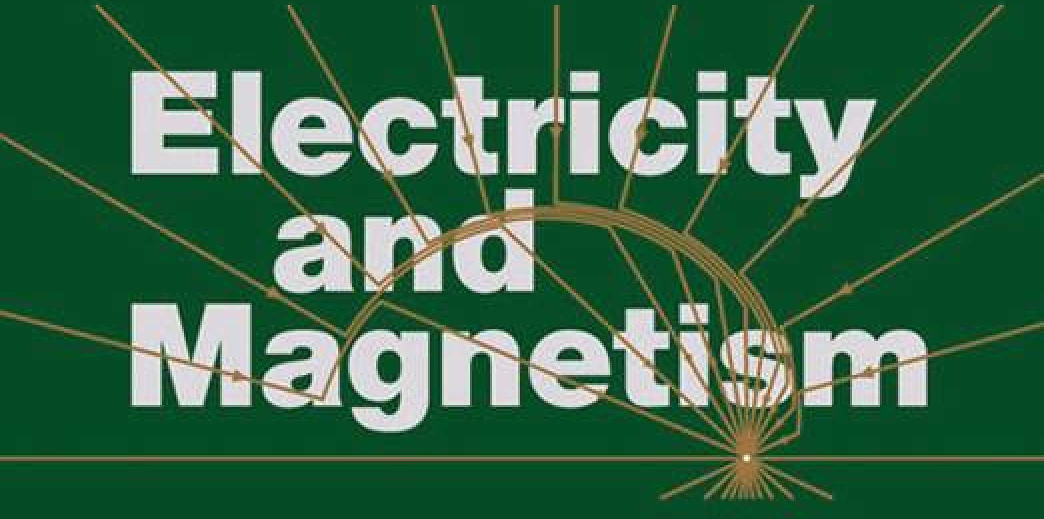
EDWARD M. PURCELL AND
DAVID J. MORIN

Electricity and Magnetism



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Electricity and Magnetism

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EDWARD M. PURCELL (1912–1997) was the recipient of many awards for his scientific, educational, and civic work. In 1952 he shared the Nobel Prize for Physics for the discovery of nuclear magnetic resonance in liquids and solids, an elegant and precise method of determining the chemical structure of materials that serves as the basis for numerous applications, including magnetic resonance imaging (MRI). During his career he served as science adviser to Presidents Dwight D. Eisenhower, John F. Kennedy, and Lyndon B. Johnson.

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ELECTRICITY AND MAGNETISM

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Preface to the third edition of Volume 2

For 50 years, physics students have enjoyed learning about electricity and magnetism through the first two editions of this book. The purpose of the present edition is to bring certain things up to date and to add new material, in the hopes that the trend will continue. The main changes from the second edition are (1) the conversion from Gaussian units to SI units, and (2) the addition of many solved problems and examples.

The first of these changes is due to the fact that the vast majority of courses on electricity and magnetism are now taught in SI units. The second edition fell out of print at one point, and it was hard to watch such a wonderful book fade away because it wasn't compatible with the way the subject is presently taught. Of course, there are differing opinions as to which system of units is "better" for an introductory course. But this issue is moot, given the reality of these courses.

For students interested in working with Gaussian units, or for instructors who want their students to gain exposure to both systems, I have created a number of appendices that should be helpful. Appendix A discusses the differences between the SI and Gaussian systems. Appendix C derives the conversion factors between the corresponding units in the two systems. Appendix D explains how to convert formulas from SI to Gaussian; it then lists, side by side, the SI and Gaussian expressions for every important result in the book. A little time spent looking at this appendix will make it clear how to convert formulas from one system to the other.

The second main change in the book is the addition of many solved problems, and also many new examples in the text. Each chapter ends with "problems" and "exercises." The solutions to the "problems" are located in Chapter 12. The only official difference between the problems and exercises is that the problems have solutions included, whereas the exercises do not. (A separate solutions manual for the exercises is available to instructors.) In practice, however, one difference is that some of the more theorem-ish results are presented in the problems, so that students can use these results in other problems/exercises.

Some advice on using the solutions to the problems: problems (and exercises) are given a (very subjective) difficulty rating from 1 star to 4 stars. If you are having trouble solving a problem, it is critical that you don't look at the solution too soon. Brood over it for a while. If you do finally look at the solution, don't just read it through. Instead, cover it up with a piece of paper and read one line at a time until you reach a hint to get you started. Then set the book aside and work things out for real. That's the only way it will sink in. It's quite astonishing how unhelpful it is simply to read a solution. You'd *think* it would do some good, but in fact it is completely ineffective in raising your understanding to the next level. Of course, a careful reading of the text, including perhaps a few problem solutions, is necessary to get the basics down. But if Level 1 is understanding the basic concepts, and Level 2 is being able to *apply* those concepts, then you can read and read until the cows come home, and you'll never get past Level 1.

