Poissonovi oklepaji. Ponovno imamo $f(\underline{q}, \underline{p}, t)$, kjer so posplošene koordinate $\underline{q}$ in impulzi $\underline{p}$ odvisni od časa. Spet je $p_i = \dfrac{\partial L}{\partial \dot{q}_i}$, $\dot{q}_i = \dfrac{\partial H}{\partial p_i}$, $\dot{p}_i = -\dfrac{\partial H}{\partial q_i}$.

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \sum_i \left(\frac{\partial f}{\partial q_i}\dot{q}_i + \frac{\partial f}{\partial p_i}\dot{p}_i \right) + \frac{\partial f}{\partial t}$$

$$= \sum_i \left(\frac{\partial f}{\partial q_i}\frac{\partial H}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial H}{\partial q_i} \right) + \frac{\partial f}{\partial t} = [f, H] + \frac{\partial f}{\partial t}$$

Definirali smo Poissonov oklepaj $[f, g] = \sum_i \left(\dfrac{\partial f}{\partial q_i}\dfrac{\partial g}{\partial p_i} - \dfrac{\partial f}{\partial p_i}\dfrac{\partial g}{\partial q_i} \right)$. Včasih se uporablja tudi oznako $\{f, g\}$.

Lastnosti. Poissonovi oklepaji imajo naslednje lastnosti:

1. $[f, \lambda g + \eta h] = \lambda[f, g] + \eta[f, h]$
2. $[f, g] = -[g, f]$
3. $[f, gh] = [f, g]h + g[f, h]$
4. $[f, f] = 0$
5. $[x, p_x] = 1$
6. $[x, p_y] = 0$

Lagrangeov formalizem za zvezno sredstvo. Na primer za struno:

$$\mathrm{d}T = \frac{1}{2}\dot{u}^2 \rho S_0 \,\mathrm{d}x = \frac{1}{2}\rho S_0 \left(\frac{\partial u}{\partial t}\right)^2 \mathrm{d}x$$

$$\mathrm{d}V = F(\mathrm{d}l - \mathrm{d}x) = F(\sqrt{\mathrm{d}x^2 + \mathrm{d}u^2} - \mathrm{d}x) \approx \frac{1}{2}F\left(\frac{\partial u}{\partial x}\right)^2 \mathrm{d}x$$

$$\mathrm{d}L = \frac{1}{2}\rho S_0 \left(\frac{\partial u}{\partial t}\right)^2 \mathrm{d}x \pm \frac{1}{2}F\left(\frac{\partial u}{\partial x}\right)^2 \mathrm{d}x \pm \rho S_0 u \,\mathrm{d}x$$

$$\mathcal{L} = \frac{\mathrm{d}L}{\mathrm{d}x} = ...$$

$$L = \int_{x_1}^{x_2} \mathcal{L}\,\mathrm{d}x = \int_{x_1}^{x_2} \mathcal{L}\left(u(x,t), u_x, u_t, x, t\right)$$

$$S = \int_0^t L\,\mathrm{d}t = \iint \mathcal{L}\,\mathrm{d}x\,\mathrm{d}t\,(\mathrm{d}y,\,\mathrm{d}z)$$

Euler-Lagrangeova enačba za tak sistem je

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathcal{L}}{\partial u_t} + \frac{\mathrm{d}}{\mathrm{d}x}\frac{\partial \mathcal{L}}{\partial u_x} - \frac{\partial \mathcal{L}}{\partial x} = 0$$

Iz tega lahko izpeljamo na primer Maxwellove zakone, valovne enačbe itd.