$u-$substitution

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24 January 2024

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Announcements

1. Homework in MyOpenMath.
2. Office hours cancelled today.

$u-$substitution: a closer look

Sometimes it is “obvious” that we need to use a $u-$ substitution when evaluating an integral, such as

$$\int_0^1 x\left(x^2-4\right)^{10}\,dx$$

For this integral, it's pretty clear that if we make the $u-$substitution

$$\begin{aligned} u =& x^2-4 \\ \frac{1}{2}du =& x\,dx \end{aligned}$$

$u-$substitution: a closer look

Then, this integral transforms into

$$\int_{-4}^{-3} \frac{1}{2} u^{10} \, du$$

which can be evaluated simply using the power rule:

$u-$substitutions can be used in other situations that aren't quite as obvious, or as a preliminary step to make an integral have a simpler form to use other techniques of integration.

For instance, let's try to use $u-$substitution to evaluate

$$\int \frac{x}{\sqrt{x-1}}\,dx$$

$u-$substitution: a closer look

$u-$substitution

Here is a general problem solving strategy for integrals involving $u-$substitution:

1. Look at the integrand to determined if there is a composition of functions of the form $f\left(g\left(x\right)\right)$.
2. Substitute $u = g\left(x\right)$ and $du = g'\left(x\right)\,dx$.
3. If there are any x's remaining in the integral after this substitution, replace them using $u = g(x)$.
4. Evaluate the integral in terms of u, if possible. If it is not possible, we might need to go back and change our $u-$substitution.
5. Write your final answer in terms of x if finding an indefinite integral.

Examples

Evaluate the following integral:

$$\int x\,(1-x)^{99}\ dx$$

Examples

Examples

Evaluate the following integral:

$$\int_{-1}^{1} t\left(1 - t^2\right)^{10}\, dt$$

Examples

Examples

Evaluate the following integral:

$$\int x\sqrt{x+1}\,dx$$

Examples

Examples

Evaluate the following integral:

$$\int \cos^3(\theta)\sin(\theta)\ d\theta$$

Examples

Examples

Evaluate the following integral:

$$\int_0^{\pi/2} \cos^3(\theta)\ d\theta$$

Examples

Examples

Evaluate the following integral:

$$\int t \sin\left(t^2\right) \cos\left(t^2\right)\, dt$$