Parametric Equations

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Announcements

1. Quizz in Canvas.
2. Exam corrections due tomorrow.
3. Final Review in Canvas.
4. Final Exam, Wednesday at 8am.

Parametric equations

How are we used to plotting equations?
Usually, we are given an equation or a function, such as

$$y = x^2 + 2x + 1$$

or

$$x^2 + y^2 = 4$$

And we solve for y and plot using a table of values (if you don't remember what the graph looks like).
In other words, we usually have 1 _____ variable and 1 _____ variable.
But in real life, very often both x and y (and other variables) depend on some other independent variable called a _____.
This parameter is very often time, but it could be other things too.

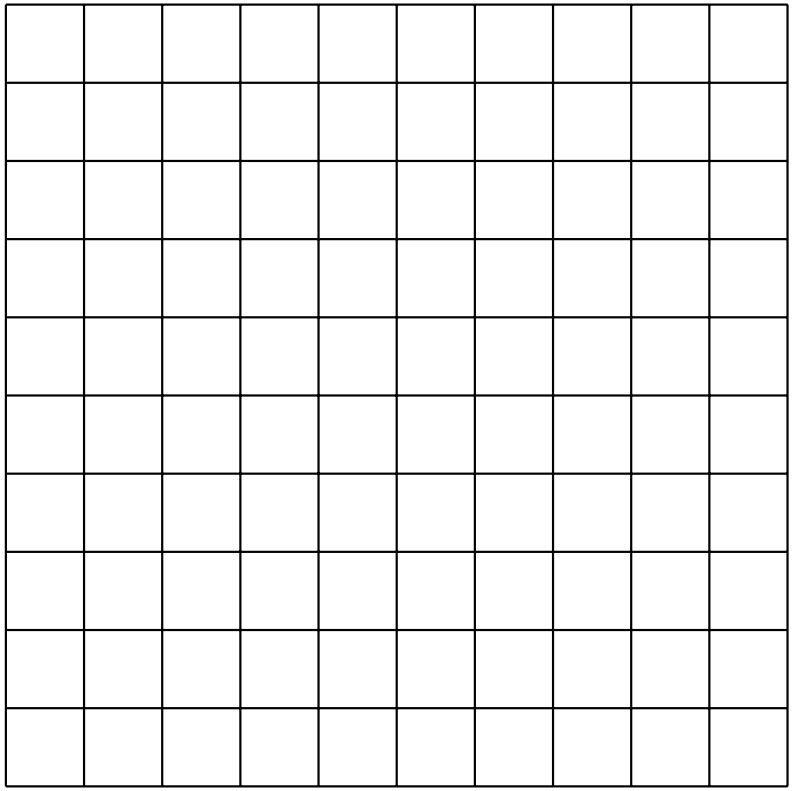
Example

You go on a trip to run some errands. Here is the data for your position at different times.

t	x	y
0	0	0
1	1	-1
2	2	-1
3	2	1
4	0	0

Assuming you traveled in straight lines between coordinates, draw a plot of the path you took on your trip.

Example



Parametric Equations

It's a fact that any curve in the $xy-$plane can be represented with _____.

Definition (Parametric curve)

If x and y are continuous functions of t on an interval I, then the equations

$$x = x(t), \quad y = y(t)$$

are called parametric equations and t is called the _____. The set of points (x, y) obtained as t varies over the interval I is called the _____ of the parametric equations. The graph of parametric equations is called a _____ or *plane curve*, and is denoted by C.

Eliminating the Parameter

Plotting parametric equations by making tables is a fine place to start, but could give us misleading results. For instance, if we try to plot

$$x(\theta) = \cos(4\theta), \quad y(\theta) = \sin(4\theta), \quad 0 \leq \theta \leq 2\pi$$

And we use a table with multiples of $\pi/4$, we might be mislead into thinking that y is never changing.

For this reason, often the best way to plot a parametric curve is to try to _____ to end up with 1 equation involving only x and y.

Example

Let's eliminate the parameter t to plot the curve given by the following parametric equations in the plane:

$$x(t) = t^2, \quad y(t) = t - 4, \quad -1 \leq t \leq 2$$

Example

Example

Eliminate the parameter θ to plot the curve given by the following parametric equations in the plane:

$$x(\theta) = 8\cos(\theta), \quad y(\theta) = 8\sin(\theta), \quad 0 \leq \theta \leq 2\pi$$

Example

Example

Let's see how you can develop parametric equations by picking a parameter to work with using the example of a cycloid:

Example