Delec v magnetnem polju. Imamo zunanje magnetno polje $\overrightarrow{B}$, velja:

$$l = 1$$

$$H = -\lambda \overrightarrow{L} \cdot \overrightarrow{B}$$

$$(\overrightarrow{L} \cdot \widehat{n}) \left|\psi, 0\right\rangle = \hbar \left|\psi, 0\right\rangle$$

Poznamo tudi časovni razvoj ψ, in sicer:

$$\left|\psi, t\right\rangle = \sin^2 \frac{\vartheta}{2} e^{-i\lambda Bt} \left|1, \text{-}1\right\rangle + \sqrt{2} \sin \frac{\vartheta}{2} \cos \frac{\vartheta}{2} \left|1, 0\right\rangle + \cos^2 \frac{\vartheta}{2} e^{i\lambda Bt} \left|1, 1\right\rangle$$

Poglejmo razvoj $\left\langle L_z, t\right\rangle$:

$$\left\langle\psi, t\right| L_z \left|\psi, t\right\rangle = \left\langle\psi, t\right| \left(-\hbar \sin^2 \frac{\vartheta}{2} e^{-i\lambda Bt} \left|1, \text{-}1\right\rangle + \hbar \cos^2 \frac{\vartheta}{2} \left|1, 1\right\rangle\right)$$

$$= \sin^2 \frac{\vartheta}{2} e^{-\lambda Bt} \left\langle 1, \text{-}1\right| - \hbar \sin^2 \frac{\vartheta}{2} e^{-i\lambda Bt} \left|1, \text{-}1\right\rangle + \cos^2 \frac{\vartheta}{2} e^{-i\lambda Bt} \left\langle 1, 1\right| \cos^2 \frac{\vartheta}{2} e^{-i\lambda Bt} \left|1, 1\right\rangle$$

$$= -\hbar \sin^4 \frac{\vartheta}{2} + \hbar \cos^4 \frac{\vartheta}{2} = \hbar \left(\cos^2 \frac{\vartheta}{2} + \sin^2 \frac{\vartheta}{2}\right) \left(\cos^2 \frac{\vartheta}{2} - \sin^2 \frac{\vartheta}{2}\right) = \hbar \cos \vartheta$$

Zdaj izračunajmo časovni razvoj L_x in L_y:

$$\left\langle L_x, t\right\rangle = \left\langle\psi, t\right| \frac{L_+ + L_-}{2} \left|\psi, t\right\rangle = \frac{1}{2} \left(\left\langle L_+, t\right\rangle + \left\langle L_-, t\right\rangle\right) = \frac{1}{2} \left(\left\langle L_+, t\right\rangle + \left\langle L_+^\dagger, t\right\rangle\right) = \mathfrak{Re}\left(\left\langle L_+, t\right\rangle\right)$$

$$\left\langle L_y, t\right\rangle = \left\langle\psi, t\right| \frac{L_+ - L_-}{2i} \left|\psi, t\right\rangle = \frac{1}{2i} \left(\left\langle L_+, t\right\rangle - \left\langle L_-, t\right\rangle\right) = \frac{1}{2i} \left(\left\langle L_+, t\right\rangle + \left\langle L_+^\dagger, t\right\rangle\right) = \mathfrak{Im}\left(\left\langle L_+, t\right\rangle\right)$$

Tako moramo izračunati samo $\left\langle L_+, t\right\rangle$:

$$\left\langle\psi, t\right| L_+ \left|\psi, t\right\rangle = ?$$

Vemo:

$$L_+ \left|1, \text{-}1\right\rangle = \sqrt{2}\hbar \left|1, 0\right\rangle$$

$$L_+ \left|1, 0\right\rangle = \sqrt{2}\hbar \left|1, 1\right\rangle$$

$$L_+ \left|1, 1\right\rangle = 0$$

To vstavimo v skalarni produkt $\left\langle\psi, t\right| L_+ \left|\psi_t\right\rangle$ (vsega nimam časa prepisati) in dobimo

$$\left\langle L_+, t\right\rangle = \hbar \sin \vartheta \cos(\lambda Bt) - i \sin \vartheta \sin(\lambda Bt)$$

Sledi:

$$\left\langle L_z, t\right\rangle = \hbar \cos \vartheta$$

$$\left\langle L_x, t\right\rangle = \hbar \sin \vartheta \cos(\lambda Bt)$$

$$\left\langle L_y, t\right\rangle = \hbar \sin \vartheta \cos(\lambda Bt)$$

Opisanemu gibanju rečemo Larmorjeva precesija.

Opomba. Ko izmerimo L_z, dobimo eno od možnih lastnih vrednosti, pri naslednjih meritvah pa dobimo le vrednost, v katero je kolapsiralo stanje pri prvotni meritvi. Izračunamo lahko tudi verjetnost za posamezno stanje.

L_z	$p = \lvert c_m\rvert^2$	ψ ob $t +$ dt
$\hbar$	$\cos^4 \frac{\vartheta}{2}$	$\left\lvert 1, 1\right\rangle$
0	$2\cos^2 \frac{\vartheta}{2}$	$\left\lvert 1, 0\right\rangle$
$-\hbar$	$\cos^4 \frac{\vartheta}{2}$	$\left\lvert 1, -1\right\rangle$