Physics of NBI heating and current drive

Lecture by P. Vincenzi

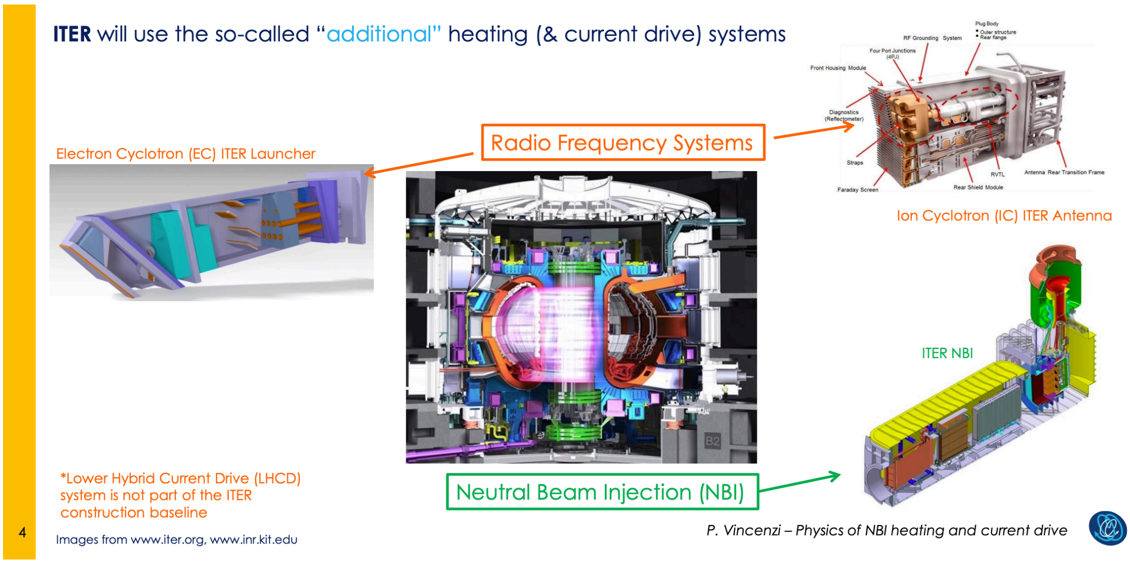
We will go through the physics of NBI.

Contents

A special acknowledgement to Chiara De Piccoli for helping me with the material in this presentation

The additional H&CD systems

Let's start with the systems for auxiliary heating power.



ICRH will contribute with 10MW while NBI with two lines.

A step back

We are talking about additional heating system.

***Additional* to…what?**

- Let's assume here we are dealing with toroidal pinches (tokamak, RFP, …). Initial considerations on stellarator devices would be a bit different.
- We will mention mainly **3 actions**: heating, current drive, and momentum injection. Spend some moments and try to think what this means in a plasma.

There is actually a fourth action that is **fueling** because we are injecting particles, but as we will see in future devices with very big volume of plasma this is negligible.

These actions clearly affects several quantities:

- There are fundamental quantities that can be linked to these actions:
  - heating→ temperature (T)
  - current drive→ current density (j)
  - momentum drive→ plasma fluid rotation (W)
- You will see that not only average values, but also radial profiles (and their time evolution!) are important to understand H&CD role.

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So we can have a local effect on these quantities thanks to the heating and current drive additional heating system.

0D power balance

So let's try to understand what is the role of heating and current drive systems in plasmas. We can start from a very simple and zero dimensional power balance equation in stationary conditions.

0D power balance in stationary conditions

$$P_{aux} + P_{\alpha} + P_{ohmic} = P_{loss}$$

P_{aux} = External heating sources
P_{α} = Alpha power (~1/5 P_{fus})
P_{ohmic} = Ohmic power
P_{loss} = Power losses (radiation, conduction)

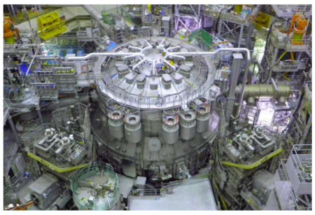
J. D. Lawson, (1957) Proc. Phys. Soc. B 70 303

In the following we have some examples of application of this formula:

Current experiments (tokamaks):

$P_\alpha \sim 0$

$\rightarrow P_{aux} + P_{ohmic} = P_{loss}$

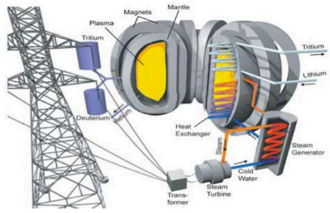


JT-60SA experiment

Future reactors:

Aiming at ignition (plasma self-sustainment)

➢ P_{ohmic} negligible, $P_{aux} = 0 \rightarrow P_\alpha = P_{loss}$



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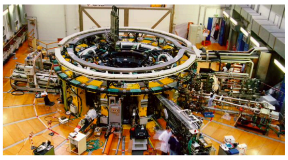
In future reactors there won't be any auxiliary power system.

In RFP no auxiliary heating and no P_α

Current experiments (RFP):

If $P_{aux} = 0$ and $P_\alpha \sim 0$

$\rightarrow P_{ohmic} = P_{loss}$

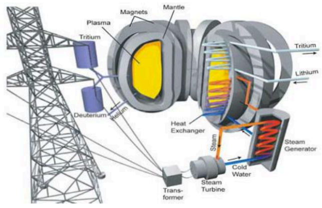


RFX-mod experiment, IT

Future reactors:

Aiming at ignition (plasma self-sustainment)

➢ P_{ohmic} negligible, $P_{aux} = 0 \rightarrow P_\alpha = P_{loss}$



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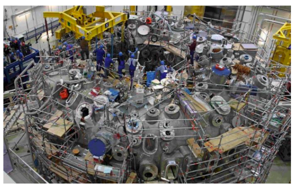
In RFX we can go to very high plasma current so high ohmic heating.

Current experiments (stellarator):

If $I_p = 0 \rightarrow P_{ohmic} = 0$ and $P_\alpha \sim 0$

$\rightarrow P_{aux} = P_{loss}$

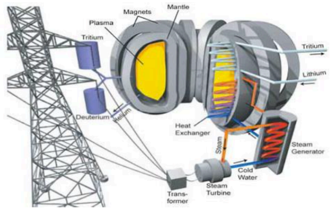


W7-X experiment, DE

Future reactors:

Aiming at ignition (plasma self-sustainment)

➢ P_{ohmic} negligible, $P_{aux} = 0 \rightarrow P_\alpha = P_{loss}$



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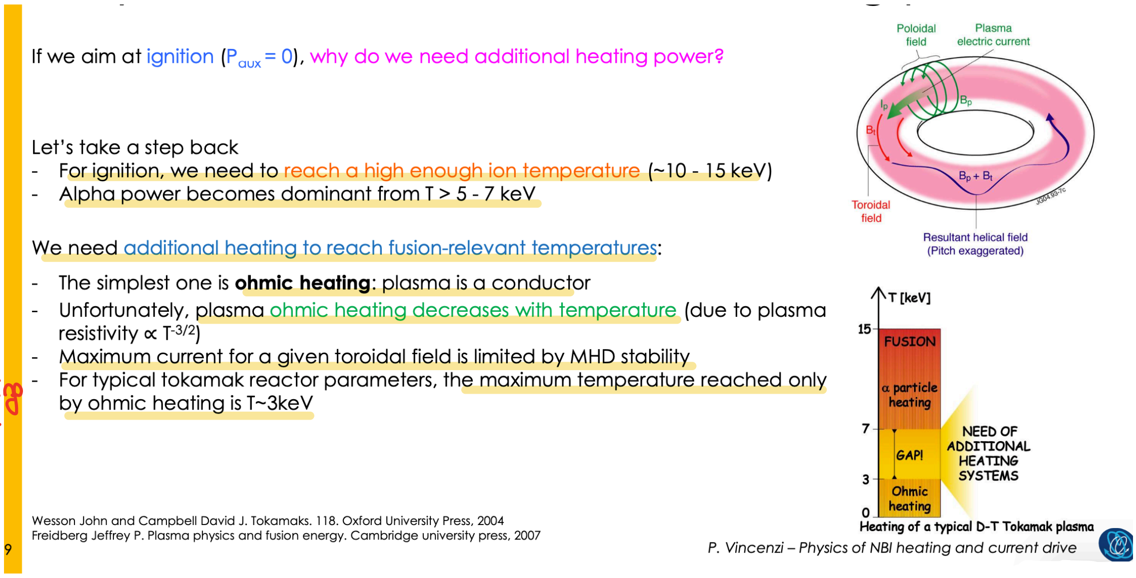
Most of stellarators want to operate without plasma current. Paux is the only source of power inside the plasma to balance the power losses.

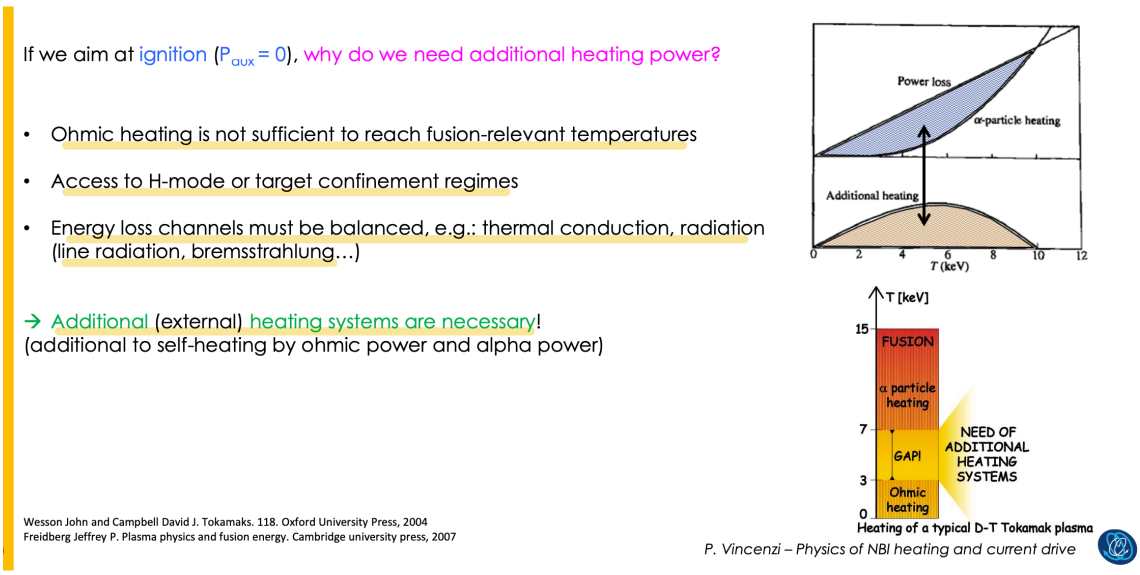
Why do we need additional heating power?

We said that in future reactors the ideal condition is to have zero external heating, so why do we need that?

need for additionae heating because ohmic heating is not high enou



To reach fusion relevant temperatures, the simplest way is to have a current in plasma because the plasma is a conductor and relying on ohmic heating. Unfortunately, the ohmic heating decreases with plasma temperature due to plasma resistivity. In addition we cannot increase as much as we want plasma current because of MHD stability.
Thus, for typical plasma parameters we can reach about 3kV with only ohmic heating. This is not enough to reach temperatures where alpha heating becomes dominant.
In addition in every phase of discharge we want to have access to high-confinement mode and to do this we have to count a lot of power to the transport.



Then there can be some unwanted or unforeseen events during the plasma (?), so we need some external heating to bring back the plasma to stationary conditions.

This is why additional heating systems are necessary also for future reactors. We need them also to control the plasma in terms of MHD.

Need additional heating
for 0D power balance
to have better performance
and to control MHD instabilities

...not only heating

Not only
to heat
the plasma
but to
sustain it

Actually, we will see that we don't only need heating for 0D power balance, but also localized heating (change of temperature radial profile) to have generally better performance and control/suppress MHD instabilities.
Moreover, additional heating systems are able not only to heat the plasma but also to sustain the plasma in many other ways, if specifically designed. These capabilities are well-known and exploited in fusion experiments.

Additional H&CD systems can drive plasma current (both RF and NBI).
It is helpful for non-inductive plasma scenarios and to ramp-up plasma current.
Current drive (CD) is achieved:
- Accelerating a plasma species (RF systems)
- Injecting fast particles, which become fast ions with a preferential toroidal direction of their velocity (tangential NBI)

Other general capabilities (different system by system):
- Torque generation
- Temperature and current profile shaping
- Plasma control (instability suppression, core impurity accumulation counteraction, assisted discharge initiation and termination...)
- Plasma fuelling
- Wall cleaning and conditioning
- Stabilize MHD modes
- ...

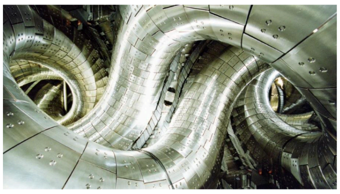
Nota: non inductive plasma scenario is when we do not rely on the central solenoid to generate the plasma but we use external heating systems to drive a current.

... and not only tokamak

We are mainly referring to tokamaks but we can use additional heating also for:

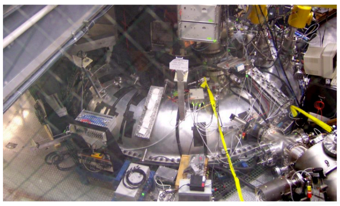
Helical devices:
The main aim is to heat the plasma (no P_{ohmic}) and balance power losses. A careful set of the systems is necessary to keep $I_p = 0$

- W7-X stellarator: ECRH, NBI, ICRH
- LHD heliotron: NBI, ICRH, ECRH
- ...



RFP devices:
H&CD systems mainly to increase performances (temperature, current)

- MST, Madison: NBI + Electron Bernstein wave current drive
- RFX-mod2? NBI?



H&CD system capabilities

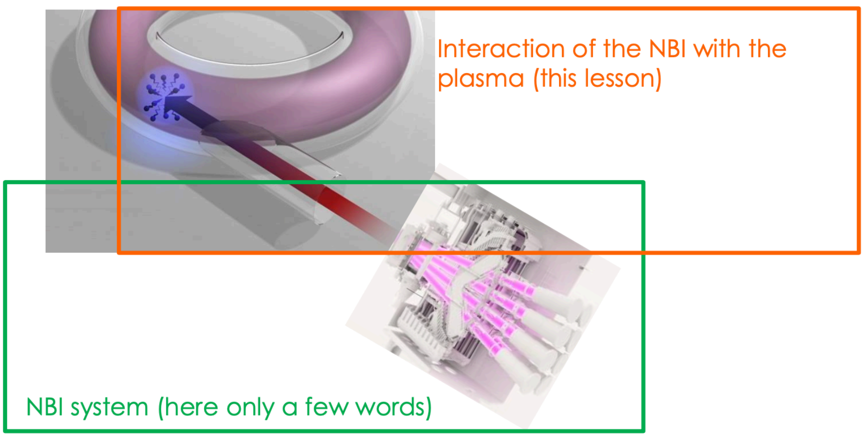
Depending on the system we can have different capabilities. There is no one system that can do everything. Thus, we use a mix of different systems.

Tasks	EC	NBI	IC
Break down & Plasma start up	X		
Plasma Current Ramp up and H-mode access	X	partially	X
Electron Heating	X	X	X
Ion Heating		X	X
Current drive	X	X	*
MHD Control (NTM & ST)	X	partially	
Fast Particle Generation		X	X
Profiles Control (locally)	X		X
Impurity Accumulation Avoidance	X	X	X
Momentum injection & Control		X	
Transport Studies	X	X	
Diagnostics	X	X	
Wall Cleaning	X		X

*fast, low frequency, ICRF waves, low efficiency

Neutral beam injection

In this lesson we will focus on **Neutral Beam Injection** (NBI) and particularly on the interaction of this system with the plasma.



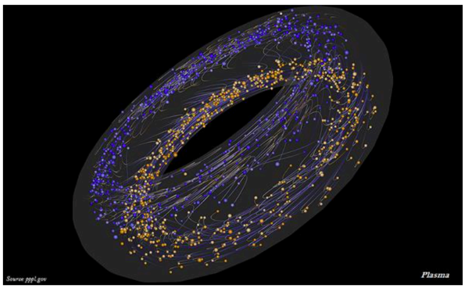
Neutral beam injection principle

Why do we consider high energy neutral particles?

why neutral particles?

- The idea is to inject high energy particles >> energy of the plasma thermal particles
- First of all, entering beam particles must be neutral otherwise they would be deflected by magnetic fields in the plasma region (and surrounding zone).
- Fast neutral particles follow straight lines (injection direction) until they are ionized by the plasma, becoming fast ions.
- Then, thanks to Coulomb collisions, the energy is transferred to the plasma (slowing down process).
- Since "fast" electrons and ions coming from NBI neutral particles have the same injection velocity, the more massive ions carry almost all the beam energy (we will only speak about fast ions).

high energy particles are slowing down after collisions



NB: slowing down process because the high energy particles are slowing down after collisions.

In this process we are generating fast ions and fast electrons. Since the most of the energy is carried out by fast ions because they are massive, we can neglect the generation of electrons when there is the ionization process.
We will talk only about fast ions but actually during the ionization process we also generate electrons.

NBI often dominant in experiments

	R_0 [m]	a [m]	I_p [MA]	B_t [T]	Installed heating power [MW]				
					P-NBI	N-NBI	ECRH	ICRH	LH
ITER	6.2	2.0	15	5.3	-	33-50	60-67	10-20	-
DTT	2.19	0.7	5.5	6	-	10	32	8	-
JT60-SA	2.97	1.17	5	2.25	24	10	7	-	-
JET	2.96	1.25	4.8	3.45	34	-	-	10	7
AUG	1.65	0.5	1.2	3.1	20	-	5	4	-
DIII-D	1.67	0.67	2.0	2.2	20	-	6	8	-
EAST	1.7	0.4	1.0	3.5	8	-	4	12	10

NBI is on fire!

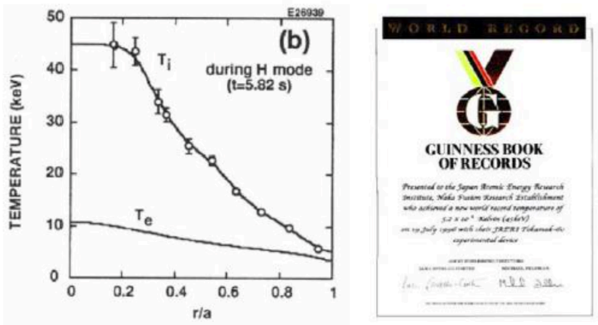


FIG. 5.12. T_i and T_e profiles for the world's highest central ion temperature in JT-60U and Guinness Book of Records on achievement of 45 keV in 1996 [5.22].

