approx 0$.

$\sin(0.01) = 0.01$
$\sin(0.1) = 0.1$
$\sin(1) = 1$

Calculator:
$\sin(0.01) = 0.00999$
$\sin(0.1) = 0.0998$
$\sin(1) = 0.841$

$\Delta f_{0.01} < 0.001$

$\Delta f_1 < 0.159$