CRANFIELD UNIVERSITY

Mateusz Golab

Forward integration of simultaneous ordinary differential equations with graphical output

School of Engineering

Software Engineering for Technical Computing

MSc THESIS

Academic Year: 2011 - 2012

Supervisor: Dr Peter Sherar, Prof Joanna Polanska

August 2012

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This thesis is submitted in partial fulfilment of the requirements for the degree of Master of Science

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ABSTRACT

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Keywords:

ODE, Runge-Kutta, Modified Midpoint, Predictor - Corrector, Web development, GWT, AppEngine, Datastore, Unit testing.

ACKNOWLEDGEMENTS

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LIST OF ABBREVIATIONS

|  |  |
| --- | --- |
| CU  GWT  ODE | Cranfield University  Google Web Toolkit  Ordinary differential equation |
|  |  |
|  |  |
|  |  |
|  |  |

# Introduction

The Introduction chapter points aims and objectives of the thesis project. All requirements of the project are explained in this chapter. Moreover, main motivators to perform this specific topic are included as well.

## Aims and objectives

The main objective of this thesis project is to develop following application :

1. Parsing equations entered by the user
2. Solving series of 1st order simultaneous ordinary differential equations (linear or non-linear) for initial value problem.
3. Presenting the solution on the 2D graph.
4. Storing and loading equations along with parameters entered by the user.

## Motivation

In general, two main motivators of this project are :

1. Discovering most efficient numerical methods for solving series of ordinary differential equations.
2. Familiarise with AJAX applications development and Google AppEngine along with Datastore.

# Literature review

This chapter demonstrates knowledge about recommended practical numerical methods for solving ODEs. Presented literature review gives an insight into major types of numerical routines taking into account efficiency and accuracy. What is more, technologies supporting AJAX applications development along with App Engine are also presented in this chapter. It is important to perform detailed research about technologies essential to develop the project.

## ODE numerical routines

Describes types of practical numerical methods for solving ODEs including Runge-Kutta methods, Bulirsch-Stoer, Rosenbrock metods and predictor-corrector rmethods.

### Runge-Kutta methods

Runge-Kutta methods propagate a solution over an interval by combining the data from several Euler-style steps. Each step involves one evaluation of the right-hand of the function .

|  |  |
| --- | --- |
|  | (2‑1) |

Thenusing the information obtained to match a Taylor series expansion up to higher order.

Developing higher – order methods made Runge-Kutta competitive with the other numerical methods in many cases. It is usually the fastest method when moderate accuracy is required (≤ ) and evaluation of the function is not too expensive. There are few kinds of Runge-Kutta methods : 2nd order method, (called midpoint method), 4th order method and also method with adaptive stepsize. [Numerical recipes]

#### 4th Order Runge-Kutta

The most often used Runge-Kutta method is fourth – order formula . In general it is superior to 2nd order method, however high order does not always mean high accuracy. 4th order method requires four evaluations of the function. [numerical recipes]

|  |  |
| --- | --- |
|  | (2‑2) |

|  |  |
| --- | --- |
|  | (2‑3) |

|  |  |
| --- | --- |
|  | (2‑4) |

|  |  |
| --- | --- |
|  | (2‑5) |

|  |  |
| --- | --- |
|  | (2‑6) |



Figure 1 4th Order Runge-Kutta method

During single step, the derivative is evaluated four times. Once at initial point, (1) twice at midpoints (2)(3) and once at trial endpoint (4). Final function value is calculated on the basis of these derivatives. [numerical recipes]

Each step in the sequence of steps is treated in an identical manner, so prior behaviour of the solution is not used in its propagation. Such approach is mathematically proper, since any point along the trajectory of an ODE can be an initial point. [numerical recipes]

#### Runge-Kutta with adaptive stepsize

### Richardson extrapolation

#### Burlish – Stoer

### Rosenbrock

### Predictor- Corrector

#### Adams-Bashforth-Moulton

## Technologies

### AJAX approach

### Google Web Toolkit

### AppEngine

### Datastore

# Methodologies chosen

## Prototyping

## Test Driven Development

## AJAX

## Versioning

# Design

## Architecture

## Design patterns

## Technologies applied

# Testing

## Test driven development approach

## Testing methods

### Unit testing

### Integration testing

### System testing

### Cross – browser testing

# Implementation

## Parser

## Solver

## Graph viewer

## Datastore connector

# 

# Results

## Results validation and verification

## Application’s outputs

# Discussion and conclusion

## Solvers correctness

## Limitations

## Problems faced

## Quality of implementation

## Conclusion

## Future work

REFERENCES

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APPENDICES

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Appendix Title (Use Heading 7)

Appendix Section (Use Heading 8)

Appendix Subsection (Use Heading 9)

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1. Select the **References** tab on the Ribbon then click on **Insert Caption**
2. Click **New Label**. Type **Figure\_Apx** then click **OK**
3. You now have two labels for figures, called **Figure** and **Figure\_Apx**  
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2. Select the **Insert Table of Figures** command on the **References** tab of the Ribbon
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   **Note:** Word will put a single blank line between the original and new lists preventing it from appearing as one seamless list. However if you select the blank paragraph between the tables you can hide it by opening the Font dialog box from the Home tab and selecting **Hidden**.
4. Click after the List of Tables and repeat for the Caption Label ‘Table\_Apx’