Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2020-0073

Vol. **30**, No. 1, pp. 67-96
July 2021

Adaptive Order WENO Reconstructions for the Semi-Lagrangian Finite Difference Scheme for Advection Problem

Jiajie Chen[1], Xiaofeng Cai[1], Jianxian Qiu[2] and Jing-Mei Qiu[1,*]

[1] *Department of Mathematical Sciences, University of Delaware, Newark, DE, 19717, USA.*
[2] *School of Mathematical Sciences and Fujian Provincial Key Laboratory of Mathematical Modeling and High-Performance Scientific Computing, Xiamen University, Xiamen, Fujian 361005, P.R. China.*

Received 28 April 2020; Accepted (in revised version) 21 October 2020

Abstract. We present a new conservative semi-Lagrangian finite difference weighted essentially non-oscillatory scheme with adaptive order. This is an extension of the conservative semi-Lagrangian (SL) finite difference WENO scheme in [Qiu and Shu, JCP, 230 (4) (2011), pp. 863-889], in which linear weights in SL WENO framework were shown to not exist for variable coefficient problems. Hence, the order of accuracy is not optimal from reconstruction stencils. In this paper, we incorporate a recent WENO adaptive order (AO) technique [Balsara et al., JCP, 326 (2016), pp. 780-804] to the SL WENO framework. The new scheme can achieve an optimal high order of accuracy, while maintaining the properties of mass conservation and non-oscillatory capture of solutions from the original SL WENO. The positivity-preserving limiter is further applied to ensure the positivity of solutions. Finally, the scheme is applied to high dimensional problems by a fourth-order dimensional splitting. We demonstrate the effectiveness of the new scheme by extensive numerical tests on linear advection equations, the Vlasov-Poisson system, the guiding center Vlasov model as well as the incompressible Euler equations.

AMS subject classifications: 65

Key words: Semi-Lagrangian, weighted essentially nonoscillatory, WENO adaptive order reconstruction, finite difference, mass conservation, Vlasov-Poisson, incompressible Euler.

1 Introduction

In this paper, we propose a conservative semi-Lagrangian (SL) finite difference (FD) weighted essentially non-oscillatory adaptive order (WENO-AO) scheme for the advec-

*Corresponding author. *Email addresses:* jiajie@udel.edu (J. Chen), xfcai@udel.edu (X. Cai), jxqiu@xmu.edu.cn (J. Qiu), jingqiu@udel.edu (J.-M. Qiu)

