Algebra and Trigonometry

Stewart

Sections	
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Chapter 0 Prerequisites

0.2 Real Numbers

1. Real Numbers Introduction
2. Properties of Real Numbers

$\vdots$

6. Sets and Intervals

0.2.6 Sets and Intervals

Questions 41 – 66

$A = \{1, 2, 3, 4, 5, 6, 7\}$

$B = \{2, 4, 6, 8\}$

$C = \{7, 8, 9, 10\}$

41. (a) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

(b) $A \cap B = \{2, 4, 6\}$

42. (a) $B \cup C = \{2, 4, 6, 7, 8, 9, 10\}$

(b) $B \cap C = \{8\}$

43. (a) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(b) $A \cap C = \{7\}$

44. (a) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(b) $A \cap B \cap C = \emptyset$

$A = \{x \mid x \ge -2\}$ $B = \{x \mid x < 4\}$

$C = \{x \mid -1 < x \le 5\}$

45. (a) $B \cup C = \{x \mid x \le 5\}$

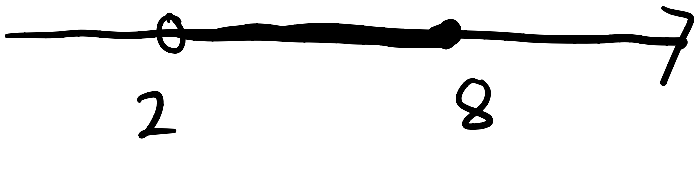
(b) $B \cap C = \{x \mid -1 < x < 4\}$

46. (a) $A \cap C = \{x \mid -1 < x \le 5\}$

(b) $A \cap B = \{x \mid -2 \le x < 4\}$

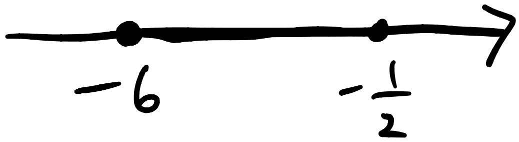
47. $(-3, 0) = \{x \mid -3 < x < 0\}$

48. $(2, 8] = \{x \mid -2 < x \le 8\}$



49. $[2, 8) = \{x \mid 2 \leq x < 8\}$

50. $[-6, -\frac{1}{2}] = \{x \mid -6 \leq x \leq -\frac{1}{2}\}$



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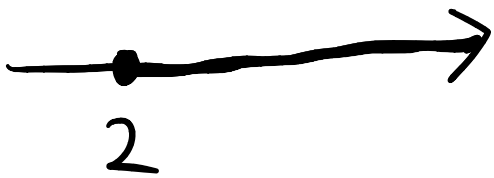
51. $[2, \infty) = \{x \mid x \geq 2\}$



52. $(-\infty, 1) = \{x \mid x < 1\}$

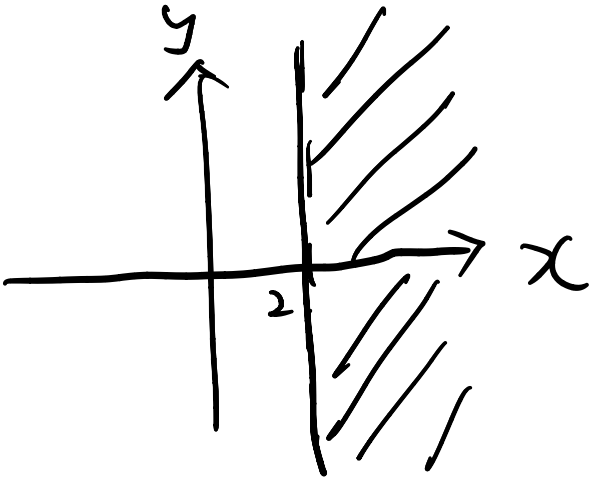
53. $x \leq 1 = x \in (-\infty, 1]$

54. $1 \leq x \leq 2 = x \in [1, 2]$

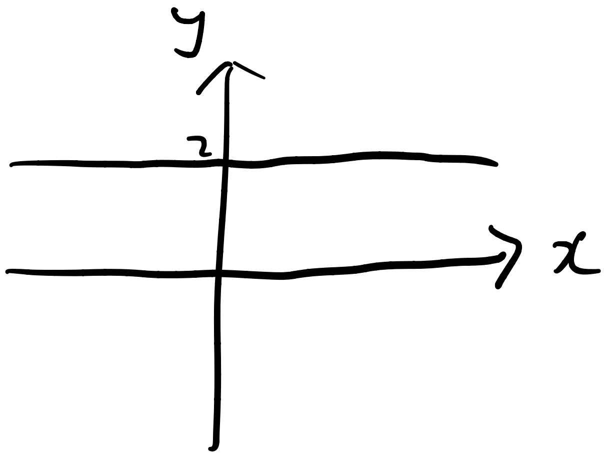
Chapter 1 Equations and Graphs

1.1 The Coordinate Plane

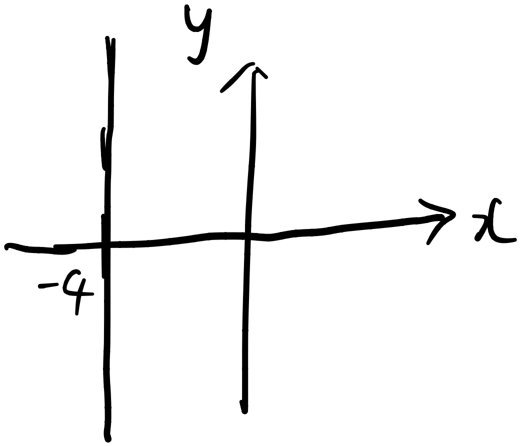
9. $\{(x,y) \mid x \geq 2\}$



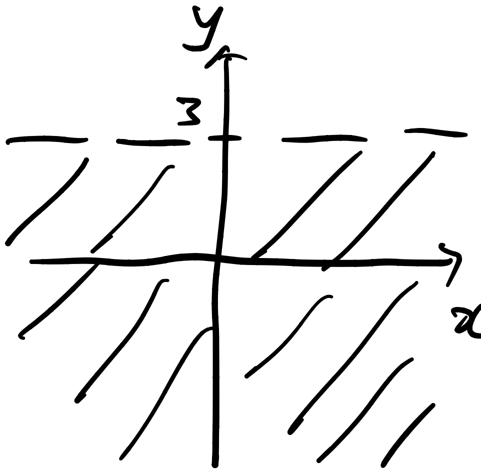
10. $\{(x,y) \mid y = 2\}$



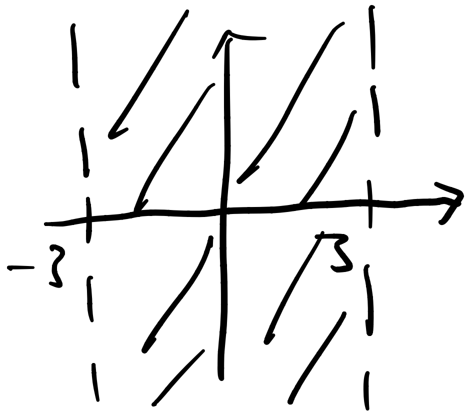
11. $\{(x,y) \mid x = -4\}$



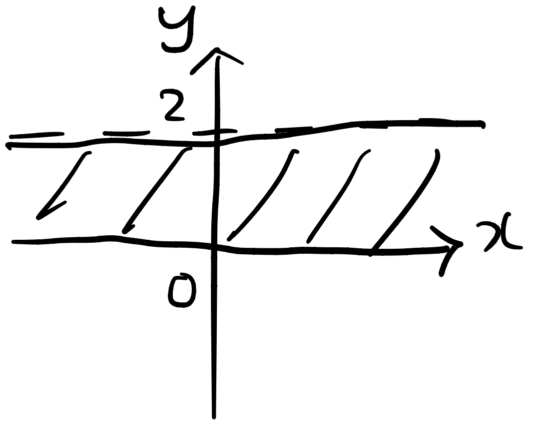
12. $\{(x,y) \mid y < 3\}$



13. $\{(x,y) \mid -3 < x < 3\}$



14. $\{(x,y) \mid 0 \leq y \leq 2\}$



Distance and Midpoint

21. $P1 = (0, 2)$

$P2 = (3, 0)$

$$\text{Distance} = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$= \sqrt{2^2 + (-3)^2}$$

$$= \sqrt{13}$$

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{0+3}{2}, \frac{2+0}{2} \right)$$

$$= \left(\frac{3}{2}, 1 \right)$$

1.1 The Coordinate Plane

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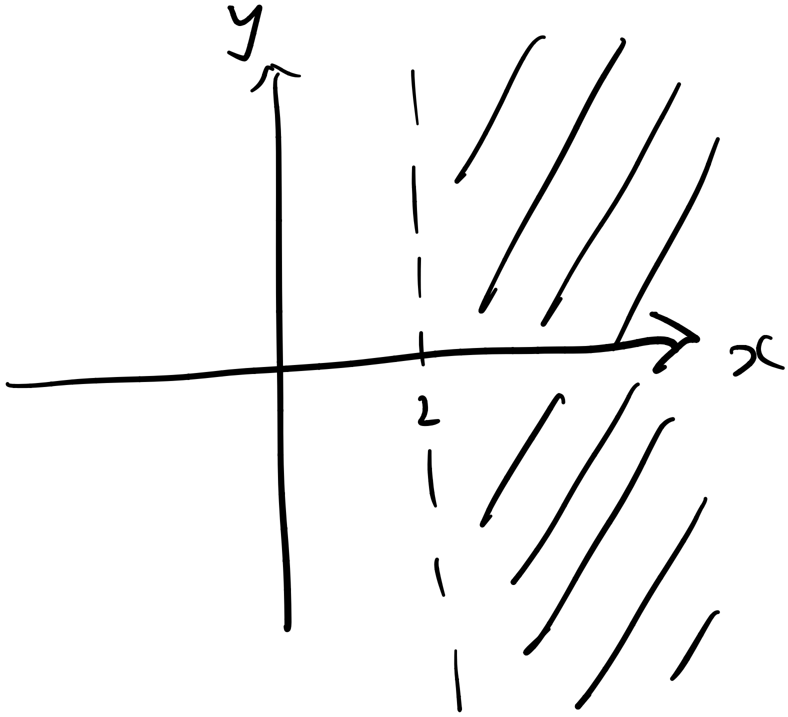
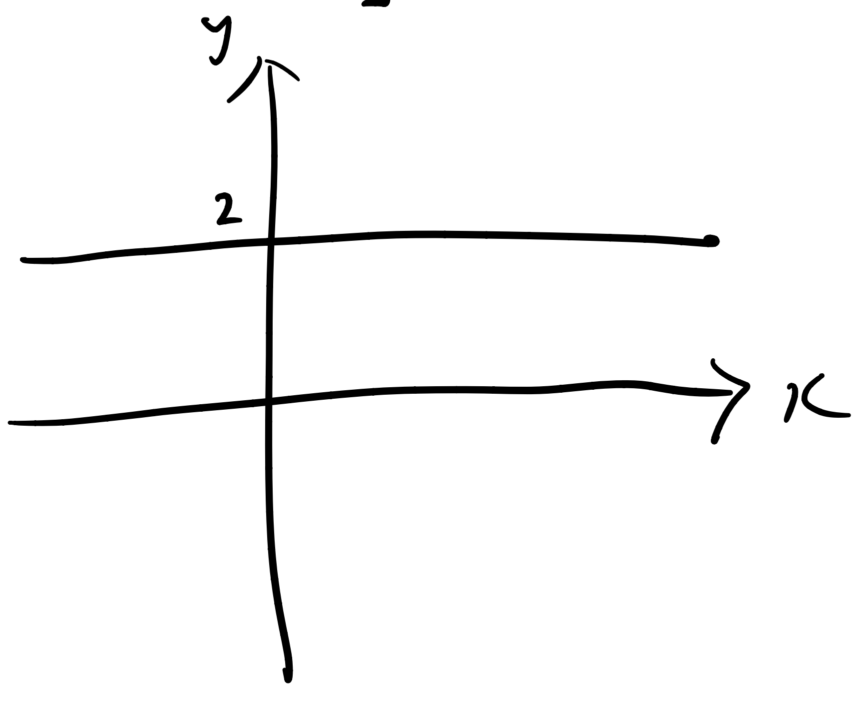
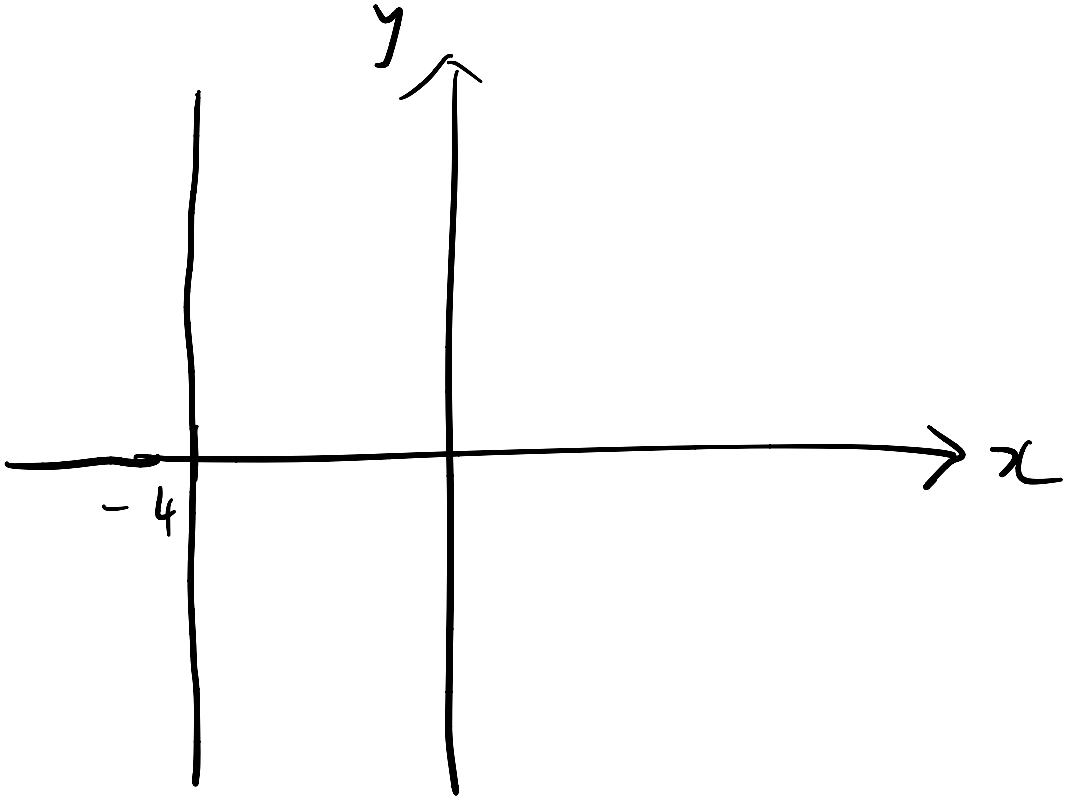
9. $\{(x, y) \mid x \geq 2\}$



10. $\{(x, y) \mid y = 2\}$



11. $\{(x, y) \mid x = -4\}$



1.2 Graphs of Equations in Two Variables; Circles

Graphing Equations by Plotting Points

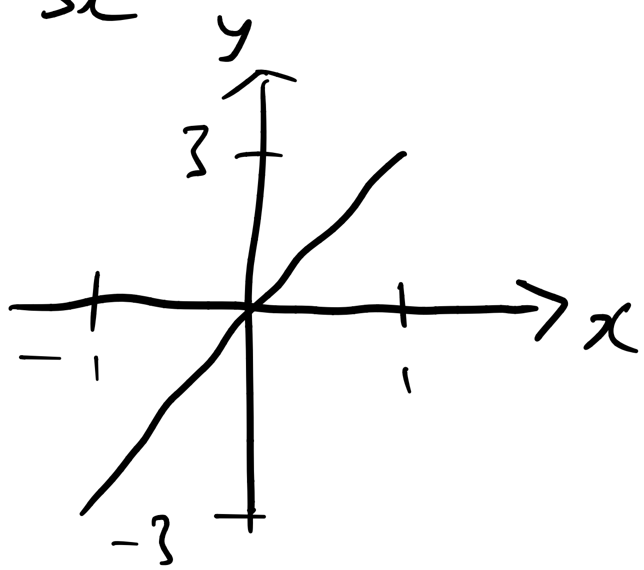
Intercepts

Circles

Symmetry

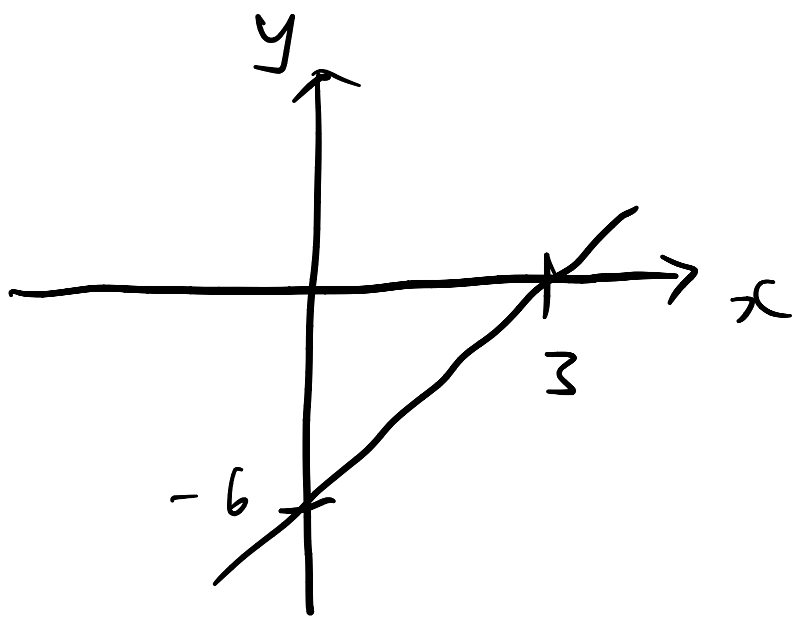
Graphing Equations

15. $y = 3x$



19. $2x - y = 6$

$y = 2x - 6$



49. $y = x^2 - 5$ Intercepts

x-intercept: $0 = x^2 - 5$

$x^2 = 5$

$x = \pm\sqrt{5}$

y-intercept: $y = 0 - 5$

$y = -5$

55. $4x^2 + 25y^2 = 100$

x-intercept: $4x^2 + 25(0) = 100$

$4x^2 = 100$

$x^2 = 25$

$x = \pm 5$

y-intercept: $4(0) + 25y^2 = 100$

$25y^2 = 100$

$y^2 = 4$

$y = \pm 2$

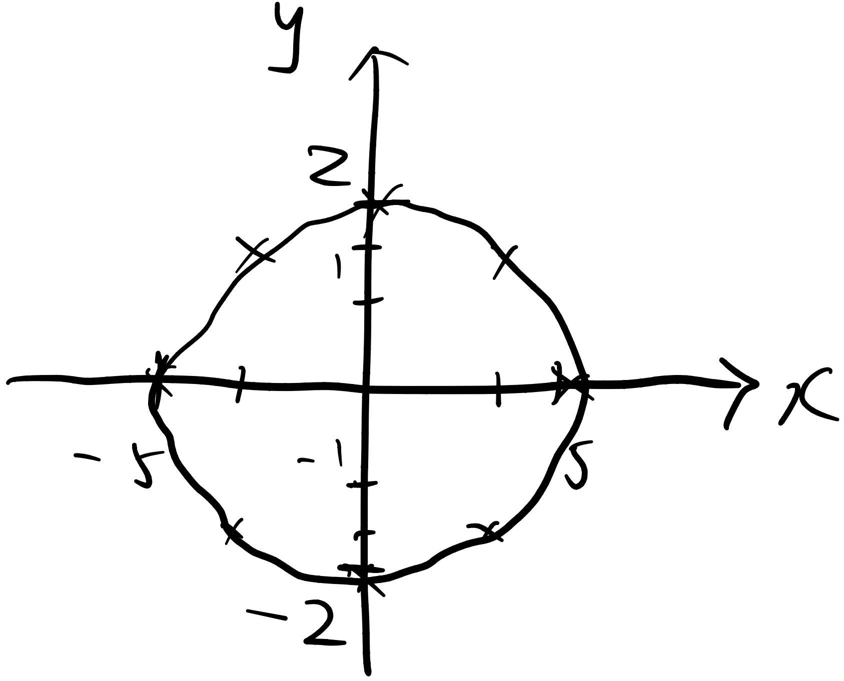
$x = 3$, $4(9) + 25y^2 = 100$

$25y^2 = 64$

$y^2 = \frac{64}{25}$

$y = \pm \frac{8}{5}$

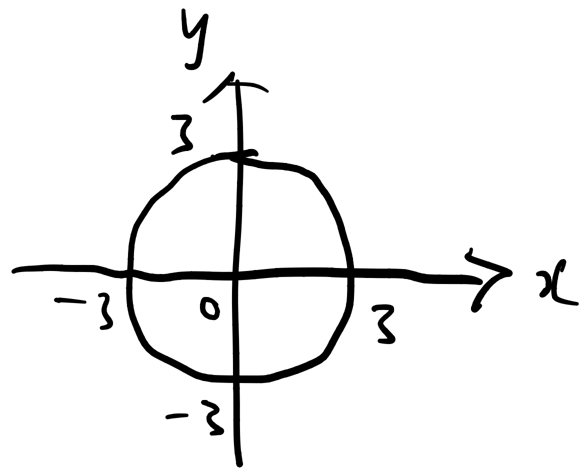
$x = -3$, $y = \pm \frac{8}{5}$


y
2
1
x
-5
-1
5
-2

Graphing Circles

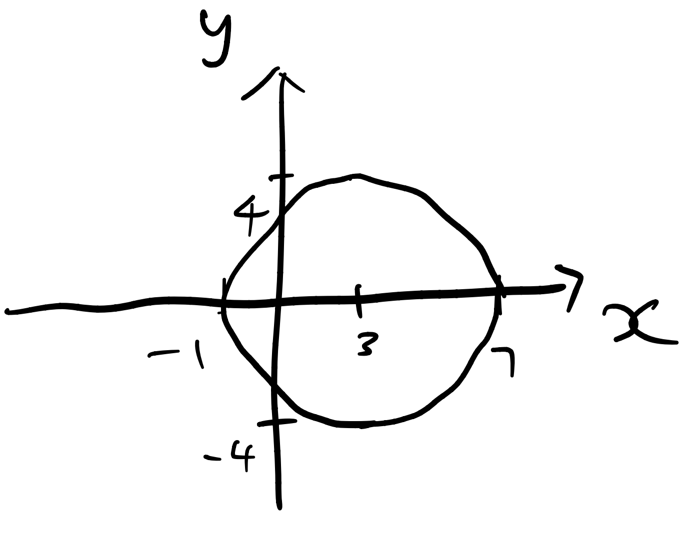
67. $x^2+y^2=9$

$x^2+y^2=3^2$



69. $(x-3)^2+y^2=16$

$(x-3)^2+y^2=4^2$



when $y=0$, $x=-1,7$

when $x=3$, $y=-4,4$

73. $C(-3, 2)$, $r = 5$

$$(-3+3)^2 + (2-2)^2 = 5^2$$

$$\therefore (x+3)^2 + (y-2)^2 = 5^2$$

Symmetry

95. $y = x^4 + x^2$

if symmetric, $(x, y) = (-x, -y)$

$y = (-x)^4 + (-x)^2$

$-y = x^4 + x^2$

Eq 1 $\neq$ Eq 2

$\therefore$ not symmetric w.r.t. the origin

97. $y = x^3 + 10x$

$(-y) = (-x)^3 + 10(-x)$

$-y = -x^3 - 10x$

$y = x^3 + 10x$

$\because$ Eq 1 $=$ Eq 2

$\therefore$ Symmetric w.r.t. the origin

99. $x^4y^4 + x^2y^2 = 1$

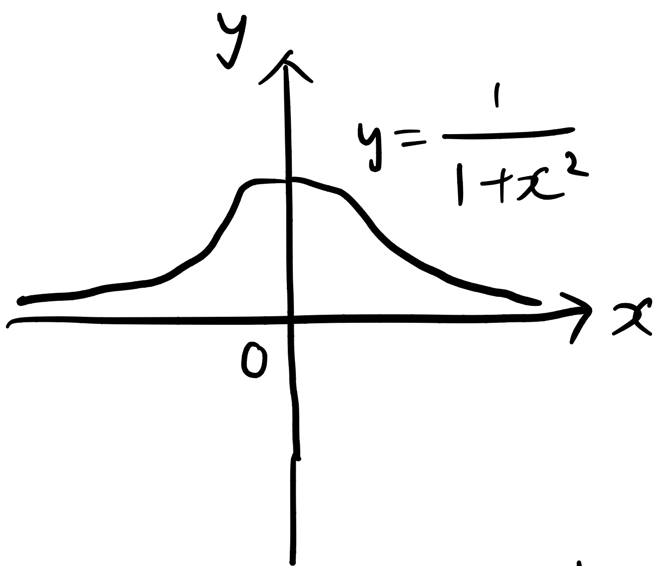
$(-x)^4(-y)^4 + (-x)^2(-y)^2 = 1$

$x^4y^4 + x^2y^2 = 1$

Eq 1 = Eq 2

$\therefore$ Symmetric w.r.t. the origin

101.



symmetric with respect to y-axis

$(x, y) = (-x, y)$

1.2 Graphs of Equations in Two Variables; Circles

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1. 2, 3

$2y = x + 1$

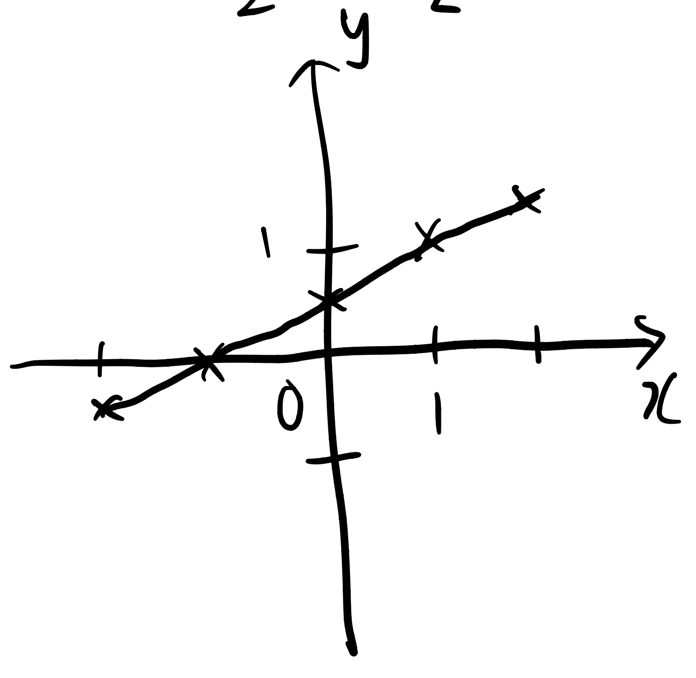
$2(3) = (2) + 1$

$6 \neq 3$

$\therefore$ No the point is not on the graph.

