

More Taylor Series!

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6 May 2024



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Announcements

- 1 Exam corrections, due Friday
- 2 New homework, due Friday
- 3 Final Exam, Wednesday May 15, 8:00am - 10:30am.
- 4 Office hours today are 12pm - 1pm.

A reminder of Taylor Series

Here's what we talked about last time! Suppose $f(x)$ is a function that has as many derivatives at the point a as we desire. Then the _____ for f centered at a is given by

If $a = 0$, we call the series the _____ for f .

Example

Find the Maclaurin series for the function $f(x) = e^x$. Also find the radius of convergence and interval of convergence.

Example

Example

Find the Taylor series centered at $a = 1$ for the function $f(x) = \ln(x)$. Also find the radius of convergence and interval of convergence.

Example

Taylor Polynomials and the Remainder

The partial sums associated with a Taylor series is called a _____.

Definition (Taylor Polynomial)

If f has n derivatives at $x = a$, then the n th Taylor Polynomial for f at a is

$$p_n(x) =$$

We also define the n th remainder as

$$R_n(x) =$$

Example

Find $p_0(x)$, $p_1(x)$, $p_2(x)$, and $p_3(x)$ at $a = 0$ for the function $f(x) = e^x$. Also write down the corresponding remainders.

Example

Example

Find $p_2(x)$ at $a = -1$ for the function $f(x) = \sqrt{5+x}$. What is the corresponding remainder?

Example

Another way to write the remainder term

Let's finish up by seeing another way to write the remainder term $R_n(x)$ for a function f that has $n + 1$ derivatives at the point a .

Another way to write the remainder term