Alternating Series

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Announcements

1. Homework in M.O.M.
2. Office hours today 10am - 11am.
3. New project in Canvas.

Alternating Series

So far, the tests we've used only work on series with ________________ terms. But in real life, there are series that have both positive and negative terms. A special type of sequence of this sort is called an ________________.

Definition (Alternating Series)

Any series whose terms alternate between positive and negative values is called an alternating series. An alternating series can be written in the forms:

Example

Which of the following are alternating series?

1. $\sum_{n=1}^{\infty} (-1)^n$
2. $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^{n-1}$
3. $\sum_{n=1}^{\infty} \frac{\sin\left(\frac{(2n+1)\pi}{2}\right)}{n}$

Alternating series test

To show how we can determine the convergence or divergence of an alternating series, let's look at a specific alternating series, the alternating harmonic series:

Alternating series test

Alternating series test

Theorem (The Alternating Series Test)

An alternating series of the form

$$\sum_{n=1}^{\infty} (-1)^n b_n \text{ or } \sum_{n=1}^{\infty} (-1)^{n+1} b_n$$

converges if:

1.
2.

Example

Which of the following series converge:

1. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2}$
2. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{e^n}$
3. $\sum_{n=1}^{\infty} (-1)^n \ln(n)$

The Remainder of an Alternating Series

Let's see if we can get a bound on the remainder

$R_N = \sum_{n=1}^{\infty} (-1)^n b_n - \sum_{n=1}^{N} (-1)^n b_n$ if the series converges.

Example

What is the remainder if we use

$$\sum_{n=0}^{5} (-1)^n \frac{1}{(2n+1)!}$$

to approximate $\sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)!}$? (This is $\sin(1)$, by the way).

Absolute vs Conditional Convergence

We've shown now that $\sum\limits_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ converges, but we also know that

$$\sum_{n=1}^{\infty} \left| \frac{(-1)^{n+1}}{n} \right| = \sum_{n=1}^{\infty} \frac{1}{n}$$

diverges (why)?

A series $\sum\limits_{n=1}^{\infty} a_n$ that converges, but for which $\sum\limits_{n=1}^{\infty} |a_n|$ diverges is called ________________.

A series $\sum\limits_{n=1}^{\infty} a_n$ that converges, and for which $\sum\limits_{n=1}^{\infty} |a_n|$ converges is called ________________.

Example

Is the series

$$\sum_{n=1}^{\infty} \left(\frac{-1}{4} \right)^n$$

conditionally convergent, absolutely convergent, or divergent?

A useful theorem

A fact that is useful is that if $\sum_{n=1}^{\infty} |a_n|$ is an absolutely convergent series, then the original series $\sum_{n=1}^{\infty} a_n$ is also convergent.