Semi-Lagrangian implicit Bhatnagar-Gross-Krook collision model for the finite-volume discrete Boltzmann method

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In order to increase the accuracy of temporal solutions, reduce the computational cost of time marching, and improve the stability associated with collisions for the finite-volume discrete Boltzmann method, an advanced implicit Bhatnagar-Gross-Krook (BGK) collision model using a semi-Lagrangian approach is proposed in this paper. Unlike existing models, in which the implicit BGK collision is resolved either by a temporal extrapolation or by a variable transformation, the proposed model removes the implicitness by tracing the particle distribution functions (PDFs) back in time along their characteristic paths during the collision process. An interpolation scheme is needed to evaluate the PDFs at the traced-back locations. By using the first-order interpolation, the resulting model allows for the straightforward replacement of $f_\alpha^{\mathrm{eq},n+1}$ by $f_\alpha^{\mathrm{eq},n}$ no matter where it appears. After comparing the proposed model with the existing models under different numerical conditions (e.g., different flux schemes and time-marching schemes) and using the proposed model to successfully modify the variable transformation technique, three conclusions can be drawn. First, the proposed model can improve the accuracy by almost an order of magnitude. Second, it can slightly reduce the computational cost. Therefore, the proposed scheme improves accuracy without extra cost. Finally, the proposed model can significantly improve the $\Delta t/\tau$ limit compared to the temporal interpolation model while having the same $\Delta t/\tau$ limit as the variable transformation approach. The proposed scheme with a second-order interpolation is also developed and tested; however, that technique displays no advantage over the simple first-order interpolation approach. Both numerical and theoretical analyses are also provided to explain why the developed implicit scheme with simple first-order interpolation can outperform the same scheme with second-order interpolation, as well as the existing temporal extrapolation and variable transformation schemes.

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I. BACKGROUND

Since its earliest development more than three decades ago [1–4], the lattice Boltzmann method (LBM) has gained a prominent role in the simulations of a large variety of complex flows across a broad range of scales, from macroscopic turbulence, all the way down to nanoscale flows of biological interest, and lately, even subnuclear flows [5]. Its success is supported by two important features. First, physically, the LBM can inherently solve problems over a wide range of length scales beyond the strict hydrodynamic regime [6]. The behavior of hydrodynamics at macroscales is basically a low-dimensional asymptotic limit of the infinite-dimensional sequence of kinetic moments associated with the Boltzmann equation that is rooted in the microscale kinetics. By capturing the high-order moments in the Boltzmann equation, the low-order moments in macroscale hydrodynamics emerge naturally from the underlying microdynamics [7]. This is why the LBM is regarded as a mesoscale technique with both upwards (to the continuum) and downwards (to the atomistic) multiscale capabilities. Second, numerically, the LBM can achieve second-order accuracy in space with only a first-order numerical scheme [8]. The reason for this is that the advection term $\boldsymbol{e}_\alpha \cdot \nabla f_\alpha$ in the LBM is linear ($\boldsymbol{e}_\alpha$ before the gradient is constant). Due to the linear advection, the LBM can couple the discretizations of all three dimensions: the microscopic velocity ($\boldsymbol{e}$), space ($\boldsymbol{x}$), and time (t). By doing this, the variables that are being advected (in this case, the particle distribution functions) will stop exactly at a grid point after each advection step. According to the definition of the Courant-Friedrichs-Lewy (CFL) number, the CFL of the microscopic velocities in the LBM becomes 1 globally, which gives rise to a universal second-order accuracy in space.

Although this unique multidimensional coupling mechanism is an important asset of the LBM, it also brings with it a substantial challenge. Since the LBM couples the discretizations of all three dimensions, this limits the freedom to choose a different way of individually discretizing any of the three dimensions, which is especially restrictive for the spatial dimension $\boldsymbol{x}$. Therefore, the mesh, which is the result of discretizing the space, has to copy the lattice structure (a lattice tells how the velocity is discretized), and also has to be uniform (in order to achieve $CFL = 1$ location-wise) and

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