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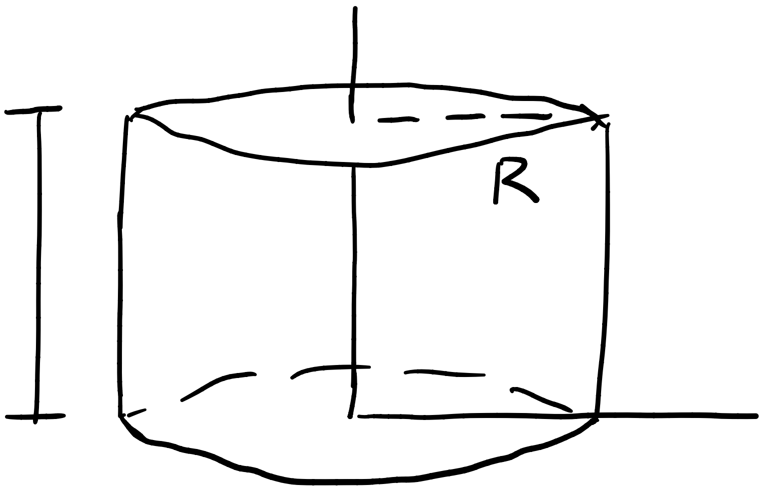

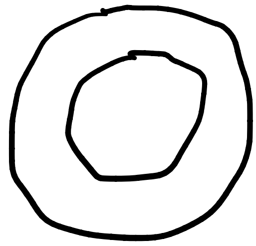

At point r, amount of chemical is $\frac{k}{1+r^2}$ g/m^3 $\times$ $\pi r^2 D$ m^3.

$$\Rightarrow y = \frac{\pi k D r}{1+r^2} \text{ g/m} \times r \text{ m}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi k D r_i}{1+r_i^2} \Delta r$$

$$= \pi k D \int_0^R \frac{r}{1+r^2} dr$$

$$= \frac{\pi k D}{2} \ln(1+R^2) \text{ grams}$$

Let $u = 1+r^2$. $du = 2r\,dr$.

$$\int_0^R \frac{r}{1+r^2} dr$$

$$= \int_0^R \frac{1}{2u} du$$

$$= \frac{1}{2} \ln|u| \Big|_0^R$$

$$= \frac{1}{2} \left(\ln|1+R^2| - \ln|1+0| \right)$$

$$= \frac{\ln(1+R^2)}{2}$$