Apply the partial fractions method to decompose

(a) $\dfrac{x^2-4x+4}{x^2-8x}$

(b) $\dfrac{x^2}{(x+1)^4}$

(c) $\dfrac{2x+2}{(4x^2+1)^2}$

(d) $\dfrac{x^3-3x^2+7x-1}{(x^2-1)^2}$

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(a) $x^2-8x \overline{)\,x^2-4x+4}$ $\quad 1$

$\qquad\qquad\quad x^2-8x$

$\qquad\qquad\quad 4x+4$

$$\frac{x^2-4x+4}{x^2-8x} = 1+\frac{4x+4}{x^2-8x}$$

$$= 1+\frac{4x+4}{x(x-8)}$$

$$= 1+\frac{A}{x}+\frac{B}{x-8}$$

$$\frac{4(0)+4}{0-8} = A \Rightarrow A = -\frac{1}{2}$$

$$\frac{4(8)+4}{8} = B \Rightarrow B = \frac{9}{2}$$

$$\therefore \frac{x^2-4x+4}{x^2-8x} = 1+\frac{36}{x}+\frac{4}{x-8}$$

(b) $\dfrac{x^2}{(x+1)^4} = \dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}+\dfrac{C}{(x+1)^3}+\dfrac{D}{(x+1)^4}$

$$x^2 = A(x+1)^3+B(x+1)^2+C(x+1)+D$$

$x=-1 \Rightarrow D=1$

$x=0 \Rightarrow 0 = A+B+C+1$

$x=1 \Rightarrow 1 = 8A+4B+2C+1$

$x=2 \Rightarrow 4 = 27A+9B+3C+1$

$\Rightarrow 1 = 6A+2B-1$

$6A+2B=2$

$4 = 24A+6B-2$

$4A+B=1$

$\Rightarrow 2A = 0$

$A=0$

$\Rightarrow B=1$

$\Rightarrow 0 = 0+1+C+1$

$C=-2$

$$\therefore \frac{x^2}{(x+1)^4} = \frac{1}{(x+1)^2} - \frac{2}{(x+1)^3} + \frac{1}{(x+1)^4}$$

(c) $$\frac{2x+2}{(4x^2+1)^2} = \frac{Ax+B}{4x^2+1} + \frac{Cx+D}{(4x^2+1)^2}$$

$\frac{2x+2}{(4x^2+1)^2}$ is already in decomposed form

$$\frac{(-1)^3-3(-1)^2+7(-1)-1}{(-2)^2} = B \Rightarrow B = \frac{-1-3-7-1}{4} = -3$$

(d) $$\frac{x^3-3x^2+7x-1}{(x^2-1)^2}$$

$$= \frac{x^3-3x^2+7x-1}{(x+1)^2(x-1)^2}$$

$$= \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$$

$$= \frac{1}{x+1} - \frac{3}{(x+1)^2} + \frac{1}{(x-1)^2}$$

$$\frac{1-3+7-1}{2^2} = D \Rightarrow D = 1$$

$$x^3-3x^2+7x-1 = A(x+1)(x-1)^2 + B(x-1)^2 + C(x-1)(x+1)^2 + D(x+1)^2$$

$x=0$,

$A+B-C+D=-1$

$A-3-C+1=-1 \Rightarrow A-C=1$

$A-C=1$ $\quad A+C=1$

$Ax^3+Cx^3=x^3 \Rightarrow 2A=2$

$A=1$

$\Rightarrow C=0$