More Ellipses

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Announcements

1. Homework due tonight.
2. Project.
3. No office hours today.

General Ellipse

Let's look at the equation of a general ellipse centered at the point (h, k).

Facts about General Ellipses

STANDARD FORMS OF THE EQUATION OF AN ELLIPSE WITH CENTER (*H*, *K*)

The standard form of the equation of an ellipse with center $(h,\ k)$ and major axis parallel to the *x*-axis is

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

where

- $a > b$
- the length of the major axis is $2a$
- the coordinates of the vertices are $(h \pm a, k)$
- the length of the minor axis is $2b$
- the coordinates of the co-vertices are $(h, k \pm b)$
- the coordinates of the foci are $(h \pm c, k)$, where $c^2 = a^2 - b^2$. See Figure 7**a**

Facts about General Ellipses

The standard form of the equation of an ellipse with center (h, k) and major axis parallel to the y-axis is

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

where

- $a > b$
- the length of the major axis is $2a$
- the coordinates of the vertices are $(h, k \pm a)$
- the length of the minor axis is $2b$
- the coordinates of the co-vertices are $(h \pm b, k)$
- the coordinates of the foci are $(h, k \pm c)$, where $c^2 = a^2 - b^2$. See Figure 7**b**

Facts about General Ellipses

Just as with ellipses centered at the origin, ellipses that are centered at a point (h, k) have vertices, co-vertices, and foci that are related by the equation $c^2 = a^2 - b^2$. We can use this relationship along with the midpoint and distance formulas to find the equation of the ellipse in standard form when the vertices and foci are given.

