Trig Substitution, Part II

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Announcements

1. Homework
2. Office hours, 10am - 11am

Integrals involving $\sqrt{x^2 - a^2}$

It should be no surprise that we can integrate functions of the form $\sqrt{x^2 - a^2}$ as well:

PROBLEM-SOLVING STRATEGY

Problem-Solving Strategy: Integrals Involving $\sqrt{x^2 - a^2}$

1. Check to see whether the integral cannot be evaluated using another method. If so, we may wish to consider applying an alternative technique.
2. Substitute $x = a\sec\theta$ and $dx = a\sec\theta\tan\theta d\theta$. This substitution yields

$$\sqrt{x^2 - a^2} = \sqrt{(a\sec\theta)^2 - a^2} = \sqrt{a^2(\sec^2\theta - 1)} = \sqrt{a^2\tan^2\theta} = |a\tan\theta|.$$

   For $x \geq a$, $|a\tan\theta| = a\tan\theta$ and for $x \leq -a$, $|a\tan\theta| = -a\tan\theta$.
3. Simplify the expression.
4. Evaluate the integral using techniques from the section on trigonometric integrals.
5. Use the reference triangles from Figure 3.9 to rewrite the result in terms of x. You may also need to use some trigonometric identities and the relationship $\theta = \sec^{-1}\left(\frac{x}{a}\right)$. (*Note*: We need both reference triangles, since the values of some of the trigonometric ratios are different depending on whether $x \geq a$ or $x \leq -a$.)

Example

Evaluate

$$\int \sqrt{2x^2 - 8}\, dx$$

Example

Example

Find

$$\int_2^3 \sqrt{2x^2 - 8}\, dx$$

Completing the square

Before we look at the next example, we need to discuss how to *complete the square*. (Something that technically is supposed to be taught in precalc, but hardly ever is).
We would like to rewrite

$$ax^2 + bx + c$$

in the form

$$a(x - h)^2 + k.$$

Let's figure out what h and k need to be.

Completing the square

Completing the square

Find

$$\int \frac{2}{\sqrt{x^2+2x}}dx$$

Completing the square

Examples

Evaluate

$$\int \frac{dx}{\sqrt{1+9x^2}}$$

Examples