str 1

a) $\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 1 & 0 & 1 \\ 3 & 2 & 1 & 2 & 1 \\ 0 & 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 2 & 2 \end{vmatrix}$ odp 24

b) $\begin{vmatrix} 1 & 2 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 3 & 2 & 1 & 2 & 1 \\ 0 & 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 2 & 2 \end{vmatrix}$ -18

c) $\begin{vmatrix} 1 & 2 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 2 & 2 \end{vmatrix}$ -18

d) $\begin{vmatrix} 1 & 2 & 0 & 0 & 0 & 0 \\ 3 & 5 & 0 & 0 & 0 & 0 \\ 2 & 1 & 3 & 2 & 3 & 3 \\ 1 & 2 & 1 & 1 & 5 & 4 \\ 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 3 & 2 \end{vmatrix}$ -1

e) $\begin{vmatrix} 1 & 2 & 1 & 0 & 0 & 0 \\ 3 & 5 & 1 & 0 & 0 & 0 \\ 2 & 1 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 5 & 4 \\ 0 & 0 & 0 & 3 & 2 & 1 \\ 0 & 0 & 0 & 1 & 3 & 2 \end{vmatrix}$ -28

2) Wyznacz macierz odwrotną do macierzy A i sprawdź poprawność wyniku mnożeniem $A \cdot A^{-1}$

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & -2 & -1 \\ 1 & 3 & -1 \end{bmatrix} \qquad \det A = 9$$

Dopełnienia algebraiczne

$$A_{11} = \begin{vmatrix} -2 & -1 \\ 3 & -1 \end{vmatrix} = 5 \qquad A_{12} = -\begin{vmatrix} 0 & -1 \\ 1 & -1 \end{vmatrix} = -1$$

$$A_{13} = \begin{vmatrix} 0 & -2 \\ 1 & 3 \end{vmatrix} = 2 \qquad A_{21} = -\begin{vmatrix} 1 & 0 \\ 3 & -1 \end{vmatrix} = 1$$

$$A_{22} = \begin{vmatrix} 2 & 0 \\ 1 & -1 \end{vmatrix} = -2 \qquad A_{23} = -\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = -5$$

$$A_{31} = \begin{vmatrix} 1 & 0 \\ -2 & -1 \end{vmatrix} = -1 \qquad A_{32} = -\begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} = 2$$

$$A_{33} = \begin{vmatrix} 2 & 1 \\ 0 & -2 \end{vmatrix} = -4$$

$$A^{-1} = \frac{1}{9}\begin{bmatrix} 5 & 1 & -1 \\ -1 & -2 & 2 \\ 2 & -5 & -4 \end{bmatrix}$$

$$A \cdot A^{-1} = \frac{1}{9}\begin{pmatrix} 2 & 1 & 0 \\ 0 & -2 & -1 \\ 1 & 3 & -1 \end{pmatrix}\begin{pmatrix} 5 & 1 & -1 \\ -1 & -2 & 2 \\ 2 & -5 & -4 \end{pmatrix} = \frac{1}{9}\begin{pmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Zad 3 Metoda macierzy odwrotnej   str 2

$$AX = B \qquad |A| \neq 0$$

$$A^{-1} \cdot A \cdot X = A^{-1} \cdot B$$

$$\underline{I} \cdot X = A^{-1} \cdot B$$

$$X = A^{-1} \cdot B$$

a)
$$\begin{cases} x - y = 0 \\ y + z = 1 \\ x + y - z = 2 \end{cases} \qquad \begin{array}{ccc} A & X & B \\ \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix} & \begin{bmatrix} x \\ y \\ z \end{bmatrix} = & \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \end{array}$$

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} \xrightarrow{k_2 + k_1} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & -1 \end{vmatrix} = -3$$

$$A^{-1} = -\frac{1}{3} \begin{bmatrix} -2 & -1 & -1 \\ 1 & -1 & -1 \\ -1 & -2 & 1 \end{bmatrix}$$

$$X = A^{-1} B = -\frac{1}{3} \begin{bmatrix} -2 & -1 & -1 \\ 1 & -1 & -1 \\ -1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} =$$

$$= -\frac{1}{3} \begin{bmatrix} -3 \\ -3 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Zd 4A

str 3

$$\left[\begin{array}{cccc|cccc} 1 & -1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 2 & -1 & 4 & 2 & 0 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ -2 & 1 & -2 & -1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow[w_4+2w_1]{\substack{w_2-2w_1 \\ w_3-3w_1}} \left[\begin{array}{cccc|cccc} 1 & -1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 3 & -5 & -3 & -3 & 0 & 1 & 0 \\ 0 & -1 & 2 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow[w_4+w_2]{w_3-3w_2}$$