$y = \frac{1}{2}x + \frac{1}{2}$

x	y	(x, y)
-2	$-\frac{1}{2}$	$(-2, -\frac{1}{2})$
-1	0	$(-1, 0)$
0	$\frac{1}{2}$	$(0, \frac{1}{2})$
1	1	$(1, 1)$
2	$\frac{3}{2}$	$(2, \frac{3}{2})$



9. $y = 3 - 4x$

$(0, 3)$, $3 = 3 - 4(0)$

$3 = 3$

$\therefore$ Yes

$(4, 0)$, $0 = 3 - 4(4)$

$0 \neq -13$

$\therefore$ No

$(1, -1)$, $-1 = 3 - 4(1)$

$-1 = -1$

$\therefore$ Yes

1.3 Lines

9. $P(-1,2)$, $Q(0,0)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-2}{0-(-1)} = \frac{-2}{1} = -2$$

25. $P(2,3)$, $m=5$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 5(x-2)$$

$$y - 3 = 5x - 10$$

$$y - 5x + 7 = 0$$

$$5x - y - 7 = 0$$

29. $P_1(2,1)$, $P_2(1,6)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-1}{1-2} = \frac{5}{-1} = -5$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -5(x - 2)$$

$$-5x + 10 - y + 1 = 0$$

$$5x + y - 11 = 0$$

23. $m = 3$, $c = -2$

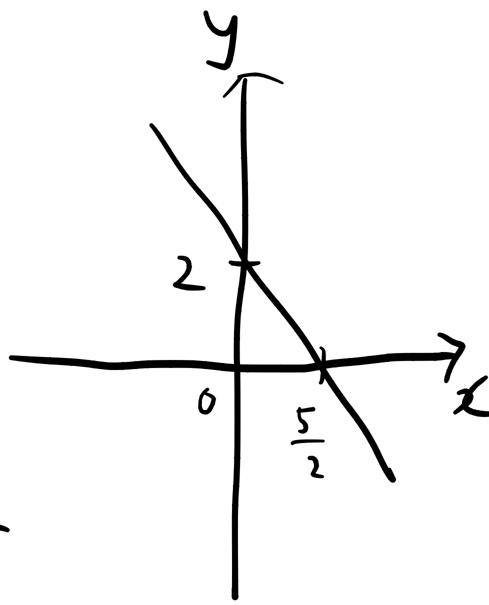
$$y = 3x - 2$$

61. $$4x + 5y = 10$$

$$5y = -4x + 10$$

$$y = -\frac{4}{5}x + 2$$

$\therefore$ slope $= -\frac{4}{5}$, y-intercept $= 2$



Vertical and Horizontal Lines

35. $P(1, 3)$, $m = 0$

$$y = mx + c$$
$$= 0 + c$$
$$= c$$

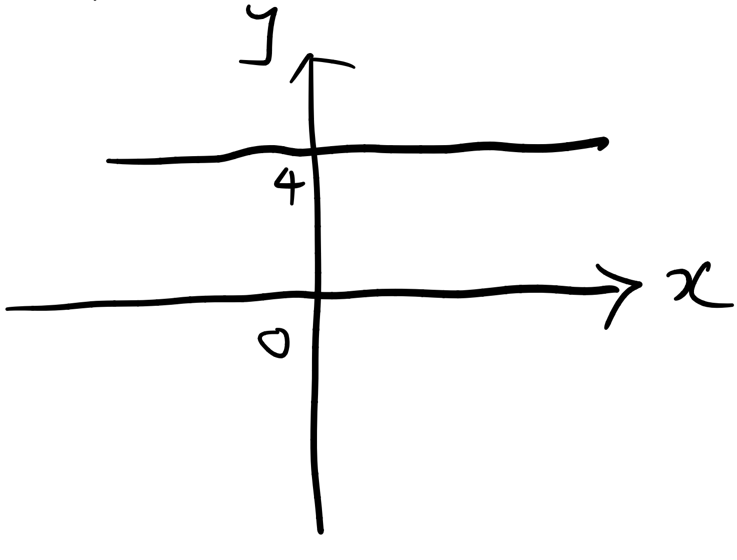
$$y = 3$$

37. $P(2, -1)$, m: undefined

$$x = 2$$

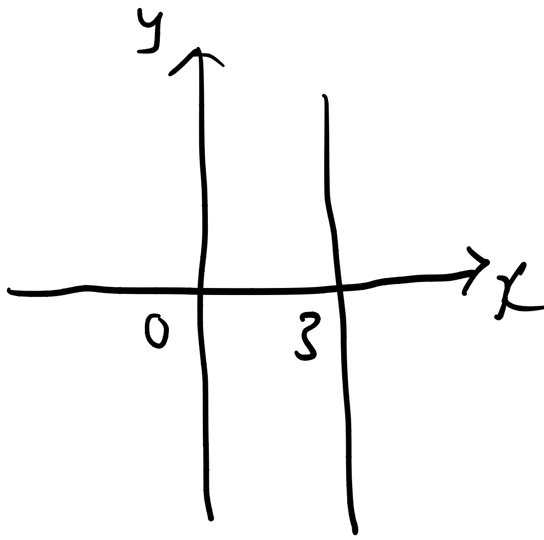
63. $y = 4$

$\therefore$ Slope $= 0$, y-intercept $= 4$



65. $x = 3$

$\therefore$ Slope $=$ undefined, no y-intercept



67. $5x+2y-10=0$

x-intercept: $5x+2(0)-10=0$

$5x=10$

$x=2$

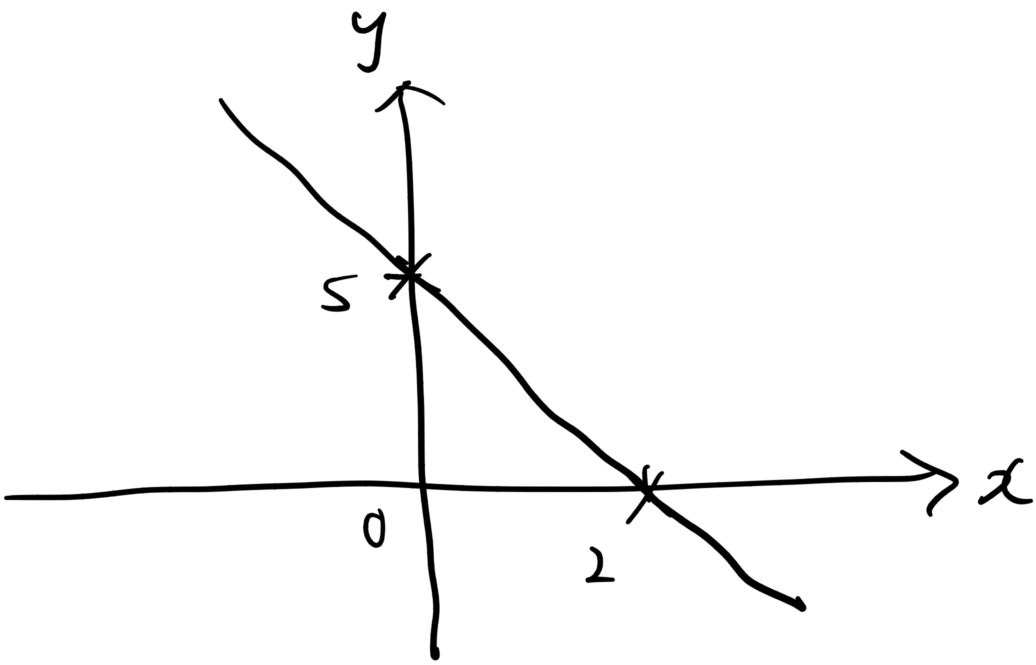
$P_1\ (2, 0)$

y-intercept: $5(0)+2y-10=0$

$2y=10$

$y=5$

$P_2\ (0, 5)$



43. $P(1, -6)$, parallel to the line

$$x+2y=6$$

$$x+2y=6$$

$$2y=-x+6$$

$$y=-\frac{1}{2}x+3$$

$$m=-\frac{1}{2}$$

$$y-y_1=m(x-x_1)$$

$$y-(-6)=-\frac{1}{2}(x-1)$$

$$y+6=-\frac{1}{2}x+\frac{1}{2}$$

$$\frac{1}{2}x+y+6-\frac{1}{2}=0$$

$$\therefore x+2y+11=0$$

81.

If line $AB \perp BC$, $AB \perp DA$,

$AB // CD$,

ABCD is a rectangle.

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{11-1} = \frac{2}{10} = \frac{1}{5}$$

$$m_{DA} = \frac{6-1}{0-1} = \frac{5}{-1} = -5$$

$\because m_{AB} m_{DA} = -1$

$\therefore AB \perp DA$

$$m_{BC} = \frac{8-3}{10-11} = \frac{5}{-1} = -5$$

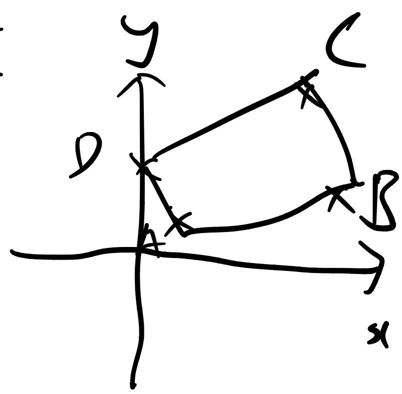
$$m_{CD} = \frac{6-8}{0-10} = \frac{-2}{-10} = \frac{1}{5}$$

$\because m_{AB} m_{BC} = -1$

$\therefore AB \perp BC$

$\because m_{AB} = m_{CD}$

$\therefore AB // CD$



$\therefore$ A, B, C and D are vertices of a rectangle.

47. $P(-1,-2)$, perpendicular to $2x+5y+8=0$

$$2x+5y+8=0$$

$$5y=-2x-8$$

$$y=-\frac{2}{5}x-\frac{8}{5}$$

$$m=-\frac{2}{5}$$

$$m_1m_2=-1$$

$$-\frac{2}{5}m_2=-1$$

$$m_2=\frac{5}{2}$$

$$y-y_1=m(x-x_1)$$

$$y-(-2)=\frac{5}{2}(x-(-1))$$

$$y+2=\frac{5}{2}(x+1)$$

$$\frac{5}{2}x+\frac{5}{2}-y-2=0$$

$$5x+5-2y-4=0$$

$$\therefore 5x-2y+1=0$$

53. $y = -2x + b$

$\therefore$ all the lines have the same slope -2

87. $T = 0.02t + 15.0$

(a) The slope 0.02 represents for every year that passes since 1950, there is a $0.02°C$ rise in surface temperature.

The T-intercept represents the surface temperature at the start of the first year : 1950 which is $15.0°C$.

(b) $T = 0.02(2050 - 1950) + 15.0$

$= 0.02(100) + 15.0$

$= 2 + 15.0$

$= 17.0°C$

1.6 Solving Other Types of Equations

Polynomial Equations

5. $x^2 - x = 0$

$x(x-1) = 0$

$\therefore x = 0, 1$

6. $3x^3 - 6x^2 = 0$

$3x^2(x-2) = 0$

$\therefore x = 0, 2$

Equations Involving Rational Expressions

27. $$\frac{1}{x-1}+\frac{1}{x+2}=\frac{5}{4}$$

$$\frac{x+2+x-1}{(x-1)(x+2)}=\frac{5}{4}$$

$$\frac{2x+1}{(x-1)(x+2)}-\frac{5}{4}=0$$

$$\frac{4(2x+1)-5(x-1)(x+2)}{4(x-1)(x+2)}=0$$

$$\frac{8x+4-5(x^2-x+2x-2)}{4(x-1)(x+2)}=0$$

Numerator:

$$=8x+4-5x^2-5x+10$$

$$=-5x^2+3x+16$$

$$4(x^2+x-2)$$

Chapter 2 Functions

Content

Example 1

$$f(x) = x^2 + 4$$

(a) x is squared and added to 4

(b) $f(3) = 9 + 4$
$= 13$

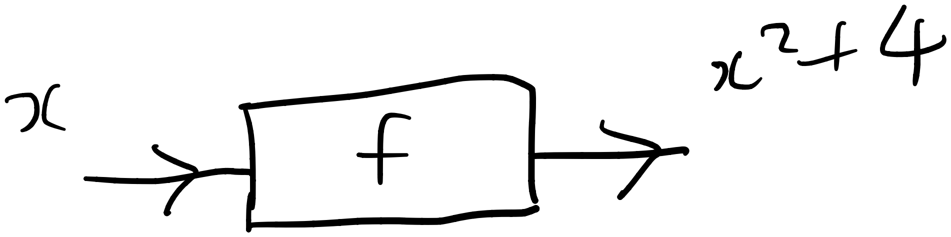
$f(-2) = 4 + 4$
$= 8$

$f(\sqrt{5})$
$= 5 + 4$
$= 9$

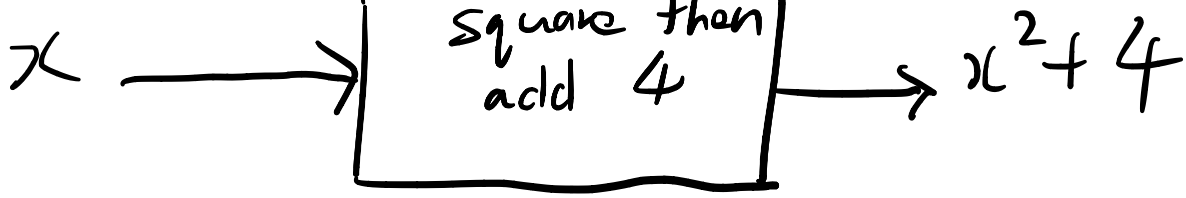
(c) domain : $x \in R$, set of all real numbers

range : $\boxed{1}$ $\{y \mid y \geq 4\}$ / $[4, \infty)$

(d)



$\boxed{1}$



Example 7 Finding Domains of Functions 16/9/23

(a) $f(x) = \dfrac{1}{x^2 - x}$

$= \dfrac{1}{x(x-1)}$

Domain: $\{x \mid x \neq 0 \text{ and } x \neq 1\}$

(b) $g(x) = \sqrt{9 - x^2}$

$9 - x^2 \geq 0$

$x^2 - 9 \leq 0$

$(x+3)(x-3) \leq 0$

$-3 \leq x \leq 3$

Domain: $\{x \mid -3 \leq x \leq 3\}$ / $[-3, 3]$

(c) $h(t) = \dfrac{t}{\sqrt{t+1}}$

$t + 1 > 0$

$t > -1$

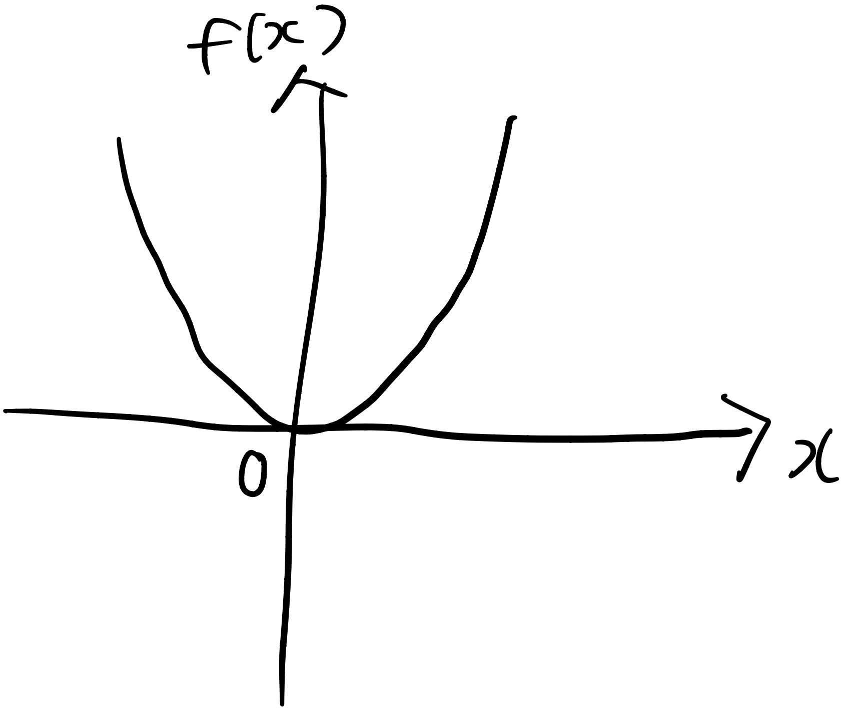
Domain: $\{t \mid t > -1\}$

/ $(-1, \infty)$

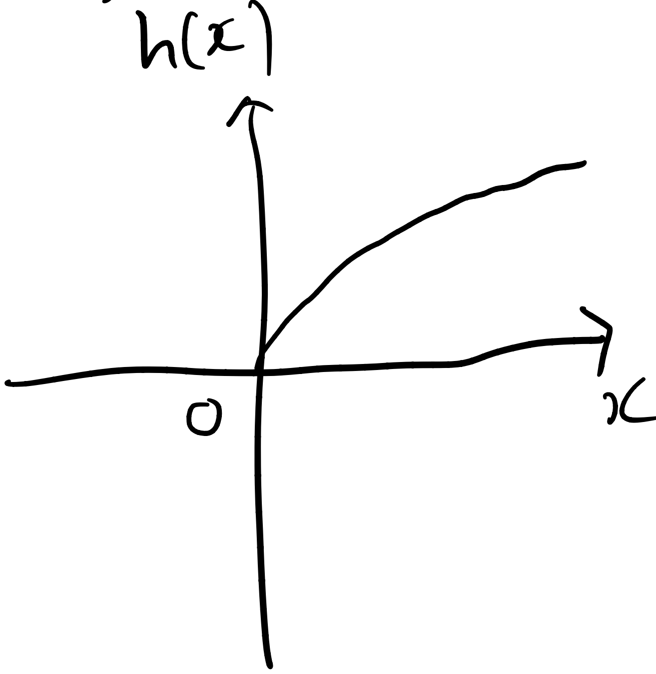
2.2.1 Graphing Functions by Plotting Points 17/9/23

Example 1

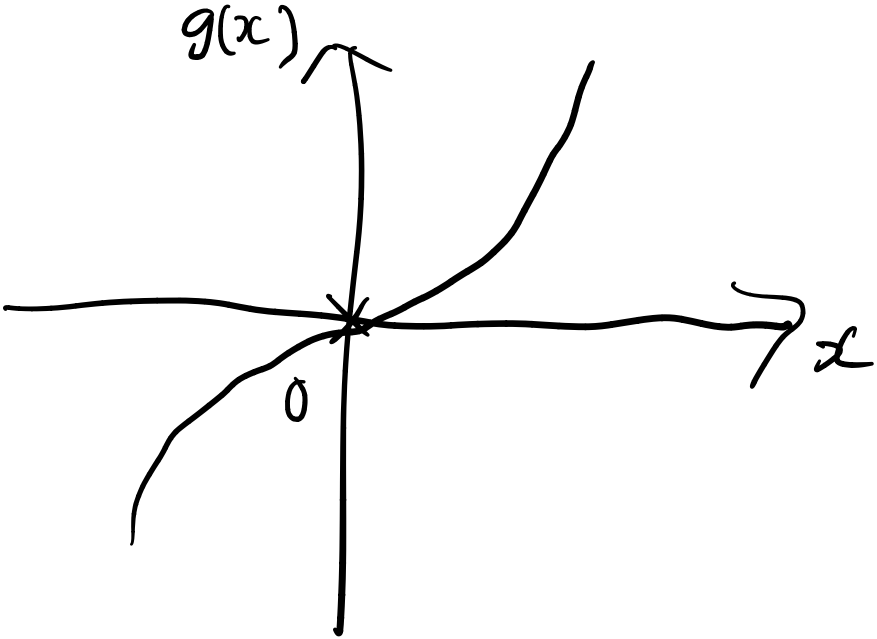
(a) $f(x) = x^2$



(c) $h(x) = \sqrt{x}$



(b) $g(x) = x^3$



2.2.3 Graphing Piecewise Defined Functions

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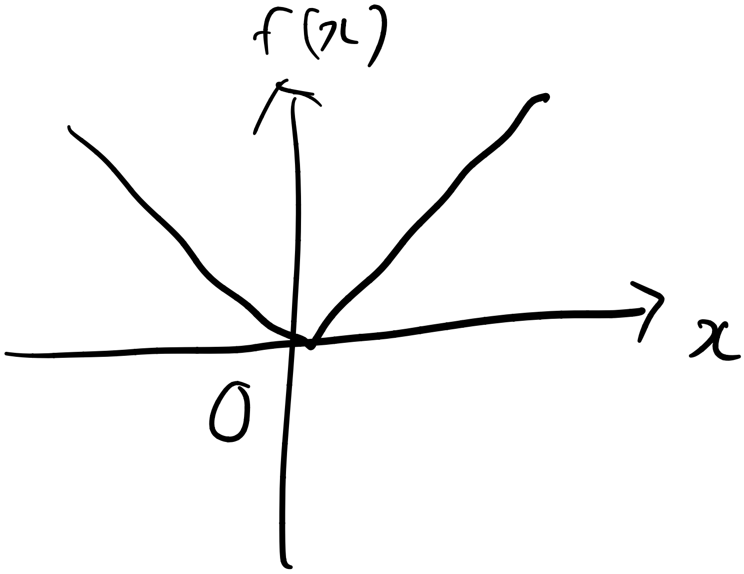
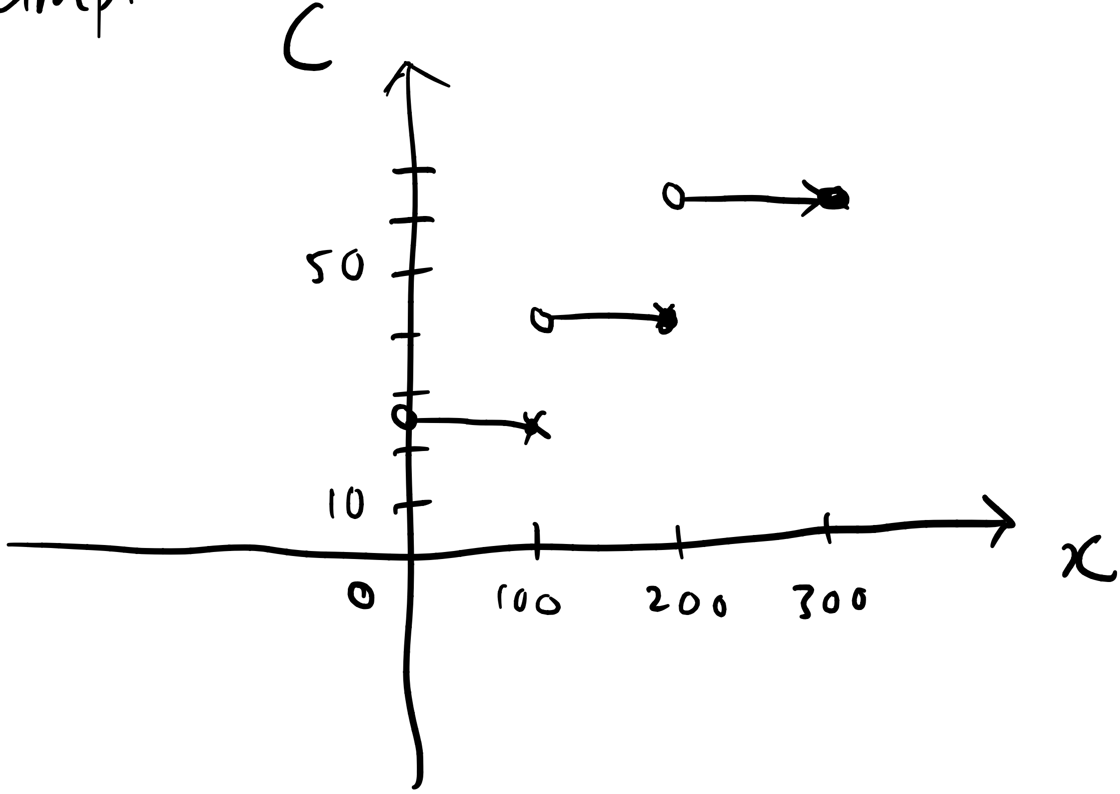
Example 5

$f(x) = |x|$



Example 7



2.3.1 Values of a Function; Domain and Range

21/9/23

Example 1

(a) $T(1) = 25^\circ F$

$T(3) = 30^\circ F$

$T(5) = 20^\circ F$

(b) $T(2)$

(c) $x = 1, 4$

(d) $1 \leq x \leq 4$

(e) $T(3) - T(1)$

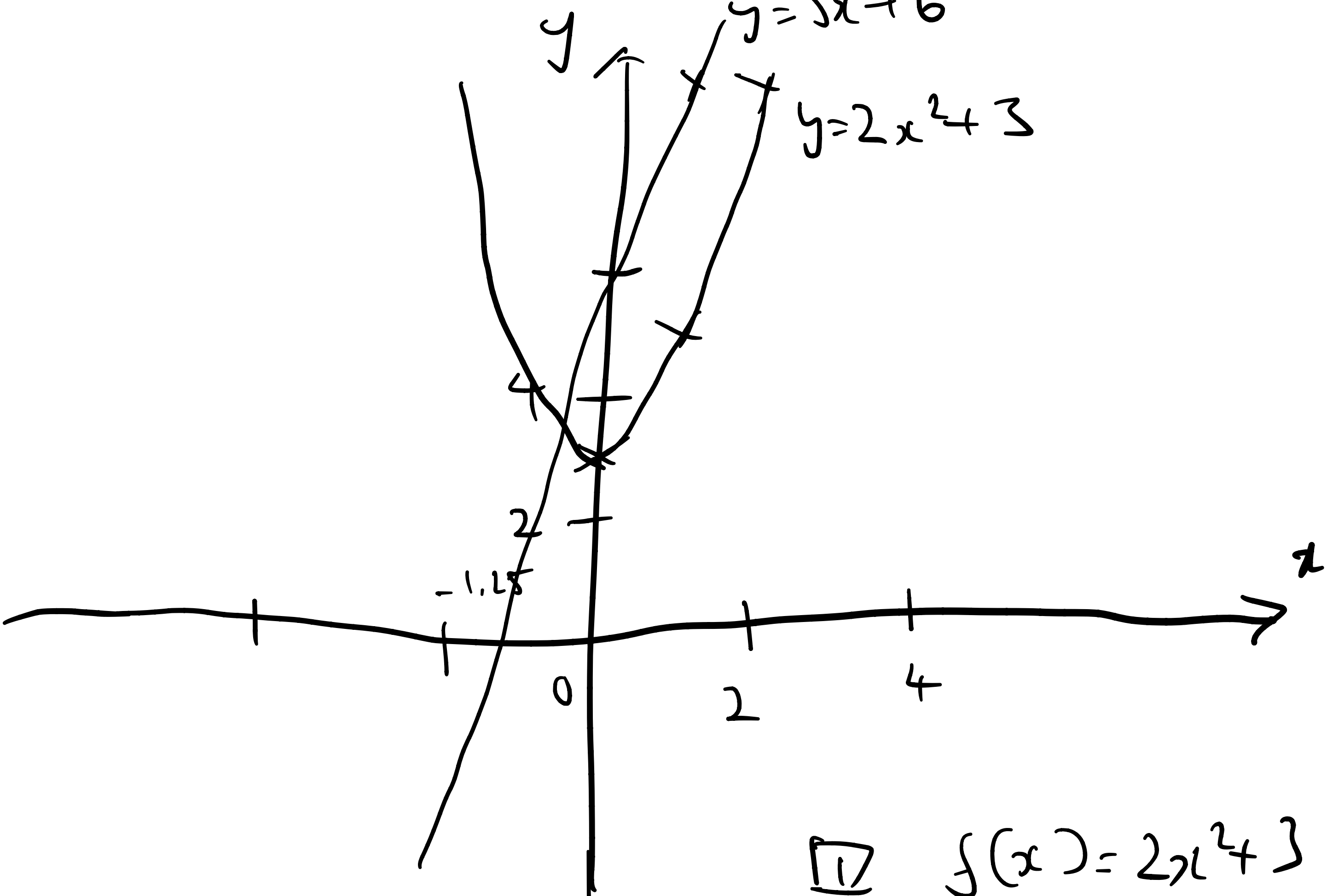
$= 30 - 25$

$= 5^\circ F$

2.3.2 Comparing Function Values: Solving Equations and Inequalities Graphically

Example 3

(a) $2x^2 + 3 = 5x + 6$



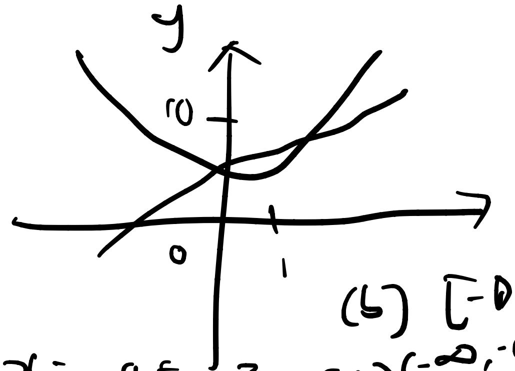
$x = -0.5$

(b) $x \geq 0.5$

(c) $x < -0.5$

Ⅲ $f(x) = 2x^2 + 3$

$g(x) = 5x + 6$



(b) $[-0.5, 3]$

(a) $x = -0.5\ ;\ 3$

(c) $(-\infty, -0.5) \cup (3, \infty)$

2.2 Exercises

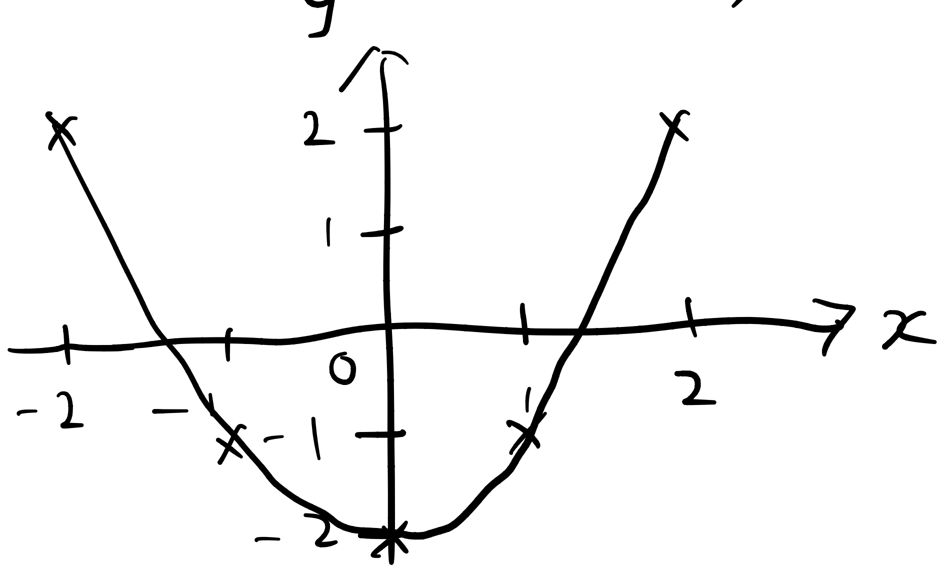
1. $(x, f(x))$

(x, x^2-2)

$(3, 7)$

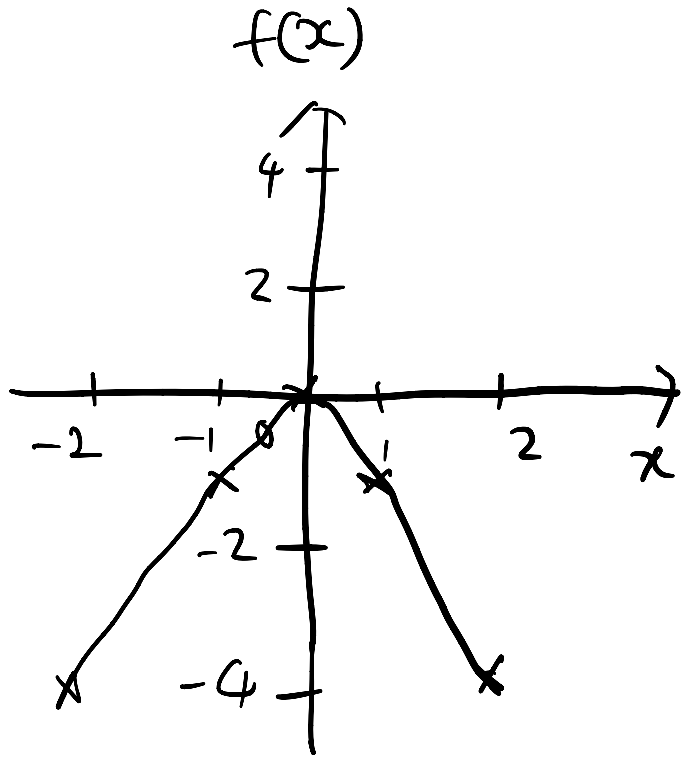
7

x	$f(x)$	(x, y)
-2	2	$(-2, 2)$
-1	-1	$(-1, -1)$
0	-2	$(0, -2)$
1	-1	$(1, -1)$
2	2	$(2, 2)$



