Quick intro to magnetic confinement… waiting for next lectures during the week

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Modeling magnetic fusion plasmas

Fusion plasma

- A fusion plasma is a fully ionized gas
- Behavior dominated by **long-range electric and magnetic fields**
- Very good conductor of electricity
  - $n_e(\text{plasma}) \approx 10^{-8} \times n_e(\text{Cu})$
  - $\sigma\ (\text{plasma}) \approx 40 \times \sigma\ (\text{Cu})$
  - Very little collisions at high temperature and low density
- Plasma shielded from DC electric fields
- DC magnetic fields can penetrate

Self-consistency in magnetized plasma

Sources (ρ, ***J***) ←→fields (***E***, ***B***)

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Models for plasma description

How magnetic fields confine charged particles ?

- Single-particle model

$$m\frac{d^2\vec{r}}{dt^2} = q(\vec{E} + \vec{v} \times \vec{B})$$

- Kinetic theory

$$f_\alpha(\vec{r},\vec{v},t) = \frac{dN_\alpha(\vec{r},\vec{v},t)}{d^3r\ d^3v}$$

- **Fluid model**

$$\begin{aligned}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{v}) &= 0 \\
\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v}\right) &= \nabla P - \underline{J} \times \underline{B} \\
\frac{\partial P}{\partial t} + \underline{v} \cdot \nabla P &= \gamma P \nabla \cdot \underline{v} \\
\frac{\partial B}{\partial t} &= \nabla \times (\underline{v} \times \underline{B})
\end{aligned}$$

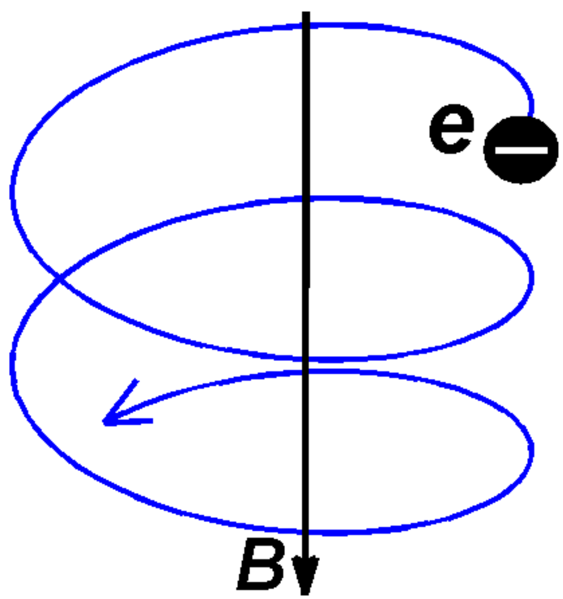
2

Single-particle motion

Single-particle motion

- Motion in **prescribed** magnetic and electric fields

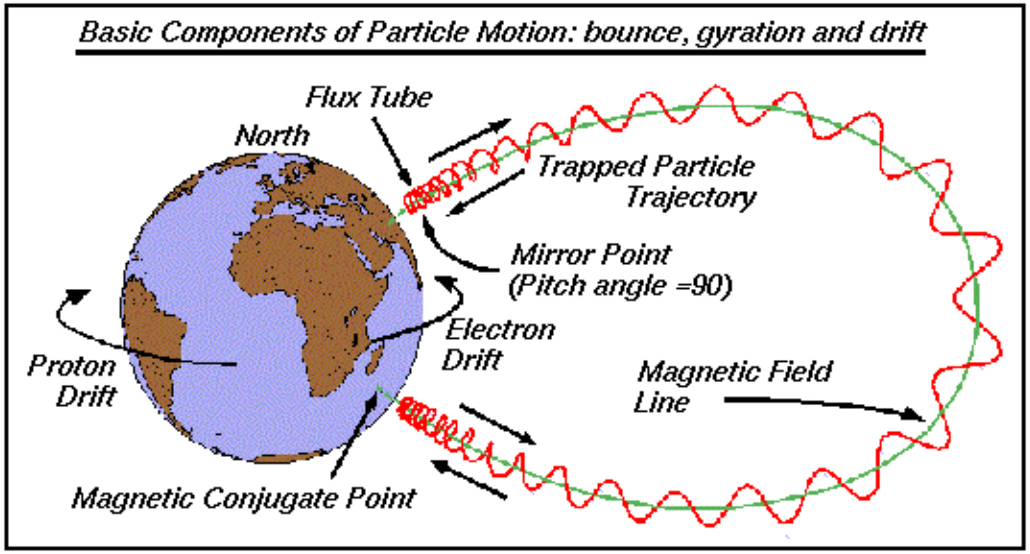
$$m\frac{d^2\vec{r}}{dt^2} = q(\vec{E} + \vec{v} \times \vec{B})$$



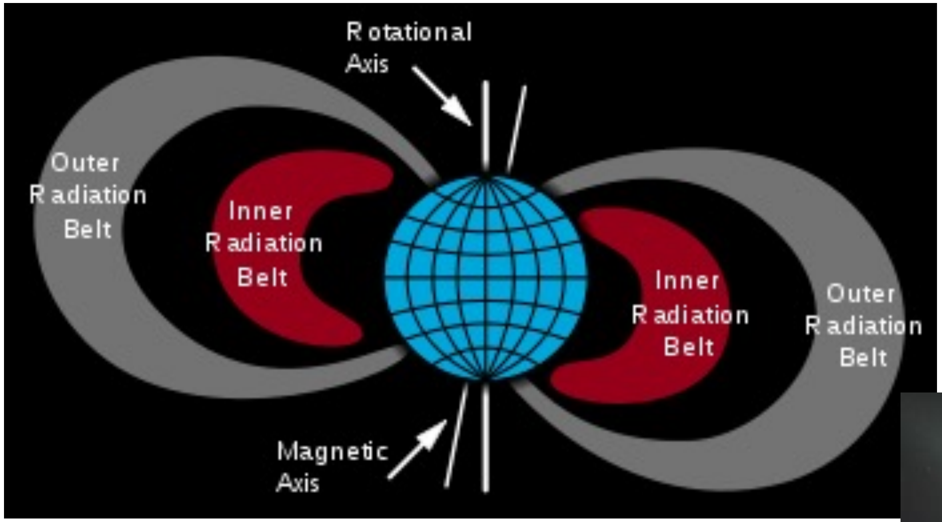
Single-particle motion

- Motion in **prescribed** magnetic and electric fields

$$m\frac{d^2\vec{r}}{dt^2} = q(\vec{E} + \vec{v} \times \vec{B})$$



Particle motion in Earth magnetic field



Single-particle model describes a variety of physics phenomena

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Need for self-consistency: the fluid model

Need for self-consistency: an example

- Equilibrium: plasma confined by background ***B***
- This magnetic field is partly produced by the plasma itself

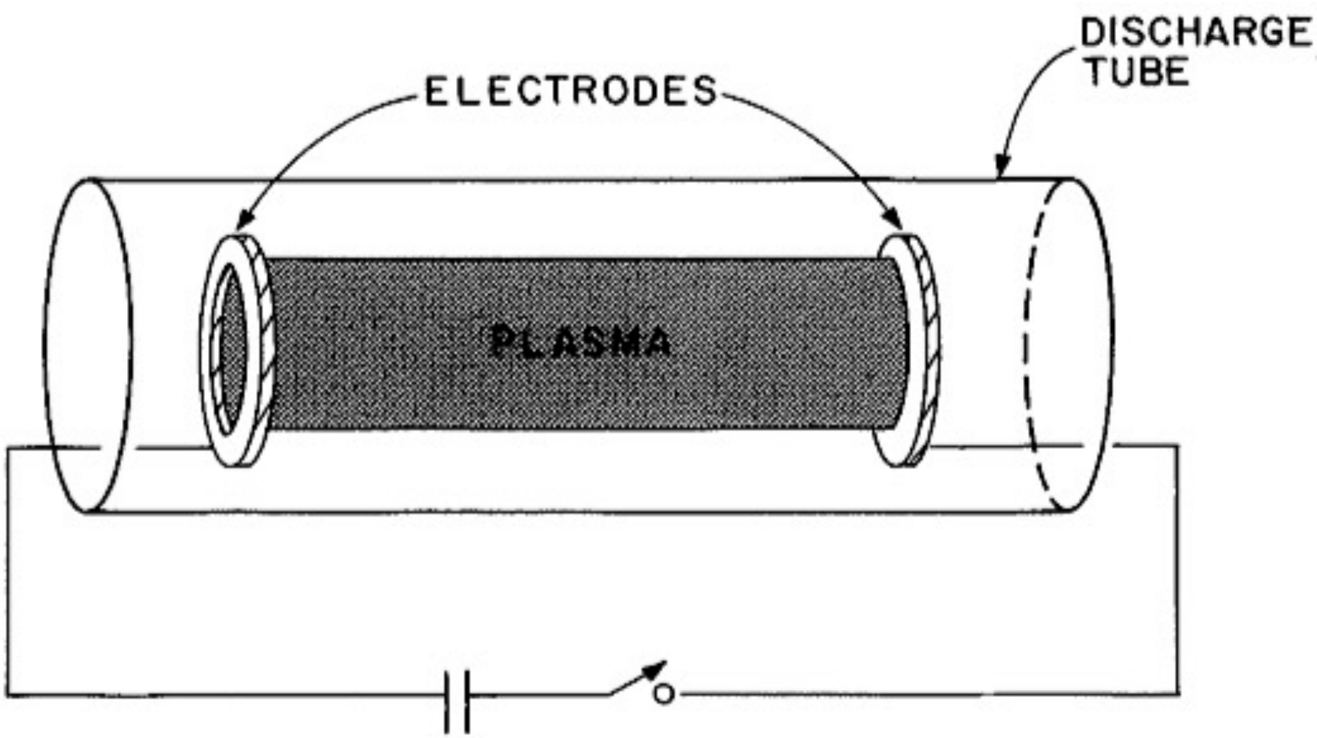
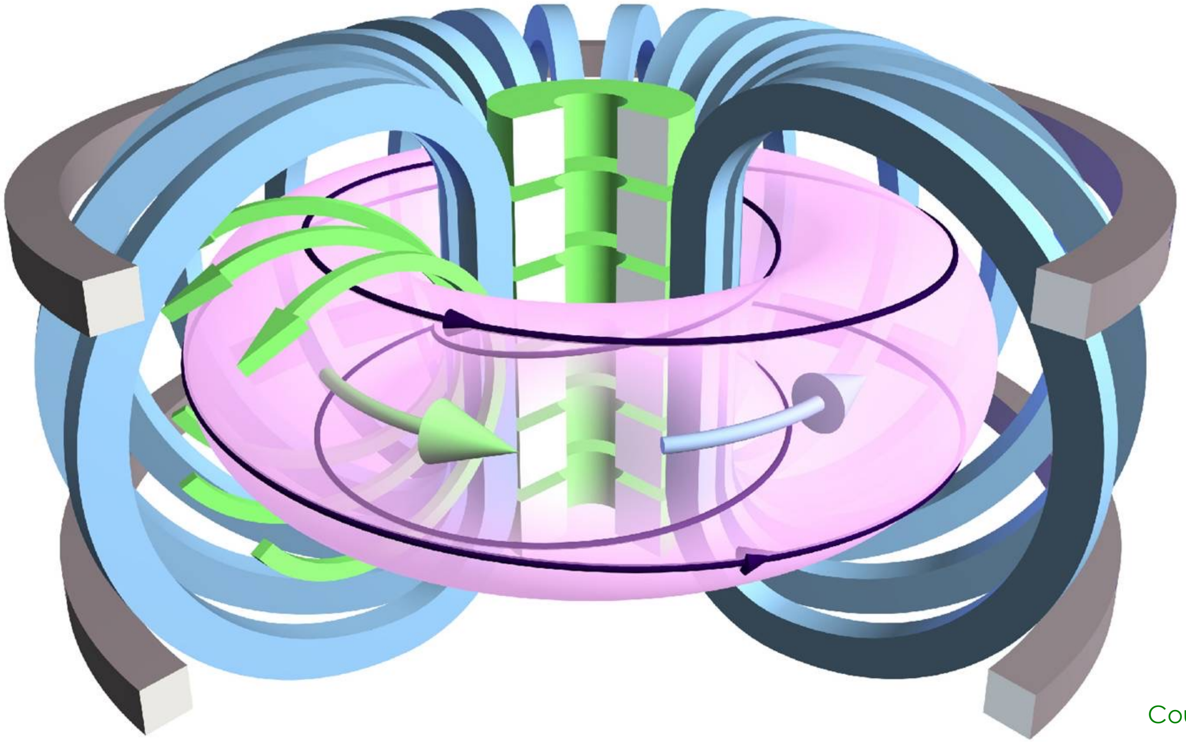


