

Compute the following limits:

$$(a) \lim_{x \rightarrow 1} \frac{x^a - 1}{x^b - 1}$$

$$(b) \lim_{x \rightarrow 0} \frac{\sin 5x}{x}$$

$$(c) \lim_{x \rightarrow 0} \frac{x^2 - 6x + 2}{x + 1}$$

$$(d) \lim_{x \rightarrow \infty} \frac{\ln(1 + e^{3x})}{2x + 5}$$

$$\begin{aligned}
 (a) \quad & \lim_{x \rightarrow 1} \frac{x^a - 1}{x^b - 1} \\
 &= \lim_{x \rightarrow 1} \frac{\frac{x^a - 1}{x - 1}}{\frac{x^b - 1}{x - 1}} \\
 &= \frac{f'(1)}{g'(1)} \\
 &= \frac{a(1)^{a-1}}{b(1)^{b-1}} \\
 &= \frac{a}{b}
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad & \lim_{x \rightarrow 0} \frac{\sin 5x}{x} \\
 & f(0) = g(0) = 0
 \end{aligned}$$

$$\begin{aligned}
 \therefore \quad & \lim_{x \rightarrow 0} \frac{\sin 5x}{x} \\
 &= \frac{5 \cos 5(0)}{1} \\
 &= 5
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad & \lim_{x \rightarrow 0} \frac{x^2 - 6x + 2}{x + 1} \\
 &= \frac{0 - 0 + 2}{0 + 1} \\
 &= 2
 \end{aligned}$$

$$(d) \quad \lim_{x \rightarrow \infty} \frac{\ln(1 + e^{3x})}{2x + 5}$$

$$\begin{aligned}
 & f(\infty) = g(\infty) = \infty \\
 & \text{and } \lim_{x \rightarrow \infty} \frac{f'(x)}{g'(x)} \text{ exists.}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \quad & \lim_{x \rightarrow \infty} \frac{\ln(1 + e^{3x})}{2x + 5} \\
 &= \lim_{x \rightarrow \infty} \frac{\frac{1}{1 + e^{3x}} \cdot 3e^{3x}}{2} \quad (L' \text{ hospital}) \\
 &= \lim_{x \rightarrow \infty} \frac{1}{2} \cdot \frac{3e^{3x}}{e^{3x}} \quad (L' \text{ hospital}) \\
 &= \frac{3}{2}
 \end{aligned}$$