A Numerical model of micro-textured sliding bearing for enhancing frictional performance and wear resistance in mechanical system

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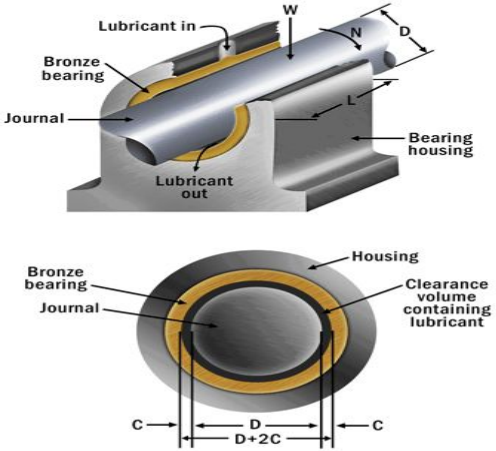
Contents :

Introduction :

- Sliding bearing refers to a bearing where two surfaces move relative to each other.
- This movement can be made easier by means of a lubricant squeezed by the motion of the components.
- It can generate sufficient pressure to separate the two surfaces, thereby reducing frictional contact and wear.

Sliding bearing -Tilt pad thrust bearing

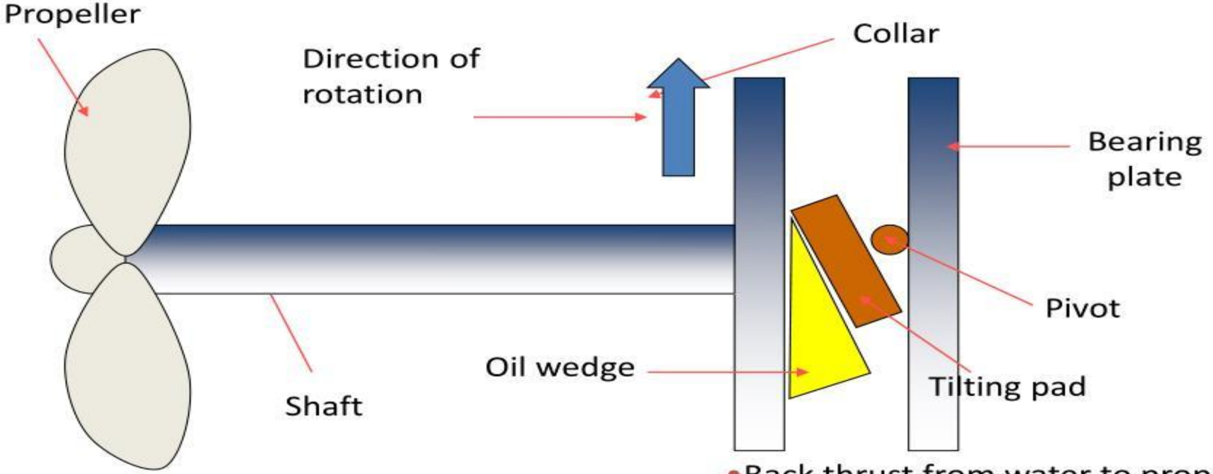


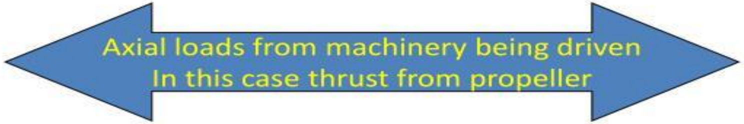
Sliding bearing - Journal bearing

PC: Waukesha ,Tilt Pad Thrust Bearings

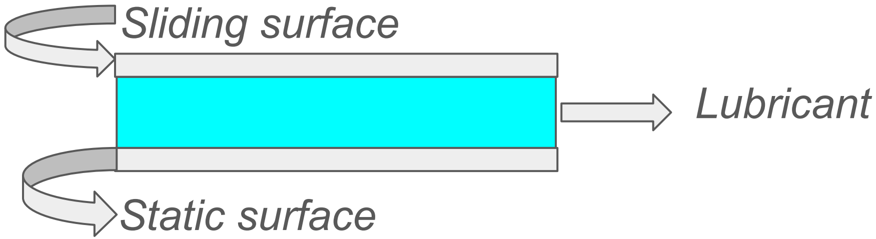
PC: Tribonet.org,Journal bearing

Introduction :





- Back thrust from water to propeller causes axial loading on the shaft
- Axial loads are opposed by pressure buildup in the wedge
- Gives a damping effect



Motivation:

Reasons to improve the sliding bearing are as follows:

- To reduce the friction and wear
- To increase the load carrying capacity

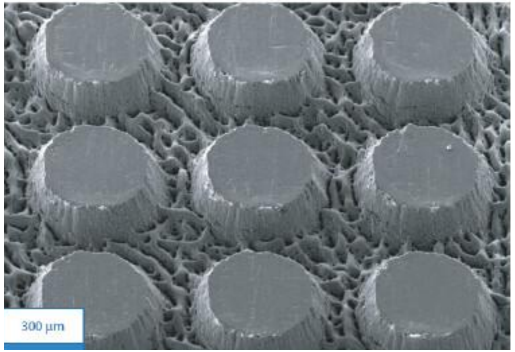
Methods to improve the performance of the sliding bearing:

- Surface texturing
- Increasing hardness of the moving surfaces

Surface texturing :

A key factor that can help in reducing friction between surfaces is surface modification.

- Surface texturing is a technique to modify the surfaces by adding distinct features to improve lubrication conditions.



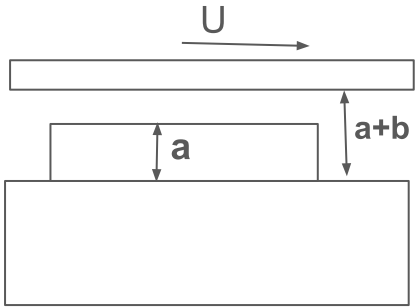
Circular textured surface

PC: Saravanan Murugayan at el. Studies on fabrication of protruded multi-shaped micro-feature array on AA 6063 by laser micromachining.

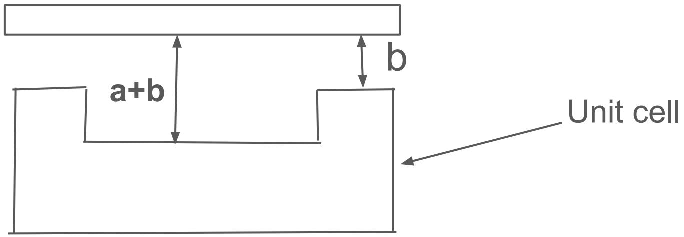
Types of texture :

U- velocity of the moving plate
a+b - distance between the two plates without texture
a- height of the texture

- Positive Texture
- Negative Texture

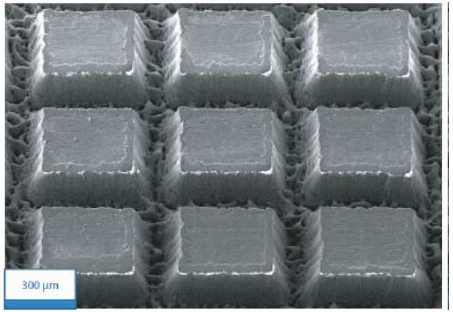


Positive surface texture

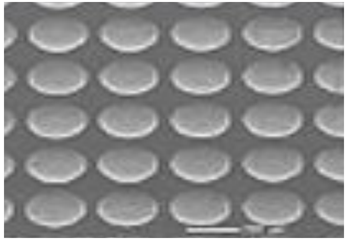


Negative surface texture

Fig: The two surface of a sliding bearing sliding parallel to each other.



Positive square cross section textures



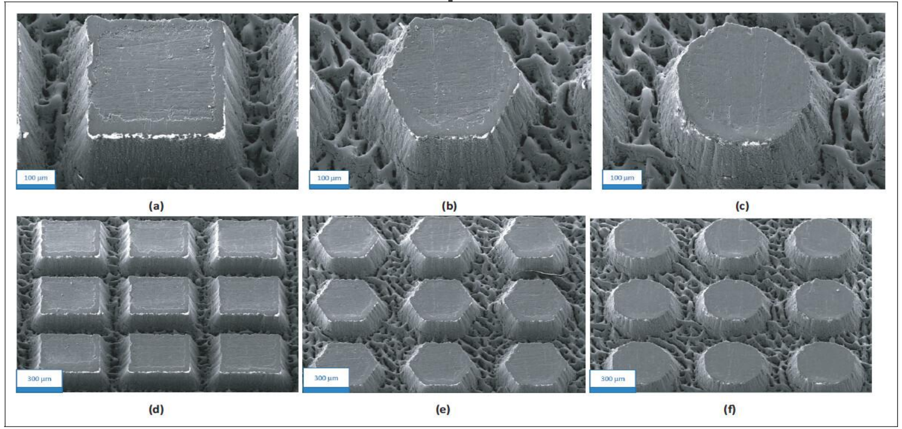
Negative square cross section textures

Reasons for doing this work :

The effectiveness of micro-textures is influenced by several key parameters :

- Height /depth
- Size
- Shape
- Spacing

In order to avoid expenses of experimental work and to reduce the development time. Numerical studies is carried out.



Distribution of square , hexagonal and circular texture

PC: Saravanan Murugayan at el. Studies on fabrication of protruded multi-shaped micro-feature array on AA 6063 by laser micromachining

Reynold's Equation :

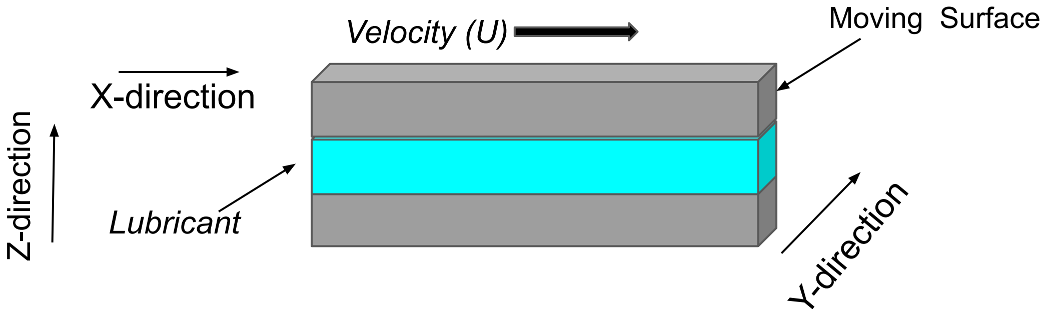
The Reynold's is a partial differential equation that describes the pressure distribution in a thin fluid in between two surfaces.

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial y}\left(h^3\frac{\partial P}{\partial y}\right)=6\mu\cdot\{(U_2-U_1)\cdot\frac{\partial h}{\partial x}+2(V_2-V_1)+(W_2-W_1)\frac{\partial h}{\partial z}\}$$

Squeezing action (bearing surfaces move perpendicular to each other)

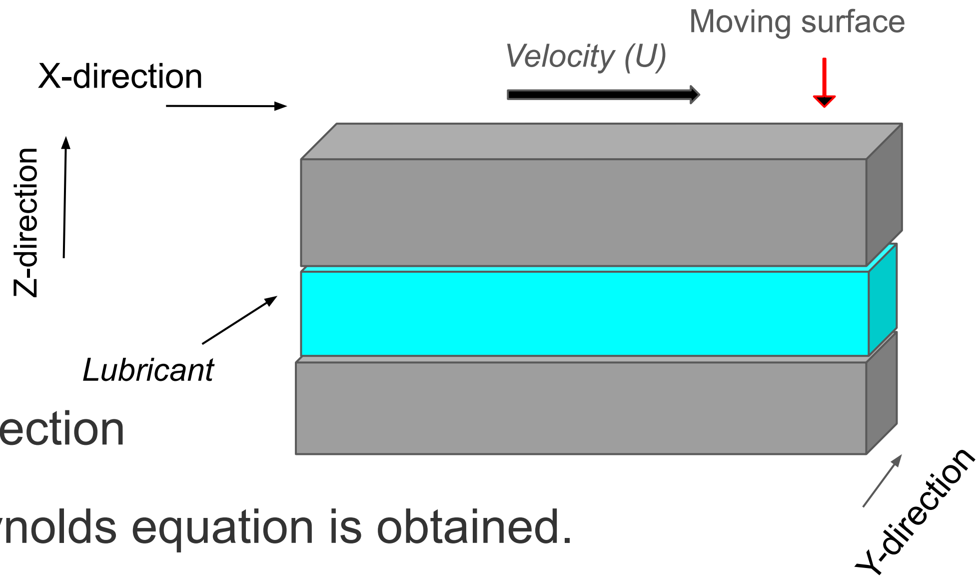
Relative velocity Wedge action

where, U_1 -velocity of the upper surface in X- direction
U_2-velocity of the lower surface in X- direction
V_1- velocity of the upper surface in Y- direction
V_2-velocity of the lower surface in Y- direction
W_1-velocity of the upper surface in Z- direction
W_2-velocity of the lower surface in Z- direction
h – height of the film thickness
P -Pressure



Assumptions :

- Constant value of viscosity
- Both rigid surface
- Newtonian fluid
- Incompressible flow
- Only upper surface slides
- No slip at boundaries
- Negligible pressure gradient in z-direction



With these assumptions a modified Reynolds equation is obtained.

$$\boxed{\frac{\partial}{\partial x}\left(h^3\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial y}\left(h^3\frac{\partial P}{\partial y}\right)=6\mu U\frac{\partial h}{\partial x}}$$

Modified 1-D Reynolds equation :

Modified Reynolds equation in 2-Dimension:

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial y}\left(h^3\frac{\partial P}{\partial y}\right)=6\mu U\frac{\partial h}{\partial x}$$

Now let us consider in 1-Dimension, we get :

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial P}{\partial x}\right)=6U\mu\frac{\partial h}{\partial x}$$

Solving this equation to find pressure distribution in a 1-Dimensional set up analytical and numerically and comparing the results obtained:

Analytical solution of modified 1-D Reynolds equation : Rayleigh step bearing

1-D Reynolds' equation is

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial P}{\partial x}\right) = 6\mu U\frac{\partial h}{\partial x}$$