Figure 5.6. Schematic diagram of a linear *Z*-pinch experiment.

Magnetic Confinement Fusion

The goal: energy production

The tool: a toroidal magnetic container, with helical magnetic field



Courtesy of S. Pinches

$$\nabla p = \vec{J} \times \vec{B}$$ force balance between magnetic and pressure forces

Fluid equations

- **Neutral gases and liquids**: fluid equations derived treating the fluid as a continuous medium and considering the dynamics of a small volume of the plasma.

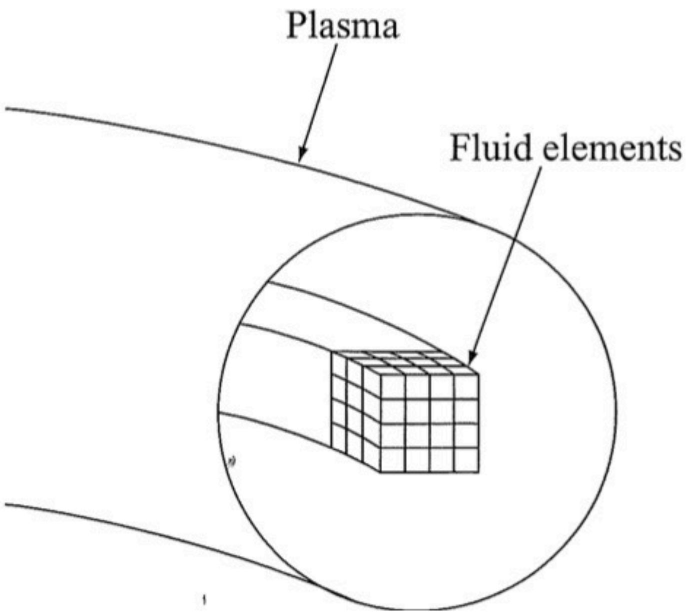
$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \rho \vec{V} = 0$$

$$\rho \frac{d\vec{V}}{dt} = \rho \vec{g} - \vec{\nabla} p + \mu \nabla^2 \vec{V} + \frac{2}{3} \mu \vec{\nabla} (\vec{\nabla} \cdot \vec{V})$$

Fluid equations in plasmas

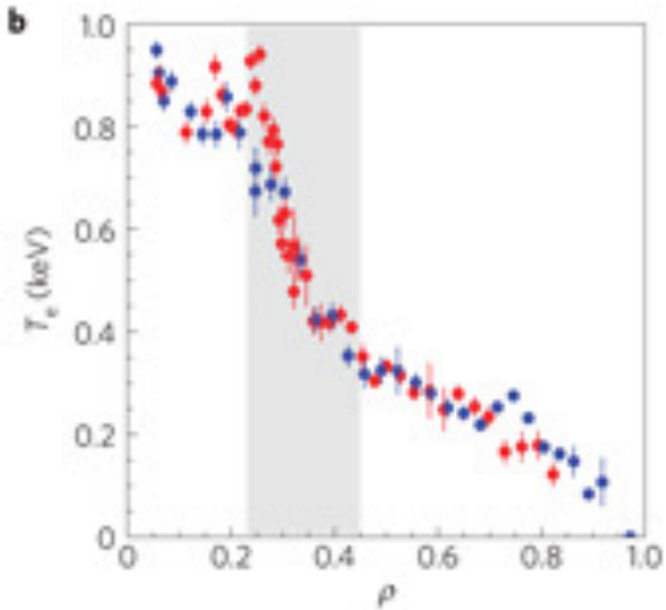
- Subdivide the plasma in a **large number of small and moving fluid elements**

- The behavior of each fluid element is described by **average macroscopic properties** of the particles contained tł



Macroscopic averages

- **Fluid description** → developing a model describing the evolution of important macroscopic plasma properties
  - $n_e(\mathbf{r},t)$, $n_i(\mathbf{r},t)$
  - $\boldsymbol{u}_e(\mathbf{r},t)$, $\boldsymbol{u}_i(\mathbf{r},t)$
  - $T_e(\mathbf{r},t)$, $T_i(\mathbf{r},t)$
  - $p_e(\mathbf{r},t)$, $p_i(\mathbf{r},t)$



- E.g.: macroscopic velocity $\boldsymbol{u}_e$
  - Average velocity of all the electrons contained in the fluid element

Size of a fluid element

- It has to be possible to define a range of sizes for each element that satisfies two conflicting requirements:
  - **The element can not be to small, otherwise too few particles inside and averaging makes little sense**
  - **Not too big, otherwise spatial accuracy lost**

- **Fusion plasma**
  - $N_e=10^{20}$ m^{-3}
  - $L=1$ m
  - $\Delta x=10^{-5}$ good resolution
  - $\Delta V=10^{-15}$ m^{-3}
  - $N_e=10^5>>1$

Is a fluid model useful for a plasma ?

- Example: **air at atmospheric pressure**
  - Molecules within each fluid element are collision dominated
  - Collisions keep molecules closely confined together
  - A molecule can not move over long distances with respect to its neighbors. It is confined in a region of the size of its mean free path
  - Molecules in each element form a well-defined cluster of particles, whose identity is maintained as the system evolves in time.

- **Coherence due to high collisionality** ➔ fluid model useful for air

Each fluid element correspond to a super-particle with mass $mn\Delta V$ and velocity u

Is a fluid model useful for a plasma ?

Fusion plasma are nearly collisionless….

- …but magnetic field acts to keep them together in the perpendicular direction.
  - The small size of the gyro-radius keep particles close to one another

The magnetic field replaces collisions in providing perpendicular coherence to the particles in a fluid element

- …but particles move freely along ***B***…
  - Need kinetic treatment
  - But fluid model often incorrect when unimportant..
  - ..not a huge problem

Two-fluid model

Conservation of mass

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{u}_e) = 0,$$
$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{u}_i) = 0. \tag{10.50}$$

Conservation of momentum

$$m_e n_e \left(\frac{\partial}{\partial t} + \mathbf{u}_e \cdot \nabla\right) \mathbf{u}_e = -e n_e (\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) - \nabla p_e - m_e n_e \overline{\nu}_{ei} (\mathbf{u}_e - \mathbf{u}_i),$$
$$m_i n_i \left(\frac{\partial}{\partial t} + \mathbf{u}_i \cdot \nabla\right) \mathbf{u}_i = e n_i (\mathbf{E} + \mathbf{u}_i \times \mathbf{B}) - \nabla p_i - m_e n_e \overline{\nu}_{ei} (\mathbf{u}_i - \mathbf{u}_e). \tag{10.51}$$

