Cylindrical Shells

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Announcements

1. Exam Thursday (Pratice in MyOpenMath)
2. Office hours today 10am - 11am

Last time

Last time we talked about the washer method to find volumes of revolution. The disk method and the washer method are very powerful for finding these volumes, but they still don't allow us to find some types of volumes of revolution.

For example, let's try to find the volume of the solid that is generated by rotating the region bounded by the graph of $f(x) = x - x^2$ and $y = 0$ about the $y-$axis.

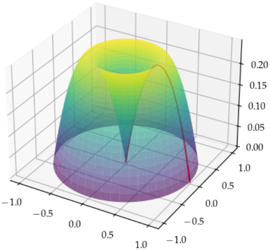
Example

Cylindrical shells

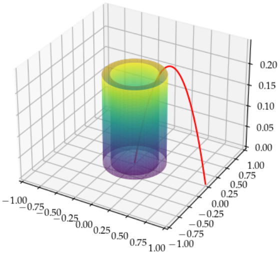
Clearly, trying to do this with the washer method is pretty awful.
It would be nice if we had an easier way to find the volume of revolution when the region is defined in terms of functions of x but rotated about the $y-$axis.
Enter: ______________.
Let's illustrate the method of cylindrical shells on that last example before we write down the general method.

Example

Find the volume of the solid that is generated by rotating the region bounded by the graph of $f(x) = x - x^2$ and $y = 0$ about the $y-$axis.



Example



0.20
0.15
0.10
0.05
0.00
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

Cylindrical shells, the general method

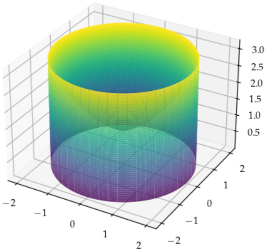
What we did in the last example is exactly how we should proceed with all cylindrical shell method problems.

Theorem (Cylindrical shell method)

Let $f(x)$ and $g(x)$ be continuous and nonnegative with $f(x) \geq g(x)$ on the interval $[a, b]$. Define R as the region bounded above by the graph of $f(x)$, below by the graph of $g(x)$ and by the lines $x = a$ and $x = b$. Then the volume of the solid generated by rotating R about the $y-$axis is given by

Example

Find the volume of the solid generated by rotating the region bounded above by the graph of $f(x) = \frac{e^x - 1}{x}$, with $f(0) = 1$, below by $y = 0$, and by the lines $x = 0$ and $x = 2$ around the $y-$axis.



Example

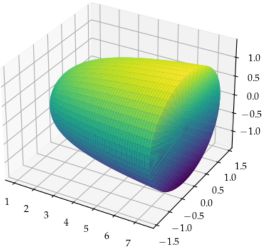
Just like with the other methods, this easily generalizes to functions of y, with appropriate changes.

Theorem (Cylindrical shell method (for functions of y))

Let $f(y)$ and $g(y)$ be continuous and nonnegative with $f(y) \geq g(y)$ on the interval $[c, d]$. Define Q as the region bounded on the right by the graph of $f(y)$, on the left by the graph of $g(y)$ and by the lines $y = c$ and $y = d$. Then the volume of the solid generated by rotating Q about the $x-$axis is given by

Example

Find the volume of the solid generated by rotating the region bounded by the curves $y = \sqrt{\ln(x)}$, $y = 0$, and $x = e^2$ about the $x-$axis.



Example

So which method should I use?

At this point you may ask, which method is the best one to use? The method you use depends heavily on the context of the problem. For instance, in the last example, we took a region defined in terms of a function of x and rotated it about the $x-$axis, something that usually we would use the disk or washer method for.

But the disk method gave us an integral that we (currently) don't know how to evaluate. So in this case, we tried the cylindrical shell method as a last ditch effort, and it happened to work out for us.

As a *rule of thumb*, if you are rotating a region that is defined in terms of functions of x about the $x-$axis, you'll ______________ use the ______________ method. If you are rotating a region defined in terms of functions of x about the $y-$axis, you'll ______________ use the ______________.

Similar statements hold for functions of y.

Example

Find the volume of the solid generated by rotating the region bounded by the graphs of $y = \left(e^x + e^{-x}\right)/2$, $y = 0$ and $x = 1$, $x = -1$ about the $x-$axis.

Example

Example

Find the volume of the solid generated by rotating the region bounded by the curves $y = x$, $y = 2 - x$ and $y = 0$ about the $x-$axis.

Example