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A High Order Semi-Lagrangian Discontinuous Galerkin Method for the Two-Dimensional Incompressible Euler Equations and the Guiding Center Vlasov Model Without Operator Splitting

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Abstract
In this paper, we generalize a high order semi-Lagrangian (SL) discontinuous Galerkin (DG) method for multi-dimensional linear transport equations without operator splitting developed in Cai et al. (J Sci Comput 73(2–3):514–542, 2017) to the 2D time dependent incompressible Euler equations in the vorticity-stream function formulation and the guiding center Vlasov model. We adopt a local DG method for Poisson's equation of these models. For tracing the characteristics, we adopt a high order characteristics tracing mechanism based on a prediction-correction technique. The SLDG with large time-stepping size might be subject to extreme distortion of upstream cells. To avoid this problem, we propose a novel adaptive time-stepping strategy by controlling the relative deviation of areas of upstream cells.

Keywords Semi-Lagrangian · Discontinuous Galerkin · Guiding center Vlasov model · Incompressible Euler equations · Non-splitting · Mass conservative · Adaptive time-stepping method

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