CONSORZIO RFX
Ricerca Formazione Innovazione

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UNIVERSITÀ DEGLI STUDI DI PADOVA

Ph.D. Programme in Fusion Science and Engineering

Advanced course on Plasma Physics and Diagnostics (AC1)

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Physics of NBI heating and current drive

P. Vincenzi*

pietro.vincenzi@igi.cnr.it

*Consorzio RFX, Padova

*CNR – ISTP, Padova

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A special acknowledgement to Chiara De Piccoli for helping me with the material in this presentation

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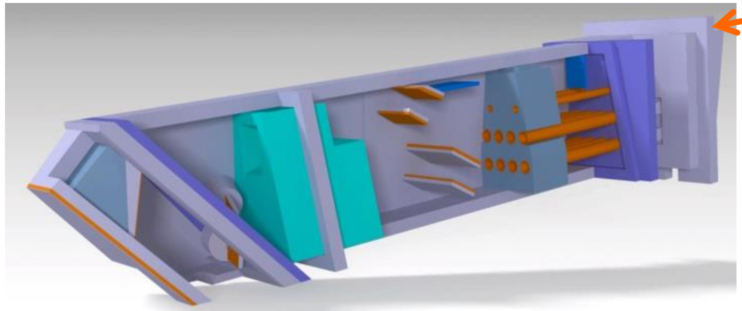
- Beam energetic particle losses
- NBI modelling techniques
- Conclusion

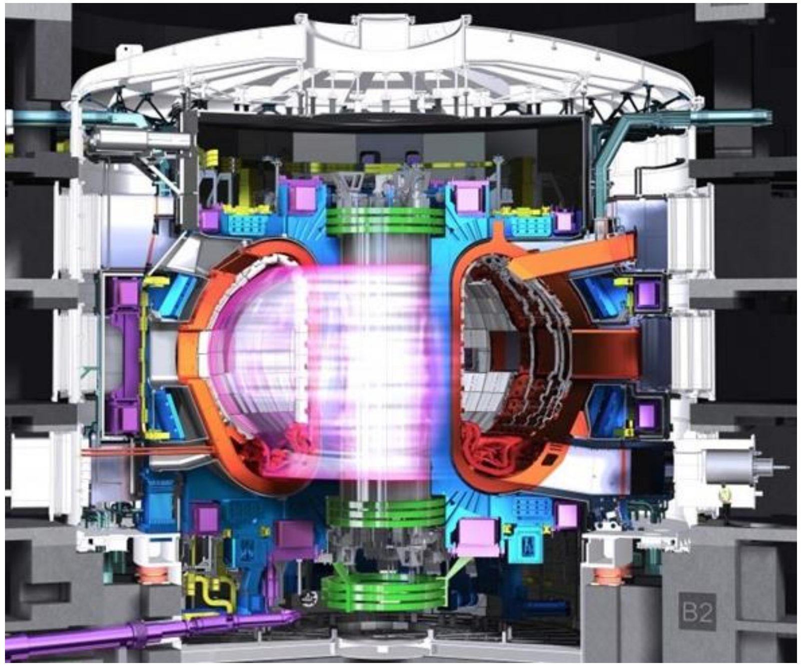
The "additional" H&CD systems

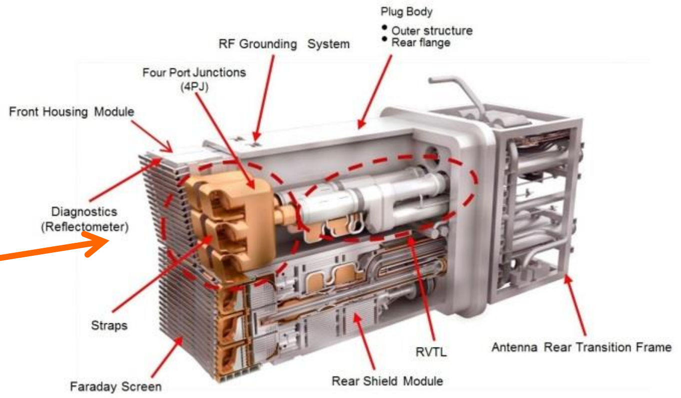
ITER will use the so-called "additional" heating (& current drive) systems

Radio Frequency Systems

Electron Cyclotron (EC) ITER Launcher

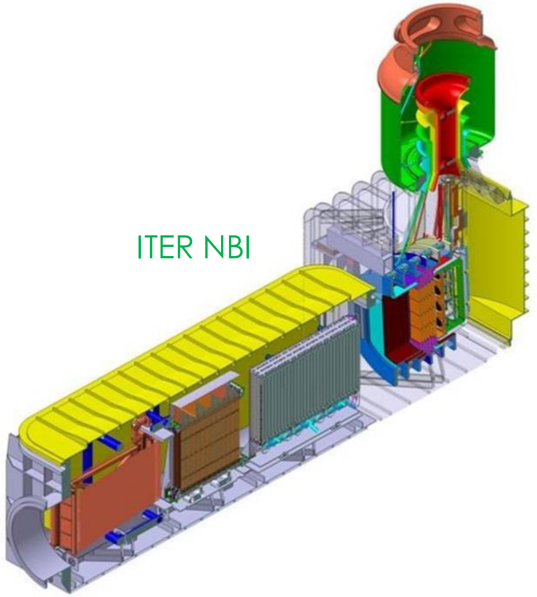






Ion Cyclotron (IC) ITER Antenna

ITER NBI



Neutral Beam Injection (NBI)

*Lower Hybrid Current Drive (LHCD) system is not part of the ITER construction baseline

Images from www.iter.org, www.inr.kit.edu

A step back…

***Additional* to…what?**

- Let's assume here we are dealing with toroidal pinches (tokamak, RFP, …). Initial considerations on stellarator devices would be a bit different.
- We will mention mainly **3 actions**: heating, current drive, and momentum injection. Spend some moments and try to think what this means in a plasma.
- There are fundamental quantities that can be linked to these actions:
    - heating→ temperature (T)
    - current drive→ current density (j)
    - momentum drive→ plasma fluid rotation (W)
- You will see that not only average values, but also radial profiles (and their time evolution!) are important to understand H&CD role.

0D power balance

0D power balance in stationary conditions

$$\mathbf{P_{aux} + P_{\alpha} + P_{ohmic} = P_{loss}}$$

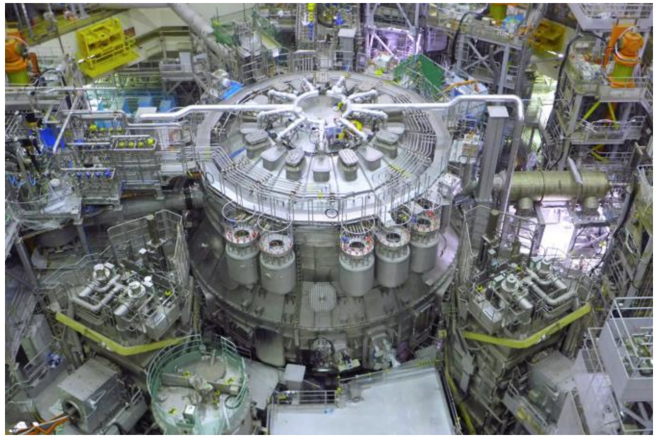
P_{aux} = External heating sources
P_{α} = Alpha power (~1/5 P_{fus})
P_{ohmic} = Ohmic power
P_{loss} = Power losses (radiation, conduction)

J. D. Lawson, (1957) Proc. Phys. Soc. B 70 303

Current experiments (tokamaks):

$P_{\alpha} \sim 0$

→ $P_{aux} + P_{ohmic} = P_{loss}$

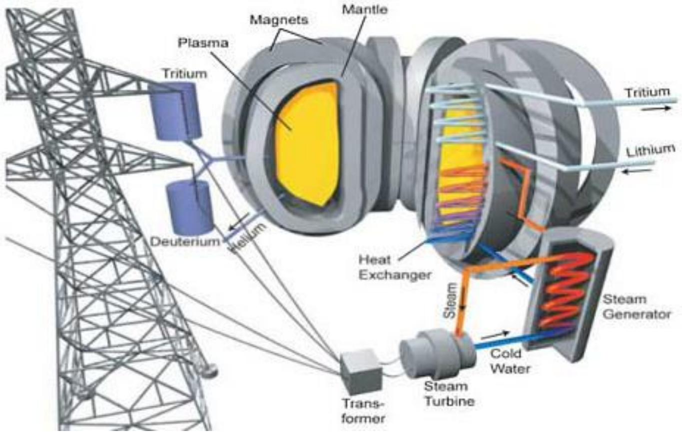


JT-60SA experiment

Future reactors:

Aiming at ignition (plasma self-sustainment)

➢ P_{ohmic} negligible, $P_{aux} = 0$ → $P_{\alpha} = P_{loss}$



0D power balance

0D power balance in stationary conditions

$$\mathbf{P_{aux} + P_a + P_{ohmic} = P_{loss}}$$

$\mathbf{P_{aux}}$ = External heating sources
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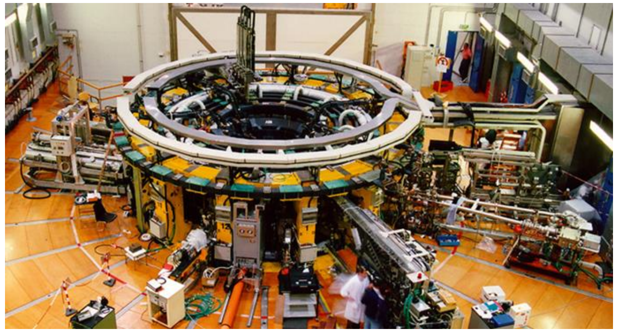
$\mathbf{P_{loss}}$ = Power losses (radiation, conduction)

J. D. Lawson, (1957) Proc. Phys. Soc. B 70 303

Current experiments (RFP):

If $P_{aux} = 0$ and $P_a \sim 0$

→ $P_{ohmic} = P_{loss}$

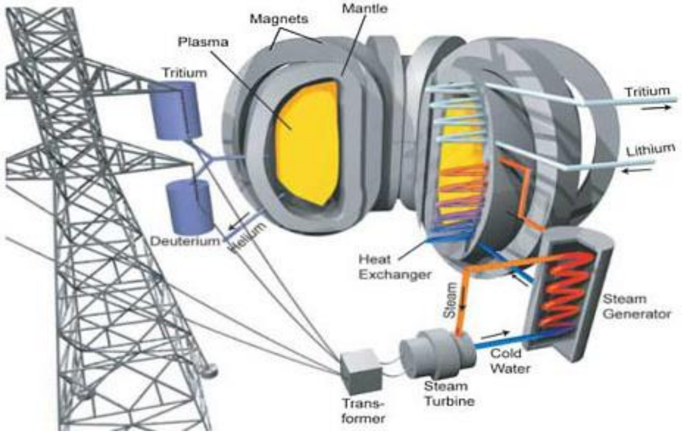


RFX-mod experiment, IT

Future reactors:

Aiming at ignition (plasma self-sustainment)

- P_{ohmic} negligible, $P_{aux} = 0$ → $P_a = P_{loss}$



0D power balance

0D power balance in stationary conditions

$$\mathbf{P_{aux} + P_a + P_{ohmic} = P_{loss}}$$

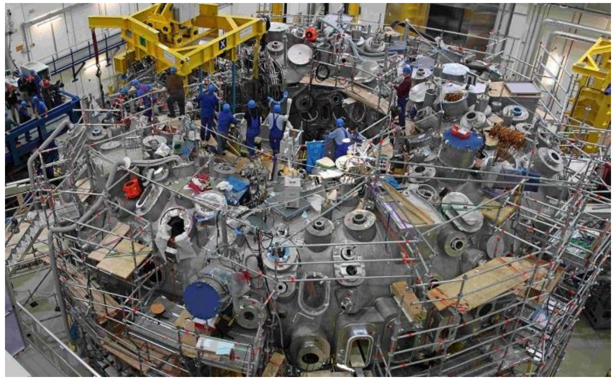
$\mathbf{P_{aux}}$ = External heating sources
$\mathbf{P_a}$ = Alpha power (~1/5 P_{fus})
$\mathbf{P_{ohmic}}$ = Ohmic power
$\mathbf{P_{loss}}$ = Power losses (radiation, conduction)

J. D. Lawson, (1957) Proc. Phys. Soc. B 70 303

Current experiments (stellarator):

If $I_p = 0 \rightarrow P_{ohmic} = 0$ and $P_a \sim 0$

$\rightarrow P_{aux} = P_{loss}$

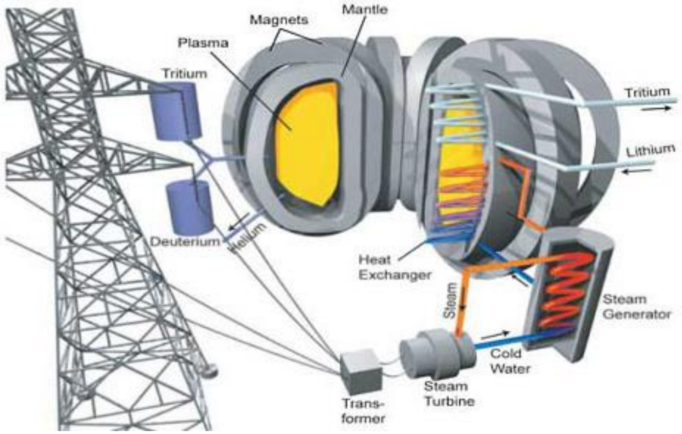


W7-X experiment, DE

Future reactors:

Aiming at ignition (plasma self-sustainment)

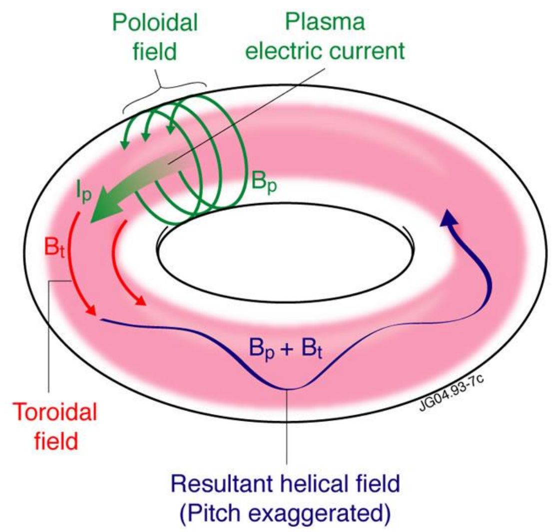
- P_{ohmic} negligible, $P_{aux} = 0 \rightarrow P_a = P_{loss}$



Why do we need additional heating power?

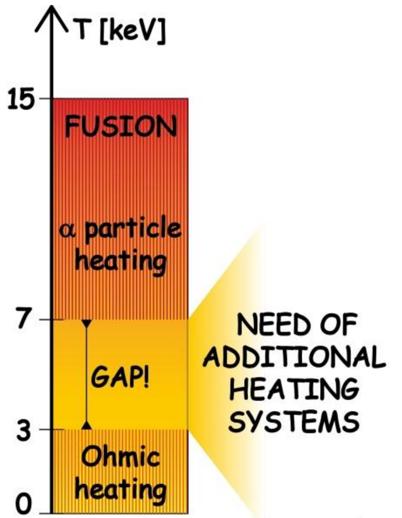
If we aim at ignition ($P_{aux} = 0$), why do we need additional heating power?

Let's take a step back
- For ignition, we need to reach a high enough ion temperature (~10 - 15 keV)
- Alpha power becomes dominant from T > 5 - 7 keV



We need additional heating to reach fusion-relevant temperatures:

- The simplest one is **ohmic heating**: plasma is a conductor
- Unfortunately, plasma ohmic heating decreases with temperature (due to plasma resistivity $\propto T^{-3/2}$)
- Maximum current for a given toroidal field is limited by MHD stability
- For typical tokamak reactor parameters, the maximum temperature reached only by ohmic heating is T~3keV



Heating of a typical D-T Tokamak plasma

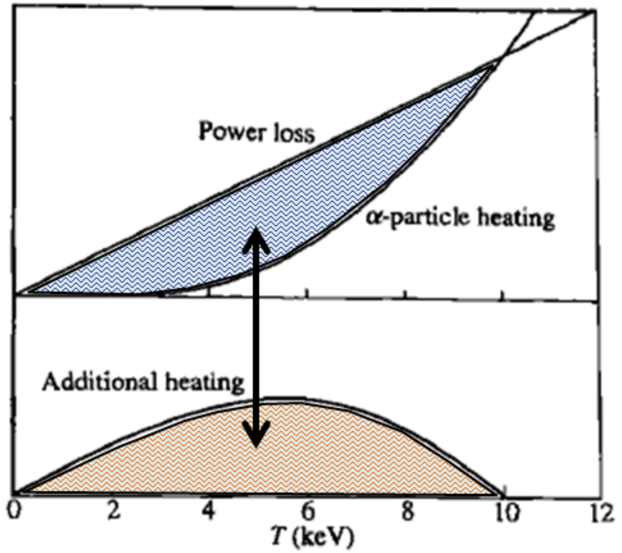
Wesson John and Campbell David J. Tokamaks. 118. Oxford University Press, 2004
Freidberg Jeffrey P. Plasma physics and fusion energy. Cambridge university press, 2007

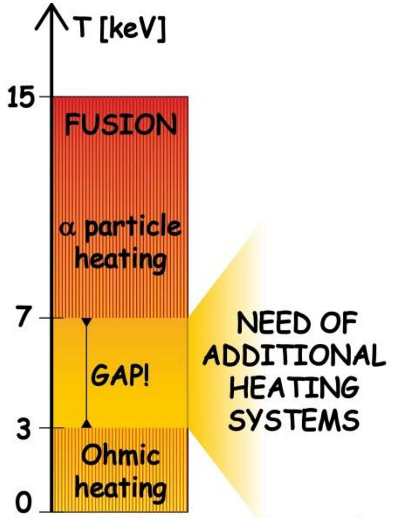
Why do we need additional heating power?

If we aim at ignition ($P_{aux} = 0$), why do we need additional heating power?

- Ohmic heating is not sufficient to reach fusion-relevant temperatures
- Access to H-mode or target confinement regimes
- Energy loss channels must be balanced, e.g.: thermal conduction, radiation (line radiation, bremsstrahlung…)

→ Additional (external) heating systems are necessary!
(additional to self-heating by ohmic power and alpha power)





Heating of a typical D-T Tokamak plasma

Wesson John and Campbell David J. Tokamaks. 118. Oxford University Press, 2004
Freidberg Jeffrey P. Plasma physics and fusion energy. Cambridge university press, 2007

…not only heating

Actually, we will see that we don't only need heating for 0D power balance, but also localized heating (change of temperature radial profile) to have generally better performance and control/suppress MHD instabilities.
Moreover, additional heating systems are able not only to heat the plasma but also to sustain the plasma in many other ways, if specifically designed. These capabilities are well-known and exploited in fusion experiments.

Additional H&CD systems can drive plasma current (both RF and NBI).
It is helpful for non-inductive plasma scenarios and to ramp-up plasma current.
Current drive (CD) is achieved:
- Accelerating a plasma species (RF systems)
- Injecting fast particles, which become fast ions with a preferential toroidal direction of their velocity (tangential NBI)

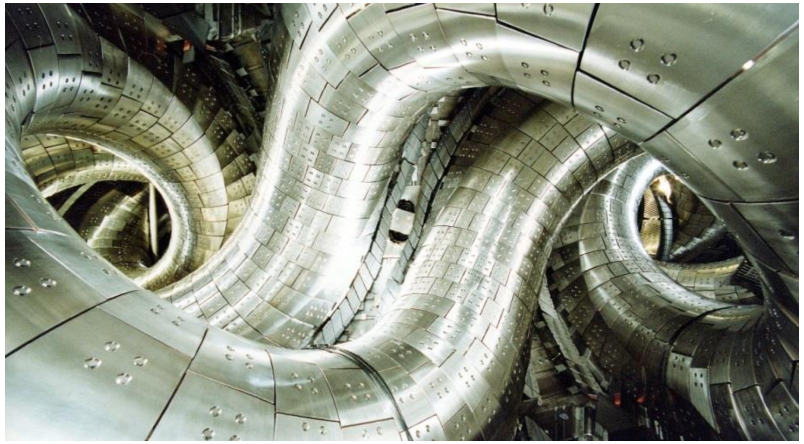
Other general capabilities (different system by system):

- Torque generation
- Temperature and current profile shaping
- Plasma control (instability suppression, core impurity accumulation counteraction, assisted discharge initiation and termination…)
- Plasma fuelling
- Wall cleaning and conditioning
- Stabilize MHD modes
- …

…and not only Tokamaks

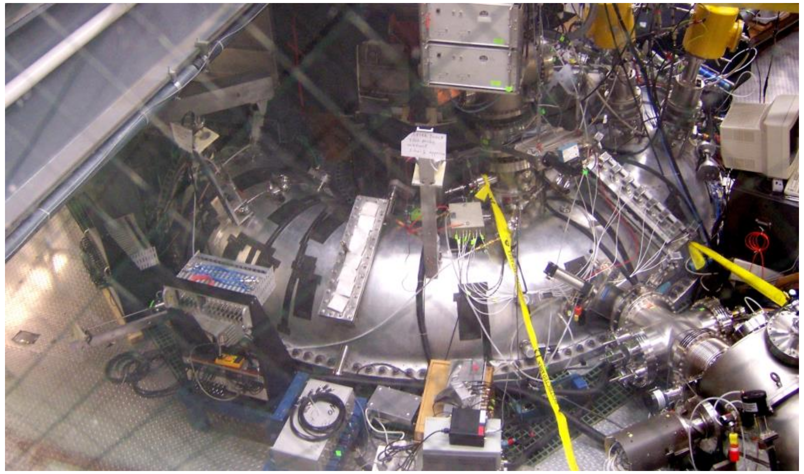
Helical devices:
The main aim is to heat the plasma (no P_{ohmic}) and balance power losses. A careful set of the systems is necessary to keep $I_p = 0$

- W7-X stellarator: ECRH, NBI, ICRH
- LHD heliotron: NBI, ICRH, ECRH
- …



RFP devices:
H&CD systems mainly to increase performances (temperature, current)

- MST, Madison: NBI + Electron Bernstein wave current drive
- RFX-mod2? NBI?



H&CD system capabilities

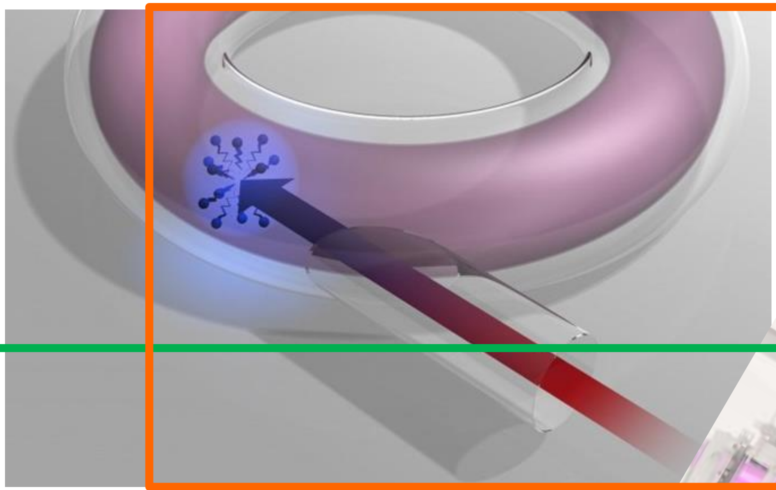
Tasks	EC	NBI	IC
Break down & Plasma start up	X		
Plasma Current Ramp up and H-mode access	X	partially	X
Electron Heating	X	X	X
Ion Heating		X	X
Current drive	X	X	*
MHD Control (NTM & ST)	X	partially	
Fast Particle Generation		X	X
Profiles Control (locally)	X		X
Impurity Accumulation Avoidance	X	X	X
Momentum injection & Control		X	
Transport Studies	X	X	
Diagnostics	X	X	
Wall Cleaning	X		X

*fast, low frequency, ICRF waves, low efficiency

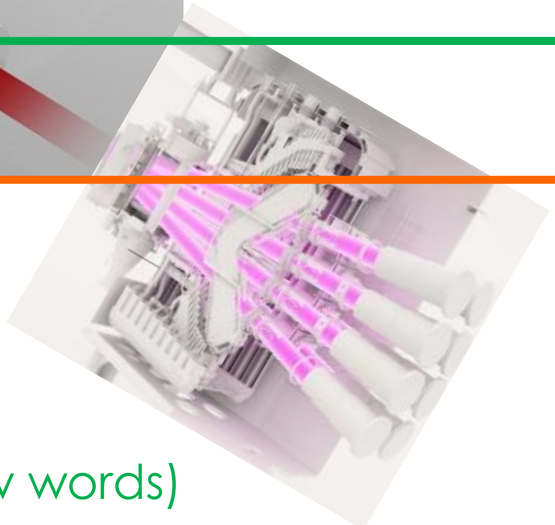
Slightly modified from G. Granucci et al., SOFT 2018

Neutral Beam Injection (NBI)

In this lesson we will focus on **Neutral Beam Injection** (NBI) and particularly on the interaction of this system with the plasma.



