Phase-only matched filtering

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From image processing work, we know that the phase information is significantly more important than amplitude information in preserving the features of a visual scene. Is the same true in the case of a matched filter? From previous work [J. L. Horner, Appl. Opt. 21, 4511 (1982)], we know that a pure phase correlation filter can have an optical efficiency of 100% in an optical correlation system. We examine this relationship between phase and amplitude in the case of alphanumeric characters, with and without noise, using a computer simulation. We compare the phase-only and amplitude-only filters to the classical matched filter using the criteria of discrimination, correlation peak, and optical efficiency. Three-dimensional plots of the autocorrelation and cross-correlation functions are presented and discussed.

I. Introduction

In a recent review paper, Oppenheim and Lim[1] examined the relative role played by phase and amplitude in the Fourier domain in the transmission of a continuous tone picture. The general finding was that the phase information is considerably more important than the amplitude information in preserving the visual intelligibility of the picture. Even more interesting, in a bizarre sort of way, was that when the phase information of one picture was combined with the amplitude information from an ensemble average of a group of unrelated pictures, the reconstructed picture was almost identical to the original picture! Similar observations concerning the importance of the phase can be made about the Kinoform,[2] which is a phase-only hologram, and about the reconstruction of atomic structure from x-ray diffraction data. Fourier synthesis of the structure from only the amplitude of the diffraction with zero phase does not reconstruct the correct atomic arrangement, whereas reconstruction from the phase data with unity amplitude does. Levi and Stark's work[3] in image restoration using only the phase information further substantiates the importance of phase.

II. Application to Matched Filtering

In a recent investigation by one of us (JLH),[4] the result was derived that the maximum optical efficency of the classical matched filter consisting of a 2-D rect function was only 44.4%. This was assuming that the filter was recorded on a medium theoretically capable of 100% diffraction efficiency, such as dichromated gelatin (DCG), in a VanderLugt-type[5] optical correlator. By optical efficiency, or Horner efficiency[6] η_H, we mean the ratio of the energy in the correlation spot to the total input signal energy. Mathematically, it is given as

$$\eta_H = \eta \cdot \frac{\int\int |f(x,y) \star g^*(x,y)|^2 dxdy}{\int\int |f(x,y)|^2 dxdy}, \qquad (1)$$

where $f(x,y)$ is any input function in real space, $g(x,y)$ is any filter function, $*$ means a complex conjugate, $\star$ means complex correlation and η_M is the medium's efficiency, which we take to be unity in this analysis. If $g(x,y) = f(x,y)$, we are, of course, performing a complex autocorrelation.

However, another idea occurred to us as a result of the above investigations. Because a phase object, such as a prism or a lens, does not consume energy when placed in a light path but merely changes the direction of the beam, much higher optical efficiencies might be achieved through the use of a phase-only filter. Why is a high optical efficiency important? As a case in point, consider that in a spaceborne or tactical application laser size and power are limited. Typically, the laser source could be an IR diode putting out ~10 mW. With a low Horner efficiency, there would consequently be very little power reaching the correlation spot. With a high η_H, however, this deficiency would be remedied.

Other investigators[7-9] have proposed composite matched filters consisting of several different sizes and orientations of a target on a single filter to cope with the scaling-rotation dependency of the matched filter. But it has been estimated[6] that η_H for these types of filter could be as low as 10^{-6}.

In this paper we analyze the phase-only filter and address such questions as: Does this filter produce a reasonably sharp correlation peak? Does it discriminate as well as a classical matched filter against targets (and noise) differing only slightly from the object recorded on the filter? How does its performance com-

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pare with an amplitude-only filter or with the classical matched filter? We answer these questions by means of computer simulation.

III. Experiment

The VanderLugt optical correlator is the ideal system in which to use a phase-only filter. This is because the filter is recorded or inserted in the Fourier transform plane of the lens. The filter can be either holographically recorded as the transform of a real object or fabricated as a computer-generated hologram. In either case, the phase and amplitude components can be manipulated separately.