Integrating both sides w.r.t. x:

$$h^3\frac{\partial P}{\partial x} = 6\mu U h + c_1$$

$$\frac{\partial P}{\partial x} = \frac{6\mu U h}{h^3} + \frac{c_1}{h^3}$$

$$\frac{\partial P}{\partial x} = \frac{6\mu U}{h^2} + \frac{c_1}{h^3}$$

Again integrating both sides w.r.t. x:

$$P(x) = 6\mu U\int\frac{1}{h^2}\,dx + c_1\int\frac{1}{h^3}\,dx + c_2$$

Boundary conditions:

$$P|_{x=a} = 0$$

$$P|_{x=c} = P_c - P_a$$

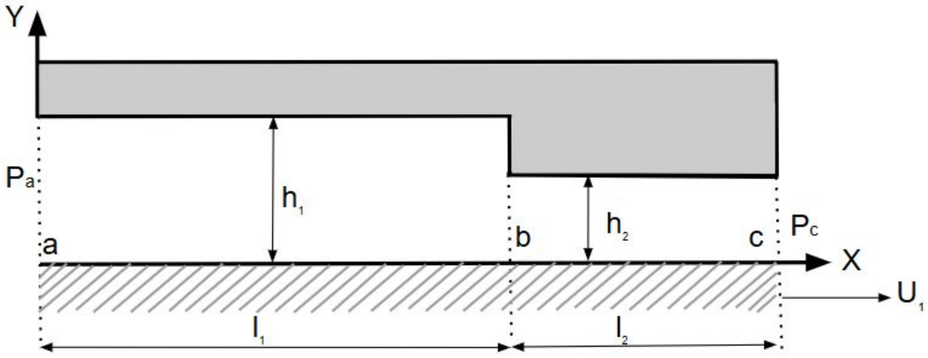


Fig. : Rayleigh Step Slider Bearing

Input values:

h_1=250 x 10^(-3) %maximum film thickness in Millimeter(mm)

h_2=133.976 x 10^(-3) %minimum film thickness in Millimeter(mm)

U_1=1x10^3 % velocity of moving plate in mm/s

μ=0.188x10^(-3) % viscosity in KPa

L=12.5; %length of the unit cell in mm

l_1 = 8.975

l_2 = 3.525

Analytical solution of modified 1-D Reynolds equation : Rayleigh step bearing

$$P|_{x=a} = 6\mu U \int_a^a \frac{1}{h(x)^2}\, dx + c_1 \int_a^a \frac{1}{h(x)^3}\, dx + c_2$$
$$0 = 0 + 0 + c_2$$
$$\Rightarrow c_2 = 0$$

Also,

$$P|_{x=c} = P_c - P_a \quad and$$
$$P|_{x=c} = 6\mu U \int_a^c \frac{1}{h(x)^2}\, dx + c_1 \int_a^c \frac{1}{h(x)^3}\, dx + c_2$$
$$\Rightarrow P_c - P_a = 6\mu U \int_a^c \frac{1}{h(x)^2}\, dx + c_1 \int_a^c \frac{1}{h(x)^3}\, dx$$

Solving for c_1:

$$c_1 = \frac{(P_c - P_a) - 6\mu U \int_a^c \frac{1}{h(x)^2}\, dx}{\int_a^c \frac{1}{h(x)^3}\, dx}$$

From figure (1)

$$h(x) = \begin{cases} h_1, & a \le x \le b \\ h_2, & b < x \le c \end{cases}$$

For $x \in [a, b]$:

$$P(x) = 6\mu U \frac{(x-a)}{h_1^2} + c_1 \frac{(x-a)}{h_1^3}$$

Analytical solution of modified 1-D Reynolds equation : Rayleigh step bearing

For $x \in]b, c]$:

$$\begin{aligned} P(x) &= 6\mu U \left[\int_a^b \frac{1}{h_1^2}\, dx + \int_b^x \frac{1}{h_2^2}\, dx \right] + c_1 \left[\int_a^b \frac{1}{h_1^3}\, dx + \int_b^x \frac{1}{h_2^3}\, dx \right] \\ &= 6\mu U \left[\frac{l_1}{h_1^2} + \frac{(x-b)}{h_2^2} \right] + c_1 \left[\frac{l_1}{h_1^3} + \frac{(x-b)}{h_2^3} \right] \end{aligned}$$

And

$$c_1 = \frac{(P_c - P_a) - 6\mu U \left(\frac{l_1}{h_1^2} + \frac{l_2}{h_2^2} \right)}{\frac{l_1}{h_1^3} + \frac{l_2}{h_2^3}}$$

Numerical solution of modified 1-D Reynolds equation :

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial P}{\partial x}\right) = 6U\mu\frac{\partial h}{\partial x}$$

The discretized form:

$$\frac{\partial h_{i,j}}{\partial x} = \frac{h_{i+0.5} - h_{i-0.5}}{\Delta x}$$

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial P_i}{\partial x}\right) = \frac{h^3_{i+0.5}\cdot P_{i+1} + h^3_{i-0.5}\cdot P_{i-1} - (h^3_{i+0.5} + h^3_{i-0.5})\cdot P_i}{\Delta x^2}$$

Putting (5) and (6) in equation (4)

$$\frac{h^3_{i+0.5}P_{i+1} + h^3_{i-0.5}P_{i-1} - (h^3_{i+0.5} + h^3_{i-0.5})P_i}{(\Delta x)^2} = 6U\mu\frac{h_{i+0.5} - h_{i-0.5}}{\Delta x}$$

Rewriting:

$$(h^3_{i+0.5} + h^3_{i-0.5})P_i = 6\mu U(h_{i+0.5} - h_{i-0.5})\Delta x + h^3_{i+0.5}P_{i+1} + h^3_{i-0.5}P_{i-1}$$

Solving for P_i:

$$P_i = \frac{-6\mu U(h_{i+0.5} - h_{i-0.5})\Delta x}{h^3_{i+0.5} + h^3_{i-0.5}} + \frac{h^3_{i+0.5}P_{i+1}}{h^3_{i+0.5} + h^3_{i-0.5}} + \frac{h^3_{i-0.5}P_{i-1}}{h^3_{i+0.5} + h^3_{i-0.5}}$$

Solving using Gauss-Seidel Iterative Scheme:

$$\boxed{P_i^{(k+1)} = \frac{-6\mu U(h_{i+0.5} - h_{i-0.5})\Delta x}{h^3_{i+0.5} + h^3_{i-0.5}} + \frac{h^3_{i+0.5}}{h^3_{i+0.5} + h^3_{i-0.5}}P_{i+1}^{(k)} + \frac{h^3_{i-0.5}}{h^3_{i+0.5} + h^3_{i-0.5}}P_{i-1}^{(k+1)}}$$

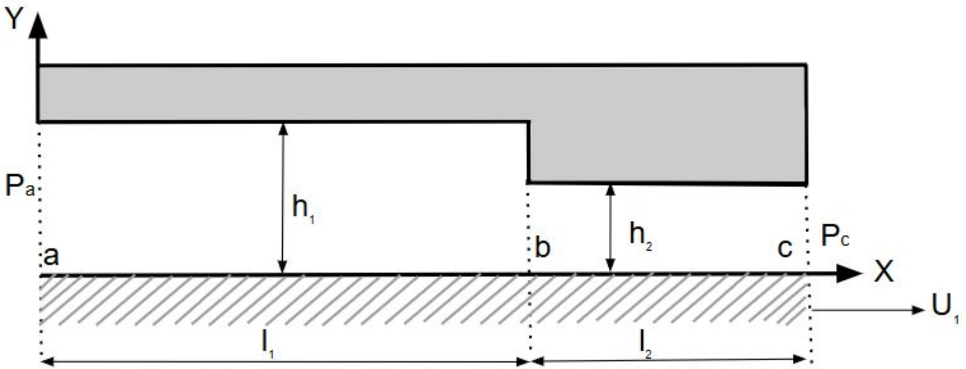


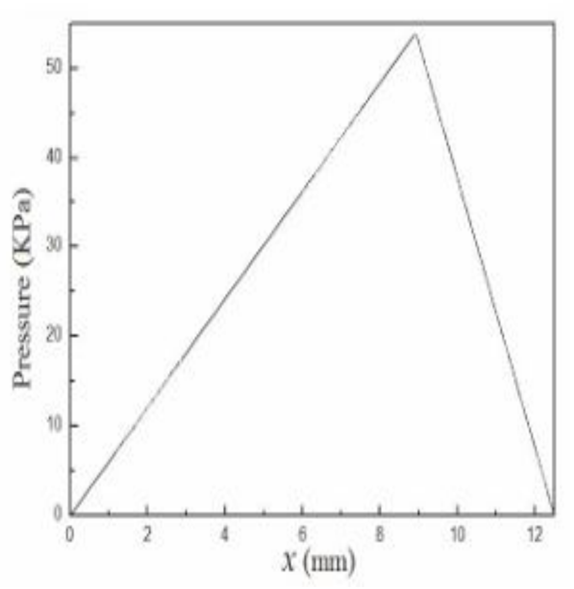
Fig. : Rayleigh Step Slider Bearing

Input values:

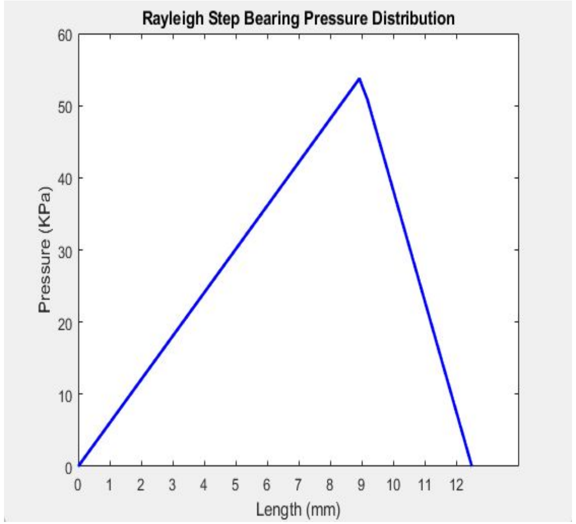
- h_1: maximum film thickness
- h_2: minimum film thickness
- L: length of the unit cell
- $[a, b]$: region before the step portion
- $]b, c]$: region of step portion

Comparing pressure distribution graphs obtained with a research paper :

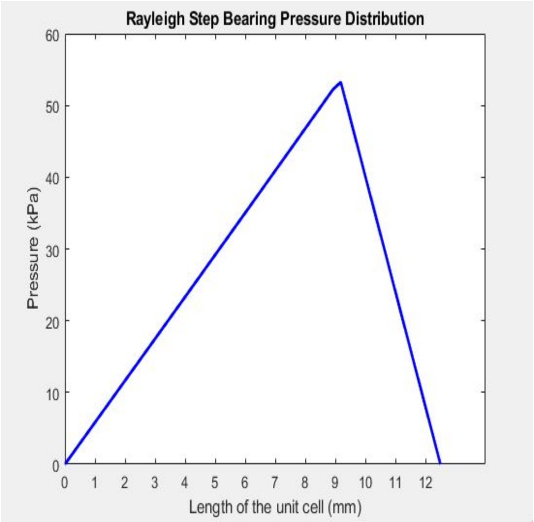
Analytical results from Feng Shen's paper



Analytical solution :



Numerical solution :



Peak Pressure obtained From Feng Shen's paper =53.7306 kPa
Peak Pressure obtained in analytical model=53.7306 kPa
Peak Pressure obtained in numerical model=53.22 kPa

Numerical model for textured surface:

Reynolds 2-D equation

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial y}\left(h^3\frac{\partial P}{\partial y}\right)=6\mu U\frac{\partial h}{\partial x} \qquad (1)$$

Now discretizing the equation,we get

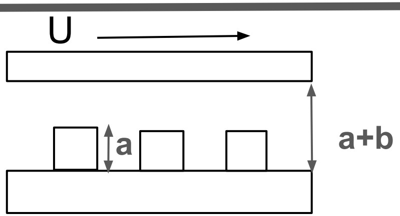
$$\frac{\partial h_{i,j}}{\partial x}=\frac{h_{i+0.5,j}-h_{i-0.5,j}}{\Delta x} \qquad (2)$$

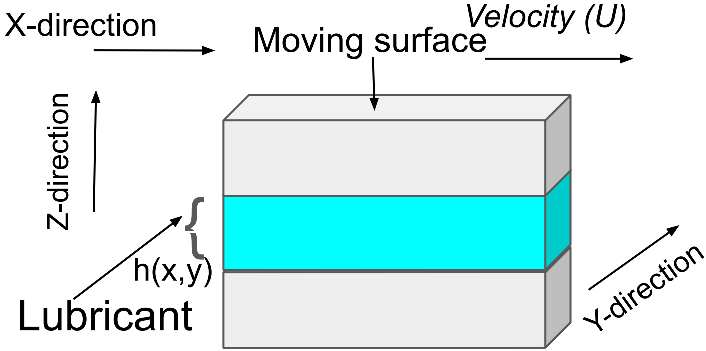
$$\frac{\partial}{\partial x}\left(h^3\frac{\partial P_{i,j}}{\partial x}\right)=\frac{h^3_{i+0.5,j}\cdot P_{i+1,j}+h^3_{i-0.5,j}\cdot P_{i-1,j}-(h^3_{i+0.5,j}+h^3_{i-0.5,j})\cdot P_{i,j}}{(\Delta x)^2} \qquad (3)$$

$$\frac{\partial}{\partial y}\left(h^3\frac{\partial P_{i,j}}{\partial y}\right)=\frac{h^3_{i,j+0.5}\cdot P_{i,j+1}+h^3_{i,j-0.5}\cdot P_{i,j-1}-(h^3_{i,j+0.5}+h^3_{i,j-0.5})\cdot P_{i,j}}{(\Delta y)^2} \qquad (4)$$

