Journal of Computational Physics 439 (2021) 110392

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

An Eulerian-Lagrangian discontinuous Galerkin method for transport problems and its application to nonlinear dynamics

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ARTICLE INFO

Article history:
Available online 4 May 2021

Keywords:
Eulerian-Lagrangian
Discontinuous Galerkin
Mass conservative
Semi-Lagrangian
Vlasov simulations
Characteristics

ABSTRACT

We propose a new Eulerian-Lagrangian (EL) discontinuous Galerkin (DG) method formulated by introducing a modified adjoint problem for the test function and by performing the integration of PDE over a space-time region partitioned by time-dependent linear functions approximating characteristics. The error incurred in characteristics approximation in the modified adjoint problem can then be taken into account by a new flux term, and can be integrated by method-of-line Runge-Kutta (RK) methods. The ELDG framework is designed as a generalization of the semi-Lagrangian (SL) DG method and classical Eulerian RK DG method for linear advection problems. It takes advantages of both formulations. In the EL DG framework, characteristics are approximated by a linear function in time, thus shapes of upstream cells are quadrilaterals in general two-dimensional problems. No quadratic-curved quadrilaterals are needed to design higher than second order schemes as in the SL DG scheme. On the other hand, the time step constraint from a classical Eulerian RK DG method is greatly mitigated, as it is evident from our theoretical and numerical investigations. Connection of the proposed EL DG method with the arbitrary Lagrangian-Eulerian (ALE) DG is observed. Numerical results on linear transport problems, as well as the nonlinear Vlasov and incompressible Euler dynamics using the exponential RK time integrators, are presented to demonstrate the effectiveness of the ELDG method.

1. Introduction

We propose a new Eulerian-Lagrangian (EL) discontinuous Galerkin (DG) method for a model transport equation in the form of

$$u_t + \nabla \cdot (\mathbf{P}(u; \mathbf{x}, t)u) = 0, \ \ (\mathbf{x}, t) \in \mathbb{R}^d \times [0, T], \tag{1.1}$$

which could come from a wide range of application fields including fluid dynamics, climate modeling, and kinetic description of plasma. There are three main classes of computational methods for solving (1.1): Lagrangian, Eulerian and semi-Lagrangian (SL). Each class of methods has their own advantages and limitations. The Lagrangian method is particle based, works efficiently for high dimensional problems, but suffers from statistical noises; while the latter two methods

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[1] Research of the first and second author is supported by NSF grant NSF-DMS-1818924, Air Force Office of Scientific Computing FA9550-18-1-0257.
[2] Research of the third author is supported by NSF grant DMS-1818467.

https://doi.org/10.1016/j.jcp.2021.110392