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### Smoothing a Piecewise Polynomial

For each of the following, find all values of  $a$  and  $b$  for which  $f(x)$  is differentiable.

$$a) f(x) = \begin{cases} ax^2 + bx + 6, & x \leq 0; \\ 2x^5 + 3x^4 + 4x^2 + 5x + 6, & x > 0. \end{cases}$$

$$b) f(x) = \begin{cases} ax^2 + bx + 6, & x \leq 1; \\ 2x^5 + 3x^4 + 4x^2 + 5x + 6, & x > 1. \end{cases}$$

$$a) f_1(x) = ax^2 + bx + 6$$

$$\lim_{x \rightarrow 0^-} f_1(x) = \lim_{x \rightarrow 0^+} f_2(x)$$

$$\lim_{x \rightarrow 0^-} f_1'(x) = \lim_{x \rightarrow 0^+} f_2'(x)$$

$$2ax + b = 10x^4 + 12x^3 + 8x + 5$$

$$\text{At } x=0, \quad b=5 \quad \therefore a = \{x \mid x \text{ is a real number}\}$$

$$b=5$$

$$(b) \lim_{x \rightarrow 1^-} f_1(x) = \lim_{x \rightarrow 1^+} f_2(x)$$

$$a+b+6 = 2+3+4+5+6$$

$$a+b = 14$$

same for derivatives,

$$2a+b = 10+12+8+5$$

$$2a+b = 35$$

$$a=21, \quad b=-7$$