9. $f(x) = -x^2$

x	$f(x)$
-2	-4
-1	-1
0	0
1	-1
2	-4



0.5 Algebraic Expressions

1. a, d and f

15. $(6x-3)+(3x+7)$

$= 9x+4$

37. $(3t-2)(7t-4)$

$= 3t(7t-4)-2(7t-4)$

$= 21t^2-12t-14t+8$

$= 21t^2-26t+8$

45. $(5x+1)^2 = 25x^2+10x+1$

67. $(x+2)(x^2+2x+3)$

$= x(x^2+2x+3)+2(x^2+2x+3)$

$= (x^3+2x^2+3x)+(2x^2+4x+6)$

$= x^3+4x^2+7x+6$

0.8 Solving Basic Equations

1. Equations Involving Fractional Expressions
2. Power Equations

① Equations Involving Fractional Expressions

49. $\frac{1}{z} - \frac{1}{2z} - \frac{1}{5z} = \frac{10}{z+1}$

$\frac{10}{10z} - \frac{5}{10z} - \frac{2}{10z} = \frac{10}{z+1}$

$\frac{3}{10z} = \frac{10}{z+1}$

$3(z+1) = 10(10z)$

$3z + 3 = 100z$

$97z = 3$

$z = \frac{3}{97}$

51. $\frac{x}{2x-4} - 2 = \frac{1}{x-2}$

$\frac{x}{2x-4} - \frac{2(2x-4)}{2x-4} = \frac{1}{x-2}$

$\frac{-3x+8}{2x-4} = \frac{1}{x-2}$

$$(-3x+8)(x-2) = 2x-4$$

$$-3x^2+14x-16 = 2x-4$$

$$3x^2-12x+12=0$$

$$(3x-6)(x-2)=0$$

$$x=2$$

$\because x \neq 2$, $\therefore$ no solution

41. $$\frac{1}{x} = \frac{4}{3x} + 1$$

$$3 = 4 + 3x$$

$$3x = -1$$

$$x = -\frac{1}{3}$$

42. $$\frac{2}{x} - 5 = \frac{6}{x} + 4$$

$$2-5x = 6+4x$$

$$9x = -4$$

$$x = -\frac{4}{9}$$

43. $$\frac{2x-1}{x+2} = \frac{4}{5}$$

$$5(2x-1) = 4(x+2)$$

$$10x - 5 = 4x + 8$$

$$6x = 13$$

$$x = \frac{13}{6}$$

44. $$\frac{2x-7}{2x+4} = \frac{2}{3}$$

$$3(2x-7) = 2(2x+4)$$

$$6x - 21 = 4x + 8$$

$$2x = 8 + 21$$

$$2x = 29$$

$$x = \frac{29}{2}$$

48. $\dfrac{12x-5}{6x+3} = 2 - \dfrac{5}{x}$

Method 1:

$$\frac{12x-5}{6x+3} = \frac{2x}{x} - \frac{5}{x}$$

$$\frac{12x-5}{6x+3} = \frac{2x-5}{x}$$

$$(12x-5)x = (2x-5)(6x+3)$$

$$12x^2 - 5x = 12x^2 - 24x - 15$$

$$19x = -15$$

$$x = -\frac{15}{19}$$

Method 2:

$$12x^2 - 5x = 2x(6x+3) - 5(6x+3)$$

$$12x^2 - 5x = 12x^2 + 6x - 30x - 15$$

$$12x^2 - 5x = 12x^2 - 24x - 15$$

equivalent

Chapter 3 Polynomials and Rational Functions

1. Quadratic Functions and Models
2. Polynomial Functions and Their Graphs
3. Dividing Polynomials
4. Real Zeros of Polynomials
5. Complex Zeros and the Fundamental Theorem of Algebra
6. Rational Functions
7. Polynomial and Rational Inequalities

15. $f(x) = x^2 + 4x + 3$

(a) $f(x) = x^2 + 4x + 3 + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2$

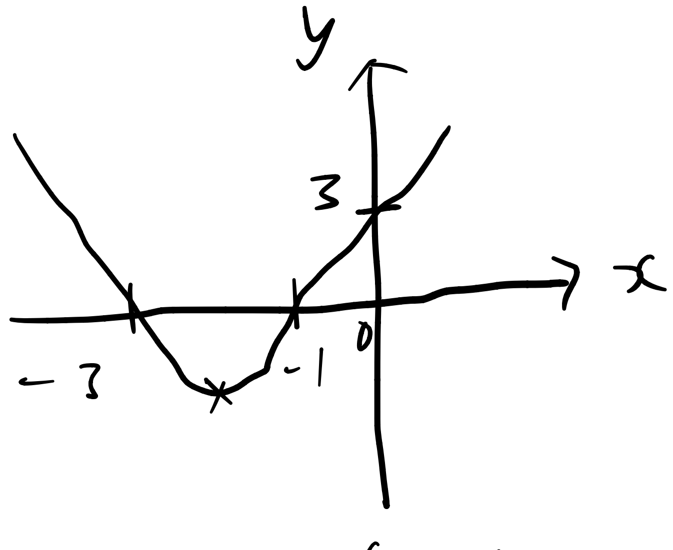
$= (x+2)^2 - 1$ $\quad (x+3)(x+1)$

(b) vertex $= (-2, -1)$

x-intercepts $= -3, -1$

y-intercepts $= 3$

(c)



$\boxed{2}$ Domain:

$(-\infty, \infty)$

(d) Domain : $\{x \mid x \in R\}$

Range: $[-1, \infty)$

27. $f(x) = 3x^2 - 6x + 1$

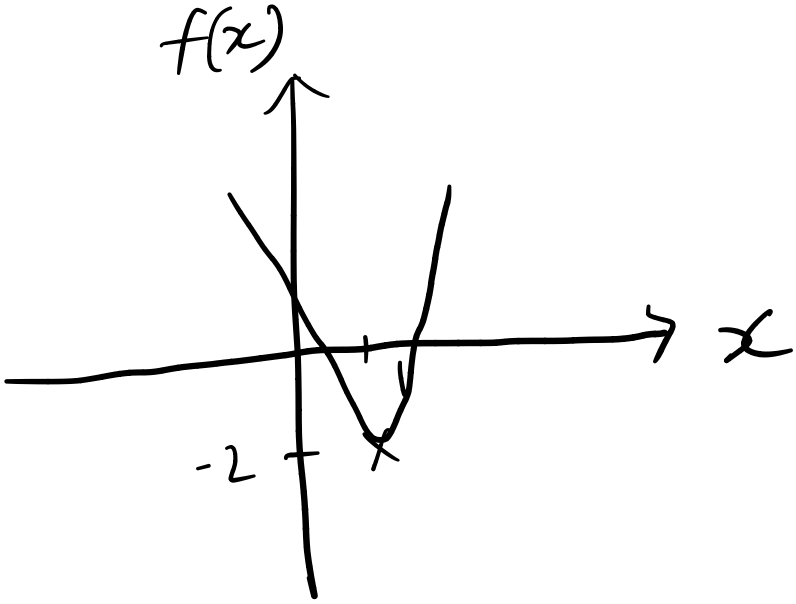
(a) $f(x) = 3\left(x^2 - 2x + \frac{1}{3}\right)$

$= 3\left(x^2 - 2x + \frac{1}{3} + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2\right)$

$= 3\left((x-1)^2 - \frac{2}{3}\right)$

$= 3(x-1)^2 - 2$

(b)



(c) minimum value: $y = -2$

3.1 Quadratic Functions and Models

1. Graphing Quadratic Functions

2. Maximum and Minimum Values

① Graphing Quadratic Functions

9. $f(x) = x^2 - 2x + 3$

(a) $f(x) = x^2 - 2x + 3$

$$= x^2 - 2x + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2 + 3$$

$$= (x-1)^2 + 2$$

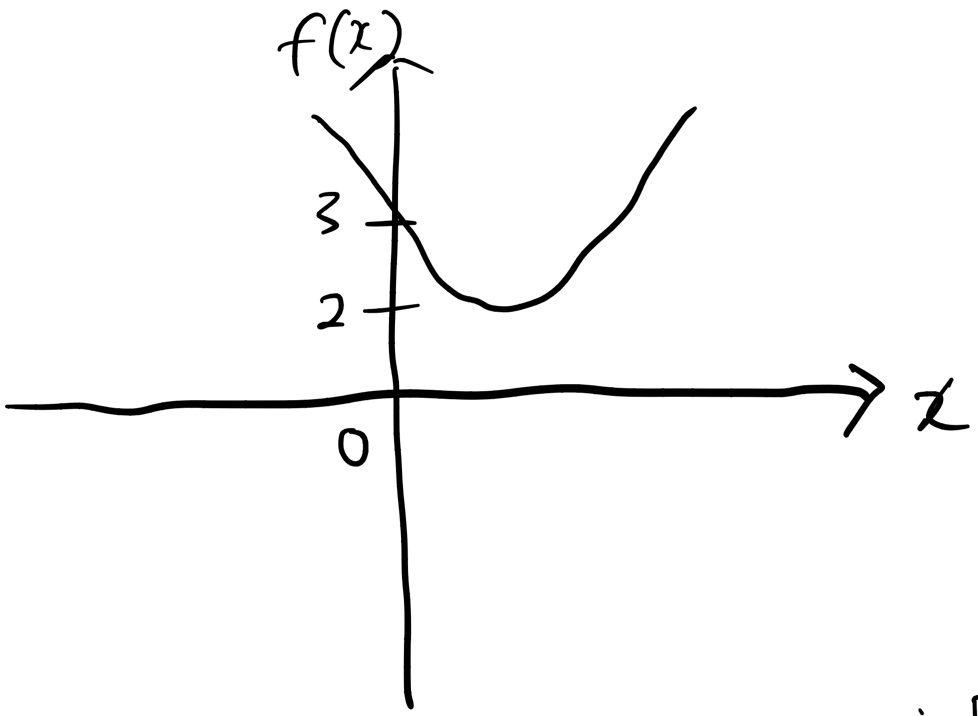
(b) Vertex: $(1, 2)$

$$b^2 - 4ac = 4 - 12$$
$$= -8 < 0$$

$\therefore$ no x-intercept

y-intercept $= 1 + 2 = 3$

(c)



(d) Domain: $(-\infty, \infty)$, Range: $[2, \infty)$

10. $f(x) = x^2 + 4x - 1$

(a) $f(x) = x^2 + 4x - 1$

$= x^2 + 4x + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 - 1$

$= (x+2)^2 - 5$

(b) vertex : $(-2, -5)$

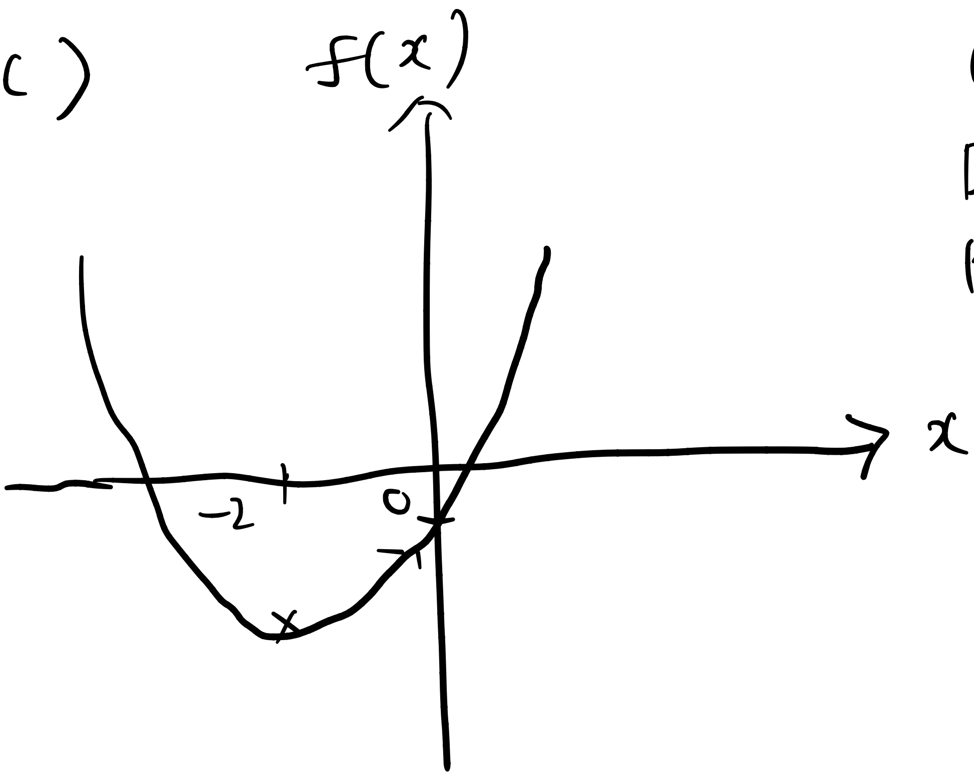
y-interecept : -1

x - interrcept : $x = \frac{-4 \pm \sqrt{16+4}}{2}$

$= \frac{-4 \pm 2\sqrt{5}}{2}$

$= -2 \pm \sqrt{5}$

(c)



(d)

Domain : $(-\infty, \infty)$

Range : $\{y \mid y \geq -5\}$

$[-5, \infty)$

11. $f(x) = x^2 - 6x$

(a) $f(x) = x^2 - 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2$

$= (x-3)^2 - 9$

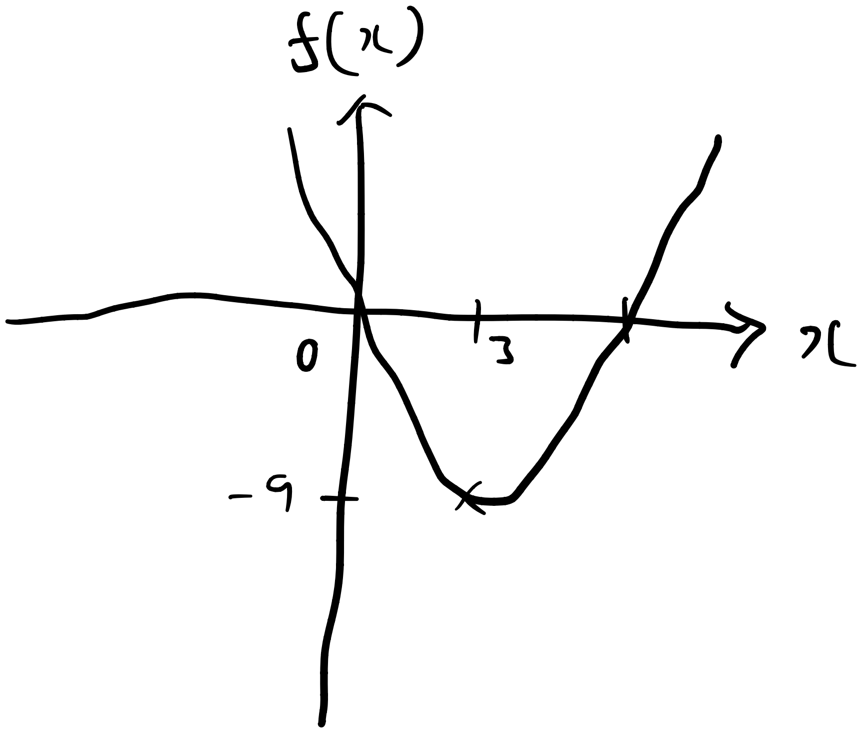
(b) vertex: $(3, -9)$

x-intercept: $x(x-6) = 0$

$x = 0, 6$

y-intercept: $y = 0$

(c)



(d) Domain: $\{x \mid x \in R\}$

Range: $[-9, \infty)$

12. $f(x) = x^2 + 8x$

(a) $f(x) = x^2 + 8x + \left(\frac{8}{2}\right)^2 - \left(\frac{8}{2}\right)^2$

$= (x+4)^2 - 16$

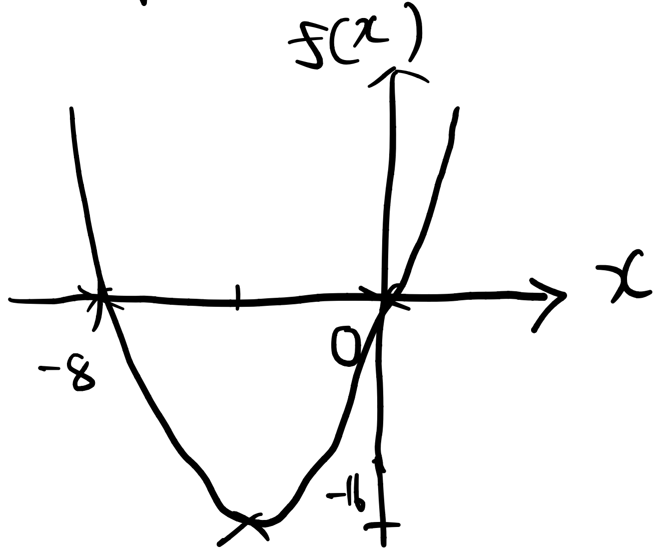
(b) Vertex: $(-4, -16)$

x-intercept: $x(x+8) = 0$

$x = 0, -8$

y-intercept: $y = 0$

(c)



(d) Domain: $(-\infty, \infty)$

Range: $[-16, \infty)$

13. $f(x) = 3x^2 + 6x$

(a) $f(x) = 3(x^2 + 2x)$

$= 3\left(x + 2x + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2\right)$

$= 3(x+1)^2 - 3$

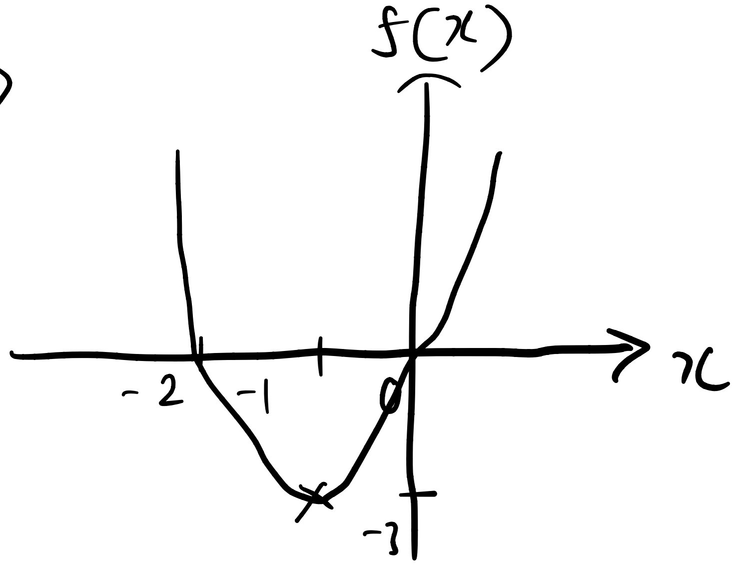
(b) Vertex: $(-1, -3)$

x-intercept: $3x(x+2) = 0$

$x = 0, -2$

y-intercept: $f(0) = 0$

(c)



(d) Domain: $(-\infty, \infty)$

Range: $[-3, \infty)$

14. $f(x) = -x^2 + 10x$

(a) $f(x) = -\left(x^2 - 10x + \left(\frac{10}{2}\right)^2 - \left(\frac{10}{2}\right)^2\right)$

$= -(x-5)^2 + 25$

(b) V: $(5, 25)$

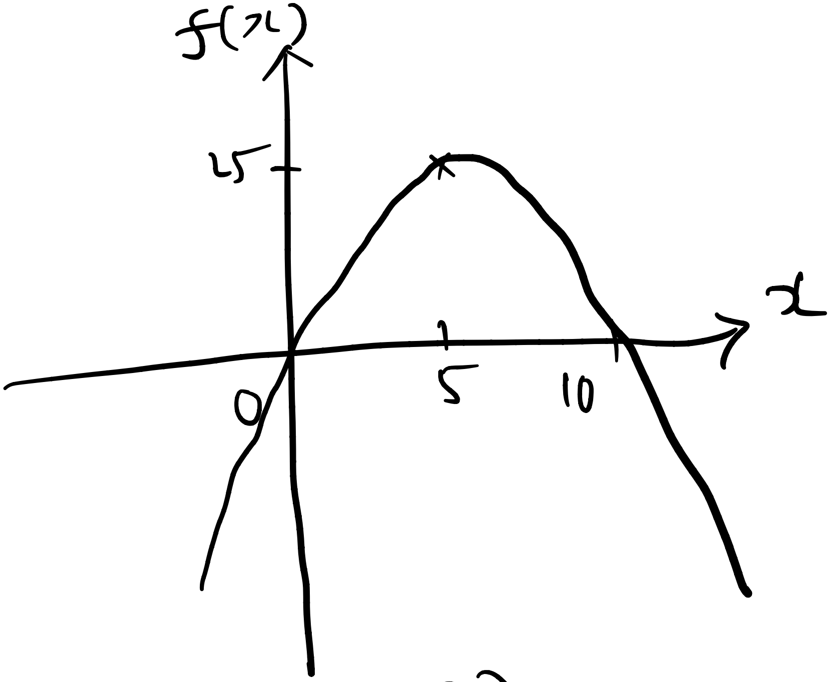
x-intercept: $-x^2 + 10x = 0$

$-x(x-10) = 0$

$x = 0, 10$

y-intercept: $f(0) = 0$

(c)



(d) Domain : $(-\infty, \infty)$

Range : $(-\infty, 25]$

15. $f(x) = x^2 + 4x + 3$

(a) $f(x) = x^2 + 4x + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 + 3$

$= (x+2)^2 - 1$

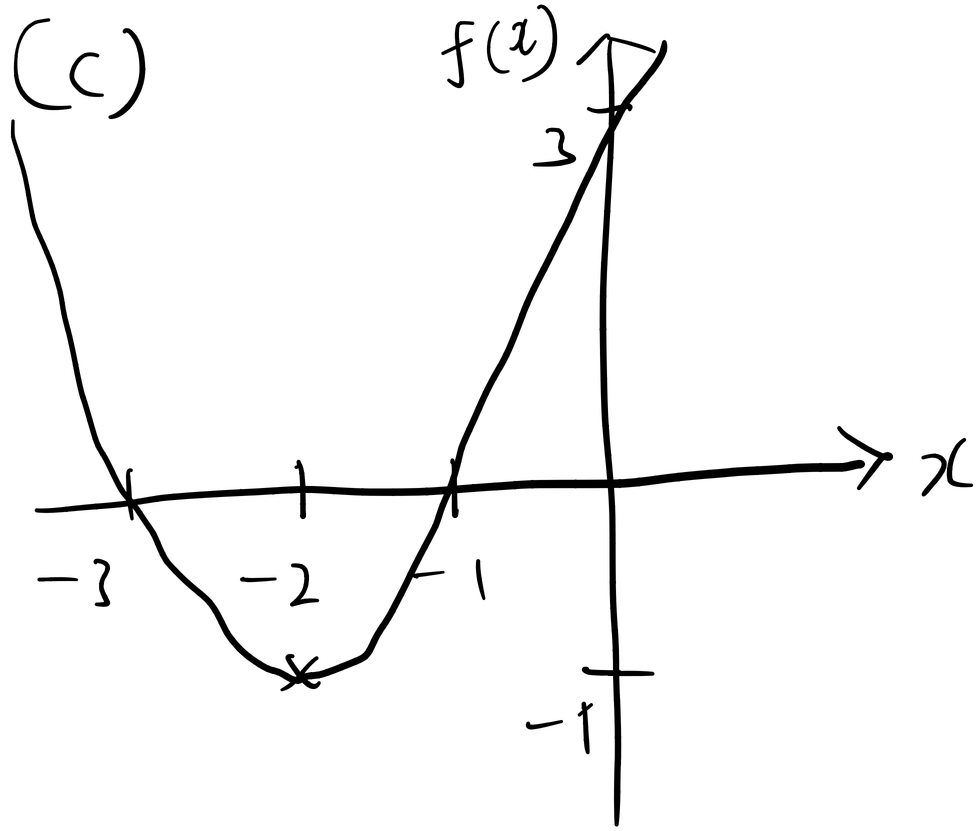
(b) Vertex : $(-2, -1)$

x-intercept : $f(x) = (x+3)(x+1)$

$x = -3, -1$

y-intercept : $f(0) = 3$

(c)



(d) Domain = $(-\infty, \infty)$

Range = $[-1, \infty)$

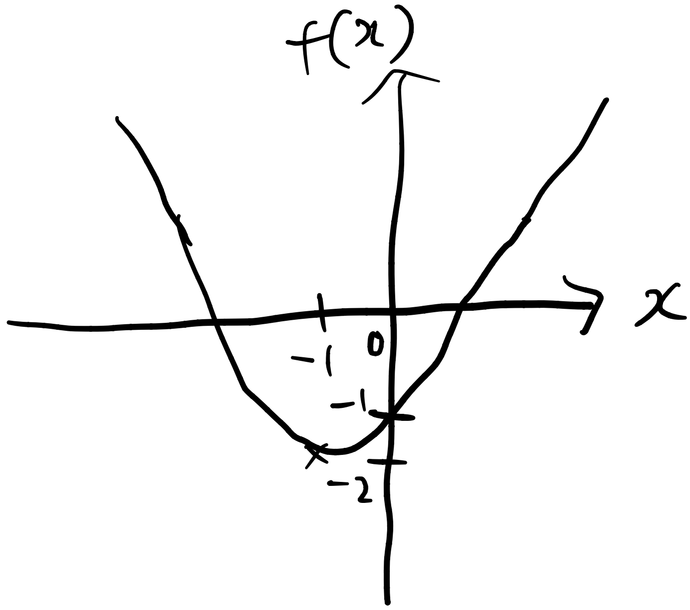
② Maximum and Minimum values

25. $f(x) = x^2 + 2x - 1$

(a) $x^2 + 2x + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2 - 1$

$= (x+1)^2 - 2$

(b)



(c) minimum value: $y = -2$

15/3

26. $f(x) = x^2 - 8x + 8$

(a) $f(x) = x^2 - 8x + \left(\frac{8}{2}\right)^2 - \left(\frac{8}{2}\right)^2 + 8$

$= (x-4)^2 - 8$

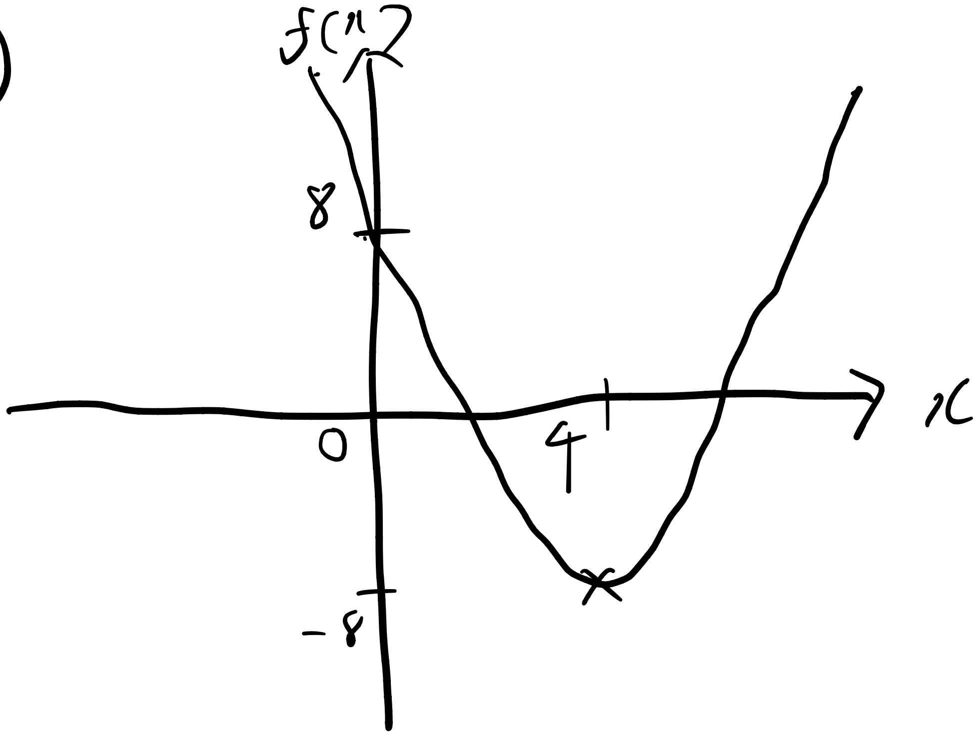
x-intercept:

$x = \dfrac{8 \pm \sqrt{64 - 32}}{2}$

$= \dfrac{8 \pm 4\sqrt{2}}{2}$

$= 4 \pm 2\sqrt{2}$

(b)



(c) Minimum value : $y = -8$

27. $f(x) = 3x^2 - 6x + 1$

$3x^2 - 6x + 1 = 0$

$x = \frac{6 \pm \sqrt{36 - 12}}{6}$

$= \frac{6 \pm \sqrt{24}}{6}$

$= \frac{6 \pm 2\sqrt{6}}{6}$

$= \frac{3 \pm \sqrt{6}}{3}$

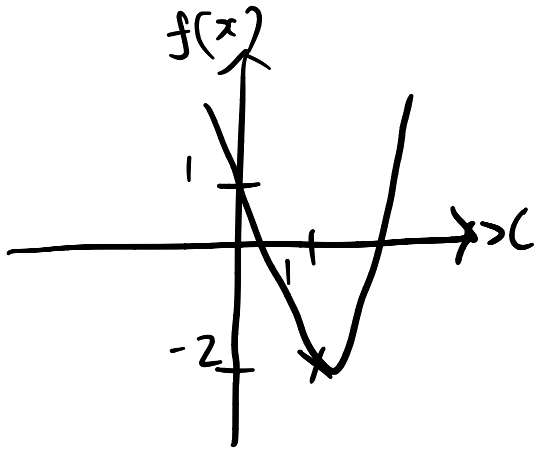
(a) $f(x) = 3\left(x^2 - 2x + \frac{1}{3}\right)$

$= 3\left(x^2 - 2x + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2 + \frac{1}{3}\right)$

$= 3\left((x-1)^2 - \frac{2}{3}\right)$

$= 3(x-1)^2 - 2$

(b)



(c) Minimum value :