$$\left[\begin{array}{cccc|cccc} 1 & -1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & -5 & -3 & 3 & -3 & 1 & 0 \\ 0 & 0 & 2 & 1 & 0 & 1 & 0 & 1 \end{array}\right] \xrightarrow{w_3:(-5)} \left[\begin{array}{cccc|cccc} 1 & -1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{3}{5} & -\frac{3}{5} & \frac{3}{5} & -\frac{1}{5} & 0 \\ 0 & 0 & 2 & 1 & 0 & 1 & 0 & 1 \end{array}\right] \xrightarrow{w_4-2w_3}$$

$$\left[\begin{array}{cccc|cccc} 1 & -1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{3}{5} & -\frac{3}{5} & \frac{3}{5} & -\frac{1}{5} & 0 \\ 0 & 0 & 0 & -\frac{1}{5} & \frac{6}{5} & -\frac{1}{5} & \frac{2}{5} & 1 \end{array}\right] \xrightarrow{w_3+3w_4} \left[\begin{array}{cccc|cccc} 1 & -1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 3 & 0 & 1 & 3 \\ 0 & 0 & 0 & -\frac{1}{5} & \frac{6}{5} & -\frac{1}{5} & \frac{2}{5} & 1 \end{array}\right]$$

$$\xrightarrow{w_4 \cdot (-5)} \left[\begin{array}{cccc|cccc} 1 & -1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 3 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 & -6 & 1 & -2 & -5 \end{array}\right] \xrightarrow{w_1-w_4} \left[\begin{array}{cccc|cccc} 1 & -1 & 2 & 0 & 7 & -1 & 2 & 5 \\ 0 & 1 & 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 3 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 & -6 & 1 & -2 & -5 \end{array}\right] \xrightarrow{w_1-2w_3}$$

$$\left[\begin{array}{cccc|cccc} 1 & -1 & 0 & 0 & 1 & -1 & 0 & -1 \\ 0 & 1 & 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 3 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 & -6 & 1 & -2 & -5 \end{array}\right] \xrightarrow{w_1+w_2} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 3 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 & -6 & 1 & -2 & -5 \end{array}\right]$$

Zad 4B

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[w_3-w_1]{w_2-2w_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & -2 & 1 & 0 \\ 0 & -3 & 1 & -1 & 0 & 1 \end{array}\right] \xrightarrow{w_3-3w_2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -3 & 1 \end{array}\right] \xrightarrow{w_1+2w_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & -1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -3 & 1 \end{array}\right] \xrightarrow{w_2\cdot(-1)}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & 5 & -3 & 1 \end{array}\right].$$

$$AX = B \qquad |A| \neq 0$$

$$A^{-1}\cdot AX = A^{-1}B$$

$$\underline{I}\,X = A^{-1}B$$

$$X = A^{-1}B.$$

Zd 5. Metoda Cramera

$$\begin{cases} 2x + 2y - z + t = 7 \\ x - y + z - t = 0 \\ x + y + z + t = 10 \\ 4x + 3y - 2z - t = 0 \end{cases} \qquad AX = B$$

Rozw. wzgl. 4 kol.

$$\det A = \begin{vmatrix} 2 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ 4 & 3 & -2 & -1 \end{vmatrix} \xrightarrow[w_4+w_1]{w_2+w_1,\ w_3-w_1} \begin{vmatrix} 2 & 2 & -1 & 1 \\ 3 & 1 & 0 & 0 \\ -1 & -1 & 2 & 0 \\ 6 & 5 & -3 & 0 \end{vmatrix} = - \begin{vmatrix} 3 & 1 & 0 \\ -1 & -1 & 2 \\ 6 & 5 & -3 \end{vmatrix} = -3(-7) - 1 \cdot 9 =$$

$$= 12$$

$$\det A_y = \begin{vmatrix} 2 & 7 & -1 & 1 \\ 1 & -2 & 1 & -1 \\ 1 & 10 & 1 & 1 \\ 4 & 0 & -2 & -1 \end{vmatrix} \xrightarrow[w_4+w_1]{w_2+w_1,\ w_3-w_1} \begin{vmatrix} 2 & 7 & -1 & 1 \\ 3 & 5 & 0 & 0 \\ -1 & 3 & 2 & 0 \\ 6 & 7 & -3 & 0 \end{vmatrix} = - \begin{vmatrix} 3 & 5 & 0 \\ -1 & 3 & 2 \\ 6 & 7 & -3 \end{vmatrix} = -3 \begin{vmatrix} 3 & 2 \\ 7 & -3 \end{vmatrix} + 5 \,*$$

