The Washer Method

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Announcements

1. Homework in MyOpenMath
2. Make sure to come to office hours if you have questions. (Friday via Zoom, 10 AM - 12 PM)
3. Quiz in Canvas today.

Other solids of revolution

Solids of revolution are a little more general than solids that always have a circular cross section. For example, let's consider finding the volume of the solid generated by rotating the region bounded by the curves

$$y = x^2,\ y = \sqrt{x}$$

about the $x-$axis.

Other solids of revolution

The washer method

In general, the cross section of a solid of revolution will not be just a circle, but instead a “washer”, or a circle with a smaller circle cut out of the center. Thankfully, it’s still pretty easy to calculate the volume of a washer: If the “big” circle has a radius of R, and the “small” (cut out) circle has a radius of r, and the washer has a width of dx, then the volume of the washer is

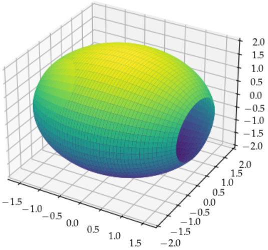
From this, it’s not hard to show the following.

Theorem (The washer method)

Suppose $f(x)$ and $g(x)$ are continuous, nonnegative functions with $f(x) \geq g(x)$ over $[a, b]$. Let R denote the region bounded above by the graph of $f(x)$ and below by the graph of $g(x)$, and by the lines $x = a$ and $x = b$. Then, the volume of the solid generated by rotating R about the $x-$axis is given by

Example

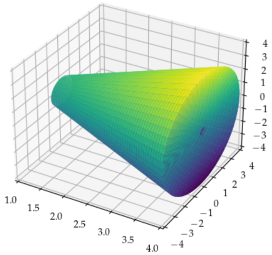
Find the volume of the "napkin ring" generated by rotating the region bounded above by $y = \sqrt{4 - x^2}$, and below by $y = 1$ about the $x-$axis.



Example

Example

Find the volume of the solid generated by rotating the region bounded above by the graph of $y = x$, below by the graph of $y = 1/x$ and on the right by $x = 4$ about the $x-$axis.



Example

The washer method, again

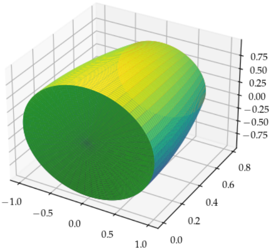
Just like when we learned the disk method, we were also able to apply it to volumes of revolution rotated about the $y-$axis, we can also apply the washer method to volumes generated by rotating regions about the $y-$axis with very little change.

Theorem (The washer method (for functions of y))

Suppose $f(y)$ and $g(y)$ are continuous, nonnegative functions with $f(y) \geq g(y)$ over $[c, d]$. Let Q denote the region bounded on the right by the graph of $f(y)$ and on the left by the graph of $g(y)$, and by the lines $y = c$ and $y = d$. Then, the volume of the solid generated by rotating Q about the $y-$axis is given by

Example

Find the volume of the solid generated by rotating the region bounded on the right by the graph of $x = \cos y$, on the left by $x = \sin y$ and below by $y = 0$ about the $y-$axis.



Example

Example

BUT WHY JUST THE $x-$ AND $y-$ AXES?????

We can even find the volume of a solid that is generated by rotating a region about any vertical or horizontal line.

Let's try to find the volume of the solid generated by rotating the region bounded by the graphs of $y = x$ and $y = x^2$ about the line $x = 2$.

Example

Next time

There's one final method that we can use to find these volumes of revolution: cylindrical shells. We'll learn how to use this method next time.