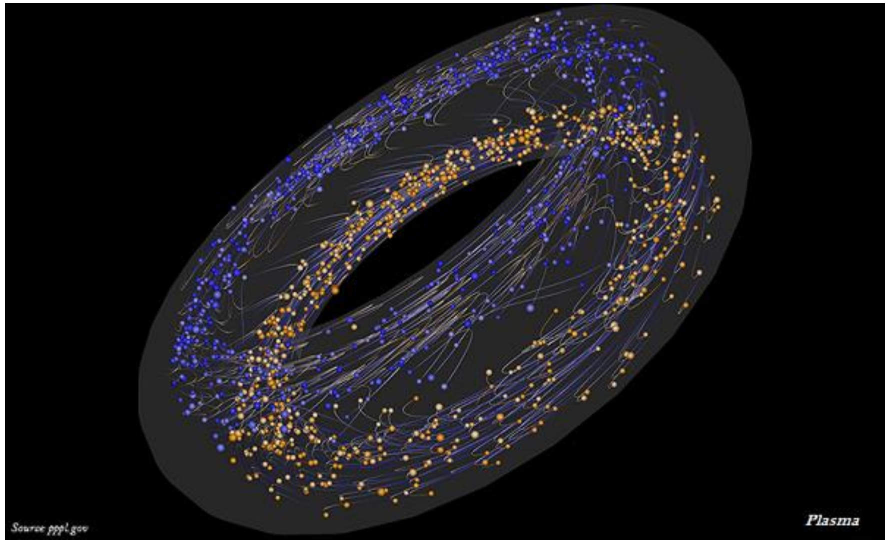
Interaction of the NBI with the plasma (this lesson)



NBI system (here only a few words)

Neutral Beam Injection principle

- The idea is to inject high energy particles >> energy of the plasma thermal particles
- First of all, entering beam particles must be neutral otherwise they would be deflected by magnetic fields in the plasma region (and surrounding zone).
- Fast neutral particles follow straight lines (injection direction) until they are ionized by the plasma, becoming fast ions.
- Then, thanks to Coulomb collisions, the energy is transferred to the plasma (slowing down process).
- Since "fast" electrons and ions coming from NBI neutral particles have the same injection velocity, the more massive ions carry almost all the beam energy (we will only speak about fast ions).



NBI often <u>dominant</u> in experiments

	R_0 [m]	a [m]	I_p [MA]	B_t [T]	Installed heating power [MW]				
					P-NBI	N-NBI	ECRH	ICRH	LH
ITER	6.2	2.0	15	5.3	-	33-50	<u>60-67</u>	10-20	-
DTT	2.19	0.7	5.5	6	-	10	<u>32</u>	8	-
JT60-SA	2.97	1.17	5	2.25	<u>24</u>	<u>10</u>	7	-	-
JET	2.96	1.25	4.8	3.45	<u>34</u>	-	-	10	7
AUG	1.65	0.5	1.2	3.1	<u>20</u>	-	5	4	-
DIII-D	1.67	0.67	2.0	2.2	<u>20</u>	-	6	8	-
EAST	1.7	0.4	1.0	3.5	8	-	4	<u>12</u>	10

NBI is on fire!

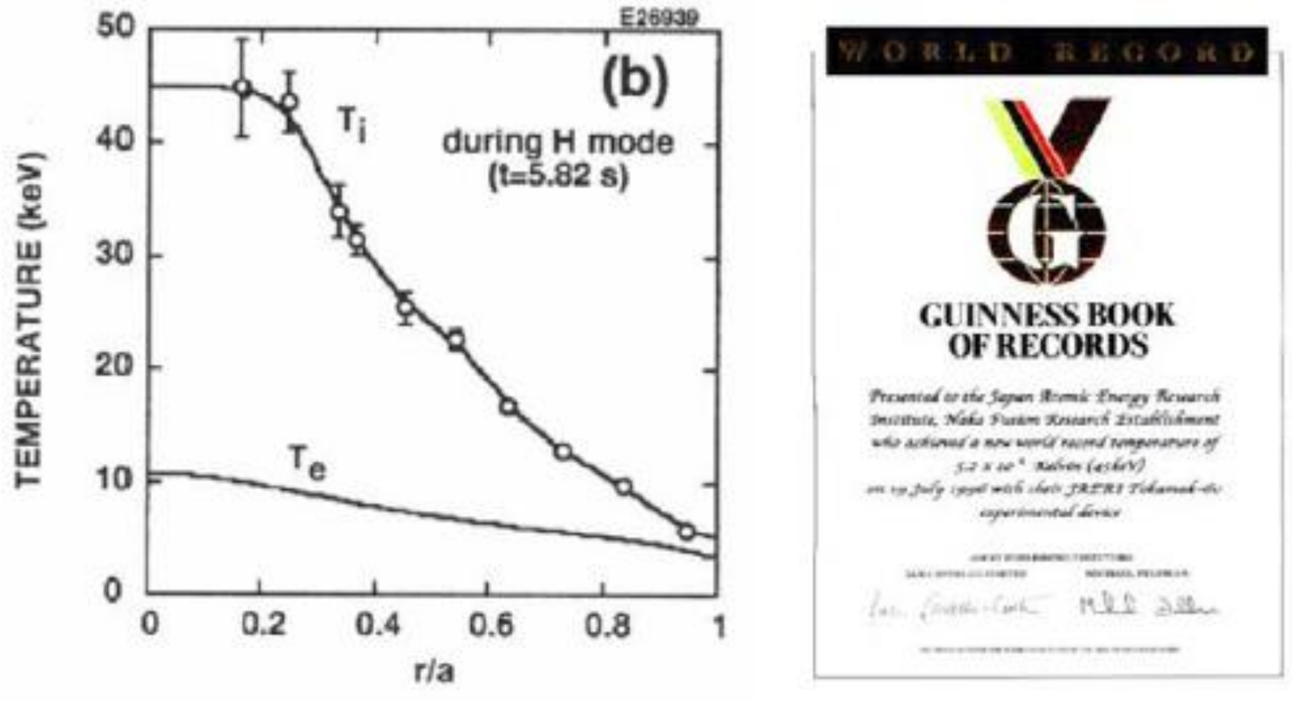


FIG. 5.12. T_i and T_e profiles for the world's highest central ion temperature in JT-60U and Guinness Book of Records on achievement of 45 keV in 1996 [5.22].

P_{NBI} = 27 MW
E_{NBI} = 92 keV (ion heating mainly)

Mitsuru Kikuchi, Karl Lackner, Minh Quang Tran, "Fusion Physics", IAEA, 2012

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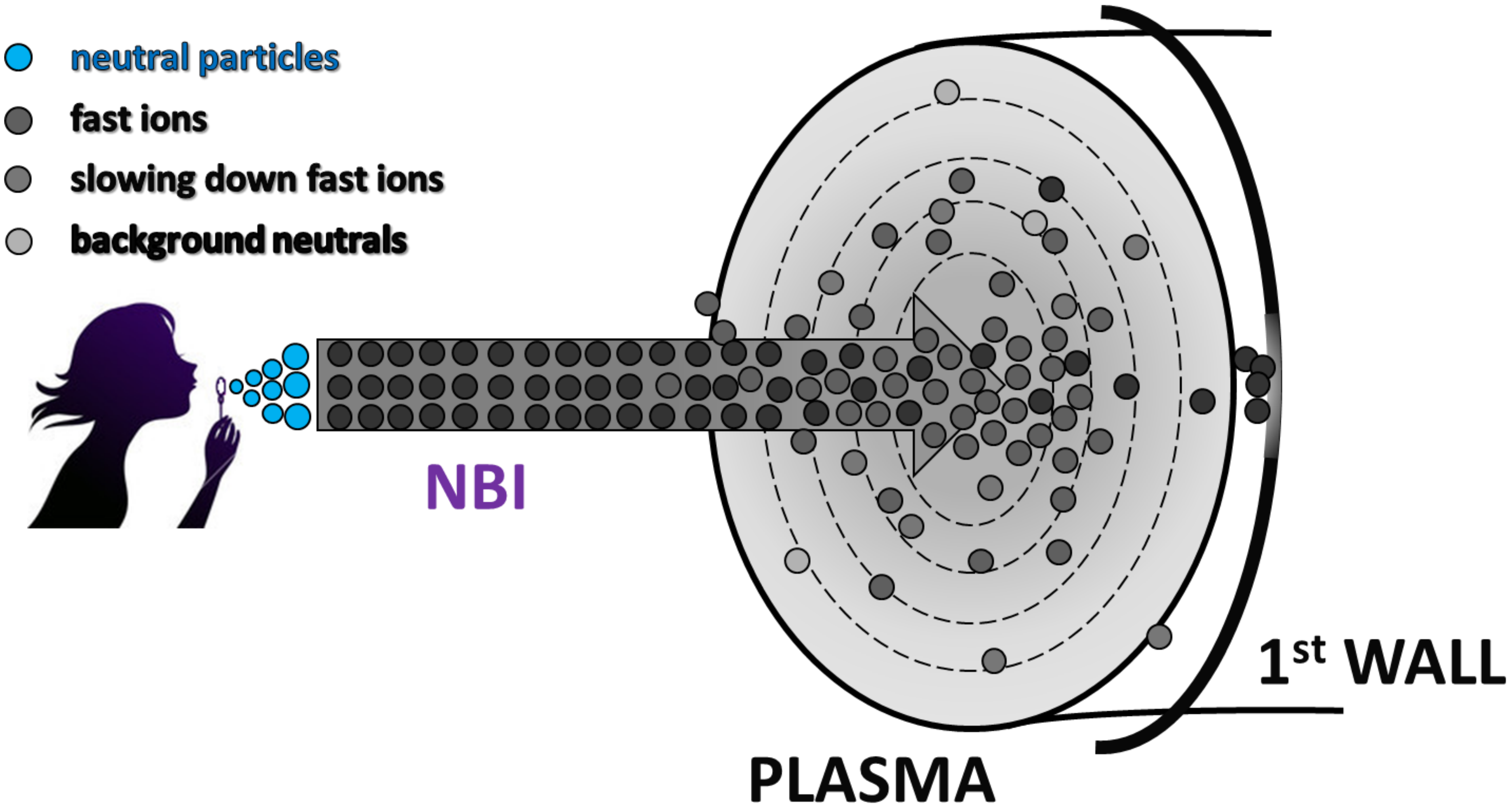
Beam generation

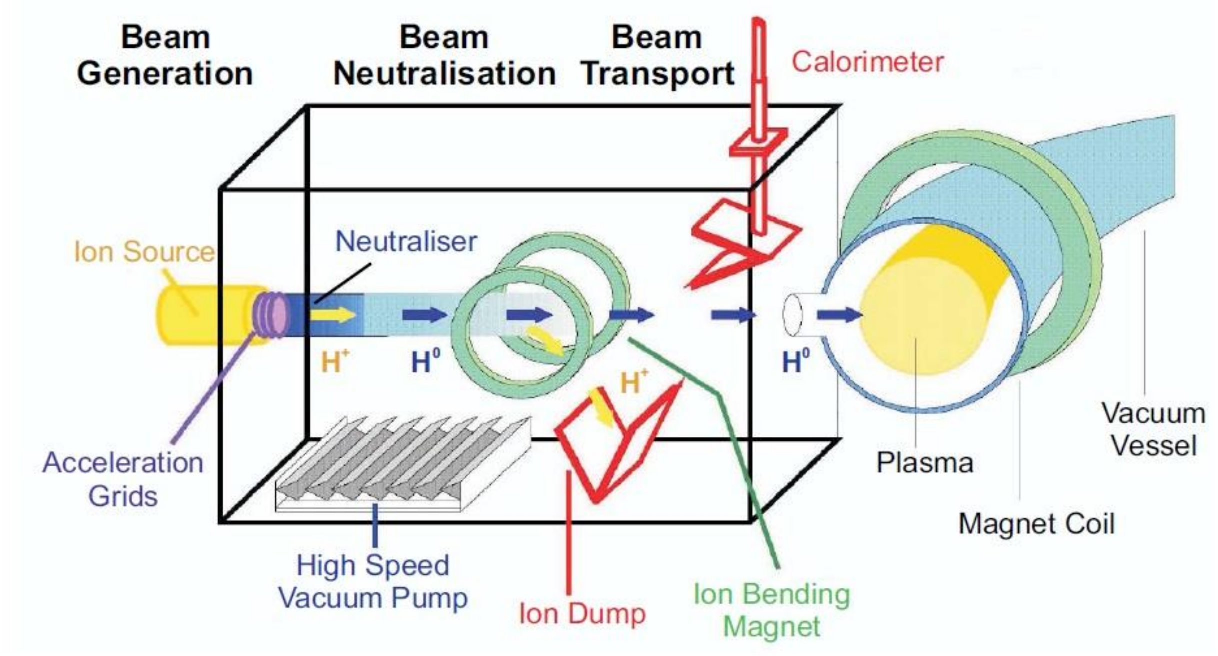


— **Neutral beam generation** — Neutral particle journey — Beam ionization — Fast particle slowing down →

Beam generation

This topic will be covered in AC3 course

The **NBI system** is composed by:
- Ion source (positive or negative)
- Acceleration grids
- Neutralizer
- Residual ion dump



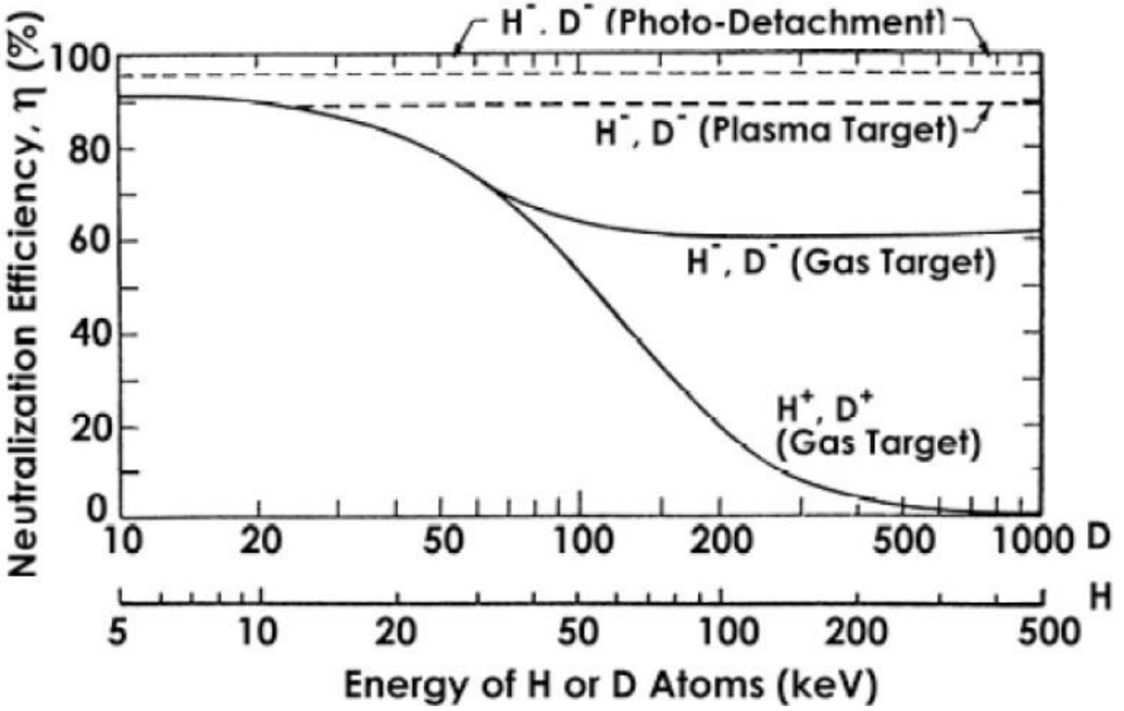
The objective of the system is to provide a stable and possibly uniform fast particle beam (usually composed by several *beamlets*) for the duration of the plasma discharge, of a given species, at a selected energy* and power, injected in a direction determined by the system geometry.

*or energies in case of positive ion source (ionized molecules)

Beam generation

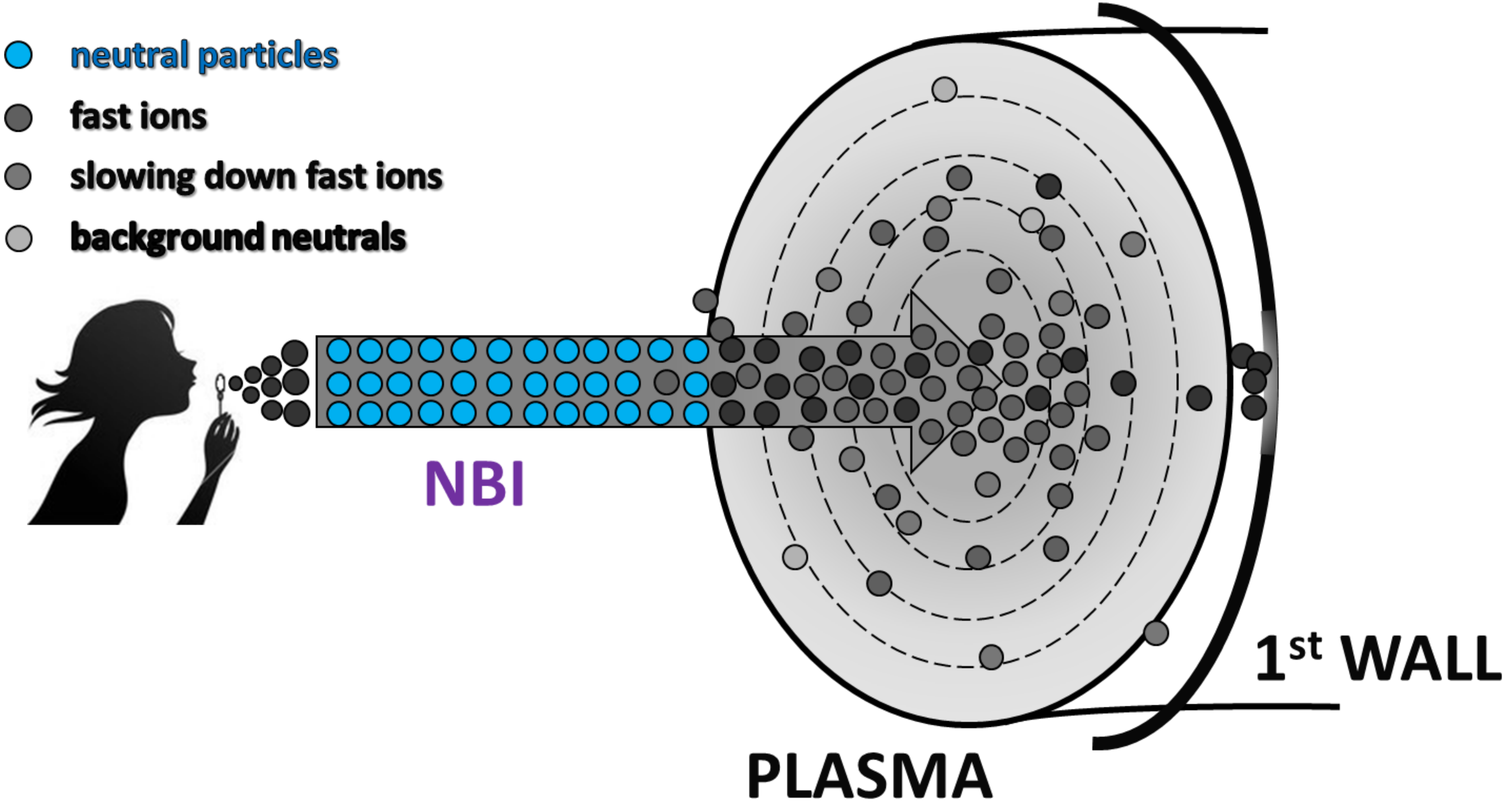
This topic will be covered in AC3 course

NBI system: negative or positive ion source depending on the designed E_{NBI}



For positive ion sources we have different energy components of the beam:
molecular ions are created (side effect), and are accelerated with the same voltage
→ when e.g. a molecular H_n is first dissociated and then ionized, each H^+ will have energy E_0/n (shorter plasma penetration)

Neutral particle journey



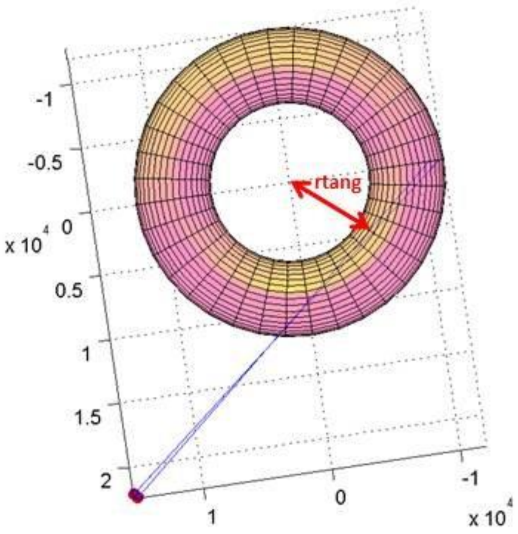
Neutral beam generation — **Neutral particle journey** — Beam ionization — Fast particle slowing down →

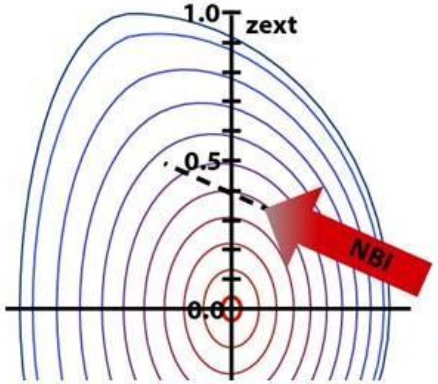
Aimed beam of fast particles

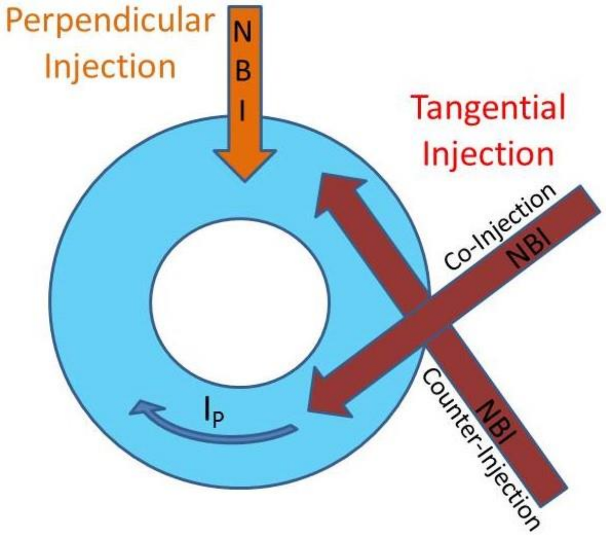
We have a beam (usually composed of different *beamlets*) of fast neutral particles travelling through the NBI system duct, passing the wall opening (port) and entering the plasma.

Relevant NBI parameters from plasma point of view are:

- Energy (E_{NBI})
- Power (P_{NBI})
- Injection geometry: tangency radius, vertical tilt, beam focus, beamlet divergence







Recently some NBI systems have been designed to be able to modulate power and energy during the discharge, within certain constraints (e.g. in DIII-D tokamak and planned for ITER and DTT)

How NBI parameters are chosen?