The overall structure of the text is essentially the same as in the second edition, although a few new sections have been added. Section 2.7 introduces dipoles. The more formal treatment of dipoles, along with their applications, remains in place in Chapter 10. But because the fundamentals of dipoles can be understood using only the concepts developed in Chapters 1 and 2, it seems appropriate to cover this subject earlier in the book. Section 8.3 introduces the important technique of solving differential equations by forming complex solutions and then taking the real part. Section 9.6.2 deals with the Poynting vector, which opens up the door to some very cool problems.

Each chapter concludes with a list of "everyday" applications of electricity and magnetism. The discussions are brief. The main purpose of these sections is to present a list of fun topics that deserve further investigation. You can carry onward with some combination of books/internet/people/pondering. There is effectively an infinite amount of information out there (see the references at the beginning of Section 1.16 for some starting points), so my goal in these sections is simply to provide a springboard for further study.

The intertwined nature of electricity, magnetism, and relativity is discussed in detail in Chapter 5. Many students find this material highly illuminating, although some find it a bit difficult. (However, these two groups are by no means mutually exclusive!) For instructors who wish to take a less theoretical route, it is possible to skip directly from Chapter 4 to Chapter 6, with only a brief mention of the main result from Chapter 5, namely the magnetic field due to a straight current-carrying wire.

The use of non-Cartesian coordinates (cylindrical, spherical) is more prominent in the present edition. For setups possessing certain symmetries, a wisely chosen system of coordinates can greatly simplify the calculations. Appendix F gives a review of the various vector operators in the different systems.

Compared with the second edition, the level of difficulty of the present edition is slightly higher, due to a number of hefty problems that have been added. If you are looking for an extra challenge, these problems should keep you on your toes. However, if these are ignored (which they certainly can be, in any standard course using this book), then the level of difficulty is roughly the same.

I am grateful to all the students who used a draft version of this book and provided feedback. Their input has been invaluable. I would also like to thank Jacob Barandes for many illuminating discussions of the more subtle topics in the book. Paul Horowitz helped get the project off the ground and has been an endless supplier of cool facts. It was a pleasure brainstorming with Andrew Milewski, who offered many ideas for clever new problems. Howard Georgi and Wolfgang Rueckner provided much-appreciated sounding boards and sanity checks. Takuya Kitagawa carefully read through a draft version and offered many helpful suggestions. Other friends and colleagues whose input I am grateful for are: Allen Crockett, David Derbes, John Doyle, Gary Feldman, Melissa Franklin, Jerome Fung, Jene Golovchenko, Doug Goodale, Robert Hart, Tom Hayes, Peter Hedman, Jennifer Hoffman, Charlie Holbrow, Gareth Kafka, Alan Levine, Aneesh Manohar, Kirk McDonald, Masahiro Morii, Lev Okun, Joon Pahk, Dave Patterson, Mara Prentiss, Dennis Purcell, Frank Purcell, Daniel Rosenberg, Emily Russell, Roy Shwitters, Nils Sorensen, Josh Winn, and Amir Yacoby.

I would also like to thank the editorial and production group at Cambridge University Press for their professional work in transforming the second edition of this book into the present one. It has been a pleasure working with Lindsay Barnes, Simon Capelin, Irene Pizzie, Charlotte Thomas, and Ali Woollatt.

Despite careful editing, there is zero probability that this book is error free. A great deal of new material has been added, and errors have undoubtedly crept in. If anything looks amiss, please check the webpage www.cambridge.org/Purcell-Morin for a list of typos, updates, etc. And please let me know if you discover something that isn't already posted. Suggestions are always welcome.

David Morin

Preface to the second edition of Volume 2

This revision of "Electricity and Magnetism," Volume 2 of the Berkeley Physics Course, has been made with three broad aims in mind. First, I have tried to make the text clearer at many points. In years of use teachers and students have found innumerable places where a simplification or reorganization of an explanation could make it easier to follow. Doubtless some opportunities for such improvements have still been missed; not too many, I hope.

