The Ratio and Root Tests

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Announcements

1. Homework
2. Office hours tomorrow: 10am - 11am
3. Project

The Ratio Test

So now that we have 2 ideas of convergence (Conditional convergence and absolute), it would be nice to have tests that tell me when a series is absolutely convergent. That's where the ratio test and the root test come into play.

Theorem (The Ratio Test)

Let $\sum_{n=1}^{\infty} a_n$ be a series with $a_n \neq 0$ for infinitely many n's, and let $R = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$.

1. *If $R < 1$, then $\sum_{n=1}^{\infty} a_n$ ______________________________.*
2. *If $R > 1$, then $\sum_{n=1}^{\infty} a_n$ _______________.*
3. *If $R = 1$, then the ratio test is _______________.*

Proof:

Example

Discuss the convergence/divergence of

1. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n!}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{(2n)!}$

Example

The Root Test

A similar test is the __________________.

The Root Test

Let $\sum_{n=1}^{\infty} a_n$ be a series and let

$$R = \lim_{n\to\infty} \sqrt[n]{|a_n|}.$$

Then,

1. If $R < 1$, then $\sum_{n=1}^{\infty} a_n$ ____________________________________.
2. If $R > 1$, then $\sum_{n=1}^{\infty} a_n$ __________________.
3. If $R = 1$, then the root test is __________________.

Example

Discuss the convergence/divergence of the following series:

1. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^n}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$
3. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$

Example

Example