Energy

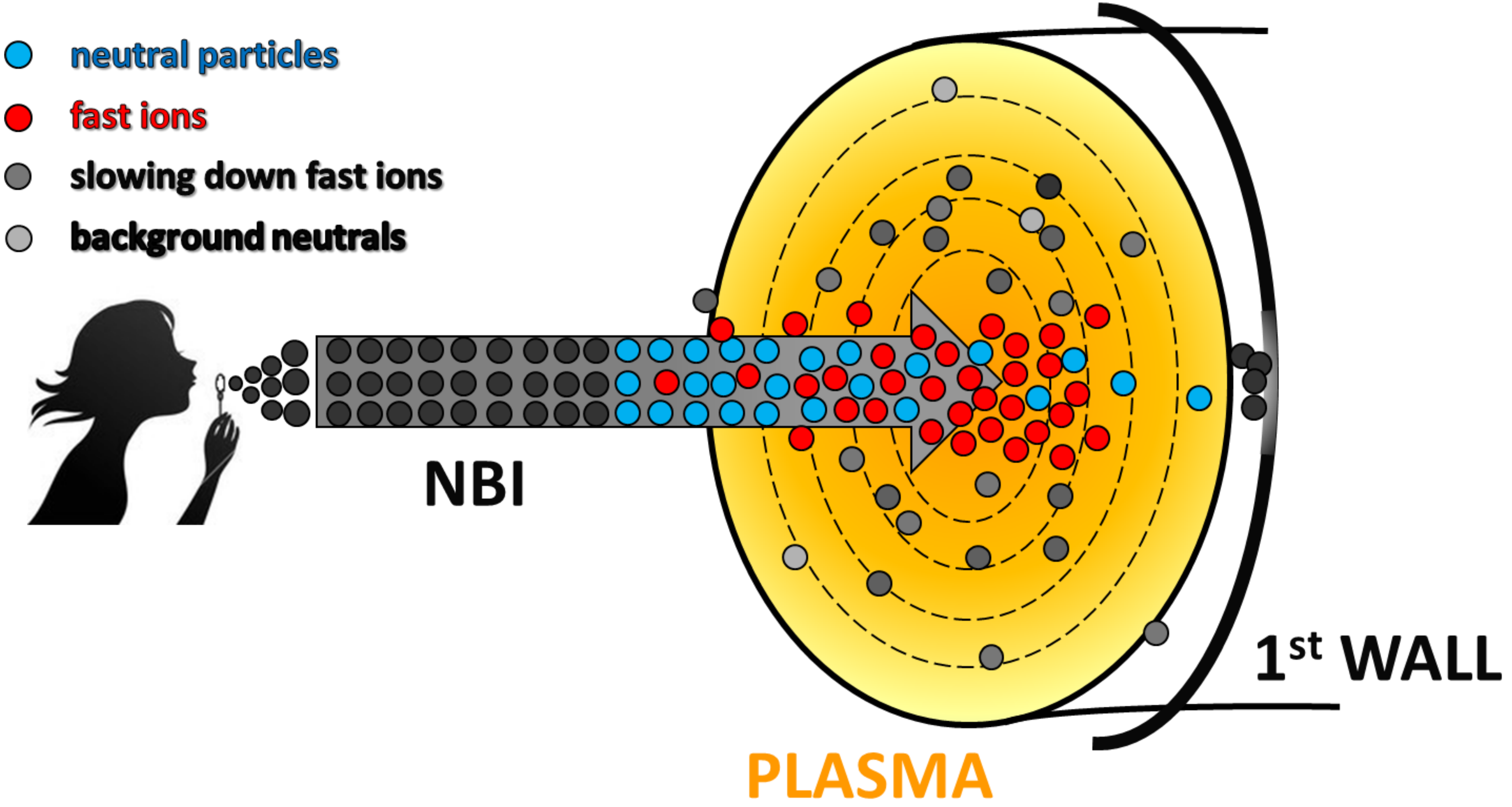
- **Low**: small plasma/low density, high input torque
- **High**: big plasma/high density, current-drive

Direction

- (Nearly) **Parallel** to magnetic field: longer path into dense plasma, current-drive/torque input
  - **Co-current** (usual choice): current drive, good coupling with plasma
  - **Counter-current**: counter torque/current to control rotation and plasma current, larger losses
- (Nearly) **Perpendicular** to magnetic field: technologically easier, shorter path in the plasma, risk of larger losses

- **On-axis** (usual choice): longer path through dense plasma, central heating
- **Off-axis**: current-drive

Neutral beam ionization



Neutral beam generation — Neutral particle journey — **Beam ionization** — Fast particle slowing down

Neutral beam ionization

Neutral particles from the beam are ionised in the plasma by mainly 3 processes:

- $H^0_{fast} + H^+ \rightarrow H^+_{fast} + H^0$ **charge exchange (CX)**
- $H^0_{fast} + H^+ \rightarrow H^+_{fast} + H^+ + e^-$ **ionization by ions (ii)**
- $H^0_{fast} + e^- \rightarrow H^+_{fast} + 2e^-$ **ionization by electrons (ie)**

The total cross section depends on the relative velocity:
$v_{rel} = |v_{beam} - v_{(ions/el.)}|$ and results:

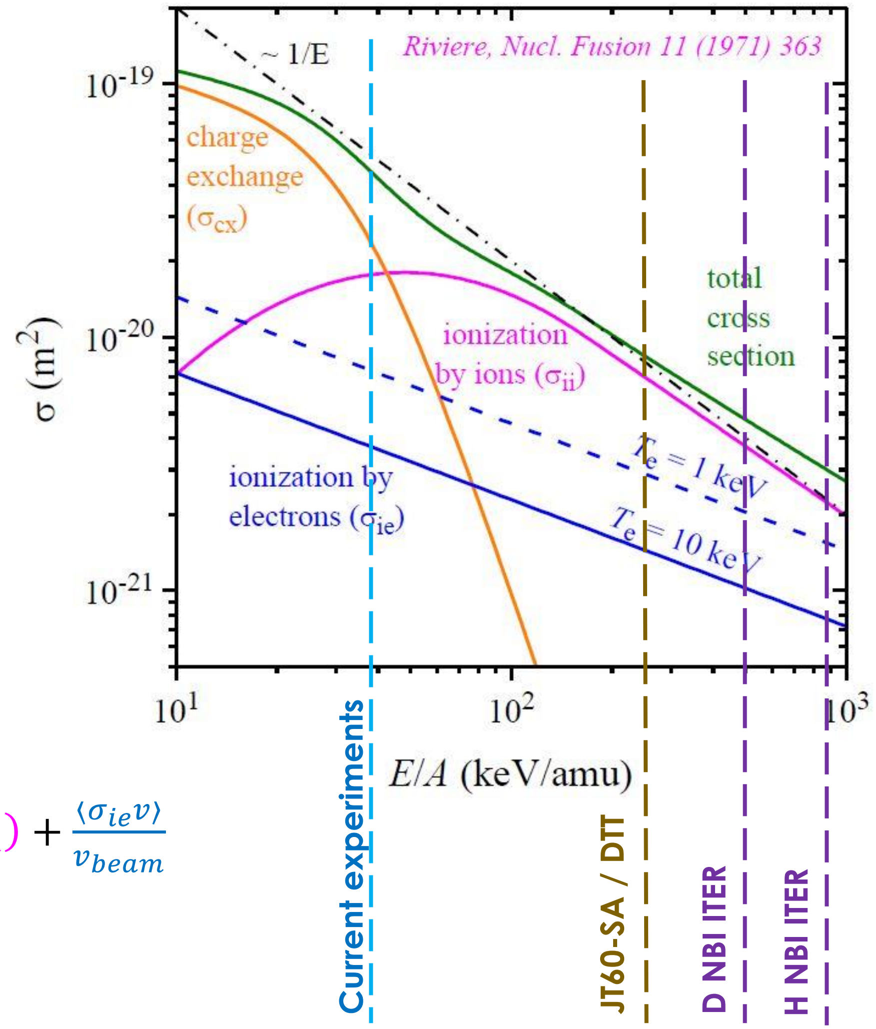
$$\sigma_{tot} = \sum_{k=cx,ii,ie} \frac{\langle \sigma_k v \rangle}{v_{beam}} \qquad \text{with} \qquad \langle \sigma_k v \rangle = \frac{\int \sigma_k(v_{rel}) v_{rel} f(v_{rel}) dv_{rel}}{\int f(v_{rel}) dv_{rel}}$$

But since $v_i \ll v_{beam} \ll v_e$ we can approximate: $\sigma_{tot} = \sigma_{cx}(v_{beam}) + \sigma_{ii}(v_{beam}) + \frac{\langle \sigma_{ie} v \rangle}{v_{beam}}$

If the *plasma rotation* is not negligible: $v_{rel} = |\vec{v}_{beam} - \vec{v}_{rotation}|$

Mean free path: $\lambda = 1/n\sigma_{tot}$ ($n = n_i = n_e$, $Z = 1$): **$\sigma_{tot} \sim 1/E_{NBI} \rightarrow \lambda \sim E_{NBI}/n$**

NBI ionization cross section



Fast ion birth

The ionization cross section is modified by two other effects:

- multi-step ionization (MSI) from **excited states**

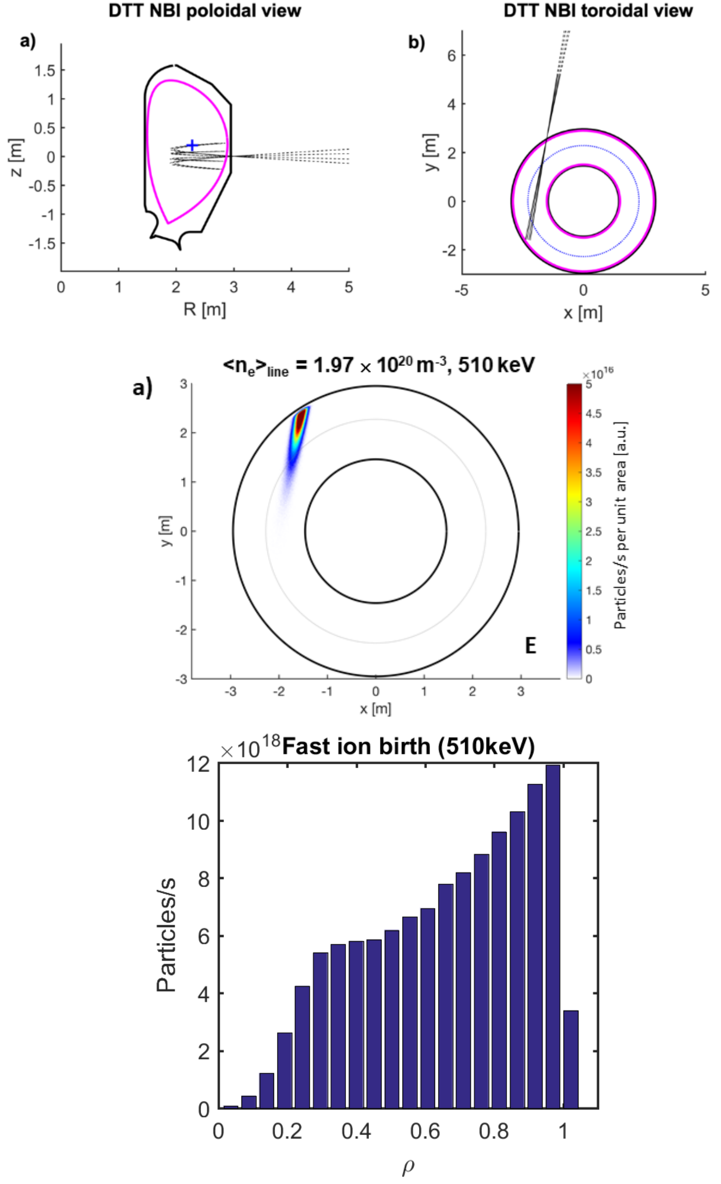
$$H^0 + (H^+ \; or \; e^-) \rightarrow H^{0*} + (H^+ \; or \; e^-) \rightarrow H^+ + e^- + (H^+ \; or \; e^-)$$

→ Enhancement factor δ: $\sigma_{tot_{MSI}} = \sigma_{tot}(1+\delta)$

→ Relevant for high E_{NBI} and high density plasmas (collisional time << excited state lifetime)
current small experiments, P-NBI: δ <0.2
ITER: δ~0.5-0.6

- **impurity** ionization (proportional to their concentration)

E.g.: DTT NBI ionization
C. De Piccoli et al., 2024, Front. Phys. 12:1492095

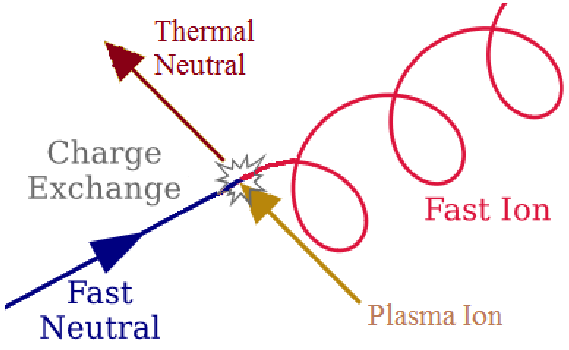


Beam halo

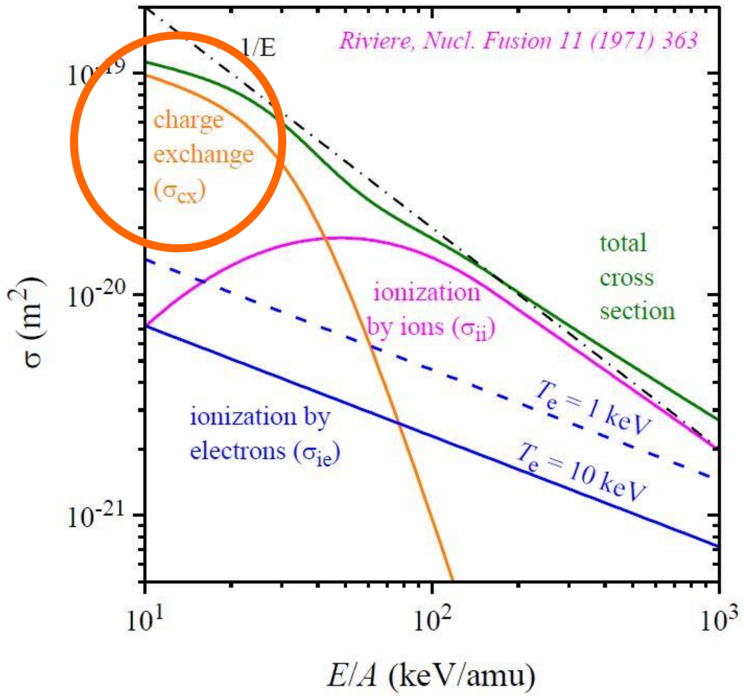
Beam halo: a side effect of CX ionization process

$$H^0_{fast} + H^+_{th} \rightarrow H^+_{fast} + \boxed{H^0_{th}}$$

creation of a thermal neutral at plasma temperature

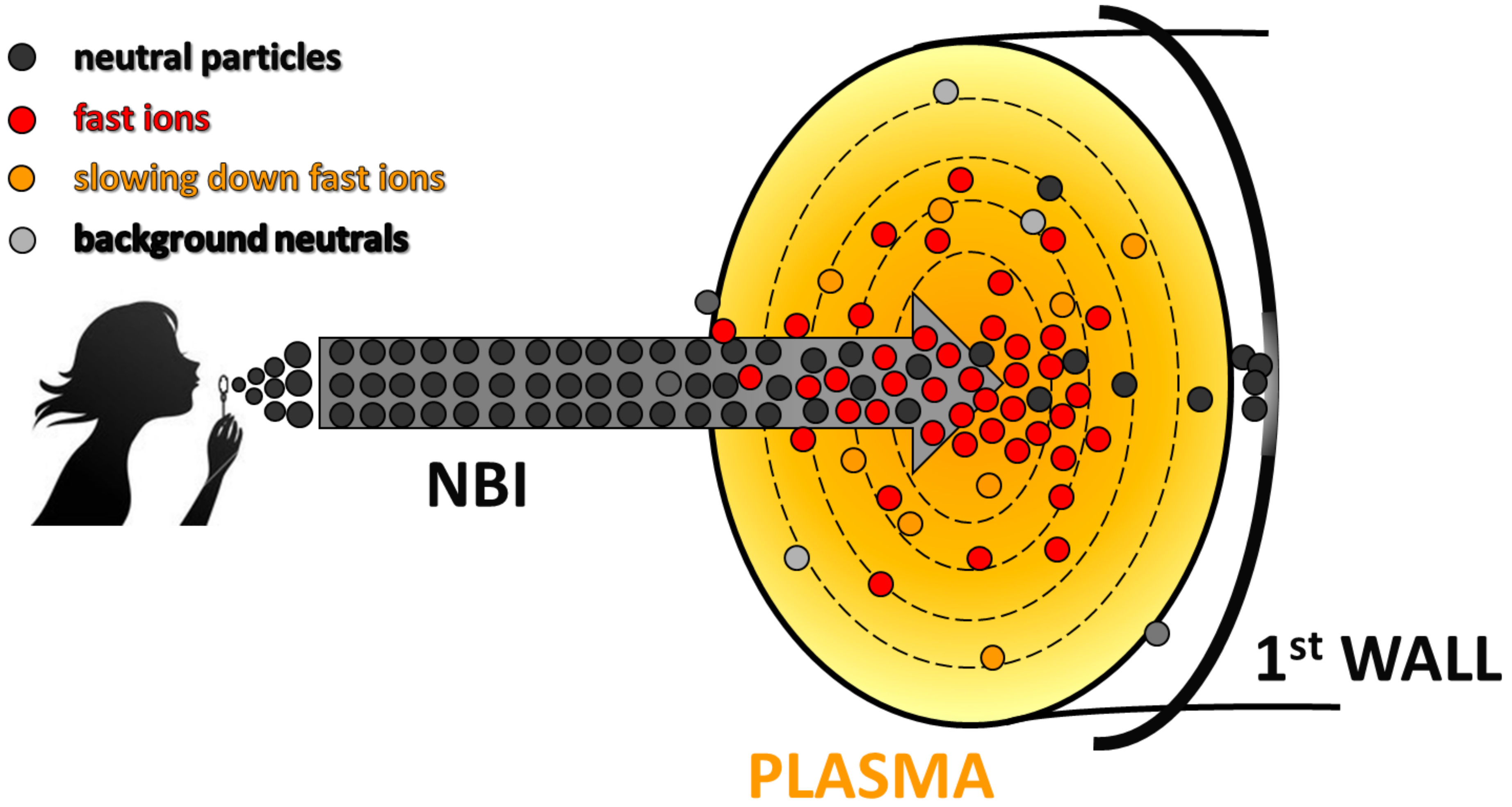


NBI ionization cross section



- Creation of a thermal neutral which travels ballistically and charge exchanges with other thermal ions, creating other neutrals (Halo)
- This process is then recursively repeated multiple times, producing fewer and fewer neutrals
- Halo of neutrals surrounding the neutral beam

Fast particle slowing down



Neutral beam generation — Neutral particle journey — Beam ionization — **Fast particle slowing down** →

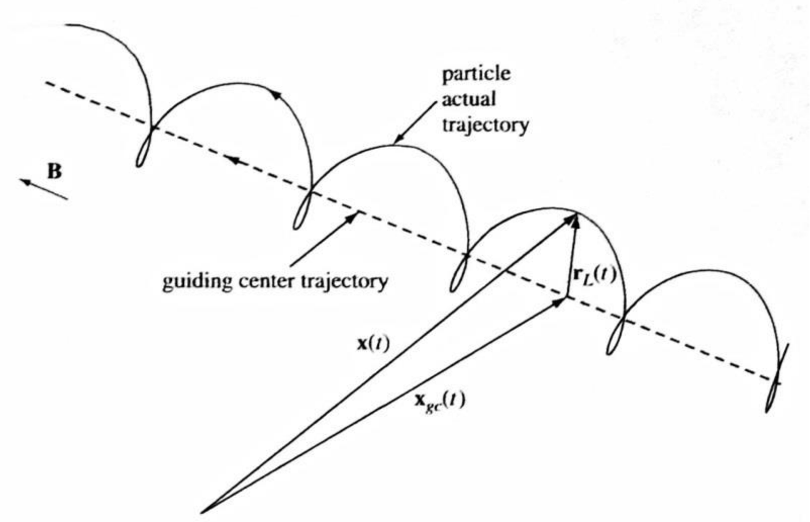
Ion orbits

After NBI ionization, newly born fast ions start experiencing the magnetic field. What are their orbits?

(refreshing single particle motion)

Charged particle motion into a homogeneous magnetic field B: the Lorentz force

Lorentz force	$m\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = q_\mathrm{e} \cdot \boldsymbol{v} \times \boldsymbol{B}$
$\parallel \boldsymbol{B}$	$\frac{\mathrm{d}v_\parallel}{\mathrm{d}t} = 0$
$\perp \boldsymbol{B}$: Larmor radius	$\rho_\mathrm{L} = \frac{mv_\perp}{q_\mathrm{e}B}$
Cyclotron frequency	$\omega_\mathrm{c} = \frac{q_\mathrm{e}B}{m}$



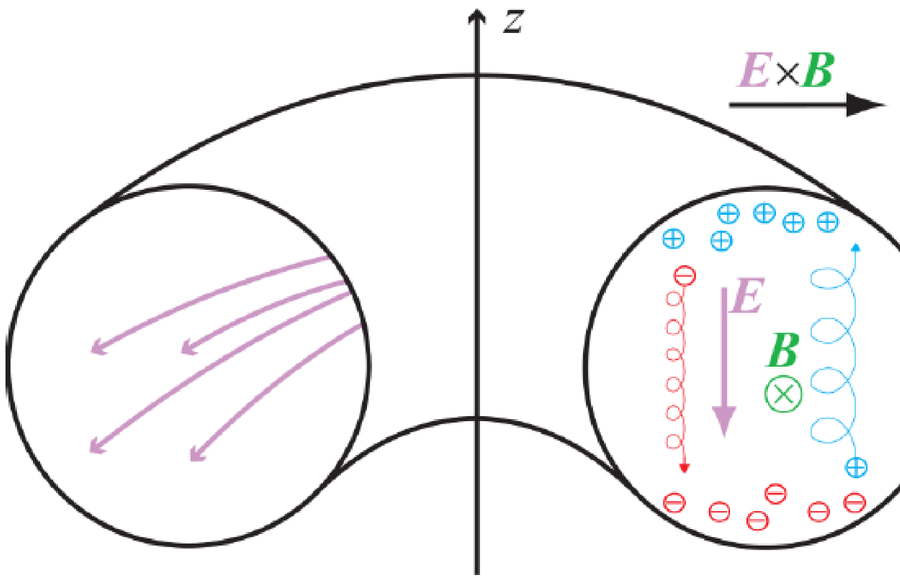
In an arbitrary magnetic field with a force F:

$$m\frac{d\vec{v}}{dt} = \vec{F} + q_e \cdot \vec{v} \times \vec{B}$$

Guiding center motion, we can split into:

- Motion along B field lines (if $|v_{//}|>0$)
- Drift motion due to the presence of force F (if F=0, we step back to Lorentz motion)

Ion motion: drifts



Different drifts due to different forces in the plasma:
- B-field gradient → Grad-B drift
- Curvature of B → Curvature drift

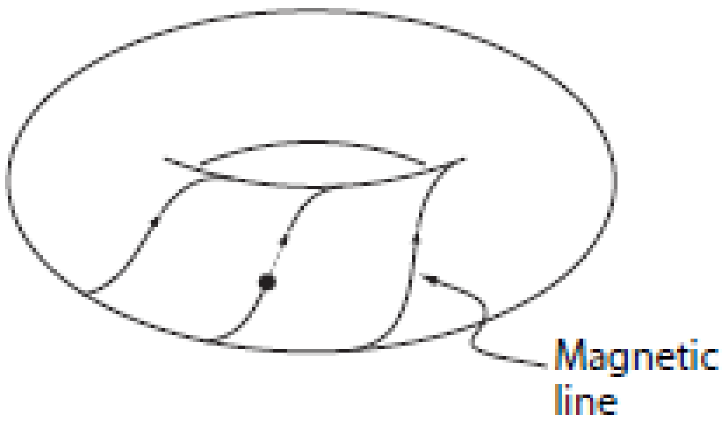
they are vertical drifts and bring to *charge separation*

- formation of *electric field* → ExB drift (outward drift for both electrons and ions)

This is the reason of the forced helical twist of the magnetic field lines in fusion experiments

(other drifts are present: e.g. diamagnetic drift…)

→ Guiding center orbits are shifted with respect to magnetic surfaces



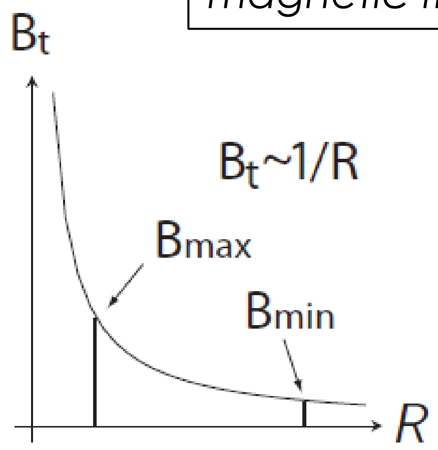
Orbit topology: passing and trapped

v_{par}, v_{perp} with respect to magnetic field

In charged particle motion kinetic energy is constant and magnetic moment an adiabatic invariant:

$$E_k = \frac{1}{2} m\left(v_{par}{}^2 + v_{perp}{}^2\right) = const$$

$$\mu = \frac{m v_{perp}{}^2}{2B} \approx const$$



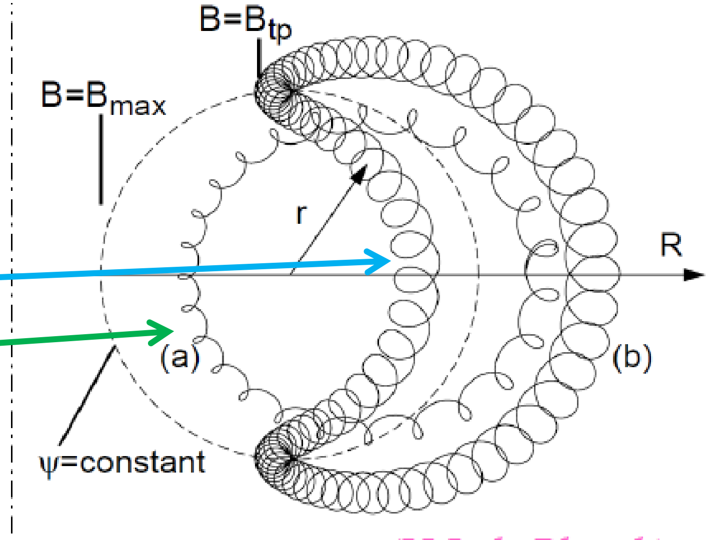
In a tokamak, the magnetic field varies as $B(R) = B_0 \frac{R_0}{R}$

In its (drifted) orbit, the fast ion experiences varying B

→ particle energy is transferred between parallel and perpendicular direction

Depending on the initial fast ion pitch ($\varepsilon = v_{par}/v$), the particle may be reflected (condition $v_{par} = 0$ at B_{max}) when travelling towards the inner side of the plasma (as in a magnetic mirror): **trapped particle (banana orbits)**

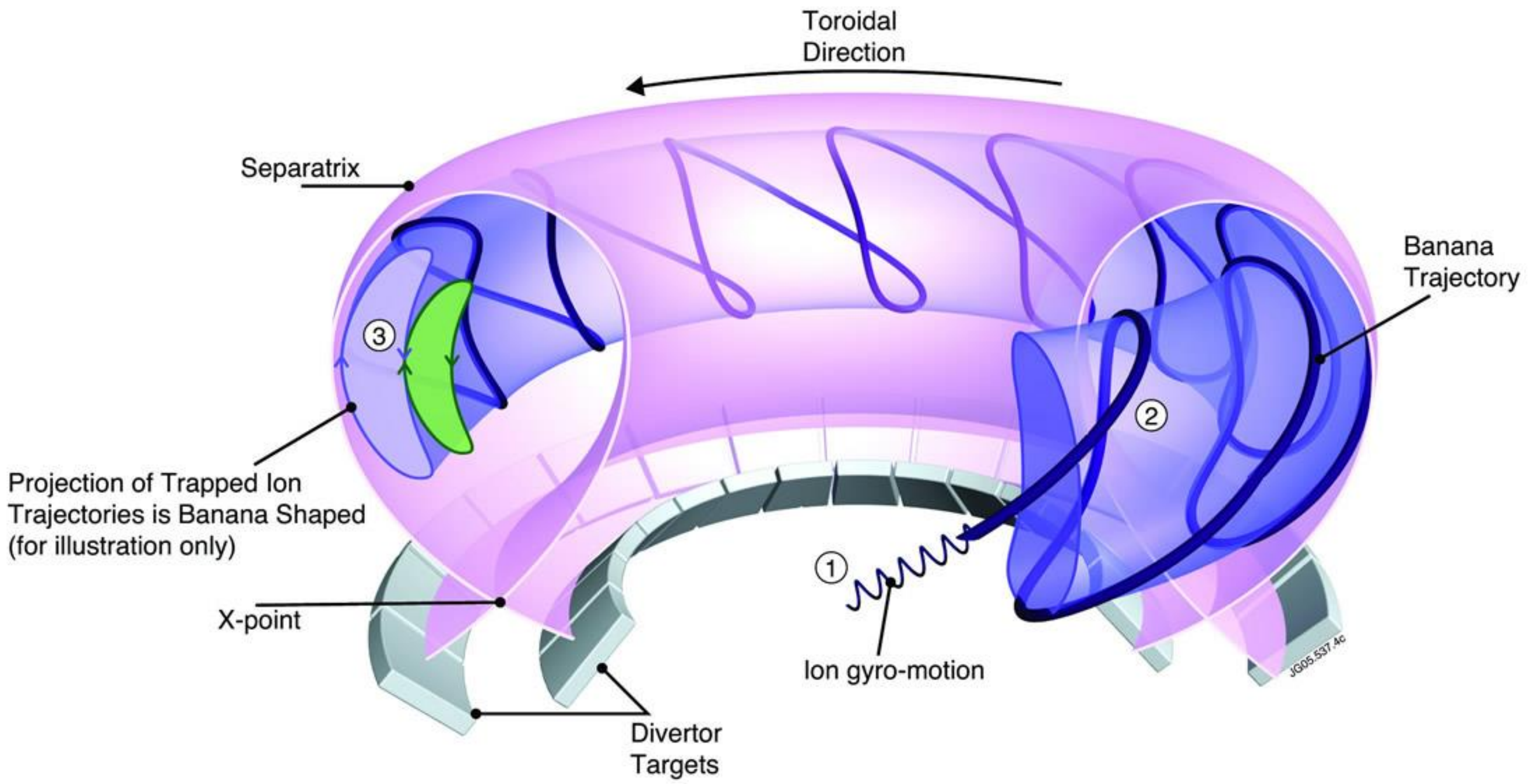
Otherwise, the fast ion is a **passing particle**



(H.J. de Blanck)

passing particle: $\frac{v_{perp}}{v_{par}} < \left(\frac{B_{max}}{B_{min}} - 1\right)^{-1/2}$

Banana orbits



Let's ask to the greengrocer: Bananas!