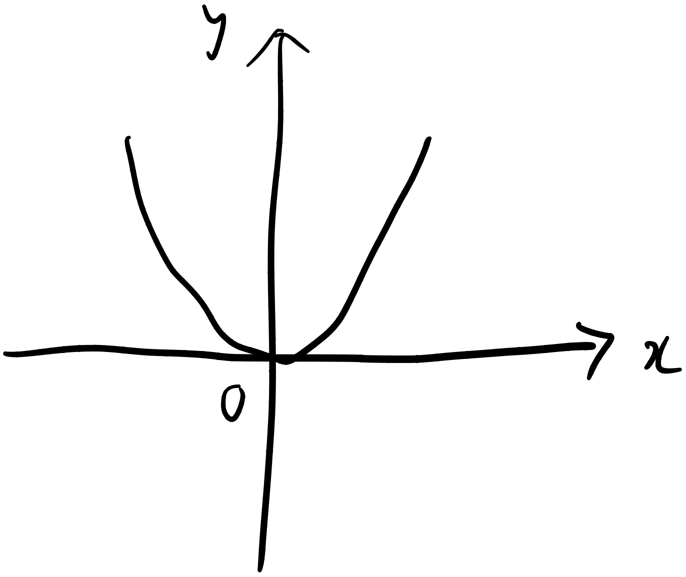
$y = -2$

3.2 Polynomial Functions and Their Graphs

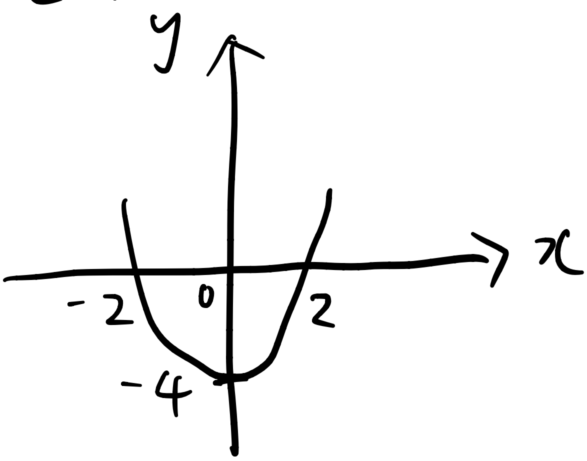
① Graphing Polynomials

② Local Extrema

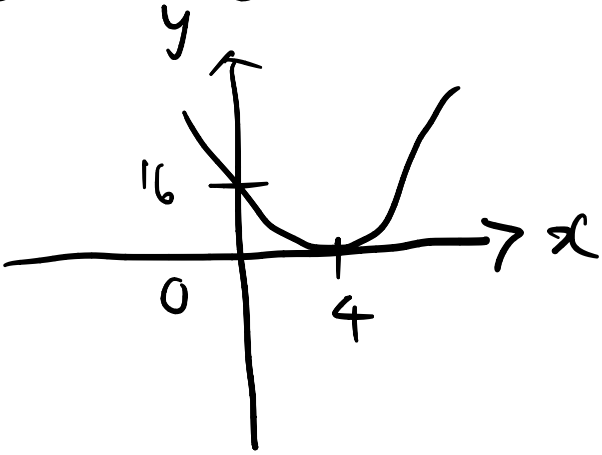
5. $y = x^2$



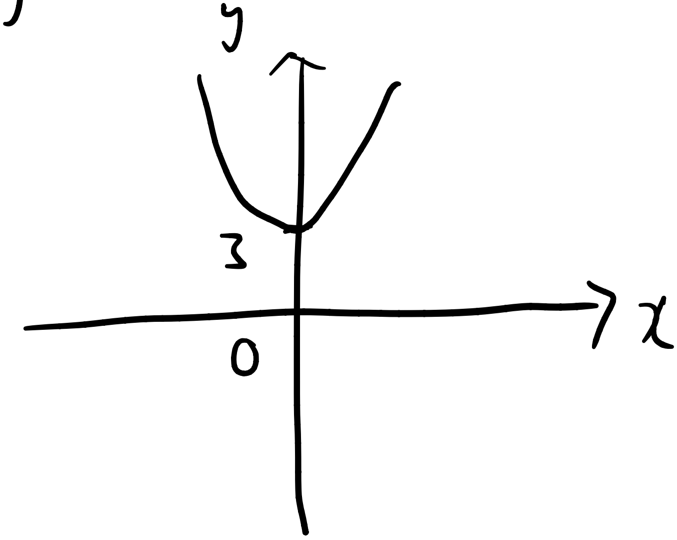
(a) $P(x) = x^2 - 4$



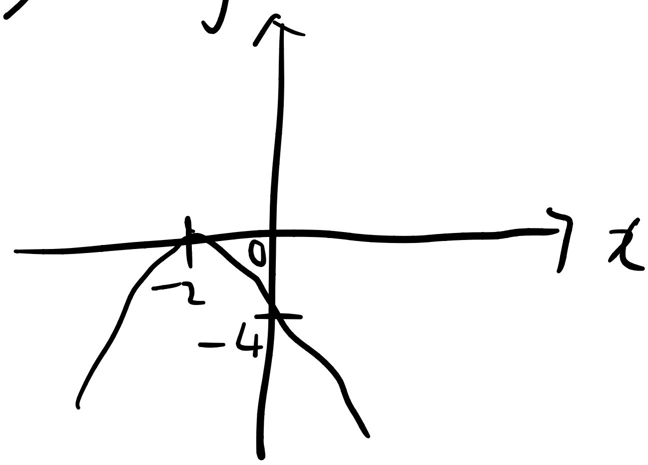
(b) $Q(x) = (x-4)^2$



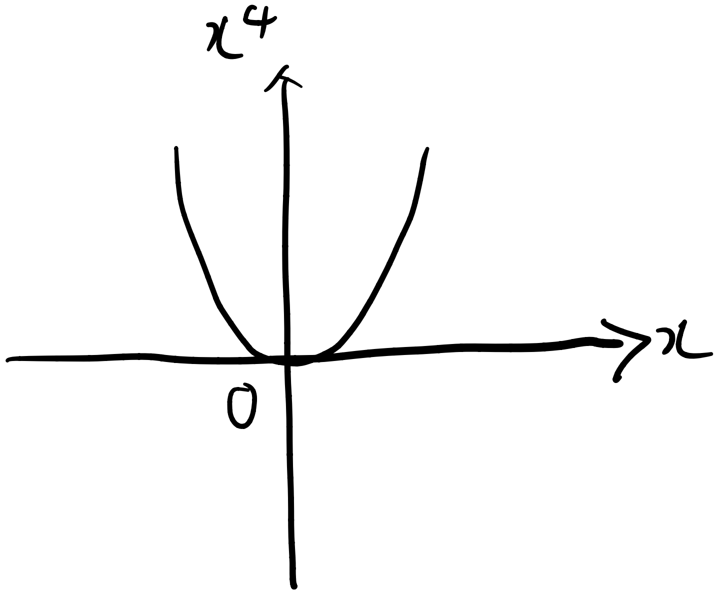
(c) $P(x) = 2x^2 + 3$



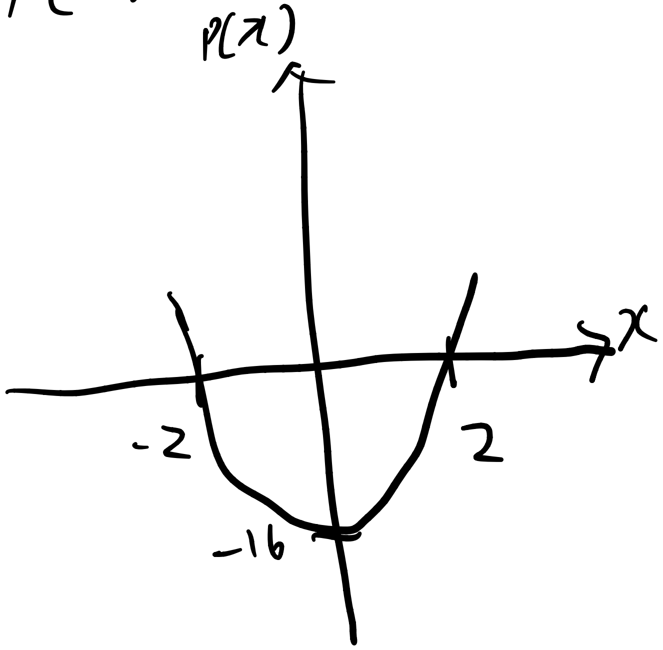
(d) $P(x) = -(x+2)^2$



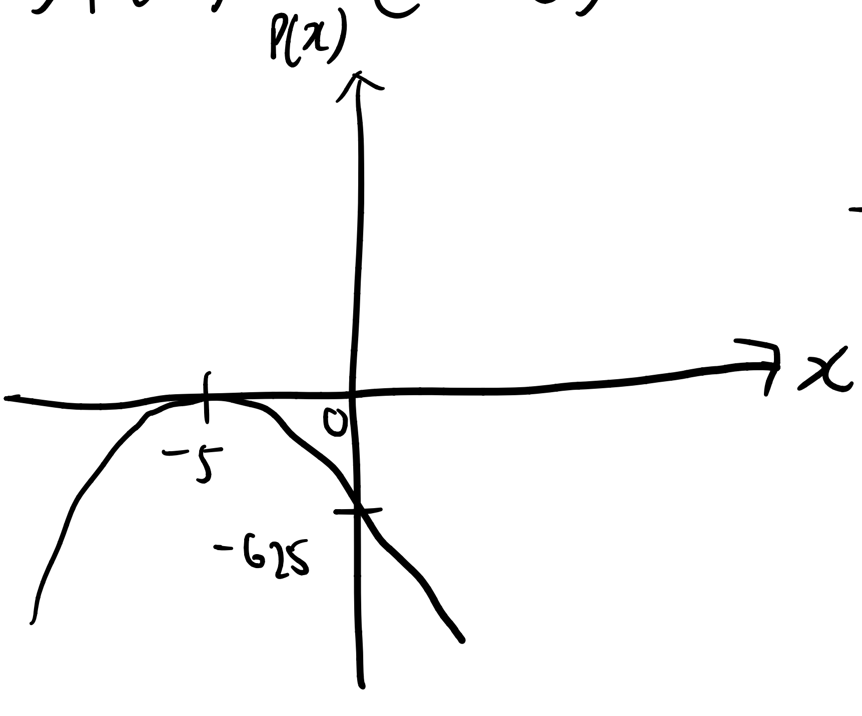
6.



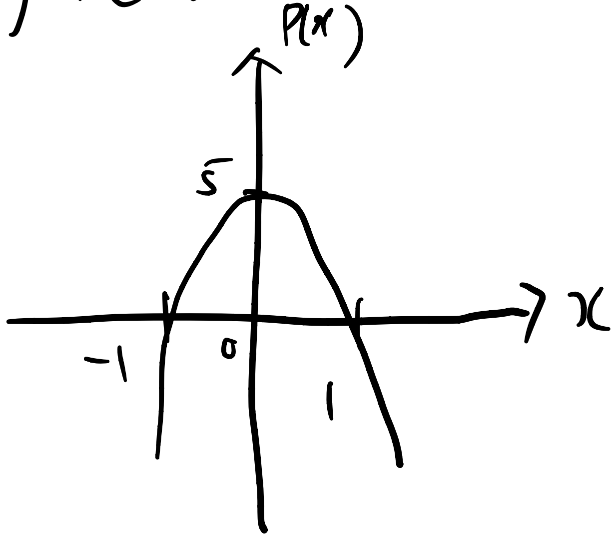
(a) $P(x) = x^4 - 16$



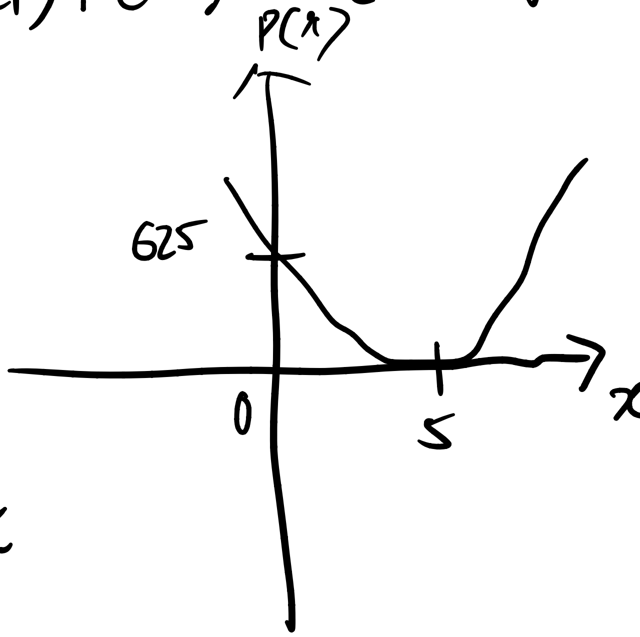
(b) $P(x) = -(x+5)^4$



(c) $P(x) = -5x^4 + 5$



(d) $P(x) = (x-5)^4$



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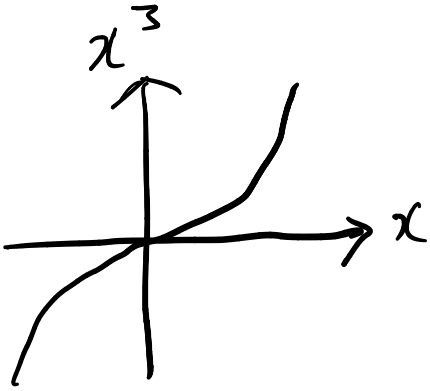
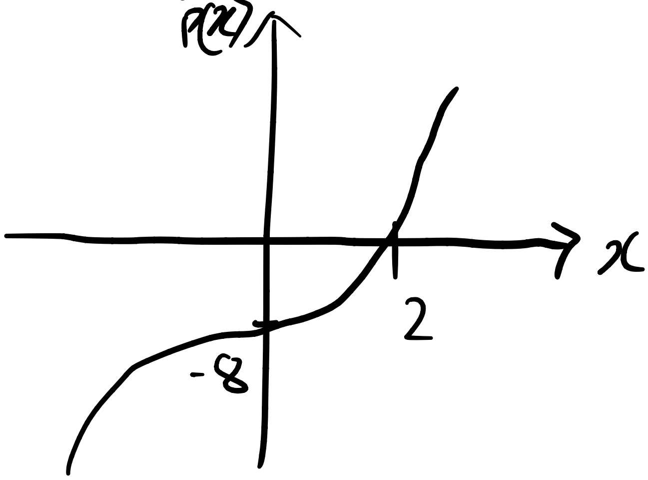
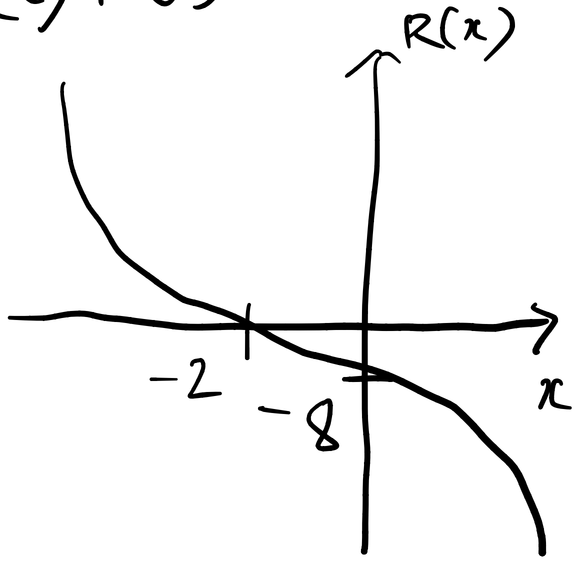
7. x^3



(a) $P(x) = x^3 - 8$

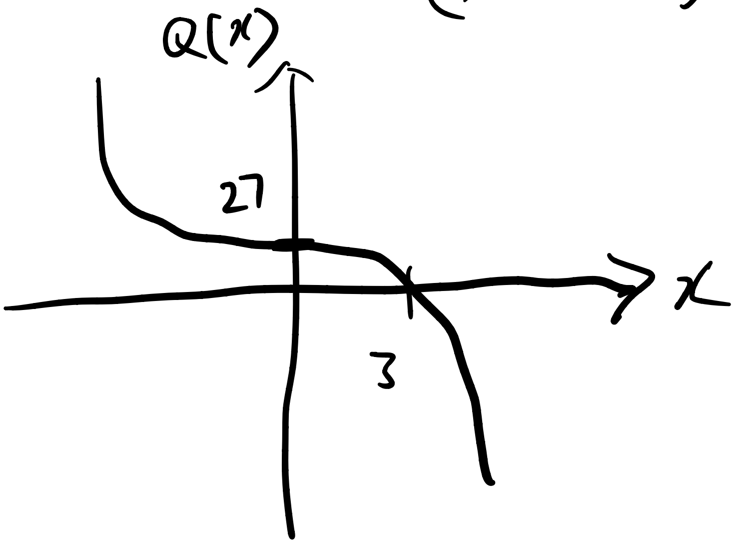


(c) $R(x) = -(x+2)^3$

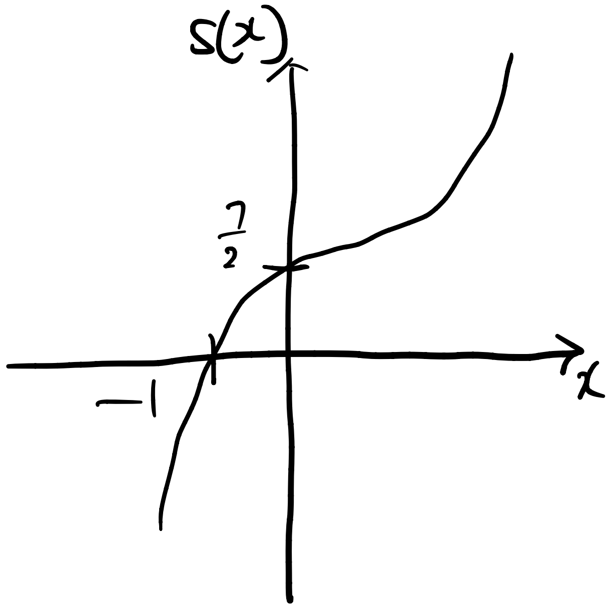


(b) $Q(x) = -x^3 + 27$

$= -(x^3 - 27)$

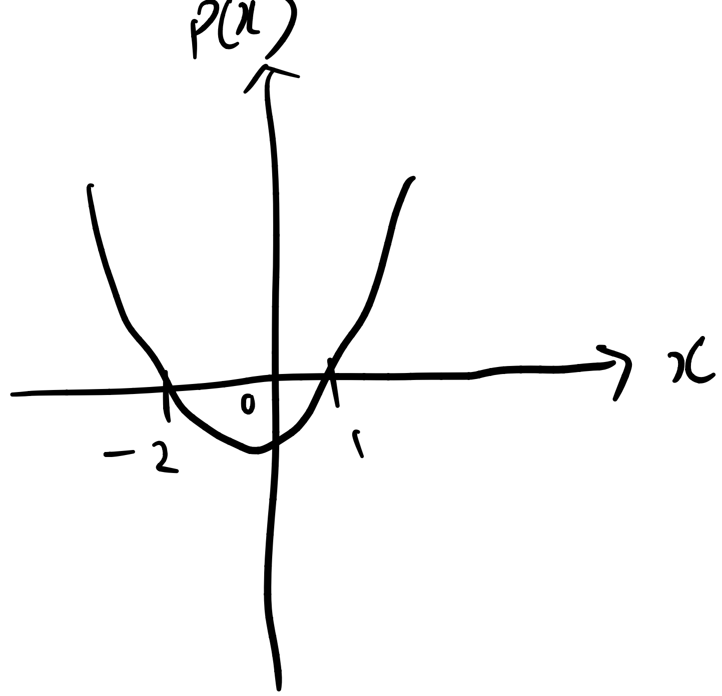


(d) $S(x) = \frac{1}{2}(x-1)^3 + 4$



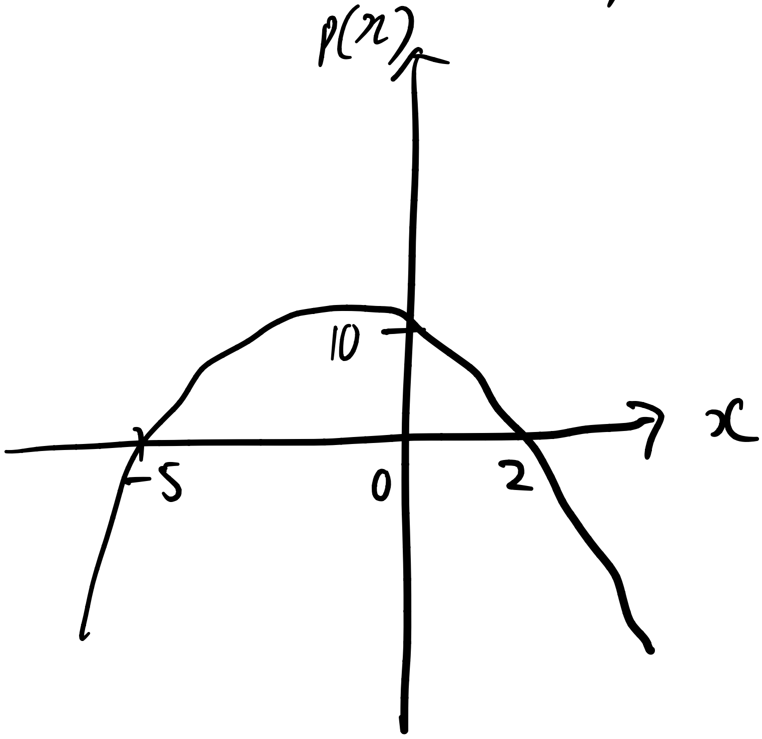
① Graphing Polynomials

15. $P(x) = (x-1)(x+2)$

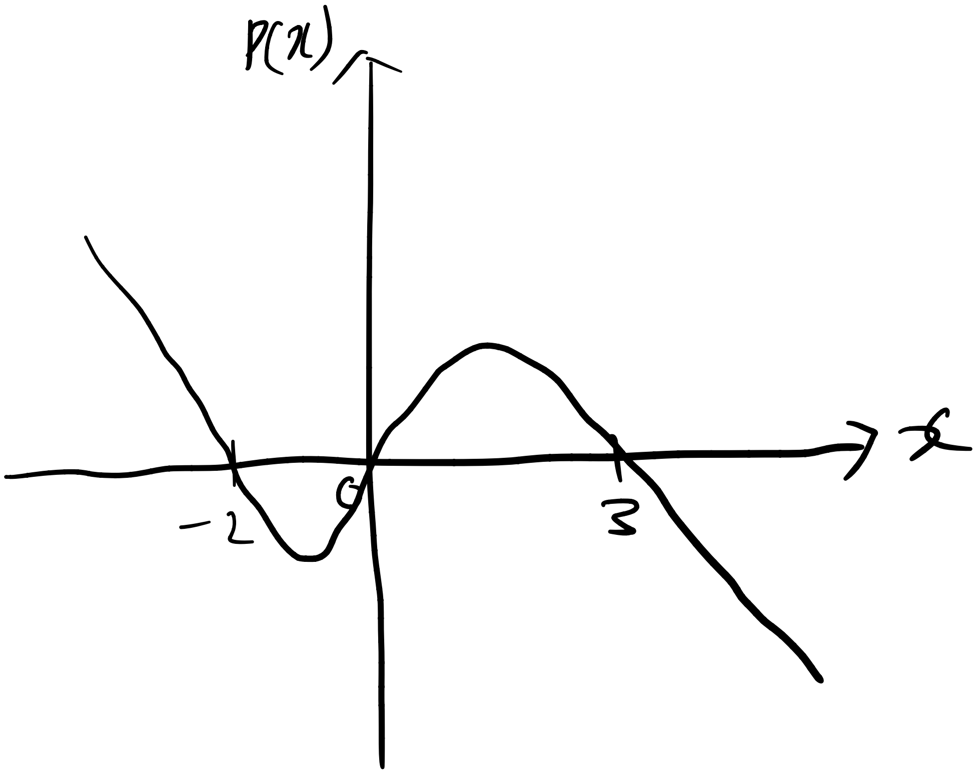


16. $P(x) = (2-x)(x+5)$

$= -(x-2)(x+5)$

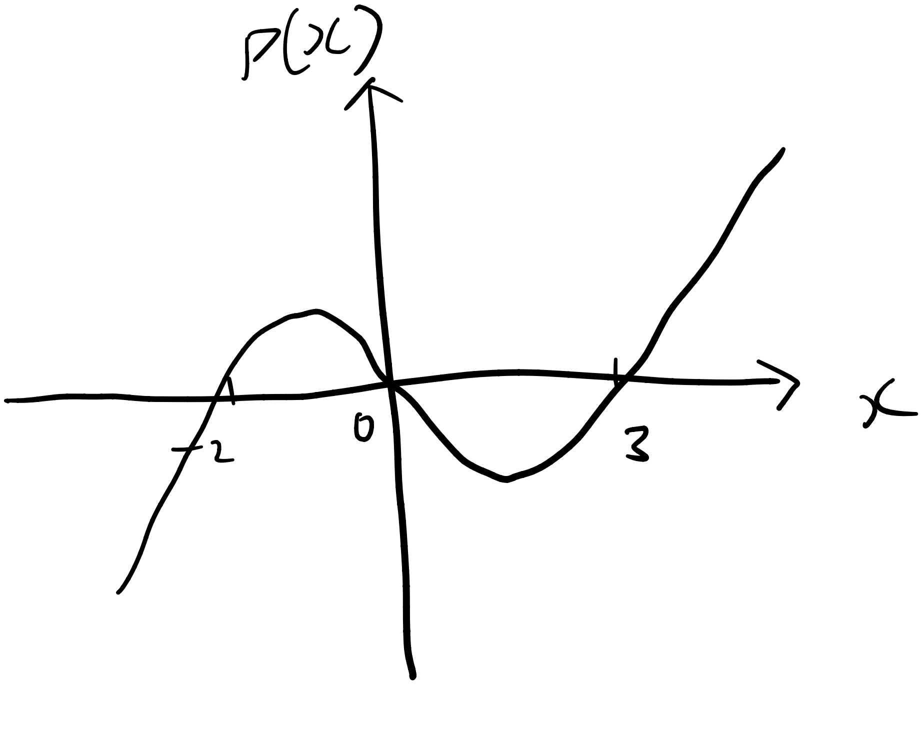


17. $p(x) = -x(x-3)(x+2)$



18. $p(x) = x(x-3)(x+2)$

$x = -2, 0, 3$



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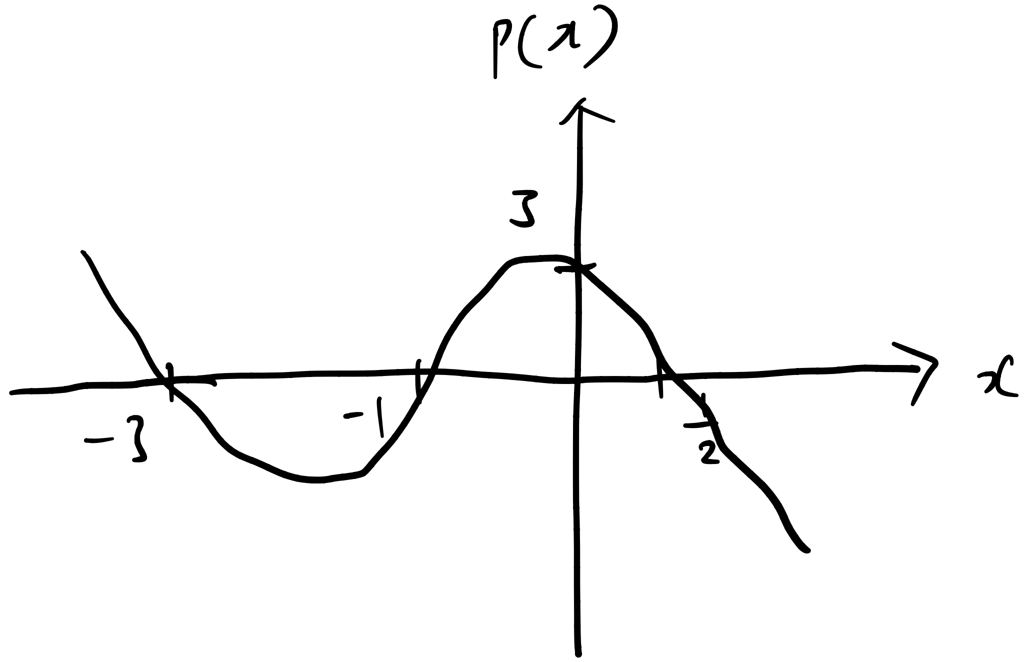
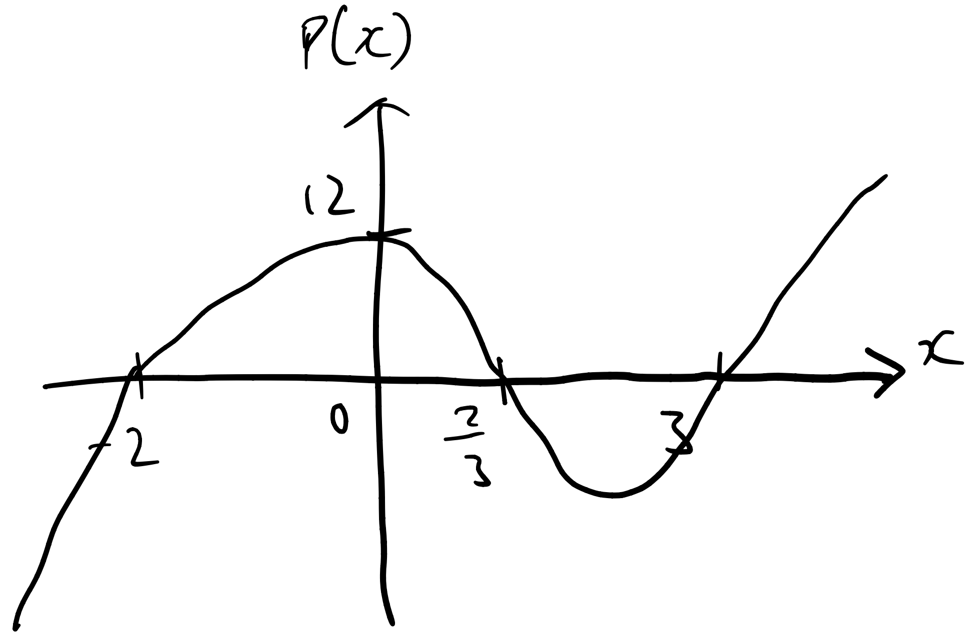
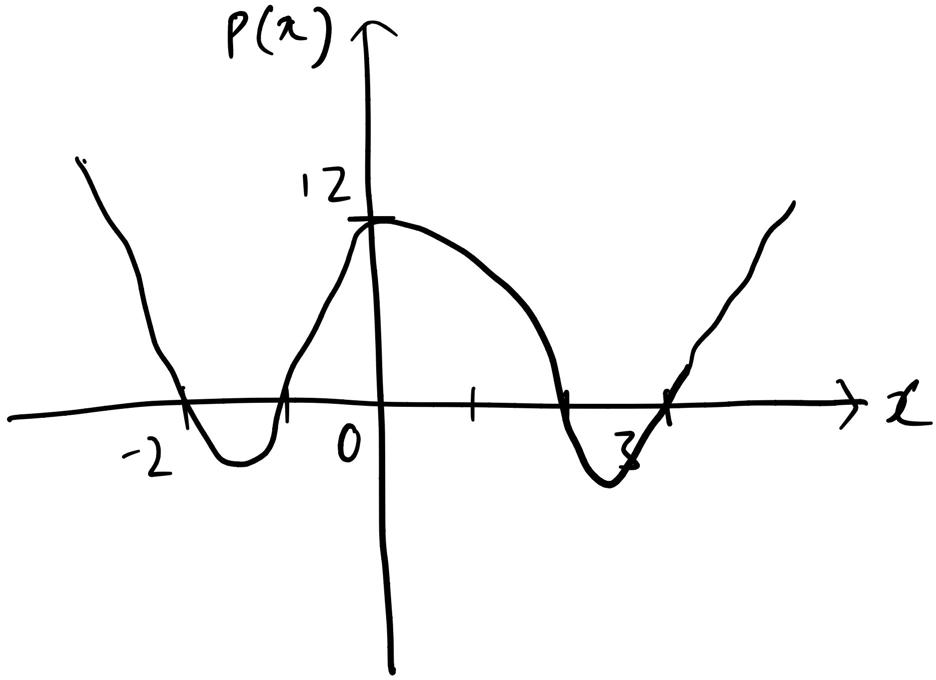
19. $P(x) = -(2x-1)(x+1)(x+3)$