A second aim was to make the book practically independent of its companion volumes in the Berkeley Physics Course. As originally conceived it was bracketed between Volume I, which provided the needed special relativity, and Volume 3, "Waves and Oscillations," to which was allocated the topic of electromagnetic waves. As it has turned out, Volume 2 has been rather widely used alone. In recognition of that I have made certain changes and additions. A concise review of the relations of special relativity is included as Appendix A. Some previous introduction to relativity is still assumed. The review provides a handy reference and summary for the ideas and formulas we need to understand the fields of moving charges and their transformation from one frame to another. The development of Maxwell's equations for the vacuum has been transferred from the heavily loaded Chapter 7 (on induction) to a new Chapter 9, where it leads naturally into an elementary treatment of plane electromagnetic waves, both running and standing. The propagation of a wave in a dielectric medium can then be treated in Chapter 10 on Electric Fields in Matter.

A third need, to modernize the treatment of certain topics, was most urgent in the chapter on electrical conduction. A substantially rewritten

Chapter 4 now includes a section on the physics of homogeneous semiconductors, including doped semiconductors. Devices are not included, not even a rectifying junction, but what is said about bands, and donors and acceptors, could serve as starting point for development of such topics by the instructor. Thanks to solid-state electronics the physics of the voltaic cell has become even more relevant to daily life as the number of batteries in use approaches in order of magnitude the world's population. In the first edition of this book I unwisely chose as the example of an electrolytic cell the one cell—the Weston standard cell—which advances in physics were soon to render utterly obsolete. That section has been replaced by an analysis, with new diagrams, of the lead-acid storage battery—ancient, ubiquitous, and far from obsolete.

One would hardly have expected that, in the revision of an elementary text in classical electromagnetism, attention would have to be paid to new developments in particle physics. But that is the case for two questions that were discussed in the first edition, the significance of charge quantization, and the apparent absence of magnetic monopoles. Observation of proton decay would profoundly affect our view of the first question. Assiduous searches for that, and also for magnetic monopoles, have at this writing yielded no confirmed events, but the possibility of such fundamental discoveries remains open.

Three special topics, optional extensions of the text, are introduced in short appendixes: Appendix B: Radiation by an Accelerated Charge; Appendix C: Superconductivity; and Appendix D: Magnetic Resonance.

Our primary system of units remains the Gaussian CGS system. The SI units, ampere, coulomb, volt, ohm, and tesla are also introduced in the text and used in many of the problems. Major formulas are repeated in their SI formulation with explicit directions about units and conversion factors. The charts inside the back cover summarize the basic relations in both systems of units. A special chart in Chapter 11 reviews, in both systems, the relations involving magnetic polarization. The student is not expected, or encouraged, to memorize conversion factors, though some may become more or less familiar through use, but to look them up whenever needed. There is no objection to a "mixed" unit like the ohm-cm, still often used for resistivity, providing its meaning is perfectly clear.

The definition of the meter in terms of an assigned value for the speed of light, which has just become official, simplifies the exact relations among the units, as briefly explained in Appendix E.

There are some 300 problems, more than half of them new.

It is not possible to thank individually all the teachers and students who have made good suggestions for changes and corrections. I fear that some will be disappointed to find that their suggestions have not been followed quite as they intended. That the net result is a substantial improvement I hope most readers familiar with the first edition will agree.

Mistakes both old and new will surely be found. Communications pointing them out will be gratefully received.

It is a pleasure to thank Olive S. Rand for her patient and skillful assistance in the production of the manuscript.

Edward M. Purcell

Preface to the first edition of Volume 2

The subject of this volume of the Berkeley Physics Course is electricity and magnetism. The sequence of topics, in rough outline, is not unusual: electrostatics; steady currents; magnetic field; electromagnetic induction; electric and magnetic polarization in matter. However, our approach is different from the traditional one. The difference is most conspicuous in Chaps. 5 and 6 where, building on the work of Vol. I, we treat the electric and magnetic fields of moving charges as manifestations of relativity and the invariance of electric charge. This approach focuses attention on some fundamental questions, such as: charge conservation, charge invariance, the meaning of field. The only formal apparatus of special relativity that is really necessary is the Lorentz transformation of coordinates and the velocity-addition formula. It is essential, though, that the student bring to this part of the course some of the ideas and attitudes Vol. I sought to develop—among them a readiness to look at things from different frames of reference, an appreciation of invariance, and a respect for symmetry arguments. We make much use also, in Vol. II, of arguments based on superposition.

Our approach to electric and magnetic phenomena in matter is primarily "microscopic," with emphasis on the nature of atomic and molecular dipoles, both electric and magnetic. Electric conduction, also, is described microscopically in the terms of a Drude-Lorentz model. Naturally some questions have to be left open until the student takes up quantum physics in Vol. IV. But we freely talk in a matter-of-fact way about molecules and atoms as electrical structures with size, shape, and stiffness, about electron orbits, and spin. We try to treat carefully a question that is sometimes avoided and sometimes beclouded in introductory texts, the meaning of the macroscopic fields **E** and **B** inside a material.