Trapped Banana orbits

- **Banana width** (half width): $\Delta r \sim \frac{m v_{par}}{e B_{pol}}$

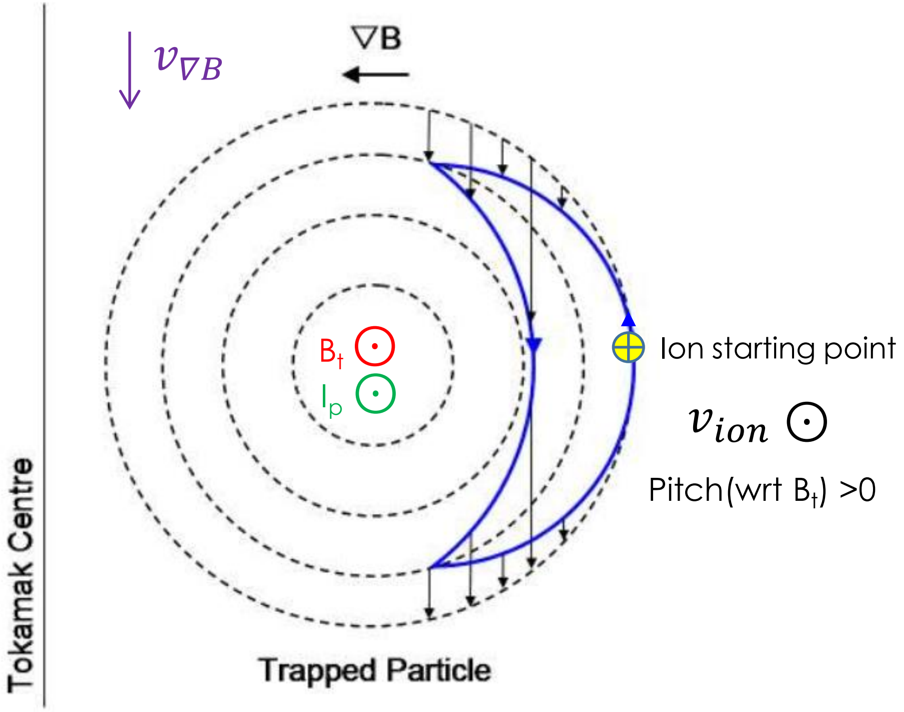
  → proportional to the mass (we consider only ions)
- Fast ions have larger bananas than thermal ions, and so trapped fast ion **banana orbits should be inward** to reduce fast ion losses.

How can I say if beam ions will have inward or outward bananas?

1) First of all, the direction of the ∇B drift should be determined (up or down). It depends on the sign of the charge.

The ∇B drift will shift the particle inwards or outwards with respect to the starting magnetic flux surface.

∇B drift (ion → q>0) $\overrightarrow{v_{\nabla B}} = \frac{m v_\perp^{\,2}}{2 q B^3} \vec{B} \times \overrightarrow{\nabla B}$



2) The second step is to determine the direction of the poloidal magnetic field $\mathbf{B_p}$ (determined by the plasma current) at the considered particle starting point.
Then we must separate the cases of particles with a velocity in the same direction of the toroidal magnetic field (positive **pitch**) or opposite (negative pitch):

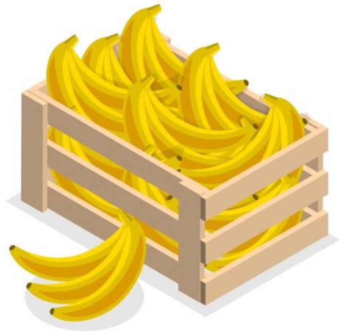
- positive pitch trapped particles will start the banana orbit in the same direction of B_p
- negative pitch particles in the opposite direction.

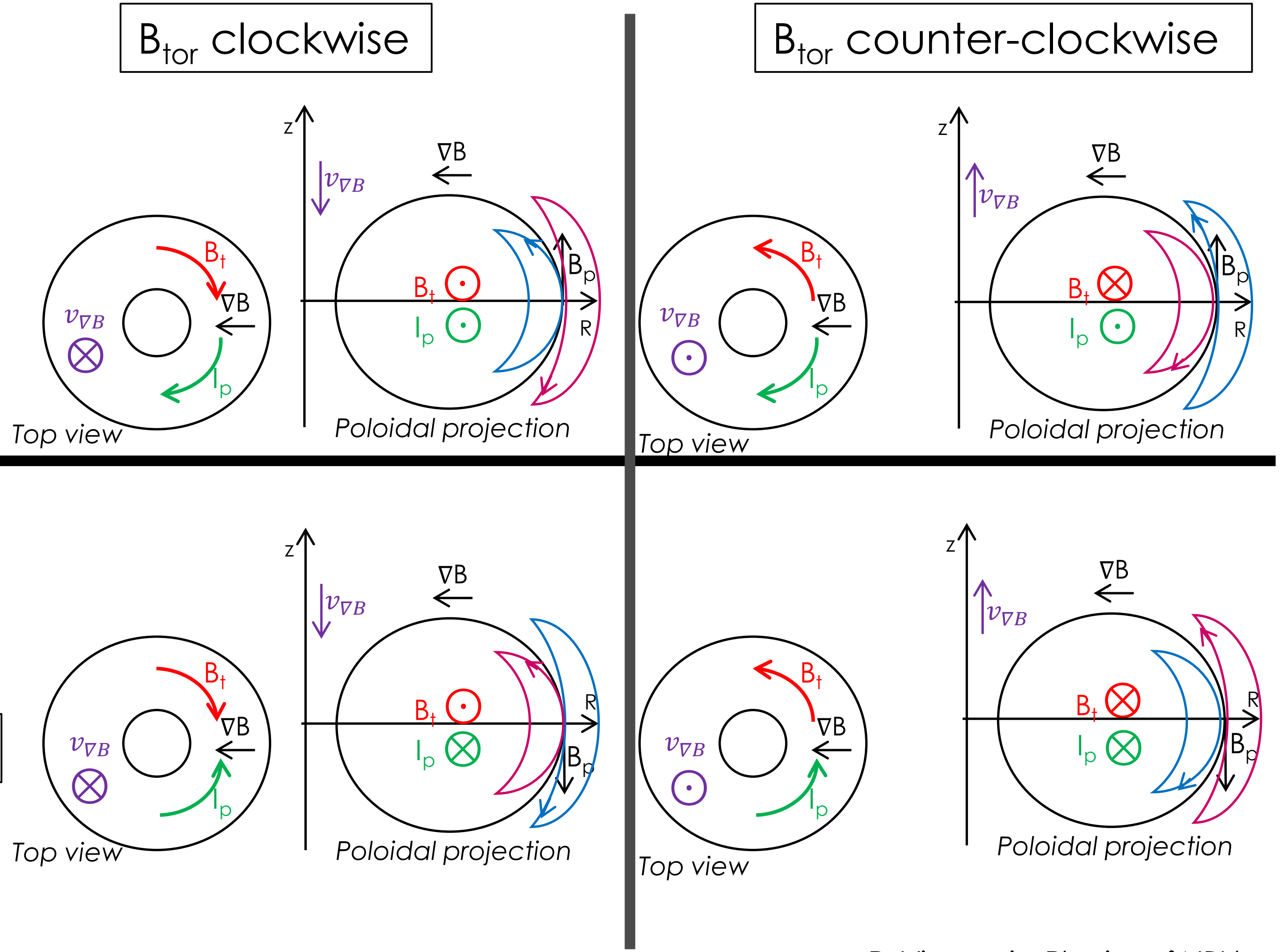
Wesson

Let's ask to the greengrocer: Bananas!

Ions, q > 0

Different magnetic configurations*



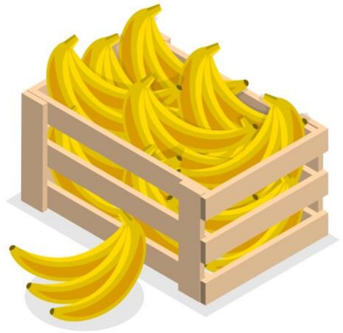


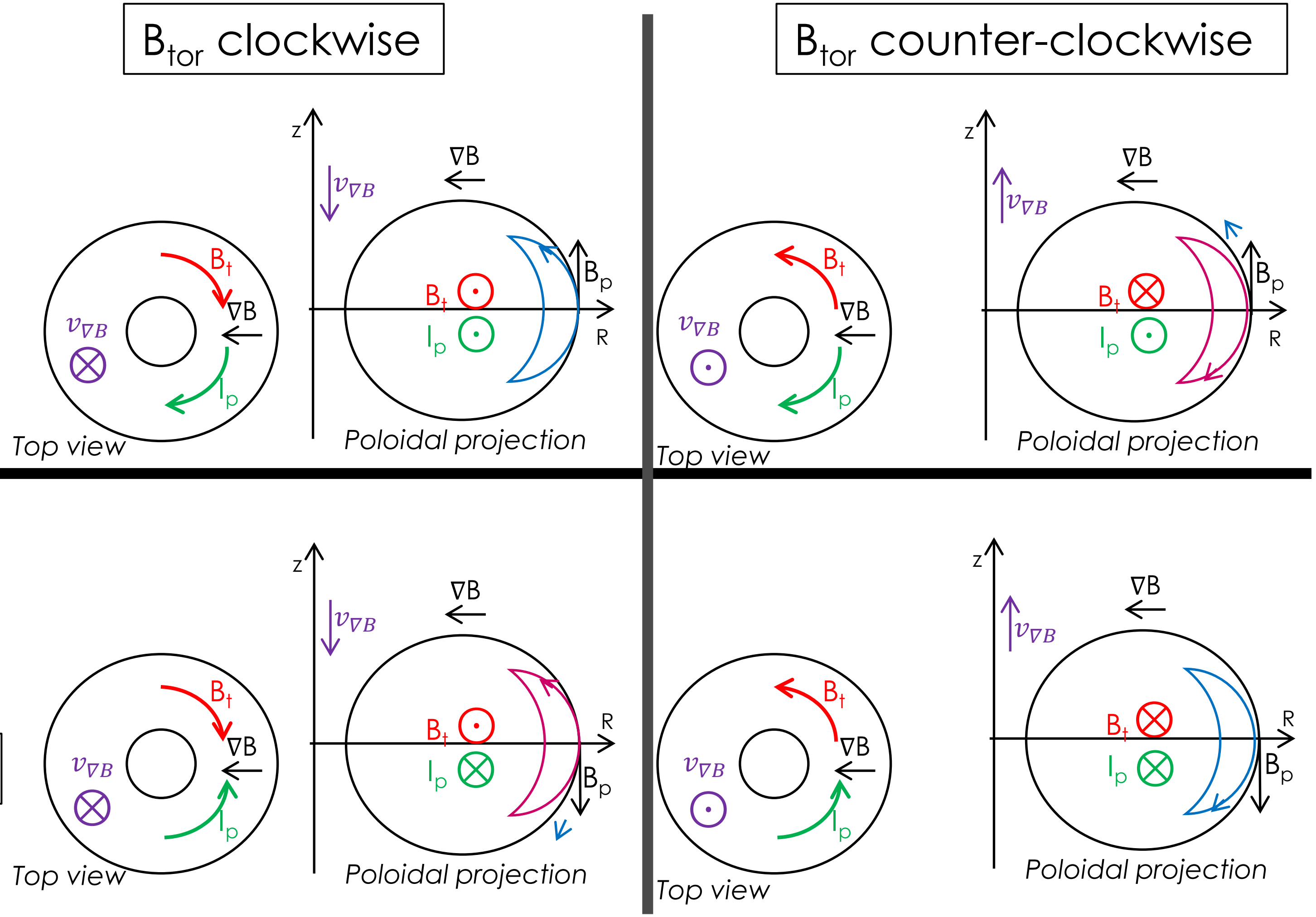
Trapped ion velocity:
Pitch(wrt B) > 0
Pitch(wrt B) < 0

*Magnetic configuration = possible combinations of B_t and I_p

Let's ask to the greengrocer: Bananas!

NBI co-current injection is preferable!*





I_p clockwise

I_p counter-clockwise

Trapped ion velocity:

Pitch(wrt B) > 0

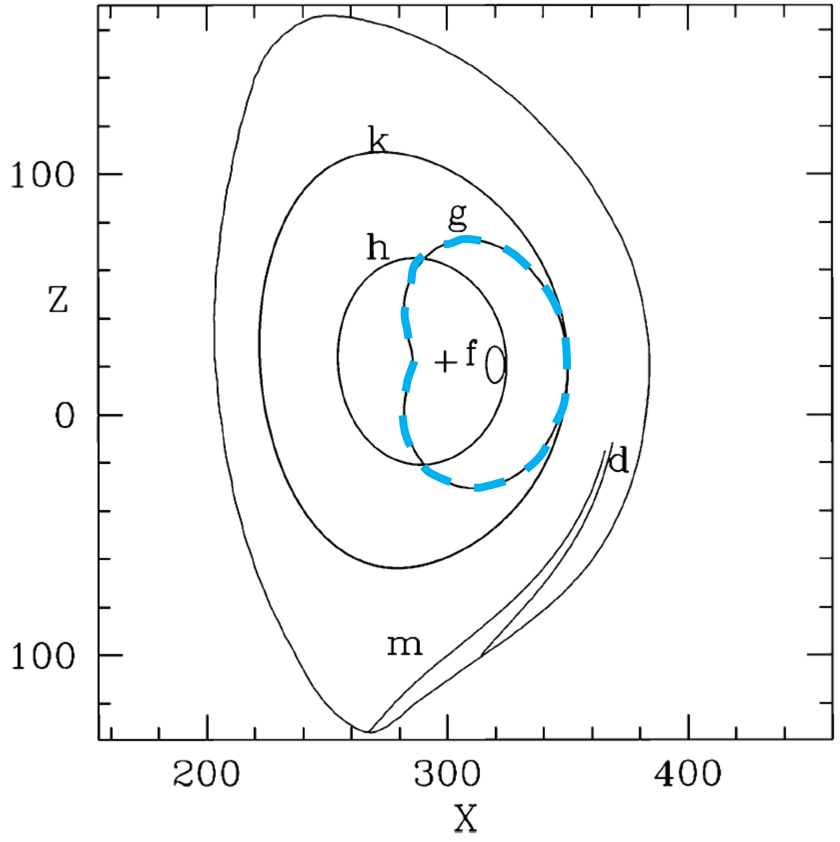
Pitch(wrt B) < 0

*because bananas point inwards always (do not mix current direction and pitch which is determined with respect to B).

Other peculiar fast ion orbits

Due to the larger energy with respect to thermal particles, fast ions can experience also other type of orbits:

- **Potato orbits**
  - trapped orbit
  - on a poloidal projection, the orbit passes close to the magnetic axis (often, encircles magnetic axis)
  - potatoes has larger width than bananas



d and m are instantly loss, not confined

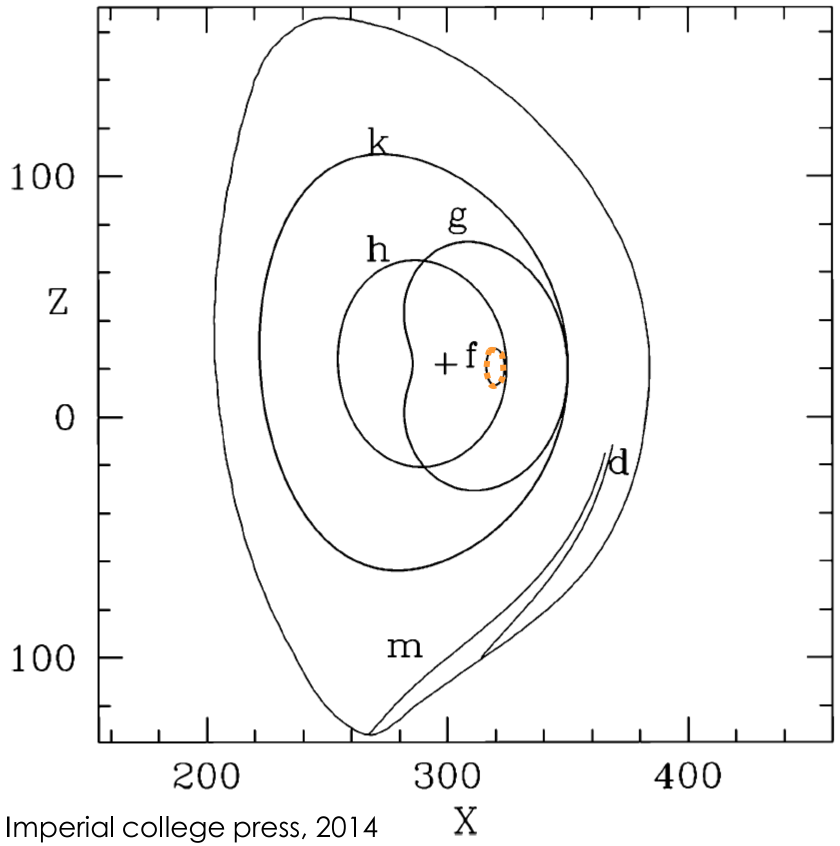
h and k are passing

R.B. White. The theory of toroidally confined plasm
W.W. Heidbrink and R.B. White, Physics of Plasmas, 27(3):030901, 2020.

Other peculiar fast ion orbits

Due to the larger energy with respect to thermal particles, fast ions can experience also other type of orbits:

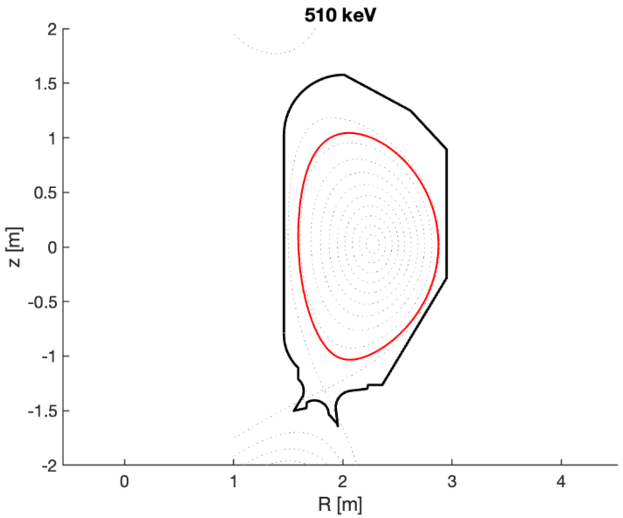
- **Potato orbits**
- **Stagnation orbits**
  - passing orbit
  - fast ion completing a toroidal orbit scarcely moving poloidally



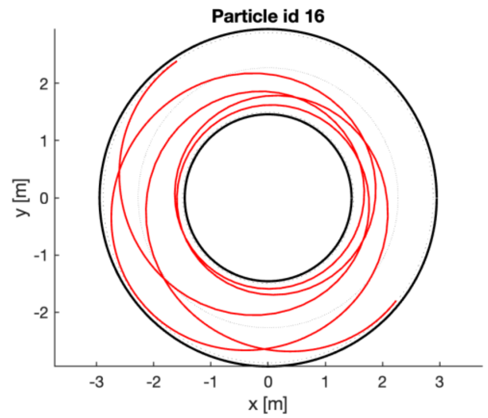
R.B. White. The theory of toroidally confined plasmas. Imperial college press, 2014
W.W. Heidbrink and R.B. White, Physics of Plasmas, 27(3):030901, 2020.

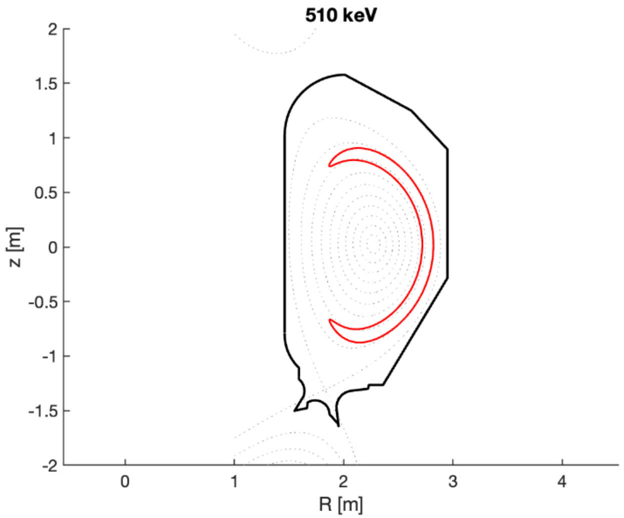
Example of DTT NBI EP orbits

ASCOT simulations (C. De Piccoli)

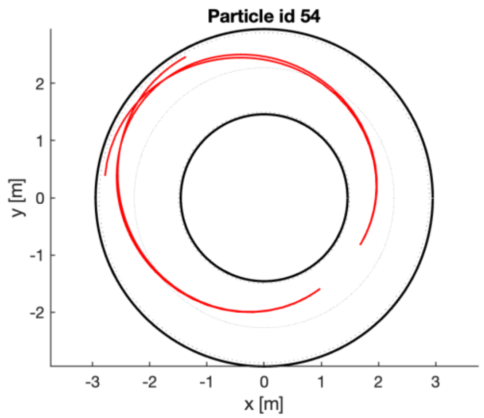


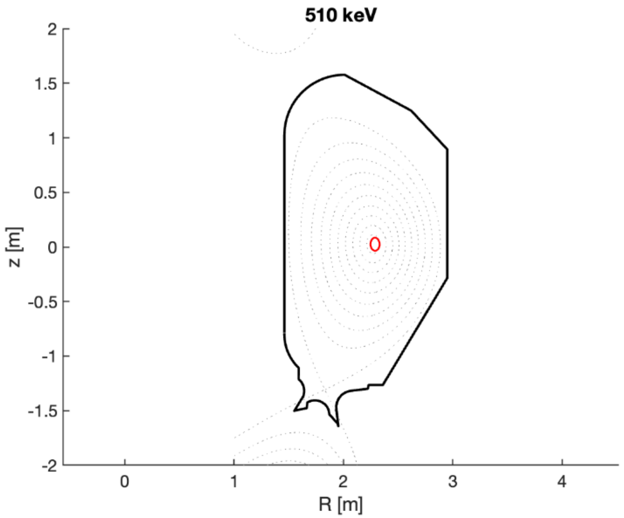
Confined passing orbit



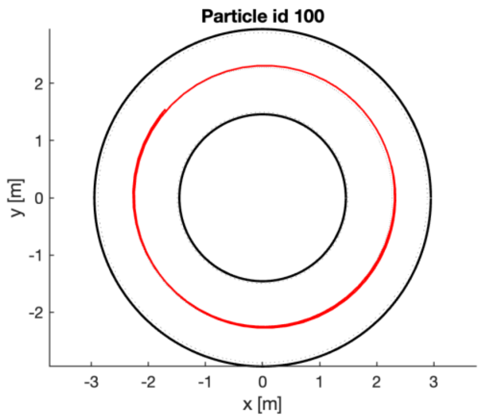


Confined trapped orbit





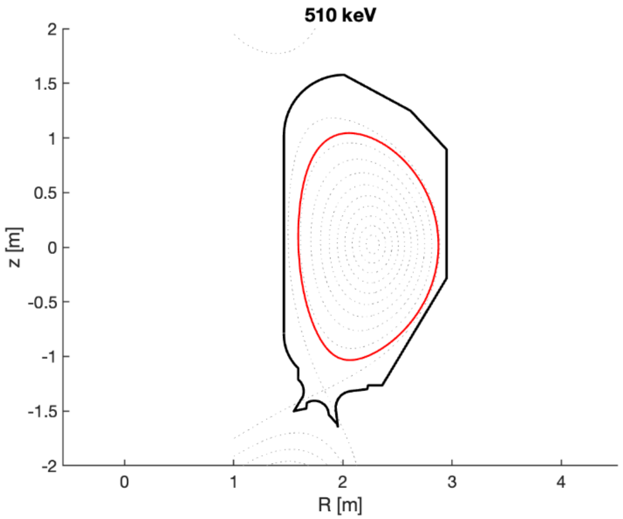
Stagnation orbit

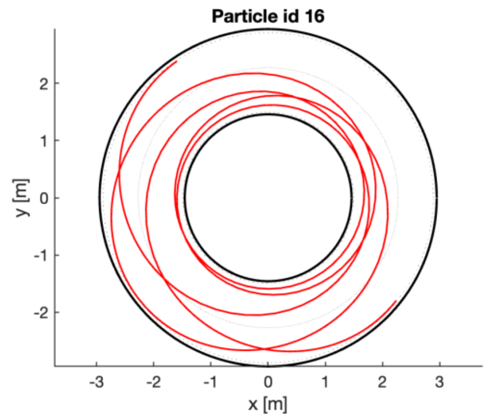


C. De Piccoli et al., 2024, Front. Phys. 12:1492095

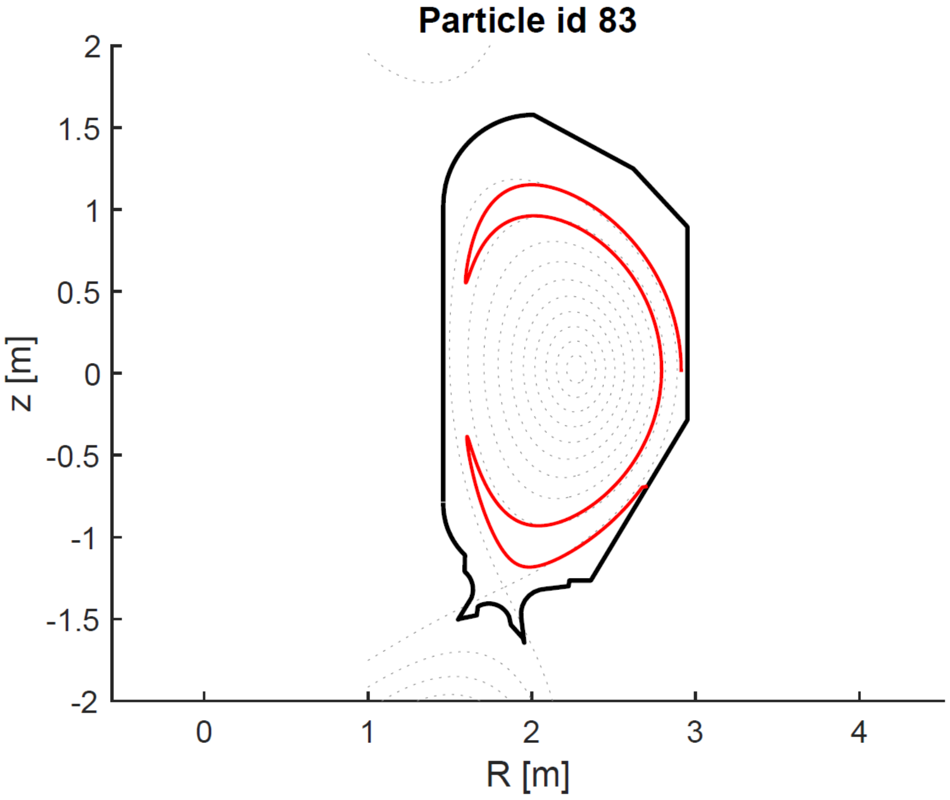
Example of DTT NBI EP orbits

ASCOT simulations (C. De Piccoli)