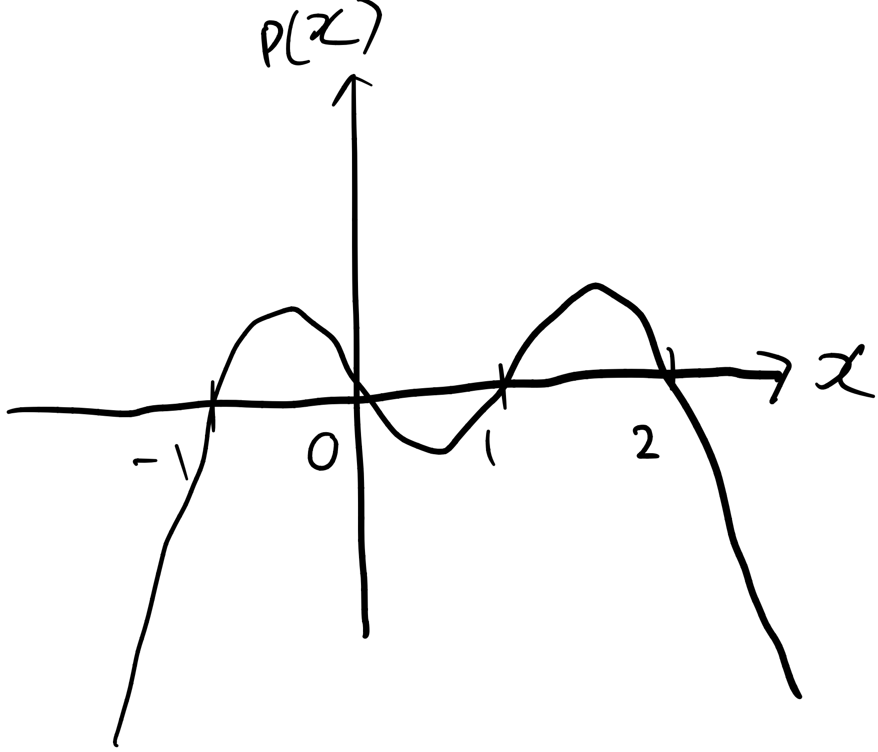
20. $P(x) = (x-3)(x+2)(3x-2)$



21. $P(x) = (x+2)(x+1)(x-2)(x-3)$

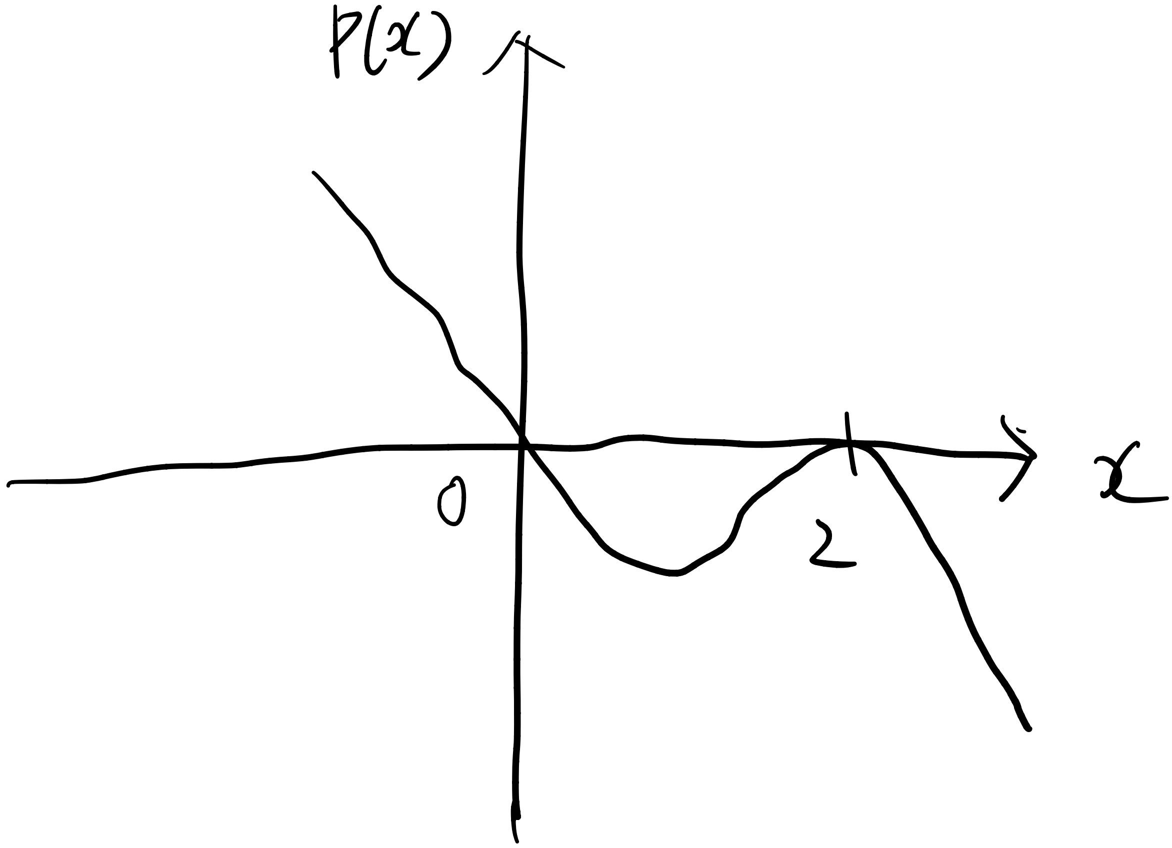


22. $P(x) = x(x+1)(x-1)(2-x) = -x(x+1)(x-1)(x-2)$



23. $P(x) = -2x(x-2)^2$

$x = 0, 2$


P(x)
0
2
x

② Local Extrema

51. $P(x) = -x^2 + 4x$

(b) $(2, 4)$

(a) $y = 0$

$x = 0, 4$

52. $P(x) = \frac{2}{9}x^3 - x^2$

$= -x^2\left(1 - \frac{2}{9}x\right)$

(a) $y = 0$

$x = 0, 4.5$

(b) $(0, 0), (3, -3)$

53. $P(x) = -\frac{1}{2}x^3 + \frac{3}{2}x - 1$

$= -\frac{1}{2}(x^3 - 3x + 2)$

(a) $x = -2, 1$

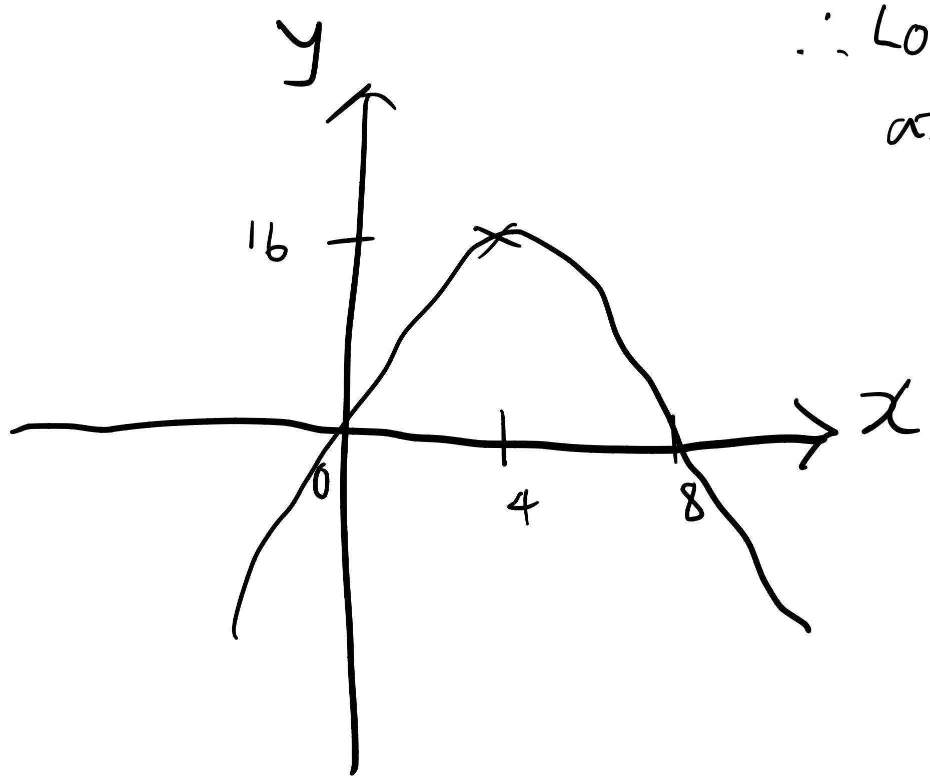
$y = -1$

(b) $(-1, -2), (1, 0)$

55. $y = -x^2 + 8x$, $[-4, 12]$ by $[-50, 30]$

$$\begin{aligned} y &= -x^2 + 8x \\ &= -(x^2 - 8x) \\ &= -\left(x^2 - 8x + \left(\frac{8}{2}\right)^2 - \left(\frac{8}{2}\right)^2\right) \\ &= -\left((x-4)^2 - 16\right) \\ &= -(x-4)^2 + 16 \end{aligned}$$

$\therefore$ Local maximum at $(4, 16)$



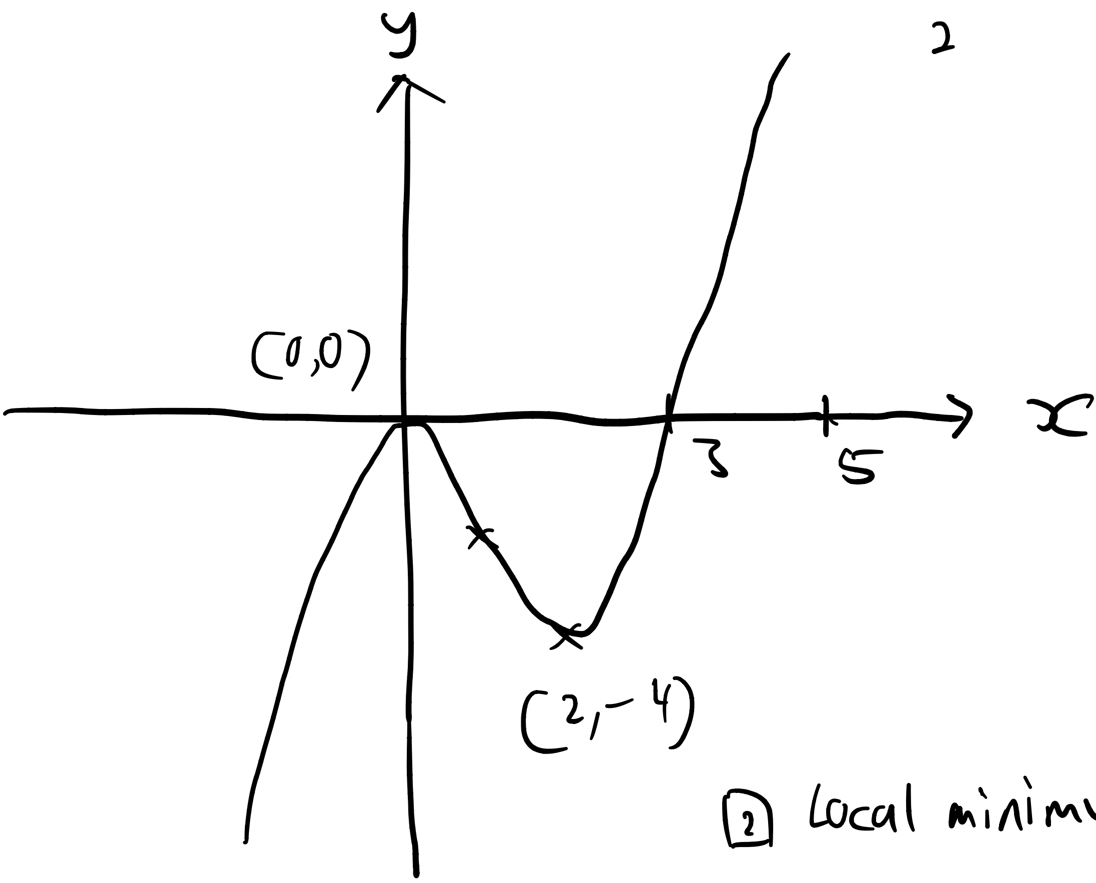
Domain: $(-\infty, \infty)$, Range: $(-\infty, 16]$

56. $y = x^3 - 3x^2$, $[-2, 5]$ by $[-10, 10]$

$y = x^2(x-3) = 0$

$x = 0, 3$

x	y
1	-2
2	-4



[2] Local minimum at $(2,-4)$

Local maximum at $(0,0)$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

3.3 Dividing Polynomials

1. Division of Polynomials
2. Remainder Theorem
3. Factor Theorem

① Division of Polynomials

3. $P(x) = 2x^2 - 5x - 7$, $D(x) = x - 2$

$$
\begin{array}{r l}
 & \phantom{2x^2-{}}2x - 1 \\
x-2 \,\big) & \overline{2x^2 - 5x - 7} \\
 & 2x^2 - 4x \\
\hline
 & - x - 7 \\
 & - x + 2 \\
\hline
 & - 9
\end{array}
$$

$$\frac{P(x)}{D(x)} = 2x - 1 + \frac{-9}{x-2}$$

4. $P(x) = 3x^3 + 9x^2 - 5x - 1$, $D(x) = x + 4$

$$
\begin{array}{r l}
 & \phantom{3x^3+{}}3x^2 - 3x + 7 \\
x+4 \,\big) & \overline{3x^3 + 9x^2 - 5x - 1} \\
 & 3x^3 + 12x^2 \\
\hline
 & - 3x^2 - 5x - 1 \\
 & - 3x^2 - 12x \\
\hline
 & \phantom{3x^3 - 3x^2 -{}} 7x - 1 \\
 & \phantom{3x^3 - 3x^2 -{}} 7x + 28 \\
\hline
 & - 29
\end{array}
$$

$$\frac{P(x)}{D(x)} = 3x^2 - 3x + 7 \frac{-29}{x+4}$$

5. $P(x) = 4x^2 - 3x - 7$, $D(x) = 2x - 1$

$$
\begin{array}{r|l}
 & 2x - \frac{1}{2} \\
\hline
2x-1 & 4x^2 - 3x - 7 \\
 & 4x^2 - 2x \\
\cline{2-2}
 & \quad -x - 7 \\
 & \quad -x + \frac{1}{2} \\
\cline{2-2}
 & \qquad -7\frac{1}{2}
\end{array}
$$

$$\frac{P(x)}{D(x)} = 2x - \frac{1}{2} + \frac{-\frac{15}{2}}{2x-1}$$

$$= 2x - \frac{1}{2} - \frac{15}{4x-2}$$

6. $P(x) = 6x^3 + x^2 - 12x + 5$, $D(x) = 3x - 4$

$$
\begin{array}{c|cccc}
\frac{4}{3} & 6 & 1 & -12 & 5 \\
 & & 8 & 12 & 0 \\
\hline
 & 6 & 9 & 0 & 5
\end{array}
$$

$$\frac{P(x)}{D(x)} = 2x^2 + 3x + \frac{5}{3x-4}$$

7. $P(x) = 2x^4 - x^3 + 9x^2, \quad D(x) = x^2 + 4$

$$
\begin{array}{r}
2x^2 - x + 1 \\
x^2 + 4 \,\overline{\big)\, 2x^4 - x^3 + 9x^2} \\
\underline{2x^4 + 8x^2} \\
-x^3 + x^2 \\
\underline{-x^3 - 4x} \\
x^2 + 4x \\
\underline{x^2 + 4} \\
4x - 4
\end{array}
$$

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$$

$$= 2x^2 - x + 1 + \frac{4x - 4}{x^2 + 4}$$

8. $P(x) = 2x^5 + x^3 - 2x^2 + 3x - 5, \quad D(x) = x^2 - 3x + 1$

$$
\begin{array}{r}
2x^3 + 6x^2 + 17x + 43 \\
x^2 - 3x + 1 \,\overline{\big)\, 2x^5 + 0 + x^3 - 2x^2 + 3x - 5} \\
\underline{2x^5 - 6x^4 + 2x^3} \\
6x^4 - x^3 - 2x^2 \\
6x^4 - 18x^3 + 6x^2
\end{array}
$$

$$
\begin{array}{r}
17x^3 - 8x^2 + 3x \\
17x^3 - 51x^2 + 17x \\
\hline
43x^2 - 14x - 5 \\
43x^2 - 129x + 43 \\
\hline
115x - 48
\end{array}
$$

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$$

$$= 2x^3 + 6x^2 + 17x + 43 + \frac{115x - 48}{x^2 - 3x + 1}$$

9. $P(x) = -x^3 - 2x + 6$, $D(x) = x + 1$

$$
\begin{array}{r}
-x^2 + x - 3 \\
x + 1 \,\overline{\big)\, -x^3 + 0 - 2x + 6} \\
-x^3 - x^2 \qquad\qquad \\
\hline
x^2 - 2x \qquad \\
x^2 + x \qquad \\
\hline
-3x + 6 \\
-3x - 3 \\
\hline
9
\end{array}
$$

$$P(x) = D(x) \cdot Q(x) + R(x)$$

$$= (x+1)(-x^2 + x - 3) + 9$$

② Remainder Theorem

39. $P(x) = 4x^2+12x+5$, $c=-1$

$$\begin{array}{r|rrr} -1 & 4 & 12 & 5 \\ & & -4 & -8 \\ \hline & 4 & 8 & -3 \end{array}$$

$P(-1) = -3$

40. $P(x) = 2x^2+9x+1$, $c=\frac{1}{2}$

$$\begin{array}{r|rrr} \frac{1}{2} & 2 & 9 & 1 \\ & & 1 & 5 \\ \hline & 2 & 10 & 6 \end{array}$$

$$\begin{aligned} P\left(\tfrac{1}{2}\right) &= 2\left(\tfrac{1}{2}\right)^2+9\left(\tfrac{1}{2}\right)+1 \\ &= \tfrac{1}{2}+\tfrac{9}{2}+1 \\ &= 6 \end{aligned}$$

41. $P(x) = x^3 + 3x^2 - 7x + 6$, $c = 2$

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -7 & 6 \\ & & 2 & 10 & 6 \\ \hline & 1 & 5 & 3 & 12 \end{array}$$

$$\begin{aligned} P(2) &= (2)^3 + 3(2)^2 - 7(2) + 6 \\ &= 8 + 12 - 14 + 6 \\ &= 12 \end{aligned}$$

③ Factor Theorem

53. $P(x) = x^3 - 3x^2 + 3x - 1, \quad c = 1$

$$P(1) = 1^3 - 3(1)^2 + 3(1) - 1$$
$$= 1 - 3 + 3 - 1$$
$$= 0$$

Factor theorem: $P(1) = 0$, $c = 1$ is a factor of $P(x)$

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 3 & -1 \\ & & 1 & -2 & 1 \\ \hline & 1 & -2 & 1 & 0 \end{array}$$

$$P(x) = (x-1)(x^2 - 2x + 1)$$

54. $P(x) = x^3 + 2x^2 - 3x - 10, \quad c = 2$

$$\begin{array}{r|rrrr} 2 & 1 & 2 & -3 & -10 \\ & & 2 & 8 & 10 \\ \hline & 1 & 4 & 5 & 0 \end{array}$$

$$P(2) = 8 + 8 - 6 - 10$$
$$= 0$$

$$P(x) = (x-2)(x^2 + 4x + 5)$$

3.4 Real Zeros of Polynomials

1. Rational Zeros
2. Integer Zeros
3. Upper and Lower Bounds

Revised subtopics

1. Zeros of a polynomial
2. Descartes' Rule of Signs

① Rational Zeros

5. $P(x) = x^3 - 4x^2 + 3$

Rational Zeros Theorem: $\pm 3, \pm 1$

6. $Q(x) = x^4 - 3x^3 - 6x + 8$

Rational Zeros Theorem: $\pm 1, \pm 2, \pm 4, \pm 8$

7. $R(x) = 2x^5 + 3x^3 + 4x^2 - 8$

$\therefore \pm \frac{1}{2}, \pm 1, \pm 2, \pm 4, \pm 8$

② Integer Zeros

15. $P(x) = x^3 + 2x^2 - 13x + 10$

Rational Zero Theorem: $\pm 1, \pm 2, \pm 5, \pm 10$

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -13 & 10 \\ & & 2 & 4 & -9 \\ \hline & 1 & 4 & -9 & 1 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -13 & 10 \\ & & -1 & -1 & 14 \\ \hline & 1 & 1 & -14 & 24 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & 2 & -13 & 10 \\ & & 2 & 8 & -10 \\ \hline & 1 & 4 & -5 & 0 \end{array}$$

$$P(x) = (x-2)(x^2 + 4x - 5)$$
$$= (x-2)(x+5)(x-1)$$

$\therefore$ Real zeros are $1, 2, -5$

16. $P(x) = x^3 - 4x^2 - 19x - 14$

Rational Zeros Theorem: $\pm 1, \pm 2, \pm 7, \pm 14$

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -19 & -14 \\ & & 1 & -3 & \\ \hline & 1 & -3 & -22 & \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -4 & -19 & -14 \\ & & 2 & -4 & \\ \hline & 1 & -2 & -23 & \end{array}$$

$$\begin{array}{r|rrrr} 7 & 1 & -4 & -19 & -14 \\ & & 7 & 21 & 14 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

$$P(x) = (x-7)(x^2+3x+2)$$
$$= (x-7)(x+2)(x+1)$$

17. $P(x) = x^3 + 3x^2 - 4$

Rational Zeros Theorem: $\pm 1, \pm 2, \pm 4$

$$\begin{array}{r|rrrr} 1 & 1 & 3 & 0 & -4 \\ & & 1 & 4 & 4 \\ \hline & 1 & 4 & 4 & 0 \end{array}$$

$$\therefore p(x) = (x-1)(x^2+4x+4)$$
$$= (x-1)(x+2)^2$$

Zeros: $-2, 1$

18. $p(x) = x^3 - 3x - 2$

Rational Zeros Theorem: $\pm 1, \pm 2$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -3 & -2 \\ & & 2 & 4 & 2 \\ \hline & 1 & 2 & 1 & 0 \end{array}$$

$$\therefore p(x) = (x-2)(x^2+2x+1)$$
$$= (x-2)(x+1)^2$$

Zeros: $-1, 2$

19. $P(x) = x^3 - 6x^2 + 12x - 8$

Rational Zeros Theorem: $\pm 1, \pm 2, \pm 4, \pm 8$

DRS: 3 or 1 positive zero, no negative zero

$$\begin{array}{c|cccc} 2 & 1 & -6 & 12 & -8 \\ & & 2 & -8 & 8 \\ \hline & 1 & -4 & 4 & 0 \end{array}$$

$$\begin{aligned} P(x) &= (x-2)(x^2-4x+4) \\ &= (x-2)(x-2)^2 \\ &= (x-2)^3 \end{aligned}$$

$\therefore$ 2

29. $P(x) = 4x^4 - 37x^2 + 9$

Rational Zero Theorem: $\pm 1, \pm 3, \pm 9, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{3}{2}, \pm \frac{3}{4}, \pm \frac{9}{2}, \pm \frac{9}{4}$

Descartes Rule of Sign: 2 or 0 positive rational zero

2 or 0 negative rational zero

$$\begin{array}{r|rrrrr} 3 & 4 & 0 & -37 & 0 & 9 \\ & & 12 & 36 & -3 & -9 \\ \hline & 4 & 12 & -1 & -3 & 0 \end{array}$$

$P(x) = (x-3)(4x^3 + 12x^2 - x - 3)$

$$\begin{array}{r|rrrr} 1/2 & 4 & 12 & -1 & -3 \\ & & 2 & 7 & 3 \\ \hline & 4 & 14 & 6 & 0 \end{array}$$

$$p(x) = (x-3)(2x-1)(4x^2+14x+6)$$
$$= (x-3)(2x-1)(4x+2)(x+3)$$

$$\therefore x = -3, -\frac{1}{2}, \frac{1}{2}, 3$$

Descartes' Rule of Signs

63. $P(x) = x^3 - x^2 - x - 3$

1 variation in sign, $\therefore$ 1 positive real zero

$P(-x) = (-x)^3 - (-x)^2 - (-x) - 3$
$= -x^3 - x^2 + x - 3$

2 variations in sign, $\therefore$ 2 or 0 negative real zeros

$\therefore$ 1 or 3 total number of real zeros possible

3. Upper and Lower Bounds

69. $p(x) = 2x^3 + 5x^2 + x - 2$; $a = -3, b = 1$

Test for Lower Bound

$$\begin{array}{r|rrrr} -3 & 2 & 5 & 1 & -2 \\ & & -6 & 3 & -12 \\ \hline & 2 & -1 & 4 & -14 \end{array}$$

$\because$ alternating signs

$\therefore$ -3 is a lower bound

Test for Upper Bound

$$\begin{array}{r|rrrr} 1 & 2 & 5 & 1 & -2 \\ & & 2 & 7 & 8 \\ \hline & 2 & 7 & 8 & 6 \end{array}$$

$\because$ non-negative entries,

$\therefore$ 1 is an upper bound

70. $p(x) = x^4 - 2x^3 - 9x^2 + 2x + 8$; $a = -3$, $b = 5$

$$\begin{array}{r|rrrrr} 5 & 1 & -2 & -9 & 2 & 8 \\ & & 5 & 15 & 30 & 160 \\ \hline & 1 & 3 & 6 & 32 & 168 \end{array}$$

$\therefore$ non-negative entries, upper bound

$$\begin{array}{r|rrrrr} -3 & 1 & -2 & -9 & 2 & 8 \\ & & -3 & 15 & -18 & 48 \\ \hline & 1 & -5 & 6 & -16 & 56 \end{array}$$

$\therefore$ alternating signs, lower bound

71. $p(x) = 8x^3 + 10x^2 - 39x + 9;\ a = -3, b = 2$

$$\begin{array}{r|rrrr} -3 & 8 & 10 & -39 & 9 \\ & & -24 & 42 & -9 \\ \hline & 8 & -14 & 3 & 0 \end{array}$$

$\therefore$ lower bound,

-3 is also a zero

$$\begin{array}{r|rrrr} 2 & 8 & 10 & -39 & 9 \\ & & 16 & 52 & 26 \\ \hline & 8 & 26 & 13 & 35 \end{array}$$

$\therefore$ upper bound

72. $p(x) = 3x^4 - 17x^3 + 24x^2 - 9x + 1;\ a = 0, b = 6$

$$\begin{array}{r|rrrrr} 0 & 3 & -17 & 24 & -9 & 1 \\ & & 0 & 0 & 0 & 0 \\ \hline & 3 & -17 & 24 & -9 & 1 \end{array}$$

$\therefore$ Alternate signs, lower bound

$$\begin{array}{r|rrrrr} 6 & 3 & -17 & 24 & -9 & 1 \\ & & 18 & 6 & 180 & \\ \hline & 3 & 1 & 30 & 171 & \end{array}$$

$\therefore$ all non-negative entries

upper bound

Descartes' Rules of Signs

63. $P(x) = x^3 - x^2 - x - 3$

$\therefore$ 1 positive real zero

$P(-x) = (-x)^3 - (-x)^2 - (-x) - 3$

$= -x^3 - x^2 + x - 3$

$\therefore$ 2 or 0 negative real zeros

$\therefore$ 1 or 3 possible total real zeros

Zeros of a Polynomial 81 - 86

81. $P(x) = 2x^4 + 3x^3 - 4x^2 - 3x + 2$

Rational Zero Theorem: $\pm 1, \pm 2, \pm \frac{1}{2}$

$$\begin{array}{r|rrrrr} 1 & 2 & 3 & -4 & -3 & 2 \\ & & 2 & 5 & 1 & -2 \\ \hline & 2 & 5 & 1 & -2 & 0 \end{array}$$

$P(x) = (x-1)(2x^3 + 5x^2 + x - 2)$

$$\begin{array}{r|rrrr} -1 & 2 & 5 & 1 & -2 \\ & & -2 & -3 & 2 \\ \hline & 2 & 3 & -2 & 0 \end{array}$$

