The Ellipse

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Announcements

1. Office hours today 10am - 11am.
2. Homework in MyOpenMath, and Projects (2 of them).

The ellipse

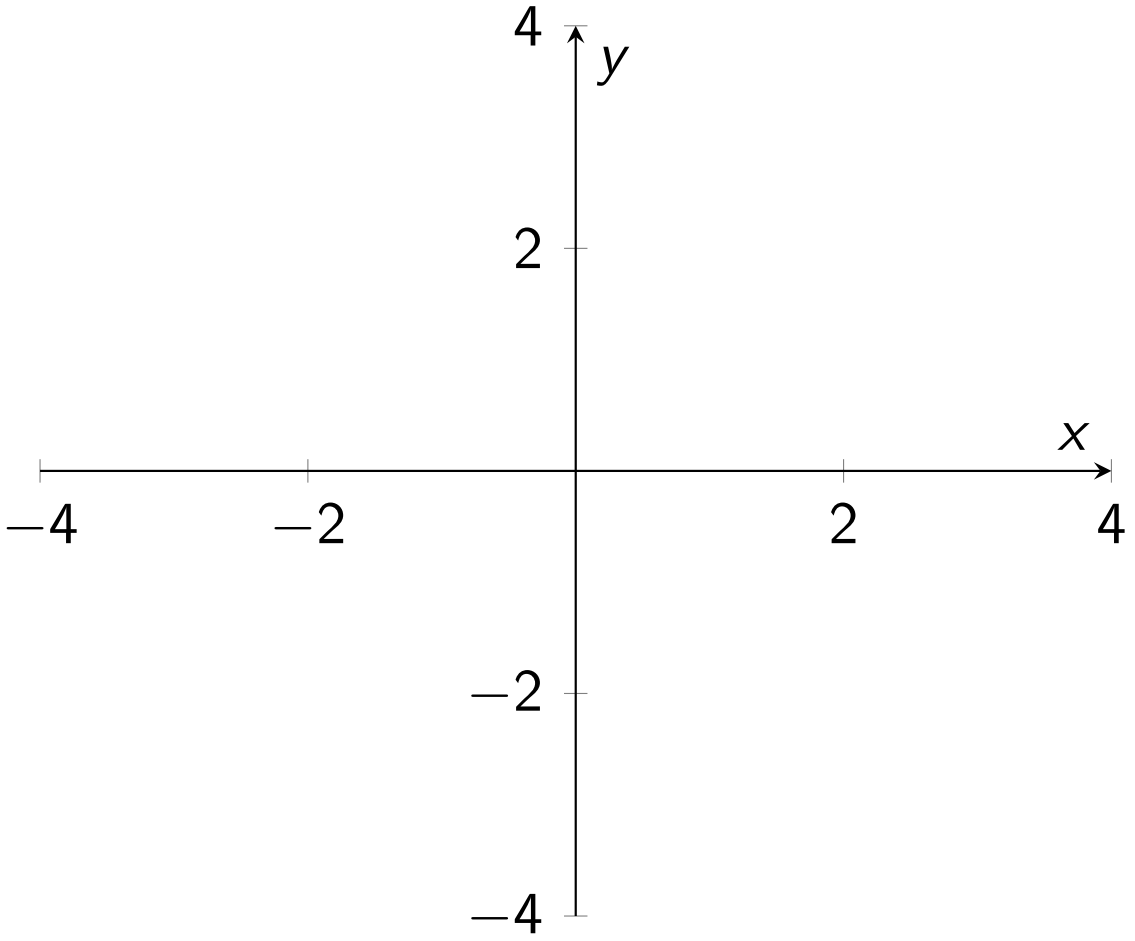
An ellipse is defined in the following strange way:

Definition (Ellipse)

Given two points F_1 and F_2 (called _____) that are not the same point in the $xy-$plane, an ellipse is the set of all (x, y) such that the sum of the distances from (x, y) to F_1 and (x, y) to F_2 is a _____ value.

The ellipse

There's a lot to unpack here, so let's get to unpacking!