In Vol. II, the student's mathematical equipment is extended by adding some tools of the vector calculus—gradient, divergence, curl, and the Laplacian. These concepts are developed as needed in the early chapters.

In its preliminary versions, Vol. II has been used in several classes at the University of California. It has benefited from criticism by many people connected with the Berkeley Course, especially from contributions by E. D. Commins and F. S. Crawford, Jr., who taught the first classes to use the text. They and their students discovered numerous places where clarification, or something more drastic, was needed; many of the revisions were based on their suggestions. Students' criticisms of the last preliminary version were collected by Robert Goren, who also helped to organize the problems. Valuable criticism has come also from J. D. Gavenda, who used the preliminary version at the University of Texas, and from E. F. Taylor, of Wesleyan University. Ideas were contributed by Allan Kaufman at an early stage of the writing. A. Felzer worked through most of the first draft as our first "test student."

The development of this approach to electricity and magnetism was encouraged, not only by our original Course Committee, but by colleagues active in a rather parallel development of new course material at the Massachusetts Institute of Technology. Among the latter, J. R. Tessman, of the MIT Science Teaching Center and Tufts University, was especially helpful and influential in the early formulation of the strategy. He has used the preliminary version in class, at MIT, and his critical reading of the entire text has resulted in many further changes and corrections.

Publication of the preliminary version, with its successive revisions, was supervised by Mrs. Mary R. Maloney. Mrs. Lila Lowell typed most of the manuscript. The illustrations were put into final form by Felix Cooper.

The author of this volume remains deeply grateful to his friends in Berkeley, and most of all to Charles Kittel, for the stimulation and constant encouragement that have made the long task enjoyable.

Edward M. Purcell

1

Electrostatics: charges and fields

Overview The existence of this book is owed (both figuratively and literally) to the fact that the building blocks of matter possess a quality called *charge.* Two important aspects of charge are *conservation* and *quantization.* The electric force between two charges is given by *Coulomb's law.* Like the gravitational force, the electric force falls off like $1/r^2$. It is *conservative,* so we can talk about the potential energy of a system of charges (the work done in assembling them). A very useful concept is the *electric field,* which is defined as the force per unit charge. Every point in space has a unique electric field associated with it. We can define the *flux* of the electric field through a given surface. This leads us to *Gauss's law,* which is an alternative way of stating Coulomb's law. In cases involving sufficient symmetry, it is much quicker to calculate the electric field via Gauss's law than via Coulomb's law and direct integration. Finally, we discuss the *energy density* in the electric field, which provides another way of calculating the potential energy of a system.

1.1 Electric charge

Electricity appeared to its early investigators as an extraordinary phenomenon. To draw from bodies the "subtle fire," as it was sometimes called, to bring an object into a highly electrified state, to produce a steady flow of current, called for skillful contrivance. Except for the spectacle of lightning, the ordinary manifestations of nature, from the freezing of water to the growth of a tree, seemed to have no relation to the curious behavior of electrified objects. We know now that electrical forces largely determine the physical and chemical properties of matter over the whole range from atom to living cell. For this understanding we have to thank the scientists of the nineteenth century, Ampère, Faraday, Maxwell, and many others, who discovered the nature of electromagnetism, as well as the physicists and chemists of the twentieth century who unraveled the atomic structure of matter.

Classical electromagnetism deals with electric charges and currents and their interactions as if all the quantities involved could be measured independently, with unlimited precision. Here *classical* means simply "nonquantum." The quantum law with its constant h is ignored in the classical theory of electromagnetism, just as it is in ordinary mechanics. Indeed, the classical theory was brought very nearly to its present state of completion before Planck's discovery of quantum effects in 1900. It has survived remarkably well. Neither the revolution of quantum physics nor the development of special relativity dimmed the luster of the electromagnetic field equations Maxwell wrote down 150 years ago.

Of course the theory was solidly based on experiment, and because of that was fairly secure within its original range of application – to coils, capacitors, oscillating currents, and eventually radio waves and light waves. But even so great a success does not guarantee validity in another domain, for instance, the inside of a molecule.