The VanderLugt correlator makes use of the so-called Wiener-Khinchine theorem to perform its correlation:

$$R_{ij} = \mathcal{F}^{-1}[F_i(\omega) \cdot F_i^*(\omega)] \tag{2}$$

with

$$F_i(\omega) = \mathcal{F}[f_i(x)], \tag{3}$$

where R_{ij} is the correlation function, and $\mathcal{F}$ is the Fourier transform operator. In the plots to follow we square Eq. (2) because in the VanderLugt correlator R_{ij} is proportional to light amplitude, and a physical detector responds to energy, which is $|R_{ij}|^2$.

The functions $f(x)$ and $F(\omega)$ are in general complex with F being represented as

$$F(\omega) = A(\omega) \exp[i\phi(\omega)]. \tag{4}$$

We define the phase-only filter to be

$$F_\phi(\omega) = \exp[i\phi(\omega)] \tag{5}$$

obtained by setting $A(\omega)$ equal to unity. We will also examine the amplitude-only filter, which is

$$F_A(\omega) = A(\omega). \tag{6}$$

To study the corrrelation properties of the filter we used a 90- × 90-point 2-D fast Fourier transform (FFT). The results were plotted using a 3-D plotting subroutine.[10] We chose the capital letters G and O to test these filters because they are fairly similar and would give an indication of each filter's discrimination ability. Figure 1 shows the actual characters. The normalization factors applied to the data were such that a VanderLugt correlator with a perfect film (linear with 100% diffraction efficiency) was simulated.

IV. Computer Simulation Results

To evaluate the hybrid matched filters proposed above, six separate correlation tests were performed, both with noise added and without. In all the tests the object recorded on the filter was the capital letter G. Figures 2–4 pertain to cases for noiseless input. They show the autocorrelations in the output plane of the input letter G with the filter G for the classical matched filter, the phase-only filter, and the amplitude-only filter, respectively. The other three tests giving the

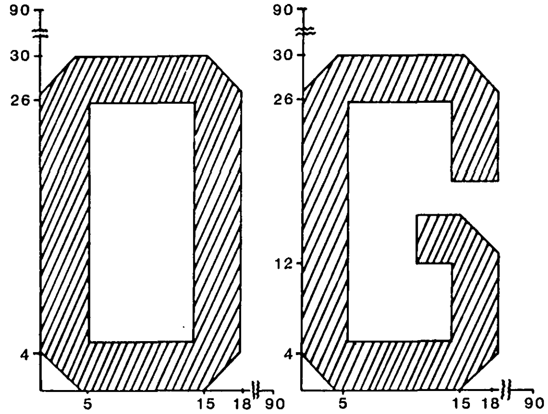

Fig. 1. Letters O and G used in correlation experiment. Numbers refer to points on the 90 × 90 input plane of the FFT. The letter O contains 356 points of unit height; the letter G 355 points.

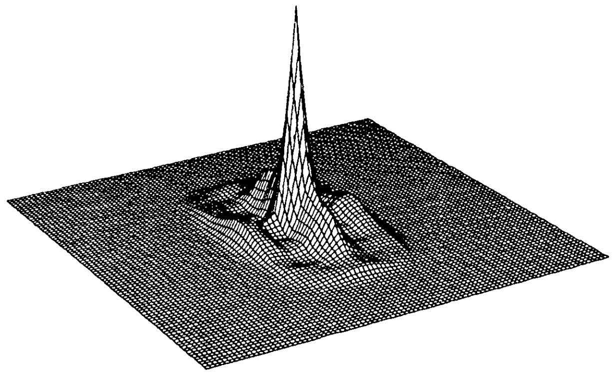

Fig. 2. Autocorrelation $|g \star g^*|^2$ using classical matched filter (full phase and amplitude).

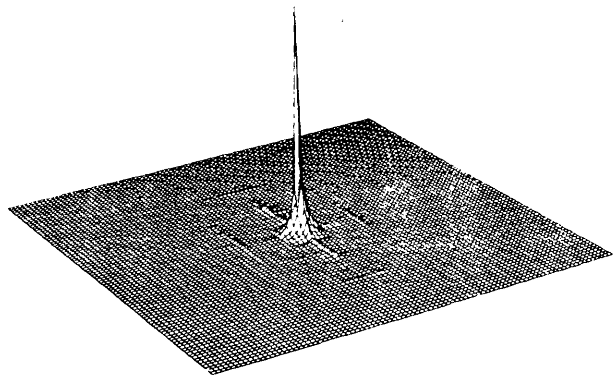

Fig. 3. Autocorrelation with phase-only filter $|g \star g_\phi^*|^2$.

