Notes for day 1

math + econ + code lectures

june 2021

Today (D1): Hedonic model

(surge pricing)

Consider supply

$$s_z(p) = \sum_i 1\{z \in \arg\max u_{iz}(p_z)\}$$

$$d_z(p) = \sum_j 1\{z \arg\min c_{jz}(p_z)\}$$

$$e_z(p) = s_z(p) - d_z(p).$$

① Smooth approximation of e_z

Replace driver's problem

$$\max_z \{u_{iz}(p_z), u_{i0}\}$$

by $$\max_z \{u_{iz}(p_z) + \sigma\varepsilon_z, u_{i0} + \sigma\varepsilon_0\}$$

Logit model $(\varepsilon_z)_{z \in \mathcal{Z} \cup \{0\}}$

are iid R.V. with Gumbel distribution.

Property $Pr\left(z \in \arg\max_z u_{iz}(p_z) + \sigma \varepsilon_{iz}\right)$

$$= \frac{\exp\left(u_{iz}(p_z)/\sigma\right)}{\sum_{z' \in \mathcal{Z} \cup \{0\}} \exp\left(u_{iz'}(p_{z'})/\sigma\right)}$$

Smooth approximation of supply is:

$$S_z^\sigma(p) = \sum_i \frac{\exp\left(u_{iz}(p_z)/\sigma\right)}{\sum_{z' \in \mathcal{Z} \cup \{0\}} \exp\left(\frac{u_{iz'}(p_{z'})}{\sigma}\right)}$$

Same thing for demand ...
excess supply function (smoothed)

$$e_z^\sigma(p) = \sum_i \frac{\exp(u_{iz}(p_z)/\sigma)}{\sum_{z'} \cdots}$$

$$- \sum_j \frac{\exp(-c_{jz}(p_z)/\sigma)}{\sum_{z'} \exp(-c_{jz'}(p_{z'})/\sigma)}$$

Solve $e_z^\sigma(p) = 0 \quad \forall z$.

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(1) when is this an optimization problem?

Can we see $e_z^\sigma(p)$ as $\partial V(p)/\partial p_z$?

If that is the case, then system rewrites as

$$\frac{\partial V}{\partial p_z}(p) = 0$$

and if V is convex, p is obtained by

$$\min_{(p_z)} V(p).$$

"optimization-style"

This works in the additive case. That is

$$u_{iz}(p_z) = u_{iz} + p_z$$

and

$$c_{iz}(p_z) = c_{iz} + p_z$$

Introduce

$$V^s(p) = \sigma \sum_i \log \sum_{z \in Z \cup \{0\}} \exp\left(\frac{u_{iz} + p_z}{\sigma}\right)$$

$$\frac{\partial V^s(p)}{\partial p_z} = \underbrace{\sum_i \frac{\exp\left(\frac{u_{iz} + p_z}{\sigma}\right)}{\sum_{z \in Z \cup \{0\}} \exp\left(\frac{u_{iz} + p_z}{\sigma}\right)}}_{S_z^\sigma(p)}$$

$$V^d(p) = \sigma \sum_j \log\left(\sum_z \exp\left(\frac{-c_{jz} - p_z}{\sigma}\right)\right)$$

$$\frac{\partial V^d(p)}{\partial p_z} = -d_z^\sigma(p)$$

In summary:

in the additive case, setting

$$V^{\sigma}(p) = V^{s}(p) + \underbrace{V^{d}(p)}$$

yields

$$\frac{\partial V^{\sigma}(p)}{\partial p_z} = s_z^{\sigma}(p) - d_z^{\sigma}(p)$$

$$\min_{p} V(p) \quad \text{yields}$$

the solution.

(gradient descent).

What about the nonadditive case?

Need to solve

$$e_Z(p) = 0.$$