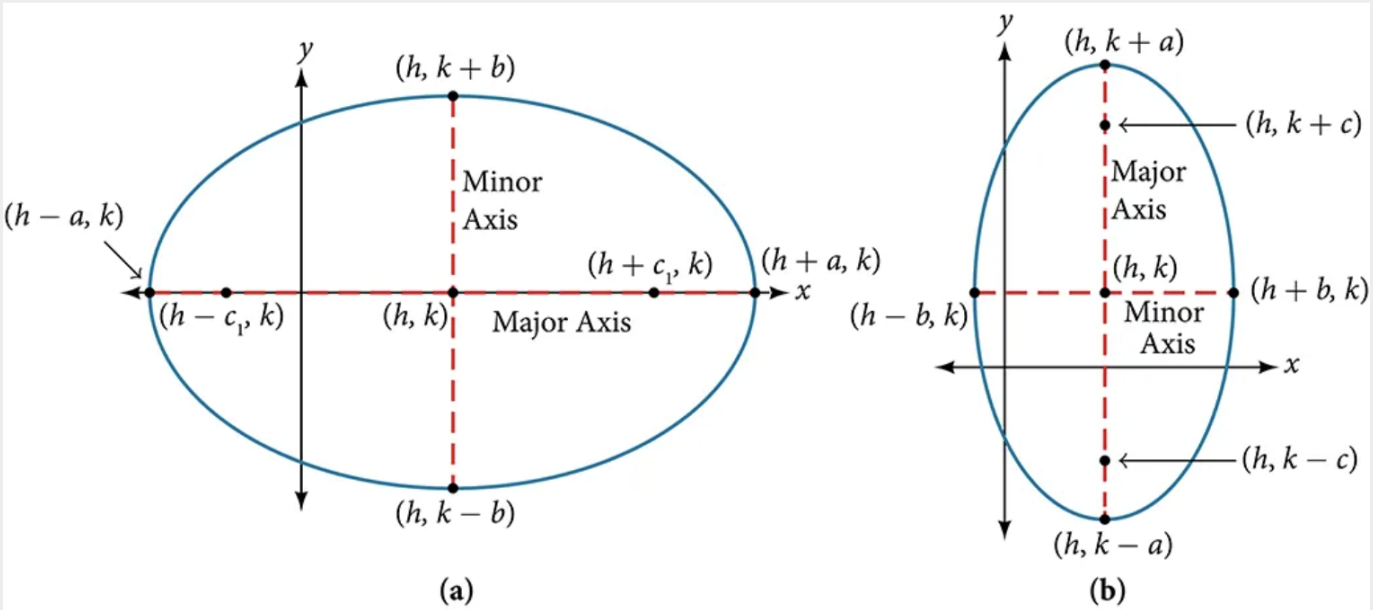
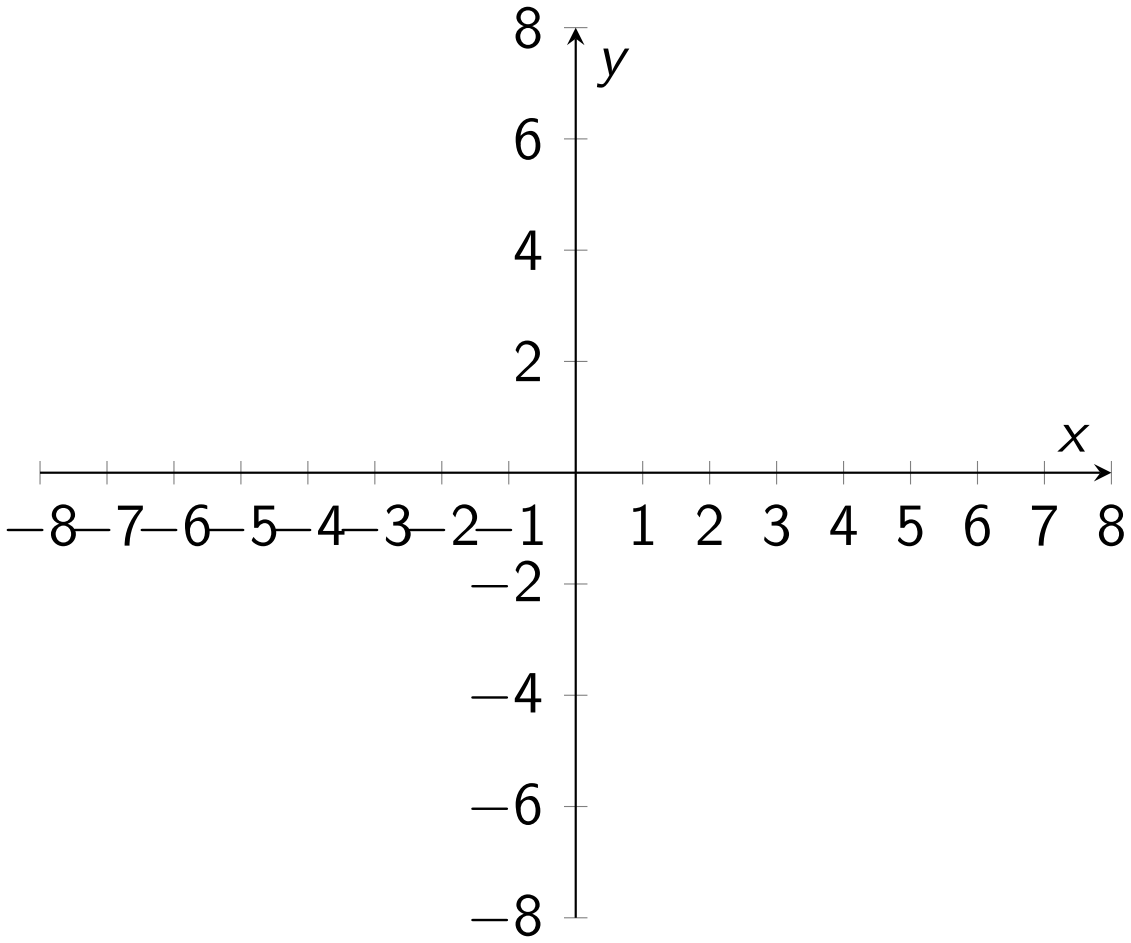


Figure 7 (a) Horizontal ellipse with center (h, k) (b) Vertical ellipse with center (h, k)