$$* \begin{vmatrix} -1 & 2 \\ 6 & -3 \end{vmatrix} = -3(-9-14) + 5(3-12) = 69 - 45 = 24$$

$$\det A_x = \begin{vmatrix} 7 & 2 & -1 & 1 \\ -2 & -1 & 1 & -1 \\ 10 & 1 & 1 & 1 \\ 0 & 3 & -2 & -1 \end{vmatrix} \xrightarrow[w_4+w_1]{w_2+w_1,\ w_3-w_1} \begin{vmatrix} 7 & 2 & -1 & 1 \\ 5 & 1 & 0 & 0 \\ 3 & -1 & 2 & 0 \\ 7 & 5 & -3 & 0 \end{vmatrix} = - \begin{vmatrix} 5 & 1 & 0 \\ 3 & -1 & 2 \\ 7 & 5 & -3 \end{vmatrix} = -5 \begin{vmatrix} -1 & 2 \\ 5 & -3 \end{vmatrix} +$$

$$+ \begin{vmatrix} 3 & 2 \\ 7 & -3 \end{vmatrix} = -5(3-10) + -9 - 14 = 35 - 23 = 12$$

$$\det A_z = \begin{vmatrix} 2 & 2 & 7 & 1 \\ 1 & -1 & -2 & -1 \\ 1 & 1 & 10 & 1 \\ 4 & 3 & 0 & -1 \end{vmatrix} \xrightarrow[w_4+w_1]{w_2+w_1,\ w_3-w_1} \begin{vmatrix} 2 & 2 & 7 & 1 \\ 3 & 1 & 5 & 0 \\ -1 & -1 & 3 & 0 \\ 6 & 5 & 7 & 0 \end{vmatrix} = - \begin{vmatrix} 3 & 1 & 5 \\ -1 & -1 & 3 \\ 6 & 5 & 7 \end{vmatrix} = 36$$

$$\det A_t = \begin{vmatrix} 2 & 2 & -1 & 7 \\ 1 & -1 & 1 & -2 \\ 1 & 1 & 1 & 10 \\ 4 & 3 & -2 & 0 \end{vmatrix} \xrightarrow[w_2+w_1,\ w_3+w_1,\ w_4-2w_2]{=} \begin{vmatrix} 2 & 2 & -1 & 7 \\ 3 & 1 & 0 & 5 \\ 3 & 3 & 0 & 17 \\ 0 & -1 & 0 & -14 \end{vmatrix} = - \begin{vmatrix} 3 & 1 & 5 \\ 3 & 3 & 17 \\ 0 & -1 & -14 \end{vmatrix} = 48$$

$$x = \frac{\det A_x}{\det A} = \frac{12}{12} = 1 \qquad y = \frac{\det A_y}{\det A} = \frac{24}{12} = 2$$

$$z = \frac{\det A_z}{\det A} = \frac{36}{12} = 3 \qquad t = \frac{\det A_t}{\det A} = \frac{48}{12} = 4$$

Zad 6 str 6.

$$\begin{cases} x - 2y + 3z = -7 \\ 3x + y + 4z = 5 \\ 2x + 5y + z = 18 \end{cases}$$

Met. eliminacji Gaussa

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & -7 \\ 3 & 1 & 4 & 5 \\ 2 & 5 & 1 & 18 \end{array}\right] \xrightarrow[w_3 - 2w_1]{w_2 - 3w_1} \left[\begin{array}{ccc|c} 1 & -2 & 3 & -7 \\ 0 & 7 & -5 & 26 \\ 0 & 9 & -5 & 32 \end{array}\right] \xrightarrow{w_2 : 7} \left[\begin{array}{ccc|c} 1 & -2 & 3 & -7 \\ 0 & 1 & -\frac{5}{7} & \frac{26}{7} \\ 0 & 9 & -5 & 32 \end{array}\right] \to$$

$$\xrightarrow{w_3 - 9w_2} \left[\begin{array}{ccc|c} 1 & -2 & 3 & -7 \\ 0 & 1 & -\frac{5}{7} & \frac{26}{7} \\ 0 & 0 & \frac{10}{7} & -\frac{10}{7} \end{array}\right] \xrightarrow{w_3 : \frac{10}{7}} \left[\begin{array}{ccc|c} 1 & -2 & 3 & -7 \\ 0 & 1 & -\frac{5}{7} & \frac{26}{7} \\ 0 & 0 & 1 & -1 \end{array}\right] \xrightarrow[w_1 + 2w_2 - 3w_3]{w_2 + \frac{5}{7}w_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right] \qquad \begin{aligned} x &= 2 \\ y &= 3 \\ z &= -1 \end{aligned}$$