Solving Trigonometric Equations

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Announcements

1. Homework in M.O.M.
2. Office hours, 10am - 11am.

Solving Trig Equations

Now we want to talk about how we might solve the equation

$$\sin^2(x) - 2\sin(x) + 1 = 0$$

We'll build up to this equation, but first let's start with some simple examples.

Example

Solve the equation

$$\sin(x) = \frac{1}{2}$$

for $0 \leq x < 2\pi$

Example

Example

Solve the equation

$$2\cos(x) - \sqrt{2} = 0$$

for $0 \leq x < 2\pi$.

Example

General Guidelines

We'll use the following general guidelines when solving more complicated trigonometric equations:

HOW TO

Given a trigonometric equation, solve using algebra.

1. Look for a pattern that suggests an algebraic property, such as the difference of squares or a factoring opportunity.
2. Substitute the trigonometric expression with a single variable, such as x or u.
3. Solve the equation the same way an algebraic equation would be solved.
4. Substitute the trigonometric expression back in for the variable in the resulting expressions.
5. Solve for the angle.

Example

Solve the equation

$$2\sin^2(x) - 1 = 0$$

for $0 \leq x < 2\pi$.

Example

Example

Solve the equation

$$\csc(x) = -2$$

for $0 \leq x < 4\pi$.

Example

Example

Sometimes we'll get solutions that don't involve special angles that we know, That's okay! We can just use a calculator and the inverse trig functions.
Solve:

$$\sin(x) = 0.7$$

for $0 \leq x < 2\pi$.

Example

Example

Time for a trickier one!
Solve

$$\sin^2(x) - \sin(x) - 1 = 0$$

for $0 \leq x < 2\pi$.

Example

Example

Solve the following equation exactly:

$$2\cos^2(x) + 3\cos(x) - 2 = 0$$

for $0 \leq x < 4\pi$.

Example

Example

Solve

$$2\sin(2x) + 1 = 0$$

for $0 \leq x < 2\pi$.