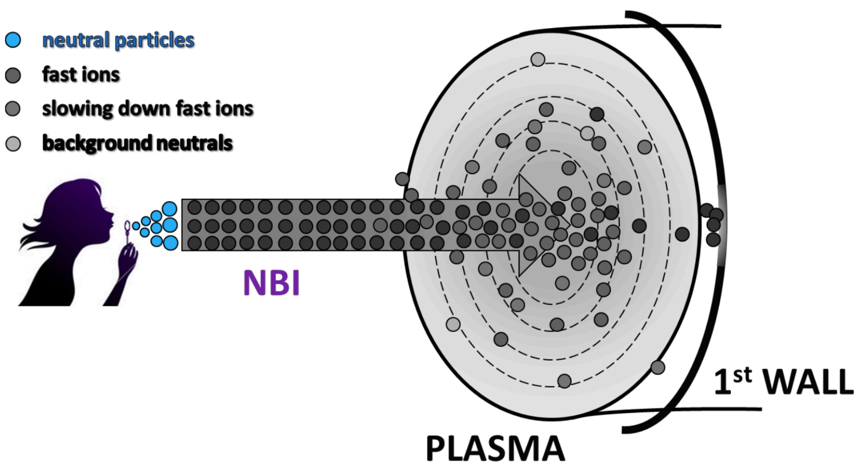
P_{NBI} = 27 MW
E_{NBI} = 92 keV (ion heating mainly)

Mitsuru Kikuchi, Karl Lackner, Minh Quang Tran, "Fusion Physics", IAEA, 2012

This is a figure from JT-60U: 45 keV of ion temperature were reached with low injection energy. We will learn that if we lower the injection energy we are heating more the ions than electrons.

Beam generation

First of all we want to have a beam of neutral particles.

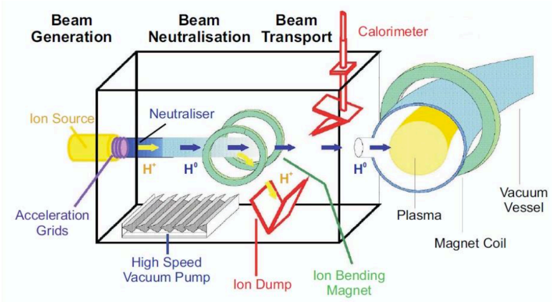


Neutral beam generation — Neutral particle journey — Beam ionization — Fast particle slowing down →

The **NBI system** is composed by:
- Ion source (positive or negative)
- Acceleration grids
- Neutralizer
- Residual ion dump

↓

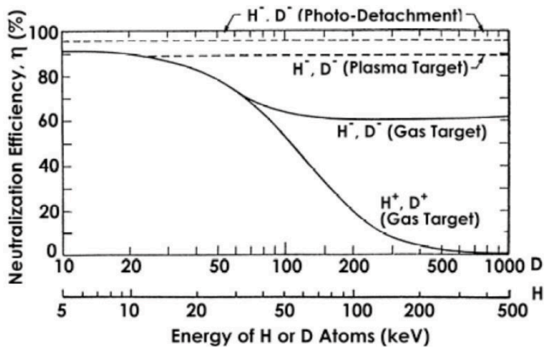
Elements of an NBI system



The objective of the system is to provide a stable and possibly uniform fast particle beam (usually composed by several *beamlets*) for the duration of the plasma discharge, of a given species, at a selected energy* and power, injected in a direction determined by the system geometry.

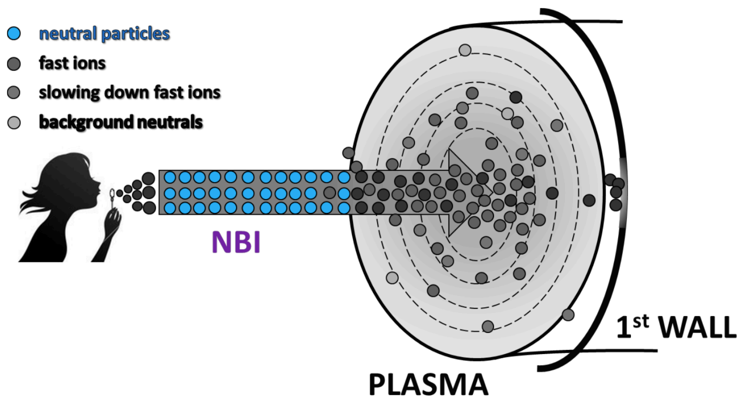
*or energies in case of positive ion source (ionized molecules)

NBI system: negative or positive ion source depending on the designed E_{NBI}



For positive ion sources we have different energy components of the beam:
molecular ions are created (side effect), and are accelerated with the same voltage
→ when e.g. a molecular H_n is first dissociated and then ionized, each H^+ will have energy E_0/n (shorter plasma penetration)

Neutral particle journey



Neutral beam generation — **Neutral particle journey** — Beam ionization — Fast particle slowing down →

Aimed beam of fast particles

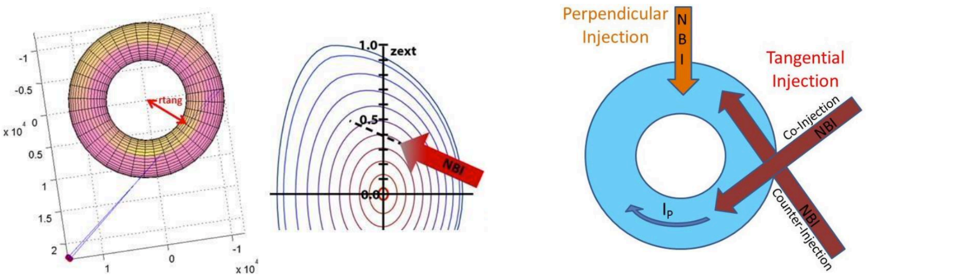
Assume to have a neutral system which is working.

→ with their own divergence

We have a beam (usually composed of different *beamlets*) of fast neutral particles travelling through the NBI system duct, passing the wall opening (port) and entering the plasma.

Relevant NBI parameters from plasma point of view are:
- Energy (E_{NBI})
- Power (P_{NBI})
- Injection geometry: tangency radius, vertical tilt, beam focus, beamlet divergence

NBI parameters



Recently some NBI systems have been designed to be able to modulate power and energy during the discharge, within certain constraints (e.g. in DIII-D tokamak and planned for ITER and DTT)

The beam is usually formed by several beamlets with their own divergence (e.g. 1280 beamlets for SPIDER).

How NBI parameters are chosen?

Energy

- **Low**: small plasma/low density, high input torque
- **High**: big plasma/high density, current-drive

Low injection energy ~ tens of kV up to 100 kV

High injection energy ~ hundreds of kV

Direction

→ increases the possibility of ionization of the beam inside the plasma

- (Nearly) **Parallel** to magnetic field: longer path into dense plasma, current-drive/torque input
  - **Co-current** (usual choice): current drive, good coupling with plasma
  - **Counter-current**: counter torque/current to control rotation and plasma current, larger losses
- (Nearly) **Perpendicular** to magnetic field: technologically easier, shorter path in the plasma, risk of larger losses

The parallel direction increases the possibility of ionizing the beam inside the plasma.

direction towards the center of the plasma

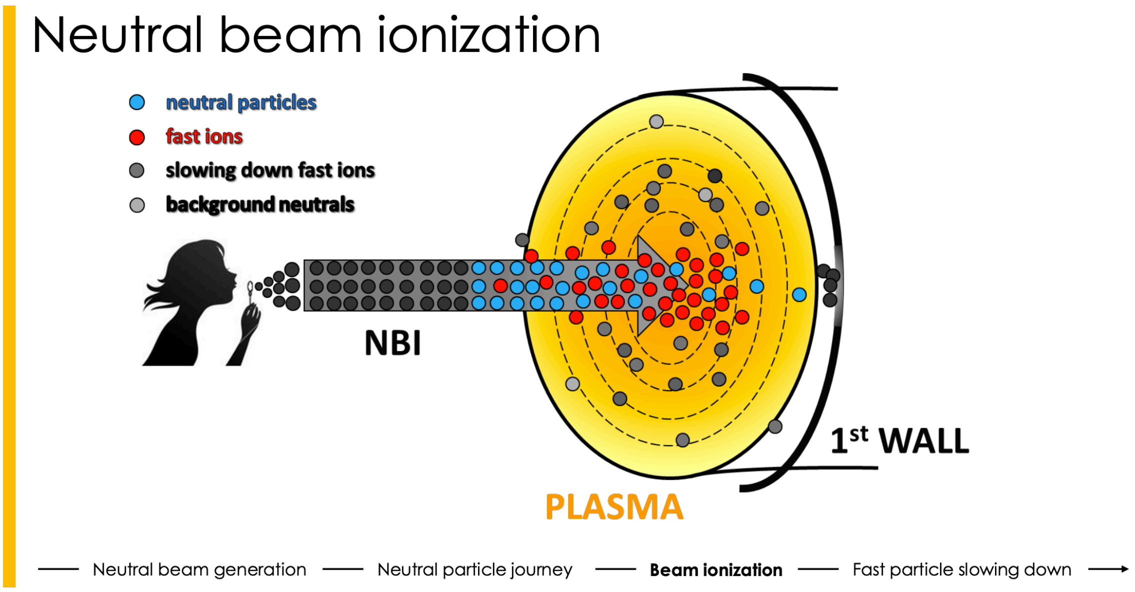
- **On-axis** (usual choice): longer path through dense plasma, central heating
- **Off-axis**: current-drive

not in the center

If we imagine the poloidal cross section of the plasma, the direction can be directed to the center of the plasma —> on axis injection.

If we are not aiming at the center of the plasma we talk about off axis injection.

Neutral beam ionization



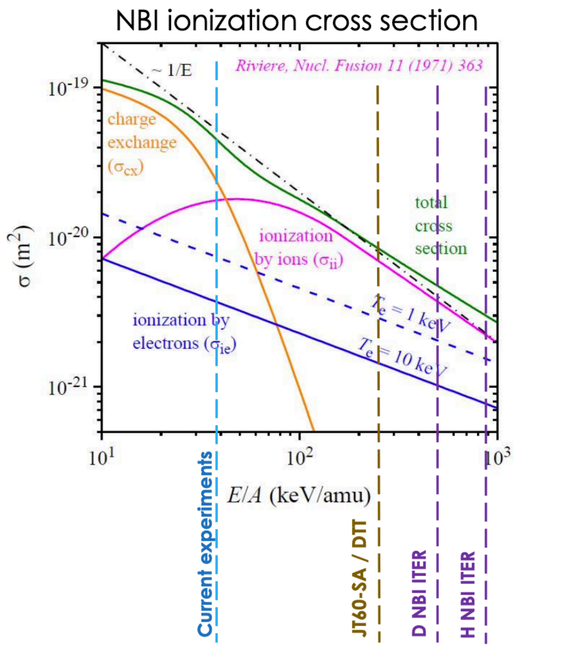
So now the beam is entering the plasma and we will have the ionization of the beam, so new fast ions are produced. How fast ions are created?

Neutral particles from the beam are ionised in the plasma by mainly 3 processes:

- $H^0_{fast} + H^+ \rightarrow H^+_{fast} + H^0$ **charge exchange (CX)**
- $H^0_{fast} + H^+ \rightarrow H^+_{fast} + H^+ + e^-$ **ionization by ions (ii)**
- $H^0_{fast} + e^- \rightarrow H^+_{fast} + 2e^-$ **ionization by electrons (ie)**

Charge exchange: fast neutral charge exchange with a slow thermal neutral, so they just exchange the charge. We will have a fast ion, an energetic ion and a thermal neutral

dominate at low injection energies



If we look at the plot, in the abscissa we have the energy of the injected particle divided by the mass unit of the particle itself (1 for H and 2 for D).

At very low injection energy - 10 kV for H beam - the charge exchange process dominates.

If you increase the energy of the injected particle, other process will dominate.

At high energies

From 100 kV the **ionization by ions** will dominate. So we have fast neutral impacting on a thermal ion, thus we will have still a thermal ion and a fast ion.

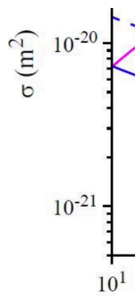
We can have the same process with an electron and as a result we will have ~~to~~ two electrons.

The **ionization with electrons** is usually lower than the other processes except at very high injection energy and very low plasma temperature. In ITER plasma we will have some parts where the ionization with electrons is relevant.

The **total cross section** takes into account these three processes. Actually, what really matters is:

The total cross section depends on the relative velocity: $v_{rel} = |v_{beam} - v_{(ions/el.)}|$ and results:

$$\sigma_{tot} = \sum_{k=cx,ii,ie} \frac{\langle \sigma_k v \rangle}{v_{beam}} \qquad \text{with} \qquad \langle \sigma_k v \rangle = \frac{\int \sigma_k(v_{rel}) v_{rel} f(v_{rel}) dv_{rel}}{\int f(v_{rel}) dv_{rel}}$$



But since $v_i \ll v_{beam} \ll v_e$ we can approximate: $\sigma_{tot} = \sigma_{cx}(v_{beam}) + \sigma_{ii}(v_{beam}) + \frac{\langle \sigma_{ie} v \rangle}{v_{beam}}$

If the *plasma rotation* is not negligible: $v_{rel} = |\vec{v}_{beam} - \vec{v}_{rotation}|$

Mean free path: $\lambda = 1/n\sigma_{tot}$ ($n = n_i = n_e$, $Z = 1$): $\sigma_{tot} \sim 1/E_{NBI} \rightarrow \lambda \sim E_{NBI}/n$

estimate how long the beam will travel inside the plasma

The mean free path is the characteristic length where the beam is absorbed in the plasma. From the plot above we can see that the total cross section goes as 1 over the energy of the beam.

$\sigma_{tot} \propto 1/E$

This is very important because then we have the mean free path that goes as E_beam/n_plasma and we can use it to estimate how long the beam will travel inside the plasma.

Fast ion birth

H^0: neutral particle

H^+ or e^-: ion or electron

H^0* : excited neutral particle

The ionization cross section is modified by two other effects:

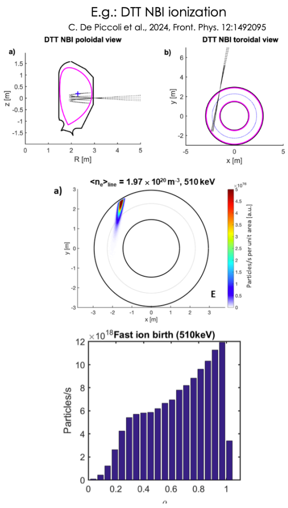
- multi-step ionization (MSI) from **excited states**

$$H^0 + (H^+ \; or \; e^-) \rightarrow H^{0*} + (H^+ \; or \; e^-) \rightarrow H^+ + e^- + (H^+ \; or \; e^-)$$

→ Enhancement factor δ: $\sigma_{tot_{MSI}} = \sigma_{tot}(1+\delta)$

→ Relevant for high E_{NBI} and high density plasmas (collisional time << excited state lifetime)
current small experiments, P-NBI: δ <0.2
ITER: δ~0.5-0.6

- **impurity** ionization (proportional to their concentration)

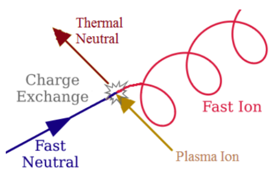


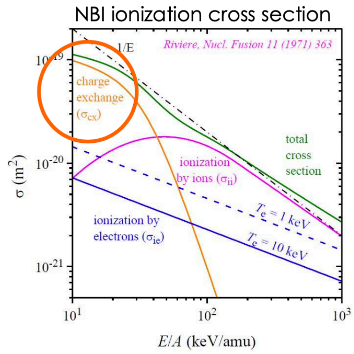
Beam halo

If we are talking about the charge exchange process, we are thinking about current experiments because the energy of the beam is quite low.

Beam halo: a side effect of CX ionization process

$$H^0_{fast} + H^+_{th} \rightarrow H^+_{fast} + \boxed{H^0_{th}}$$ creation of a thermal neutral at plasma temperature





- Creation of a thermal neutral which travels ballistically and charge exchanges with other thermal ions, creating other neutrals (Halo)
- This process is then recursively repeated multiple times, producing fewer and fewer neutrals
- Halo of neutrals surrounding the neutral beam

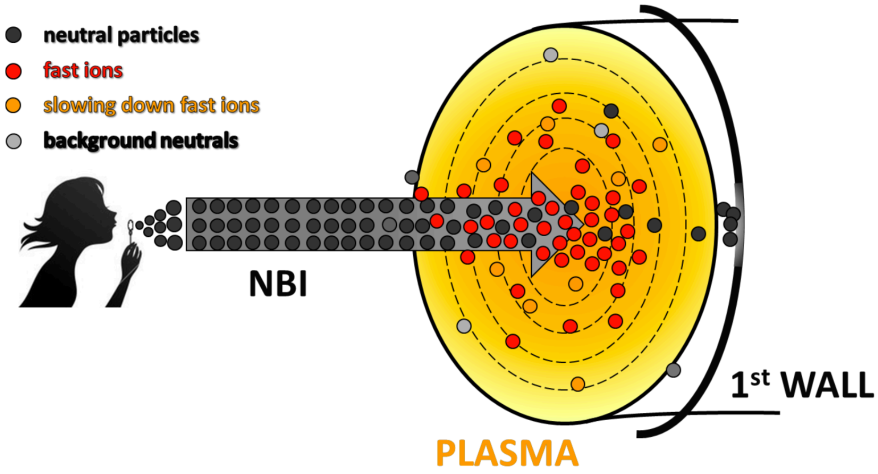
thermal neutral that travels and charge exchange with other thermal ions
↓
this create other neutrals

HALO OF NEUTRALS + NEUTRAL BEAM

Fast particle slowing down

Once we have the ionization of the beam, the newly born fast ions start to slow down, to travel and do collision inside the plasma.

ionization of the beam
↓
newly born fast ions start to slow down
↓
collision inside the plasma



Neutral beam generation — Neutral particle journey — Beam ionization — **Fast particle slowing down** →

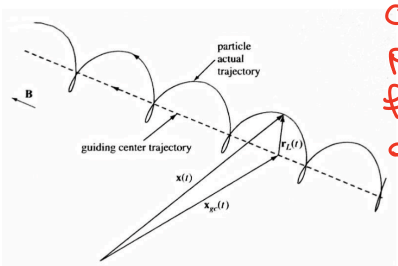
Ion orbits

After NBI ionization, newly born fast ions start experiencing the magnetic field. What are their orbits?

