CRANFIELD UNIVERSITY

Mateusz Golab

Forward integration of simultaneous ordinary differential equations with graphical output

School of Engineering

Software Engineering for Technical Computing

MSc THESIS

Academic Year: 2011 - 2012

Supervisor: Dr Peter Sherar, Prof Joanna Polanska

August 2012

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This thesis is submitted in partial fulfilment of the requirements for the degree of Master of Science

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ABSTRACT

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Keywords:

ODE, Runge-Kutta, Modified Midpoint, Predictor - Corrector, Web development, GWT, AppEngine, Datastore, Unit testing.

ACKNOWLEDGEMENTS

Click here to enter acknowledgement text

TABLE OF CONTENTS

[ABSTRACT i](#_Toc331601125)

[ACKNOWLEDGEMENTS ii](#_Toc331601126)

[LIST OF FIGURES v](#_Toc331601127)

[LIST OF TABLES vi](#_Toc331601128)

[LIST OF EQUATIONS vii](#_Toc331601129)

[LIST OF ABBREVIATIONS viii](#_Toc331601130)

[1 Introduction 1](#_Toc331601131)

[1.1 Aims and objectives 1](#_Toc331601132)

[1.2 Motivation 1](#_Toc331601133)

[2 Literature review 2](#_Toc331601134)

[2.1 ODE numerical routines 2](#_Toc331601135)

[2.1.1 Runge-Kutta methods 2](#_Toc331601136)

[2.1.2 Richardson extrapolation 4](#_Toc331601137)

[2.1.3 Rosenbrock 4](#_Toc331601138)

[2.1.4 Predictor- Corrector 4](#_Toc331601139)

[2.2 Technologies 4](#_Toc331601140)

[2.2.1 AJAX approach 4](#_Toc331601141)

[2.2.2 Google Web Toolkit 4](#_Toc331601142)

[2.2.3 AppEngine 4](#_Toc331601143)

[2.2.4 Datastore 4](#_Toc331601144)

[3 Methodologies chosen 6](#_Toc331601145)

[3.1 Prototyping 6](#_Toc331601146)

[3.2 Test Driven Development 6](#_Toc331601147)

[3.3 AJAX 6](#_Toc331601148)

[3.4 Versioning 6](#_Toc331601149)

[4 Design 7](#_Toc331601150)

[4.1 Architecture 7](#_Toc331601151)

[4.2 Design patterns 7](#_Toc331601152)

[4.3 Technologies applied 7](#_Toc331601153)

[5 Testing 8](#_Toc331601154)

[5.1 Test driven development approach 8](#_Toc331601155)

[5.2 Testing methods 8](#_Toc331601156)

[5.2.1 Unit testing 8](#_Toc331601157)

[5.2.2 Integration testing 8](#_Toc331601158)

[5.2.3 System testing 8](#_Toc331601159)

[5.2.4 Cross – browser testing 8](#_Toc331601160)

[6 Implementation 9](#_Toc331601161)

[6.1 Parser 9](#_Toc331601162)

[6.2 Solver 9](#_Toc331601163)

[6.3 Graph viewer 9](#_Toc331601164)

[6.4 Datastore connector 9](#_Toc331601165)

[7 Results 10](#_Toc331601166)

[7.1 Results validation and verification 10](#_Toc331601167)

[7.2 Application’s outputs 10](#_Toc331601168)

[8 Discussion and conclusion 11](#_Toc331601169)

[8.1 Solvers correctness 11](#_Toc331601170)

[8.2 Limitations 11](#_Toc331601171)

[8.3 Problems faced 11](#_Toc331601172)

[8.4 Quality of implementation 11](#_Toc331601173)

[8.5 Conclusion 11](#_Toc331601174)

[8.6 Future work 11](#_Toc331601175)

[REFERENCES 13](#_Toc331601176)

[APPENDICES 14](#_Toc331601177)

# LIST OF FIGURES

[Figure 1 4th Order Runge-Kutta method 4](#_Toc331601178)

LIST OF TABLES

**Nie można odnaleźć pozycji dla spisu ilustracji.**

LIST OF EQUATIONS

[(2‑1) 2](#_Toc331601179)

[(2‑2) 3](#_Toc331601180)

LIST OF ABBREVIATIONS

|  |  |
| --- | --- |
| CU  GWT  ODE | Cranfield University  Google Web Toolkit  Ordinary differential equation |
|  |  |
|  |  |
|  |  |
|  |  |

# Introduction

The Introduction chapter points aims and objectives of the thesis project. All requirements of the project are explained in this chapter. Moreover, main motivators to perform this specific topic are included as well.

## Aims and objectives

The main objective of this thesis project is to develop following application :

1. Parsing equations entered by the user
2. Solving series of 1st order simultaneous ordinary differential equations (linear or non-linear) for initial value problem.
3. Presenting the solution on the 2D graph.
4. Storing and loading equations along with parameters entered by the user.