Two facts help to explain the continuing importance in modern physics of the classical description of electromagnetism. First, special relativity required no revision of classical electromagnetism. Historically speaking, special relativity *grew out* of classical electromagnetic theory and experiments inspired by it. Maxwell's field equations, developed long before the work of Lorentz and Einstein, proved to be entirely compatible with relativity. Second, quantum modifications of the electromagnetic forces have turned out to be unimportant down to distances less than 10^{-12} meters, 100 times smaller than the atom. We can describe the repulsion and attraction of particles in the atom using the same laws that apply to the leaves of an electroscope, although we need quantum mechanics to predict how the particles will behave under those forces. For still smaller distances, a fusion of electromagnetic theory and quantum theory, called *quantum electrodynamics,* has been remarkably successful. Its predictions are confirmed by experiment down to the smallest distances yet explored.

It is assumed that the reader has some acquaintance with the elementary facts of electricity. We are not going to review all the experiments by which the existence of electric charge was demonstrated, nor shall we review all the evidence for the electrical constitution of matter. On the other hand, we do want to look carefully at the experimental foundations of the basic laws on which all else depends. In this chapter we shall study the physics of stationary electric charges – *electrostatics.*

Certainly one fundamental property of electric charge is its existence in the two varieties that were long ago named *positive* and *negative.*

The observed fact is that all charged particles can be divided into two classes such that all members of one class repel each other, while attracting members of the other class. If two small electrically charged bodies A and B, some distance apart, attract one another, and if A attracts some third electrified body C, then we always find that B repels C. Contrast this with gravitation: there is only one kind of gravitational mass, and every mass attracts every other mass.

One may regard the two kinds of charge, positive and negative, as opposite manifestations of one quality, much as *right* and *left* are the two kinds of handedness. Indeed, in the physics of elementary particles, questions involving the sign of the charge are sometimes linked to a question of handedness, and to another basic symmetry, the relation of a sequence of events, a, then b, then c, to the temporally reversed sequence c, then b, then a. It is only the duality of electric charge that concerns us here. For every kind of particle in nature, as far as we know, there can exist an *antiparticle,* a sort of electrical "mirror image." The antiparticle carries charge of the opposite sign. If any other intrinsic quality of the particle has an opposite, the antiparticle has that too, whereas in a property that admits no opposite, such as mass, the antiparticle and particle are exactly alike.

The electron's charge is negative; its antiparticle, called a *positron,* has a positive charge, but its mass is precisely the same as that of the electron. The proton's antiparticle is called simply an *antiproton*; its electric charge is negative. An electron and a proton combine to make an ordinary hydrogen atom. A positron and an antiproton could combine in the same way to make an atom of antihydrogen. Given the building blocks, positrons, antiprotons, and antineutrons,[1] there could be built up the whole range of antimatter, from antihydrogen to antigalaxies. There is a practical difficulty, of course. Should a positron meet an electron or an antiproton meet a proton, that pair of particles will quickly vanish in a burst of radiation. It is therefore not surprising that even positrons and antiprotons, not to speak of antiatoms, are exceedingly rare and short-lived in our world. Perhaps the universe contains, somewhere, a vast concentration of antimatter. If so, its whereabouts is a cosmological mystery.

The universe around us consists overwhelmingly of matter, not antimatter. That is to say, the abundant carriers of negative charge are electrons, and the abundant carriers of positive charge are protons. The proton is nearly 2000 times heavier than the electron, and very different, too, in some other respects. Thus matter at the atomic level incorporates negative and positive electricity in quite different ways. The positive charge is all in the atomic nucleus, bound within a massive structure no more than 10^{-14} m in size, while the negative charge is spread, in

[1] Although the electric charge of each is zero, the neutron and its antiparticle are not interchangeable. In certain properties that do not concern us here, they are opposite.

effect, through a region about 10^4 times larger in dimensions. It is hard to imagine what atoms and molecules – and all of chemistry – would be like, if not for this fundamental electrical asymmetry of matter.

What we call negative charge, by the way, could just as well have been called positive. The name was a historical accident. There is nothing essentially negative about the charge of an electron. It is not like a negative integer. A negative integer, once multiplication has been defined, differs essentially from a positive integer in that its square is an integer of opposite sign. But the product of two charges is not a charge; there is no comparison.

Two other properties of electric charge are essential in the electrical structure of matter: charge is *conserved,* and charge is *quantized.* These properties involve *quantity* of charge and thus imply a measurement of charge. Presently we shall state precisely how charge can be measured in terms of the force between charges a certain distance apart, and so on. But let us take this for granted for the time being, so that we may talk freely about these fundamental facts.

