Naloga. Torzijsko sklopljeni težni nihali. Zanimajo nas lastna nihanja pri majhnih odkimih okoli ravnovesne lege.

$$V = -mgl(\cos\varphi_1 + \cos\varphi_2) + \frac{1}{2}D(\varphi_2 - \varphi_1)^2$$

$$T = \frac{1}{2}m(l\dot{\varphi}_1)^2 + \frac{1}{2}m(l\dot{\varphi}_2)^2 = \frac{1}{2}ml^2(\varphi_1^2 + \varphi_2^2)$$

Iščemo ravnovesni legi:

$$\frac{\partial V}{\partial \varphi_1} = -mgl(-\sin\varphi_{10}) + \frac{1}{2}D(-2\varphi_{20} + 2\varphi_{10}) = 0$$

$$\frac{\partial V}{\partial \varphi_2} = -mgl(-\sin\varphi_{20}) + \frac{1}{2}D(2\varphi_{20} - 2\varphi_{10}) = 0$$

Enačbi seštejemo, nato odštejemo:

$$+: \quad \sin\varphi_{10} + \sin\varphi_{20} = 0$$

$$-: \quad mgl(\sin\varphi_{10} - \sin\varphi_{20}) + 2D(-\varphi_{20} + \varphi_{10}) = 0$$

Možne rešitve prve enačbe:

$$\varphi_{10} = -\varphi_{20} + 2k\pi$$

$$\varphi_{10} = -\varphi_{20} + \pi + 2k\pi$$

Označimo $\alpha = D/mgl$

$$V = V_0 + \frac{1}{2}\underline{s\varphi}\,\underline{\underline{V}}\,\underline{s\varphi}, \quad \underline{\underline{V}} = mgl\begin{bmatrix}\cos\varphi_{10} + \alpha & -\alpha \\ -\alpha & \cos\varphi_{20} + \alpha\end{bmatrix}$$

$$\underline{\underline{T}} = ml^2\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$$

Naloga. $\tilde{L} = \frac{1}{2}\underline{\dot{\eta}}\,\underline{\underline{T}}\,\underline{\dot{\eta}} - \frac{1}{2}\underline{\eta}\,\underline{\underline{V}}\,\underline{\eta}$. Uporabimo nastavek $\underline{\eta} = e^{i\omega t}\underline{a}$

$$\underline{\underline{V}}\,\underline{a} = \omega^2\underline{\underline{T}}\,\underline{a}$$

$$(\underline{\underline{V}} - \omega^2\underline{\underline{T}})\underline{a} = 0$$

$$\det(\underline{\underline{V}} - \omega^2\underline{\underline{T}}) = 0$$

Izračunamo lastne vrednosti ω in pripadajoče lastne vektorje $\underline{a}$. $\eta(t)$ je linearna kombinacija teh rešitev.