Improper Integrals

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Announcements

1. Exam corrections, Due next Tuesday
2. Office hours, every day, 10am - 11am.
3. Homework in MyOpenMath.

Example

How would we define something that looks like

$$\int_1^\infty \frac{1}{t^2}\,dt?$$

Improper integrals

This process that we just did should remind you of a limit, because we essentially just took a limit!
Sometimes, like in the example above, we are interested in integrating functions not just over a finite interval $[a, b]$, but over an infinite interval, like $[a, \infty)$. This gives rise to one type of _____.
We'll cover the other type of improper integral tomorrow. For now, let's look at some definitions.

Improper integrals

Definition (Improper Integral (infinite bound))

If a is a real number, we define

$$\int_a^{\infty} f(x)\,dx = \qquad .$$

Similarly, we define

$$\int_{-\infty}^{a} f(x)\,dx = \qquad .$$

We define

$$\int_{-\infty}^{\infty} f(x)\,dx = \qquad .$$

Improper Integrals

If the above limits exist, we say the improper integral _____.
Otherwise, the improper integral _____.

Example

Determine if the integral

$$\int_1^\infty \frac{1}{x}\,dx$$

converges.

Example

Example (Gabriel's horn)

Find the volume of revolution of the solid generated by rotating the region whose top boundary is the graph of the function

$$y = \frac{1}{x}$$

and whose lower boundary is the $x-$axis for $x \geq 1$.

Example

Example

Does the integral

$$\int_{-\infty}^{0} xe^{-x^2}\,dx$$

converge or diverge?

Example

Example

Does the integral

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$$

converge or diverge? Find its value if it converges.

Example (Important!)

Show that

$$\int_1^\infty \frac{1}{x^p}\,dx$$

Converges for $p > 1$ and diverges for $p \leq 1$.

Example