1.2 Conservation of charge

The total charge in an isolated system never changes. By *isolated* we mean that no matter is allowed to cross the boundary of the system. We could let light pass into or out of the system, since the "particles" of light, called *photons,* carry no charge at all. Within the system charged particles may vanish or reappear, but they always do so in pairs of equal and opposite charge. For instance, a thin-walled box in a vacuum exposed to gamma rays might become the scene of a "pair-creation" event in which a high-energy photon ends its existence with the creation of an electron and a positron (Fig. 1.1). Two electrically charged particles have been newly created, but the net change in total charge, in and on the box, is zero. An event that *would* violate the law we have just stated would be the creation of a positively charged particle *without* the simultaneous creation of a negatively charged particle. Such an occurrence has never been observed.

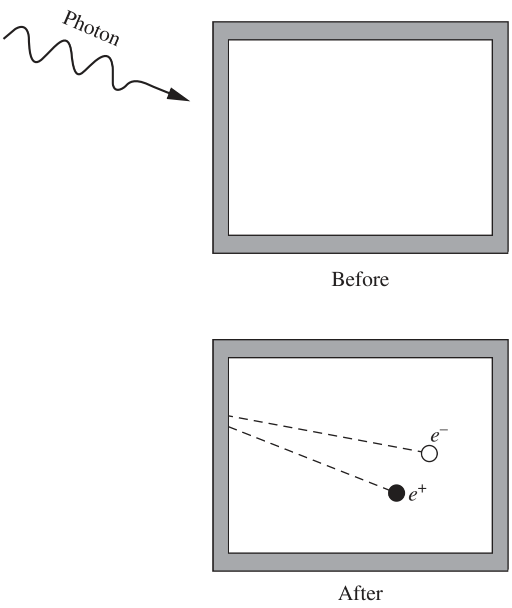

Figure 1.1.
Charged particles are created in pairs with equal and opposite charge.

Of course, if the electric charges of an electron and a positron were not precisely equal in magnitude, pair creation would still violate the strict law of charge conservation. That equality is a manifestation of the particle–antiparticle duality already mentioned, a universal symmetry of nature.

One thing will become clear in the course of our study of electromagnetism: nonconservation of charge would be quite incompatible with the structure of our present electromagnetic theory. We may therefore state, either as a postulate of the theory or as an empirical law supported without exception by all observations so far, the charge conservation law:

The total electric charge in an isolated system, that is, the algebraic sum of the positive and negative charge present at any time, never changes.

Sooner or later we must ask whether this law meets the test of relativistic invariance. We shall postpone until Chapter 5 a thorough discussion of this important question. But the answer is that it does, and not merely in the sense that the statement above holds in any given inertial frame, but in the stronger sense that observers in different frames, measuring the charge, obtain the same number. In other words, the total electric charge of an isolated system is a relativistically invariant number.

1.3 Quantization of charge

The electric charges we find in nature come in units of one magnitude only, equal to the amount of charge carried by a single electron. We denote the magnitude of that charge by e. (When we are paying attention to sign, we write $-e$ for the charge on the electron itself.) We have already noted that the positron carries precisely that amount of charge, as it must if charge is to be conserved when an electron and a positron annihilate, leaving nothing but light. What seems more remarkable is the apparently exact equality of the charges carried by all other charged particles – the equality, for instance, of the positive charge on the proton and the negative charge on the electron.

That particular equality is easy to test experimentally. We can see whether the net electric charge carried by a hydrogen molecule, which consists of two protons and two electrons, is zero. In an experiment carried out by J. G. King,[2] hydrogen gas was compressed into a tank that was electrically insulated from its surroundings. The tank contained about $5 \cdot 10^{24}$ molecules (approximately 17 grams) of hydrogen. The gas was then allowed to escape by means that prevented the escape of any ion – a molecule with an electron missing or an extra electron attached. If the charge on the proton differed from that on the electron by, say, one part in a billion, then each hydrogen molecule would carry a charge of $2 \cdot 10^{-9}e$, and the departure of the whole mass of hydrogen would alter the charge of the tank by $10^{16}e$, a gigantic effect. In fact, the experiment could have revealed a residual molecular charge as small as $2 \cdot 10^{-20}e$, and none was observed. This proved that the proton and the electron do not differ in magnitude of charge by more than 1 part in 10^{20}.

Perhaps the equality is really *exact* for some reason we don't yet understand. It may be connected with the possibility, suggested by certain

[2] See King (1960). References to previous tests of charge equality will be found in this article and in the chapter by V. W. Hughes in Hughes (1964).

theories, that a proton can, *very* rarely, decay into a positron and some uncharged particles. If that were to occur, even the slightest discrepancy between proton charge and positron charge would violate charge conservation. Several experiments designed to detect the decay of a proton have not yet, as of this writing, registered with certainty a single decay. If and when such an event is observed, it will show that exact equality of the magnitude of the charge of the proton and the charge of the electron (the positron's antiparticle) can be regarded as a corollary of the more general law of charge conservation.