Conservation of energy

$$\frac{3}{2} n_e \left(\frac{\partial}{\partial t} + \mathbf{u}_e \cdot \nabla\right) T_e + p_e \nabla \cdot \mathbf{u}_e + \nabla \cdot \mathbf{q}_e = S_e,$$
$$\frac{3}{2} n_i \left(\frac{\partial}{\partial t} + \mathbf{u}_i \cdot \nabla\right) T_i + p_i \nabla \cdot \mathbf{u}_i + \nabla \cdot \mathbf{q}_i = S_i, \tag{10.52}$$

with

$$S_e = \frac{F_e^{(\alpha)}}{4} E_\alpha n_e^2 \langle \sigma v \rangle + F_e^{(a)} S_a + \eta J^2 - C_B n_e^2 T_e^{1/2} - \frac{3}{2} n_e \overline{\nu}_{eq} (T_e - T_i),$$
$$S_i = \frac{1 - F_e^{(\alpha)}}{4} E_\alpha n_e^2 \langle \sigma v \rangle + \left(1 - F_e^{(a)}\right) S_a - \frac{3}{2} n_e \overline{\nu}_{eq} (T_i - T_e). \tag{10.53}$$

Two-fluid model

- **Collisional friction force**: result of momentum exchange collisions

Conservation of momentum

$$\begin{aligned} m_e n_e \left(\frac{\partial}{\partial t} + \mathbf{u}_e \cdot \nabla \right) \mathbf{u}_e &= -e n_e (\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) - \nabla p_e \boxed{- m_e n_e \overline{\nu}_{ei} (\mathbf{u}_e - \mathbf{u}_i)}, \\ m_i n_i \left(\frac{\partial}{\partial t} + \mathbf{u}_i \cdot \nabla \right) \mathbf{u}_i &= e n_i (\mathbf{E} + \mathbf{u}_i \times \mathbf{B}) - \nabla p_i \boxed{- m_e n_e \overline{\nu}_{ei} (\mathbf{u}_i - \mathbf{u}_e)}. \end{aligned} \tag{10.51}$$

Two-fluid model

- Rate of change of internal energy
- Compression work
- Thermal conduction

Conservation of energy

$$
\begin{aligned}
\frac{3}{2} n_{\mathrm{e}} \left(\frac{\partial}{\partial t} + \mathbf{u}_{\mathrm{e}} \cdot \nabla \right) T_{\mathrm{e}} + p_{\mathrm{e}} \nabla \cdot \mathbf{u}_{\mathrm{e}} + \nabla \cdot \mathbf{q}_{\mathrm{e}} &= S_{\mathrm{e}}, \\
\frac{3}{2} n_{\mathrm{i}} \left(\frac{\partial}{\partial t} + \mathbf{u}_{\mathrm{i}} \cdot \nabla \right) T_{\mathrm{i}} + p_{\mathrm{i}} \nabla \cdot \mathbf{u}_{\mathrm{i}} + \nabla \cdot \mathbf{q}_{\mathrm{i}} &= S_{\mathrm{i}},
\end{aligned}
\tag{10.52}
$$

with

$$
\begin{aligned}
S_{\mathrm{e}} &= \frac{F_{\mathrm{e}}^{(\alpha)}}{4} E_\alpha n_{\mathrm{e}}^2 \langle \sigma v \rangle + F_{\mathrm{e}}^{(a)} S_a + \eta J^2 - C_{\mathrm{B}} n_{\mathrm{e}}^2 T_{\mathrm{e}}^{1/2} - \frac{3}{2} n_{\mathrm{e}} \overline{\nu}_{\mathrm{eq}} (T_{\mathrm{e}} - T_{\mathrm{i}}), \\
S_{\mathrm{i}} &= \frac{1 - F_{\mathrm{e}}^{(\alpha)}}{4} E_\alpha n_{\mathrm{e}}^2 \langle \sigma v \rangle + \left(1 - F_{\mathrm{e}}^{(a)} \right) S_a - \frac{3}{2} n_{\mathrm{e}} \overline{\nu}_{\mathrm{eq}} (T_{\mathrm{i}} - T_{\mathrm{e}}).
\end{aligned}
\tag{10.53}
$$

From two-fluid to single fluid assumptions of ideal MHD

- Length scales ≫ Larmor radius

$$a >> r_{Li} >> r_{Le} \approx \lambda_{de}$$

- Frequencies << gyrofrequency
- Fluid velocity << thermal velocity
- No electron inertia
- Quasi-neutrality
- No Hall term in Ohm's law

The single fluid model

$$
\begin{aligned}
&\text{mass}: && \frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho\nabla\cdot\mathbf{v} = 0;\\
&\text{momentum}: && \rho\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \mathbf{J}\times\mathbf{B} - \nabla p;\\
&\text{Ohm's law}: && \mathbf{E} + \mathbf{v}\times\mathbf{B} = 0 \quad \text{ideal MHD},\\
& && \mathbf{E} + \mathbf{v}\times\mathbf{B} = \eta_{\parallel}\mathbf{J} \quad \text{resistive MHD};\\
&\text{energy}: && \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{p}{\rho^{\gamma}}\right) = 0;\\
&\text{Maxwell}: && \nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t},\\
& && \nabla\times\mathbf{B} = \mu_0\mathbf{J},\\
& && \nabla\cdot\mathbf{B} = 0.
\end{aligned}
$$

Plasma can be described as a single magnetized fluid

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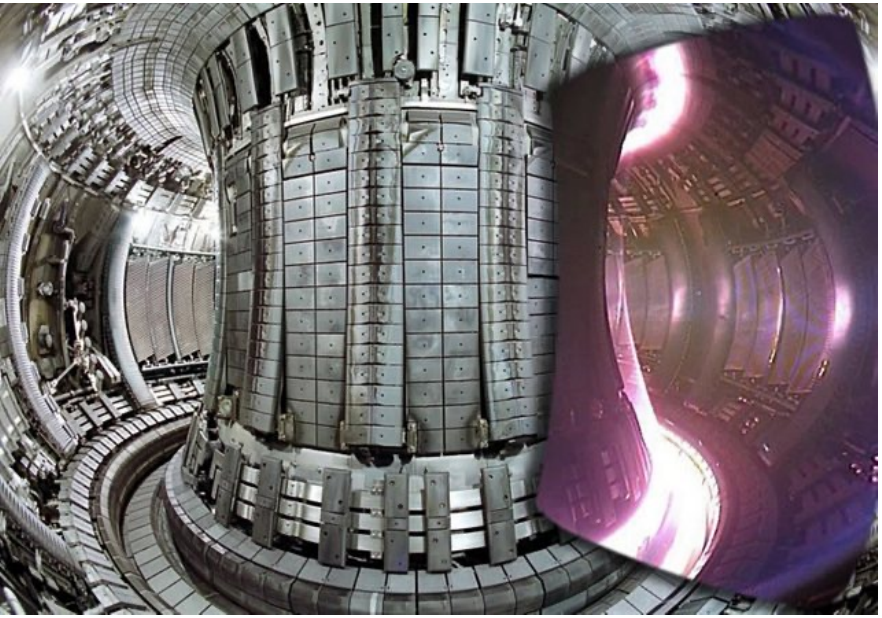
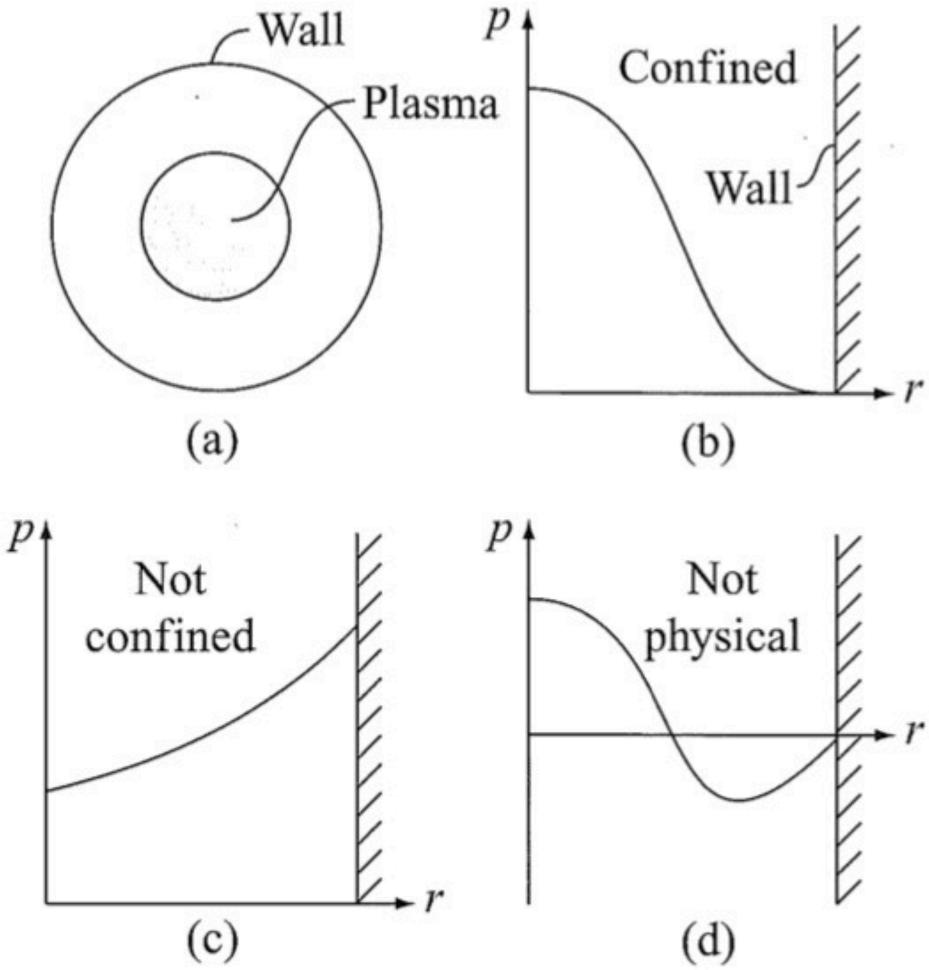
MHD equilibrium

The problem of MHD equilibrium

JET (www.efda.org)