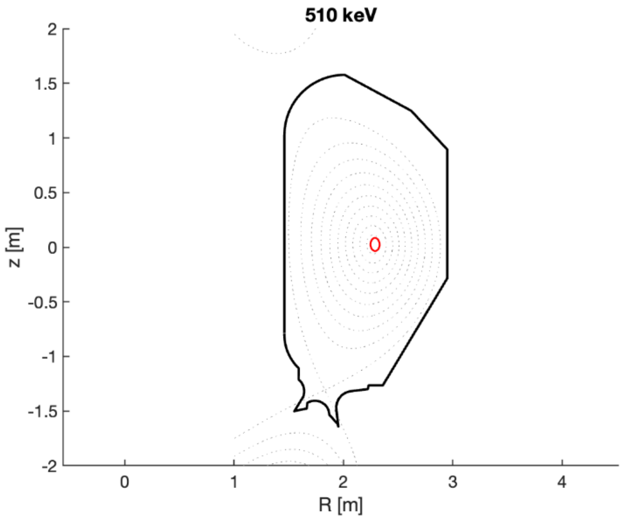


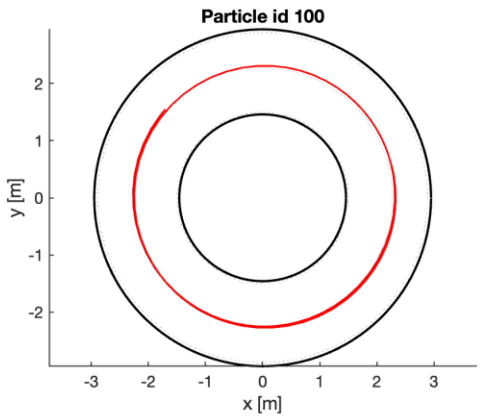


Confined passing orbit



Lost trapped orbit



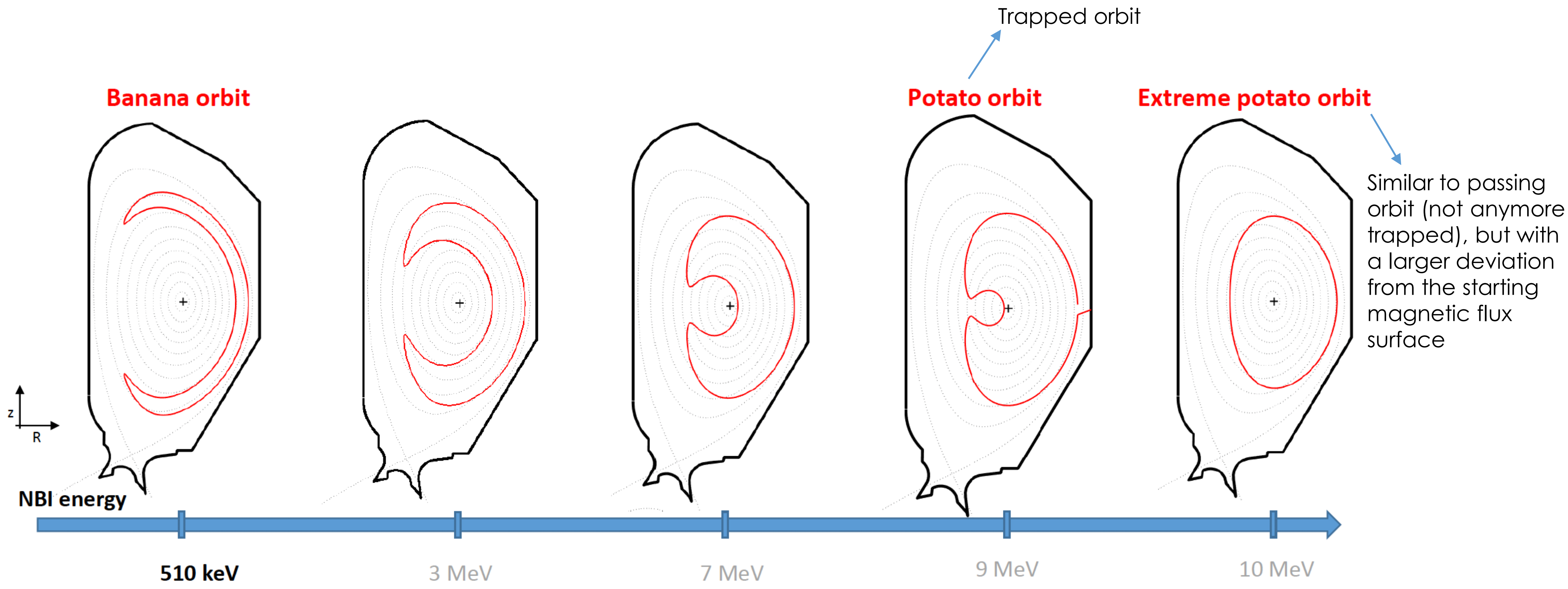


Stagnation orbit

C. De Piccoli et al., 2024, Front. Phys. 12:1492095

Example of DTT NBI EP orbits

ASCOT simulations (C. De Piccoli)



C. De Piccoli et al., 2024, Front. Phys. 12:1492095

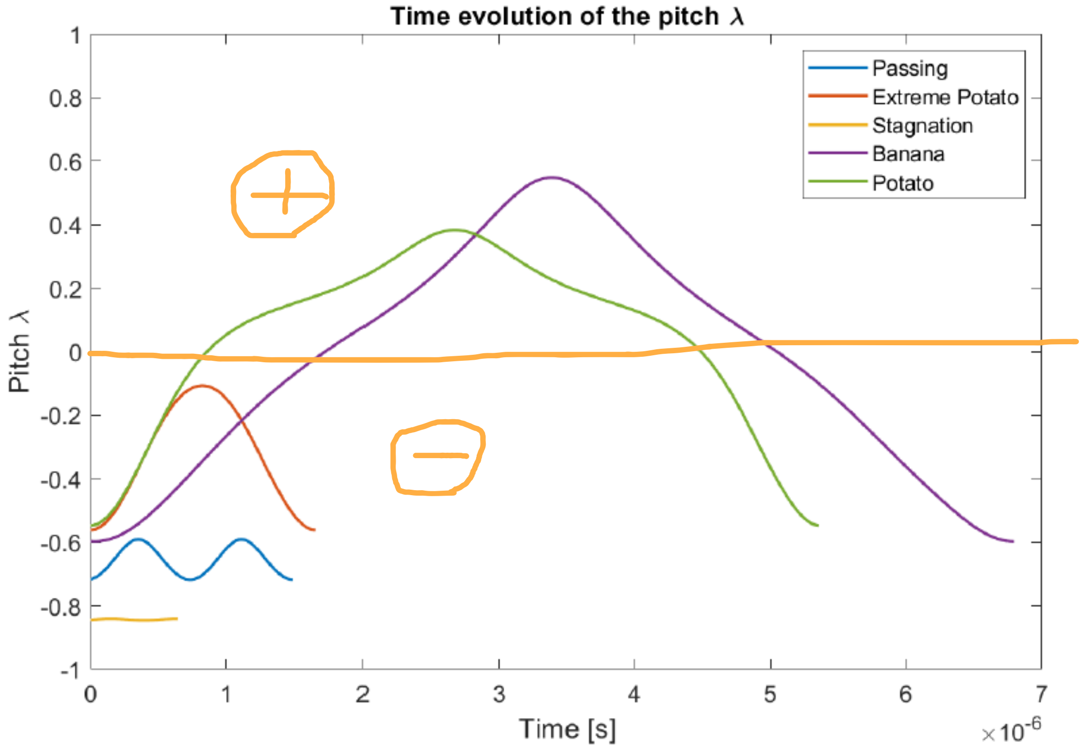
Pitch evolution of non-standard orbits

ASCOT simulations (C. De Piccoli)

Pitch: $\lambda = v_{par}/v_{tot}$ with respect to magnetic field B

Example of different orbits of imaginary* NBI EP in DTT

- Pitch changes sign → particle turning back:
  - Banana
  - Potato
- Pitch does not change sign:
  - Passing
  - Stagnation
  - Extreme potato



*EP energy increased arbitrarily to see these exotic orbits, NBI EP in DTT will be mostly passing

Fast ion slowing down and energy transfer

During their motion, fast ions transfer their energy to plasma species through Coulomb collisions (**slowing down process**):

- To ions, including impurities
- To electrons

How the energy is transferred?
(approximation valid for $v_i << v_{beam} << v_e$)

$$\frac{dE}{dx} = -\underbrace{\frac{\alpha}{E}}_{\text{to ions}} - \underbrace{\beta\sqrt{E}}_{\text{to electrons}}$$

We can define a fast ion energy corresponding to equal stopping from electrons and ions:

$$\frac{\alpha}{E} = \beta\sqrt{E}$$

And define therefore this energy as the critical energy E_c: $E_c = \left(\frac{\alpha}{\beta}\right)^{2/3} = 14.8\left[\sum_{j=species}\frac{n_j Z_j^{\,2}}{n_e A_j}\right]^{2/3} A_b T_e[keV]$

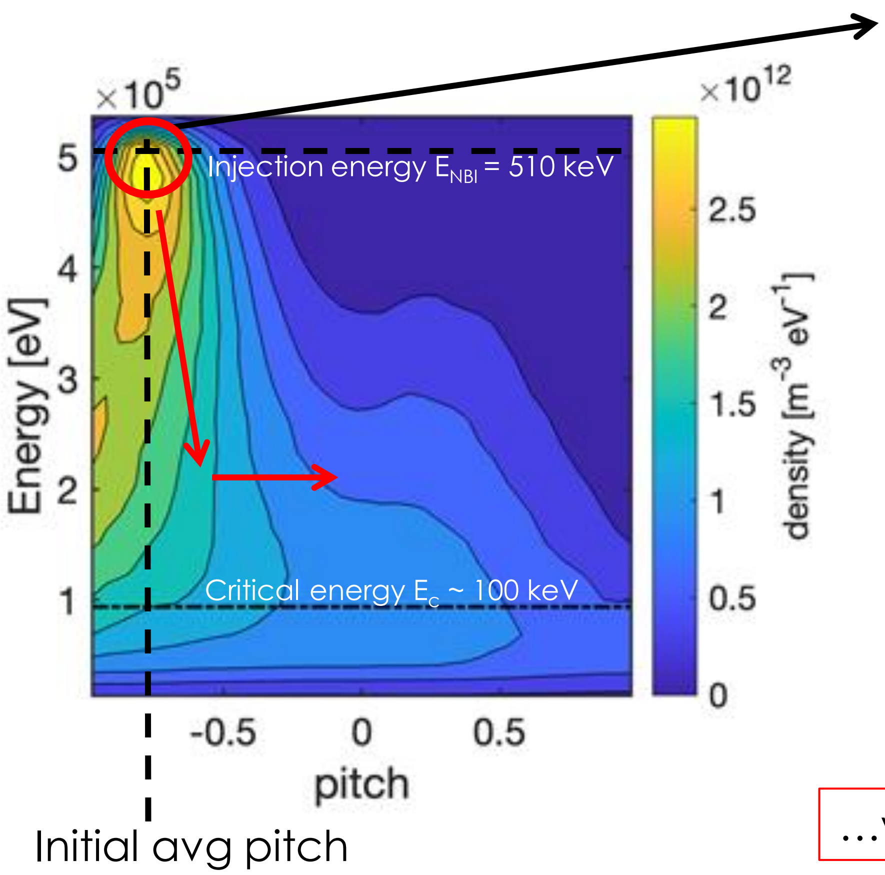
- It depends only on plasma (temperature, density, plasma species, except dependence on NBI species)
- At time t during a fast ion slowing down lasting τ_s > t:
  - $E_{fast\ ion}(t) > E_c$ dominant energy transfer to plasma electrons
  - $E_{fast\ ion}(t) < E_c$ dominant energy transfer to plasma ions

...

T H Stix. "Heating of toroidal plasmas by neutral injection". In: Plasma Physics 14.4 (1972), p. 367
Kikuchi, Lackner, Tran "fusion Physics", IAEA, p. 544

Fast ion energy transfer

In steady state conditions, we have a fast ion population composed of newly ionised particles, fast ions in the middle of slowing down, scattered, almost thermalized…
An example of the steady state numerical solution of the fast ion particle distribution function (density of ions in E+dE and ε+dε):



Fast ion slowing down:
from collision with electrons to collisions with ions

1) electron collision ($E_{fast\ ion} > E_c$): energy scattering (direction of motion not affected)

2) ion collision ($E_{fast\ ion} \approx E_c$): energy & pitch scattering (direction of motion affected)

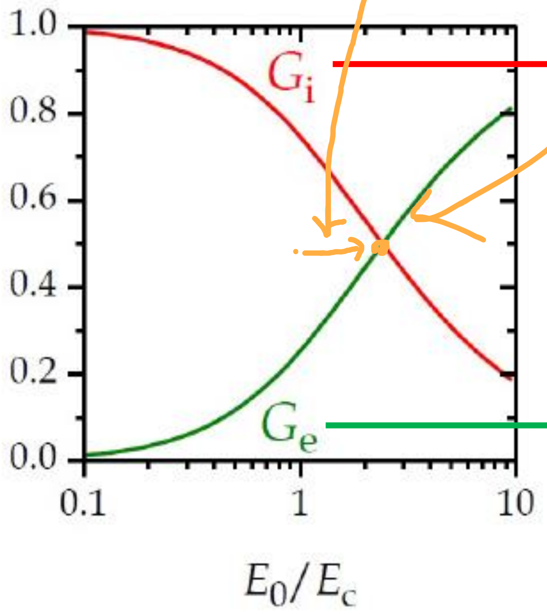
= distribution spreading in pitch at lower fast ion energy

…we will learn more about fast ion distribution function later

C. De Piccoli et al., 2024, Front. Phys. 12:1492095

Fast ion energy transfer

If we want to estimate the NBI power (for a NBI with energy E_0) going to electrons and ions we have to **integrate the energy transferred in time**:



Power fraction to ions: $G_i = -\frac{1}{E_0}\int_0^{\tau_s}\left(\frac{dE}{dt}\right)_{ion} dt$

Power fraction to electrons: $G_e = 1 - G_i$

For ***E_0 > 2.5 E_c***, electron heating dominates (future reactors, alpha heating…)

In current experiments (low E_{NBI}), ion heating is dominant, also due to lower energy components of the beam, in future reactors it will be the other way round

Slowing down time

The **time** a fast ion spends to transfer its energy (from E_0 to $E \sim E_{thermal}$, thermalization) is:

Slowing down time: $$\boxed{\tau_{sd}} = \int_{t(E_0)}^{t(E=0)} dt = \int_{E_0}^{E=0} \left[\frac{dE}{dx}\frac{dx}{dt}\right]^{-1} dE = \int_0^{E_0} \left[-\frac{dE}{dx}\sqrt{\frac{2E}{m}}\right]^{-1} dE = \boxed{= \frac{t_s}{3} \ln\left[1 + \left(\frac{E_0}{E_c}\right)^{\frac{3}{2}}\right]}$$

with the Spitzer slowing down time defined as: $$t_s[s] = \frac{\sqrt{2m_i}}{\beta} = 6.28 \times 10^{14} \cdot \frac{AT_e[eV]^{\frac{3}{2}}}{Z^2 n_e[m^{-3}] \ln\Lambda}$$

For current experiments τ_{sd}~100-200 ms, in future reactors (larger T_e, larger E_{NBI}) it will be up to seconds

Wesson, Tokamaks 3rd edition, Clarendon Press-Oxford, 2004
W. Ott, E. Speth, A. Stäbler "Slowing down of Fast Ions in Plasma: Energy Transfer, Charge Exchange Losses and Wall Sputtering.", IPP 4/161, 1997

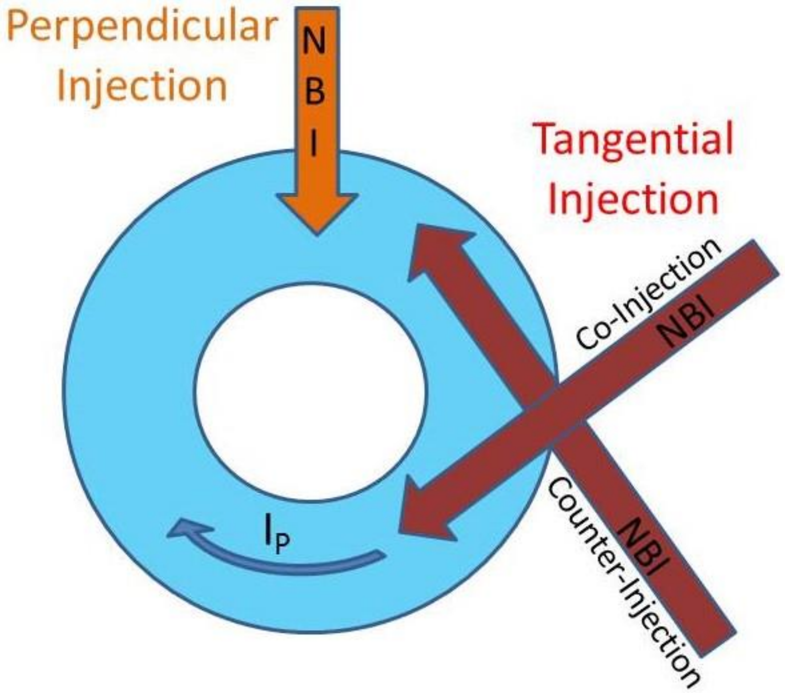
Current-drive

Inducing plasma current

In tokamaks plasma current is currently sustained mainly by the central solenoid → **pulsed, inductive operations with limited pulse length**, cycles

The generation of **non-inductive currents** (boostrap, or by external H&CD systems) could extend discharge duration:

- fully non-inductive plasma current (steady state tokamak)
- partially non-inductive (extended pulse length)



NBI, as other systems, can drive current in the plasma.

NBI must have tangential injection to drive current

NBI current drive

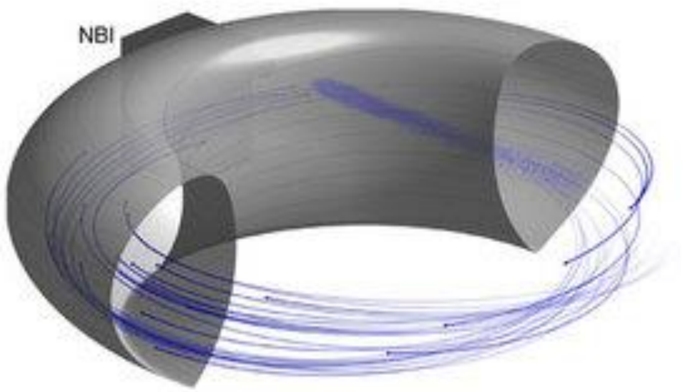
A simple estimation of the driven current

The tangentially injected fast particles, becoming fast ions, generate a circulating **fast ion current**:

$$I_{circ} = \boxed{I_0} \boxed{\frac{\langle v_{par} \rangle \tau_{sd}}{2\pi R_o}}$$

→ average number of toroidal laps ($\langle v_{//} \rangle \sim v_0/2$)

$I_0 = P_{NBI}/E_0$ injected particles



But we miss:

- correct averaging over slowing down
- pitch angle scattering
- electrons dragged by fast ions

Back electron current: could cancel all the fast ion current if we do not consider toroidal effects and if $Z_{beam} = Z_{eff}$. Luckily these conditions are not present in experiments

$$\rightarrow \quad I_{NBCD} = I_{circ}\left(1 - \frac{Z_{beam}}{Z_{eff}}\right)$$

- trapped electrons that cannot be dragged along with fast ions (this would give some current also for $Z_{beam} = Z_{eff}$)

$$\rightarrow \quad I_{NBCD} = I_{circ}\left(1 - \frac{Z_{beam}}{Z_{eff}}\left(1 - f(Z_{eff}, \varepsilon)\right)\right)$$

ε = aspect ratio

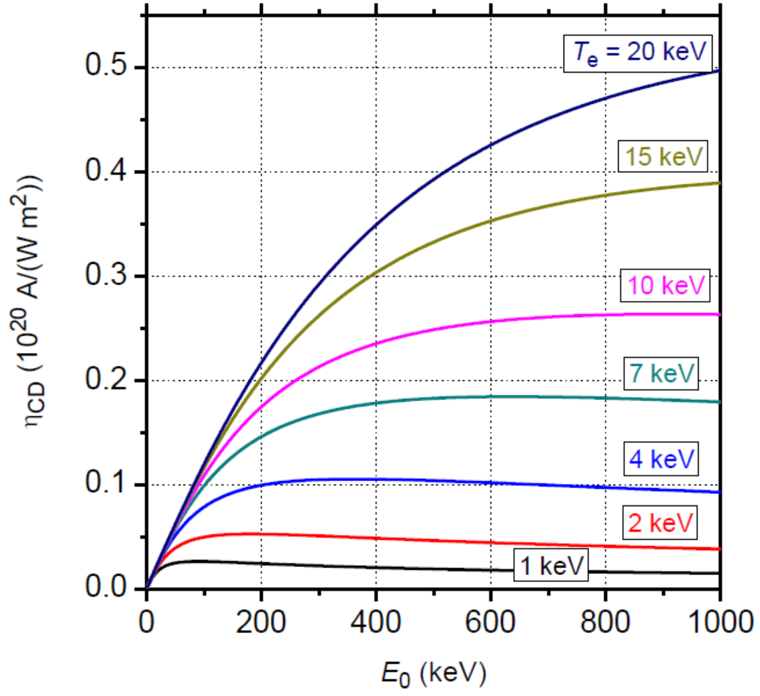
Current drive efficiency

The **efficiency** of an auxiliary current drive system can be defined as the current driven per power injected I_{CD}/P_{NBI}.