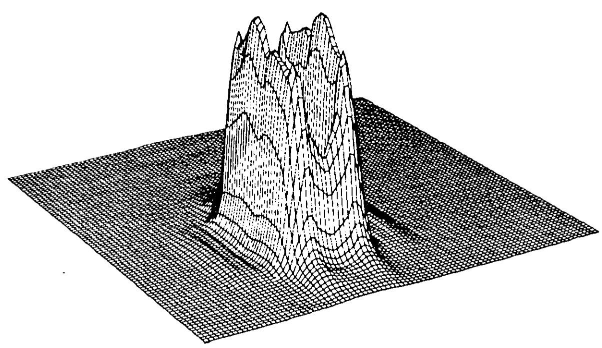

Fig. 4. Autocorrelation with amplitude-only filter $|g \star g_A^*|^2$.

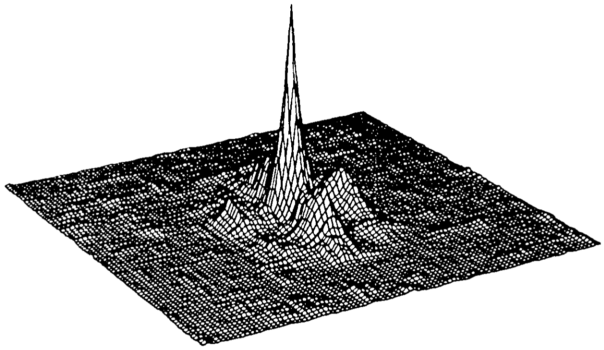

Fig. 5. Autocorrelation $|g \star g^{*}|^2$, SNR = 1, classical matched filter.

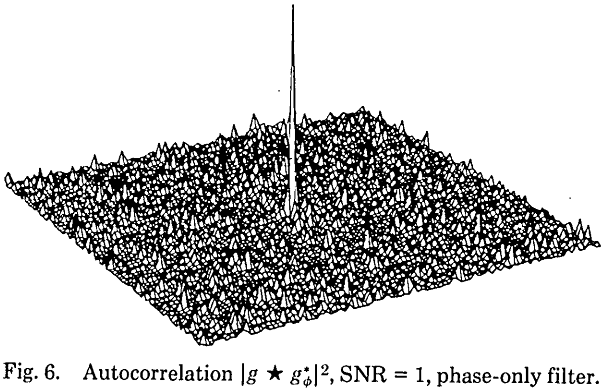

Fig. 6. Autocorrelation $|g \star g_{\phi}^{*}|^2$, SNR = 1, phase-only filter.

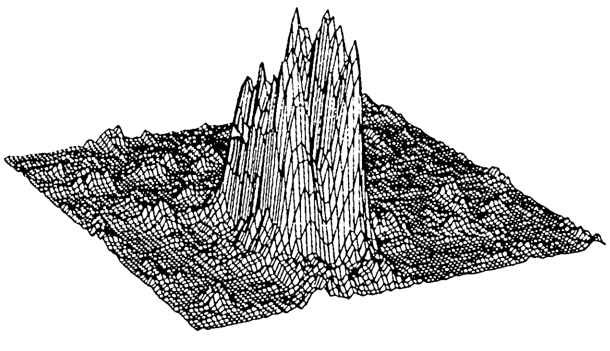

Fig. 7. Autocorrelation $|g \star g_{A}^{*}|^2$, SNR = 1, amplitude-only filter.

cross-correlations in the output plane between the input lettter O and the filter G were also calculated for the three different filters. However, their 3-D graphs are very similar to those depicted in Figs. 2–4 and so are not shown.

The same kinds of correlation test as described above were repeated with noise added to the original input signal but not the filter. This noise was Gaussian-distributed having a zero mean and a standard deviation of σ. The SNR is related to σ as SNR = $1/\sigma$. The noise values were generated by a random number subroutine in the computer program. Even though σ may have varied from run to run, for a given seed in the random number subroutine, the noise values at a given point in the input space differed only by a factor of σ. The results of some of the autocorrelation tests for the SNR = 1 case are shown in Figs. 5–7. As before, the graphs for the cross-correlation tests look quite similar to those depicted in Figs. 5–7 and so are not shown.