The MHD equilibrium

$$\begin{aligned}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{v}) &= 0 && (1)\\
\rho \left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v} \right) &= \nabla P - \underline{J} \times \underline{B} && (2)\\
\frac{\partial P}{\partial t} + \underline{v} \cdot \nabla P &= \gamma P \nabla \cdot \underline{v} && (3)\\
\frac{\partial \underline{B}}{\partial t} &= \nabla \times (\underline{v} \times \underline{B}) && (4)
\end{aligned}$$

- Plasma equilibrium (v=0 if flow << sound speed):

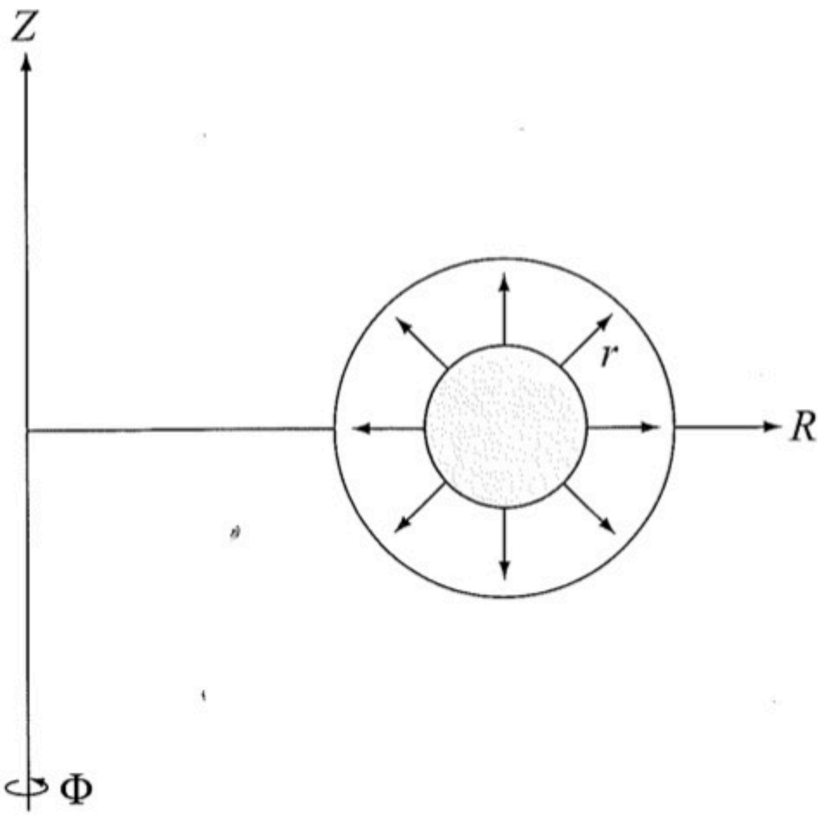
$$\frac{\partial}{\partial t} = 0 \qquad \vec{v} = 0$$

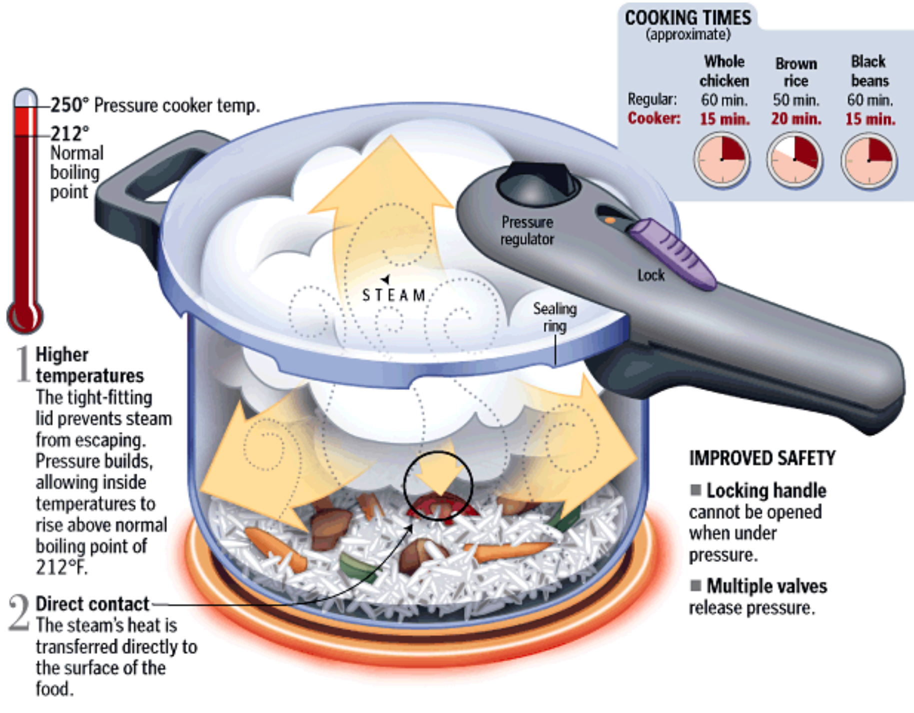
The MHD equilibrium

- MHD equilibrium in toroidal geometry has two parts
  - **RADIAL PRESSURE BALANCE**
  - **TOROIDAL FORCE BALANCE**

Radial pressure balance

- The plasma is a hot core of gas that tends to expand uniformly along the minor radius *r*



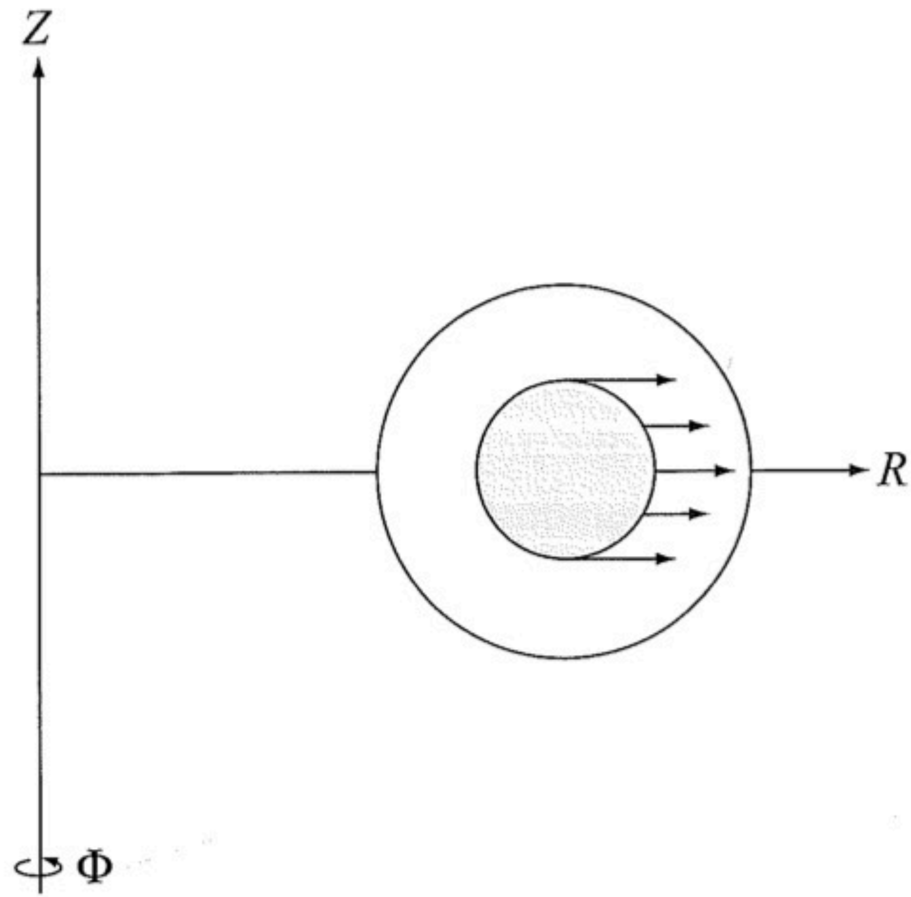


Toroidal force balance

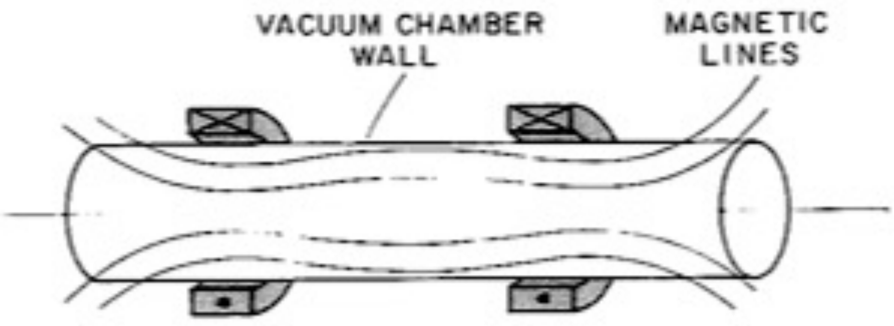
- Because of the toroidal geometry, unavoidable forces are generated by both the toroidal and poloidal B

- They tend to push the plasma outward.