But since: $I_{CD} \propto P \frac{1}{R_0 n_e}$ Cd = current drive

the **current drive efficiency** is defined as: $\eta_{CD} = R_0 n_e \frac{I_{CD}}{P}$ usually in $\left[\frac{10^{20} A}{W m^2}\right]$

- The NBI CD efficiency increases with E_{NBI} and plasma temperature: $\max(\eta_{NBCD})$ increases linearly with T_e
- 90% of $\max(\eta_{NBCD})$ at considerably lower E_{NBI}: $E_{0.9max} \sim 0.5 \cdot E_{max}$
- In current experiments $\eta_{NBCD} \sim 0.05 - 0.1$ (record in JT60-SA: 0.15)
- For future reactors (ITER, DEMO) $\eta_{NBCD} \sim 0.3 - 0.4$
- NBI has usually the highest CD efficiency



Ex. for D tangential injection

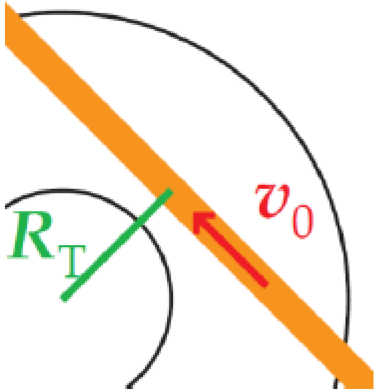
NBI is also a source of…

Source of particle N (plasma fuelling)

$$\frac{dN_{NBI}}{dt} = \frac{P_{NBI}}{eE_0}$$

- useful for core plasma fueling (efficient)
- relevant for current experiments
- negligible for future reactor-like experiments (V_{plasma} two order of magnitude larger, E_{NBI} 1 order of magnitude higher, P_{NBI} almost the same)

Source of toroidal torque (only for tangential injection)



- Due to a combination of collisional torque and j x B torque (due to fast ion current)
- Collisional torque: $M = \frac{I_0}{e} A_{beam} m_p v_0 R_T e_\phi$

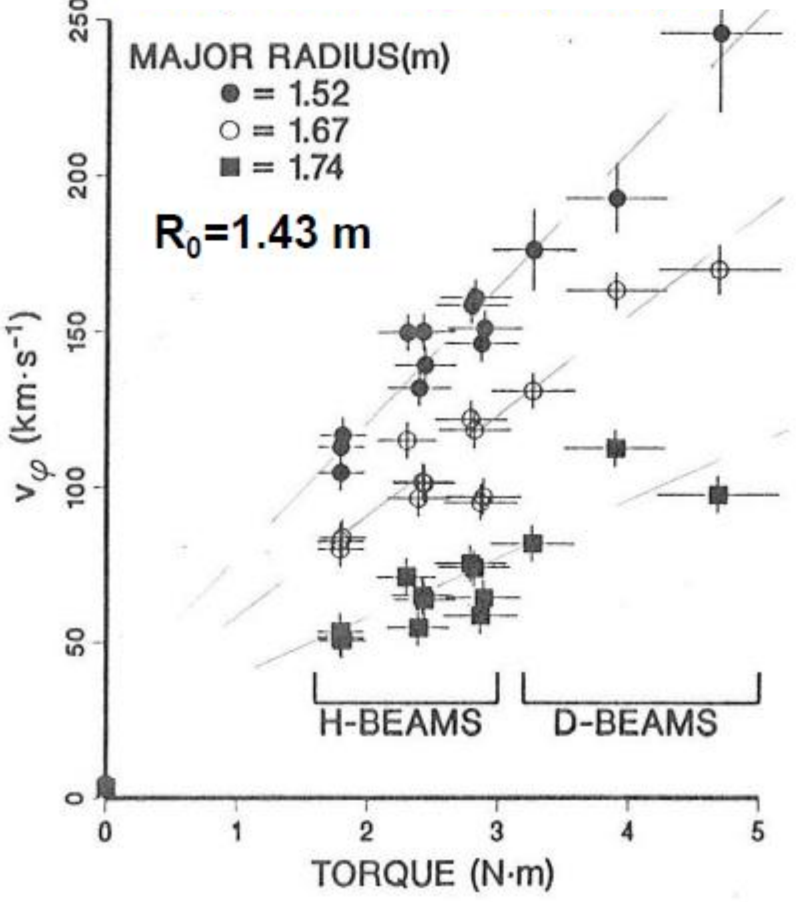
  since $I_0 = P_{NBI}/E_0$ and $v_0 \propto \sqrt{E_0/A_{beam}} \rightarrow \frac{\boldsymbol{M}}{\boldsymbol{P_{NBI}}} \propto \boldsymbol{E_0}^{-1/2}$

- Torque induces plasma rotation depending also on the momentum confinement time

F.L. Hinton, M.N. Rosenbluth, Physics Letters A 259 1999. 267–275

NBI torque and plasma rotation

DIII-D tokamak (USA), Nucl. Fus. 28 3 (1998)



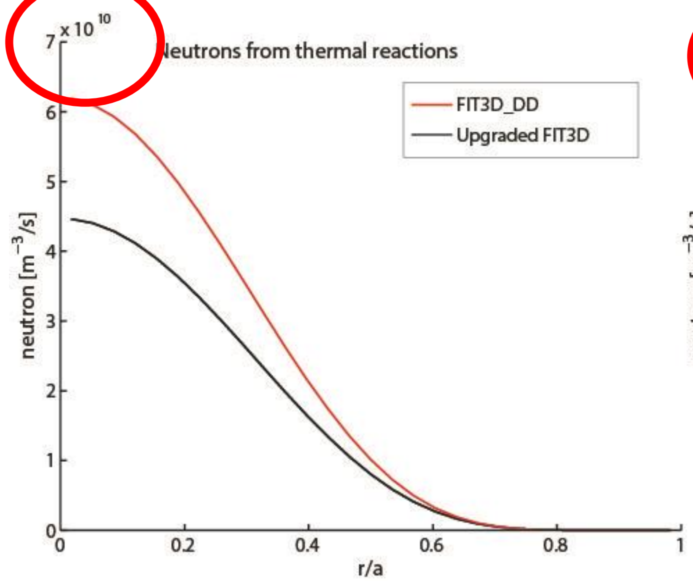
- Control of plasma rotation by NBI torque
- In case of counter-current injection, the plasma rotation can be reduced (decreasing it to condition similar to future reactors)
- plasma rotation and rotation shear (dv_{rot}/dr) can suppress turbulence and help plasma stability

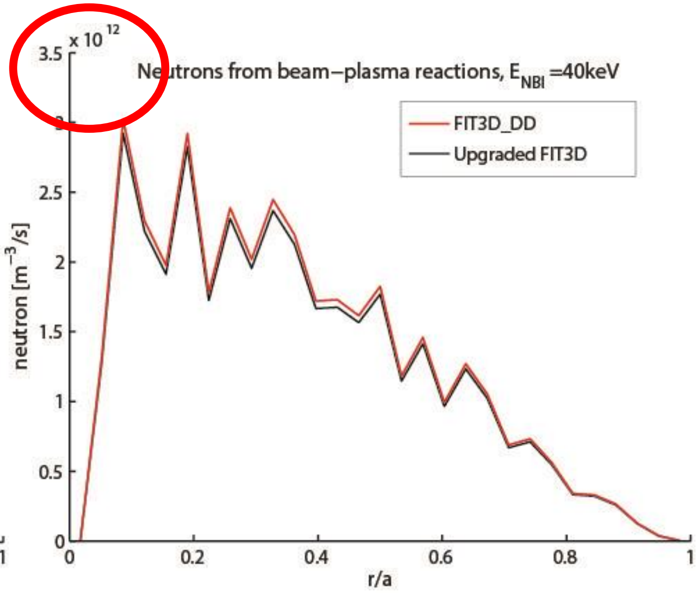
Beam-plasma fusion reactions

In case of reactive species (D-D, D-T), the interaction between NBI and plasma is a source of fusion reactions (beam-plasma reactions), additional to background plasma reactions (thermal reaction).
In world-record JET and TFTR D-T experiments, beam-plasma reactions dominated.

Simulation for D campaign of LHD experiment ("stellarator family", $P_{NBI} \sim 20$MW, $E_{NBI} = 50$-200 keV, $n_{e,0} = 3e19$ m^{-3}, $T_{e,0} = 1$keV)
P. Vincenzi et al., Plasma Phys. Control. Fusion 58 (2016) 125008

- D + D → p (3.02MeV) + T (1.01 MeV) — proton production
- D + D → n (2.45 MeV) + ^{3}He (0.82 MeV) — neutron production





Also the fast ions of the beam itself can react (beam-beam reactions)

Energetic particle physics: a few words

NBI fast ions represent a particular ion population in the plasma: **suprathermal ions** (or Energetic Particles EPs)

- NBI fast ions can be a relevant part (tens %) of plasma ions in current experiments, while they will be a small population in larger tokamaks
- NBI fast ions are different from fusion alphas not only for the energy but also for the anisotropy in velocity space
- fast ions can drive instabilities such as Alfvén Eigenmodes (AEs), if their velocity ≈ Alfvén velocity $v_A = \frac{B}{\sqrt{\mu_0 n m}}$
- fast ions can be affected by instabilities and fast ions can be lost
- Experimental findings on fast ion AEs that reduce thermal ion micro-turbulence in JET and AUG (very interesting for T_i increase and reactor performance)

A. Di Siena et al., "New high-confinement regime with fast ions in the core of fusion plasmas", arXiv:2010.14839
Mazzi, S. et al. Enhanced performance in fusion plasmas through turbulence suppression by megaelectronvolt ions. Nat. Phys. 18, 776–782 (2022

Summary: NBI is a source of…

Neutral Beam injection is a source in the plasma of:

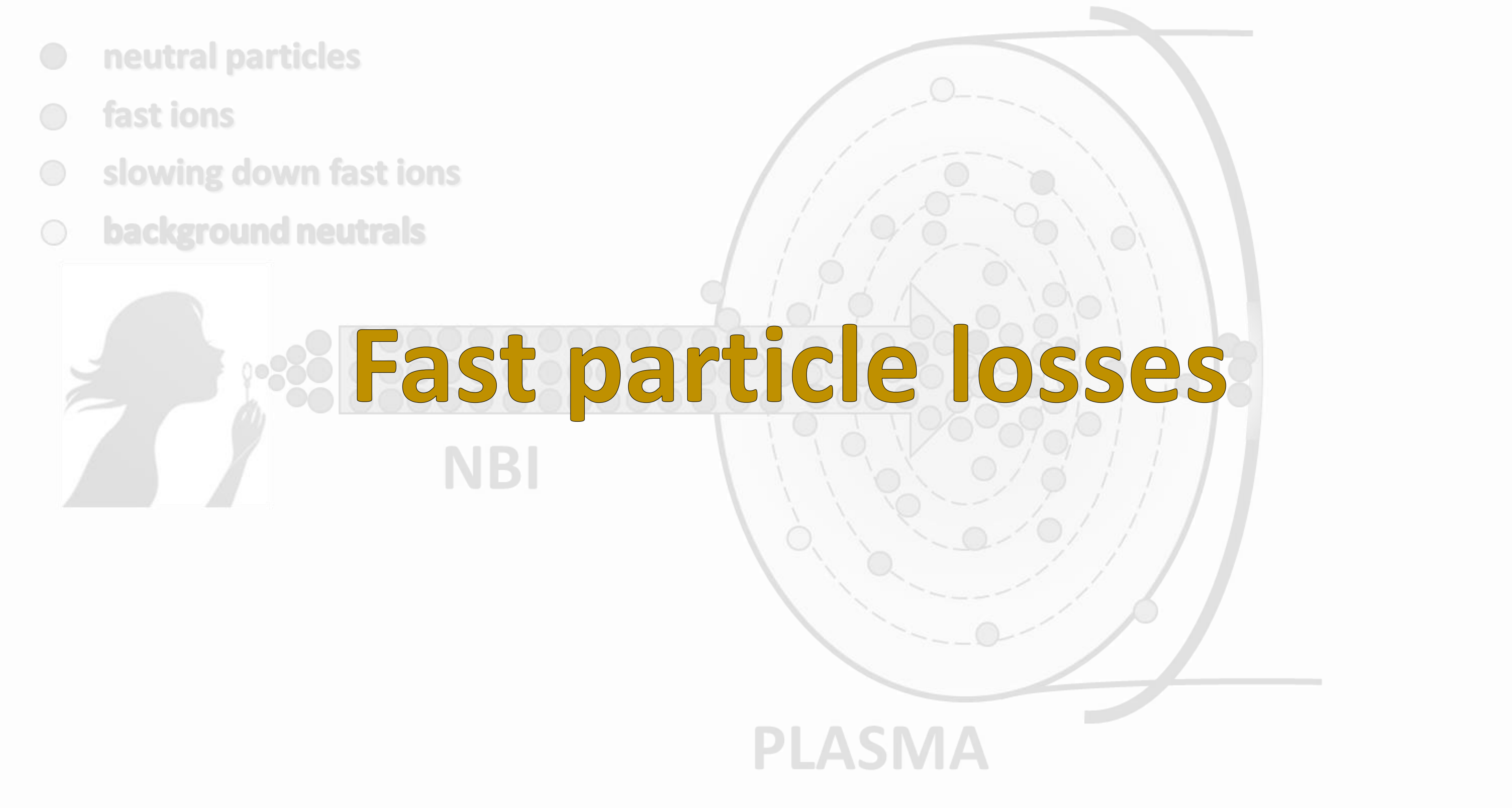
- **Energy** (→ heating)
- **Current** (in case of tangential injection)
- **Particles** (“plasma fueling”)
- **Torque** (acting on plasma rotation, in case of tangential injection)
- **Fusion reactions** (beam-plasma reactions, in case of reactive species)
- “**Beam halo**” (= thermal neutrals), in case of relevant CX ionization process
- **Fast particles** (peculiar particle population with their own **instabilities** and interaction with thermal plasma, used also for diagnostics)

NBI…towards future

		Present day	ITER/Reactors (?)
NBI technology	Energy	Low-middle (40-200 up to ~500 keV - JT60SA & DTT)	High ~1000 keV
	Power	Tens MW	Tens MW
NBI physics	Heating source	✓	✓
	Current source	✓	✓
	Particle source	✓	Negligible
	Torque source	✓	✓
	Fusion reactions (beam-plasma) source	✓	Relatively low

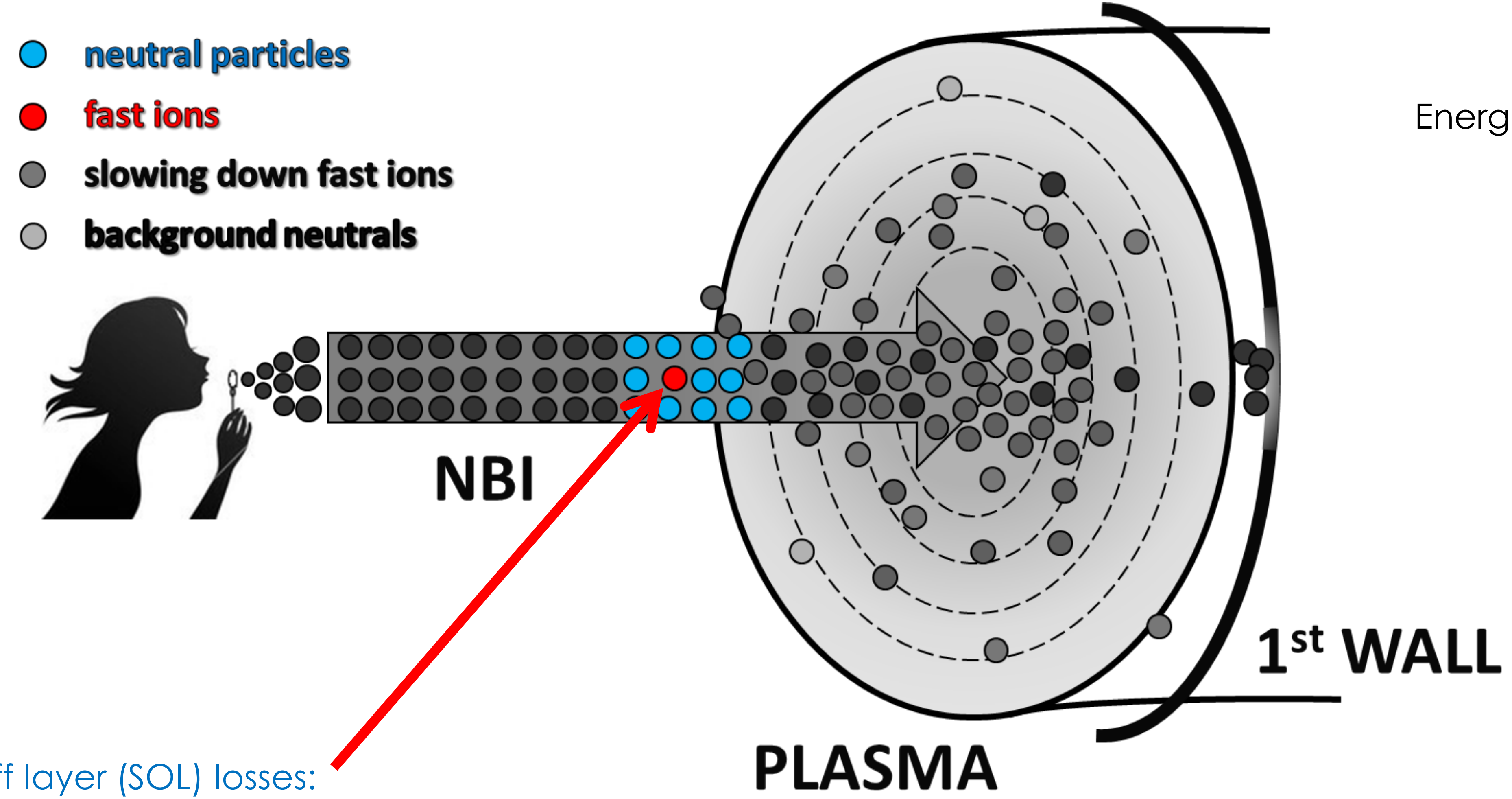
Contents

Fast particle losses



Fast particle losses

Energetic particle losses



Energetic particle losses:

Scrape-off layer

First orbit

Charge-exchange

Orbit

Shine-through

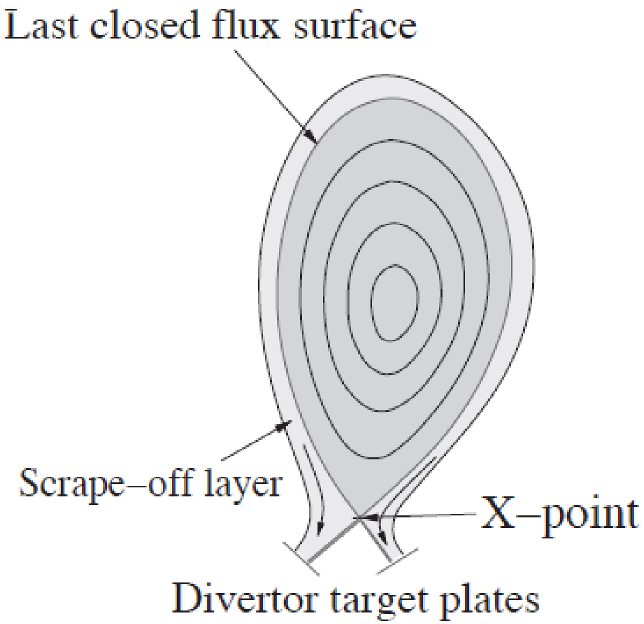
Scrape-off layer (SOL) losses:

Fast ions born inside the tokamak chamber but outside confined plasma

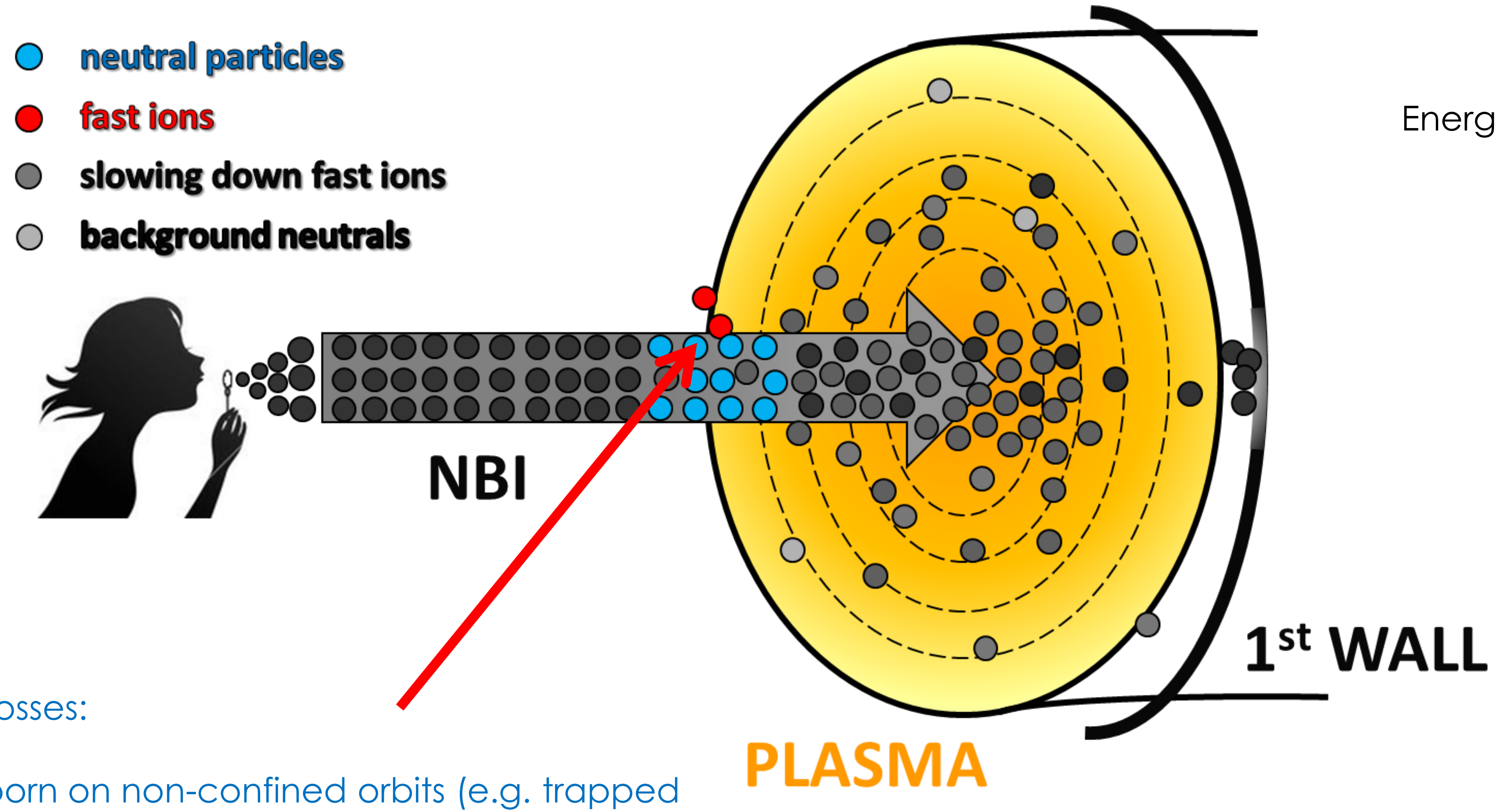
Scape-off layer losses

Scrape-off layer (SOL) losses:

- SOL is characterized by open magnetic flux surfaces
- If a fast neutral particle is ionized in the SOL, its orbit will collide with divertor plates
- In this region we find neutral particles (e.g. gas puff) and ions entering/escaping from the plasma. It is usually difficult to estimate SOL density: if it is known, SOL losses can be easily estimated by numerical modelling



Energetic particle losses



Energetic particle losses:

Scrape-off layer

First orbit

Charge-exchange

Orbit

Shine-through

First orbit losses:

Fast ions born on non-confined orbits (e.g. trapped outward bananas)

First orbit losses

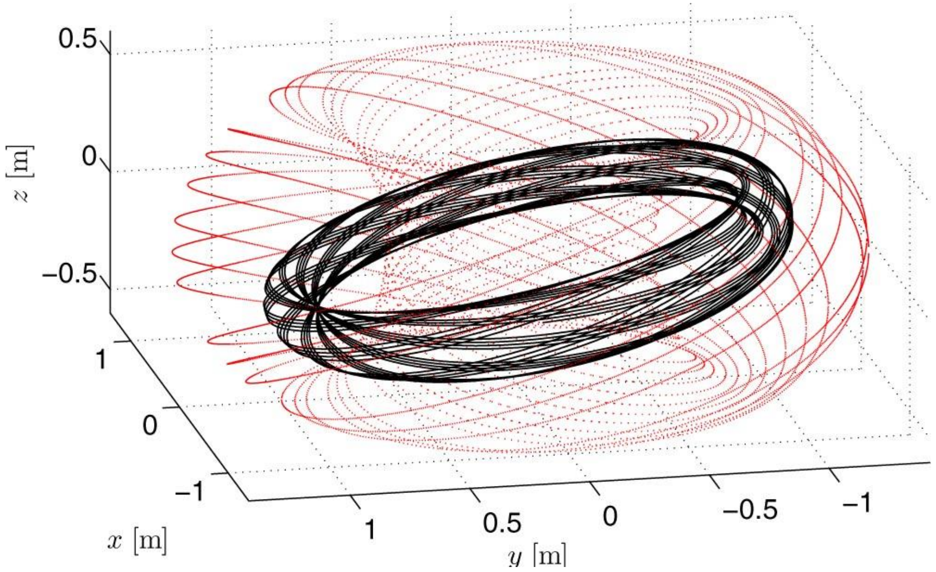
Fast ions born on non-confined orbits causes first orbit losses

- Most of fist orbit losses are due to banana orbits, when part of the banana orbit cross the separatrix (*as we have seen before when describing beam fast ion banana orbits*)

- A particular case with very high first orbit losses is represented by **NBI counter-current injection**, because banana orbits move fast ions outwards

- Depending on plasma parameters and injection direction, these losses can be relevant

- An analytical formulation is available to estimate first orbit losses

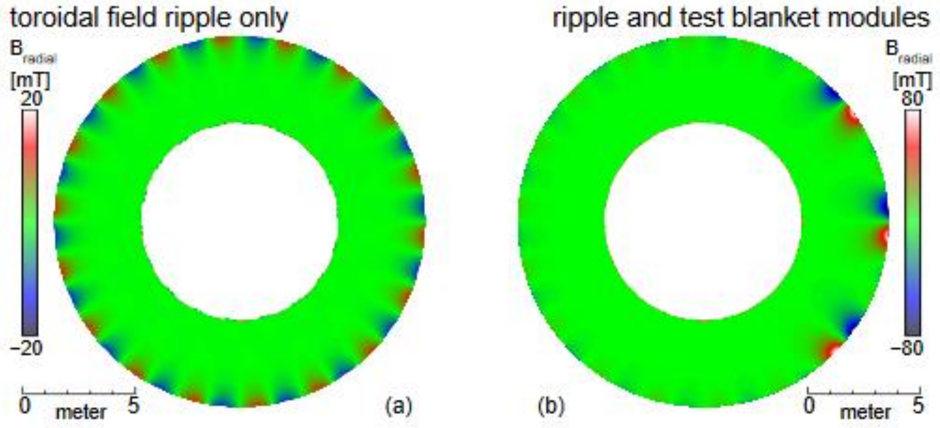


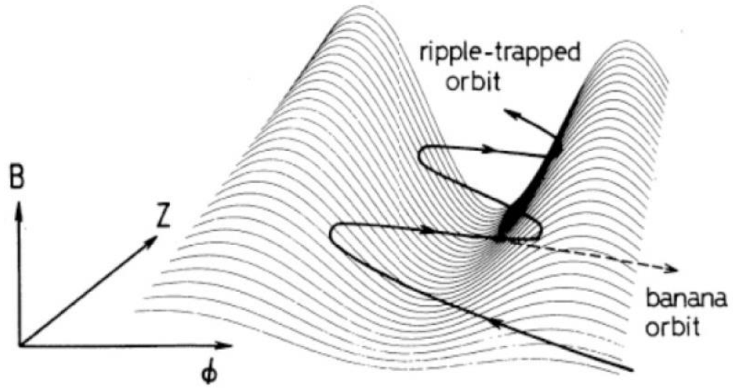
NBI particle trajecory in 3D for MAST
[D. Pfefferlé et al., 2014 Nucl. Fusion **54** 064020]