## Motivation

In general, two main motivators of this project are :

1. Discovering most efficient numerical methods for solving series of ordinary differential equations.
2. Familiarise with AJAX applications development and Google AppEngine along with Datastore.

# Literature review

This chapter demonstrates knowledge about recommended practical numerical methods for solving ODEs. Presented literature review gives an insight into major types of numerical routines taking into account efficiency and accuracy. What is more, technologies supporting AJAX applications development along with App Engine are also presented in this chapter. It is important to perform detailed research about technologies essential to develop the project.

## ODE numerical routines

Describes types of practical numerical methods for solving ODEs including Runge-Kutta methods, Bulirsch-Stoer, Rosenbrock metods and predictor-corrector rmethods.

### Runge-Kutta methods

Runge-Kutta methods propagate a solution over an interval by combining the data from several Euler-style steps. Each step involves one evaluation of the right-hand of the function .

|  |  |
| --- | --- |
|  | (2‑) |

Thenusing the information obtained to match a Taylor series expansion up to higher order.

Developing higher – order methods made Runge-Kutta competitive with the other numerical methods in many cases. It is usually the fastest method when moderate accuracy is required (≤ ) and evaluation of the function is not too expensive. There are few kinds of Runge-Kutta methods : 2nd order method, (called midpoint method), 4th order method and also method with adaptive stepsize. [Numerical recipes]

#### 4th Order Runge-Kutta

The most often used Runge-Kutta method is fourth – order formula . In general it is superior to 2nd order method, however high order does not always mean high accuracy. 4th order method requires four evaluations of the function. [numerical recipes]

|  |  |
| --- | --- |
|  | (‑) |

|  |  |
| --- | --- |
|  | (2‑3) |

|  |  |
| --- | --- |
|  | (2‑4) |

|  |  |
| --- | --- |
|  | (2‑5) |

|  |  |
| --- | --- |
|  | (2‑6) |



Figure 4th Order Runge-Kutta method

During single step, the derivative is evaluated four times. Once at initial point, (1) twice at midpoints (2)(3) and once at trial endpoint (4). Final function value is calculated on the basis of these derivatives. [numerical recipes]

Each step in the sequence of steps is treated in an identical manner, so prior behaviour of the solution is not used in its propagation. Such approach is mathematically proper, since any point along the trajectory of an ODE can be an initial point. [numerical recipes]

#### Runge-Kutta with adaptive stepsize

The purpose of adaptive stepsize method is to achieve predetermined accuracy in the solution with minimum computational effort. It is possible to face very smooth interval , while performing Runge-Kutta steps. Few great strides instead of small steps should speed through such undifferentiated interval, what may result in significant gain in efficiency. The idea of adaptive stepsize method is to control the size of the step and increase it when possible maintaining required level of accuracy. It is important to estimate truncation error to control accuracy level while increasing step size. Obviously the calculation of this information will add to the computational overhead, however it is profitable investment in terms of efficiency.

With 4th order Runge-Kutta , the

### Richardson extrapolation

#### Burlish – Stoer

### Rosenbrock

### Predictor- Corrector

#### Adams-Bashforth-Moulton

## Technologies

### AJAX approach

### Google Web Toolkit

### AppEngine

### Datastore

# Methodologies chosen

## Prototyping

## Test Driven Development

## AJAX

## Versioning

# Design

## Architecture

## Design patterns

## Technologies applied

# Testing

## Test driven development approach

## Testing methods

### Unit testing

### Integration testing

### System testing

### Cross – browser testing

# Implementation

## Parser

## Solver

## Graph viewer

## Datastore connector

# 

# Results

## Results validation and verification

## Application’s outputs

# Discussion and conclusion

## Solvers correctness

## Limitations

## Problems faced

## Quality of implementation

## Conclusion

## Future work

REFERENCES

Insert list of references here

APPENDICES

Whilst Heading 1 to Heading 6 can be used to number headings in the main body of the thesis, Heading styles 7–9 have been modified specifically for lettered appendix headings with Heading 7 having the ‘Appendix’ prefix as shown below.

Appendix Title (Use Heading 7)

Appendix Section (Use Heading 8)

Appendix Subsection (Use Heading 9)

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If you have chosen to include chapter numbers in your captions then follow the instructions given here to apply the same format to the captions in your appendices. This section explains how to caption the figures and tables in your Appendices, assuming that Heading 7 is numbered “Appendix A” and that the Figures and Tables are going to be labelled ‘Figure A-1’, ‘Figure A-2’, ‘Table B-1’ etc.

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   Repeat for table captions.
4. In the **Caption** box, type your caption text
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2. Select the **Insert Table of Figures** command on the **References** tab of the Ribbon
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4. Click after the List of Tables and repeat for the Caption Label ‘Table\_Apx’