The Geometric Series and the Divergence Test

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1. Homework in MyOpenMath

2. Office hours, 10am - 11am

3. Free coffee and cookies on Fridays in MEC at noon.

The Geometric Series

Now let's investigate a very special type of a series: a series in which the individual terms form a geometric sequence will be called a
________________ and has the form

$$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + ar^3 + \dots$$

Let's see what we can say about a series like this.

The Geometric Series

The Geometric Series

We've shown

Theorem (Geometric Series)

The series

$$a + ar + ar^2 + ar^3 + \ldots = \sum_{n=0}^{\infty} ar^n$$

____________________ *if* $|r| < 1$ *and* ____________________ *if* $|r| \geq 1$.

Example

What is

$$.123123123\ldots$$

as a fraction?

Example

Example

What is the area of the Sierpinski Triangle?

Example

The Divergence Test

It would be nice if, given a series, there was a quick way to tell if the series was divergent. Thankfully, we have the divergence test for that!

Theorem (The Divergence Test)

If $\lim_{n\to\infty} a_n \neq 0$, *then the series*

$$\sum_{n=1}^{\infty} a_n$$

______________.

Note!

Example

Which of the following series can we immediately say diverges?

1. $$\sum_{n=1}^{\infty} \frac{1}{n!}$$

2. $$\sum_{n=2}^{\infty} \frac{n+1}{n-1}$$

3. $$\sum_{n=1}^{\infty} n^{\frac{1}{n}}$$

Example