

# Finite Volume HWENO Schemes for Nonconvex Conservation Laws

Xiaofeng Cai<sup>1</sup> · Jianxian Qiu<sup>2</sup> · Jingmei Qiu<sup>1</sup>

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**Abstract** Following the previous work of Qiu and Shu (SIAM J Sci Comput 31: 584–607, 2008), we investigate the performance of Hermite weighted essentially non-oscillatory (HWENO) scheme for nonconvex conservation laws. Similar to many other high order methods, we show that the finite volume HWENO scheme performs poorly for some nonconvex conservation laws. We modify the scheme around the nonconvex regions, based on a first order monotone scheme and a second entropic projection, to ensure entropic convergence. Extensive numerical tests are performed. Compare with the earlier work of Qiu and Shu which focuses on 1D scalar problems, we apply the modified schemes (both WENO and HWENO) to one-dimensional Euler system with nonconvex equation of state and two-dimensional problems.

**Keywords** Nonconvex conservation laws · Finite volume HWENO scheme · Entropy solution · Entropic projection

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✉ Jingmei Qiu  
jingqiu@math.uh.edu

Xiaofeng Cai  
xfcai@math.uh.edu

Jianxian Qiu  
jxqiu@xmu.edu.cn

<sup>1</sup> Department of Mathematics, University of Houston, Houston, TX 77204, USA

<sup>2</sup> School of Mathematical Sciences and Fujian Provincial Key Laboratory of Mathematical Modeling and High-Performance Scientific Computing, Xiamen University, Xiamen 361005, Fujian, People's Republic of China