First orbit losses

Also magnetic field ripple can cause immediate orbit losses(collisionless process)

- Magnetic field perturbation due to the finite number of toroidal field coils.
- It can be mitigated with ferritic inserts.
- The ripple modifies the trajectories of fast ions causing localized losses (wall hot spots)
- Both NBI fast ions and alphas (for reactors) will suffer this issue
- Mitigated in ITER, negligible in DTT [Spizzo et al., NF 2021]

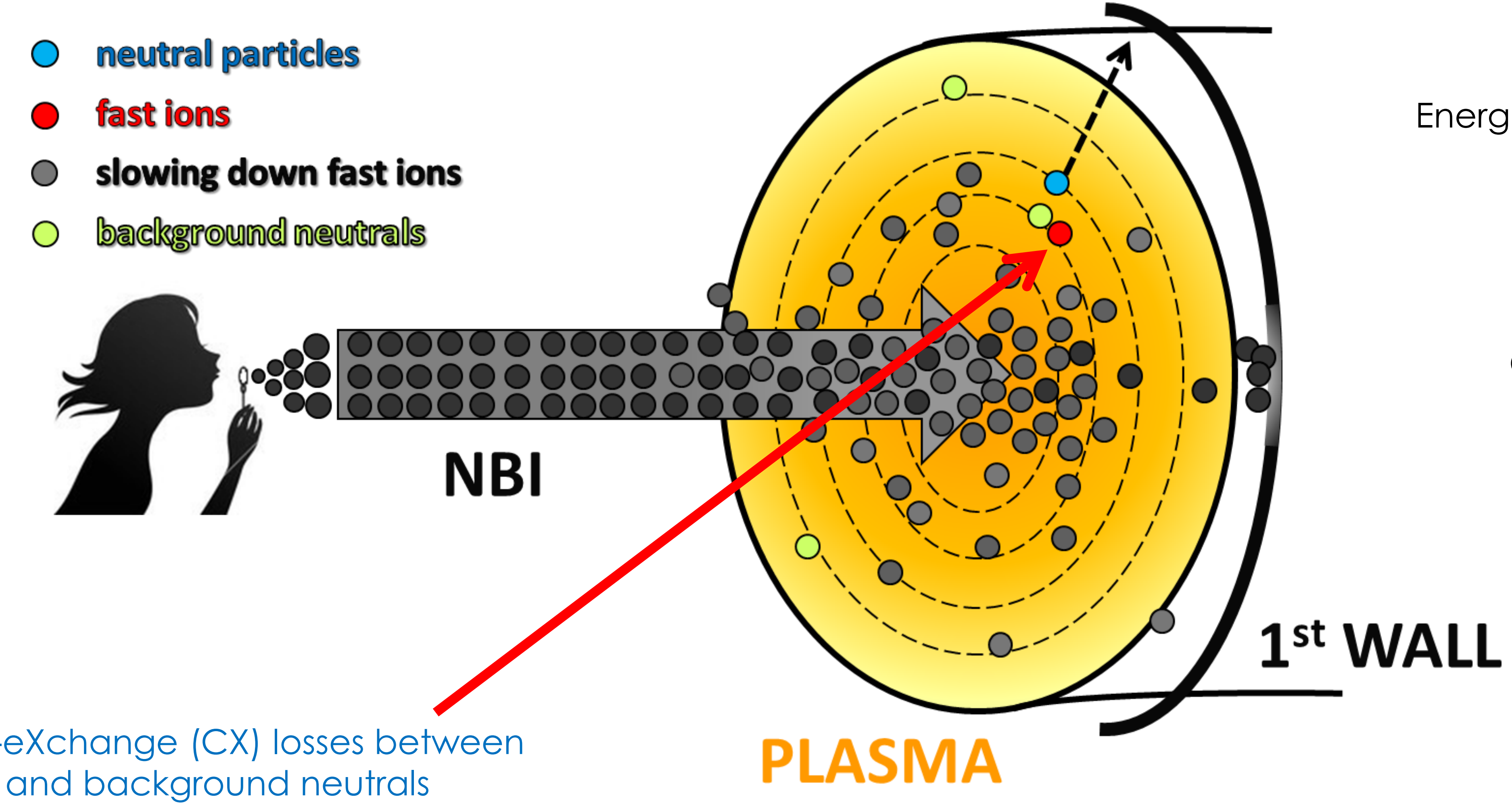




Ex. of collisionless trapping of a banana particle

Example for alphas in ITER
[G. J. Kramer, R. B. White, R. Nazikian, H. L. Berk, "Fusion-born alpha particle ripple loss studies in ITER", *22nd IAEA Fusion Energy Conf.*, 2008-Oct.]

Energetic particle losses



Energetic particle losses:

Scrape-off layer

First orbit

Charge-exchange

Orbit

Shine-through

Charge-eXchange (CX) losses between fast ions and background neutrals

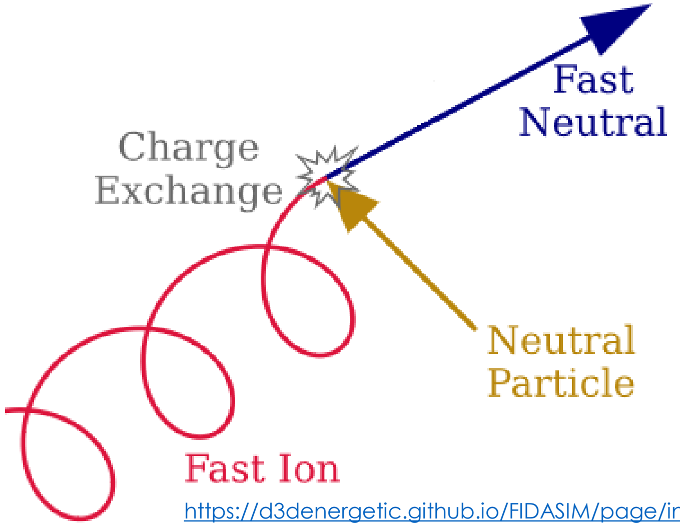
$H^{+}_{fast} + H^{0} \rightarrow H^{0}_{fast} + H^{+}$

Charge-eXchange (CX) losses

Losses due to Charge eXchange (CX) reaction between a fast ion and a neutral particle in the plasma (*pay attention, it is a different reaction wrt to NBI fast particle ionization by CX, but beam halo can contribute to these losses*)

$$H^{+}_{fast} + H^{0} \rightarrow H^{0}_{fast} + H^{+}$$

Fast neutrals escape the plasma, with lower probability of being ionized



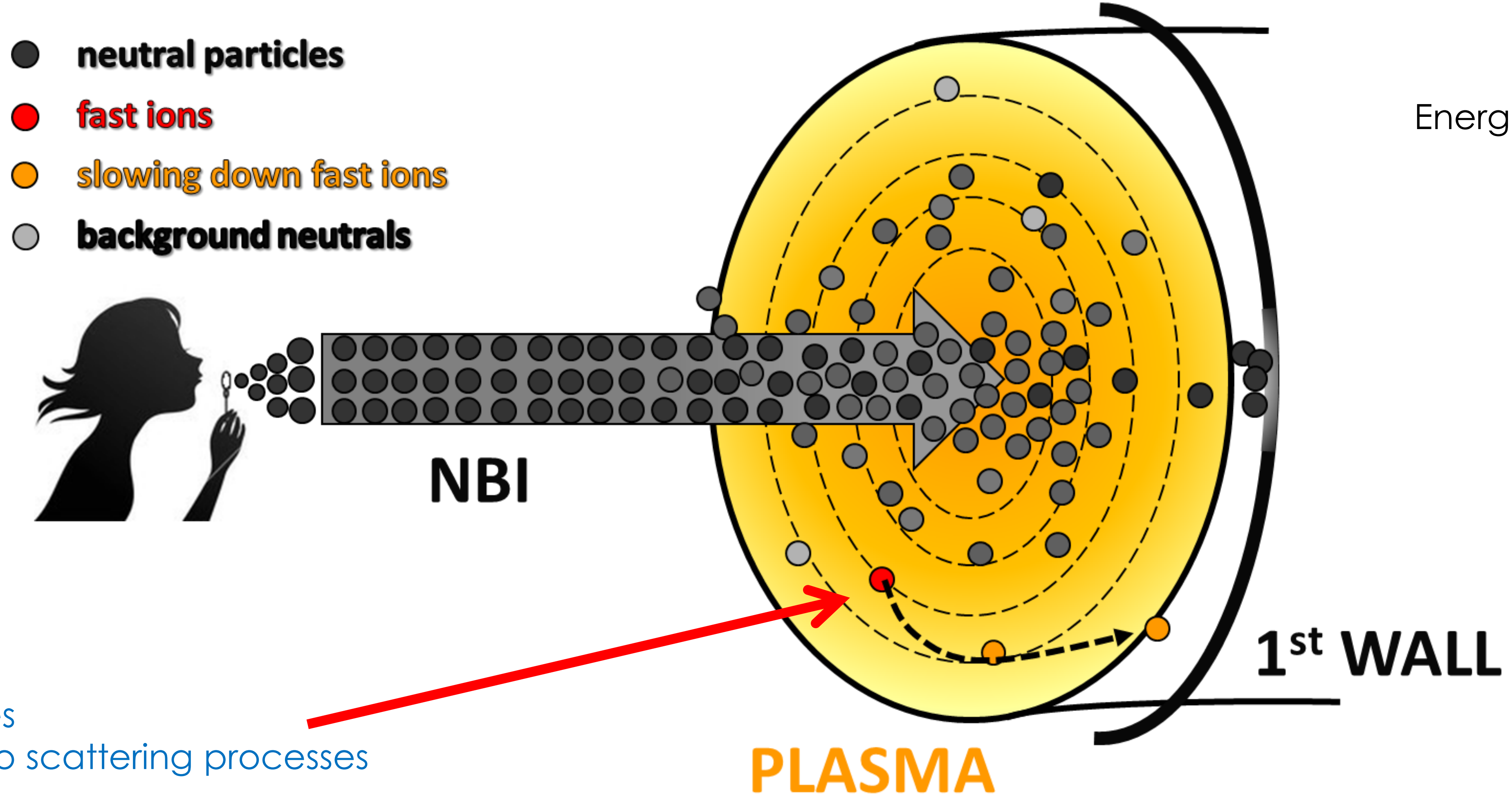
https://d3denergetic.github.io/FIDASIM/page/index.html

Where do the neutrals come from?

- Gas puff (cold neutrals)
- Pellet injection
- CX NBI ionization (thermal neutrals = beam halo)

- In small experiments CX losses can be up to 20-30% of the injected NBI power
- H^{0}_{fast} can then be re-ionized before leaving the plasma, broadening the power deposition profile
- In future reactors we expect very high temperature and density, therefore small neutral density in the plasma and low CX fast ion losses

Energetic particle losses



Energetic particle losses:

Scrape-off layer

First orbit

Charge-exchange

Orbit

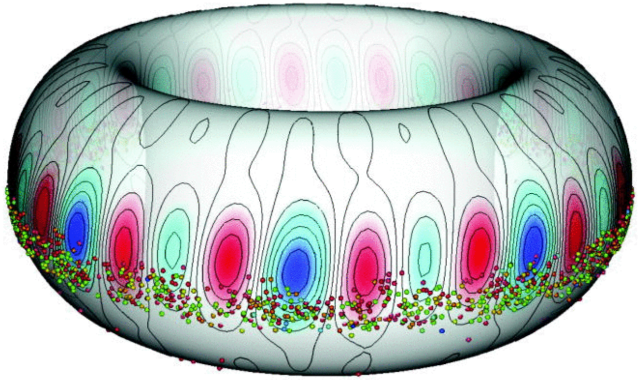
Shine-through

Orbit losses
e.g. due to scattering processes

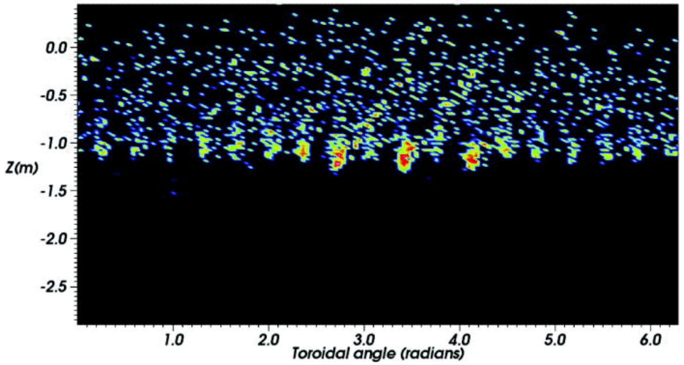
Orbit losses

Similar to first orbit losses but happening for confined fast ions that move into unconfined orbits:

- Losses can be due to pitch-angle scattering: after a collision the fast ions move into a unconfined orbit
- Since the pitch-angle scattering is stronger with ion-ion collisions, these losses are more probable when $E_{fast\ ion} \approx< E_c$
- Orbit losses are more probable for edge fast ions
- Orbit losses can be a result also of fast ion redistribution due to turbulence and instabilities

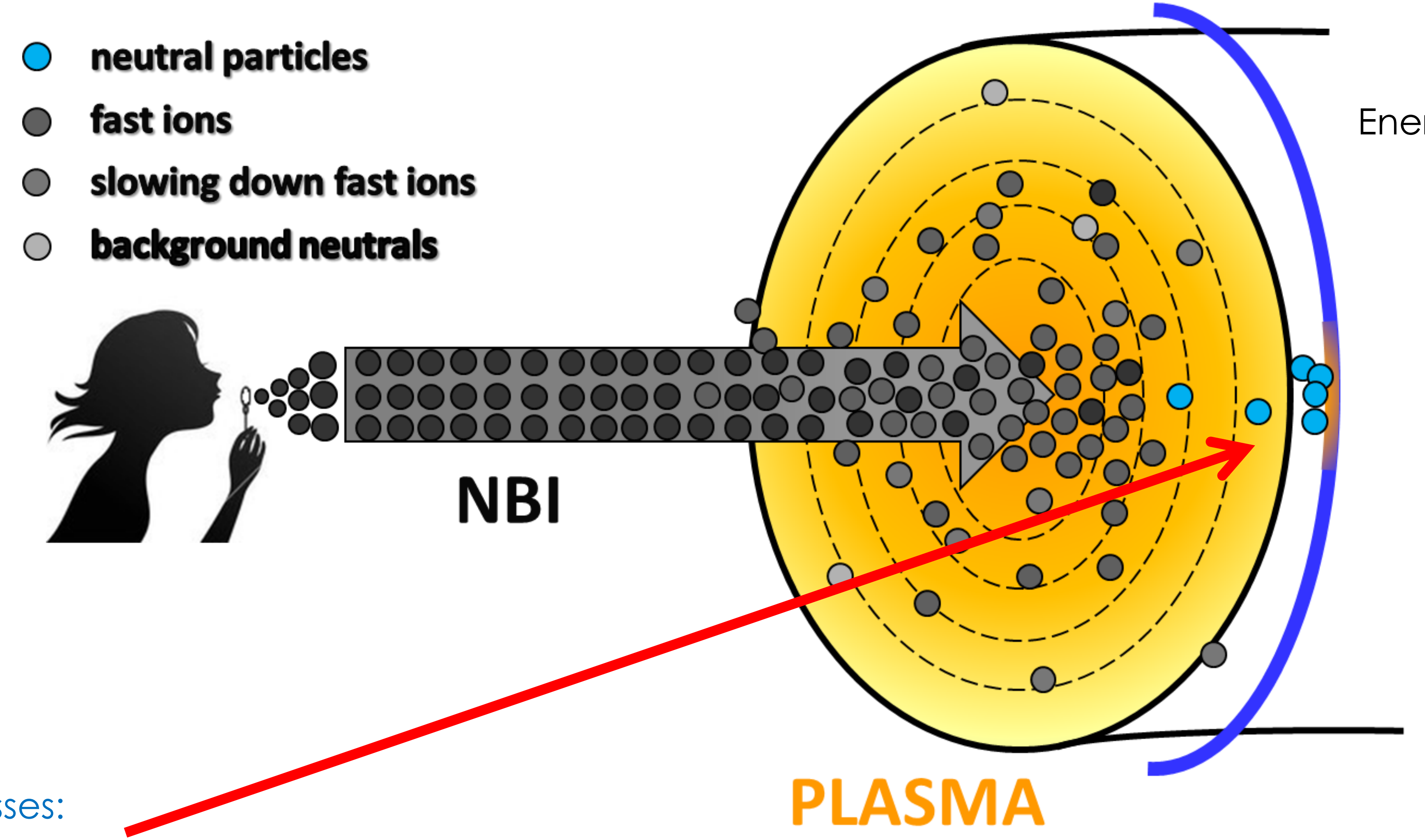


(a)



(b)

Energetic particle losses



Energetic particle losses:

Scrape-off layer

First orbit

Charge-exchange

Orbit

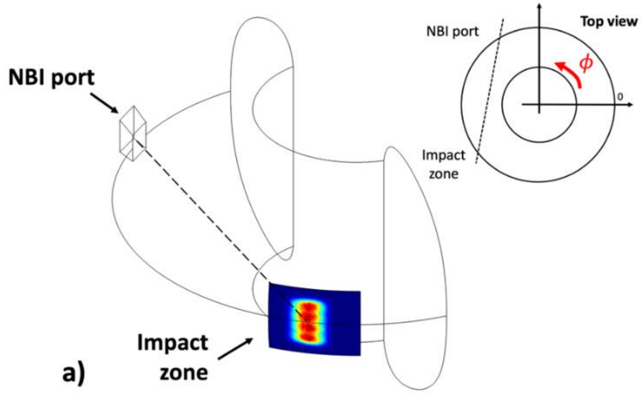
Shine-through

Shine through losses:

part of the beam not ionized passing through plasma

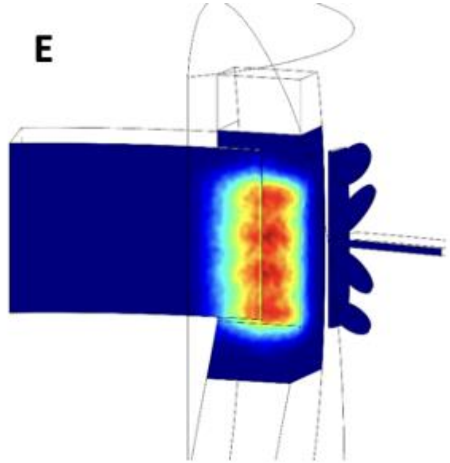
Shine-through losses

The fraction of the neutral beam, which is not ionised in the plasma and passes through it colliding with the first wall causes the so-called shine-through losses.



- shine-through losses can cause localized wall hot spots with the risk of exceeding the first wall thermal load limit (sometimes the wall is strengthened explicitly by armours)
- Shine-through increases exponentially with $\sim E_{NBI}/n$
- Usually, a lower limit on plasma density is set for NBI operation due to shine-through
- In current experiments this is one of the highest loss channels for NBI (up to 5 - 10% or more); for ITER and future reactors it will be relevant only in low-density phases (e.g. ramp-up/down phase)

DTT case



DTT: C. De Piccoli et al., 2024, Front. Phys. 12:1492095
ITER: P Vincenzi et al., submitted to NF 2024
DEMO: P. Vincenzi et al., Nuclear Materials and Energy 18 (2019), 188-192

Energetic particle losses

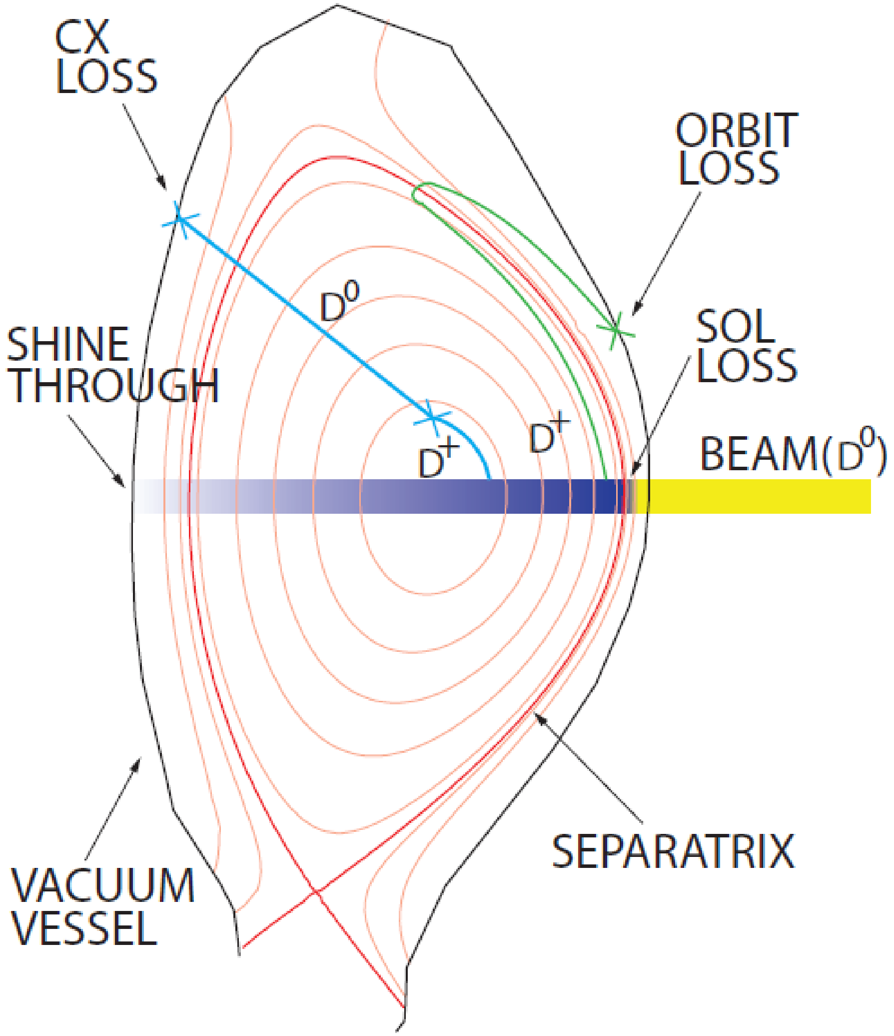


Image courtesy of M. Cavedon, IPP (Master thesis in Physics, 2012, Unipd)

Contents

Measuring NBI effectiveness is hard

Fast ion-related quantities are difficult to measure, e.g.:

Heating:
- Indirect from estimation of NBI losses
- profile: indirectly from temperature measurements and modelling

Total driven current:
- comparing the loop voltage between a current-drive discharge and reference discharge, matching the relevant plasma profiles (as much as possible)

Driven current profile:
- using Motional Stark effect (MSE): local direction of B → at best q(r) profile → j(r) profile
- forward modelling on MSE measurements

Fast ion distribution function:
- measuring the fast neutrals escaping the plasma generated by CX processes between fast ions and background neutrals (e.g. FIDA)

More information on diagnostics and NBI used as diagnostic in another lesson (?)

