derived Boolean operations

Recall that a Boolean algebra is an algebraic system A consisting of five operations:

1. two binary operations: the meet $\wedge$ and the join $\vee$,
2. one unary operation: the complementation $'$, and
3. two nullary operations (constants): 0 and 1.

From these operations, define the following "derived" operations (on A): for $a, b \in A$

1. (subtraction) $a - b := a \wedge b'$,
2. (symmetric difference or addition) $a \Delta b$ (or $a + b$) $:= (a - b) \vee (b - a)$,
3. (conditional) $a \to b := (a - b)'$,
4. (biconditional) $a \leftrightarrow b := (a \to b) \wedge (b \to a)$, and
5. (Sheffer stroke) $a|b := a' \wedge b'$.

Notice that the operators $\to$ and $\leftrightarrow$ are dual of $-$ and Δ respectively.

It is evident that these derived operations (and indeed the entire theory of Boolean algebras) owe their existence to those operations and connectives that are found in logic and set theory, as the following table illustrates:

symbol \ operation	**Boolean**	**Logic**	**Set**
$\vee$ or $\cup$	join	logical or	union
$\wedge$ or $\cap$	meet	logical and	intersection
$'$ or $\neg$ or $^{\complement}$	complement	logical not	complement
0	bottom element	falsity	empty set
1	top element	truth	universe
$-$ or $\setminus$	subtraction		set difference
Δ or $+$	symmetric difference		symmetric difference (`http://planetmath.org/SymmetricDifference`)
$\to$	conditional	implication	
$\leftrightarrow$	biconditional	logical equivalence	
$\mid$	Sheffer stroke	Sheffer stroke	

$Some of the elementary properties of these derived Boolean operators are:$ 1. = - a 0 a and - a a = - 0 a = - a 1 = 0 , 2. (A , + , ∧ , 0 , 1) is a ring (a Boolean ring), 3. all Boolean operations can be defined in terms of the Sheffer stro