Pontryagin Duality and Self-Dual Groups

Dip Saha

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Abstract

Topological groups blend the algebraic structure of groups with the topological structure of spaces. A topological group is a mathematical object where the group operations of multiplication and inversion are continuous with respect to the topology of the group. The study of these structures allows for the application of both algebraic and topological methods, making them a critical area of study in various fields such as modern mathematics, including harmonic analysis, number theory, and algebraic geometry.

Pontryagin Duality theorem is one of the most important results in the context of locally compact abelian groups. Understanding this theorem is the main motivation of this project.

The project delves into the essential definitions, properties, and examples of topological groups, along with key lemmas and theorems in the area of homogeneity, separation conditions and morphisms.

We studied characters on a locally compact abelian groups and Pontryagin duality theorem. A significant part of the project is to understand the locally compact and totally disconnected group $\Omega_{\mathfrak{a}}$ consisting of $\mathfrak{a}$-adic numbers, $\mathfrak{a}$-adic integers and construct their Pontryagin duals explicitly. For some special sequences of $\mathfrak{a}$ the above groups have self duals.