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A conservative semi-Lagrangian HWENO method for the Vlasov equation ☆

Xiaofeng Cai [a], Jianxian Qiu [b], Jing-Mei Qiu [c,*]

[a] School of Mathematical Sciences, Xiamen University, Xiamen, Fujian, 361005, PR China
[b] School of Mathematical Sciences and Fujian Provincial Key Laboratory of Mathematical Modeling & High-Performance Scientific Computing, Xiamen University, Xiamen, Fujian, 361005, PR China
[c] Department of Mathematics, University of Houston, Houston, 77204, United States

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ABSTRACT

In this paper, we propose a high order conservative semi-Lagrangian (SL) finite difference Hermite weighted essentially non-oscillatory (HWENO) method for the Vlasov equation based on dimensional splitting. HWENO was first proposed for solving nonlinear hyperbolic problems by evolving both function values and its first derivative values (Qiu and Shu (2004) [23]). The major advantage of HWENO, compared with the original WENO, lies in its compactness in reconstruction stencils.

There are several new ingredients in this paper. Firstly we propose a *mass-conservative* SL HWENO scheme for a 1-D equation by working with a flux-difference form, following the work of Qiu and Christlieb (2010) [25]. Secondly, we propose a proper splitting for equations of partial derivatives in HWENO framework to ensure local mass conservation. The proposed fifth order SL HWENO scheme with dimensional splitting has been tested to work well in capturing filamentation structures without oscillations when the time step size is within the Eulerian CFL constraint. However, when the time stepping size becomes larger, numerical oscillations are observed for the 'mass conservative' dimensional splitting HWENO scheme, as there are extra source terms in equations of partial derivatives. In this case, we introduce WENO limiters to control oscillations. Classical numerical examples on linear passive transport problems, as well as the nonlinear Vlasov–Poisson system, have been tested to demonstrate the performance of the proposed scheme.

1. Introduction

This paper focuses on a mass conservative semi-Lagrangian scheme with high order Hermite weighted essentially non-oscillatory (HWENO) reconstruction for the Vlasov–Poisson (VP) simulations based on dimensional splitting. The VP system, arising from collisionless plasma applications, reads as follows,

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f + \mathbf{E}(t, \mathbf{x}) \cdot \nabla_{\mathbf{v}} f = 0, \tag{1.1}$$

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* Corresponding author.
E-mail addresses: xfcai89@126.com (X. Cai), jxqiu@xmu.edu.cn (J. Qiu), jingqiu@math.uh.edu (J.-M. Qiu).

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