Termini za ustne izpite. 12. ter 13. 6 in 1. ter 2. 7.

Majhna nihanja. Imamo vektor odmika $\underline{\eta}$.

$$\tilde{L} = \frac{1}{2}\underline{\dot{\eta}}^T \underline{\underline{T}}\dot{\eta} - \frac{1}{2}\underline{\eta}^T \underline{\underline{V}}\eta$$

$$\sum_j T_{ij}\ddot{\eta}_j + \sum_j V_{ij}\eta_j = 0, \ \forall i$$

Lastna nihanja. $\eta_i = \alpha a_i e^{i\omega t}, \ \omega \in \mathbb{R}$ To vstavimo v enačbo, da dobimo

$$\sum_j V_{ij}a_j - \omega^2 \sum_j T_{ij}a_j = 0$$

$$\underline{\underline{V}}\underline{a} = \omega^2 \underline{\underline{T}}\underline{a} = \lambda \underline{\underline{T}}\underline{a}, \ \omega^2 \geq 0$$

Poseben primer. $\underline{\underline{T}} = T\underline{\underline{I}}$.

$$(\underline{\underline{V}} - \lambda T \underline{\underline{I}})\underline{a} = (\underline{\underline{V}} - \tilde{\lambda} I)\underline{a} = 0$$

Pokažemo lahko, da so lastni vektorji $\underline{a_k}$ ortogonalni. Iz normiranih vektorjev $\underline{a_k}$ sestavimo matriko $\underline{\underline{A}}$.

$$\underline{\underline{A}} = \begin{bmatrix} a_{11} & a_{21} & a_{31} & \dots & a_{n1} \\ a_{12} & \ddots & & & \\ a_{13} & & \ddots & & \\ \vdots & & & \ddots & \\ a_{1n} & & & & a_{nn} \end{bmatrix} = (\underline{a_1}, \underline{a_2}, ...\underline{a_n})$$

Velja:

$$\underline{\underline{A}}^T \underline{\underline{A}} = \underline{\underline{A}}\underline{\underline{A}}^T = \underline{\underline{T}}$$

$$\underline{\underline{A}}^T \underline{\underline{V}}\underline{\underline{A}} = \underline{\underline{\tilde{\Lambda}}}$$

tu je $\underline{\underline{\tilde{\Lambda}}}$ diagonalna matrika oblike

$$\underline{\underline{\tilde{\Lambda}}} = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix}$$

Poleg tega velja tudi

$$\underline{\underline{A}}^T \underline{\underline{T}}\underline{\underline{A}} = \underline{\underline{I}}$$

$$\underline{\underline{A}}^T \underline{\underline{V}}\underline{\underline{A}} = \underline{\underline{A}}^T \underline{\underline{T}}\underline{\underline{A}}\underline{\underline{\Lambda}} = \underline{\underline{\Lambda}}$$

Normalne koordinate. Od prej:

$$\tilde{L} = \frac{1}{2}\underline{\dot{\eta}}\underline{\underline{T}}\dot{\eta} - \frac{1}{2}\underline{\eta}\underline{\underline{V}}\eta$$

Ker je η linearna kombinacija lastnih vektorjev, zapišemo $\eta = \underline{\underline{A}}\underline{\alpha}$. Sledi:

$$\tilde{L} = \frac{1}{2}\underline{\dot{\alpha}}^T \underline{\dot{\alpha}} - \frac{1}{2}\underline{\alpha}^T \underline{\underline{\lambda}}\underline{\alpha}$$

Hamiltonov formalizem. $L = T - V$;

$$L = L(\underline{q}, \underline{\dot{q}}, t)$$

$$p_i = \frac{\partial L}{\partial \dot{q}_i}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q_i} = 0$$

Definiramo $H = \sum_i p_i \dot{q}_i - L = T + V = H(\underline{q}, \underline{p}, t)$

$$\mathrm{d}H = \sum_i (\dot{q}_i \,\mathrm{d}p_i + p_i \,\mathrm{d}\dot{q}_i) - \sum_i \left(\frac{\partial L}{\partial q_i}\,\mathrm{d}q_i + \frac{\partial L}{\partial \dot{q}_i}\,\mathrm{d}\dot{q}_i\right) - \frac{\partial L}{\partial t}\,\mathrm{d}t$$

Dobimo Hamiltonove enačbe:

$$\dot{q}_i = \frac{\partial H}{\partial p_i}$$

$$\dot{p}_i = -\frac{\partial H}{\partial q_i}$$

$$\frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}$$

Kajti $\mathrm{d}H = \frac{\partial H}{\partial q_i}\,\mathrm{d}q_i + \frac{\partial H}{\partial p_i}\,\mathrm{d}p_i + \frac{\partial H}{\partial t}\,\mathrm{d}t$

Nabit delec v elektromagnetnem polju.

$$\vec{F} = e(\vec{E} + \vec{v} \times \vec{B})$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0, \quad F_i = \frac{\partial V}{\partial q_i} + \frac{\partial}{\partial t}\frac{\partial V}{\partial \dot{q}_i}, \quad V(\underline{\underline{q}}, \underline{\underline{\dot{q}}}, t)$$

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t}$$

$$\nabla \times \vec{E} = -\nabla \times \frac{\partial \vec{A}}{\partial t} = -\frac{\mathrm{d}}{\mathrm{d}t}\nabla \times \vec{A} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{F} = e\left[-\nabla\phi - \frac{\partial \vec{A}}{\partial t} + \vec{v} \times (\nabla \times \vec{A})\right]$$

$$H = \frac{(\vec{p} - e\vec{A})^2}{2m} + V + e\phi$$