2. izpit 2023 Podane imamo sledeče podatke:

$$\partial_x T = \frac{\Delta T}{L}$$

$$\overrightarrow{j} = (j_x, 0)$$

$$\varphi(y) = \frac{y}{H}\Delta\varphi$$

Zanimata nas $j_x(y)$ in P.

$$\overrightarrow{j} = -\underline{\underline{\lambda}}\nabla T$$

V lastnem sistemu je $\underline{\underline{\lambda}} = \begin{pmatrix} \lambda_\perp & 0 \\ 0 & \lambda_\parallel \end{pmatrix}$, hkrati vemo:

$$\overrightarrow{j} \cdot \widehat{e}_y = -(\underline{\underline{\lambda}}\nabla T) \cdot \widehat{e}_y = 0$$

$$\widehat{e}_y^T \left(\lambda \partial_x T \widehat{e}_x + \lambda \partial_y T \widehat{e}_y\right) = 0$$

Vemo: $\hat{e}_i^T \lambda \hat{e}_j = \lambda_{ij}$

$$\lambda_{xy}\frac{\Delta T}{L} + \lambda_{yy}\partial_y T = 0$$

$$\partial_y T = -\frac{\lambda_{xy}}{\lambda_{yy}}\frac{\Delta T}{L}$$

$$j_x = \overrightarrow{j} \cdot \widehat{e}_x = -\widehat{e}_x^T\left(\lambda\frac{\Delta T}{L}\widehat{e}_x + \lambda\partial_y T\widehat{e}_y\right) =$$

$$= -\left(\lambda_{xx}\frac{\Delta T}{L} - \lambda_{xy}\frac{\lambda_{xy}}{\lambda_{yy}}\frac{\Delta T}{L}\right) = -\frac{\lambda_{xx}\lambda_{yy} - \lambda_{xy}^2}{\lambda_{yy}}\frac{\Delta T}{L} = \frac{\det(\lambda)}{\lambda_{yy}}\frac{\Delta T}{L}$$

$\det(\lambda)$ je enaka v lastnem sistemu, torej hitro vidimo $\det(\lambda) = \lambda_\perp\lambda_\parallel$. λ_{yy} dobimo tako, da v lastnem sistemu zapišemo vektorja $\widehat{e}_x$ in $\widehat{e}_y$. Tedaj je $\lambda_{yy} = \widehat{e}_y^T \lambda^{\text{last.}} \widehat{e}_y = \lambda_\perp \sin^2\varphi + \lambda_\parallel \cos^2\varphi$.
Sledi

$$j_x(\varphi) = -\frac{\Delta T}{L}\frac{\lambda_\perp\lambda_\parallel}{\lambda_\perp\sin^2\varphi + \lambda_\parallel\cos^2\varphi}$$

Iz tega lahko hitro izrazimo $j_x(y)$.

$$P = -\int j_x \mathrm{d}S = \frac{\Delta T}{L} r \lambda_\parallel \lambda_\perp \int_{-H/2}^{H/2} \frac{\mathrm{d}y}{\lambda_\perp\sin^2\varphi + \lambda_\parallel\cos^2\varphi}$$

Pri tem smo označili $\mathrm{d}S = r\mathrm{d}y$ - predpostavimo, da je plošča pravokotna s stranicama r in H. Zdaj bomo vzeli novo spremenljivko, in sicer je $y = \dfrac{H}{\Delta\varphi}\varphi$. Konstante pred integralom bomo združili v konstanto α.

$$P = \alpha\int_{-\Delta\varphi}^{\Delta\varphi}\frac{\mathrm{d}\varphi}{\lambda_\perp\sin^2\varphi + \lambda_\parallel\cos^2\varphi}$$

Vzamemo substitucijo $u = \tan\varphi$, $\mathrm{d}u = \dfrac{\mathrm{d}\varphi}{\cos^2\varphi}$

$$P = \alpha\int_{-\tan(\Delta\varphi/2)}^{\tan(\Delta\varphi/2)}\frac{1/\lambda_\perp}{u^2 + \frac{\lambda_\parallel}{\lambda_\perp}}$$

$$P = \frac{\Delta T \cdot S}{L} \cdot \frac{2\sqrt{\lambda_\perp\lambda_\parallel}}{\Delta\varphi}\arctan\left(\sqrt{\frac{\lambda_\perp}{\lambda_\parallel}}\tan\left(\frac{\Delta\varphi}{2}\right)\right)$$

V limiti $\lambda_\perp = \lambda_\parallel$ preverimo:

$$P = \frac{\Delta T \cdot S}{L}\lambda$$

Pismena vaja, 1987 (v učbeniku vaja 192) Imamo palico, ki jo zvijemo. V nekem koordinatnem sistemu je

$$\mu = \begin{pmatrix} \mu_2 & & \\ & \mu_2 & \\ & & \mu_1 \end{pmatrix}$$

Tu sta μ_2 in μ_1 pravokotna na smer palice, μ_1 pa je vzporedna. Vklopimo navor, ki je tangenten na enega od koncev palice. Zanima nas navor na palico (označimo s $\overrightarrow{T}$).

$$\overrightarrow{T} = \overrightarrow{p_m} \times \overrightarrow{B}$$

$$\overrightarrow{M} = \underline{\underline{\chi}}\overrightarrow{H} + \mathcal{O}(H^2) \approx \frac{\mathrm{d}\overrightarrow{p_m}}{\mathrm{d}V}\overrightarrow{H}$$

$$\mathrm{d}\overrightarrow{T} = \underline{\underline{\chi}}\overrightarrow{H} \times \overrightarrow{B}\,\mathrm{d}V$$

$$\overrightarrow{H} = \frac{1}{\mu_0}\overrightarrow{B}, \quad \underline{\underline{\chi}} = (\underline{\underline{\mu}} - I)$$

Vemo: $I\overrightarrow{B} \times \overrightarrow{B} = 0$

$$\mathrm{d}\overrightarrow{T} = (\underline{\underline{\mu}}\overrightarrow{B}) \times \overrightarrow{B}\,\frac{1}{\mu_0}\,S\mathrm{d}l$$