$|\sin(b) - \sin(a)|$ **vs.** $|b - a|$

Use what we've learned about the mean value theorem to compare the values of $|\sin(b) - \sin(a)|$ and $|b - a|$.

$|\sin(b) - \sin(a)|$ **vs.** $|b - a|$

Use what we've learned about the mean value theorem to compare the values of $|\sin(b) - \sin(a)|$ and $|b - a|$.

5/8/25

Let $b > a$. $\Rightarrow$ $b - a > 0$

$f(x) = \sin x \Rightarrow f'(x) = \cos x$

$$\min_{a \leq x \leq b} f'(x) \leq \frac{f(b) - f(a)}{b - a} = f'(c) \leq \max_{a \leq x \leq b} f'(x)$$

$$\Rightarrow \quad -1 \quad \leq \quad f'(c) \quad \leq \quad 1$$

$$\Rightarrow \quad |f'(c)| \leq 1$$

$$\frac{|f(b) - f(a)|}{|b - a|} \leq 1$$

$$\therefore \quad |\sin(b) - \sin(a)| \leq |b - a|$$