1 Introduction

The principles of minimization and maximization are among the most widely used tools to formulate mathematical models that govern the equilibrium configurations of physical systems. In these notes, we will develop the basic mathematical analysis of nonlinear optimization principles on infinite-dimensional function spaces - a topic known as the ”calculus of variations”, for reasons we will explain as soon as we present the basic ideas. The mathematical techniques developed to handle such optimization problems are fundamental in many areas of mathematics and its applications. The calculus of variations deals with finding extrema and, in this sense, it can be considered a branch of optimization. However, the problems and techniques in this branch are markedly different from the problems and techniques dealing with extrema of functions of several variables because of the nature of the domain over the quantities to be optimized.A functional is a mapping from a set of functions to the real numbers. The calculus of variations deals with finding extrema for functionals as opposed to functions. In the premilinaries part we will derive some useful theorems and results that will be useful throughout the subject. We develop a necessary condition for a function to yield an extremum for a functional. The centerpiece of the chapter is a second-order differential equation, the Euler-Lagrange equation, which plays a role analogous to the gradient of a function. We first motivate the analysis by reviewing the necessary conditions for functions to have local extrema. Then we will go for the existence of minimizers and in the last section we take the application of the calculus of variations.