Zestaw (wielomiany, liczby zespolone)

①

Zad 1 a) $(1-i)^3 (\sqrt{3}+i)^4 (1-i\sqrt{3})^5$

Sprowadzamy do postaci wykładniczej liczby zespolone

$(1-i\sqrt{3})^5 = \left(2e^{-i\frac{\pi}{3}}\right)^5 = 2^5 e^{-\frac{5}{3}\pi i} = 2^5 e^{\frac{2}{3}\pi i}$

$|z| = 2 \cdot \cos\alpha = \frac{1}{2}, \ \sin\alpha = -\frac{\sqrt{3}}{2} \Rightarrow \alpha = -\frac{\pi}{3}$

$(1-i)^3 = (\sqrt{2})^3 e^{-i\frac{3}{4}\pi}$

$(\sqrt{3}+i)^4 = 2^4 e^{\frac{2\pi}{3}i}$

$(1-i)^3(\sqrt{3}+i)^4(1-i\sqrt{3})^5 = 2^{\frac{3}{2}} e^{-i\frac{3}{4}\pi} \cdot 2^4 e^{i\frac{2}{3}\pi} \cdot 2^5 e^{-i\frac{5}{3}\pi} =$

$= 2^{10} \cdot \sqrt{2}\, e^{i\frac{\pi}{4}} = 2^{10} \cdot \sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = 2^{10}(1+i)$

b) $\left(\frac{1-i\sqrt{3}}{1-i}\right)^{100} = \frac{2^{100} e^{(-i\frac{\pi}{3})100}}{2^{50} e^{(-i\frac{\pi}{4})100}} = 2^{50}\frac{e^{-i\pi\frac{100}{3}}}{e^{-i\pi \cdot 25}} = 2^{50} e^{-i\pi\frac{25}{3}} =$

$= 2^{50} e^{-i\frac{\pi}{3}} = 2^{50}\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = 2^{49}(1-i\sqrt{3})$

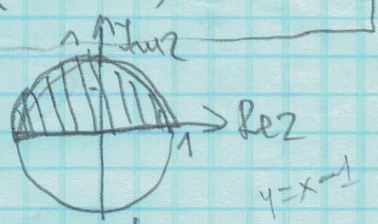
c) $(1+i)^{50}(\sqrt{3}-i)^{60} = 2^{\frac{50}{2}} e^{i\frac{50}{4}\pi} \cdot 2^{60} \cdot e^{-i10\pi} =$

$1+i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = \sqrt{2}e^{i\frac{\pi}{4}}$

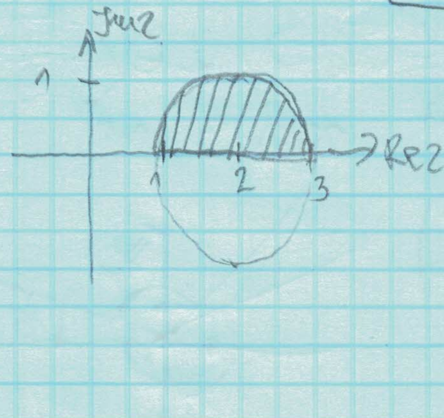
$\sqrt{3} - i = 2\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = 2e^{-i\frac{\pi}{6}}$

$= 2^{85} e^{i\frac{25}{2}\pi} = 2^{85} e^{i\frac{\pi}{2}} = \boxed{2^{85} \cdot i}$

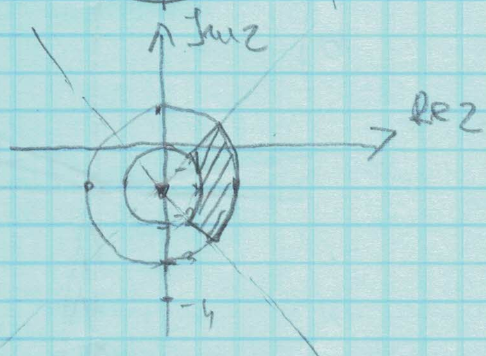
Zad 2 a)



b)



c)



Zad 3

$z = x + iy = |z|e^{i\alpha}$ $\quad z^3 = |z|^3 e^{i3\alpha}$

$z^3 = |z|^3(\cos 3\alpha + i\sin 3\alpha)$

$\text{Arg}\, z^3 = 3\alpha$

a) $0 < \arg z^3 < \frac{\pi}{2}$

$0 + 2k\pi < 3\alpha < \frac{\pi}{2} + 2k\pi$

$0 + \frac{2}{3}k\pi < \alpha < \frac{\pi}{6} + \frac{2}{3}k\pi$

$k=0 \Rightarrow 0 < \alpha < \frac{\pi}{6}$

$k=1 \Rightarrow \frac{2}{3}\pi < \alpha < \frac{\pi}{6} + \frac{2}{3}\pi$

$\frac{2}{3}\pi < \alpha < \frac{5}{6}\pi$

$k=2 \Rightarrow \frac{4}{3}\pi < \alpha < \frac{\pi}{6} + \frac{4}{3}\pi$

$\frac{4}{3}\pi < \alpha < \frac{3}{2}\pi$

b) $\frac{\pi}{2} < \arg z^3 < \pi$

$\frac{\pi}{2} + 2k\pi < 3\alpha < \pi + 2k\pi$

$\frac{\pi}{6} + \frac{2}{3}k\pi < \alpha < \frac{\pi}{3} + \frac{2}{3}k\pi$

podstawiamy $k = 0, 1, 2$, otrzymujemy 3 przedziały jak w poprzednim zadaniu

$k=0 \quad \frac{\pi}{6} < \alpha < \frac{\pi}{3}$

$k=1 \quad \frac{\pi}{6} + \frac{2}{3}\pi < \alpha < \frac{\pi}{3} + \frac{2}{3}\pi$

$k=2 \quad \frac{\pi}{6} + \frac{4}{3}\pi < \alpha < \frac{\pi}{3} + \frac{4}{3}\pi$