$P(x) = (x-1)(x+1)(2x^2 + 3x - 2)$

$= (x-1)(x+1)(2x-1)(x+2)$

3.5 Complex Zeros and the Fundamental Theorem of Algebra

① Complete Factorisation

② Finding Complex Zeros

② Finding Complex Zeros

37. $Q(x) = x^4 + 2x^2 + 1$

$$x^2+1=0$$
$$x^2=-1$$
$$x=\pm i$$

$$= (x^2+1)^2$$
$$= ((x+i)(x-i))^2$$
$$= (x+i)^2(x-i)^2$$

Zeros: $-i, i$
each of multiplicity 2

37. Zeros: $1+i, 1-i$

Complete Factorisation Theorem:

$$p(x) = a(x-c_1)(x-c_2)\cdots(x-c_n)$$

$$p(x) = a(x-(1+i))(x-(1-i))$$
$$= a(x-1-i)(x-1+i)$$
$$= a((x-1)^2 - i^2)$$
$$= a((x-1)^2+1)$$
$$= a(x^2-2x+1+1)$$
$$= a(x^2-2x+2)$$

Let $a=1$, $\therefore p(x) = x^2-2x+2$

Complex Zeros Come in Conjugate Pairs

41. Zeros : $2, i$

Conjugate Zeros Theorem

Zeros : $2, i, -i$

$$P(x) = (x-2)(x-i)(x+i)$$
$$= (x-2)(x^2-i^2)$$
$$= (x-2)(x^2+1)$$
$$= x^3+x-2x^2-2$$
$$\therefore P(x) = x^3-2x^2+x-2$$

Linear and Quadratic Factors

67. $P(x) = x^4+8x^2-9$

(a) $P(x) = (x^2-1)(x^2+9)$
$= (x+1)(x-1)(x^2+9)$

(b) $P(x) = (x+1)(x-1)(x+\sqrt{3}i)$
$(x-\sqrt{3}i)$

$x^2+9=0$
$x^2=-3$
$x=\pm\sqrt{3}i$

① Complete Factorisation

7. $P(x) = x^4 + 4x^2$

(a) $P(x) = x^4 + 4x^2$

$= x^2(x^2+4)$

$x^2 = 0$

$x = 0$

$x^2 + 4 = 0$

$x^2 = -4$

$x = \pm\sqrt{-4}$

$= \pm\sqrt{4i^2}$

$= \pm 2i$

$\therefore 0, -2i, 2i$

(b) $P(x) = x^2(x+2i)(x-2i)$

8. $P(x) = x^5 + 9x^3$

$= x^3(x^2+9)$

$= x^3(x-3i)(x+3i)$

$x^2+9=0$

$x^2 = -9$

$x = \pm 3i$

$\therefore 0, \pm 3i$

Multiplicity: 3, 1, 1

9. $P(x) = x^3 - 2x^2 + 2x$

Rational Zeros Theorem: $\pm 1, \pm 2$

$$\begin{array}{r|rrr} 1 & 1 & -2 & 2 \\ & & 1 & -1 \\ \hline & 1 & -1 & 1 \end{array}$$

$$\begin{array}{r|rrr} 2 & 1 & -2 & 2 \\ & & 2 & 0 \\ \hline & 1 & 0 & 2 \end{array}$$

$\therefore$ upper bound

$$\begin{array}{r|rrr} -1 & 1 & -2 & 2 \\ & & -1 & 3 \\ \hline & 1 & -3 & 5 \end{array}$$

$\therefore$ Lower bound

(b)
$$P(x) = x(x^2 - 2x + 2)$$
$$= x(x - 1 - i)(x - 1 + i)$$

(a)
$\therefore$ Zeros: $0, 1+i, 1-i$

Multiplicity: $1, 1, 1$

$x^2 - 2x + 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{2 \pm \sqrt{4 - 8}}{2}$$
$$= \frac{2 \pm \sqrt{4i^2}}{2}$$
$$= 1 \pm i$$

10. $P(x) = x^3 + x^2 + x$

(a) $P(x) = x(x^2 + x + 1)$

$x^2 + x + 1 = 0$

$x = \frac{-1 \pm \sqrt{1-4}}{2}$

$= \frac{-1 \pm \sqrt{-3}}{2}$

$= \frac{-1 \pm \sqrt{3}i}{2}$

Zeros: $x = 0, \frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}$

(b)

$P(x) = x\left(x + \frac{1-\sqrt{3}i}{2}\right)\left(x + \frac{1+\sqrt{3}i}{2}\right)$

② Finding Complex Zeros

47. $P(x) = x^3 + 2x^2 + 4x + 8$

Rational Zero Theorem : $\pm 1, \pm 2, \pm 4, \pm 8$

$$\begin{array}{r|rrrr} 1 & 1 & 2 & 4 & 8 \\ & & 1 & 3 & 7 \\ \hline & 1 & 3 & 7 & 15 \end{array}$$

$\because$ non-negative entries

$\therefore$ 1 is an upper bound

$$\begin{array}{r|rrrr} -1 & 1 & 2 & 4 & 8 \\ & & -1 & -1 & -3 \\ \hline & 1 & 1 & 3 & 5 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 4 & 8 \\ & & -2 & 0 & -8 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

$x^2 + 4 = 0$

$x^2 = -4$

$x = \pm 2i$

$p(x) = (x+2)(x^2+4)$

$= (x+2)(x+2i)(x-2i)$

48. $P(x) = x^3 - 7x^2 + 17x - 15$

Rational Zero Theorem: $\pm 1, \pm 3, \pm 5, \pm 15$

$$
\begin{array}{r|rrrr}
1 & 1 & -7 & 17 & -15 \\
 & & 1 & -6 & 11 \\
\hline
 & 1 & -6 & 11 & -4
\end{array}
$$

$$
\begin{array}{r|rrrr}
3 & 1 & -7 & 17 & -15 \\
 & & 3 & -12 & 15 \\
\hline
 & 1 & -4 & 5 & 0
\end{array}
$$

$P(x) = (x-3)(x^2 - 4x + 5)$

$= (x-3)(x-(2+2i))(x-(2-2i))$

$= (x-3)(x-2-2i)(x-2+2i)$

$x^2 - 4x + 5 = 0$

$x = \dfrac{4 \pm \sqrt{11 - 20}}{2}$

$= \dfrac{4 \pm \sqrt{4i^2}}{2}$

$= 2 \pm 2i$

$\therefore 3,\ 2+2i,\ 2-2i$

49. $P(x) = x^3 - 2x^2 + 2x - 1$

Rational Zeros Theorem : ± 1

$$\begin{array}{r|rrrr} 1 & 1 & -2 & 2 & -1 \\ & & 1 & -1 & 1 \\ \hline & 1 & -1 & 1 & 0 \end{array}$$

$P(x) = (x-1)(x^2 - x + 1)$

$x^2 - x + 1 = 0$

$x = \dfrac{1 \pm \sqrt{1-4}}{2}$

$= \dfrac{1 \pm \sqrt{3}\,i}{2}$

$\therefore 1, \ \dfrac{1+\sqrt{3}i}{2}, \ \dfrac{1-\sqrt{3}i}{2}$

50. $P(x) = x^3 + 7x^2 + 18x + 18$

Rational Zeros Theorem : $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

$$\begin{array}{r|rrrr} 2 & 1 & 7 & 18 & 18 \\ & & 2 & 18 & 72 \\ \hline & 1 & 9 & 36 & 90 \end{array}$$

$\therefore$ upper bound

$$\begin{array}{r|rrrr} 1 & 1 & 7 & 18 & 18 \\ & & 1 & 8 & 26 \\ \hline & 1 & 8 & 26 & 44 \end{array}$$

$$
\begin{array}{r|rrrr}
-3 & 1 & 7 & 18 & 18 \\
 & & -3 & -12 & -18 \\
\hline
 & 1 & 4 & 6 & 0
\end{array}
$$

$x^2+4x+6=0$

$$x = \frac{-4 \pm \sqrt{16-24}}{2}$$

$$= \frac{-4 \pm \sqrt{8i^2}}{2}$$

$$= -2 \pm \sqrt{2}i$$

$$\therefore P(x) = (x-(-3))(x^2+4x+6)$$

$$= (x+3)(x+2-\sqrt{2}i)(x+2+\sqrt{2}i)$$

51. $P(x) = x^3 - 3x^2 + 3x - 2$

Rational Zeros Theorem: $\pm 1, \pm 2$

$$
\begin{array}{r|rrrr}
1 & 1 & -3 & 3 & -2 \\
 & & 1 & -2 & 1 \\
\hline
 & 1 & -2 & 1 & -1
\end{array}
$$

$$
\begin{array}{r|rrrr}
2 & 1 & -3 & 3 & -2 \\
 & & 2 & -2 & 2 \\
\hline
 & 1 & -1 & 1 & 0
\end{array}
$$

$x^2-x+1=0$

$$x = \frac{1 \pm \sqrt{1-4}}{2}$$

$$= \frac{1 \pm \sqrt{3}i}{2}$$

$$P(x) = (x-2)(x^2-x+1)$$

$$\therefore \quad 2, \quad \frac{1+\sqrt{3}i}{2}, \quad \frac{1-\sqrt{3}i}{2}$$

52. $P(x) = x^3 - x - 6$

Descartes' Rule of Signs: 1 positive real zero
2 or 0 negative real zero

Rational Zeros Theorem: $\pm 1, \pm 2, \pm 3, \pm 6$

$$\begin{array}{c|cccc} 1 & 1 & 0 & -1 & -6 \\ & & 1 & 1 & 0 \\ \hline & 1 & 1 & 0 & -6 \end{array} \qquad \begin{array}{c|cccc} 2 & 1 & 0 & -1 & -6 \\ & & 2 & 4 & 6 \\ \hline & 1 & 2 & 3 & 0 \end{array}$$

$x^2 + 2x + 3 = 0$

$$x = \frac{-2 \pm \sqrt{4-12}}{2} = \frac{-2 \pm 2\sqrt{2}i}{2} = -1 \pm \sqrt{2}i$$

$$P(x) = (x-2)(x^2+2x+3)$$
$$= (x-2)(x+1-\sqrt{2}i)(x+1+\sqrt{2}i)$$

$\therefore$ Zeros: $2, -1+\sqrt{2}i, -1-\sqrt{2}i$

53. $P(x) = 2x^3 + 7x^2 + 12x + 9$

RZT: $\pm 1, \pm 3, \pm 9, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}$

DRS: No positive real zero, 3 or 1 negative real zero

$$\begin{array}{c|cccc} -3 & 2 & 7 & 12 & 9 \\ & & -6 & -3 & -27 \\ \hline & 2 & 1 & 9 & -18 \end{array}$$

$$\begin{array}{r|rrrr} -9 & 2 & 7 & 12 & 9 \\ & & -18 & 99 & -999 \\ \hline & 2 & -11 & 111 & -981 \end{array}$$ Lower bound

$$\begin{array}{r|rrrr} -\frac{9}{2} & 2 & 7 & 12 & 9 \\ & & -9 & 9 & \\ \hline & 2 & -2 & 21 & \end{array}$$

$$\begin{array}{r|rrrr} -\frac{3}{2} & 2 & 7 & 12 & 9 \\ & & -3 & -6 & -9 \\ \hline & 2 & 4 & 6 & 0 \end{array}$$

$$P(x) = (2x-3)(2x^2+4x+6)$$

$$= (2x-3)(x+1-\sqrt{2}i)(x+1+\sqrt{2}i)$$

$$4x^2+8x+12=0$$

$$x^2+2x+3=0$$

$$x = \frac{-2 \pm \sqrt{4-12}}{2}$$

$$= \frac{-2 \pm 2\sqrt{2}i}{2}$$

$$= -1 \pm \sqrt{2}i$$

$\therefore -\frac{3}{2}, -1+\sqrt{2}i, -1-\sqrt{2}i$

3.6 Rational Functions

① Graphing Rational Functions

Table of Values

9. $r(x) = \dfrac{x}{x-2}$

(a)

x	$r(x)$
1.5	-3
1.9	-19
1.99	-199
1.999	-1999

x	$r(x)$
2.5	5
2.1	21
2.01	201
2.001	2001

$r(1.5) = \dfrac{1.5}{1.5-2}$

$= -3$

$r(1.9) = \dfrac{1.9}{1.9-2}$

$= -19$

$r(1.99) = \dfrac{1.99}{1.99-2}$

$= -199$

$r(1.999) = -1999$

$r(2.5) = \dfrac{2.5}{2.5-2}$

$= 5$

$r(2.1) = \dfrac{2.1}{2.1-2}$

$= 21$

$r(2.01) = \dfrac{2.01}{2.01-2}$

$= 201$

$r(2.001) = \dfrac{2.001}{2.001-2}$

$= 2001$

(b) $r(x) \to -\infty$, as $x \to 2^-$

$r(x) \to \infty$, as $x \to 2^+$

(c)

Table 3

x	$r(x)$
10	2
50	1.042
100	1.020
1000	1.002

Table 4

x	$r(x)$
-10	0.833
-50	0.962
-100	0.980
-1000	0.998

$r(10) = \frac{10}{10-2}$

$= 2$

$r(50) = \frac{50}{50-2}$

$= 1.042$

$r(100) = \frac{100}{100-2}$

$= 1.020$

$r(1000) = \frac{1000}{1000-2}$

$= 1.002$

$r(-10) = \frac{-10}{-10-2}$

$= 0.833$

$r(-50) = \frac{-50}{-50-2}$

$= 0.962$

$r(-100) = \frac{-100}{-100-2}$

$= 0.980$

$r(-1000) = \frac{-1000}{-1000-2}$

$= 0.998$

$r(x) \to 1$, as $x \to \infty$

$r(x) \to 1$, as $x \to -\infty$

Graphing Rational Functions Using Transformations

15. $s(x) = \dfrac{3}{x+1}$

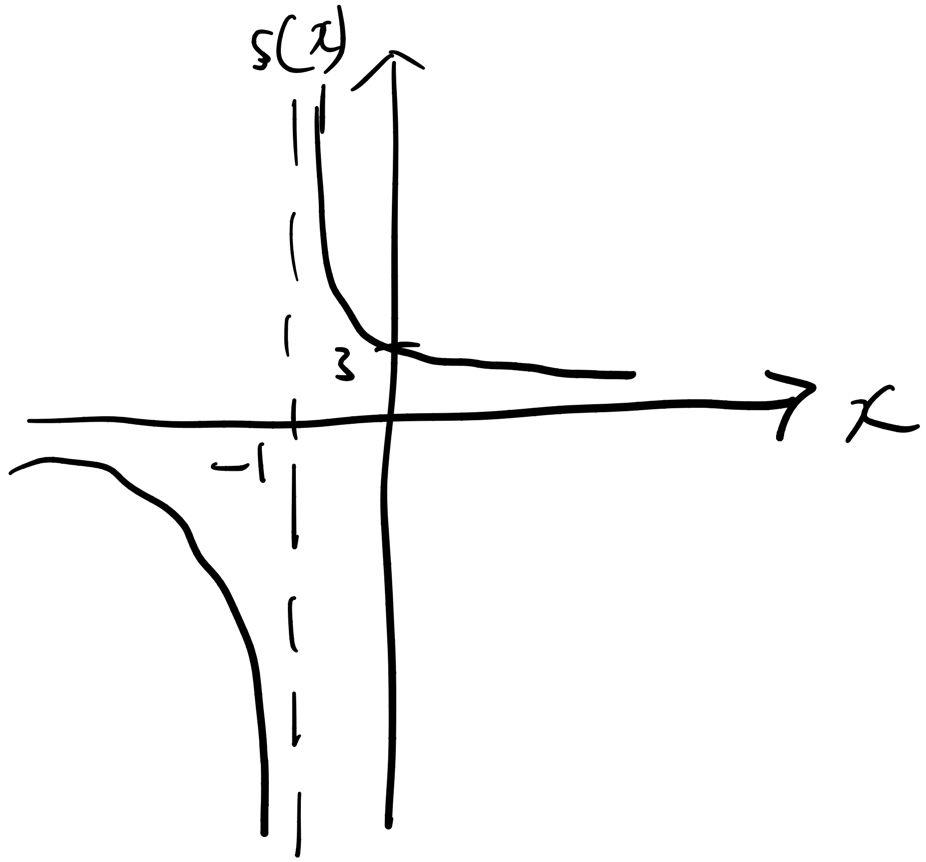
Domain: $\{x \mid x \neq -1\}$

Range: $\{y \mid y \neq 0\}$

$y = \dfrac{1}{x}$

$f(x) = \dfrac{1}{x}$

$$s(x) = \frac{3}{x+1}$$
$$= 3\left(\frac{1}{x+1}\right)$$
$$= 3f(x+1)$$



17. $t(x) = \dfrac{2x-3}{x-2}$

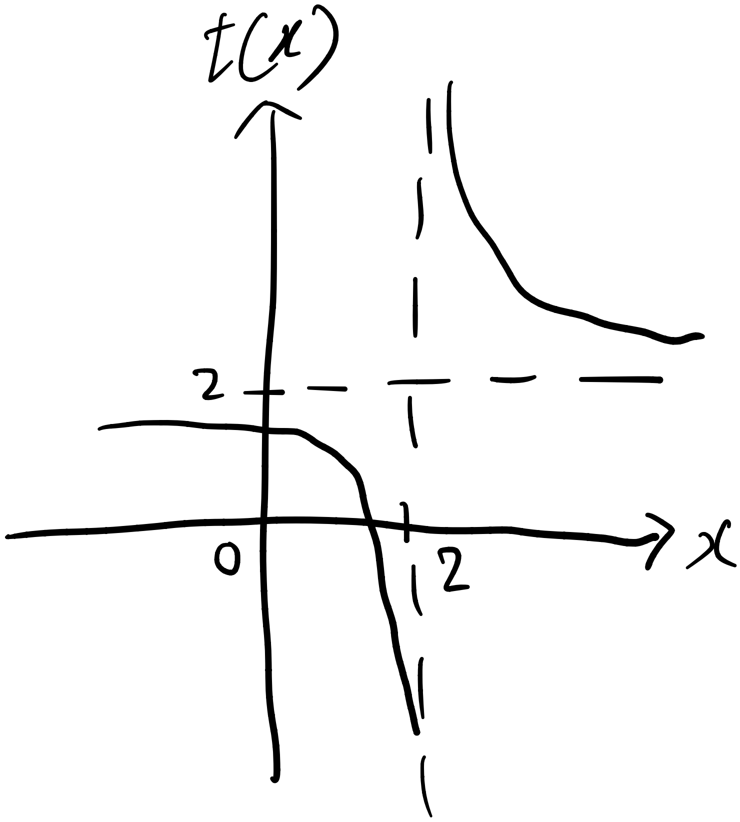
$y = \dfrac{1}{x}$

$f(x) = \dfrac{1}{x}$

$$\begin{array}{c|cc} 2 & 2 & -3 \\ & & 4 \\ \hline & 2 & 1 \end{array}$$

$t(x) = 2 + \dfrac{1}{x-2}$

$t(x) = 2 + f(x-2)$



Graphing Rational Functions

45. $r(x) = \dfrac{3x^2 - 12x + 13}{x^2 - 4x + 4}$ $\qquad r(x) = \dfrac{3x^2 - 12x + 13}{(x-2)^2}$

$$= 3 + \frac{1}{x^2 - 4x + 4}$$

$$= 3 + \frac{1}{(x-2)^2}$$

if $f(x) = \frac{1}{x}$

$$= 3 + f((x-2)^2)$$

$x \to 2^-$

x	$r(x)$
1	4
1.4	5.78
1.8	28
1.9	103

$x \to 2^+$

x	$r(x)$
3	4
2.4	9.25
2.1	103
2.01	10003

$r(2.1) = 103$

$r(2.01) = 10003$

$r(1) = \dfrac{3 - 12 + 13}{1} = 4$

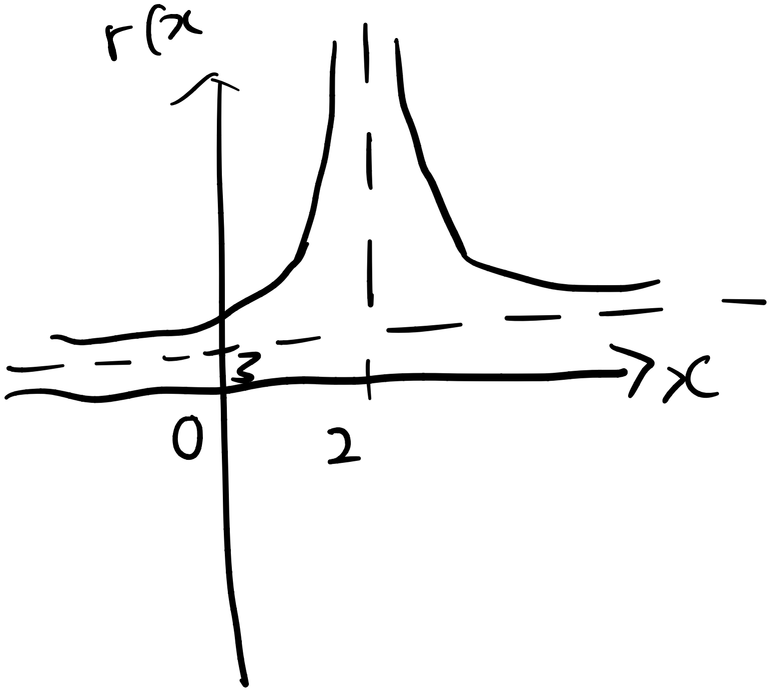
$r(1.4) = 5.78$

$r(1.8) = 28$

$r(1.9) = 103$

$r(3) = \dfrac{27 - 36 + 13}{1} = 4$

$r(2.4) = 9.25$



$$r(x) = \frac{3 - \frac{12}{x} + \frac{13}{x^2}}{1 - \frac{4}{x} + \frac{4}{x^2}}$$

As $x \to \infty$

$r(x) = \frac{3}{1} = 3$

33. $r(x) = \dfrac{3x+1}{4x^2+1}$

$$= \frac{3 + \frac{1}{x}}{4x + \frac{1}{x}}$$

$4x^2 + 1 = 0$

$4x^2 = -1$

$x^2 = -\frac{1}{4}$

$x = \pm \frac{1}{2} i$

As $x \to \infty$, $r(x) \to 0$

As $x \to -\infty$, $r(x) \to 0$

$\therefore$ Horizontal asymptote : 0,

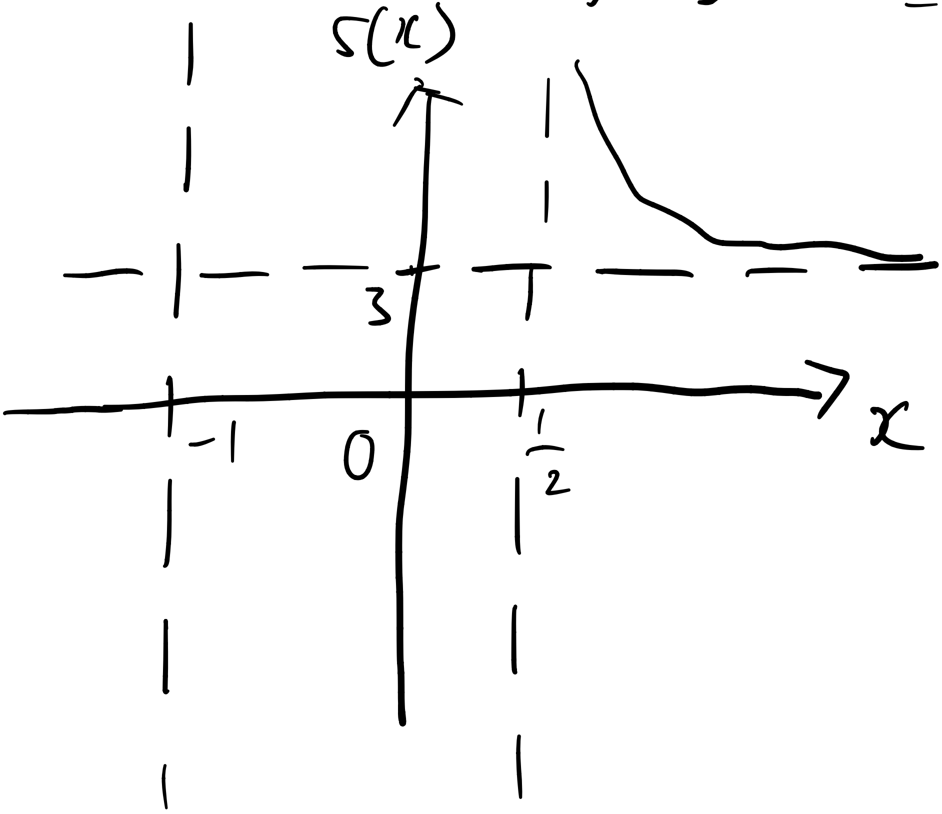
No vertical asymptote since $4x^2+1 > 0$ for all x

35. $s(x) = \dfrac{6x^2+1}{2x^2+x-1}$

$= \dfrac{6x^2+1}{(2x-1)(x+1)}$

Vertical asymptote: $x = \frac{1}{2}, -1$

Horizontal asymptote: as $x \to \infty$, $s(x) = 3$



$s\left(\frac{3}{8}\right) =$

$s(x) = \dfrac{6 + \frac{1}{x^2}}{2 + \frac{1}{x} - \frac{1}{x^2}}$

$s\left(\frac{3}{4}\right)$

$= \dfrac{6\left(\frac{9}{16}\right)+1}{2\left(\frac{9}{16}\right)+\left(\frac{3}{4}\right)-1}$

$= \dfrac{\frac{27}{8}+\frac{8}{8}}{\frac{7}{8}}$

$= 5$

$s\left(\frac{5}{8}\right)$

$= \dfrac{6\left(\frac{25}{64}\right)+\frac{32}{32}}{2\left(\frac{25}{64}\right)+\left(\frac{25}{64}\right)-\frac{64}{64}}$