Putting equation (2), (3), and (4) into equation (1), we get:

$$\frac{h^3_{i+0.5,j}\cdot P_{i+1,j}+h^3_{i-0.5,j}\cdot P_{i-1,j}-(h^3_{i+0.5,j}+h^3_{i-0.5,j})\cdot P_{i,j}}{(\Delta x)^2}$$
$$+\frac{h^3_{i,j+0.5}\cdot P_{i,j+1}+h^3_{i,j-0.5}\cdot P_{i,j-1}-(h^3_{i,j+0.5}+h^3_{i,j-0.5})\cdot P_{i,j}}{(\Delta y)^2}$$
$$=6\mu U\frac{h_{i+0.5,j}-h_{i-0.5,j}}{\Delta x} \qquad (5)$$





"h(x,y)" - thickness of the lubrication
P - pressure
μ - coefficient of viscosity
U- velocity of the moving plate
h=a (when there is surface texture)
h=a+b (when there is no texture)
b - height of the texture

Numerical model for textured surface:

Rearranging equation (5) ,we get

$$P_{i,j} = \frac{h^3_{i,j+0.5} \cdot P_{i,j+1}}{\left(h^3_{i+0.5,j} + h^3_{i-0.5,j}\right) + \left(h^3_{i,j+0.5} + h^3_{i,j-0.5}\right)} + \frac{h^3_{i,j-0.5} \cdot P_{i,j-1}}{\left(h^3_{i+0.5,j} + h^3_{i-0.5,j}\right) + \left(h^3_{i,j+0.5} + h^3_{i,j-0.5}\right)} +$$

$$\frac{h^3_{i+0.5,j} \cdot P_{i+1,j}}{\left(h^3_{i+0.5,j} + h^3_{i-0.5,j}\right) + \left(h^3_{i,j+0.5} + h^3_{i,j-0.5}\right)} + \frac{h^3_{i-0.5,j} \cdot P_{i-1,j}}{\left(h^3_{i+0.5,j} + h^3_{i-0.5,j}\right) + \left(h^3_{i,j+0.5} + h^3_{i,j-0.5}\right)}$$

$$- \frac{6\mu U\left(h_{i+0.5,j} - h_{i-0.5,j}\right)\Delta x}{\left(h^3_{i+0.5,j} + h^3_{i-0.5,j}\right) + \left(h^3_{i,j+0.5} + h^3_{i,j-0.5}\right)}$$

$$P_{i,j} = AP_{i,j+1} + BP_{i,j-1} + CP_{i+1,j} + DP_{i-1,j} - E(6\mu U)$$

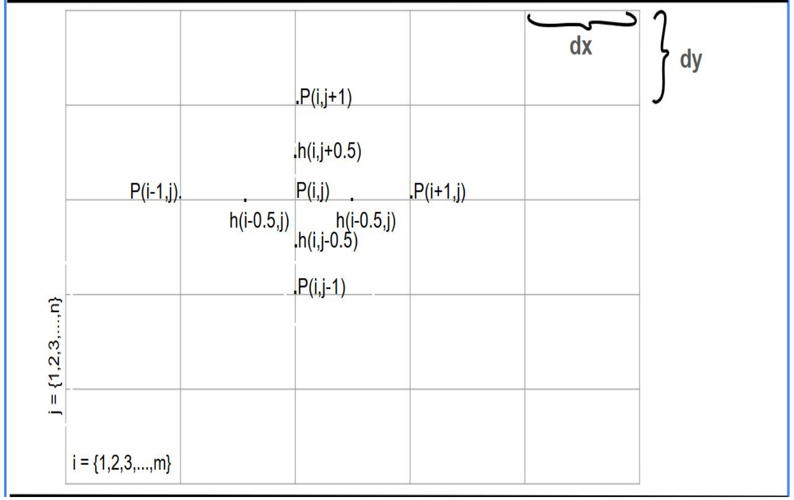
Solving using Gauss Seidel Iterative scheme

$$\boxed{P^{(k+1)}_{i,j} = A\,P^{(k)}_{i,j+1} + B\,P^{(k+1)}_{i,j-1} + C\,P^{(k)}_{i+1,j} + D\,P^{(k+1)}_{i-1,j} - E \cdot (6\mu U)}$$

Applying successive over relaxation

$$\boxed{{P_{i,j}}^{(k+1)} = (1-\omega)P^{(k)}_{i,j} + \omega\, {P_{i,j}}^{(k+1)}}$$

Showing How Points are Represented in Staggered Grid Method



Initial and boundary conditions:

Initial condition :

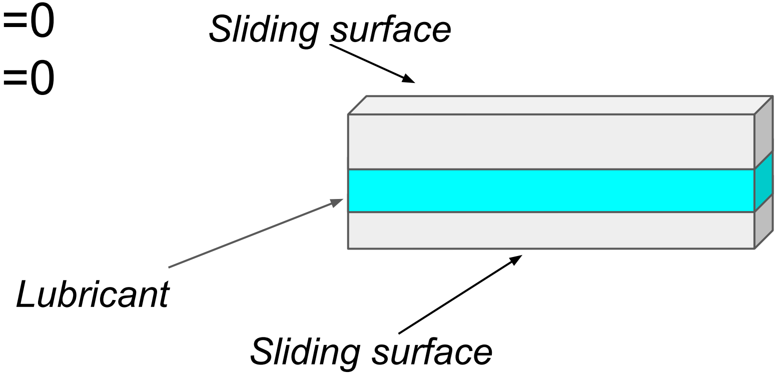
Pressure(P(i,j))=0

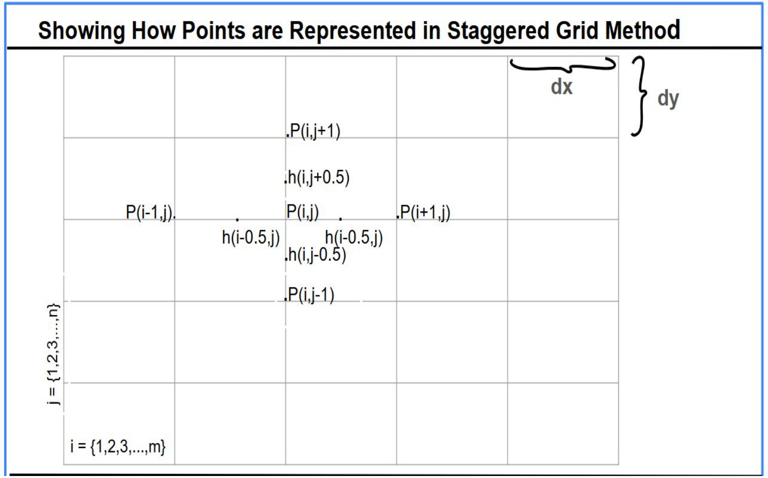
Periodic boundary condition :

P(1,j) = P(m,j)

P(i,1)=0

P(i,n)=0

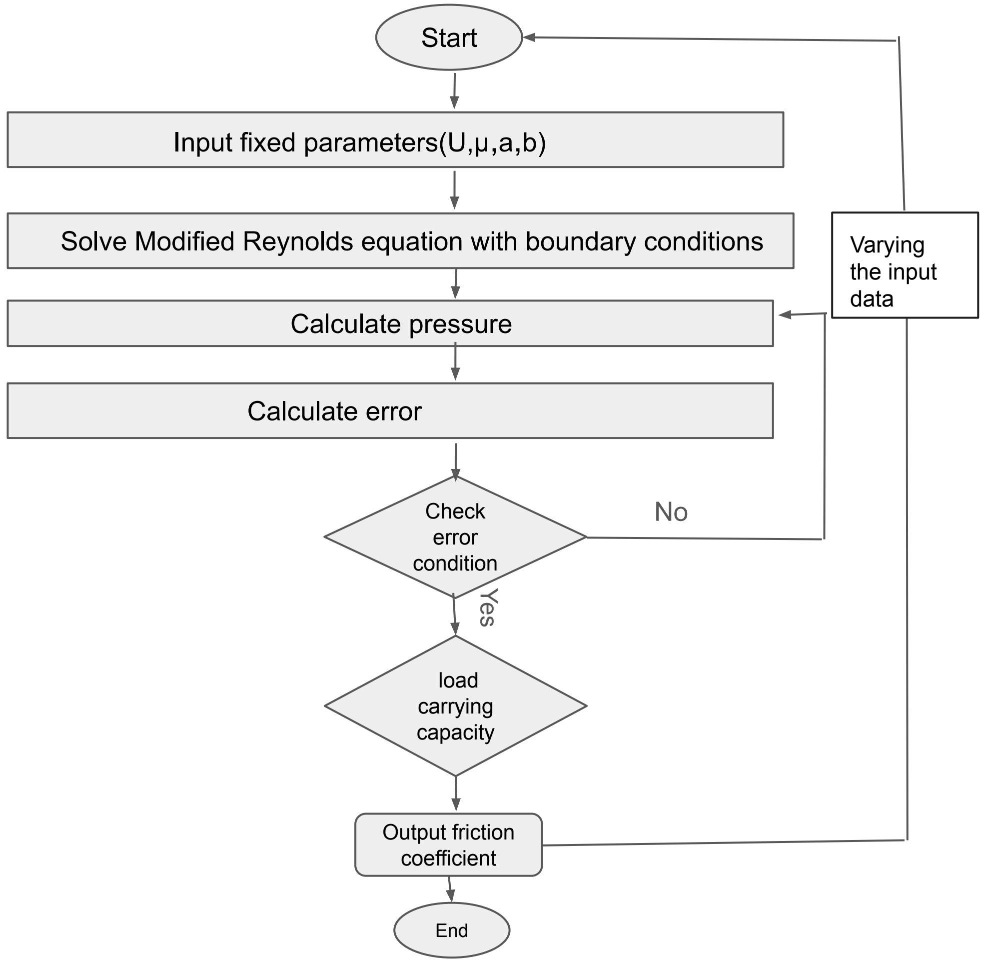




Input data :

- Coefficient of viscosity
- Velocity of the moving plate
- Number of texture
- Height/depth of the texture
- Film thickness

Methodology :



Calculation for

- Load support: $W = \int_0^L \int_0^B P(x,y)\,\mathrm{d}y\,\mathrm{d}x$
- Coefficient of friction

  f =F/W

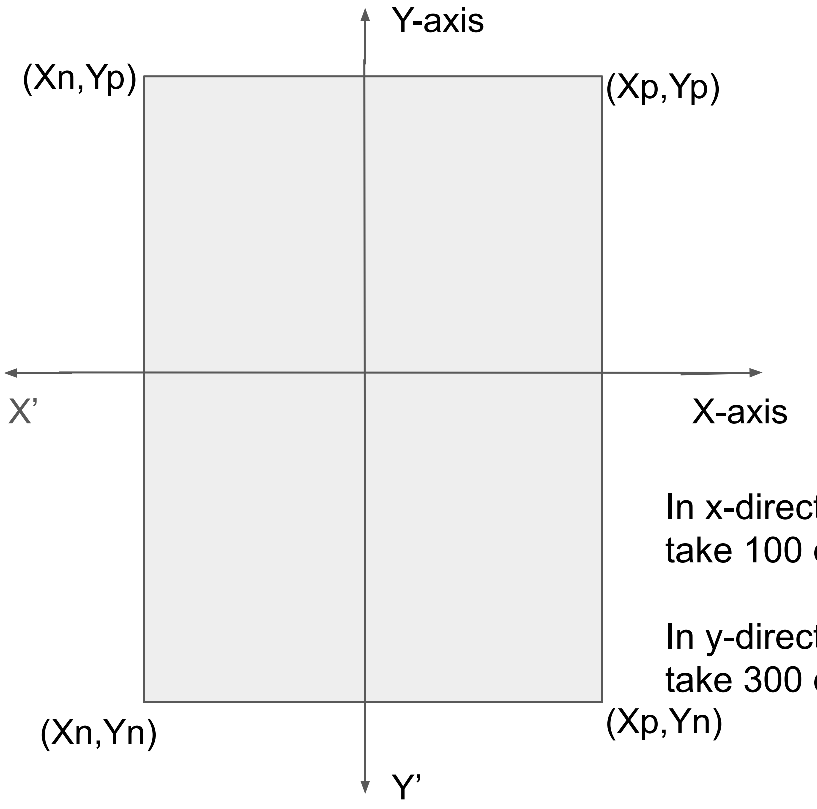
where F is the average shear stress on the fluid and F= $\mu \cdot u \cdot \left(\frac{d}{b} + \frac{1-d}{a+b}\right)$

Error condition :

Error : $\sum_i \sum_j \left| \frac{P_{i,j}^{(k+1)} - P_{i,j}^{(k)}}{P_{i,j}^{(k+1)}} \right| < \varepsilon$

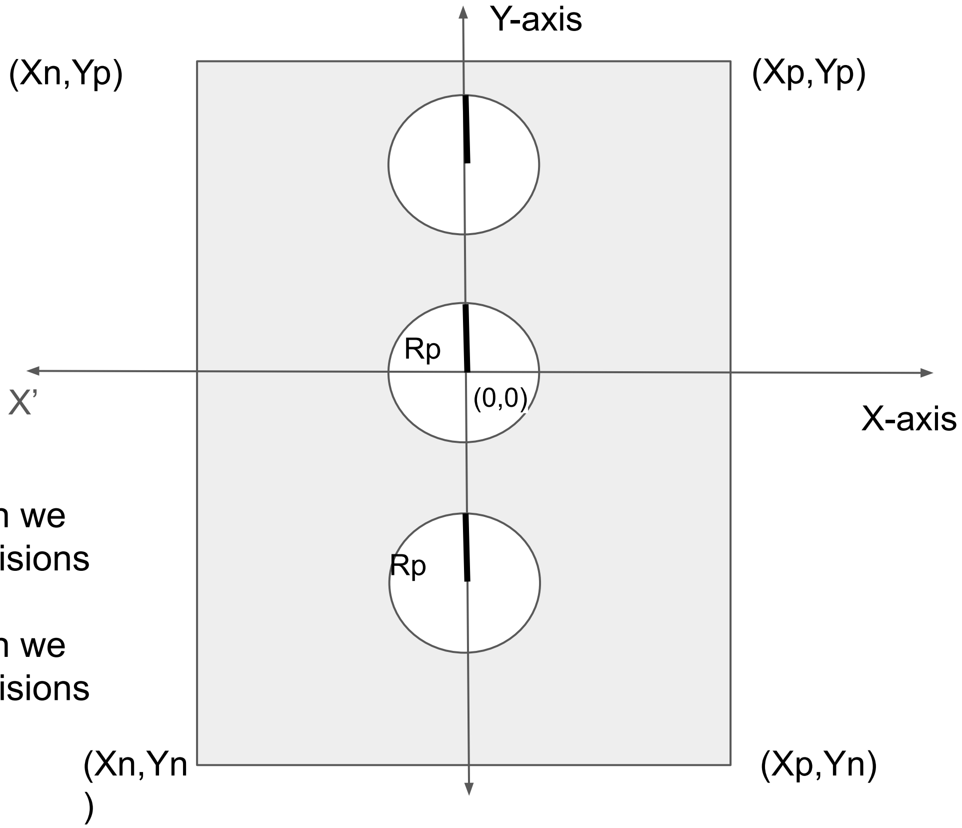
If Error <1e-5 .We proceed to next step.

