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ORIGINAL PAPER

Comparison of Semi-Lagrangian Discontinuous Galerkin Schemes for Linear and Nonlinear Transport Simulations

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Abstract
Transport problems arise across diverse fields of science and engineering. Semi-Lagrangian (SL) discontinuous Galerkin (DG) methods are a class of high-order deterministic transport solvers that enjoy advantages of both the SL approach and the DG spatial discretization. In this paper, we review existing SLDG methods to date and compare numerically their performance. In particular, we make a comparison between the splitting and non-splitting SLDG methods for multi-dimensional transport simulations. Through extensive numerical results, we offer a practical guide for choosing optimal SLDG solvers for linear and nonlinear transport simulations.

Keywords Semi-Lagrangian (SL) · Discontinuous Galerkin (DG) · Transport simulations · Splitting · Non-splitting · Comparison

Mathematics Subject Classification 65M60 · 65M25

1 Introduction

Semi-Lagrangian (SL) discontinuous Galerkin (DG) methods are a class of high-order transport solvers that enjoy the computational advantages of both the SL approach and the DG spatial discretization. In this paper, we conduct a systematic comparison for several existing SLDG methods in the literature by considering aspects including the accuracy,

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