CRANFIELD UNIVERSITY

Mateusz Golab

Forward integration of simultaneous ordinary differential equations with graphical output

School of Engineering

Software Engineering for Technical Computing

MSc THESIS

Academic Year: 2011 - 2012

Supervisor: Dr Peter Sherar, Prof Joanna Polanska

August 2012

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Supervisor: Dr Peter Sherar, Prof Joanna Polanska

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This thesis is submitted in partial fulfilment of the requirements for the degree of Master of Science

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ABSTRACT

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Keywords:

ODE, Runge-Kutta, Modified Midpoint, Predictor - Corrector, Web development, GWT, AppEngine, Datastore, Unit testing.

ACKNOWLEDGEMENTS

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LIST OF ABBREVIATIONS

|  |  |
| --- | --- |
| CU  GWT  ODE | Cranfield University  Google Web Toolkit  Ordinary differential equation |
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# Introduction

The Introduction chapter points aims and objectives of the thesis project. All requirements of the project are explained in this chapter. Moreover, main motivators to perform this specific topic are included as well.

## Aims and objectives

The main objective of this thesis project is to develop following application :

1. Parsing equations entered by the user
2. Solving series of simultaneous ordinary differential equations (linear or non-linear).
3. Presenting the solution on the 2D graph.
4. Storing and loading equations along with parameters entered by the user.

## Motivation

In general, two main motivators of this project are :

1. Discovering most efficient numerical methods for solving series of ordinary differential equations.
2. Familiarise with AJAX applications development and Google AppEngine along with Datastore.

# Literature review

This chapter demonstrates knowledge about recommended practical numerical methods for solving ODE’s. Presented literature review gives an insight into major types of numerical routines taking into account efficiency and accuracy. What is more, technologies supporting AJAX applications development along with App Engine are also presented in this chapter. It is important to perform detailed research about technologies essential to develop the project.

## ODE numerical routines

Describes types of practical numerical methods for solving ODE’s including Runge-Kutta, Modified midpoint, Bulirsch-Stoer, Rosenbrock metods and predictor-corrector rmethods.

### 4th Order Runge-Kutta

4th Order Runge-Kutta method propagate solution over an interval by combining the information from four Euler-style steps ( . Each step involves one evaluation of the right-hand of the and then using the information obtained to match a Taylor series expansion up to higher order.

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|  | (2‑7) |

### Modified Midpoint

### Richardson extrapolation

#### Burlish – Stoer

### Rosenbrock

### Predictor- Corrector

#### Adams-Bashforth-Moulton

## Technologies

### AJAX approach

### Google Web Toolkit

### AppEngine

### Datastore

# Methodologies chosen

## Prototyping

## Test Driven Development

## AJAX

## Versioning

# Design

## Architecture

## Design patterns

## Technologies applied

# Testing

## Test driven development approach

## Testing methods

### Unit testing

### Integration testing

### System testing

### Cross – browser testing

# Implementation

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## Solver

## Graph viewer

## Datastore connector

# 

# Results

## Results validation and verification

## Application’s outputs

# Discussion and conclusion

## Solvers correctness

## Limitations

## Problems faced

## Quality of implementation

## Conclusion

## Future work

REFERENCES

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APPENDICES

Whilst Heading 1 to Heading 6 can be used to number headings in the main body of the thesis, Heading styles 7–9 have been modified specifically for lettered appendix headings with Heading 7 having the ‘Appendix’ prefix as shown below.

Appendix Title (Use Heading 7)

Appendix Section (Use Heading 8)

Appendix Subsection (Use Heading 9)

Creating captions in Appendices

If you have chosen to include chapter numbers in your captions then follow the instructions given here to apply the same format to the captions in your appendices. This section explains how to caption the figures and tables in your Appendices, assuming that Heading 7 is numbered “Appendix A” and that the Figures and Tables are going to be labelled ‘Figure A-1’, ‘Figure A-2’, ‘Table B-1’ etc.

You will have to create new, separate labels that look like the ‘Figure’ and ‘Table’ labels you used in the main body of your thesis.

1. Select the **References** tab on the Ribbon then click on **Insert Caption**
2. Click **New Label**. Type **Figure\_Apx** then click **OK**
3. You now have two labels for figures, called **Figure** and **Figure\_Apx**  
   Repeat for table captions.
4. In the **Caption** box, type your caption text
5. Click **Numbering**. Tick **Include chapter numbering** and choose **Heading 7** from the drop-down list of styles and click **OK** twice
6. Your caption should look something like this:

**Figure\_Apx A‑1 This is the caption text for a Figure in the Appendix**

1. Delete the extraneous ‘\_Apx’ from the caption label so it reads:  
   **Figure A‑1 This is the caption text for a Figure in the Appendix**  
   **TIP:** Instead of deleting each ‘\_Apx’ individually use **Find & Replace** to modify all the labels at once.

Creating Lists of Figures and Tables for Appendices

This template already includes a List of Figures and a List of Tables, however you will have to create two new lists for the ‘Figure\_Apx’ and the ‘Table\_Apx’ labels.

1. Place the insertion point on a blank row after the existing List of Figures
2. Select the **Insert Table of Figures** command on the **References** tab of the Ribbon
3. Set the **Caption Label** box to ‘**Figure\_Apx**’ and click **OK**  
   **Note:** Word will put a single blank line between the original and new lists preventing it from appearing as one seamless list. However if you select the blank paragraph between the tables you can hide it by opening the Font dialog box from the Home tab and selecting **Hidden**.
4. Click after the List of Tables and repeat for the Caption Label ‘Table\_Apx’