

Eulerjevi koti. φ je kot precesije:

$$R_\varphi = \begin{bmatrix} \cos \varphi & \sin \varphi & 0 \\ -\sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

 θ je kot nutacije:

$$R_\theta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{bmatrix}$$

 ψ je kot zasuka:

$$R_\psi = \begin{bmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Vrtenje telesa tedaj opišemo kot

$$\vec{\omega} = \dot{\varphi} \hat{k} + \dot{\theta} \hat{x}'' + \dot{\psi} \hat{k}' = \omega'_x \hat{x}' + \omega'_y \hat{y}' + \omega'_z \hat{k}'$$

Izraziti moramo bazne vektorje (\hat{k}, \hat{x}'') :

$$k = R_\psi R_\theta R_\varphi \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \dots = \begin{bmatrix} \sin \psi \sin \theta \\ \cos \psi \sin \theta \\ \cos \theta \end{bmatrix}$$

$$\hat{x}'' = R_\psi R_\theta \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \dots = \begin{bmatrix} \cos \psi \\ -\sin \psi \\ 0 \end{bmatrix}$$

$$\omega'_x = \dot{\varphi} \sin \psi \sin \theta + \dot{\theta} \cos \psi$$

$$\omega'_y = \dot{\varphi} \cos \psi \sin \theta - \dot{\theta} \sin \psi$$

$$\omega'_z = \dot{\varphi} \cos \theta + \dot{\psi}$$

Kinetična energija simetrične vrtavke.

$$T = \frac{1}{2} \vec{\omega} \underline{\underline{J}} \vec{\omega}$$

$$\underline{\underline{J}} = \begin{bmatrix} J_{\parallel} & & \\ & J_{\perp} & \\ & & J_{\perp} \end{bmatrix}$$

Stožec na nagnjeni podlagi.

$$J_x = J_y = \int (y^2 + z^2) dm$$

Uporabimo cilindrične koordinate:

$$x = r \cos \varphi, \quad y = r \sin \varphi, \quad dV = r dr d\varphi dz$$

$$J_x = \int_0^{2\pi} \int_0^R \int_0^h \rho (r^2 \sin^2 \varphi + z^2) r dr d\varphi dz$$