That notwithstanding, we now know that the *internal* structure of all the strongly interacting particles called *hadrons* – a class that includes the proton and the neutron – involves basic units called *quarks,* whose electric charges come in multiples of $e/3$. The proton, for example, is made with three quarks, two with charge $2e/3$ and one with charge $-e/3$. The neutron contains one quark with charge $2e/3$ and two quarks with charge $-e/3$.

Several experimenters have searched for single quarks, either free or attached to ordinary matter. The fractional charge of such a quark, since it cannot be neutralized by any number of electrons or protons, should betray the quark's presence. So far no fractionally charged particle has been conclusively identified. The present theory of the strong interactions, called *quantum chromodynamics,* explains why the liberation of a quark from a hadron is most likely impossible.

The fact of charge quantization lies outside the scope of classical electromagnetism, of course. We shall usually ignore it and act as if our point charges q could have any strength whatsoever. This will not get us into trouble. Still, it is worth remembering that classical theory cannot be expected to explain the structure of the elementary particles. (It is not certain that present quantum theory can either!) What holds the electron together is as mysterious as what fixes the precise value of its charge. Something more than electrical forces must be involved, for the electrostatic forces between different parts of the electron would be repulsive.

In our study of electricity and magnetism we shall treat the charged particles simply as carriers of charge, with dimensions so small that their extension and structure is, for most purposes, quite insignificant. In the case of the proton, for example, we know from high-energy scattering experiments that the electric charge does not extend appreciably beyond a radius of 10^{-15} m. We recall that Rutherford's analysis of the scattering of alpha particles showed that even heavy nuclei have their electric charge distributed over a region smaller than 10^{-13} m. For the physicist of the nineteenth century a "point charge" remained an abstract notion. Today we are on familiar terms with the atomic particles. The graininess of electricity is so conspicuous in our modern description of nature that we find a point charge less of an artificial idealization than a smoothly varying distribution of charge density. When we postulate such smooth charge distributions, we may think of them as averages over very large numbers of elementary charges, in the same way that we can define the macroscopic density of a liquid, its lumpiness on a molecular scale notwithstanding.

1.4 Coulomb's law

As you probably already know, the interaction between electric charges at rest is described by Coulomb's law: two stationary electric charges repel or attract one another with a force proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance between them.

We can state this compactly in vector form:

$$\mathbf{F}_2 = k\frac{q_1 q_2 \hat{\mathbf{r}}_{21}}{r_{21}^2}. \tag{1.1}$$

Here q_1 and q_2 are numbers (scalars) giving the magnitude and sign of the respective charges, $\hat{\mathbf{r}}_{21}$ is the unit vector in the direction[3] from charge 1 to charge 2, and $\mathbf{F}_2$ is the force acting on charge 2. Thus Eq. (1.1) expresses, among other things, the fact that like charges repel and unlike charges attract. Also, the force obeys Newton's third law; that is, $\mathbf{F}_2 = -\mathbf{F}_1$.

The unit vector $\hat{\mathbf{r}}_{21}$ shows that the force is parallel to the line joining the charges. It could not be otherwise unless space itself has some built-in directional property, for with two point charges alone in empty and isotropic space, no other direction could be singled out.

If the point charge itself had some internal structure, with an axis defining a direction, then it would have to be described by more than the mere scalar quantity q. It is true that some elementary particles, including the electron, do have another property, called *spin*. This gives rise to a magnetic force between two electrons in addition to their electrostatic repulsion. This magnetic force does not, in general, act in the direction of the line joining the two particles. It decreases with the inverse fourth power of the distance, and at atomic distances of 10^{-10} m the Coulomb force is already about 10^4 times stronger than the magnetic interaction of the spins. Another magnetic force appears if our charges are moving – hence the restriction to stationary charges in our statement of Coulomb's law. We shall return to these magnetic phenomena in later chapters.

Of course we must assume, in writing Eq. (1.1), that both charges are well localized, each occupying a region small compared with r_{21}. Otherwise we could not even define the distance r_{21} precisely.

