1. Summary of net and gross premiums

Def 1.1 (Age of death)

Define X, a random variable, to be the age of death of a newborn. X is assumed to be a continuous, non-negative random variable.
We have:

- $F_X(x) = P(X \leq x)$,
- $S(x) = 1 - F_X(x) = P(X \geq x)$ - survival function, probability that a newborn will survive to x,
- $f_x(x) = F'_X(x)$
  Let us also define (x) to be a life aged x, usually meant as a person aged x years.

Def 1.2 (Future lifetime)

Define $T(x)$ to be the future lifetime of (x), the amount of time that a person aged x will live starting now. $T(x)$ is also a continuous, non-negative random variable, similar to X.
We have:

- $G(t) = G_{T(x)}(t) = P(T(x) \leq t)$

Def 1.3 (Probability symbols)

Define the following:

- ${}_tp_x = P(T(x) \geq t) = P(X - x \geq t | X \geq x)$
- ${}_tq_x = 1 -_t p_x = G(t)$
- $K(x)$ - curtate future lifetime of (x), $k(x) = \lfloor T(x) \rfloor$,
- ${}_{t|u}q_x = P(t \leq T(x) \leq t + u) =_{t+u} q_x -_t q_x$ - probability that (x) will survive t years, and die within the following u years.
  Let us also define a convention regarding p and q functions:
- ${}_1p_x = p_x$,
- ${}_1q_x = q_x$.

Def 1.4 (Force of mortality)

Define μ_x to be the force of mortality at age x. $\mu_x = -\frac{S'(x)}{S(x)}, \mu_x \geq 0$.

Theorem 1.1 (Relationships)

The following equalities are true:

- $F(x) = \int_0^{\infty} f(s)ds = 1 - s(x) = 1 - \exp(-\int_0^x \mu_s ds)$,
- $f(x) = F'(x) = -S'(x) = \mu_x \exp(-\int_0^x \mu_s ds)$,
- $S(x) = 1 - F(x) = 1 - \int_0^{\infty} f(s)ds = \exp(-\int_0^x \mu_s ds)$,
- $\mu_x = \frac{F'(x)}{1-F(x)} = \frac{f(x)}{\int_x^{\infty} f(s)ds} = -\frac{S'(x)}{S(x)}$.

Def 1.5 (UDD)

The uniform distribution of deaths (UDD) assumption assumes the following:

$$S(x+1) = (1-t)S(x) + tS(x+1), 0 \leq t \leq 1.$$

It also implies that

$$_tq_x = tq_x, 0 \leq t \leq 1,$$

and that $K(x)$ and $S(x)$ are independent.

Def 1.6 (Life insurance products)

Typical life insurance products:

1. Whole life insurance,
2. n-year term insurance,
3. n-year pure endearment,
4. n - year endearment,
5. annuities,
6. unit - united life insurance.
   We have:

- $T = T(x)$ - insurances payable at the moment of death,
- $K = K(x)$ - insurances payable at the end of year of death.

Def 1.7 (Net single premium)

Let A_x, and $\overline{A}_x$ indicate the life insurance benefit of 1 payable at the end of the year of death, and at the time of death, respectively. These two cases correspond to discrete, and continuous models. Let b_t be the benefit function, v_t be the discount function. Let $Z = Z_t = b_t v_t$.
Then the net single premium is defined as

$$NSP = E[Z] = \overline{A}_x = \int_0^{\infty} v_t g(t)dt.$$

Usually we take $b_t = 1$, and $v_t = v^t$, with a fixed v. Then the NSP takes the following form

$$NSP = \int_0^\infty v^t {}_tp_x \mu_{x+t} dt,$$

with $v = e^{-\delta}$.
In the discrete case, substitute $t = k + 1$, and $v = \frac{1}{1+i}$, then we have

$$NSP = \sum_{k=0}^\infty v^{k+1} {}_kp_x q_{x+k}.$$

Now let's explicitly define the life insurance benefits:

$$\begin{aligned} \overline{A}_x &= \int_0^\infty v^t {}_tp_x \mu_{x+t} dt, \\ \overline{A}_{x:\overline{n}|} &= \int_0^n v^t {}_tp_x \mu_{x+t} dt \end{aligned}$$

Def 1.8 (Life annuity)

A life annuity is a series of periodic or continuous payments provided that the insured is alive.
We have Y - present value of future whole life annuity

$$\begin{aligned} Y &= \int_0^T v^t dt = \bar{a}_{\overline{T}|}, \\ v &= e^{-\delta}, \\ \bar{a}_{\overline{T}|} &= \frac{1 - v^T}{\delta}. \end{aligned}$$