Area between curves

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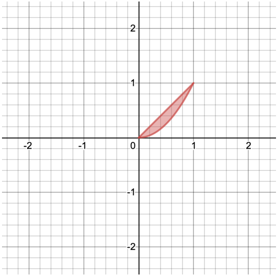
Announcements

1. Homework due tonight! in MyOpenMath!
2. Quiz on Friday.
3. Office hours: M - F, 10am - 11am.

Area between curves

We know that we arrived at the definition of the definite integral by considering the area under the curve given by $y = f(x)$. We can use the definite integral to find more general areas. For instance, we can find the area bounded by the two curves

$$y = x, \quad \text{and} \quad y = x^2$$



Let's see how!

Area between curves

Assume for now that for $a \leq x \leq b$ we have $f(x) \geq g(x)$. Then, to find the area between $f(x)$ and $g(x)$, let's look at a sketch of what's going on:

Area between curves

Area between curves

The previous slides showed that if $f(x) \geq g(x)$ for $a \leq x \leq b$, then the area between $f(x)$ and $g(x)$ is given by:

Example

Find the area bounded by the graphs of the functions $f(x) = x$ and $g(x) = x^4$.

Example

Example

Let $n > 0$. Find the area bounded by the graphs of the functions $f(x) = x$ and $g(x) = x^n$ with $x \geq 0$. What is the limit of this area as $n \to \infty$?

Example

When the curves switch

We've only considered so far the area between two curves where one function is always greater than the other function, but what about if we want to find the area between two curves where the "top curve" switches? Well, that's simply given by

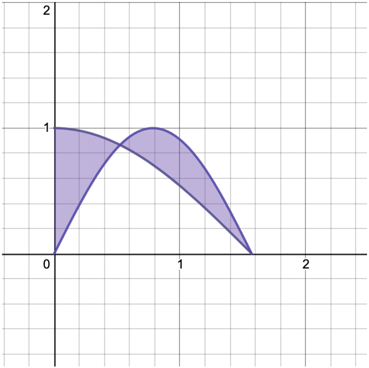
$$\int_a^b |f(x) - g(x)| \, dx$$

Example

Find the area between the curves

$$f(x) = \sin(2x) \quad \text{and} \quad g(x) = \cos(x)$$

between $x = 0$ and $x = \pi/2$.



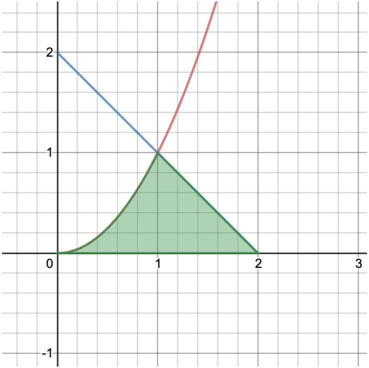
Example

A different example

Let's consider finding the area bounded by the curves given by the functions

$$f(x) = x^2, \quad g(x) = 2 - x, \quad h(x) = 0$$

for $0 \leq x \leq 2$.



A different example

Parting word

We'll see a simpler way to tackle that last example in the next class.
Don't forget about homework!