Dinamika togega telesa.

$$\underline{\underline{J}} = \sum_i m_i (Ir^2 - \overrightarrow{r} \otimes \overrightarrow{r})$$

$$\overrightarrow{r} \otimes \overrightarrow{r} = \begin{pmatrix} x^2 & xy & xz \\ yx & y^2 & yz \\ zx & zy & z^2 \end{pmatrix}$$

Vsoto ocenimo z integralom.

$$\underline{\underline{J}} = \int \mathrm{d}m \begin{pmatrix} x^2 & xy & xz \\ yx & y^2 & yz \\ zx & zy & z^2 \end{pmatrix}$$

$$T = \frac{1}{2} \overrightarrow{\omega}^T \underline{\underline{J}} \overrightarrow{\omega}$$

V lastnem sistemu velja

$$\underline{\underline{J}} = \begin{pmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{pmatrix}$$

V tem sistemu je

$$\overrightarrow{L} = J_x \omega_x' \hat{i}' + J_y \omega_y' \hat{j}' + J_z \omega_z' \hat{k}'$$

$$dotvctL = J_x \dot{\omega}_x' \hat{i}' + J_x \dot{\omega}_x' (\overrightarrow{\omega} \times \hat{i}') + J_y \dot{\omega}_y' \hat{j}' + J_y \dot{\omega}_y' (\overrightarrow{\omega} \times \hat{j}') + J_z \dot{\omega}_z' \hat{k}' + J_z \dot{\omega}_z' (\overrightarrow{\omega} \times \hat{k}')$$

Tako dobimo Eulerjeve enačbe:

$$M_x' = J_x \dot{\omega}_x' - (J_y - J_z) \omega_y' \omega_z'$$
$$M_y' = J_y \dot{\omega}_x' - (J_z - J_x) \omega_z' \omega_x'$$
$$M_z' = J_z \dot{\omega}_x' - (J_x - J_z) \omega_x' \omega_y'$$

Prosta simetrična vrtavka. $J_x = J_y = J \neq J_z$, $\overrightarrow{M} = 0$ Uporabimo Eulerjeve enačbe:

$$0 = J \dot{\omega}_x' + (J - J_z) \omega_y' \omega_z'$$

$$0 = J \dot{\omega}_y' + (J_z - J) \omega_z' \omega_x'$$

$$0 = J_z \dot{\omega}_z'$$

Iz tretje enačbe dobimo $\omega_z' = \omega_0$. Uvedemo novo spremenljivko $\xi = \omega_x' + i\omega_y'$, $\dot{\xi} = \dot{\omega}_x' + i\dot{\omega}_y'$. Seštejemo prvi dve enačbi:

$$J\dot{\xi} + J\omega_0 i\xi - J_z \omega_0 \xi = 0$$

$$\frac{\mathrm{d}\xi}{\xi} = -i\omega_0 \left(1 - \frac{J_z}{J} \mathrm{d}t\right)$$

$$\xi = Ce^{-i\omega_z \left(1 - \frac{J'}{J}\right) t}$$

Označimo $\Omega_p = \omega_z \left(1 - \dfrac{J_z}{J}\right)$

$$\omega_x' = |C| \cos(\Omega_p t)$$

$$\omega_y' = |C| \sin(\Omega_p t)$$

$$\omega_z' = \omega_0$$

Navoj v ležajih pravokotne plošče. Pravokotnik naj ima stranici a in b.

$$J_x = \int (y^2 + z^2)\mathrm{d}m = \iint y^2 \frac{m}{ab}\,\mathrm{d}x\,\mathrm{d}y$$

$$= \int_{-b/2}^{b/2}\int_{-a/2}^{a/2} y^2 \frac{m}{ab}\,\mathrm{d}x\,\mathrm{d}y = ... = \frac{mb^2}{12}$$

$$J_y = ... = \frac{ma^2}{12}$$

$$J_z = \frac{m}{12}(b^2 + a^2)$$

$$J_{xy} \int -xy\,\mathrm{d}m = 0$$

(Liha funkcija na simetričnem intervalu)

$$\overrightarrow{\omega} = \omega_0 \frac{a}{\sqrt{a^2+b^2}} i' + \omega_0 \frac{b}{\sqrt{a^2+b^2}} j'$$

$$M'_x = M'_y = 0$$

$$M'_z = \frac{mab\omega_0^2}{12(a^2+b^2)}(a^2 - b^2)$$

Vstavimo v Eulerjeve enačbe. V posebnem primeru, ko je $a = b$, lahko izračunamo silo F, kajti $M_z = Fr$, $r = \dfrac{a}{2\sqrt{a^2+b^2}}$. Tedaj je

$$F = \frac{mb\,\omega_0^2}{6\sqrt{a^2+b^2}}$$