(refreshing single particle motion)

Charged particle motion into a homogeneous magnetic field B: the Lorentz force

Lorentz force	$m\frac{d\boldsymbol{v}}{dt} = q_e \cdot \boldsymbol{v} \times \boldsymbol{B}$
$\parallel \boldsymbol{B}$	$\frac{dv_\parallel}{dt} = 0$
$\perp \boldsymbol{B}$: Larmor radius	$\rho_L = \frac{mv_\perp}{q_e B}$
Cyclotron frequency	$\omega_c = \frac{q_e B}{m}$



circular motion of the particle around the B field and if $v = v_\parallel$ also along the B field

In an arbitrary magnetic field with a force F:

$$m\frac{d\vec{v}}{dt} = \vec{F} + q_e \cdot \vec{v} \times \vec{B}$$

Guiding center motion, we can split into:

- Motion along B field lines (if $|v_{//}|>0$)
- Drift motion due to the presence of force F (if F=0, we step back to Lorentz motion)

two types of motion:
- along the B field
- drift motion

The particle will have a circular motion around the magnetic line and if it has a parallel velocity to the B field it will also travel along the B field. Thus we can split the motion of a charged particle in a B field in two: a motion along the B field line if we have a parallel velocity and a drift motion due to the presence of force F.

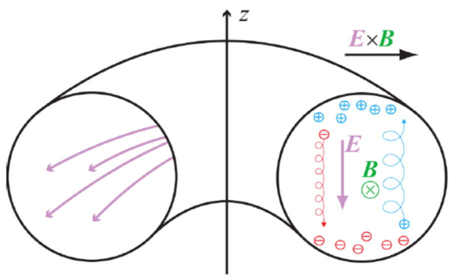
Ion motion: drifts

Different type of drift

Different drifts due to different forces in the plasma:
- B-field gradient → Grad-B drift
- Curvature of B → Curvature drift

they are vertical drifts and bring to *charge separation*

dependence on charge sign



They depend on the charge sign so they will cause a drift in a opposite direction of ions with respect to the electrons. These drifts lead to a separation of charges in our plasma and lead to the:

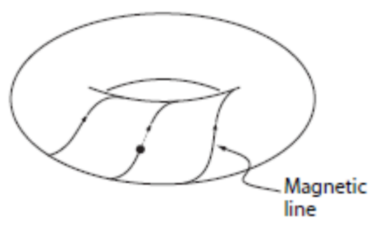
- formation of *electric field* → ExB drift (outward drift for both electrons and ions)

Both electrons and ions, due to the *ExB* drift, will be shifted towards the external part of the plasma.
This is the reason why we want the B field lines to have some poloidal turn in the tokamak because if these particles are forced to follow the B field lines which are also turning in poloidal direction, this can counteract the external drift that brings the particles outside.

This is the reason of the forced helical twist of the magnetic field lines in fusion experiments

(other drifts are present: e.g. diamagnetic drift...)

→ Guiding center orbits are shifted with respect to magnetic surfaces



Orbit topology: passing and trapped

Let's now analyze the orbits of fast ions starting from the constant of motion of these particles. We neglect the collisions: not very strange because fast ions are so fast that they can do some toroidal turns without experiencing any collisions. So they are collision-less if we look at them in a few turns in a toroidal device.

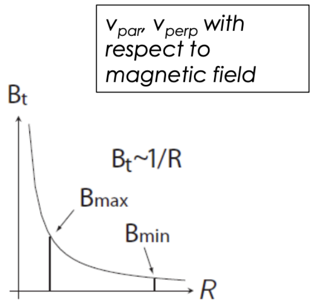
orbit of fast ions → collisionless because they are so fast that they cannot experience any collisions

In charged particle motion kinetic energy is constant and magnetic moment an adiabatic invariant:

$E_k = \frac{1}{2}m({v_{par}}^2 + {v_{perp}}^2) = const$

$\mu = \frac{m{v_{perp}}^2}{2B} \approx const$

In a tokamak, the magnetic field varies as $B(R) = B_0\frac{R_0}{R}$



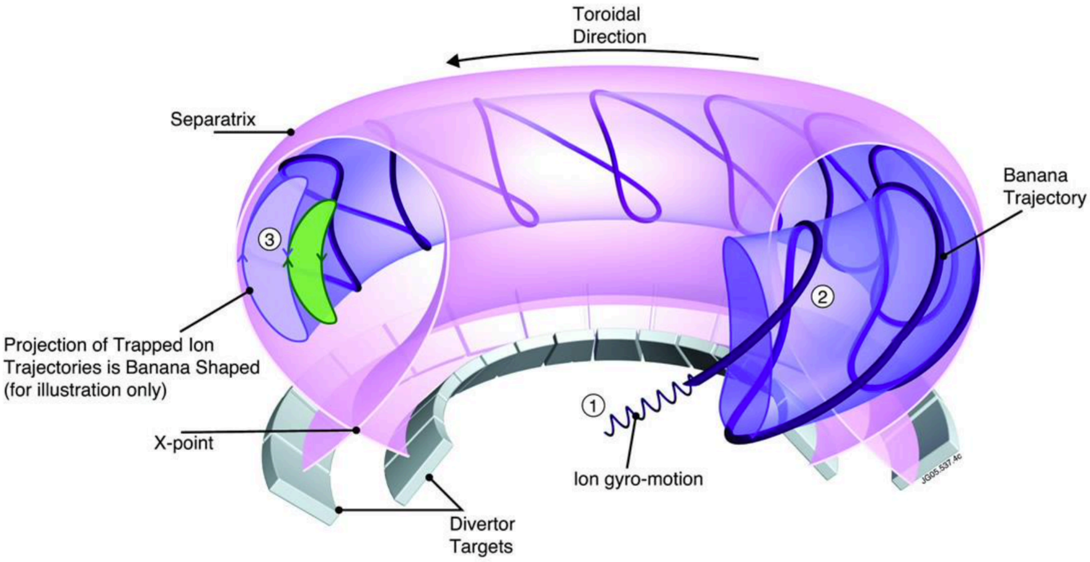
At the external part of the tokamak the magnetic field is lower than in the inner part following the relation $B(R) = B_0\frac{R_0}{R}$.

If we want to conserve the two quantities *v_par* and *v_perp* and we consider a particle that travels along the magnetic field lines - so it will experience the magnetic field in the outer part but also in the inner part which is larger - we end up in a situation which is closer to the magnetic mirror. Indeed, from the conservation of the magnetic moment, if B is increasing, the *v_perp* (perpendicular with respect to the toroidal field) must increase to have the conservation of the quantity. The perpendicular velocity will increase stealing some velocity from the parallel direction because the kinetic energy is constant - indeed if we are increasing the *v_perp* to conserve the magnetic moment, the *v_par* must decrease. If the parallel velocity decreases up to zero it means that the particle is stopped, it cannot follow anymore the magnetic field line, so along the B field lines it has a turning point and it cannot go on and experience areas in the plasma where the B field is larger.

magnetic mirror

Banana orbits

The particle is not really turning back along the B field, it continues to follow the toroidal direction but it cannot go more inside in the poloidal plane. We see a section of the poloidal plane in the picture, the particle is restricted to stay in the low field side of the machine. This generate the so-called banana orbits.



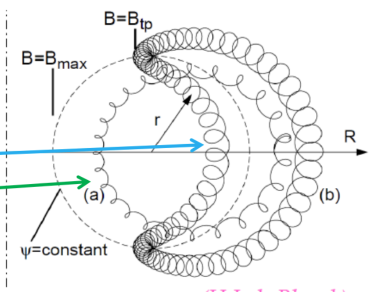
So we have a trapped particle, that cannot experience the full travel along the B field lines, and at a certain point in the poloidal direction. It does not complete a full circle, but where the B field is too high, it just turns back.
This happens when the initial parallel velocity is not enough. In the picture below there are passing orbits with the gyro-motion represented by the full circle, in this case the parallel velocity is decreasing where the B field is larger but it does not go to zero because it has enough parallel velocity at the beginning. The banana orbit instead is a trapped particle, so when it goes close to the max of the B field the parallel velocity goes to zero and it turns back. These are the two orbits that we can have.

In its (drifted) orbit, the fast ion experiences varying B
→ particle energy is transferred between parallel and perpendicular direction

Depending on the initial fast ion pitch ($\varepsilon = v_{par}/v$), the particle may be reflected (condition $v_{par} = 0$ at B_{max}) when travelling towards the inner side of the plasma (as in a magnetic mirror): **trapped particle (banana orbits)**

Otherwise, the fast ion is a **passing particle**

passing particle: $\frac{v_{perp}}{v_{par}} < \left(\frac{B_{max}}{B_{min}} - 1\right)^{-1/2}$



(H.J. de Blanck)

Let's ask to the greengrocer: Bananas!

Why the particles are following this shape of orbit? It is reported the banana width - that is how big is the radial excursion in the plasma for the trapped particles - proportional to the initial parallel velocity (parallel wrt to the toroidal B field).

Since fast ions have higher kinetic energy than thermal ions, they will have larger banana orbits.

Trapped Banana orbits

- **Banana width** (half width): $\Delta r \sim \frac{m v_{par}}{e B_{pol}}$

  → proportional to the mass (we consider only ions)
- Fast ions have larger bananas than thermal ions, and so trapped fast ion **banana orbits should be inward** to reduce fast ion losses.

We can have two types of banana orbits: we can have an inward banana (the green one in the picture above) or an outward banana (the blue one). This is a very important difference because in an inward banana the trapped particles will be confined in the plasma (if they start in a confined place they will stay there). For the outward banana which closes on the side of the external plasma, the particles can go out of the plasma and we can loose particles. How to determine if we have an inward banana or outward?

How can I say if beam ions will have inward or outward bananas?

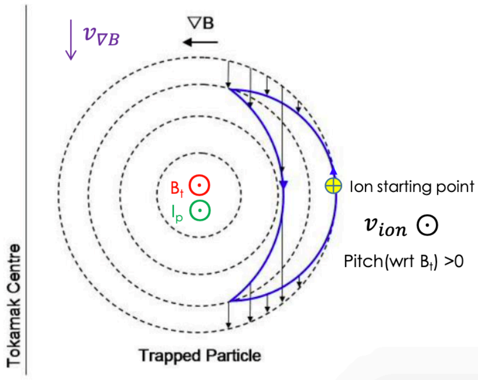
1) First of all, the direction of the ∇B drift should be determined (up or down). It depends on the sign of the charge.

The ∇B drift will shift the particle inwards or outwards with respect to the starting magnetic flux surface.

∇B drift (ion → q>0) $\overrightarrow{v_{\nabla B}} = \frac{m v_{\perp}^{2}}{2 q B^{3}} \vec{B} \times \overrightarrow{\nabla B}$

2) The second step is to determine the direction of the poloidal magnetic field B_p (determined by the plasma current) at the considered particle starting point.
Then we must separate the cases of particles with a velocity in the same direction of the toroidal magnetic field (positive **pitch**) or opposite (negative pitch):

- positive pitch trapped particles will start the banana orbit in the same direction of B_p
- negative pitch particles in the opposite direction.

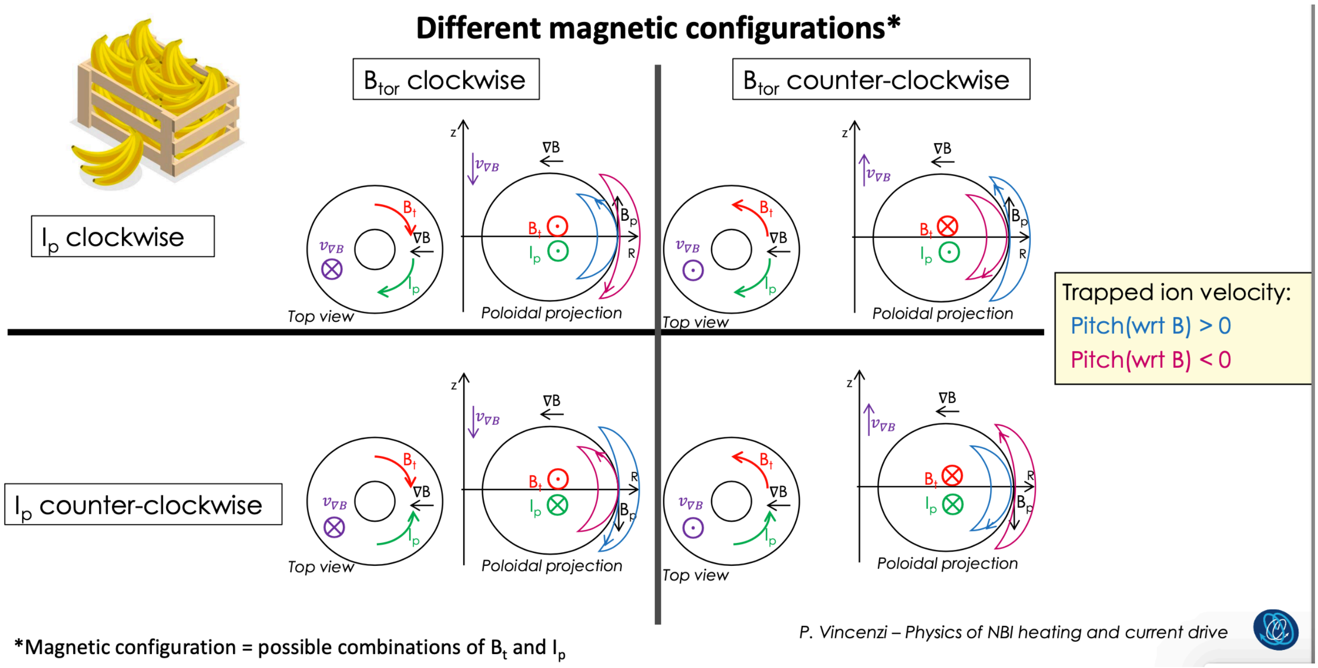


1) The gradB usually points towards the center of the tokamak because the toroidal magnetic field increases going towards the inner part of the tokamak. The gradB drift depends on the charge of the particles - it is different for electrons and ions, and depends on *B x gradB.* If you want to understand the direction of the velocity of the gradB drift, we have to use the right hand rule. For instance, in the plot we have a toroidal magnetic field which is pointing out from the paper; the gradB is towards the inner side of the tokamak so the gradB drift is directed downwards. So the arrows in

the plot are pointing down. This means that the particle that would follow the circle is experiencing the gradB that brings it down to the other circles.

2) The other important thing is that we have an ion that is starting where it is indicated in yellow. Will it follow the banana orbit going up or down? This depends on the direction of the poloidal B field that is determined by the plasma current. In this case, the plasma current is in the same direction of the toroidal magnetic field, so we have a plasma current which is going out from the screen and the poloidal B field is counterclockwise. It means that the ion will follow the same direction of the poloidal B field in the case that it is injected in the same direction of the B field.

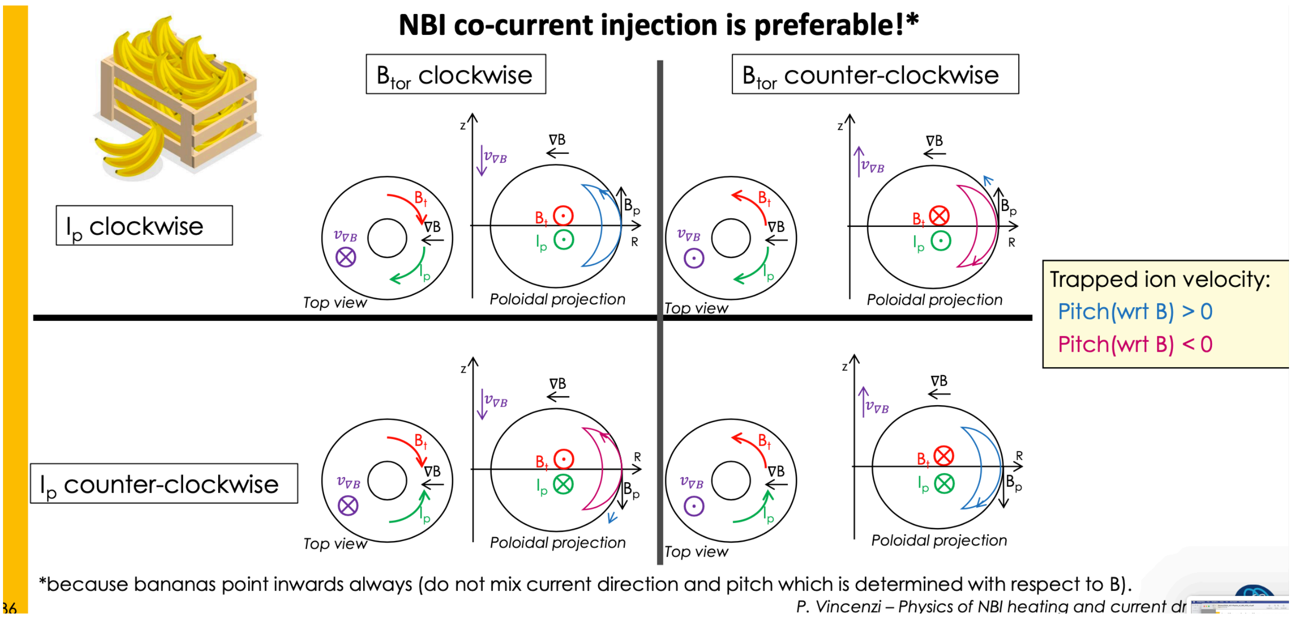
In the slide below we have all the cases.



Let's see the first plot on the left. The toroidal field and the plasma current are going towards us. So imagine to have a NBI system that is injecting particles in the same direction of the toroidal field. In this case the particles will have a positive pitch that is the ratio between the parallel velocity wrt the B field and the total velocity. If we have a beam that is firing in the other way, the particles will have a negative pitch because the parallel velocity is negative (because it is taken wrt the B field) and the particles will not follow the direction of the poloidal B field, but the opposite.

For each configuration (for configuration we mean the different coNBInation of the toroidal B field and the plasma current) we have different cases. We will have the inward or outward bananas depending on the pitch of the particle if at the beginning the particle is going in the same direction of the toroidal B field or in the other.

If we want to design a NBI system that has trapped particles doing inward bananas, in the first case we want a particle with a pitch > 0 so injecting particles in the same direction of the toroidal B field and the same of the plasma current. In second case we must have a pitch < 0 so inject particles in the opposite direction of the toroidal B field that means in the same direction of the plasma current. In the third case, if we want to have an inward banana, we must inject particles with negative pitch so in the opposite direction of the B field and in the same of the plasma current. In the last case, we have an inward banana for the positive pitch so we have to inject particles in the same direction of the B field and the same of the plasma current.



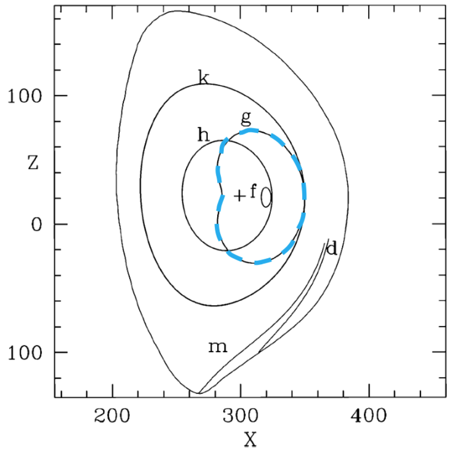
To be sure to have inward bananas, which minimize the fast particle losses, an NBI should inject particles always in the same direction of the plasma current, so co-current injection is the choice.

