$r(\varphi) = \dfrac{1}{1 + 2\frac{|\varphi|}{\pi} + \frac{\varphi^2}{\pi^2}}$, $\overrightarrow{B}(0,0,0) =?$

$$d\overrightarrow{B}(0,0,0) = \frac{\mu_0 I}{4\pi} \frac{d\overrightarrow{r} \times \overrightarrow{r}}{|\overrightarrow{r}|^3}$$

$$d\overrightarrow{r} = r'(\varphi)\hat{e}_r + r(\varphi)\hat{e}_\varphi$$

$$d\overrightarrow{r} \times \overrightarrow{r} = r^2 \hat{e}_\varphi \times \hat{e}_r = r^2 e_z$$

$$d\overrightarrow{B} = \hat{e}_z \frac{r^2(\varphi)}{r^3(\varphi)} = \frac{1}{r(\varphi)}\hat{e}_z = \hat{e}_z \left(1 + 2t + t^2\right) dt$$

$$B_z = \frac{\mu_0 I}{4\pi} \int_0^1 (1 + t^2)\, dt = \frac{7\mu_0 I}{6}$$

V nadaljnje bomo označevali $\partial_x = \dfrac{\partial}{\partial x}$.

$\cos\vartheta = a + b\cos 3\varphi$, ploskev je enakomerno nabita, $r = \text{konst.}$ $\overrightarrow{E}(0,0,0) =?$

Zaraadi simetrije pričakujemo, da bo polje kazalo z smeri $\hat{e}_z$.

$$d\overrightarrow{E} = -\frac{de}{4\pi\varepsilon_0 r^2}\hat{r}$$

$$de = \sigma dS = \sigma r^2 d(\cos\vartheta) d\varphi$$

$$E = E_z = |E|\,\hat{r}\cdot\hat{z} = |E|\cos\vartheta$$

$$dE = \frac{-\sigma\cos\vartheta\, d(\cos\vartheta)\, d\varphi}{4\pi\varepsilon_0}$$

$$\cos\vartheta \in [a + b\cos 3\varphi, 1]$$

$$E = -\frac{\sigma}{4\pi\varepsilon_0 a^2} \int_0^{2\pi} \int_{a+b\cos 3\varphi}^1 \cos\vartheta d(\cos\vartheta) d\varphi =$$

$$= -\frac{\sigma}{4\pi\varepsilon_0} \int_0^{2\pi} \frac{u^2}{2}\Big|_{a+b\cos 3\varphi}^1 d\varphi = -\frac{\sigma}{4\pi\varepsilon_0} \int_0^{2\pi} \frac{1 - (a + b\cos 3\varphi)^2}{2} d\varphi$$

$$= \frac{-\sigma}{8\pi\varepsilon_0} \int_0^{2\pi} 1 - (a + b\cos 3\varphi)^2 d\varphi$$

Pomožni izračuni:

$$\int_0^{2\pi} \cos 3\varphi d\varphi = 0$$

$$\int_2^{2\pi} \cos^2 3\varphi d\varphi = \pi$$

$$E = -\frac{\sigma(2 - 2a^2 - b^2)}{8\varepsilon_0}$$

Vektorski produkti v sferičnih koordinatah:

$$\hat{e}_r \times \hat{e}_\vartheta = e_\varphi$$

$$\hat{e}_\varphi \times \hat{e}_r = e_\vartheta$$

$$\hat{e}_\vartheta \times \hat{e}_\varphi = e_r$$

Skalarni produkti z navpišnico v sferičnih koordinatah:

$$\hat{e}_r \cdot \hat{z} = \cos\vartheta$$

$$\hat{e}_\varphi \cdot \hat{z} = 0$$

$$\hat{e}_\vartheta \cdot \hat{z} = -\sin\vartheta$$