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A fourth-order conservative semi-Lagrangian finite volume WENO scheme without operator splitting for kinetic and fluid simulations

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Abstract

In this paper, we present a fourth-order conservative semi-Lagrangian (SL) finite volume (FV) weighted essentially non-oscillatory (WENO) scheme without operator splitting for two-dimensional linear transport equations with applications in kinetic models including the nonlinear Vlasov–Poisson system, the guiding center Vlasov model and the incompressible Euler equation in the vorticity-stream function formulation. To achieve fourth-order accuracy in space, two main ingredients are proposed in the SL FV formulation. Firstly, we introduce a so-called cubic-curved quadrilateral upstream cell and applying an efficient clipping method to evaluate integrals on upstream cells. Secondly, we construct a new WENO reconstruction operator, which recovers a P^3 polynomial from neighboring cell averages. Mass conservation is accomplished with the mass conservative nature of the reconstruction operator and the SL formulation. A positivity-preserving limiter is applied to maintain the positivity of the numerical solution wherever appropriate. For nonlinear kinetic models, the SL scheme is coupled with a fourth-order Runge–Kutta exponential integrator for high-order temporal accuracy. Extensive benchmarks are tested to verify the designed properties.

Keywords: Vlasov systems; Non-splitting scheme; Semi-Lagrangian; WENO reconstruction; Mass conservation; High-order accuracy

1. Introduction

Semi-Lagrangian (SL) schemes are popular for solving transport equation which can be found in many areas of applications, such as climate modeling [1,2] and kinetic description of plasma [3–5]. We are concerned with solving a transport equation in the form of

$$u_t + \nabla_{\mathbf{x}} \cdot (\mathbf{a}(u, \mathbf{x}, t)u) = 0, \tag{1}$$

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