# Other Trigonometric Integrals

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# Announcements

1. Exam corrections due Tuesday.
2. Homework in MyOpenMath (One due Wednesday, one due next Monday)

# Tangent and secant

Instead of showing you all of the cases that we can encounter involving $\tan(x)$ and $\sec(x)$, let's jump to the general strategy, because the process is largely the same as before:

**PROBLEM-SOLVING STRATEGY**

**Problem-Solving Strategy: Integrating $\int \tan^k x \sec^j x \, dx$**

To integrate $\int \tan^k x \sec^j x \, dx$, use the following strategies:

1. If $j$ is even and $j \geq 2$, rewrite $\sec^j x = \sec^{j-2} x \sec^2 x$ and use $\sec^2 x = \tan^2 x + 1$ to rewrite $\sec^{j-2} x$ in terms of $\tan x$. Let $u = \tan x$ and $du = \sec^2 x \, dx$.
2. If $k$ is odd and $j \geq 1$, rewrite $\tan^k x \sec^j x = \tan^{k-1} x \sec^{j-1} x \sec x \tan x$ and use $\tan^2 x = \sec^2 x - 1$ to rewrite $\tan^{k-1} x$ in terms of $\sec x$. Let $u = \sec x$ and $du = \sec x \tan x \, dx$. (*Note*: If $j$ is even and $k$ is odd, then either strategy 1 or strategy 2 may be used.)
3. If $k$ is odd where $k \geq 3$ and $j = 0$, rewrite $\tan^k x = \tan^{k-2} x \tan^2 x = \tan^{k-2} x(\sec^2 x - 1) = \tan^{k-2} x \sec^2 x - \tan^{k-2} x$. It may be necessary to repeat this process on the $\tan^{k-2} x$ term.
4. If $k$ is even and $j$ is odd, then use $\tan^2 x = \sec^2 x - 1$ to express $\tan^k x$ in terms of $\sec x$. Use integration by parts to integrate odd powers of $\sec x$.

# A few special cases

Here are a few special cases that don't quite follow those rules:

1. $\int \sec^2 x \, dx =$
2. $\int \sec(x)\tan(x) \, dx =$
3. $\int \tan(x) \, dx =$
4. $\int \sec(x) \, dx =$

# Example

Evaluate

$$\int \sec^4(x)\tan(x)\,dx$$

# Example

# Example

Evaluate

$$\int \tan^5(x)\sec(x)\,dx$$

# Example

# Example

Evaluate

$$\int \tan^3(x)\, dx$$

# Example

# Power Reduction Formula

Let's develop a power reduction formula for $\int \sec^n(x)\,dx$ if $n \geq 3$ is odd.

(This will really help us with the next example).

$$I_n = \int \sec^n(x)\,dx$$

# Example

# Example

Evaluate

$$\int \tan^4(x)\sec(x)\,dx$$

# Example