Some key features

Major axis

Minor axis

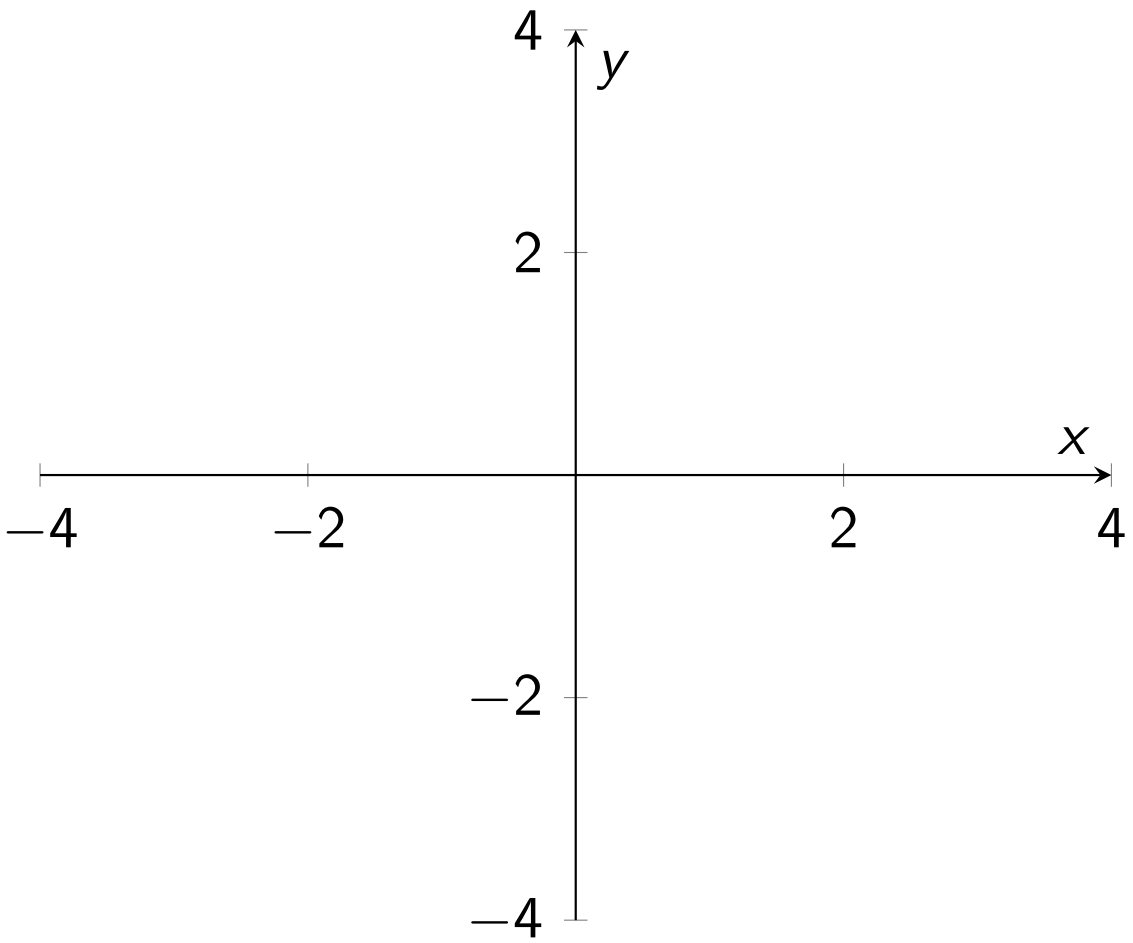
Vertex

Co-vertex

Center of ellipse

The foci always lie on the _____ axis.

Deriving the formula for an Ellipse



Deriving the formula for an Ellipse

Facts about Ellipses centered at the Origin

STANDARD FORMS OF THE EQUATION OF AN ELLIPSE WITH CENTER (0,0)

The standard form of the equation of an ellipse with center $(0, 0)$ and major axis on the *x-axis* is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where

- $a > b$
- the length of the major axis is $2a$
- the coordinates of the vertices are $(\pm a, 0)$
- the length of the minor axis is $2b$
- the coordinates of the co-vertices are $(0, \pm\, b)$
- the coordinates of the foci are $(\pm c, 0)$, where $c^2 = a^2 - b^2$. See Figure 6 **a**

Facts about Ellipses centered at the Origin

The standard form of the equation of an ellipse with center $(0, 0)$ and major axis on the *y-axis* is

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

where

- $a > b$
- the length of the major axis is $2a$
- the coordinates of the vertices are $(0, \pm\, a)$
- the length of the minor axis is $2b$
- the coordinates of the co-vertices are $(\pm b, 0)$
- the coordinates of the foci are $(0, \pm\, c)$, where $c^2 = a^2 - b^2$. See Figure 6 **b**

Note that the vertices, co-vertices, and foci are related by the equation $c^2 = a^2 - b^2$. When we are given the coordinates of the foci and vertices of an ellipse, we can use this relationship to find the equation of the ellipse in standard form.

