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"""Convenience functions built on top of `grad`."""
from future import absolute import
import itertools as it
import autograd.numpy as np
from autograd.core import grad, getval
from six.moves import map
def multigrad(fun, argnums=0):
     """Takes gradients wrt multiple arguments simultaneously."""
    original fun = fun
     def combined arg fun(multi arg, *args, **kwargs):
         extra args list = list(args)
         for argnum ix, arg ix in enumerate(argnums):
              extra_args_list[arg_ix] = multi_arg[argnum_ix]
         return original_fun(*extra_args_list, **kwargs)
     gradfun = grad(combined arg fun, argnum=0)
     def gradfun rearranged(*args, **kwargs):
         multi arg = tuple([args[i] for i in argnums])
         return gradfun(multi_arg, *args, **kwargs)
     return gradfun_rearranged
def grad and aux(fun, argnum=0):
     """Builds a function that returns the gradient of the first output and the
     (unmodified) second output of a function that returns two outputs."""
     def grad and aux fun(*args, **kwargs):
         saved aux = []
         def return val save aux(*args, **kwargs):
              val, aux = fun(*args, **kwargs)
              saved aux.append(aux)
              return val
         gradval = grad(return_val_save_aux, argnum)(*args, **kwargs)
         return gradval, saved aux[0]
     return grad and aux fun
def value and grad(fun, argnum=0):
     """Returns a function that returns both value and gradient. Suitable for use
     in scipy.optimize"""
     def double val fun(*args, **kwargs):
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val = fun(*args, **kwargs)
          return val, getval(val)
     gradval and val = grad and aux(double val fun, argnum)
     def value and grad fun(*args, **kwargs):
          gradval, val = gradval and val(*args, **kwargs)
          return val, gradval
     return value and grad fun
def elementwise grad(fun, argnum=0):
     """Like 'jacobian', but produces a function which computes just the diagonal
    of the Jacobian, and does the computation in one pass rather than in a loop.
     Note: this is only valid if the Jacobian is diagonal. Only arrays are
    currently supported."""
     def sum_output(*args, **kwargs):
          return np.sum(fun(*args, **kwargs))
     return grad(sum output, argnum=argnum)
def jacobian(fun, argnum=0):
     """Returns a function that computes the Jacobian of `fun`. If the input to
     'fun' has shape (in1, in2, ...) and the output has shape (out1, out2, ...)
     then the Jacobian has shape (out1, out2, ..., in1, in2, ...). Only arrays
    are currently supported."""
    # TODO: consider adding this to 'autograd.grad'. We could avoid repeating
    # the forward pass every time.
     def jac_fun(*args, **kwargs):
          arg in = args[argnum]
         output = fun(*args, **kwargs)
          assert isinstance(getval(arg in), np.ndarray), "Must have array input"
          assert isinstance(getval(output), np.ndarray), "Must have array output"
         jac = np.zeros(output.shape + arg_in.shape)
          input slice = (slice(None),) * len(arg in.shape)
          for idxs in it.product(*list(map(range, output.shape))):
              scalar fun = lambda *args, **kwargs : fun(*args, **kwargs)[idxs]
              jac[idxs + input_slice] = grad(scalar_fun, argnum=argnum)(*args, **kwargs)
          return jac
     return jac fun
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"""Builds a function that returns the exact Hessian-vector product.

def hessian vector product(fun, argnum=0):

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The returned function has arguments (*args, vector, **kwargs), and takes
     roughly 4x as long to evaluate as the original function."""
    fun grad = grad(fun, argnum)
     def vector dot grad(*args, **kwargs):
          args, vector = args[:-1], args[-1]
          return np.dot(vector, fun grad(*args, **kwargs))
     return grad(vector_dot_grad, argnum) # Grad wrt original input.
def hessian(fun, argnum=0):
     """Returns a function that computes the exact Hessian.
     The Hessian is computed by calling hessian vector product separately for
     each row. For a function with N inputs, this takes roughly 4N times as
     long as a single evaluation of the original function."""
     hvp = hessian vector product(fun, argnum)
     def hessian fun(*args, **kwargs):
          arg_in = args[argnum]
         directions = np.eye(arg_in.size) # axis-aligned directions.
         hvp_list = [hvp(*(args+(direction,)), **kwargs) for direction in directions]
          return np.array(hvp list)
```

return hessian fun