**Separation of Variables**

**Quiz:** Separation of Variables.
What is the general solution to the ODE $dy/dx = 2y + 1$? (Use separation of variables.)

   Think about your answer and then look at the choices.

$$\frac{dy}{dx} = 2y + 1$$

$$\int \frac{1}{2y+1} \, dy = \int 1 \, dx$$

$$\frac{\ln|2y+1|}{2} = x + C_1$$

$$\ln|2y+1| = 2x + 2C_1$$

$$2y + 1 = e^{2x} \cdot e^{2C_1}$$

$$2y = C_2 e^{2x} - 1$$

$$y = C e^{2x} - \frac{1}{2}$$

**Is it Separable?**

**Quiz:** (Is it Separable?.)
Is $y' + xy = x$ separable?

Think about your answer and then look at the choices.

27/2/25

$$y' = x(1 - y)$$

$$\frac{1}{1-y} \, dy = x \, dx$$

$\therefore$ yes

# 18.03SC Practice Problems 1

## Introduction, natural growth and decay, review of logarithm

An African government is trying to come up with a good policy regarding the hunting of oryx. They are using the following model: the oryx population has a natural growth rate of $k$ years$^{-1}$, and there is assumed a constant harvesting rate of $a$ oryxes/year.



Figure 1: A picture of an *Oryx gazella*, popularly known as a Gemsbok. Inhabitant of southern Africa. One of the least threatened species of oryx.

**Tasks:**

**1.** Write down a model for the oryx population. [First step: choose symbols and units.]

**2.** Suppose $a = 0$. What is the *doubling time*?

**3.** Find the general solution of this equation.

**4.** Check that the proposed solution satisfies the ODE.

**5.** There is a constant solution. Find it.

Does the way the solution depends upon $k$ and $a$ make sense? That is, do the units come out right? Does it behave right when $a$ is large or small? When $k$ is large or small?

**6.** Graph the constant solution (equilibrium) and some others, and comment on what they signify. Why, in this case, do we say the constant solution is "unstable?"

Notice that for initial values less than the equlibrium, the solutions stop having meaning in terms of the real-world situation they are modeling when they become negative (of course, this is true for all initial values, but for somewhat different reasons). In these cases, predict the time $t_e$ at which oryxes will become extinct. For example, suppose that $x_0$, the population of oryxes at time $t = 0$, is half the equilibrium. For this initial condition what is $t_e$? Your answer should depend on $k$.

27/2/25

18.03SC Differential Equations
Fall 2011

1.  $\dfrac{dy}{dt} = ky - a$

2.  $\dfrac{dy}{dt} = ky$

$\displaystyle\int \frac{1}{y}\,dy = \int k\,dt$

$\ln y = kt + C$

$y = e^{kt+C}$

$y = e^{kt} \cdot e^{C}$

$\Rightarrow y = Ce^{kt}$

At time $t=0$, $y = C$

$2C = Ce^{kt}$

$e^{kt} = 2$

$kt = \ln 2$

$t = \dfrac{\ln 2}{k}$

3. $\dfrac{dy}{dt} = ky - a$

$$\int \dfrac{1}{ky-a}\, dy = \int 1 \, dt$$

$$\dfrac{1}{k} \cdot \ln|(ky-a)| = t + C$$

$$\ln|ky - a| = kt + C$$

$$ky - a = Ce^{kt}$$

$$y = \dfrac{Ce^{kt} + a}{k}$$

$$\Rightarrow y = Ce^{kt} + \dfrac{a}{k}$$

4. $\dfrac{dy}{dt} = ky - a$

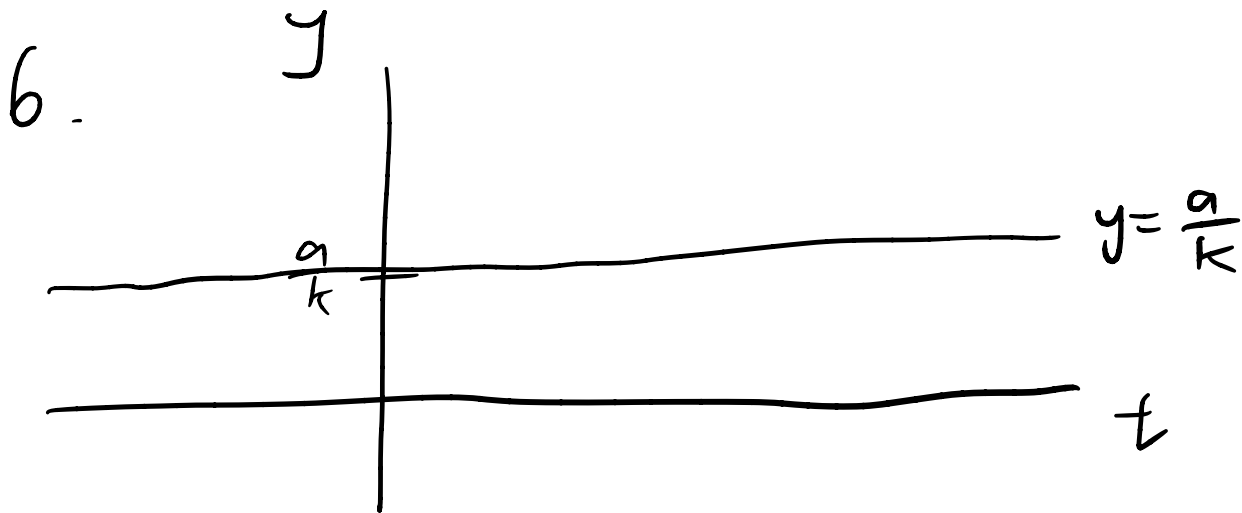LHS: $\dfrac{dy}{dt} = kCe^{kt}$

RHS: $kCe^{kt} + a - a$
$= kCe^{kt}$

$\therefore$ yes

5. $\dfrac{dy}{dt} = 0$, $ky - a = 0$

$$y = \dfrac{a}{k}$$

when $a$ is large, population $y$ is large. $\Rightarrow$ More oryx have to be harvested for growth to be constant

When $k$ is large, population $y$ is small. $\Rightarrow$ Growth $k$ is large, so population $y$ is smaller with constant harvest rate $a$.

6.



$y = \dfrac{a}{k}$

a. $\dfrac{dy}{dx} = y^2$ , $y(0) = 1$

$$\int \frac{1}{y^2} dy = \int 1 \, dx$$

$$\frac{y^{-1}}{-1} + C = x$$

$$-\frac{1}{y} = x - C$$

$$y = -\frac{1}{x - C}$$

$\therefore y = -\dfrac{1}{x - 1}$

$y(0) = 1$ , $1 = -\dfrac{1}{-C}$

$-C = -1$

$C = 1$

b.

$$\frac{dy}{dx} = y^2$$

$$\therefore \quad y = -\frac{1}{x - c}$$