Find $\int \tan^4 \theta \, d\theta$.

Find $\int \tan^4 \theta \, d\theta$.

$$\sin^2 \theta + \cos^2 \theta = 1$$
$$\Rightarrow \tan^2 \theta + 1 = \sec^2 \theta$$

$\int \tan^4 \theta \, d\theta$

$u = \tan \theta \Rightarrow du = \sec^2 \theta \, d\theta$

$= \int (\sec^2 \theta - 1) \tan^2 \theta \, d\theta$

$= \int \sec^2 \theta \tan^2 \theta \, d\theta - \int \tan^2 \theta \, d\theta$

$= \int u^2 \, du - \int \sec^2 \theta - 1 \, d\theta$

$= \dfrac{u^3}{3} + C_1 - \left(\tan \theta - \theta + C_2 \right)$

$= \dfrac{\tan^3 \theta}{3} - \tan \theta + \theta + C$