Journal of Computational Physics 354 (2018) 529–551

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

# A high order semi-Lagrangian discontinuous Galerkin method for Vlasov–Poisson simulations without operator splitting

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A R T I C L E I N F O

*Article history:*
Received 8 September 2017
Received in revised form 24 October 2017
Accepted 25 October 2017
Available online 27 October 2017

*Keywords:*
Semi-Lagrangian
Discontinuous Galerkin
Vlasov–Poisson
Non-splitting
Mass conservative
Positivity-preserving

A B S T R A C T

In this paper, we develop a high order semi-Lagrangian (SL) discontinuous Galerkin (DG) method for nonlinear Vlasov–Poisson (VP) simulations without operator splitting. In particular, we combine two recently developed novel techniques: one is the high order non-splitting SLDG transport method (Cai et al. (2017) [4]), and the other is the high order characteristics tracing technique proposed in Qiu and Russo (2017) [29]. The proposed method with up to third order accuracy in both space and time is locally mass conservative, free of splitting error, positivity-preserving, stable and robust for large time stepping size. The SLDG VP solver is applied to classic benchmark test problems such as Landau damping and two-stream instabilities for VP simulations. Efficiency and effectiveness of the proposed scheme is extensively tested. Tremendous CPU savings are shown by comparisons between the proposed SL DG scheme and the classical Runge–Kutta DG method.



## 1. Introduction

This paper focuses on development of a class of high order semi-Lagrangian discontinuous Galerkin (SLDG) methods for Vlasov–Poisson (VP) simulations without operator splitting. This is a continuation of our previous research effort on a high order non-splitting SLDG method for solving linear transport equations [4]. The VP system, arising from plasma applications, is known as a fundamental model for collisionless plasmas with a negligible magnetic field. It reads as follows,

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f + \mathbf{E}(\mathbf{x}, t) \cdot \nabla_{\mathbf{v}} f = 0, \tag{1.1}$$

and

$$\mathbf{E}(\mathbf{x}, t) = -\nabla_{\mathbf{x}} \phi(\mathbf{x}, t), \quad -\Delta_{\mathbf{x}} \phi(\mathbf{x}, t) = \rho(\mathbf{x}, t), \tag{1.2}$$

where $\mathbf{x}$ and $\mathbf{v}$ are coordinates in phase space $(\mathbf{x}, \mathbf{v}) \in \mathbb{R}^3 \times \mathbb{R}^3$, $\mathbf{E}$ is the electric field, $\phi$ is the self-consistent electrostatic potential and $f(t, \mathbf{x}, \mathbf{v})$ is probability distribution function which describes the probability of finding a particle with velocity

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1 Research is supported by NSF grant NSF-DMS-1620047.
2 Research supported by NSF grant NSF-DMS-1522777 and Air Force Office of Scientific Computing FA9550-12-0318.

https://doi.org/10.1016/j.jcp.2017.10.048