Facts about Ellipses centered at the Origin

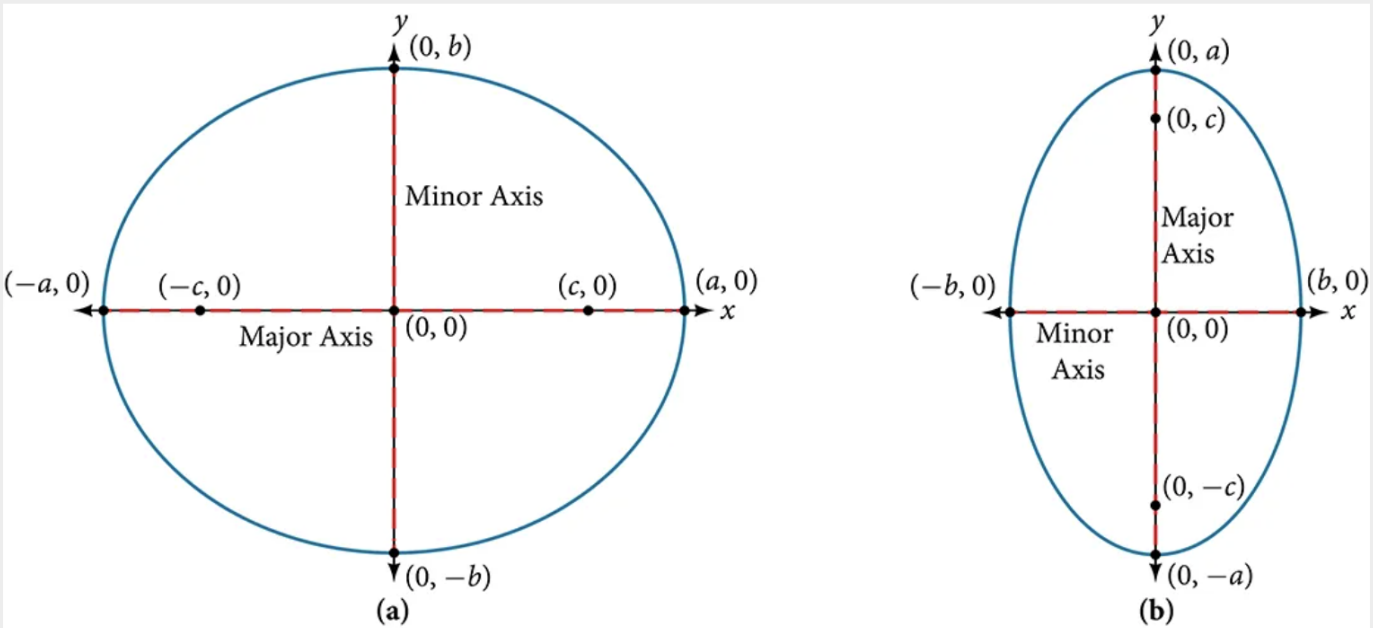
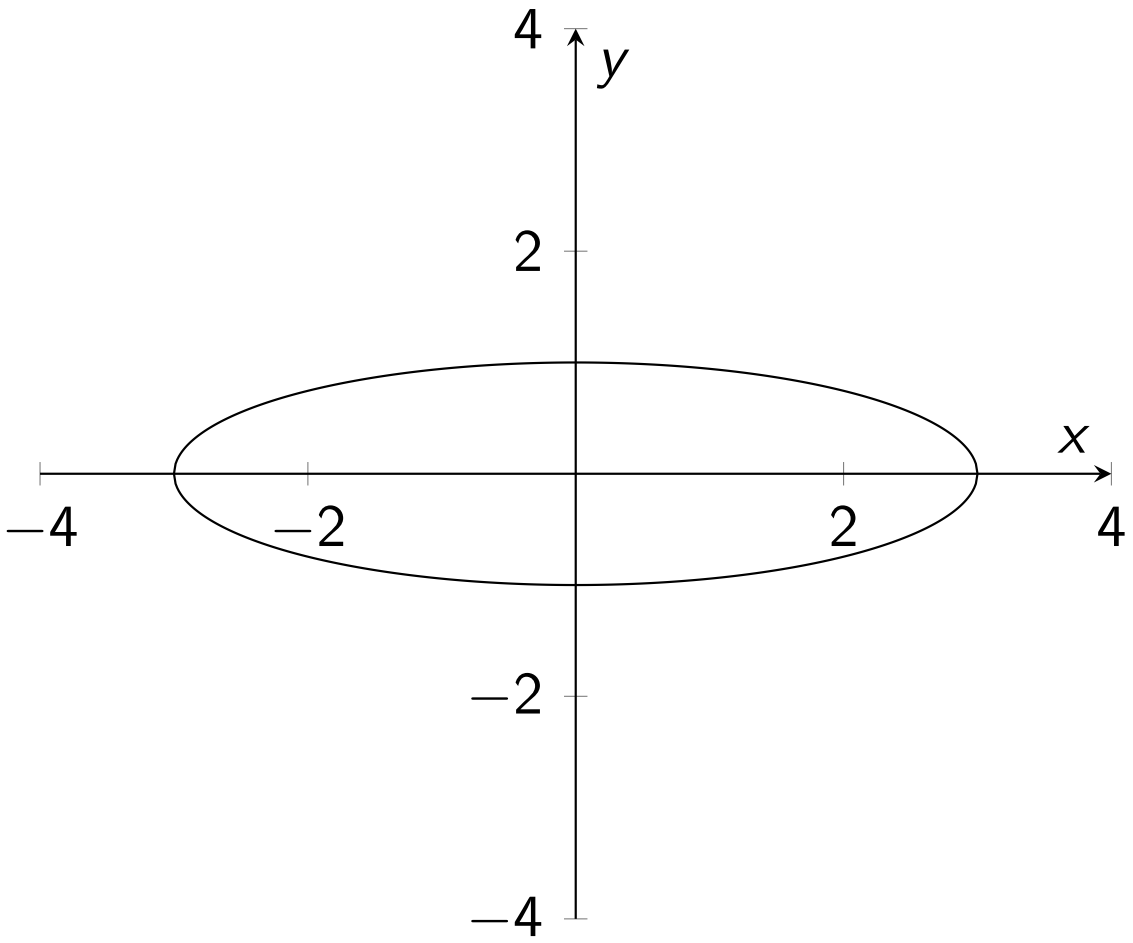