Other peculiar fast ion orbits

We will show now some possible orbits of fast ions. In the picture, we have the poloidal projection, k is the projection of a passing particle orbit so as h; d and m are lost particles in a orbit not confined. But we can have also exotic orbit, like **potato orbits**. In this case, the particles are still trapped because they are reflected and cannot complete a full poloidal turn and the width of the orbit is very big.

Due to the larger energy with respect to thermal particles, fast ions can experience also other type of orbits:
- Potato orbits
  - trapped orbit
  - on a poloidal projection, the orbit passes close to the magnetic axis (often, encircles magnetic axis)
  - potatoes has larger width than bananas

characteristics of POTATO ORBITS

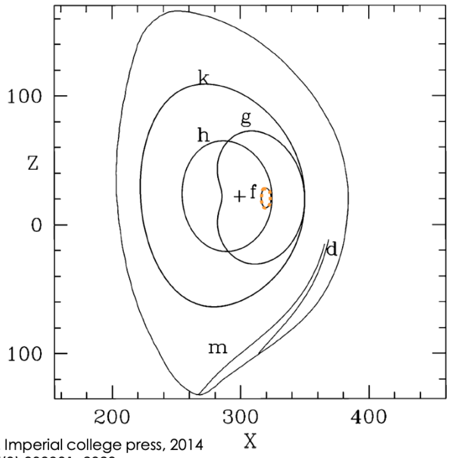


R.B. White. The theory of toroidally confined plasm
W.W. Heidbrink and R.B. White, Physics of Plasmas, 27(3):030901, 2020.

enzi – Physics of NBI heating and current drive

Another orbit is represented by the **stagnation orbit.**

Due to the larger energy with respect to thermal particles, fast ions can experience also other type of orbits:
- **Potato orbits**
- **Stagnation orbits**
  - passing orbit
  - fast ion completing a toroidal orbit scarcely moving poloidally



R.B. White. The theory of toroidally confined plasmas. Imperial college press, 2014
W.W. Heidbrink and R.B. White, Physics of Plasmas, 27(3):030901, 2020.

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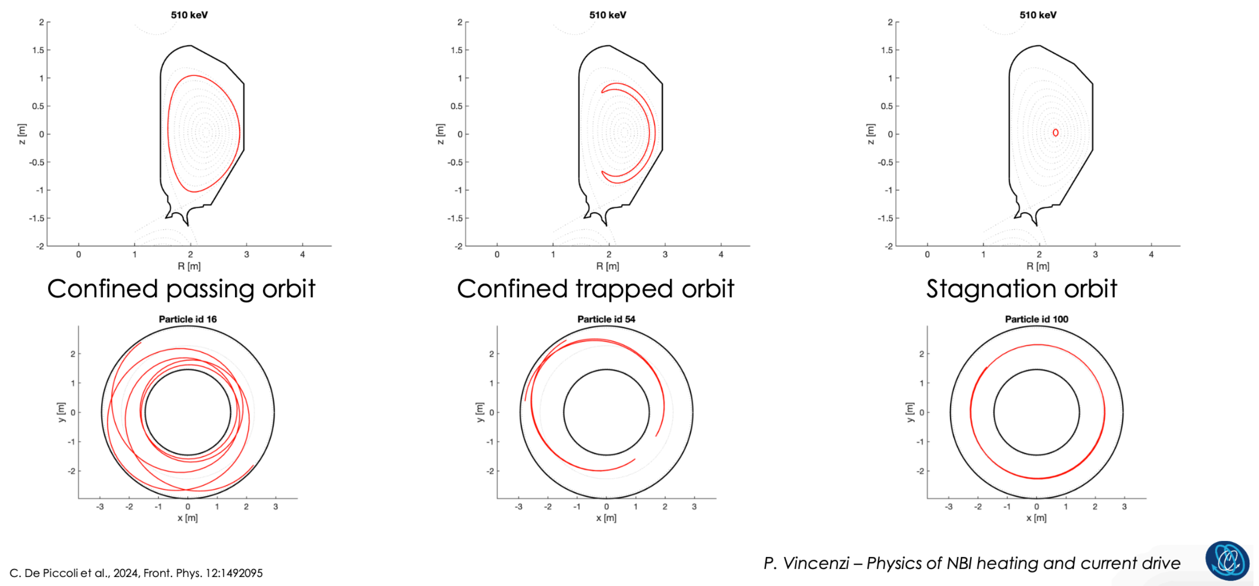
Example of DTT NBI EP orbits

These are cases for DTT.

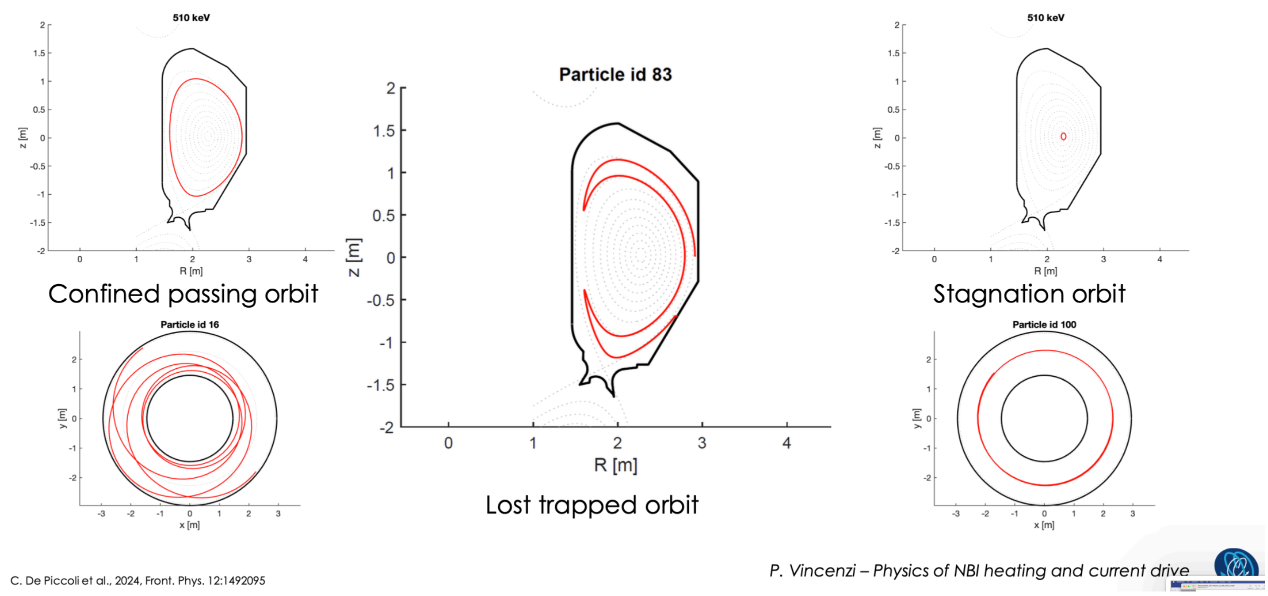
First case: we have a passing orbit that can complete a full toroidal turn. In the poloidal plane, the particle can experience the full B field.

Second case: banana orbit where the turning points are on the inner side so the parallel velocity is still high but not enough to complete a full turn.

Third case: stagnation orbit where the particle does a complete toroidal turn.



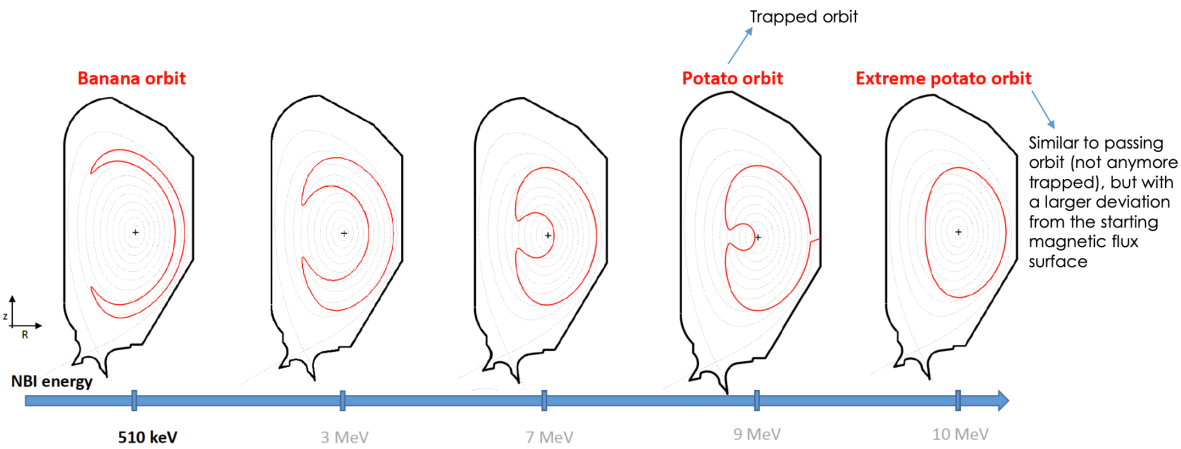
Fourth case: lost trapped orbit that is a banana orbit that goes outside and crashes into the wall, so we lost the particle. We cannot close the banana shape that crashes into the wall.



This is very important because, when we inject particles and they are newly born fast ions, they have time to do few toroidal turns before experiencing collisions. The collision probability is quite low for fast ions, so if the particle is born in an unconfined orbit is lost immediately like in 90 millisecond, so that's why it's important to study and to design the beam so that we don't have this kind of lost orbit.

Example of DTT NBI EP orbits

This is an imaginary exercise so we increased the energy of the particle of the DDT machine from the 510kV which is the nominal injection energy to like 10MeV. It is just to show that increasing the particle energy the banana width depends on the parallel velocity, it increases with the kinetic energy of the particle.



So if we increase the injection energy we are increasing also the width of the banana and we can increase the width of the banana with the energy of the particle until we reach a potato orbit. So you can see the difference: at a certain point the banana encloses the magnetic axis, so it becomes a potato orbit. These are so exotic, it's very difficult to see this kind of orbits in realities but it's just to say that we can approximate these with different formulas. If you continue to increase the particle of the energy, at a certain point you can close the banana having the extreme potato orbit but it becomes a passing orbit with a very large drift because a passing orbit would almost follow the flux surface. You can see from the plot that these particles have a very very strong drift because it drifts from the almost very external magnetic flux to a very inner one so that's why we call it extreme potato orbit.

Pitch evolution of non-standard orbits

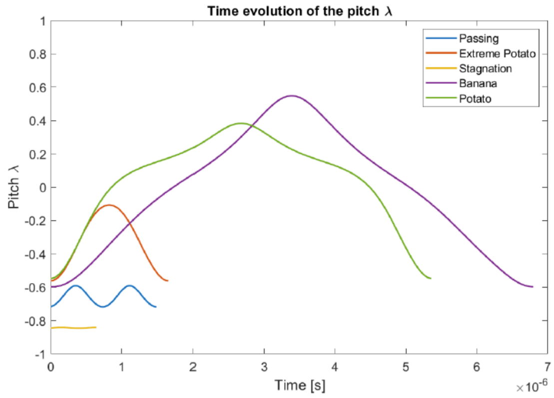
If you remember it's very important to know the pitch of the newly born fast ions. When a fast ion is born, we want to know its pitch so the ratio between the velocity parallel to toroidal field line divided by the total velocity.

Pitch: $\lambda = v_{par}/v_{tot}$ with respect to magnetic field B

Example of different orbits of imaginary* NBI EP in DTT

- Pitch changes sign → particle turning back:
  - Banana
  - Potato
- Pitch does not change sign:
  - Passing
  - Stagnation
  - Extreme potato

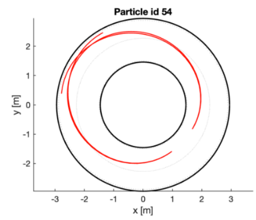
*EP energy increased arbitrarily to see these exotic orbits, NBI EP in DTT will be mostly passing



This is an example for this imaginary NBI fast ions in DTT.

Let's look to the purple curve: it is a banana orbit. We do expect that the parallel velocity at a certain point changes sign because if you look at the image on the right, you see that it starts and goes then it turns back.

Confined trapped orbit



So at a certain point the parallel velocity changes sign and you can see it the plot above because the pitch of a banana orbit is changing sign, so it starts from a negative pitch (in DTT we have the NBI that inject particles with a negative pitch) and at a certain point of its orbit it goes with a positive pitch and then turns back and goes back to the negative pitch. It means a trapped particle that changes sign due to the conservation of the magnetic moment and also the kinetic energy.

We can have also potato orbits in green that are trapped particles, so it means that at a certain point the pitch of the particle is changing sign so again it starts from negative, then it goes to positive and then it is negative.

There are orbits which do not change the pitch of the sign which are the passing orbits. The normal passing one is the one in blu, so it stays negative. The yellow one is the stagnation orbit. We have just said that we have a little excursion in the poloidal plane and the pitch almost doesn't change, so it stays the same because the particle is not moving poloidally. And also the extreme potato is almost a passing orbit and you can see that pitch is not changing.

So these are the orbits of the particles.

Fast ion slowing down and energy transfer

Now let's try to quantify the slowing down and the energy transfer during these orbits. We include collision now (coulomb collisions) in this orbit. Fast ions can transfer the energy to plasma species, which are ions, including also impurities, and electrons of the plasma. So, how is the energy transfer? The one below is the equation, which describes the energy transfer from a fast ion to ions, which is the first term, and to electrons. And you can see that depends on the energy of the particle in a very different way.

> During their motion, fast ions transfer their energy to plasma species through Coulomb collisions (**slowing down process**):
> - To ions, including impurities
> - To electrons
>
> How the energy is transferred?
> (approximation valid for $v_i \ll v_{beam} \ll v_e$)
>
> $$\frac{dE}{dx} = -\underbrace{\frac{\alpha}{E}}_{\text{to ions}} - \underbrace{\beta\sqrt{E}}_{\text{to electrons}}$$

Already from this equation, we can see that if the fast ion energy is very high, the first term goes almost to zero. So we don't give energy to plasma ions, but mainly we give energy to plasma electrons. If the energy of the fast ion is low, then the energy to the electrons can be neglected, while the energy transfer to the ions is very important.

> We can define a fast ion energy corresponding to equal stopping from electrons and ions:
>
> $$\frac{\alpha}{E} = \beta\sqrt{E}$$
>
> And define therefore this energy as the critical energy E_c: $E_c = \left(\frac{\alpha}{\beta}\right)^{2/3} = 14.8\left[\sum_{j=species}\frac{n_j Z_j^2}{n_e A_j}\right]^{2/3} A_b T_e[keV]$
>
> - It depends only on plasma (temperature, density, plasma species, except dependence on NBI species)
> - At time t during a fast ion slowing down lasting τ_s > t:
>   - $E_{fast\ ion}(t) > E_c$ dominant energy transfer to plasma electrons
>   - $E_{fast\ ion}(t) < E_c$ dominant energy transfer to plasma ions

α and β are coefficient to describe the plasma. From this equation above, if we compare the two terms, we have the energy at which the stopping, so the energy transfer to

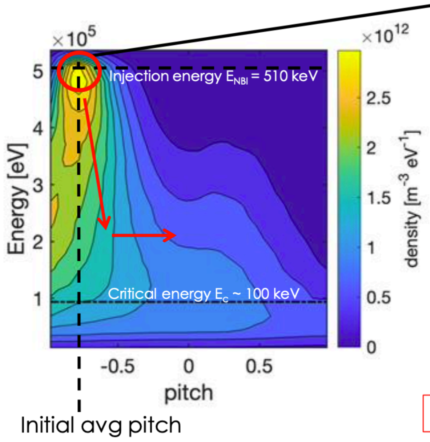
electron and ions is equal. And we call this particular energy as the critical energy: it is the energy when fast ion is transferring energy equally to electron and ions in the plasma.
And as you can see from the formula, this does not depend to the injection energy of the beam, but it depends only on parameters of the plasma and in particular it is proportional to the temperature of the plasma. This is very important because depending on the temperature of the plasma, given a fixed injection energy of the beam, I am giving more energy either to plasma ions or plasma electrons.
So if the initial fast ion energy is much larger than the critical energy, then for most of the time the fast ion is giving energy to plasma electrons.
If the fast ion energy is lower than the critical energy, the fast ion is transferring energy more to the plasma ions.

Fast ion energy transfer

The picture represents the distribution function of the fast ion in plasma. And it is given as a function of the pitch (remember that the pitch is the velocity parallel to toroidal B field the total velocity of the fast ion) which more or less give the direction of the beam.

In steady state conditions, we have a fast ion population composed of newly ionised particles, fast ions in the middle of slowing down, scattered, almost thermalized…
An example of the steady state numerical solution of the fast ion particle distribution function (density of ions in E+dE and ε+dε):

Fast ions accelerated from Coulomb collisions (it happens…)



Fast ion slowing down:
from collision with electrons to collisions with ions

1) electron collision ($E_{fast\ ion}>E_c$): energy scattering (direction of motion not affected)

2) ion collision ($E_{fast\ ion} \approx E_c$): energy & pitch scattering (direction of motion affected)

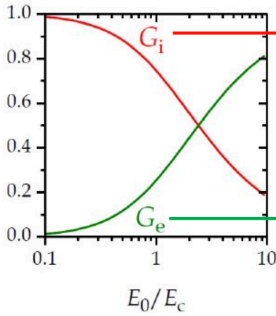
= distribution spreading in pitch at lower fast ion energy

…we will learn more about fast ion distribution function later

So negative pitch is in the opposite direction of the toroidal magnetic field. Positive pitch are fast ions in the same direction of the toroidal magnetic field.
Zero is a perpendicular injection, a perpendicular direction, which means that we don't have parallel velocity. And on the ordinate there is the energy of the particle. So we are

injecting fast ions with the beam and we are in stationary conditions. So there is the source of the beam, which injects particles at the injection energy.
This is a DTT case, so the injection energy is 510 keV. You can see that actually there is a large density of particles at this injection energy. This is the source of the beam in the distribution function. And also, in DTT the beam is injecting particles in the opposite direction of the toroidal magnetic field. The pitch is negative, so the source here is characterized by the injection energy of the beam and the initial negative pitch.
So we have quite localized source because the beam is uniform, so it injects with the same energy and with the same pitch as approximation.
Then we have Coulomb collisions. At the beginning, since the critical energy for DTT is around 100 keV - the dashed line - the fast ions will interact mainly with electrons, so will lose energy to plasma electrons.
Since plasma electrons have very low mass, plasma electrons cannot change the direction of the massive fast ions, so the pitch is almost conserved in the first part of the slowing down, because the collisions are with electrons. But they lose energy, they lose energy to plasma electrons. Decreasing the energy, the collision with the ions start to take place and become even dominant, so closer to the critical energy, larger is the contribution of collisions of fast ion collisions with the ions and in this case, the pitch is affected because the fast ions are changing the direction of their motion due to collision to thermal ions. So you can see that in the picture there is a spread of the distribution function in pitch because fast ions are changing the direction of motion when colliding with the thermal ions at energy closer to the critical energy.