$= \dfrac{\frac{107}{32}}{\frac{36}{64}}$

$= \dfrac{214}{36}$

$= \dfrac{107}{18}$

53. $r(x) = \dfrac{(x-1)(x+2)}{(x+1)(x-3)}$

Vertical Asymptote : $x = -1, 3$

$r(x) = \dfrac{x^2+x-2}{x^2-2x-3}$

Horizontal Asymptote : $y = 1$

x-intercepts : $x = -2, 1$

$x \to -1^+$

when $x = -0.9$,

$r(-0.9) = \dfrac{(-)(+)}{(+)(-)} = +$

$x \to -1^-$

$r(-1.1) = \dfrac{(-)(+)}{(-1)(-)} = -$

$x \to 3^+$

$r(3.1) = \dfrac{(+)(+)}{(+)(+)}$

$= +$

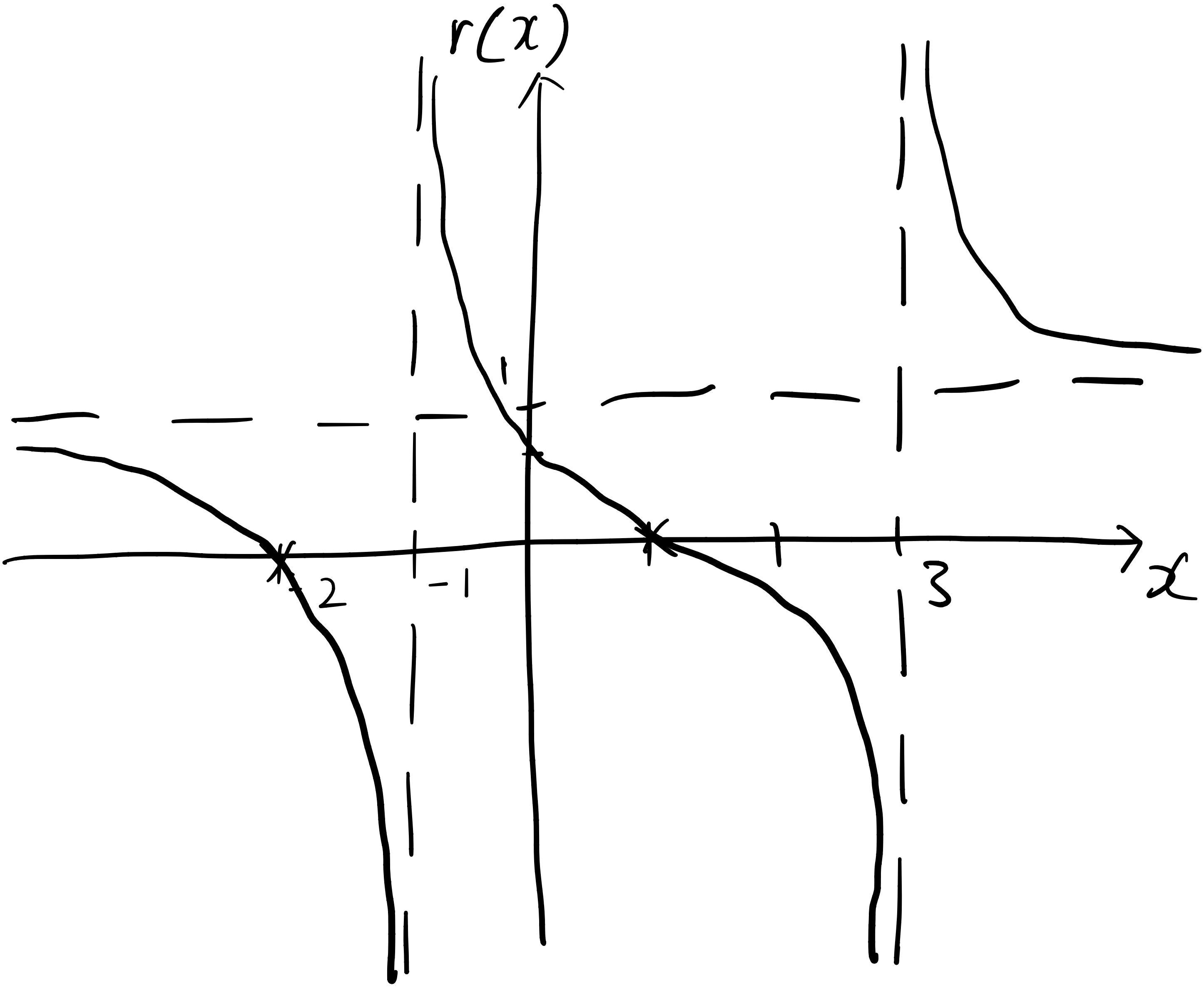
$x \to 3^-$

$r(2.9) = \dfrac{(+)(+)}{(+)(-)}$

$= -$

y-intercept : $r(0) = \dfrac{(-1)(2)}{(1)(-3)}$

$= \dfrac{2}{3}$



Domain : $\{x \mid x \neq -1, 3\}$

Range : $\{y \mid y \in R\}$

59. $r(x) = \dfrac{x^2 - 2x + 1}{x^2 + 2x + 1}$

$$= \frac{(x-1)^2}{(x+1)^2}$$

Vertical Asymptote: $x = -1$

Horizontal Asymptote: $y = \frac{1}{1} = 1$

y-interrcept: $r(0) = \dfrac{0 - 0 + 1}{0 + 0 + 1} = 1$

x-intercept: $x - 1 = 0$

$x = 1$

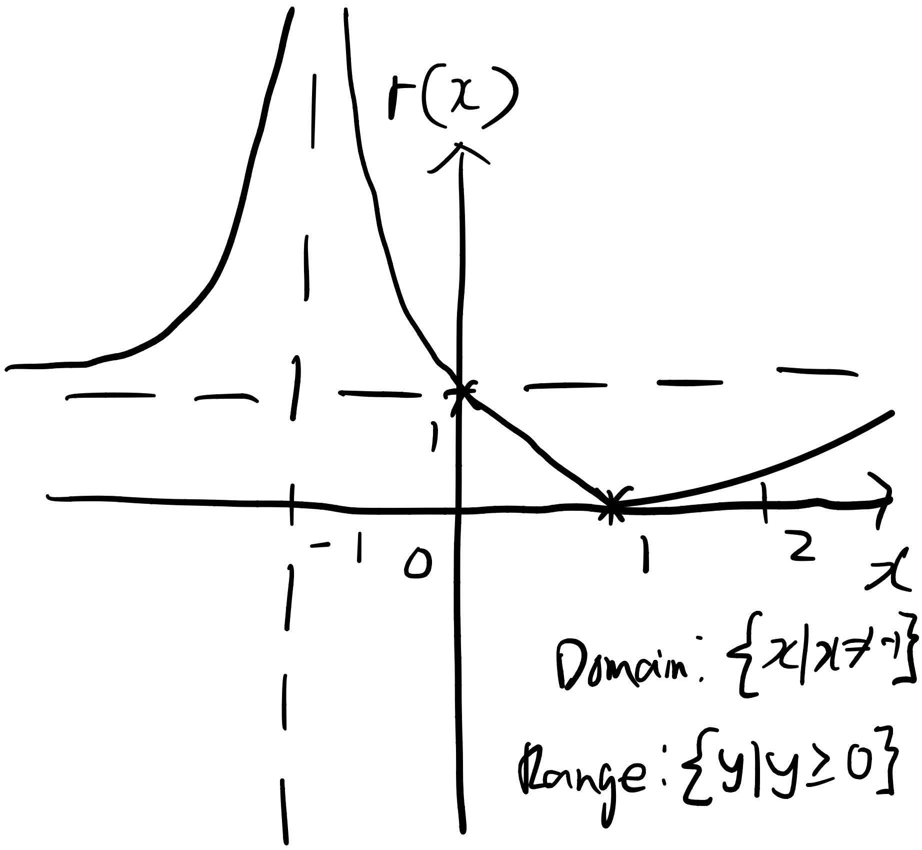
$x \to -1^-$

$r(-1.1) = +$

$x \to -1^+$

$r(-0.9) = +$

x	$r(x)$
2	1/9
3	1/4
4	9/25
1.2	0.008



Domain: $\{x \mid x \neq -1\}$

Range: $\{y \mid y \geq 0\}$

Common Factors in Numerator and Denominator

63. $r(x) = \dfrac{x^2+4x-5}{x^2+x-2}$

$= \dfrac{(x+5)(x-1)}{(x+2)(x-1)}$

$= \dfrac{x+5}{x+2} \quad x \neq 1$

Vertical asymptotes: $x = -2$

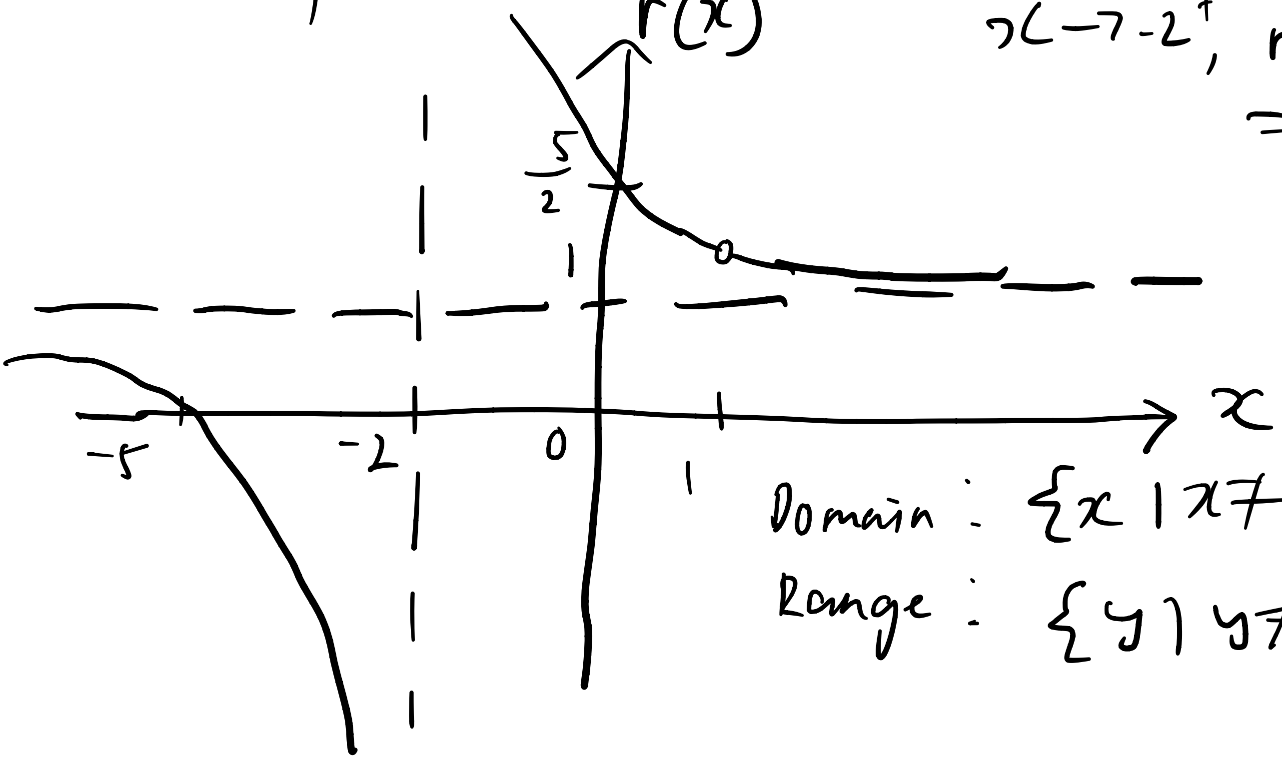
Horizontal asymptotes: $y = \frac{1}{1} = 1$

y-interrept: $y = \frac{5}{2}$

x-interrept: $x = -5$

$x \to -2^-$, $r(-2.1) = \dfrac{2.9}{-0.1}$

$x \to -2^+$, $r(-1.9) = \dfrac{3.1}{0.1}$



Domain: $\{x \mid x \neq -2, 1\}$

Range: $\{y \mid y \neq 1, 2\}$

① Graphing Rational Functions

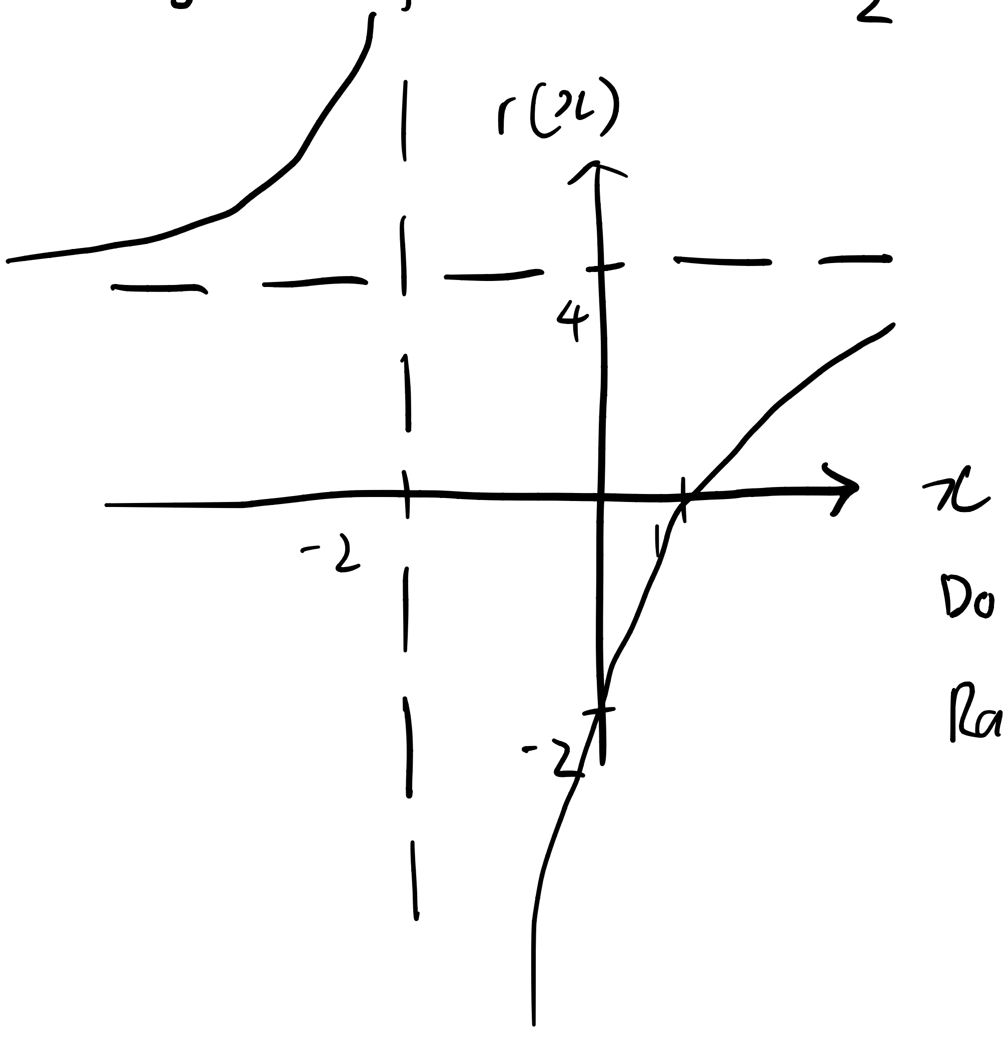
43. $r(x) = \dfrac{4x-4}{x+2}$

Vertical asymptote: $x = -2$

Horizontal asymptote: $y = \frac{4}{1} = 4$

x-intercept: $x = 1$

y-intercept: $y = \dfrac{-4}{2} = -2$



$x \to -2^{+}, y \to -\infty$

$x \to -2^{-}, y \to \infty$

Domain: $\{x \mid x \neq -2\}$

Range: $\{y \mid y \neq 4\}$

44. $r(x) = \dfrac{2x+6}{-6x+3}$

vertical asymptote: $x = \frac{1}{2}$

Horizontal asymptote: $y = -\frac{1}{3}$

$x \to \frac{1}{2}^-,\ y \to \infty$

$x \to \frac{1}{2}^+,\ y \to -\infty$

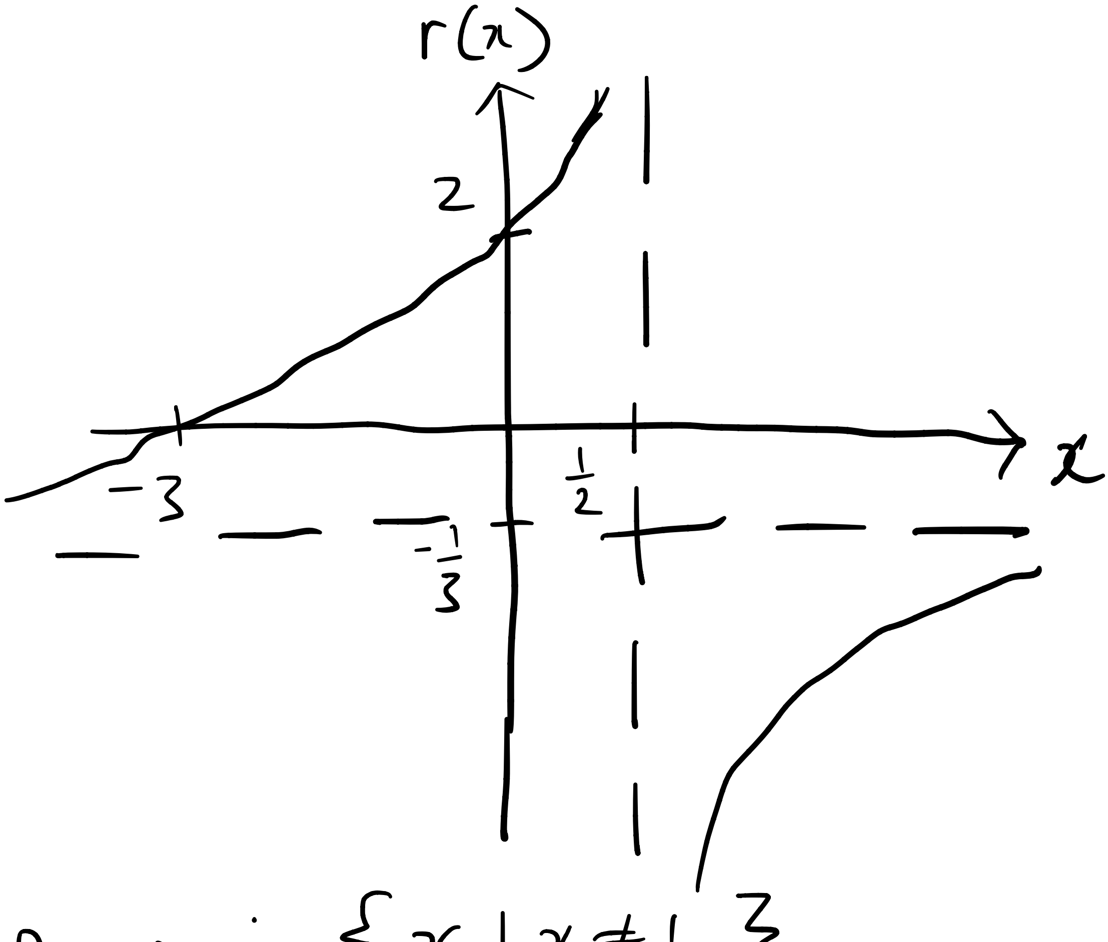
$x = \frac{3}{4},\ y = \dfrac{\frac{6}{4} + \frac{24}{4}}{-\frac{9}{2} + \frac{6}{2}}$

$= \frac{15}{2} \div -\frac{3}{2}$

$= -5$

x-intercept: $x = -3$

y-intercept: $y = 2$



Domain: $\{x \mid x \neq \frac{1}{2}\}$

Range: $\{y \mid y \neq -\frac{1}{3}\}$

45. $r(x) = \dfrac{3x^2-12x+13}{x^2-4x+4}$

$= \dfrac{3x^2-12x+13}{(x-2)^2}$

$x \to 2^-, y \to \infty$

$x \to 2^+, y \to \infty$

Vertical Asymptote : $x = 2$

Horizontal Asymptote : $y = \frac{3}{1} = 3$

x-intercept : $3x^2-12x+13 = 0$

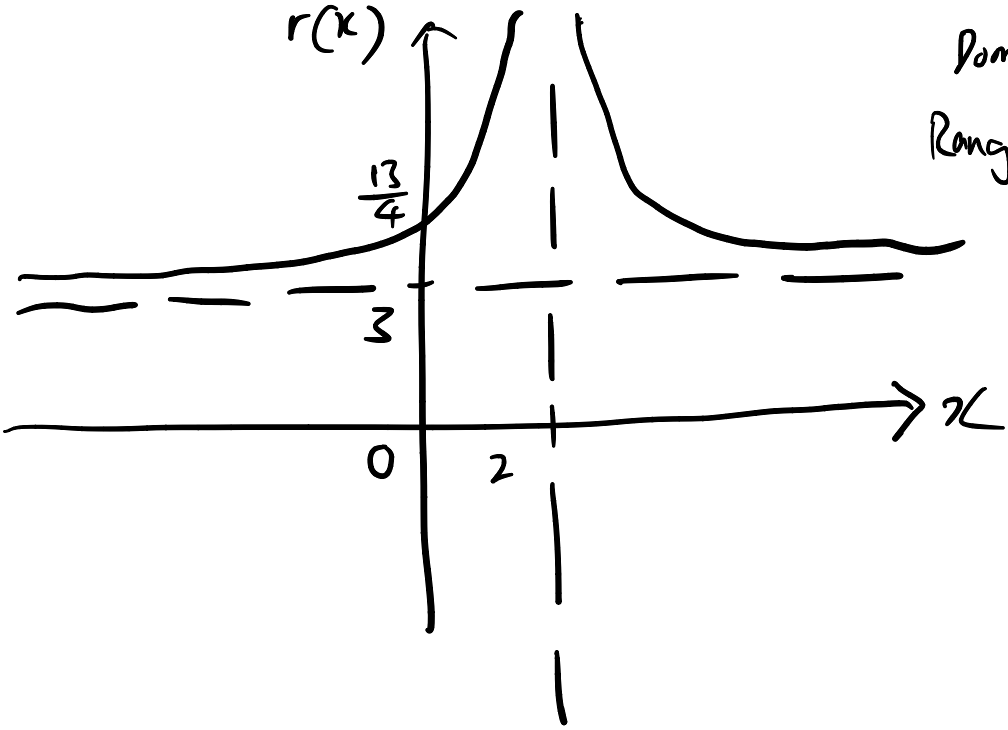
$$x = \frac{12 \pm \sqrt{144-156}}{6}$$

$$= \frac{12 \pm \sqrt{12 i^2}}{6}$$

$$= \frac{12 \pm 2\sqrt{3} i}{6}$$

$$= \frac{6 \pm \sqrt{3} i}{3}$$

y-intercept : $r(0) = \frac{13}{4}$



Domain: $\{x \mid x \neq 2\}$
Range: $\{y \mid y > 3\}$

46. $r(x) = \dfrac{-2x^2 - 8x - 9}{x^2 + 4x + 4}$

$= \dfrac{-2x^2 - 8x - 9}{(x+2)^2}$

As $x \to -2^-$, $y \to -\infty$
As $x \to -2^+$, $y \to \infty$

Vertical asymptote: $x = -2$

Horizontal asymptote: $y = \frac{-2}{1} = -2$

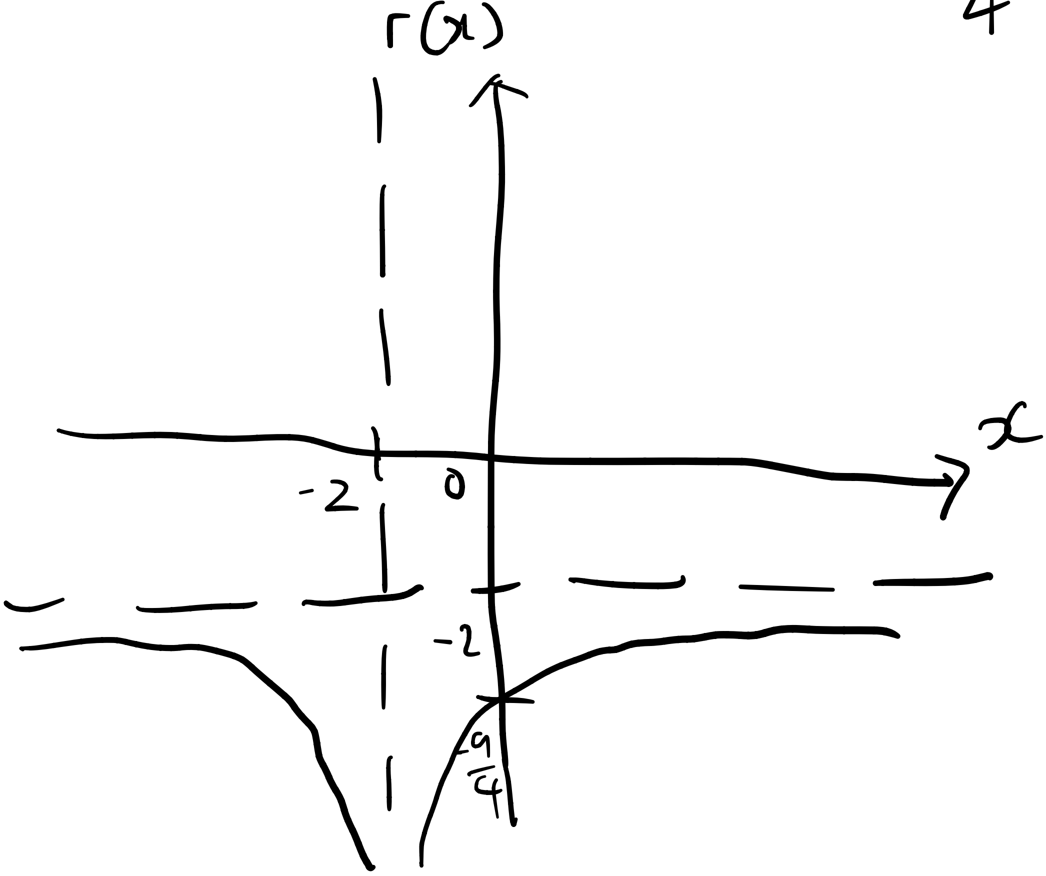
x-intercept: $-2x^2 - 8x - 9 = 0$

$x = \dfrac{8 \pm \sqrt{64 - 72}}{-4}$

$= \dfrac{-8 \pm 2\sqrt{2}i}{4}$

$= \dfrac{-4 \pm \sqrt{2}i}{2}$

y- intercept: $r(0) = \frac{-9}{4} = -\frac{9}{4}$



Domain: $\{x \mid x \neq -2\}$

Range: $\{y \mid y < -2\}$

47. $r(x) = \frac{-x^2+8x-18}{x^2-8x+16}$

$= \frac{-x^2+8x-18}{(x-4)^2}$

Vertical asymptote: $x = 4$

y- asymptote: $r(x) = \frac{-1+\frac{8}{x}-\frac{18}{x^2}}{1-\frac{8}{x}+\frac{16}{x^2}}$

as $x \to \infty$, $r(x) \to \frac{-1}{1} = -1$

as $x \to 4^-$, $r(x) \to \infty$

$x = 3.9$, $r(x) = 3399$

as $x \to 4^+$, $r(x) \to \infty$

$x = 4.1$, $r(x) = 3599$

$-x^2 + 8x - 18 = 0$

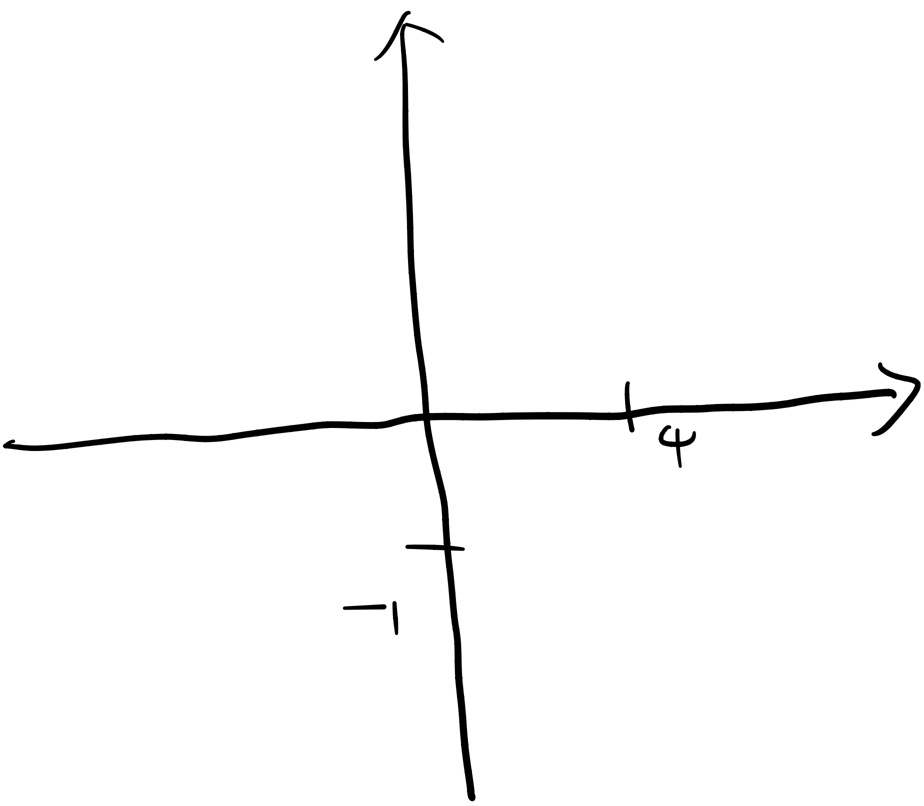
$x^2 - 8x + 18 = 0$

$x = \frac{8 \pm \sqrt{64 - 72}}{2}$

$= \frac{8 \pm \sqrt{-8}}{2}$

$= \frac{8 \pm 2\sqrt{2}i}{2}$

$= 4 \pm \sqrt{2}i$



3.7 Polynomial and Rational Inequalities

① Polynomial Inequality

② Rational Inequality

Polynomial Inequalities

7. $x^3+4x^2 \geq 4x+16$

$x^3+4x^2-4x-16 \geq 0$

Rational zero theorem: $\pm1, \pm2, \pm4, \pm8, \pm16$

1	1	4	−4	−16
		1	5	1
	1	5	1	−15

2	1	4	−4	−16
		2	12	16
	1	6	8	0

$f(x) = (x-2)(x^2+6x+8)$

$= (x-2)(x+4)(x+2)$

$(x-2)(x+2)(x+4) \geq 0$

$-$ $+$ $-$ $+$

-4 -2 2

$\therefore -4 \leq x \leq -2, \; x \geq 2$

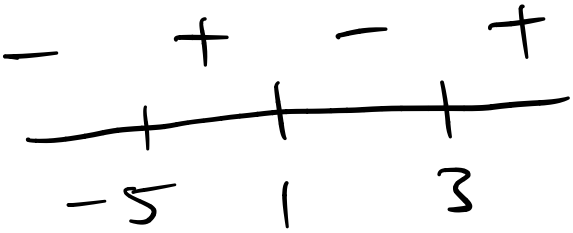
13. $x^3+x^2-17x+15 \geq 0$

RZT: $\pm1, \pm3, \pm5, \pm15$

$$\begin{array}{r|rrrr} 1 & 1 & 1 & -17 & 15 \\ & & 1 & 2 & -15 \\ \hline & 1 & 2 & -15 & 0 \end{array}$$

$$(x-1)(x^2+2x-15) \geq 0$$

$$(x-1)(x+5)(x-3) \geq 0$$



$$\therefore \ -5 \leq x \leq 1, \ x \geq 3$$

Rational Inequalities

27. $\dfrac{x-3}{2x+5} \geq 1$

$x-3 \geq 2x+5$

$-x-8 \geq 0$

$-x \geq 8$

$x \leq -8$

$\therefore -8 \leq x < \dfrac{5}{2}$

$\dfrac{x-3}{2x+5} - 1 \geq 0$

$\dfrac{x-3-(2x+5)}{2x+5} \geq 0$

$f(-10) = \dfrac{2}{-15}$

$\dfrac{-x-8}{2x+5} \geq 0$

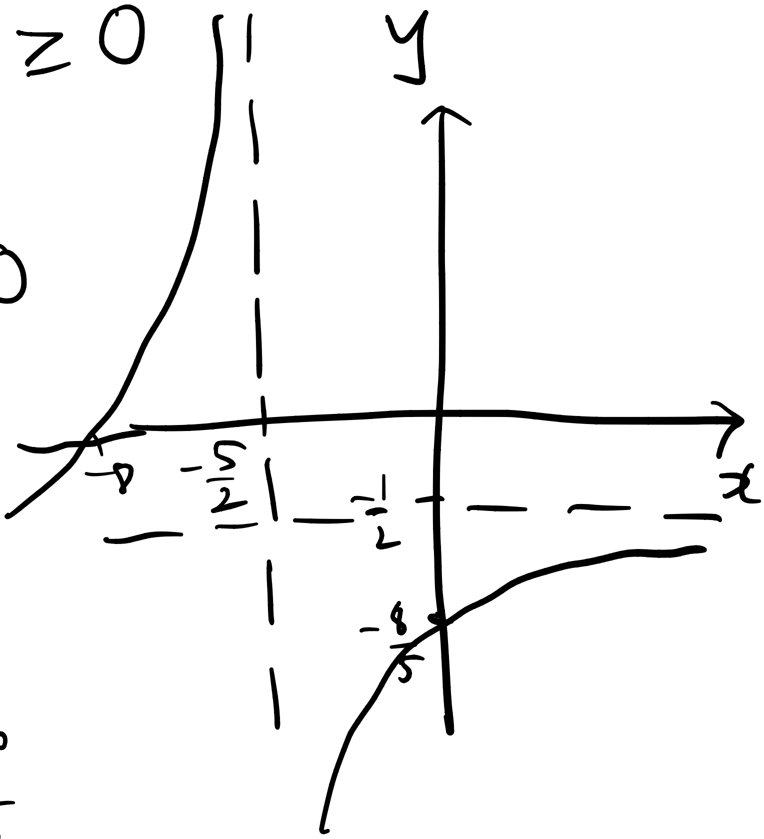
$f(10) = \dfrac{-18}{25}$

$=$

$\dfrac{-1-\frac{8}{x}}{2+\frac{5}{x}}$

$x \to -\frac{5}{2}^{+}, f(-2) = -6$

$x \to -\frac{5}{2}^{-}, f(-3) = 5$



23. $\dfrac{x^2+2x-3}{3x^2-7x-6} > 0$

Horizontal asymptote : $y = \frac{1}{3}$

$$\frac{(x+3)(x-1)}{(3x+2)(x-3)} > 0$$

$-3 \qquad -\frac{2}{3} \qquad 1 \qquad 3$

$+ \qquad - \qquad + \qquad - \qquad +$

$\therefore \ (-\infty, -3) \cup (-\frac{2}{3}, 1) \cup (3, \infty)$

24. $\dfrac{x-1}{x^3+1} \geq 0$

$+ \qquad - \qquad +$

$-1 \qquad 1$

$x^3+1=0$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & 0 & 1 \\ & & -1 & 1 & -1 \\ \hline & 1 & -1 & 1 & 0 \end{array}$$

