Volumes of Revolution: The Disk Method

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Announcements

1. Homework!
2. Office hours!

Volumes of revolution

One type of solid that comes up a lot in real life is a solid that is generated by rotating a region about an axis.
One example of this is the unit sphere: it can be generate by rotating a upper half circle of radius 1 and the area underneath it about the $x-$axis.
Really, any object that has circular cross sections can be generated in this manner. Because of this, what we learned last week will apply to these ______________. The volume of a solid of revolution will be called a ______________.
Let's look at a specific example before moving on to the general method.

Example

Find the volume of the solid generated by rotating the region bounded by the curves

$$y = \frac{1}{x},\, x = 1,\, x = 2,\, y = 0$$

about the $x-$axis by using the method of slicing.

Example

The disk method

The main difference between what we're talking about today, and what we talked about last time, is that we know beforehand with a solid of revolution that the cross-sectional areas are going to be circles.
Let's look at a general curve, and the volume that results from it when we rotate the region of interest about the $x-$axis:

The disk method

The disk method

We have shown the following

Theorem (The disk method)

Let $f(x)$ be continuous and non-negative. Let R be the region bounded above by the graph of $f(x)$, below by the $x-$axis and by the curves $x = a$ and $x = b$. Then, the volume of the solid that is generated by rotating R about the $x-$axis is given by

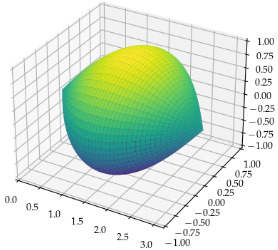
Example (Volume of a circular cone)

Find the volume of the solid that is generated by rotating the region in the first quadrant that is bounded above by $y = -\frac{r}{h}x + r$ about the $x-$axis, where r and h are constants.

Example

Example

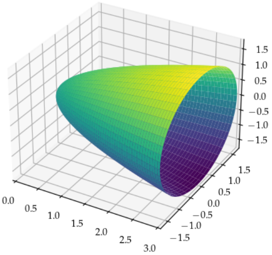
Find the volume of the solid that is generated by rotating the region bounded by the curves $y = \sin(x)$, $y = 0$, $x = 0$ and $x = \pi$ about the $x-$axis.



Example

Example

Find the volume of the solid generated by rotating the region bounded by the curves $y = \sqrt{x}$, $y = 0$, $x = 0$ and $x = 3$ about the $x-$axis.



Example

The disk method for functions of y

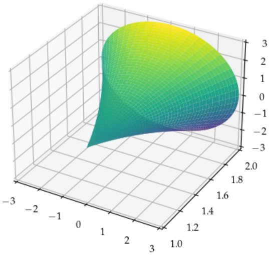
We can use this exact same method to find the volume of solids of revolution generated by rotating regions about the $y-$axis. When we do this, we need to make sure to write our functions in terms of ______________. This will give us the following rule:

Theorem (Disk method (in terms of y))

Let $g(y)$ be continuous and nonnegative. Defined Q to be the region bounded on the right by the graph of $g(y)$, on the left by $x = 0$, and between $y = c$ and $y = d$. Then the volume of the solid generated by rotating Q around the $y-$axis is given by

Example

Find the volume of the solid generated by rotating the region bounded by the curves $y = \sqrt{1+x}$, $x = 0$, and $y = 2$ about the $y-$axis.



Example

Next time

Next time we'll talk about how to find the volume of even more complicated volumes of revolution using the *washer method* (of which the disk method is just a special case).
Make sure to get started on the homework!