Matrix groups and their homogeneous spaces

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Abstract

Matrix groups play a crucial role in understanding the structure and properties of symmetries in mathematics. These groups consist of matrices that, under matrix multiplication, form a group. Matrix groups are extensively studied in areas like geometry, physics, and representation theory, where they are used to represent transformations that preserve structures such as distances or angles. This project delves into the algebraic and geometric properties of matrix groups, along with their associated Lie algebras, which provide insights into the infinitesimal symmetries of these groups.

This project explores the theory of *matrix groups*, which are sets of invertible matrices closed under matrix multiplication and inversion. Matrix groups form a crucial link between linear algebra and abstract algebra, providing a concrete framework to study continuous symmetries and transformations. Motivated by an interest in the algebraic and geometric structures of these groups, this project presents a focused study on their fundamental properties.

The discussion includes several classical matrix groups, such as the general linear group $\mathrm{GL}(n, \mathbb{R})$, the special linear group $\mathrm{SL}(n, \mathbb{R})$, the orthogonal group $\mathrm{O}(n)$, the special orthogonal group $\mathrm{SO}(n)$, and the symplectic group $\mathrm{Sp}(n)$. Key properties, subgroup structures, and geometric interpretations are explored, laying the groundwork for further studies in Lie groups, harmonic analysis, and representation theory.