

22/11/24

1. a) $A = \langle 1, 3 \rangle$, $B = \langle 3, 4 \rangle$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

$$3 + 12 = \sqrt{1^2 + 3^2} \sqrt{3^2 + 4^2} \cos \theta$$

$$\cos \theta = \frac{15}{\sqrt{10} 3\sqrt{5}}$$

$$= \frac{15}{15\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ$$

$$\begin{aligned} \text{(i)} \quad |\vec{A}| \cos \theta &= \sqrt{10} \left(\frac{\sqrt{2}}{2} \right) \\ &= \frac{2\sqrt{5}}{2} \\ &= \sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad |\vec{B}| \cos \theta &= 3\sqrt{5} \left(\frac{\sqrt{2}}{2} \right) \\ &= \frac{3}{2}\sqrt{10} \end{aligned}$$

$$\begin{aligned}
 b) \quad \cos \theta &= \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} \\
 &= \frac{9 + 20 + 0}{\sqrt{3^2 + 5^2 + 7^2} \sqrt{3^2 + 4^2}} \\
 &= \frac{29}{\sqrt{83} \cdot 5} \\
 &= 0.6366 \\
 \theta &= 50.46^\circ
 \end{aligned}$$

$$\begin{aligned}
 A &= \langle 3, 5, 7 \rangle \\
 B &= \langle 3, 4, 0 \rangle
 \end{aligned}$$

$$|\vec{A}| \cos \theta = 5.8$$

$$2. \quad A = \langle a, 2 \rangle, \quad B = \langle 1, 3 \rangle$$

$$|A| \cos \theta = 0 \quad \text{only when}$$

$$\cos \theta = 0, \quad \theta = 90^\circ$$

$$\vec{A} \cdot \vec{B} = 0$$

$$a + 6 = 0$$

$$\therefore a = -6$$

$$|A| \cos \theta < 0$$

$$\cos \theta < 0$$

$$\frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} < 0$$

$$\vec{A} \cdot \vec{B} < 0$$

$$a + 6 < 0$$

$$a < -6$$

$$3. \quad \theta = 90^\circ$$