Stožec na nagnjeni podlagi.

$$J = J_x = J_y = ... = \frac{hR^2\pi}{5}\left(h^2 + \frac{R^2}{4}\right)\frac{m}{V} = \frac{3}{5}m\left(h^2 + \frac{R^2}{4}\right)$$

$$J' = J_z = \int (x^2+y^2)\,\mathrm{d}m = \int_0^{2\pi}\int_0^h\int_0^{\frac{R}{h}z} r^3\rho\,\mathrm{d}r\,\mathrm{d}z\,\mathrm{d}\varphi = 2\pi\rho\int_0^h\left(\frac{r^4}{4}\right)\Big|_0^{\frac{R}{h}z}\mathrm{d}z = ... = \frac{3}{10}mR^2$$

Gibanje stožca bomo izračunali z Lagrangeovim formalizmom, zato poiščimo vezi za Eulerjeve kote.

1. $\vartheta = \frac{\pi}{2} - \alpha = \text{konst.}$
2. $\sqrt{h^2+R^2}\dot{\varphi} + R\dot{\psi} = 0$

$$\dot{\psi} = -\frac{1}{\sin\alpha}\dot{\varphi}$$

$$T = \frac{1}{2}J\left(\dot{\varphi}^2\sin^2\vartheta + \dot{\vartheta}^2\right) + \frac{1}{2}J'\left(\dot{\varphi}\cos\vartheta + \dot{\psi}\right)^2 =$$

$$= \dot{\varphi}^2\left(\frac{1}{2}J\cos\alpha + \frac{1}{2}J'\left(\sin^2\alpha - 2 + \frac{1}{\sin^2\alpha}\right)\right) = \frac{1}{2}J_{eff}\dot{\varphi}^2$$

$$V = mg\frac{3}{4}h(-\cos\alpha\sin\beta\cos\varphi + \sin\alpha\cos\beta)$$

(višina težišča v odvisnosti od kota φ).

$$L = \frac{1}{2}J_{eff}\dot{\varphi}^2 - mg\frac{3}{4}h\left(\cos\alpha\sin\beta\cos\varphi + \sin\alpha\cos\beta\right)$$

$$\frac{\partial L}{\partial\varphi} = \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial\dot{\varphi}}J_{eff}\ddot{\varphi} = -mg\frac{3}{4}h\cos\alpha\sin\beta\sin\varphi$$

Za $\varphi \ll 1$:

$$-mg\frac{3}{4}h\cos\alpha\sin\beta\varphi = J_{eff}\ddot{\varphi}$$

Dobimo nihanje s kotno hitrostjo $\omega = \frac{3}{4}\frac{mgh\cos\alpha\sin\beta}{J_{eff}}$, pri čemer je

$$J_{eff} = J\sin^2\left(\frac{\pi}{2}-\alpha\right) + J'\left(\cos\left(\frac{\pi}{2}-\alpha\right) - \frac{1}{\sin\alpha}\right)^2 = ... = \frac{h^2}{h^2+R^2}m\left(\frac{9}{10}h^2 + \frac{3}{20}R^2\right)$$

Mala nihanja.

$$L = T - V = \frac{1}{2}\sum_{ij} w_{ij}(\underline{q})q_iq_j - V$$

Stabilna ravnovesna lega:

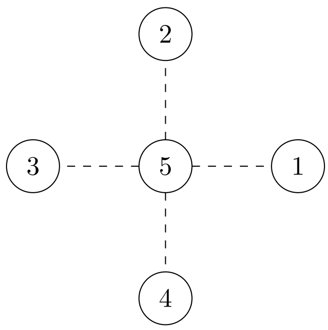
$$\frac{\partial V}{\partial q_i}\Big|_{\underline{q}^*} = 0, \quad \frac{\partial^2 V}{\partial q_i\partial q_j} > 0$$

$$\eta_i = q_i - q_i^*$$

$$L = \frac{1}{2}\sum_{ij} w_{ij}(\underline{q}^0)\dot{\eta}_i\dot{\eta}_j - V(\underline{q}^0) - \frac{1}{2}\sum_{ij}\frac{\partial^2 V}{\partial q_i\partial q_j}$$

$$L = \frac{1}{2}\dot{\eta}\underline{\underline{T}}\dot{\eta} + \eta\underline{\underline{V}}\eta$$

Naloga. Pet mas, povezanih z vzmetmi.



$$T = \sum_{i=1}^{5} \frac{1}{2} m \dot{z}_i^2$$

$$V = \sum_{i=1}^{4} k \left(\sqrt{l^2 + (z_5 - z_j)^2} + l_0^2 \right)^2 = \sum_{i=0}^{4} k \left(l \sqrt{1 + \left(\frac{z_5 - z_i}{l} \right)^2} - l_0 \right)$$

Upoštevamo $(1+x)^n \approx 1 + nx$ za dovolj majhne x.

$$V = \sum_{i=1}^{4} \frac{1}{2} k \left(\frac{1}{2} (z_5 - z_i)^2 + l_0^2 \right)^2$$

$$= V_0 + \frac{1}{2} \sum_{ij} V_{ij} z_i z_j$$

$$\underline{\underline{V}} \underline{a} = \omega^2 \underline{\underline{T}} \underline{a} = \omega^2 m \underline{a}$$

$$V = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 1 & -1 \\ -1 & -1 & -1 & -1 & -4 \end{bmatrix}$$

Imamo lastne vektorje

$$a_1 = \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \\ 0 \end{pmatrix}, \quad \underline{\underline{V}} a_1 = \tilde{k} a_1 = \omega_1^2 m \underline{a_1}$$

$$\omega_1 = \sqrt{\frac{k}{m}}$$

Podobno dobimo za $a_2 = (1, -1, -1, 1, 0)$ in $a_3 = (1, -1, 1, -1, 0)$. Imamo še dva lastna vektorja:

$$a_4 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad \underline{\underline{V}} a_4 = 0 \to \omega_4 = 0$$

Pri a_4 gre v bistvu za translacijo.

$$a_5 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ -\alpha \end{pmatrix}, \quad \underline{\underline{V}} a_5 = \lambda a_5$$

Imamo dve možnosti za λ, in sicer $\lambda = 0$ in $\lambda = 4$. Če je $\lambda = 0$, smo spet dobili translacijo, za $\lambda = 4$ pa dobimo:

$$\omega_5 = \sqrt{\frac{4k}{m}}$$