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HERMITE WENO SCHEMES WITH STRONG STABILITY PRESERVING MULTI-STEP TEMPORAL DISCRETIZATION METHODS FOR CONSERVATION LAWS*

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Abstract

Based on the work of Shu [SIAM J. Sci. Stat. Comput, 9 (1988), pp.1073-1084], we construct a class of high order multi-step temporal discretization procedure for finite volume Hermite weighted essential non-oscillatory (HWENO) methods to solve hyperbolic conservation laws. The key feature of the multi-step temporal discretization procedure is to use variable time step with strong stability preserving (SSP). The multi-step temporal discretization methods can make full use of computed information with HWENO spatial discretization by holding the former computational values. Extensive numerical experiments are presented to demonstrate that the finite volume HWENO schemes with multi-step discretization can achieve high order accuracy and maintain non-oscillatory properties near discontinuous region of the solution.

Mathematics subject classification: 65M06.
Key words: Multi-step temporal discretization; Hermite weighted essentially non-oscillatory scheme; Uniformly high order accuracy; Strong stability preserving; Finite volume scheme.

1. Introduction

In this paper, we construct a class of high order multi-step temporal discretization procedure for finite volume HWENO (Hermite weighted essential non-oscillatory) methods to solve hyperbolic conservation laws:

$$\begin{cases} u_t + \nabla \cdot F(u) = 0, \\ u(x,0) = u_0(x). \end{cases} \tag{1.1}$$

In recent years, WENO (weighted essentially non-oscillatory) schemes have been designed as a class of high order finite volume or finite difference schemes to solve hyperbolic conservation laws with the property of maintaining both uniform high order accuracy and an essentially

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