It should be noted that all plots in Figs. 2–7 are normalized to unity by the plotting subroutine.

In another computer simulation we allowed the filter to change in a gradual linear fashion from the amplitude-only condition to the phase-only condition. That is, we allowed $A(\omega)$ in Eq. (4) to change linearly so that $F(\omega)$ gradually evolved from Eq. (6) to Eq. (5). The results showed an approximately linear progression in all characteristics from the pure amplitude filter to the pure phase filter.

V. Discussion of Results

How well do the phase-only filter and amplitude-only filter perform compared with the classical matched filter? To see this for the noiseless case, consider the absolute values of the peak heights of the correlation spots listed in column 3 of Table I. In autocorrelation the classical matched filter has an absolute value of unity for its peak height, whereas the phase-only filter has a value of 57.6 and the amplitude-only filter has a value of 0.29. Consequently, if we have a VanderLugt correlator set up with a given laser source, and we replace the classical matched filter with a phase-only filter, we could expect an increase of 57.6× in the detector output voltage. Replacement with an amplitude-only filter, on the other hand, would decrease the detector output voltage by 0.29×. These results are, of course, related to the Horner efficiency η_H in column 5: the phase-only filter has 100% efficiency, while the classical and amplitude-only filters have efficiencies of only 27%.

A correlator is frequently used to identify characters. This implies that it must be able to discriminate between closely similar characters, such as a capital O and a capital G. From column 4 of Table I we see that for the classical matched filter there is only an 8% difference in its ability to discriminate the O from the G and only a 6% difference for the amplitude-only filter. However, the phase-only filter with a 39% difference demonstrates that its ability to discriminate is considerably better.

The real test of any filter is its performance in the presence of noise or what is now called its robustness.

Table I. Correlation Results for Noise-Free Inputs [a]

Fig.	Correlation	Peak height	Δ(%)	η_H(%)
2	$\|g \star g^{*}\|^2$	1.00		27.2
3	$\|o \star g^{*}\|^2$	0.92	−8	27.7
4	$\|g \star g_{\phi}^{*}\|^2$	57.6		100
5	$\|o \star g_{\phi}^{*}\|^2$	35.3	−39	100
6	$\|g \star g_{A}^{*}\|^2$	0.29		27.2
7	$\|o \star g_{A}^{*}\|^2$	0.27	−6	27.3

[a] Peak height refers to the absolute maximum value of the correlation spot. Δ is the percentage difference between the peaks of the autocorrelation and the cross-correlation functions. η_H is the Horner efficiency.

Table II. Correlation Results for the Signal with Additive Noise [a]

Fig.	Correlation	SNR = 1 Peak height	Δ(%)	η_H(%)	SNR = 2 Peak height	Δ(%)	η_H(%)	SNR = 4 Peak height	Δ(%)	η_H(%)
8	$\lvert g \star g^* \rvert^2$	0.95		1.2	0.97		3.9	0.99		10.8
	$\lvert o \star g^* \rvert^2$	0.87	−8	1.2	0.89	−8	3.9	0.90	−9	10.9
9	$\lvert g \star g_\phi^* \rvert^2$	51.8		100	54.7		100	56.1		100
	$\lvert o \star g_\phi^* \rvert^2$	30.9	−40	100	33.1	−39	100	34.2	−39	100
10	$\lvert g \star g_A^* \rvert^2$	0.32		1.2	0.29		3.9	0.29		10.8
	$\lvert o \star g_A^* \rvert^2$	0.32	0	1.2	0.29	0	3.9	0.28	−3	10.9

[a] SNR is the signal-to-noise ratio in the input plane. Peak height, Δ, and η_H are as defined in Table I.