The value of the constant k in Eq. (1.1) depends on the units in which r, $\mathbf{F}$, and q are to be expressed. In this book we will use the International System of Units, or "SI" units for short. This system is based on the

[3] The convention we adopt here may not seem the natural choice, but it is more consistent with the usage in some other parts of physics and we shall try to follow it throughout this book.

meter, kilogram, and second as units of length, mass, and time. The SI unit of charge is the *coulomb* (C). Some other SI electrical units that we will eventually become familiar with are the volt, ohm, ampere, and tesla. The official definition of the coulomb involves the magnetic force, which we will discuss in Chapter 6. For present purposes, we can define the coulomb as follows. Two like charges, each of 1 coulomb, repel one another with a force of $8.988 \cdot 10^9$ newtons when they are 1 meter apart. In other words, the k in Eq. (1.1) is given by

$$k = 8.988 \cdot 10^9 \; \frac{\mathrm{N\,m^2}}{\mathrm{C^2}}. \tag{1.2}$$

In Chapter 6 we will learn where this seemingly arbitrary value of k comes from. In general, approximating k by $9 \cdot 10^9\ \mathrm{N\,m^2/C^2}$ is quite sufficient. The magnitude of e, the fundamental quantum of electric charge, happens to be about $1.602 \cdot 10^{-19}$ C. So if you wish, you may think of a coulomb as defined to be the magnitude of the charge contained in $6.242 \cdot 10^{18}$ electrons.

Instead of k, it is customary (for historical reasons) to introduce a constant ϵ_0 which is defined by

$$k \equiv \frac{1}{4\pi\epsilon_0} \quad \Longrightarrow \quad \boxed{\epsilon_0 \equiv \frac{1}{4\pi k} = 8.854 \cdot 10^{-12} \; \frac{\mathrm{C^2}}{\mathrm{N\,m^2}}} \quad \left(\text{or} \; \frac{\mathrm{C^2\,s^2}}{\mathrm{kg\,m^3}}\right). \tag{1.3}$$

In terms of ϵ_0, Coulomb's law in Eq. (1.1) takes the form

$$\boxed{\mathbf{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2 \hat{\mathbf{r}}_{21}}{r_{21}^2}} \tag{1.4}$$

The constant ϵ_0 will appear in many expressions that we will meet in the course of our study. The 4π is included in the definition of ϵ_0 so that certain formulas (such as Gauss's law in Sections 1.10 and 2.9) take on simple forms. Additional details and technicalities concerning ϵ_0 can be found in Appendix E.

Another system of units that comes up occasionally is the *Gaussian* system, which is one of several types of cgs systems, short for centimeter–gram–second. (In contrast, the SI system is an mks system, short for meter–kilogram–second.) The Gaussian unit of charge is the "electrostatic unit," or esu. The esu is defined so that the constant k in Eq. (1.1) *exactly* equals 1 (and this is simply the number 1, with no units) when r_{21} is measured in cm, **F** in dynes, and the q values in esu. Figure 1.2 gives some examples using the SI and Gaussian systems of units. Further discussion of the SI and Gaussian systems can be found in Appendix A.

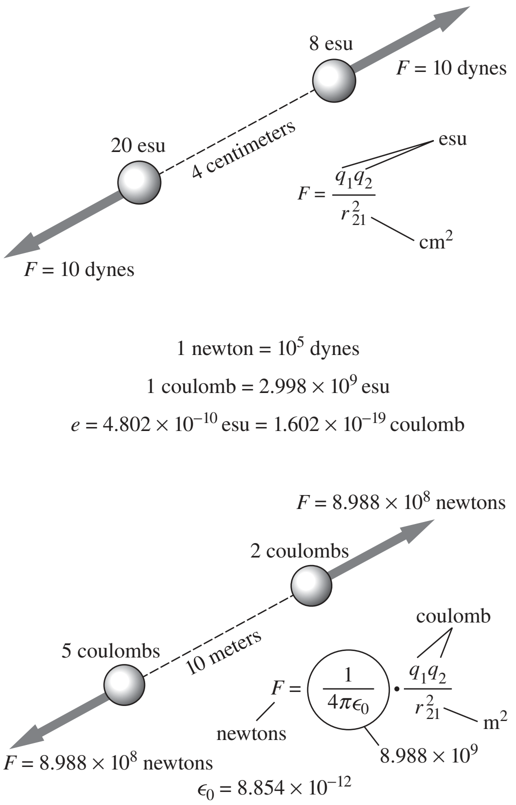

Figure 1.2.
Coulomb's law expressed in Gaussian electrostatic units (top) and in SI units (bottom). The constant ϵ_0 and the factor relating coulombs to esu are connected, as we shall learn later, with the speed of light. We have rounded off the constants in the figure to four-digit accuracy. The precise values are given in Appendix E.