Example

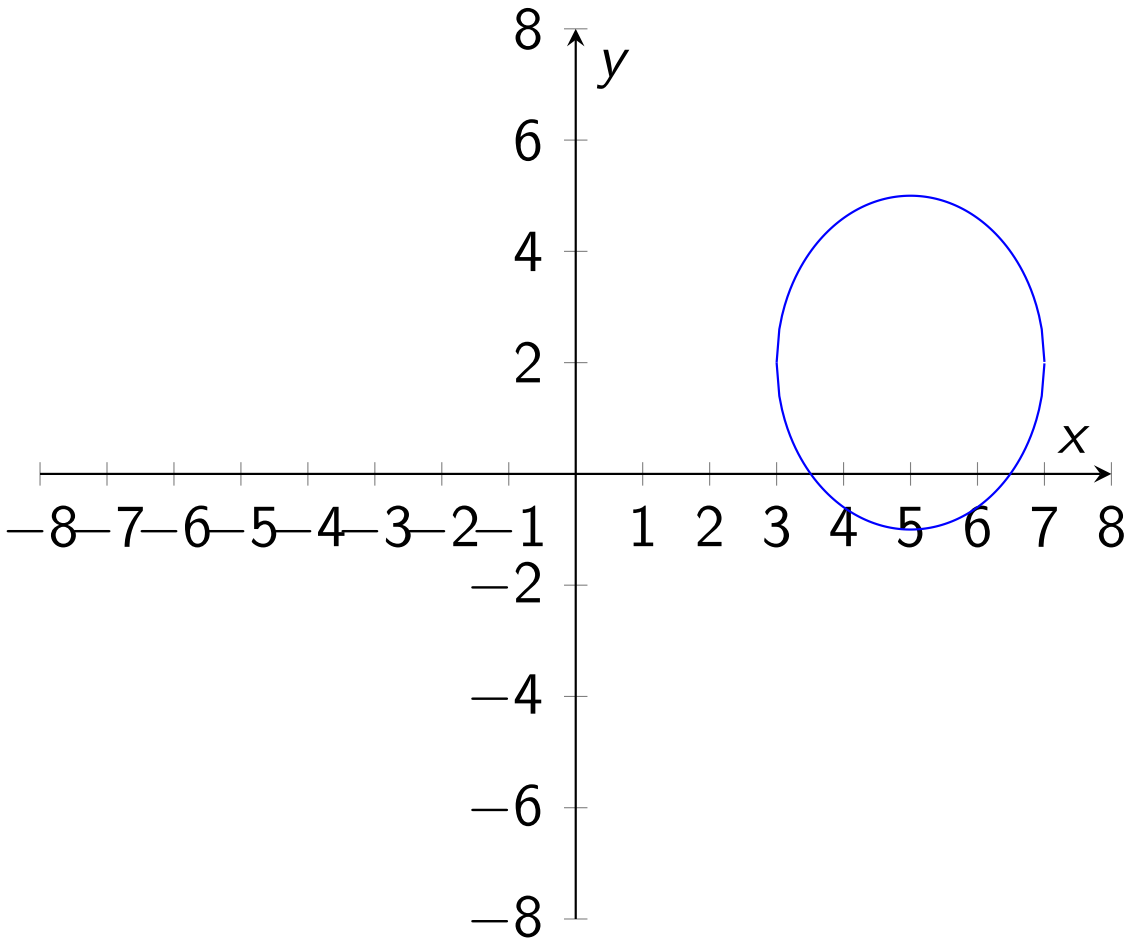
Let's plot the ellipse given by the equation

$$\frac{(x-1)^2}{16} + \frac{(y+2)^2}{4} = 1$$



Example

What is the equation of the ellipse with this graph:



Example

Completing the square

Is the following equation the equation of an ellipse?

$$9x^2 + 36x + 4y^2 - 32y + 100 = 36$$

What about

$$9x^2 + 36x - 4y^2 - 32y + 100 = 36$$

Let's look at how we can see!

Completing the square

Let's begin with the first example.

$$9x^2 + 36x + 4y^2 - 32y + 100 = 36$$

Example

Example

Example

Now let's check if

$$9x^2 + 36x - 4y^2 - 32y + 100 = 36$$

is the equation of an ellipse.