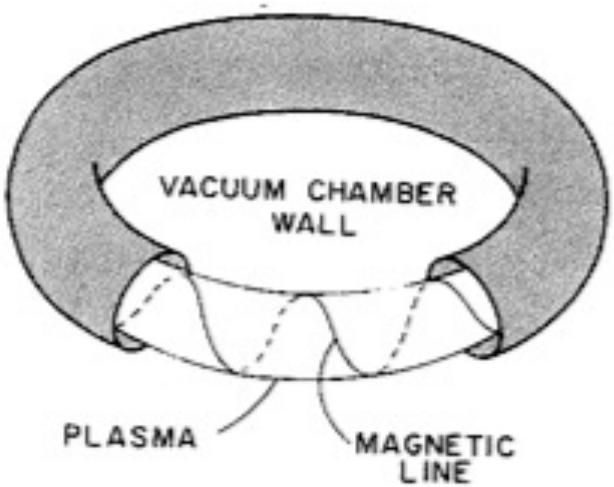
- Need to be balanced



Linear vs. toroidal configurations

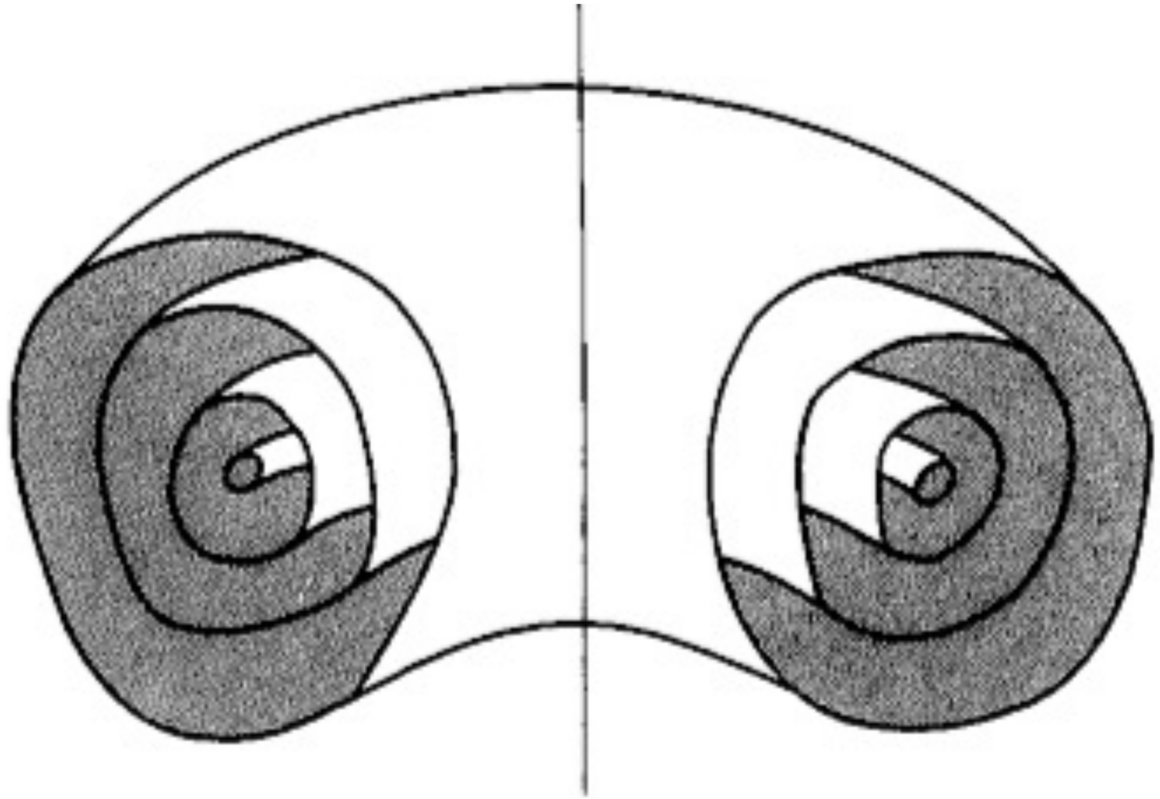


(a)



Magnetic flux surfaces

$$\mathbf{B} \cdot \nabla p = 0$$



Magnetic field perpendicular to pressure gradient

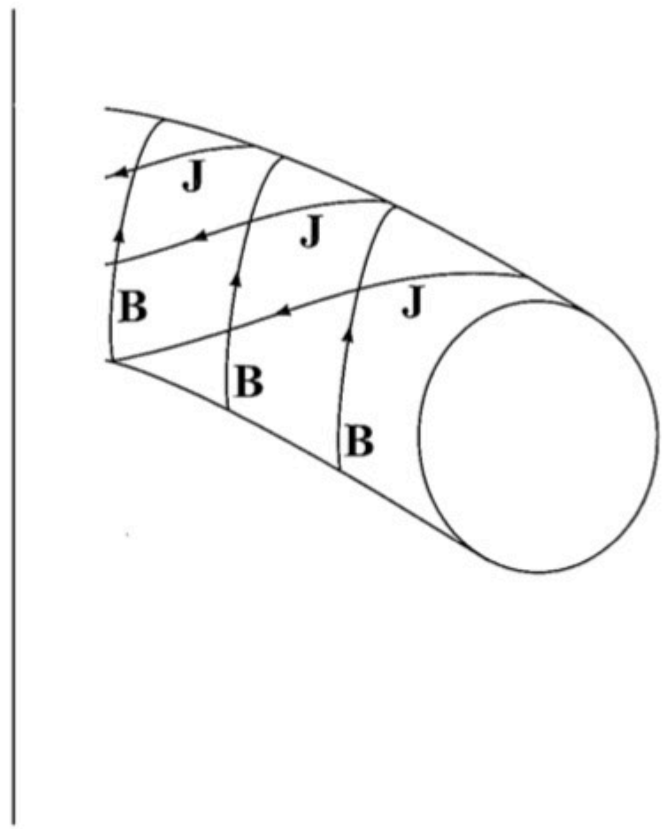
Pressure is constant on magnetic flux surfaces

- Important for experimental measurements

Pressure is constant on magnetic flux surfaces

Current, magnetic and pressure surfaces

The angle between ***J*** and ***B*** is in general arbitrary



$$\vec{J} \bullet \nabla p = 0$$

Current density perpendicular to pressure gradient

MHD equilibrium equation

$$\boxed{\vec{J} \times \vec{B} = \nabla p}$$

MHD equilibrium equation

$$\boxed{\vec{J} \times \vec{B} = \nabla p}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\boxed{\frac{1}{\mu_0}(\nabla \times \vec{B}) \times \vec{B} = \nabla p}$$

MHD equilibrium equation

$$\boxed{\vec{J} \times \vec{B} = \nabla p}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\boxed{\frac{1}{\mu_0}(\nabla \times \vec{B}) \times \vec{B} = \nabla p}$$

$$\vec{B} \times (\nabla \times \vec{B}) = \frac{1}{2}\nabla B^2 - (\vec{B} \cdot \nabla)\vec{B}$$

MHD equilibrium equation

$$\boxed{\vec{J} \times \vec{B} = \nabla p}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\boxed{\frac{1}{\mu_0}(\nabla \times \vec{B}) \times \vec{B} = \nabla p}$$

$$\vec{B} \times (\nabla \times \vec{B}) = \frac{1}{2}\nabla B^2 - (\vec{B} \cdot \nabla)\vec{B}$$

$$\boxed{\nabla\left(p + \frac{B^2}{2\mu_0}\right) - (\vec{B} \cdot \nabla)\vec{B} = 0}$$

Radial pressure balance

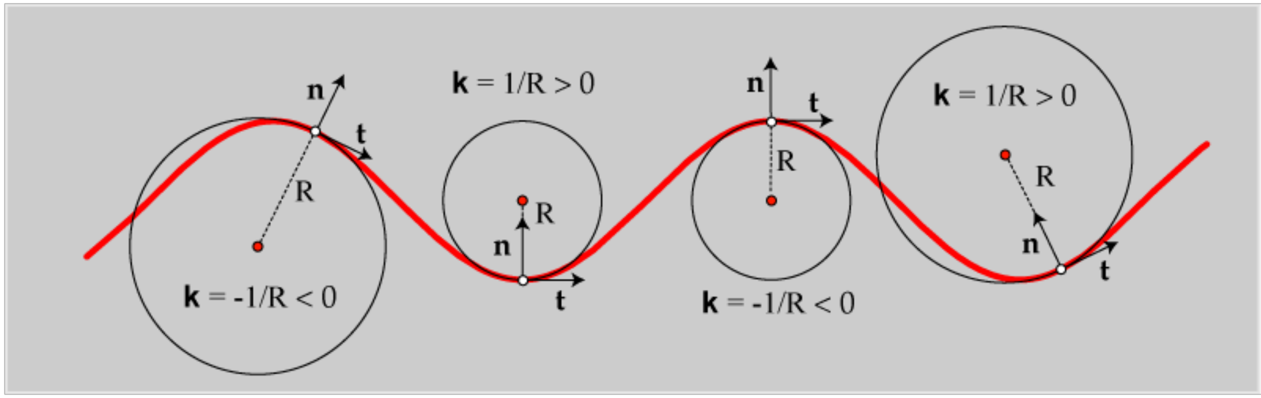
$$\nabla_\perp \left(p + \frac{B^2}{2\mu_0} \right) - \frac{B^2}{\mu_0} (\hat{b} \cdot \nabla) \hat{b} = 0$$

$$\hat{b} = \frac{\vec{B}}{B}$$

$$\nabla_\perp = \nabla - \hat{b}(\hat{b} \cdot \nabla)$$

Curvature

$$\vec{\kappa} = \hat{b}(\hat{b} \cdot \nabla) = -\frac{\vec{R}_C}{R_C^2}$$



Radial pressure balance

Magnetic field provide two radial force terms:

$$\nabla_\perp\left(p+\boxed{\frac{B^2}{2\mu_0}}\right)\boxed{-\frac{B^2}{\mu_0}(\hat{b}\cdot\nabla)\hat{b}=0}$$