Representation Of Only The Unit Cell In X and Y- Coordinates

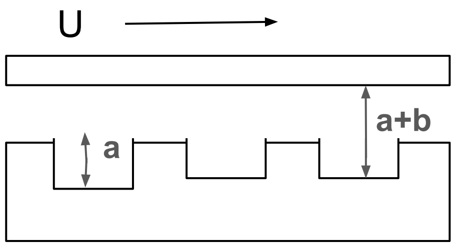


In x-direction we take 100 divisions

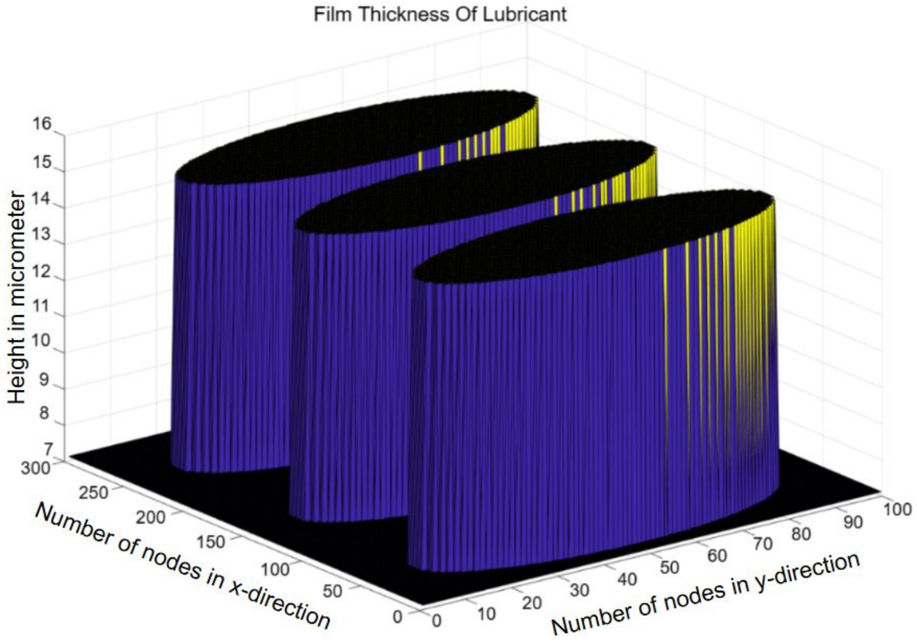
In y-direction we take 300 divisions

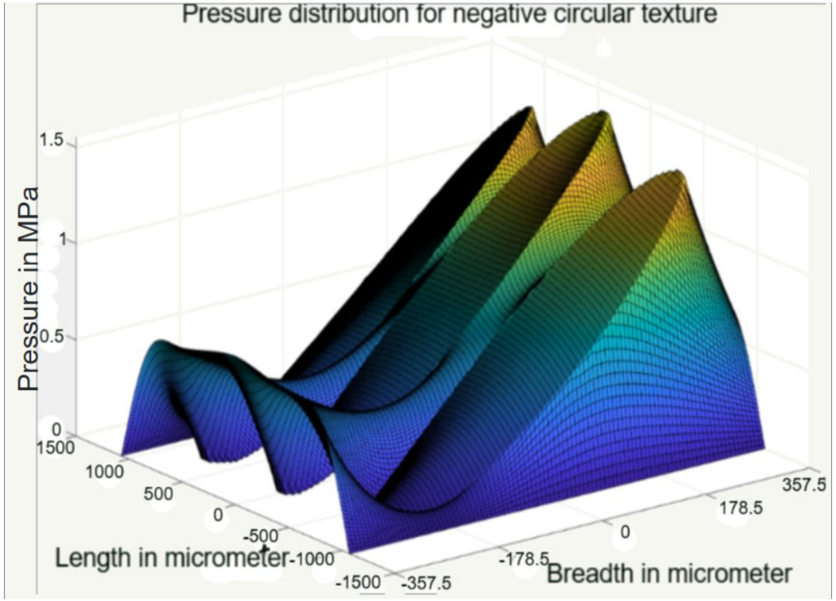


Negative circular texture:

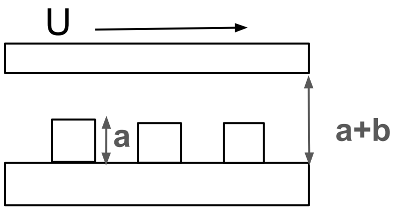


a = 7.8 ; Texture Depth in micrometer
b = 7.0 ; Minimum Film thickness in micrometer
μ =41.98 x10^(-9) MPa*s Viscosity
U = 6649704.76 Micrometer per second Velocity of moving plate
Unit cell length =2145 Micrometer
Unit cell breadth = 715 Micrometer
Radius=275 Micrometer
Average load support =**0.6404 MPa**
Coefficient of friction = **0.04456**

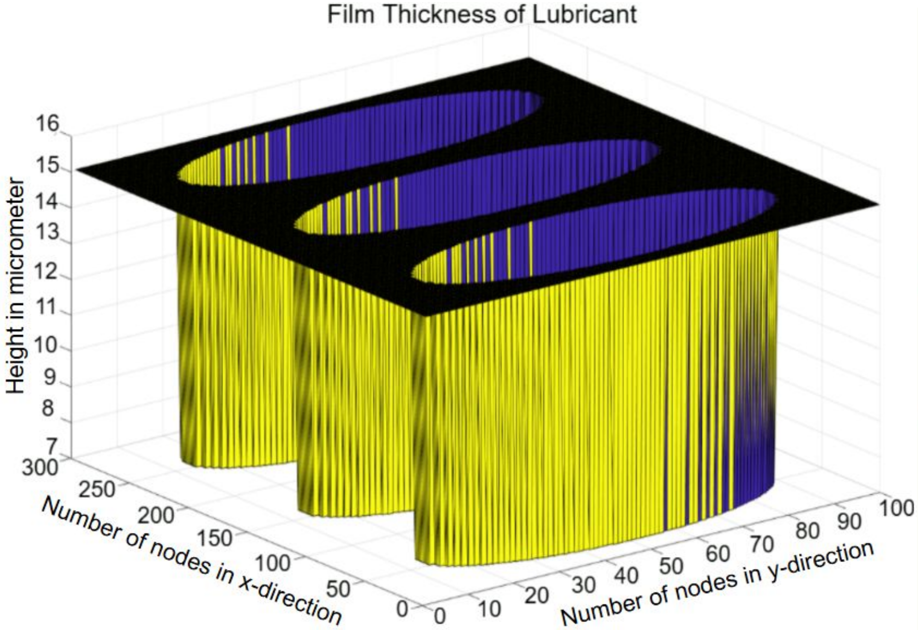


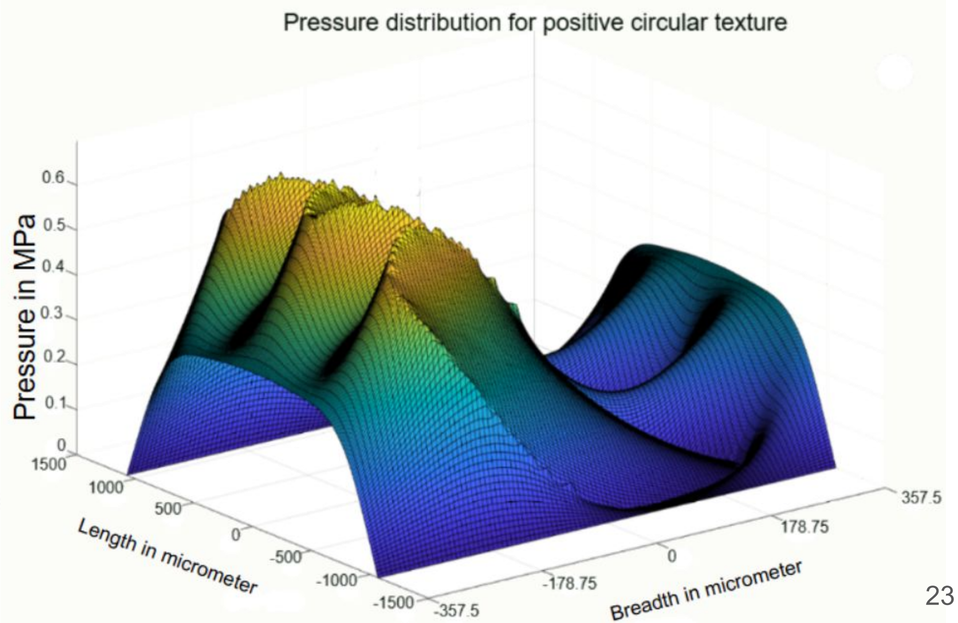


Positive circular texture:

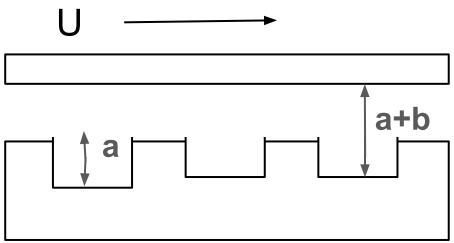


a = 7.8 ; Texture Depth in micrometer
b = 7.0 ; Minimum Film thickness in micrometer
μ =41.98 x10^(-9)MPa*s viscosity
U = 6649704.76 Micrometer per second Velocity of moving plate
Radius=275 Micrometer
Average load support =**0.2727 MPa**
Coefficient of friction =**0.1045**

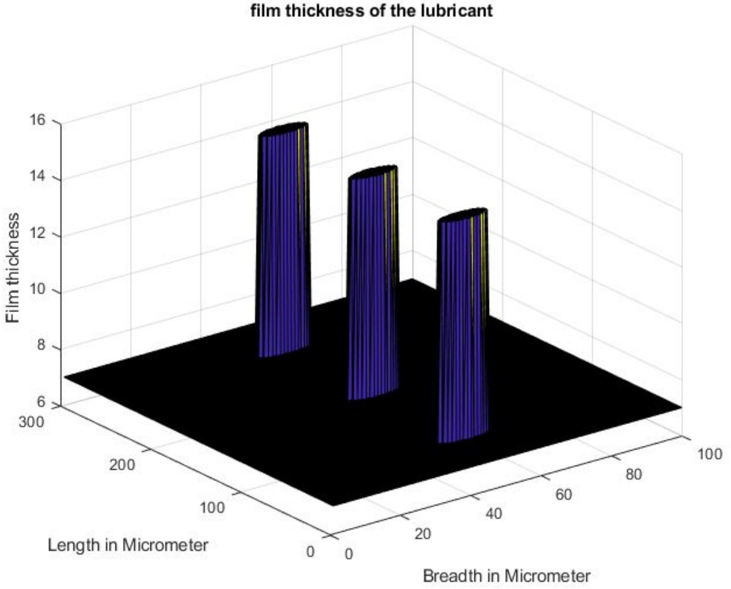


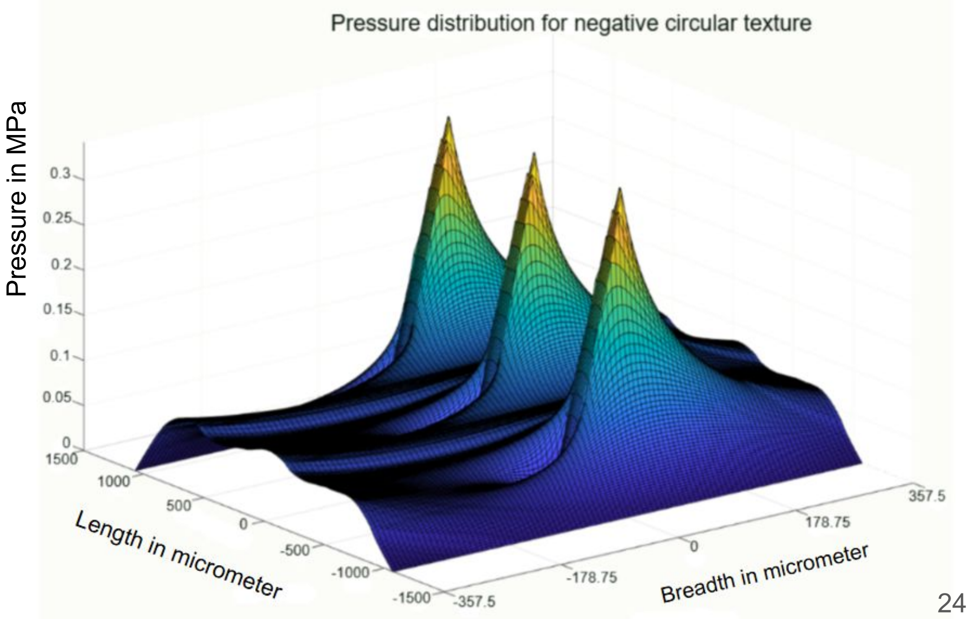


Small negative circular texture :