NBI modelling

Estimating quantities related to NBI-plasma interaction requires **modelling**.
NBI modelling is used for data interpretation and for predictions (e.g. for design of future machines)

NBI modelling requires two steps:

1. Modelling the injection and ionization of energetic neutral particles

⇨ **Beam deposition codes**

Example of pure deposition codes: BBNBI, NEMO…

M. Schneider et al., 2015, 42nd EPS on Plasma Physics, P2.159
NEMO: M. Schneider et al 2011 Nucl. Fusion 51 063019
BBNBI: Asunta O et al 2015 Comput. Phys.Commun. 188 33–46

NBI modelling

Estimating quantities related to NBI-plasma interaction requires modelling.
NBI modelling is used for data interpretation and for predictions (e.g. for design of future machines)

NBI modelling requires two steps:

1. Modelling the injection and ionization of energetic neutral particles

   ⇨ **Beam deposition codes**

2. Modelling the dynamics (slowing down) of NBI ions in the plasma

   ⇨ **Fokker-Planck solvers** (↔ fast ion distribution function)

Example of pure Fokker-Planck solvers: ASCOT, SPOT, RISK…

M. Schneider et al., 2015, 42nd EPS on Plasma Physics, P2.159
ASCOT: E. Hirvijoki et al, Computer Physics Communications, pp. 1310-1321, 2014
SPOT: M. Schneider et al., Nucl. Fusion 49 (2009) 125005
RISK: M. Schneider et al., Nucl. Fusion 55 (2015) 013003

NBI modelling

NBI codes can have different approaches (impacting on precision and time consumption):

a. (**Semi-**) **Analytical** solutions / simplified modelling / scaling laws:

- Beam ionization solved numerically in small steps on straight line(s) representing the beam
- Simple beam geometry (pencil-like)
- Power deposition (total, to electrons, to ions), current-drive, momentum source calculated using analytical formulae
- Fast simulation time
- Stand-alone codes or modules of integrated codes (e.g. transport suites)

E.g.: PENCIL (JETTO suite), Rabbit (stand-alone), METIS (integrated), NEMO-RISK (CRONOS suite)

- 3D beam shape but analytical ionization calculation: BTR

PENCIL: M. Stubberfield and M.L. Watkins, Multiple Pencil Beam, JET-DPA(06)/87, 1987
Rabbit: M. Weiland et al 2018 Nucl. Fusion 58 082032
METIS: J.F. Artaud et al 2018 Nucl. Fusion 58 105001
NEMO: M. Schneider et al 2011 Nucl. Fusion 51 063019
BTR: E. Dlougach et al Appl. Sci. 2022, 12, 8404

NBI modelling

A typical problem for fast ion modelling is the estimation of the fast ion distribution function.
In general, the time evolution of the distribution function can be described by the Fokker-Planck equation:

$$\frac{df}{dt} = C + S + L$$

- C is the collision term (fast ion – background plasma or fast ion – fast ion)
- S is the source term (beam ionization)
- L is a loss term (e.g. fast ion losses)

3 Many approximations can be done[1,2], and usually we are interested of stationary solutions (df/dt=0).

[1]Rome J A 1976 Nucl. Fusion 16 55
[2]R. Koch (2010) Fusion Science and Technology, 57:2T

NBI modelling

A typical problem for fast ion modelling is the estimation of the fast ion distribution function.
In general, the time evolution of the distribution function can be described by the Fokker-Planck equation:

$$\frac{df}{dt} = C + S + L$$

In order to find an analytical solution for f, we assume[1]:

- Fast ion bounce frequency >> collision frequency
  - In this approximation we have the following constant of motion:
    - Fast ion kinetic energy $E = \frac{1}{2}mv^2$
    - Toroidal canonical angular canonical momentum $P_\phi = mRv_{par} + q\psi$
    - Magnetic moment (adiabatic invariant) $\mu = \frac{mv_{perp}{}^2}{2B_{tot}}$

  → f(E, P_φ, μ)
  - A point in the constant of motion phase space (E, P_φ, μ) represents an entire orbit
  - Collisions will move points in the phase space ("kicks")

- Still, the equation is too complex to be solved analytically

[1]Rome J A 1976 Nucl. Fusion 16 55

NBI modelling

A typical problem for fast ion modelling is the estimation of the fast ion distribution function.
In general, the time evolution of the distribution function can be described by the Fokker-Planck equation:

$$\frac{df}{dt} = C + S + L$$

In order to find an analytical solution for f, we assume[1]:

Angular toroidal momentum

- f is independent of P_φ, i.e. the fast ions remains close to a given flux surface during the slowing down ("small banana width approximation) – hardly true for trapped energetic particles

- neglecting trapped particles when performing the integral around an entire bounce orbit

[1]Rome J A 1976 Nucl. Fusion 16 55

NBI modelling

The resulting analytic solution of the Fokker-Planck equation is:

$$f(v,\xi,t) = U\left[t - \frac{\tau_s}{3} ln\left(\frac{{v_0}^3 + {v_c}^3}{v^3 + {v_c}^3}\right)\right] \frac{\tau_s}{2\pi(v^3 + {v_c}^3)} \left[\frac{{v_0}^3 + {v_c}^3}{v^3 + {v_c}^3}\right]^{-\frac{\tau_s}{3\tau_{cx}}} \times \sum_{n=0}^{\infty} \left(n + \frac{1}{2}\right) P_n(\xi) P_n(\xi_0) \left[\frac{{v_0}^3 + {v_c}^3}{v^3 + {v_c}^3} \frac{v^3}{{v_0}^3}\right]^{\frac{m_i n(n+1)\langle Z\rangle}{6 m_{fast\,ion}[Z]}} U(v_0 - v)$$

- U is the step function
- P_n are Legendre polynomials
- ξ is the pitch (ξ_0 the initial pitch)
- τ_s the slowing down time
- v_0 the initial fast ion velocity, v_c the critical velocity $v_c = \sqrt{\frac{2E_c}{m_{fast\,ion}}}$
- τ_{cx} the charge exchange time

$$[Z] \equiv \sum_j n_j Z_j^2 (m_i/m_j) / \sum_j n_j Z_j$$

$$\langle Z \rangle \equiv \sum_j n_j Z_j^2 / \sum_j n_j Z_j$$

Rome J A 1976 Nucl. Fusion 16 55 (equation 4.17)

NBI modelling

The resulting analytic solution of the Fokker-Planck equation is:

$$f(v,\cancel{\xi},t) = U\left[t - \frac{\tau_s}{3} ln\left(\frac{{v_0}^3 + {v_c}^3}{v^3 + {v_c}^3}\right)\right] \frac{\tau_s}{2\pi(v^3 + {v_c}^3)} \left[\frac{{v_0}^3 + {v_c}^3}{v^3 + {v_c}^3}\right]^{-\frac{\tau_s}{3\tau_{cx}}} \times \cancel{\sum_{n=0}^{\infty} \left(n + \frac{1}{2}\right) P_n(\xi) P_n(\xi_0) \left[\frac{{v_0}^3 + {v_c}^3}{v^3 + {v_c}^3} \frac{v^3}{{v_0}^3}\right]^{\frac{m_i n(n+1)\langle Z\rangle}{6 m_{fast\ ion}[Z]}}} U(v_0 - v)$$

=1/2

Approximations:

1. Neglect fast ion velocity distribution anisotropy
   - n=0 → P0(x) = 1
   - Legendre polynomials are even functions → n=1 term is zero

   → No dependence from the pitch ξ

Rome J A 1976 Nucl. Fusion 16 55 (equation 4.17)

NBI modelling

The resulting analytic solution of the Fokker-Planck equation is:

$$f(v,\cancel{\xi},\cancel{t}) = \cancel{U\left[t-\frac{\tau_s}{3}ln\left(\frac{{v_0}^3+{v_c}^3}{v^3+{v_c}^3}\right)\right]}\frac{\tau_s}{2\pi(v^3+{v_c}^3)}\left[\frac{{v_0}^3+{v_c}^3}{v^3+{v_c}^3}\right]^{-\frac{\tau_s}{3\tau_{cx}}}\times\underbrace{\cancel{\sum_{n=0}^{\infty}\left(n+\frac{1}{2}\right)P_n(\xi)P_n(\xi_0)\left[\frac{{v_0}^3+{v_c}^3}{v^3+{v_c}^3}\frac{v^3}{{v_0}^3}\right]^{\frac{m_i n(n+1)\langle Z\rangle}{6m_{fast\ ion}[Z]}}}}_{=1/2}U(v_0-v)$$

Approximations:

1. Neglect fast ion velocity distribution anisotropy

2. Stationary solution ($t \gg \tau_s$)

   → No time dependence

Rome J A 1976 Nucl. Fusion 16 55 (equation 4.17)

NBI modelling

The resulting analytic solution of the Fokker-Planck equation is:

$$f(v,\cancel{\xi},\cancel{t}) = \cancel{U\left[t-\frac{\tau_s}{3}ln\left(\frac{{v_0}^3+{v_c}^3}{v^3+{v_c}^3}\right)\right]}\frac{\tau_s}{2\pi(v^3+{v_c}^3)}\cancel{\left[\frac{{v_0}^3+{v_c}^3}{v^3+{v_c}^3}\right]^{-\frac{\tau_s}{3\tau_{cx}}}}\times\cancel{\sum_{n=0}^{\infty}\left(n+\frac{1}{2}\right)P_n(\xi)P_n(\xi_0)\left[\frac{{v_0}^3+{v_c}^3}{v^3+{v_c}^3}\frac{v^3}{{v_0}^3}\right]^{\frac{m_i n(n+1)\langle Z\rangle}{6m_{fast\ ion}[Z]}}}U(v_0-v)$$

=1/2

Approximations:

1. Neglect fast ion velocity distribution anisotropy

2. Stationary solution ($t \gg \tau_s$)

3. Neglect background neutrals
   → $\tau_{cx} = \infty$

Rome J A 1976 Nucl. Fusion 16 55 (equation 4.17)

NBI modelling

The resulting analytic solution of the Fokker-Planck equation is:

$$f(v,\cancel{\xi},\cancel{t}) = \cancel{U\left[t - \frac{\tau_s}{3} ln\left(\frac{{v_0}^3 + {v_c}^3}{v^3 + {v_c}^3}\right)\right]} \frac{\tau_s}{2\pi(v^3 + {v_c}^3)} \cancel{\left[\frac{{v_0}^3 + {v_c}^3}{v^3 + {v_c}^3}\right]^{-\frac{\tau_s}{3\tau_{cx}}}} \times \underbrace{\cancel{\sum_{n=0}^{\infty}\left(n + \frac{1}{2}\right) P_n(\xi) P_n(\xi_0) \left[\frac{{v_0}^3 + {v_c}^3}{v^3 + {v_c}^3} \frac{v^3}{{v_0}^3}\right]^{\frac{m_i n(n+1)\langle Z\rangle}{6 m_{fast\ ion}[Z]}}}}_{=1/2} U(v_0 - v)$$

Approximations:

1. Neglect fast ion velocity distribution anisotropy
2. Stationary solution ($t \gg \tau_s$)
3. Neglect background neutrals
   → $\tau_{CX} = \infty$

Resulting:

$$f(v) = \frac{\tau_s}{2\pi(v^3 + {v_c}^3)} \times \frac{1}{2} U(v_0 - v) = \frac{\tau_s}{4\pi(v^3 + {v_c}^3)} U(v_0 - v)$$

- Commonly used in fast NBI modelling tools (e.g. METIS)

Rome J A 1976 Nucl. Fusion 16 55 (equation 4.17)

NBI modelling

The resulting analytic solution of the Fokker-Planck equation is:

$$f(v,\cancel{\xi},\cancel{t}) = \cancel{U\left[t-\frac{\tau_s}{3}ln\left(\frac{{v_0}^3+{v_c}^3}{v^3+{v_c}^3}\right)\right]}\frac{\tau_s}{2\pi(v^3+{v_c}^3)}\cancel{\left[\frac{{v_0}^3+{v_c}^3}{v^3+{v_c}^3}\right]^{-\frac{\tau_s}{3\tau_{cx}}}}\times\cancel{\sum_{n=0}^{\infty}\left(n+\frac{1}{2}\right)P_n(\xi)P_n(\xi_0)\left[\frac{{v_0}^3+{v_c}^3}{v^3+{v_c}^3}\frac{v^3}{{v_0}^3}\right]^{\frac{m_i n(n+1)\langle Z\rangle}{6m_{fast\ ion}[Z]}}}U(v_0-v)$$

=1/2

Approximations:

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Resulting:

$$f(v) = \frac{\tau_s}{2\pi(v^3+{v_c}^3)}\times\frac{1}{2}U(v_0-v) = \frac{\tau_s}{4\pi(v^3+{v_c}^3)}U(v_0-v)$$

- Beam particle density: $n_b f(v) = \frac{P}{eE_0}\frac{\tau_s}{4\pi(v^3+{v_c}^3)}U(v_0-v)$

Rome J A 1976 Nucl. Fusion 16 55 (equation 4.17)

NBI modelling

NBI codes can have different approaches (impacting on precision and time consumption):

a. (**Semi-**) **Analytical** solutions / simplified modelling / scaling laws:

b. **Monte Carlo** solvers (e.g. orbit following in phase space):

- Initialization of a random test particle following a given distribution
- Each Monte Carlo test particle represents N real particles, through weights
- Random test particles are followed until particle thermalization (e.g. E > 3/2 local T_i) or loss
- Detailed beamline/tokamak geometries
- Detailed physics (e.g. collisions, impurities, orbit drifts, finite Larmor radius effects, 3D magnetic fields, …)
- Long computational time
- Usually stand-alone codes, or used in detailed transport modelling

E.g.: BBNBI & ASCOT (stand-alone), NUBEAM (TRANSP suite), SPOT (CRONOS suite) etc…

BBNBI: Asunta O et al 2015 Comput. Phys.Commun. 188 33–46
ASCOT: E. Hirvijoki et al, Computer Physics Communications, pp. 1310-1321, 2014
NUBEAM: Pankin A. et al., 2004 Comput. Phys. Commun. 159 157–84
SPOT: M. Schneider et al., Nucl. Fusion 49 (2009) 125005

NBI modelling

Fast codes using **analytical solutions** of fast ion Fokker-Planck equation, well suitable for sensitivity studies.
Simplified approach

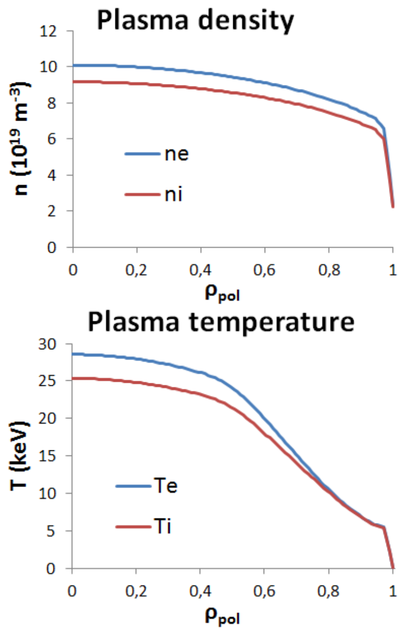
vs

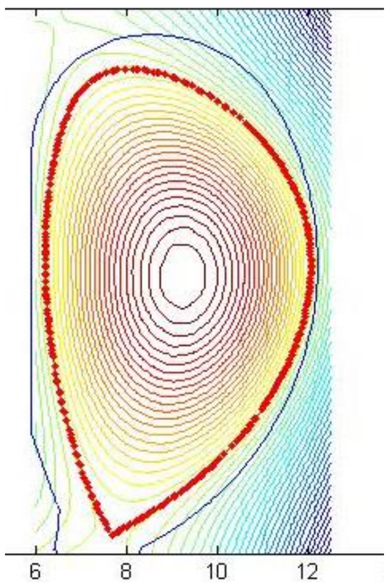
Orbit-following **Monte Carlo codes** solving kinetic equation for fast ions, which can take into account fast ions orbit effects.
Long simulation time, but detailed

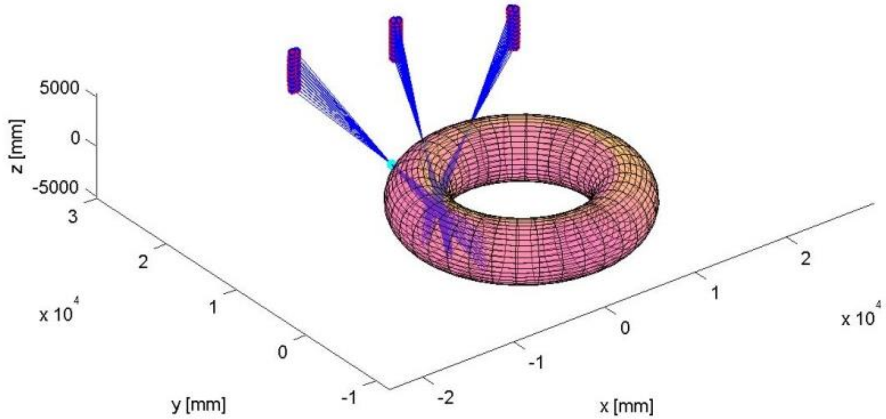
NBI modelling

Typical **inputs** for NBI codes:

- Plasma profiles and characteristics (isotope, impurities, neutrals…)
- Plasma magnetic equilibrium (and MHD)
- Machine geometry (wall, size…)
- Injection and beam geometry (injection line, beamlets, divergence…)
- Beam parameters (energy, power, isotope)





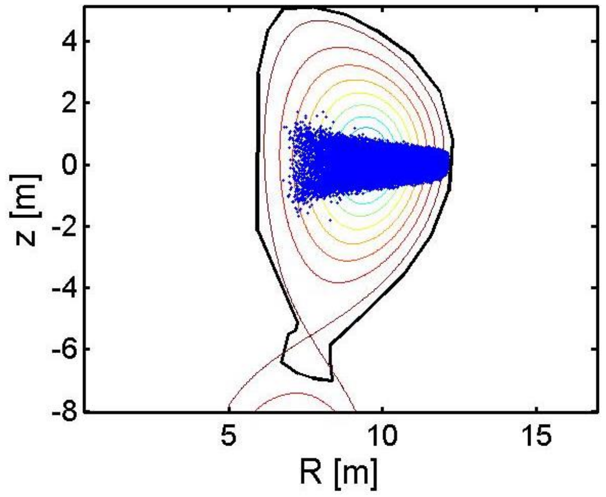


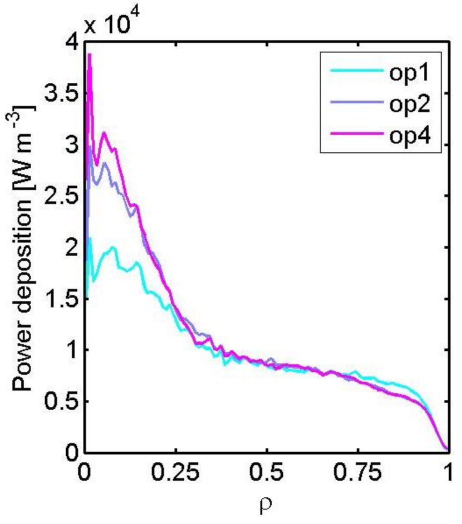
NBI modelling

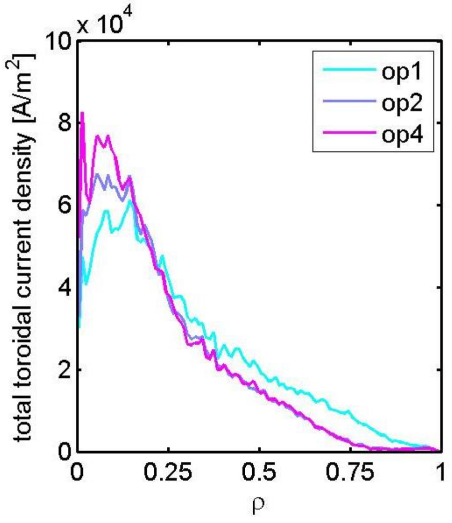
Typical **outputs** of NBI codes:

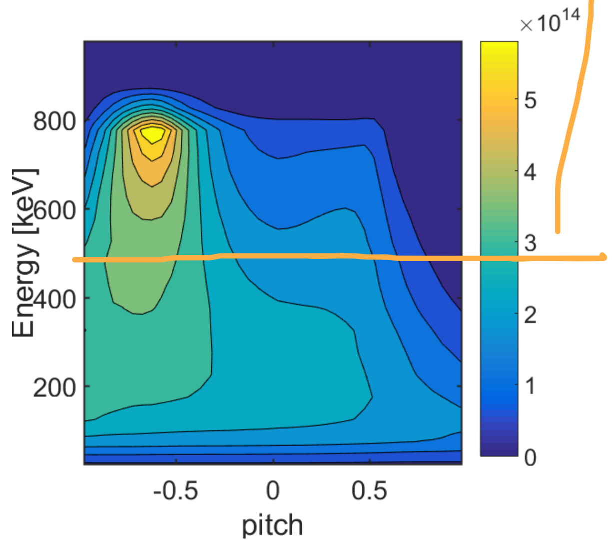
- Beam ionization and shine-through losses
- Power deposition
- Orbit losses
- Current-drive
- Input torque
- Fast ion distribution function
- …

Critical temp





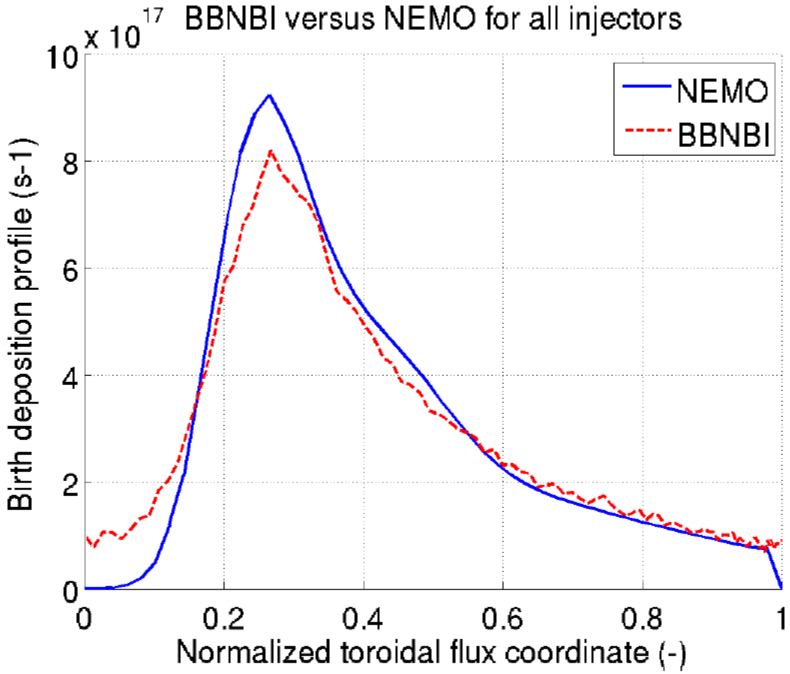


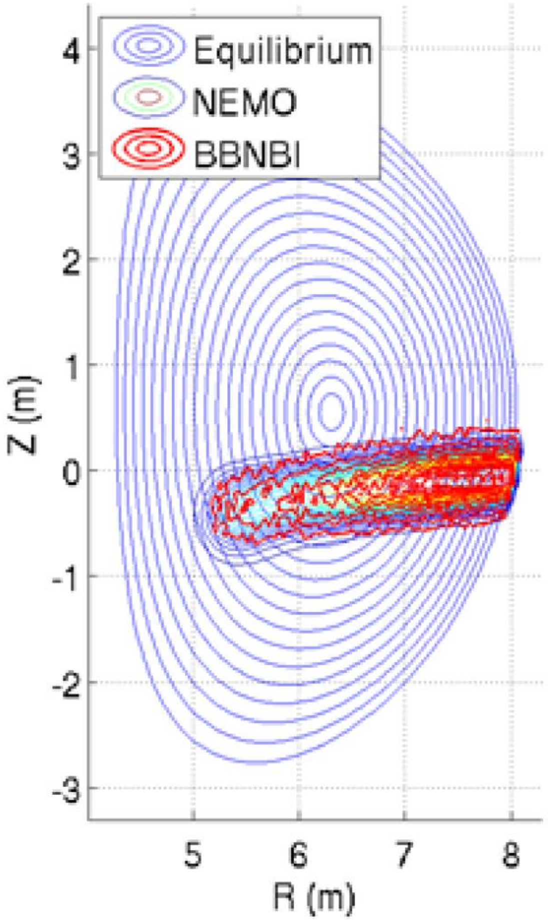


NBI code benchmark

Different solutions (i.e. codes) of the same problem must agree if the inputs are the same.
NBI models and numerical codes are benchmarked towards other codes to prove their reliability.
Here an example for ITER studies:

Beam deposition codes:
BBNBI (Monte Carlo) vs NEMO (narrow-beam model)





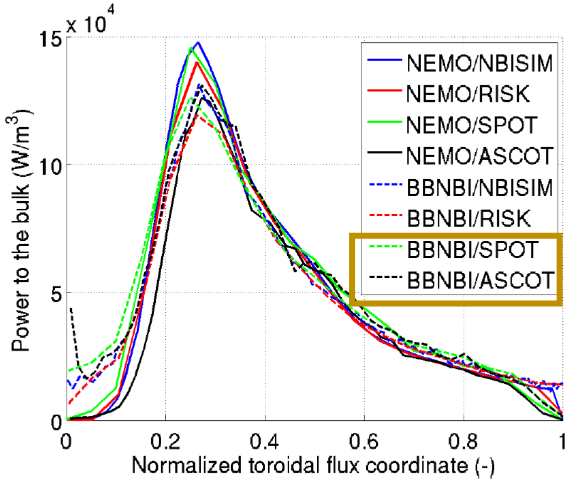
M. Schneider et al., 2015, 42nd EPS on Plasma Physics, P2.159
Shine-trhough benchmark among BBNBI, NEMO, METIS in P. Vincenzi et al., submitted to NF 2024

NBI code benchmark

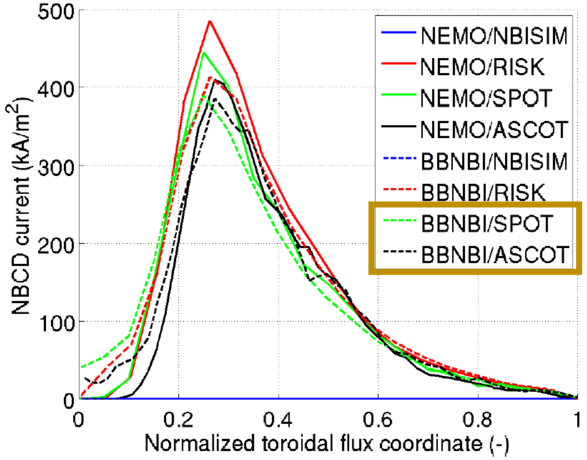
Different solutions (i.e. codes) of the same problem must agree if the inputs are the same.
NBI models and numerical codes are benchmarked towards other codes to prove their reliability.
Here an example for ITER studies:

Some Fokker-Planck solvers (they can be combined with deposition codes):

- NBISIM* is a simple 1D analytic model
- RISK* is a 2D Fokker-Planck code that combines finite elements and an eigenfunction expansion
- ASCOT and SPOT are Monte Carlo codes with a high level of accuracy including orbit width effects



Highest accuracy



*They use the so-called zero-banana-width limit

M. Schneider et al., 2015, 42nd EPS on Plasma Physics, P2.159

Contents

In the end

NBI works only because:

- …the cross sections for CX and ionization between an ion beam and a neutral gas are just adequate for efficient neutralization
- …the cross sections for ionization are just adequate for an efficient beam ionization in the plasma
- …the collision frequency in the plasma is just adequate for slowing down of fast ions and energy transfer

The lucky coincidence, as from C. Hopf, IPP Garching, in his former Ph.D. lesson

Bibliography

This presentation has been prepared with material from:

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…and all the material cited in the slides!