Diferencialne enačbe. Imamo tri plasti, ki drsijo ena mimo druge. Med njimi deluje viskoznost s koeficientom γ.

$$m\dot{v}_1 = \gamma(v_2 - v_1)$$
$$m\dot{v}_2 = \gamma(v_3 - v_2) + \gamma(v_1 - v_2) = \gamma(v_3 - 2v_2 + v_1)$$
$$m\dot{v}_3 = \gamma(v_2 - v_3)$$

Sistem zapišemo v matrični obliki:

$$\dot{\vec{v}} = \tilde{A}\vec{v}$$
$$\tilde{A} = \frac{\gamma}{m}\begin{bmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{bmatrix} = \frac{\gamma}{m}A$$

Diagonalizirajmo A:

$$\det(A - \lambda I) = (-1-\lambda)\left((-2-\lambda)(1-\lambda) - 1\right) - (-1-\lambda) = ... = -\lambda(\lambda+1)(\lambda+2)$$

Iammo lastne vrednosti $\lambda_1 = 0$, $\lambda_2 = -1$, $\lambda_3 = -3$. Izračunamo še pripadajoče lastne vektorja:

$$\lambda_1 = 0: \quad \vec{u_1}(t) = u_0\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\lambda_2 = -1: \quad \vec{u_2}(t) = u_0 e^{-\gamma t/m}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\lambda_3 = -3: \quad \vec{u_3}(t) = u_0 e^{-3\gamma t/m}\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

Sledi: $\vec{v}(t) = a\vec{u_1}(t) + b\vec{u_2}(t) + cu_3(t)$

Kolokvij 2019. Imamo kroglo z gostoto toplotnih izvorov q. Velja $q = kT^2$. Zanima nas toplotni tok, ki izhaja iz krogle.

$$\frac{\mathrm{d}W}{\mathrm{d}t} = qV - jS$$
$$mc\frac{\mathrm{d}T}{\mathrm{d}t} = qV - \lambda\frac{D}{s}T$$
$$\dot{T} = \frac{k}{\rho c}T^2 - \frac{\lambda}{k\rho c}\frac{3}{R}T$$

Imamo enačbo oblike

$$\dot{T} = A(T^2 - BT)$$
$$A\dot{T} = A^2\left((T - \frac{B}{2})^2 - \frac{B^2}{4}\right) = \left(A\left(T - \frac{B}{2}\right)\right)^2 - \frac{A^2B^2}{4}$$

Vzamemo novo spremenljivko u:

$$\dot{u} = u^2 - \left(\frac{AB}{2}\right)^2 = u^2 - u_0^2$$
$$\mathrm{d}t = \frac{\mathrm{d}u}{u^2 - u_0^2}$$
$$t = \begin{cases} -\frac{1}{u_0}\mathrm{artanh}\frac{u}{u_0} + t_0 & |u| < u_0 \\ -\frac{1}{u_0}\mathrm{arcoth}\frac{u}{u_0} + t_0 & |u| > u_0 \end{cases}$$

Poiščemo lahko inverz:

$$u_1 = -u_0\tanh\left[(t - t_0)u_0\right]$$
$$u_1 = -u_0\coth\left[(t - t_0)u_0\right]$$

Majhna nihanja. Pri majhnih nihanjih imamo posplošene koordinate $\underline{q}$, v katerih mora veljati:

$$\frac{\partial V}{\partial q_i} = 0$$

Konstruiramo matriki $\underline{\underline{V}}$ in $\underline{\underline{T}}$, kjer je

$$V_{jk} = \partial_j \partial_k V$$

$$T_{jk} = \partial_j \partial_k T$$

Nato poiščemo lastne frekvence ω in pripadajoče lastne vektorje, da velja

$$\det(\underline{\underline{V}} - \omega^2 \underline{\underline{T}}) = 0$$

Tedaj je

$$\underline{q}(t) = \mathrm{Span}\{\underline{q}_{last.} e^{i\omega t}\}$$