If we want to estimate the NBI power (for a NBI with energy E_0) going to electrons and ions we have to **integrate the energy transferred in time**:



Power fraction to ions: $G_i = -\frac{1}{E_0}\int_0^{\tau_s}\left(\frac{dE}{dt}\right)_{ion} dt$

Power fraction to electrons: $G_e = 1 - G_i$

For $E_0 > 2.5\ E_c$, electron heating dominates (future reactors, alpha heating…)

In current experiments (low E_{NBI}), ion heating is dominant, also due to lower energy components of the beam, in future reactors it will be the other way round

Now if we want to understand how much NBI power, so how much the fast ions from the beam are transferring to electrons and ions, we should integrate the equation we have seen before during the slowing down time, so the times the fast ion takes to start from the initial energy to the thermal energy of the plasma - this is the slowing down time.

So you can see that depending on the ratio of the initial fast ion energy to the critical energy we have a different contribution, a different ratio of power going to electrons which is this green line and power going to the ion which is the red one.
For very high injection energy with respect to the critical energy we have most of the power - the green line - going to electrons, while very little energy to ions.
If we have very low injection energy with respect to the critical energy most of the power will be given to ions and not to electrons.

So there is a very subtle difference in what we have seen because the condition $\frac{\alpha}{E} = \beta \sqrt(E)$ is the energy at which fast ion are giving the same energy to plasma electrons and to plasma ions. So if you want to look at the plot above at 1, so when the energy of the fast ion is equal to the critical energy, one should expect that we have the same power given to electron ions. But this is not the case because we are integrating in time. So this depends on how much time my fast ion spend in the first part of the process and how much time the fast ion spend in the other part of the process. Integration in time of the equation gives this 2.5 factor, so actually we have the same power going to ions and electrons when the injection energy is 2.5 the critical of energy.

This is only as an approximation because the critical energy depends on the electron temperature, it is linear with the electron temperature of the plasma and you know that the temperature of the plasma is not uniform in the plasma - it is lower in the edge and higher in the core. So also the critical energy depends on the radial dependence of the electron temperature, so it is lower in the edge of the plasma and it is higher in the core of the plasma. Depending where a fast ion spend most of the time we can have a different absorption of energy from the thermal plasma electrons or thermal plasma ions.

Slowing down time

this is the formula for the slowing down time which can be calculated from the same equation of the variation of the energy of a fast ion in the space.

The **time** a fast ion spends to transfer its energy (from E_0 to $E \sim E_{thermal}$, thermalization) is:

Slowing down time: $\boxed{\tau_{sd}} = \int_{t(E_0)}^{t(E=0)} dt = \int_{E_0}^{E=0} \left[\frac{dE}{dx}\frac{dx}{dt}\right]^{-1} dE = \int_0^{E_0} \left[-\frac{dE}{dx}\sqrt{\frac{2E}{m}}\right]^{-1} dE = \boxed{= \frac{t_s}{3} \ln\left[1 + \left(\frac{E_0}{E_c}\right)^{\frac{3}{2}}\right]}$

with the Spitzer slowing down time defined as: $t_s[s] = \frac{\sqrt{2m_i}}{\beta} = 6.28 \times 10^{14} \cdot \frac{AT_e[eV]^{\frac{3}{2}}}{Z^2 n_e[m^{-3}]\ln\Lambda}$

For current experiments τ_{sd}~100-200 ms, in future reactors (larger T_e, larger E_{NBI}) it will be up to seconds

We reported this because it is true that the slowing down time depends on the E0 which is the initial energy of the fast ion, but it depends on the initial fast ion energy only through a logarithm. So it is a very little dependence, while it has a strong dependence on the Spitzer slowing down time, which depends on plasma parameters like the electron temperature and the density of the plasma. So the slowing down time is mainly given by the plasma characteristics and less by the energy of the fast ion.

Thus, the slowing down time is the time a fast ion takes to give all these energies, so from the ejection energy to the average energy of the plasma. At the end of the slowing down time the fast ion becomes a normal thermal ion like every other ion in the plasma. And for current experiments the slowing down times is about 100 or 200ms, while in future reactors this time will be up to seconds like in ITER.

Current-drive

Let's now talk of another thing which can happen during the slowing down time that is the **induction of plasma current** which is referred to as current drive. This is important to sustain the generation of the plasma current inside the tokamak for instance to reach the fully non inductive plasma current. This is a scenario which is of interest for ITER in the second phase of operation and it is also of interest for JT60SA or it is also of interest of the partially non inductive so not fully non inductive but only a part of the plasma current is generated by external current drive system and in this case by NBI. This is for instance also what jet experimented.

Inducing plasma current

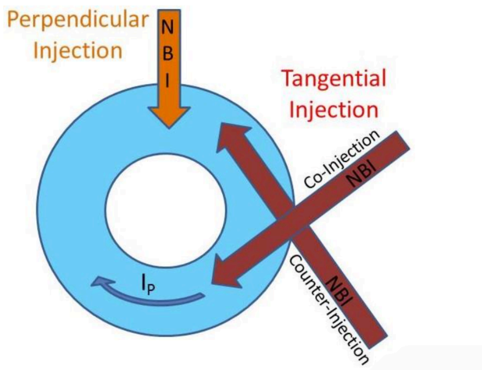
In tokamaks plasma current is currently sustained mainly by the central solenoid → **pulsed, inductive operations with limited pulse length**, cycles

The generation of **non-inductive currents** (boostrap, or by external H&CD systems) could extend discharge duration:

- fully non-inductive plasma current (steady state tokamak)
- partially non-inductive (extended pulse length)

NBI, as other systems, can drive current in the plasma.

NBI must have tangential injection to drive current



So first of all to drive some current in the plasma, we must have a tangential injection, so either in one way or the other but of course with perpendicular injection we are not generating any toroidal the current.

NBI Current-drive

We reported here a very simple estimation of the current drive just to understand how the beam which can be seen in the figure can generate plasma current.

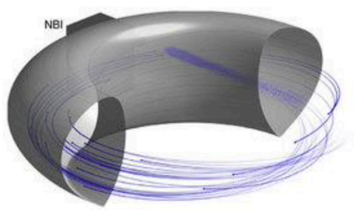
A simple estimation of the driven current

The tangentially injected fast particles, becoming fast ions, generate a circulating **fast ion current**:

$$I_{circ} = \boxed{I_0}\boxed{\frac{\langle v_{par}\rangle \tau_{sd}}{2\pi R_o}}$$

→ average number of toroidal laps ($\langle v_{//}\rangle \sim v_0/2$)

$I_0 = P_{NBI}/E_0$ injected particles



But we miss:

- correct averaging over slowing down
- pitch angle scattering
- electrons dragged by fast ions
  - Back electron current: could cancel all the fast ion current if we do not consider toroidal effects and if $Z_{beam} = Z_{eff}$. Luckily these conditions are not present in experiments → $I_{NBCD} = I_{circ}\left(1 - \frac{Z_{beam}}{Z_{eff}}\right)$
- trapped electrons that cannot be dragged along with fast ions (this would give some current also for $Z_{beam} = Z_{eff}$)

$$\rightarrow \quad I_{NBCD} = I_{circ}\left(1 - \frac{Z_{beam}}{Z_{eff}}\left(1 - f(Z_{eff}, \varepsilon\right)\right)$$

ε = aspect ratio

We have a population of fast ions which are traveling in the same way toroidally and this of course generates a current. I_{circ} is actually the circulating current.

In this very simple estimation we are missing some parts, some points. First of all we are **not correctly averaging** over the slowing down time because I_{circ} depends on the parallel velocity but of course during the slowing down time it changes. We are not taking into account the **pitch angle scattering**, so collision processes that changes the pitch of the particles which of course affect the parallel velocity and also we are not taking into account that some **electrons** in the plasma are **dragged** by the fast ions and this of course can screen the generated fast ion current. We call this the back electron current. This actually can totally cancel the fast ion current if the charge of the beam, the z of the beam, matches exactly the effective charge of the plasma. So in this case if the charge of the beam is matching exactly the effective charge of the plasma, we have zero current-drive. Usually this is not the case in present experiments.

Another thing is that not all the electrons can be dragged by the fast ions because we also have trapped electrons and trapped electrons cannot follow the full toroidal turn because at a certain point they go back, then they turn back again and they cannot really follow the fast ions. So if we take into account all these elements, we have a net neutral beam current-drive due to the fast ion motion.

Current drive efficiency

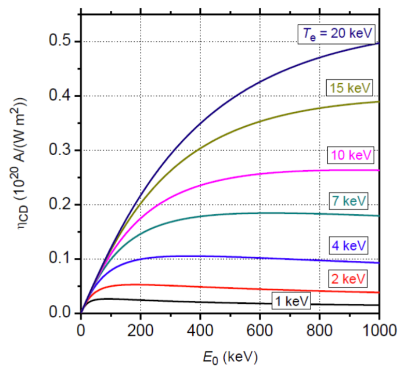
Here it is summarized the dependence of the current drive from NBI.

The **efficiency** of an auxiliary current drive system can be defined as the current driven per power injected I_{CD}/P_{NBI}.

But since: $I_{CD} \propto P \frac{1}{R_0 n_e}$

the **current drive efficiency** is defined as: $\eta_{CD} = R_0 n_e \frac{I_{CD}}{P}$ usually in $\left[\frac{10^{20} A}{W m^2}\right]$

- The NBI CD efficiency increases with E_{NBI} and plasma temperature: max(η_{NBCD}) increases linearly with T_e
- 90% of max(η_{NBCD}) at considerably lower E_{NBI}: $E_{0.9max} \sim 0.5 \cdot E_{max}$
- In current experiments $\eta_{NBCD} \sim 0.05 - 0.1$ (record in JT60-SA: 0.15)
- For future reactors (ITER, DEMO) $\eta_{NBCD} \sim 0.3 - 0.4$
- NBI has usually the highest CD efficiency



Ex. for D tangential injection

Usually the current drive is proportional to the injecting power and it is inversely proportional to the major radius of the machine and the plasma density. So we can define this called **current drive efficiency** as the current drive multiplied by the major radius, the density of the plasma divided by the injected particle. In order to take into account that the current drive has this dependence, the current drive efficiency is given in these units we can see in the slide above.

dependence of the current drive

The first dependence is that the NBI current drive efficiency increases with the NBI energy, so as you can see in the plot you have the NBI energy on the x axis and usually if you don't get the very low plasma temperature condition, increasing the energy of the beam also the efficiency of the current drive increases. Actually there is a plateau in the curve, so that's why we were referring to the plasma electron temperature condition and increasing also the electron temperature of the plasma, this also increases a lot the current drive efficiency.

There are reported also some numbers for current experiments and future reactors.

NBI is also a source of…

The last two things that NBI can do as a source to the plasma is **plasma fueling**, so being a source of particle.

> **Source of particle N (plasma fuelling)**
>
> $$\frac{dN_{NBI}}{dt} = \frac{P_{NBI}}{eE_0}$$
>
> eq. of the number of particles injected by the beam
>
> - useful for core plasma fueling (efficient)
> - relevant for current experiments
> - negligible for future reactor-like experiments (V_{plasma} two order of magnitude larger, E_{NBI} 1 order of magnitude higher, P_{NBI} almost the same)

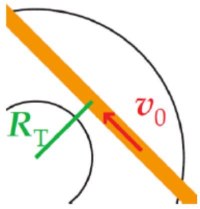
The first equation above is the equation of the number of particles injected by the beam. This is actually a fueling source which has to be taking account in current experiments where the plasma volume is quite low that is not relevant for future reactor like experiments because in the future experiments we will have large injection energy, plasma fueling is reduced and also the volume of the plasma will be very big. So we should have a very large injective power to have a relevant effect.

The last effect contributing inside a plasma is the **source** of a **toroidal torque**.

Source of toroidal torque (only for tangential injection)

- Due to a combination of collisional torque and j x B torque (due to fast ion current)
- Collisional torque: $M = \frac{I_0}{e} A_{beam} m_p v_0 R_T e_\phi$

  since $I_0 = P_{NBI}/E_0$ and $v_0 \propto \sqrt{E_0/A_{beam}} \rightarrow \frac{M}{P_{NBI}} \propto E_0^{-1/2}$
- Torque induces plasma rotation depending also on the momentum confinement time



F.L. Hinton, M.N. Rosenbluth, Physics Letters A 259 1999. 267–275

P. Vincenzi – Physics of NBI heating and current drive

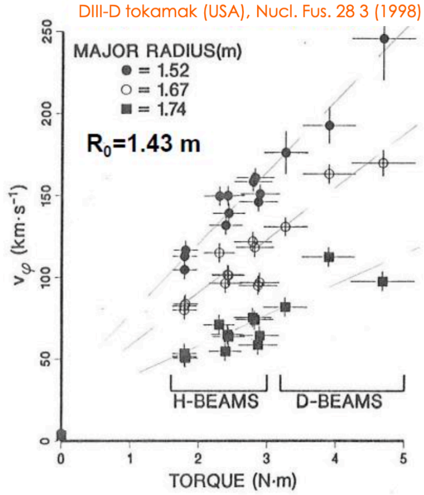
This is of course only for tangential injection and this is a coNBInation of collisional torque - so really fast ions that are colliding with the plasma mass which is mainly ions - and this gives an input torque generating rotation. And also there is another force that can generate a rotation which is the *JxB,* so we refer *JxB* torque. J is the plasma current due to fast ions.

We have seen that we can generate plasma current with the fast ions, JxB is a force so we have a tangential injection with a certain tangency radius, so a force due to the JxB torque so we can have also a rotation due to the JxB torque.

NBI torque and plasma rotation

This is just to show that the beam can really generate a rotation in plasma.

DIII-D tokamak (USA), Nucl. Fus. 28 3 (1998)



- Control of plasma rotation by NBI torque
- In case of counter-current injection, the plasma rotation can be reduced (decreasing it to condition similar to future reactors)
- plasma rotation and rotation shear (dv_{rot}/dr) can suppress turbulence and help plasma stability

On the abscissa you have the injected torque which due to the mass of the injected particles is larger for D-beams than for hydrogen beams and on the y axis we have the effect on the rotation of the plasma in the DIII-D tokamak. You can see that an increase in the torque injected by the neutral beam, you increase also the rotation velocity of the plasma.

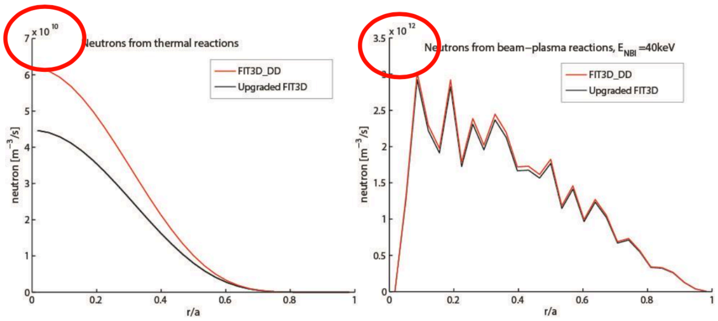
Beam plasma fusion reactions

There is another effect which is related to the injection of neutral beam in plasmas. It is the generation of some very specific fusion reactions which are the beam plasma fusion reactions.

In case of reactive species (D-D, D-T), the interaction between NBI and plasma is a source of fusion reactions (beam-plasma reactions), additional to background plasma reactions (thermal reaction).
In world-record JET and TFTR D-T experiments, beam-plasma reactions dominated.

Simulation for D campaign of LHD experiment ("stellarator family", P_{NBI} ~ 20MW, E_{NBI} = 50-200 keV, $n_{e,0}$ = 3e19 m^{-3}, $T_{e,0}$ =1keV)
P. Vincenzi et al., Plasma Phys. Control. Fusion 58 (2016) 125008

- D + D → p (3.02MeV) + T (1.01 MeV) — proton production
- D + D → n (2.45 MeV) + ^{3}He (0.82 MeV) — neutron production



Also the fast ions of the beam itself can react (beam-beam reactions)

These are fast ions that collide and make fusion with the thermal ions and all fast ions can make fusion among fast ions. So this is another possibility.
Actually if you followed the JET record shot of fusion power, most of the fusion power was given by beam plasma reactions, so the effect of the beam plasma.

Energetic particle physics: few words

This is a summary of what NBI particles, what an NBI source can do in a plasma and which effects are present when we are injecting a neutral beam in a plasma.

NBI fast ions represent a particular ion population in the plasma: **suprathermal ions** (or Energetic Particles EPs)

- NBI fast ions can be a relevant part (tens %) of plasma ions in current experiments, while they will be a small population in larger tokamaks
- NBI fast ions are different from fusion alphas not only for the energy but also for the anisotropy in velocity space
- fast ions can drive instabilities such as Alfvén Eigenmodes (AEs), if their velocity ≈ Alfvén velocity $v_A = \frac{B}{\sqrt{\mu_0 n m}}$
- fast ions can be affected by instabilities and fast ions can be lost
- Experimental findings on fast ion AEs that reduce thermal ion micro-turbulence in JET and AUG (very interesting for T_i increase and reactor performance)

A. Di Siena et al., "New high-confinement regime with fast ions in the core of fusion plasmas", arXiv:2010.14839
Mazzi, S. et al. Enhanced performance in fusion plasmas through turbulence suppression by megaelectronvolt ions. Nat. Phys. 18, 776–782 (2022

1) First of all, you generate suprathermal ion population which is called energetic particle population. In current experiments this population can be also a relevant part of the plasma - like around 10% of the plasma in some experiments - but in future experiments this will be negligible but still it will have a lot of effects in the plasma.

2) The NBI fast ions are different from fusion alphas. Fusion alphas are still fast ions but they are born from fusion reactions. The main difference is that fusion alphas have a uniform distribution in the pitch angle, so they don't have a preferential toroidal velocity when they are born, while neutral beam fast ions have preferential parallel velocity.

3) Fast ions in general can drive some instabilities, like Alfven eigenmodes when we match the Alfven velocity which depends on the magnetic field of the plasma and the density of the plasma.

4) The motion of the fast ions can be affected by MHD instability, so some fast ions can be lost - not only by collisions, by orbits that are not confined but also by MHD activity of the plasma.

5) There are also some recent studies in the last year that show that fast ions are stabilizing some turbulence in the plasma, so they are not only driving instabilities but they can also stabilize some other instabilities.

Summary: NBI is a source of…

Neutral Beam injection is a source in the plasma of:

- **Energy** (→ heating)
- **Current** (in case of tangential injection)
- **Particles** ("plasma fueling")
- **Torque** (acting on plasma rotation, in case of tangential injection)
- **Fusion reactions** (beam-plasma reactions, in case of reactive species)
- "**Beam halo**" (= thermal neutrals), in case of relevant CX ionization process
- **Fast particles** (peculiar particle population with their own **instabilities** and interaction with thermal plasma, used also for diagnostics)

This is a summary of what a NBI can do in plasma.

