

2d 4) a)  $z^4 + 1 = (z^4 + z^2 + 1) - z^2 = (z^2 + 1) - (\sqrt{2}z) = (3)$   
 $= (z^2 - \sqrt{2}z + 1)(z^2 + \sqrt{2}z + 1) = (z + \frac{1-i}{2}\sqrt{2})(z + \frac{1+i}{2}\sqrt{2})$   
 $\cdot (z - \frac{1-i}{2}\sqrt{2})(z - \frac{1+i}{2}\sqrt{2})$

b)  $z^6 - 1 = (z^3 + 1)(z^3 - 1)$   
 wielomiany  $z^3 + 1 = (z + 1)(z - \frac{1}{2} + i\frac{\sqrt{3}}{2})(z - \frac{1}{2} - i\frac{\sqrt{3}}{2})$   
 $z^3 - 1 = (z - 1)(z^2 + z + 1) = (z - 1)(z + \frac{1}{2} + i\frac{\sqrt{3}}{2})(z + \frac{1}{2} - i\frac{\sqrt{3}}{2})$

2d 5

a)  $\sqrt[4]{-8+8\sqrt{3}i}$   $z_0 = -8+8\sqrt{3}i$   $|z_0| = 16$   $\arg z_0 = \frac{2\pi}{3}$

$\sqrt[4]{-8+8\sqrt{3}i} = \sqrt[4]{16} \left( \cos \frac{2\pi/3 + 2k\pi}{4} + i \sin \frac{2\pi/3 + 2k\pi}{4} \right), k=0,1,2,3$

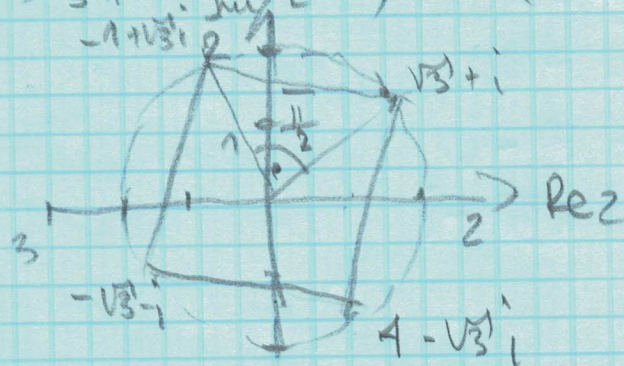
$= 2 \left( \cos \left( \frac{\pi}{6} + \frac{k\pi}{2} \right) + i \sin \left( \frac{\pi}{6} + \frac{k\pi}{2} \right) \right), k=0,1,2,3$

$z_0 = 2 \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) = 2 \left( \frac{\sqrt{3}}{2} + \frac{1}{2}i \right) = \sqrt{3} + i$

$z_1 = 2 \left( \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right) = 2 \left( -\frac{1}{2} + i\frac{\sqrt{3}}{2} \right) = -1 + \sqrt{3}i$

$z_2 = 2 \left( \cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} \right) = 2 \left( -\frac{\sqrt{3}}{2} - \frac{1}{2}i \right) = -\sqrt{3} - i$

$z_3 = 2 \left( \cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right) = 2 \left( \frac{1}{2} - i\frac{\sqrt{3}}{2} \right) = 1 - \sqrt{3}i$



2d 5) b)

$\sqrt[6]{1}$   $z_0 = 1$   $|z_0| = 1$   $\arg z_0 = 0$   
 $\sqrt[6]{1} = \left\{ \sqrt[6]{1} \left( \cos \frac{0+2k\pi}{6} + i \sin \frac{0+2k\pi}{6} \right), k=0,1,2,3,4,5 \right\}$

$z_0 = 1 \left( \cos 0 + i \sin 0 \right) = 1$

$z_1 = 1 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) = \frac{1}{2} + i\frac{\sqrt{3}}{2}$

$z_2 = 1 \left( \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right) = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$

$z_3 = 1 \left( \cos \pi + i \sin \pi \right) = -1$

$z_4 = 1 \left( \cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right) = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

$z_5 = 1 \left( \cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right) = \frac{1}{2} - i\frac{\sqrt{3}}{2}$