PRESSURE **TENSION**

Magnetic field exerts pressure and tension

Magnetic pressure: Θ-pinch

- Configuration with pure toroidal field

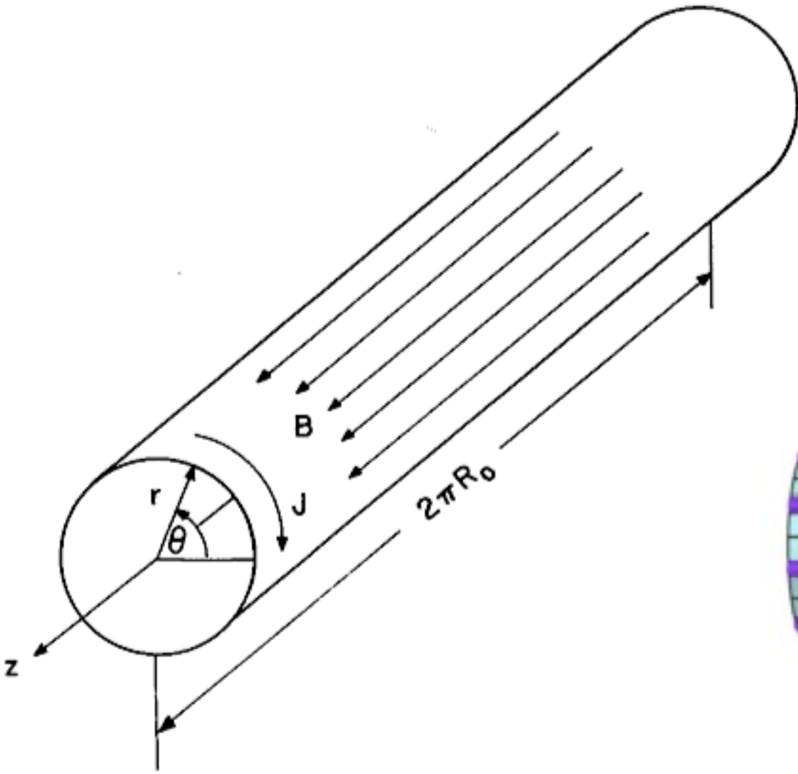
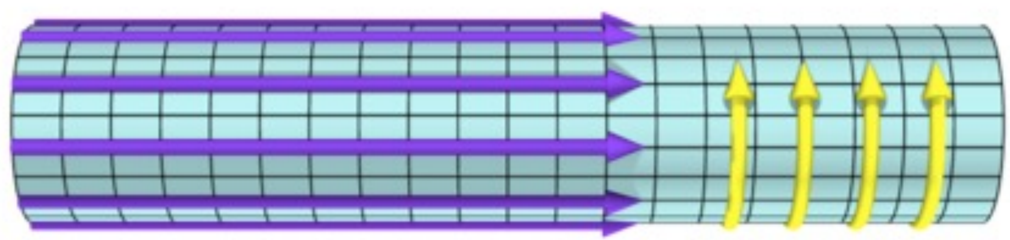


Figure 5.1. Linear θ-pinch geometry.

$$\mathbf{J} \times \mathbf{B} = \nabla p$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$\nabla \cdot \mathbf{B} = 0$$



A simple example: Θ-pinch

- **MAGNETIC** + **KINETIC** pressure = **CONSTANT** in the plasma
- Plasma confined **by the pressure of the applied magnetic field**

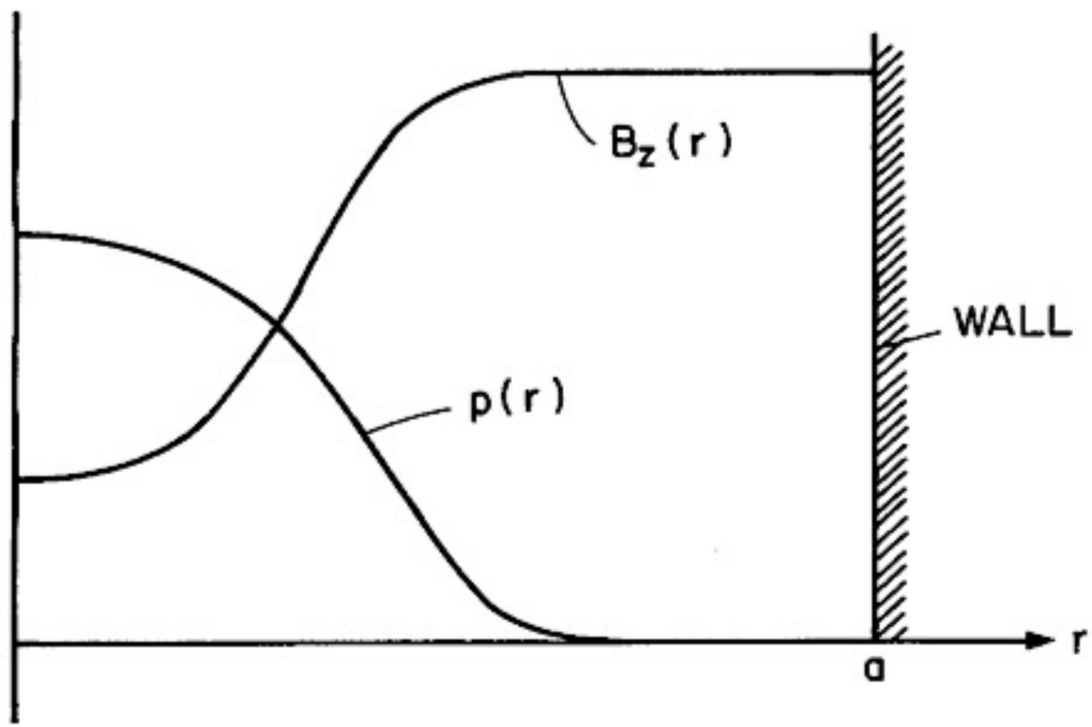
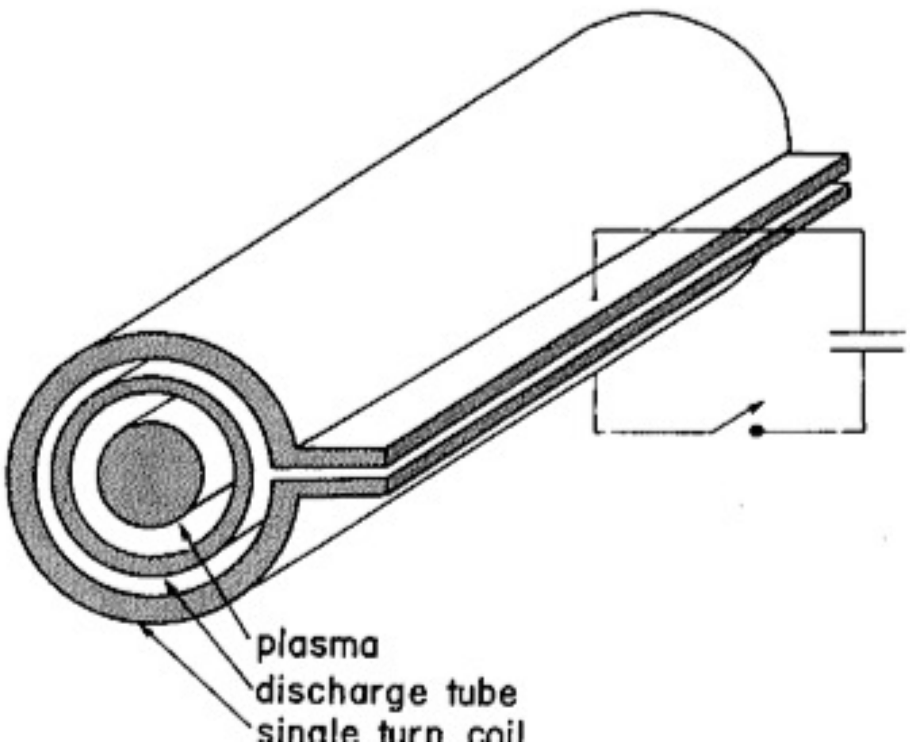
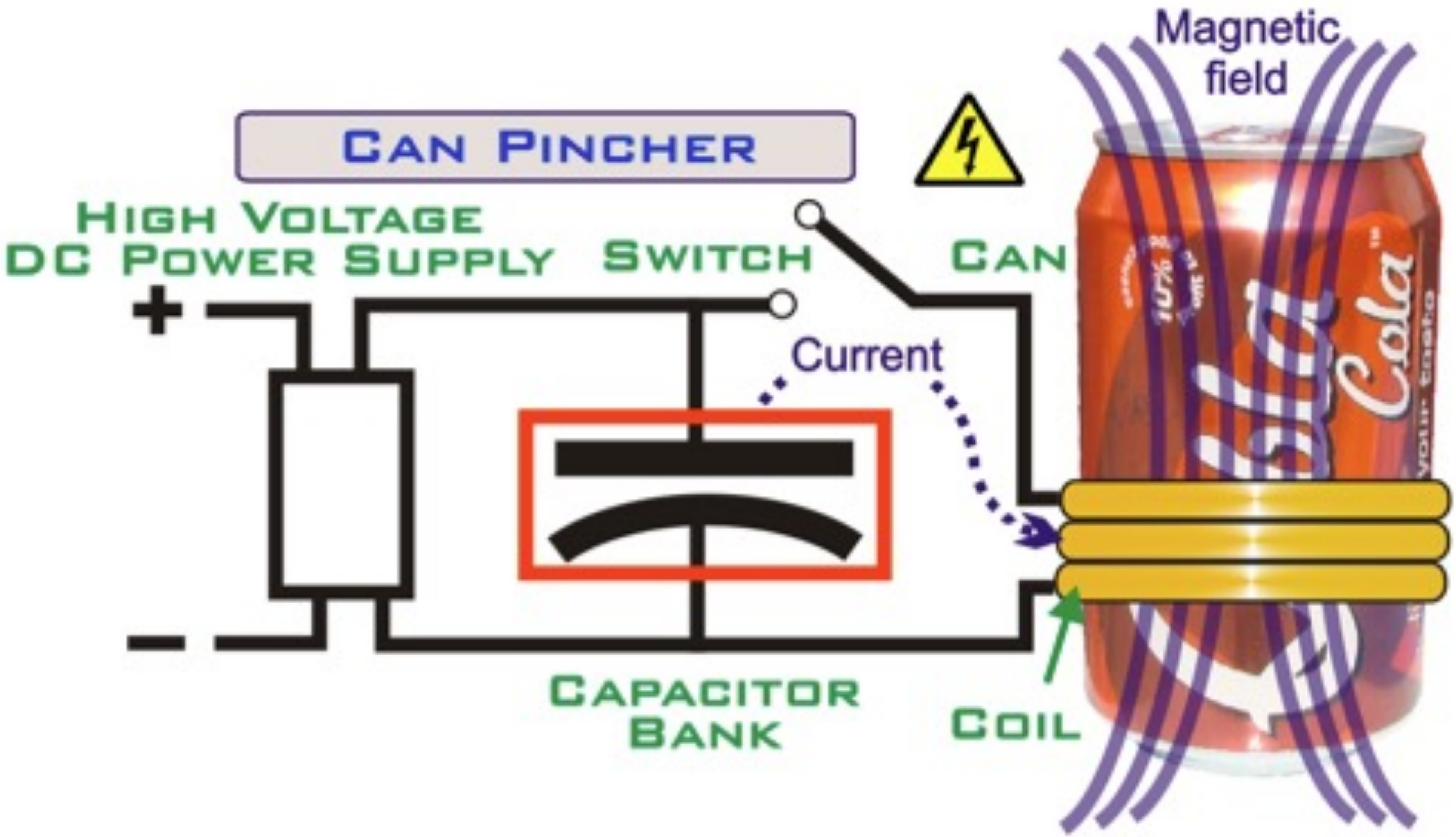