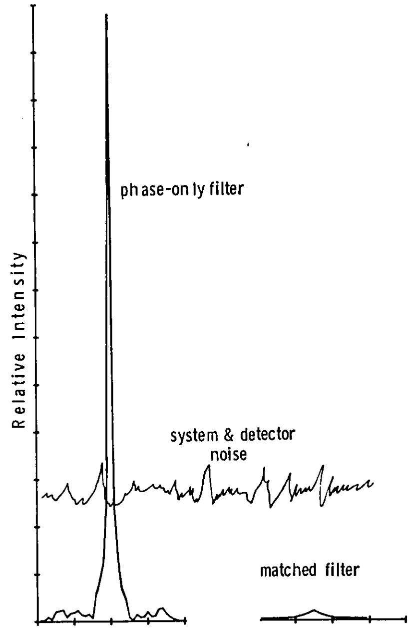

Fig. 8. Comparison of autocorrelation outputs for the phase-only and classical matched filters with SNR = 4.

Table II shows the performances of the three filters in the presence of three different amounts of white, additive, zero-mean, Gaussian noise with SNRs of 1, 2, and 4 (that is, σ = 1.0, 0.5, and 0.25, respectively). The same noise array was used in all the runs. However, as the SNR varied, the noise array was multiplied by the constant factor σ. As far as peak height is concerned, the trend is similar to the noise-free case, with the phase-only filter producing a much larger peak height compared with the other two filters. We also see that the discrimination ability of the phase-only filter holds up very well in the presence of noise; Δ has the same value as in the noise-free case (−39%) and remains practically constant as the SNR varies from 1 to 4. We conclude that the phase-only filter is a robust filter.

It should be noted that the results of Table II will vary slightly if different starting seeds are used in the random number generator. This is because a different seed will generate a different noise array. In fact, for several trials with different noise arrays, but with the same σ = 1, the cross-correlation peak for the classical matched filter was sometimes found to be slightly larger than the autocorrelation peak.

As a further comparison of the relative merits of the performance of the phase-only and classical matched filters in the presence of noise, consider Fig. 8. Here we plot the autocorrelation outputs for these filters in the vicinity of their peaks along one dimension in the correlation plane for SNR = 4. The ratio of the two peak heights is ~57:1 (56.1/0.99). A hypothetical system noise line is shown. The level of this system noise is attributable to the detector and the optical components of the system itself.[11]

However, in a low power optical correlator system, it would be the absolute values of the peaks relative to the total noise that would be the most important factor. Under these conditions the matched filter's signal would be undetectable. Hence the phase-only filter would be a much better choice.

VI. Conclusions

We have examined several new types of modified matched filter, namely, the phase-only and amplitude-only ones. We tested them by computer simulation side by side with the classical matched filter using alphabet characters as the input objects. Comparing the phase-only filter to the matched filter, we find at SNR = 1, for example (see Table II), that the former's optical efficiency η_H is 83 times higher (100%/1.2%), that its absolute peak energy in the correlation plane is 54.5 times greater (51.8/0.95), and that its ability to discriminate is 5 times better (40%/8%). By contrast, the amplitude-only filter is a dismal failure on all the above criteria. The shape of its response as shown in Figs. 4 and 7 is also quite broad and has multiple peaks.

Another advantage of the phase-only filter is the absence of sidelobes, as can be observed by comparing Figs. 2 and 3 (no noise) or 5 and 6 (SNR = 1). If a weak target is present and one is using the classical matched filter, the presence of additional sidelobes could deceive one into thinking that more than one target exists.

With reference to Fig. 8, one may wonder whether the fact that the central peak of the phase-only filter is higher than that of the classical matched filter violates any assumptions used in deriving the formulations of the matched filter. That is, have we gotten something for nothing? The answer is no. The price we pay is the decrease in the phase-only filter's SNR (85:1) in the correlation plane compared with that of the matched filter (500:1) (compare Figs. 5 and 6). The classical matched filter still has a higher SNR in the output plane.

Consequently, we conclude from our rather limited tests that the phase-only filter performs better on all counts than the other two filters. We have also verified that in matched filtering the phase information is considerably more important than the amplitude information. This is consistent with the results from other areas of scientific research, as referred to in Sec. I, for which a similar conclusion has been drawn.

On the other hand, we are not implying that the phase-only filter should replace the classical matched filter under all circumstances. Depending upon the application, for example, when a low level of discrimination is required or when discrimination is not important, the classical matched filter may be the better choice.

Finally, we raise an interesting question for the reader to ponder: Would applying the phase-only principle to the class of smart composite matched filters[6–8] improve their optical efficiency while at the same time preserve their desirable scaling-rotation invariant properties?

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