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High order semi-Lagrangian discontinuous Galerkin method coupled with Runge-Kutta exponential integrators for nonlinear Vlasov dynamics

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ABSTRACT

In this paper, we propose a semi-Lagrangian discontinuous Galerkin method coupled with Runge-Kutta exponential integrators (SLDG-RKEI) for nonlinear Vlasov dynamics. The commutator-free Runge-Kutta (RK) exponential integrators (EI) were proposed by Celledoni, et al. (FGCS, 2003). In the nonlinear transport setting, the RKEI can be used to decompose the evolution of the nonlinear transport into a composition of a sequence of linearized dynamics. The resulting linearized transport equations can be solved by the semi-Lagrangian (SL) discontinuous Galerkin (DG) method proposed in Cai, et al. (JSC, 2017). The proposed method can achieve high order spatial accuracy via the SLDG framework, and high order temporal accuracy via the RK EI. Due to the SL nature, the proposed SLDG-RKEI method is not subject to the CFL condition, thus they have the potential in using larger time-stepping sizes than those in the Eulerian approach. Inheriting advantages from the SLDG method, the proposed SLDG-RKEI schemes are mass conservative, positivity-preserving, have no dimensional splitting error, perform well in resolving complex solution structures, and can be evolved with adaptive time stepping sizes. We show the performance of the SLDG-RKEI algorithm by classical test problems for the nonlinear Vlasov-Poisson system, as well as the Guiding center Vlasov model. Though that it is not our focus of this paper to explore the SLDG-RKEI scheme for nonlinear hyperbolic conservation laws that develop shocks, we show some preliminary results on schemes' performance on the Burgers' equation.

1. Introduction

In this paper, we consider the following two nonlinear Vlasov models. The first is the nonlinear Vlasov-Poisson system

$$f_t + v f_x + E(x,t) f_v = 0, \tag{1.1}$$

$$E(x,t) = -\phi_x, \ -\phi_{xx}(x,t) = \rho(x,t). \tag{1.2}$$

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