Syllabus day/Review of Calc I

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22 January 2024

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Announcements

1. Welcome to Calc II!
2. Syllabus quiz in Canvas.
3. Homework assigned and due next Monday (mostly review).

Syllabus

Let us take 10 minutes to review Canvas and the syllabus!

Review of Calc I (Derivatives)

Remember that Calculus I (or most of Calculus I) is about finding the derivative, which is a generalization of the slope of a straight line. We're going to freely use all of the rules for finding derivatives that we developed last semester, including:

1. The power rule: $\frac{d}{dx}(x^a) =$
2. The sum/difference rule: $\frac{d}{dx}(f(x) \pm g(x)) =$
3. The product rule: $\frac{d}{dx}(f(x)g(x)) =$
4. The quotient rule: $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) =$
5. The chain rule: $\frac{d}{dx}(f(g(x))) =$

Review of Calc I (Derivatives)

We'll also freely use the derivatives of elementary functions we know:

1. $\frac{d}{dx}\sin(x) =$
2. $\frac{d}{dx}\cos(x) =$
3. $\frac{d}{dx}\tan(x) =$
4. $\frac{d}{dx}e^x =$
5. $\frac{d}{dx}a^x =$
6. $\frac{d}{dx}\ln(x) =$

Reminder of the definition

Given a function $f(x)$ that is defined on an open interval containing the point a, we say f is differentiable at a if

$$\lim_{h \to 0}$$

exists.

We call $f'(a)$ the above limit. If f is differentiable at every point in an interval, we say f is differentiable on the interval.

Review problem

Find the derivatives of the following functions:

1. $f(x) = x^2 + 2^x$
2. $g(x) = e^{x^2}$
3. $h(x) = \frac{x^2-4}{3+2x^2}$

Review problem

Review problem

A farmer finds that if she plants 30 trees per acre, each tree will yield 60 bushels of fruit. She estimates that for each additional tree planted per acre, the yield of each tree will decrease by 3 bushels. How many trees should she plant per acre to maximize her harvest?

Review problem

Recall the definition of the definite integral:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x$$

where $x_k = x_0 + k\Delta x$, $x_0 = a$, $x_n = b$ and $\Delta x = (b-a)/n$.

The definite integral $\int_a^b f(x)\ dx$ is simply the signed area under the curve defined by $y = f(x)$.

Special note!

The definition of the definite integral is actually more general than what we've presented. Really, we can define:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$$

where $x_k = x_0 + k\Delta x$, $x_0 = a$, $x_n = b$ and $\Delta x = (b-a)/n$, and x_k^* is *any* point in $[x_{k-1}, x_k]$.

Review of Calc I (Integrals)

We will make some use the definition of the definite integral, but more often we will use the Fundamental theorem of Calculus:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $F(x)$ is an antiderivative of $f(x)$ on the interval $[a, b]$, which means $F'(x) = f(x)$ for each x in $[a, b]$. If $F'(x) = f(x)$ for all x in an interval, we write

$$\int f(x)\,dx = F(x) + C$$

Integration rules

Much of this semester is going to be about finding new and better ways to take antiderivatives to use in the fundamental theorem of calculus, but here are a few basic rules to get us started:

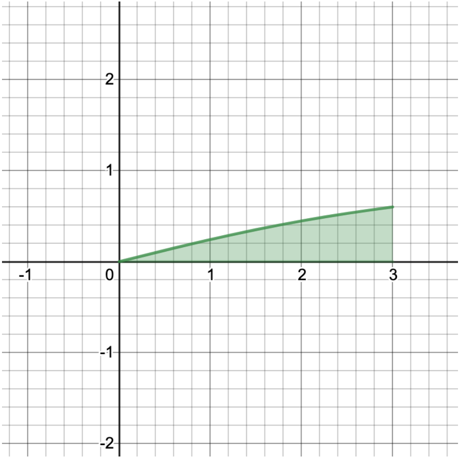
1. $\int x^n \, dx =$
2. $\int f(x) \pm g(x) \, dx =$
3. $\int cf(x) \, dx =$
4. $\int f(g(x))g'(x) \, dx =$

where $F(x)$ is an antiderivative of $f(x)$.

Integration example

Find

$$\int_0^3 \frac{x}{\sqrt{16+x^2}}\,dx$$



Integration example

Integration example

Find

$$\int_0^4 \sqrt{16 - x^2}\, dx$$

Integration example

Integration example

Evaluate

$$\int_{-1}^{1} 4xe^{-10x^2}\, dx$$