It is a **source of energy**, so this is the heating of the plasma; it is a **source of current** in case of tangential injection; it is a **source of particles** - this is always true, the only thing is that plasma fueling from NBI will be negligible in future devices -; it is a **source of torque** - if the neutral beam injector injects particle in tangential direction, so it can acts on the plasma rotation -; it can be a source of **fusion reactions** - the beam plasma reactions and the beam-beam reactions in case of reactive species; you can have the effect of the **beam halo** - this is the effect of the charge exchange ionization process and you have then a source of fast particles, so you generate a population of fast particles.

NBI… towards future

		Present day	ITER/Reactors (?)
NBI technology	Energy	Low-middle (40-200 up to ~500 keV - JT60SA & DTT)	High ~1000 keV
	Power	Tens MW	Tens MW
NBI physics	Heating source	✓	✓
	Current source	✓	✓
	Particle source	✓	Negligible
	Torque source	✓	✓
	Fusion reactions (beam-plasma) source	✓	Relatively low

This is a summary of what NBI can do in present day with the main differences. In present day devices, usually the energy of the beam is quite low except the JT-60SA and the design beam for DTT which is around 500 kV, while in ITER it will be 1MeV.

The power is almost very similar in present day devices and in ITER - tens of MW.

The beam will be a heating source, a current drive source in every experiments, it is a particle source but it will be negligible in very big experiments like ITER also due to the high energy, so you are injecting less particle with same power.

It is a source of torque which due to the low energy is very effective in the present day devices, it will be less effective but still a source in the ITER plasma.

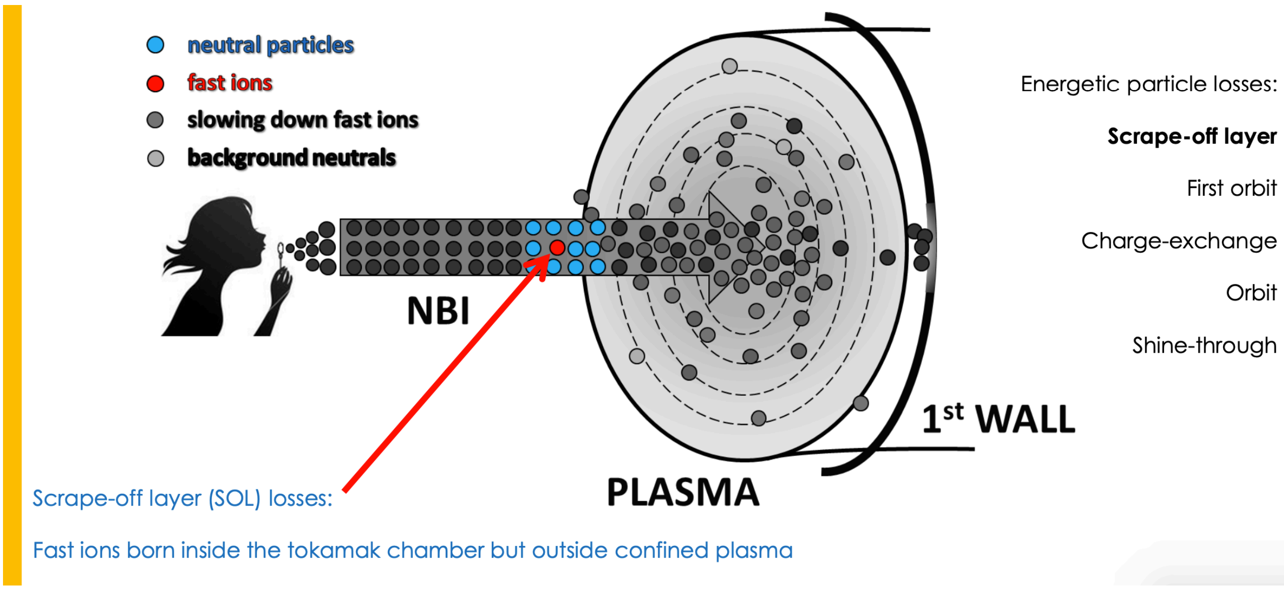
It is in the current devices also a fusion reactions source, while for ITER it will be present but it will be a very negligible source of fusion.

Beam energetic particle losses

In the last part of the lesson, we will talk about the beam energetic particle losses, so the loss processes, affecting the fast ions and also the neutral beam particles. There are also some modeling techniques to simulate all the physics things we have explored.

Fast particle losses

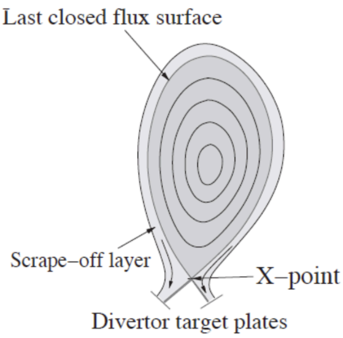
We start from the edge or the outer part of the plasmas and then we go inside the plasma, following an ideal path of a beam particle.



So the first kind of losses we can encounter are called **scrape-off layer losses**. The scrape-off layer is the outer part, so the non-confined part of the plasma, which is at the edge in the toroidal chamber. We have first ions that are born inside the tokamak, but outside the confined plasma. And then, as we have seen before for the fast ion orbits, if particles are born in non-confined part of the plasma they are lost.

Scrape-off layer losses

In the scrape-off layer, a fast ion is lost because the magnetic flux surfaces are not closed and we still have a bit of plasmas. So the density of the plasma in this scrape-off layer is not negligible and we can have some ionization processes.

> Scrape-off layer (SOL) losses:
>
> - SOL is characterized by open magnetic flux surfaces
> - If a fast neutral particle is ionized in the SOL, its orbit will collide with divertor plates
> - In this region we find neutral particles (e.g. gas puff) and ions entering/escaping from the plasma. It is usually difficult to estimate SOL density: if it is known, SOL losses can be easily estimated by numerical modelling
>
> 
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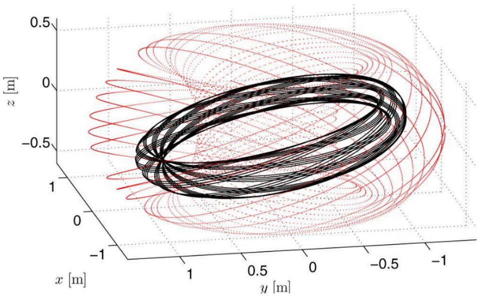
Usually we don't have a lot of scrape-off layer losses because if we have a high enough injection energy of the beam, a particle will just travel through this very thin region and will ionize inside the plasma. The only case where scrape-off layer losses can become significant is when we have a large density in the scrape-off layer, which is very unusual, or a very low injection energy. Because, as you remember, if we have low injection energy, then the cross-section of the ionization process increases, and so we have more ionization processes.

First orbit losses

When we have fast ions that are born in the plasma, as we were discussing before with the orbit theory of fast ions, we can have fast ions that are born inside the confined plasma region, but are born on non-confined orbits like the trapped lost.
In this case, we call this first orbit losses, or even prompt losses, because they are immediately lost.

Fast ions born on non-confined orbits causes first orbit losses

- Most of fist orbit losses are due to banana orbits, when part of the banana orbit cross the separatrix (*as we have seen before when describing beam fast ion banana orbits*)
- A particular case with very high first orbit losses is represented by **NBI counter-current injection**, because banana orbits move fast ions outwards
- Depending on plasma parameters and injection direction, these losses can be relevant
- An analytical formulation is available to estimate first orbit losses

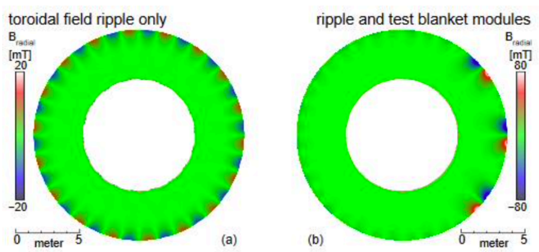


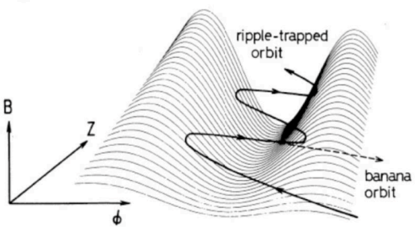
NBI particle trajecory in 3D for MAST
[D. Pfefferlé et al., 2014 Nucl. Fusion **54** 064020]

So they usually do one, even not one toroidal turn, and they are lost. This is a collision-less process, and at the beginning of the fast ion life, the collisions are not really taking place, because the time fast ions spend to do a toroidal turn is so little that a collision with thermal ions or electrons is unlikely. So if a fast ion is born in a non-confined orbit then it is immediately lost.

Also magnetic field ripple can cause immediate orbit losses(collisionless process)

- Magnetic field perturbation due to the finite number of toroidal field coils.
- It can be mitigated with ferritic inserts.
- The ripple modifies the trajectories of fast ions causing localized losses (wall hot spots)
- Both NBI fast ions and alphas (for reactors) will suffer this issue
- Mitigated in ITER, negligible in DTT [Spizzo et al., NF 2021]





Ex. of collisionless trapping of a banana particle

Example for alphas in ITER
[G. J. Kramer, R. B. White, R. Nazikian, H. L. Berk, "Fusion-born alpha particle ripple loss studies in ITER", *22nd IAEA Fusion Energy Conf.*, 2008-Oct.]

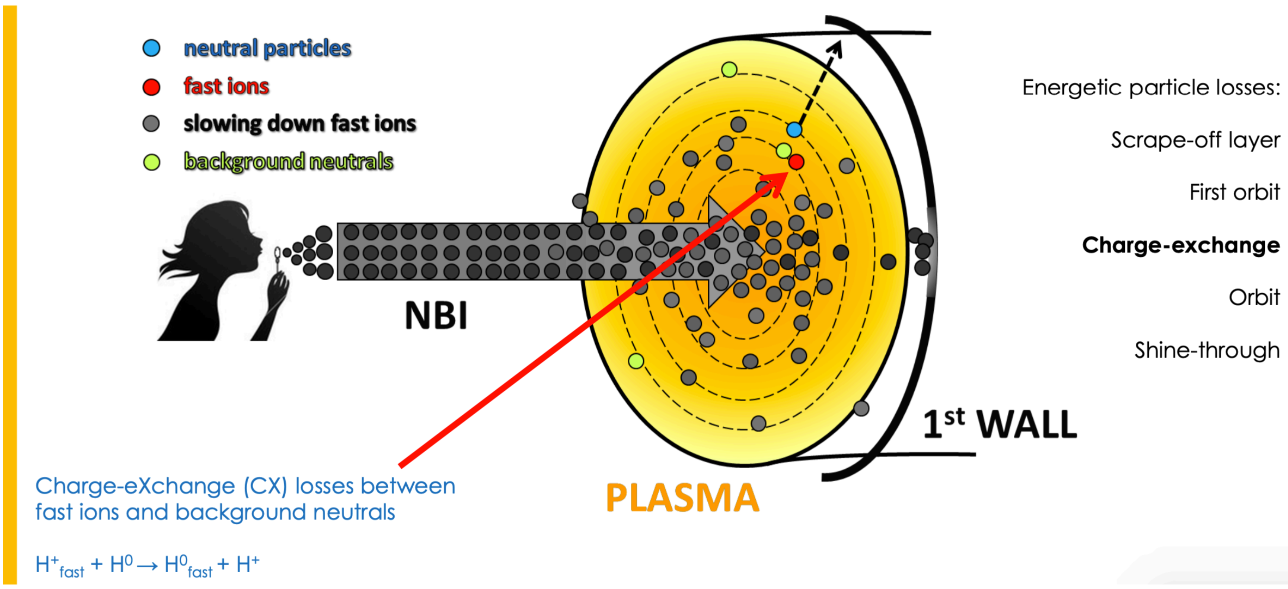
We are always thinking about a toroidal magnetic field, which depends on the major radius of the plasma, and it's axisymmetric in the two dimensions, but we can also have 3D effects on the magnetic field. So the magnetic field is not the same at every toroidal angle. And an example of this are the toroidal field ripples.

Since we have a field number of toroidal coils, the magnetic field generated is not really uniform in the toroidal angle, but it depends on the number of toroidal magnetic coils. This is a 3D effect that may have a significant impact on the confinement of plasma ions, and in particular on the confinement of plasma fast ions, because the fast ions have larger orbits, as we have seen before. So it means that they experience larger part of the plasma, and if there are some irregular magnetic field, we can also have a specific loss of fast ions due to the toroidal field.

In the picture of the slide above on the right, it is shown a ripple trapped orbit. We would have a banana orbit, but the magnetic field is changing also locally due to the ripple of the magnetic field coils. So the particle is reflected also in this well of the magnetic field, and the particles start to drift towards the edge of the plasma field until it is lost. In ITER they quantify this effect, and they have added some ferritic insert within the toroidal field coils in order to mitigate this effect. And with this mitigation, we almost cancel the effect - in ITER now is almost negligible.

For DTT, there has been also an evaluation of this effect, which is present in any device with a finite number of toroidal coils, and it is shown that it is negligible.

Charge exchange losses

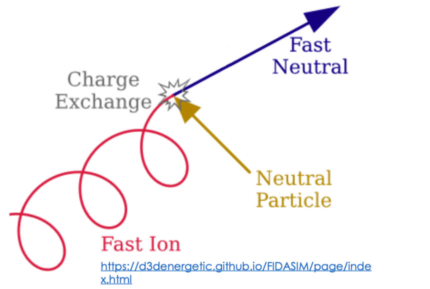


Another source of losses of fast ions is the charge exchange. The process is the same as we have seen for the ionization, but now we don't have a neutral energetic particle, but we have a fast ion, so with high energy.

Losses due to Charge eXchange (CX) reaction between a fast ion and a neutral particle in the plasma (*pay attention, it is a different reaction wrt to NBI fast particle ionization by CX, but beam halo can contribute to these losses*)

$$H^+_{fast} + H^0 \rightarrow H^0_{fast} + H^+$$

Fast neutrals escape the plasma, with lower probability of being ionized



Where do the neutrals come from?
- Gas puff (cold neutrals)
- Pellet injection
- CX NBI ionization (thermal neutrals = beam halo)

- In small experiments CX losses can be up to 20-30% of the injected NBI power
- H^0_{fast} can then be re-ionized before leaving the plasma, broadening the power deposition profile
- In future reactors we expect very high temperature and density, therefore small neutral density in the plasma and low CX fast ion losses

So the charge exchange is the opposite way of what we can see for ionization. We have the fast ion, which collides with a neutral particle, that for any reason is present in the plasma (we will see later why the neutral particle should be in the plasma) and as a result, we have a charge exchange between these two particles, and so we have a fast neutral generated by this reaction, and a thermal ion.
Actually here in this reaction we are not losing a charge, we are just losing the energy, because if we are generating a fast - so it means high energy - energetic neutral, the energetic neutral will travel with high velocity, because it is a fast particle, without experiencing the magnetic field, so it is not confined anymore, it just travels straight, and it will collide with the wall. Thus, with the reaction we are losing the energy, we are not losing a charge, because of this generation of the fast neutral.

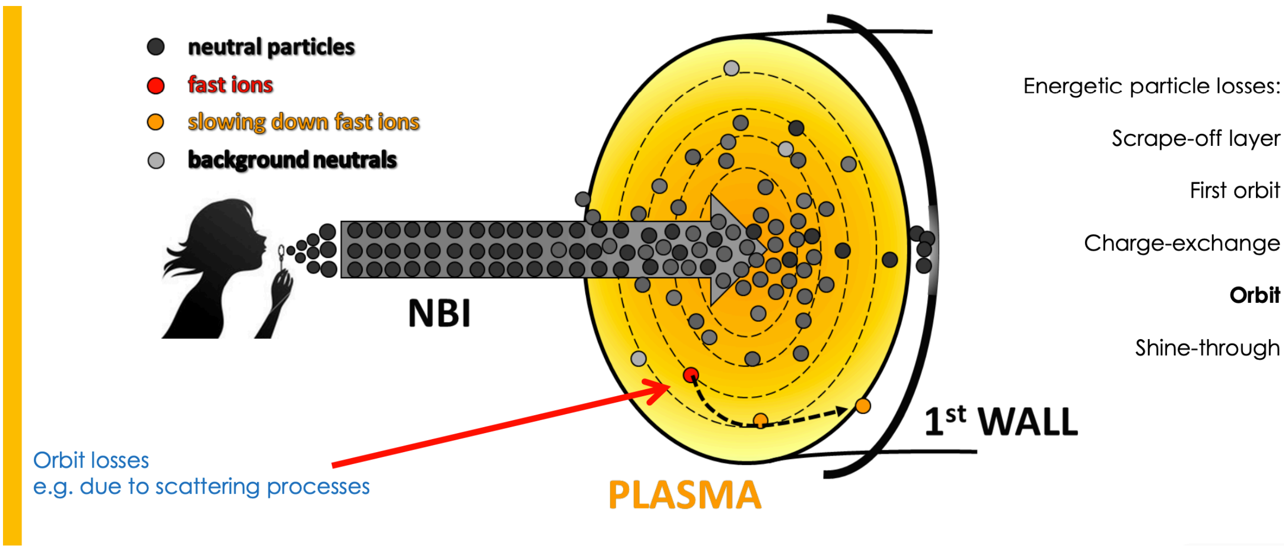
Where do the neutrals come from in the plasma? Because you see that this loss is driven by the presence of neutral particle inside the plasma. It could be cold neutrals that are coming from the gas puff, so the fueling of the plasma. It can be from pellet injection, that generates neutrals inside the plasma, or as we have seen before, it can be neutrals coming from the charge exchange process in the NBI ionization. We have seen before that it is the generation of the beam halo, so a cloud of thermal neutrals close to the position of the NBI ionization.

When fast ions are traveling in the plasma, and pass again in the region of the NBI ionization, they can meet the thermal neutrals generated by beam halo process.

In the small experiment the thermal neutrals can live quite long inside the plasma, because they have less collisions, the density of the plasma is low and these kind of losses can be even 20 or 30% of the injected NBI powers, so it is a lot.
In future reactors, actually, a neutral particle inside the plasma is very unlikely, because the density will be higher, the temperature of the plasma will be higher, and so the life of a neutral particle inside the reactor plasma will be very, very small. So we will not have many charge exchange losses.

