Inverse Trig Functions

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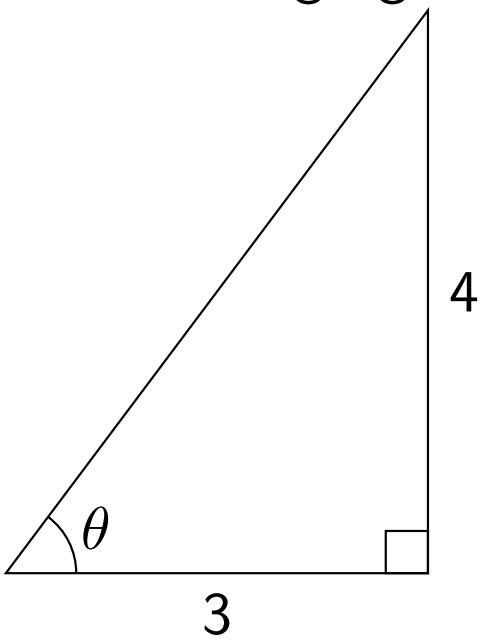
Announcements

1. Homework in MyOpenMath
2. Exam 2 Friday after Spring Break.

Inverse Trigonometric Functions

The main question we want to answer today is this: What is the angle θ in the following right triangle?



Certainly we can find $\sin(\theta), \cos(\theta)$ and $\tan(\theta)$, but how can we say what θ is if we know those values?

Let's go to a graph!

Let's go look at a graph of $\sin(\theta)$ to see if we can answer the question on the previous slide.

Inverse Functions

Let's talk about general functions for a second:

The need for restrictions

So, in order for a function to have an inverse, that function needs to be one-to-one.
Question: Are any of the trig functions one-to-one?
So apparently we need to make a _____ on the domain of the trig functions to turn them into one-to-one functions. Let's look at how that's typically done.

Desmos!

Notation

The inverse function of the function $f(x) = \sin(x)$ that has domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is the function $f^{-1}(x) =$ _____. This function has a domain that is identical to the range of $\sin(x)$. Similarly, the range of $\arcsin(x)$ is the domain of our restricted sin function, so _____.

Example

Let's take a second and think how we can restrict the domain of $\cos(x)$to turn it into a one-to-one function. Then do the same with $\tan(x)$.

Example

Find

1. $\arcsin\left(\frac{1}{2}\right)$
2. $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$
3. $\arccos\left(\frac{1}{2}\right)$
4. $\arctan(1)$.

Example

Given that $\sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{2-\sqrt{3}}}{2}$, write a relation involving inverse sine.

Example

Now let's see how we can use a calculator to answer the question that we opened class with:
What is the angle θ in the following right triangle?