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A semi-Lagrangian discontinuous Galerkin (DG) – local DG method for solving convection-diffusion equations

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ABSTRACT

In this paper, we propose an efficient high order semi-Lagrangian (SL) discontinuous Galerkin (DG) method for solving linear convection-diffusion equations. The method generalizes our previous work on developing the SLDG method for transport equations [5], making it capable of handling additional diffusion and source terms. Within the DG framework, the solution is evolved along the characteristics; while the diffusion term is discretized by the local DG (LDG) method and integrated along characteristics by implicit Runge-Kutta methods together with source terms. The proposed method is named the 'SLDG-LDG' method and enjoys many attractive features of the DG and SL methods. These include the uniformly high order accuracy (e.g. third order) in space and in time, compact, mass conservative, and stability under large time stepping size. An L^2 stability analysis is provided when the method is coupled with the first order backward Euler discretization. Effectiveness of the method are demonstrated by a group of numerical tests in one and two dimensions.

1. Introduction

In this paper, we are concerned with solving the time dependent convection-diffusion problems in the form of

$$\begin{cases} u_t + \nabla_{\mathbf{x}} \cdot (\mathbf{a}(\mathbf{x},t)u) = \epsilon \Delta u + g, \quad \mathbf{x} \in \Omega, \quad t > 0, \\ u(\mathbf{x},0) = u_0(\mathbf{x}), \quad \mathbf{x} \in \Omega \end{cases} \tag{1.1}$$

with $\epsilon \geq 0$. For the scope of our current research, we assume the velocity field $\mathbf{a}(\mathbf{x},t)$ to be continuous with respect to $\mathbf{x}$ and t.

A popular computational method for finding approximate solutions to transport dominant problems in the form (1.1) is the semi-Lagrangian (SL) method, which has a long history in computational fluid dynamics, e.g. for convection-diffusion problems [21,16], climate modeling [15,18,9], plasma simulations [17], as well as linear and Hamilton-Jacobi equations [10]. For transport dominant problems, the method is designed via tracking the characteristics forward or backward in time, thus

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