Figure 6 (a) Horizontal ellipse with center $(0,0)$ (b) Vertical ellipse with center $(0,0)$

Example

Write the equation of the following ellipse and identify the key features of the ellipse.



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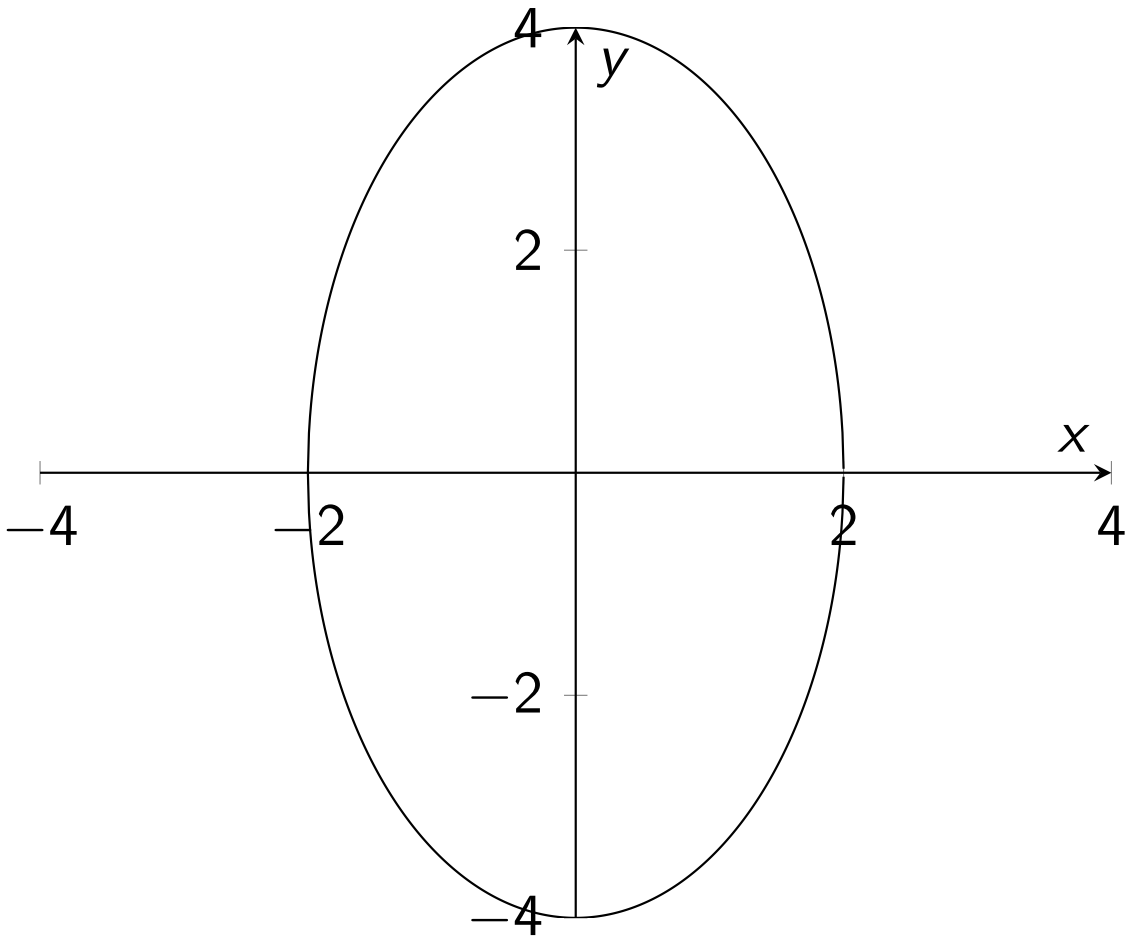

Example