Problem 1

$$y'' - 3y' + 2y = 0$$

Guess $y = e^{rt}$.

$$\Rightarrow r^2 e^{rt} - 3re^{rt} + 2e^{rt} = 0$$

$$(r^2 - 3r + 2)e^{rt} = 0$$

$$r^2 - 3r + 2 = 0$$

$$(r-2)(r-1) = 0$$

$$\therefore r = 1, 2$$

$$y = c_1 e^t + c_2 e^{2t}$$

Problem 2

$$y'' + 2y' - 3y = 0 \quad, \quad y(0) = 1 \quad, \quad y'(0) = -1$$

$$\Rightarrow r^2 + 2r - 3 = 0$$

$$(r+3)(r-1) = 0$$

$$\therefore r = -3, 1$$

$$\Rightarrow y = c_1 e^{-3t} + c_2 e^t$$

$$y' = -3c_1 e^{-3t} + c_2 e^t$$

$$\Rightarrow 1 = c_1 + c_2$$

$$-1 = -3c_1 + c_2$$

$$\Rightarrow 2 = 4c_1 \qquad 1 = \frac{1}{2} + c_2$$

$$c_1 = \frac{1}{2} \qquad c_2 = \frac{1}{2}$$

$$\therefore y = \frac{1}{2} e^{-3t} + \frac{1}{2} e^t$$

Problem 3

$$y = c_1 + c_2 e^{-5x}$$

$$\Rightarrow r = -5, 0$$

$$p(r) = r(r+5)$$

$$= r^2 + 5r$$

$$\Rightarrow y'' + 5y' = 0$$

Problem 4

$$y = c_1 e^{5x} + c_2 e^{-5x}$$

$$\Rightarrow p(r) = (r-5)(r+5)$$

$$= r^2 - 25$$

$$\Rightarrow y'' - 25y = 0$$

Problem 5

$$y = c_1 + c_2 x \quad \Rightarrow \quad p(r) = r^2$$

$$\Rightarrow \quad 1 = e^{rx} \qquad \Rightarrow y'' = 0$$

$$\therefore \ r = 0$$

Problem 6

$$y'' - 4y = 0$$

$$\Rightarrow r^2 - 4 = 0$$

$$r^2 = 4$$

$$r = \pm 2$$

$$\Rightarrow y = c_1 e^{2x} + c_2 e^{-2x}$$

Problem 7

$$2y'' - 3y' = 0$$

$$y'' - \frac{3}{2}y' = 0$$

$$\Rightarrow p(r) = r^2 - \frac{3}{2}r = r\left(r - \frac{3}{2}\right)$$

$$r = 0, \frac{3}{2}$$

$$\Rightarrow y = c_1 + c_2 e^{\frac{3}{2}x}$$

Problem 8

$$4y'' - 12y' + 9y = 0 \qquad \Rightarrow y = c_1 e^{\frac{1}{2}x} + c_2 t e^{\frac{3}{2}x}$$

$$y'' - 3y' + \frac{9}{4}y = 0$$

$$\Rightarrow r^2 - 3r + \frac{9}{4} = 0$$

$$\left(r - \frac{3}{2}\right)^2 = 0 \quad \therefore r = \frac{3}{2}$$
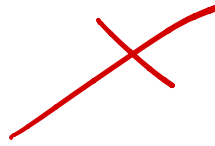
Problem 9

$$y^{(4)} - 8y'' + 16y = 0$$

$$\Rightarrow P(r) = r^4 - 8r^2 + 16 = 0$$

$$(r^2 - 4)(r^2 - 4) = 0$$

$$r^2 = 4$$

$$r = \pm 2$$

$$\therefore \quad y = c_1 e^{2x} + c_2 e^{-2x} \quad \times$$

Problem 10

$$y'' + 2y' + 2y = 0$$

$$\Rightarrow r^2 + 2r + 2 = 0$$

$$r = \frac{-2 \pm \sqrt{4-8}}{2}$$

$$= -1 \pm i$$

$$\Rightarrow e^{(-1+i)x}, e^{(-1-i)x}$$

$$= e^{-x}(\cos x + i\sin x) = e^{-x}(\cos x - i\sin x)$$

$$\therefore y = c_1 e^{-x}\cos x + c_2 e^{-x} i\sin x$$

Problem 11

$$y'' - 2y' + 5y = 0$$

$$\Rightarrow r^2 - 2r + 5 = 0$$

$$r = \frac{2 \pm \sqrt{4 - 20}}{2}$$

$$= 1 \pm 2i$$

$$\Rightarrow y = c_1 e^x \cos 2x + c_2 e^x i \sin 2x$$

$$1 = c_1, \quad c_1 = 1$$
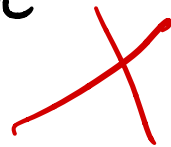
$$y' = -2c_1 e^x \sin 2x + 2 c_2 e^x i \cos 2x$$

$$-1 = 2 c_2 i$$

$$c_2 = -\frac{1}{2i}$$

$$= \frac{1}{2} i$$

$$\therefore \quad y = e^x \cos 2x - \frac{1}{2} e^x \sin 2x$$

Problem 12

$$y'' - 4y' + 4y = 0$$

$$\Rightarrow r^2 - 4r + 4 = 0$$

$$r = \frac{4 \pm \sqrt{16 - 16}}{2} = 2$$

$$\Rightarrow y = Ce^{2x}$$

$$1 = C, \therefore C = 1$$

$$y' = 2Ce^{2x}$$

$$1 = 2C$$

$$C = \frac{1}{2}$$

$$\therefore y = e^{2x} + \frac{1}{2}e^{2x}$$

$$= \frac{3}{2}e^{2x} \quad \times$$