Orbit losses

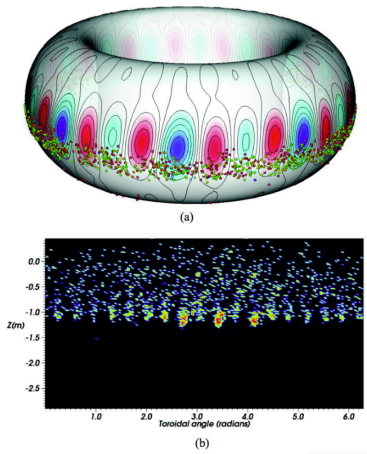
We can have orbit losses due to scattering processes, due to collisions. We have the fast ion, which is in a confined orbit, it undergoes a collision with a thermal ion, for instance, and as a result, it goes into an unconfined orbit and it is lost.



This is, of course, more relevant when the fast ions are affected by the so-called pitch angle scattering, so when the fast ion energy is already lower and closer to the critical energy of the plasma, because pitch angle scattering actually can change the direction and can change the orbit of the fast ions. That's why we are referring to the critical energy.

Similar to first orbit losses but happening for confined fast ions that move into unconfined orbits:

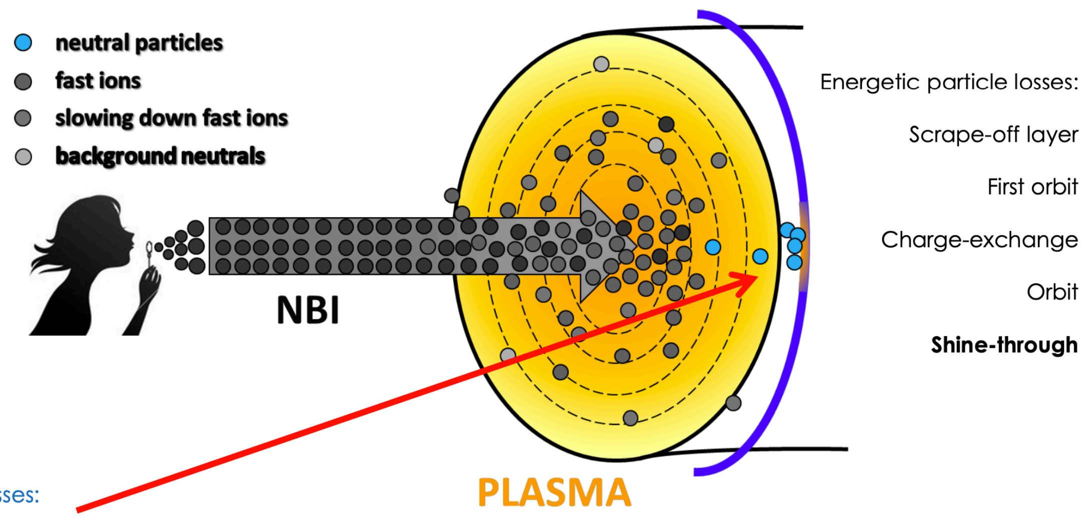
- Losses can be due to pitch-angle scattering: after a collision the fast ions move into a unconfined orbit
- Since the pitch-angle scattering is stronger with ion-ion collisions, these losses are more probable when $E_{fast\ ion} \approx< E_c$
- Orbit losses are more probable for edge fast ions
- Orbit losses can be a result also of fast ion redistribution due to turbulence and instabilities



Orbit losses then are more probable for edge fast ions, because if a fast ion is closer to the edge and it changes a bit its orbit, it is closer to the border of the plasma, to the edge of the plasma, so it can go outside the confined plasma region. And orbit losses can be also a result of fast ion redistribution due to turbulences and instabilities that are present in the plasma.

Shine through losses

The last source of losses for beam particles is the shine through losses. This is a bit different because it's not a loss of a fast ion, but it is directly the loss of beam neutral energetic particles that are not ionized in the plasma and they are just traveling straight and they collide with the other part with the machine on the other side.

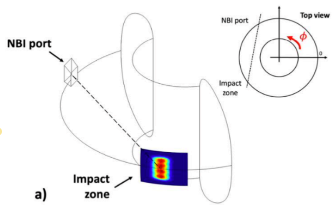


Shine through losses:

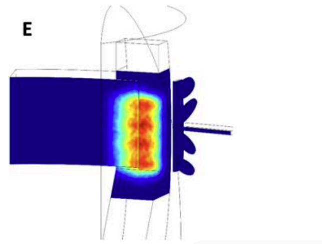
part of the beam not ionized passing through plasma

In the slide below there are some figures where we have the injection of the beam from the NBI port.

The fraction of the neutral beam, which is not ionised in the plasma and passes through it colliding with the first wall causes the so-called shine-through losses.



DTT case



- shine-through losses can cause localized wall hot spots with the risk of exceeding the first wall thermal load limit (sometimes the wall is strengthened explicitly by armours)
- Shine-through increases exponentially with $\sim E_{NBI}/n$
- Usually, a lower limit on plasma density is set for NBI operation due to shine-through
- In current experiments this is one of the highest loss channels for NBI (up to 5 - 10% or more); for ITER and future reactors it will be relevant only in low-density phases (e.g. ramp-up/down phase)

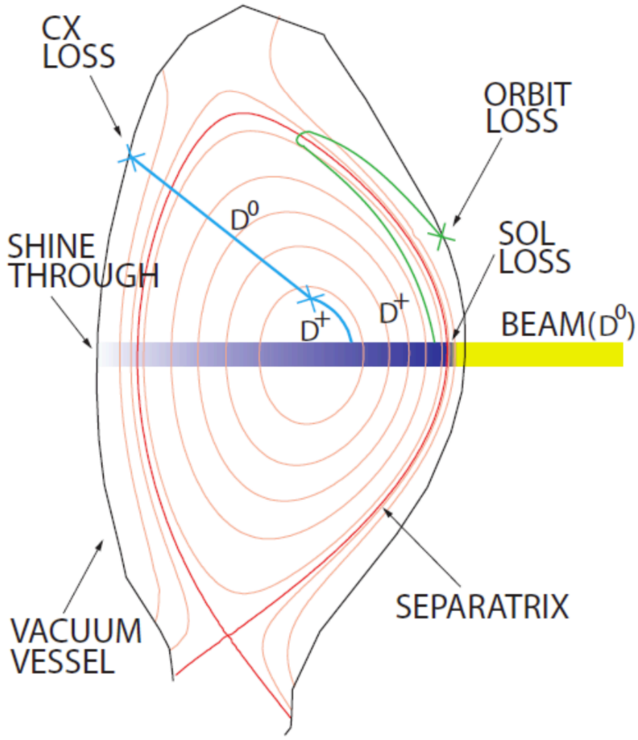
The dashed line is the straight line of the beam path inside the tokamak. And if the beam is not ionized completely inside the plasma, in the figure below it is shown the NBI footprint we see on the opposite wall. This can pose a risk for the machine safety because this generates a wall hot spot. So you can see some deposited power on the first wall structures. And since this loss depends on the ionization process, it means that the shine through increases exponentially with the energy of the beam divided by the density of the plasma. So larger is the NBI energy and lower is the density of the plasma, higher will be the shine through losses.

Usually for a given NBI energy, there is a minimum plasma density to be met in order to safely operate the NBI: also for ITER, this has been calculated. So it is defined a density of the plasma, which of course depends on the injection energy, that is the minimum density for a safe operation of the beam without exceeding the allowed power that can be deposited on the plasma facing components in the footprint of the beam.

In the current experiments, since we have low density, usually the shine through losses can be around 5-10% or even more. For ITER plasmas, it would be relevant only in low density phases. So if we are thinking about the plasmas that will produce out of fusion energy, in this case, the shine through will be negligible, almost zero. But if you want to use the NBI during, for instance, the ramp-up and ramp-down phases, in this case, the low density plasma can result in the shine through losses.

Energetic particle losses

This below is a sketch of the possible losses of the beam.



For charge exchange, you can see the path of a fast ion that is following the magnetic field lines, it takes an electron from a thermal neutral, it becomes a fast neutral, and it just travels ballistically to the wall.

NBI modeling techniques

We can now go through the modeling techniques.

Measuring NBI effectiveness is hard

So the fast ion-related quantities, so everything we have talked about, for instance, current drive, or torque, etc, it's very difficult to measure.

For instance, for the heating, we have an indirect estimation from the NBI losses, so if we can quantify the NBI losses, we can also have evaluation of how much power we are providing to the plasma. We can also see the effect of the heating on the temperature profiles, from temperature measurements, but this is only an indirect measurement of the NBI effect.

Fast ion-related quantities are difficult to measure, e.g.:

Heating:
- Indirect from estimation of NBI losses
- profile: indirectly from temperature measurements and modelling

Total driven current:
- comparing the loop voltage between a current-drive discharge and reference discharge, matching the relevant plasma profiles (as much as possible)

Driven current profile:
- using Motional Stark effect (MSE): local direction of B → at best q(r) profile → j(r) profile
- forward modelling on MSE measurements

Fast ion distribution function:
- measuring the fast neutrals escaping the plasma generated by CX processes between fast ions and background neutrals (e.g. FIDA)

Similar is for the total driven current, because the only way to understand how much current is driving my beam is to compare the loop voltage, so we can say the strength of the solenoid. We have to compare between a current drive discharge and a reference discharge. So if we can reproduce the same plasma with the same parameters, one plasma using the NBI to generate the current, the other plasma with the solenoid generating the current - this is actually difficult - we can compare the two, so we can understand how much current is providing my beam.

The current drive profile is also difficult to measure, so one possibility is to use the motional stark effect diagnostic, and this measures the local direction of the magnetic field, so we can reconstruct the information, the safety factor profile - the Q - and retrieve finally the current density profile.

Last, the first ion distribution function, so the characteristics of the fast ion population. This can be done by measuring the fast ion escaping the plasma, for instance by charge exchange reactions, and there are diagnostics that do this.

NBI modelling

It's very difficult to measure the NBI related quantities, so to do this we have to go through numerical modeling. Numerical modeling for the interaction between the beam and plasma is used for data interpretation or for prediction for future experiments or future machine.

Estimating quantities related to NBI-plasma interaction requires **modelling**.
NBI modelling is used for data interpretation and for predictions (e.g. for design of future machines)

NBI modelling requires two steps:

1. Modelling the injection and ionization of energetic neutral particles

⇨ **Beam deposition codes**

Modelling of ionization

Example of pure deposition codes: BBNBI, NEMO…

The NBI modeling requires two steps: the first step is the modeling of the ionization, so the generation of the newly born fast ion population. This is done with a so-called beam deposition code and you may have heard about BBNBI, about NEMO, many others.

2. Modelling the dynamics (slowing down) of NBI ions in the plasma

⇨ **Fokker-Planck solvers** (↔ fast ion distribution function)

Example of pure Fokker-Planck solvers: ASCOT, SPOT, RISK…

M. Schneider et al., 2015, 42nd EPS on Plasma Physics, P2.159
ASCOT: E. Hirvijoki et al, Computer Physics Communications, pp. 1310-1321, 2014
SPOT: M. Schneider et al., Nucl. Fusion 49 (2009) 125005
RISK: M. Schneider et al., Nucl. Fusion 55 (2015) 013003

The second part is when we have a population of fast ions, so we have the source of the ionized beam particles and it is the modeling of the slowing down process, and for this we have to use the Fokker-Planck equation solvers.

Example of Fokker-Planck equation solvers are, for instance, Ascot, Spot, and risk, but there are many more.
All these NBI codes can be beam deposition codes - so taking the ionization process or slowing down process -, they can have different approaches. So they can use analytical or semi-analytical simplified modeling, also including some scaling laws, and in this case the beam ionization is solved numerically with some approximations.

Usually this is done small steps and approximating the beam as straight lines, so not taking into account the divergence of the beam, the focus of the beam, and many other geometrical effects.

NBI codes can have different approaches (impacting on precision and time consumption):

a. (**Semi-**) **Analytical** solutions / simplified modelling / scaling laws:

- Beam ionization solved numerically in small steps on straight line(s) representing the beam
- Simple beam geometry (pencil-like)
- Power deposition (total, to electrons, to ions), current-drive, momentum source calculated using analytical formulae
- Fast simulation time
- Stand-alone codes or modules of integrated codes (e.g. transport suites)

E.g.: PENCIL (JETTO suite), Rabbit (stand-alone), METIS (integrated), NEMO-RISK (CRONOS suite)

- 3D beam shape but analytical ionization calculation: BTR

PENCIL: M. Stubberfield and M.L. Watkins, Multiple Pencil Beam, JET-DPA(06)/87, 1987
Rabbit: M. Weiland et al 2018 Nucl. Fusion 58 082032
METIS: J.F. Artaud et al 2018 Nucl. Fusion 58 105001
NEMO: M. Schneider et al 2011 Nucl. Fusion 51 063019
BTR: E. Dlougach et al Appl. Sci. 2022, 12, 8404

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We have a power deposition, a current drive, a momentum source, calculating in the analytic formula. This has the advantage of very fast simulation time, and examples of this are like PENCIL, which is used for JET, RABBIT in ASDEX upgrade, METIS, which is an integrated plasma simulator, which takes into account the beam effect with the simplified formulas, and NEMO-RISK. There is also another code which is used a lot in the ITER design, which takes into account the 3D beam shape, and it is called the BTR.

A typical problem for the fast ion modeling is the estimation of the fast ion distribution function. Fast ion distribution function can be pictured as the figure we have seen before, and to evolve the distribution function of fast ions, we must solve the Fokker-Planck equation. So the Fokker-Planck equation is quite simple, it gives the evolution in time of the fast ion distribution function, which depends on many variables, like the position and the velocity of the fast ions, and this depends on a collisional term, which is C. So of course collisions can modify my fast ion distribution function, for instance they can change the pitch, they can change the energy of the fast ions.

S is the source term, which is the beam ionization, so I have a beam continuously injecting new particles that become fast ions. Of course if we switch off the beam, S goes to zero. And N is the loss term, so it's all the loss processes we have seen before.

A typical problem for fast ion modelling is the estimation of the fast ion distribution function.
In general, the time evolution of the distribution function can be described by the Fokker-Planck equation:

$$\frac{df}{dt} = C + S + L$$

- C is the collision term (fast ion – background plasma or fast ion – fast ion)
- S is the source term (beam ionization)
- L is a loss term (e.g. fast ion losses)

Many approximations can be done[1,2], and usually we are interested of stationary solutions (df/dt=0).

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[1]Rome J A 1976 Nucl. Fusion 16 55
[2]R. Koch (2010) Fusion Science and Technology, 57:2T

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So we can do many approximations to have analytical representation of the Fokker Planck solution.

Usually to find an analytical solution for the fast ion distribution function, we can assume some approximations.

In order to find an analytical solution for f, we assume[1]:

- Fast ion bounce frequency >> collision frequency
    - In this approximation we have the following constant of motion:
        - Fast ion kinetic energy $E = \frac{1}{2}mv^2$
        - Toroidal canonical angular canonical momentum $P_\phi = mRv_{par} + q\psi$
        - Magnetic moment (adiabatic invariant) $\mu = \frac{m{v_{perp}}^2}{2B_{tot}}$

    → f(E, P_φ, μ)
    - A point in the constant of motion phase space (E, P_φ, μ) represents an entire orbit
    - Collisions will move points in the phase space ("kicks")
- Still, the equation is too complex to be solved analytically

[1]Rome J A 1976 Nucl. Fusion 16 55

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The fast ion bounce frequency is the pulse frequency for trapped particles, it is the frequency between turning points of the banana orbit, and this is much larger of the collision frequency. If we assume this, we can have the constant of motion above.
So recall what we have said before, if we neglect collisions, because the bounce frequency - so the time the orbit follow the banana is much more than the collision frequency - we can consider the constant of motion of the particles, like the kinetic energy, the magnetic moment, and we also add the toroidal canonical angular momentum.

So neglecting collisions, we can take into account the conservation of these quantities, and we can define the fast ions with three parameters, which ~~is~~ are kinetic energy, the toroidal canonical angular momentum, and the magnetic moment. A point in the distribution function defined by these three variables represents an entire orbit, because we are neglecting collisions, so if we identify fast ions with these three parameters, we are identifying a fast ion orbit.

Of course, which is the effort of collisions? Collisions will move points, so they will change these three parameters, so we have kicks in this phase space due to the collisions.

This approximation is not enough to have an analytical solution of the Fokker-Planck equation.

A typical problem for fast ion modelling is the estimation of the fast ion distribution function.
In general, the time evolution of the distribution function can be described by the Fokker-Planck equation:

$$\frac{df}{dt} = C + S + L$$

In order to find an analytical solution for f, we assume[1]:

- f is independent of P_φ, i.e. the fast ions remains close to a given flux surface during the slowing down ("small banana width approximation) – hardly true for trapped energetic particles
- neglecting trapped particles when performing the integral around an entire bounce orbit

[1]Rome J A 1976 Nucl. Fusion 16 55

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Other assumption to have this analytical solution is that the distribution function f is independent of the toroidal magnetic angular momentum. This means that the fast ions remain close to a given flux surface during the slowing down, and this is also called the small banana width approximation - this means that it's neglecting the angular momentum. This is a strong approximation that is hardly true for trapped energetic particles, because we have seen that they have quite large bananas, and therefore this cannot be true with this approximation.

1)

Another very strong approximation which works if we have mainly passing particles is to neglect the trapped particles. If we have a neutral beam injecting particles tangentially with a very strong parallel velocity in the toroidal magnetic field, we can neglect the trapped particles, and this helps if we want to find an analytical solution of the Fokker-Planck equation.

2)

The resulting analytic solution of the Fokker-Planck equation is:

$$f(v,\xi,t) = U\left[t - \frac{\tau_s}{3} ln\left(\frac{{v_0}^3 + {v_c}^3}{v^3 + {v_c}^3}\right)\right] \frac{\tau_s}{2\pi(v^3 + {v_c}^3)} \left[\frac{{v_0}^3 + {v_c}^3}{v^3 + {v_c}^3}\right]^{-\frac{\tau_s}{3\tau_{cx}}} \times \sum_{n=0}^{\infty} \left(n + \frac{1}{2}\right) P_n(\xi) P_n(\xi_0) \left[\frac{{v_0}^3 + {v_c}^3}{v^3 + {v_c}^3} \frac{v^3}{{v_0}^3}\right]^{\frac{m_i n(n+1)\langle Z\rangle}{6m_{fast\,ion}[Z]}} U(v_0 - v)$$

- U is the step function
- P_n are Legendre polynomials
- ξ is the pitch (ξ_0 the initial pitch)
- τ_s the slowing down time
- v_0 the initial fast ion velocity, v_c the critical velocity $v_c = \sqrt{\frac{2E_c}{m_{fast\,ion}}}$
- τ_{cx} the charge exchange time

$$[Z] \equiv \sum_j n_j Z_j^2 (m_i/m_j) / \sum_j n_j Z_j$$

$$\langle Z\rangle \equiv \sum_j n_j Z_j^2 / \sum_j n_j Z_j$$

Rome J A 1976 Nucl. Fusion 16 55 (equation 4.17)

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If we do all this approximation, we still have a very complicated definition, a very complicated solution of the fast ion distribution function, which in this case is given by three parameters: the velocity of the fast particle, the pitch of the particle where ξ_0 is the initial pitch, and the time where we are considering this particle.