Figure 5.2. Equilibrium profiles for a θ pinch.

Experimental Θ-pinch

- Θ-pinch devices **among the first experiments to be realized**
- **End-losses severe problem**
- A Θ-pinch can not be bent into a toroidal equilbrium




Can Pincher
High Voltage
DC Power Supply
Switch
Can
Magnetic
field
Current
Capacitor
Bank
Coil
+
-

Z-pinch

- **Purely poloidal field**
- All quantities are only functions of *r*

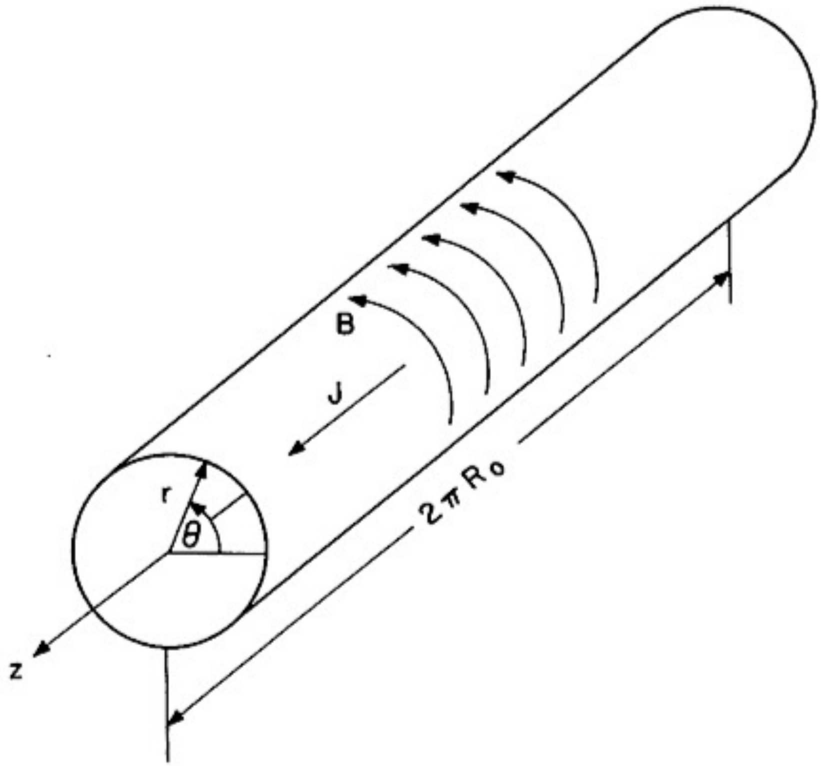


Figure 5.4. Linear *Z*-pinch geometry.

Z-pinch

- In contrast to the Θ-pinch, for a Z-pinch it is the tension force and not the magnetic pressure gradient that provides radial confinement of the plasma

$$\frac{d}{dr}\left(p + \frac{B_\theta^2}{2\mu_0}\right) + \frac{B_\theta^2}{\mu_0 r} = 0 \qquad (5.15)$$

- The Bennet pinch satisfies the Z-pinch equilibrium

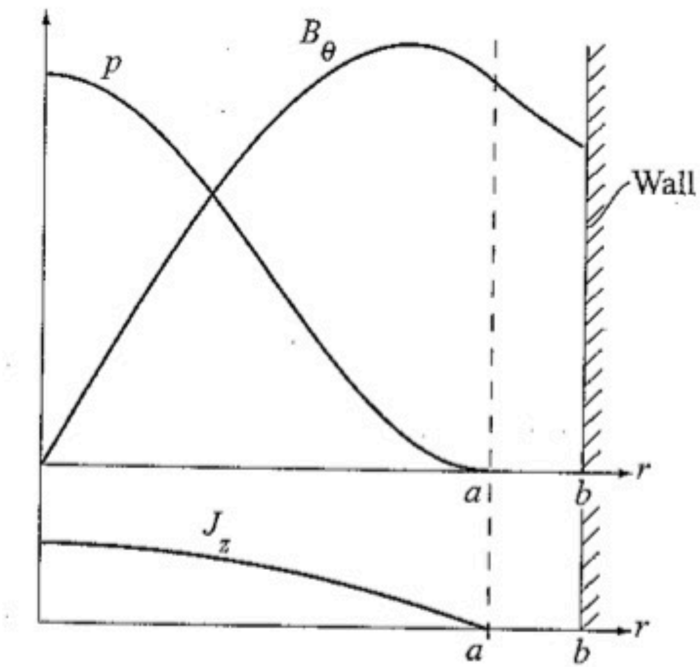
Willard Harrison Bennett (far right) with colleagues at the U.S. Naval Research Laboratory, working on the Störmertron tube

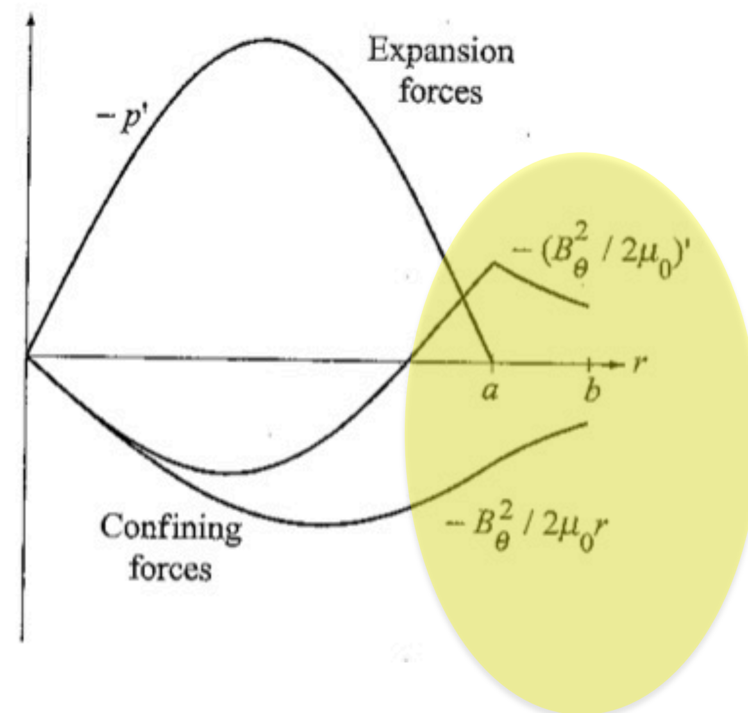
$$B_\theta = \frac{\mu_0 I_0}{2\pi} \frac{r}{r^2 + r_0^2}$$

$$J_z = \frac{I_0}{\pi} \frac{r_0^2}{(r^2 + r_0^2)^2}$$

$$p = \frac{\mu_0 I_0^2}{8\pi^2} \frac{r_0^2}{(r^2 + r_0^2)^2}$$

Z-pinch





Tension force acts inwards at the edge providing radial pressure balance.