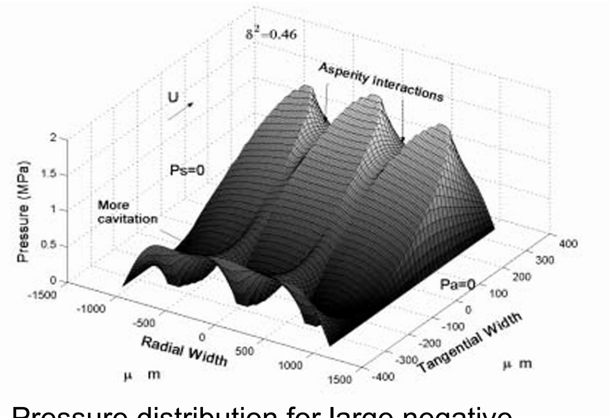


a = 7.8 ; Texture Depth in micrometer
b = 7.0 ; Minimum Film thickness in micrometer
μ =41.98 x10^(-9)MPa*s ;Viscosity
U = 6649704.76 Micrometer per second ;Velocity of moving plate
Radius =50 Micrometer
Average load support =**0.0668MPa**
Coefficient of friction = **0.2745**

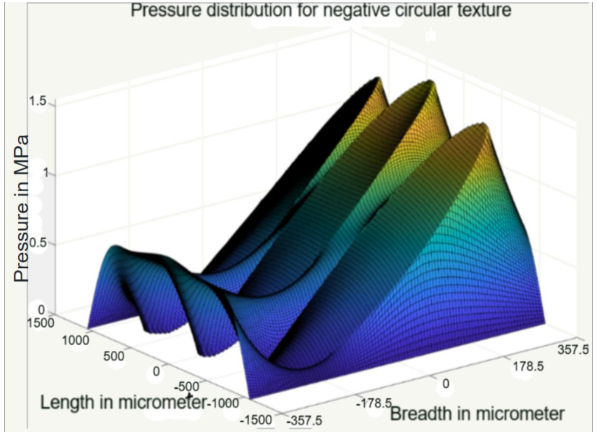




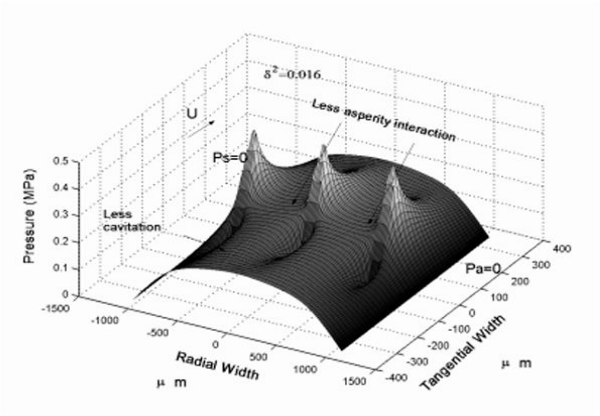
Comparing with sample results for circular texture distribution:



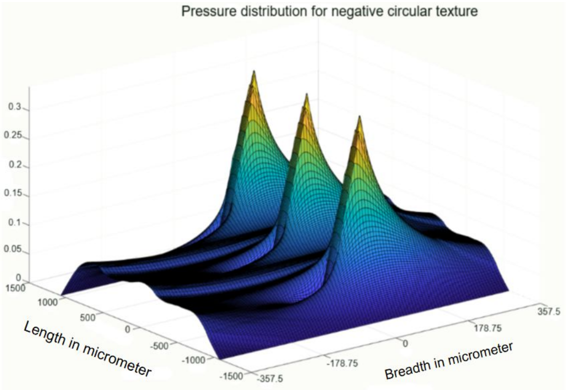
Pressure distribution for large negative radial texture



Pressure distribution obtained



Pressure distribution for small negative radial texture



Pressure distribution obtained

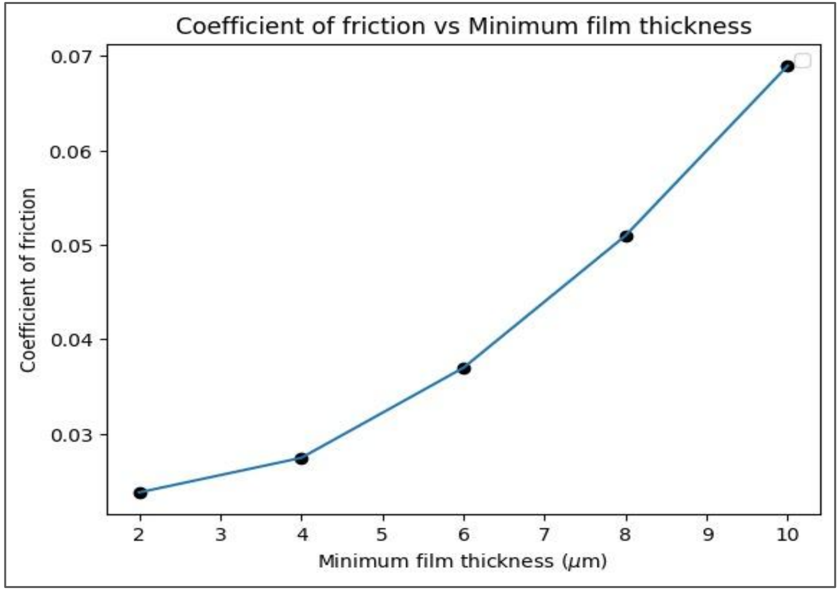
Comparing the results:

Property	Wide positive texture	Wide negative texture	Small negative texture	Small positive texture
Radius	275μm	275μm	50μm	50μm
Coefficient of friction	0.1045	0.04456	0.2745	0.6117
Average load support	0.2727MPa	0.6404MPa	0.0668MPa	0.0309 MPa

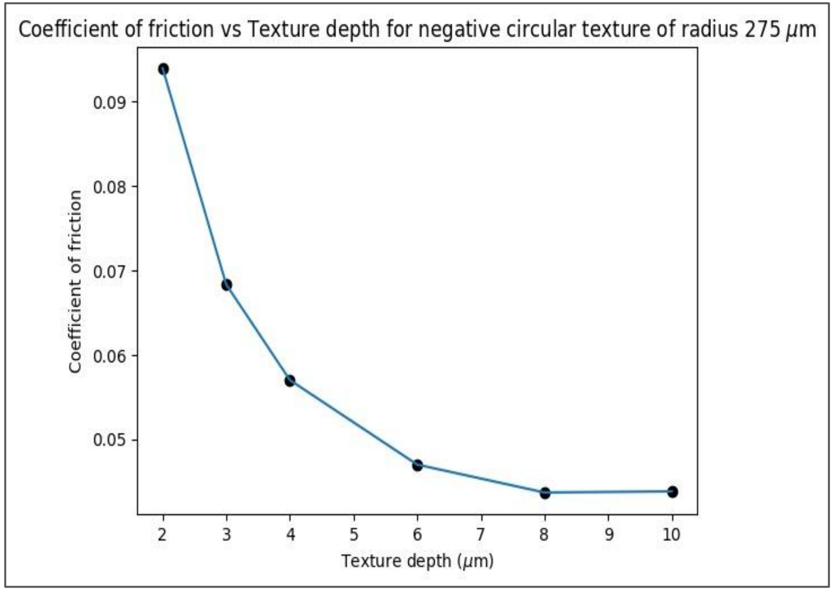
From the above data wide negative circular texture gives the least coefficient of friction

Effects of texture depth and minimum film thickness on coefficient of friction:

a = 7.8 ; Texture Depth in micrometer
μ =41.98 x10^(-9)MPa*s; Viscosity
U = 6649704.76 micrometer per second ; Velocity of moving plate



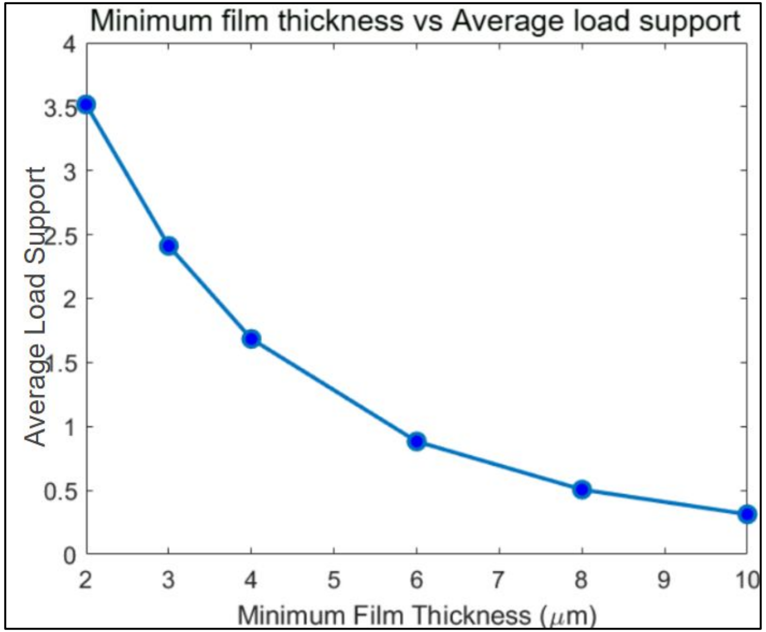
b = 7.0 ; Minimum Film thickness in micrometer
μ =41.98 x10^(-9)MPa*s ; Viscosity
U = 6649704.76 micrometer per second ; Velocity of moving plate



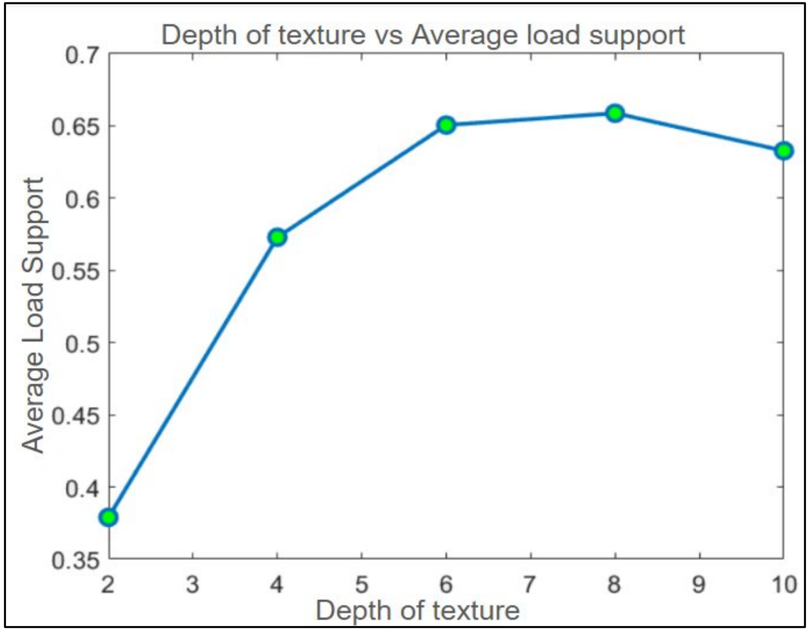
Note : 1. Coefficient of friction increases with increase in minimum film thickness.
2.Increasing texture depth reduces the coefficient of friction

Effects of texture depth and minimum film thickness on average load support :

a = 7.8 ; Texture Depth in Micrometer
μ =41.98x10^(-9)MPa*s ; Viscosity
U = 6649704.76 micrometer per second ;Velocity of moving plate



b = 7.0 ; Minimum Film thickness in Micrometer
μ =41.98 x10^(-9)MPa*s ; Viscosity
U = 6649704.76 micrometer per second ; Velocity of moving plate

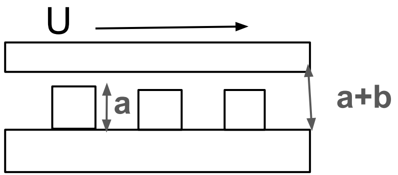


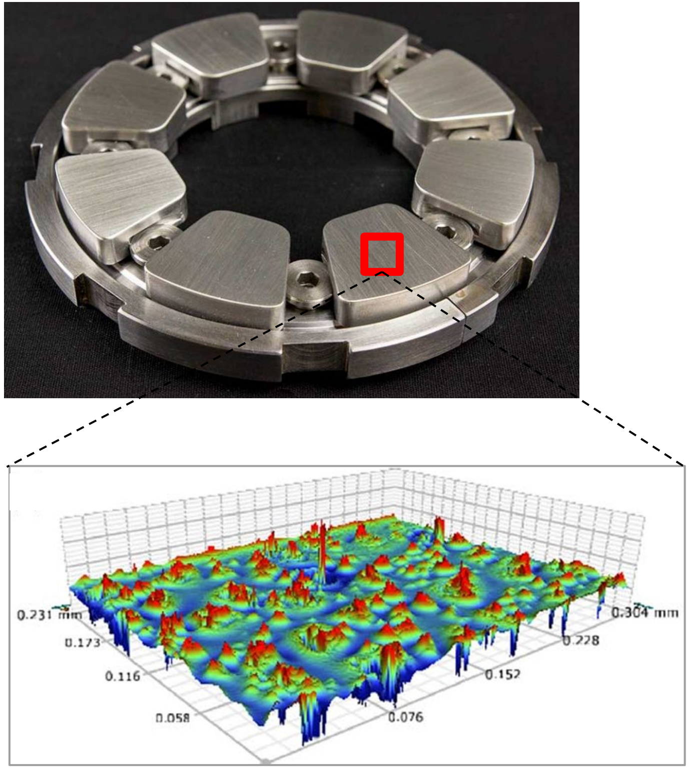
Note : 1. Average load support decreases as increasing the minimum film thickness
2.Increasing texture depth increases average load support

Modelling Real Engineering Surface:

- In real-world applications, surfaces are never perfectly smooth. When the fluid film thickness becomes comparable to the roughness of the surfaces, textures come into contact.

- The contact factor helps model the load carried by these textures, which is not captured by the standard Reynolds equation.





Factors:

Contact factor(Φ_c) : A parameter that accounts for the effect of surface roughness and asperity contact on pressure build up in lubricant.

Pressure flow factor(Φ_x, Φ_y) : The pressure flow factor is a parameter that accounts for the effect of surface roughness on the pressure-driven flow.

Shear stress factor(Φ_s) : The shear stress factor is related to the shear stress acting on the lubricant film, which arises due to the relative motion of the surfaces and is influenced by the viscosity of the lubricant.