$(x+1)(x^2-x+1)=0$

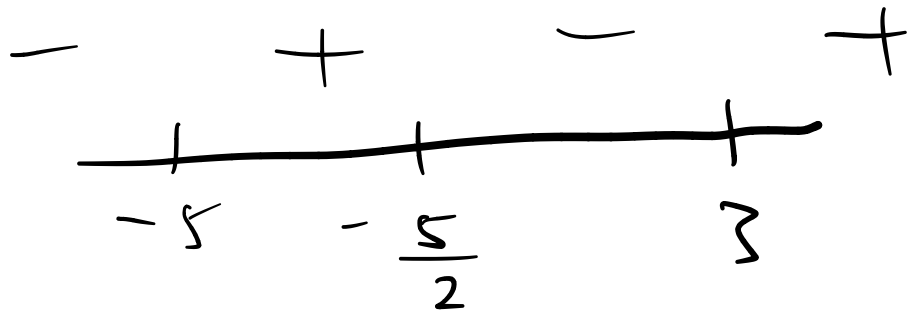
$x = \dfrac{1 \pm \sqrt{1-4}}{2}$

$= \dfrac{1 \pm \sqrt{3}i}{2}$

$\therefore (-\infty, -1) \cup [1, \infty)$

① Polynomial Inequalities

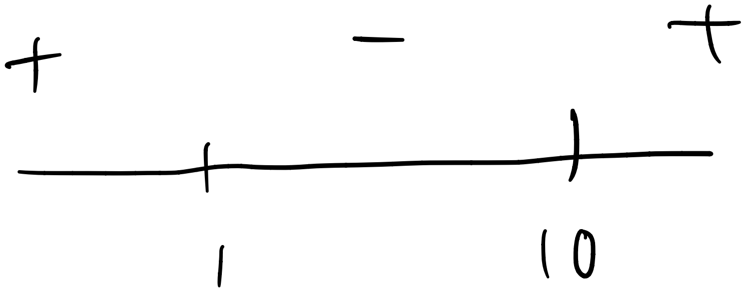
3. $(x-3)(x+5)(2x+5) < 0$



$\therefore \quad (-\infty, -5) \cup \left(-\frac{5}{2}, 3\right)$

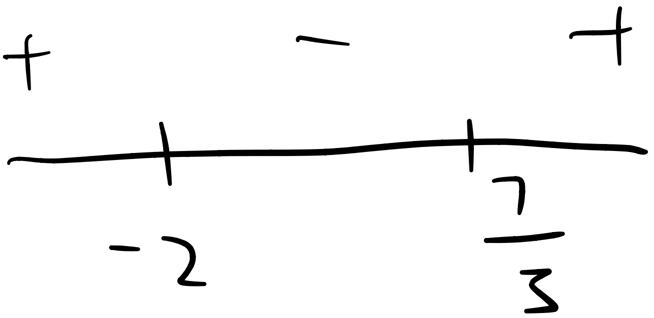
② Rational Inequalities

17. $\dfrac{x-1}{x-10} < 0$



$\therefore\ 1 < x < 10$ / $(1, 10)$

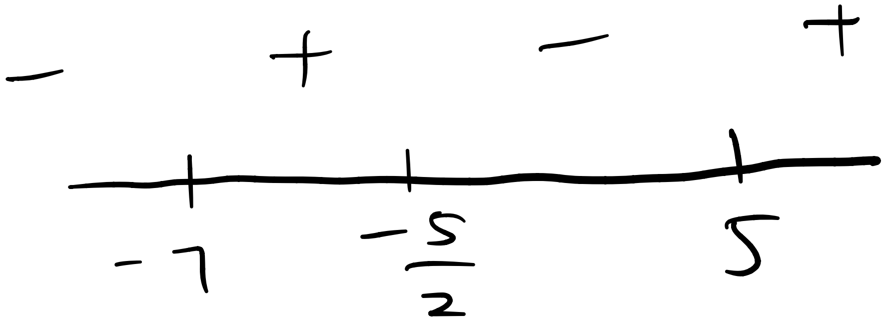
18. $\dfrac{3x-7}{x+2} \le 0$



$\therefore\ -2 < x \le \dfrac{7}{3}$ $\boxed{2}$ $\left(-2, \dfrac{7}{3}\right]$

19. $$\frac{2x+5}{x^2+2x-35} \geq 0$$

$$\frac{2x+5}{(x+7)(x-5)} \geq 0$$

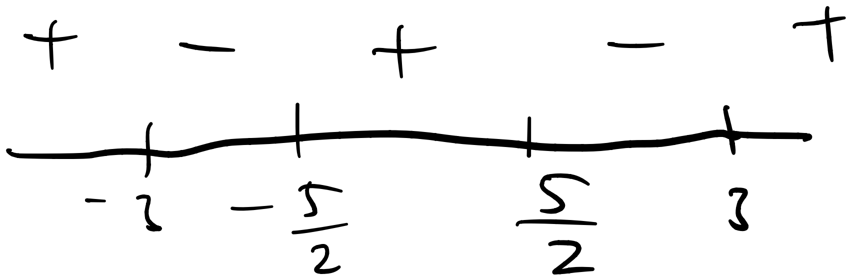


$\therefore$ $(-7, -\frac{5}{2}] \cup (5, \infty)$

20. $$\frac{4x^2-25}{x^2-9} \leq 0$$

$\therefore$ $(-3, -\frac{5}{2}] \cup [\frac{5}{2}, 3)$

$$\frac{(2x+5)(2x-5)}{(x+3)(x-3)} \leq 0$$



Domain of a Function 41 – 44

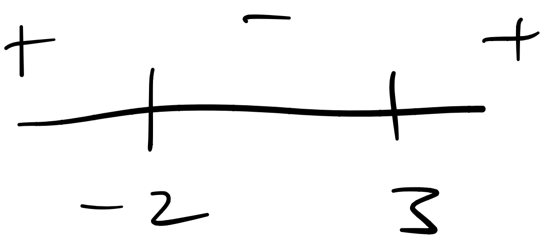
41. $f(x) = \sqrt{6 + x - x^2}$

square root has to be non-negative,

$$6 + x - x^2 \geq 0$$

$$x^2 - x - 6 \leq 0$$

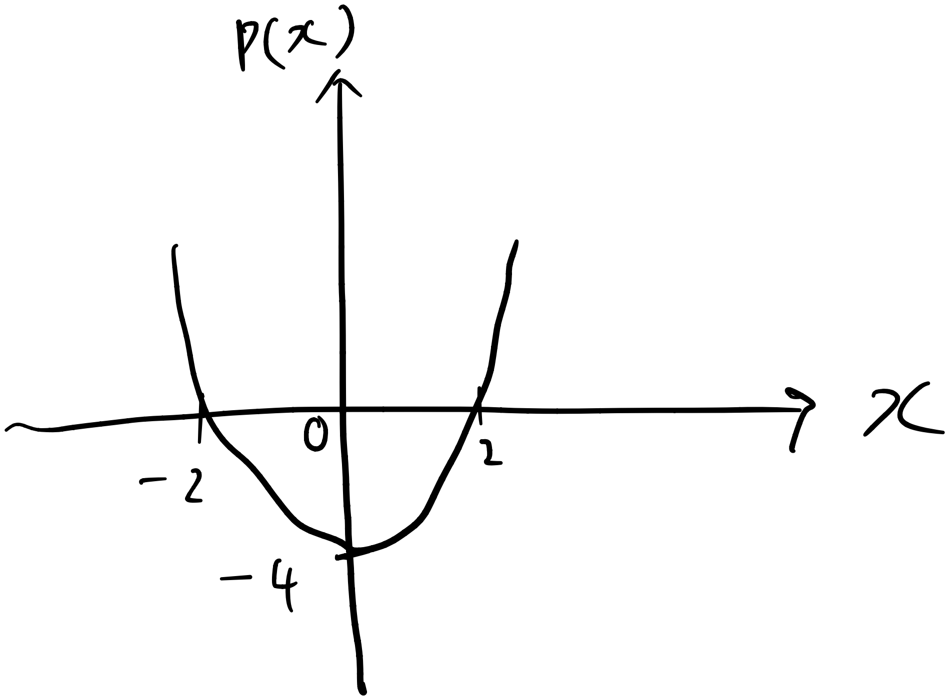
$$(x-3)(x+2) \leq 0$$



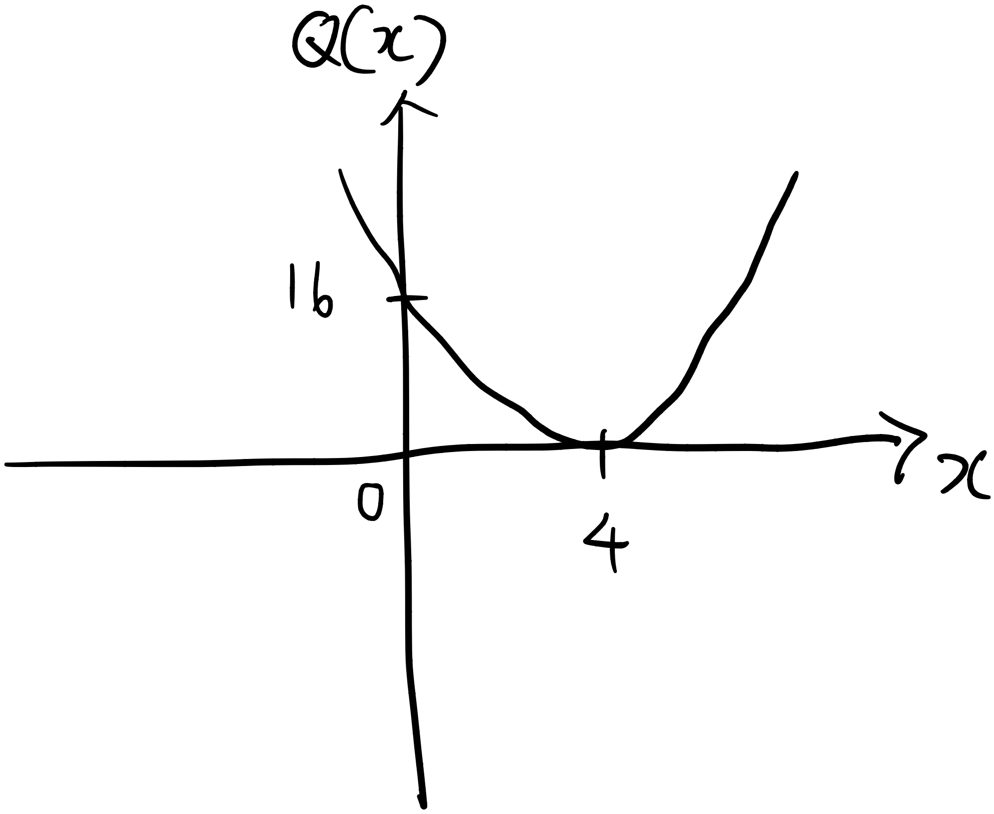
$\therefore$ Domain : $[-2, 3]$

3.2 Polynomial Functions and Their Graphs

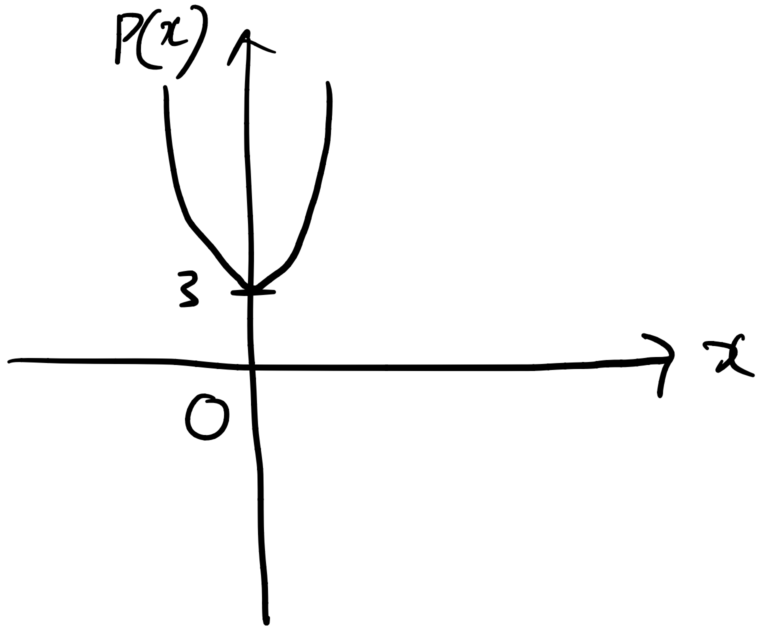
5. (a) $P(x) = x^2 - 4$



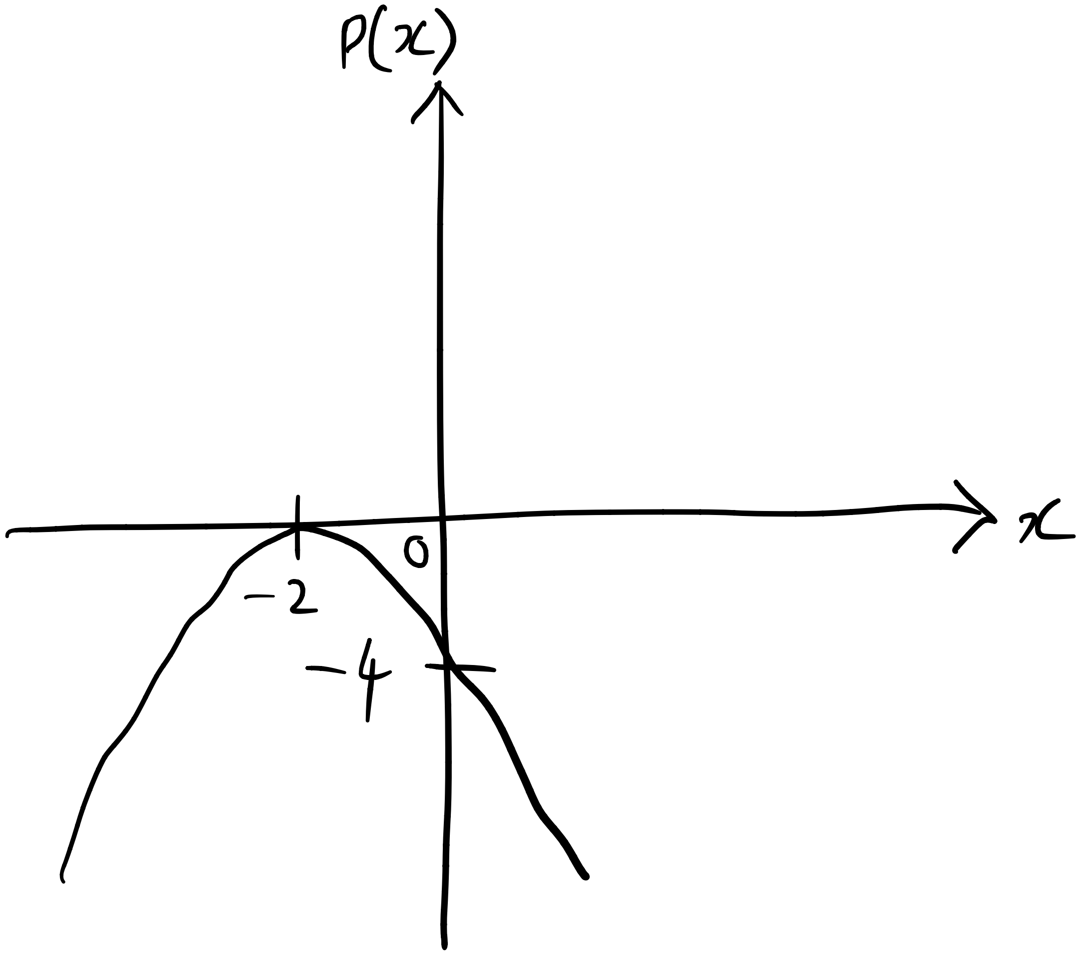
(b) $Q(x) = (x-4)^2$



(c) $P(x) = 2x^2 + 3$



(d) $P(x) = -(x+2)^2$



11. $R(x) = -x^5 + 5x^3 - 4x$

(a) $x \to -\infty,\ y \to \infty$

$x \to \infty,\ y \to -\infty$

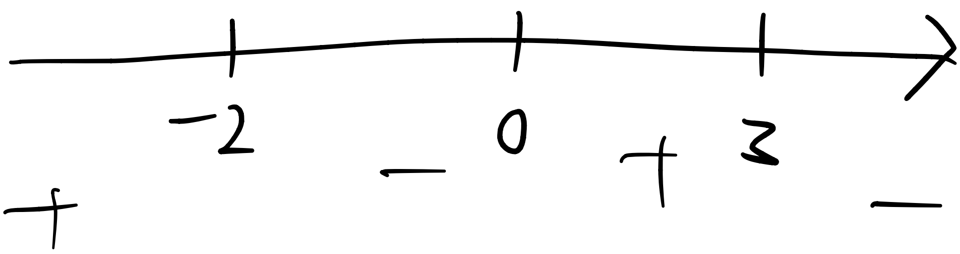
(b) IV

45. $P(x) = 3x^3 - x^2 + 5x + 1$; $Q(x) = 3x^3$

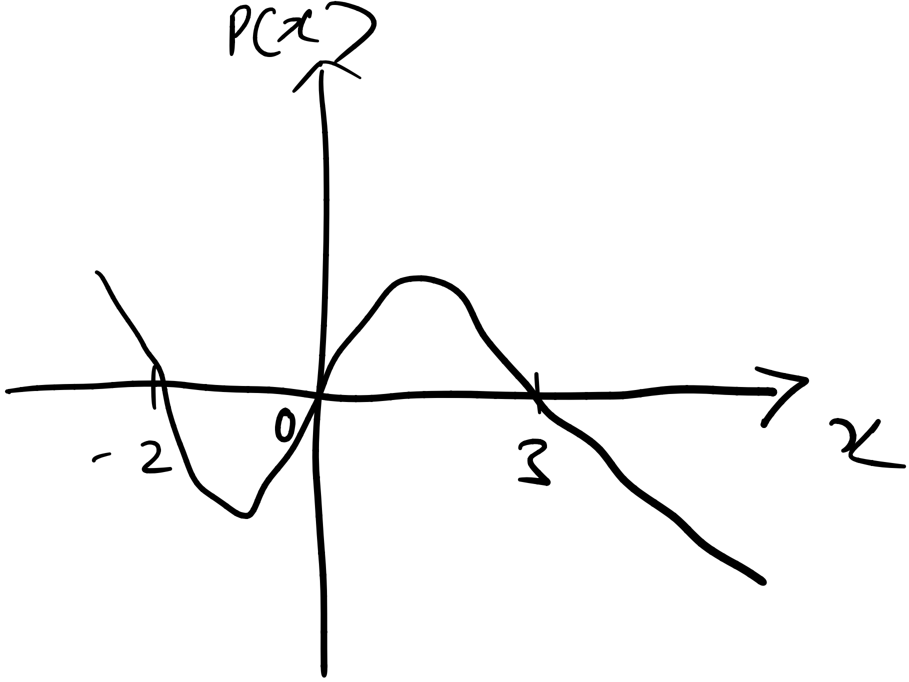
$$P(x) = 3x^3\left(1 - \frac{1}{3x} + \frac{5}{3x^2} + \frac{1}{3x^3}\right)$$

17. $P(x) = -x(x-3)(x+2)$

Zeros: $-2,\ 0,\ 3$



x	$P(x)$
-3	18
-1	-4
1	6
4	-24



65. $y = x^3 - x^2 - x$

$= x(x^2 - x - 1)$

67. $y = x^4 - 5x^2 + 4$

3.3 Dividing Polynomials

3. $P(x) = 2x^2 - 5x - 7$, $D(x) = x - 2$

$$
\begin{array}{r|l}
 & 2x - 1 \\
\hline
x - 2 & 2x^2 - 5x - 7 \\
 & 2x^2 - 4x \\
\hline
 & \quad -x - 7 \\
 & \quad -x + 2 \\
\hline
 & \qquad -9
\end{array}
$$

$$\frac{P(x)}{D(x)} = 2x - 1 - \frac{9}{x-2}$$

19. $\dfrac{x^3 + 2x + 1}{x^2 - x + 3}$

$$
\begin{array}{r|l}
 & \qquad\qquad x + 1 \\
\hline
x^2 - x + 3 & x^3 + 0x^2 + 2x + 1 \\
 & x^3 - x^2 + 3x \\
\hline
 & \qquad x^2 - x + 1 \\
 & \qquad x^2 - x + 3 \\
\hline
 & \qquad\qquad -2
\end{array}
$$

$$P(x) = D(x) \cdot Q(x) + R(x)$$
$$= (x^2 - x + 3)(x + 1) - 2$$

31. $\dfrac{x^3 - 8x + 2}{x + 3}$

$$\begin{array}{r|rrrr} -3 & 1 & 0 & -8 & 2 \\ & & -3 & 9 & -3 \\ \hline & 1 & -3 & 1 & -1 \end{array}$$

$$\frac{x^3 - 8x + 2}{x + 3} = (x+3)(x^2 - 3x + 1) - 1$$

39. $p(x) = 4x^2 + 12x + 5, \quad c = -1$

$$\begin{array}{r|rrr} -1 & 4 & 12 & 5 \\ & & -4 & -8 \\ \hline & 4 & 8 & -3 \end{array}$$

$$4x^2 + 12x + 5 = (x+1)(4x+8) - 3$$

$$\begin{aligned} p(-1) &= 4(-1)^2 + 12(-1) + 5 \\ &= 4 - 12 + 5 \\ &= -3 \end{aligned}$$

53. $P(x) = x^3 - 3x^2 + 3x - 1$, $c = 1$

$$
\begin{aligned}
P(1) &= 1^3 - 3(1)^2 + 3(1) - 1 \\
&= 1 - 3 + 3 - 1 \\
&= 0 \text{ (shown)}
\end{aligned}
$$

57. $P(x) = x^3 + 2x^2 - 9x - 18$, $c = -2$

$$
\begin{aligned}
P(-2) &= (-2)^3 + 2(-2)^2 - 9(-2) - 18 \\
&= -8 + 8 + 18 - 18 \\
&= 0
\end{aligned}
$$

$$
\begin{array}{r|rrrr}
-2 & 1 & 2 & -9 & -18 \\
 & & -2 & 0 & 18 \\
\hline
 & 1 & 0 & -9 & 0
\end{array}
$$

$$
\begin{aligned}
P(x) &= (x^2 - 9)(x - (-2)) \\
&= (x+2)(x+3)(x-3)
\end{aligned}
$$

63. Degree 3, zeros: $-1, 1, 3$

$$
\begin{aligned}
P(x) &= (x+1)(x-1)(x-3) \\
&= (x^2 - 1)(x - 3) \\
&= x^3 - x - 3x^2 + 3 = x^3 - 3x^2 - x + 3
\end{aligned}
$$

67. Degree 4, zeros $-2, 0, 1, 3$ coef. of x^3 is 4

$$P(x) = (x+2)(x)(x-1)(x-3)$$
$$= x(x-1)(x-3)(x+2)$$
$$= (x^2-x)(x^2-x-6)$$
$$= x^4-x^3-6x^2-x^3+x^2+6x$$
$$= x^4-2x^3-5x^2+6x$$

$$P(x) = -2x^4+4x^3+10x^2-12x$$

3.4 Real Zeros of Polynomials

15. $P(x) = x^3 + 2x^2 - 13x + 10$

$$P(2) = 8 + 8 - 26 + 10 = 0$$

$$P(5) = 125 + 50 - 65 + 10 = 170$$

$$P(10) = 1000 + 200 - 130 + 10 \neq 0$$

$$P(1) = 1 + 2 - 13 + 10 = 0$$

$$P(-1) = -1 + 2 + 13 + 10 \neq 0$$

$$P(-2) = -8 + 8 + 26 + 10 \neq 0$$

$$P(-5) = -125 + 50 + 65 + 10 = 0$$

$$P(-10) = -1000 + 200 + 130 + 10 \neq 0$$

$$P(x) = (x-1)(x-2)(x+5)$$

29. $p(x) = 4x^4 - 37x^2 + 9$

$p\left(\frac{3}{2}\right) \neq 0$

$p(3) = 324 - 333 + 9 = 0$

$$\begin{array}{r|rrrrr} -3 & 4 & 0 & -37 & 0 & 9 \\ \hline & & -12 & 36 & 3 & -9 \\ \hline & 4 & -12 & -1 & 3 & 0 \end{array}$$

$p(x) = (x-3)(4x^3 - 12x^2 - x + 3)$

$$\begin{array}{r|rrrr} 3 & 4 & -12 & -1 & 3 \\ \hline & & 12 & 0 & -3 \\ \hline & 4 & 0 & -1 & 0 \end{array}$$

$p(x) = (x-3)(x+3)(4x^2 - 1)$

$= (x-3)(x+3)(2x+1)(2x-1)$

45. $p(x) = 3x^3 + 5x^2 - 2x - 4$

$p(4) = 3(64) + 5(16) - 2(4) - 4$

$= 192 + 80 - 8 - 4$

$\neq 0$

$p(2) = 3(8) + 5(4) - 2(2) - 4$

$= 24 + 20 - 4 - 4 \neq 0$

$p(-2) = 3(-8) + 5(4) - 2(-2) - 4$

$= -24 + 20 + 4 - 4$

$\neq 0$

$p(-4) = 3(-64) + 5(16) + 8 - 4 \neq 0$

$p(1) = 3 + 5 - 2 - 4 \neq 0$

$p(-1) = -3 + 5 + 2 - 4 = 0$

$p\left(\frac{2}{3}\right) = 3\left(\frac{8}{27}\right) + 5\left(\frac{4}{9}\right) - 2\left(\frac{2}{3}\right) - 4$

$= \frac{8}{9} + \frac{20}{9} - \frac{4}{3} - \frac{36}{9}$

$= \frac{28}{9} - \frac{12}{9} - \frac{36}{9} \neq 0$

$p\left(-\frac{2}{3}\right) = 3\left(-\frac{8}{27}\right) + 5\left(\frac{4}{9}\right) - 2\left(-\frac{2}{3}\right) - 4$

$= -\frac{8}{9} + \frac{20}{9} + \frac{4}{3} - \frac{36}{9}$

$= \frac{12}{9} + \frac{12}{9} - \frac{36}{9}$

$$P\left(\frac{4}{3}\right) = 3\left(\frac{64}{27}\right) + 5\left(\frac{16}{9}\right) - 2\left(\frac{4}{3}\right) - 4$$

$$= \frac{64}{9} + \frac{80}{9} - \frac{24}{9} - \frac{36}{9}$$

$$\neq 0$$

$$P\left(-\frac{4}{3}\right) = 3\left(-\frac{64}{27}\right) + 5\left(\frac{16}{9}\right) - 2\left(-\frac{4}{3}\right) - 4$$

$$= -\frac{64}{9} + \frac{80}{9} + \frac{24}{9} - \frac{36}{9}$$

$$= \frac{16}{9} + \frac{24}{9} - \frac{36}{9} \neq 0$$

$$P\left(\frac{1}{3}\right) = 3\left(\frac{1}{27}\right) + 5\left(\frac{1}{9}\right) - 2\left(\frac{1}{3}\right) - 4$$

$$= \frac{3}{27} + \frac{15}{27} - \frac{18}{27} -$$

$$\neq 0$$

$$P\left(-\frac{1}{3}\right) = -\frac{3}{27} + \frac{15}{27} -$$

$$\neq 0$$

$$x = \frac{-2 \pm \sqrt{4+48}}{6}$$

$$= \frac{-2 \pm \sqrt{52}}{6}$$

$$= \frac{-2 \pm 2\sqrt{13}}{6}$$

$$= \frac{-1 \pm \sqrt{13}}{3}$$

$$\begin{array}{r|rrrr} -1 & 3 & 5 & -2 & -4 \\ & & -3 & -2 & 4 \\ \hline & 3 & 2 & -4 & 0 \end{array}$$

$$P(x) = (x+1)(3x^2 + 2x - 4)$$

$$\therefore -1, \frac{-1 \pm \sqrt{13}}{3}$$

63. $p(x) = x^3 - x^2 - x - 3$

1 positive real zero

$$p(-x) = (-x)^3 - (-x)^2 - (-x) - 3$$
$$= -x^3 - x^2 + x - 3$$

either 2 or 0 negative zeros

$\therefore$ 3 or 1 real zeros

69. $P(x) = 2x^3 + 5x^2 + x - 2$; $a = -3, b = 1$

$$\begin{array}{r|rrrr} -3 & 2 & 5 & 1 & -2 \\ & & -6 & 3 & -12 \\ \hline & 2 & -1 & 4 & -14 \end{array}$$

$\therefore$ alternate signs, lower bound

$$\begin{array}{r|rrrr} 1 & 2 & 5 & 1 & -2 \\ & & 2 & 7 & 8 \\ \hline & 2 & 7 & 8 & 6 \end{array}$$

$\therefore$ no negative entry, upper bound

81. $P(x) = 2x^4 + 3x^3 - 4x^2 - 3x + 2$

$$
\begin{array}{r|rrrrr}
2 & 2 & 3 & -4 & -3 & 2 \\
 & & 4 & 14 & 20 & 34 \\
\hline
 & 2 & 7 & 10 & 17 & 36
\end{array}
$$

$$
\begin{array}{r|rrrrr}
1 & 2 & 3 & -4 & -3 & 2 \\
 & & 2 & 5 & 1 & -2 \\
\hline
 & 2 & 5 & 1 & -2 & 0
\end{array}
$$

$$P(x) = (2x^3 + 5x^2 + x - 2)(x+1)$$

$$
\begin{array}{r|rrrr}
-1 & 2 & 5 & 1 & -2 \\
 & & -2 & -3 & 2 \\
\hline
 & 2 & 3 & -2 & 0
\end{array}
$$

$$P(x) = (x+1)(x-1)(2x^2 + 3x - 2)$$
$$= (x+1)(x-1)(2x-1)(x+2)$$

Quadratic Formula