Infinite Series

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Announcements

1. Homework/project.

Example

Let

$$S_k = 1 + \frac{1}{2} + \ldots + \frac{1}{2^k}$$

Let's try to find

1. A "nicer" way to write S_k and
2. $\lim_{k\to\infty} S_k$

Example

Last time

In the last example, we found the limit of a sequence defined by

$$S_k = \sum_{n=1}^{k} \left(\frac{1}{2}\right)^n$$

When taking the limit of a sequence like this, it might make sense to write something like

$$\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$$

Today we're going to start with things that look like

$$\sum_{n=1}^{\infty} a_n$$

and try to define what this means. The object above is called an

________________.

Infinite Series Definition

An infinite series is an expression of the form

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots$$

For each positive integer k, the sum

$$S_k = \sum_{n=1}^{k} a_n$$

is called the ______________________________ of the series. These partial sums form a sequence $\{S_k\}$. If the sequence of partial sums converges to a real number S, we say the infinite series _______________ to S and write

$$\sum_{n=1}^{\infty} a_n = S.$$

If the sequence of partial sums diverges, we say the infinite series

_______________.

Example

Find the sequence of partial sums to evaluate the following series:

1. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$
2. $\displaystyle\sum_{n=0}^{\infty} (-1)^n$
3. $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right)$
4. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+n}$

Example

Algebraic Properties of Series

Suppose $\sum_{n=1}^{\infty} a_n$ converges to A and $\sum_{n=1}^{\infty} b_n$ converges to B, and c is a real number. Then we have

1. $\sum_{n=1}^{\infty} a_n \pm b_n =$
2. $\sum_{n=1}^{\infty} ca_n =$

Example