

Gaussian width 1

$$\left\{ \begin{array}{l} r(g_1\cos(\alpha_1)\dots\cos(\alpha_{n-1})+g_2\sin(\alpha_2)\cos(\alpha_3)\dots\cos(\alpha_{n-1})+g_3\sin(\alpha_2)\cos(\alpha_3)\dots\cos(\alpha_{n-1})+\dots+g_{n-1}\sin(\alpha_{n-1}))\rightarrow\max_{r,\alpha_1,\dots,\alpha_{n-1}}, \\ 0\leq r\leq 1, \\ 0\leq \alpha_1\leq 2\pi, \\ \frac{-\pi}{2}\leq \alpha_2\leq \frac{\pi}{2}, \\ \dots \\ \frac{-\pi}{2}\leq \alpha_{n-1}\leq \frac{\pi}{2}. \end{array} \right.$$

In [205...

```
from scipy.optimize import minimize
import numpy as np
import seaborn as sns
from matplotlib import pyplot as plt
%matplotlib inline
```

In [197...

```
def gen_vector(m, n):
    g = np.random.normal(0, 1, m * n)
    return g

def f(x, g):
    n = len(x)
    sum = 0
    for i in range(n):
        cur = 1
        if i != 0:
            cur = np.sin(x[i])
            for j in range(i, n - 1):
                cur *= np.cos(x[j + 1])

        else:
            cur = 1
            for j in range(n - 1):
                cur *= np.cos(x[j + 1])
        sum += g[i] * cur
    return -x[0] * sum

def width(g):
    n = len(g)
    x0 = np.zeros(n)
    bounds = [(0, 1), (0, 2 * np.pi)]
    for i in range(2, n):
        bounds.append((-np.pi / 2, np.pi / 2))
    res = minimize(f, x0, args=(g), tol=1e-6, bounds=bounds)
    return -f(res.x, (g))
```

In [211...

```
x = []
y = []
for i in range(2, 101):
    sum = 0
    cnt = 0
    for j in range(10):
        cur = width(gen_vector(i, 1))
        if cur != 0:
            cnt += 1
            sum += cur
    x.append(i)
    y.append(sum / cnt)
```

In [212...

```
plt.figure(figsize=(10, 5))
sns.scatterplot(x, y)
```

/home/matfu/anaconda3/lib/python3.8/site-packages/seaborn/_decorators.py:36: FutureWarning: Pass the following variables as keyword args: x, y. From version 0.12, the only valid positional argument will be `data`, and passing other arguments without an explicit keyword will result in an error or misinterpretation.

warnings.warn(

Out[212...

