Kompleksni integral. Bodi $f\colon \mathcal{D} \to \mathbb{C}$ in $\mathcal{K}$ orientirana krivulja. Bodi $\gamma\colon [a,b] \to \mathbb{C}$ parametrizacija te krivulje.
Tedaj je

$$\int_{\mathcal{K}} f(z)dz = \int_a^b f(\gamma(t))\dot{\gamma}dt$$

1. naloga: Bodi $f(z) = z^n$ za nek $n \in \mathbb{N}$, krivulja $\mathcal{K}$ pa enotska krožnica.

$$\gamma(t) = e^{it};\ t \in [0, 2\pi]$$

$$\int_{\mathcal{K}} f(z)dz = \int_0^{2\pi} ie^{it(n+1)}dt = \frac{1}{n+1}e^{it(n+1)}\Big|_0^{2\pi} = 0$$

To velja za vsako celo število, razen za $n = -1$. V tem primeru pa velja:

$$\int_{\mathcal{K}} \frac{\mathrm{d}z}{z} = \int_0^{2\pi} e^{-it}\left(ie^{it}\right)dt = \int_0^{2\pi} i\mathrm{d}t = 2\pi i$$

2. naloga: $\int_{\mathcal{K}} \frac{\sin z}{z+1}dz$, kjer je $\mathcal{K} = \{z \in \mathbb{C}, |z| = 2\}$

$$I = \int_{\mathcal{K}} \frac{f(z)}{z-(-1)}dz = 2\pi i f(-i) = 2\pi \sinh(1)$$

3. naloga: $\oint_{\mathcal{K_R}} \frac{dz}{z^2+9}$, kjer je $\mathcal{K} = \{z \in \mathbb{C}; |z| = R\}$, za $R = 2$ in $R = 4$.

$$\oint_{\mathcal{K}_R} \frac{dz}{(z-3i)(z+3i)} = 0 \text{ za } R = 2, \text{ saj ni nobene singularnosti.}$$

$$\oint_{\mathcal{K}_4} = \frac{dz}{(z-3i)(z+3i)} = \oint_{\mathcal{K}_4}\left[\frac{A}{z-3i} + \frac{B}{z+3i}\right]dz$$

$$= \oint_{\mathcal{K}_4} \frac{dz}{6i(z-3i)} - \oint_{\mathcal{K}_4} \frac{1}{6i(z+3i)} = \frac{\pi}{3} - \frac{\pi}{3} = 0$$

Kajti

$$\oint_{\mathcal{K}} \frac{f(\zeta)}{\zeta - z}dz = 2\pi i f(-1)$$

4. naloga: Izračunaj

$$\oint_{\mathcal{K}} \frac{dz}{(1+z)(z-1)^3},$$

kjer je $\mathcal{K} = \left\{z \in \mathbb{C}, |z-1| = \frac{3}{2}\right\}$

$$\oint_{\mathcal{K}} \frac{dz}{(1+z)(z-1)^3} = \oint_{\mathcal{K}} \frac{\left(\frac{1}{z+1}\right)}{(z-1)^3} = \frac{2\pi i}{2!}\left(\frac{1}{1+z}\right)_{z=-1}^{z=0} = \frac{i\pi}{4}$$

5. naloga: $\oint_{\partial\mathcal{D}} \frac{e^z}{z(1-z)^3}$, kjer je $\mathcal{D} = \{z \in \mathbb{C}; |z| \leq 3\}$

$$\oint_{\partial\mathcal{D}} \frac{e^z}{z(1-z)^3} = \oint_{L_1} \frac{\frac{e^z}{(1-z)^3}}{z}dz + \oint_{L_2} \frac{\frac{e^z}{z}}{(1-z)^3}dz = 2\pi i\left(\frac{e^0}{(1-0)^3} - \frac{\left(\frac{e^z}{z}\right)''\Big|_{z=1}}{2!}\right)$$

$$= ... = 2\pi i\left(1 - \frac{e}{2}\right)$$