Experimental Z-pinch

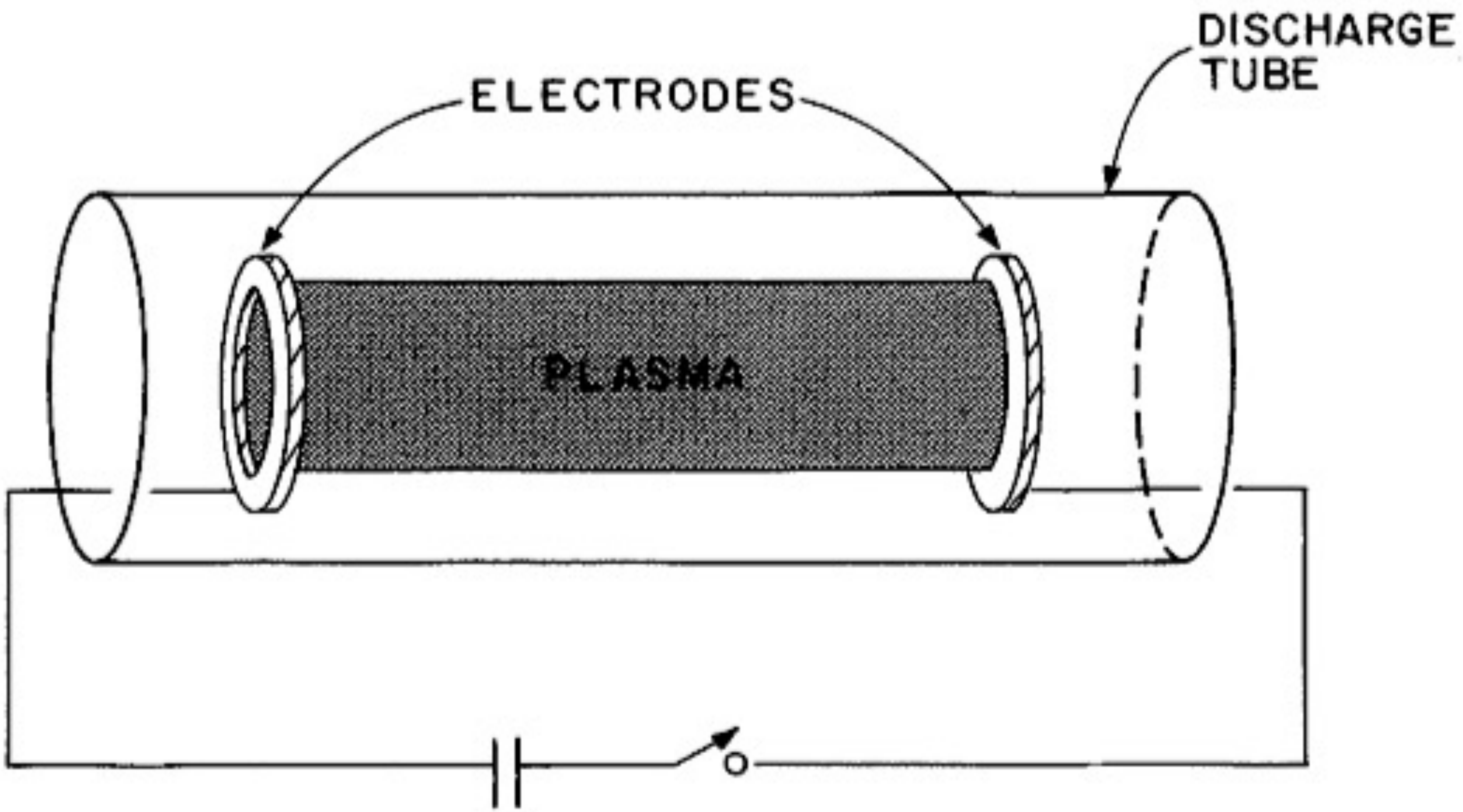


Figure 5.6. Schematic diagram of a linear Z-pinch experiment.

Z- and Theta-pinches are at the basis of many toroidal confinement concepts

General screw pinch

Though the momentum equation is non-linear, the Θ-pinch and Z-pinch forces add as a linear superposition

$$\frac{d}{dr}\left(p+\frac{B_p^2}{2\mu_0}+\frac{B_t^2}{2\mu_0}\right)+\frac{B_p^2}{\mu_0 r}=0$$

One is free to specify two functions, e.g. B_p(r) and B_t(r)

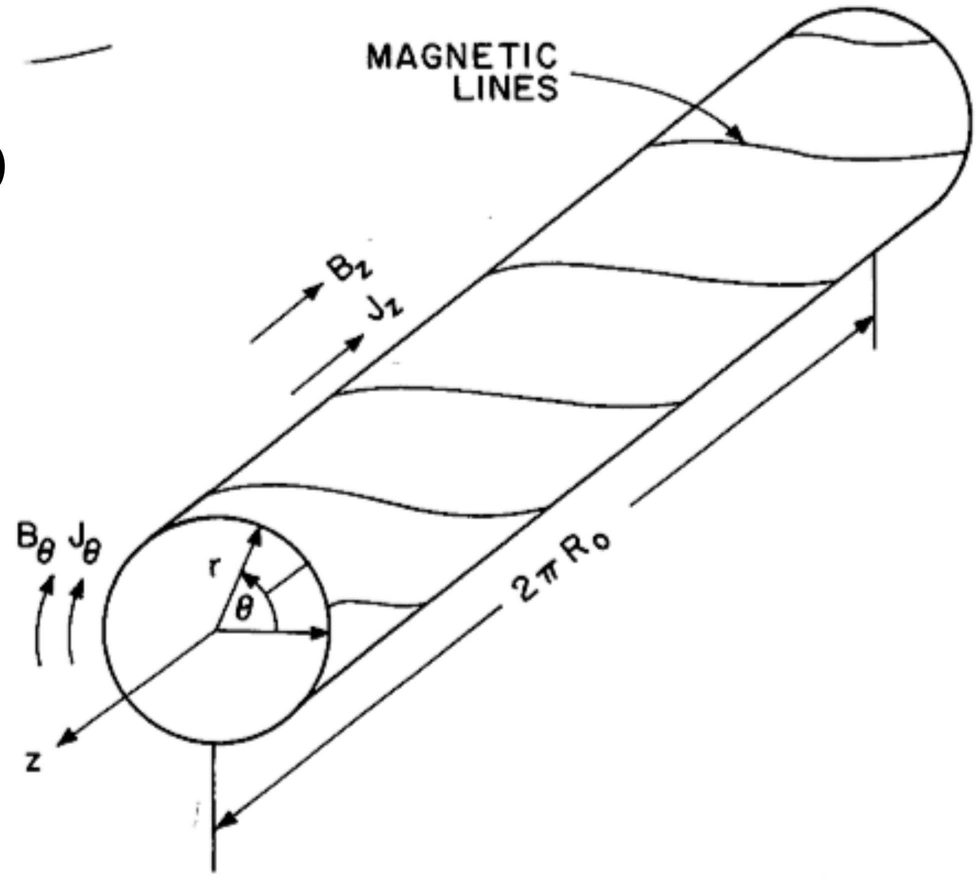
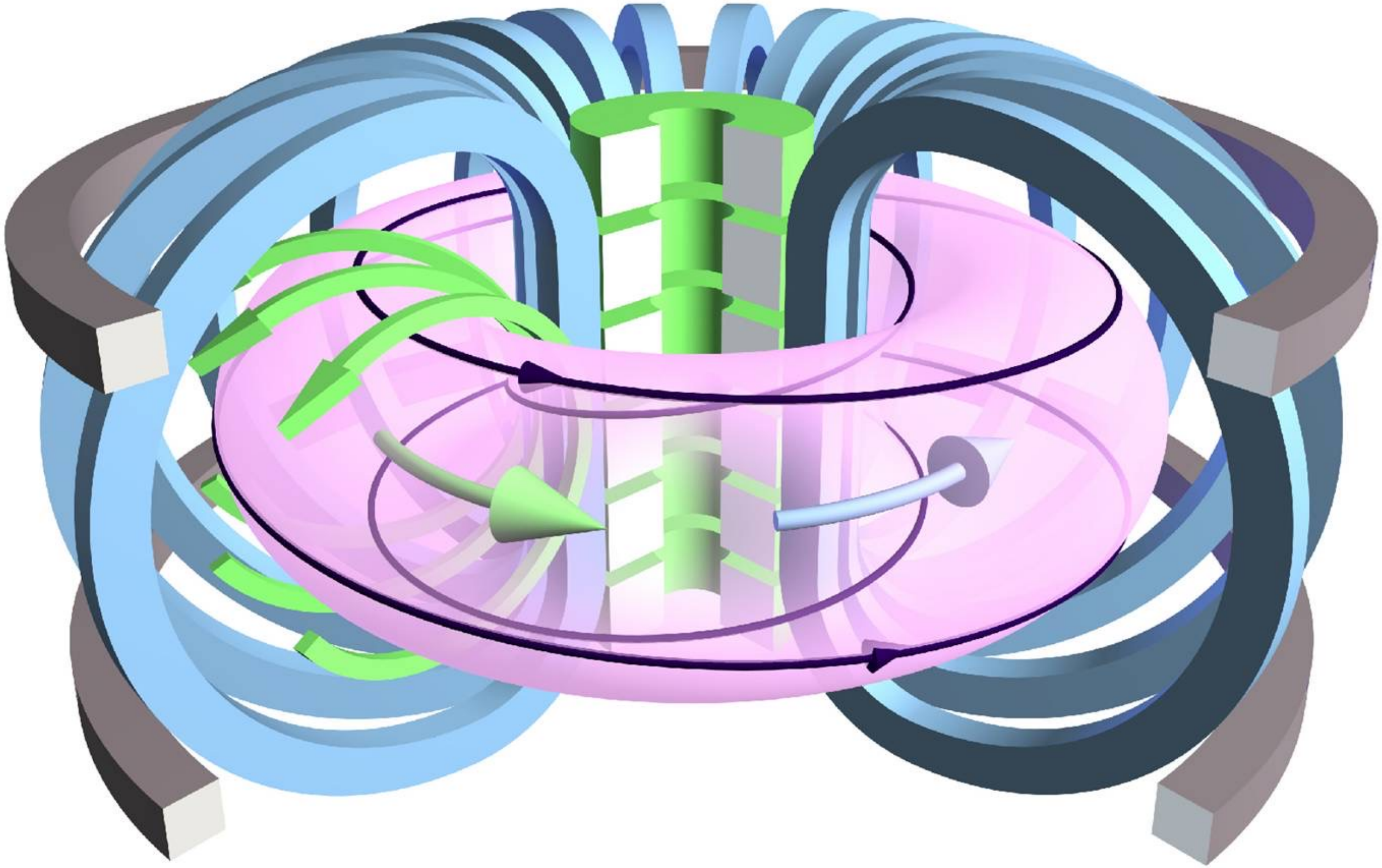


Figure 5.7. General screw-pinch geometry.

The tokamak



Reversed Field Pinch: the low field approach

- **The RFP configuration is similar to a tokamak**:
  - it is toroidal
  - a toroidal electrical current is driven in a plasma embedded in a toroidal magnetic field: pinch effect.
  - ….but the **applied toroidal field is 10x weaker** than in a tokamak