Composite Roughness(σ) : Composite roughness refers to the irregularities and small-scale variations found on a physical surface

Modified Reynolds equation :

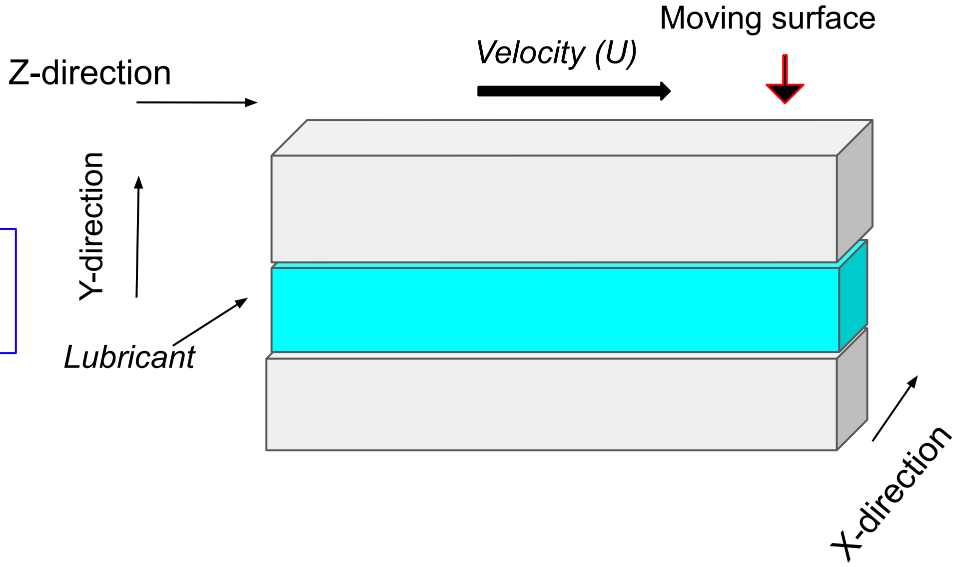
$$\frac{\partial}{\partial x}\left(h^3\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial y}\left(h^3\frac{\partial P}{\partial y}\right)=6\mu U\frac{\partial h}{\partial x}$$

Equation used in this semester

$$\frac{\partial}{\partial x}\left(\phi_x\frac{h^3}{\mu}\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial y}\left(\phi_y\frac{h^3}{\mu}\frac{\partial P}{\partial y}\right)=6U\phi_c\frac{\partial h}{\partial x}+6U\sigma\frac{\partial \phi_s}{\partial x}$$

Symbol	Description
ϕ_x	Pressure flow factor in x direction
ϕ_z	Pressure flow factor in z direction
ϕ_c	Contact factor
ϕ_s	Shear stress factor
h	Film thickness
σ	Composite roughness
μ	Coefficient of viscosity
U	Velocity of the moving plate

Table 1: List of Symbols



Modified Reynolds equation :

Modified Reynolds with contact factor

$$\frac{\partial}{\partial x}\left(\phi_x \frac{h^3}{\mu}\frac{\partial P}{\partial x}\right) + \frac{\partial}{\partial y}\left(\phi_y \frac{h^3}{\mu}\frac{\partial P}{\partial y}\right) = 6U\phi_c\frac{\partial h}{\partial x} + 6U\sigma\frac{\partial \phi_s}{\partial x} \qquad (1)$$

Discretized Form

Left-Hand Side (LHS)

$$\frac{\partial}{\partial x}\left(\phi_x \frac{h^3}{\mu}\frac{\partial P}{\partial x}\right) = \frac{1}{\mu\Delta x}\left[\phi_{x,i+\frac{1}{2},j}h^3_{i+\frac{1}{2},j}\cdot\frac{P_{i+1,j}-P_{i,j}}{\Delta x} - \phi_{x,i-\frac{1}{2},j}h^3_{i-\frac{1}{2},j}\cdot\frac{P_{i,j}-P_{i-1,j}}{\Delta x}\right]$$

$$\frac{\partial}{\partial y}\left(\phi_y \frac{h^3}{\mu}\frac{\partial P}{\partial y}\right) = \frac{1}{\mu\Delta y}\left[\phi_{y,i,j+\frac{1}{2}}h^3_{i,j+\frac{1}{2}}\cdot\frac{P_{i,j+1}-P_{i,j}}{\Delta y} - \phi_{y,i,j-\frac{1}{2}}h^3_{i,j-\frac{1}{2}}\cdot\frac{P_{i,j}-P_{i,j-1}}{\Delta y}\right]$$

Right-Hand Side (RHS)

$$6U\phi_c\frac{\partial h}{\partial x} = 6U\cdot\phi_{c,i,j}\cdot\frac{h_{i+1/2,j}-h_{i-1/2,j}}{\Delta x}$$

$$6U\sigma\frac{\partial \phi_s}{\partial x} = 6U\sigma\cdot\frac{\phi_{s,i+1/2,j}-\phi_{s,i-1/2,j}}{\Delta x}$$

Therefore equation (1) becomes,

$$\begin{aligned}&\frac{1}{\Delta x}\left[\phi_{x,i+\frac{1}{2},j}\,h^3_{i+\frac{1}{2},j}\cdot\frac{P_{i+1,j}-P_{i,j}}{\Delta x} - \phi_{x,i-\frac{1}{2},j}\,h^3_{i-\frac{1}{2},j}\cdot\frac{P_{i,j}-P_{i-1,j}}{\Delta x}\right]\\ &+\frac{1}{\Delta y}\left[\phi_{y,i,j+\frac{1}{2}}\,h^3_{i,j+\frac{1}{2}}\cdot\frac{P_{i,j+1}-P_{i,j}}{\Delta y} - \phi_{y,i,j-\frac{1}{2}}\,h^3_{i,j-\frac{1}{2}}\cdot\frac{P_{i,j}-P_{i,j-1}}{\Delta y}\right]\\ &= 6U\mu\cdot\phi_{c,i,j}\cdot\frac{h_{i+1/2,j}-h_{i-1/2,j}}{\Delta x} + 6U\mu\sigma\cdot\frac{\phi_{s,i+1/2,j}-\phi_{s,i-1/2,j}}{\Delta x}\end{aligned}$$

Symbol	Description
ϕ_x	Pressure flow factor in x direction
ϕ_z	Pressure flow factor in z direction
ϕ_c	Contact factor
ϕ_s	Shear stress factor
h	Film thickness
σ	Composite roughness
μ	Coefficient of viscosity
U	Velocity of the moving plate

Table 1: List of Symbols

Modified Reynolds equation :

Rearranging all terms containing $P_{i,j}$:

$$-\left(\frac{\phi_{x,i+\frac{1}{2},j}h^3_{i+\frac{1}{2},j}}{\Delta x^2}+\frac{\phi_{x,i-\frac{1}{2},j}h^3_{i-\frac{1}{2},j}}{\Delta x^2}+\frac{\phi_{y,i,j+\frac{1}{2}}h^3_{i,j+\frac{1}{2}}}{\Delta y^2}+\frac{\phi_{y,i,j-\frac{1}{2}}h^3_{i,j-\frac{1}{2}}}{\Delta y^2}\right)P_{i,j}$$

$$=\frac{\phi_{x,i+\frac{1}{2},j}h^3_{i+\frac{1}{2},j}}{\Delta x^2}P_{i+1,j}-\frac{\phi_{x,i-\frac{1}{2},j}h^3_{i-\frac{1}{2},j}}{\Delta x^2}P_{i-1,j}+\frac{\phi_{y,i,j+\frac{1}{2}}h^3_{i,j+\frac{1}{2}}}{\Delta y^2}P_{i,j+1}-\frac{\phi_{y,i,j-\frac{1}{2}}h^3_{i,j-\frac{1}{2}}}{\Delta y^2}P_{i,j-1}$$

$$+6U\mu\left(\frac{h_{i+\frac{1}{2},j}-h_{i-\frac{1}{2},j}}{\Delta x}\phi_{c,i,j}+\sigma\frac{\phi_{s,i+\frac{1}{2},j}-\phi_{s,i-\frac{1}{2},j}}{\Delta x}\right)$$

Rearranging the terms , we get

$$P_{i,j}=\frac{\begin{gathered}\frac{\phi_{x,i+\frac{1}{2},j}\,h^3_{i+\frac{1}{2},j}}{\Delta x^2}P_{i+1,j}+\frac{\phi_{x,i-\frac{1}{2},j}\,h^3_{i-\frac{1}{2},j}}{\Delta x^2}P_{i-1,j}\\+\frac{\phi_{y,i,j+\frac{1}{2}}\,h^3_{i,j+\frac{1}{2}}}{\Delta y^2}P_{i,j+1}+\frac{\phi_{y,i,j-\frac{1}{2}}\,h^3_{i,j-\frac{1}{2}}}{\Delta y^2}P_{i,j-1}\\-6U\mu\left(\phi_{c,i,j}\frac{h_{i+\frac{1}{2},j}-h_{i-\frac{1}{2},j}}{\Delta x}+\sigma\frac{\phi_{s,i+\frac{1}{2},j}-\phi_{s,i-\frac{1}{2},j}}{\Delta x}\right)\end{gathered}}{\begin{gathered}\frac{\phi_{x,i+\frac{1}{2},j}\,h^3_{i+\frac{1}{2},j}}{\Delta x^2}+\frac{\phi_{x,i-\frac{1}{2},j}\,h^3_{i-\frac{1}{2},j}}{\Delta x^2}\\+\frac{\phi_{y,i,j+\frac{1}{2}}\,h^3_{i,j+\frac{1}{2}}}{\Delta y^2}+\frac{\phi_{y,i,j-\frac{1}{2}}\,h^3_{i,j-\frac{1}{2}}}{\Delta y^2}\end{gathered}}$$

Modified Reynolds equation :

$$\Rightarrow P_{i,j} = \frac{\begin{aligned}&\phi_{x,i+\frac{1}{2},j}\, h^3_{i+\frac{1}{2},j}\, \Delta y^2\, P_{i+1,j} + \phi_{x,i-\frac{1}{2},j}\, h^3_{i-\frac{1}{2},j}\, \Delta y^2\, P_{i-1,j}\\ &+ \phi_{y,i,j+\frac{1}{2}}\, h^3_{i,j+\frac{1}{2}}\, \Delta x^2\, P_{i,j+1} + \phi_{y,i,j-\frac{1}{2}}\, h^3_{i,j-\frac{1}{2}}\, \Delta x^2\, P_{i,j-1}\\ &- 6U\mu\Delta x\Delta y^2 \left(\phi_{c,i,j} h_{i+\frac{1}{2},j} - h_{i-\frac{1}{2},j} + \sigma\phi_{s,i+\frac{1}{2},j} - \phi_{s,i-\frac{1}{2},j}\right)\end{aligned}}{\begin{aligned}&\phi_{x,i+\frac{1}{2},j}\, h^3_{i+\frac{1}{2},j}\, \Delta y^2 + \phi_{x,i-\frac{1}{2},j}\, h^3_{i-\frac{1}{2},j}\, \Delta y^2\\ &+ \phi_{y,i,j+\frac{1}{2}}\, h^3_{i,j+\frac{1}{2}}\, \Delta x^2 + \phi_{y,i,j-\frac{1}{2}}\, h^3_{i,j-\frac{1}{2}}\, \Delta x^2\end{aligned}}$$

Using Gauss Seidel Iterative scheme,to solve it.

$$P^{(k+1)}_{i,j} = \frac{\begin{aligned}&\phi_{x,i+\frac{1}{2},j}\, h^3_{i+\frac{1}{2},j}\, \Delta y^2\, P^{(k)}_{i+1,j}\\ &+ \phi_{x,i-\frac{1}{2},j}\, h^3_{i-\frac{1}{2},j}\, \Delta y^2\, P^{(k+1)}_{i-1,j}\\ &+ \phi_{y,i,j+\frac{1}{2}}\, h^3_{i,j+\frac{1}{2}}\, \Delta x^2\, P^{(k)}_{i,j+1}\\ &+ \phi_{y,i,j-\frac{1}{2}}\, h^3_{i,j-\frac{1}{2}}\, \Delta x^2\, P^{(k+1)}_{i,j-1}\\ &- 6U\mu\Delta x\Delta y^2 \left(\phi_{c,i,j}(h_{i+\frac{1}{2},j} - h_{i-\frac{1}{2},j}) + \sigma(\phi_{s,i+\frac{1}{2},j} - \phi_{s,i-\frac{1}{2},j})\right)\end{aligned}}{\begin{aligned}&\phi_{x,i+\frac{1}{2},j}\, h^3_{i+\frac{1}{2},j}\, \Delta y^2\\ &+ \phi_{x,i-\frac{1}{2},j}\, h^3_{i-\frac{1}{2},j}\, \Delta y^2\\ &+ \phi_{y,i,j+\frac{1}{2}}\, h^3_{i,j+\frac{1}{2}}\, \Delta x^2\\ &+ \phi_{y,i,j-\frac{1}{2}}\, h^3_{i,j-\frac{1}{2}}\, \Delta x^2\end{aligned}}$$

Initial and boundary conditions:

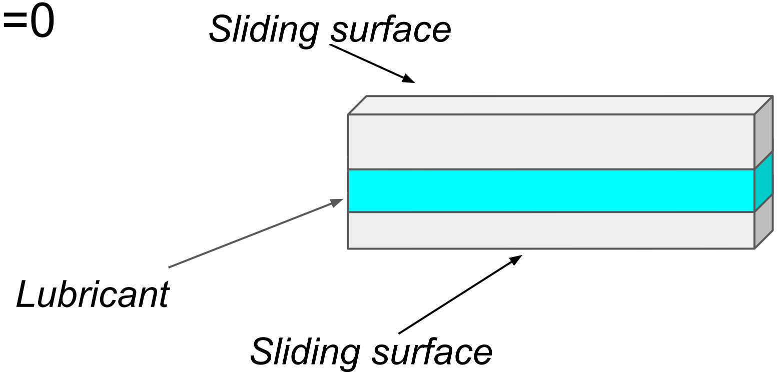
Initial condition : Pressure(P(i,j))=0

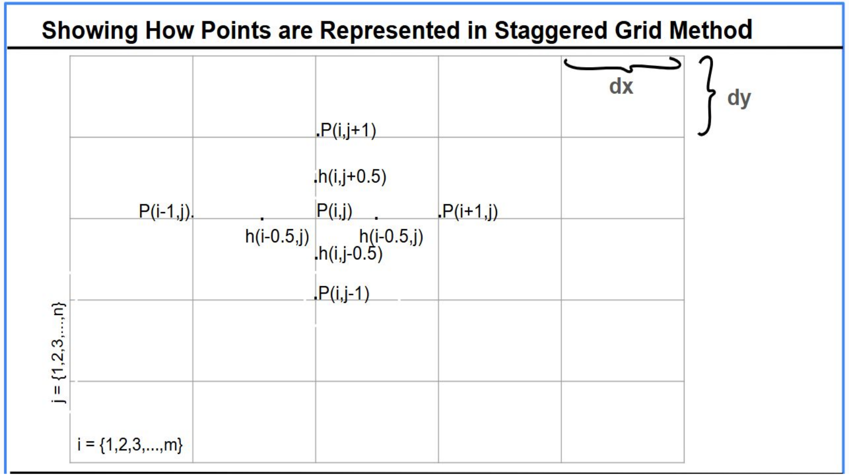
Periodic boundary condition :

P(1,j) = P(m,j)

P(i,1)=0

P(i,n)=0

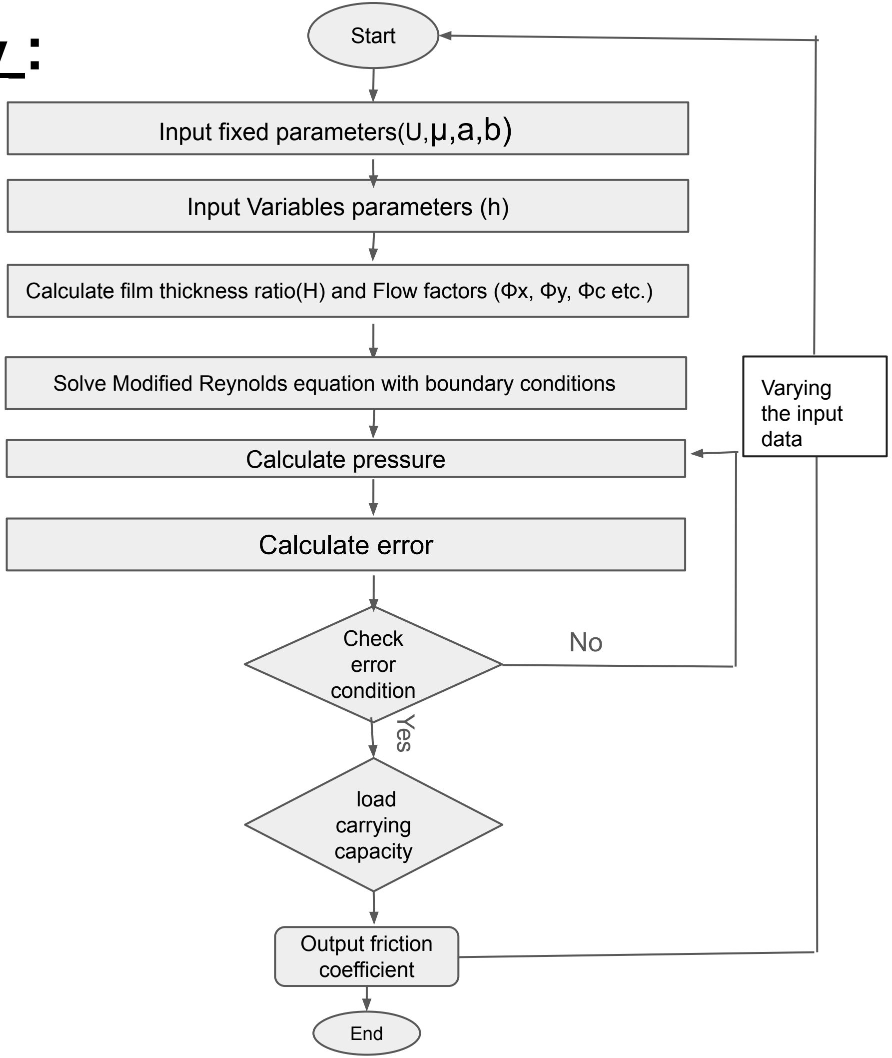




Input data :

- Coefficient of viscosity
- Velocity of the moving plate
- Number of texture
- Height/depth of the texture
- Film thickness

Methodology :



Calculation for

- Load support: $W = \int_0^L \int_0^B P(x,y)\,\mathrm{d}y\,\mathrm{d}x$
- Coefficient of friction
  f =F/W

where F is the average shear stress on the fluid and F= $\mu \cdot u \cdot \left(\frac{d}{b} + \frac{1-d}{a+b}\right)$

Error condition :

Error = $\sum_i \sum_j \left(\frac{P^{(k+1)}(i,j) - P^{(k)}(i,j)}{P^{(k)}(i,j)}\right)$

If Error <1e-5 .We proceed to next step.

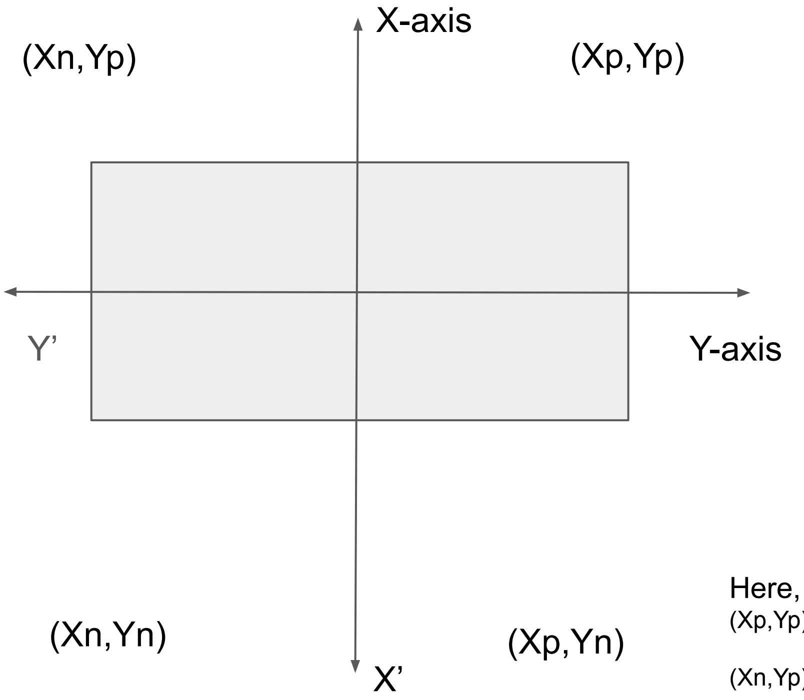
The roughness effects on lubricant flow can be attributed to four factors Φ_x, Φ_z, Φ_s and Φ_c. All these factors depends on film thickness ratio (H=h/σ).

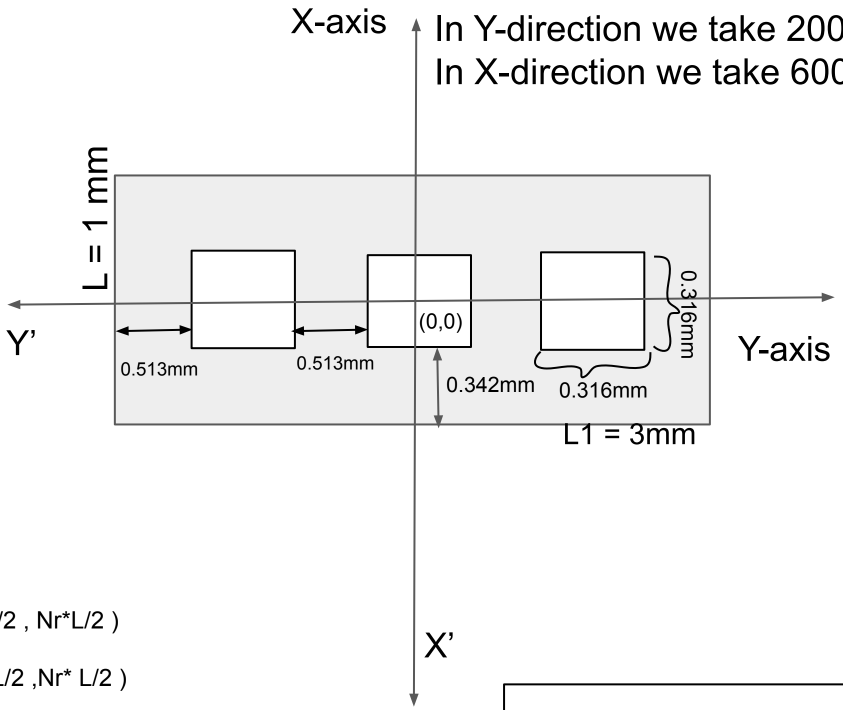
$$\phi_x = \phi_y = 1 - 0.9e^{-0.56H}$$

$$\phi_s = \begin{cases} 1.899H^{0.98}e^{-0.92H+0.05H^2}, & \text{if } H \leq 5 \\ 1.126e^{-0.25H}, & \text{if } H > 5 \end{cases}$$

$$\phi_c = \begin{cases} e^{-0.6912+0.782H-0.304H^2+0.0401H^3}, & \text{if } 0 \leq H < 3 \\ 1, & \text{if } H \geq 3 \end{cases}$$

Representation Of The Unit Cell In X and Y- Coordinates





In Y-direction we take 200 divisions
In X-direction we take 600 divisions

Here,
(Xp,Yp) = (L/2 , Nr*L/2)

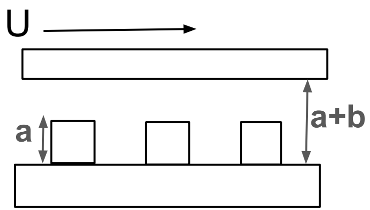
(Xn,Yp) = (-L/2 ,Nr* L/2)

(Xn,Yn) = (-L/2 , -Nr*L/2)

(Xp,Yn) = (L/2 , -Nr*L/2)

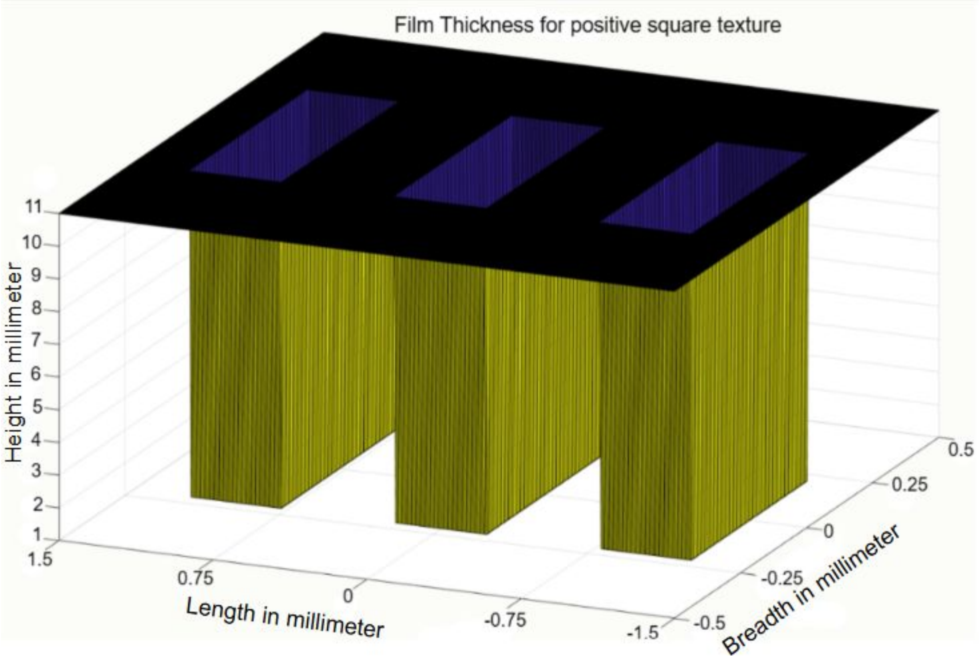
Texture area fraction (d)= 0.1
Number of texture(Nr)=3

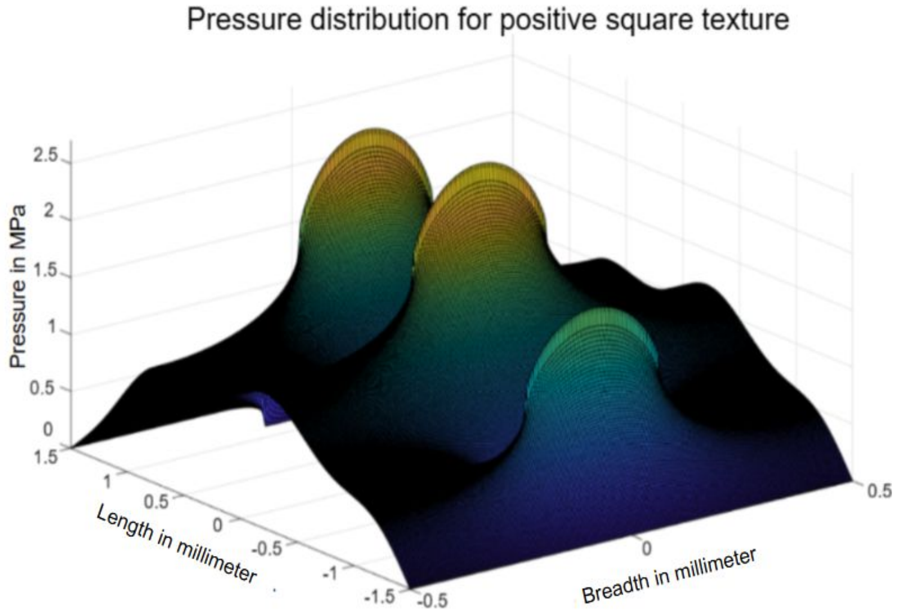
Positive square Texture:



Texture area fraction (d)= 0.1
Number of texture(Nr)=3

Viscosity = 1.21e-7 MPa.s
Velocity(U) = 1670 mm/s
Minimum film thickness = 10^{-2} mm
Depth of the texture = 10^{-3} mm
Average load support =0.822MPa
Coefficient of friction = 0.04468565
Composite roughness =0.546e-3mm

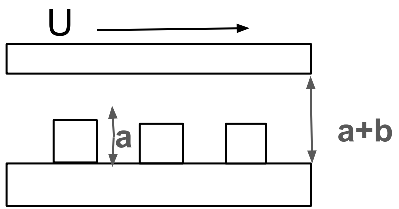


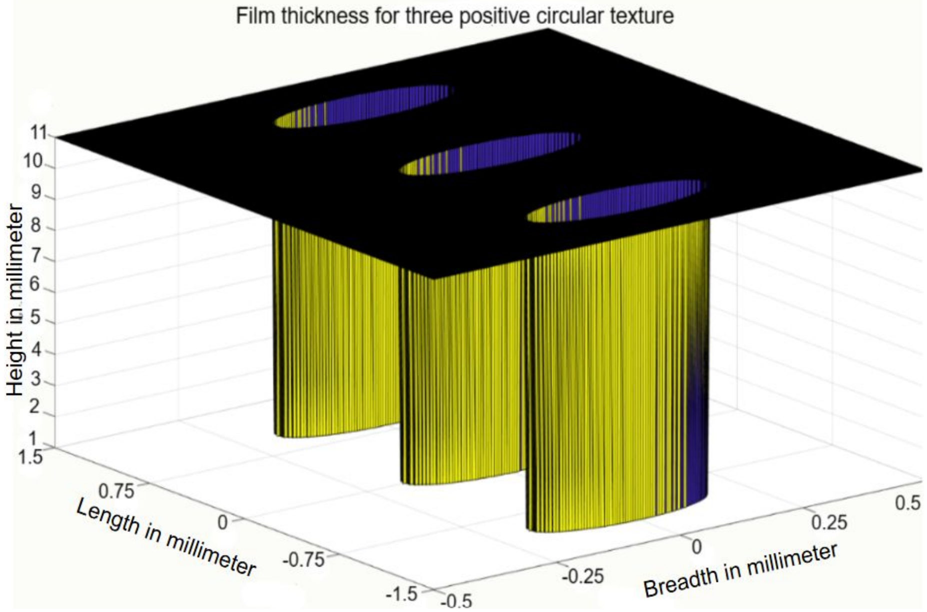


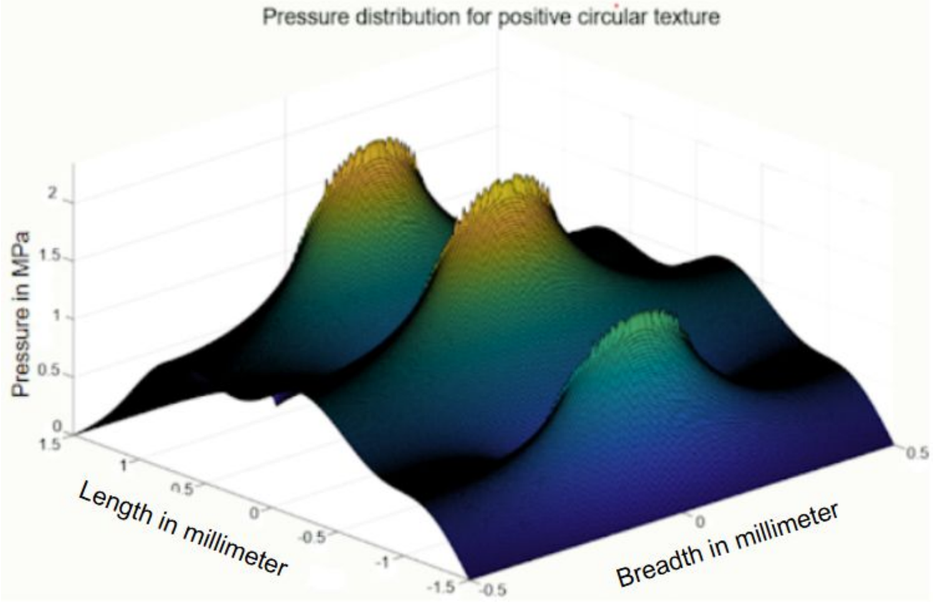
Positive circular texture:

Texture area fraction (d)= 0.1
Number of texture(Nr)=3

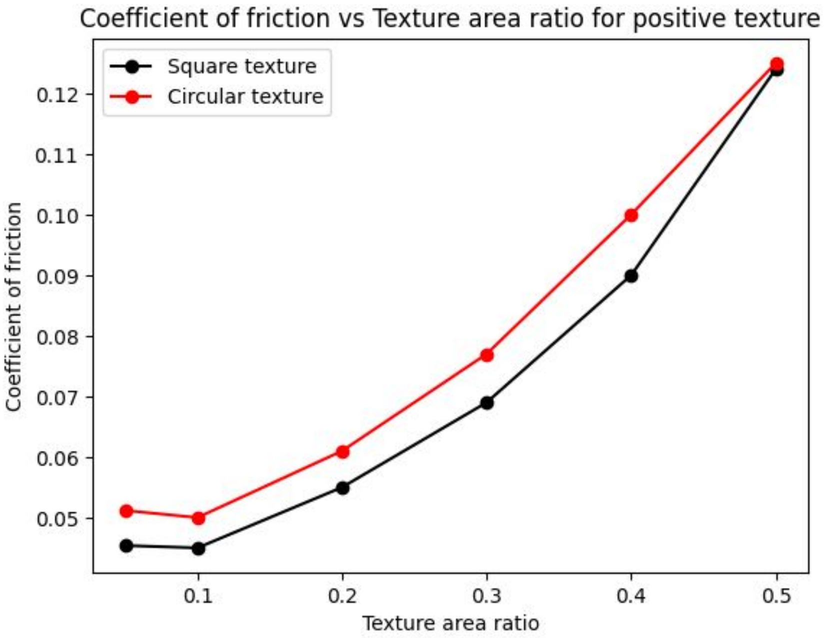
Viscosity = 1.21e-7 MPa.s
Velocity(U) = 1670 mm/s
Minimum film thickness = 10^{-2} mm
Depth of the texture = 10^{-3} mm
Average load support =0.7324MPa
Coefficient of friction =0.50162
Composite roughness =0.546e-3mm

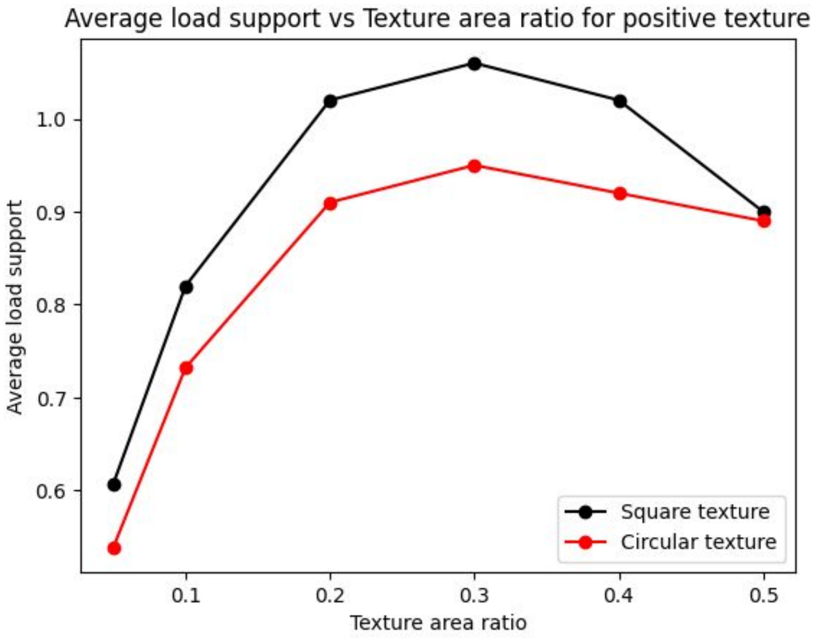






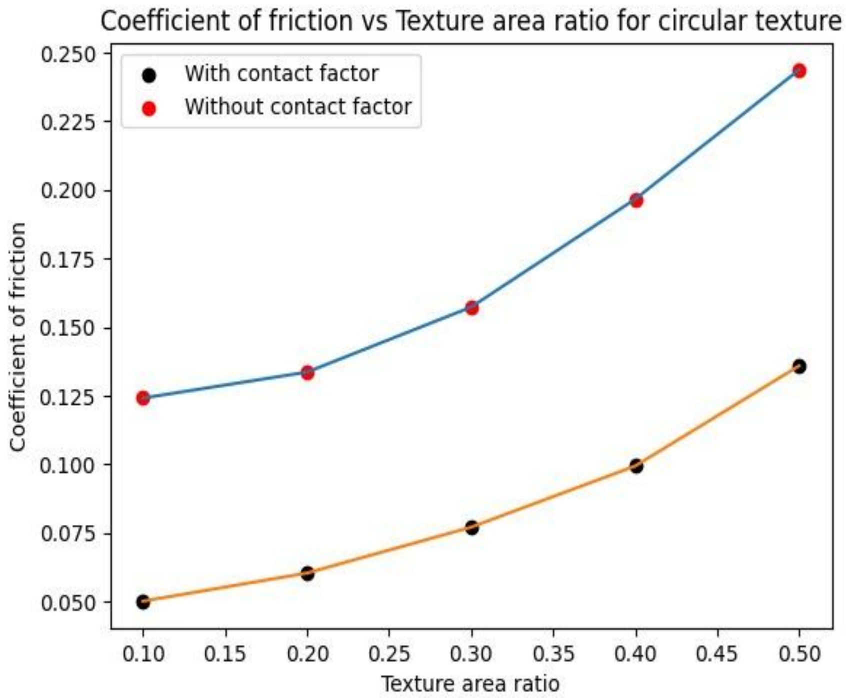
Comparing the results obtained for positive circular textured and square textured model

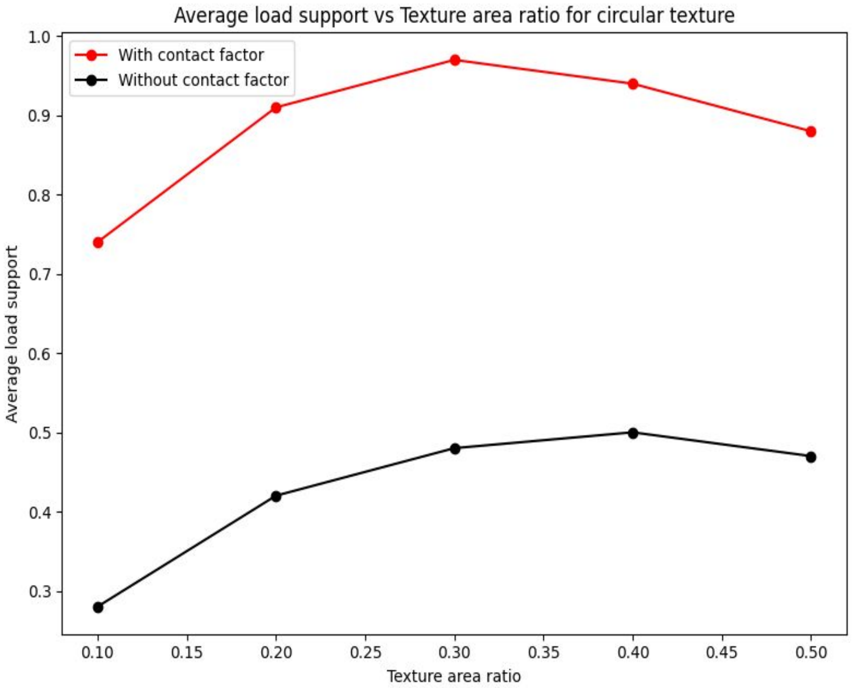




Note : 1. Coefficient of friction increases with increase in texture area ratio.
2.Increasing texture area ratio increases the average load support upto an optimum point

Comparing the results obtained for positive circular textured model with contact factor and without contact factor





Note : 1.Adding contact factor improve the accuracy of the coefficient of friction and lubrication models used in the study

Conclusion :

1. From the above data, the wide negative circular texture gives the least coefficient of friction (0.04456) and the highest average load support (0.6404 MPa), making it the most effective among the textures studied.

2. It is observed that the coefficient of friction increases with minimum film thickness. As the film thickness increases from 2 µm to 10 µm, the friction coefficient rises steadily, reaching its highest value at the maximum thickness. This indicates that for the given texture depth (7.8 µm), viscosity (41.98×10^{-9} MPa·s), and velocity (6649704.76 µm/s), thinner lubricant films result in lower friction.

3. It is observed that the coefficient of friction decreases with increasing texture depth for the negative circular texture of radius 275 µm. As the texture depth increases from 2 µm to around 8 µm, the friction reduces significantly, after which it tends to stabilize. For the given conditions minimum film thickness of 7.0 µm, viscosity of 41.98×10^{-9} MPa·s, and plate velocity of 6649704.76 µm/.Deeper textures contribute to lower friction, suggesting an optimal range of texture depth beyond which further increase yields minimal benefit.

4. It is evident that average load support decreases with increasing minimum film thickness.At lower film thickness values , the load support is at its maximum, while it gradually declines to below 0.5 units as the film thickens to 10 µm. This indicates that thinner films are more effective in supporting load under the given texture depth of 7.8 µm, viscosity of 41.98×10^{-9} MPa·s, and sliding velocity of 6649704.76 µm/s.

5. Average load support initially increases with texture depth, reaching a peak around 7–8 µm, and then starts to decline slightly beyond that. This implies that for a minimum film thickness of 7 µm, there exists an optimal texture depth (8 µm) that maximizes load carrying capacity.

6. As the texture area ratio increases, the coefficient of friction increases for both textures, with square texture consistently showing lower friction. For average load support, it increases up to a peak around 0.3 texture area ratio and then decreases.

7. To improve the accuracy of the coefficient of friction and lubrication models used in the study, new dimensionless factors such as the contact factor, pressure flow factor,and shear stress factor were introduced.

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