Tcx is the typical time between charge exchange reactions.

To simplify this huge equation, we can still do some approximation.

The resulting analytic solution of the Fokker-Planck equation is:

$$f(v,\xi,t) = U\left[t - \frac{\tau_s}{3} ln\left(\frac{{v_0}^3 + {v_c}^3}{v^3 + {v_c}^3}\right)\right] \frac{\tau_s}{2\pi(v^3 + {v_c}^3)} \left[\frac{{v_0}^3 + {v_c}^3}{v^3 + {v_c}^3}\right]^{-\frac{\tau_s}{3\tau_{cx}}} \times \underbrace{\cancel{\sum_{n=0}^{\infty}\left(n + \frac{1}{2}\right) P_n(\xi) P_n(\xi_0) \left[\frac{{v_0}^3 + {v_c}^3}{v^3 + {v_c}^3}\frac{v^3}{{v_0}^3}\right]^{\frac{m_i n(n+1)\langle Z\rangle}{6 m_{fast\,ion}[Z]}}}}_{=1/2} U(v_0 - v)$$

Approximations:

1. Neglect fast ion velocity distribution anisotropy
   - n=0 → P0(x) = 1
   - Legendre polynomials are even functions → n=1 term is zero

   → No dependence from the pitch ξ

1) First of all, we can neglect the fast ion velocity distribution anisotropy, what does it mean? It means that we neglect the dependence of the fast ion distribution function from the pitch, which is not true, because we have seen that the source has a very strong pitch dependence, that the fast ion process has a very strong pitch dependence.

If we want to simplify, to arrive to an analytical solution of the Fokker-Planck equation describing the slowing down of a fast ion, we can do this approximation: we neglect the dependence on the pitch, this means that all the terms cancelled becomes 1/2.

The resulting analytic solution of the Fokker-Planck equation is:

$$f(v,\xi,t) = \cancel{U\left[t - \frac{\tau_s}{3} ln\left(\frac{{v_0}^3 + {v_c}^3}{v^3 + {v_c}^3}\right)\right]} \frac{\tau_s}{2\pi(v^3 + {v_c}^3)} \left[\frac{{v_0}^3 + {v_c}^3}{v^3 + {v_c}^3}\right]^{-\frac{\tau_s}{3\tau_{cx}}} \times \underbrace{\cancel{\sum_{n=0}^{\infty}\left(n + \frac{1}{2}\right) P_n(\xi) P_n(\xi_0) \left[\frac{{v_0}^3 + {v_c}^3}{v^3 + {v_c}^3}\frac{v^3}{{v_0}^3}\right]^{\frac{m_i n(n+1)\langle Z\rangle}{6 m_{fast\,ion}[Z]}}}}_{=1/2} U(v_0 - v)$$

Approximations:

1. Neglect fast ion velocity distribution anisotropy
2. Stationary solution ($t >> \tau_s$)

   → No time dependence

2) Second approximation is that we are searching for stationary solution, so we are looking at a time which is much larger than the slowing down time - we want that everything is stationary - and if we do this, due to the delta function, we can neglect the first term.

3) The third approximation we can do is that if we expect there are very few neutrals inside the plasma - this is true for instance for ITER -, we can neglect the charge exchange losses due to the presence of neutrals inside the plasma, so we can put the charge exchange time to infinity - this is the time we have to wait for a charge exchange process to happen.

If we put to infinity, the term in blu can be neglected, so we can finally have a fast ion distribution function which depends only on velocity, because we have neglected the dependence on the pitch, we have neglected the dependence on the current.

The resulting analytic solution of the Fokker-Planck equation is:

$$f(v,\cancel{\xi},\cancel{t}) = \cancel{U\left[t-\frac{\tau_s}{3}ln\left(\frac{{v_0}^3+{v_c}^3}{v^3+{v_c}^3}\right)\right]}\frac{\tau_s}{2\pi(v^3+{v_c}^3)}\cancel{\left[\frac{{v_0}^3+{v_c}^3}{v^3+{v_c}^3}\right]^{-\frac{\tau_s}{3\tau_{cx}}}}\times\underbrace{\cancel{\sum_{n=0}^{\infty}\left(n+\frac{1}{2}\right)P_n(\xi)P_n(\xi_0)\left[\frac{{v_0}^3+{v_c}^3}{v^3+{v_c}^3}\frac{v^3}{{v_0}^3}\right]^{\frac{m_i n(n+1)\langle Z\rangle}{6m_{fast\ ion}[Z]}}}}_{=1/2}U(v_0-v)$$

Approximations:
1. Neglect fast ion velocity distribution anisotropy
2. Stationary solution ($t \gg \tau_s$)
3. Neglect background neutrals
   → $\tau_{cx} = \infty$

This formula below is for the fast ion distribution function as a function of the velocity of the fast ion, which is v, vc is the critical velocity, and now you can understand where it comes from and which approximation has been done that are quite strong.

Resulting:

$$f(v)=\frac{\tau_s}{2\pi(v^3+{v_c}^3)}\times\frac{1}{2}U(v_0-v)=\frac{\tau_s}{4\pi(v^3+{v_c}^3)}U(v_0-v)$$

- Commonly used in fast NBI modelling tools (e.g. METIS)

Rome J A 1976 Nucl. Fusion 16 55 (equation 4.17)

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This is actually used in some analytical tools, for instance METIS. It has so strong approximation that if you want to do some NBI modeling, you cannot really rely on this model here.
So this was for the analytical solution.

Another possibility is to use **Monte Carlo solvers.** In this case, we have an initialization of a random test particle, which follows a given distribution, and each Monte Carlo test particle represents a number of real particles through some weights in the code, and this random test particles is followed inside the plasma until the full particle thermalization, which means that the particle of the fast ion energy is close to the local energy of the plasma, so the local ion temperature, for instance, or in case it is lost.
So we have these codes that follow this random test particles, either thermalization or loss process.

b. Monte Carlo solvers (e.g. orbit following in phase space):

- Initialization of a random test particle following a given distribution
- Each Monte Carlo test particle represents N real particles, through weights
- Random test particles are followed until particle thermalization (e.g. E > 3/2 local T_i) or loss
- Detailed beamline/tokamak geometries
- Detailed physics (e.g. collisions, impurities, orbit drifts, finite Larmor radius effects, 3D magnetic fields, …)
- Long computational time
- Usually stand-alone codes, or used in detailed transport modelling

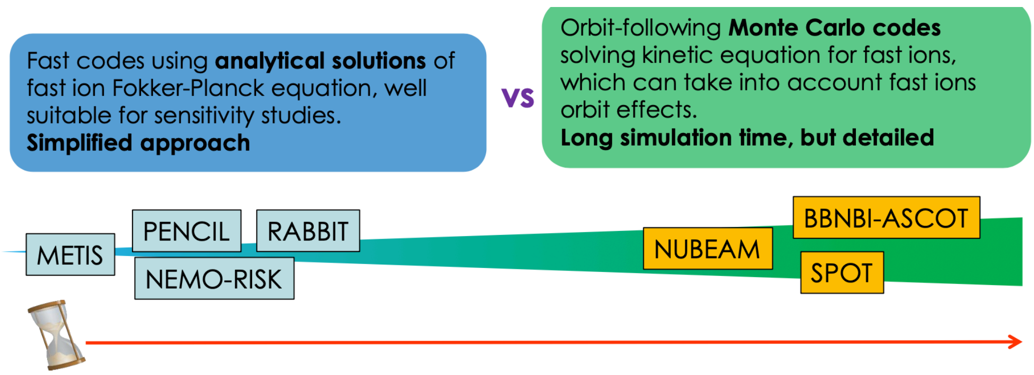
E.g.: BBNBI & ASCOT (stand-alone), NUBEAM (TRANSP suite), SPOT (CRONOS suite) etc…

BBNBI: Asunta O et al 2015 Comput. Phys.Commun. 188 33–46
ASCOT: E. Hirvijoki et al, Computer Physics Communications, pp. 1310-1321, 2014
NUBEAM: Pankin A. et al., 2004 Comput. Phys. Commun. 159 157–84
SPOT: M. Schneider et al., Nucl. Fusion 49 (2009) 125005

And usually in these Monte Carlo solvers, since the initial test particles come from the ionization of the beam, you can use a very detailed beam geometry - so taking into account the beamlets, taking into account the divergence of the single beamlets, taking into account the focus of the complete beam. You can include a lot of these, much more than in the analytical solution. So you can take into account the collisions with the plasma ions and electrons, collision with impurities, all the orbits drift, all the Larmour radius effects, also the gyromotion of the particle, the effect of the 3D magnetic fields, but at the cost of a very long computational time.

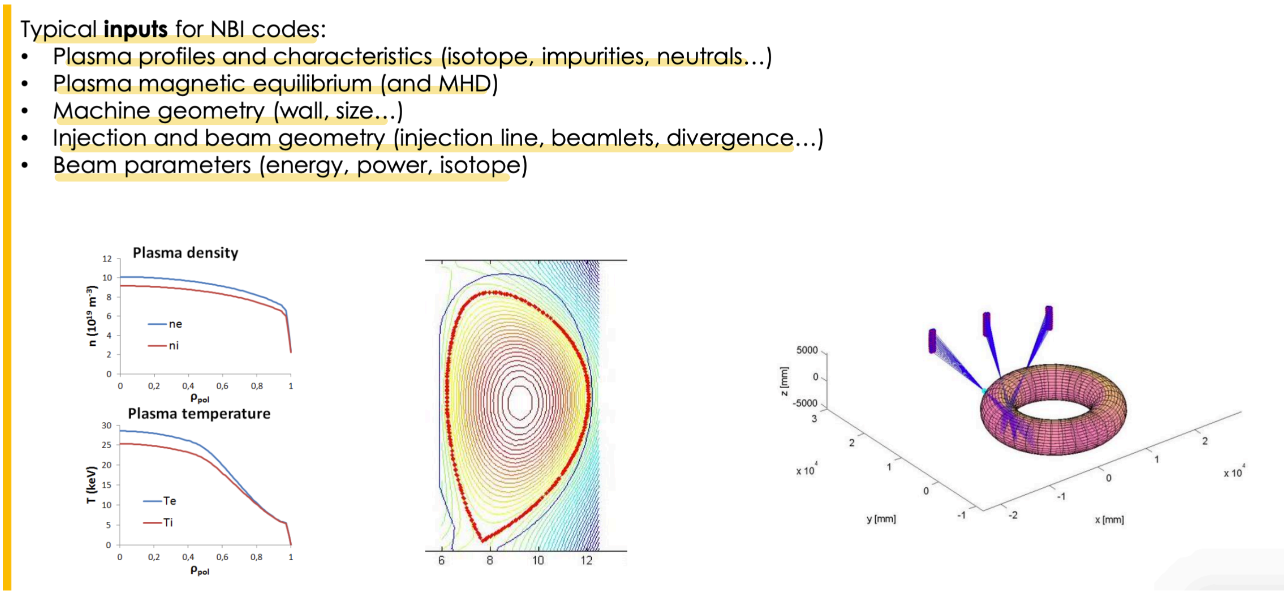
So usually these codes are used standalone, they are not integrated in a full plasma simulation, like for instance what METIS is doing or they are used for detailed transport modeling, which is a very high computational time.

We can find in the slide below a summary of the two approaches: the fast approach with analytical solutions or simplified approach, and the orbit following Monte Carlo codes, which solves in much more details the kinetic equation for fast ions, they take into account the fast ion orbit effects.



The picture showed for the orbit, they have been done with the ASCOT code, they just follow particles in the real plasma, but of course the simulation time is much longer.

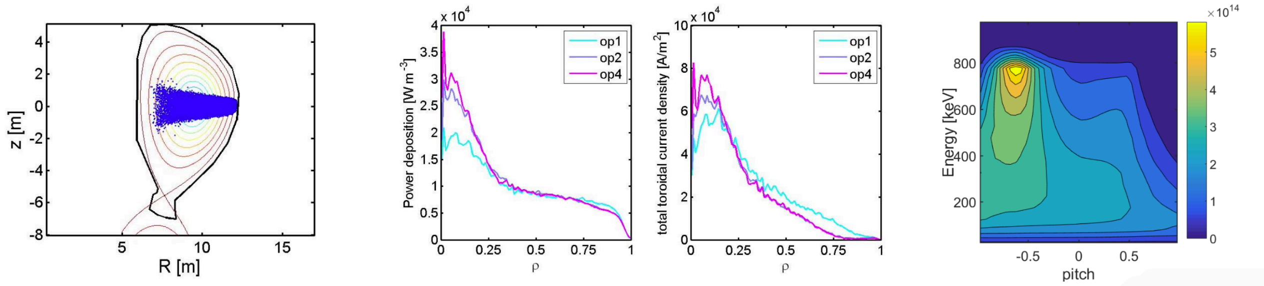
The typical inputs for an NBIs code in general are:



You must provide the plasma profile and the characteristics, so which plasma you are simulating, which impurities you expect, if there are neutrals, you must have the plasma magnetic equilibrium, and if there are some 3D effects you may use the 3D non axisymmetric magnetic equilibrium, the machine geometry of course (so the wall, the size of the machine, the radius), the beam and injection geometry, so the injection line, the divergence of the beamlets, and the beam parameters like the energy, the power, the isotope you are injecting.

Typical output of NBI codes. You can solve all the things listed that are very difficult to measure for NBI plasma interaction, but with modeling you can have a result. For instance, the beam ionization and shine through losses, the power deposition, also power deposition to plasma ions, how much to plasma electrons, and also among plasma ions, how much to each species, because NBI can be deposit powers to plasma impurities.
You can model the orbit losses, you can model the current drive, you can model the input torque, and a result on plasma rotation.

Typical **outputs** of NBI codes:
- Beam ionization and shine-through losses
- Power deposition
- Orbit losses
- Current-drive
- Input torque
- Fast ion distribution function
- …



Just as an exercise, if we look at the figure on the right, this is the fast ion distribution function, pitch on the abscissa and energy on the ordinate.

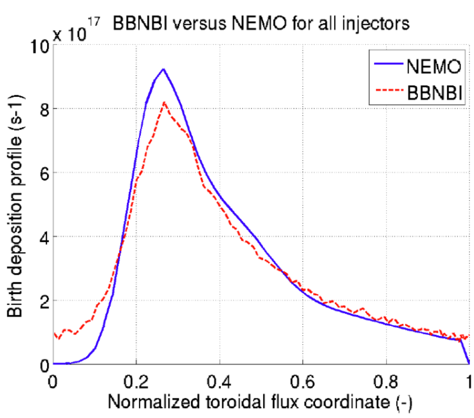
Can you identify the initial energy, injection energy of the beam? It is 800keV - there is a strong source, it also have negative pitch, so it means that it is injected in the opposite direction of the magnetic field. And we also expect that the plasma is quite hot, because if you see the pitch angle scattering starts quite soon in energy, so it means that the critical energy should be quite high, some hundreds or something around here.

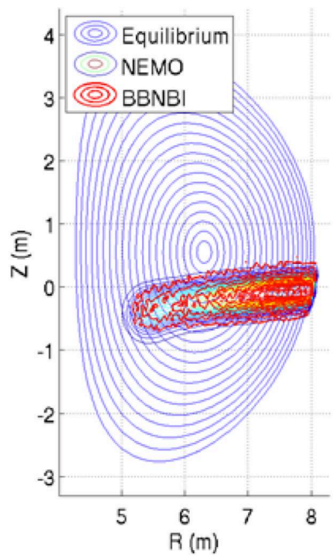
NBI code benchmark

In literature you can find a lot of papers comparing different codes and comparing also different models, like analytical models versus Monte Carlo models.

Different solutions (i.e. codes) of the same problem must agree if the inputs are the same.
NBI models and numerical codes are benchmarked towards other codes to prove their reliability.
Here an example for ITER studies:

Beam deposition codes:
BBNBI (Monte Carlo) vs NEMO (narrow-beam model)





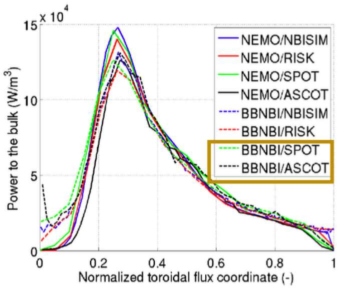
M. Schneider et al., 2015, 42nd EPS on Plasma Physics, P2.159
Shine-trhough benchmark among BBNBI, NEMO, METIS in P. Vincenzi et al., submitted to NF 2024

The plots are are comparing BBNBI and NEMO for the ionization of the beam.

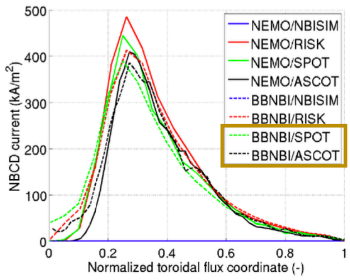
NEMO is an analytical deposition ionization solver, BBNBI is Monte Carlo solver, so you see there are some differences, but the two codes agree quite well. BBNBI is a bit slower, NEMO is much faster, and you can see that also in the 2D poloidal projection, the two codes agree, so the cloud of newly fast ions is very similar.

In the plots below we are looking at slowing down, so we are coupling a beam deposition code, which could be BBNBI or NEMO, to a slowing down code, which can be, for instance, ASCOT or SPOT. And again, the agreement was shown.

> Different solutions (i.e. codes) of the same problem must agree if the inputs are the same.
> NBI models and numerical codes are benchmarked towards other codes to prove their reliability.
> Here an example for ITER studies:
>
> Some Fokker-Planck solvers (they can be combined with deposition codes):
> - NBISIM* is a simple 1D analytic model
> - RISK* is a 2D Fokker-Planck code that combines finite elements and an eigenfunction expansion
> - ASCOT and SPOT are Monte Carlo codes with a high level of accuracy including orbit width effects



Highest accuracy



*They use the so-called zero-banana-width limit

M. Schneider et al., 2015, 42nd EPS on Plasma Physics, P2.159

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Summary

> NBI works only because:
>
> - …the cross sections for CX and ionization between an ion beam and a neutral gas are just adequate for efficient neutralization
> - …the cross sections for ionization are just adequate for an efficient beam ionization in the plasma
> - …the collision frequency in the plasma is just adequate for slowing down of fast ions and energy transfer

We have the cross section for discharge exchange and ionization between an ion beam and an neutral gas inside the NBI system, and this is just sufficient for an efficient neutralization.

Then, when the beam enters the plasma, the cross section for ionization of the neutral beam are just adequate for an efficient beam ionization in the plasma, and then the collision frequency in the plasma is just adequate for slowing down time of fast ion energy transfer within a typical first time life inside the machine. So we have a combination of these three effects inside the system to generate a beam for the beam ionization and for the slowing down process that makes the neutral beam heating possible.

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…and all the material cited in the slides!