Sequences

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Announcements

1. Homework due tonight.
2. Homework/Project in Canvas.
3. Exam corrections due Tuesday.

Definition (Infinite Sequence)

A _______________ $\{a_n\}$is an _______________ of numbers of the form

The subscript n is called the _______________ and usually (but not always) begins at $n =$. Each number a_n is called a _______________ in the sequence.

Example

Write out the first 4 terms in the following examples of sequences:

1. $a_n = \frac{1}{n}$ with $n \geq 1$ (Explicit formula).
2. $b_{n+2} = b_{n+1} + b_n$ with $n \geq 0$ and $b_0 = 1$ and $b_1 = 1$. (Recurrence Relation)
3. $c_n = \frac{1}{2^n}$, $n \geq 3$
4. $d_n = d_{n-1} + 5$, with $n \geq 1$, $d_1 = 2$ (Arithmetic)
5. $e_{n+1} = \frac{1}{3} e_n$, with $n \geq 0$, $e_0 = 2$ (Geometric)

Example

Example

Find an explicit formula for the sequences

1. $$\frac{1}{3}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{5}{7}, \frac{3}{4}, \ldots$$

2. $$4, 5, 7, 11, 19, 35, \ldots$$

The Limit of a Sequence

We'll be interested in what happens to a sequence as $n \to \infty$. If a sequence "settles down" to 1 number as $n \to \infty$, then we'll call that number the ________________ of the sequence.

Definition (Limit of a sequence)

If $\{a_n\}_{n=n_0}^{\infty}$ is a sequence of real numbers, we call L the limit of the sequence $\{a_n\}_{n=n_0}^{\infty}$ if, given $\epsilon > 0$ there is some integer N such that if $n \geq N$, then

$$|a_n - L| < \epsilon.$$

If the sequence $\{a_n\}_{n=n_0}^{\infty}$ has a limit L, then we write

$$\lim_{n\to\infty} a_n = L$$

and also say that the sequence converges to the limit L.

Example

Show, using the definition of the limit of a sequence, that

$$\lim_{n \to \infty} \frac{1}{2^n} = 0.$$

Example

Example

Show that the sequence $b_n = \frac{n-1}{n}$ for $n \geq 1$ converges to 1 using the definition of the limit of a sequence.